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THREE ESSAYS ON MARKET FRICTIONS AND FINANCIAL INTERMEDIATION

Bialova Ina

UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT DE FINANCE

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THÈSE DE DOCTORAT

présentée à la

Faculté des Hautes Études Commerciales
de l'Université de Lausanne

pour l'obtention du grade de
Docteure ès Sciences Économiques, mention « Finance »

par

Ina BIALOVA

Co-directeurs de thèse
Prof. Norman Schürhoff
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Jury

Prof. Felicitas Morhart, Présidente
Prof. Diane Pierret, experte interne
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IMPRIMATUR

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Abstract

The thesis is composed of three papers on market frictions and financial intermediation. It explores the impact that different forms of market frictions have on the choice of capital structure of financial institutions or on the fragility of the financial system.

Chapter 1 proposes a model of how securitisation can alleviate investment distortions that arise within banks if the latter have a high level of risky debt. In a securitisation a bank will retain an equity-like tranche of the securitised portfolio as a commitment to proper asset selection and monitoring. The model suggests that banks with a high deposit base benefit from securitisation more from the perspective of reduction of investment distortions. The model predicts that for assets with high default recovery value the bank decreases its screening and monitoring of the assets when the retention by the bank exceeds certain threshold. However, for assets with low default recovery value a higher retention leads to better screening.

Chapter 2 analyses how a risk of a fire sale liquidation affects the debt structure of financial institutions. A fire sale occurs when a financial institution needs to sell some of its assets, but the potential buyers of these assets are themselves financially constrained. I show in a theoretical framework that when a bank holds an asset with a small set of potential buyers, it will prudently choose to borrow less of short-term secured debt. Thus, a possibility of a fire sale liquidation disciplines the bank when the liquidity of its assets is low. However, if a financial institution holds an asset that has a large set of potential buyers, such financial institution will borrow too much of short-term secured debt thus increasing the risk of a fire sale spillover.

In chapter 3 I evaluate the resilience of a financial system to shocks when financial institutions are connected to each other through interbank loans. When one or several banks go bankrupt, they default on their interbank liabilities, which can in turn put its banking counter parties in financial destress. I consider a financial system in which a shock can propagate both through the network of interbank loans, but also through the centralised asset market. First, I find that the higher is the number of interbank connections each bank has, the higher is the resilience of the system to the shocks. Second, a network structure where a few banks have a high number of connections (core) while the rest of the banks are connected mainly to those banks in the core, is more vulnerable to collapses and to declines of asset market price.

Résumé

La thèse est composée de trois papiers sur les frictions de marché et intermédiation financiére.

La thèse est composée de trois articles sur les frictions de marché et l'intermédiation financière. Elle explore l'impact que les différentes formes de frictions de marché ont sur le choix de la structure du capital des institutions financières ou sur la fragilité du système financier.

Le chapitre 1 propose un modèle expliquant comment la titrisation d'actifs peut atténuer les distorsions d'investissements dans les banques qui détiennent un niveau élevé de dette risquée. Lors d'une titrisation, une banque émettrice détient une tranche dite junior du portefeuille titrisé afin de garantir son engagement dans la sélection et le contrôle des actifs sous-jacents. Le modèle suggère que les banques avec une large base de dépôts bénéficient plus de la titrisation grâce à une réduction des distorsions des investissements. Le model prédit que pour les actifs avec une large valeur de recouvrement en défaut les banques diminuent leurs efforts de sélection et de contrôle des actifs une fois que leur taux de rétention dépasse un certain niveau. Par contre, pour les actifs avec une faible valeur de recouvrement une rétention plus élevée induit une meilleure sélection des actifs.

Le chapitre 2 analyse comment un risque de liquidation de type " fire sale" affecte la structure de la dette des institutions financières. Un " fire sale " a lieu quand une institution financière a besoin de vendre une partie de ses actifs, mais les acheteurs potentiels sont eux-mêmes financièrement contraints. Je montre théoriquement que si une banque détient un actif qui a un petit nombre des acheteurs potentiels, elle choisira prudemment d'emprunter moins de dette à court terme garantie. Ainsi, la possibilité de liquidation en " fire sale " discipline les banques si la liquidité des actifs est faible. Par contre, si une banque détient un actif avec un grand nombre des acheteurs potentiels, elle empruntera trôp de dette à court terme garantie ce qui augmente le risque de " fire sale ".

Dans le chapitre 3 j'évalue la résilience du système financier à des chocs lorsque les banques sont connectées l'une à l'autre à travers des emprunts interbancaires. Quand une ou plusieurs banques font faillite, elles sont en défaut de paiement sur leurs dettes interbancaires, ce qui à son tour peut amener leurs contreparties bancaires à une détresse financière. J'analyse un système financier dans lequel un choc peut se propager non seulement à travers le réseau des emprunts interbancaires mais aussi à travers le marché centralisé des actifs. Le modèle suggère que plus le nombre de contreparties de chaque banque dans le système est grand, plus la résilience du système face aux chocs sera élevée. De plus, un réseau bancaire dans lequel un petit nombre des banques possède un grand nombre de contreparties (core) tandis que d'autres banques sont principalement connectées aux banques du " core " est plus vulnérable aux chocs et à la baisse des prix de marchés des actifs.

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Introduction

The global financial crisis of 2007-2009 highlighted multiple sources of fragility of the financial system. Some sources of fragility bear a systemic nature which means that a financial institution, which would otherwise be financial sound, can be brought into financial destress because of the events unrelated to its own economic activity, for example because of a fire sale externality or default cascades due to network of interbank exposures. Fire sale externality can arise if one or a group of financial institutions has to sell a large amount of financial assets, which pushes the market price of these assets down. Given that the asset side of banks' balance sheet is marked to market, and that banks face leverage constraints, a large exogenously given price decline can force other banks, which would have otherwise been financially sound, to de-lever and sell their assets. Additional asset sale pushes the asset prices further down. A downward price spiral occurs propagating losses to financial institutions across the financial sector. Fire sale spillover occurs because banks hold same or similar assets and therefore the shock is propagated through the decline in the market asset prices, but not because banks have direct exposures one to another. Contrary to that, network default cascades occur due to the direct exposures that banks have against each other, i.e. interbank loans. If one bank collapses, it defaults on all its debt that it owes to other banks, pushing its banking counter parties under financial stress. If one or several of these counter parties in turn default on their own interbank liabilities, a cascade of defaults due to network effects begins. Chapter 2 and 3 are dedicated to these two sources of systemic fragility.

The latest global financial crisis also highlighted problems related to securitisation, a process when a bank sells the loans it issued to a separate legal entity which structures and tranches these loan portfolios into asset backed securities (ABS) which are further sold to investors. Securitisation allows banks to free up capital for new loans and investments, and share risk with the rest of the financial participants who are willing to bear that risk. There is a problem however, which is that the quality of the loans that a bank sells is not observable by the buyers of the ABS. As a result, banks might have incentives to sell loans of poor quality and keep loans of better quality to themselves. Poor screening of the sub-prime mortgage borrowers is one of the causes of the financial crisis of 2007-2009.

In chapter 1 of the thesis I contribute to the analysis of costs and benefits of securitisation. First, I show that securitisation can reduce bank's investment distortions due to a presence of high level of risky debt if the bank retains an equity-like tranche of a securitised portfolio as a commitment to proper loan screening. Second, I analyse the optimal level of bank's retention in the securitised portfolios.

Securitisation, through a sale of assets to a special purpose vehicle, limits the claims of the buyers of the asset backed security only to the assets that back the ABS, but not to other assets of the bank. As a result, the set of priority claimants that benefit from new investments before shareholders decreases, and a bank undertakes positive net present value (NPV) projects that a non-securitising bank would have forgone. To illustrate a mechanism I develop a model where securitisation is accompanied by a moral hazard problem. The costly effort that the bank puts into loan screening is unobservable, and so is the quality of loans. Because of this moral hazard problem an originating bank retains an equity-like tranche of the securitised portfolio as a commitment to loan screening. The model suggests that a higher retention by the bank leads to higher effort to screen the loans if the loan recovery value in default is low.

For loans with high default recovery rates, the relationship between bank effort and the size of the retained tranche is hump-shaped because an increase in the amount of loans that back the ABS increases the expected payoff of the ABS holders in default, if the default recovery rate is high. If the level of bank retention exceeds certain threshold, the sensitivity of the cost of ABS to effort decreases, and the bank chooses to screen less.

In chapter 2 I analyse how banks adjust their capital structure in anticipation of a possible fire sale spillover. I build a model in which bank capital structure and asset liquidity are intertwined, causing excessive leverage spirals. Banks issue short-term secured debt to finance asset purchases. When market prices of pleadgeable assets decrease, banks are unable to roll over their short-term debt and are forced to liquidate assets. This creates a fire sale spillover. In anticipation of fire sales banks adjust their ex-ante capital structure. If the market liquidity of an asset is high, the price decline in a fire sale liquidation is low. As a result, high market liquidity of pledgeable asset decreases the cost of a potential fire sale for a bank and creates incentives to take too much of the short-term secured debt increasing the risk of forced deleveraging in the future. When the liquidity of pleadgeable asset is low, in case of a fire sale liquidation banks anticipate a higher price decline. The possibility of a costly asset liquidation disciplines the banks and they borrow less. As a result, the possibility of a fire sale decreases. Therefore, the model suggests that the anticipation of a fire sale spillover disciplines financial institutions only if the liquidity of their pledgeable assets is low. Compared to social optimum, banks raise too much debt as they do not internalise the impact of their leverage choice on equilibrium asset prices. The model supports regulatory minimum capital requirements, while it puts in question policies based on regulatory price support. The latter, while being efficient ex-post, ex-ante creates incentives for the banks to borrow too much of the short-term secured debt increasing the chance of a fire-sale spillover.

In chapter 3 I propose a model of contagion in a financial network where a shock that hits a bank or a group of banks can propagate in the financial system though two channels: direct network of interbank exposures and a fire sale externality. When some banks in the system are hit by a shock and are unable reimburse their interbank liabilities, it reduces financial soundness of its banking parties and their the ability to reimburse their own liabilities. A bank unable to honour its liabilities sells its assets in a centralised market to the banks in the network who have resources to buy the assets of liquidated banks. If few banks default and the rest of banks have enough of resources to acquire the assets of insolvent banks, the asset price will remain high. As a consequence, a higher asset price increases the default recovery value, which in turn improves the ability of banks to support the asset price. The findings suggest that a more densely connected network is able to sustain a higher level of asset prices, thus limiting the spread of default cascades in the system. Core-periphery network is more prone to systemic collapse and decline in asset market price than a non-concentrated network.

Chapter 1

The Bright Side of Securitisation

1.1 Introduction

Securitisation is a controversial topic. On the one hand, it is an important financial innovation that allows banks to free up capital to invest in new projects, and to transform illiquid loans into liquid marketable securities (Gorton and Metrick (2012)). On the other hand, securitisation is largely blamed for the role it played in the financial crisis. For example, securitisation was blamed was reducing bank lending standards. Keys et al. (2010) find that the default rate of a loan portfolio which is easier to securitise was 0.5% – 1% higher than that of a loan portfolio with a similar risk profile but with lesser ease of securitisation. Before the crisis banks largely used securitisation to engage in regulatory arbitrage rather than risk sharing: they would sell the loans to a special purpose vehicle (SPV), thus laying the risk off the balance sheet, and then provide credit or liquidity guarantees for the SPV (Acharya et al. (2013)). As these guarantees were not included in the calculation of the minimum capital requirements, the capital charges of banks were lower, while their risk exposure did not change. Securitisation has also been blamed for creating funding fragility: some ABS were of short-term maturity, while the assets backing these securities were long-term. This effectively led to a collapse of the short-term asset-backed funding (Acharya et al. (2013)).

In this paper I contribute to the analysis of costs and benefits of securitisation. First, I show that if banks retain an equity-like tranche of a securitised portfolio, securitisation can mitigate the debt overhang problem because it limits the set of priority claimholders that benefit from the future investment before the shareholders. Second, I analyse the optimal size of a tranche that banks should retain.

Securitisation is an off-balance sheet debt financing.[1] A bank sells a portfolio of loans to a SPV which restructures, tranches into senior (debt-like) and junior (equity-like) tranches, and then sells the securities backed by a pool of loans to investors. The loan originating bank usually retains some interest in the sold portfolio, such as a junior tranche (Gorton and Metrick (2013). The existing literature (Chiesa (2014), Ayotte and Gayon (2010)) suggests that the true sale of assets that securitisation entails provides protection for the buyers of the ABS against the bankruptcy of the loan originator, bankruptcy remoteness. In this paper I argue that the sale of assets to the SPV also reduces debt overhang because it creates a segregation

[1] I do not analyse synthetic securitisation, when the risk transfer is achieved by purchasing of credit derivatives, while the exposures remains on the balance sheet of the loan originator.

of liabilities.

The debt overhang problem arises from the agency conflict between shareholders and debtholders, because the former are the residual claimholders (Myers (1977)). When debt is risky, a company acting in the interest of shareholders might forgo positive NPV projects because otherwise mainly the existing debtholders would benefit from the investment. Empirical literature finds evidence that debt overhang has a first-order effect on investment policy. For example, Favara et al. (2017) look at sample of non-financial firms in distress. They document that firms with lower expected shareholder payoff in default invest less and in riskier projects. The debt overhang can be even more pronounced for financial firms, because bank leverage is much higher than the leverage of non-financial firms.[2] I show that securitisation helps alleviate the debt overhang problem because the claims of the ABS holders are limited only to the assets that back the ABS. If these assets do not perform well, the ABS holders seize them and recover their residual value in default but have no claim against the rest of the assets of the loan originator. As existing debtholders do not benefit from future investment, a larger value from these investments accrues to the shareholders, and the bank undertakes projects that would have otherwise been forgone.

To illustrate this mechanism, I build a model featuring a deposit bank that faces demand for loans of a given size, the bank is a price-taker and faces exogenously given loan rates. Bank deposits are not enough to finance the loans, and the bank has to raise additional debt. It can issue either on-balance sheet or off-balance sheet debt, but not a combination of the two types. Deposits are insured by the government, which imposes a minimum capital requirement on the bank.

Raising additional debt is accompanied by a moral hazard problem. The bank can increase the quality of loans that it finances by more intense screening, but the effort to screen is costly. Because the effort is unobservable, the bank cannot commit to a certain level of screening unless it is incentive-compatible. Hence, if it issues the off-balance sheet debt, the prospective buyers of these ABS impose a minimum retention of an equity-like tranche of the securitised portfolio as a commitment to proper loan screening. The market observes the fraction of the securitised portfolio retained by the bank and correctly infers the effort the bank will optimally exert. The debt is fairly priced.

When the bank issues off-balance sheet debt, in case of default the ABS holders seize the loans that back their security, and have no claim against other assets. The bank has different incentives to screen the securitised loans compared to the loans held till maturity for two reasons. First, the prospective ABS buyers price their claim only as a function of the effort put in screening the securitised portfolio, as their expected payoff does not depend on the profit from loans retained by the bank until maturity. Second, putting more effort into screening the securitised loans than in the loans held till maturity benefits the depositors, but not the shareholders. If the securitised portfolio defaults but the loans held till maturity pay off, the shareholders are reimbursed right after depositors. However, if the securitised portfolio pays off, but loans held do not, the shareholders are reimbursed only after the ABS holders and the depositors. Therefore, the agency conflict between shareholders and depositors alters

[2]The anecdotal evidence supports this claim: a senior bank executive, when talking about rise of shadow lending, mentioned that after the crisis banks were busy cleaning up their balance sheets, rather than issuing new loans, while nonbank financial institutions did not have anything to clean up and they picked up the lending.

the screening incentives of the bank.

In the model the debt overhang problem arises because the bank knows that in the future it might face a more attractive investment opportunity, but this opportunity is uncertain. If a fraction of the existing loans is financed with securitisation, a larger value from the new project will accrue to the shareholders. The bank anticipates how its current financing decision and effort will affect the future investment and chooses what type of debt to raise today and how much screening effort to put. In my model the debt overhang does not result in a reduced quantity of loans financed, but in a higher minimum return that the bank will require on a loan.

The model yields the following results. First, issuing off-balance sheet debt results in a higher shareholder value than issuing on-balance sheet debt if the level of deposits is sufficiently high compared to the amount of loans the bank needs to finance. If deposits are low, the bank has to raise a large amount of the off-balance sheet debt. The prospective ABS buyers will therefore require a higher retention, but the ability of the bank to retain is limited by the low level of deposits and equity. The cost of the off-balance sheet debt rises, and the bank switches to the on-balance sheet funding. Lower debt overhang implies that a securitising bank has a lower required return on the investments, and it can offer a lower interest rate to its borrowers compared to a non-securitising bank. This result is driven only by liability segregation, and not by regulatory arbitrage or costly equity issuance: in the framework I propose the bank puts the same amount of equity regardless of the form of debt financing it chooses.

This result is consistent with the empirical literature on securitisation and the cost of debt for bank borrowers. Nadauld and Weisbach (2012) document that spreads on corporate loans originated by securitisation-active banks are 11 basis points lower than spreads on corporate loans issued by other banks, holding other factors constant. Similarly, Guner (2006) finds that "the average yield spread on loans originated by active loan sellers is about 20 basis points lower than the average spread on loans originated by moderate loan sellers". The model presented in this papers offers a novel rationale for these results: securitisation decreases the agency conflict between shareholders and depositors, thus mitigating the debt overhang problem.

Second, I show that the bank puts weakly less effort into screening a securitised portfolio of loans compared to a portfolio of loans held until maturity, even when the bank retains a junior tranche of a securitised portfolio and the ABS is fairly priced. The result is due to the agency conflict between shareholders and depositors: shareholders do not benefit from the states of the world when securitised loans payoff but loans held till maturity do not, as the profit is captured by depositors. Whether the bank chooses to put as much effort into screening the securitised portfolio as in the retained portfolio or less depends on the loan characteristics, such as recovery value in default, the cost of screening, and the size of a junior tranche of a securitised portfolio retained by the bank. This result offers a novel rationale to why the existing empirical evidence on the presence of moral hazard in securitisation is mixed. On the one hand, Keys et al. (2010), and Bord and Santos (2015) find that a securitised portfolio of loans has a higher default rate compared to a portfolio of loans with similar risk characteristics but retained on the balance sheet of a bank. On the other hand, Benmelech et al. (2012) do not find convincing evidence of the moral hazard problem in corporate loan securitisation. While Benmelech et al. (2012) argue that "the securitisation of corporate loans is fundamentally different from securitisation of other asset classes because securitised loans are fractions of syndicated loans", I suggest that the differences in findings might also result from the expected recovery in default of the collateral asset, the cost of screening of a particular loan type that a bank incurs, and the size of the junior tranche retained by the bank.

Third, higher retention of a junior tranche of a securitised portfolio does not always increase effort intensity of the bank. Consider the following hypothetical example: when the ABS holders buy \$100 debt that promises to pay \$200 upon maturity, with this debt the bank finances a unit of loans of \$100 value. However, to be able to raise this debt, the bank has to retain a junior claim on this portfolio, which means that \$100 ABS claim is backed by 1.5 units of loans, and the bank puts \$50 of its on-balance sheet funds to finance the additional 0.5 unit of loans. If loans promise \$180, and in default the borrower recovers 0.4 of the promised payoff, in case of default the ABS holders seize 1.5 units of loans and recover the $1.5 \times 0.4 \times 180 = \$114$. Therefore, if the default recovery rate is high, a higher retention decreases the sensitivity of the cost of debt to the effort of the bank: the debt becomes less risky not because the bank exercises more effort, but because it is backed by a larger portfolio of loans with high recovery value. The relationship between the screening effort and retention in this case is hump-shaped: first effort increases with the retention, but then over-collateralisation effect kicks in, and the effort decreases. If the recovery rate is low, a higher retention does not improve the payoff of the ABS holders in default and it signals bank commitment to exert effort.

The assumption that the moral hazard problem is resolved through the retention of a minimum tranche of a securitised portfolio is critical to the claim that securitisation can alleviate debt overhang. Before the crisis banks did not retain a junior tranche of loan portfolios they securitised. Theory as well as anecdotal evidence (Winton and Yerramilli (2015)) points that banks were committing to screening with their reputation: even though the market cannot observe the quality of the underlying collateral, over time it observes the realised default rates and can punish the bank in the future for not exerting enough effort in the past, which creates incentives for banks to bail out the securitised portfolio if it does not perform well. This commitment mechanism only aggravates the debt overhang problem. However, when banks commit to screening by retaining a junior tranche, which implies risk sharing between the bank and the market rather than complete risk transfer to the market, the debt is fairly priced and there is no reason for banks to bail out the ABS holders in bad states of the world.

Literature review. The paper relates to several strands of literature. First, it contributes to the literature on agency conflict between shareholders and debt holders which oscillates mainly around the optimal maturity and seniority structure that could mitigate the problem of debt overhang (Myers (1977), Hart and Moore (1995), Diamond and He (2012), Stulz and Johnson (1985), Hackbarth and Mauer (2011), Jensen and Meckling (1976), Leland (1998), Sundaresan and Wang (2006), Halov and Heider (2011)). Stulz and Johnson (1985) argue that if newly issued debt is secured it reduces the debt overhang because issuance of secured debt reduces the cost of borrowing for a firm, as opposed to unsecured debt, and the benefit of cost reduction accrues to the shareholders. In contrast, I show that securitisation decreases debt overhang if it was used to finance past investments. Admati et al. (2018) suggest that firms are unwilling to issue additional equity, and they prefer to increase rather then decrease their leverage because leverage reduction transfers wealth from shareholders to existing creditors. Goncharenko et al. (2018) study the impact of issuance of contingent convertible bonds (CoCos) on the debt overhang of banks. They argue that issuing CoCos might be suboptimal for banks, who anticipate equity issuance in the future, because future equity issuance increases the value of CoCos. Favara et al. (2017) empirically document the presence of debt overhang in non-financial firms. They find that if shareholders of financially distressed firms expect to receive less in bankruptcy, the firms invest less (debt overhang) and make riskier investments (risk-shifting) then financially distressed firms in which shareholders expect to receive a higher payoff in bankruptcy.

