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MARKET SHARE ANALYSIS AND NON-COOPERATIVE GAME THEORY WITH APPLICATIONS IN SWISS HEALTH INSURANCE

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UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

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THÈSE DE DOCTORAT

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pour l'obtention du grade de
Docteure ès Sciences Actuarielles

par

Dalit DAILY-AMIR

Directeur de thèse
Prof. Hansjörg Albrecher

Jury

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and have found it to meet the requirements for a doctoral thesis.
All revisions that I or committee members
made during the doctoral colloquium
have been addressed to my entire satisfaction.

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Lausanne, July 2019 *Dalit Daily-Amir*

Preface

An effective pricing process is an essential component of any insurance company base strategy and is an ongoing discussion issue. Actuaries are expected to determine the correct price for a product for which the costs are uncertain and various matters beside the expected loss, like competition and profit optimization, need to be addressed and take into consideration. Questions like "how the insurer can fix his premium in order to maximize his expected profits?", "how the insurer can incorporate competition into premium decision?", "how does the premium affects the demand of the policyholders?" are being investigated. This thesis attends to the above questions and suggests some results on the optimization of premium calculation and demand. It is based on three papers that have been accepted or submitted for publication during my PhD studies.

Chapter 1 gives a survey on classical pricing methods as well as price optimization, game theory basic concepts and its' applications in insurance. Chapter 2 presents an extension to a game-theoretic pricing model under competition for non-life insurance companies presented in Dutang et al. (2013). In this paper, we define an asymmetric information game where not all the insurers have full information about their competitors and we examine the effects on the equilibrium premium. The results support the intuitive one may have on the effects of uncertainty on the equilibrium premiums. Chapter 3 examines the effect of different factors such as price, group affiliation and service level on the annual changes of the insurers' market shares. Through a linear model with two-sided lognormally distributed errors and using a published data, we test several hypotheses concerning the main motives for these changes in market shares. The results suggest that the premium related variables are significant in explaining annual market share changes. In Chapter 4, we develop a demand function for the mandatory health insurance in Switzerland based on the findings of Chapter 3. We adapt the operation profit as the objective function of the insurer as presented in Dutang et al. (2013), and develop a pricing model to optimize premiums. Unlike Dutang et al. (2013), the new model is constructed such that the initial market share becomes a relevant factor for the equilibrium premiums. We compare the results with the equilibrium premiums calculated with the model presented in Dutang et al. (2013), and we assess the effect of the initial market share on the equilibrium premiums through sensitivity tests.

Contents

Chapter 1

Introduction

The appropriate pricing of an insurance contract is essential for the existence of insurance and the premium level is a strategic decision that reflects on the objectives of the insurance company. Before introducing a new insurance product, the insurance company often deals with questions like: "can we set a competitive price for the product?" and "would it be possible to find a reinsurance cover for the new product?". For non-life insurance products, actuaries mainly use traditional cost-based approaches to determine the technical premium including loss models, Generalized Linear Models (GLM) and credibility theory (see e.g. Herzog (1994), Mikosch (2006) and Ohlsson (2010)). The technical premium calculation accumulates loadings on the expected loss, to compensate for the uncertainty related to the data and the model. Moreover, other costs such as sales commissions, legal expenses and administrative cost are taken in consideration. In many cases, the insurer will buy a reinsurance cover as a protection against extreme claim events, and the related fees are included in the premium calculation, as well. Reinsurance also allows the insurer to reduce the reserves, to diversify his portfolio and to underwrite larger risks.

Once the technical premium has been determined, the insurer can perform a price optimization in order to meet with his base directives, be it profit maximizing, market share increase or minimizing the required reserve level. Although the practice of price optimization is common in many industries to determine the price of a product, its implementation in the insurance market has induced deep concerns among all stake-holders, i.e., regulators, consumers and insurers. The main concern is the fear that such an optimization will lead to discrimination of certain customers (CAS (2015)).

Introducing competition into insurance pricing means that the offered price is not only reflecting the insurer's own costs and profit objectives, but is relative to what is offered by the competitors. A web-user customer can easily compare insurance products, their prices, and their fit to his needs, using search and compare applications easily found on-line. As a result, insurance companies need to offer increasingly competitive premiums, while the claim amounts and other related costs keep increasing (Valkenburg & Bosschaart (2018)). Dutang et al. (2013) suggest a one-period pricing model that incorporates the competition in premium calculations based on non-cooperative game-theory concepts.

In this chapter, we present the theoretical foundations of non-life insurance pricing, as well as the price optimization classical model. We then describe general concepts of game theory and some insurance pricing models based on cooperative and non-cooperative game theory concepts. Finally, we detail the contribution of this thesis.

1.1. Theoretical foundations of insurance pricing

Various techniques and methods are developed and used to price different insurance products, see for instance Herzog (1994), Kaas et al. (2001), Mikosch (2006) and Parodi (2014), on which some of the following considerations are based. The pricing process starts with estimating the risk that the insurer underwrites, offering compensation to the policyholder for faced damages. *Exposure unit* is the size of the potential loss from insuring the risk and *ratemaking* is the assessment of the proper unit rate to charge for that insurance coverage. The assessment of the potential loss in the pricing process is named *will costing.* Contrary to the pricing of other products for which the actual cost is known, in case of an insurance product the real loss of underwriting the insurance is unknown until the contract period has expired, so building a loss model is essential for pricing. Two models are mainly applied by actuaries to estimate the aggregate expected loss:

1. **The individual risk model** estimates the aggregate loss as the sum of losses from all the individual policies issued by the insurer, i.e

$$S = \sum_{i=1}^{n} X_i,$$

   where S is the aggregate loss, n is the number of policies and X_i is the cost arising from policy i. Note that X_i equals zero for policies without claims. The individual risk model is typically based on the assumptions that there are a finite number of risks n, that only one claim is possible for each risk i and that loss events as well as their severities are independent. These assumptions make the individual risk model suitable for various applications in life insurance.

2. **The collective risk model** is estimating the aggregate loss as the sum of all claims

arising from a risk during one period of time, usually one year, such that,

$$S = \sum_{j=1}^{N} Y_j$$

where S is the aggregate loss, N is a random variable (r.v.) denoting the number of claims during the period and $(Y_j)_{j\geq 1}$ are positive i.i.d random variables denoting the amount of the jth claim with generic r.v. Y. We assume that the r.v. N is independent of $(Y_j)_{j\geq 1}$. The expected loss $E(S)$ is then equal to the expected number of claims $E(N)$ times the expected claim size $E(Y)$, i.e.

$$E(S) = E(N)E(Y)$$

and the variance of the aggregate loss then easily follows as

$$Var(S) = Var(Y)E(N) + E(Y)^2 Var(N).$$

The collective risk model is popular in non-life insurance.

In cases where a large amount of reliable data is available, and in order to better measure the individual risk of a policyholder, the insurance company assigns a personal rate according to individual characteristics like age, profession, education and residential area rather than in a one-size-fits-all approach. Once the risk profile is determined, the insurer uses rating factors that are measurable proxies of the risk factors to fix the premium. The most popular technique to select rating factors is the Generalized Linear Models, see e.g. de Jong & Heller (2008), although in recent years advanced techniques, like neural networks are being used as well (Spedicato et al. (2018)).

1.1.1. Loading and methods to calculate the technical premium

The ***pure premium*** is the amount equal to the insurer's expected loss under the risk, and risk costing is an essential part of insurance pricing. Nevertheless, other factors need to be included in the calculation of the premium charged from the insureds. The ***technical premium*** is defined as the amount that will cover costs and expenses and achieve the profit target without taking into account commercial considerations on competition and marketing. Anderson et al. (1994) summarize the five approaches to calculate the technical premium described by The General Insurance Rating Issues Working Party (GRIP):

- ***Tariff,*** where a rating agency like the regulator decides the rates or approves them.

- ***Qualitative,*** when data is insufficient so that statistical estimation needs to be combined with a subjective analysis.

- ***Cost plus*** is the most popular method for cases where there is enough information. Cost plus is based on data analysis for a quantitative assessment plus loadings.

- ***Distribution*** is used mainly in personal line insurance. In this method, the estimated demand is combined in the insurer's objective function in order to obtain a price optimization.

- ***Industrial*** is used by big insurers and aims to optimize the operational efficiency and enjoy the scale with the industrialisation of the pricing process over several classes of business.

The most popular method to estimate the technical premium is the cost plus and according to this approach, the technical premium is a compound of five elements:

1. The expected loss, which is the mean of the aggregate loss distribution.

2. Loading for uncertainty: to compensate for the uncertainty arising from poor or limited data and model uncertainty. The sum of the expected loss and the loading is considered as the ***risk premium***.

3. Other costs like commissions to agents and claim expenses. The claim expenses might include allocated loss expenses for cases where these fees are not included in the claim amount (e.g. lawyer fees and claim management fees) and unallocated loss adjustment expenses like administration, accounting and IT.

4. Investment income that represents the return on investment the insurer earns between the premium payment and claim payment. As the premium is usually paid at the beginning of the insurance period while claim payments occur later in time, the insurer can invest the money under some investment restrictions imposed by the regulation authorities.

5. Profit is the return on capital that the shareholders expect to have on their capital investment. The insurance company is obliged to hold a certain level of reserves to limit its ruin probability. The capital raised from the shareholders of the insurance company comes with a cost, the Cost of Capital (CoC). Insurers add the CoC to the insurance pricing in order to ensure that the shareholders receive the expected return on their capital investment.

1.1.2. Principles of premium calculation

The *pure risk premium* is insufficient as in the long run ruin is unavoidable even in the case of considerable (though finite) initial reserves. Actuaries use various principles as *risk measures* to calculate premiums and add a loading on the pure premium to compensate the uncertainty risk and avoid ruin with high probability. In the following section, we denote the premium calculated for risk X by Π_X. Among discussed desirable properties for premium calculation principles are:

- *Non-negative loading* to ensure that the premium will not be less than the expected claims, $\Pi_X \geq E(X)$,
- *Additivity* requires that, for independent risks, the combined premium to insure the risks will be equal to the sum of premiums to insure the risks individually, $\Pi_{X_1+X_2} = \Pi_{X_1} + \Pi_{X_2}$. For *Subadditivity* property, $\Pi_{X_1+X_2} \leq \Pi_{X_1} + \Pi_{X_2}$.
- *Scale invariance:* If $Z = aX$ is a scaled version of a risk X with scaling factor $a > 0$, the premium for Z should be, $\Pi_Z = a\Pi_X$. This property is also known by *positive homogeneity.*
- *Translation invariance:* For $Z = X + b$, where b is a real constant, $\Pi_Z = \Pi_X + b$
- *Monotonicity:* For two risks, X_1 and X_2, if $F_{X_1}(x) \leq F_{X_2}(x)$, the premium for risk X_1 should be greater or equal to the premium for risk X_2, $\Pi_{X_1} \geq \Pi_{X_2}$, for all x.
- *No ripoff* requires that the premium should be smaller or equal to the maximum possible claim amount X_{max} such that $\Pi_X \leq X_{max}$.

Various principles are suggested for premium calculations (see e.g Gerber (1974), Kaas et al. (2001), Dickson (2005) and Albrecher et al. (2017) for surveys). Each of the principles fulfils part or all the properties and in general, the insurer decides which properties are more relevant for each risk and uses the appropriate one to calculate the premiums. The most popular principles include:

The pure premium principle is defined as $\Pi_X = E(X)$.

The expected value principle where

$$\Pi_X = (1 + \Theta)E(X),$$

for some $\Theta > 0$. This is a popular and simple way to add a risk loading. However, it assigns the same loading for every risk with equal expected loss without accounting for the volatility of the risk X. It is preferred in cases where the insurer has a limited reliable information to estimate higher moments of the risk.

The variance principle

$$\Pi_X = E(X) + \alpha_v Var(X)$$

and the **standard deviation principle**

$$\Pi_X = E(X) + \alpha_s \sqrt{Var(X)}$$

account for the volatility where $\alpha_v > 0$ and $\alpha_c > 0$ are chosen constants. The two principles differ in their properties and they are widely used, depending on the context.

The zero utility principle is based on the idea of equating the expected utility of the insurer with and without accepting the risk X. For a utility function u, one gets

$$E(u(W)) = E(u(W + \Pi_X - X)),$$

where W is the initial surplus of the insurer which itself can be random or deterministic. The utility function is usually non-decreasing and concave, meaning that a larger risk is compensated by a higher premium and it reflects the risk aversion of the insurer.
In case the insurer has an exponential utility function $u(x) = -e^{-\beta x}$ and the initial capital is deterministic, one can determine the premium to be

$$\Pi_X = \beta^{-1} \log E[\exp(\beta X)],$$

which does not depend on the capital level of the insurer. Here $\beta > 0$ denotes the risk aversion coefficient.

The Esscher principle is defined as

$$\Pi_X = \frac{E[Xe^{hX}]}{E[e^{hX}]}$$

for some chosen $h > 0$. The Esscher transform can be viewed as the pure premium for a risk $\tilde{X}$ with density g which is an expected weighted version of the density f of the original risk X. This transform means that the pure premium of the scaled risk $\tilde{X}$ includes a loading on the pure premium of the original risk X.

The risk adjusted premium principle scales the risk X such that the expected value of the scaled risk $\tilde{X}$, is higher than the original risk as in the Esscher principle. Using *proportional hazards (PH) transform*, it is defined as,

$$\Pi_X = \int_0^\infty [Pr(X > x)]^{1/\rho} dx = \int_0^\infty [1 - F(x)]^{1/\rho} dx,$$

where ρ is called the (risk-averse) index. In contrast to the Esscher principle, it is also applicable for heavy-tailed risks. Since it is additive when a risk is split into layers, one can use it for insurance layer pricing and determine an optimal reinsurance plan. The concavity of g ensures the coherence of this principle.

A more general form of this principle is the **distortion principle**, where g is a concave, non-decreasing function with $g(0) = 0$ and $g(1) = 1$. The **distortion principle** is determined by

$$\Pi_X = \int_0^\infty g(1 - F_X(y))dy,$$

where $g(1 - F(x))$ is a risk-adjusted survival function such that it assigns higher probabilities than the observed ones to larger values of loss. Wang (2000) suggested a distortion operator, known as the Wang transform, i.e.,

$$g(u)_\alpha = \Phi(\Phi^{-1}(u) + \alpha),$$

where α is the *market price of risk*. The g_α is concave for positive α and convex for negative α. The corresponding risk-adjusted premium excluding expenses is calculated as the mean under the distorted probability function. The Wang transform can be applied to any probability distribution of the risk. For instance, if risk X has a normal distribution $N(\mu, \sigma^2)$, the distorted risk X^* is normally distributed with $N(\mu + \alpha\sigma, \sigma^2)$.

The *value at risk* (VaR) and the *expected shortfall* (ES) are both *risk measures* that can in fact be seen as distortion function measures. The VaR_α is defined as the smallest number l for which the probability that the loss L exceeds l, is smaller or equal to $(1 - \alpha)$, i.e.

$$VaR_\alpha(L) = \inf\{l \in R : P(L > l) \leq 1 - \alpha\}.$$

The ES_α is defined as the expected value of the loss given it is greater than the VaR_α, i.e.

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 VaR_u(L)du.$$

Note that for the same level of confidence $\alpha < 1$, $ES_\alpha > VaR_\alpha$.
The distortion function defining the VaR_α is

$$g(u) = \begin{cases} 0 & \text{if } 0 \leq u \leq 1-\alpha, \\ 1 & \text{if } (1-\alpha) < u \leq 1. \end{cases}$$

The distortion function defining the ES_α is

$$g(u) = \begin{cases} \text{u / (1-}\alpha\text{)} & \text{if } 0 \leq u \leq 1-\alpha, \\ 1 & \text{if } (1-\alpha) < u \leq 1. \end{cases}$$

A risk measure is defined as a *coherent risk measure* when it satisfies the properties of monotonicity, subadditivity, positive homogeneity, and translational invariance (Artzner et al. (1999)). While the VaR does not fulfil the subadditivity property of a coherent risk measure, the ES is coherent and satisfies all the four properties.

1.1.3. Credibility theory

The mentioned premium principles are all based on the past claim experience of the insurer in order to calculate the expected loss $E(X)$. If the insurer has insufficient data, credibility theory combines the individual claims experience with the experience of other related portfolios. The *credibility premium* (CP_j) is defined as a weighted average of the client j's risk premium X_j and the collective risk premium $\overline{X}$:

$$CP_j = z_j X_j + (1 - z_j)\overline{X}$$

where $z_j \in [0, 1]$ is the *credibility factor*. The credibility factor z_j indicates the 'credibility' level of the portfolio's own claim history. A credibility factor close to one is usually selected when the insurer has an extensive experience in the portfolio and this experience displays a small level of variation. When this experience is large enough and the likelihood of having a specific relative error in the individual mean loss is smaller than a certain threshold, the insurer assigns a credibility factor of one, meaning *full credibility*. The other extreme position, where $z_j = 0$, means that the insurer fixes the same risk premium $\overline{X}$ to everyone. This decision is reasonable when the portfolio is homogeneous and all the groups have the same expected claim size. Otherwise, "good" risks will be overcharged and may leave, while "bad" risks will be undercharged.
Different methods are suggested to manage credibility and the way to calculate the credibility factor. Bühlmann (1967) developed one of the most popular respective models. See

Bühlmann & Gisler (2005) for many extensions of the classical model.

Let us denote $X_{j,t}$ as the loss of client j in year t where $j = (1, 2, \ldots, J)$ and $t = (1, 2, \ldots, T)$. The average loss of client j is determined by

$$\overline{X_j} = \frac{1}{T} \sum_{t=1}^{T} X_{j,t},$$

and the average loss for the entire portfolio is

$$\overline{X} = \frac{1}{JT} \sum_{j=1}^{J} \sum_{t=1}^{T} X_{j,t}.$$

We then set the following assumptions:

(i) For each risk j, there exists a parameter Θ_j related to the risk j that indicates the random deviation of $X_{j,t}$ from the overall mean loss m. The parameters Θ_j are i.i.d.

(ii) For each risk j and time t, there exists a parameter $\Theta_{j,t}$ related to the risk j at time t, that indicates the random deviation of $X_{j,t}$ from the average loss $\overline{X_j}$. The parameters $\Theta_{j,t}$ are i.i.d.

The loss $X_{j,t}$ can then be divided such that

$$X_{j,t} = m + \Theta_j + \Theta_{j,t}$$

with $E(\Theta_j) = E(\Theta_{j,t}) = 0$, $Var(\Theta_{j,t}) = s^2$ and $Var(\Theta_j) = a$. The parameter $a > 0$ reflects the heterogeneity level of the portfolio and s^2 is a measure for the homogeneity within group j.
The Bühlmann model determines the credibility factor by minimizing the mean-squared error (MSE) of the best unbiased predictor of $X_{j,T+1}$. With the credibility premium equal to $z_j \cdot X_j + (1 - z_j) \cdot \overline{X}$, the Bühlmann credibility factor for all the clients is defined by

$$z = \frac{aT}{aT + s^2}.$$

This credibility factor has some asymptotic properties such that:

- If the portfolio is homogeneous such that the expected claim amounts of the entire portfolio are i.i.d, the parameter $a \to 0$ and then $z \to 0$. In this case, $\overline{X}$ is optimal to estimate the risk premium.

- If the parameter $a \to \infty$, then $z \to 1$. In this case, the claim experience of the other risks in the portfolio does not give information about the risk j.

- If $T \to \infty$, the insurer has an extensive experience with risk j and the credibility factor is then $z \to 1$.

- If $s^2 \to \infty$, the variability of the individual risk j is extremely high and the insurer can not rely on the individual claim experience in calculating the risk premium. In that case, $z \to 0$.

In order to apply the model, one needs to estimate the parameters m, s^2 and a with unbiased estimators. For the overall mean m, the natural estimator is the sample average $\hat{m} = \overline{X}$. The parameter s^2 can be estimated with the unbiased

$$\hat{s}^2 = \frac{1}{J(T-1)} \sum_{j=1}^{J} \sum_{t=1}^{T} (X_{j,t} - \overline{X_j})^2,$$

and for a, the unbiased estimator

$$\hat{a} = \frac{1}{J-1} \sum_{j=1}^{J} (\overline{X_j} - \hat{m})^2 - \frac{\hat{s}^2}{T}.$$

Credibility theory is mainly applied in collective insurance contracts. Client j might represent, for example, a company that buys insurance for its employees. The Bühlmann-Straub model is an extension that incorporates an additional parameter, $P_{j,t}$, to represent the volume of the risk of client j in year t. We define

$$P_j = \sum_{t=1}^{T} P_{j,t} \quad and \quad P = \sum_{j=1}^{J} P_j$$

as the total volume of risk for client j and the overall volume of risk, respectively. The credibility factor for client j is

$$z_j = \frac{aP_j}{aP_j + s^2}.$$

The estimated credibility premium is set by

$$\hat{P}_{j,T+1}(z_j \frac{1}{P_j} \sum_{j=1}^{J} X_{j,t} + (1 - z_j)m),$$

where $\hat{P}_{j,T+1}$ is the estimated volume of risk for client j at time $T + 1$.

The *limited-fluctuation credibility* approach is another method to calculate the credibility premium. Let M denote the *manual premium*, a preconceived estimate of the predicted loss that is determined by some past experience and underwriting expertise, μ as the mean of the loss and D as the observed value of the loss based on the experience with a certain risk group. The credibility premium is calculated with,

$$CP = zD + (1 - z)M,$$

where $z \in [0, 1]$ is the *credibility factor*. According to the *limited-fluctuation credibility* approach, when the insurer has a required minimum of the experience data, it will be given full credibility, meaning $z = 1$. The minimum required data for full credibility is called *the standard for full credibility*. It can be computed for different loss measures such as the claim frequency, the claim severity and the aggregate loss. For given values of k and α, when the probability of observing the loss measure is within $100k\%$ of the mean is at least $1 - \alpha$, full credibility is achieved and the insurer can set the credibility factor $z = 1$. When the risk group is not sufficiently large, the *standard for full credibility* is cannot be achieved, and the value of $z < 1$ needs to be determined. For a loss measure W with mean μ_W, the basic assumption for calculating z, is that the probability of zW within the interval $[z\mu_W - k\mu_W, z\mu_W + k\mu_W]$ is equal $1 - \alpha$.
Other credibility models are used by actuaries. More details can be found in e.g., Kaas et al. (2001) and Schmidli (2017).

1.1.4. Other considerations in pricing

Commercial considerations

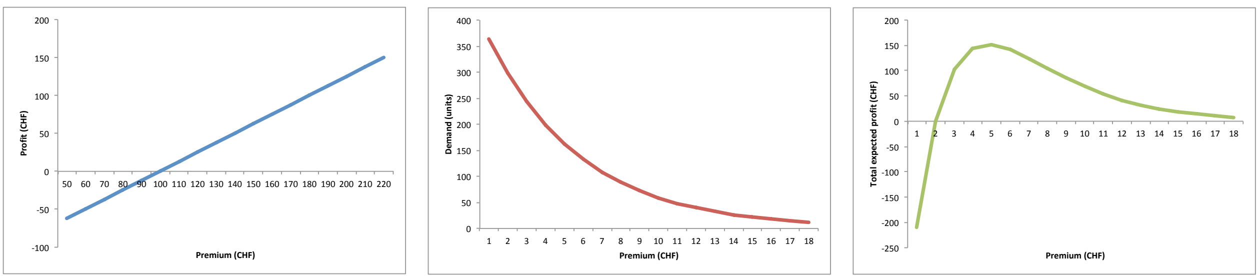
The actual premium set by the insurer is a strategic decision and accounts for considerations like regulatory constraints, competition and the relationship with other products sold by the insurer. For example, according to the Swiss law, insurers who offer mandatory health insurance cannot charge premiums above the level that covers their costs. These insurers usually offer a complementary plan on which they can make a profit. When these insurers determine the premium level for the mandatory plan, they might consider the profits they can gain from the complementary plan.

Capital considerations

To ensure that the insurer is able to fulfil his commitments to the policyholders and to remain solvent with high probability (usually 99.5%), the regulator imposes a minimum level of capital on the insurer, the ***Solvency capital requirement*** (SCR). In general, insurers hold higher levels of capital than the required regulatory one in order to protect themselves from insolvency, to attain a high rating from credit-rating agencies and to attract investors and clients. The investors require a minimum return level that will be considerably higher than the risk-free rate. Accordingly, the insurer incorporates the need for a certain level of profitability towards an additional loading on the premium. The VaR and the ES measures are being used to calculate solvency capital requirements (SCR) according to regulations set by the law. The European Solvency II framework uses the VaR with $\alpha = 0.995$, and the Swiss Solvency Test (SST) defines the SCR as the ES with $\alpha = 0.99$. For typical risks, the SCR under the SST corresponds to approximately VaR at level 99.6% to 99.8%, depending on the underlying distribution (see FOPI (2006)).

1.2. Price optimization

When the insurer has sufficient reliable data, he may consider a price optimization and setting the premium at a level that will maximize the expected profits under certain constraints such as regulation price rules, market share change constraints and maximum difference from competitors' prices. The goal is to determine which contract to offer to which client and at what price in order for a company to maximize its profit besides achieving other strategic objectives, like to increase its market share, to maintain a certain reserve level and to avoid ruin. The following price optimization approach (see Parodi (2014)) is simplified and gives a general idea of this concept. For an insurance product being sold for the same premium P to all customers as is the case in the Swiss mandatory health insurance, according to the basic price theory, the profit per unit $\pi(P)$ is a linear function of the premium (see Figure 1.1a). The demand function, $D(P)$ is a decreasing function of the premium, for instance like the one defined in Figure 1.1b and the total expected profit (TEP) is then determined by $TEP(P) = \pi(P) \cdot D(P)$. Usually, the TEP is maximized for a price $P^{\star}$, as illustrated in Figure 1.1c. Note that in this case, we assume a homogeneous portfolio of policyholders with similar price elasticity parameter.



(a) Expected profit per policy (b) Demand function (c) Total expected profit

Figure 1.1: Premium versus profit and demand

Krikler et al. (2004) describe a price optimization process for a car insurance portfolio. They used a price test method to estimate the price elasticity of policyholders and to calculate the demand. The price optimization process in their study resulted in 10% growth in the insurance profits.

Harshova et al. (2018) develop a model of price optimization for renewal policies. They define the objective as maximization of the expected future premium income from renewal policies. For a portfolio of N policies, where each policyholder $i \in (1, \ldots, N)$ pays a current premium P_i and his corresponding percentage of premium increase is δ_i, the renewal probability is defined by $\Psi_i(P_i, \delta_i)$. The premium volume in case of complete renewal is then

$$V^{\star} = \sum_{i=1}^{N} P_i^{\star} = \sum_{i=1}^{N} P_i(1 + \delta_i).$$

The random number of renewed policies N_R can be calculated with

$$N_R = \sum_{i=1}^{N} I_i,$$

where $I_1, \ldots, I_N$ are independent Bernoulli random variables with

$$P\{I_i = 1\} = \Psi_i(P_i, \delta_i), \quad 1 \leq i \leq N.$$

The premium volume at renewal is defined by

$$V_R = \sum_{i=1}^{N} I_i P_i(1 + \delta_i),$$

and the objective function by

$$E(V_R) = \sum_{i=1}^{N} P_i(1 + \delta_i)E(I_i) = \sum_{i=1}^{N} P_i(1 + \delta_i)\Psi_i(P_i, \delta_i).$$

As the $P_1 \ldots P_N$ are known, the optimization is done only with respect to δ_i's. The model includes two additional objectives:

(i) Minimization of the variance of the renewal premium volume

$$Var(V_R) = \sum_{i=1}^{N} (P_i(1 + \delta_i))^2 \Psi_i(P_i, \delta_i)(1 - \Psi_i(P_i, \delta_i)).$$

(ii) Maximization of the expected premium difference

$$\tau = \sum_{i=1}^{N} P_i \delta_i \Psi_i(P_i, \delta_i).$$

The optimization is performed under a constraint on the minimum renewal rate and a min-max constraint on the renewal premium $P_i^{\star}$. Through some numerical examples, Harshova et al. (2018) illustrate the complexity and the importance of the price optimization process. The model ignores the effects of competition on the premium volume and assumes that the competitors' prices do not affect the renewal rate.
Other contributions on price optimization are mainly focused on some ethical issues like price discrimination and the regulations on this practice, see e.g. CAS (2015) and Schwartz & Harrington (2015).

Demand function

In general, the demand is a function of the price and is measured in quantity. However, in insurance price optimization, the demand function reflects the tendency of a client to purchase the insurance cover by the probability to select a specific contract from a set of possible alternatives. Discrete choice models are linked with utility maximization (McFadden (1973)) and set a framework to calculate the probability to select a particular alternative as a function of observed factors that relate to the set of choices and to the preferences of the decision maker. One of the models presented in McFadden (1973), is a multinomial logit probability choice model. Assuming all decision makers have the same preferences, the probability to choose alternative i from a set of I possible choices is

$$P_i = \frac{e^{\beta_i Z}}{1 + \sum_{j=1}^{I-1} e^{\beta_j Z}},$$

where β_i are a set of coefficients corresponding to alternative i and Z is a set of explanatory variables.

Dutang et al. (2013) adapt the model to calculate the lapse probability, p_j^k, as the probability of a policyholder to switch from insurer j to k. Given the price vector $x = (x_1, \ldots, x_I)$ denoting the prices offered by each of I insurers in the market, they define p_j^k by the multinomial logit model

$$p_{j\to k}(x) = lg_j^k(x) = \begin{cases} \frac{1}{1+\sum\limits_{j\neq l} e^{f_j(x_j,x_l)}} & \text{if } j = k, \\ \frac{e^{f_j(x_j,x_l)}}{1+\sum\limits_{j\neq l} e^{f_j(x_j,x_l)}} & \text{if } j \neq k, \end{cases}$$

where the summation is over the I insurers and f_j is a price sensitivity function of insurer j.

