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Competitive Balance and Revenue Sharing in Sports Leagues With Utility-Maximizing Teams

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Abstract

This paper develops a contest model of a professional sports league in which clubs maximize a weighted sum of profits and wins (utility maximization). The model analyzes how more win-oriented behavior of certain clubs affects talent investments, competitive balance, and club profits. Moreover, in contrast to traditional models, the authors show that revenue sharing does not always reduce investment incentives due to the dulling effect. The authors identify a new effect of revenue sharing called the "sharpening effect." In the presence of the sharpening effect (dulling effect), revenue sharing enhances (reduces) investment incentives and improves (deteriorates) competitive balance in the league.

Keywords

competitive balance, contest, invariance proposition, revenue sharing, team sports league, utility maximization

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Introduction

Existing models of team sports leagues primarily assume that club owners maximize either profits (El-Hodiri & Quirk, 1971; Fort & Quirk, 1995; Falconieri, Palomino, & Sákovics, 2004; Grossmann & Dietl, 2009; Szymanski & Késenne, 2004) or wins (Késenne, 2000, 2006; Vrooman, 2007; Zimbalist, 2003). These assumptions are restrictive and not supported by evidence. In contrast, empirical evidence from North American major leagues and European leagues supports the assumption that clubs trade-off profits and wins (e.g., Atkinson, Stanley, & Tschirhart, 1988; Garcia-del-Barrio & Szymanski, 2009).

Given this evidence, we present a contest model of a sports league in which club owners maximize an objective function given by a weighted sum of profits and winning percentage. As compared to previous analyses, this model may be useful to develop more general propositions. In particular, the model can shed light on the controversy surrounding the famous invariance proposition (IP) of sports economics. According to the IP, which was introduced by Rottenberg (1956), changes in property rights, such as the introduction of a reserve clause, will not alter the allocation of playing talent within a sports league and therefore will have no impact on competitive balance. El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995) have extended the IP in their models to gate revenue sharing by showing that revenue sharing has no effect on player allocation within a league. This result is of huge importance to professional team sports in general and league managers in particular because revenue sharing has been introduced as a means to increase competitive balance. The optimal level of competitive balance is crucial for overall demand and total revenues in professional sports, as fans tend to prefer competitions with uncertain outcomes.

The IP with respect to revenue sharing was originally developed under the assumptions of purely profit-maximizing clubs and Walrasian conjectures (El-Hodiri & Quirk, 1971; Fort & Quirk, 1995). In their models, Késenne (2000, 2005) and Vrooman (2007, 2008) show that the IP does not hold in a league with purely win-maximizing clubs. Moreover, Szymanski and Késenne (2004) provide a model that contradicts the IP even under the assumption of purely profit-maximizing clubs. They show that under contest-Nash conjectures, revenue sharing does not increase but rather decreases competitive balance (see also Dietl & Lang, 2008; Vrooman, 2008). This result is driven by the so-called dulling effect of revenue sharing. According to the dulling effect, revenue sharing reduces the incentives for clubs to invest in playing talent because each club has to share some of the resulting marginal benefits of its talent investment with the other clubs in the league.[1] Lang, Grossmann, and Theiler (in press) confirm the dulling effect of revenue sharing in a two-club league consisting of a pure profit-maximizing club and a club that is owned by a so-called sugar daddy. Sugar daddies invest enormous amounts of money into clubs and become actual club owners with full control. They seem not to take the resulting financial losses into account because the utility derived from sporting success appears to compensate for the financial losses. Finally, Dietl, Lang, and Werner (2009) show that in mixed leagues with one pure profit-maximizing club and one pure win-maximizing club, revenue sharing decreases competitive balance as well.

Our model can be of interest to competition authorities and legislators because it derives new insights regarding the effect of revenue sharing on investment incentives and competitive balance. In contrast to previous models, our analysis shows that revenue sharing does not necessarily reduce incentives to invest in playing talent. We identify a new effect of revenue sharing called the "sharpening effect," which has the opposite effect of the well-known dulling effect. With our model, we can determine the conditions under which the sharpening effect or the dulling effect is at work. We show that in the presence of the sharpening effect (dulling effect), revenue sharing enhances (reduces) investment incentives and improves (deteriorates) competitive balance in the league. Moreover, we determine the conditions under which the IP holds even under contest-Nash conjectures. Finally, our model analyzes how a more win-oriented behavior of certain clubs affects talent investments, competitive balance, and club profits.

The remainder of the paper is structured as follows. The Basic Model section develops the model of a team sports league with utility-maximizing clubs. In the section on The Effect of Revenue Sharing in a League with Utility-Maximizing Clubs, we introduce a revenue-sharing arrangement and analyze its effect on talent investment and competitive balance. Finally, the Conclusion section summarizes the key findings and concludes.

The Basic Model

Notation and Assumptions

We model a two-club league[2] in which both clubs participate in a noncooperative game and independently invest a certain amount $x_i > 0$ in playing talent. The club objective function is such that clubs maximize a weighted sum of profits and wins.[3] The win percentage w_i of club i is characterized by the contest-success function (CSF), which maps the vector (x_1, x_2) of talent investment onto probabilities for each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests.[4] The win percentage of club $i = 1, 2$ in this imperfectly discriminating contest is then given by

$$w_i(x_i, x_j) = \frac{x_i}{x_i + x_j}, \tag{1}$$

with $i, j = 1, 2,\ i \neq j$. Given that win percentages must sum to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$. In our model, we adopt the contest-Nash conjectures $dx_i/dx_j = 0$ and compute the derivative of Equation 1 with respect to x_i as $\partial w_i/\partial x_i = x_j/(x_i + x_j)^2$. The so-called Walrasian conjectures $\partial x_i/\partial x_j = -1$ have been applied in the traditional literature (El-Hodiri & Quirk, 1971; Fort & Quirk, 1995; Rascher, 1997) for leagues with a fixed supply of talent. These conjectures indicate that clubs internalize that due to the fixed amount of talent, a one-unit increase of talent hired at one club implies a one-unit reduction of talent at the other club. The recent literature, however, proposes the use of the contest-Nash conjectures $\partial x_i/\partial x_j = 0$ to characterize noncooperative behavior between clubs (Szymanski, 2003, 2004; Szymanski & Késenne, 2004). For a discussion regarding the Walrasian and Nash conjectures, see Szymanski (2004), Eckard (2006), and Fort Quirk (2007).

The uncertainty of outcome is measured by the competitive balance in the league. One way of measuring competitive balance is through the ratio of win percentages, which is also called win ratio (Hoehn & Szymanski, 1999; Vrooman, 2007, 2008). Without loss of generality, we define the win ratio by the ratio of club 1's win percentage and club 2's win percentage:

$$\mathrm{WR}(x_1, x_2) = \frac{w_1(x_1, x_2)}{w_2(x_1, x_2)}. \tag{2}$$

Note that the win ratio WR equals one in a fully balanced league. A win ratio that is lower or higher than 1 thus indicates a league with a lower degree of competitive balance.

As in Szymanski (2003, p. 1164), we specify the revenue function of club $i = 1, 2$ as

$$R_i(x_i, x_j) = m_i w_i(x_i, x_j) - \frac{b}{2} w_i(x_i, x_j)^2, \tag{3}$$

where $b > 0$ characterizes the effect of competitive balance on club revenues and $m_i > 0$ represents the market size or drawing potential of club i. Without loss of generality, we assume throughout this paper that club 1 is the large-market club, while club 2 is the small-market club such that $m_1 > m_2$.[5]

It is important to mention that club i's revenues initially increase with winning until the maximum is reached for $w_i^{'} \equiv m_i/b$. By increasing the win percentage above $w_i^{'}$, club i's revenues start to decrease because excessive dominance by one team is detrimental to club revenues. This reflects the uncertainty of outcome hypothesis; the higher b is, the more important is competitive balance and the sooner revenues start to decrease due to the dominance by one team.