Next, the paper relates to the literature on securitisation, as well as on screening and monitoring incentives associated with it (Gorton and Pennacchi (1995), Parlour and Winton (2013), Winton and Yerramilli (2015), Chiesa (2008), Hartman-Glaser et al. (2012), Gorton and Metrick (2013), Chemla and Henessy (2014), Duffie (2008)). DeMarzo and Duffie (1999) characterise an optimal security design when a security issuer has a private information regarding the distribution of cash flows of the underlying asset. They conclude that the optimal tranche to be sold to investors consists of a senior claim against the pool of loans. Parlour and Winton (2013) study loan monitoring incentives when a bank sells a loan, or buys a credit default swap (CDS). They find that riskier loans are monitored excessively, while safer credits are insufficiently monitored. Winton and Yerramilli (2015) argue that in case of originate and distribute model of securitisation, when a loan originator does not retain any fraction of the securitised portfolio, loan screening can be maintained with reputational commitment: though the market does not observe the quality of collateral, over time it observes the realised default rate of underlying loans and infers the effort that the loan originator put in the past. As a result, the market can punish the loan originator for poor loan screening by not buying the ABS in the future. Building on this literature, I assume that the market imposes on a loan originator a retention of a junior tranche of the securitised portfolio to align bank incentives with those of the ABS buyers. The focus of this paper is different: I show that if a bank commits to screening with minimum retention, can reduce the debt overhang.

The empirical evidence of the presence of moral hazard in securitisation is mixed. On the one hand, Keys et al. (2010), Bord and Santos (2015) find that a securitised portfolio of loans has a higher default rate than a portfolio of loans with similar risk characteristics but retained on the balance sheet of a bank. On the other hand, Benmelech et al. (2012) do not find evidence for the moral hazard in securitisation for the case of collateralised loan obligations.

The paper is organised as follows. Section 2 presents the general model setup, Section 3 describes the optimisation problem of the bank that issues off-balance sheet debt. Section 4 shows the optimal investment decision of a bank issuing on-balance sheet debt. Section 5 presents the results. Section 6 concludes.

1.2 Model description

There are three periods, $(0, 1, 2)$. The economy consists of a representative bank and a continuum of borrowers of measure 1. I assume that the demand for loans is inelastic, but the quantity of loans is fixed and normalised to 1.

At $t = 0$ the bank lends one unit, and is promised to be repaid R_0 at $t = 2$. The face value of debt R_0 is reimbursed with probability p_0, and with probability $(1 - p_0)$ the bank recovers nR_0, where $n \in (0, 1)$ is the default recovery rate. At $t = 0$ the bank also knows that with a probability $\frac{1}{2}$ it will have a better investment opportunity at $t = 1$. Because the future project is uncertain, the bank will also invest in the project today.

At $t = 1$, if the investment opportunity arrives, the bank lends a unit, and the loan pays off

R_1 at $t = 2$. The quantity of loans the bank finances across periods is the same. Similarly to $t = 0$ project, the project at $t = 1$ is reimbursed with a probability p_1, and with the probability $(1 - p_1)$ the bank recovers nR_1.

The bank can increase the quality of the loan by putting more effort into loan screening and monitoring. The effort is costly and unobservable. The cost, associated with a probability of success p_t, for $t = 0, 1$, is given by γp_t^2, where γ accounts for the cost of effort.

I assume that at $t = 0$ the bank has exogenously given deposits in the amount k. Deposits are insured by the government. Because of the government insurance deposits are costless for the bank, and it has no incentives to hold equity.[3] To correct for this friction, the government imposes a minimum capital requirement on the bank. I assume that the minimum requirement binds. This assumption is based on the empirical observation that despite the fact that banks hold capital buffers in excess of the minimum required level, when banks face tighter minimum requirements they replenish the buffers but go no further (Bahaj and Malherbe (2016)).[4]

For each unit of loans financed, the bank must put g units of equity. I assume that the minimum equity requirement is calculated on the basis of the total loans that the bank financed. This assumption ensures that the decision of the bank to issue off-balance sheet debt instead of the on-balance sheet debt is not driven by the amount of capital they are required to provide. Equity issuance is costless.

At $t = 0$ the bank does not have enough deposits and equity to finance the project, $k+g < 1$. As such, a bank has to raise more debt: it can raise on-balance sheet unsecured debt, on-balance sheet secured debt, or off-balance sheet debt. The three options are mutually exclusive. The debt that a bank issues matures when the loan is reimbursed.[5]

At $t = 1$ the bank has enough deposits to finance the unit of loans, and there is no need to issue external debt. This assumption, while simplifies the model, does not alter the results: securitisation helps to alleviate the debt overhang if it was used to finance projects in the past. If the bank faces an attractive investment opportunity and the loans in the past are all financed through on-balance sheet debt, financing future projects with securitisation does not provide the benefits analysed here. As such, assuming that banks won't have enough deposits at $t = 1$ to finance the projects would complicate the model with no additional benefits.

A bank maximises the expected shareholder value. Given that shareholders are reimbursed after all other claimholders, the expected equity value is:

$$E(V) = E(\max(R_0 + R_1 - D, 0)) - cost_{screening} - equity, \tag{1.1}$$

where R_0 and R_1 is the payoff from the investments in each state of the world, D is the face value of debt due, and *equity* is the amount of equity shareholders invest.

The reimbursement of the claimholders in bankruptcy of a depository institution is different from a non-depository institution because deposits have a priority in bankruptcy. For example,

[3]Even in the presence of a deposit insurance premium, if it is mis-priced, the bank has no incentives to put equity.

[4]The interpretation to such behavior is that violating minimum requirements is costly for a bank, therefore the excess capital serves as a buffer against potential shocks. In addition, the fact that in pre-crisis period banks where actively doing regulatory arbitrage also supports the assumption of binding capital requirements (Acharya, Schnabl, and Suarez (2013)).

[5]Note that this model excludes a possibility for repo financing: repo financing involves posting of liquid collateral, whose value does not depend on the screening effort of a bank. This is not the case in this model.

in the US, under the term "depositor preference", priority in bankruptcy was granted to all deposits in 1993 (Birchler (2000)). This means that if a bank goes bankrupt, the Federal Deposit Insurance Corporation (FDIC) seizes the assets of a bank, reimburses the deposits first, and all other creditors afterwards. As such, unsecured debt holders are reimbursed after the depositors.[6]

The issuance of the off-balance sheet debt is achieved through securitisation: the bank can sell a fraction of loans to a SPV, which structures it into the debt-like (senior) and equity-like (junior) tranches. The tranches are further sold to external investors, which I will further refer to as the market. The market recognises that there is a moral hazard problem: the effort that the bank puts into screening the loans is unobservable, and the bank has incentives to shirk. Therefore, the market will require the bank to hold some skin in the game in the securitised portfolio as a commitment to proper screening. This skin in the game translates into the minimum retention of an equity-like tranche of a portfolio that the market imposes on the bank.

The model builds on the following key assumptions:

1. The market disciplines the bank by imposing a minimum retention of a securitised portfolio on the loan originator. ABSs are always fairly priced.

2. The credit guarantees for the special purpose vehicles are included in the minimum capital requirements.

3. Banks do not provide implicit recourse.

Credit guarantees are an explicit ex-ante guarantee provided by the loan originator saying that it will ensure that the ABS holders will either receive the face value of the security, or a given fraction of it. After the financial crisis of 2008-2009 banks are required to include the off-balance sheet credit guarantees in the calculation of Tier 1 capital requirements, which, from the incentive point of view, is identical to bringing these guarantees on the balance sheet of a bank, and improving the incentives to properly screen and monitor the loans. The inclusion of the credit and liquidity guarantees into the calculation of the minimum requirements is equivalent to the minimum retention of a junior tranche of a portfolio by the bank, as modelled in this paper.

The implicit recourse means that even in the absence of explicit ex-ante contractual guarantees to support the SPV, ex-post a bank might choose to bail-out the ABS holders out of reputational concerns.[7] The theories that try to reconcile the zero-retention of a securitised portfolio by a loan originator with the maintained effort to screen loan applicants suggest that in case of zero retention the bank commits to proper loan screening with its reputation: even though the market cannot observe the quality of the loans that back the ABS, over time it observes the default rates of the loans and infers the effort that the bank put into screening (Winton and Yerramilli (2015)). As a result, the market can punish the bank by not buying its ABS in the future. In such framework banks might choose to bail out the SPV if the loans that back the security do not perform well in order to preserve their reputation. This does

[6] For the issuance of the secured debt, I assume that the holders of the secured debt can seize the asset that collateralises their claim, but the remaining of their claim is reimbursed after the depositors.

[7] See Gorton and Metrick (2013) for the discussion of the issue.

not alleviate the debt overhang problem, and can even aggravate it. However, in the model I propose the bank commits to loan screening through the retention of a junior tranche. As such, there are no reasons to bail out the SPV, because the ABS is fairly priced and the bank shares the risk with the ABS holders, instead of transferring it completely from its balance sheet.

In the context of the model I keep the quantity of financed loans constant. Therefore, the degree of debt overhang is measured by the minimum level of R_0 that a bank will require to undertake the projects: the lower is the minimum R_0, the lower is the price that the bank will charge on the loan. Debt overhang does not result in the decrease in the quantity of lending, but in a higher rate that the bank will require on the loan to undertake the investment. The risk-taking incentives of the bank are measured by the level of effort that it optimally puts in loan screening, which determines the probabilities p_0 that loans will payoff. I first present the optimisation problem of a securitising bank, that is, a bank that finances a fraction of its loans through off-balance sheet debt. Then I present the model of a non-securitising bank.

1.3 Securitising bank

A bank that finances a fraction of loans through off-balance sheet debt, sells this fraction of the loans to a special purpose vehicle (SPV). The SPV tranches the loans into senior (debt-like) and junior (equity-like) tranches and sells them to the market. As will be shown below, a bank will then retain the junior tranche of a securitised portfolio as a commitment to properly screen loans. The sale of the loans to a special purpose vehicle ensures that if the bank defaults, the holders of the Asset Backed Securities (ABS) will seize the assets that secure their claim and will recover the value of these assets. However, if the asset that secures the ABS defaults, the holders of the ABS cannot be residual claim holders for the rest of the bank's assets.

The bank's effort is unobservable, and as such is unobservable the quality of assets that are sold to the SPV and that back the ABS. To align the incentives of the bank, the market imposes the retention of a junior tranche of a securitised portfolio. The assumption that the alignment of incentives occurs through the retention by a loan originator of a junior tranche of the portfolio is based on the existing literature on optimal security design.[8]

Therefore, the asset side of a securitising bank is composed of a junior tranche of a securitised portfolio of loans, and of a portfolio of loans held on the balance sheet until their maturity. In what follows I use the term *retained loans* (loans hold) when referring to the loans that are held on the balance sheet of a bank until their maturity. By *securitised portfolio of loans* I mean loans that were financed through securitisation: these loans are sold to the SPV, tranched, and the asset backed securities are distributed to the investors. Denote a_0 the fraction of the junior tranche of a securitised portfolio in the total amount of bank assets. The fraction $(1 - a_0)$ of the on-balance sheet funds is invested in the retained loans.

The incentives of the bank to screen and monitor the portfolio of loans which is securitised are different from the incentives to screen the retained portfolio for two reasons. First, the market discipline, which arises when markets impose minimum retention, concerns only the loans financed through securitisation, but not the loans retained on the balance sheet. Second, the differences in incentives come from the priority of the depositors in default.

[8] See Gorton and Metrick (2012) for the survey of the literature, as well as Innes (1990), and Duffie (2008).

Second point merits an additional explanation. If loans financed through securitisation default, but loans retained on the balance sheet payoff, the ABS holders get only the residual value of collateral, and have no claim against the profit of the loans retained. However, in the states of the world when loans retained default, but the securitised loans do not, depositors are reimbursed after the ABS holders, but before the shareholders of the bank.[9] As a result, banks do not have incentives to put more effort into screening securitised loans, than loans retained on the balance sheet, because depositors will reap the profit from the securitised portfolio after the ABS holders but before the shareholders.

Therefore, while the loans that the bank finances are homogenous, the probability that securitised loans payoff might be different from the probability that retained loans pay off. I denote p_h the probability of success of a portfolio of loans retained on the balance sheet (loans hold), and p_s the probability of success of a securitised portfolio of loans. The off-balance sheet debt (senior tranche of the asset backed securities) and on-balance sheet liabilities of a securitising bank are presented in Figure 1.1, where the off-balance sheet debt is shown with a dotted line.

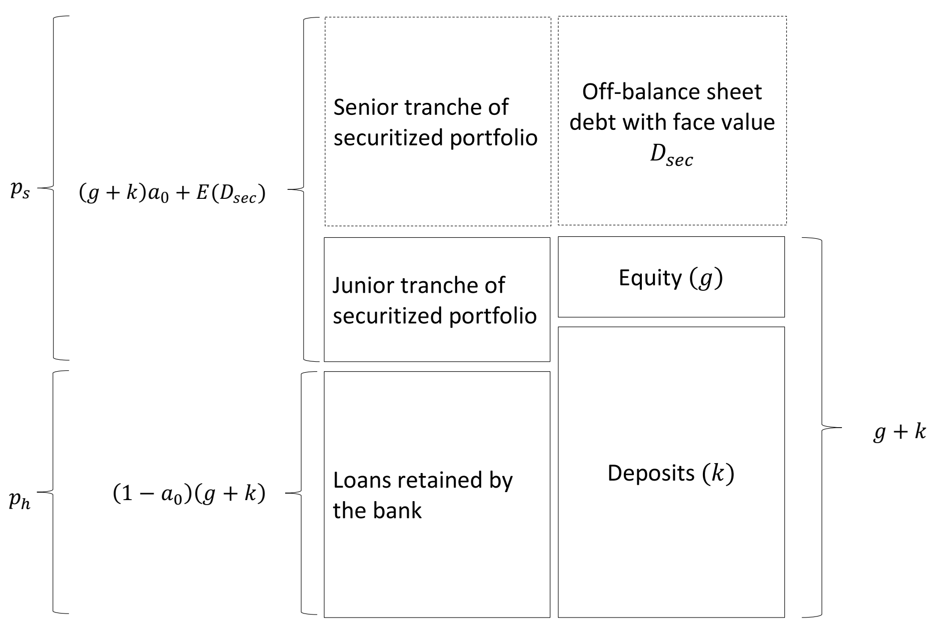

Figure 1.1: A $t = 0$ structure of a balance sheet of a bank that securitises a fraction of its assets, together with the off-balance sheet liabilities, where $E(D_{sec})$ is the expected value of the off-balance sheet debt with the face value D_{sec}.

For the ease of exposition, I first present an optimisation problem of the bank that invests only once at $t = 0$. Then I show how one-period model builds in a two-period framework, when the bank might finance loans at $t = 1$.

[9]Among the financial assets typically held by banks, only qualified financial contracts, such as repos and derivatives, are exempt from automatic stay. Therefore, depositors have no priority claim against them.

1.3.1 One-time lending bank

As explained before, the bank has different incentives to screen securitised portfolio versus non-securitised portfolio. Under the conjecture that $p_s \leq p_h$, the bank puts more effort into screening retained loans than securitised loans. Because the loans in the two portfolios are homogenous, putting more effort into screening is equivalent to reducing the possible states of the world when the loan defaults. The expected shareholder value has the following functional form:

$$\begin{aligned} V^{sec} = max_{p_s,p_h} \Big(& p_s(R_0 - k - D_{sec}) + (p_h - p_s)\max(0, R_0(1-a_0)(g+k) - k) \\ & - (\gamma p_s)^2(a_0(k+g) + (1-k-g)) - (\gamma p_h)^2(1-a_0)(k+g) - g\Big), \end{aligned} \tag{1.2}$$

where p_s reflects the states of the world where securitised loans and loans held til maturity pay off, and $p_h - p_s$ reflects the states of the world where only retained loans pay off. If securitised loans default, ABS holders seize the collateral in the amount $(a_0(g+k) + (1-k-g))$. The quantity of the retained loans is $(1-a_0)(k+g)$. Therefore, if securitised loans default, the bank receives $R_0(1-a_0)(k+g)$ and it has to reimburse only the deposits k. This payoff structure demonstrates the *state-contingent* nature of the off-balance sheet debt: in the states of the world when the asset that collateralises the off-balance sheet debt does not payoff, the bank do not have to reimburse the holders of the off-balance sheet debt.

The functional form of the bank objective function under the conjecture that $p_s > p_h$ is presented in the Appendix .2.[10]

The incentives of the bank to screen and monitor the securitised portfolio of loans are aligned in the following way: the bank chooses the value of the junior tranche of the securitised portfolio that it will retain. It makes an offer to the market (d_{sec}, D_{sec}), where d_{sec} is the present value of the off-balance sheet debt with the promised value D_{sec}. The market observes the offer, and the amount of the junior tranche that the bank retained, $a_0(k+g)$, and it correctly infers the effort that the bank will optimally exercise given. If the offer allows the market to break even in expectation, it accepts it, if not, the offer is rejected and the bank forgoes the investment opportunity. The bank anticipates the market reaction, and makes such an offer (d_{sec}, D_{sec}) that the prospective ABS holders break even: $d_{sec} = E(D_{sec})$. Once the bank raised the debt, it chooses the level of effort that maximises expected shareholder value.

For the prospective buyers of the ABS, their expected payoff is the following:

$$d_{sec} = E(D_{sec}) = p_s D_{sec} + (1-p_s)\min\left(nR_0(d_{sec} + a_0(k+g)), D_{sec}\right). \tag{1.3}$$

The r.h.s. of the equation states that with the probability p_s debt holders recover the full amount of the promised payment, with the probability $(1-p_s)$ the market recovers the residual value of the collateral, nR_0 per loan, and the quantity of loans that back the ABS is $d_{sec} + a_0(k+g)$, where $a_0(k+g)$ is the quantity of loans that the bank financed with its own on-balance sheet funds to be able to commit to properly screen the securitised portfolio. The bank needs to raise $1-k-g$ today, so the present value of debt $d_{sec} = 1-k-g$, and the

[10] Numerically I do not find an interior solution $1 \geq p_s > p_h > 0$.

promised value of debt D_{sec} has the following functional form:

$$D_{sec} = \frac{1}{p_s}(1 - g - k - (1 - (1 - a_0)(g + k))n(1 - p_s)R_0). \tag{1.4}$$

I solve only for the case when the debt is risky.

The cost of the securitised debt increases as the recovery rate in default n decreases:

$$\frac{\partial D_{sec}}{\partial n} = -(1 - (1 - a_0)(g + k))(1 - p_s)R_0 < 0. \tag{1.5}$$

At the same time, the cost of debt decreases in the value of junior tranche retained, a_0:

$$\frac{\partial D_{sec}}{\partial a_0} = -\frac{n(1 - p_s)R_0(g + k)}{p_s} < 0. \tag{1.6}$$

However, if default recovery rate n is low (zero in the extreme case), retaining a higher amount of junior tranche does not reduce the cost of debt: $\frac{\partial D_{sec}}{\partial a_0}|_{n=0} = 0$.

This implies that for higher default recovery rate n the bank can reduce the sensitivity of the cost of debt to the effort intensity by retaining a higher fraction of the junior tranche, a_0. In other words, if the bank increases a_0, and the default recovery rate n is high, the ABS holders in default seize a large pool of loans with high recovery value, which limits their losses. As a result, they care less about the effort intensity the bank chooses. This logic is not true for low default recovery rate: if n is low, retaining more assets does not improve significantly the payoff in default, and therefore by retaining a higher fraction of the junior tranche of the securitised portfolio the bank cannot reduce the sensitivity fo the cost of debt to the effort intensity. The sensitivity of the cost of debt to the effort intensity is critical in determining the relationship between the optimal screening effort of the bank and the amount of a junior tranche the bank retained, a_0. When default recovery rate is low, the bank effort increases in a_0. If the default recovery rate is high, the relationship between the optimal effort and amount of junior tranche is hump shaped. The sketch of the proof is the following. The optimal effort to screen loans held till maturity is determined by the first order condition:

$$\frac{\partial V^{sec}}{\partial p_h} = 2(1 - a_0)(g + k)y^2 p_h + \max(0, R_0(1 - a_0)(g + k) - k). \tag{1.7}$$

The f.o.c. implies that an interior solution for p_h exists only when $R_0(1 - a_0) - k > 0$. In the opposite case, $p_h = p_s$. Under the conjecture $R_0(1 - a_0) - k > 0$ the optimal level of effort to screen loans held till maturity decreases in a_0:

$$p_h^{opt} = \frac{(1 - a_0)(g + k)R_0 - k}{2(1 - a_0)y^2(g + k)} \Rightarrow \frac{\partial p_h^{opt}}{\partial a_0} = -\frac{k}{2(1 - a_0)^2 y^2(g + k)} < 0. \tag{1.8}$$

The result implies that the more the bank retains of a junior tranche, the less loans it holds till maturity. As the amount of loans held till maturity decreases, so do the incentives to screen these loans. As for the optimal effort to screen securitised loans, p_s, I find numerically that the it increases in a_0.[11]

$$\frac{\partial p_s^{opt}}{\partial a_0} > 0 \tag{1.9}$$

If one level of effort increases in a_0 and the other one decreases, there exists such $\bar{a}_0$ that for $a_0 > \bar{a}_0$ effort to screen securitised loans is greater than the effort to screen loans held till maturity: $p_s > p_h$. However, screening securitised loans above loans hold till maturity benefits the depositors and not the shareholders. Therefore, for $a_0 > \bar{a}_0$ the constraint binds: $p_s = p_h$.[12] The figure 1.2 shows the change in the optimal screening effort as the value of junior tranche retained by the bank increases.

