Scottish Journal of Political Economy

Scottish Journal of Political Economy, DOI:10.1111/sjpe.12060, Vol. 62, No. 1, February 2015

SPILLOVERS IN SPORTS LEAGUES WITH PROMOTION AND RELEGATION

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Abstract

This paper analyzes spillover effects in sports leagues that are embedded in a system of promotion and relegation. Based on a contest model of a professional sports league with a top division and a second division, we show that league prizes and club efficiencies have opposing effects; while a stronger second division that offers a higher league prize leads to a more balanced top division, the opposite is true for a stronger second division whose clubs become more cost efficient. Moreover, we demonstrate that a higher second-division prize induces a lower investment level, but higher profits in the top division, while higher club efficiency in the second division leads to both a lower investment level and lower profits in the top division. These results have important policy implications for the organization of sports leagues.

I Introduction

Peter J. Sloane's 1971 article on the economics of professional football is an attempt to provide a 'theoretical framework with respect to the objectives of football clubs and the nature of competition under which they operate' (p. 122). One of the most important peculiarities of professional team sports in general and professional football in particular is the organization of market entry. In North America, for example, professional team sports leagues are organized as 'closed shops' in the sense that new teams cannot enter the league without the permission of existing teams. In European football and many other sports around the world, market entry of new teams is usually organized through a tiered system of promotion and relegation. Within this system, the weakest teams of the first division are replaced by the strongest teams of the second division at the beginning of each season.

This system of market entry and exit has important implications for the structure of competition within each league. Sloane (1971) argues that promotion and relegation are a 'stimulus to improve performance as relegation to a lower division may, amongst other things, lead to a financial loss' (p. 125). Ross and Szymanski (2002), Szymanski and Valletti (2005), and Jasina and

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Rotthoff (2012) develop theoretical models to analyze the economic differences between open and closed leagues. Based on their models, the authors show that overall spending on player talent is higher in open than in closed leagues because the prospect of promotion and relegation enhances competition within the top and lower divisions. Noll (2002) analyzes data from English football (soccer) to empirically support these findings. Finally, Dietl *et al.* (2008) examine how a system of promotion and relegation affects the over-investment problem in professional team sports. They find that clubs invest more when they play in an open league compared with a closed league. Moreover, the over-investment problem within open leagues increases with the revenue differential between leagues.

Our paper has a different focus because we analyze the vertical spillover effects between divisions that are generated through such a system of promotion and relegation. Particularly, we are interested in the top division and we seek to examine how different characteristics of the second division affect the competition within the top division.

Based on a contest model of a professional sports league with a top division and a second division, we show that changes in league prizes do not affect competitive balance if the two divisions are not connected via a system of promotion and relegation. Surprisingly, if divisions are embedded in a system of promotion and relegation, league prizes and club efficiencies have opposing effects on competitive balance; while a stronger second division that offers a higher league prize leads to a more balanced top division, the opposite is true for a stronger second division, whose clubs become more cost efficient. In the latter case, competitive balance decreases in the top division. Moreover, we demonstrate that a higher second-division prize induces a lower investment level but higher profits in the top division, while higher club efficiency in the second division leads to both a lower investment level and lower profits in the top division.

These results have important policy implications for the organization of sports leagues. Suppose that a sports league planner has a total prize that can be split between the two divisions, and if the league planner is interested in a balanced league in the top division, s/he should choose a low prize spread. In contrast, if the league planner prefers high talent investments and high profits in the top division, s/he should choose a high prize spread. These results highlight the trade-off a league planner faces between a balanced league on one hand and high talent investment and club profits on the other hand.

The remainder of this paper is organized as follows. Section II presents the model setup. Section III derives the optimization problem and Section IV presents the results. Finally, Section V discusses the results and concludes the paper.

II Model Setup

We model a European football league that is organized hierarchically in ascending divisions, offering a system of promotion and relegation. Our model covers two periods and consists of two divisions denoted by division A and division B. Each division contains two clubs. After the first period, the loser in division A is relegated to division B and is replaced by the winner from that division. We assume that the revenue (per period) of each division is exogenously given with V^A and V^B, denoting the prize of division A and B, respectively. Division A is the top division which offers a higher prize than the second division B, i.e., $V^A > V^B$ due to higher merchandising potential.

We further assume that clubs i and j start in the first period in division A competing for the first-division prize V^A. The first-period winner receives the prize V^A, remains in division A and competes in the second period against the promoted club from division B. The defeated club from division A receives nothing, is relegated to the second division and competes in the second period against the defeated club from division B. Clubs k and k' start in the first period in division B and compete for the second-division prize V^B. The first-period winner receives the prize V^B, is promoted to division A and competes in the second period against the first-period winner of division A. The defeated club from division B receives nothing, remains in this division and competes in the second period against the relegated club from division A.