Let the random variable $N_j(x)$ represent the number of policies issued by insurer j in the next period. It equals the sum of the renewed policies and the new policyholders coming from other insurers. The expected number of policies for insurer j is then given by

$$E\left(N_j(x)\right) = n_j \times lg_j^j(x) + \sum_{j\neq l} n_l \times lg_j^l(x) = n_j \times p_{j\to j}(x) + \sum_{l\neq l} n_l \times p_{j\to k}(x).$$

The demand in the model, $D_j(x)$, is an approximation to the expected market share $E\left(N_j(x)\right)/n$ such that

$$D_j(x) = \frac{n_j}{n}(1 - \beta_j(\frac{x_j}{m_j(x)} - 1)),$$

where β_j is the price elasticity coefficient and

$$m_j(x) = \frac{1}{I-1}\sum_{j\neq l} x_k$$

is the average price of the competitors.

In Daily-Amir et al. (2019), we develop a demand function in terms of market share based on a linear function. They calibrate the price elasticity coefficients performing a linear regression using a published dataset on the Swiss mandatory health insurance market. Let $MS_{j,t-1}$ be the initial market share of insurer j, $\beta_j > 0$ the price elasticity coefficient, $m_j(x)$ the average premium of the competitors, then the demand function is calculated by

$$D_j(x) = MS_{j,t-1} - \beta_j \cdot (\frac{x_j}{m_j(x)} - 1).$$

This demand function is a sum of the initial market share and the change resulting from the relative difference between the insurer's premium and the market premium. As $\beta_j > 0$, if insurer j sets a premium higher than the market premium, he will lose market share and vice versa.

1.3. Life insurance pricing concepts

Health and life insurance aims to cover the policyholder in case of illness, accident or death. Different policies include a cover in case of partial or total, permanent or non-permanent income loss, expenses related to hospitalization, medical care and surgery and a payment (lump-sum or annuity) to the survivor of the insured person. There are various types of life insurance policies, including:

- the pure endowment policy in which the insured person receives the payment of the face amount in case he is alive at maturity,
- the endowment policy includes a payment in both cases when the insured is alive at the end of the contract, or for premature death,
- the term insurance policy that grants the payment of the face amount in case of death during the duration of the policy,
- a whole life cover that guaranties the payment of the face amount at death and lasts until the death of the insured,
- the annuity (deferred or not-deferred), where the policyholder transfers a single premium to the insurer in the time of the inception of the policy, and receives periodical payments for a fixed number of years or as long as he lives.

The pure premium (P_x) calculation of the life insurance cover is based on *the equivalence principle* such that at the time of inception, the present value of future premiums should be equal to the present value of future benefits. Define:

- x as the age of the insured at inception of the policy,
- n as the policy duration,
- k as the duration of the premium payments,
- v as a financial discount factor,
- q_x as the probability of death between age x and $x + 1$,
- l_x as the number of lives at age x,
- d_x as the number of death between age x and $x + 1$,
- $[T\,]_x$ as the death benefits at age x,
- $[L\,]$ as the benefits in case of survival at maturity.

One can determine the probability of surviving at least n years for a person age x by ${}_np_x = l_{x+n}/l_x$, and the *actuarial discount factor* by ${}_nE_x = v^n {}_np_x$. The equivalence principle implies

$$\sum_{t=0}^{k-1} v^t l_{x+t} P_{x+t} = \sum_{t=0}^{n-1} v^{t-1} d_{x+t} [T\,]_x + v^n l_{x+n} [L\,].$$

For each type of contract, the actuary adapts the different components of the equivalence equation and determines the pure premium.

Health insurance pricing combines methods from both the life and non-life insurance pricing models. The life insurance aspects include, e.g., disability annuities and long term care cover and survivor modelling. The non-life insurance methods are used in the estimation of the claim frequency and claim severity.

1.4. Game theory and equilibrium concepts

Game theory deals with strategic relations between decision makers called players. Each player has an objective function that represents his preferences among multiple choice alternatives and the constraints related to this set of optional choices. Players are assumed to take rational decisions and to optimize (maximize or minimize) their objective function. A non-trivial situation is one where each of the objective functions depends on a set of the players' decisions and not only on one player's action. A good overview of different games can be found in Osborne & Rubinstein (2006), on which parts of this section are based.

1.4.1. History of game theory

The concept of game theory dates back to at least the book Cournot (1838), written by the French mathematician A. Cournot. He presents a duopoly market competition, as each competitor has to decide his production levels simultaneously, estimating the other producer's decision. The total production level affects the market price of the product. The basic assumptions of the model include:

(i) there are at least two producers,

(ii) all producers have the same marginal costs and produce exactly the same product,

(iii) the producers do not cooperate and decide simultaneously on the production levels.

The book presents a conceptual solution that is a version of the Nash equilibrium (see below). In 1883, J. Bertrand, another French mathematician, puts Cournot's model in doubt, claiming that it is not a real equilibrium, since "whatever the common price adopted, if one of the owners, alone, reduces his price, he will, ignoring any minor exceptions, attract all of the buyers, and thus double his revenue if his rival lets him do so" (Bertrand (1883)). The resulting Bertrand model is based on the assumptions that

(i) the clients will always choose the lowest price, and the firm that offers the lower price will gain the entire market,

(ii) if more than one firm offers the lowest price, then the customers will randomly choose one or the other,

(iii) the firms do not cooperate and they set their prices simultaneously, and

(iv) all firms have the same marginal costs.

This behaviour of the market will result in all firms offering the same price which is equal to the marginal cost. The model's result is known as the *Bertrand Paradox* which shows that already with two competitors, the equilibrium price will become the marginal price. In contrast to the paradox, in real life situations it is expected that only in a market with a large number of competitors, the product price will be close to the marginal cost.
Between 1921 - 1927, Emile Borel published a series of papers where he developed the first modern model of a mixed strategy and the minimax solution for duo games with three or five available strategies. Hotelling (1929) expanded the Bertrand model by presenting differentiated products in a duopoly. His model introduced the concept that the consumers will choose the product not just because of its price but also considering qualities such as product design, customer service and brand name. This model resolves the *Bertrand Paradox* to some extent.

Game theory research as we know it today, started with the publication of the 'Theory of Games and Economic Behaviour' by John von Neumann and Oskar Morgenstern (1944) and the introduction of the utility function concept. Since then, many researchers contributed to the development of the field. Game theory is increasingly applied in different scientific and real life situations, including insurance, economics, business, political science, computer science, auction theory, biology and psychology. Its importance is demonstrated by researchers of game theory, who received the Nobel Memorial Prize, including Paul A. Samuelson in 1970, Nash, Selten and Harsanyi in 1994, Thomas Schelling and Robert Aumann on 2005, Leonid Hurwicz, Eric Maskin and Roger Myerson in 2007, Alvin E. Roth and Lloyd S. Shapley in 2012 and Jean Tirole in 2014.

1.4.2. Type of games

The following types of games are proposed to describe various circumstances:

- A *finite game* is one where each player has a finite number of options to choose from; if the set of options is infinite, it is called an *infinite game*.

- A *repeated game*, is one that includes a long period of interaction, and is played repeatedly. In such a game the players include in their decision process the effect of their action on the other players' future behaviour. It is possible in such a game to encounter phenomena such as cooperation, threats and revenge. The game can be played a finite or infinite number of iterations and equilibrium results for the two types of repeated game can be very different. While in an *infinitely repeated game*, the optimal action is to cooperate and play a more 'social' game strategy, in the *finite repeated game*, with a defined number of moves, the solution is a repeated action of a single game.

- A *complete information game* is the case where the players have all the information about each other including their identities, the available actions and their objective functions. They do not know, however, which actions each of the other players will take. In situations where at least one player does not have the full information about the other players, the game is defined as an *asymmetric information game*, also known as *Bayesian Game*.

- In an *imperfect information game* players may have a full information of the other players, but they cannot see the actions chosen by them. A game where all players have full information about each other's options and actions is called a *perfect information game*.

1.4.3. Cooperative game theory and solution concepts

Cooperative game theory describes games where the players can cooperate and take decisions with full trust and collaboration. Cooperative games with transferable payoff include:

- Players: $N = \{1, 2, \ldots, n\}$ is finite and fixed set of players with individual payoff function $\phi = \{\phi_1, \ldots, \phi_n\}$.

- Coalitions: $C \subseteq N$, where ρ is a coalition partition $\rho = \{C_1, \ldots, C_k\}$. Note that $\emptyset$ is an empty coalition, N is the *grand coalition* and the set of all coalitions is 2^N.

- Characteristic function: $\nu(C)$ assigns a value/payoff to each coalition which can be shared by the i members of the coalition, with no restrictions on the members' share.

The main solution concept of cooperative game is *the Core* (Gillies (1959)). According to this concept, *the Core* includes all the outcomes where the *grand coalition* and the equilibrium payoffs set $\phi^\star$ are:

1. Pareto-efficient: $\sum_{i \in N} \phi_i^\star = \nu(N)$ meaning that a player i can increase his payoff only at the expense of another player j.

2. Superadditivity: the grand coalition is efficient, i.e. $\sum_{C \in \rho} \nu(C) \leq \nu(N)$.

3. Individual rationality: all players receive a payoff which is not less than the individual payoff, i.e. $\nu(i) \leq \phi_i^\star$ for all i.

4. Coalition rationality: the sum of the individual payoffs $\phi_i^\star$ within a coalition C, is equal or larger than the coalition payoff $\nu(C)$, i.e $\sum_{i \in C} \phi_i^\star \geq \nu(C)$ for all C.

Generally, a cooperative game is suitable for situations in which the payoff of the cooperation is larger than the sum of groups/individuals payoffs. Shapley (1953) defines some properties for an acceptable allocation and suggests a 'fair' solution how to split the payoff gain from the cooperation between the players and to allocate each player his average marginal contribution to the payoff gain. The *Shapley value* is Pareto-efficient, it assigns the same payoff to any two symmetric players, and it satisfies monotonicity and assigns zero payoff to dummy players.
In cooperative games without transferable payoff, the coalition is assigned a payoff and there may be restrictions on the split between the members of the group (see e.g. Shapley & Shubik (1953)). Other references for cooperative games and solutions can be found in e.g. Aumann (1989) and Luce & Raiffa (1957).

1.4.4. Non-cooperative game theory and equilibrium concepts

In many situations, the players cannot, or do not want to, cooperate. In this case, each player will make his own decision without collaborating with the other players. These games belong to *non-cooperative game theory* in which we consider two structures of games:

- A *static game* includes three identified elements; the players, available strategies for each player, and the subsequent payoffs. Each of the players will take decisions based on available strategies and payoffs. The players have only a priori information and they take decisions simultaneously, without knowing the other players' actions. These types of games can be represented with a multidimensional payoff table, in which each combination of the player decisions gives a different payoff to all the players.

- A *dynamic game* in which each player can take a decision in different time frames and not only in the beginning of the game. This type of game is represented by a decision tree, which includes the different decisions at each stage for each player and their resulting payoffs. It is used to model complex, repeated games with a challenging mathematical optimization process.

Two main solution and equilibrium concepts are:

Nash Equilibrium

In 1950, John Nash presented the *static game* solution concept in his paper 'Non-Cooperative Games' Nash (1950), that later came to be known as the Nash Equilibrium. The Nash Equilibrium is a solution to a game, where the decisions of the players are taken simultaneously. Every player tries to predict his competitors' decisions, and an equilibrium point is achieved where no player has an incentive to change his decision, given the other players' decisions. In other words, each player maximizes his utility given the other players choices. In his book, Nash also proved that such an equilibrium exists in a *finite game* with a finite number of players. Applying the solution in practice gives rise to some complexities, like the existence of more than one equilibrium point to a game, or the strong assumption that all players are completely aware of other participants' options and priorities (i.e *complete information game*). Selten (1965) suggested the existence of the *sub-game perfect Nash equilibrium.* He proved that any game that can be split into sub-games, like a *repeated game*, will have a *sub-game perfect Nash equilibrium.* In Harsanyi (1966), Harsanyi gave a definition to distinguish between cooperative and non-cooperative games. During the years 1967-1968, Harsanyi (1968) extended the Nash equilibrium concept to *incomplete information games* for situations that include uncertainty on the other players' payoffs. In order to model these type of games, Harsanyi used a Bayesian prior distribution.

Stackelberg Equilibrium

Another game solution can be found when dealing with non-simultaneous decisions. Von Stackelberg (1934) presented an equilibrium concept in a duopoly game, where one of the players, the leader, makes a decision first, and then the other player (*the follower*) reacts. In an oligopoly market, the model assumes that the leader chooses a price and the followers then react with a Nash equilibrium, given the leader's choice.

Many other game solution concepts, mostly Nash equilibrium refinements, have been developed in the last decades, e.g. coalition-proof Nash equilibrium (Bernheim et al. (1987)) and correlated equilibrium (Aumann (1974)). The mathematical formulation of Nash and Stackelberg equilibriums is given in Chapters 2 and 4.

1.4.5. Status quo and insurance

According to the classical model of decision making, agents are expected to behave rationally and choose the alternative with best outcome on the basis of well-defined preferences (von Neumann & Morgenstern (1944)). Friedman (1948) analyzed choices involving risk and proposed a utility function composed with both concave and convex segments to explain both risk-averse and risk-loving behaviour of the decision makers. However, much evidence on a systematic breach from this concept has challenged the validity of the utility theory model (Thaler (1980)). Kahneman & Tversky (1979) develop an alternative model, the prospect theory, in which instead of evaluating the final state of wealth, a value is given to earnings and losses and probabilities are weighted. They suggest a value function that is convex and steep for losses and concave for gains. As humans tend to assess probabilities with some distortion, the model corrects these distortions by weighted probabilities that are, for most cases, lower than their true value. For the range of low probabilities, humans tend to overestimate the true probabilities. This distortion may support the attractiveness of insurance and the willingness of policyholders to pay premiums higher than the expected loss.

One of the most common deviations from the expected rational behaviour, is the high preference for the status quo choice. Insurance is one of many markets in which decision makers tend to avoid change and stay with the same insurance contract. Many researchers study status quo in insurance. Samuelson & Zeckhauser (1988) find evidence for a significant level of status quo in health plan choice and were the first to refer to this phenomenon as the "status quo bias". See as well Strombom et al. (2002) for status quo in health plan decisions. Johnson et al. (1993) performed a series of tests to examine biases in probability valuation and perceptions of loss and the effect they might have on policyholders' insurance choice. The results of the study show that the clients have a false perception of risk and distorted premiums and benefit evaluations. Frank & Lamiraud (2009) study the Swiss health insurance market and find that status quo preference is higher when more options are available to the policyholder and the longer a policyholder stayed with the same insurer. Sinaikoa & Hirth (2011) study employee health plan choices in case where the set of possible plans offered by their employer includes a dominated plan. They find evidence for a status quo bias where a significant number of employees chose to stay with their current plan although it is dominated by the other available options. For further research on status quo in health insurance, see e.g. Krieger & Felder (2013) and Marquis & Holmer (1996). These studies on the status quo bias support the findings of low price elasticity coefficients in the Swiss health insurance market (see Chapter 4).

1.5. Applications of game theory in insurance pricing

Game theory deals with situations where the expected utility of the players is a function not only of their decision but the decision of their competitors. Insurance is considered as a competitive market, and clients that look into reducing the premium payments, compare the offered insurance contracts. An insurer that does not consider his competitors' premiums in his premium decision, might set such a high price that he will lose market share and find himself out of the business. Game theory seems then a natural choice for modelling the insurance pricing process. Borch (1962) suggests a model for the automobile insurance based on cooperative game theory ideas and proved that groups of policyholders with different risk profiles can benefit from reduced prices, if they cooperate. He suggests a model to calculate the "fair" premium of each risk group. Other contributions for applications of cooperative game theory to insurance include e.g. Lemaire (1980, 1991).

Various models based on non-cooperative game theory were suggested as insurance pricing models. Taylor (1986) investigates the insurance cycle in Australia and develops a pricing model under competition to calculate a premium strategy for J periods. He defines the total expected profit for the Jth period, E, as the objective function to maximize, such that

$$E = \sum_{j=1}^{J} v^{j-1/2} q_j (p_j - \pi_j),$$

where v is the discount factor, q_j is the level of exposure insured by the insurer in year j, p_j is the premium per unit of exposure in year j, and π_j is the break-even premium. The level of exposure, q_j, is defined as a demand function related to the insurer's previous year's exposure, q_{j-1}, the insurer's premium p_j and the market premium $\overline{p_j}$:

$$q_j = q_{j-1} f(p_j, \overline{p_j}).$$

The optimal pricing strategy for the insurer, is the premium vector $(p_1, \ldots, p_J)$ that maximizes E. Taylor (1986) sets certain restrictive assumptions on the demand function:

(i) the demand function is stationary over the insurance periods,

(ii) the demand in period j is proportional to the previous period demand,

(iii) the market premium is not sensitive to the insurer's premium.

The last assumption restricts the model to a market with no leaders but a group of smaller insurers. Taylor (1986) presents an exponential and constant elasticity demand function and calculates the resulting premium that will maximize the insurer's expected profit. The results deviate from what might be considered as obvious choice and does not support following the competition when premium levels are low. The optimal premium strategy depends on the price elasticity of demand and the return rate the shareholders expect to receive for their investment. When insurance prices drop and insurers underwrite contracts incurring loss, the expected time until the prices will increase above the break-even point affect the optimal decision as well. Taylor concludes that, generally, underwriting insurance with loss is rarely an optimal alternative that will benefit the longer term profitability of the insurer.

Emms & Haberman (2005) extend the model of Taylor (1986) and develop a continuous form model based on deterministic and stochastic optimal control theory to calculate the premium which maximizes the expected wealth of the insurer at time T. As in Taylor's model, the market premium does not change by the insurer's price decision. An important assumption of this model is that all policyholders are charged the current premium rate p. For insurer i set:
w as the wealth,
α the rate of return to the shareholders for the capital investment,
q as the exposure level at time t,
p as the premium per unit of exposure,
π is the break-even premium,
$\overline{p}$ as the market average premium per unit of exposure,

The state equations are

$$dq = q \log g(p/\overline{p})dt$$

for the exposure process where g is the demand function and,

$$dw = -\alpha w dt + q(p - \pi)dt$$

for the wealth process. The market premium $\overline{p}$ is assumed to be a random process with finite mean and the distribution of the claim size, represented by the break-even premium, π, is left un-defined. The objective function to maximize is the expected wealth at the end of time T given a state $S(0)$ at time $t = 0$,

$$V = (E(w(T)) \mid S(0)).$$

Emms & Haberman (2005) calculate the optimum premium $p_i^\star$ for various demand functions. They find that optimal strategies vary and depend on the demand function. A withdrawal from the market, setting a premium above break-even or loss-leading can all

turn out to be optimal. With a linear demand function when the loss ratio $\pi_i/\overline{p}$ is sufficiently small or the mean contract length is sufficiently large, a loss-leading premium strategy arises as optimal.
Extensions for this model can be found in Emms et al. (2007), Emms & Haberman (2009) and Emms (2007, 2011).
In the above-mentioned models, a basic assumption is that the insurer plays a game with a virtual player, the market, as his decision does not affect the market premium. In recent years, some studies suggested models in which the market premium is influenced by the insurer's premium decision such that insurers consider their competitors premium decisions for their own premium decision. Emms (2012) adapts the control theory ideas presented in previous models and suggests a model with the market premium as the average premium of insurer i's competitors. Each insurer aims at maximizing his own terminal wealth. A result of a two player game in a finite market suggests that the actuarial premium cycle is a result of the competition with limited demand and entry of new players to the market. The suggested model is complex and one needs to solve multiple coupled optimization problems in order to determine the Nash equilibrium premium. Other models based on the same methods of control theory, e.g. Wu & Pantelous (2017) and Boonen et al. (2018) suggest variations to the pricing model of Emms (2012).
Another model that extends the Taylor's insurer against market setup to a game where the market premium depends on the insurer's competition premium decisions is presented in Dutang et al. (2013). They use non-cooperative game theory concepts to develop a one-period pricing model under competition in non-life insurance markets. Albrecher & Daily-Amir (2017) extend the model to an asymmetric information game setting where not all insurers share information about their parameters with their competitors. Battulga et al. (2018) develop the one-period pricing model into a repeated game by adding a transition probability matrix that describes insureds' switching probabilities between insurers and reflects economic factors. The model considers a set of I insurers and an uninsured state as the available strategies for the policyholders, such that

$$P_{i,j}(k) = \begin{pmatrix} p_{1,1}(k) & p_{1,2}(k) & \cdots & p_{1,I+1}(k) \\ p_{2,1}(k) & p_{2,2}(k) & \cdots & p_{2,I+1}(k) \\ \vdots & \vdots & \ddots & \vdots \\ p_{I,1}(k) & p_{I,2}(k) & \cdots & p_{I,I+1}(k) \\ p_{I+1,1}(k) & p_{I+1,2}(k) & \cdots & p_{I+1,I+1}(k) \end{pmatrix}$$

where $p_{i,j}(k)$ is the probability to switch from insurer i to insurer j in period k. The portfolio size of insurer i in period k, $N_i(k)$, is calculated as the sum of the renewals and the new clients arriving from other insurers

$$N_i(k) = N_i(k-1)p_{i,i}(k) + \sum_{j=1, j\neq i}^{I+1} N_j(k-1)p_{j,i}(k) = \sum_{j=1}^{I+1} N_j(k-1)p_{j,i}(k)$$

where $N_i(0) = n_i$. For a discount factor v, the adapted objective function from Dutang et al. (2013) is defined as the present value of the operating profit for m periods of the game, i.e;

$$\mathcal{O}_i^m(x) = \frac{v^k N_i(k)}{n} \sum_{j=1}^{m} \Big(1 - \beta_i(k)\Big(\frac{x_i^k}{m_i(x^k)} - 1\Big)\Big)(x_i^k - \pi_i(k)),$$

where n is the total number of possible customers, and for period k, $m_i(x^k)$ is the average premium of insurer i's competitors, $\beta_i(k)$ is the price elasticity coefficient and $\pi_i(k)$ is the break-even premium. The extended model considers the effects of the economic situation on the break-even premium and on price elasticity coefficients, where clients can be more or less sensitive to premium differences.

1.6. Contributions in this thesis

Chapter 2 gives an extension to a game-theoretic pricing model under competition for non-life insurance companies presented in Dutang et al. (2013). We define a game where not all the insurers have full information about their competitors and examine the effects of this asymmetric information on the equilibrium premium. We calculate Bayesian Nash equilibria as well as Bayesian Stackelberg equilibria and illustrate the sensitivity of equilibrium prices to different configurations and degrees of information asymmetry through various numerical examples. The results indicate that Nash premiums are lower than the Stackelberg premiums, and that the uncertainty in an asymmetric information game is normally compensated by higher premiums. We get evidence that in a market with more players, the equilibrium price is more sensitive to uncertainty than in a market with just a few competitors. We also demonstrate how, depending on the situation, insurers do or do not have incentives to share their own information with their opponents. The obtained quantitative results support the perception one may have on the influence of incomplete information on prices.

Chapter 3 starts with an overview of the Swiss mandatory health insurance market. We examine the effect of different factors such as price, group affiliation and service level on the annual changes of the insurers' market shares. A linear model with two-sided lognormally distributed errors is developed and using a published dataset for the years 2002 to

2015, we test several hypotheses concerning the main motives for changes in the insurers' market shares. The results suggest that the difference between the insurer's premium and the market premium is the main factor that affects the market share changes. We define a second premium related variable as the relative difference between the insurer's premium change and the market premium change and it turns out to have a smaller yet significant impact on the market share changes. Service level and group affiliation appear not to be significant in explaining annual market share changes.

Chapter 4 introduces a demand function for the mandatory health insurance in Switzerland based on findings from Chapter 3. A game with the operation profit as an objective function of the insurer is considered as a pricing model to optimize premiums. While in the model from Dutang et al. (2013), the initial market share does not affect the equilibrium premiums, the new developed pricing model is constructed such that the initial market share becomes a relevant factor for the equilibrium premiums. A numerical illustration demonstrates the calculated Nash equilibrium premiums as well as Stackelberg equilibrium premiums. The levels of price elasticity coefficients of policyholders in the Swiss mandatory health insurance are evaluated using a published dataset and support the market dynamics where policyholders have relatively low sensitivity to price differences resulting in low switching rates. Accordingly, the calculated equilibrium premiums are equal to the maximum allowed premium in most cases. We compare the results with the equilibrium premiums calculated with the model presented in Dutang et al. (2013), and we assess the effect of the initial market share on the equilibrium premiums through sensitivity tests.

Chapter 2

On Effects of Asymmetric Information on Non-Life Insurance Prices under Competition

ABSTRACT

We extend a game-theoretic model of Dutang et al. (2013) for non-life insurance pricing under competition among insurers and investigate the effects of asymmetric information on the equilibrium premium of insurance companies. We study Bayesian Nash equilibria as well as Bayesian Stackelberg equilibria and illustrate the sensitivity of equilibrium prices with respect to model assumptions in numerical examples.
The chapter is based on the paper Albrecher, H. and Daily-Amir, D. (2017), published in International Journal of Data Analysis Techniques and Strategies 9, 287- 299.

2.1. Introduction

The pricing of an insurance policy is a classical research topic. In practice, insurance companies use various approaches including general principles of premium calculation (often based on moments of the claim distribution), credibility theory and generalised linear models (GLM). Game theory concepts have been suggested as a way to evaluate the strategic options available to the insurance companies with respect to competition and the premiums choices. Although applications of game theory in insurance have a long history, the potential of this approach has not yet been fully exploited, particularly with respect to non-cooperative games. The classical reference is Rothschild and Stiglitz (1976), where insurance firms offer contracts with different premium and deductible. The model demonstrates that there might be no equilibrium point in this type of competition in the insurance market. However, when an equilibrium exists, high-risk clients will choose full coverage, where low risk clients will choose partial coverage. Lemaire (1980) developed a number of insurance applications based on cooperative game theory concepts and implemented them for different situations. Emms & Haberman (2005) present a model based on control theory in which they use a demand function to describe the number of underwriting policies, and the objective function of the insurer is the expected terminal wealth. Their findings include two pricing strategies; (i) set a low premium, which creates a loss in order to gain a large market share, followed by higher premium that produces profit, (ii) market withdrawal strategy, which means that the insurer leaves the market and does not compete. This model does, however, not cover the situation of a very competitive insurance market, with few strong insurance companies. Emms (2012) describes an extension using game theory concepts. Other papers dealing with game theory concepts for the insurance market include e.g. Taylor (1986), Polborn (1998), Rees *et al.* (1999) and Powers and Shubik (2006).

In this paper, we adopt a one-period model of Dutang et al. (2013)based on a non-cooperative game among non-life insurers (see Section 2.2 for details) to the situation where players do not have full information on their competitors and we investigate the effects on resulting premium equilibria. In particular, we study Bayesian Nash equilibria and Bayesian Stackelberg equilibria through a numerical implementation for several different scenarios.

Section 2.2 introduces the model assumptions and the considered solution concepts. Section 2.3 then presents and interprets the numerical results. Finally, Section 3.4 concludes.

2.2. One-period model and solution concepts

Let us assume a market with I insurers competing for a fixed number n of policyholders with homogeneous risks. The insurance contracts are issued for one period (one year). Each insurer j sets his premium x_j at time 0, choosing from a possible 'action' set A_j. After x_j is set by all the insurers, each policyholder can decide either to renew his policy with the present insurer or switch to a competitor. It is assumed that there is no price distinction between new insurance contracts and renewed ones as the insurers offer the same product to all clients. Furthermore, the insurers do not cooperate to set their prices. In Dutang et al. (2013) it was assumed that the lapse rate of the policyholders as a function of the different prices x_j can be estimated through a multinomial logit function which motivates the demand function

$$D_j = 1 - \beta_j(\frac{x_j}{m_j(x)} - 1)$$

of insurer j. This demand function gives an approximation for the rate of renewed policies, where $\beta_j > 0$ is a price sensitivity (elasticity) parameter representing the lapse behavior of the policyholders relative to company j and $m_j(x) = \frac{1}{I-1}\sum_{j\neq l} x_k$ is the average price of the competitors. For each insurer, the objective is to maximize the expected operational profit

$$\mathcal{O}_j(x) = D_j(x_j - \pi_j), \tag{2.1}$$

where π_j is the break-even premium calculated as a weighted average of the actuarial premium $\overline{a}_{j,0}$ of insurer j (based on the individual claim experience of each insurer) and the market premium $\overline{m}_0$ (based on the collective claim experience of the entire market). In addition, the model includes a solvency constraint imposed by the insurance regulator with respect to the choice of premium x_j as well as possible general premium restrictions, i.e. $x_j \in [\underline{x}\ , \overline{x}]$. For the scenarios implemented in the next section, realistic magnitudes for the solvency constraint and such bounds on the premium turn out to be automatically fulfilled by the obtained equilibrium prices, so we will not consider these constraints further in the present paper.

As insurance companies will typically only have limited information about their competitors, in this paper we extend the model of Dutang et al. (2013) to the case of asymmetric information and we will apply the concept of *Bayesian games* to determine the resulting premiums. In a Bayesian game, players only have prior beliefs about some characteristics of other players, so player j is assumed to be of 'type' i (t_{ji}) with probability $p(t_{ji})$. The characteristic for which the information is not symmetric is either assumed to be the elasticity parameter β or the internal break-even premium in this paper.
Harsanyi (1966) established a solution to such an incomplete information game, where each player 'plays' against all the different types of other players at the same time and maximizes the expected value of the objective function with respect to the (subjective) probabilities of the various types of the other players. A *Bayesian Nash equilibrium* (BNE) is hence a set of actions $x^\star = (x_1^\star, \dots, x_M^\star)$ such that

$$\forall x_j \in A_j : E(\mathcal{O}_j(x_j^\star(t_j), x_{-j}^\star(t_{-j}))) \geq E(\mathcal{O}_j(x_j(t_j), x_{-j}^\star(t_{-j}))) \tag{2.2}$$

for all players j and all types t_j, where player j knows his own type, so that the index $\cdot_{-j}$ refers to all the possible types of the other players, and M is the number of all the considered types in the game. As shown in Dutang et al. (2013), the objective function ensures the existence of a unique Nash equilibrium, so that the same holds true for the BNE $x^\star$.