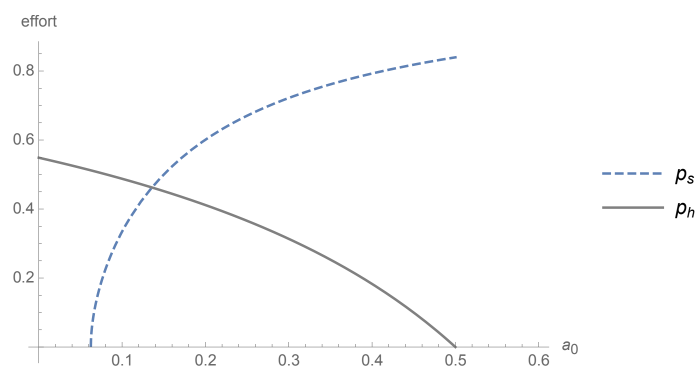

Figure 1.2: Optimal effort to screen the securitised loans, p_s, and loans retained till maturity, p_h, as a function of the size of junior tranche retained by the bank a_0, $n = 0.4, g = 0.1, R_0 = 1.6$.

Therefore, for $a_0 > \bar{a}_0$, the bank will either increase the effort intensity for the both types of portfolios to the effort p_s, which means that $p_{opt} = p_s$, or it will decrease the optimal effort on the both types of portfolios to the level p_h, $p_{opt} = p_h$. The lemma below summarises the optimal effort taking when the constraint $p_s \leq p_h$ is binding.

Proposition 1. *There exists such level of parameters $(\bar{k}, \bar{g}, \bar{n})$ that when the constraint $p_s < p_h$ binds:*

- *For $g + k < \bar{g} + \bar{k}$ the bank puts more effort on both types of portfolios, $p_{opt} = p_s$, and the effort intensity increases in the size of junior tranche a_0 retained by the bank;*
- *When $g + k > \bar{g} + \bar{k}$*

[11] The optimal screening effort of securitised portfolio is the highest root of the following polynomial of second degree:

$$\begin{aligned} F(p_s) = & nR_0(1-(1-a_0)(g+k)) - (1-k-g) + (a_0 - (1-g-k)n - a_0(g+k)n)p_sR_0 - \\ & - 2(1-(1-a_0)(k+g))p_s^2y^2. \end{aligned}$$

[12] The level $\bar{a}_0$ solves $p_s = p_h$ where

$$\begin{cases} p_h = \frac{(1-a_0)(g+k)R_0 - k}{2(1-a_0)y^2(g+k)} \\ F(p_s) = 0. \end{cases}$$

- *For $n < \bar{n}$ the bank puts more effort on both types of portfolios, $p_{opt} = p_s$, and the effort intensity increases in the size of the junior tranche retained by the bank;*
- *For $n > \bar{n}$ there exists such $\bar{a}_0$ that for $a_0 < \bar{a}_0$ the optimal effort is $p_{opt} = p_s$, and for $a_0 > \bar{a}_0$ the optimal effort is $p_{opt} = p_h$.*

The proposition 1 implies that if the recovery rate on the loan is high, the relationship between the optimal effort and the size of the tranche is hump-shaped. The economic intuition relates to the sensitivity of the cost of debt to the size of the junior tranche retained by the bank, presented in equation 1.6. When recovery rate n is high, and the bank has enough of equity and deposits $g + k$, then higher retention reduces the cost of debt for the bank. Once the retention becomes sufficiently high, the market discipline weakens, and the bank reduces its screening to the level of loans held till maturity.

1.3.2 Securitising bank, two periods

When the bank faces an investment opportunity at $t = 1$, it already has loans in place, some of which are financed through securitisation, and some are financed with the on-balance sheet funds (deposits and equity). At $t = 1$ the bank chooses the optimal level of effort as a function of R_1, and of the effort taken at $t = 0$.

The realisations of the payoffs R_1 and R_0 are i.i.d. The expected equity value of a securitising bank at $t = 1$ under the conjecture that $p_{s,0} \leq p_{h,0}$ is:

$$
\begin{aligned}
V_{t=1}^{Sec} = \max_{p_1} \Big(p_s \Big(& p_1 \left(R_0 + R_1 - (1-g) - k - D_{0,s}\right) \\
& + (1-p_1)\max\left(0, nR_1 + R_0 - (1-g) - k - D_{0,s}\right) \Big) \\
& + (p_h - p_s)\max\Big(0, p_1 \left(R_0(k+g)(1-a_0) + R_1 - (1-g) - k\right) \\
& + (1-p_1)\max\left(0, nR_1 + R_0 k(1-a_0) - (1-g) - k\right) \Big) \\
& + (1-p_h)p_1\max\Big(0, R_0 n(1-a_0)(k+g) + R_1 - (1-g) - k\Big) \\
& - (\gamma p_s)^2 \left((k+g)a_0 + d_{0,sec}\right) - (\gamma p_h)^2 (k+g)(1-a_0) - (\gamma p_1)^2 - 2g \Big),
\end{aligned} \tag{1.10}
$$

where $d_{0,s}$ is the present value of a securitised debt with the face value D_s, defined in the equation 1.3. p_s and p_h are the probabilities the securitised loans and loans held until maturity payoff. k is the amount of deposits the bank had at $t = 0$, $(1 - g)$ is the quantity of deposits the bank receives at $t = 1$. Note the state-contingent nature of the off-balance sheet debt: in the states of the world $(1 - p_s)$, when the securitised portfolio does not pay off, the bank does not reimburse the ABS holders. In these states of the world, a bank loses the fraction $a_0(k+g)$ of the defaulted assets, which is seized by the ABS holders, but it does not share with them the profit from a new investment R_1.

The optimal level of effort is determined by the first order condition: $\frac{\partial V_{t=1}^{Sec}}{\partial p_1} = 0$.

Securitising bank at $t = 0$

The optimisation problem of a securitising bank at $t = 0$ has two components: the shareholder value if there is no investment at $t = 1$, and the shareholder value if the bank will invest at $t = 1$:

$$V_0^{sec} = \frac{1}{2}V^{sec} + \frac{1}{2}V_{t=1}^{Sec}$$

where $Vsec$ is the expected equity value if the bank does not invest at $t = 1$, defined in 1.2. $V_{t=1}^{Sec}$ is the expected value if the investment opportunity at $t = 1$ materialises, equation 1.10. The optimal levels effort to screen securitised loans and loans held till maturity are defined by the first order condition:

$$\begin{cases} \frac{\partial V_0^{sec}}{\partial p_s} = 0 \\ \frac{\partial V_0^{sec}}{\partial p_h} = 0 \end{cases}$$

Note that the optimal effort p_s is chosen for a given level of D_s, and in the derivative of V_0^{sec} w.r.t. p_s D_s is taken as a constant. The functional form of D_s (equation 9) is plugged in the first order condition after. The optimal level of securitisation a_0 takes into account the impact on all the variables that it will have. I solve the model numerically: for given values (R_0, n, k, g) I find the optimal p_s and p_h for all a_0 defined on a grid, and then select a_0 that gives the highest value. I present the results in section 5 where I compare the optimal investment policy of a securitising versus a non-securitising bank.

1.4 Investment decision of a non-securitising bank

I solve the model backwards: first, I find the optimal investment decision at $t = 1$ taking the optimal decisions of time $t = 0$ as given. Then I solve for the optimal investment decisions at $t = 0$, accounting for the fact that with a probability of 0.5 the bank will invest at $t = 1$ and the optimal effort choice at $t = 0$ will affect the optimal effort choice at $t = 1$.

I first present the optimal decision making if the bank issues unsecured debt.

1.4.1 Non-securitising bank at $t = 1$

At $t = 1$ the bank already holds loans from the previous period with the face value R_0, that will be reimbursed with a probability p_0 determined by the effort the bank put at $t = 0$. On the liability side, it holds deposits k, unsecured debt with face value D_{unsec}, plus the deposits in the amount $(1 - g)$ that it receives at $t = 1$.

The manager of the bank maximises the expected shareholder value w.r.t. p_1. The value function of a such bank is composed of the expected return on the loans, minus the debt claims, and the cost of loan screening. I assume that if both loans default, shareholders get nothing. The realisations of the payoff R_0 and R_1 are i.i.d, and the expected shareholder value at $t = 1$ is:

$$
\begin{aligned}
V_{t=1}^{NoSec} =& \max_{p_1}\Big(p_1 p_0\left(R_0 + R_1 - k - D_{unsec} - (1-g)\right) \\
&+ p_1(1-p_0)\max\left(0, nR_0 + R_1 - k - D_{unsec} - (1-g)\right) \\
&+ p_0(1-p_1)\max\left(0, nR_1 + R_0 - k - D_{unsec} - (1-g)\right) \\
&- (\gamma p_0)^2 - (\gamma p_1)^2 - 2g\Big),
\end{aligned} \tag{1.11}
$$

where the index $NoSec$ stands for "non-securitising bank". The limited liability ensures that in case of default of one loan or of both, the payoff of shareholders is non-negative, hence the term $max\left(0, nR_0 + R_1 - k - D_{unsec} - (1-g)\right)$. $2g$ is the total amount of equity that shareholders invest over two periods, $1+k$ is the total amount of deposits, γp_0^2 is the cost of screening per unit of loans. $V_{t=1}^{NoSec}$ must be non-negative, so that shareholders break through.

Non-securitising bank at $t = 0$

At $t = 0$ the bank knows what p_1 will be as a function of the probability p_0 and the promised return R_1. This investment will take place only with a probability $\frac{1}{2}$. At $t = 0$ the optimisation problem of a non-securitising bank is:

$$
V_{t=0}^{NoSec} = \max_{p_0}\left(\frac{1}{2}\Big(p_0(R_0 - k - D_{unsec}) - (\gamma p_0)^2 - g\Big) + \frac{1}{2}V_{t=1}^{NoSec}(p_0)\right), \tag{1.12}
$$

where the expression $\frac{1}{2}\Big(p_0(R_0 - k - D_{unsec}) - (\gamma p_0)^2 - g\Big)$ is the expected payoff to shareholders if a bank invests only at $t = 0$, and $V_{t=1}^{NoSec}$ is the payoff if the bank invests at $t = 1$, equation 1.11.

In the presence of moral hazard the unsecured debt with present value $1-k-g$ is priced as follows. First the bank makes a take-it-or-leave-it offer to the market to buy a debt with the face value $\bar{D}_{unsec}$, and present value d_{unsec}, where $d_{unsec} = 1-k-g$. The market observes the offer, the level of deposits k, and the promised payoff on the investment R_0, and it correctly infers the effort that the bank will choose to put into loan screening. If the offer $(d_{unsec}, \bar{D}_{unsec})$ allows the prospective debt buyers to break even in expectation, they will buy the debt. If not, the offer is rejected and the bank is left without additional funding. The bank anticipates this, and will such an odder (d_{unsec}, D_{unsec}) such that the debt is fairly priced. After the debt $\bar{D}_{unsec}$ is raised, the bank chooses the optimal level of effort. As a result, the bank does not have incentives to shirk, and the unsecured debt is fairly priced.

From the technical perspective the time sequence presented above means that the bank maximises the shareholder value w.r.t. p_0 for a given value of D_{unsec}. p_0 and D_{unsec} are jointly determined by the following system of equations:

$$
\begin{cases}
\frac{\partial V_{t=0}(R_1, R_0, D_{unsec})}{\partial p_0} = 0 \\
1 - k - g = E(D_{unsec})
\end{cases}
$$

$E(D_{unsec})$ is determined by the prospective buyers of the unsecured debt who take into account the probability with which they will be reimbursed in full, and the amount they receive if the bank goes bankrupt. Unsecured debtholders are reimbursed after the depositors in bankruptcy. Therefore, the face value of debt D_{unsec} with the present value equal to $1-k-g$ must satisfy the following equality:

$$\begin{aligned}1-k-g = E(D_{unsec}) =& \frac{1}{2}\left(p_0 D_{unsec} + (1-p_0)\min(nR_0 - k, D_{unsec})\right) \\ &+ \frac{1}{2}\Big(p_0 p_1 D_{unsec} + (1-p_0)p_1 \min(D_{unsec}, R_1 + nR_0 - (1-g) - k) \\ &+ (1-p_1)p_0 \min(D_{unsec}, R_0 n R_1 - (1-g) - k, D_{unsec}) \\ &+ (1-p_1)(1-p_0)\min(n(R_1+R_0) - (1-g) - k, D_{unsec})\Big).\end{aligned} \tag{1.13}$$

The first line in 1.13 accounts for the expected payoff if no investment is done at $t = 1$, and the rest is the expected payoff if a bank finances the project at $t = 1$, and all funding comes from deposits. I solve the model numerically. The results are presented below, when I compare the optimal decisions of a securitising bank versus a non-securitising bank.

The optimisation problem of a bank that issues secured debt differs from the one presented above only in the reimbursement of the secured claim holders in default, which affects the cost of debt for a bank. I present the details in the Appendix .9.

1.5 Securitising versus non-securitising bank: results

The numerical solution of the model yields the following results.

First, the bank will issue off-balance sheet securitised debt if the amount of deposits is sufficiently high compared to the amount of loans the bank needs to finance. If the amount of available deposits is low, securitised debt becomes too costly for the bank for the following reason. The yield on the off-balance sheet securitised debt ($\frac{D_{sec}}{d_{sec}}$) decreases in the amount of junior tranche retained by the bank $(1-k-g)a_0$:

$$\frac{\partial \frac{D_{sec}}{d_{sec}}}{\partial (1-k-g)a_0} = -\frac{a_0^2 n(1-p_s)R_0}{p_s(a_0(1-g-k))^2} < 0.$$

When deposits k decrease, the yield increases. To maintain the same yield, the bank has to increase the fraction a_0 of funds invested in the junior tranche. When k is low, the bank is unable to retain more than $g+k$ of the junior tranche, and ultimately the yield rises. Figure 1.3 shows the fraction of the on-balance sheet funds (deposits and equity) that the bank must invest in the junior tranche of a securitised portfolio as a function of the deposits k.

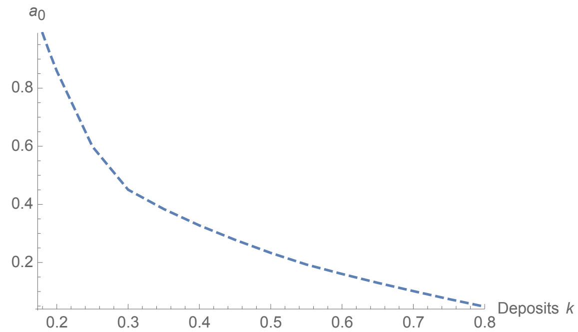

Figure 1.3: The fraction of the on-balance sheet funds that the bank invests in the junior tranche of a securitised portfolio as a function of deposits k, $n = 0.3, g = 0.1, R_0 = 1.7$.

Overall, the higher is k, the bigger is the difference in the equity value between a securitising and a non-securitising bank. Figure 1.4 shows the expected shareholder value for the bank issuing off-balance sheet debt and for the bank issuing on-balance sheet debt, when deposits are high ($k = 0.8$) versus when deposits are low ($k = 0.4$) .

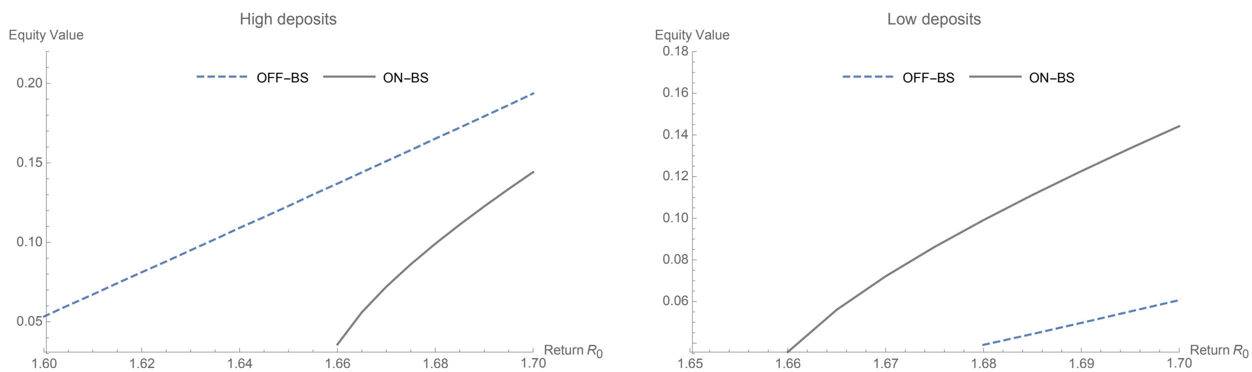

Figure 1.4: Equity value of a bank issuing off-balance sheet ("OFF-BS") debt versus equity value of a bank issuing unsecured on-balance sheet ("ON-BS") debt as a function of the payoff R_0 and of the level of deposits k, $g = 0.1, n = 0.3$

The fact that for the same level of R_0 the securitising bank has a higher expected shareholder value means that it can offer lower rates to its borrowers. The result is consistent with the empirical evidence that securitisation reduces the cost of funding for bank borrowers: Nadauld and Weisbach (2012) document that "spreads on Term Loan B facilities originated by securitisation-active banks are 11 basis points lower than spreads on facilities issued by other banks, holding other factors constant." Similarly, Guner (2006) finds that "the average yield spread on loans originated by active loan sellers is about 20 basis points lower than the average spread on loans originated by moderate loan sellers". In contrast to the existing literature the present model offers a novel theoretical explanation of why we observe a lower cost of loans: securitisation decreases the debt overhang associated with the agency conflict between depositors and shareholders.

Note that in figure 1.4 the bank equity value does not reach zero: in this framework the bank needs to realise a positive profit on the loans in order to exercise effort. This is consistent with the literature on bank competition and screening incentives (Allen and Gale (2004), Vives (2010)). While the securitising bank is able to offer lower rates, it still needs to extract profit from the investment in order to have incentives to put effort into screening.

Third, I find that the securitising bank has no incentives to put more effort into screening the securitised portfolio of loans compared to the loans held till maturity: $p_s \leq p_h$. The effect is due to the fact that the holders of the ABS have no claim against the profit from the loans retained on the balance sheet, but the reverse is not true: if retained portfolio of loans does not perform well, depositors benefit from the profit of the securitised portfolio before the shareholders. Thus, the model suggests that even when the market efficiently incentivises the bank to put effort into loan screening, the agency conflict between shareholders and depositors shifts the incentives of banks to screen a securitised portfolio as properly as a retained portfolio of loans. Whether the bank chooses to put as much effort into screening the securitised portfolio as in the retained portfolio depends on the loan characteristics, such as default recovery rate and the cost of screening.

This result offers a novel rationale to why the existing empirical evidence on the presence of moral hazard in securitisation is mixed. On the one hand, Keys et al. (2010), and Bord and Santos (2015) find that a securitised portfolio of loans has a higher default rate compared to a portfolio of loans with similar risk characteristics but retained on the balance sheet of a bank. On the other hand, Benmelech et al (2012) do not find convincing evidence of the moral hazard in corporate loan securitisation. While Benmelech et al (2012) argue that "the securitisation of corporate loans is fundamentally different from securitisation of other asset classes because securitised loans are fractions of syndicated loans", I suggest that differences in findings might also arise because bank incentives to screen depend on the individual loan characteristics, such as expected loan recovery value in default, and the cost of screening of each type of loan.

Lastly, I find that the effort intensity does not necessarily increase with the increase in the amount of junior tranche retained by the originating bank. Higher retention leads to higher effort if the recovery rate on loans is low. If the recovery rate on loans is high, the relationship between the minimum retention and effort intensity is hump-shaped, as illustrated in the graph b of the figure 1.5.

High recovery rate alters the optimal effort in the following way. First, it reduces the incentives of the bank to differentiate the screening effort for securitised loans versus loans held till maturity. When a securitised portfolio defaults, the bank has only $(1 - a_0)(k + g)$ of loans from the unit 1 that it granted at $t = 0$, therefore as a_0 increases, the benefit of effort differentiation decreases. Second, from the equation 1.14, that determines the cost of the off-balance sheet debt, it follows that higher a_0 gives a higher expected recovery value in default to the ABS holders, reducing the cost of debt.

$$d_0 = D_0 p_s + (1 - p_s) R_0 n(d_0 + a_0(k + g)). \tag{1.14}$$

If the default recovery rate n is high, higher minimum retention makes the off-balance sheet debt less sensitive to the screening effort, because in default ABS holders receive a large amount of assets with high recovery value. As a result, the market discipline loosens, and the bank reduces the effort to screen the both types of the loan portfolios. Thus, a high default recovery rate leads to the *over-collateralisation* of the off-balance sheet debt at the expense of the depositors. To illustrate the point better, consider the following hypothetical example.