By assuming a competitive labor market and following the sports economic literature, the market clearing cost of a unit of talent, denoted by c, is the same for every club. The cost function of club $i = 1, 2$ is thus given by $C(x_i) = cx_i$, where c is the marginal unit cost of talent.

The profit function of club $i = 1, 2$ is given by revenues minus costs and yields

$$\pi_i(x_i, x_j) = R_i(x_i, x_j) - C(x_i) = \frac{x_i\left[(m_i - \frac{b}{2})x_i + m_i x_j\right]}{(x_i + x_j)^2} - cx_i, \tag{4}$$

with $i,j = 1, 2,\ i \neq j$.

As mentioned above, the objective function of club i is given by a weighted sum of one's own profits and wins; it is defined as

$$u_i(x_i, x_j) = \pi_i(x_i, x_j) + \gamma_i w_i(x_i, x_j), \tag{5}$$

where $\gamma_i \geq 0$ is the "win preference," which characterizes the weight club owner i puts on winning in the objective function. A higher parameter γ_i thus reflects that club owner i becomes more win-oriented and less profit-oriented. As in Rascher (1997) and Késenne (2007), we refer to this objective function as the utility function of club i.[6]

Moreover, note that we have two dimensions of heterogeneity in our model. On the one hand, clubs differ with respect to their market size and on the other hand, clubs differ regarding their win preference. In the following sections, we analyze the interaction effects of these two dimensions of heterogeneity.

Equilibrium Analysis

In this section, we solve the model and determine the equilibrium. Each club i maximizes its objective function and thus solves the following maximization problem:

$$\max_{x_i \geq 0}\left\{u_i(x_i, x_j) = \frac{x_i\left[(m_i - \frac{b}{2})x_i + m_i x_j\right]}{(x_i + x_j)^2} - cx_i + \gamma_i \frac{x_i}{x_i + x_j}\right\}. \tag{6}$$

with $i,j = 1, 2$ and $i \neq j$. The solution to the above maximization problem is presented in Lemma 1.

Lemma *1*: The equilibrium investment and win percentage of club i are given by

$$x_i^* = \frac{(\gamma_i + m_i)^2(\gamma_j + m_j)(m_1 + \gamma_1 + m_2 + \gamma_2 - b)}{c(m_1 + \gamma_1 + m_2 + \gamma_2)^3},$$

$$w_i^* = \frac{m_i + \gamma_i}{m_1 + \gamma_1 + m_2 + \gamma_2},$$

with $i,j = 1, 2$ and $i \neq j$.

Proof: See Appendix A1.

To guarantee positive equilibrium investments, we assume that either the clubs' market sizes or the win preferences are sufficiently large such that $m_1 + \gamma_1 + m_2 + \gamma_2 > b$. Lemma 1 shows that ceteris paribus, the win percentage of club i increases with either a higher win preference γ_i or a larger market size m_i: that is, $\partial w_i^*/\partial\gamma_i > 0$ and $\partial w_i^*/\partial m_i > 0$. The opposite holds true if the market size m_j or the win preference γ_j of the other club increases: that is, $\partial w_i^*/\partial\gamma_j < 0$ and $\partial w_i^*/\partial m_j < 0$.

A comparison of the equilibrium investments of the two clubs leads to the following proposition.

Proposition 1: The small-market club invests more than the large-market club if and only if $m_2 + \gamma_2 > m_1 + \gamma_1$.
Proof: Straightforward and therefore omitted.

Note that in our model, it is possible that the small-market club invests more in equilibrium and, as a consequence, is the dominant team that has a higher win percentage than the large-market club. This outcome occurs if the objective function of the small-market club has a sufficiently high win preference parameter. In this case, the win preference compensates for the lower market size such that marginal revenue is higher for the small-market club than for the large-market club, ceteris paribus. However, if the sum of market size and win preference of the large-market club is larger than (equal to) the sum of market size and win preference of the small-market club, then the former invests more than (the same as) the latter.

The Effect on Competitive Balance

From the equilibrium win percentages (w_1^*, w_2^*) of Lemma 1, we derive the win ratio in equilibrium in a league with utility-maximizing clubs as

$$\mathrm{WR}^* = \frac{m_1 + \gamma_1}{m_2 + \gamma_2},$$

and we establish the following proposition.

Proposition 2 (i): Ceteris paribus, competitive balance in a league with utility-maximizing clubs decreases if the dominant team j becomes more win-oriented and competitive balance increases if the underdog i becomes more win-oriented until $\gamma_i < \gamma_i' \equiv m_j - m_i + \gamma_j$ *with* $i,j = 1, 2$ and $i \neq j$.
(ii) A league with utility-maximizing clubs is more balanced than a league with pure profit-maximizing clubs if and only if the win preference of the

small-market club is within the interval $\gamma_2 \in (\gamma_2^{min}, \gamma_2^{max}) = (\gamma_1 m_2 / m_1, m_1(m_1 + \gamma_1)/m_2 - m_2)$.

Proof: Straightforward and therefore omitted.

Part (i) of the proposition shows that the effect of a more win-oriented behavior on competitive balance depends on which club is the dominant team in equilibrium. It is clear that a more win-oriented behavior of the dominant team *j* produces an even less balanced league. On the other hand, the league becomes more balanced if the underdog *i* increases its win preference until the league is perfectly balanced for $\gamma_i = \gamma_i^{'}$. By increasing the win preference above $\gamma_i^{'}$, the former underdog becomes the dominant team and competitive balance starts to decrease.

In part (ii), we compare a league with utility-maximizing clubs to the benchmark league with pure profit-maximizing clubs ($\gamma_1 = \gamma_2 = 0$). In the benchmark league, the win ratio is given by $m_1/m_2 > 1$. We know that in this league, the large-market club is the dominant team in equilibrium, while the small-market club is the underdog. If the difference in the market size of the two clubs increases (decreases), the win ratio increases (decreases): thus, the league becomes less (more) balanced. This result is well known in the sports economics literature (Fort & Quirk, 1995; Szymanski, 2003; Vrooman, 1995). However, if the club owner of at least one club becomes more win-oriented (i.e., $\gamma_1 > 0$ and/or $\gamma_2 > 0$), then the league may become more or less balanced than in the benchmark case.

In particular, the small-market club must have a sufficiently high win preference with $\gamma_2 > \gamma_2^{min}$ to guarantee that the league with utility-maximizing clubs is more balanced than the benchmark league.[7] If the win preference of the small-market club attains the upper threshold $\gamma_2 = \gamma_2^{max}$, the league with utility-maximizing clubs is characterized by the same degree of competitive balance as the benchmark league. The difference is that the small-market club is the dominant team and the large-market club the underdog in equilibrium. By increasing the win preference of the small-market club above γ_2^{max}, the league becomes less balanced than in the benchmark case.

The Effect on Club Profits

In this section, we determine how the win preferences affect aggregate club profits in leagues in which one club is a profit maximizer and the other club is a utility maximizer.[8] We establish the following proposition and differentiate two cases. In case (i), the large-market club is a pure profit maximizer and the small-market club is a utility maximizer: that is, $\gamma_1 = 0$ and $\gamma_2 > 0$. In case (ii), the opposite holds true: that is, $\gamma_1 > 0$ and $\gamma_2 = 0$.