While the BNE is an intuitively appealing solution concept in a competitive environment, in some markets there may be a clear market leader who takes his decision first, and the other companies will take that premium choice into account to then choose their own optimal premium. This approach was formalized by Stackelberg (1934) (originally in a duopoly setup). In an oligopoly market, the model assumes that the leader chooses a price and the followers then react with a Nash equilibrium model, given the leader's choice. Assume without loss of generality that Player 1 is the leader, then the Stackelberg equilibrium is the vector $x^\star = (x_1^\star, \dots, x_I^\star)$ if $x_1^\star$ solves the subproblem

$$\forall j, x_j \in X_j : \mathcal{O}_1(x_1^\star, x_{-1}^\star(x_1)) \geq \mathcal{O}_1(x_1, x_{-1}^\star(x_1)), \tag{2.3}$$

and $x_{-1}^\star(x_1)$ is the Nash Equilibrium for all other (types of) players, given the leader's choice $x_1^\star$.

2.3. Numerical implementation and results

In the following, we calculate Bayesian Nash premiums and Bayesian Stackelberg premiums according to the setup of Section 2.2 under some particular choices for model parameters.

2.3.1. Bayesian Nash Equilibrium Premiums

In order to illustrate the effect of the information asymmetry with respect to the case of full information, we start with the situation that only one insurer (Player 1) has unshared information.

Bayesian Nash Premiums for three companies

In general, K_i possible types for Player i lead to $\sum_{i=1}^{I} K_i$ equations, each one maximizing the choice of one type according to (2.1). Assume now a market with three insurance companies (players), where the price sensitivity (elasticity) parameter β_1 of Insurer 1 is unknown to the other players, and based on their beliefs they assign probabilities $p(t_{1i})$ for Player 1 to have price sensitivity parameter β_{1i} (cf. 2.1). For simplicity we assume that these values and their probabilities coincide for Insurer 2 and 3. If they assign two possible values for β_{1i}, then (2.2) consists of four equations (two types for Player 1, one type for Player 2 and Player 3) and the equilibrium premiums are the solution of the linear equation system

$$\begin{aligned}
4\beta_{11}x_{11} - (1+\beta_{11})(x_2+x_3) - 2\beta_{11}\pi_1 &= 0\\
4\beta_{12}x_{12} - (1+\beta_{12})(x_2+x_3) - 2\beta_{12}\pi_1 &= 0\\
4\beta_2 x_2 - (1+\beta_2)\big[p(t_{11})(x_{11}+x_3) + p(t_{12})(x_{12}+x_3)\big] - 2\beta_2\pi_2 &= 0\\
4\beta_3 x_3 - (1+\beta_3)\big[p(t_{11})(x_{11}+x_2) + p(t_{12})(x_{12}+x_2)\big] - 2\beta_3\pi_3 &= 0.
\end{aligned}$$

The break-even premiums $\pi_j = w_j\overline{a}_{j,0} + (1-w_j)\overline{m}_0$ are chosen corresponding to the loss model and the respective individual claim experience. For the present purpose and for the purpose of comparison, we stick to the setup of Dutang et al. (2013) and assume $\overline{a}_{j,0} = (1.1, 1.15, 1.05)$ and $\overline{m}_0 = 1.1$ as well as weighting parameters $w_j = (1/3, 1/3, 1/3)$, i.e. $\pi_j = (1.1, 1.117, 1.083)$.

Figure 2.1 shows the resulting BNE premiums for the case where $\beta_1 \in \{1, 5\}$ with probability 1/2 each, compared with the case of four types $\beta_1 \in \{0.75, 1.25, 4.75, 5.25\}$ with probability 1/4 each (the constant values $\beta_2 = 3.8$ and $\beta_3 = 4.6$ for Player 2 and 3 are known to all market participants). Note that for both distributions of β_1 we have $E(\beta_1) = 3$ and $Var(\beta_1) = 4$. The results in Figure 2.1 indicate that higher uncertainty about the type of Player 1 (more possible types for Player 1) results in higher equilibrium premiums for the other players, although the first two moments of β_1 do not change.

Figure 2.1: BNE Premiums with two or four types for Player 1

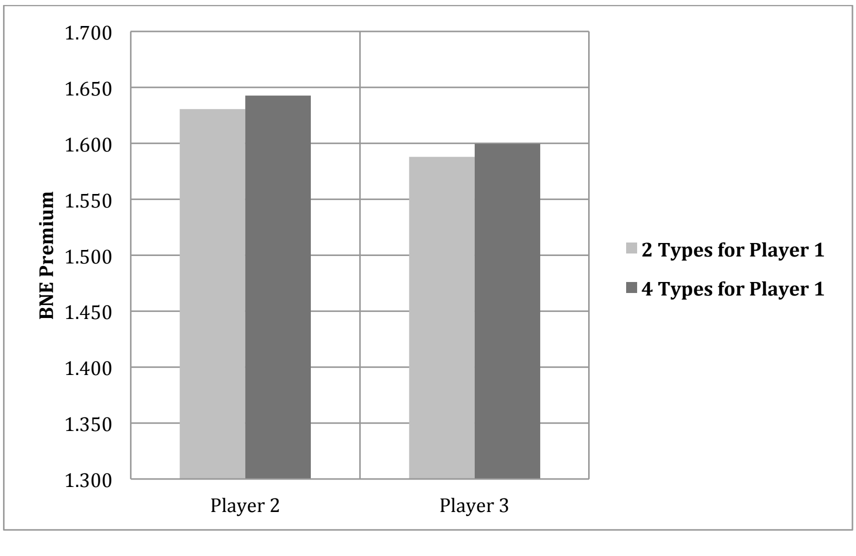
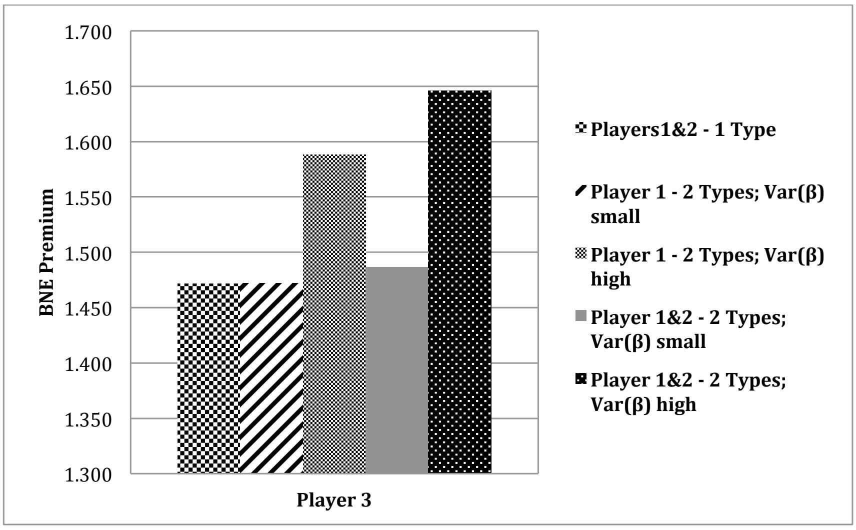


Figure 2.2 shows – from the viewpoint of Player 3 – the influence on the BNE premium of

the number of other players with unknown β_i as well as the value of $Var(\beta_i)$. We depict the following five scenarios for comparison:

- a full information game where the price sensitivity parameter is $\beta_1 = 3$ (and known to all players),
- a Bayesian game where Player 1 has two possible types $\beta_1 \in \{2.75, 3.25\}$,
- a Bayesian game where Player 1 has two possible types $\beta_1 \in \{1, 5\}$,
- a Bayesian game where Player 1 and Player 2 both have two possible types $\beta_1 \in \{2.75, 3.25\}$, $\beta_2 \in \{3, 4.6\}$ (the expected value for the previously constant value of β_2 stays the same),
- a Bayesian game where Player 1 and Player 2 both have two possible types $\beta_1 \in \{1, 5\}$ and $\beta_2 \in \{1.8, 5.8\}$ (same expected value, higher variance)

Figure 2.2: BNE Premiums for Player 3 - various games



The results show that the additional uncertainty spread (for Player 3) over several opponents changes the respective equilibrium premium only slightly while the variance of the price sensitivity has a larger (and the major) effect on the BNE premiums.

It is also of interest to see to what extent it is an advantage or disadvantage in this setup for one particular player to not communicate transparently his β-value. To that end, let us compare the BNE premium of Player 1 (having elasticity parameter $\beta_1 = 3$) in the following three situations:

- a full information game where the price sensitivity parameter $\beta_1 = 3$

- a Bayesian game where Player 1 has $\beta_1 = 3$, but the other players only know $\beta_1 \in \{1, 3, 5\}$
- a Bayesian game where Player 1 has $\beta_1 = 3$, but the other players only know $\beta_1 \in \{0.55, 3, 5.45\}$.

Figure 2.3: BNE Premiums with varying degree of uncertainty about Player 1

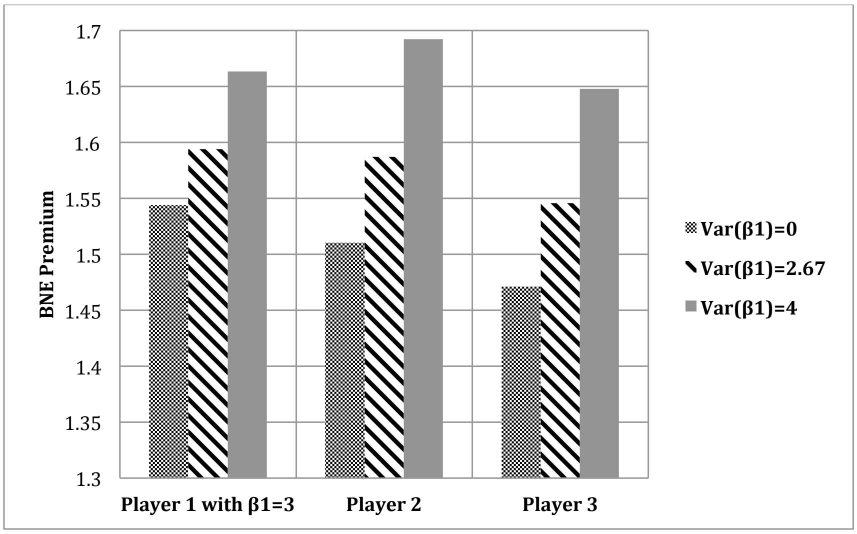


Figure 2.3 illustrates that a higher uncertainty about Player 1 causes Player 2 and Player 3 to choose higher premiums. In turn, Player 1 (being of type $\beta_1 = 3$) also has to 'rationally' (in the sense of Nash equilibria) react by choosing a higher premium. Table 2.1 depicts the actual values of the objective function for each player (which anticipates the resulting market share). One can see that it can be an advantage for Player 1 not to disclose his β_1 value to the competitors, since his objective function value increases with additional uncertainty (the increase is slightly higher for Player 1 than for the other players).

	Var(β_1)=0	Var(β_1)=2.67	Var(β_1)=4
Player 1 ($\beta_1 = 3$)	0.397	0.468	0.570
Player 2	0.391	0.451	0.527
Player 3	0.454	0.523	0.611

Table 2.1: Objective function values corresponding to Figure 2.3

However, what if Player 1 is of type $\beta_1 = 1$? Table 2.2 compares the objective function values for this case between the full information game and the game where the other players only knows $\beta_1 \in \{1, 3, 5\}$ with probability 1/3. Here Player 1 of type $\beta_1 = 1$ turns out to have an incentive to communicate his true (low) β value to his opponents. So one interpretation may be that the disclosure of information is beneficial if the value of β is under-estimated by competitors, but not if there is symmetric uncertainty.

	Var(β_1)=0	Var(β_1)=2.67
Player 1 ($\beta_1 = 1$)	0.956	0.659

Table 2.2: Objective function values with $\beta_1 = 1$

Up to now, the uncertainty in the game was about the price sensitivity parameter β of competitors. Let us now investigate a situation where Player 1 considers reducing the used value of the break-even premium to, say, 80% of its real value (e.g. in order to gain market share). A natural question is whether this information should be passed on to the competitors or not. Figure 2.4 gives equilibrium premiums (and Table 2.3 the respective objective function values) for the following situations:

- using a low π_1-value and disclosing it to the other players (left column in Fig. 2.4): as may be expected, this results in a substantially reduced premium and objective function value.

- asymmetric information scenario, where the other players know that either Player 1 uses the (true) higher break-even premium (probability 0.5) or the reduced one (probability 0.5). In case Player 1 actually uses the higher π_1, the BNE premium as well as the objective function value is lower than for the complete information scenario, so it is better to communicate this to the competitors. However, in case Player 1 uses the smaller π_1, he has an incentive not to tell the competitors about it (in other words, creating uncertainty can be an advantage), cf. Table 2.3. For the other players, even if Player 1 uses the smaller value, not knowing this with certainty is an advantage in terms of their objective function value, and they will also use higher premiums in that case.

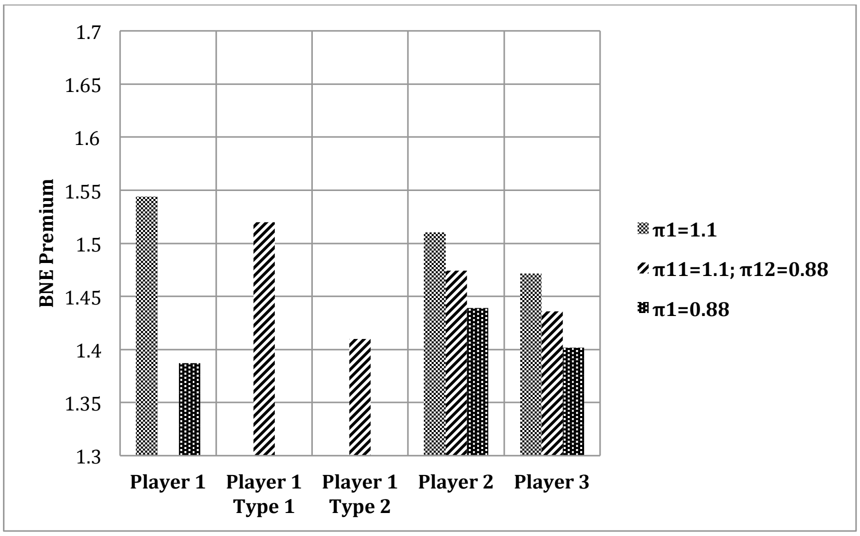
Bayesian Nash premiums for seven companies

Let us now look at the effect of the number of players on the resulting premiums. For that purpose, add four insurers to the original game in such a way that the game consists now of three insurers with the same characteristics as Insurer 2 before, and three insurers with the same characteristics as Insurer 3 before, but the total number of policyholders stays the

	π_1=1.1	$\pi_1 \in \{0.88, 1.1\}$	π_1=0.88
Player 1	0.397		0.307
Player 1 ($\pi_1 = 1.1$)		0.364	
Player 1 ($\pi_1 = 0.88$)		0.339	
Player 2	0.391	0.337	0.283
Player 3	0.454	0.392	0.330

Table 2.3: Objective function values

Figure 2.4: BNE Premiums for two possible π_1



same. We choose $\pi_j = (1.1, 1.117, 1.083, 1.117, 1.083, 1.117, 1.083)$ and $\beta_2 = \beta_4 = \beta_6 = 3.8$, $\beta_3 = \beta_5 = \beta_7 = 4.6$ throughout, and calculate the resulting premiums for a

- a full information game with $\beta_1 = 3$,
- two possible types for Player 1 with $\beta_1 \in \{2.75, 3.25\}$, specifications as before
- two possible types for Player 1 with $\beta_1 \in \{1, 5\}$, specifications as before.

The resulting BNE premiums for Insurer 1 of type $\beta_1 = 3$ and for Insurer 2 are given in Figure 2.5 and 2.6, respectively. The results suggest that in a full information game, higher competition due to additional players in the market result in only slightly reduced premiums whereas in an incomplete information game, the higher the uncertainty, in a market with more players the resulting premiums will be reduced much more significantly.

Figure 2.5: BNE Premiums for Player 1

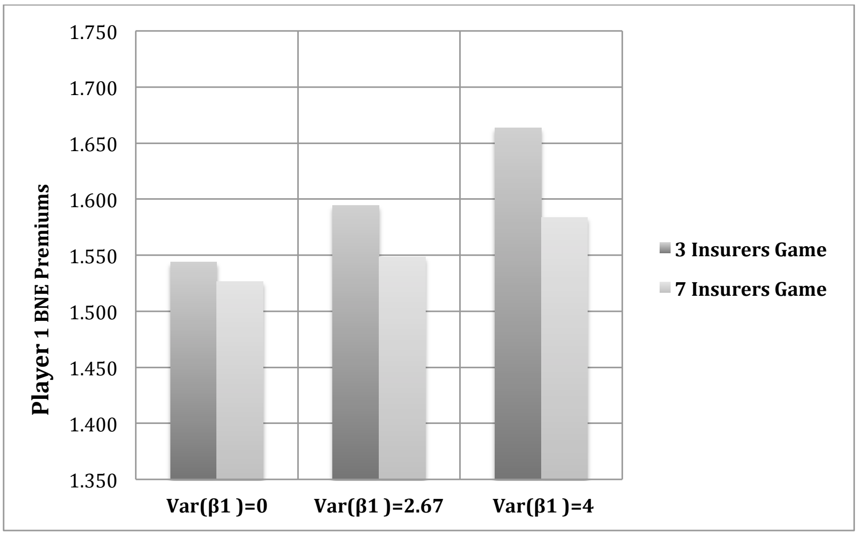
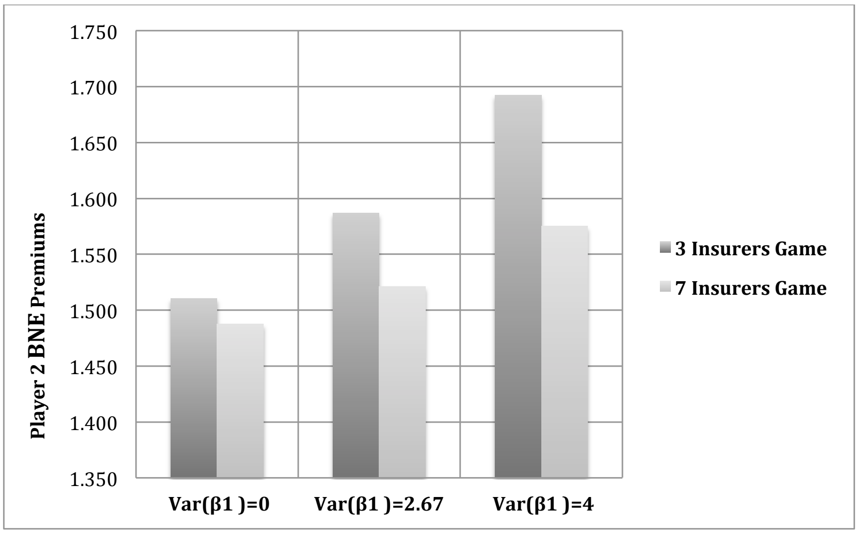


Figure 2.6: BNE Premiums for Player 2



2.3.2. Stackelberg equilibrium premiums for three companies

Let us now assume that there are three players, and Player 1 is the leader as his price sensitivity parameter β is the lowest, which suggests that his policyholders are less sensitive to a possible price increase. We assume that the elasticity parameter β_2 of Player 2 is not known to the other players, they instead use probabilities $p(t_{2i})$ for Player 2 to use β_{2i}. If they assume two possible values, then the expected value of the objective function of the leader, Player 1 is:

$$E(\mathcal{O}_1) = \Big[p(t_{21})(1-\beta_1(\frac{2x_1}{x_{21}+x_3}-1)) \\ + p(t_{22})(1-\beta_1(\frac{2x_1}{x_{22}+x_3}-1))\Big](x_1-\pi_1). \quad (2.4)$$

Once Player 1 sets his premium $x_1^\star$ by maximizing (2.4), the other players calculate their corresponding Bayesian Nash premiums and the optimal solution is obtained by finding the zeroes of their respective derivatives, i.e.

$$\begin{aligned} 4\beta_{21}x_{21} - (1+\beta_{21})(x_1+x_3) - 2\beta_{21}\pi_2 &= 0 \\ 4\beta_{22}x_{22} - (1+\beta_{22})(x_1+x_3) - 2\beta_{22}\pi_2 &= 0 \\ 4\beta_3 x_3 - (1+\beta_3)\big[p(t_{21})(x_1+x_{21}) + p(t_{22})(x_1+x_{22})\big] - 2\beta_3\pi_3 &= 0. \end{aligned}$$

First, we calculate the Stackelberg equilibria for a full information game with $\pi_j = (1.1, 1.167, 1.083)$ and $\beta_j = (3, 3.8, 4.6)$. Table 2.4 presents the equilibrium premiums and the objective function values for the different insurers:

j	$x_j^\star$	$\mathcal{O}_j$
Player 1	1.792	0.446
Player 2 ($\beta_2 = 3.8$)	1.624	0.578
Player 3	1.581	0.668

Table 2.4: Stackelberg premiums and objective function values

Figure 2.7 shows the Stackelberg premiums of Insurer 1, the leader, as a function of $Var(\beta_2)$, for constant $E(\beta_2) = 3.8$. One sees that in general the Stackelberg equilibrium premiums are much higher than the BNE premiums and Insurer 1 will react to additional uncertainty about Insurer 2 with higher premiums.

To verify the incentive of Insurer 2 to communicate the information about his β_2-value to his competitors, in Figure 2.8 we compare the value of the objective function for the two types of Insurer 2 with $\beta_{21} = 2.8$ and $\beta_{22} = 4.8$. One sees that if Insurer 2 is of Type 1, he has an incentive to communicate his information to the other insurers (as in a full information game he has a higher objective function value), whereas if Insurer 2 is of Type 2, he has no incentive to communicate his type and prefers to keep the information asymmetric.

Figure 2.7: Stackelberg Bayesian premiums for Player 1

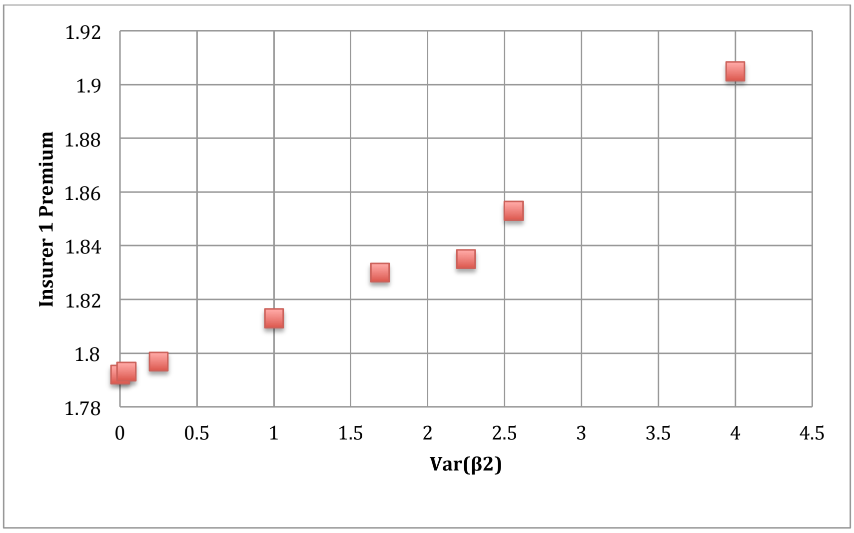
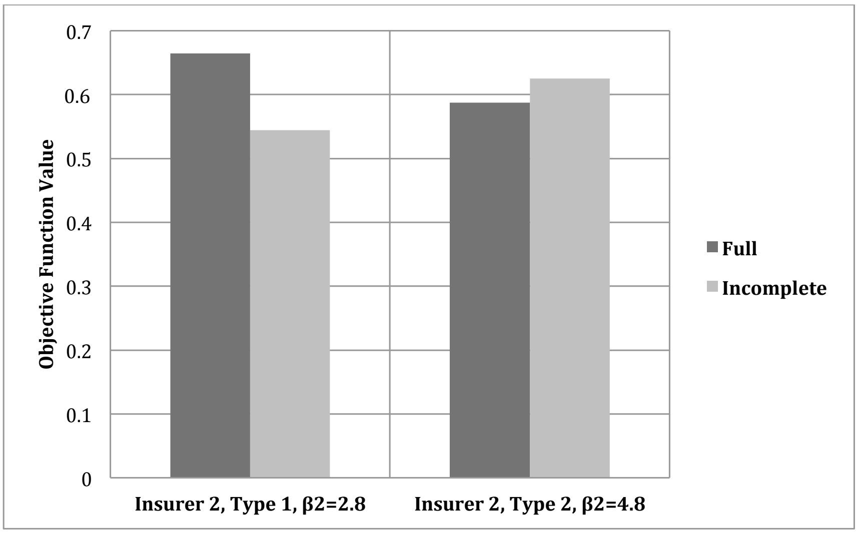


Figure 2.8: Stackelberg Bayesian premiums for Player 2



2.4. Conclusion

In this paper, we deal with the effects of asymmetric information about risk profiles on the pricing mechanism of a competitive insurance market. We use Bayesian Nash and Stackelberg equilibrium concepts to quantify these effects in an extension of the game-theoretic model of Dutang et al. (2013). We numerically illustrate some scenarios in which uncertainty can be beneficial, and others where market participants have incentives to be transparent to their competitors. For the cases considered in this paper, Stackelberg premiums resulting from the presence of a market leader are higher than Nash premiums, and the uncertainty in a Bayesian game is typically compensated by increased premiums. In a market with more participants (competitors), we find the equilibrium points to be more sensitive to uncertainty than in a market with fewer players. Clearly, the findings in this paper rely on the particular choice of model, objective function and equilibrium concept. However, a number of important market features have been taken into account, and the obtained results quantify within this framework some intuitive causal relations in a quantitative way. It is left for future research to study similar questions for other such choices.

Appendix: Additional illustrations of equilibrium premiums

Case I

We are interested in testing the equilibrium premiums when one player has uncertainty about his price elasticity coefficient. We calculate the Nash equilibrium premiums for a game with asymmetric information in three situations:

- G1: a full information game with $\beta_j = (3, 3.8, 4.6)$,
- G2: three possible types for Player 1 with $\beta_1 \in \{1, 3, 5\}$,
- G3: three possible types for Player 2 with $\beta_2 \in \{1.8, 3.8, 5.8\}$,
- G4: three possible types for Player 3 with $\beta_3 \in \{2.8, 4.6, 6.6\}$.

The variance is fixed such that $Var(\beta_j) = 2.67$, the probabilities of each type are equal such that the expected value of the price sensitivity coefficients are $E(\beta_j) = (3, 3.8, 4.6)$. The Nash equilibrium premiums are presented in Table 2.5. In the three games, the player with uncertain β increases his premiums less than his competitors who face the uncertainty. For instance, when the uncertainty is about Player 1, the competitors increase their premiums by 5% compared with only 3% raise in the premium of Player 1. The smaller raise allows him to increase his demand compared with his demand in a full information game. We note similar results when the uncertainty is about the other two players with lower premium change for higher expected values of the β.

	$x_1^\star$	$x_2^\star$	$x_3^\star$	D_1	D_2	D_3
G1	1.544	1.511	1.471	0.893	0.992	1.169
G2	1.594	1.587	1.546	0.947		
G3	1.576	1.530	1.501		1.021	
G4	1.561	1.527	1.481			1.186

Table 2.5: Nash equilibrium premiums and the demand function value

Case II

We now want to test the effect of small uncertainty about the price elasticity coefficient on the incentive of the insurer to become transparent. We construct three games with three possible slightly different values of β_j with equal probabilities such that:

- G1: a full information game with $\beta_j = (3, 3.8, 4.6)$,
- G2: three possible types for Player 1 with $\beta_1 \in \{2.99, 3, 3.02\}$,
- G3: three possible types for Player 2 with $\beta_2 \in \{3.79, 3.8, 3.81\}$,
- G4: three possible types for Player 3 with $\beta_3 \in \{4.59, 4.6, 4.61\}$.

We compare the objective function values for each type to verify if the player with the unknown parameter has an incentive to expose his information to his competitors and present the transparency incentives in Table 2.6. Naturally, for such a low level of uncertainty about the β_j coefficient, the resulted equilibrium premiums and the objective function value are very close, e.g., for Player 1 with $\beta_1 = 2.99$, the objective function value for full information game is greater by 0.00013 than with asymmetric information. Contrarily, if Player 1 has $\beta_1 = 3.01$, in full information game, the objective function value is smaller by 0.00014 than in the asymmetric information game. For Player 1 with $\beta_1 = 3$, the difference is even smaller, nonetheless, the objective function value is greater for the asymmetric information game, so a rational player who wants to maximize his utility, will keep the information for himself, and will not be transparent. Same situation applies for the other two players. For all the insurers, when their price sensitivity coefficient is even slightly smaller than the expected value, they already have an incentive to be transparent with their competitors. When their price sensitivity coefficient is equal or greater than the expected value, they have no incentive to be transparent.