When the ABS holders buy \$100 debt that promises to pay \$200 upon maturity, the bank finances a unit of loans of \$100 value. However, to be able to raise this debt, the bank has to retain a junior claim on this portfolio, which means that \$100 ABS claim is backed by 1.5 units of loans, and the bank put \$50 of its on-balance sheet funds to finance the additional 0.5 unit of loans. If loans promise \$180, and in default the borrower recovers 0.4 of the promised payoff, in case of default the ABS holders seize 1.5 units of loans and recover the $1.5 \times 0.4 \times 180 = \$114$. Therefore, if the default recovery rate is high, a higher retention decreases the sensitivity of the cost of debt to the effort of the bank: the debt becomes less risky not because the bank exercises more effort, but because it is backed by a larger portfolio of loans with high recovery value. As a result, the bank screens less.

When the recovery rate is low, larger pool of assets that backs the ABS does not increase significantly the payoff in default of the ABS holders. In this case, higher retention reflects the *skin in the game* of the bank, rather then an attempt to reduce the cost of debt through over-collateralisation at the expense of the depositors.

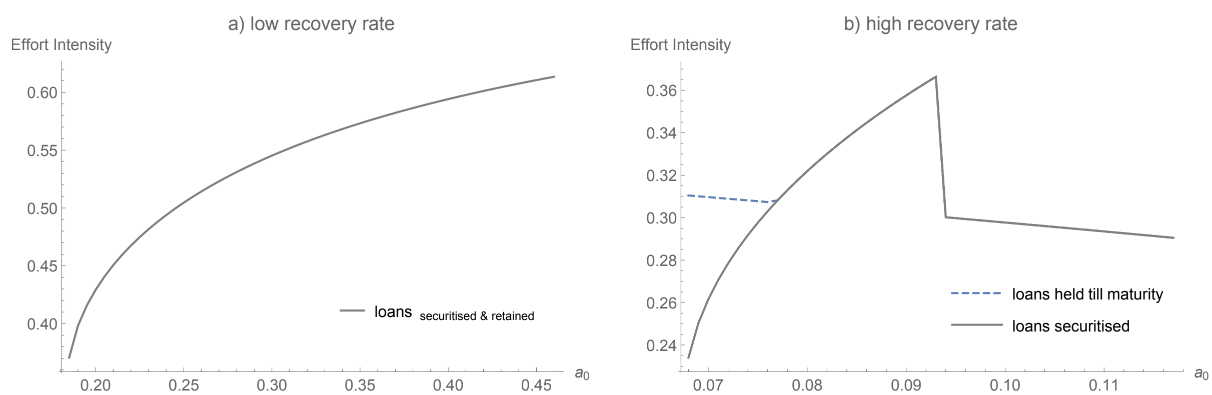

Figure 1.5: Effort intensity to screen a securitised portfolio versus a portfolio hold till maturity as a function of the size of the junior tranche of the securitised portfolio a_0 retained by the bank, $g = 0.1, k = 0.7$.

The graph b in the figure 1.5 shows that when retention a_0 is very low, the bank puts strictly greater effort into screening the retained portfolio than the securitised one. For low levels of a_0 the benefit of effort differentiating is positive. As the degree of retention a_0 increases, the benefits of effort differentiation decrease. At the same time, there is a region when higher retention increases screening incentives of the both types of loan portfolios. However, once a_0 passes a certain threshold, the effect of over-collateralisation kicks in, and the screening effort of the two portfolios decreases.

1.6 Conclusion

In this paper I present a model that shows that securitisation can reduce debt overhang of banks. Debt overhang arises because of the agency conflict between shareholders and debtholders: given that shareholders are residual claim holders in bankruptcy, they might either forgo some positive NPV projects, because otherwise the benefits will accrue to the debtholders only. Securitisation creates a segregation of liabilities: the buyers of the ABS hold the claim only against the collateral that backs the security, but not against any other asset of the bank. If a bank used securitisation in the past, a larger fraction of the profit from new investments accrues to the shareholders and the banks undertakes the projects that would have been forgone otherwise. As a result, a securitising banks offer lower interest rates to their borrowers.

Commitment to loan screening with the retention of a junior tranche of a securitised portfolio is critical for the mechanism described in this paper to work. If the bank does not retain a junior tranche and commits with its reputation, as described by Winton and Yerramilli (2015), it has incentives for banks to bail out the SPV, and the debt overhang becomes even more severe. The model predicts that if a bank has a high deposit base, it is more likely to finance additional loans with securitisation. The model also predicts that the relationship between the effort intensity to screen loans and the size of the junior tranche retained by the bank can be hump-shaped. This prediction holds for loans with high recovery value. For loans with low recovery value larger retention by the bank leads to higher effort intensity.

Chapter 2

Bank Capital Structure with a Fire Sale Externality

2.1 Introduction

Short-term funding is an important source of financing for financial institutions, and it played an important role in the amplification of the financial crisis of 2007-2009. While in normal times banks can roll-over their short-term debt, if the market is hit by a shock, a contraction of short-term funding can follow. Thus, the short-term funding of asset backed securities contracted by 1.4 trillion (Krishnamurthy et al. (2014)), which is sizable given that the aggregate value of the assets of US bank holding companies was 16 trillion in 2009 (Hanson et al. (2011)).[1]

Following a short-term funding dry-up in 2007-2008, banks, unable to rollover their short-term liabilities, had to partially liquidate their assets. When many institutions are forced to de-lever at the same time, and other market participants are financially constrained and unable to support the asset prices, a fire sale spillover can occur. According to some estimations the losses of the financial system caused by a fire sale spillover amounted up to 25 cents on each dollar of exogenous decline in asset prices in the peak of the crisis of 2007-2008.[2] The fire sale friction, highlighted by this crisis, sparked a vivid discussion on how to measure and limit the systemic risks of financial institutions. For example, Duffie and Skeel (2012) questioned whether repo transactions and derivative contracts should be exempt from the automatic stay in bankruptcy. They suggest that repo transactions with liquid collateral should be exempt from the automatic stay, while repos backed by a less liquid collateral should be subject to the automatic stay. In this paper I show that when banks hold less liquid pleadgeable asset, a possibility of a future fire sale liquidation disciplines the banks and they borrow less of short-term secured debt, because less liquid collateral carries a higher cost of a fire sale liquidation for a bank. In case of liquid collateral banks borrow too much of short-term secured debt, thus increasing a possibility of a fire sale liquidation.

To illustrate the mechanism, I develop a model of an industry equilibrium, in which banks need to raise external financing in order to invest in a financial asset with uncertain payoff.

[1]The repo markets also experienced a run: tri-party repo with ABS collateral collapsed from 196bn to 14bn (Krishnamurthy et al. (2014)), with the repo haircut index rising from 0% to 45% over the course of 2007-2008 (Gorton and Metrick (2012)).

[2]See Duarte and Eisenbach (2015).

First a bank issues equity and unsecured debt to acquire the first unit of the asset. Then it can use the acquired asset as collateral and raise short-term (ST) secured debt.[3] This allows a bank to buy more units of the asset. A margin haircut that applies to the ST secured debt limits the total amount of ST funding that a bank can raise via collateralization. The optimal level of each form of debt is a trade-off between its costs and benefits. The unsecured long-term debt reduces the need of costly external equity, but increases the probability of default of a bank. Issuing short-term secured debt allows a bank to buy more assets, but might lead to the partial fire sale of assets because banks might be unable to rollover their debt.

After a bank issues the ST and LT debt, the value of the bank's asset is realized. If the asset value is too low and a bank cannot reimburse its debt holders, the bank is in default and sells its assets in the secondary market. The natural buyers of the asset are the other banks whose realized asset value is high. If the price of a collateralizable asset increases, banks can roll-over the entirety of their ST debt. However, if the market price of the asset declines, banks find themselves unable to roll-over their ST debt. This happens because banks are subject to a time-varying borrowing constraint, which arises endogenously because the asset side of bank's balance sheet is marked to market. When the market price of the asset decreases, the asset side of the balance sheet shrinks, and a capacity of a bank to borrow against collateral decreases. Therefore, if the market price of the asset goes down, a bank might be forced to de-lever.

The natural buyers of the asset are the other banks in the industry whose realized asset value is high. The market price, determined in the market clearing, produces a spillover on banks' collateral value and ultimately on their debt capacity: if a large number of banks defaults and a price drop is large, other banks can exceed their borrowing capacity and will be forced to partially liquidate their asset. Banks rationally take capital structure decisions in anticipation of a potential fire sale.

The model draws a link between the bank capital structure and the asset liquidity and bears three main implications.

First, the level of secondary market liquidity of an asset ? defined throughout the paper as the degree to which an asset preserves its value when it changes its ownership? affects to a large extent the financing incentives of a bank. High asset liquidity increases the set of potential buyers, even if all of them are financially constrained. This implies that even in periods of crisis and fire sale liquidations a higher market price is sustained due to the presence of a larger set of potential buyers. This reduces the cost of a fire sale liquidation for a bank and provides an incentive to take the maximum short-term secured debt. For any negative market price change banks are unable to roll-over the entire debt and have to liquidate their assets at a fire-sale. From a regulatory perspective, requiring a higher margin haircut for assets with high liquidity can help make short-term debt less risky.

Second, the expected market price of the collateral asset affects the level and composition of bank debt. Higher market price of the asset lowers the cost of a fire-sale spillover for a bank, hence the incentives to take more short-term secured debt are higher. This suggests that the policy of a lender of last resort (LOLR) to support market price of financial assets provides incentives for banks to take more short-term debt ex-ante, thus increasing the risk of a fire-sale spillover and the need for a LOLR intervention ex-post.

[3]This newly acquired asset can be again posted as collateral to raise new funding, but the total amount of collateralized debt is limited by the margin haircut.

The effect of the expected market price on the optimal level of LT unsecured debt is twofold. On the one hand, higher market price of the asset increases the total value of a bank in default, so, ceteris paribus, the cost of LT unsecured debt should be lower. However, as price goes up, banks issue more ST secured debt. This means that while the total payoff to claim holders in default increases with the increase in asset price, it is the ST secured claim holders who ripe the benefits. In anticipation of this, unsecured debt holders require a higher yield on debt and banks find it optimal to reduce the level of LT unsecured debt as it becomes too expensive.

Third, when banks anticipate high added value from acquisitions of the assets of defaulted banks, they preserve some ?spare borrowing capacity?: they choose to issue less LT unsecured debt today, to be able to finance the acquisition of the assets with a debt issue in the future. With high asset liquidity the potential buyers expect to extract more value from the acquisition, if they themselves remain solvent. This provides incentives for the banks to be more prudent and to reduce the probability of their own default by substituting the LT unsecured debt with equity. The empirical prediction of this result echoes the one of Morellec (2001) and Weiss and Wruck (1998): higher asset liquidity increases the debt capacity of a firm only when the debt is secured by highly liquid asset.

Finally, I analyze the social planner's optimal choice of debt. In the model the social planner (SP) maximizes the total lending by banks and engodenizes the impact that banks leverage choice has on the equilibrium asset price. I find that SP takes less debt than what banks in a competitive equilibrium (CE) would take.This is a numerical result. This result suggests that imposing minimum capital requirements on the non-depository financial institutions is desirable from the social planner point of view.

Literature review. The present paper relates to a strand of literature on bank leverage, asset liquidity, and financial fire sales. Shleifer and Vishny (1992) pioneered the idea of the fire sale liquidation. They develop a model in which they show why an asset can be sold below its economic value. An asset can be liquidated below its economic value if the natural buyers of this asset are themselves financially constrained. The present paper is related to the model by Bernardo et al. (2015), who study the idea of an asset liquidity and debt capacity in a framework of rational expectations equilibrium: firms choose their debt as a function of price, which is determined in the market equilibrium depending on the supply of and demand for the assets. They show that the degree of asset liquidity can both increase and decrease the optimal level of debt firms choose depending on the degree of the debt overhang problem within a firm. They also analyze the impact that the strength of debt holder's rights has on the optimal leverage choice. The present paper differs from the paper of Bernardo et al. (2015) along the following dimensions. I study the optimal choice of leverage of financial firms, compared to a non-bank firm in the model of Bernardo et al. (2015). The financial firms face a fire sale friction that comes with their choice of leverage structure. Second, I analyze the optimal choice of short-term secured and long-term unsecured debt, contrary to a one type of debt in Bernardo et al. (2015). Acharya and Viswanathan (2011) explain liquidity dry-ups in a model with a fire sale friction and inability of banks to roll-over an exogenous level of short-term debt due to the risk-shifting problem. Contrary to their paper, I analyze how the capital structure of banks is ex-ante affected by a possibility of future fire sales, with bank's leverage decision and fire-sale friction arising endogenously.

The present model contributes to the analysis as to how asset liquidity affects the borrowing capacity of a firm. Williamson (1988) argues that higher asset liquidity increases the level of debt that banks will be able to take through the impact that the asset liquidity will have on the recovery value of assets in default. On the other hand, Morellec (2001) and Weiss and Wruck (1998) suggest that higher asset liquidity can increase the debt capacity of a firm only when the debt is secured by highly liquid asset. In this paper I show that once fire sale cost is endogenously determined, high asset liquidity makes the cost of taking secured debt lower, but higher secured debt increases the cost for the unsecured debt. Thus the latter goes down.

Li and Ma (2016) endogenize asset fire sales, bank run and the effect of contagion in a global game framework, but take the liability side of the balance sheet as endogenously given, while I focus on the choice of liability structure of banks. Diamond and Rajan (2011) argue that banks' anticipation of fire sales leads to even deeper fire sales and to a credit freeze. Acharya et al. (2011) show how the amount of liquidity that banks hold is affected by the prospects of potential fire sales: with high prices the pledgeability of risky assets is high, and there is little incentive to hold cash. Hence, a sudden adverse shock leads to a situation when financial institutions overall have little liquid assets to support the falling prices.

On the empirical side, Gan (2007) and Leary and Roberts (2014) document the impact of collateral on debt capacity and the impact of an industry level of leverage on the leverage choice of individual firms. Gropp and Heider (2010) document that bank's leverage is determined by similar factors than the leverage of non-financial firms.

This paper is structured as follows. Section 2 describes the model, Section 3 provides equilibrium results for the benchmark case without the fire sale externality. Section 4 provides equilibrium results of the economy with a fire sale externality, and Section 5 contains a comparative analysis of the impact a fire sale has on the optimal leverage decisions of banks. Section 6 concludes.

2.2 Model description

2.2.1 Informal model description

This is the model of rational expectations equilibrium. At date 0, there is a continuum of banks, and each bank raises external funding to acquire an asset. To acquire the first asset, banks can raise either external equity or long-term unsecured debt. External equity is costly, hence it can be optimal for a bank to raise some unsecured long-term debt. Once banks have an asset, they can use it as collateral to raise short-term secured debt. The maximum amount that banks can raise in the form of short-term debt is limited by a margin haircut.

After banks choose the optimal capital structure, the asset value is realized. The asset value is bank-specific, and if that asset is transferred in the hands of bond-holders, they do not have the expertise needed to extract the payoff from the asset. Hence, if the asset value is not enough to pay off the debt holders, the bank is liquidated and its asset is sold in the centralized market.

The short-term nature of the secured debt and the marked-to-market balance sheet of banks gives rise to a time-varying borrowing constraint. The level of secured debt cannot exceed the borrowing capacity of a bank. However, because the balance sheet is marked to market, the borrowing capacity changes with the changes in the market price of the asset. Hence, if the market price goes down, the borrowing capacity of a bank decreases. This has implications for the short-term debt: if a bank took too much debt in the previous period and the borrowing capacity decreased, it cannot rollover the entire amount of debt, and is forced to de-lever. The partial sale of the asset, entailed by such deleveraging, creates additional supply of the asset in the market and reduces its market price further down. In turn, this decreases the borrowing capacity of banks, pushing them to sell more of their asset. Thus, a fire sale spillover occurs.

A fraction of solvent banks with a high realized asset value are willing to acquire the asset in the secondary market. The rational expectations setting implies that the price that clears the market equals the price conjectured by the banks.

When banks choose the optimal capital structure mix, they also take into account the following elements:

- The market price of the asset. It determines not only the cost of a fire sale spillover, but also the bank value in liquidation and consequently the cost of the unsecured debt.

- The expected value from the acquisition of the asset in the secondary market. The amount banks can raise to finance the acquisition of assets in the secondary market is limited by the total debt banks already hold on its balance sheets. Therefore, if the expected value from the acquisition is high, banks have an incentive to reduce the level of debt they take ex-ante, to take advantage of attractive acquisition opportunities later.

I solve the model in the following way. First, I find the optimal level of each type of debt as a function of the expected market price of the asset. As each bank is infinitesimally small, it does not take into account the impact it might have on the market price, which means that it takes it as given. The optimal level of debt corresponds to the industry choice of debt, because I consider only symmetrical equilibria when all banks are ex-ante identical. Given the industry choice of debt, I determine the supply and demand for the assets and find the level of price that clears the market. In equilibrium, the level of price conjectured by banks equals the level of price that clears the market. Next, I compute the optimal level of both secured and unsecured debt for the market clearing price. This gives equilibrium quantities.

2.2.2 Formal model description

There is a continuum of risk-neutral banks of measure 1, and 3 periods, $0, 1, 2$. At $t = 0$ a bank can invest in a project with an uncertain payoff, V_i. The asset is the same across all banks, but the realized payoff will be different, because it depends on the screening and monitoring skills of a manager of each bank. At $t = 0$ a bank knows that the payoff of the asset is uniformly distributed over the interval $(0, 2)$, and its price at $t = 0$ is $P_0 = 1$.[4] The value of the asset becomes known at period $t = 1$, and the payoff takes place at $t = 2$. At $t = 0$ a bank decides how much debt to take to finance the acquisition of the asset.

A bank finances the acquisition of the asset by issuing long-term unsecured debt and equity. The number of units that bank buys with unsecured debt and equity is normalized to 1.

The face value of unsecured debt is D. The debt-holders are risk-neutral, hence they will provide to the bank the expected value of their payoff, denoted D_0. The remaining $(1 - D_0)$ is financed by issuing equity. The cost of issuing one unit of equity is s. No additional equity issuance is possible in next periods.

[4] I assume that there is an unlimited perfectly inelastic supply of the asset.

Next, a bank can buy more of the asset, by posting the acquired unit of the asset as collateral. With an asset of value equal to 1, a bank can raise $(1-h) \times 1$ of funds, where h is a margin haircut. I assume a bank cannot post as collateral the assets that will be acquired in the future: this obliges a bank to raise funds for the first unit with equity and unsecured debt. Having borrowed $(1-h)$, a bank can acquire $(1-h)$ units of the asset and post it again as collateral, and so on. Therefore, the total amount of the collateral debt a bank takes is:

$$H_0 = 1 \times \sum_{i=1}^{M}(1-h)^i \leq \sum_{i=1}^{\infty}(1-h)^i = \frac{1-h}{h} \tag{2.1}$$

where M corresponds to the number of times that banks posts newly acquired asset as collateral. It follows from the equation 2.1 that the maximum amount of the collateralized debt a bank can issue is limited. The limit equals the amount a bank can raise by collateralising an infinite number of times.

I solve only for the equilibria when short-term debt is risk-free, hence the amount of debt that bank receives equals to the face value: $H_0 = H$. A sufficient condition for that is that $P_1 \geq 1-h$, where $P_1$is the equilibrium market price at period 1.

Therefore, at $t = 0$ the total face value of bank's debt is the sum of secured and unsecured debt it issued, $(H_0 + D)$. The amount of equity is $(1 - D_0)$. The total number of the asset purchased is $(H_0 + 1)$, and the quantity of the asset posted as collateral is $\frac{H_0}{1-h}$.

A bank optimally chooses H_0 and D that maximize the expected firm value. The optimal level of each type of debt sets its marginal cost equal to marginal benefit. The benefit of issuing long-term unsecured debt is the reduction of the cost of external equity financing, and the cost of issuing unsecured debt is increase in the bank's default probability. The benefit of taking short-term secured debt is the possibility to buy more units of the asset. The cost is a threat of fire sale liquidation in case the market asset price goes down.

At period $t = 1$ banks learn the payoff of their asset. A fraction of the banks are unable to repay their debt and their assets are sold in the secondary market at price P_1 per unit of the asset, where P_1 is an equilibrium market asset price.

At $t = 2$ the asset of each bank pays V_i, and all claimholders are reimbursed.

The timeline of the decisions is presented in the picture below.

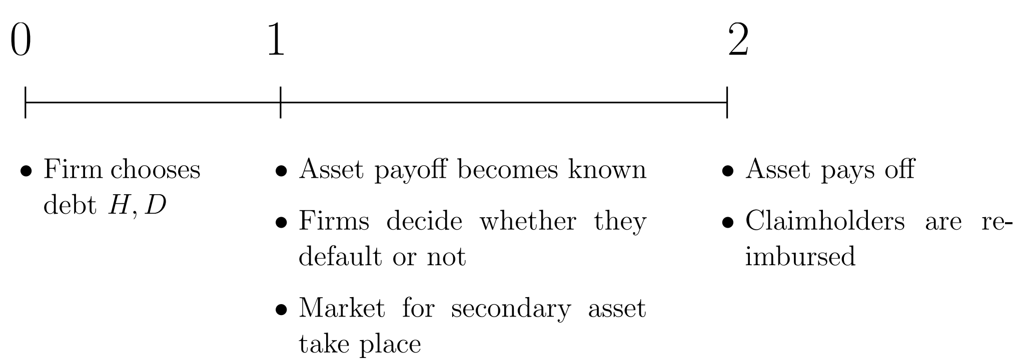

Figure 2.1: Model timeline.