The investments in playing talent of club $\alpha \in I = \{i, j, k, k'\}$ that plays against club $\beta \in I$ with $\alpha \neq \beta$ in period $t \in \{1,2\}$ in division $L \in \{A,B\}$ are denoted by $x^{t,L}_{\alpha,\beta}$ with associated talent costs $C_\alpha(x^{t,L}_{\alpha,\beta}) = \frac{c_\alpha}{2}(x^{t,L}_{\alpha,\beta})^2$ with $0 < c_i \leq c_j < c_k \leq c_{k'}$. We interpret the cost parameter c_α as a measure for a club's cost efficiency: ceteris paribus, a larger value of c_α implies higher marginal costs. At a more cost-efficient club, each unit of playing talent generates lower costs. Thus, a higher c_α denotes a club with lower cost efficiency. Moreover, costs are strictly convex due to the quadratic term. Thus, ceteris paribus, marginal costs increase for a higher level of playing talent.[1]

The probability that club α wins against club β is characterized by the contest-success function (CSF). We employ the Tullock CSF (see Tullock, 1980), which is the most widely used functional form of a CSF in sporting contests. The probability of club α's success playing against club β in this imperfectly discriminating contest is thus given by[2]

$$p^{t,L}_{\alpha,\beta} = \frac{x^{t,L}_{\alpha,\beta}}{X^{t,L}_{\alpha,\beta}}, \tag{1}$$

where $X^{t,L}_{\alpha,\beta} \equiv x^{t,L}_{\alpha,\beta} + x^{t,L}_{\beta,\alpha}$ denotes aggregate talent investments. Given that the win probabilities must sum to one, we obtain the adding-up constraint: $p^{t,L}_{\beta,\alpha} = 1 - p^{t,L}_{\alpha,\beta}$. For notational clarity, we use subscripts α, $\beta \in I$ to characterize clubs, while the superscripts L denotes the division, with $L \in \{A,B\}$ and t stands for the period, with $t \in \{1,2\}$.

[1] See Grossmann *et al.* (2008), who provide an economic explanation for the assumption of convexity in sports contests.

[2] For an axiomatization of the Tullock CSF, see Skaperdas (1996) and Clark and Riis (1998). Alternative functional forms are the difference-form CSF (Hirshleifer, 1989), the probit CSF (Lazear and Rosen, 1981; Dixit, 1987), and the value weighted CSF (Runkel, 2006).

The uncertainty of outcome is measured by the competitive balance in the division. One way of measuring competitive balance is through the ratio of win percentages, which is also called win ratio (Hoehn and Szymanski, 1999; Vrooman, 2007, 2008). Competitive balance in period t between club α and β in division L is thus given by

$$CB^{t,L}_{\alpha,\beta} \equiv \frac{p^{t,L}_{\alpha,\beta}}{p^{t,L}_{\beta,\alpha}} = \frac{x^{t,L}_{\alpha,\beta}}{x^{t,L}_{\beta,\alpha}}. \tag{2}$$

Note that competitive balance $CB^{t,L}_{\alpha,\beta}$ equals one in a fully balanced division. A level of competitive balance that is lower or higher than one thus indicates a division with a lower degree of competitive balance.

Expected overall profits of club i are given by[3]

$$\pi^A_i = p^{1,A}_{i,j}(V^A + \pi^{2,A}_i) + (1 - p^{1,A}_{i,j})\pi^{2,B}_i - \frac{c_i}{2}(x^{1,A}_{i,j})^2. \tag{3}$$

With probability $p^{1,A}_{i,j}$ club i wins against club j in period one and obtains the first-division prize V^A. Club i then remains in division A, competes in period two against the promoted club from division B and receives an expected second-period payoff of $\pi^{2,A}_i$. With probability $1 - p^{1,A}_{i,j}$ club i loses against club j and is relegated to division B without receiving a prize in period one. Then, club i competes in the second period against the defeated club of division B, obtaining an expected second-period payoff of $\pi^{2,B}_i$. Investment costs are given by $\frac{c_i}{2}(x^{1,A}_{i,j})^2$.

If club i remains in division A in period 2, its expected overall second-period profits are given by

$$\pi^{2,A}_i = p^{1,B}_{k,k'}\underbrace{\left(p^{2,A}_{i,k}V^A - \frac{c_i}{2}(x^{2,A}_{i,k})^2\right)}_{=\pi^{2,A}_{i,k}} + p^{1,B}_{k',k}\underbrace{\left(p^{2,A}_{i,k'}V^A - \frac{c_i}{2}(x^{2,A}_{i,k'})^2\right)}_{=\pi^{2,A}_{i,k'}}$$

With probability $p^{1,B}_{k,k'}$, the division B club k is promoted to division A subsequent to a win in period 1 and will play against club i in period 2. The expected second-period profits in this case are $\pi^{2,A}_{i,k} = p^{2,A}_{i,k}V^A - \frac{c_i}{2}(x^{2,A}_{i,k})^2$. Analogously, with probability $p^{1,B}_{k',k}$, the division B club k' is promoted to division A subsequent to a win in period 1 and will play against club i in period 2. The expected second-period profits in this case are $\pi^{2,A}_{i,k'} = p^{2,A}_{i,k'}V^A - \frac{c_i}{2}(x^{2,A}_{i,k'})^2$.