Type	T1	T2	T3
Insurer 1	✓	✗	✗
Insurer 2	✓	✗	✗
Insurer 3	✓	✗	✗

Table 2.6: Transparency incentive

Case III

Until now, we calculated equilibrium premiums for situations with symmetric probabilities for being each of the possible types of the player with uncertainty. We construct three games with asymmetric information about Player 1, with changing probabilities of the two types:

- G1: two possible types for Player 1 with $\beta_1 \in \{3, 3.5\}$,
- G2: two possible types for Player 1 with $\beta_1 \in \{2.75, 3.25\}$,
- G3: two possible types for Player 1 with $\beta_1 \in \{2.5, 3\}$.

For all games, we calculate the equilibrium premiums when the probabilities $P(\beta_{11})$ and $P(\beta_{12})$ are changing. The Nash equilibrium premiums for Player 2 and Player 3 are presented in Table 2.7. The results support previous findings about the effect of the asymmetric information on the equilibrium premiums. For situations with the same $E(\beta_1)$, the equilibrium premiums are higher for higher $Var(\beta_1)$. Furthermore, comparing equilibrium premiums of cases with equal $Var(\beta_1)$, the higher the $E(\beta_1)$, the lower the equilibrium premiums.

Game	G1: $\beta_1 \in \{3, 3.5\}$				G2: $\beta_1 \in \{2.75, 3.25\}$				G3: $\beta_1 \in \{2.5, 3\}$			
$P(\beta_{11})$	$E(\beta_1)$	$Var(\beta_1)$	$x_2^\star$	$x_3^\star$	$E(\beta_1)$	$Var(\beta_1)$	$x_2^\star$	$x_3^\star$	$E(\beta_1)$	$Var(\beta_1)$	$x_2^\star$	$x_3^\star$
1	3.000	0.000	1.511	1.471	2.750	0.000	1.525	1.486	2.500	0.000	1.544	1.503
0.75	3.125	0.047	1.505	1.466	2.875	0.047	1.519	1.479	2.625	0.047	1.535	1.495
0.5	3.250	0.063	1.499	1.460	3.000	0.063	1.512	1.472	2.750	0.063	1.527	1.487
0.25	3.375	0.047	1.493	1.454	3.125	0.047	1.505	1.466	2.875	0.047	1.519	1.479
0	3.500	0.000	1.488	1.449	3.250	0.000	1.498	1.459	3.000	0.000	1.511	1.471

Table 2.7: Equilibrium premiums for games with asymmetric probabilities

Case IV

For two games with asymmetric information about Player 1, with $E(\beta_1) = 3$, $Var(\beta_1) = 0.5$ and probabilities $P(\beta_{11}) = 1/3$ and $P(\beta_{12}) = 2/3$ with:

- G1: two possible types for Player 1 with $\beta_1 \in \{2, 3.5\}$,
- G2: two possible types for Player 1 with $\beta_1 \in \{4, 2.5\}$,

the Nash equilibrium premiums for Player 2 and Player 3 are:

- G1: $x_2^\star = 1.521$, $x_3^\star = 1.481$,
- G2: $x_2^\star = 1.518$, $x_3^\star = 1.478$.

Although the expected value and the variance of β_1 are equal, the equilibrium premium are slightly different. In G1, where $\beta_{12} > \beta_{11}$ and $P(\beta_{12}) > P(\beta_{11})$, the equilibrium premiums are slightly higher that in G2, where $\beta_{12} < \beta_{11}$ and $P(\beta_{12}) > P(\beta_{11})$.

We then determine the equilibrium premium for G1 and G2 with seven players, where $\pi_j = (1.1, 1.117, 1.083, 1.117, 1.083, 1.117, 1.083)$ and $\beta_2 = \beta_4 = \beta_6 = 3.8$, $\beta_3 = \beta_5 = \beta_7 = 4.6$. The Nash equilibrium premiums for Player 2 and Player 3 are:

- G1: $x_2^\star = 1.492$, $x_3^\star = 1.446$,
- G2: $x_2^\star = 1.491$, $x_3^\star = 1.445$.

One can note the same effect as with three players, however, the difference between the equilibrium premiums, for each player in the two games, is 0.001 compared with 0.003 in the three players game. As in a previous example, with more players, the effect of the uncertainty is smaller.

For other variations, with different parameters, we identify similar effects of the asymmetric information on the equilibrium premiums. Below we give two more examples:

Case V

For $\pi_j = (1.1, 1.117, 1.083)$, $\beta_1 \in \{2, 4\}$ and $\beta_2 \in \{2.8, 4.8\}$ with changing probabilities and $\beta_3 = 4.6$, the Nash equilibrium premiums are presented in Table 2.8.

Game	G1	G2	G3	G4	G5
$P(\beta_{11})$	0.25	0.5	0.75	0.25	0.75
$P(\beta_{21})$	0.25	0.5	0.75	0.75	0.25
Nash equilibrium premiums					
$x_{11}^\star$	1.649	1.693	1.742	1.692	1.694
$x_{12}^\star$	1.466	1.502	1.543	1.502	1.503
$x_{21}^\star$	1.563	1.608	1.658	1.588	1.629
$x_{22}^\star$	1.453	1.493	1.537	1.475	1.511
$x_3^\star$	1.451	1.498	1.551	1.487	1.511

Table 2.8: Three players games with asymmetric information about β_1 and β_2

Smaller expected value of the price elasticity coefficient and higher uncertainty, result in higher premiums.

Case VI

For $\beta_j = (3, 3.8, 4.6)$, $\pi_1 \in \{0.8, 1.4\}$ with changing probabilities, $\pi_2 = 1.117$ and $\pi_3 = 1.083$, the Nash equilibrium premiums are presented in Table 2.9.

$P(\pi_{11})$	0	0.25	0.333	0.5	0.667	0.75	1
$x^\star_{11}$		1.426	1.415	1.394	1.373	1.362	1.330
$x^\star_{12}$	1.758	1.726	1.715	1.694	1.673	1.662	
$x^\star_2$	1.608	1.559	1.543	1.511	1.478	1.462	1.413
$x^\star_3$	1.566	1.519	1.503	1.471	1.440	1.424	1.376
OB_{11}		0.764	0.746	0.710	0.674	0.656	0.604
OB_{12}	0.242	0.207	0.196	0.174	0.153	0.143	

Table 2.9: Three players games with asymmetric information about π_1

One can note, that if Player 1 has $\pi_1 = 0.8$, his objective function is equal to 0.604 in the full information game. The objective function value in the asymmetric game has higher value, meaning that a rational Player 1 will keep the uncertainty about his break-even premium.

Chapter 3

Analysis of insurers' market share in the Swiss mandatory health insurance market

ABSTRACT

In the mandatory health insurance market in Switzerland a range of insurers offer policies that differ in characteristics like premium and service level. In this paper, we give an overview of the Swiss health insurance market and analyse the relationship between these characteristics and the changes of the insurers' market shares. We develop a linear model with two-sided lognormally distributed errors and use a publicly available data on the Swiss mandatory health insurance market for the years 2002 to 2015 to test several hypotheses concerning main drivers for changes in market shares. The results suggest that market share changes are particularly linked to the difference between the insurer's premium and the market premium. In addition, the difference to the previous year's premium also has an impact on the market share while the service level as well as group affiliation turn out not be significant in explaining annual market share changes.

This chapter is based on the manuscript Daily-Amir et al. (2019), which is submitted for publication.

3.1. Introduction

Health insurance systems around the world face major challenges including increasing expenditures and a growing number of old people leading to a growth of premiums. According to the OECD Health Statistics published in 2017 (Groninger & Lacher (2017)), the average expenditure for health-related services in 2016 was 9% of the GDP in the OECD countries. In Switzerland, it increased from 9.4% in 1999 to 12.4% in 2016 and, in the US, it amounted to 17.2% in 2016. While the GDP in Switzerland only grew by 90% and the population by 23% from 1990 to 2014, the healthcare costs grew by 165% in the same period.

In this paper, we are interested in better understanding the development of health insurers' size and to link this evolution to the pricing decisions in the mandatory health insurance market in Switzerland. We formulate a model for the impact of the premium decisions on the individual insurer's market share and test it on publicly available data. In fact, insurers operate in a competitive and highly regulated market that went through a process of consolidation and a major reduction of the number of players over the last years. There are numerous studies in different countries regarding the influence of price changes, service level, number of available insurance plans, the health situation and the age of the insureds on the health plan consumer's choice. Schut et al. (2018) compare the switching behaviour of insureds in Germany and the Netherlands. They show that in Germany, insureds react more sensitively to an increase of premium than in the Netherlands. They also find that older policyholders are less sensitive to increasing premiums than younger ones. Christiansen et al. (2016) analyse the policyholders' switching behaviour in the German private health insurance market. Their findings show that a premium change and its adjustment frequency relate to the switching behaviour of customers between insurers. Schmitz & Ziebarth (2017) use field data to test the effect of price framing on the switching rates in the German health insurance market. They find that presenting differences between the insurer's premium and a federal reference premium in absolute Euro values instead of percentage points of the gross salary, results in increasing switching rates. In the Dutch health insurance market, Boonen et al. (2016) test the effect of premium and quality rating of the insurer on the switching decision of the insureds. Other studies such as Strombom et al. (2002) and Goldman et al. (2004) also investigate the effect of price on the insureds' health insurance choice. Schmeiser et al. (2014) study the perception of risk factors and gender-related price differences in several European countries and insurance products including Switzerland and health insurance. A game-theoretic approach to model pricing decisions and lapse rates of policyholders as a function of the different players' premium decisions is suggested by Dutang et al. (2013), see also Albrecher (2016) and Albrecher & Daily-Amir (2017) for including asymmetric information in the analysis.

Studies on decision making find that more options to choose from is linked with inertia. Decision makers tend to avoid taking a decision and stay with the current choice (see e.g. Samuelson & Zeckhauser (1988)). Eling & Kiesenbauer (2011) and Hellier et al. (2003) suggest the number of different insurance models and the rate of complementary insurance holders as explanatory variables to the switching behaviour of the insureds. Frank

& Lamiraud (2009) discover that the large number of plans, offered by a large number of players in the Swiss health insurance market, contributes to the relatively low switching rate and large premium differences. They find evidence that policyholders who stay with the same health plan provider for longer periods are less likely to switch plans, representing a certain degree of inertia. In another study on German health insurance, Wuppermann et al. (2014) test the effect of the number of available health insurance plans and their respective premium differences on the premium sensitivity of the insureds. They also find that insureds are less likely to switch to a lower priced plan when they have more plans to choose from.

In terms of linking customer satisfaction and loyalty to their service provider, conclusions are unclear. For example, Abraham et al. (2006) report that they did not find a connection between health plan satisfaction and switching behaviour, for which high switching costs are suggested as a possible reason. In their studies Staudt & Wagner (36) and Mau et al. (2018), the authors analyse customer loyalty, the development of relationships and purchase in the non-life insurance market in Switzerland. They link purchase behaviour to the services and channels used.

Among other factors to explain switching behaviour, Browne & Hofmann (2013) find evidence that low-risk policyholders are more likely to change health plans. In the Spanish health insurance market, Pinquet et al. (2011) find evidence that insufficient information about the available insurance plans cause the insureds to lapse.

In this paper, we formulate and test several hypotheses concerning the influence of premiums, group affiliation and service level on the market share of Swiss insurers. We use data covering a period of 14 years, from 2002 to 2015, containing portfolio size, premiums and service level for each insurer in each of the 26 Swiss cantons. Applying linear regression models, the results suggest that the relative difference between insurer's premium and market premium represent a significant factor to explain changes in market share. The difference between the relative annual change in the premium and the relative annual change in the market premium is another significant factor while our study does not indicate a significant impact on market share changes from service level and group affiliation. We also establish that those sensitivities vary considerably among cantons. We find that residuals in the regression are rather two-sided lognormally distributed than normally distributed, and develop a corresponding statistical procedure for this case, which may be interesting in its own right.

The rest of this paper is organised as follows: Section 2 provides an overview of the Swiss Health insurance market, Section 3 then describes the available dataset, discusses limitations, states the model and the hypotheses. Section 4 contains the results and Section 5 concludes. Details on the methodology for our linear model with lognormal residuals is given in the appendix.

3.2. The Swiss health insurance market

Buying a basic health insurance policy is compulsory for all Swiss residents since 1996 with the introduction of the Swiss Health Insurance Federal Law (Swiss Confederation, 1994). LAMal intended to deal with the increasing costs on health while ensuring a high quality health system, promoting freedom of choice and solidarity. LAMal determines a mandatory health insurance for all Swiss residents (Swiss Health Insurance Benefits Ordinance 832.102) as a basic homogeneous cover with defined benefits (Health Insurance Benefits Ordinance 832.112), imposes reserve levels to ensure the financial stability of the insurers, forces the acceptance of every person as a client without screening and ensures the flexibility to change the insurance plan without switching costs (Theurl (1999)). In terms of organization, although highly regulated by the state, insurance plans are offered by private insurance companies. The law sets limits on premium discounts between plans (e.g. for deductibles) and forces the insurers to set premiums for each level of deductible in a way that the cover of the insurance expenses is done without profit. The government employs a risk adjustment scheme and transfers capital between insurers in order to balance the financial situation across insurers. In that way, insurers facing higher medical expenses receive capital from insurers with a less risky pool of policyholders.

The governance of the health care system is done on a federal level by both the Swiss Federal Office of Public Health (SFOH) and the Federal Department of Home Affairs (FDHA), and on a cantonal level by the cantonal Department of Public Health. The SFOH defines the mandatory health insurance benefit basket, regulates the insurers and approves their premiums annually. The FDHA defines up to three regions within a single canton for the premiums and sets the maximum differences between them (Ordinance of the Federal Department of Home Affairs on Premium Regions 832.106). The cantons supervise the hospitals and finance part of their expenses. The federal government together with the cantons subsidy the health insurance premiums for low-income households. The conditions and the level of the financial aid vary from canton to canton. In 2015, almost 27% of the insureds received some level of financial support in the premium payments. The cantons transfer the financial aid directly to the health insurers and the policyholders pay

the difference.
The billing system includes two standard methods. The indirect claim settlement, known as "tier garant", is the main billing method. Under this method, the care giver (e.g. doctors and physiotherapists) sends the medical bill directly to the policyholder. After controlling and paying the bill, the policyholder passes a reimbursement demand to his insurer that will compensate the policyholder with the bill amount minus the deductible and the participation amounts. In the direct claim settlement, known as "tier payant", the doctor sends the bill directly to the insurer. The insurer pays the bill and issues an invoice to the policyholder for the deductible and the participation amounts. Usually, policyholders can choose between the two billing methods, however, some insurers do not allow a "tier payant" method for the purchase of medications. In such cases, the policyholders need to pay for medications themselves and wait for the reimbursement. For low-income households, this might present financial difficulties so these customers may choose other insurers.

The 26 Swiss cantons differ considerably in terms of economic, political and demographic characteristics. Table 3.1 summarises some characteristics for the most populated cantons in each linguistic region with Zurich (ZH), Bern (BE) and Aargau (AG) from the German speaking region, Vaud (VD) and Geneva (GE) from the French speaking region and Ticino (TI) as the Italian speaking region as well as Switzerland (CH) as a whole. For example, in 2016, the rate of population living in urban areas was 100% in Geneva and only 74.6% in Bern. The number of physicians in private practice per 100,000 people was more than twice as large in Geneva than in Bern. Differences like these lead to variations among the cantons in the market structure, premium levels, competition and switching rates.

An overview of the Swiss health insurance market is published every year by SFOH and key financial figures for the years 1998 to 2015 are reported in Table 3.2. One can observe the reduction of the number of insurers, the large growth in premiums and expenditures and the operating results alternating through years with positive and negative results. In the following subsections, we present the development since the introduction of the mandatory health insurance law.

3.2.1. Market structure

The Swiss health insurance market is heterogeneous with insurance portfolio sizes ranging from very small regional insurers with a customer base of less than 5,000 up to big insurers with a customer base of over 500,000 insureds. Like other health insurance markets, the

Canton	AG	BE	GE	TI	VD	ZH	CH
Residents (2016)	663,462	1,026,513	489,524	354,375	784,822	1,487,969	8,419,550
Urban population in % (2016)	85.1	74.6	100.0	92.0	89.6	99.3	84.6
Unemployment rate in % (annual average 2017)	3.15	2.59	5.28	3.38	4.52	3.54	3.19
Physicians in private practice per 100 000 people (2017)	167	220	376	219	244	257	219
Hospital beds per 1000 people (2017)	4.5	4.8	5.2	5.4	4.8	4.2	4.5
Debt of the cantons in CHF per inhabitant (2016)	3,542	6,735	31,504	10,300	4,382	5,236	7,640
Social assistance rate (2016)	2.2	4.2	5.7	2.8	4.8	3.2	3.3
Higher education rate (2016)	22.6	24.0	28.0	29.1	25.9	22.6	28.7

Source: Federal Statistical Office, 2018

Table 3.1: Economic and demographic data for selected cantons and Switzerland

Swiss market underwent a consolidation process since the introduction of LAMal (Robinson (2004)). In 1996, the number of insurers offering health insurance was 145 and has reduced significantly in the last 20 years to a total of only 58 in 2015 (see Table 3.3) due to mergers and acquisitions of companies. The number of insurers with a customer base of up to 5,000 reduced significantly from 90 in 1996 to 11 in 2015, whereas the number of insurers with a customer base greater than 100,000 has increased. Froidevaux & Kilchenmann (2016) suggest that insurers who offer low premiums receive many new affiliation demands, but as the state of health of newly insureds is not known, these insurers have to substantially increase their reserves to cover the unknown risk. They mention that this risk has put several insurers into financial difficulties in the past, among them mainly small and medium sized ones that targeted regional communities. Over the long run, some of these insurers have then been absorbed by larger competitors.

Groups of insurers often have more flexibility and are able to offer larger premium ranges for different members in the group. Additionally, a group of insurers can consolidate the business by restructuring the group. In 2015, more than 50% of the insurers were either part of a group or related to a group by a collaboration contract, and these groups of insurers amount to 80% of the market (Froidevaux & Kilchenmann (2016)). As a result, the market share (in terms of number of insureds) of the insurers with more than 100,000 customers increased from 82% in 1996 to 92% in 2015 (see Figure 3.1). Figure 3.2 illustrates the development of market shares from 1998 to 2015 of the eight largest insurers as of 2015. The insurers' market shares largely differ among cantons. As an illustration, Figure 3.3

Year	Number of insurers	Average number of insureds (1'000)	Average premium per insured (CHF)	Gross expenses per insured (CHF)	Total Premium (CHF mio.)	Gross expenses (CHF mio.)	Total operating result (CHF mio.)
1998	118	7,247	1,754	1,935	12,708	14,024	-0.03
1999	109	7,271	1,793	2,011	13,034	14,621	-49.39
2000	101	7,265	1,850	2,131	13,442	15,478	-305.95
2001	99	7,301	1,917	2,244	13,997	16,386	-789.7
2002	93	7,345	2,091	2,328	15,355	17,096	-223.67
2003	93	7,373	2,281	2,431	16,820	17,924	399.73
2004	92	7,384	2,442	2,592	18,030	19,140	514.14
2005	85	7,436	2,487	2,736	18,496	20,348	171.42
2006	87	7,478	2,583	2,755	19,315	20,603	490.95
2007	87	7,538	2,612	2,863	19,689	21,579	178.66
2008	86	7,616	2,586	2,984	19,692	22,722	-755.32
2009	81	7,709	2,611	3,069	20,125	23,656	-471.61
2010	81	7,780	2,834	3,123	22,051	24,292	224.51
2011	63	7,863	3,005	3,171	23,631	24,932	587.67
2012	61	7,953	3,075	3,257	24,458	25,901	915.88
2013	60	8,046	3,105	3,471	24,984	27,926	-141.2
2014	60	8,147	3,172	3,515	25,845	28,639	295.74
2015	58	8,245	3,289	3,653	27,119	30,122	-606.89

Table 3.2: Key figures on the Swiss health insurance market from 1998 to 2015

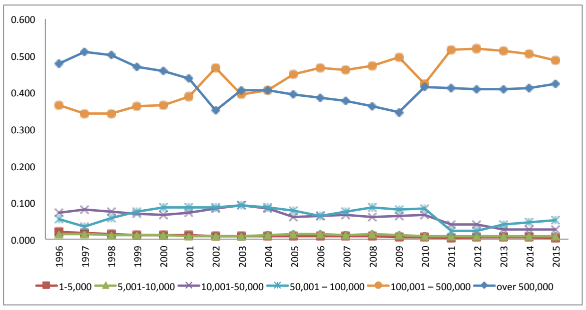


Figure 3.1: Development of the market share by insurer size

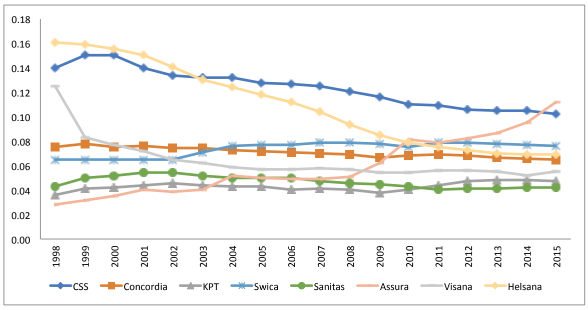
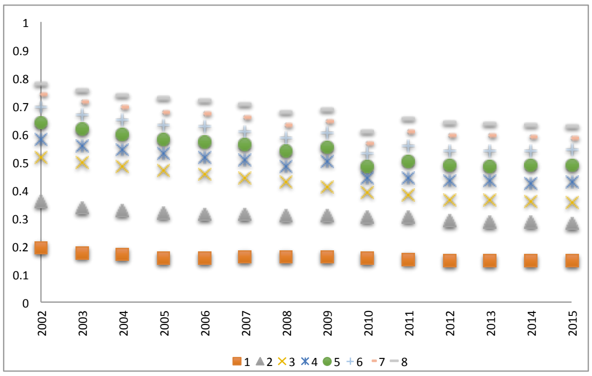


Figure 3.2: Development of the market share of the eight largest insurers

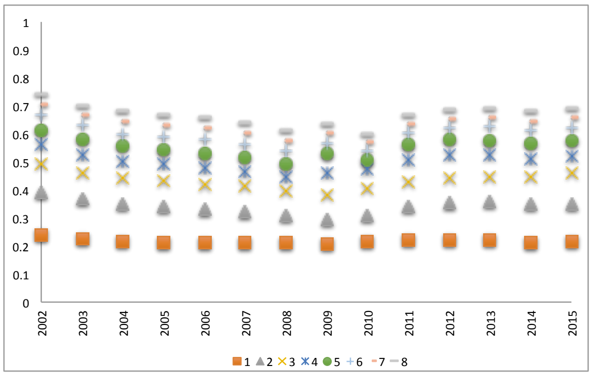
shows the market shares in six cantons of the eight largest insurers in the respective canton per year between 2002 and 2015 where the largest insurer in each year is noted by 1, the second largest by 2 and the eighth largest insurer by the number 8. In GE, the total market share of the eight largest insurers grew from about 40% of the market in 2002 to 60% in 2015. In ZH, the market share is distributed more equally between companies. In VD, during the first years, the market share was more equally distributed, but in recent years, big insurers have captured large market shares, so that market is mainly dominated by a few bigger insurers today.

3.2.2. Premium development 1998-2015

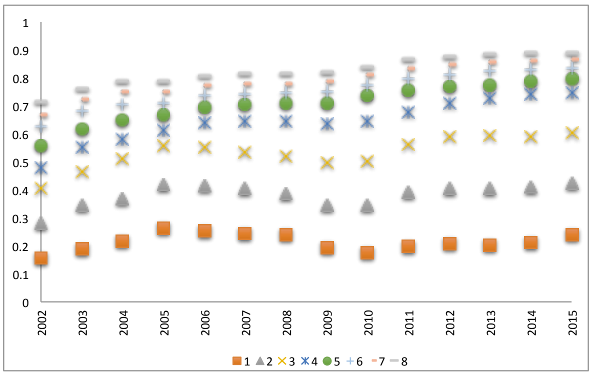
Naturally, the premium size may drive the insureds to switch their insurance plan. Since health expenditures keep growing the insurers have to raise the premiums (see Table 3.2). The calculated average yearly growth (CAGR) is 3.68% during 1998-2015 where the yearly



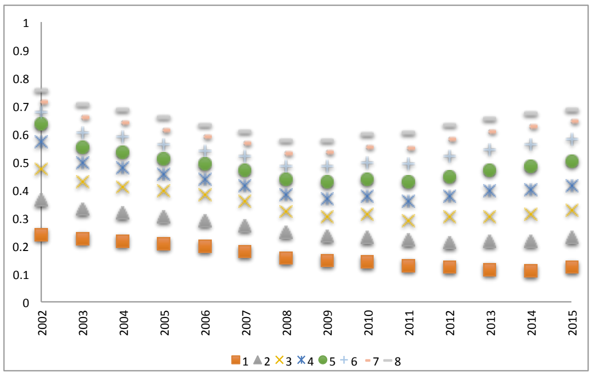
(a) AG



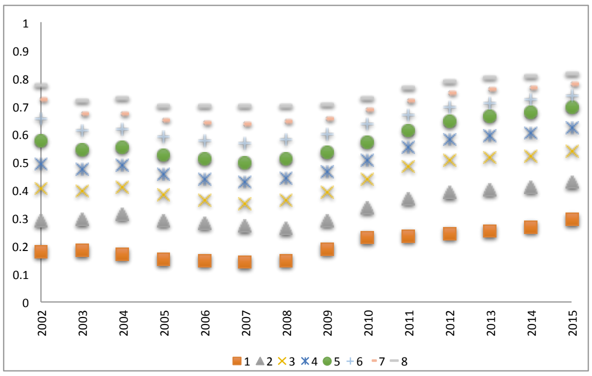
(b) BE



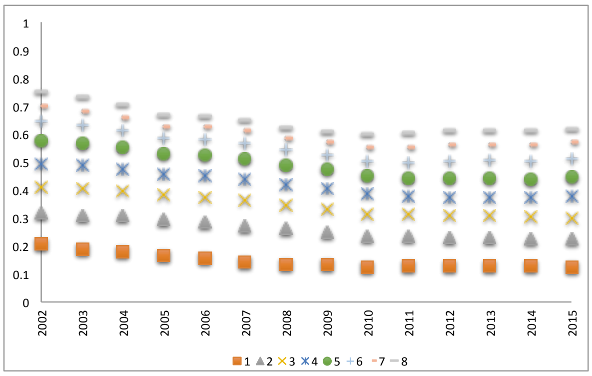
(c) GE



(d) TI



(e) VD



(f) ZH

Figure 3.3: Market shares of eight largest insurers in different cantons 2002-2015

Number of customers	Up to 5,000	5,001-10,000	10,001-50,000	50,001-100,000	100,001-500,000	Over 500,000	Total
1996	90	14	20	6	12	3	145
1997	76	14	20	4	11	4	129
1998	64	13	21	6	10	4	118
1999	56	12	19	8	10	4	109
2000	48	11	19	9	10	4	101
2001	43	9	23	9	11	4	99
2002	33	10	25	9	13	3	93
2003	32	10	26	9	12	4	93
2004	32	11	24	9	12	4	92
2005	27	13	20	8	13	4	85
2006	28	14	20	7	14	4	87
2007	27	13	21	8	14	4	87
2008	26	13	19	9	15	4	86
2009	22	10	20	8	17	4	81
2010	19	10	22	9	16	5	81
2011	13	10	14	3	18	5	63
2012	14	8	13	3	18	5	61
2013	14	8	10	5	18	5	60
2014	14	8	10	5	18	5	60
2015	11	9	10	6	17	5	58
$\Delta 1996-2015$	-88%	-35%	-50%	0%	42%	67%	-60%

Table 3.3: Number of insurers by portfolio size (number of customers)

average premium raised from CHF 1,754 in 1998 to CHF 3,289 in 2015. In 2008, after five years of positive operating results and a reduction by 1% of required reserve rate (expressed as a percentage of the premium volume) by the authorities, the premium level has been reduced by 1% (see Comparis, 2008). However, in the years 2002, 2003 and 2010 the average growth in premiums was much higher than in the other years. One explanation for this increase is the negative balance between the collected premiums and the paid benefits in the years prior to the premium jump (see Table 3.2).

The population is aging with life expectancy increasing in Switzerland from 78.9 years in 1996 to 82.9 years in 2015 (World Bank, 2017) and the proportion of adults aged 26 or more in the population is growing (cf. Figure 3.4), causing increasing health expenses.

The premiums are fixed by the insurers for each premium region in each canton. In each region, all residents of the same age group (below 18 years, 18-25 years and 26 plus years) pay the same premium per insurance plan and chosen deductible at the same insurer. Regulations do not allow premium discounts in the mandatory insurance scheme. To increase the choice for insureds and in addition to the deductible levels (CHF 300, 500, 1,000, 1,500, 2,000 and 2,500 for adults), insurers offer restricted models. The two main restricted models are the Health Maintenance Organization model (HMO) in which the policyholder agrees to always first consult a specified doctor based at the HMO centre and the family doctor model in which policyholders consult first their predefined family doctor who will refer them to a specialist if needed. Other models include the "Telmed" model in which the policyholder has to call a hotline for a first consultation and then will be referred to further treatment if needed. The purpose of these models is to limit medical care within a predefined network of doctors in order to lower health expenses and to offer lower premiums. According to SFOH data, in 1998 about 66% of the insureds in Switzerland had the standard basic insurance (with accident coverage and with the lowest possible deductible, which is CHF 300 since 2004) whereas in 2015 only 21.1% were covered by this insurance model. Against a background of ever-rising premiums, the reduction in this share is linked to the fact that policyholders change towards a more restrictive plan to benefit from lower premiums. The proportion of insureds choosing the standard model (including accident coverage and deductible of CHF 300) per canton is available for the years 2001 to 2015 and is presented in Figure 3.5.