2.2.3 Bank's borrowing capacity and fire sale externality

I define a borrowing capacity of a bank in a similar way proposed by Shleifer and Vishny (2010).

A bank that holds E_0 units of cash can buy E_0 units of assets at price $P_0 = 1$. In the case of this model this cash includes the value of equity and long-term debt. Using secured source of funding, a bank can raise additional $E_0(1-h)$ of funds and acquire more assets. The total amount of collateralized funding is a sum of a geometric series that is limited from above:

$$H_0 = E_0 \sum_{i=1}^{M} (1-h)^i \leq E_0 \frac{1-h}{h} \tag{2.2}$$

By adding E_0 on both sides of the expression 2.2, the following inequality is obtained:

$$E_0 + H_0 \leq \frac{E_0}{h} \Rightarrow$$

$$h \leq \frac{E_0}{E_0 + H_0} \tag{2.3}$$

where $E_0 + H_0$ is the total value of assets on the bank's balance sheet.

The expression 2.3 has the following interpretation: a firm can borrow short-term against collateral as long as the ratio of equity divided by total market value of the assets is above the margin haircut. At $t = 0$ a bank cannot exceed its borrowing capacity by construction, because it actively chooses its level of debt at this period and at this point the constraint translates in the fact that a bank cannot collateralize more then infinite number of times.

However, everything changes in the next period: at $t = 1$ the asset price moves from $P_0 = 1$ to P_1 where P_1 is determined in the market clearing. A bank already holds on its balance sheet the level H_0 of secured debt from the previous period and needs to roll over it. Following the change in the market price, the market value of equity and long-term debt becomes

$$E_1 = P_1 \times (E_0 + H_0) - H_0 = E_0 \times P_1 + H_0\Big(P_1 - 1\Big)$$

where $P_1(E_0 + H_0)$ is the marked-to-market value of the assets of a bank at $t = 1$.[5]

In $t = 1$ the borrowing capacity of a bank is limited in a similar way as in expression 2.3:

$$h \leq \frac{E_1}{P_1 \times (H_0 + E_0)} = \frac{E_0 + H_0(1 - \frac{1}{P_1})}{E_0 + H_0} \tag{2.4}$$

From the expression 2.4 it follows that when price increases from one period to the next, the borrowing capacity also increases to the previous period. When price goes down, the capacity shrinks.

If at $t = 1$ the borrowing capacity is exceeded, which means the inequality 2.4 is not satisfied, a bank has to sell a part of its assets.

Below I explore conditions for H_0 and P_1 under which a borrowing capacity of a bank at $t = 1$ increases or decreases. If a borrowing capacity increases, a bank can raise more

[5]Strictly speaking, the market value is $P_1 \frac{E_0+H_0}{P_0}$ where $\frac{E_0+H_0}{P_0}$ is the number of assets acquired in period $t = 0$. But $P_0 = 1$, hence everything simplifies to $P_1(E_0 + H_0)$.

short-term secured financing. If it decreases, it might be forced to partially de-lever.

Let m be the minimum number of the asset a bank has to sell if it exceeds its borrowing constraint, or $(-m)$ be the maximum number of an asset a bank is able to acquire due to the increased borrowing capacity. In both cases the expression 2.4 is satisfied with equality. The value of secured debt H_1 and of long-term liabilities E_1 at $t = 1$ are respectively:

$$H_1 = H_0 - m \times P$$

$$E_1 = (1 + H_0 - m) * P_1 - H_1 = (1 + H_0 - m) * P_1 - (H_0 - m \times P_1)$$

where $1 + H_0 - m$ is the number of the asset on the balance sheet after a forced sale if $m > 0$ or after an acquisition if $m < 0$.

Plugging these two terms into 2.4 and setting it to equality, I obtain the following expression:

$$h = \frac{E_1}{H_1 + E_1} = \frac{(1 + E_0) * P_1 - E_0}{P_1(1 + E_0 - m)}$$

Therefore,

$$m = \frac{(h-1)(H_0 + 1)P_1 + H_0}{hP_1}$$

Next I analyze the sign of m and determine the conditions under which the borrowing capacity of a bank expands ($m < 0$) or contracts ($m > 0$):

1. A firm faces an increased borrowing capacity, meaning it can acquire more assets and $m < 0$, when it chooses H_0 as a function of price P_1 such that:

$$\begin{cases} P_1 \in (1, 2) \\ H_0 \in (0, \frac{1-h}{h}) \end{cases}$$

$$\begin{cases} P_1 \in (1 - h, 1) \\ H_0 \in \left(0, \frac{P_1 - hP_1}{hP_1 - P_1 + 1}\right) \end{cases}$$

   This has the following interpretation: when the asset price P_1 goes up compared to the previous period, no fire sale spillover will occur regardless of the level of collateralized debt taken by banks. However, when the price movement is negative, $P_1 < P_0 = 1$, then only if the bank was prudent and took a low level of secured debt, it can avoid a need to de-lever and consequently a fire-sale liquidation.

2. A firm has to sell m units of its assets when the price P_1 is low, and a bank chooses a high level of H_0:

$$\begin{cases} P_1 \in (1 - h, 1) \\ H_0 \in \left(\frac{P_1 - hP_1}{hP_1 - P_1 + 1}, \frac{1-h}{h}\right) \end{cases}$$

   This means that if the price movement is negative, a bank might be forced to sell a part of its assets only if it unprudently issued too much short-term secured debt. A bank cannot rollover its debt even though H_0, if held to maturity $t = 2$, is risk-free.

2.2.4 Market for the assets of bankrupt firms

After the asset value realizes, a fraction of banks that are unable to payoff their debt obligations, will liquidate their assets in a secondary market. The demand for these assets comes from a fraction of banks that hold an asset with high realized value. Their highest valuation of the asset comes from their expertise. When a bank j goes bankrupt, and a bank i acquires its assets, the acquiring bank gets a value of nV_i from the acquisition, $n \in (0, 1)$. The coefficient n corresponds to a degree of asset liquidity, or asset redeployability, the degree to which a new acquirer can extract value from the new asset. In the context of financial industry it could correspond to the debt management and debt collection skills of an acquiring bank. Acquiring bank pays price P_1 for a unit of the asset, hence the added value from the acquisition is $(nV_i - P_1)$. Therefore, the buyers will be banks with the asset value V_i s.t. $V_i \geq \frac{P_1}{n}$.

Next, as long as $V_i \geq \frac{P_1}{n}$, firms will be willing to acquire an unlimited amount of assets because it will increase the firm value: there is no more uncertainty at $t = 1$.

In order to define the demand of acquiring banks for the assets on the secondary markets, I make the following assumptions:

1. Financing of the new acquisitions of assets happens entirely through debt. This assumption is supported by the empirical evidence that banks finance their asset acquisitions mainly by issuing debt (Adrian and Shin (2014)).

2. Banks are limited in the amount they can borrow because the total amount of debt they hold cannot exceed the existing total value of their assets.[6] Thus, the debt banks previously took limits the amount that the acquiring banks can borrow for asset purchase.

3. Only firms with a positive added value from the acquisition can obtain financing via unsecured debt at $t = 1$, which is equivalent to financing a positive NPV project.[7] This means that if the price goes down and banks have to sell some of their assets, the banks that have a negative NPV from acquisition of the asset in the secondary market, $V_i \leq \frac{P_1}{n}$, cannot issue unsecured debt at $t = 1$ and use this cash to payoff a part of short-term secured debt which they cannot rollover.[8]

To define the demand for the liquidated assets, two separate cases have to be considered: when a fire sale spillover occurs and when it does not. If a fire sale liquidation takes place, the

[6] The quantity of the asset the bank will acquire is not included in the calculation. This is the same argument that banks cannot post as collateral the assets they will acquire in the future.

[7] If price P_1 is such that firms face an additional collateral requirement, firms will be selling and buying at the same time. Firms with a positive demand for the liquidated assets of other firms when they have to partially sell their assets, are strictly better off using debt they obtained first to post more collateral, thus avoiding the need to sell their own asset, and then to use the rest of the money to buy the assets on the market. For each unit of asset sold a bank looses V in period 3 but when it buys one unit of asset, it acquires only nV. However, I exclude such possibility assuming that the unsecured debt can be used only for the sake of new acquisitions

[8] At $t = 1$ there is no uncertainty any more with regards to the asset payoff V_i, therefore either firms cannot obtain funding because their current debt is already higher then the realized asset value, or the debt they obtain is risk-free. An unsecured risk-free debt is an unrealistic feature of the banking sector, that is why it is excluded. This means that banks with the asset value $V_i < \frac{P_1}{n}$ have no access to any cash to cover the additional collateral requirement, and have to sell a fraction of their assets.

capacity of a bank to borrow unsecured debt will be reduced because the number of assets a bank holds on its balance sheet will be smaller:

$$V(H_0 + 1 - m) \geq H_0 - m \times P_1 + D + l_{FS} \times P_1 \tag{2.5}$$

The inequality indicates that the total amount of debt a bank can hold cannot exceed its current book value of the balance sheet. The l.h.s. of the equation is the total value of assets after a quantity m has been sold to satisfy the borrowing constraint. The r.h.s. is the total debt a bank holds, plus the amount of new issues of unsecured debt it needs for the acquisition of l unit of the asset, lP_1.[9] The quantity mP_1 corresponds to the value of short-term secured debt a bank paid off using fire sale proceeds. For banks with the realized asset value $nV > P_1$, the added value from the acquisition is positive, hence they will be willing to acquire the maximum number of assets they can. This means that the actual demand for the asset is such a level of l that sets 2.5 to equality:

$$l_{FS} = \frac{V(H_0 + 1 - m) - (H_0 + D - m)}{P_1} \tag{2.6}$$

In the absence of a sale liquidation, the bank can raise $(-m \times P_1)$ units by issuing secured debt because it has a spare borrowing capacity, and it can also issue long-term unsecured debt such that the total amount of debt a bank holds cannot exceed the book value of the assets. The demand in this case has the following functional form:

$$l_{NoFS} = \frac{V(H_0 + 1) - (H_0 + D - m \times P_1)}{P_1} - m = \frac{V(H_0 + 1) - (H_0 + D)}{P_1}$$

Market clearing

he supply and demand of the asset are deterministic, because there is a continuum of agents of measure 1 and by the low of large numbers the fraction of banks that will be entirely liquidated is $\frac{\hat{V}_{def}}{2-0}$, where $\hat{V}_{def}$ is a default threshold, the asset value below which banks choose to default, and $(2 - 0)$ is the interval over which the asset value is uniformly distributed. Hence the total supply of the asset in the secondary market consists of the number of assets of insolvent banks, plus the number of the asset sold by solvent banks due to their inability to rollover their short-term debt:

$$Supply = \frac{1}{2}\left(\int_0^{\hat{V}_{def}} (H_0 + 1)dV + \left(\int_{\hat{V}_{def}}^{2} mdV \right) * I_{sell} \right)$$

where $\hat{V}_{def}$ is a default threshold, and I_{sell} is an indicator function that takes value of 1 when agents are required to partially liquidate their assets.

The total demand for the liquidated assets when a *fire sale occurs* consists of the number of banks willing to acquire the asset multiplied by the demand for the asset by an individual bank. The number of banks willing to acquire the asset is deterministic: $\frac{1}{2}(1 - max(\frac{P_1}{n}, \frac{D+H_0-mP_1}{H_0+1-m}))$,

[9] Note that the number of assets that will be acquired in the future is not taken into account in the l.h.s. of the expression 2.5. This goes in line with an earlier assumption that one cannot pledge assets which are not yet acquired to expand its borrowing capacity.

where $\frac{P_1}{n}$ is the threshold above which banks will be willing to acquire the asset, $\frac{D+H_0-mP_1}{H_0+1-m}$ is the threshold above which banks will be able to raise financing for the acquisition. Only banks with realized asset value above both thresholds will be willing and able to acquire the asset. Hence, the total demand is the following:

$$Demand = \frac{1}{2}\left(\int_{max\left(\frac{P_1}{n}, \frac{D+H_0-mP_1}{H_0+1-m}\right)}^{2} ldV \right)$$

where l is defined in equation 2.6.

For the case when a fire sale occurs the market clearing condition is the following:

$$\int_0^{Vdef} (H_0+1)\, dV + \int_{V_{def}}^{2} m\, dV = \int_{\frac{P}{n}}^{2} \frac{V(H_0-m+1)-(D+H_0-mP_1)}{P_1}\, dV$$

where the l.h.s. corresponds to the supply of assets, and the r.h.s. is the demand. The supply of assets is increased by the additional amount m of asset units banks will have to sell to payoff a fraction of ST debt they could not roll over. On the other hand, the demand is limited because of this fire sale. This puts additional downward pressure on price.

When no fire sale occurs, the fraction of banks who will acquire the assets is $\frac{1}{2}(1 - max(\frac{P_1}{n}, \frac{D+H_0}{H_0+1}))$. In this case the market clearing condition take the following form:

$$\frac{1}{2}\int_0^{V_{def}} (H_0+1)\, dV = \frac{1}{2}\int_{\frac{P}{n}}^{2} \frac{(H_0+1)V-(D+H_0)}{P_1}\, dV$$

where the l.h.s. corresponds to the supply of assets, the supply consists only of the assets of insolvent banks.

Rational Expectations Equilibrium

When banks decide on an optimal level of secured and unsecured debt, they conjecture a certain level of price $\tilde{P}_1$ that will clear the market in the next period. As all banks are identical ex-ante, they take the same level of debt. This means that individual bank's leverage decision corresponds to the industry level of secured and unsecured debt. A price $\bar{P}_1$ is the level of price of the asset that clears the market at $t = 1$. A rational expectations equilibrium framework implies that the price that banks conjecture corresponds to the price that will realize in the next period: $\tilde{P}_1 = \bar{P}_1$. Banks perfectly anticipate the price of the next period and consequently whether there will be a fire sale spillover or not.

2.2.5 Liquidation decision

A bank takes a decision to liquidate its assets to maximize the payoff to shareholders. This happens for values V_i s.t.

$$V_{def} \times (H_1+1) - (H_1+D) \leq \max\Big(0, P_1(H_1+1) - (H_1+D)\Big)$$

The l.h.s. of the expression is the payoff that shareholders get if bank does not go bankrupt: it is the total value of the assets minus the face value of debt a bank holds. The right hand side reflects the position of shareholders in bankruptcy: they are the last to be reimbursed, after secured and unsecured debt holders, but their liability is limited.

I consider only case when unsecured debt is risky. This means that in bankruptcy shareholders get nothing, therefore the default decision simplifies to the following:

$$V_{def} \times (H_1 + 1) - (H_1 + D) \leq 0 \Rightarrow V_{def} = \frac{H_1 + D}{H_1 + 1}$$

Lemma 2.2.1. *When the total debt is risky, which means that $D + H_1 > P_1 * (H_1 + 1)$, where $H_1 + 1$ is a number of assets at $t = 1$, the default threshold in a no-fire-sale-liquidation case is*

$$\hat{V}_{def}^{NoFS} = \frac{D + H_0}{H_0 + 1}$$

In the presence of fire sale liquidation, the default threshold is

$$\hat{V}_{def}^{FS} = \frac{D + H_1}{H_1 + 1} = \frac{D + H_0 - mP_1}{H_0 + 1 - m}$$

The proof of lemma 2.2.1 is in the Appendix .3.

2.2.6 Objective function: expected firm value

As mentioned earlier, banks choose D and H_0 such as to maximize their expected firm value at $t = 0$. The expected firm value is composed of the following elements:

- The expected firm value in solvency;
- The expected firm value in insolvency;
- The expected value of the option to acquire assets of bankrupt firms;
- The expected loss from partial liquidation of assets, if any;
- The cost of issuing equity (this item reduces the firm value).

$$\begin{aligned}
E(V) &= V_0 \\
&= \frac{1}{2}\Bigg((H_0 + 1)\int_{\hat{V}_{def}}^{2} V dV + (H_0 + 1)\int_{0}^{\hat{V}_{def}} P_1 dV + \int_{max\left(\frac{P}{n}, \frac{H_1+D}{H_1+1}\right)}^{2} l(nV - P_1)dV \\
&\quad - \left(\int_{\hat{V}_{def}}^{2} m(V - P_1)dV\right) * I_{FS}\Bigg) - s(1 - D_0)
\end{aligned} \tag{2.7}$$

where I_{FS} corresponds to an indicator function, equal 1 when a bank has to partially liquidate its asset.

2.3 Benchmark economy: no fire sale friction

A fire sale friction arises because banks issue short-term debt and their balance sheet is marked to market. In order to quantify the impact of a fire sale friction on the equilibrium capital and debt structure of banks, I construct a benchmark economy. In this economy banks in a similar way finance the asset acquisition by equity and unsecured debt, and they issue secured debt to be able to buy more of the asset. However, secured debt is long term. The absence of need to rollover the short-term debt at $t = 1$ removes the friction that creates a fire sale friction, but keeps other elements of the model the same: the price at $t = 1$ is still determined in the market clearing, and if banks take too much debt, its equilibrium level will be low. However, no deleveraging takes place.

The expected firm value is composed of the expected value in insolvency, expected firm value in solvency, expected value from the acquisition of assets in secondary market, minus the cost of issuing equity:

$$V_0 = \frac{1}{2}\Big(\int_{\hat{V}_{def}}^{2} V dV + \int_{0}^{\hat{V}_{def}} P_1 dV + \int_{\frac{P}{n}}^{2} \frac{V(H+1)-(H+D)}{P_1}(nV - P_1)dV\Big) - s(1-D_0)$$

The details of the solution are in Appendix .4.[10]

As the major cost of taking secured debt is absent, it is intuitive that banks choose to take the maximum secured debt they can:

$$H_{opt} = \sum_{i=1}^{\infty}(1-h)^i = \frac{1-h}{h}$$

As presented in the Appendix .4, the equilibrium price does not depend on the level of margin haircut. As Figure 2.2 demonstrates, P_1 is increasing in the degree of asset liquidity n. Higher asset liquidity means a higher fraction of firms will be willing to buy the assets on the secondary market, increasing the demand for the assets.

[10]The acquisition threshold for the assets in the secondary market is $V_{acquis} = max\Big(\frac{P}{n}, \frac{D+H}{H+1}\Big)$. However, when $V_{acquis} = \frac{D+H}{H+1}$, I prove analytically that there is no solution with and without firesale spillover when $s \leq 0.5$. Empirically, no solution has been found for a range of parameter s even above this threshold for models with and without firesale externality.

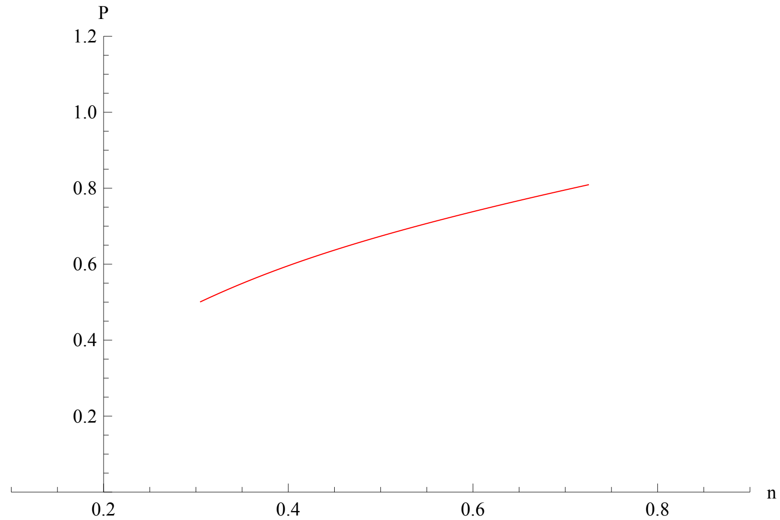

Figure 2.2: The equilibrium level of price P as a function of the degree of asset liquidity, $h = 0.5$, $s = 0.3$.

Higher expected liquidation price feeds back into the leverage decision of firms: they take more unsecured debt D. As Williamson (1988) pointed out, the higher is the expected recovery in default, the less is the cost of debt, the more debt banks are willing to take. In turn, a higher level of debt increases a number of defaulted banks, and the supply of assets in the secondary market goes up. This pushes the price down limiting to a certain extent the impact that higher price has on leverage choice.

Figure 2.3 demonstrates that the optimal level of debt increases in the asset liquidity when the latter is low, and decreases when the latter is high. When liquidity is low, the option to acquire the assets in the secondary market becomes less valuable, which lowers the opportunity cost of unsecured debt. And the opposite, high asset liquidity promises higher return from asset acquisition in the secondary market, and banks reduce the level of unsecured debt.[11] This allows banks to have some spare capacity to borrow unsecured debt at $t = 1$ to finance the acquisition of the assets of insolvent banks.

This result demonstrates that the asset liquidity can both increase and decrease the capacity of firms to take unsecured debt, which unites the views of Bernardo et al. (2015) with the ones of Morellec (2001). The result is not specific for financial firms: within the framework of the benchmark economy there is nothing that differentiates financial firms from non-financial ones.