Proposition 3 (i): Suppose that $\gamma_1 = 0$ and $\gamma_2 > 0$. Aggregate club profits decrease when the small-market club becomes more win-oriented (i.e., γ_2 increases).

(ii) Suppose that $\gamma_1 > 0$ and $\gamma_2 = 0$. Aggregate club profits increase when the large-market club becomes more win-oriented (i.e., γ_1 increases) if and only if the market size of the large-market club is sufficiently large.

Proof: See Appendix A2.

In the proof of Proposition 3, we have normalized the market-size parameters to $m_1 \equiv m$, $m_2 \equiv 1$ with $m > 1$ and we have set $b = 1$. The intuition behind the result in part (i) is as follows. If the small-market club becomes more win-oriented, then the win percentage of the small-market club increases, whereas the win percentage of the large-market club decreases (see discussion after Lemma 1). It follows that the revenues of the small-market club increase, while the revenues of the large-market club decrease through a higher win preference of the small-market club: that is, $\partial R_2^*/\partial\gamma_2 > 0$ and $\partial R_1^*/\partial\gamma_2 < 0$.[9] Moreover, the small-market club increases its investment in playing talent, which induces higher costs for this club in equilibrium. The increase in revenues, however, cannot compensate for the increase in costs such that profits of the small-market club decrease. The large-market club, on the other hand, decreases or increases its talent investment, that is, $\partial x_1^*/\partial\gamma_2 \gtreqless 0 \Leftrightarrow m(m-1) \gtreqless \gamma_2^2 - 1$. But even if the large-market club's costs decrease due to smaller investments, club profits decrease as well because the lower costs cannot compensate for the lower revenues. Because profits of both types of clubs decrease, aggregate club profits also decrease.

In part (ii), the large-market club is a utility maximizer, while the small-market club is a pure profit-maximizer. In contrast to part (i), a higher win preference γ_1 yields higher revenues for the large-market club due to a higher win percentage in equilibrium. The opposite holds true for the small-market club. Moreover, talent investment and thus costs are always higher for the large-market club, whereas talent investments are lower for the small-market club if and only if the market size of the large-market club is sufficiently large with $m > m' \equiv 2 - \gamma_1$. Even though costs may decrease for the small-market club, the loss in revenues is so substantial that the profits of the small-market club always decrease. In contrast, the profits of the large-market club increase if the market size of the large-market club is sufficiently large such that $(m + \gamma_1)[m(m + \gamma_1 - 2) - 4\gamma_1] > \gamma_1$ is satisfied. In this case, higher revenues compensate for higher costs. If the market size of the large-market club further increases above another threshold given by $m'' \equiv 1/2(3 - \gamma_1 + [(\gamma_1 + 1)\ \ (\gamma_1 + 9)]^{1/2}) > m'$, the higher profits of the large-market club compensate for the lower profits of the small-market club and aggregate club profits increase.

In a league in which both clubs are utility maximizers (i.e., $\gamma_1 > 0$ and $\gamma_2 > 0$), a higher win preference γ_2 for the small-market club always yields lower profits for both clubs. However, the effect of a higher win preference γ_1 for the large-market club on club profits is ambiguous.

The Effect of Revenue Sharing in a League With Utility-Maximizing Clubs

Equilibrium Analysis

In this section, we integrate a gate revenue-sharing arrangement into our model and analyze its effects in a league with utility-maximizing clubs. The sharing of gate revenues plays an important role in the redistribution of revenues and has long been accepted as an exemption from antitrust law (Fort & Quirk, 1995; Szymanski, 2003). The basic idea of this cross-subsidization policy is to redistribute revenues from large-market clubs to small-market clubs because large-market clubs have a higher revenue-generating potential than do small-market clubs.

In its simplest form, gate revenue sharing allows the visiting club to retain a share of the home club's gate revenues. The after-sharing revenues of club i are given by $\hat{R}_i = \alpha R_i + (1-\alpha)R_j$, with $i,j = 1,2$ and $i \neq j$. Note that the share of revenues that is assigned to the home team is given by the parameter $\alpha \in (1/2, 1]$, while $(1-\alpha)$ is assumed to be the share of revenues received by the away team.[10]

Thus, the utility of club i in a league with utility-maximizing clubs is given by $\hat{u}_i = \hat{R}_i - cx_i + \gamma_i w_i$. Maximizing utility $\hat{u}_i$ yields the following maximization problem of club $i = 1,2$:

$$\max_{x_i \geq 0} \left\{ \alpha \frac{x_i\left[(m_i - \frac{b}{2})x_i + m_i x_j\right]}{(x_i + x_j)^2} + (1-\alpha)\frac{x_j\left[(m_j - \frac{b}{2})x_j + m_j x_i\right]}{(x_i + x_j)^2} - cx_i + \gamma_i \frac{x_i}{x_i + x_j} \right\}, \quad (7)$$

with $i,j = 1,2,\ i \neq j$. The corresponding first-order conditions are computed as

$$\frac{\partial \hat{u}_i(x_1, x_2)}{\partial x_i} = \left(\alpha \frac{\partial R_i}{\partial w_i} - (1-\alpha)\frac{\partial R_j}{\partial w_j} + \gamma_i \right) \frac{\partial w_i}{\partial x_i} - c = 0, \quad (8)$$

with $\frac{\partial w_j}{\partial x_i} = -\frac{\partial w_i}{\partial x_i}$. Rearranging the first-order conditions yields

$$\frac{\partial \hat{u}_1(x_1, x_2)}{\partial x_1} = \left(\gamma_1 + \alpha(m_1 - b) - (1-\alpha)m_2 + \frac{bx_2}{x_1 + x_2} \right) \frac{x_2}{(x_1 + x_2)^2} - c = 0,$$

$$\frac{\partial \hat{u}_2(x_1, x_2)}{\partial x_2} = \left(\gamma_2 + \alpha(m_2 - b) - (1-\alpha)m_1 + \frac{bx_1}{x_1 + x_2} \right) \frac{x_1}{(x_1 + x_2)^2} - c = 0,$$

We determine the equilibrium win percentages in the following lemma.

Lemma 2: In a league with a revenue-sharing arrangement, the equilibrium win percentage of club i is given by

$$\hat{w}_i^* = \frac{\gamma_i + \alpha(m_i - b) - (1-\alpha)m_j + b}{(m_1 + m_2)(2\alpha - 1) + 2b(1-\alpha) + \gamma_1 + \gamma_2}, \quad (9)$$

with $i,j = 1,2,\ i \neq j$.

Proof: See Appendix A3.

From Lemma 2, we compute the equilibrium win ratio in a league with a revenue-sharing arrangement as:

$$\widehat{\mathrm{WR}}^* = \frac{\hat{w}_1^*}{\hat{w}_2^*} = \frac{\gamma_1 + \alpha(m_1 - b) - (1-\alpha)m_2 + b}{\gamma_2 + \alpha(m_2 - b) - (1-\alpha)m_1 + b} \gtreqless 1. \tag{10}$$

As in a league without revenue sharing, the small-market club invests more in equilibrium and consequently has a higher win percentage than the large-market club if and only if the sum of the market size and win preference for the small-market club is larger than that for the large-market club: that is, $m_2 + \gamma_2 > m_1 + \gamma_1$.[11] In this case, we obtain $\widehat{\mathrm{WR}}^* < 1$. If, however, $m_2 + \gamma_2 \leq m_1 + \gamma_1$, then the large-market club does not invest less than the small-market club, that is, $\widehat{\mathrm{WR}}^* \leq 1$.