If club i is relegated to division B in period 2, its expected overall second-period profits are given by

$$\pi^{2,B}_i = p^{1,B}_{k',k}\underbrace{\left(p^{2,B}_{i,k}V^B - \frac{c_i}{2}(x^{2,B}_{i,k})^2\right)}_{=\pi^{2,B}_{i,k}} + p^{1,B}_{k,k'}\underbrace{\left(p^{2,B}_{i,k'}V^B - \frac{c_i}{2}(x^{2,B}_{i,k'})^2\right)}_{=\pi^{2,B}_{i,k'}}$$

The interpretation is similar to above.

[3] Note that we characterize profits for club i only. Profits of the other clubs are derived in a similar way.

We solve our model as an "aggregative game" (Corchon, 1994).[4] That is, we assume the payoffs to depend only on individual talent investments and an aggregate of all talent investments. The clubs choose talent investments independently by taking the aggregate talent investment in the division as given. Nevertheless, aggregate investment will be endogenously determined in equilibrium. In economic theory, such a type of behavior usually is referred to as aggregate-taking behavior (ATB).[5] The assumption of ATB is reasonable because it is plausible that the clubs have an idea of some aggregate (or average) investment level in the division rather than the investment level of individual clubs. Moreover, the equilibrium based on ATB is a reasonable approximation to the Nash equilibrium (see, for example, Hefti, 2011, Grossmann and Dietl, 2014 and Hefti *et al.*, 2014). Finally, ATB leads to high analytical tractability and allows to solve problems that would not be tractable otherwise.

III Optimization Problem

We apply backward induction to solve for the subgame-perfect equilibrium in this two-period game. We first compute the equilibrium in the second period, then calculate the equilibrium in the first period.

Period 2: In this subsection, we derive the second-period equilibrium outcomes of club i if it plays against club k in division A. The equilibrium outcomes in the other cases are calculated analogously. The expected second-period profits of club i are given by

$$\pi_{i,k}^{2,A} = p_{i,k}^{2,A} V^A - \frac{c_i}{2}(x_{i,k}^{2,A})^2 = \frac{x_{i,k}^{2,A}}{X_{i,k}^{2,A}} V^A - \frac{c_i}{2}(x_{i,k}^{2,A})^2,$$

where $X_{i,k}^{2,A} = x_{i,k}^{2,A} + x_{k,i}^{2,A}$ represent aggregate talent investments. Based on ATB, club i maximizes its profits with respect to $x_{i,k}^{2,A}$ by taking aggregate talent investments as given so that the first-order condition for club i is given by:

$$\frac{\partial \pi_{i,k}^{2,A}}{\partial x_{i,k}^{2,A}} = \frac{1}{X_{i,k}^{2,A}} V^A - c_i x_{i,k}^{2,A} = 0.$$

The first-order condition for club k can be derived in a similar way. We establish the following lemma:[6]

Lemma 1: Suppose that club i plays against club k in the second period in division A, then in equilibrium,

[4] Although in many cases they are not referred to explicitly as "aggregative games", such an aggregative structure is very common in economic models. See e.g., Alós-Ferrer and Ania (2005) for a comprehensive overview of different examples (Cournot oligopoly, rent seeking, tragedy of the commons, etc).

[5] For aggregate games and ATB, see, e.g., Alós-Ferrer and Ania (2005), Possajennikov (2003) and Jensen (2010).

[6] Note that the equilibrium investments and profits for other matchups in division A or B are derived analogously.

(i) club i's talent investment in the second period is $\hat{x}_{i,k}^{2,A} = \left(\frac{c_k}{c_i}\frac{V^A}{c_i+c_k}\right)^{1/2}$,
(ii) the second-period competitive balance between club i and k is $CB_{i,k}^{2,A} = \frac{c_k}{c_i}$,
(iii) the second-period expected profit of club i is $\hat{\pi}_{i,k}^{2,A} = \frac{c_k V^A}{2(c_i+c_k)}$.

Proof: The proof is straightforward by combining the corresponding first-order conditions. Notice that the second-order conditions for a maximum are satisfied. □

The lemma shows that competitive balance in period 2 does not depend on the division prizes; it depends only on the clubs' relative cost efficiency. This result is standard in the contest literature.[7] However, as we will see below, first-period competitive balance depends on the division prizes due to the existence of spillover effects from one division to the other.

Period 1: In this subsection, we exemplarily derive the first-period optimization problem of club i in division A. Plugging second-period equilibrium profits into (3), we obtain expected overall profits of club i as

$$\pi_i^A = p_{i,j}^{1,A}(V^A + \hat{\pi}_i^{2,A}) + (1 - p_{i,j}^{1,A})\hat{\pi}_i^{2,B} - \frac{c_i}{2}(x_{i,j}^{1,A})^2$$

Based on ATB, club i maximizes its expected overall profits π_i^A with respect to $x_{i,j}^{1,A}$ by taking aggregate talent investments $X_{i,j}^{1,A} = x_{i,j}^{1,A} + x_{j,i}^{1,A}$ as given. The setup of the optimization problems of the other clubs and the problem solving is derived analogously.