[11] Recall that the expected value of the option to acquire the assets in the secondary market is the following:

$$V_{acquis} = \frac{1}{2} \int_{\frac{P}{n}}^{2} \frac{((H+1)V - (D+H))(nV - P)}{P} dV$$

It is obvious that it is decreasing in D and increasing in n.

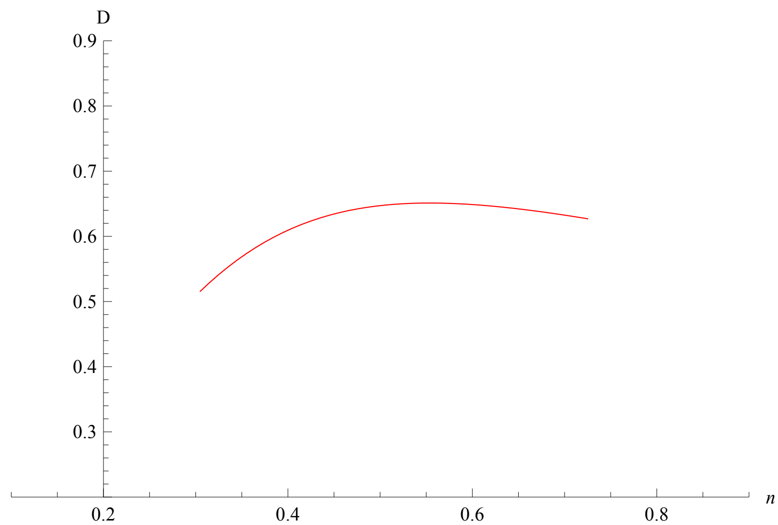

Figure 2.3: The optimal level of unsecured debt D as a function of asset liquidity n, $h = 0.5$, $s = 0.3$.

2.4 Economy with a fire sale friction

When fire sale externality is present, the functional form of the expected firm value, as well as the market supply and demand of the assets are different depending on whether a spillover takes place or not.[12]

Therefore, I consider three possible cases:

- The asset price goes up from its initial level $P_0 = 1$: $P_1 \in (1, 2)$, no fire sale spillover occurs, and banks can borrow even more against collateral at period $t = 1$.
- The asset price goes down, but no fire sale spillover occurs because banks precociously took low level of secured debt H_0 : $H_0 < \frac{P_1 - hP_1}{1 - P_1 + hP_1}$.
- The asset goes down, and a fire sale spillover occurs because banks took a high level of secured short-term debt H_0: $H_0 > \frac{P_1 - hP_1}{1 - P_1 + hP_1}$.

In what follows, I present the equilibrium solution of each of three cases.

2.4.1 Market price goes up

When price is high, $P_1 \in (1, 2)$ banks can borrow more against collateral because a favorable price movement creates additional borrowing capacity for banks.

Because no cost is associated with taking secured debt (see the proof in Appendix .5), banks will take as much collateralized debt as they can:

$$H_{opt} = \frac{1 - h}{h}$$

[12]This is a rational expectations equilibrium, hence banks will perfectly anticipate whether there is a fire sale spillover or not.

High liquidation price makes taking unsecured debt cheaper, and in this case banks choose only debt financing, no equity: $D_0 = 1$.[13] However, the total level of debt taken by banks is too high and there is no price within the range $(1,2)$ that can support the supply of the assets by failed banks. Therefore, when price is conjectured to be high, banks tend to take too much debt, and the market breaks down.

2.4.2 Market price goes down, and no fire sale spillover occurs

The important elements of this case is that even though the price movement is negative, banks prudently had taken a level of secured debt which is low enough to avoid a forced partial liquidation of their assets. For this to be true, secured debt cannot exceed the level has to be within the interval $H_0 \in (0, \frac{P_1 - hP_1}{1-P_1+hP_1})$. I find that once D is set at its optimal level determined by the FOC, the optimal level of H_0 is a corner solution:

$$H_{opt} = \begin{cases} \frac{P_1 - hP_1}{1-P_1+hP_1}, \\ 0 \end{cases}$$

There are no such parameters (n, s, h) for which it would be optimal to have zero secured debt, and a risky level of unsecured debt.[14] This means that it is always optimal to have at least $\frac{P_1 - hP_1}{1-P_1+hP_1}$ of collateralized debt. The necessary and sufficient condition for the level $H_{opt} = \frac{P_1 - hP_1}{1-P_1+hP_1}$ to be optimal is described in Attachment .6.

This outcome implies that when the cost of a fire sale is too high, banks take the maximum short-term debt that allows them to avoid a need to de-lever. In this case $m = 0$, which means that there is no additional borrowing capacity for banks at $t = 1$ to buy the assets of distressed banks against collateral. Banks finance the acquisition of additional units of assets through the issue of unsecured debt.

2.4.3 Market price goes down, and fire sale spillover occurs

The detailed solution is presented in the Appendix .6.

A fire sale spillover occurs only when banks take a high level of secured debt: $H_0 \in \left(\frac{P_1 - hP_1}{1-P_1+hP_1}, \frac{1-h}{h}\right)$.

Once D is at the optimal level determined by the FOC, the optimal level of H_0 is either at its minimum or maximum levels:

$$H_{opt} = \begin{cases} \frac{1-h}{h} \\ \frac{P_1 - hP_1}{1-P_1+hP_1} \end{cases}$$

When secured debt is at the upper limit, $H_{opt} = \frac{1-h}{h}$, a fire sale liquidation happens despite the fact that banks perfectly anticipate this liquidation.

[13] The optimal level of unsecured debt, determined by the first order condition, is so high that the expected value of unsecured debt, D_0 is always greater then 1, which is not possible, because the number of units a bank buys with equity and debt is normalized to 1. That is why $D_0 = 1$ is a corner solution.

[14] I consider only the cases when the unsecured debt is risky. I find that whenever there is an equilibrium with a risky level of debt, banks always choose the risky level over the risk-free level.

When secured debt is at the lower boundary, $H_0 = \frac{P-hP}{1-P+hP}$, then the number of assets a bank has to sell is zero. This case becomes identical to the one described in the subsection 2.4.2.

2.5 Equilibrium results

There is a unique equilibrium in this model, but there are two outcomes of the equilibrium depending on the level of parameters.

In the first equilibrium outcome, the price goes down, banks take the maximum level of secured debt and a fire sale spillover follows.

In the second outcome, the asset price goes down, banks take the intermediary level of secured debt that protects against negative price movements. And no fire sale spillover occurs.

Figure 2.4 puts in perspective the optimal choice of secured debt in both equilibrium outcomes, compared to the benchmark economy with no fire sale externality.[15]

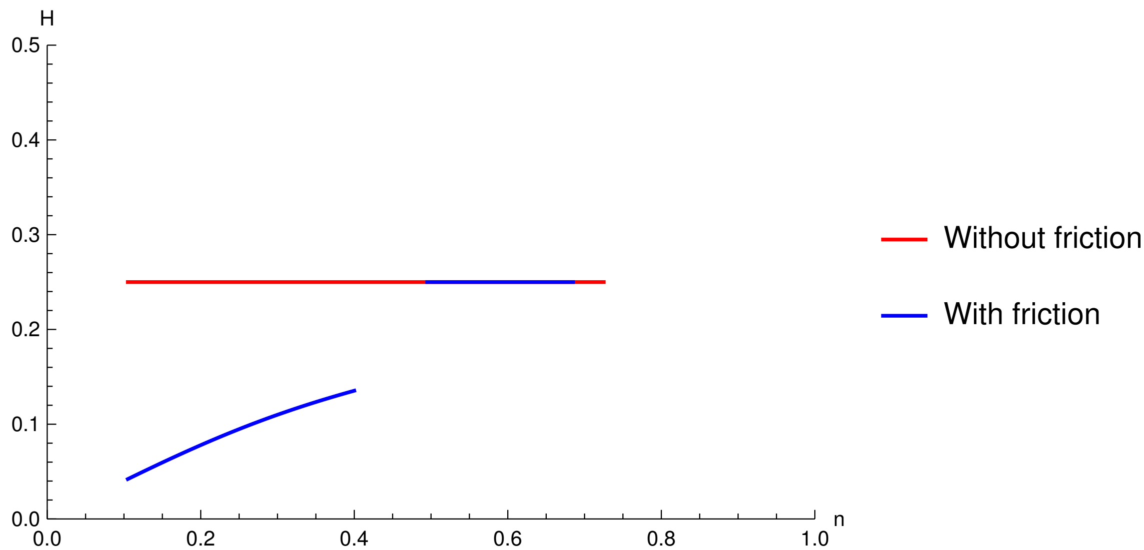

Figure 2.4: The equilibrium level of secured debt H, for the economy with the friction and for the benchmark economy without the friction, $h = 0.8$, $s = 0.3$.

The figure above illustrates a more general point: when asset liquidity is high, banks choose maximum secured debt, the choice followed by a fire sale liquidation. It is the same level of secured debt that would have been taken in the economy with no fire sale friction, and it depends only on the margin haircut: $H_{max} = \frac{1-h}{h}$. High asset liquidity supports the price of the asset in the secondary market, lowering the cost of a fire sale liquidation for a bank.

When the degree of asset liquidity is low, banks cut the level of unsecured debt: they take the maximum level that allows them to avoid a fire sale spillover: $H_{intermed} = \frac{P_1-hP_1}{1-P_1+hP_1}$. When asset liquidity is low, the cost of a partial liquidation of assets is high, and banks choose to avoid it. As demonstrated by figure 2.4, this optimal level increases with the increase in asset

[15]In the benchmark economy, the secured debt is long-term and there is no binding constraint at the intermediary period that forces banks to partially liquidate their assets. Nor can banks borrow more against collateral at the intermediary period.

liquidity. This is due to the fact that higher asset liquidity increases the equilbrium asset price (see figure 2.6), and higher price reduces the tightness of the constraint that might bind in period 1. This allows banks to take more secured debt and avoid a costly fire sale liquidation.

Figure 2.5 shows the set of parameters that determine which of the equilibrium outcomes occurs. The level of margin haircut must be very high for the equilibrium with a fire sale liquidation to exist. This is because I consider only risk-free collateralized debt, and I solve only for the parameter values for which the price is sufficiently high to ensure that secured debt is risk-free.[16] For the case when banks take intermediary level of collateralized debt, the range of values of the margin haircut that maintain the risk-free debt is much larger.

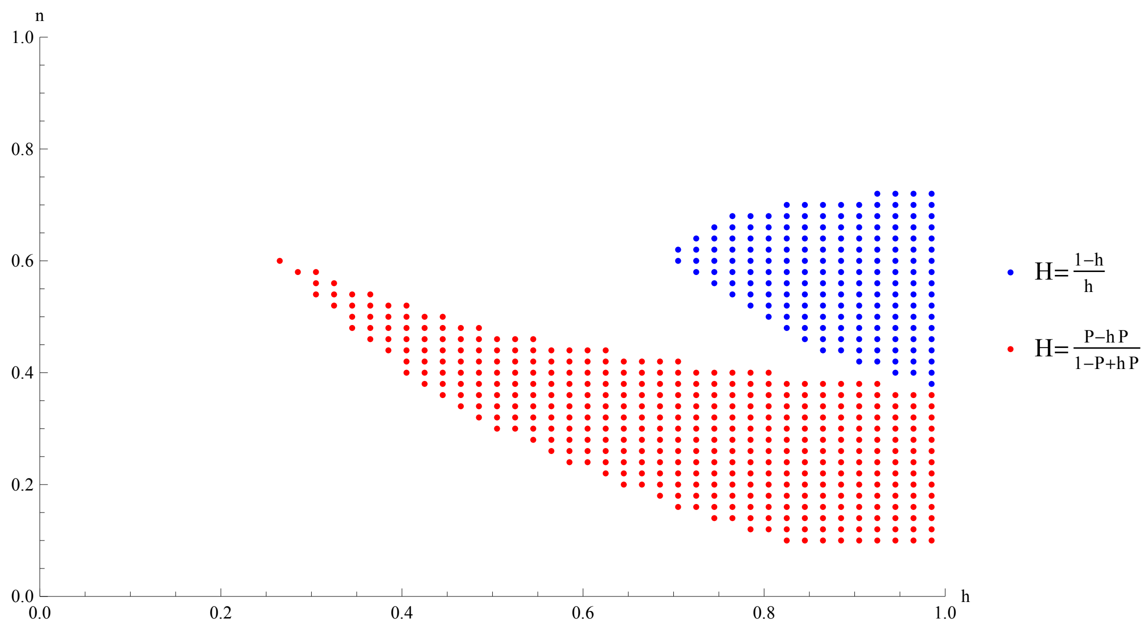

Figure 2.5: The range of values of degree of asset liquidity and haircut, for which banks choose the maximum debt possible, $H_0 = \frac{1-h}{h}$, and the value for which fire sale is zero, $H_0 = \frac{P-hP}{1-P+hP}$. The cost of equity is set to be 30%, $s = 0.3$.

The figure 2.6 compares the price in the economy with the fire-sale friction, against the benchmark economy without the friction. When the asset liquidity is low, on the graph below $n < 0.4$, and banks avoid a fire sale liquidation, the price is the same in the economy with the friction and in the economy without the friction. When the asset liquidity is above$n > 0.5$ and a fire sale spillover happens, this lowers the equilibrium price compared to the benchmark.

[16]Even though this choice limits the range of parameters of the margin haircut for which equilibrium exists, it allows to study the equilibrium in a tractable way. Therefore, it is intuitive that the margin haircut has to be very high to ensure that when banks take the maximum collateralized debt, it remains risk-free despite the fire sale liquidation that this choice of debt engenders.. The case when collateralized debt is risky will be an extension of the present model.

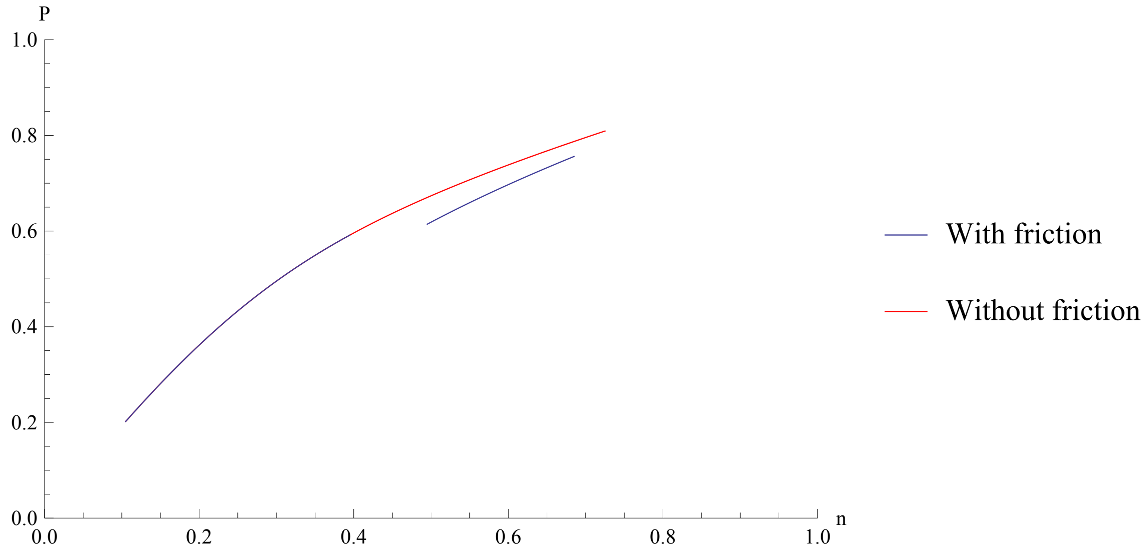

Figure 2.6: The equilibrium level price P_1, for the economy with the fire sale friction against the benchmark economy without the friction, $h = 0.8$, $s = 0.3$.

Figure 2.7 show the optimal level of unsecured debt D as a function of asset liquidity. Again, I compare the equilibrium outcomes in the economy with spillover against the benchmark economy without the spillover. The graph illustrates how banks change their capital structure in response to a fire sale externality. There are two major results.

- When asset liquidity is low and banks reduce their secured debt, they take an unsecured debt which is much higher then the level of the benchmark economy with no spillover.
- When asset liquidity is high and banks choose maximum level of secured debt, they cut the level of unsecured debt.

The economic mechanism behind this optimal outcomes is the following.

1. Low asset liquidity increases the cost of a fire sale liquidation. This gives incentives for a bank to cut the secured debt it issues. A lower level of secured debt leaves a higher payoff to risky debt holders in bankruptcy, because they are paid off after the secured claim holders. The cost of risky debt becomes lower.
2. Lower asset liquidity decreases the expected value from the option to acquire assets in the secondary market. Lower value from the acquisition option lowers the opportunity cost of unsecured debt: the more risky debt a bank issues at$t = 0$, the less it will be able to borrow at $t = 1$ to finance the acquisition of the assets.

These channels provide incentives for the bank to take more risky debt when asset liquidity is low.

Further, figure 2.7 demonstrates that the reduction in the risky debt for high asset liquidity is more dramatic then its increase for low liquidity. This highlights one more channel that affects the optimal level of unsecured debt: level of equilibrium price. The model suggests that optimal level of unsecured debt banks take is very sensitive to the level of asset price. Some numerical simulations give the result that a 1-percent decrease in price leads to a 2-percent decrease in the optimal level of unsecured debt. This, in addition to the other two channels, explain a dramatic fall in the optimal level of unsecured debt for high level of asset liquidity.

This channels suggest that even when banks cannot substitute one type of debt for the other to finance their assets, the two types of debt do become substitues because they jointly affect the probability of default of a bank, the cost of unsecured debt, and on the industry level, of the asset prices. A lower level of secured debt leaves more "space" for a bank to take more unsecured debt. In this context secured and unsecured debt are substitutes. Hence, banks substitute secured debt for unsecured when asset liquidity is low, and vice versa. Moreover, this substitution is the result of the two separate equilibrium outcomes. Within each outcome, the level of unsecured debt generally increases in the degree of asset liquidity.[17]

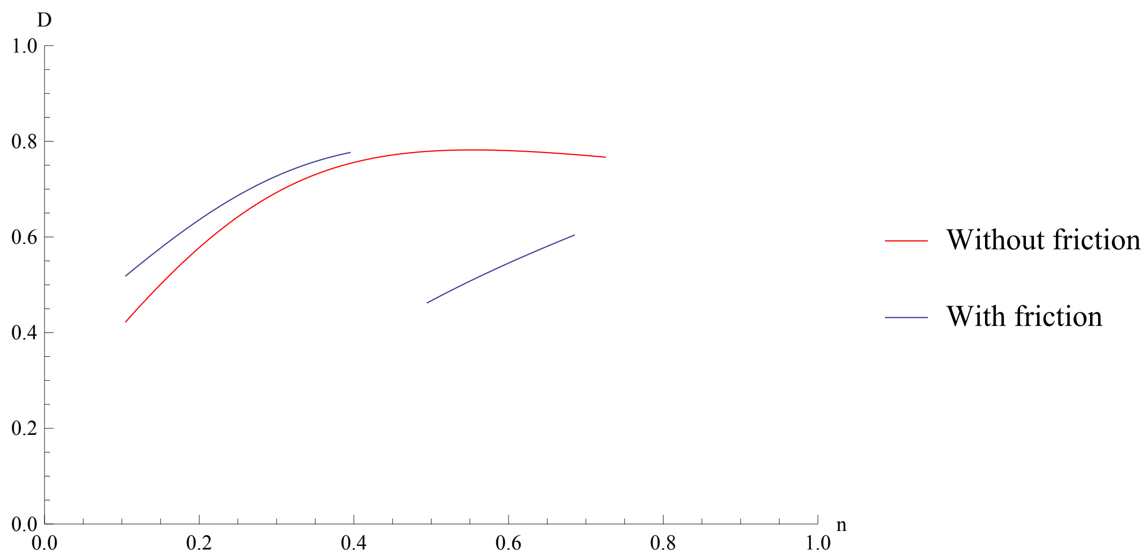

Figure 2.7: The equilibrium level of risky unsecured debt D, for the economy with the friction and for the benchmark economy without the friction, $h = 0.8$, $s = 0.3$.

This channels suggest that even when banks cannot substitute one type of debt for the other to finance their assets, the two types of debt do become substitues because they jointly affect the probability of default of a bank, the cost of unsecured debt, and on the industry level, of the asset prices. A lower level of secured debt leaves more "space" for a bank to take more unsecured debt. In this context secured and unsecured debt are substitutes. Hence, banks substitute secured debt for unsecured when asset liquidity is low, and vice versa. Moreover, this substitution is the result of the two separate equilibrium outcomes. Within each outcome, the level of unsecured debt generally increases in the degree of asset liquidity.[18]

Figure 2.8 shows the equilibrium default rate of banks. While locally, it increases in asset liquidity, once n passes above the threshold and equilibrium outcome changes, the default probability falls. Ex-ante, when banks take the maximum level of secured debt and a fire sale spillover occurs, the probability of default increases, all other things being equal. However, the

[17]Numerically I find cases when debt decreases in asset liquidity. This is the case for high levels of asset liquidity.

[18]Numerically I find cases when debt decreases in asset liquidity. This is the case for high levels of asset liquidity.

anticipation of an increase in the default probability and of decrease in the equilibrium price forces banks to reduce their issue of unsecured debt. As illustrated above, this decrease is so dramatic that in equilibrium the probability of default decreases compared to the benchmark with no fire sale friction.