Regarding the effect of revenue sharing on club revenues, we compute the partial derivative of club i's marginal revenue $\mathrm{MR}_i = \partial \hat{R}_i / \partial x_i$ with respect to the revenue-sharing parameter α as:

$$\frac{\partial \mathrm{MR}_i}{\partial \alpha} = \frac{x_j}{(x_1 + x_2)^2}(m_1 + m_2 - b) \gtreqless < 0, \tag{11}$$

with $i, j = 1, 2,\ i \neq j$. We derive that a higher degree of revenue sharing (i.e., a lower parameter α) has a positive effect on club i's marginal revenue if $b > m_1 + m_2$, while it has a negative effect on marginal revenue if $b < m_1 + m_2$. In the case that $b = m_1 + m_2$, revenue sharing has no effect on marginal revenue.

To further analyze the effect of revenue sharing on competitive balance, we derive the partial derivative of the win ratio $\widehat{WR}^*$ as:

$$\frac{\partial \widehat{WR}^*}{\partial \alpha} = \frac{[b - (m_1 + m_2)][(m_1 + \gamma_1) - (m_2 + \gamma_2)]}{(\gamma_2 + \alpha(m_2 - b) - (1-\alpha)m_1 + b)^2} \gtreqless 0. \tag{12}$$

In equilibrium, the effect of revenue sharing on the win ratio and the incentives to invest depends on how revenue sharing affects marginal revenue (i.e., $b \gtreqless m_1 + m_2$) as well as on which club is the dominant team in equilibrium (i.e., $m_1 + \gamma_1 \gtreqless m_2 + \gamma_2$).

The Effect of Revenue Sharing on Investment Incentives and Competitive Balance

In this section, we analyze the effects of revenue sharing in a league with utility-maximizing clubs on investment incentives and on competitive balance. We establish the following proposition.

Proposition 4 (i): Sharpening effect: If $b > m_1 + m_2$, more revenue sharing increases the amount of talent hired by each club and produces a more balanced league if the league is not fully balanced in equilibrium. In the case that both clubs have equal playing strength in equilibrium, the IP holds.

(ii) Dulling effect: If $b < m_1 + m_2$, more revenue sharing reduces the amount of talent hired by each club and produces a less balanced league if the league is not fully balanced in equilibrium. In the case that both clubs have equal playing strength in equilibrium, the IP holds.

(iii) Invariance proposition: If $b = m_1 + m_2$, more revenue sharing has no effect on equilibrium investments and on competitive balance.

Proof: See Appendix A4.

Table 1 summarizes the results of Proposition 4. In contrast to the existing literature,[12] part (i) of this proposition shows that revenue sharing does not necessarily reduce incentives to invest in playing talent. If $b > m_1 + m_2$, then revenue sharing has a positive effect on marginal revenue for both clubs and more revenue sharing enhances incentives to invest in playing talent. It follows that both clubs will increase the amount of talent hired in equilibrium. Hence, we identify a new effect of revenue sharing that we call the "sharpening effect." Note that this sharpening effect of revenue sharing has the opposite effect of the dulling effect.[13]

In the presence of the sharpening effect, a revenue-sharing arrangement proves to be an efficient instrument for improving competitive balance in an unbalanced league. We explain the intuition behind this result as follows. If the large-market club is the dominant team in equilibrium (i.e., $\widehat{WR^*} > 1$),[14] then the positive effect of revenue sharing on marginal revenue is stronger for the underdog (i.e., small-market club) than for the dominant team (i.e., large-market club) due to the logit formulation of the CSF. As a consequence, the sharpening effect of revenue sharing is more pronounced for the underdog than for the dominant team, because the marginal impact on the dominant team's revenues of an increase in talent investment by the underdog is greater than the marginal impact on the underdog's revenues of an increase in talent investment by the dominant team. As a result, the small-market club will increase its investment level relatively more than the large-market club such that the league becomes more balanced through revenue sharing.

If, however, the small-market club is the dominant team in equilibrium (i.e., $\widehat{WR^*} < 1 \Leftrightarrow m_1 + \gamma_1 < m_2 + \gamma_2$), then the positive effect of revenue sharing on marginal revenue is stronger for the large-market club than for the small-market club. In this case, the sharpening effect of revenue sharing is stronger for the large-market club. Again, the underdog (in this case, the large-market club) will increase its investment level relatively more than the dominant team (in this case, the small-market club) such that the league becomes more balanced through revenue sharing.

In the case that the league is already perfectly balanced (i.e., both clubs have equal playing strength in equilibrium such that $\widehat{WR^*} = 1$), the (marginal) sharpening effect of revenue sharing is equally strong for both clubs. As a consequence, both

Table 1. Effect of Revenue Sharing on Competitive Balance

	Large-Market Club is Dominant Team	Fully Balanced Competition	Small-Market Club is Dominant Team
$b > m_1 + m_2$	CB increases	IP holds	CB increases
$b < m_1 + m_2$	CB decreases	IP holds	CB decreases
$b = m_1 + m_2$	IP holds	IP holds	IP holds

clubs will marginally increase their investment level at an equal rate and competitive balance will not be altered through revenue sharing such that the IP holds.

The integration of a win preference parameter γ_i for club i allows that the case in which revenue sharing has a positive effect on marginal revenue is a feasible equilibrium outcome. Without a win preference parameter, the parameter constellation $b > m_1 + m_2$ would not constitute an equilibrium. This parameterization implies that in equilibrium, the win percentage $\hat{w}_1^*$ of the large-market club and/or the win percentage $\hat{w}_2^*$ of the small-market club are higher than the revenue-maximizing win percentages $w_1^{'} = m_1/b$ and/or $w_2^{'} = m_2/b$. In this case, the marginal revenue of club 1 and/or club 2 would be negative, which is not feasible in equilibrium. The negative marginal revenue, however, can be compensated by additional marginal revenue through the integration of a win preference parameter γ_i. Due to this additional effect with respect to the marginal revenue of investment, the parameter constellation $b > m_1 + m_2$ is feasible in equilibrium.

Part (ii) posits that each club reduces the amount of talent hired in equilibrium if revenue sharing has a negative effect on marginal revenue of both clubs in equilibrium. That is, in this case, the well-known dulling effect of revenue sharing is present. If revenue sharing has a negative effect on marginal revenue a revenue-sharing arrangement will worsen the competitive balance in an already unbalanced league. With a similar argumentation as above, this dulling effect is more pronounced for the underdog than for the dominant team, because the marginal impact on the dominant team's revenues of a decrease in talent investment by the underdog is greater than the marginal impact on the underdog's revenues of a decrease in talent investment by the dominant team. If the large-market club is the dominant team in equilibrium, then the small-market club will reduce its investment level relatively more than the large-market club such that the league becomes less balanced through revenue sharing. This replicates the result of Szymanski and Késenne (2004).

If, however, the small-market club is the dominant team in equilibrium, then the dulling effect of revenue sharing is stronger for the large-market club than for the small-market club. In this case, the large-market club will reduce its investment level relatively more than the small-market club. As a result, the league becomes again less balanced through revenue sharing. In the case that the league is already perfectly balanced, the (marginal) dulling effect is equally strong for both clubs such that both clubs will marginally decrease their investment level at an equal rate. As a consequence, competitive balance will not be altered through revenue sharing, and the IP holds again.

Part (iii) shows that revenue sharing has no effect on talent investments, and hence, it does not change the level of competitive balance in the league if revenue sharing has no effect on marginal revenue. As a result, the IP with respect to revenue sharing, which has been derived only under Walrasian conjectures, holds even under contest-Nash conjectures.