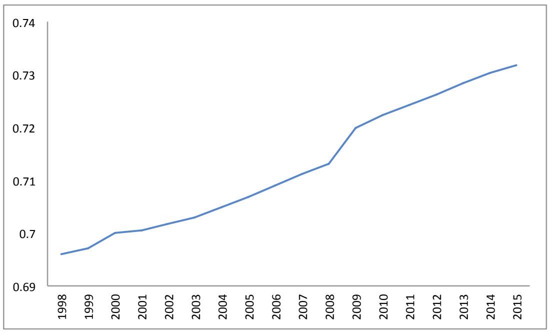


Figure 3.4: Share of the population aged 26 or more

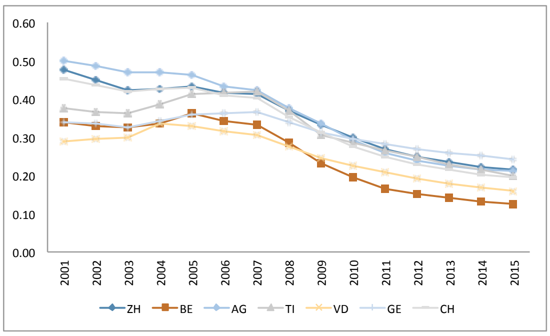


Figure 3.5: Percentage of insureds with the standard model

As mentioned above, insurers perform a premium differentiation by offering several levels of deductible for the three age groups and models with restrictions. Employees covered by accident insurance through their employer can buy the basic insurance without accident cover. These options result in many variations of the basic health insurance premiums (Colombo (2001)). The average premium is calculated for each insurer as the total premium income divided by the number of insured from the data published by the SFOH. Although the law aims at increasing the competition between the insurers, the difference between the highest premium and the lowest one has not changed much (Figure 3.6) and the maximum average premium is approximately twice as large as the lowest one. Leu et al. (2009) suggest that the premium differences among insurers are a result of poor risk adjustment schemes. Figure 3.7 shows the premium differences between the cantons. Leu et al. (2009) find a high positive correlation between the density of physicians and the premium level, which explains to some extent the premium differences across cantons (see Table 3.1).

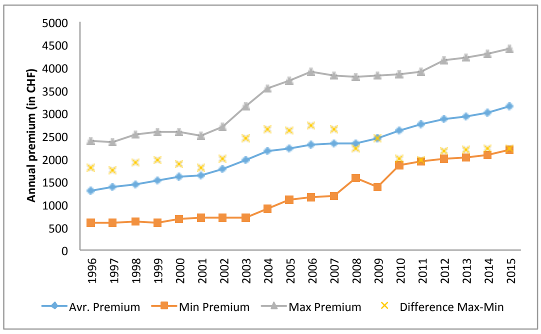


Figure 3.6: Premium range in Switzerland, 1998-2015

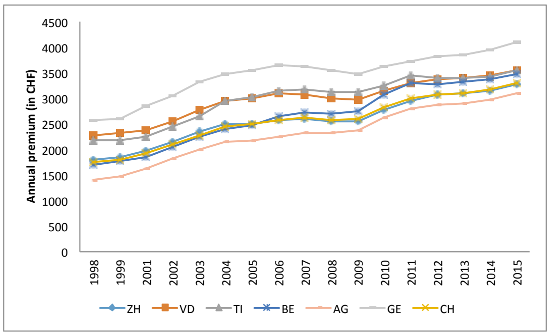
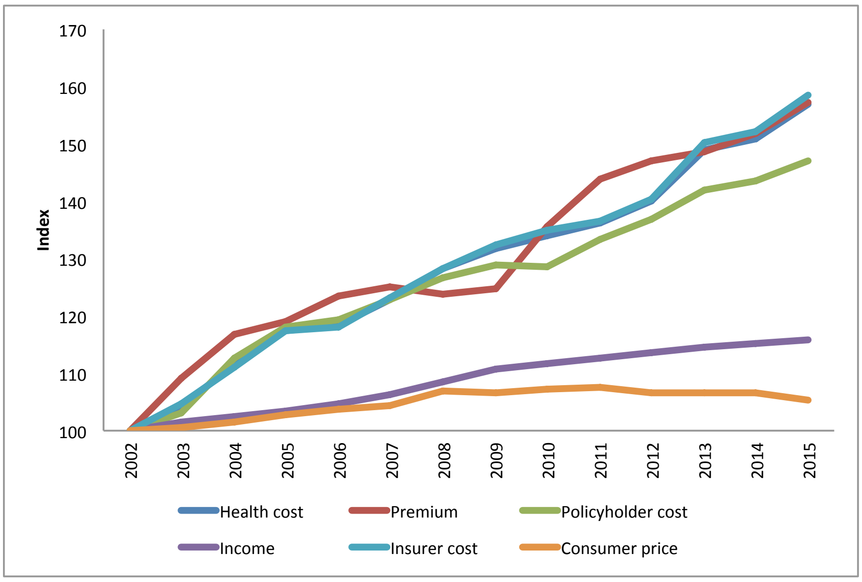


Figure 3.7: Average premium per canton, 1998-2015

The effect of the deductible and inflation on the premium level

New drugs, new treatments, new technology and the increase in life expectancy are some of the reasons that are related to the constant raise in health costs. This phenomenon is called 'medical inflation'. In the Swiss mandatory health insurance, policyholders share the health costs with the insurer. Since 2004, the minimum deductible is CHF 300 and adults can choose between six options of franchise deductible (CHF 300, 500, 1000, 1500, 2000 and 2500). Adults pay a retention fee as well, above the deductible, of 10% of any treatment up to a maximum CHF 700 per year and participate by CHF 15 per day of hospitalization. Children and young adults, contribute reduced amounts. Figure 3.8 shows the economic index development of:

- Health cost index: the evolution of gross benefits (with participation in the expenses of the insureds) per insured,
- Premium index: the evolution of the average premium,
- Policyholder cost index: the evolution of costs for policyholders including premiums and participation in the expenses (only takes into account the invoices announced to the insurers),
- Income index: the evolution of the nominal wage,
- Insurer cost index: the evolution of the net benefits (without participation in the expenses of insureds) per insured,
- Consumer price index (CPI): the evolution in prices of goods and services in Switzerland.



Source: Mandatory health insurance statistics 2016, Federal office of health

Figure 3.8: Index rate in the health insurance sector

One can note that while the development in the premium level and the insurer's cost follow the increase in health cost, the CPI and the nominal wage, are significantly lower. Since the deductible levels stay fixed for many years, the insurers experience even higher increase in the health costs, due to the *leverage effect* (Klugman et al. (2008)). This increase in costs, is transferred to the policyholders through increasing premiums.

3.2.3. Switching behaviour of the policyholders

In view of the strict regulation on the insurers and a defined catalogue of benefits, one might expect a strong price competition and high switching rates of the insureds to ensure paying the lowest possible premium. This could then be followed by convergence of the prices so that the difference between the lowest and the highest premiums would shrink over the years. However, as seen in Figure 3.6, the premium range does not change a lot over the years and the yearly estimated switching rate of policyholders published by SFOH are between $6.5\% - 13\%$ (see Figure 3.9). Note that in case of a merger between insurers, the insureds are counted as a new entries, although they do not have actively changed. Hence, the actual switching rates are expected to be even lower. Figure 3.9 also suggests a relationship between the average annual premium change and the switching rate.

Each year in September, the SFOH publishes the premiums for the coming year. At that time, each insured can compare the change in premium of the own contract with the average change of all insurers. Policyholders have time until the end of November to change their provider. The results of a survey among policyholders concerning their switching behaviour (Federal Statistical Office, 2007) suggest that 77% of the ones that changed their insurer, did so due to premium increases, with higher switching rates among those with a higher education level (5.5% against 3.6%). The linguistic region has an effect on the switching rates as well, with a 8% rate in the French-speaking region and only a 2.9% switching rate in the Italian-speaking region. In 2010, the consumer website `bonus.ch` did a survey among 3,700 insureds, where 80% of the customers who changed their insurer, did so in order to reduce their premium.

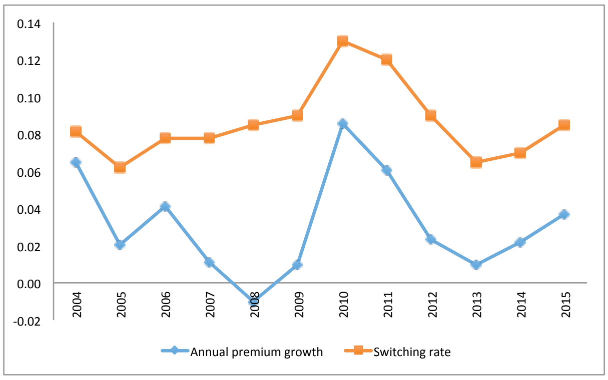


Figure 3.9: Switching rates and annual premium growth from 2004 to 2015

Ortiz (2011) investigates three data sets of Swiss health insurance plans and their prices for the period from 2004 to 2010. His findings suggest that many policyholders do not perform a sufficient price optimization when switching the insurer, resulting in choosing insurers with relatively high premiums. Wilson and Price (2010) suggest that even in a transparent simple market, the ability of consumers to compare correctly between different suppliers is rather limited.

The satisfaction level in the health insurance market in Switzerland is very high (79%) and insureds do not switch insurers when they are satisfied with their current one (Thomson et al. (2013)). In the above-mentioned survey from 2010 by `bonus.ch`, 4% of the insureds that changed their insurer declared that they did so due to a lack of satisfaction with the services.

In addition to the basic mandatory cover, insureds can buy a complementary insurance. Dormont et al. (2009) investigate the effect of existing complementary insurance on the switching behaviour. The results show some evidence that insureds who purchase a complementary insurance are more reluctant to switch the basic insurance provider. These results are supported by the survey from 2010 done by `bonus.ch`, in which 13% of insureds responded that the main reason for them to stay with their current insurer is having a complementary plan.

3.3. Hypotheses, available data and model assumptions

3.3.1. Hypotheses and variables

We now present our hypotheses about the effect of selected factors on the insurer's market share and define the dependent variable $DMS_{i,t,c}$ as the absolute year-to-year difference of an insurer's market share with the one of the previous year, where i indicates the insurer, t the year and c the canton.

Premiums: Colombo (2001) conducted a study on the switching behaviour of the health insurance customers in Switzerland and his findings suggest that while service level appears as minor parameter supporting switching decisions, the premium level is the main motivation for customers to switch their health plan. Nevertheless, the results of that study show that in order to reduce their premium level, policyholders prefer to change their health insurance plan from the standard basic one to more restrictive ones (like HMO) from the same insurer. Following these findings and the surveys mentioned in Section 3.2.3, we want to test two different variables associated with the premiums. First, we suggest that the annual changes in market share of insurers depend on the relative difference between the insurer's premium and the market premium denoted by $RDPM_{i,t,c}$. Our hypothesis (H1) is:

(H1): A *lower* premium than the market premium results in an *increasing* market share.

Secondly, we define $DRPC_{i,t,c}$ as the absolute difference between the insurer's relative annual change in the premium and the one of the market premium. We assume a positive relationship between premium increase and switching decisions:

(H2): A *higher* relative annual increase in premium than in the market premium results in a *decreasing* market share.

Satisfaction: Although a satisfied customer is not necessarily a loyal customer, Berry (1995) claims that a good service level promotes the relationship with customers in service companies such as insurance. Anderson et al. (1994) find a positive effect of satisfaction and performance. We define an explanatory binary variable for the satisfaction of the policyholders $SL_{i,t}$. Our hypothesis (H3) is:

(H3): Insurers with *better* customer satisfaction level yield an *increasing* market share.

Group: Companies that are members of a group have advantages such as knowledge sharing, reduced administrative expenses from scale effects and stronger marketing. Following various research findings such as Cummins and Xie (2008), we test whether belonging to a group supports the insurer and allows to attract and retain more customers. We define a binary variable ($\mathrm{GR}_{i,t}$) representing if the insurer is part of an insurance group or not. Our hypothesis (H4) is:

(H4): Belonging to a group results in an *increasing* market share.

Canton: As discussed in Section 3.2.3, the switching behaviour of the policyholders is related to demographic parameters such as language and education level. The language spoken is strongly linked to the cantons, i.e. we want to test the hypothesis that in different cantons, the independent variables affect the market share of the insurers differently. Nevertheless, as different insurers operate in the different cantons, we cannot quantify these differences so we will test this assumption qualitatively (H5).

We summarise the variables introduced above in Table 3.4:

Variable	Description	Type
$DMS_{i,t,c}$	Absolute year-to-year difference in market share	Number
$RDPM_{i,t,c}$	Relative difference between the insurer's premium and the market premium	Number
$DRPC_{i,t,c}$	Difference between the insurer's and the market's relative annual change of the premium	Number
$SL_{i,t}$	Satisfaction level of the customers	Binary: low, high
$GR_{i,t}$	Group affiliation	Binary: yes, no

Table 3.4: Description of the variables

3.3.2. Available Data

Data description

The Swiss Federal Statistical Office publishes yearly reports, including data of the health insurance system in Switzerland. The available data for the years 2002-2015 include the premiums for different insurance models offered by the insurers in the different cantons, the size of the portfolio of each insurer per canton, the percentage of insureds with the basic standard model per canton and general statistics about the insured population including age and sex. For each of the 26 cantons and each year, we consider the data from insurers with market share greater than 1% neglecting smaller insurers. We analyse a dataset containing a total of 6,117 data points with an accumulated market share from 80% held by 16 insurers in Neuchâtel (NE) in 2003 to over 97% held by 22 insurers in Schaffhausen (SH) in 2014. The number of retained insurers per canton and per year varies from 10 in the canton Appenzell Innerrhoden (AI) in 2002 (total market share of 96%) to 25 insurers in BE in 2008 (total market share of 95%). More specifically, in GE and VD, the number of insurers with market share greater than 1% decreased by three between the years 2002 to 2015 while in the German cantons, the number increased by three. In TI, the number increased from 17 insurers in 2002 to 22 in 2008, before decreasing back to 17 in 2015.

Data preparation

In our work, we only consider the basic standard insurance model including accident cover and a deductible of CHF 300 for the group of adults aged 26+ years. We use the published premiums for the chosen insurance model and the portfolio size for each insurer, each year and canton. From the data, we calculate:

- The Market Share ($MS_{i,t,c}$) per insurer, year and canton as the size of an insurer's customer base divided by the population in the canton.
- The Market Premium ($MP_{i,t,c}$) for insurer i as the weighted average premium (with market shares) of all the other insurers in the market per year and canton.
- The absolute year-to-year change in market share $DMS_{i,t,c}= MS_{i,t,c} - MS_{i,t-1,c}$ as the difference between the current year market share and the one of the previous year per insurer, year and canton.
- The relative difference between the insurer's premium and the market premium ($RDPM_{i,t,c}$) as the difference between the current year premium ($P_{i,t,c}$) and the current market premium ($MP_{i,t,c}$) divided by the current market premium, i.e. $RDPM_{i,t,c} = (P_{i,t,c} - MP_{i,t,c})/MP_{i,t,c}$. As this is a relative difference variable, we can ignore the inflation and use the nominal premium values given in the dataset.
- The difference between the relative annual change of the insurer's premium and the relative annual change in the market premium ($DRPC_{i,t,c}$) per insurer, year and canton, which is defined by

$$DRPC_{i,t,c} = [(P_{i,t,c} - P_{i,t-1,c})/P_{i,t-1,c}] - [(MP_{i,t,c} - MP_{i,t-1,c})/MP_{i,t-1,c}].$$

As mentioned in Section 3.2.3, the policyholder can compare the annual change in his premium with the average annual change in the canton. These changes are based on the actual premium levels without considering the inflation, so we ignore the inflation effect when calculating the variable $DRPC_{i,t,c}$ values.

Annual consumer satisfaction reports are published by `comparis.ch` and `bonus.ch` and include a calculated grade for each of the ranked insurers based on customer satisfaction. Generally, the policyholders are satisfied with their insurers and give relatively high grades for the service level with averages ranging from 4.80 to 5.03 out of 6 over the years. We assigned a binary satisfaction level variable ($SL_{i,t}$) with 0 (=low) for all non-ranked insurers and defined it as our baseline. The ranked insurers are all given a ranking grade of 1 (=high). Since the source for available satisfaction reports varies over the years and not all insurers are ranked, the reliability of these data is lower. This is why we remain with only using a two-level rating.

The group affiliation variable ($GR_{i,t}$) is binary and is assigned using information from insurers' websites and reports. Our baseline value is zero for insurers that are not affiliated to a group.

Outliers: As in 2011, one group of insurers restructured the group and reduced the number of insurers from 15 to only four. This event led to considerable increase in the market share of the four remaining insurers. We consider these data points for 2011 as outliers and we remove them (29 data points) from the dataset.

Data limitations

Before proceeding with the analysis, we lay out the important limitations of our data as follows:

- Information about market events such as mergers and acquisitions are not included as this information is not fully available. Nevertheless, such events may have an important impact on the market share.
- Our data includes only the total number of insureds per year per insurer per canton mixing all available insurance models so we use the market share ($MS_{i,t,c}$) as an estimation of the insurers' cantonal market share. This is relevant since for the premium level we remain with using the standard model premium for reference.
- The yearly switching rates between insurers are unknown and there is no information about the number of policyholders that change their insurance model within the same

insurer.

Descriptive statistics

Table 3.5 presents descriptive statistics of the variables for the entire dataset. We give the average monthly premium, the mean of the market share for each year which ranges between 5.2% − 6.3% and basic statistics (the mean, minimum, maximum and standard deviation) of the dependent variable $DMS_{i,t,c}$, the explanatory variables $RDPM_{i,t,c}$ and $DRPC_{i,t,c}$ as well as $SL_{i,t}$ and $GR_{i,t}$.

We suggest that some of extremal values in $DMS_{i,t,c}$ can be explained by specific events in the market like mergers and acquisitions of insurers, reduction in financial reserves below the required level which results in an important raise in premiums or the restructuring of an insurance group. The minimum and maximum yearly values of $DMS_{i,t,c}$ are in 2010 and 2011 with both years having high switching rates (see Figure 3.9). The year 2010 was a year with high increases in premiums while between 2010 and 2011 the number of insurers has reduced from 81 to 63. These events partially explain the extremal values of $DMS_{i,t,c}$ in 2010 and 2011.

The minimum yearly values of $RDPM_{i,t,c}$ are between -0.182 and -0.245. Some insurers offer an attractive premium with average $RDPM_{i,t,c}$ over all cantons and all years of -12%. Other insurers set premiums above average with an average value of $RDPM_{i,t,c}$ of 14%. Groups of insurers often offer different premium ranges for the basic health insurance. For example, in one group, two insurers may offer different premiums for the basic standard model with one being significantly higher than the market premium and the other one being lower than the market premium.

Reduction in financial reserves below the required level can result in an important raise in premiums in the following years. The largest observed $DRPC_{i,t,c}$ is 32% in 2013 when an insurer fell below the statuary reserve ratio and had to increase premiums by up to 34% in canton Nidwald (NW) while the market premium increased by only 2%. Unexpected increases in claims or administration costs can be another reason for a significant rise of premiums. Such has happened in 2012 in canton Zug (ZG) with one insurer raising premiums by 20%. In 2015, the minimum value of $DRPC_{i,t,c}$ is -22% for an insurer that reduced premiums in the canton Obwalden (OW) by 16% while the market premium increased by 6%.

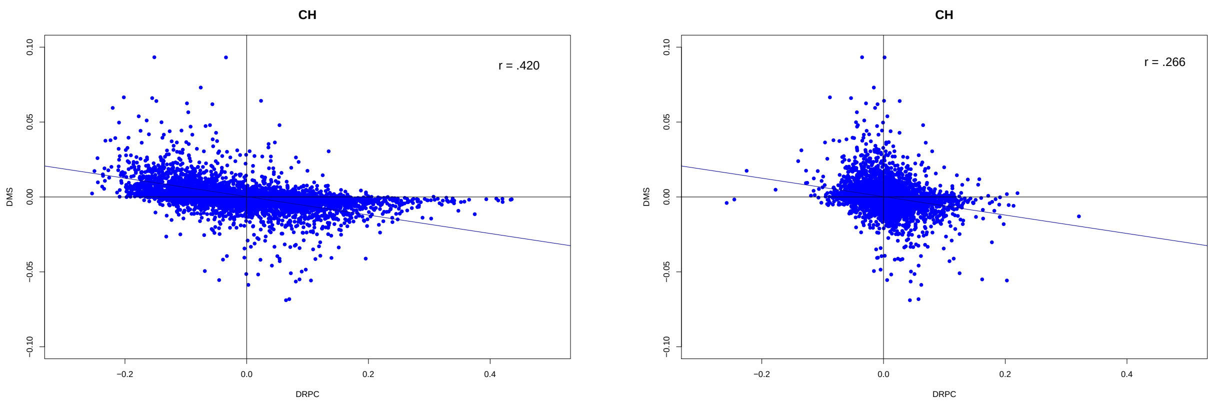
Cooperation between insurers in the same group and mergers and acquisitions might be

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Average premium	225.76	247.43	258.06	266.68	280.22	288.03	287.98	296.83	322.66	345.49	355.94	362.68	371.36	389.33
$MS_{i,t,c}$														
Mean	0.063	0.061	0.060	0.058	0.057	0.055	0.053	0.053	0.052	0.055	0.054	0.053	0.053	0.054
StD	0.072	0.069	0.068	0.066	0.065	0.063	0.061	0.060	0.059	0.061	0.060	0.060	0.059	0.060
$DMS_{i,t,c}$														
Min	-0.057	-0.051	-0.039	-0.030	-0.052	-0.041	-0.034	-0.046	-0.069	-0.068	-0.042	-0.025	-0.017	-0.056
Mean	-0.001	-0.001	0.000	-0.001	0.000	-0.001	0.000	0.001	-0.001	0.001	0.000	0.000	0.000	0.000
StD	0.010	0.008	0.009	0.006	0.007	0.006	0.009	0.011	0.013	0.012	0.007	0.005	0.005	0.008
Max	0.067	0.063	0.066	0.044	0.031	0.026	0.073	0.064	0.093	0.093	0.031	0.020	0.021	0.038
$RDPM_{i,t,c}$														
Min	-0.254	-0.226	-0.209	-0.182	-0.203	-0.196	-0.210	-0.220	-0.224	-0.216	-0.194	-0.207	-0.236	-0.250
Mean	-0.011	-0.007	-0.003	-0.010	-0.009	-0.006	-0.011	-0.008	-0.004	0.006	0.003	0.003	0.001	0.007
StD	0.099	0.086	0.080	0.080	0.084	0.090	0.093	0.098	0.099	0.108	0.104	0.099	0.096	0.099
Max	0.338	0.348	0.394	0.352	0.421	0.434	0.435	0.420	0.317	0.375	0.358	0.365	0.337	0.410
$DRPC_{i,t,c}$														
Min	-0.258	-0.115	-0.177	-0.073	-0.135	-0.140	-0.127	-0.083	-0.245	-0.064	-0.102	-0.069	-0.092	-0.225
Mean	0.006	0.013	0.010	-0.004	0.004	0.006	0.003	0.008	0.011	0.013	0.007	0.005	0.003	0.008
StD	0.052	0.050	0.041	0.031	0.036	0.031	0.024	0.032	0.044	0.044	0.035	0.031	0.026	0.033
Max	0.220	0.206	0.172	0.178	0.174	0.152	0.073	0.151	0.156	0.203	0.191	0.321	0.088	0.203
$SL_{i,t}$														
High	0.773	0.822	0.823	0.816	0.814	0.806	0.779	0.799	0.743	0.850	0.829	0.771	0.803	0.764
$GR_{i,t}$														
Yes	0.526	0.538	0.568	0.577	0.589	0.595	0.605	0.622	0.625	0.686	0.707	0.714	0.732	0.731
Observations	388	398	412	418	431	439	458	458	456	420	457	462	467	453

Table 3.5: Basic statistics of the variables: $DMS_{i,t,c}$, $RDPM_{i,t,c}$, $DRPC_{i,t,c}$, $SL_{i,t}$ and $GR_{i,t}$

related to extremal values as well. In 2003, a ZH based insurer joined an insurers' group and while its market share reduced in the canton by 30%, another insurer from the group increased its portfolio size by a factor of five from 4,900 insureds in 2002 to 24,500 in 2003. In 2009, a merger of a Luzern based insurer with another insurance company increased the market share by 3.69 in the canton.

When we plot $RDPM_{i,t,c}$ and $DRPC_{i,t,c}$ versus $DMS_{i,t,c}$ for all Switzerland (Figures 3.10a and 3.10b), one can observe that the relationship is not linear. One possible explanation is that the plot mixes data points from all cantons and that the influence of the variables on the market shares differs in different cantons, in a superposition of different slopes. The value of the correlation coefficient between $DMS_{i,t,c}$ and $RDPM_{i,t,c}$ is -0.420 and between $DMS_{i,t,c}$ and $DRPC_{i,t,c}$ the value is -0.266 for the $6,117$ data points. Figure 3.11 presents the relationship between $RDPM_{i,t,c}$ and $DMS_{i,t,c}$ for six selected cantons. Indeed, on the canton level the relationship turns out to be more linear, with the slopes differing across cantons. In fact, French-speaking cantons have a steeper slope than others, indicating stronger reactions by insureds in those regions. A similar picture occurs when plotting $DRPC_{i,t,c}$ versus $DMS_{i,t,c}$ (Figure 3.12), with again more pronounced slopes for the French cantons.



(a) $RDPM_{i,t,c}$ versus $DMS_{i,t,c}$ (b) $DRPC_{i,t,c}$ versus $DMS_{i,t,c}$

Figure 3.10: The relationship between DMS and explanatory variables

Overall, the distribution of the $SL_{i,t}$ values through all 26 cantons is shown in Figure 3.13a. The variation in the distribution between the cantons can been seen with ZH and TI having over 85% data points of high ranked insurers and only 15% of low ranked insurers. BE has more than 28% data points of lower ranked insurers. One explanation could be that the insureds in BE are more sensitive and critical to service level. Figure 3.13b shows the percentage of data points of insurers with group affiliation per year from 2002 to 2015

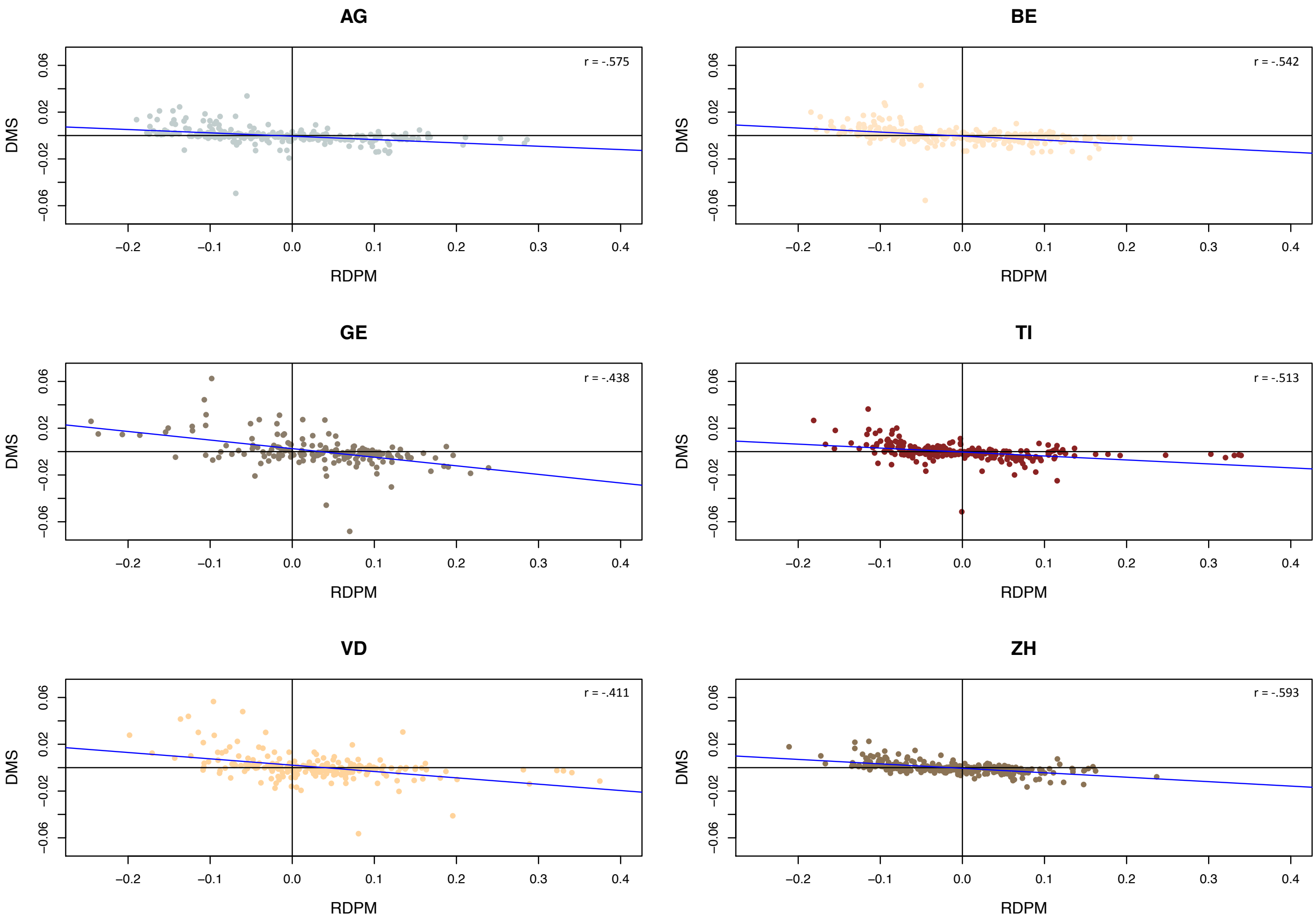


Figure 3.11: RDPM versus DMS in selected cantons

(variable $GR_{i,t}$). The percentage of insurers that are not affiliated with a group reduced during the years from 47% in 2002 to 27% in 2015.