To solve these optimization problems and derive the equilibrium, we henceforth assume that division B clubs k and k' are symmetric with respect to their cost efficiency, i.e., $c^B \equiv c_k = c_{k'}$, but division A clubs i and j are still asymmetric, i.e., $c_i < c_j$. Nevertheless, there are still two kinds of asymmetries: intra-division and inter-division asymmetry. First, clubs within division A are asymmetric with respect to their cost efficiency. Second, divisions A and B are asymmetric with respect to the corresponding clubs' cost efficiencies and the league prizes. In Section 'Investment levels and expected club profits', we additionally assume that there is only inter-division asymmetry but intra-division symmetry.

IV Results

Existence and uniqueness of equilibrium

We derive the following proposition:

Proposition 1: A unique subgame-perfect equilibrium exists.

[7] Note that conventional (non-ATB) Nash strategies would provide a qualitatively similar result. Competitive balance is $CB_{i,k}^{2,A} = (c_k/c_i)^{1/2}$ in a Nash equilibrium. In this case, competitive balance is also independent of the contest prize.

Proof: See Appendix □

In the appendix, we demonstrate that there exists a unique equilibrium and derive the first-period investments $\hat{x}_{i,j}^{1,A}$ and $\hat{x}_{j,i}^{1,A}$ in division A in equilibrium. Moreover, we show that clubs in division B invest the same amount in equilibrium with $\hat{x}_{k,k'}^{1,B} = \hat{x}_{k',k}^{1,B}$ due to the symmetry within division B.

For further analyses, it is useful to introduce club i's first-order condition in a reduced form as follows

$$\frac{1}{X_{i,j}^{1,A}}\left(V^A + \frac{c^B}{2(c_i + c^B)}V^A - \frac{c^B}{2(c_i + c^B)}V^B\right) = c_i x_{i,j}^{1,A}, \tag{4}$$

which has a familiar interpretation: It states that the marginal revenue of increasing the first-period investment level $x_{i,j}^{1,A}$ must be equal to its marginal costs. The marginal revenue has several components. On one hand, increasing the investment level increases the probability to gain the division A prize V^A in period 1. In this case, club i stays in division A and receives an expected second-period profit of $\frac{c^B}{2(c_i+c^B)}V^A$. On the other hand, increasing the first-period investment level decreases the probability to be relegated to division B and to obtain an expected second-period profit of $\frac{c^B}{2(c_i+c^B)}V^B$. The latter has a negative effect on marginal revenue because it represents forgone profits. In addition, the first-order condition shows that the expected second-period profits lead to interesting spillover effects that would be absent without the connectedness between division A and division B.

Competitive balance

Next, we focus on competitive balance in division A and analyze the spillovers from division B into division A. Therefore, we assume that clubs in division A have different cost efficiencies with $c_i < c_j$ throughout this subsection. We establish the following proposition:

Proposition 2:

(i) First-period competitive balance $CB_{i,j}^{1,A}$ in division A is given by

$$CB_{i,j}^{1,A} \equiv \frac{\hat{x}_{i,j}^{1,A}}{\hat{x}_{j,i}^{1,A}} = \frac{c_j(c_j + c^B)\left(2c_i V^A + 3c^B V^A - c^B V^B\right)}{c_i(c_i + c^B)\left(2c_j V^A + 3c^B V^A - c^B V^B\right)} > 1.$$

(ii) Division A becomes more balanced in the first period if
(iia) the prize in division B increases
(iib) the cost efficiency of clubs in division B decreases (i.e., c^B increases).

Proof: See Appendix. □

Part (i) of the proposition shows that club i has larger first-period investments than club j leading to an unbalanced division A with $CB_{i,j}^{1,A} > 1$. Part

(ii) shows that there are spillover effects from division B into division A that act in non-trivial ways: Division A becomes more balanced when either the strength of division B increases (via a higher prize) or the strength of division B decreases (via less cost-efficient clubs).

The intuition of Part (ii) is as follows: (iia) A higher division B prize V^B implies that the marginal revenue of investment decreases for both division A clubs i and j because a higher division B prize represents larger forgone expected second-period profits in case of a relegation (see left-hand side of (4)). However, the negative effect on marginal revenue is quantitatively larger for the more cost efficient club i than for club j. This is true because club i has larger expected second-period profits than club j if it is relegated so that the former, club i, will react with a stronger decrease in the investment level than the latter, club j. Thus, a higher division B prize V^B induces an increase in competitive balance in division A.

(iib) Less cost-efficient division B clubs (i.e., a higher c^B) imply that club i's expected second-period profits $\frac{c^B}{2(c_i+c^B)}V^A$ and $\frac{c^B}{2(c_i+c^B)}V^B$ increase (see left-hand side of (4)). Yet, larger expected second-period profits affect the marginal revenue of investment for club i in two opposing ways: on one hand, there is a positive impact on marginal revenue for club i because this club receives higher expected profits if it stays in division A. On the other hand, there is a negative impact on marginal revenue for club i due to higher forgone profits if club i is relegated to division B.