Conclusion

In this paper, we develop a contest model of a sports league and introduce a more general objective function for club owners by assuming that clubs maximize a weighted sum of profits and wins. This approach differs from previous analyses of sports leagues, which primarily assume either pure profit-maximizing and/or win-maximizing clubs. Evidence from the real world of major sports leagues, however, suggests that clubs trade-off profits and wins.

Our model provides new insights regarding the effect of revenue sharing on investment incentives as well as determines the conditions under which revenue sharing increases or decreases competitive balance. The model also analyzes how more win-oriented behavior of certain clubs affects talent investment, competitive balance, and club profits. In particular, we show that the small-market club will be the dominant team in equilibrium and will invest more than the large-market club if the small-market club has a sufficiently high preference for winning. In this case, the resulting incentive effect to invest in playing talent compensates for the size effect. The effect of more win-oriented behavior of certain clubs on the competitive balance in the league is ambiguous and depends on market-size parameters and win preferences. We further show that aggregate club profits decrease with a more win-oriented behavior on the part of the small-market club in a league in which the large-market club is a pure profit-maximizer. On the other hand, in a league in which the small-market club is a pure profit-maximizer, aggregate club profits may increase through a more win-oriented behavior on the part of the large-market club.

Regarding the effect of revenue sharing, our analysis shows that revenue sharing may enhance incentives to invest in playing talent. Thus, we identify a new effect of revenue sharing called the "sharpening effect," which has the opposite effect of the well-known dulling effect. As a consequence, revenue sharing may increase or decrease competitive balance, or it may have no effect on competitive balance such that the IP holds. The effect of revenue sharing on competitive balance depends on (a) which club has a higher win percentage and hence is the dominant team in equilibrium and (b) whether the sharpening or dulling effect of revenue sharing is at work.

The sharpening effect is present if revenue sharing has a positive effect on marginal revenue, while the dulling effect is present if revenue sharing has a negative effect on marginal revenue. We find the sharpening or dulling effect to be more pronounced for the underdog than for the dominant team in equilibrium. In the presence of the sharpening effect (dulling effect), revenue sharing will improve (deteriorate) competitive balance if the league is not yet fully balanced. This holds true independent of which club is the dominant team in equilibrium. In the case in which the league is already fully balanced in equilibrium (i.e., both clubs have the same win percentage), revenue sharing has no effect on competitive balance, and the IP holds. The IP also holds if revenue sharing has no effect on marginal revenue, independent of whether the league is already fully balanced.

An interesting avenue for further research in this area is the analysis of salary restrictions (caps and floors). A salary cap (floor) puts an upper (lower) bound on a club's payroll and have been introduced as a measure to improve competitive balance in sports leagues. Salary restrictions are widely applied in professional sports leagues all over the world. In the National Hockey League (NHL), for example, each team had to spend between US$ 34.3 million and 50.3 million on player salaries in the 2007-2008 season. In the National Football League (NFL), the salary cap in 2009 is approximately US$ 128 million per team, whereas the salary floor was 87.6% of the salary cap, which is equivalent to US$ 112.1 million. The Australian Football League (AFL) also operates with a combined salary cap and floor: for 2009, the salary cap was fixed at A$ 7.69 million, the floor at 7.12 million.[15] Our model framework can be extended to analyze the effect of such salary restrictions on competitive balance, talent investment, and club profits in sports leagues with utility-maximizing clubs.

Appendix A1

Proof of Lemma 1

The first-order conditions for the maximization problem (6) are given by[16]

$$\frac{\partial u_1(x_1,x_2)}{\partial x_1} = \frac{x_2}{(x_1+x_2)^2}\left(m_1+\gamma_1-\frac{bx_1}{x_1+x_2}\right)-c=0,$$
$$\frac{\partial u_2(x_1,x_2)}{\partial x_2} = \frac{x_1}{(x_1+x_2)^2}\left(m_2+\gamma_2-\frac{bx_2}{x_1+x_2}\right)-c=0,$$

Subtraction of club 2's FOC from club 1's FOC yields

$$\frac{\partial u_1(x_1,x_2)}{\partial x_1}-\frac{\partial u_2(x_1,x_2)}{\partial x_2}=\frac{1}{\left(x_1^*+x_2^*\right)^2}\left[x_2^*(m_1+\gamma_1)-x_1^*(m_2+\gamma_2)\right]=0.$$

Hence, in equilibrium it must hold that

$$x_1^* = x_2^*\frac{m_1+\gamma_1}{m_2+\gamma_2}. \tag{A1}$$

Substituting $x_1^* = x_2^*\frac{m_1+\gamma_1}{m_2+\gamma_2}$ into the FOC of club 2 yields

$$\frac{\frac{m_1+\gamma_1}{m_2+\gamma_2}}{x_2^*\left(\frac{m_1+\gamma_1}{m_2+\gamma_2}+1\right)^2}\left(m_2+\gamma_2-\frac{b}{\frac{m_1+\gamma_1}{m_2+\gamma_2}+1}\right)=c.$$

Solving for x_2^*, we derive

$$x_2^*=\frac{(\gamma_2+m_2)^2(\gamma_1+m_1)(m_1+\gamma_1+m_2+\gamma_2-b)}{c(m_1+\gamma_1+m_2+\gamma_2)^3}.$$

Analogously, we can calculate the equilibrium investment x_1^* of club 1 given by

$$x_1^*=\frac{(\gamma_1+m_1)^2(\gamma_2+m_2)(m_1+\gamma_1+m_2+\gamma_2-b)}{c(m_1+\gamma_1+m_2+\gamma_2)^3}.$$

Substituting (x_1^*, x_2^*) into Equation 1 yields (w_1^*, w_2^*) as stated in Lemma 1.

Appendix A2

Proof of Proposition 3

ad (i): Suppose that $m_1 = m$ and $m_2 = 1$ with $m > 1$ and $b = 1$. Moreover, consider a league in which the large-market club is a pure profit-maximizer and the small-market club has a positive win preference, that is, $\gamma_1 = 0$ and $\gamma_2 > 0$. In this scenario, equilibrium talent investments are given by

$$(x_1^*, x_2^*)=\left(\frac{m^2(m+\gamma_2)(1+\gamma_2)}{c(1+m+\gamma_2)^3},\frac{m(m+\gamma_2)(1+\gamma_2)^2}{c(1+m+\gamma_2)^3}\right).$$

The partial derivatives of talent investments with respect to the win preference parameter γ_2 yield

$$\frac{\partial x_1^*}{\partial \gamma_2}=\frac{m^2(1+m(m-1)-\gamma_2^2)}{c(1+\gamma_2+m)^4}>0\Leftrightarrow 1+m^2>\gamma_2^2+m,$$
$$\frac{\partial x_2^*}{\partial \gamma_2}=\frac{(1+\gamma_2)m(1+\gamma_2+2\gamma_2 m+2m^2)}{c(1+\gamma_2+m)^4}>0,$$

for all $c > 0$, $m > 1$, and $\gamma_2 > 0$.