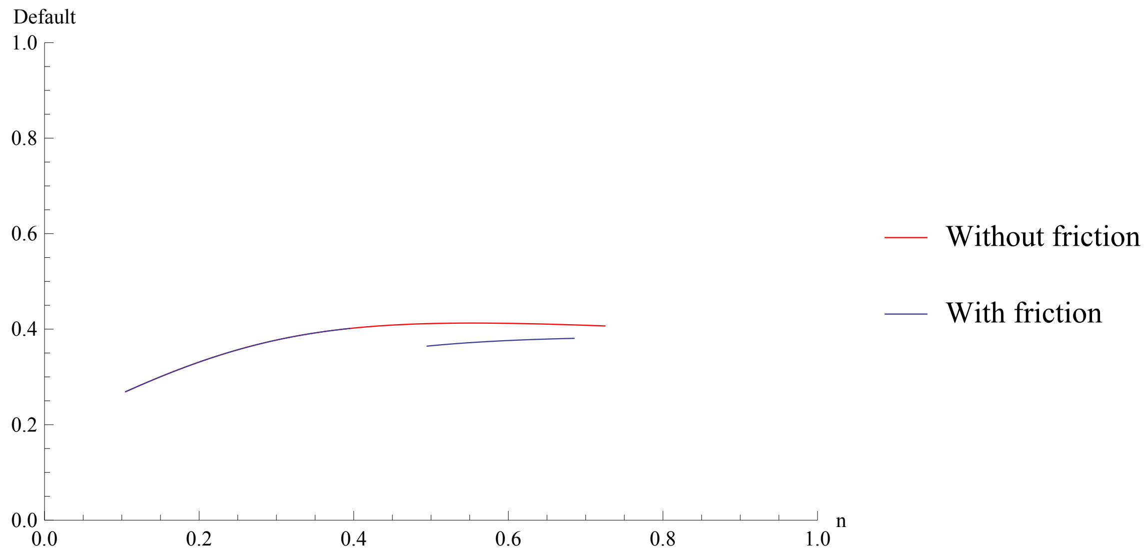

Figure 2.8: The equilibrium level of default probability of a bank, for the economy with the friction and for the benchmark economy without the friction, $h = 0.8$, $s = 0.3$.

2.6 Social planner and policy implications

In this section I explore the optimal leverage choice when social planner (SP) seeks to maximize the expected value of banks. The difference to the case of competitive equilibrium analyzed previously is that social planner takes into account the impact that the leverage choice of firms will have on the asset price.

2.6.1 Social planner in the benchmark economy with no fire sale externality

To solve the social planner's problem, I first find the price as a function of unsecured and collateralized debts in the market clearing condition [19]:

$$\begin{cases} Demand = \int_{\frac{P_1}{n}}^{2} \frac{V(H_0+1)-(H_0+D)}{P_1} dV \\ Supply = \int_0^{V_{def}} (H_0+1) dV \end{cases}$$

$$\Rightarrow P_1 = \frac{n\left(\sqrt{(n-1)^2(D+H_0)^2 - 4(D-1)(H_0+1)} + D(-n) + D - H_0 n + H_0\right)}{H_0+1}$$

Then I plug the above expression for P into the expected firm value (equation 2.3) and find the optimal level of unsecured debt D and collateralized debt H using the first order condition.

[19] Market clearing condition gives two possible roots for P_1 that would clear the market. However, it is easy to show that the other root does not satisfy the conjecture of this case that $P < \frac{D+H}{H+1} < \frac{P}{n}$.

In this way I optimize the total size of the financial sector: the bankruptcy costs, perceived as a loss by an individual bank, is just a transfer of value from one owner to another. As illustrated in the figure 2.9, the level of price in a SP economy can be both higher and lower then the price in a competitive economy (CE) equilibrium. There is a threshold of the level of asset liquidity above which the CE price becomes higher then in the SP economy.[20]

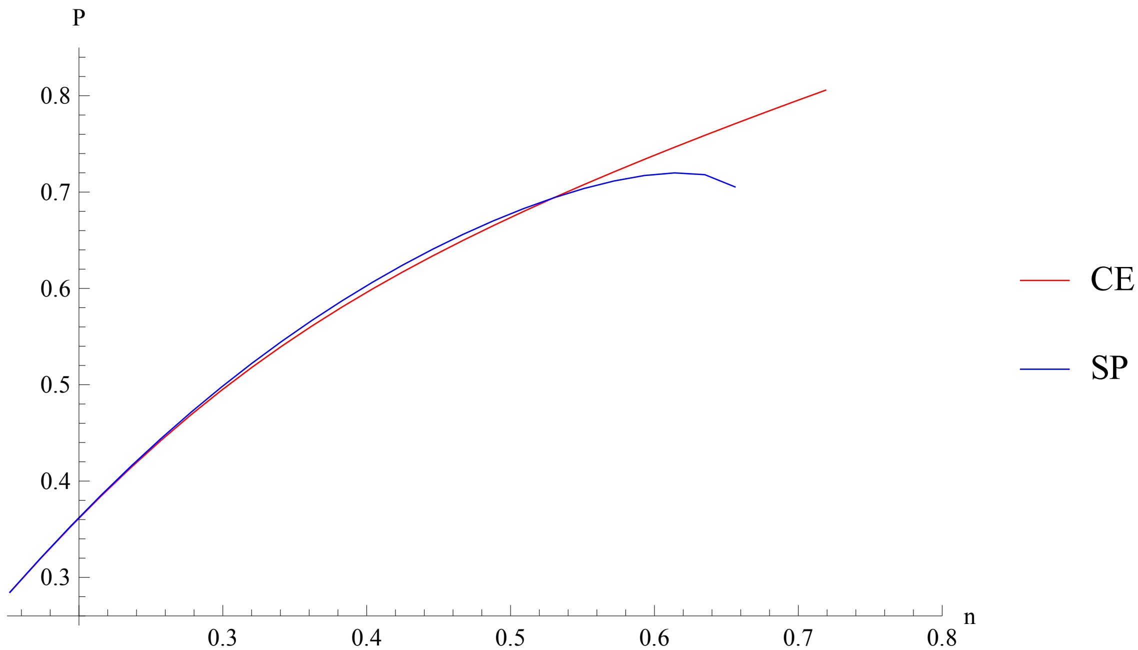

Figure 2.9: The equilibrium level of price P for the economy with social planner and in case of an economy with competitive equilibrium, $s = 0.3$ and $h = 0.75$.

Surprisingly enough, in a social planner's economy when the price is below the level of a competitive equilibrium and the asset liquidity n is high, the optimal level of debt is above the level of a competitive equilibrium, as demonstrated in figure 2.10. This means that when asset liquidity is high, and there is no fire sale friction, the social planner prefers to take more risk, then what agents would choose in a CE. Even though I do not show a graph for the default rate, it follows the same dynamics as the level of unsecured debt.

[20] I solve the case for three different levels of s, and I find that the higher is s, the higher is the threshold above which price of a competitive equilibrium becomes higher then price in a social planner economy.

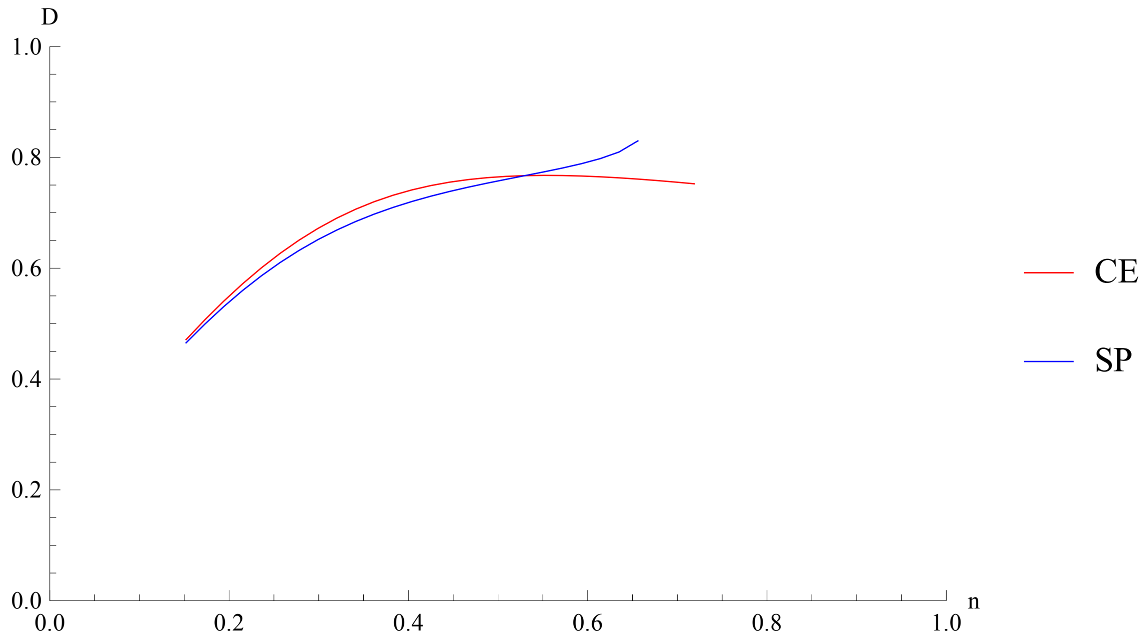

Figure 2.10: The equilibrium level of long-term unsecured debt D for the economy with social planner and in case of an economy with competitive equilibrium, $s = 0.3$ and $h = 0.75$.

The explanation lies in one of the opportunity cost of unsecured debt, the acquisition option. When social planner consider the transfer of financial assets from one bank to another, then the total level of debt actually *increases* this value. This is because asset price decreases in the level of unsecured debt. Lower price gives a higher expected value of the option. Therefore, when little value is lost when assets change their owner, SP chooses a higher higher level of unsecured debt, compared to the CE. This means that the total marginal cost of debt becomes lower when the asset liquidity is high, and social planner chooses a higher level of risk in the economy. As to the level of collateralized debt, in all the numerical values tried, I have not found any case when the regulator would take a level of debt lower then the maximum possible. This is very intuitive, because collateralized debt increases the size of banks, but in this framework there is no cost of a fire sale spillover. means lower price of the assets.

This result bridges two opposing views on the socially optimal level of corporate debt. Lorenzoni (2008) argues that socially optimal level is lower then the level of a competitive equilibrium, because agents do not internalize the impact that their leverage decision has on the equilibrium price. On the other hand, in the model of Gale and Gottardi (2015) taxes paid by a firm, as will as bankruptcy costs, are just transfer of assets from one hands to the other. Therefore, in the SP economy the leverage that firms take should be higher. I show that by introducing the concept of asset liquidity you can have both higher and lower level of debt in a SP economy.

2.6.2 Fire sale externality and regulation

Figure 2.11 demonstrates, for given values of the cost of equity and margin haircut, that in the economy with a fire sale friction SP takes the level of unsecured debt which is lower then the level of a CE. Even though this result is numerical and holds only for 2 values of the parameter, I build a three-dimensional graph where D is a function of both h and n, and I numerically verify for a larger set of parameter values that the risky debt in a CE is higher then the risky debt in the SP economy.

The result has implications for the regulatory intervention. I do not model explicitly bank's deposits in order to abstract from such frictions as deposit insurance or implicit guarantees. The only friction modeled here is a fire sale spillover. The model suggests that once the friction is introduced in the economy, the socially optimal level of debt is below the level of a competitive economy.[21] Therefore, the minimum capital requirements even for non-depository institutions can be desirable.

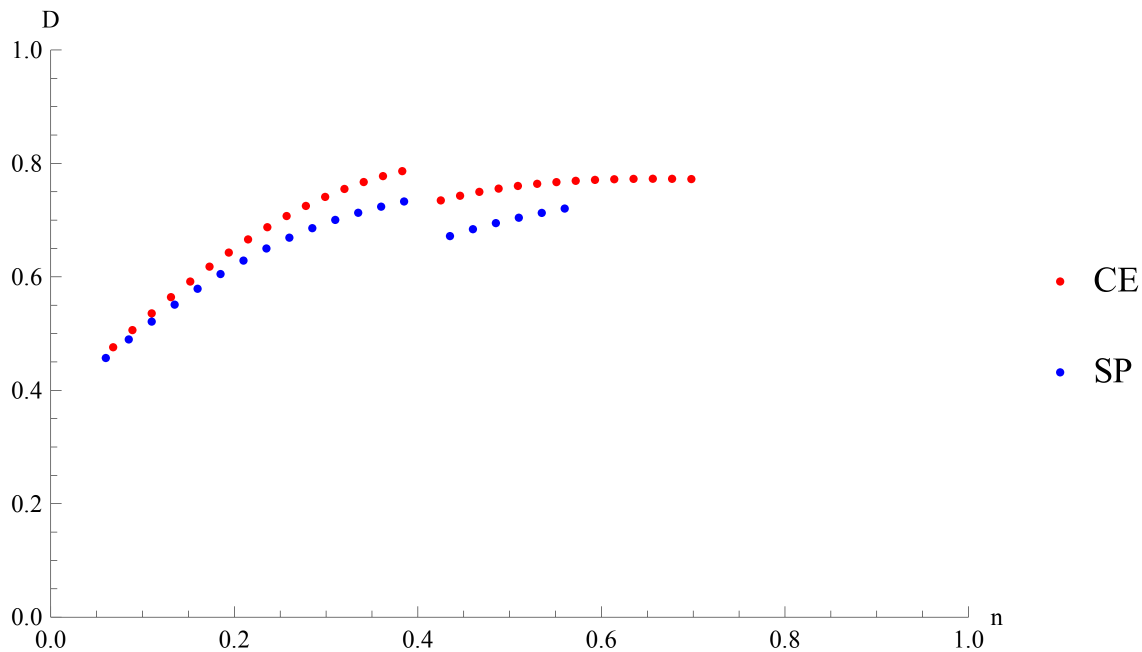

Figure 2.11: The equilibrium level of the probability of default for the economy with social planner and in case of an economy with competitive equilibrium, $s = 0.3$ and $h = 0.9$.

The policy of a price commitment by a regulator are viewed as a desirable ex-ante regulatory policy by some researchers.[22] In the presence of a multiple equilibrium outcomes this might be indeed a useful tool to influence the expectations of the financial players regarding the future asset prices. However, the present framework highlight the incentives that such policy gives to banks to take more risky debt, which can increase their default risk and thus increase the supply of the assets in the secondary market. In addition, this policy might turn out to be not only increasing the risk of banks, but will necessitate the actual intervention in the asset market by a regulator. Hence, without putting a limit on the amount of collateralized debt banks can take, the actual ex-post intervention of the regulator in the secondary market seem to be highly likely.

[21] As explained in the previous section, this does not always hold for the economy without the friction.

[22] See Kuong (2015) for the details

2.7 Conclusion

This paper investigates the impact of a fire sale externality on the capital and liability structure of banks when bank's capital structure and market price of assets are endogenously determined in the market equilibrium. I show that in response to the externality banks reduce their total debt. However, they either reduce the long-term unsecured or short-term collateralized debt, depending on the level of asset liquidity. The model predicts that higher asset liquidity encourages banks to take more short-term collateralized debt to the detriment of a long-term unsecured debt, and this increases the risk of a fire sale spillover. Further, I find that in a social planner's economy the level of bank's debt is lower then in the competitive economy. The results put in question such regulatory policy as price commitment and support the policy of minimum capital requirements for non-depository institutions.

The model so far assumes no aggregate uncertainty and risk-free collateralized debt. Hence, introducing these two features will be useful to complement and extend the results of the present model.

Chapter 3

Network Topology and a Fire Sale Externality

Introduction

The financial crisis of 2008-2009 highlighted the role of interconnectedness of a financial system in the propagation of shocks from one institution to another. When a financial institution goes bankrupt, it defaults on its obligations towards other banks that are its counter parties, which might deteriorate their own solvency. Therefore, a collapse of a large financial institution can lead to a cascade of defaults because banks have credit exposure towards each other. For example, Duarte and Jones (2017) find that in the midst of the financial crisis of 2009 "network default spillovers can amplify initial losses by up to 25 percent". As a result, a growing body of literature studies how the structure of the interbank connections, network topology, affects the degree to which a shock propagates within a financial system.[1] Network resilience though is mainly analysed as a function of two parameters: the degree of bank interconnectedness and the strength of a shock that hits the system. When the shock is low, a densely connected network serves as a shock absorber, because the losses are divided among a larger number of banks, thus reducing the losses of individual banks and increasing the resilience of a network. Once the shock passes a certain threshold, a densely connected network serves no more as a shock absorber, but as a shock amplifier. This defines a "robust but fragile" feature of the financial system.[2]

However, the empirical literature finds that it is difficult to generate a strong contagion merely through spillover of interbank losses.[3] These papers suggest that more than one source of contagion is necessary to generate substantial systemic losses as observed during the recent financial crisis of 2007-2009. Additional source of contagion can be fire sale spillover, that occurs when assets are sold at a price below its economic value because the natural buyers of these assets are themselves financially constrained.

The purpose of this paper is to analyse the resilience of a financial system to shocks in the presence of two sources of contagion: a network of interbank liabilities and a fire sale externality. I propose a model in which banks have exogenously given liabilities towards each

[1] See Acemoglu et al. (2015), Bernard et al. (2017), Gai and Kapadia (2010)

[2] Acemoglu et al. (2015), Gai and Kapadia (2017)

[3] See Glasserman and Young (2015), Degryse and Ngyen (2004), Georg (2013).

other. Apart for interbank borrowing, banks are funded with deposits and equity. On the asset side, each bank has loans to real economy, loans to other banks, and cash. An exogenous shock hits the system, and some banks are unable to reimburse their interbank liabilities, which reduces the ability of their counter parties to reimburse their own liabilities, creating a default cascade. These banks have to sell their financial assets in a centralised market. The natural buyers of these assets are banks from the network who have enough cash to acquire the liquidated assets. The higher is the asset price, the higher is the ability of banks to payoff their interbank loans, limiting the scale of default cascade and losses in the financial system. On the other hand, low asset price enhances the default cascade.

To bypass the need to make strong assumptions on the topology of the financial network, I simulate networks with different node degrees and study numerically the resilience of financial networks as the degrees of connectivity changes. The model yields the following results.

First, in a non-concentrated network, in which node degree is binomially distributed, higher network connectivity leads to a higher resilience. This result differs from the one in Gai and Kapadia (2010) and Acemoglu et al. (2015) and can be attributed to the role that endogenously formed liquidation asset price plays. Higher connectivity results in a higher diversification of counter party risk and enables banks which are not hit by the shock to support the asset liquidation price. Higher asset price allows banks who have to sell their assets to reimburse higher fraction of their liabilities, limiting the losses of its counter-parties. In the empirical simulations that I perform the network with the highest degree of connectivity resists to the higher level of shock, and has the lowest frequency of systemic collapse as opposed to networks with lower degree of connectivity.

Second, I analyse the resilience of a network with a core-periphery structure, a network topology observed empirically in the financial sector. While the resilience of the network increases with the degree of connectivity, a concentrated network is more prone to systemic collapse, and is less able to support the asset market price compared to a non-concentrated network because nodes with a high number of connections represents points of fragility in the system. The results support the recent regulatory policy to increase regulatory requirements for the large systemically important financial institutions.

Lastly, the model highlights the importance of the asset policy of price support on behalf of a regulator in a core-periphery network, as this network topology is more prone to a fire sale discounts and default cascade, and policy of asset price support can limit the degree of shock contagion in the network.

Related literature

First, this paper relates to the literature on network topology. Allen and Gale (2000) were the first to pioneer the idea that different network structures could either aggravate or attenuate the crisis in the financial system, and that a more densely connected network offers the advantage of risk diversification, thus increasing the network resilience to shocks. Eisenberg and Noe (2001) propose an elegant framework to determine a clearing system, a payment vector that determines the payment each participant in the system will receive from its counter parties, if some banks in the system are hit by a shock. They show that the payment vector exists and is unique. I use their approach to compute the payments that banks make to each other following a shock. Gai and Kapadia (2010) use network simulations to evaluate system fragility without imposing any assumptions on a particular network topology. The present paper differs from Gai and Kapadia (2010) in that I incorporate a centralised market for the assets of defaulted banks and evaluate how two sources of shock propagation affect the resilience of the financial system. Elliott et al. (2014) analyze networks as a function of the degree of integration and diversification, where integration is thought of as the fraction of firm's assets held by other firms in the network, i.e. the lower is the fraction of assets hold by other firms, the lower is the integration. Diversification refers to the number of counterparties a firm has. Contrary to the present paper, in the model of Elliott et al. (2014) there is no secondary market for assets of liquidated banks, and the asset liquidation price is assumed to be zero.

Bernard et al. (2017) analyse an optimal regulatory intervention in Eisenberg-Noe framework and show that regulator's credibility not to bail out financial institutions decreases in exogenousl given liquidation costs and in the degree of shock propagation, which depends on network topology and strength of the shock. Gai and Kapadia (2011) introduce repo lending in the framework of and Gai and Kapadia (2010) and estimate how the shocks to the margin haircuts can affect the resilience of a financial network.

Acemoglu et al. (2015) demonstrate the "robust yet fragile" feature of a highly interconnected financial system: for low level of shocks, highly interconnected network serves as a shock absorber, however for strong shocks a densely connected network serves as a shock propagator and poorly connected network is more resilient. While they show analytically the robust but fragile nature of financial networks, they restrict their attention to regular networks, in which interbank liabilities of each institution equal its interbank assets. In contrast, I do not make an assumption of a regularity of a network and allow for interbank assets to be endogenously determined as a function of simulated interbank liabilities. Caballero and Simsek (2013) demonstrate how uncertainty about the creditworthiness of other financial institutions leads to the freeze in the trading of assets in the secondary market and to a fire sale liquidation. Glasserman and Young (2015) in a fairly general framework develop bounds on the losses of banks due to network contagion.