3.3.3. Regression models

To test our hypotheses $(H1)$ - $(H4)$, we first construct a set of linear models $(R1)$. As the number of data points is limited for each canton, we define $(R1)$ with single regressions testing our hypotheses about the different variables one by one such that the difference in market share $(DMS_{i,t,c})$ depends on the relative difference between the premium and the market premium $(RDPM_{i,t,c})$, the difference between the relative annual change in the premium and the market premium $(DRPC_{i,t,c})$, the service level $(SL_{i,t})$ and the group affiliation $(GR_{i,t})$. To analyse our assumption on different effects of variables on the market shares in different cantons, $(H5)$, we will run the regression for the six selected cantons separately. We define $(R1)$ as:

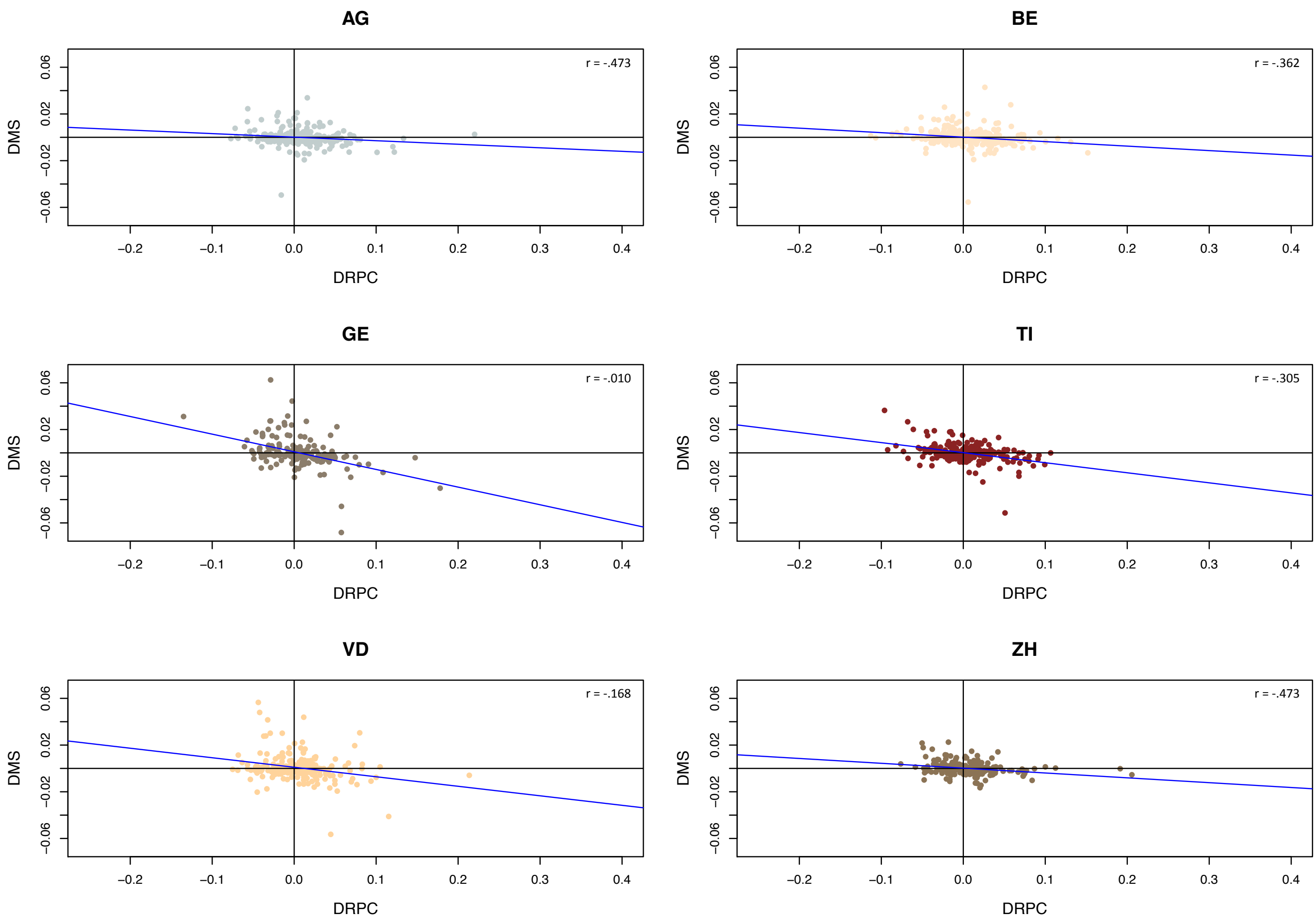


Figure 3.12: DRPC versus DMS in selected cantons

$$DMS_{i,t,c} = \beta_0 + \beta_j \cdot \mathrm{VAR}_{i,t,c} + \epsilon_{i,t,c} \text{ with } \mathrm{VAR}_{i,t,c} \in \{RDPM_{i,t,c},\, DRPC_{i,t,c},\, GR_{i,t},\, SL_{i,t}\} \quad (R1)$$

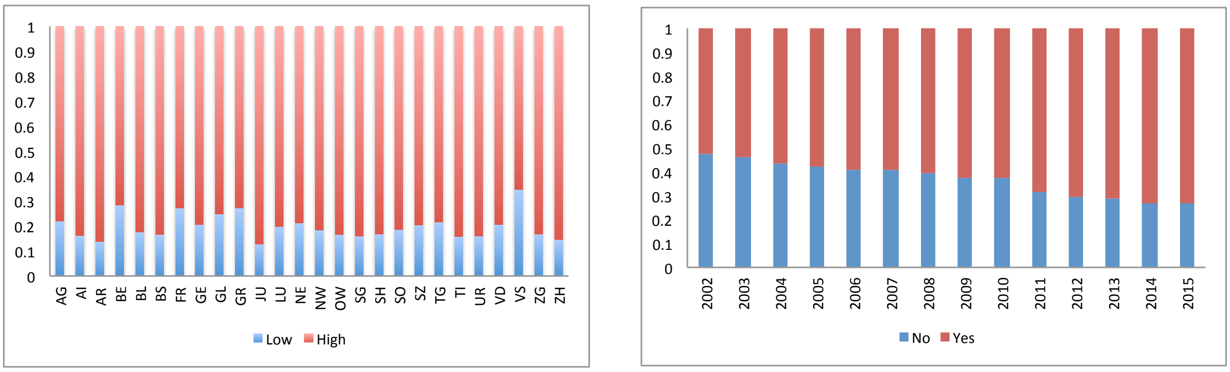
where β_0 is the intercept, β_j is the coefficient for the independent continuous variables $RDPM_{i,t,c}$, $DRPC_{i,t,c}$ and the binary categorial variables $SL_{i,t}$ and $GR_{i,t}$ and $\epsilon_{i,t,c}$ are error terms.

Secondly, we propose a multivariable linear model ($R2$) including both continuous premium related variables such that:

$$DMS_{i,t,c} = \beta_0 + \beta_1 \cdot \mathrm{RDPM}_{i,t,c} + \beta_2 \cdot \mathrm{DRPC}_{i,t,c} + \epsilon_{i,t,c} \quad (R2)$$

where β_0 is the intercept, β_j is the coefficient for the independent continuous variables $RDPM_{i,t,c}$ and $DRPC_{i,t,c}$ and $\epsilon_{i,t,c}$ are error terms.

Figure 3.14 shows the histogram of $DMS_{i,t,c}$ for the entire dataset together with a normal



(a) Satisfaction distribution per canton (b) Group distribution per year

Figure 3.13: Group distribution and satisfaction distribution

and a two-sided lognormal fit. Clearly, the normality assumption is violated in this case, and the two-sided lognormal assumption seems to be a much better description for the data. For a quantitative comparison in terms of AIC on the resulting residuals, we refer to Section 3.4. We therefore suggest to perform the linear regressions ($R1$) and ($R2$) with two-sided lognormally distributed residuals. The corresponding needed statistical methodology is given in the appendix. In Section 4, we compare the results and Q-Q plots of the residuals from the regressions with normal residuals and with the two-sided lognormal residuals. We will see that for individual cantons the normality assumption is better fulfilled, however, the regression with two-sided lognormally distributed residuals turns out to fit better in these cases as well.

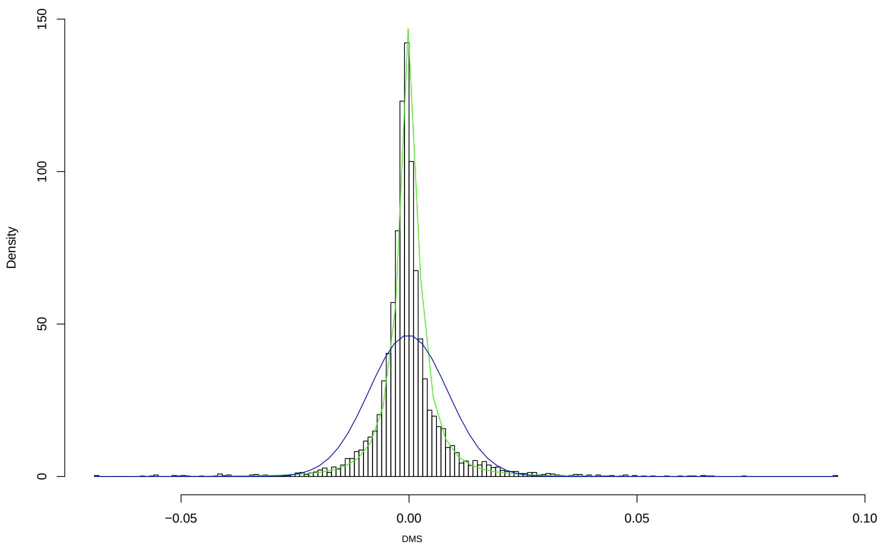


Figure 3.14: Distribution of the dependent variable $DMS_{i,t,c}$ with normal (blue) and two-sided lognormal (green) fit

3.4. Testing the regression models and results

First, we test the classical regression with normally distributed residuals. The results for model ($R1$) with normal residuals are presented in Table 3.6 including the coefficients with their standard deviation and the adjusted R^2 as criterion for goodness of fit. They show that the explanatory variables, $RDPM_{i,t,c}$ and $DRPC_{i,t,c}$, are very significant in the six cantons and for Switzerland. Service level and group affiliation are not significant explanatory variables in most of the cantons.

We also conduct a linear regression analysis with normal residuals for a multivariate regression model ($R2$) with $RDPM_{i,t,c}$ and $DRPC_{i,t,c}$ as explanatory variables in the selected cantons and for the entire dataset and we report the results in Table 3.7 including the variable coefficients (β), the significance level with standard deviation (in brackets), the standardised coefficients ($Std.\beta$), the adjusted R^2 and number of observations. Both explanatory variables are very significant, nevertheless a comparison of their coefficients shows that the standardised coefficients of $RDPM_{i,t,c}$ are higher than the ones of $DRPC_{i,t,c}$ which suggests that a change in $RDPM_{i,t,c}$ has a higher effect on $DMS_{i,t,c}$ than a change in $DRPC_{i,t,c}$. The coefficient values vary as a function of the canton with highest values in GE and lowest in AG. The adjusted R^2 values are higher than in the set of single regressions and are between 16.8% in AG and 35% in GE and ZH.

The analysis of the residuals from the regression with the entire dataset of Switzerland (see Figure 3.15a) confirms that the normality assumption is not met in this case, so the validity of the results is non-satisfactory. For the individual cantons (see Figure 3.15b for TI as an example), the results show a better agreement to the normality assumption.

In order to enhance the credibility of the findings, we conduct the linear regression ($R2$) with two-sided lognormal residual distribution (according to the procedure outlined in the appendix) and we report the results in Table 3.8 including the variable coefficients and their significance. We verify the significance levels of the explanatory variables with N_{sim} = 5,000 (see appendix).

The analysis of the residuals in the case of the entire dataset of Switzerland (Figure 3.16a) confirms the excellent fit of the two-sided lognormal model with AIC =-47,257 compared with -42,120 under the normality assumption. Figure 3.16b shows the Q-Q plot of the residuals for TI (as an example for the performance on the cantonal level) and confirms a better fit than the model with normal residuals (compare with Figure 3.15b). The two-

	AG	BE	GE	TI	VD	ZH	CH
$RDPM_{i,t,c}$	-.0285 *** (.0039)	-.0343 *** (.0044)	-.0734 *** (.0101)	-.0338 *** (.0049)	-.0542 *** (.0078)	-.0380 *** (.0034)	-.0381 *** (.0011)
Intercept	-.0006 (.0004)	-.0005 (.0004)	.0025 *** (.0009)	-.0004 (.0004)	.0021 *** (.0007)	-.0006 ** (.0003)	-.0003 *** (.0001)
Adjusted R^2	.1552	.1681	.2220	.1523	.1730	.3215	.1759
$DRPC_{i,t,c}$	-.0304 *** (.0115)	-.0384 *** (.0118)	-.1512 *** (.0238)	-.0861 *** (.0127)	-.0815 *** (.0202)	-.0414 *** (.0090)	-.0616 *** (.0029)
Intercept	.0001 (.0004)	.0001 (.0004)	.0009 (.0009)	.0002 (.0004)	.0010 (.0007)	.0002 (.0003)	.0002 ** (.0001)
Adjusted R^2	.0207	.0320	.1775	.1478	.0634	.0698	.0704
Satisfaction Level (baseline: Low)							
High	.0010 ** (.0005)	.0009 ** (.0005)	-.0006 (.0011)	.0003 (.0006)	-.0005 (.0009)	.0004 (.0004)	.0010 *** (.0001)
Intercept	.0005 * (.0005)	.0003 (.0005)	-.0002 (.0011)	.0001 (.0006)	.0003 (.0009)	.0003 (.0009)	.0004 *** (.0001)
Adjusted R^2	.0118	.0108	-.0039	-.0032	-.0033	-.0001	.0080
Group affiliation (baseline: No)							
Yes	-.0004 (.0008)	-.0015 * (.0008)	.0001 (.0021)	.0012 (.0009)	-.0012 (.0016)	-.0007 (.0006)	-.0014 *** (.0002)
Intercept	.0002 (.0007)	.0008 (.0006)	.0001 (.0018)	-.0010 (.0008)	.0014 (.0013)	.0005 (.0005)	.0007 *** (.0002)
Adjusted R^2	-.0028	.0081	-.0055	.0025	-.0019	.0009	.0062
Observations	283	293	184	259	228	268	6,117

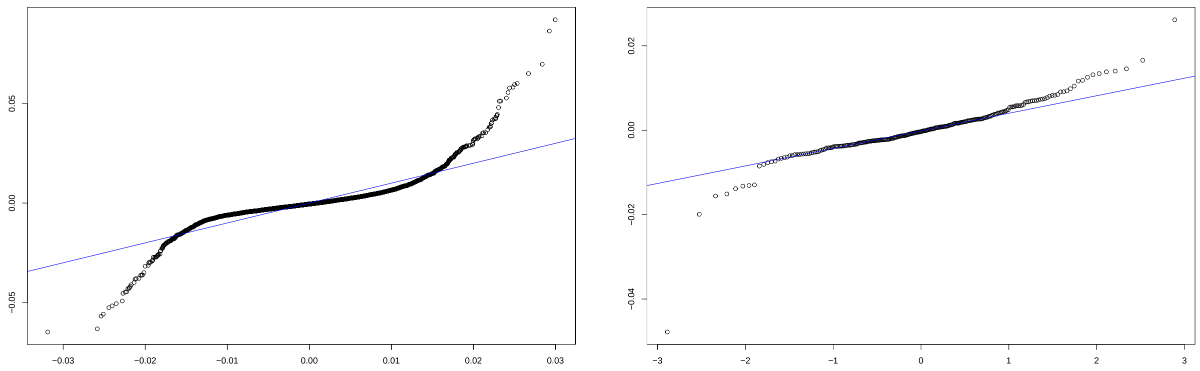
Significance codes: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Table 3.6: Single variable regression results ($R1$) with normally distributed residuals

	AG	BE	GE	TI	VD	ZH	CH
$RDPM_{i,t,c}$	-.0275 *** (.0040)	-.0328 *** (.0044)	-.0652 *** (.0093)	-.0285 *** (.0047)	-.0496 *** (.0078)	-.0360 *** (.0033)	-.0348 *** (.0011)
$Std.\beta$	-.3832	-.3948	-.4224	-.3326	-.3845	-.5388	-.3831
$DRPC_{i,t,c}$	-.0195 *** (.0108)	-.0270 ** (.0109)	-.1293 *** (.0214)	-.0721 *** (.0122)	-.0583 *** (.0189)	-.0274 *** (.0077)	-.0448 *** (.0027)
$Std.\beta$	-.1000	-.1322	-.3650	-.3255	-.1860	-.1792	-.1932
Intercept	-.0004 (.0004)	-.0003 (.0004)	.0030 (.0010)	.0000 (.0004)	.0020 (.0010)	-.0004 (.0003)	.0000 (.0001)
Adjusted R^2	.1620	.1824	.3496	.2519	.2029	.3503	.2118
Observations	283	293	184	259	228	268	6,117

The reported values show the regression coefficients with standard deviations in brackets and the standardised values with significance code.
Significance codes: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Table 3.7: Multivariate regression results ($R2$) with normally distributed residuals



(a) CH (b) TI

Figure 3.15: QQ-plot of normally distributed residuals versus theoretical quantiles

	AG	BE	GE	TI	VD	ZH	CH
$RDPM_{i,t,c}$	-.0150 ***	-.0140 ***	-.0412 ***	-.0101 ***	-.0192 ***	-.0204 ***	-.0164 ***
$DRPC_{i,t,c}$	-.0033	-.0215 ***	-.0925 ***	-.0337 **	-.0236 **	-.0195 ***	-.0175 ***
Intercept	-.0008	-.0006	.0015	-.0006	.0004	-.0006	-.0006
Observations	283	293	184	259	228	268	6,117

The reported values show the regression coefficients with significance code.
Significance codes: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$; $N_{\text{sim}} = 5,000$

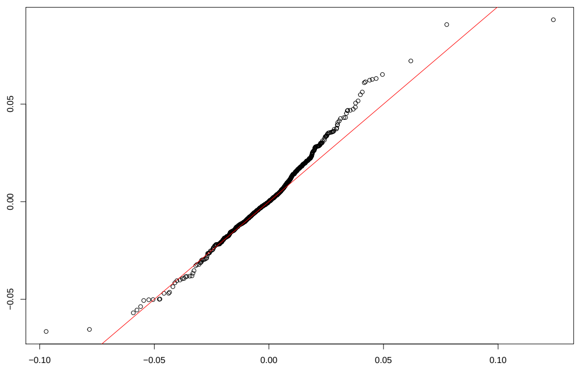
Table 3.8: Multivariate regression results with two-sided lognormal residuals

sided lognormal model fits the data better in the other individual cantons as well.

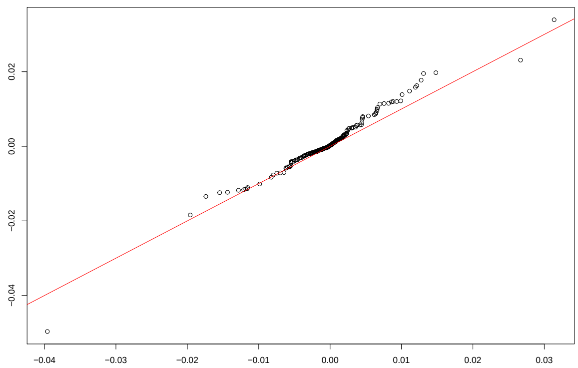
When considering the results from the regressions with normality assumption and two-sided lognormal assumption, we notice that:

- $\mathbf{RDPM_{i,t,c}}$, the relative difference between the insurer's and the market premium, is the most significant explanatory variable with very high significance level in all cantons and with the aggregated data for all Switzerland in both regressions. The coefficient values of $RDPM_{i,t,c}$ are always negative which indicates the negative relation between the dependent variable $DMS_{i,t,c}$ and the explanatory variable $RDPM_{i,t,c}$. The coefficient values with normal residuals are always higher (in absolute value) than the ones with the two-sided lognormal residuals.

- $\mathbf{DRPC_{i,t,c}}$, the difference between the insurer's and market annual premium change, is a significant explanatory variable with negative coefficient values, meaning that a larger value of the difference between the relative change in annual premium and the relative change of the market premium causes a diminution in market share.



(a) CH



(b) TI

Figure 3.16: QQ-plot of Two-sided lognormally distributed residuals versus theoretical quantiles

However, for the model with two-sided lognormal residuals, in AG the $DRPC_{i,t,c}$ is not a significant explanatory variable and the coefficient value is only -.0033 compared to -.0195 under the assumption of normal residuals.

- **The intercept** of the multivariate regression ($R2$) has positive values in GE and VD and negative values in the German cantons. As the number of insurers in GE and VD reduced along the years of our study, naturally, even with zero values of the explanatory variables, the insurers' market shares grow and vice-versa for the German cantons.

We summarise the results in the following: **The relative difference between the insurer's and the market premium** is the most significant variable and the coefficients of $RDPM_{i,t,c}$ have negative value. The results support our hypothesis ($H1$) that a lower premium than the market premium results in increasing market share. Further, **the difference between the relative annual change in the insurer's premium and the relative annual change in the market premium** is a significant explanatory variable with negative coefficient values. The results support hypothesis ($H2$) that higher relative annual increase in premium than the market premium results in decreasing market shares. **Satisfaction level** is not a significant variable and the results do not support hypothesis ($H3$). The **group affiliation** does not affect the dependent variable so this result does not support our hypothesis ($H4$). The regression results are different among the cantons implying that factors influence the market shares differently in different cantons, which supports hypothesis ($H5$). Since the operating insurers vary between the cantons, we could not quantify these effects through a categorial canton variable.

3.5. Concluding remarks

The Swiss health insurance market has changed dramatically in the years since the introduction of the mandatory health insurance in 1996. The number of insurers offering health insurance plans decreased extensively from 145 insurers in 1996 to 58 in 2015 and the annual premiums increased from an average annual premium of CHF 1,917 in 1996 to CHF 3,286 in 2015. We analyse the effect of different variables such as premium, satisfaction level and group affiliation on the market shares of the insurers in the different cantons. We define a model with the market share as dependent variable and four explanatory variables and test hypotheses using data of the mandatory health insurance for the years from 2002 to 2015.

Our regression results support the hypothesis that the difference between the insurer's premium and the market premium is strongly negatively correlated to the market share. We get confirmation for the hypothesis that the premium change from one year to another is important as well. The results do not support the hypothesis about satisfaction. One explanation can be that customers in Switzerland are in general satisfied with their insurer and the small differences in satisfaction levels do not affect the market shares much. The hypothesis that belonging to a group of insurers results in a higher market share is not supported in our findings. Additional investigation of other explanatory variables such as age, education level, health status and possession of complementary insurance is suggested for future research according to the availability of such data. As the available panel dataset is growing every year with the publication of new data, it will be possible to add explanatory variables without the risk of overfitting.

Appendix: A regression model with two-sided lognormal residuals and its estimation

Data sometimes exhibit a linear relation between variables to a satisfactory degree, but instead of normally distributed residuals, the latter follow another distribution. For non-normally distributed residuals within the context of risk modelling see e.g. Prettenthaler et al. (2012). In the context of the application in this paper, it turns out that two-sided lognormally distributed residuals provide an excellent fit. Therefore, we develop a regression model which can be fitted to data that exhibit two-sided lognormally distributed residuals.

More specifically, let us consider a p-dimensional vector of independent variates X and a real-valued response variable Y. We propose the model

$$Y = \beta X + \varepsilon_\mu,$$

where β is a p-dimensional vector of slopes and ε_μ is a random variable with density

$$f_\mu(x) = \frac{1}{\Phi(1)\sqrt{8\pi}(|x| + e^{\mu-1})} e^{-(\ln(|x|+e^{\mu-1})-\mu)^2/2},$$

where Φ is the standard normal cumulative distribution function. Such a density arises naturally when considering the density of the transformation $e^{N+\mu}$, where N is a standard normal random variable, and then reflecting it at its mode, and finally centering it such that the symmetry point is the origin. Observe that a direct consequence of this construction is that it has mean zero. Another consequence is that the tails on both sides of the origin are heavy-tailed, with lognormal behaviour. We hence denote the above density as a *two-sided lognormal* density, with the name carrying over for the variable ε_μ, the cumulative distribution function $F_\mu(x) = \int_{-\infty}^{x} f_\mu(y)y$, and so on.

We are interested in the estimation of the slope parameter vector β above. For the fitting of a Gaussian linear model, least squares are used, which coincides with maximum likelihood estimation. Here, we adopt the maximum likelihood approach as well, but now the geometrical interpretation of least squares is lost, due to the form of the density f_μ. Given some observed covariates $x = (x_1, \ldots, x_n)$, where each x_i is a p-dimensional vector, and variates $y = (y_1, \ldots, y_n)$, where each y_i is a real number, the likelihood of the model for the parameters β and μ is given by

$$L(\beta, \mu | x, y) = \prod_{i=1}^{n} f_\mu(y_i - \beta x_i),$$

and the maximum likelihood estimates are

$$(\widehat{\beta}, \widehat{\mu}) = \arg\max_{(\beta,\mu)} L(\beta, \mu | x, y),$$

which can be computed numerically.

Testing significance between nested models

Whenever two nested models, with resulting maximum likelihood estimators

$$(\widehat{\beta}_0, \widehat{\mu}_0), \quad (\widehat{\beta}_1, \widehat{\mu}_1), \quad \dim(\widehat{\beta}_0) = q < p = \dim(\widehat{\beta}_1),$$

are fitted, it is of importance to know the significance of the additional parameters in $\widehat{\beta}_1$

with respect to the more basic model. The way this is done for Gaussian errors is by regarding the simpler model as true, and under this assumption determining the distribution of the extra parameters in the more complicated model. The resulting distribution helps to assess how likely it is to see parameters of the magnitude that were estimated (or even larger), obtaining in such a way a p-value. Presently, the distribution of the extra parameters given the simpler model is complicated to obtain, so we use Monte Carlo simulation.
Furthermore, given an estimator $\widehat{\beta}_1$, testing for the significance of a single entry of the vector, say, $\hat{\beta}_1(i)$, $i \in \{1, \ldots, p\}$, is equivalent to fitting a model without that entry, and comparing that simpler model with the full model. That is, regarding the $q = p - 1$-dimensional model as true, and determining the distribution of $\hat{\beta}_1(i)$ for the p-dimensional model, such that we can say how likely it is to observe a value such as $\hat{\beta}_1(i)$, or of larger magnitude. Since this is the most common and directly interpretable way of assessing significance, we describe the Monte Carlo algorithm only for $q = p - 1$. Analogous algorithms can be deduced for $q < p - 1$, but significance intervals must then be replaced by suitable significance regions.

Procedure:

1. Fit the two-sided lognormal regression model (M_1) with p slope parameters to the data (x, y). Let the resulting maximum likelihood estimator be denoted by

$$(\widehat{\beta}_1, \widehat{\mu}_1) = (\widehat{\beta}_1(1), \ldots, \widehat{\beta}_1(p), \widehat{\mu}_1).$$

2. Fit the two-sided lognormal regression model (M_0) with $p-1$ slope parameters which is formed by deleting the i-th slope parameter from the model in the previous step to the data (x, y). Let the resulting maximum likelihood estimator be denoted by

$$(\widehat{\beta}_0, \widehat{\mu}_0) = (\widehat{\beta}_0(1), \ldots, \widehat{\beta}_0(p-1), \widehat{\mu}_0).$$

3. Simulate N_{sim} times from model M_0, that is, create

$$y_j^{\text{sim}} = \widehat{\beta}_0\, x_{\setminus i} + \varepsilon_{\widehat{\mu}_0}, \quad x_{\setminus i} := x \setminus x_i, \quad j = 1, \ldots, N_{\text{sim}}.$$

4. Fit N_{sim} p-dimensional (full) two-sided lognormal regression models to each simulated response, resulting in the replicated estimators $(\widehat{\beta}_1^j, \widehat{\mu}_1^j)$, $j = 1, \ldots, N_{\text{sim}}$, and

denote the empirical distribution function of $\widehat{\beta}_1^j(i),\ j = 1, \ldots, N_{\text{sim}}$, by $\widehat{F}_{\beta_1(i)}(x)$, $x \in \mathbb{R}$.

5. Define the p-value of the parameter $\widehat{\beta}_1(i)$ as $p = \widehat{F}_{\beta_1(i)}(-|\widehat{\beta}_1(i)|) + 1 - \widehat{F}_{\beta_1(i)}(|\widehat{\beta}_1(i)|)$.

Goodness of fit

Having fitted and chosen a model, it is customary to look at the residuals $r_k = y_k - \widehat{\beta}x$, $k = 1, \ldots n$, as a goodness of fit diagnostic. Analogous to the Gaussian linear models, we look for homogeneous dispersion of the residuals, with their distribution now being two-sided lognormal rather than normal, we suggest here a QQ-plot of $\Phi^{-1}(F_{\widehat{\mu}}(r_k)),\ k = 1, \ldots n$, against theoretical standard normal quantiles, and visually expect a straight line when the fit is adequate.