The profit of club $i = 1, 2$ is given by

$$\pi_1^* = \frac{m^2(1+m(2m+1)+\gamma_2(2m+1))}{2(m+\gamma_2+1)^3},$$
$$\pi_2^* = \frac{(1+\gamma_2)(m(3+\gamma_2-2\gamma_2^2)+(1+\gamma_2)^2-2m^2\gamma_2)}{2(m+\gamma_2+1)^3}.$$

The partial derivatives of club profits with respect to the win preference parameter γ_2 yield:

$$\frac{\partial\pi_1^*}{\partial\gamma_2} = -\frac{m^2(1+2m(m+\gamma_2)+\gamma_2)}{(m+\gamma_2+1)^4} < 0,$$
$$\frac{\partial\pi_2^*}{\partial\gamma_2} = -\frac{m((1+\gamma_2)^2+m^2(1+2\gamma_2)+m(\gamma_2(2\gamma_2+1)-1)}{(m+\gamma_2+1)^4} < 0,$$

for all $c > 0$, $m > 1$, and $\gamma_2 > 0$. This means that profits of the small-market club and the large-market club always decrease with a higher win preference γ_2. It follows that aggregate club profits also decrease. This completes the proof of the proposition.

ad (ii): Suppose that $m_1 = m$ and $m_2 = 1$ with $m > 1$ and $b = 1$. Moreover, consider a league in which the large-market club has a positive win preference and the small-market club is a pure profit-maximizer, that is, $\gamma_1 > 0$ and $\gamma_2 = 0$. In this scenario, equilibrium talent investments are given by

$$\left(x_1^*, x_2^*\right) = \left(\frac{(m+\gamma_1)^3}{c(1+m+\gamma_1)^3}, \frac{(m+\gamma_1)^2}{c(1+m+\gamma_1)^3}\right).$$

The partial derivatives of talent investments with respect to the win preference parameter γ_1 yield

$$\frac{\partial x_1^*}{\partial\gamma_1} = \frac{3(\gamma_1+m)^2}{c(1+\gamma_1+m)^4} > 0,$$
$$\frac{\partial x_2^*}{\partial\gamma_1} = \frac{(2-\gamma_1-m)(\gamma_1+m)}{c(1+\gamma_1+m)^4} > 0 \Leftrightarrow m < 2-\gamma_1.$$

The profit of club $i = 1, 2$ is given by

$$\pi_1^* = \frac{(m+\gamma_1)[(m+\gamma_1)(1-m-\gamma_1)+2(1+m+\gamma_1)(m^2+\gamma_1(m-1)]}{2(m+\gamma_1+1)^3},$$
$$\pi_2^* = \frac{1+3(m+\gamma_1)}{2(m+\gamma_1+1)^3}.$$

The partial derivative of club 1's profits with respect to the win preference parameter γ_1 yields:

$$\frac{\partial \pi_1^*}{\partial \gamma_1} = \frac{(m+\gamma_1)[m(m+\gamma_1-2)-4\gamma_1]-\gamma_1}{(m+\gamma_1+1)^4} > 0$$
$$\Leftrightarrow (m+\gamma_1)[m(m+\gamma_1-2)-4\gamma_1] > \gamma_1.$$

The inequality is satisfied for m sufficiently large.

The partial derivative of club 2's profits with respect to the win preference parameter γ_1 yields:

$$\frac{\partial \pi_2^*}{\partial \gamma_1} = -\frac{3(m+\gamma_1)}{(m+\gamma_1+1)^4} < 0,$$

for all $c > 0$, $m > 1$, and $\gamma_2 > 0$.

The partial derivative of aggregate club profits with respect to the win preference parameter γ_1 is given by

$$\frac{\partial(\pi_1^*+\pi_2^*)}{\partial \gamma_1} = \frac{m(m+\gamma_1-3)-4\gamma_1}{(m+\gamma_1+1)^3} > 0 \Leftrightarrow m(m+\gamma_1-3) > 4\gamma_1.$$

The last inequality is satisfied for $m > m'' \equiv 1/2\left(3-\gamma_1+[(\gamma_1+1)(\gamma_1+9)]^{1/2}\right)$. This completes the proof of the proposition.

Appendix A3

Proof of Lemma 2

Rewriting the first-order conditions, we obtain:

$$\frac{\partial \hat{u}_1(x_1,x_2)}{\partial x_1} = \frac{x_2}{(x_1+x_2)^3}\left[(x_1+x_2)\left(\underbrace{\gamma_1 - m_2(1-\alpha)+\alpha m_1 - b\alpha}_{\equiv r}\right)+bx_2\right]$$
$$-c = 0,$$
$$\frac{\partial \hat{u}_2(x_1,x_2)}{\partial x_2} = \frac{x_1}{(x_1+x_2)^3}\left[(x_1+x_2)\left(\underbrace{\gamma_2 - m_1(1-\alpha)+\alpha m_2 - b\alpha}_{\equiv s}\right)+bx_1\right]$$
$$-c = 0.$$

Combining both equations and rearranging yields

$$(x_1+x_2)(x_2 r - x_1 s + bx_2 - bx_1) = 0.$$

In equilibrium (x_1^*, x_2^*), it must hold:

$$x_1^* = \frac{r+b}{s+b} x_2^* = \frac{\gamma_1 - m_2(1-\alpha) + \alpha(m_1 - b) + b}{\gamma_2 - m_1(1-\alpha) + \alpha(m_2 - b) + b} x_2^*,$$

This implies that

$$\hat{w}_i^* = \frac{x_i^*}{x_1^* + x_2^*} = \frac{\gamma_i + \alpha(m_i - b) - (1-\alpha)\mathrm{m}_j + b}{(m_1 + m_2)(2\alpha - 1) + 2b(1-\alpha) + \gamma_1 + \gamma_2},$$

with $i,j = 1,2$ and $i \neq j$. This completes the proof of the lemma.

Appendix A4

Proof of Proposition 4

We divide the proof in two parts: In (a), we show how revenue sharing affects the clubs' investment incentives and in (b) we analyze the effect of revenue sharing on competitive balance.

(a) The effect of revenue sharing on investment incentives

We claim that the effect of more revenue sharing on talent investments depends on how revenue sharing affects marginal revenue in equilibrium. In this proof, we will show that a higher degree of revenue sharing (i) increases equilibrium investment of each club if $b > m_1 + m_2$, (ii) decreases equilibrium investment of each club if $b < m_1 + m_2$, and (iii) has no effect on equilibrium investment of each club if $b = (m_1 + m_2)$.

To prove this claim, we derive the total differential of the first-order conditions $\frac{\partial \hat{u}_1}{\partial x_1} = 0$ and $\frac{\partial \hat{u}_1}{\partial x_2} = 0$:

$$\frac{\partial^2 \hat{u}_1}{\partial x_1^2} dx_1 + \frac{\partial^2 \hat{u}_1}{\partial x_1 \partial x_2} dx_2 + \frac{\partial^2 \hat{u}_1}{\partial x_1 \partial \alpha} d\alpha = 0,$$
$$\frac{\partial^2 \hat{u}_2}{\partial x_2 \partial x_1} dx_1 + \frac{\partial^2 \hat{u}_2}{\partial x_2^2} dx_2 + \frac{\partial^2 \hat{u}_2}{\partial x_2 \partial \alpha} d\alpha = 0.$$

For notational convenience, we write: $\frac{\partial^2 \hat{u}_1}{\partial x_1^2} = \hat{u}_{11}$, $\frac{\partial^2 \hat{u}_1}{\partial x_2 \partial x_1} = \hat{u}_{12}$, $\frac{\partial^2 \hat{u}_1}{\partial \alpha \partial x_1} = \hat{u}_{1\alpha}$, and $\frac{\partial^2 \hat{u}_2}{\partial x_2^2} = \hat{u}_{22}$, $\frac{\partial^2 \hat{u}_2}{\partial x_1 \partial x_2} = \hat{u}_{21}$, and $\frac{\partial^2 \hat{u}_2}{\partial \alpha \partial x_2} = \hat{u}_{2\alpha}$. Moreover, $R_i^{'} = \frac{\partial R_i}{\partial w_i}$ and $R_i^{''} = \frac{\partial^2 R_i}{\partial w_i}$ for $i = 1,2$.