Nevertheless, the positive impact on club i's marginal revenue always dominates the negative impact because division A offers a higher prize than division B, i.e., $V^A > V^B$. As a consequence, the overall effect of less cost efficient division B clubs on club i's marginal revenue is positive so that this club increases its investment level in period 1. The same is true for the other division A club j. However, the positive impact on marginal revenue is quantitatively larger for the less cost-efficient club j than for club i as $\partial \frac{c^B}{2(c_i+c^B)} / \partial c^B < \partial \frac{c^B}{2(c_j+c^B)} / \partial c^B$. As a result, club j will react with a stronger increase in the investment level than club i. The result is an increase in competitive balance in division A if the cost efficiency of the division B clubs decreases.

Corollary 1: Division A becomes more balanced in the first period if the prize in this division decreases.

Proof: See Appendix. □

The intuition of Corollary (1) is as follows: A higher division A prize has a positive impact on the marginal revenue of investment for both division A clubs i and j because the expected first-period profits V^A as well as the expected second-period profits increase. However, the impact on club i's expected second-period profits $\frac{c^B}{2(c_i+c^B)}(V^A - V^B)$ is larger than the impact on club j's expected second-period profits $\frac{c^B}{2(c_j+c^B)}(V^A - V^B)$ so that the former will react with a stronger increase in the investment level than the latter. The result is a decrease in competitive balance in division A through a higher prize V^A.

The following table summarizes the comparative statics of Proposition (2) and Corollary (1).[8]

	V^A	V^B	c^b
$CB^{1,A}_{i,j}$	+	–	–

Investment levels and expected club profits

In this subsection, we additionally assume that division A clubs i and j are also symmetric with respect to their cost efficiency, i.e., $c^A \equiv c_i = c_j$ but there is still inter-division heterogeneity, i.e., $c^A < c^B$. The reason for this procedure is that we now are able to analyze the investment levels and expected overall profits as well as its comparative statics in detail.

For symmetric division A and symmetric division B clubs, first-period talent investments and overall profits for the division A clubs are given by

$$\hat{x}^{1,A}_{i,j} = \hat{x}^{1,A}_{j,i} = \left(\frac{c^A V^A + 0.5c^B(3V^A - V^B)}{2c^A(c^A + c^B)}\right)^{1/2} \text{ and}$$
$$\hat{\pi}^A_i = \hat{\pi}^A_j = \frac{2c^A V^A + 3c^B(V^A + V^B)}{8(c^A + c^B)}. \tag{5}$$

We establish the following proposition:

Proposition 3:

(i) First-period investments of division A clubs increase if
- (ia) the division A prize increases, i.e., $\partial\hat{x}^{1,A}_{i,j}/\partial V^A > 0$,
- (ib) the division B prize decreases, i.e., $\partial\hat{x}^{1,A}_{i,j}/\partial V^B < 0$,
- (ic) the cost efficiency of the division A clubs increases, i.e., $\partial\hat{x}^{1,A}_{i,j}/\partial c^A < 0$,
- (id) the cost efficiency of the division B clubs decreases, i.e., $\partial\hat{x}^{1,A}_{i,j}/\partial c^B > 0$.

(ii) First-period profits of division A clubs increase if
- (iia) the division A prize increases, i.e., $\partial\hat{\pi}^{1,A}_i/\partial V^A > 0$
- (iib) the division B prize increases, i.e., $\partial\hat{\pi}^{1,A}_i/\partial V^B > 0$
- (iic) the cost efficiency of the division A clubs increases, i.e., $\partial\hat{\pi}^{1,A}_i/\partial c^A < 0$
- (iid) the cost efficiency of the division B clubs decreases, i.e., $\partial\hat{\pi}^{1,A}_i/\partial c^B > 0$.

Proof: The proof is straightforward and therefore omitted. □

[8] Recall that a more balanced league is characterized through a lower level of $CB^{1,A}_{i,j}$.

Part (i) of the proposition shows that the effects of league prizes and cost efficiencies on the investment levels of the top-division clubs is as one would expect: higher top-division prizes incentivize the clubs in this division to invest more, while higher second-division prizes reduce investment incentives in the top division. The opposite is true with respect to efficiency: lower cost efficiency of the top-division clubs induce an decrease in investment incentives, while lower cost efficiency of the second-division clubs increase investment incentives of the top-division clubs.

Part (ii) of the proposition highlights the effect of league prizes and cost efficiencies on profits of the top-division clubs. It shows that higher league prizes in the top and second division lead to higher club profits in the top division. In addition, more cost efficient clubs in division A also lead to higher club profits in this division, while the opposite is true regarding the cost efficiency of the second-division clubs.

The following table summarizes the comparative statics of Proposition (3).

	V^A	V^B	c^A	c^B
$\hat{x}_{i,j}^{1,A}$	+	−	−	+
$\hat{\pi}_i^{1,A}$	+	+	−	+

Policy implications

Suppose that a sports league planner has a total prize V that can be split between division A and B, i.e., $V = V^A + V^B$, depending on the league planner's objective, we now examine the optimal prize split. Due to the uncertainty of outcome hypothesis,[9] a league planner might be interested in a balanced league. Second, the planner might be interested in a high league quality measured by investments in playing talent in order to increase the attractiveness of matchups. Third, the planner might be interested in maximizing club profits.