Chapter 4

A game-theoretic health insurance pricing model

ABSTRACT

We present and compare two non-cooperative games as pricing models for insurance companies. We apply the models using a dataset of the Swiss mandatory health insurance to determine price elasticity coefficients and analyze the resulting Nash equilibria for the premiums. We also consider a Stackelberg solution to study equilibrium premiums for a market where a few players dominate the market. We evaluate the influence of the model parameters on the equilibrium premiums through a set of sensitivity tests. The calculated price elasticity parameters demonstrate the low switching rates in the health insurance market in Switzerland and as a result, the estimated equilibrium premiums equal the largest allowed value. We suggest an extended model that incorporates the cost of capital in the objective function and present corresponding numerical results. The premiums calculated with the extended model reflect the additional cost loadings.
This chapter is based on the manuscript Daily-Amir (2019), which will be submitted for publication.

4.1. Introduction

Game theory concepts have been applied to insurance in general and to premium calculation in particular for many years. Rothschild & Stiglitz (1976) investigate an insurance contract, where insurers offer different premiums and deductibles. The customers can choose freely any of these contracts taking into consideration their own risk perception, and the typical assumption is that customers will want to maximize their own expected utility. Insurance companies, however, do not have the knowledge about the individual risk perception of their clients, so they compete in a market with imperfect and asymmetric information. The model demonstrates that an equilibrium point does not necessarily exist in this type of competition. However, when an equilibrium exists, low risk clients prefer a partial coverage and high risk clients prefer the full coverage. Lemaire (1980) introduced insurance applications for cooperative game theory and implemented them for various situations. Taylor (1986, 1987) examined the optimal premium when the market premium changes and found that during economic downturn, the strategy to follow the market premium might not be optimal. In his work, he assumed that the market premium is independent of the individual insurer premium.

The relationship between the Law of Large Number (LLN) and the number of insurers in an oligopoly market was investigated by Powers & Shubik (1998). Using the Cournot model, they presented a one period game where each player proposes a price he is willing to pay for the insurance and each insurer decides the level of risk he is willing to underwrite. The aggregate risk taken by insurers is subject to solvency regulations. A central clearing house then matches the insurers' offers and the clients' needs. The results of that study suggest a direct relationship between the LLN and the number of insurers.

Jordan & Rothwell (2009) used the winner's curse notion of auction theory to determine the premium of an insurance policy. In this framework, firms bid by setting a premium on an insurance product. As the real risk of that product is a stochastic process, insurers will determine the price by calculating the expected loss. The winner's curse theory suggests that the lowest winning premium offered is likely to underestimate the real loss and therefore it is likely to be 'cursed' by lower profit than expected.

Emms & Haberman (2005) present a model based on control theory in which they use a demand function to describe the number of underwriting policies, and an objective function of the insurer is the expected wealth. Their findings include two pricing strategies; the first one is to set a low premium, which creates a loss in order to gain a large market share, followed by a higher premium which produces profit, and the second one is a market withdrawal strategy, which means that the insurer leaves the market and does not compete. This model is limited in the sense that it applies only for a very competitive insurance market, with few strong insurance companies. Emms (2007, 2012) describes an extension to the Emms-Haberman model, using game theory concepts. The model is still based on a demand function and an objective function of the insurers, and presents a pricing strategy for the insurers.

Dutang et al. (2013) suggest a game theory solution for a non-cooperative competition among non-life insurers. They construct a one-period model and test their model with a numerical example. An extension to their work for an asymmetric information game is presented in Albrecher & Daily-Amir (2017). Battulga et al. (2018) extend the one-period

model and develop a multi-period model using a transition probability matrix which reflects on various economic circumstances.
Boonen et al. (2016) investigate the effect of various parameters as price, service level, age and education on policyholders' switching behaviour in the Dutch health insurance market. They suggest to model the insureds' switching decisions with a logit function of the decision parameters. Their results support the assumption that switching decisions depend on these parameters. Boonen et al. (2018) suggest a differential game as a pricing model and investigate the dynamics of insurers' equilibrium premium in a competitive non-life insurance market. They show how competition influences the premium process. Asimit & Boonen (2018) study a set of Pareto-optimal insurance contracts using cooperative game theory concepts, where insurers share the insured risk. They find a closed-form solution for different preferences that the insurance companies might have and investigate the resulting optimal contracts.

Insurance companies sell a product that is basically a commitment to compensate the policyholder in case of loss. Insurance companies need to raise capital to support their commitment to the policyholders and the regulator imposes a certain level, the Solvency Capital Requirement (SCR), in order to ensure that the insurance company has sufficient reserves to cover the liabilities from underwriting insurance policies up to a defined level of confidence (e.g. 99.5%). Pantelous & Passalidou (2016) describe an optimization model to determine a fair insurance price and develop a non-linear pricing model. While their model includes parameters such as break-even premium, market premium, portfolio size, the insurer's elasticity of demand and inflation, they give an additional consideration to the influence of the reserve level as a parameter in the insurer's premium decision.
Fuether models suggested by Rees et al. (1999), Polborn (1998), Powers & Shubik (2006), Wu & Pantelous (2017) and others will not be covered in this work.

In this paper, we introduce a demand function for health insurance in Switzerland based on findings from Daily-Amir et al. (2019). We develop a game with an objective function in terms of the operation profit of the insurer, and we calculate Nash equilibrium as well as Stackelberg equilibrium premiums. The estimated level of price elasticity coefficients for health insurance in Switzerland demonstrates the low sensitivity of policyholders to price differences and the low switching rates. This phenomenon results in insurers setting the maximum allowed premium. We compare the results with the premiums calculated for the one-period model introduced in Dutang et al. (2013) and perform parameter sensitivity tests.
The rest of this paper is organised as follows: Section 4.2 provides a description of the

one-period pricing model including the definition of two demand functions and the derived objective functions with a set of constraints. Section 4.3 explains the solution concepts used, Section 4.4 then reports the results of a numerical example using a published dataset of the health insurance in Switzerland, and Section 4.6 concludes.

4.2. One-period pricing model

In this chapter, we describe in detail a one-period pricing model for a market with I insurers who compete for n policyholders. Insurance contracts are issued at time t for one period (usually one year). After the insurers take their premium decisions, the n policyholders react by either renewing their policy or by switching the insurer and buying the insurance coverage from another one. The model assumes that the insurers offer the same product to all policyholders, there is no price distinction between renewal policies and new policies, and the insurers do not cooperate in their premium decisions. The model includes a demand function, a loss model and a definition of an objective function with constraints. In many aspects, we follow the model presented in Dutang et al. (2013) as we want to compare the results of the numerical illustration with the results in Dutang et al. (2013).

In the following subsections, the insurer index will be denoted by $i \in \{1, \ldots, I\}$. The price vector $(x_1, \ldots, x_I) \in \mathbb{R}^I$ represents the premiums, where x_i is the premium decision of insurer i and $m_i(x) = \frac{1}{I-1} \sum_{k \neq i} x_k$ is the market premium calculated as an average price of the competitors.

4.2.1. The demand function

We define the demand function $D_i(x)$ as the the market share $(MS_{i,t})$ of insurer i and suggest two models:

In the first model we adopt the approach introduced in Daily-Amir et al. (2019), and use a linear model to calculate the difference in market share for insurer i (DMS_i) as a function of the relative difference between the premium and the market premium $(RDPM_i)$ such that:

$$RDPM_i = (\frac{x_i}{m_i(x)} - 1)$$

and

$$DMS_i = MS_{i,t} - MS_{i,t-1} = b_i \cdot RDPM_i + \epsilon_i,$$

(R1)

where B_i is the coefficient for the independent continuous variable $RDPM_i$ and ϵ_i are residual error terms. The $RDPM_i$ coefficients B_i will have a negative value, verifying the negative relation between changes in market shares and the relative difference of the insurer's premium and the market premium. We define the demand of insurer i as

$$D_i(x) = MS_{i,t-1} - \beta_i \cdot (\frac{x_j}{m_j(x)} - 1),$$

with price elasticity coefficients $\beta_i = -b_i$. The model does not restrict the market share, so theoretically it may assume values outside $[0\%, 100\%]$. However, it will turn out that the numerical values obtained in the subsequent study do not get close to these bounds in any case.

As a second alternative, we use the linear model

$$RDMS_i = (MS_{i,t} - MS_{i,t-1})/MS_{i,t-1} = a_i \cdot RDPM_i + \epsilon_i, \tag{R2}$$

where a_i is the coefficient for the independent continuous variable $RDPM_i$ and ϵ_i are residual error terms. We calculate the price elasticity coefficients β_i for the model suggested by Dutang et al. (2013) as a second demand function $\widetilde{D}_i(x)$:

$$\widetilde{D}_i(x) = \frac{n_i}{n}(1 - \beta_i(\frac{x_i}{m_i(x)} - 1)) = MS_{i,t-1}(1 - \beta_i(\frac{x_i}{m_i(x)} - 1)),$$

with price elasticity coefficients $\beta_i = -a_i$.

Whereas Daily-Amir et al. (2019) perform a linear regression for all insurers per canton, we implement the model per insurer and per canton as we want to calculate the price elasticity coefficients for each insurer. Implementing the multivariate regression model suggested in Daily-Amir et al. (2019), it turns out that the coefficients of the second variable $DRPC$ (which is the difference between the relative annual change in the premium and the market premium) are close to zero, hence the effect of the variable on the market share is negligible here.

4.2.2. The loss model

For the loss model we adapt the model presented in Dutang et al. (2013). Let Y_j be the aggregate claim size for policyholder j, M_j the number of claims, $Z_{j,l}$ the random variable of the claim size and n the total number of policies issued. The portfolio size for insurer i is denoted by $N_i(x)$, and S_i is his aggregate loss. With that in mind, note that:

$$Y_j = \sum_{l=1}^{M_j} Z_{j,l}$$

and

$$S_i(x) = \sum_{j=1}^{N_i(x)} Y_j = \sum_{j=1}^{N_i(x)} \sum_{l=1}^{M_j} Z_{j,l} = \sum_{l=1}^{\widetilde{M_i(x)}} Z_l$$

with

$$\widetilde{M_i(x)} = \sum_{j=1}^{N_i(x)} M_j$$

(the number of claims M_j is i.i.d. and independent from the claim size $Z_{j,l}$).

The model considers a Lognormal distribution $\mathcal{LN}(\mu_1, \sigma_1^2)$ for the claim size r.v. $Z_{i,l}$ and two different types of distribution for the number of claims M_j: (i) Poisson distribution, $\mathcal{P}(N_i(x)\lambda)$, (ii) Negative Binomial distribution, $\mathcal{NB}(N_i(x)r, p)$.
As Y_j are assumed to be independent and identically distributed random variables, the distribution of the aggregate loss $S_i(x)$ is a compound distribution. In the model, the notation PLN will be used for the Poisson-Lognormal distribution and NBLN for Negative-Binomial-Lognormal distribution.

4.2.3. The objective function

We choose the *operating profit* (OP) as the objective function for the insurers to maximize. OP is defined as the difference between the *operating revenues* and the *operating expenses*. In our model, we adopt the procedure presented in Dutang et al. (2013) in calculating the *expected operating profit*. We define it as the difference between the premiums income and expenses (including payments to settle policyholders' claims and other operating expenses like administrative fees and marketing costs). This approach for calculating the OP means that it only includes policies issued at the beginning of the insurance period, without previous years outstanding claims demand. We multiply the demand by the difference between the policy's price x_i and the break-even price π_i per unit of exposure, which includes the underwriting expenses. The first objective function is then

$$OP_i = D_i(x) \cdot (x_i - \pi_i) = \big(MS_{i,t-1} - \beta_i \cdot (\frac{x_j}{m_j(x)} - 1)\big) \cdot (x_i - \pi_i), \tag{4.1}$$

and the second one

$$\widetilde{OP_i} = \widetilde{D}_i(x) \cdot (x_i - \pi_i) = MS_{i,t-1}(1 - \beta_i(\frac{x_i}{m_i(x)} - 1))(x_i - \pi_i). \tag{4.2}$$

Tax and interest payments paid by the insurers are not incorporated in the OP computations, and neither are the capital costs.

4.2.4. The solvency constraint function

Article 101 of the Solvency II Directive states that "The Solvency Capital Requirement (SCR) shall be calibrated so as to ensure that all quantifiable risks to which an insurance undertaking is exposed are taken into account. It shall cover existing business, as well as the new business expected to be written over the following 12 months [...]. It shall correspond to the Value at Risk (VaR) of the basic own funds of an insurance undertaking subject to a confidence level of 99.5% over a one year period". While under the Solvency II, health insurers in the EU countries are required to have an absolute minimum of capital (2.5 mio Euro), in Switzerland, there is no absolute minimum capital requirement. The Swiss Solvency Test (SST) defines similar solvency capital requirements where the calculated SCR should be equal to the ES with 99% confidence level. As for the lognormal distribution (the defined distribution of the claim size in the loss model), the ES with 99% confidence level is approximately equal to the VaR with 99.68%, we consider it as a good enough approximation and we adapt the method in Dutang et al. (2013) who suggest an approximation for the SCR using a q-quantile (in this case 99.5%) of the aggregate claim amount of n_i i.i.d. risks such that,

$$SCR_i = k_{995} \cdot \sigma(Y) \cdot \sqrt{n_i},$$

where k_{995} is the number of standard deviation needed to ensure 99.5% confidence level and $\sigma(Y)$ is the standard deviation of the expected loss. This SCR represents only renewal and new written policies without previous years business. The solvency constraint function is defined as

$$g_2^i(x_i) = \frac{K_i + n_i(x_i - \pi_i)(1 - e_i)}{k_{995}\sigma(Y)\sqrt{n_i})} - 1. \tag{4.3}$$

The minimum price vector which will satisfy the solvency constraint, can be calculated with

$$x_i \geq \frac{(k_{995}\sigma(Y)\sqrt{n_i} - K_i)}{n_i(1 - e_i)} + \pi_i, \tag{4.4}$$

where K_i is the initial SCR_i and e_i is the expenses value.

4.2.5. The min-max price constraints

We impose the same constraints on the minimum and maximum prices that the insurer can ask for and use the constraints defined in Dutang et al. (2013). The minimum price constraint is set as $\underline{x} = E(Y)/(1 - e_{min})$ and the maximum price constraint is, $\overline{x} = 3E(Y)$. The min-max constraints functions are then;

$$\begin{aligned} g_2^i(x_i) &= x_i - \underline{x} \geq 0, \\ g_3^i(x_i) &= \overline{x} - x_i \geq 0. \end{aligned} \tag{4.5}$$

4.2.6. The expected underwriting results

Once the insurers set their equilibrium prices ($x_i^\star$), the policyholders choose their insurer. The underwriting results are calculated with

$$UWR_i(x^\star) = D_i(x^\star) \cdot x_i^\star \cdot (1 - e_i) - S_i(x^\star), \tag{4.6}$$

where $D_i(x^\star)$ is the market share and $S_i(x^\star) = \sum_{j=1}^{D_i(x)} Y_j$.

4.3. Solution concepts

4.3.1. Nash equilibrium solution concept

In this model, the main solution concept considered is the Nash equilibrium, where all the insurers take their decision simultaneously.

Definition (Nash equilibrium): For a game with I players, where $\mathcal{O}_i$ is the objective function of player i, and action set X_i, the Nash equilibrium solution is the vector $x^\star =$

$(x_1^\star, \dots, x_I^\star)$, if for all $i = 1, \dots, I, x^\star$ solves the subproblem

$$\forall i, x_i \in X_i : \mathcal{O}_i(x_i^\star, x_{-i}^\star) \geq \mathcal{O}_i(x_i, x_{-i}^\star).$$

The equilibrium premium of insurer i, $x_i^\star$, solves the following Karush-Kuhn-Tucker conditions, see e.g. Facchinei & Kanzow (2009):

$$\begin{aligned} &\nabla_{x_i}\mathcal{O}_i(x^\star) + \sum_{1\leq k\leq 3} \lambda_k^{i\star} \nabla_{x_i} g_i^k(x_i^\star) = 0, \\ &\lambda^{i\star} \geq 0, \;\; g_i(x_i^\star) \geq 0, \;\; g_i(x_i\star)^T \lambda^{i\star} = 0, \end{aligned} \tag{4.7}$$

where $\lambda^{i\star} \in \mathbb{R}^3$ are Lagrange multipliers. The complementary condition in Equation 4.7, $g_i(x_i^\star)^T \lambda_i^\star = 0$, denotes that the kth constraint g_k is either inactive ($g_k(x^\star) > 0$) but then $\lambda_i^\star = 0$, or active ($g_k(x^\star) = 0$).

For the case that the constraints are inactive, the premium equilibrium for insurer i will verify $\nabla_{x_i}\mathcal{O}_i(x_i^\star) = 0$. Maximizing the objective function (4.1), implies that the Nash equilibrium solution - when the constraint functions are inactive - will be a function of the sensitivity parameters β_i, the break-even premiums π_i, the initial market share $MS_{i,t-1}$ and the number of insurers I. We get the following linear system of equations:

$$2\beta_i x_i^\star - (MS_{i,t-1} + \beta_i) \cdot m_i^\star = \beta_i \pi_i, \tag{4.8}$$

which is equivalent to

$$2\beta_j x_j^\star - (MS_{i,t-1} + \beta_i) \frac{1}{I-1} \sum_{k \neq i} x_k^\star = \beta_i \pi_i,$$

or solving the linear system $M_\beta x = v$, where

$$M_\beta = \begin{pmatrix} 2\beta_1 & -\frac{MS_{i,t-1}+\beta_1}{I-1} & \cdots & \\ -\frac{MS_{i,t-1}+\beta_2}{I-1} & 2\beta_2 & -\frac{MS_{i,t-1}+\beta_2}{I-1} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{MS_{i,t-1}+\beta_I}{I-1} & -\frac{MS_{i,t-1}+\beta_I}{I-1} & \cdots & 2\beta_I \end{pmatrix} \text{ and } v = \begin{pmatrix} \beta_1\pi_1 \\ \vdots \\ \beta_I\pi_I \end{pmatrix}. \tag{4.9}$$

With the objective function (4.2) when the constraints are inactive, the NE price solves the equation system

$$2\beta_i x_i^\star - (1+\beta_i)\tfrac{1}{I-1}\textstyle\sum_{k\neq i} x_k^\star = \beta_i \pi_i,$$

or solving the linear system $M_\beta x = v$, where,

$$M_\beta = \begin{pmatrix} 2\beta_1 & -\frac{1+\beta_1}{I-1} & \cdots & \\ -\frac{1+\beta_2}{I-1} & 2\beta_2 & -\frac{1+\beta_2}{I-1} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1+\beta_I}{I-1} & -\frac{1+\beta_I}{I-1} & \cdots & 2\beta_I \end{pmatrix} \text{ and } v = \begin{pmatrix} \beta_1\pi_1 \\ \vdots \\ \beta_I\pi_I \end{pmatrix}. \tag{4.10}$$

The above structure of the objective function and the settings imply that the Nash equilibrium solution - when the constraints functions are inactive - will be a function only of the sensitivity parameters, β_i, the break-even premiums π_i and the number of insurers I, but not a function of the initial market shares.

Properties of the equilibrium

Rosen (1965) proved that the following requirements are sufficient to ensure the existence and uniqueness of a NE:

1. The action set (the price vector) $X_i \in R = [\underline{x}, \overline{x}]$, which is nonempty, compact and convex set.

2. The function $x_i \to \mathcal{O}_i(x)$ is a strictly concave and differentiable function. With the parameter values of the models, both objective functions are strictly concave. So, we ensure the existence and uniqueness of the NE.

4.3.2. Stackelberg solution concept

The Stackelberg model is based on games with one or more leaders and a few followers. The Stackelberg equilibrium fits a market where decisions are made sequentially. First, the market leader takes a decision, and then the followers take their decision as a function of the one of the leader.

Definition (Stackelberg equilibrium): *For a game with I players, one leader and $I-1$ followers, where $\mathcal{O}_i$ is the objective function of player i, and action set X_i, the Stackelberg equilibrium is the vector $x^\star = (x_1^\star, \ldots, x_I^\star)$, if $x_1^\star$ solves the subproblem*

$$\forall i, x_i \in X_i : \mathcal{O}_1(x_1^\star, x_{-1}^\star) \geq \mathcal{O}_1(x_1, x_{-1}^\star),$$

with $x_{-1}^\star(x_1)$ being the Nash equilibrium for $I-1$ players, given the leader's choice.

4.4. Numerical implementation

We apply a numerical test of the one-period model to a game with both objective functions OP_i and $\widetilde{OP_i}$ for both loss models PLN and $NBLN$ and calculate the resulted NE premium for each insurer with three and eight players in two cantons, Geneva (GE) and Bern (BE).

4.4.1. Model parameters, assumptions and price elasticity coefficient estimates

Price elasticity coefficients

To evaluate the price elasticity coefficients β_i, we use a dataset presented in Daily-Amir et al. (2019) by implementing the linear regressions $R1$ and $R2$. The yearly reports with the data are published by the Swiss Federal Statistical Office and contain the information on the premiums of the mandatory health insurance plan for each insurer active in the twenty-six cantons, the portfolio size of each insurer per canton and the solvency ratio of the insurer. Including the data from insurers with market share greater than 1% and neglecting smaller insurers, the dataset includes 6,117 data points for all Switzerland with an accumulated market share between 80% and 97% per year and canton. The number of maintained insurers is between 10 to 25 per year and canton. From the data, we calculate for each insurer, year and canton:

- The Market Share ($MS_{i,t,c}$) as the insurer's portfolio size divided by the population in the canton.

- The Market Premium ($MP_{i,t,c}$) for insurer i as the average premium of all the competitors in the market per year and canton.

- The absolute year-to-year change in market share $DMS_{i,t,c}$= $MS_{i,t,c} - MS_{i,t-1,c}$ as the difference between the market share at time t and the previous year's premium.

- The relative year-to-year change in market share $RDMS_{i,t,c}= (MS_{i,t,c}-MS_{i,t-1,c})/MS_{i,t-1,c}$ as the difference between the current and the previous year's market shares divided by the previous year's market share.

- The relative difference between insurer i's premium and the market premium ($RDPM_{i,t,c}$) as the difference between the premium ($P_{i,t,c}$) and the market share ($MP_{i,t,c}$) divided by the market premium, i.e. $RDPM_{i,t,c} = (P_{i,t,c} - MP_{i,t,c})/MP_{i,t,c}$.

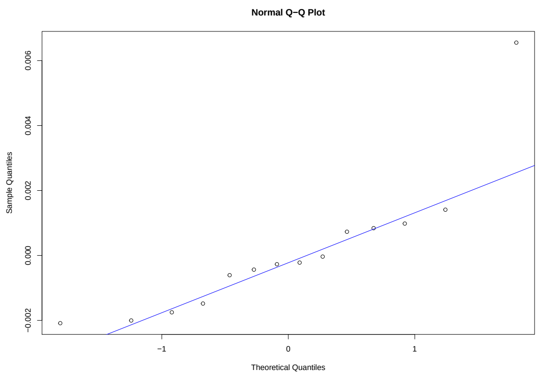
We perform linear regressions R1 and R2 for the six largest insurers that offer the health insurance plan in two selected cantons, Bern (BE), a German-speaking canton and Geneva (GE), a French-speaking canton, and we add two other insurers to a total of eight per canton. All regressions include 14 data points, one for each year, from 2002 to 2015. Table 4.1 gives the regression results including the B_i and A_i coefficients and the adjusted R^2 coefficient as a measure of the goodness of fit. Figure 4.1 shows an example of the residuals distribution for Insurer 4 for both regression models in GE and BE and reasonably confirms the normality assumption of the linear model. The adjusted R^2 values vary, while Insurer 6 having the best fit from 46% for model $R2$ in GE up to 86.5% for the same model in BE. The regression results for Insurer 1 in GE do not support the assumption of a relation between the market share changes and the premium difference. $RDPM_{i,t,c}$ coefficients are negative and in most cases very significant implying a negative relationship between the market share change and the relative difference between the insurer's premium and the market premium.

We take the insurers' market shares for 2015 in each canton from the dataset and determine the break-even premiums π_i by assigning a value of $\pi_i = 1.05$ (representing 5% markup on the expected loss $E(Y_j)$ to the insurer who offers the lowest premium) and adjusting for the other insurers multiplying by the corresponding insurers 2015's premiums ratio. The total market share for the eight insurers in GE and BE is 88% and 61%, respectively.
We present the insurers' portfolio size, market share, break-even premium and the solvency ratio for the year 2015, as well as the coefficient values for the two models in Table 4.2. Comparing the β_i for the model $\widetilde{OP_i}$ with the numerical illustration in Dutang et al. (2013) who examine a case with β_i between 3 and 4.6, we observe significantly lower values. The low values of the price elasticity coefficient can be related to the low switching rates in Switzerland and the reduction to less than half in the number of insurers during the years of the dataset, see e.g. Daily-Amir et al. (2019). One can note that the coefficients for the same insurer vary among the cantons, with higher price elasticity coefficients in GE than in BE, which implies that policyholders in GE are more sensitive to premium differences. The highest coefficient per model and canton is 8-10 times larger than the smallest one,

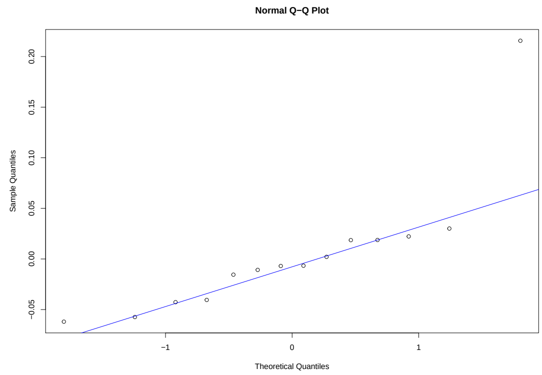
Table 4.1: Linear regression results - GE

Canton	GE				BE			
Model	R1		R2		R1		R2	
Insurer	b_i	Adjusted R^2	a_i	Adjusted R^2	b_i	Adjusted R^2	a_i	Adjusted R^2
1	-.0467 (.0377)	0.0370	-.5170 (.4351)	0.0286	-.0182*** (.0046)	0.5096	-.5932*** (.1092)	0.6705
2	-.0290*** (.0132)	0.2143	-.7239** (.3293)	0.2150	-.0089*** (.0031)	0.3395	-.3020*** (.0972)	0.3820
3	-.0640*** (.0232)	0.3204	-2.4417*** (.8632)	0.3334	-.0211* (.0111)	0.1568	-1.4845* (.7778)	0.1588
4	-.0419*** (.0127)	0.4116	-1.2248*** (.3972)	0.3781	-.0609*** (.0075)	0.8211	-1.1882*** (.1652)	0.7836
5	-.0592*** (.0155)	0.4943	-.3434*** (.1087)	0.3906	-.0484** (.0206)	0.2432	-.8150** (.3516)	0.2380
6	-.2856*** (.0592)	0.6137	-3.0255*** (.8399)	0.4610	-.0834*** (.0145)	0.6969	-.9507*** (.0999)	0.8649
7	-.2421** (.1167)	0.1909	-2.3195*** (.8553)	0.3122				
8	-.2553*** (.0811)	0.3887	-1.6088*** (.4911)	0.4101				
9					-.0886*** (.0340)	0.2926	-.8878** (.3557)	0.2719
10					-.0437** (.0197)	0.2187	-.1750** (.0850)	0.1882

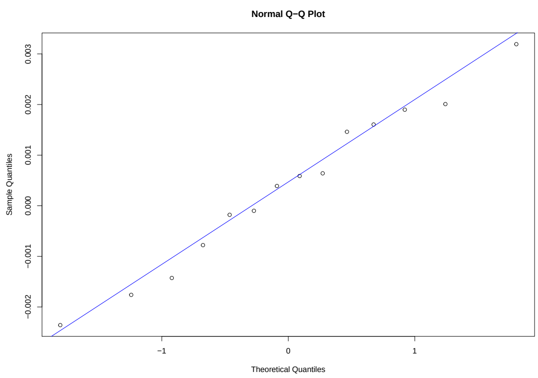
The reported values show the regression coefficients with significance code and standard deviation in brackets.
Significance codes: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$



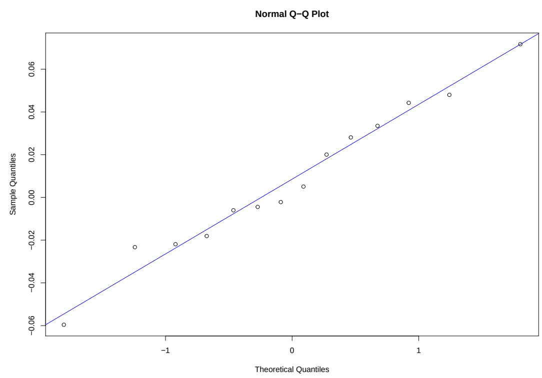
(a) OP_i - GE



(b) $\widetilde{OP_i}$ - GE



(c) OP_i - BE



(d) $\widetilde{OP_i}$ - BE

Figure 4.1: Residual QQ-plot for Insurer 4

Canton	GE					BE					
Insurer	n_i	MS_i	π_i	$\beta_i\ (OP_i)$	$\beta_i\ (\widetilde{OP_i})$	n_i	MS_i	π_i	$\beta_i\ (OP_i)$	$\beta_i\ (\widetilde{OP_i})$	ρ_i
1	63796	0.1435	1.4049	0.0467	0.5170	15448	0.0153	1.4950	0.0182	0.5932	1.17
2	11048	0.0248	1.4224	0.0290	0.7239	25439	0.0252	1.3322	0.0089	0.3020	2.51
3	21949	0.0494	1.3759	0.0640	2.4417	31335	0.0310	1.1864	0.0211	1.4845	0.87
4	11004	0.0247	1.3952	0.0419	1.2248	33931	0.0336	1.2806	0.0609	1.1882	1.18
5	106397	0.2393	1.0500	0.0592	0.3434	112389	0.1112	1.0500	0.0484	0.8150	1.13
6	81064	0.1823	1.3996	0.2856	3.0255	45703	0.0452	1.3555	0.0834	0.9507	1.45
7	16797	0.0378	1.3944	0.2421	2.3195						1.66
8	81587	0.1835	1.3521	0.2553	1.6088						0.79
9						132859	0.1315	1.2112	0.0886	0.8878	0.79
10						220059	0.2177	1.3368	0.0437	0.1750	2.09

Table 4.2: Initial portfolio size, initial market share, break-even premium and price elasticity coefficients per insurer in GE and BE

which suggests a significant difference in the policyholders' price sensitivity among the insurers. Insurer 5 from GE has the highest market share in the canton and small price elasticity coefficients, the same situation applies for Insurer 10 from canton BE. In both cantons, there is one insurer with market share larger than 20% and two to three insurers with market share between 10-20%. The other insurers have market shares smaller than 5%. The largest break-even premium is about 35-40% higher than the lowest one. The solvency ratio ranges from 79% for Insurers 8 and 9 to 251% for Insurer 2.