The total differential of the first-order conditions from above can also be written as

$$\begin{bmatrix} \hat{u}_{11} & \hat{u}_{12} \\ \hat{u}_{21} & \hat{u}_{22} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = \begin{bmatrix} -\hat{u}_{1\alpha} \\ -\hat{u}_{2\alpha} \end{bmatrix} d\alpha, \tag{A2}$$

where

$$\hat{u}_{11} = \left(\alpha R_1' - (1-\alpha)R_2' + \gamma_1\right)\left(\frac{-2w_2}{(x_1+x_2)^2}\right) + \left(\alpha R_1'' + (1-\alpha)R_2''\right)\left(\frac{w_2^2}{(x_1+x_2)^2}\right),$$
$$\hat{u}_{12} = \left(\alpha R_1' - (1-\alpha)R_2' + \gamma_1\right)\left(\frac{w_1-w_2}{(x_1+x_2)^2}\right) - \left(\alpha R_1'' + (1-\alpha)R_2''\right)\left(\frac{w_1w_2}{(x_1+x_2)^2}\right),$$
$$\hat{u}_{21} = \left(\alpha R_2' - (1-\alpha)R_1' + \gamma_2\right)\left(\frac{w_2-w_1}{(x_1+x_2)^2}\right) - \left(\alpha R_2'' + (1-\alpha)R_1''\right)\left(\frac{w_1w_2}{(x_1+x_2)^2}\right),$$
$$\hat{u}_{22} = \left(\alpha R_2' - (1-\alpha)R_1' + \gamma_2\right)\left(\frac{-2w_1}{(x_1+x_2)^2}\right) + \left(\alpha R_2'' + (1-\alpha)R_1''\right)\left(\frac{w_1^2}{(x_1+x_2)^2}\right),$$
$$\hat{u}_{1\alpha} = \left(R_1' + R_2'\right)\frac{w_2}{x_1+x_2} = (m_1+m_2-b)\frac{w_2}{x_1+x_2},$$
$$\hat{u}_{2\alpha} = \left(R_1' + R_2'\right)\frac{w_1}{x_1+x_2} = (m_1+m_2-b)\frac{w_1}{x_1+x_2}.$$

Note that in equilibrium it must hold that

$$\alpha R_1' - (1-\alpha)R_2' + \gamma_1 = \frac{c(x_1+x_2)}{w_2} > 0 \text{ and } \alpha R_2' - (1-\alpha)R_1' + \gamma_2 = \frac{c(x_1+x_2)}{w_1} > 0.$$

Applying Cramer's Rule to Equation A2, we derive

$$\frac{dx_1}{d\alpha} = \frac{\hat{u}_{12}\hat{u}_{2\alpha} - \hat{u}_{22}\hat{u}_{1\alpha}}{\hat{u}_{11}\hat{u}_{22} - \hat{u}_{12}\hat{u}_{21}} \quad \text{and} \quad \frac{dx_2}{d\alpha} = \frac{\hat{u}_{21}\hat{u}_{1\alpha} - \hat{u}_{11}\hat{u}_{2\alpha}}{\hat{u}_{11}\hat{u}_{22} - \hat{u}_{12}\hat{u}_{21}}. \tag{A3}$$

In order to ensure a maximum, we need the stability condition $\hat{u}_{11}\hat{u}_{22} - \hat{u}_{12}\hat{u}_{21} > 0$. Therefore, the denominator has to be positive (see, e.g., Dixit, 1986; Szymanski & Késenne, 2004).

The sign of the numerator depends on how revenue sharing affects marginal revenue. We differentiate three cases:

Part (i): Assume that $b > m_1 + m_2$. In this case, $\hat{u}_{1\alpha} < 0$ and $\hat{u}_{2\alpha} < 0$.

(ia) If club 1 is the dominant team in equilibrium, that is, $w_1 > w_2$, then $\hat{u}_{12} > 0$ and thus the numerator $\hat{u}_{12}\hat{u}_{2\alpha} - \hat{u}_{22}\hat{u}_{1\alpha}$ of $\frac{dx_1}{d\alpha}$ is negative. It follows that $\frac{dx_1}{d\alpha} < 0$, that is, revenue sharing induces the dominant team (club 1) to increase its investment. Because revenue sharing increases competitive balance,[17] the underdog (club 2) has to increase its investment as well, that is, $\frac{dx_2}{d\alpha} < 0$.

(ib) If club 2 is the dominant team in equilibrium, that is, $w_2 > w_1$, then $\hat{u}_{21} > 0$ and thus the numerator $\hat{u}_{21}\hat{u}_{1\alpha} - \hat{u}_{11}\hat{u}_{2\alpha}$ of $\frac{dx_2}{d\alpha}$ is negative. It follows that $\frac{dx_2}{d\alpha} < 0$, that is, revenue sharing induces the dominant team (club 2) to increase its investment. Because revenue sharing increases competitive balance, the underdog (club 1) has to increase its investment as well, that is, $\frac{dx_1}{d\alpha} < 0$.

Part (ii): Assume that $b < m_1 + m_2$. In this case, $\hat{u}_{1\alpha} > 0$ and $\hat{u}_{2\alpha} > 0$.

(iia) If club 1 is the dominant team in equilibrium, that is, $w_1 > w_2$, then $\hat{u}_{12} > 0$ and thus the numerator $\hat{u}_{12}\hat{u}_{2\alpha} - \hat{u}_{22}\hat{u}_{1\alpha}$ of $\frac{dx_1}{d\alpha}$ is positive. It follows that $\frac{dx_1}{d\alpha} > 0$, that is, revenue sharing induces the dominant team (club 1) to decrease its investment. Because revenue sharing decreases competitive balance,[18] the underdog (club 2) has to decrease its investment as well, that is, $\frac{dx_2}{d\alpha} > 0$.

(iib) If club 2 is the dominant team in equilibrium, that is, $w_2 > w_1$, then $\widehat{u_{21}} > 0$ and thus the numerator $\hat{u}_{21}\hat{u}_{1\alpha} - \hat{u}_{11}\hat{u}_{2\alpha}$ of $\frac{dx_2}{d\alpha}$ is positive. It follows that $\frac{dx_2}{d\alpha} > 0$, that is, revenue sharing induces the dominant team (club 2) to decrease its investment. Because revenue sharing decreases competitive balance, the underdog (club 1) has to decrease its investment as well, that is, $\frac{dx_1}{d\alpha} > 0$.

Part (iii): Assume that $b = m_1 + m_2$. In this case, $\hat{u}_{1\alpha} = 0$ and $\hat{u}_{2\alpha} = 0$. It immediately follows that the numerator is 0 and thus $\frac{dx_1}{d\alpha} = \frac{dx_2}{d\alpha} = 0$. That is, revenue sharing has no effect on talent investments and on competitive balance. This completes the proof of part (iii) of this proposition.

(b) The effect of revenue sharing on competitive balance

Part (i): Assume that $b > m_1 + m_2$: We claim that a higher degree of revenue sharing increases competitive balance if either the small-market club or the large-market club is the dominant team in equilibrium. In the case that both clubs have equal playing strength in equilibrium, the IP holds.