Given our focus on the top division and the spillovers from a lower division to this top division, we concentrate on competitive, talent investment and club profits in division A. If the league planner prefers a balanced league in the top division A, s/he should choose a *low* prize spread $V^A - V^B$ according to Proposition (2) and Corollary (1). In contrast, if the league planner prefers high talent investments in the top division, s/he should choose a *high* prize spread $V^A - V^B$ according to Parts (ia) and (ib) in Proposition (3). If the league planner prefers high profits in the top division, s/he should also choose a high prize spread $V^A - V^B$ as $\partial\hat{\pi}_i^A/\partial V^A > \partial\hat{\pi}_i^A/\partial V^B$ according to equation (5).

[9] According to the so-called "uncertainty of outcome" hypothesis (Rottenberg, 1956), fans prefer to attend games with an uncertain outcome and enjoy close championship races. For empirical contributions that analyze the relation between competitive balance and match attendance, see Downward and Dawson (2000), Borland and MacDonald (2003), and Szymanski (2003).

Thus, the league planner faces a trade-off between a balanced league on the one hand or high talent investment and club profits on the other hand.

V Conclusion

In his 1971 paper, Peter J. Sloane has provided important foundations for the economic theory of professional sports leagues. We have tried to advance these foundations by modeling the spillover effects in sports leagues that are organized trough a tiered system of promotion and relegation. In particular, we have shown how the allocation of prizes between vertical leagues and the club efficiency in the lower division affect competitive balance, talent investment, and club profits in the top division. Our main results are summarized in Table 1.

The table shows that increasing the division A prize leads to a less balanced division A, higher talent investments (quality) and higher profits in division A. In contrast, increasing the division B prize leads to a more balanced division A, lower talent investments (quality) and higher profits in division A. Finally, more cost efficient division B clubs lead to a less balanced division A, lower talent investments (quality) and lower profits in division A. In sum, our results highlight a major trade-off: Higher (lower) prize spreads between the top- and second division increase (decrease) talent investment and club profits, but decrease (increase) competitive balance in the top division.

Our model yields testable comparative-static results. In particular, our model might help to predict how prize changes affect competitive balance, talent investments and club profits.

Finally, our simple model may serve as a basic framework to further analyze spillover effects in sports leagues that are embedded in a system of promotion and relegation. There is a broad range of further applications and model extensions. For instance, an interesting avenue for further research could be the extension of our model to integrate the consumers and to examine welfare implications of spillovers. Furthermore, interesting extensions would be to endogenize the league prizes and to extend the model to more than two clubs per division.

Table 1
Comparative Statics in Division A

	Division *A* prize	Division *B* prize	Division *B* efficiency
Balance	–	+	–
Talent investment	+	–	–
Profits	+	+	–

Appendix A

A.1 Proof of Proposition 1

Plugging second-period equilibrium profits into (3), we obtain overall expected profits of club i as

$$
\begin{aligned}
\pi_i =& p_{i,j}^{1,A}(V^A + \hat{\pi}_i^{2,A}) + (1 - p_{i,j}^{1,A})\hat{\pi}_i^{2,B} - \frac{c_i}{2}(x_{i,j}^{1,A})^2 \\
=& \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\left(V^A + \frac{x_{k,k'}^{1,A}}{X_{k,k'}^{1,A}}\frac{c_k V^A}{2(c_i + c_k)} + \frac{x_{k',k}^{1,A}}{X_{k',k}^{1,A}}\frac{c_{k'} V^A}{2(c_i + c_{k'})}\right) \\
&+ \left(1 - \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\right)\left(\frac{x_{k',k}^{1,B}}{X_{k',k}^{1,B}}\frac{c_k V^B}{2(c_i + c_k)} + \frac{x_{k,k'}^{1,B}}{X_{k,k'}^{1,B}}\frac{c_{k'} V^B}{2(c_i + c_{k'})}\right) - \frac{c_i}{2}(x_{i,j}^{1,A})^2
\end{aligned}
$$

Anticipating that the assumption of symmetry between club k and k' holds such that $c^B \equiv c_k = c_{k'}$, we get $\hat{x}_{k,k'}^{1,A} = \hat{x}_{k',k}^{1,A}$ and $\hat{x}_{k,k'}^{1,B} = \hat{x}_{k',k}^{1,B}$ in equilibrium. Therefore, we obtain

$$
\begin{aligned}
\pi_i =& \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\left(V^A + \frac{c^B V^A}{4(c_i + c^B)} + \frac{c^B V^A}{4(c_i + c^B)}\right) \\
&+ \left(1 - \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\right)\left(\frac{c^B V^B}{4(c_i + c^B)} + \frac{c^B V^B}{4(c_i + c^B)}\right) - \frac{c_i}{2}(x_{i,j}^{1,A})^2
\end{aligned}
$$