Loss model and solvency parameters

For the loss model parameters and solvency constraint parameters are all taken from Dutang et al. (2013):

1. The parameters for the loss model, we choose $E(Y_j) = 1$ for the aggregate loss Y_j of policyholder j. The standard deviation for the PLN model is $\sigma(Y)_{PLN} = 4.472$, and for the NBLN model we choose $\sigma(Y)_{NBLN} = 10.488$.

2. A solvency coefficient $k_{995} = 3$ has been chosen. This is the number of standard deviations required to ensure the 99.5% confidence level in the Solvency II framework and is based on the Lognormal distribution of the claims' size. The full institution for this choice can be found in Dutang et al. (2013).

3. The initial capital $K_i = k_{995}\sigma(Y)\sqrt{n_i}\rho_i$ per unit of exposure for the two loss models is estimated with the initial solvency coverage ratio ρ_i as reported in Table 4.2. The minimum value for the price vector, which will satisfy the solvency constraint, can be calculated with (4.4). With the expense value $e_i = 15\%$ for all insurers and the calculated initial capital K_i, the solvency price constraints are presented in Table 4.3 for both loss models PLN and NBLN. As Insurer 2 has a very high solvency ratio $\rho_2 = 2.51$, his solvency constraint price is much lower than the one of the other insurers. Insurer 5, with low break-even premium, has relatively low solvency constraint prices as well. Insurer 3 with low initial solvency coverage and relatively high break-even premium in GE, has relatively high solvency price constraints in the canton. In BE, where his break-even premium is low, even with low initial solvency ratio, Insurer 3 has medium size solvency price constraints.

4. The minimum and maximum price constraints, as presented in Section 4.2.5, are

$$\underline{x} = \frac{E(Y)}{(1 - e_{min})} = \frac{1}{1 - 0.15} = 1.1765, \ and \ \overline{x} = 3E(Y) = 3.$$

Canton	Insurer Model	1	2	3	4	5	6	7	8	9	10
GE	PLN	1.3944	1.1958	1.3898	1.3687	1.0435	1.3747	1.3144	1.3640		
	NBLN	1.3803	0.8930	1.4085	1.3333	1.0348	1.3414	1.2075	1.3799		
BE	PLN	1.4736	1.1829	1.1988	1.2655	1.0437	1.3223			1.2204	1.2999
	NBLN	1.4450	0.9833	1.2135	1.2454	1.0352	1.2779			1.2326	1.2507

Table 4.3: Solvency constraints on prices

Note that from the above results, the minimum price constraint on $\underline{x}$ is fulfilled automatically with the solvency price presented in Table 4.3 except for Insurer 5 and for Insurer 2 for the NBLN loss model.

4.4.2. Nash equilibrium results

We start with assessing equilibrium premiums for the two objective functions, OP_i and $\widetilde{OP}_i$ in a three players game. We choose the three insurers with the highest coefficients (Game I), i.e. Insurers 6, 7, 8 in GE and Insurers 3, 4, 6 in BE. From the results of Game I in GE (see Table 4.4), we observe that the range of NE premiums' is higher for OP_i than for $\widetilde{OP}_i$. In the same game with objective function OP_i, in which the market share influences the equilibrium premiums, Insurer 7, who starts with low market share, offers premiums lower by almost 20% than his competitors although he has the lowest price elasticity coefficient. This strategy results in him increasing his market share by 4.4%, while the other insurers lose market share. With objective function $\widetilde{OP}_i$, where the market share is not a parameter, Insurer 6 with the highest price elasticity coefficient, offers the lowest premium. Nevertheless, the premium is not low enough for him to increase much his market share (1% increase) and Insurer 8 who offers the highest premium, loses only 1% market share as he benefits from much lower price elasticity coefficient than his competitors. In BE (see Table 4.5), where the price elasticity coefficients are lower than in GE, the NE premiums are equal to the maximum possible premium for all the insurers such that $x_i^\star = 3$ for all i. As a direct result, the market shares do not change.

Model/Insurer	$x_6^\star$	$x_7^\star$	$x_8^\star$	ΔMS_6	ΔMS_7	ΔMS_8	$x_6^\star/\pi_6$	$x_7^\star/\pi_7$	$x_8^\star/\pi_8$
OP_i	2.873	2.374	2.931	-0.023	0.044	-0.029	2.053	1.703	2.094
$\widetilde{OP}_i$	2.437	2.532	2.691	0.01	0.00	-0.01	1.741	1.816	1.343

Table 4.4: Game I: NE premiums, expected market shares and markup ratio - GE

Model/Insurer	$x_3^\star$	$x_4^\star$	$x_6^\star$	ΔMS_3	ΔMS_4	ΔMS_6	$x_3^\star/\pi_3$	$x_4^\star/\pi_4$	$x_6^\star/\pi_6$
OP_i	3	3	3	0	0	0	2.5286	2.3427	2.2132
$\widetilde{OP}_i$	3	3	3	0	0	0	2.5286	2.3427	2.2132

Table 4.5: Game I: NE premiums, expected market shares and markup ratio - BE

We then calculate NE premiums for the three largest insurers in each canton (Game II), i.e. Insurers 5, 6 and 8 in GE and Insurers 5, 9 and 10 in BE. The three largest insurers in GE hold together approximately 60% of the market, while it is 46% for BE. In both cantons, the insurers set the maximum constraint premium (see Tables 4.6 and 4.7). We get the same results for other combinations of insurers in both cantons.

Model/Insurer	$x_5^\star$	$x_6^\star$	$x_8^\star$	ΔMS_5	ΔMS_6	ΔMS_8	$x_5^\star/\pi_5$	$x_6^\star/\pi_6$	$x_8^\star/\pi_8$
OP_i	3	3	3	0	0	0	2.8571	2.1434	2.2188
$\widetilde{OP}_i$	3	3	3	0	0	0	2.8571	2.1434	2.2188

Table 4.6: Game II: NE premiums, expected market shares and markup ratio- GE

Model/Insurer	$x_5^\star$	$x_9^\star$	$x_{10}^\star$	ΔMS_5	ΔMS_9	ΔMS_{10}	$x_5^\star/\pi_5$	$x_9^\star/\pi_9$	$x_{10}^\star/\pi_{10}$
OP_i	3	3	3	0	0	0	2.5286	2.3427	2.2132
$\widetilde{OP}_i$	3	3	3	0	0	0	2.5286	2.3427	2.2132

Table 4.7: Game II: NE premiums, expected market shares and markup ratio - BE

With our model settings, the loss model does not influence the equilibrium premium directly but through the solvency minimum premium constraints. Since the equilibrium premiums are higher than the minimum solvency constraints, the results of the game are the same for both loss models.

Secondly, we calculate the NE premiums in a game with eight insurers in both cantons and for the two objective functions (Game III). As in the three players game, since the price elasticity coefficients β_i, are relatively low, the equilibrium price is equal to the maximum allowed premium, $x_i^\star = \overline{x} = 3$ for all i in both GE and BE. Table 4.8 presents the set of β_i-values for each insurer in GE and BE such that the equilibrium price with no constraints, i.e. the solution for the (4.9) and (4.10) is equal to $\overline{x} = 3$ for all insurers. The calculated β_i-values are higher than the real values for most of the insurers in both cantons and both models. With these values, any increase in the price elasticity of any of the insurers will result in premiums smaller than the maximum value 3.

Canton	Insurer / Model	1	2	3	4	5	6	7	8	9	10
GE	OP_i	0.2699	0.0472	0.0912	0.0463	0.3681	0.3418	0.0706	0.3340		
	$\widetilde{OP_i}$	1.8808	1.9016	1.8472	1.8693	1.5385	1.8746	1.8684	1.8205		
BE	OP_i	0.0305	0.0453	0.0513	0.0586	0.1711	0.0825			0.2205	0.3927
	$\widetilde{OP_i}$	1.9933	1.7988	1.6542	1.7448	1.5385	1.8242			1.6771	1.8037

Table 4.8: Price elasticity coefficients insuring NE premiums without constraints equal the maximum premium

With equilibrium premiums for all insurers equal to the maximum premium constraint and much higher than the solvency constraint, all insurers will keep their market share unchanged and will increase the solvency ratio as the premiums are significantly higher than the break-even premiums.

4.4.3. Stackelberg equilibrium premiums

Calculating the Stackelberg equilibrium, we consider a game with 8 insurers in canton GE. Insurer 5 is set as the leader, and the other insurers are the followers. The leader sets his price, and the others follow by optimizing their objective function with respect to the leader's decision. Figure 4.2 shows the value of $\widetilde{OP_i}$ for Insurer 5 as a function of his premium decision $x_5 \in (\underline{x}, \overline{x})$.
Evidently, if Insurer 5 wants to maximize his objective function, he should set his premium to the maximum possible value, i.e. $x_5^\star = 3$. As Insurer 5 has a very low price elasticity coefficient in the $\widetilde{OP_i}$ model, only if β_5 is multiplied by a factor higher than 6, the objective function $\widetilde{OP_i}$ will be maximized for a price slightly lower than the maximum premium.

4.4.4. Parameter sensitivity analysis

In this subsection, a sensitivity analysis is conducted for the canton GE by changing values of different parameters and calculating the resulted equilibrium prices. We analyse the effect of:

(i) multiplying by 3 the price elasticity parameter β_i,

(ii) decreasing the break-even premium π_j by 20%,

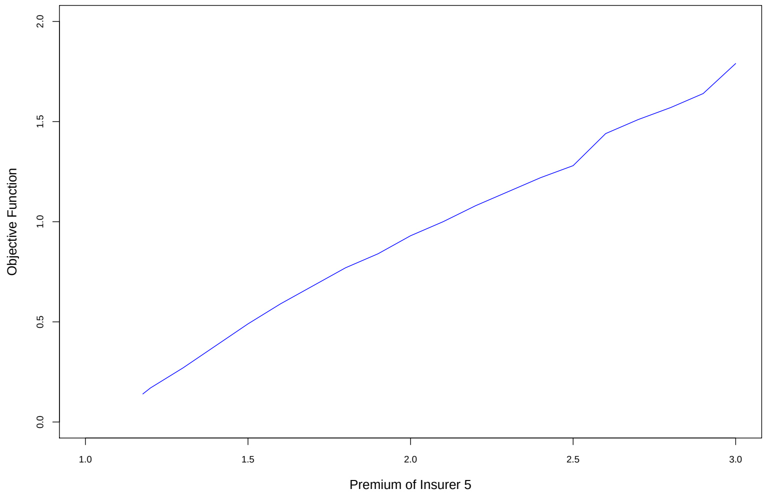


Figure 4.2: $\widetilde{OP_i}$ for Insurer 5 as a function of his premium decision

(iii) starting with an equal initial market share for all insurers.

(iv) starting with an identical price elasticity parameter β and break-even premium π for all insurers.

Price elasticity coefficient analysis

We would like to investigate the effect of the price elasticity coefficient on the equilibrium prices. The calculated price elasticity coefficients indicate low sensitivity of policyholders to price difference between insurers. Hence, calculated equilibrium premiums are equal in most cases to the maximum premium constraint. We determine the equilibrium premiums for a game with the three insurers with highest β_i coefficients (Game I), the three insurers with highest market shares (Game II) and the eight insurers (Game III) in GE, assuming their β_i-coefficients are tripled. One can note that with tripled β_i values, the insurers reduce their prices below the maximum premium allowed. In a three players game, the equilibrium premiums are lower by 33-42% than in the game with the real β_i values (see Tables 4.9 and 4.10). The NE premiums for a game with eight insurers (Game III) are below the maximum price constraint for all the insurers as presented in Table 4.11. The premiums calculated for model OP_i are higher than the ones calculated for model $\widetilde{OP_i}$. The highest equilibrium premiums equals 2.912 for Insurer 5 who has relatively low price elasticity coefficient, the largest market share and the lowest break-even premium. Insurer 7 who has relatively high price elasticity coefficient, small market share and medium break-even premium sets the smallest premium of 1.935.

With tripled price elasticity coefficients for the insurers in Game I, the equilibrium premiums are similar to the ones calculated in the numerical illustration in Dutang et al. (2013)

Model/Insurer	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	1.695	1.588	1.694
$\widetilde{OP_i}$	1.612	1.631	1.655

Table 4.9: Game I: NE premiums with tripled price elasticity parameter - GE

where the NE prices where in the range of 1.353-1.884 depending on the parameters.

Model/Insurer	$x_5^\star$	$x_6^\star$	$x_8^\star$
OP_i	3	1.984	1.998
$\widetilde{OP_i}$	2.472	1.944	2.009

Table 4.10: Game II: NE premiums with tripled price elasticity parameter - GE

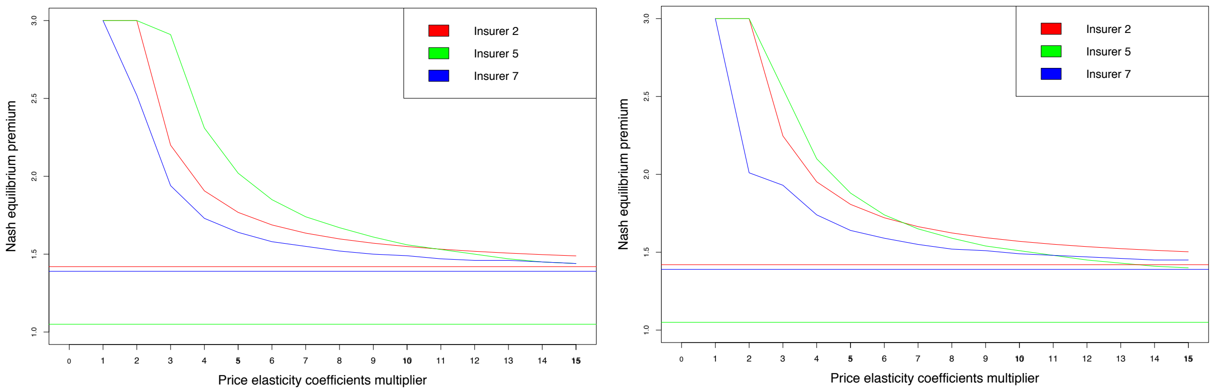
Insurer	$x_1^\star$	$x_2^\star$	$x_3^\star$	$x_4^\star$	$x_5^\star$	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	2.8928	2.1998	2.1482	2.0927	2.9118	2.1118	1.9352	2.1186
$\widetilde{OP_i}$	2.4122	2.2467	1.9102	2.0529	2.5537	1.8949	1.9259	1.9693

Table 4.11: Game III: NE premiums with tripled price elasticity parameter - GE

We then estimate the NE premium as a function of the price elasticity coefficient multiplier. We multiply the coefficient values by 2 to 15 in a game with the 8 insurers in GE. Figures 4.3(a) and 4.3(b) show the NE premiums for insurers 2, 5 and 7 as a function of the price elasticity coefficient multiplier for the two models. The break-even premiums of each insurer are presented as a horizontal line. All the insurers reduce premiums with higher β_i coefficients and one can note the convergence of the equilibrium prices towards the break-even premium. In both models, Insurer 5 who starts with highest market share, smallest break-even premium and low price elasticity coefficient, sets the highest premiums until coefficients multiplied by 12 in the OP_i model and by 6 in the $\widetilde{OP_i}$ model. Then, Insurer 5 takes advantage of his low break-even premium and sets the lowest equilibrium premium that gives his a margin of 33% above the break-even premium. At the same conditions, the other insurers set premiums that allow them only 3-5% margin. Premium levels in the OP_i model are higher than the ones in the $\widetilde{OP_i}$ model.

Break-Even premium analysis

The break-even premium π is a parameter in the objective function. Any increase in the break-even premium will result in insurers increasing the prices up to the maximum price constraint, whereas any decrease in the break-even premium will result in decreasing of



(a) OP_i model (b) $\widetilde{OP_i}$ model

Figure 4.3: Price elasticity coefficient multiplier versus NE premiums and break-even premiums for Insurers 2, 5 and 7

prices up to the minimum price constraint. Since in most cases we test, the NE premium is the maximum premium constraint, we want to test if break-even premiums lower by 33% result in equilibrium premiums lower than the maximum price. In a game with three players (see Table 4.12), the NE premiums are lower by 33% meaning that the insurers transfer the decreasing costs to the policyholders.

Model/Insurer	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	1.9247	1.5909	1.9635
$\widetilde{OP_i}$	1.6330	1.6965	1.8030

Table 4.12: Game I: NE premiums with reduced break-even premium - GE

Market size analysis

A significant difference between the two models is that while in the $\widetilde{OP_i}$ model presented in Dutang et al. (2013), the initial market share does not influence the equilibrium premiums, and it is an influencing parameter in the OP_i model. In order to analyze the effect of the initial market share in the OP_i model on the equilibrium premiums, we construct several further games and calculate the Nash and Stackelberg equilibrium premiums. First, we define a game in which all players start with an equal market share. By construction of the model with objective function $\widetilde{OP_i}$, the insurers' initial market share has no direct effect on the equilibrium premiums. For the OP_i model, in a game with equal initial market share, all the insurers fix the maximum premium, so $x_i^\star = 3$ for all i both in 3 and 8 players' games.

Secondly, we construct a game where all insurers start with the same price elasticity coefficient $\beta_i = 3.05$ for the $\widetilde{OP_i}$ model and 0.26 for the OP_i model and the same break-even premium $\pi_i = 1.35$ for all insurers. The chosen price elasticity coefficient is twice the average one, and the break-even premium is the average break-even premium of the eight insurers. As the initial market share is not an influencing parameter on the equilibrium premiums in the $\widetilde{OP_i}$ model, with equal $\beta_i = 3.05$ and $\pi_i = 1.35$, all insurers will fix the same price $x_i^\star = 2.0085$, which reflects almost 50% markup on the break-even premium. The NE premiums for the OP_i model are reported in Table 4.13 and one can note that the premium levels are linearly related to the initial market share, where Insurer 5, who starts with the highest market share, sets the highest premium and loses 6.76% of the market. Insurers 2 and 4 with lowest market shares, set the lowest premiums and gain over 4.3% of market share. The average premium for the OP_i model is significantly higher than the one for the $\widetilde{OP_i}$ model.

Insurer	$x_1^\star$	$x_2^\star$	$x_3^\star$	$x_4^\star$	$x_5^\star$	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	2.4639	1.9761	2.0793	1.9757	2.8375	2.6175	2.0307	2.6221
$MS_{i,t-1}$	0.1435	0.0248	0.0494	0.0247	0.2393	0.1823	0.0378	0.1835
$MS_{i,t}$	0.1256	0.0685	0.0803	0.0685	0.1717	0.1443	0.0748	0.1449

Table 4.13: NE premiums with equal β_i and π_i - GE

For the Stackelberg equilibrium, in the $\widetilde{OP_i}$ model with equal $\beta_i = 3.05$ and $\pi_i = 1.35$, Insurer 5 will fix his premium level on $x_5^\star = 1.8150$ in order to maximize his objective function, while all the followers will fix the same price, $x_i^\star = 1.6445$. Figure 4.4 shows the objective function value OP_5 of Insurer 5 for the OP_i model. Insurer 5 will maximize his utility if he will set his premium at $x_5^\star = 1.86$. The premiums of his followers are presented in Table 4.14. While the followers set a premium as a function of their initial market share, Insurer 5 can benefit from being a leader and set a lower premium to ensure his position as the market leader with 21.25% market share compared with 17.17% in the Nash settings.

Insurer	$x_1^\star$	$x_2^\star$	$x_3^\star$	$x_4^\star$	$x_5^\star$	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	1.8235	1.5074	1.5741	1.5072	1.8600	1.9234	1.5427	1.9264
$MS_{i,t-1}$	0.1435	0.0248	0.0494	0.0247	0.2393	0.1823	0.0378	0.1835
$MS_{i,t}$	0.1232	0.0592	0.0724	0.0591	0.2125	0.1442	0.0662	0.1448

Table 4.14: Stackelberg premiums with equal β_i and π_i, Insurer 5 as leader - GE

As the leader sets a lower premium in a game with Stackelberg settings than in a game

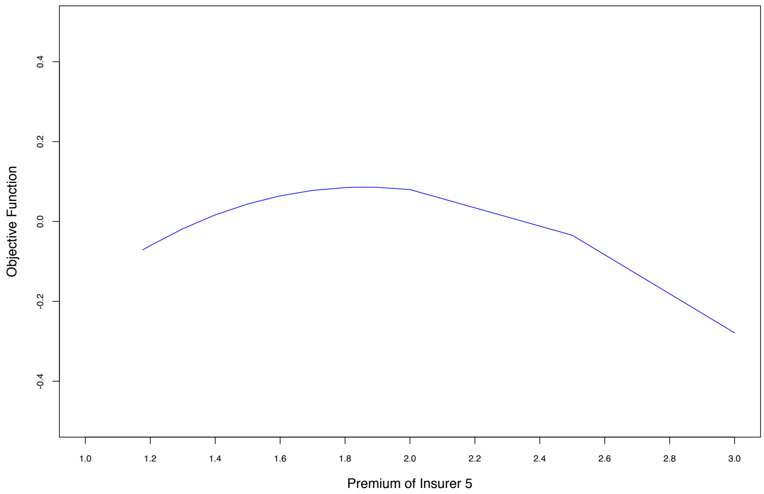


Figure 4.4: OP_5 for Insurer 5 as a function of his premium decision, $\beta_i = 3.05$ and $\pi_i = 1.35$

with Nash settings, the followers adjust and reduce premiums as well.

4.4.5. Additional game settings

If for example, the objective of Insurer 5 is to become the cheapest insurer in order to maintain his position as the market leader and to increase his market share, he might decide to quit the game and to set his premium as the lowest possible, i.e. $x_5^\star = 1.1765$. In this case, Insurer 5 does not consider his opponents premiums and still has a 12% markup on his break-even premium. The other insurers who are still playing the game, set their equilibrium premiums lower than the maximum as well. We present the equilibrium premiums for this scenario in Table 4.15. The results demonstrate how the very low elasticity parameter of Insurer 1 allows him to set the highest premium even if the other insurers reduce their premiums. One can note that in a game with eight insurers, even if one decides to reduce the premium considerably, the other insurers will react and reduce premiums as well. Nevertheless, they still set significantly higher premiums than the lowest.

Insurer	$x_1^\star$	$x_2^\star$	$x_3^\star$	$x_4^\star$	$x_5^\star$	$x_6^\star$	$x_7^\star$	$x_8^\star$
OP_i	3.0000	2.4828	2.3885	2.2395	1.1765	2.2835	1.8462	2.3321
$\widetilde{OP_i}$	3.0000	2.9795	2.1075	2.4845	1.1765	2.0449	2.1359	2.2907

Table 4.15: Equilibrium premiums with Insurer 5 setting $x_5^\star = \underline{x}$ - GE

According to the published reports including the insurers' premiums, in reality indeed Insurer 5 sets the lowest premium in GE. Even with strict premium regulations, Insurers 1 and 2 set the highest premiums, which in 2015, are 35% higher than the one of Insurer 1.

4.5. Cost of capital and an extended model

Insurance companies sell a product that is basically a commitment to compensate the policyholder for losses. This commitment imposes the insurance companies to raise capital to support their liabilities to the policyholders. The raised capital from shareholders has a cost associated with it, the *Cost of Capital* (CoC). The CoC depends on the financing structure of the company. It can be either the cost of debt or the cost of equity. Companies that are using a combination of equity and debt use the *Weighted Average Cost of Capital*, known as WACC. As insurance companies usually have small or no long-term debt, the CoC is the *risk premium* the investors demand in order to invest in the company. The CoC is a function of the risk: the higher the risk, the higher the cost. We denote the rate related to the CoC by r_{CoC}.

Kielholz (2000) suggests to incorporate the cost of capital in the pricing decision for the policy, by using measures of *economic profit* (also known as the Economic Value Added - EVA). He claims that "only by targeting economic profit as a decision metric can insurers maximize shareholder value". The *economic profit* is a measure used to evaluate the company's financial performance. It is defined as the difference between the *net operating profit after tax* and the CoC needed to finance the company's operations.
SwissRe (2005) also suggests that a company creates value to its shareholders only if the return generated (the economic profit) is greater than the expected CoC. Generating a positive economic profit enables the company to insure its independence and to ensure a market value that is higher than the book value.

In view of the above, one may want to adapt the objective function of the different models to the *economic profit* instead of the *operating profit* (OP) leading to

$$\mathcal{O}_i(x) = OP - r_{CoC}SCR.$$

With the settings and model definitions in Dutang et al. (2013), the *economic profit* is then defined as

$$\mathcal{O}_i(x) = \frac{n_i}{n}(1 - \beta_i(\frac{x_i}{m_i(x)} - 1))(x_i - \pi_i) - r_{CoC} \cdot k_{995} \cdot \sigma(Y) \cdot \sqrt{n_i}.$$

When the constraints are inactive, the NE premiums vector, $x^\star$ solves the equation system

$$(1+\beta_i)\cdot m_i\cdot n_i + n_i\cdot\beta_i(\pi_i - 2x_i) + \frac{0.5\cdot\beta_i\cdot r_{CoC}\cdot k_{995}\cdot\sigma(Y)\cdot n_i^{0.5}}{(1+\beta_i-\beta_i(x_i/m_i))^{0.5}} = 0.$$

One can observe that the Nash equilibrium premiums (when the constraints are inactive) depend on the price sensitivity parameter β_i, on the break-even premium π_i, the initial portfolio size n_i, the standard deviation of the loss model $\sigma(Y)$, the r_{CoC} and the k_{995}. For a basic numerical illustration, we adapt the chosen parameters and assumptions from Dutang et al. (2013) and calculate the Nash equilibrium premiums. We then compare the results with Dutang et al. (2013). The chosen parameters are:

- Break-even premium: for the PLN model $\pi_j = (1.170, 1.202, 1.136)$. and for the NBLN model $\pi_j = (1.247, 1.285, 1.231)$.
- Price elasticity coefficients: $\beta_i = (3, 3.8, 4.6)$.
- The minimum and maximum price constraints using an expense rate of 15%, $\underline{x} = 1.1765$, and $\overline{x} = 3$.
- A solvency coefficient $k_{995} = 3$.
- The $r_{CoC} = 0.06$, based on an average CoC rate values in the insurance market in the last few years and the rate set by the regulator in Europe and Switzerland.

The Nash equilibrium premiums for the two loss models are shown in Table 4.16.

Model/Insurer	$x_1^\star$	$x_2^\star$	$x_3^\star$	$x_1^\star/\pi_j$	$x_2^\star/\pi_j$	$x_3^\star/\pi_j$
PLN	1.651	1.621	1.568	1.411	1.349	1.380
NBLN	1.778	1.748	1.697	1.426	1.360	1.378

Table 4.16: Nash equilibrium premiums and markup ratio

As intuitively expected, Insurer 1 who has the lowest price sensitivity parameter β_i and an average break-even premium π_i, sets the highest price.

In general, the Nash equilibrium premiums are 35-42% higher than the break-even premium π_i. The equilibrium price vector in Dutang et al. (2013) is $x_i^\star = (1.544, 1.511, 1.471)$. As expected, the additional loading of the cost of capital results in increased premiums. For PLN model, the premiums are increased by 6-7% and for NBLN model, the premiums are

approximately 15% higher.
The Nash equilibrium results are much higher than the solvency requirements and satisfy $x_i^\star \in [\underline{x}, \overline{x}]$.
Further examination of the extended model with other demand functions and sensitivity tests are left for future research.

4.6. Discussion and conclusions

The one-period model presented in Dutang et al. (2013) presents a model for pricing an insurance product in a regulated market with solvency constraints, using non-cooperative game theory. In this paper, we suggest a supplementary demand function for this pricing model which is constructed such that the initial market share has an impact on the resulting equilibrium premiums. We perform a numerical test for both pricing models using a published dataset on the Swiss mandatory health insurance market to estimate the price elasticity coefficient of the insurers. First, we calculate these coefficients for each insurer in two different cantons and use them in the pricing model to calculate the equilibrium premiums. The low price elasticity coefficients support the market experience that switching rates of policyholders are low. As a result, in most cases, the equilibrium premiums equal the maximum allowed price in the two pricing models. When we triple the value of the coefficients, the insurers reduce their premiums. The computed equilibrium premiums with the two models are comparable. Future research with other datasets might help to determine which model is more accurate.
One may also consider other objective functions for different insurance markets. For example, as in the Swiss mandatory health insurance, the law restricts any profit and the premiums set by the insurers should cover only their expenses, the insurers might consider maximizing their market share so that they can enjoy profits from selling complementary insurance products.
In non-life insurance, incorporating the cost of capital into the premium consideration and choosing the *economic profit* as an objective function might improve the accuracy of the model. As a first step in that direction, we presented here an example of the *economic profit* as an objective function of the insurer with numerical illustrations.

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