We derive

$$\frac{\partial \widehat{WR^*}}{\partial \alpha} = \frac{[b - (m_1 + m_2)][(m_1 + \gamma_1) - (m_2 + \gamma_2)]}{(\gamma_2 + \alpha(m_2 - b) - (1 - \alpha)m_1 + b)^2}.$$

The sign of $\frac{\partial \widehat{WR^*}}{\partial \alpha}$ only depends on $m_1 + \gamma_1 \gtreqless m_2 + \gamma_2$. Note that

$\frac{\partial MR_1}{\partial \alpha} = \frac{x_2}{(x_1 + x_2)^2}(m_1 + m_2 - b) < 0$ and $\frac{\partial MR_2}{\partial \alpha} = \frac{x_1}{(x_1 + x_2)^2}(m_1 + m_2 - b) < 0.$

It follows that a higher degree of revenue sharing (i.e., a lower parameter α) implies higher marginal revenue for both clubs.
We differentiate three cases:

1. Assume that $m_1 + \gamma_1 = m_2 + \gamma_2$. In this case, it is easy to see that revenue sharing has no effect on competitive balance and the IP holds, because $\frac{\partial \widehat{WR^*}}{\partial \alpha} = 0$.
2. Assume that $m_1 + \gamma_1 > m_2 + \gamma_2$. In this case, the large-market club 1 invests more in talent and thus has a higher win percentage than the small-market club 2 in equilibrium. Furthermore, $\left|\frac{\partial MR_1}{\partial \alpha}\right| < \left|\frac{\partial MR_2}{\partial \alpha}\right|$ because $x_1 > x_2$, such that the positive effect of revenue sharing on marginal revenue is stronger for the small-market club. Therefore, $\widehat{WR^*} > 1$ decreases and competitive balance increases if revenue sharing increases.
3. Assume that $m_1 + \gamma_1 < m_2 + \gamma_2$. In this case, the small-market club 2 invests more in talent and thus has a higher win percentage than the large-market club 1 in equilibrium. Furthermore, $\left|\frac{\partial MR_2}{\partial \alpha}\right| < \left|\frac{\partial MR_1}{\partial \alpha}\right|$ because $x_2 > x_1$, such that the positive effect of revenue sharing on marginal revenue is stronger for the large-market club. Therefore, $\widehat{WR^*} < 1$ increases and competitive balance increases if revenue sharing increases.
4. Part (ii): Assume that $b < m_1 + m_2$. We claim that a higher degree of revenue sharing decreases competitive balance if either the small-market club or the large-market club is the dominant team in equilibrium. In the case that both clubs have equal playing strength in equilibrium, the IP holds.

As in the proof of Proposition 4, the sign of $\frac{\partial \widehat{WR^*}}{\partial \alpha}$ only depends on $m_1 + \gamma_1 \gtreqless m_2 + \gamma_2$. Note that $\frac{\partial MR_1}{\partial \alpha} > 0$ and $\frac{\partial MR_2}{\partial \alpha} > 0$ if $b < m_1 + m_2$. It follows that a higher degree of revenue sharing (i.e., a lower parameter α) implies higher marginal revenue for both clubs.
Again, we differentiate three cases:

1. Assume that $m_1 + \gamma_1 = m_2 + \gamma_2$. In this case, it is easy to see that revenue sharing has no effect on competitive balance and the IP holds, because $\frac{\partial \widehat{WR^*}}{\partial \alpha} = 0$.
2. Assume that $m_1 + \gamma_1 > m_2 + \gamma_2$. In this case, the large-market club 1 invests more in talent and thus has a higher win percentage than the small-market club 2 in equilibrium. Furthermore, $\frac{\partial \mathrm{MR}_1}{\partial \alpha} < \frac{\partial \mathrm{MR}_2}{\partial \alpha}$ because $x_1 > x_2$, such that the

negative effect of revenue sharing on marginal revenue is stronger for the small-market club. Therefore, $\widehat{WR^*} > 1$ increases even more and competitive balance decreases if revenue sharing increases.

3. Assume that $m_1 + \gamma_1 < m_2 + \gamma_2$. In this case, the small-market club 2 invests more in talent and thus has a higher win percentage than the large-market club 1 in equilibrium. Furthermore, $\frac{\partial \mathrm{MR}_2}{\partial \alpha} < \frac{\partial \mathrm{MR}_1}{\partial \alpha}$ because $x_2 > x_1$, such that the negative effect of revenue sharing on marginal revenue is stronger for the large-market club. Therefore, $\widehat{WR^*} < 1$ decreases even more and competitive balance decreases if revenue sharing increases.

This completes the proof of the proposition.

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Notes

1. See also Cyrenne (2009).
2. According to Vrooman (1995), the "strength of the two-team model derives from its simplicity and efficiency in dealing with the questions of talent polarization." See also Szymanski and Kesenne (2004) and Vrooman (2007, 2008), who conduct their analysis in a two-club league.
3. See also Rascher (1997), who assumes that clubs maximize a linear combination of profits and wins. However, the crucial difference with respect to our model is that Rascher applies Walrasian conjectures and assumes a fixed supply of talent in the league (see also Késenne, 2007). Lang et al. (in press) present a welfare analysis of a sports league and assume that a sugar daddy club owner maximizes a linear

combination of profits and wins. However, they do not find the sharpening effect of revenue sharing.

4. The logit CSF was generally introduced by Tullock (1980) and was subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). See Dietl, Franck, and Lang (2008) and Fort and Winfree (2009) for analyses of the CSF's discriminatory power in sporting contests.
5. Note that $m_1 > m_2$ "reflects the possibility that team 1 may be able to generate a higher revenue from a given level of success" than team 2 (Szymanski, 2003, p. 1164).
6. Note that Sloane (1971) was the first to suggest that the owner of a sports club actually maximizes utility, which may include inter alia playing success and profits.
7. Note that if the win preference of the small-market club equals $\gamma_2 = \gamma_2^{min}$, then the clubs' win percentages in the league with utility-maximizing clubs correspond to those in the league with profit-maximizing clubs.
8. Regarding the effect on utility, one can show that the utility of club i increases with its win preference parameter γ_i and decreases with the win preference parameter γ_j of the other club. The effect on aggregate utility in the league, however, is ambiguous and depends on the parameters (γ_i, m_i). In particular, in the case of $\gamma_1 > 0$ and $\gamma_2 = 0$, aggregate utility in the league always increases if the large-market club becomes more win-orientated, whereas in the case of $\gamma_1 = 0$ and $\gamma_2 > 0$, the effect on aggregate utility is ambiguous if the small-market club becomes more win-orientated.
9. Note that the revenue function of club $i = 1, 2$ is a strictly increasing function on the interval $w_i \in [0, 1]$ for $b = 1$.
10. Grossmann, Dietl, and Lang (2010) analyze the effects of revenue sharing in a dynamic model of a sports league. For an analysis of a pool-revenue sharing arrangement, see for example, Dietl, Lang, and Rathke (2011). Moreover, Palomino and Sákovics (2004) provide an explanation for the difference in revenue sharing rules between the U.S. and the European sports leagues.
11. Note that this condition does not depend on the revenue-sharing parameter α.
12. See Szymanski (2003), Szymanski and Késenne (2004), Cyrenne (2009), Dietl et al. (2009), and Lang et al. (in press).
13. The dulling effect describes the well-known result in sports economics that revenue sharing reduces the incentive to invest in playing talent (see Szymanski & Késenne, 2004).
14. Remember that $\widehat{WR^*} > 1$ holds if and only if $m_1 + \gamma_1 > m_2 + \gamma_2$.
15. The data are taken from the collective bargaining agreements of the respective leagues.
16. It is easy to verify that the second-order conditions for a maximum are satisfied.
17. See part (b) of the proof below.
18. See part (b) of the proof below.

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