Then, the FOC of club i is

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial x_{i,j}^{1,A}} =& \frac{1}{X_{i,j}^{1,A}}\left(V^A + \frac{c^B V^A}{4(c_i + c^B)} + \frac{c^B V^A}{4(c_i + c^B)}\right) \\
&- \frac{1}{X_{j,i}^{1,A}}\left(\frac{c^B V^B}{4(c_i + c^B)} + \frac{c^B V^B}{4(c_i + c^B)}\right) - c_i x_{i,j}^{1,A}
\end{aligned}
$$

It is easy to see that the second-order condition for a maximum is satisfied. Note that $X_{i,j}^{1,A} = X_{j,i}^{1,A}$ such that club i's first-order condition reduces to:

$$
V^A + \frac{c^B V^A}{2(c_i + c^B)} - \frac{c^B V^B}{2(c_i + c^B)} = c_i x_{i,j}^{1,A} X_{i,j}^{1,A} \tag{6}
$$

which is equivalent to club i's first-order condition in reduced form presented in (4). Symmetrically, club j's first-order condition is

$$
V^A + \frac{c^B V^A}{2(c_j + c^B)} - \frac{c^B V^B}{2(c_j + c^B)} = c_j x_{j,i}^{1,A} X_{i,j}^{1,A} \tag{7}
$$

Combining the two first-order conditions (6) and (7), we get

$$
\frac{V^A + \frac{c^B V^A}{2(c_i + c^B)} - \frac{c^B V^B}{2(c_i + c^B)}}{V^A + \frac{c^B V^A}{2(c_j + c^B)} - \frac{c^B V^B}{2(c_j + c^B)}} = \frac{c_i x_{i,j}^{1,A}}{c_j x_{j,i}^{1,A}} \tag{8}
$$

Solving the last equation for $x_{j,i}^{1,A}$, we get

$$x_{j,i}^{1,A}(x_{i,j}^{1,A}) = \frac{c_i}{c_j} \frac{V^A + \frac{c^B V^A}{2(c_j+c^B)} - \frac{c^B V^B}{2(c_j+c^B)}}{V^A + \frac{c^B V^A}{2(c_i+c^B)} - \frac{c^B V^B}{2(c_i+c^B)}} x_{i,j}^{1,A}$$

Using the last equation and $X_{i,j}^{1,A} = x_{i,j}^{1,A} + x_{j,i}^{1,A}$ in equation (6), we obtain the first-period investment of club i

$$\hat{x}_{i,j}^{1,A} = \left(\frac{c_i V^A + 1.5 c^B V^A - 0.5 c^B V^B}{c_i(c_i + c^B)\left(1 + \frac{c_i(c_i+c^B)\left(2c_j V^A + 3c^B V^A - c^B V^B\right)}{c_j(c_j+c^B)(2c_i V^A + 3c^B V^A - c^B V^B)}\right)} \right)^{1/2} \tag{9}$$

and symmetrically for club j

$$\hat{x}_{j,i}^{1,A} = \left(\frac{c_j V^A + 1.5 c^B V^A - 0.5 c^B V^B}{c_j(c_j + c^B)\left(1 + \frac{c_j(c_j+c^B)(2c_i V^A + 3c^B V^A - c^B V^B)}{c_i(c_i+c^B)\left(2c_j V^A + 3c^B V^A - c^B V^B\right)}\right)} \right)^{1/2}. \tag{10}$$

To prove the existence of the equilibrium, we also need to solve the optimization problems of club k and k'. Plugging second-period equilibrium profits into the profit function of club k, we obtain the expected overall profit of club k as follows:

$$\begin{aligned}
\pi_k =& p_{k,k'}^{1,B}(V^B + \widehat{\pi}_k^{2,A}) + (1 - p_{k,k'}^{1,B})\widehat{\pi}_k^{2,B} - \frac{1}{2}c^B(x_{k,k'}^{1,A})^2 \\
=& \frac{x_{k,k'}^{1,B}}{X_{k,k'}^{1,B}}\left(V^B + \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_i V^A}{2(c_i + c^B)} + \frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_j V^A}{2(c_j + c^B)}\right) \\
&+ \left(1 - \frac{x_{k,k'}^{1,B}}{X_{k,k'}^{1,B}}\right)\left(\frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_i V^B}{2(c_i + c^B)} + \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_j V^B}{2(c_j + c^B)}\right) - \frac{1}{2}c^B(x_{k,k'}^{1,A})^2
\end{aligned}$$

Applying ATB, club k's first-order condition is

$$\begin{aligned}
\frac{\partial \pi_k}{\partial x_{k,k'}^{1,A}} =& \frac{1}{X_{k,k'}^{1,B}}\left(V^B + \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_i V^A}{2(c_i + c^B)} + \frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_j V^A}{2(c_j + c^B)}\right) \\
&- \frac{1}{X_{k,k'}^{1,B}}\left(\frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_i V^B}{2(c_i + c^B)} + \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_j V^B}{2(c_j + c^B)}\right) - c^B x_{k,k'}^{1,A} = 0.
\end{aligned}$$

It is easy to see that the second-order condition for a maximum is satisfied. Considering that $x_{k,k'}^{1,A} = x_{k',k}^{1,A}$ holds in equilibrium due to the symmetry of club k and k', the first-order condition reduces to

$$x_{k,k'}^{1,A} = \left(\frac{V^B + \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_i V^A}{2(c_i+c^B)} + \frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_j V^A}{2(c_j+c^B)} - \frac{x_{j,i}^{1,A}}{X_{j,i}^{1,A}}\frac{c_i V^B}{2(c_i+c^B)} - \frac{x_{i,j}^{1,A}}{X_{i,j}^{1,A}}\frac{c_j V^B}{2(c_j+c^B)}}{2c^B} \right)^{1/2}$$

Simplifying the last expression, we can derive the optimal investment of club k in equilibrium, i.e., $\hat{x}_{k,k'}^{1,A}(\hat{x}_{i,j}^{1,A}, \hat{x}_{j,i}^{1,A})$, as a function of the positive equilibrium values $\hat{x}_{i,j}^{1,A}$ and $\hat{x}_{j,i}^{1,A}$. Then, it is easy to see that $\hat{x}_{k,k'}^{1,A} = \hat{x}_{k',k}^{1,A} > 0$:

$$\hat{x}_{k,k'}^{1,A}(\hat{x}_{i,j}^{1,A},\hat{x}_{j,i}^{1,A}) = \hat{x}_{k',k}^{1,A}(\hat{x}_{i,j}^{1,A},\hat{x}_{j,i}^{1,A}) = \frac{1}{2}[\frac{\left(c_i(c_j+c^B)V^A + (c_i+c^B)(c_j+2c^B)V^B\right)\hat{x}_{i,j}^{1,A}}{c^B(c_i+c^B)(c_j+c^B)\left(\hat{x}_{i,j}^{1,A}+\hat{x}_{j,i}^{1,A}\right)} + \frac{\left(c_j(c_i+c^B)V^A + (c_j+c^B)(c_i+2c^B)V^B\right)\hat{x}_{j,i}^{1,A}}{c^B(c_i+c^B)(c_j+c^B)\left(\hat{x}_{i,j}^{1,A}+\hat{x}_{j,i}^{1,A}\right)}]^{1/2} > 0$$

Therefore, we obtain a unique equilibrium with first-period investments $\hat{x}_{i,j}^{1,A}, \hat{x}_{j,i}^{1,A}, \hat{x}_{k,k'}^{1,A}, \hat{x}_{k',k}^{1,A}$ and second-period investments according to Lemma (1).

A.2 Proof of Proposition 2

Recall that club i has a higher cost efficiency than club j, i.e., $c_i < c_j$.

(i) Dividing (9) by (10), it is easy to verify that competitive balance is given by

$$CB_{i,j}^{1,A} \equiv \frac{\hat{x}_{i,j}^{1,A}}{\hat{x}_{j,i}^{1,A}} = \frac{c_j(c_j+c^B)\left(2c_iV^A + 3c^BV^A - c^BV^B\right)}{c_i(c_i+c^B)\left(2c_jV^A + 3c^BV^A - c^BV^B\right)}$$

and $CB_{i,j}^{1,A} > 1$.

(iia) First, we show that a larger prize in division B increases competitive balance in division A in period 1:

$$\frac{\partial CB_{i,j}^{1,A}}{\partial V^B} = \frac{2(c_i-c_j)c_jc^B\left(c_j+c^B\right)V^A}{c_i(c_i+c^B)\left(2c_jV^A + 3c^BV^A - c^BV^B\right)^2} < 0$$

(iib) Next, we show that competitive balance in period 1 in division A increases for a larger c^B:

$$\frac{\partial CB_{i,j}^{1,A}}{\partial c^B} = \frac{c_j(c_j-c_i)(V^A-V^B)(2c_ic_jV^A + (c^B)^2(V^B-3V^A))}{c_i(c_i+c^B)^2\left(2c_jV^A + 3c^BV^A - c^BV^B\right)^2}$$

Then, it is easy to show that

$$\frac{\partial CB_{i,j}^{1,A}}{\partial c^B} < 0 \Leftrightarrow 2c_ic_jV^A + (c^B)^2(V^B-3V^A) < 0$$

$$\Leftrightarrow 2\underbrace{\frac{c_ic_j}{c^Bc^B}}_{<1} < 3 - \underbrace{\frac{V^B}{V^A}}_{<1}$$

The last inequality holds, as the left-hand side is smaller than 2 and the right-hand side is larger than 2. Therefore, competitive balance in division A increases, i.e., $CB_{i,j}^{1,A}$ gets closer to the value one, if clubs in division B are less cost efficient, i.e., for a larger value of c^B.

A.3 Proof of Corollary 1

Recall that club i has a higher cost efficiency than club j, i.e., $c_i < c_j$. The effect of a larger prize in division A on competitive balance in period 1 is as follows:

$$\frac{\partial CB_{i,j}^{1,A}}{\partial V^A} = \frac{2(c_j - c_i)c_j c^B (c_j + c^B) V^B}{c_i(c_i + c^B)(2c_j V^A + 3c^B V^A - c^B V^B)^2} > 0$$

Therefore, a larger prize in division A decreases competitive balance in division A in period 1.

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Date of receipt of final manuscript: 29 August 2014