Degryse and Nguyen (2007) as well as Boss et al. (2004) empirical analyse the structure of banking network for the Belgian and Austrian financial systems respectively. They document a shift in the network structure from equally distributed links to a core-periphery, in which "a few "money-center banks" are linked together and linked to otherwise disconnected banks".[4]

Next, the present paper relates to the literature on a fire sale externality. Shleifer and Vishny (1992) pioneered the idea of the fire sale liquidation and show that an asset can be solved below its economic value if the natural buyers of this asset are themselves financially constrained. Diamond and Rajan (2011) argue that banks' anticipation of fire sales leads to even deeper fire sales and to a credit freeze. Acharya et al. (2011) show how the amount of liquidity that banks hold is affected by the prospects of potential fire sales: with high prices the pledgeability of risky assets is high, and there is little incentive to hold cash. Hence, a sudden adverse shock leads to a situation when financial institutions overall have little liquid assets to support the falling prices. Bernardo et al. (2015) study the impact of a fire sale prices on the debt capacity of a firm in a framework of a rational expectations equilibrium. Higher liquidation price of an asset increases the borrowing capacity of a firm because debtholders expect to receive more in bankruptcy.

This paper is organised as follows. Section 1 presents the model. Section 2 contains numer-

[4] Degryse and Nguyen (2007)

ical simulations for network with binomial degree distribution. Section 3 presents numerical simulations for a core-periphery network with exponential degree distribution. Section 4 concludes.

3.1 Model description

There are N banks in the system which are connected to each other through interbank borrowing links. There are 3 periods, 0, 1, and 2.

At $t = 0$ interbank borrowing connections are established. The endogenous network formation is out of the scope of this paper, and it is assumed that the network of liabilities is exogenously given. On the asset side, a bank holds cash, short-term claims against other banks and a portfolio of loans to the real economy. On the liability side, the bank is financed with deposits, short-term borrowing from other banks and equity. Deposits are senior claims and have priority in bankruptcy over the interbank debt. All banks are of the same size. Table 3.1 shows the composition of the bank's balance sheet.

Assets	Liabilities
Cash C_i	Deposits D_i^{dep}
Interbank Assets A_i^{IB}	Interbank borrowing D_i^{IB}
Loans A_i^{Loans}	Equity Eq_i

Table 3.1: Composition of the bank's balance sheet.

The composition of the liabilities is exogenously given. On the asset side the amount of cash is as well exogenous. The level of interbank assets A_i^{IB} and loans A_i^{Loans} are endogenously determined: given that I simulate the interbank liabilities, the interbank assets are determined as a function of the simulated liabilities. Loans to the real economy are a balancing item that ensures that bank's total liabilities equal total assets.

At $t = 1$ two events take place: banks have to reimburse their interbank liabilities, and the system is hit by a shock. When a shock hits a system, a given number of randomly selected banks experiences a decrease in the value of their assets and default on their interbank liabilities. Further, I also consider system-wide shocks, when all banks simultaneously face a shock to their assets.

At $t = 2$ the loans to the real economy pay off. At $t = 1$ the value of a unit of such a loan is 1. Therefore, A_i^{Loans} corresponds both to the total value and to the quantity of loans held by a bank i.

3.1.1 Network structure

In order to simulate a network, I simulate the interbank liabilities: with a given probability n_c a bank i borrows from a bank j, where n_c reflects the degree of network connectivity. The network of interbank liabilities is represented by a directed graph, in which a node is a bank, and a link pointing from node i to node j shows that bank i took a loan from bank j. Therefore, the *out-degree* of a directed graph, which corresponds to the number of links that go out of a node, gives the total number of banks from which a given bank has borrowed. The *in-degree* corresponds to the number of links that point to a node and shows to how many banks a given bank has lent money. The expected number of connections, in - and out- degree, for each bank is

$$E(degree) = n_c(N-1)$$

Note that the number of average connections increases in the number of banks in the system. Even if the probability to create a connection is low, $c_i = 0.08$, as the number of potential counter parties grows, so does the average number of connections each bank has. Figure 3.1 illustrates two type of network topologies: one is a fully connected network, $c_i = 1$, the other is poorly connected, $c_i = 0.08$.

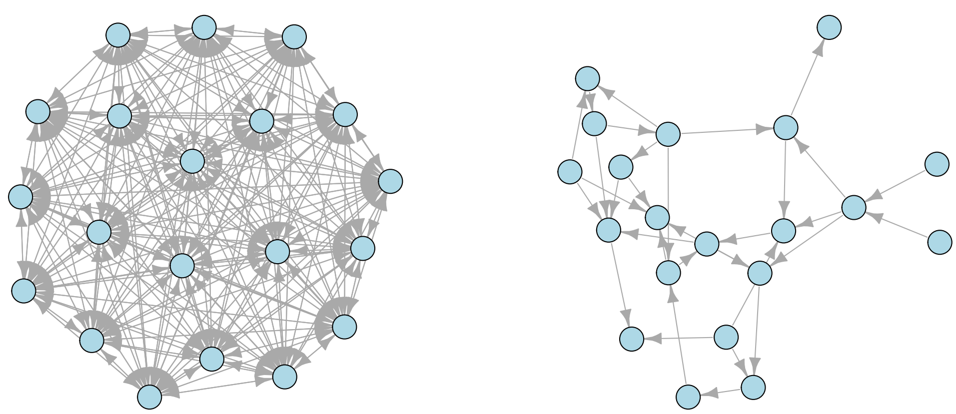

Figure 3.1: Two network topologies for interbank liabilities, one is fully connected network, the other is weakly connected, number of banks is $N = 20$.

Two banks can hold mutual claims against each other. These claims are not netted out. The interbank assets of each bank are endogenously determined given the structure of interbank liabilities. The liabilities of each bank are evenly distributed across its counter parties. However, the interbank assets of each bank are not evenly distributed for the following reason. Consider a bank 1 has out-degree of 3 which means it borrowed from 3 different banks. A bank 2 has out-degree of 4. Let bank 3 be a lender both to bank 1 and bank 2. Given that all banks are of the same size the balance sheet size can be normalised to 100. The total value of the interbank assets is 20. As a result, bank 1 borrowed $\frac{20}{3}$ from each of its counter parties. Bank 4 borrowed $\frac{20}{4}$ from each of its counter parties. Bank 3, as a counter party of both banks, has thus a claim of $\frac{20}{3}$ against bank 1 and a claim of $\frac{20}{4}$ against bank 2.

Consider bank j that is hit by a shock and defaults on its liabilities. Denote d_j^{out} as out-degree of bank i. A bank i has a claim against bank j which is worth 0 after the default of bank j. Bank i is able to honour all its liabilities if its available cash at $t = 1$ is greater then the liabilities it has to pay at $t = 1$. Its available cash is composed of the cash it hold from period 0 plus the amount of interbank claims which are reimbursed by its financial counter parties. The bank has to reimburse its interbank liabilities D_i^{IB}. Therefore, the solvency condition of a bank i is determined as

$$\begin{cases} C_i + \overline{A_i^{IB}} - D_i^{IB} > 0 \\ C_i + \overline{A_i^{IB}} + A_i^{Loans} \geq D_i^{IB} + D_i^{dep} \end{cases} \tag{3.1}$$

where $\overline{A_i^{IB}} = A_i^{IB} - \frac{D_j^{IB}}{d_j^{out}}$ and denotes the interbank assets bank i received following the default of bank j. The first expression in 3.1 defines the ability of a financial institution to meet its short-term liabilities given the resources they have at $t = 1$. Second inequality ensures that the reimbursement of the short-term interbank liabilities is not done at the expense of the depositors. Note that though deposits are short-term claims, when they are guaranteed by the government, the depositors have no incentives to run on a bank. Therefore, the second assumption is equivalent to assuming that short-term creditors are not reimbursed at the expense of the government that guaranteed the deposits.

If the solvency condition in system 3.1 is violated, bank j is insolvent and has to sell its loans in the centralised market to raise cash and payoff its liabilities. For simplicity I assume that the bank sells all the loans, no partial liquidation is allowed.[5] Denote P the equilibrium market price of such an asset, $P \in (0, 1)$. The upper and lower boundaries for the price are motivated by the fact that natural buyers of this asset won't pay more than the economic value of these loans, which is 1. At the same time if the demand for such loans is weak, the equilibrium price can go below 0 and the loans are sold at a fire sale price. The market clearing condition that determines P is described below.

After the loan portfolio is sold at a price P, the depositors are reimbursed first as priority claimholders. Financial counter parties are reimbursed proportionally to their claims hold against a defaulted bank, in equal shares of equal priority of the short-term unsecured creditors. Let $\overline{D_i^{IB}}$ be the interbank liabilities that bank i is able to reimburse in bankruptcy:

$$\overline{D_i^{IB}} = \min\left(D_i^{IB}, \max\left(0, C_i + \overline{A_i^{IB}} + PA_i^{Loans} - D_i^{dep}\right)\right) \tag{3.2}$$

where $\max\left(0, C_i + \overline{A_i^{IB}} + PA_i^{Loans} - D_i^{dep}\right)$ reflects the residual payoff that is left to interbank lenders after depositors are reimbursed. It is bounded by 0 reflecting the limited liability of the shareholders: in bankruptcy the losses to shareholders are limited by their equity stake in the company. The minimum is taken to make sure that debtholders do not receive a payment greater than the face value of their claims.

I formulate the problem using the approach developed by Eisenberg and Noe (2001). Let p be the clearing vector: it reflects the proportion of interbank claims each bank is able to reimburse, where p_i is a proportion of the interbank claims D_i^{IB} a bank i can reimburse. Then equation 3.3 can be rewritten in the following way:

$$p_i D_i^{IB} = \min\left(D_i^{IB}, \max\left(0, C_i + \sum_{j=1, j\neq i}^{N} p_j A_{i,j}^{IB} + PA_i^{Loans} - D_i^{dep}\right)\right) \tag{3.3}$$

[5] While this assumption inflates the supply of the assets and thus decreases the equilibrium price, this does not alter the results: with a partial liquidation one would need a greater shock to generate the same price discount, but this would not modify the comparative statics across different degrees of network connectivity.

where $p_j A_{i,j}^{IB}$ is the proportion of a claim $A_{i,j}^{IB}$ a bank i receives from bank j, given the fraction of the liabilities bank j can reimburse, p_j. If a bank i has no claim against bank j, then $A_{i,j}^{IB} = 0$.

The clearing vector for the entire system, written in a matrix form, is the following:

$$p(D^{IB})' = \min\left(D^{IB}, \max\left(0, C + A^{IB}p + PA^{Loans} - D^{dep}\right)\right) \tag{3.4}$$

where p is the payment vector, D^{IB} is a vector of interbank liabilities where element i denotes total interbank liabilities of a bank i. A^{IB} is a matrix of interbank assets: the element $A_{i,j}^{IB}$ denotes the amount bank j owes to bank i. The matrix has 0 on the diagonal. A^{Loans} is the vector of total loans each bank holds, D^{dep} is a vector of deposits.

3.1.2 Market clearing

The market price P of the loans is determined in the market clearing condition. Though I do not impose that all banks hold the same type of loans, I assume that the characteristics of these loans, i.e. risk and return, yield the same price. If the price of a portfolio of loans of one insolvent bank is below the price of a loan portfolio of another insolvent bank, then natural buyers of these loans will shift their demand for loans with lower price, until the prices on both types of loan portfolios are the same. Therefore, the supply of all these loans can be pulled together.

The demand for these loans comes from the banks in the network who have available resources to purchase the loans. I assume that banks are not strategic in their decisions and that they won't hold cash in the anticipation of even stronger fire sale discounts: banks will buy the loans if they have extra cash and if the market price of these loans is not above their economic value, $P \leq 1$.

Let M be a subset of banks who went bankrupt and liquidated their assets, with the number of elements m. $N - M$ is therefore a subset of banks that are solvent, with the number of elements being $N - m$. Total asset supply can be written as

$$Supply = \sum_{i \in M} A_i^{Loans} \tag{3.5}$$

The ability of a solvent bank to acquire these loans is determined by how much cash it holds, and the amount of loans it was repaid by other banks, minus the interbank liabilities this bank has itself:

$$Demand_j = C_j + \overline{A}_j^{IB} - D_j^{IB}$$

Total available cash to purchase the liquidated loans writes as:

$$Demand^{Total} = \sum_{j \in (N-M)} \left(C_j + \overline{A}_j^{IB} - D_j^{IB}\right)$$

The equilibrium price is the level of price that sets demand equal to supply. The market clearing condition has the following form:

$$P \sum_{i \in M} A_i^{Loans} = \sum_{j \in (N-M)} \left(C_j + \overline{A}_j^{IB} - D_j^{IB} \right) \tag{3.6}$$

If $\sum_{i \in M} A_i^{Loans} < C_i + \overline{A}_i^{IB} - D_i^{IB}$, then the price is $P = 1$ and only a fraction of solvent banks with available cash acquires the liquidated assets. Note however, that the demand for the assets depends on the amount of interbank liabilities solvent institutions have received, $\overline{A}_j^{IB} = A^{IB} p$, where p is itself a function of the liquidation price P, as can be seen from the equation 3.4. The higher is the liquidation price of the assets, the higher is the payment vector p. At the same time, a higher payment vector enables solvent banks to have enough cash to acquire assets of the liquidated banks and support the asset price.

The endogenous determination of an asset price given the balance sheet structure of the financial institutions is a modelling feature that differs from the models proposed by Acemoglu et al. (2015) and Gai and Kapadia (2010), who assume exogenously given level of liquidation price, which is independent of the ability of the financial system to absorb the financial assets at their economic value.

3.2 Numerical simulations

I calibrate the model and simulate possible network connections in order to evaluate the impact of the centralised asset market on a network stability. The network is simulated using Poisson distribution: each pair of nodes in the network is connected with a probability n_c. The higher is n_c, the higher is the network connectivity. I consider a network of 50 banks. The calibration parameters are presented in the Table 3. The higher the ratio of interbank liabilities towards other liabilities, the lower is the resilience of any network because banks have a higher exposure to other banks and they become more sensitive to the shocks that hit other financial institutions. Given that the main purpose of this paper is to compare the resilience of different network structures to shock of the same size, with all other things being equal, the size of liabilities w.r.t. to the total size of the balance sheet is not important.

Parameter	Description	Baseline calibration
N	number of banks in the network	50
C	bank's cash	2% of the total assets
A^{IB}	interbank assets	endogenously determined
A^{Loans}	loans to real economy	balancing item
D^{IB}	interbank liabilities	35% of the total liabilities
D^{dep}	deposits	61% of the total liabilities
E	bank equity	4% of the total liabilities
c_i	degree of network connectivity	$(0.08, 0.12, 0.16, 0.20, 1)$
$E(degree)$	average degree	$(3.92, 5.88, 7.84, 9.8, 49)$

Table 3.2: Model parameter calibration

For each scenario I simulate 500 networks, and compute the average market price, as well as the average number of defaults generated by the spillover.

3.2.1 Simulation results with idiosyncratic shocks

For each degree of connectivity I assume that f number of randomly picked banks fail following an exogenous shock, $f \in (2 : 20)$. Figures 3.2 and 3.3 show the total number of defaults and the equilibrium asset market price, following an exogenous shock where a number f of banks go bankrupt and default on interbank liabilities. The horizontal axis ”Stressed Banks” shows the number f of initially distressed banks. The vertical axes contain respectively the number of total defaults (including exogenously defaulted f banks), and the equilibrium market price that result from exogenous f defaults. Figure 3.2 shows that the higher is the connectivity of the network, the greater is the ability of the system to support the asset price, and hence the ability of the banks concerned by the initial defaults to reimburse their liabilities.

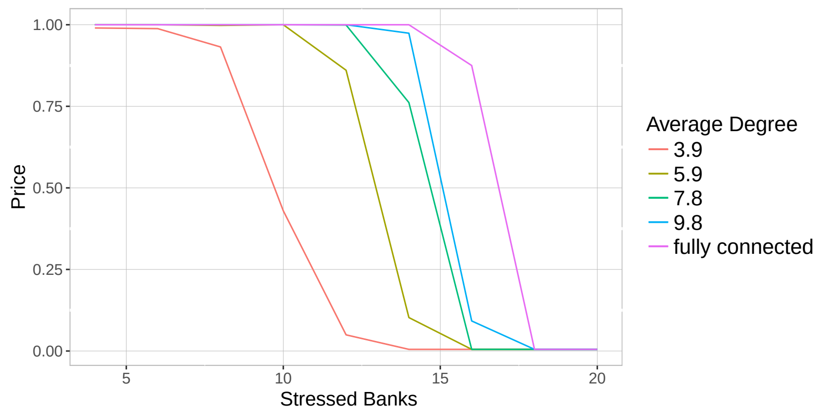

Figure 3.2: The average equilibrium asset price.

The equilibrium asset price in the least connected network decreases faster then in more connected networks, and this affects the average dynamics of defaults in the system. When the liquidation price is lower, financial counter parties suffer greater losses, and their ability to reimburse their own liabilities reduces, as demonstrated in the equation 3.3. Figure 3.3 shows that the number of defaults rises faster for less connected networks.

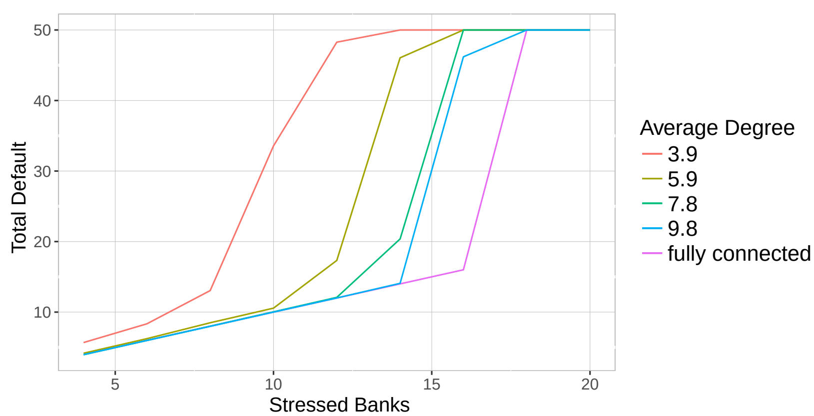

Figure 3.3: The average number of total defaults in the system, including stressed banks that went bankrupt because of an exogenous shock.

The numerical simulations suggest that the higher is the network connectivity, the higher is the liquidation price and lower is the average number of defaults in the system. Therefore, higher network connectivity increases the network resilience. Acemoglu et al. (2015) define network resilience as an expected performance of a financial network following f negative shocks.[6] Their analytical result is based on the assumption that the asset liquidation price across all network topologies is the same. As demonstrated in figure 3.2, the asset price weakly increases in the degree of connectivity. Therefore, as numerical simulations suggest, once the asset liquidation price is endogenised network resiliency increases with the network connectivity. Another reason for why I do not find the same results as in Acemoglu et al.

[6] Formally, Acemoglu et al. (2015) define network resilience as an expected social surplus in the economy following f negative shocks. The social surplus increases in the liquidation price, and decreases in the number of defaults in the system.

(2015) is that they restrain their attention to regular networks, networks in which the total interbank assets of any bank in the network equal its interbank liabilities. Thus, there are no net borrowers and net lenders. In the numerical simulations presented in this paper no such assumption is made: any bank in the network can be either net borrower or net lender in the interbank market. Lastly, Acemoglu et al. (2015) show that a network, in which there is a subset of banks with very high exposure towards each other and a very low exposure towards the rest of the system, is more resilient than ring or fully connected networks. Numerical simulations method might not account for any particular network configuration, and therefore might miss out this result.

A system wide stress, when assets of all banks are simultaneously hit by a shock, yield similar results: a more connected network is more resilient than a poorly connected network. The results of the simulations are presented in the Appendix .10.

3.3 Core-periphery network topology

In this section I present the methodology used to simulate a core-periphery network, as well as the simulation results. This network type is of particular because it is documented empirically that financial networks exhibit a core-periphery structure.[7]

To simulate such a network, I follow the algorithm suggested by Newman et al. (2001). I assume that the *in* and *out* degree of each node follows an exponential distribution. Further, I assume that the *in* and *out* degrees of each node are positively correlated, with a correlation ρ.[8] Figure 3.4 offers an example of such network.

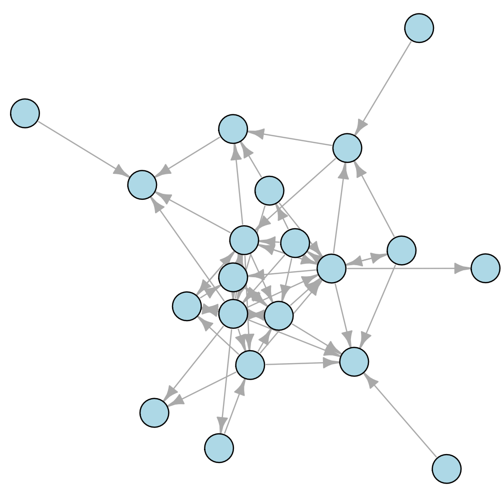

Figure 3.4: Core-periphery network topology, the number of banks is $N = 20$.

Given that in this simulation I would like to capture a feature that a bank that is actively lending is also actively borrowing, which implies a strong positive correlation between the

[7]See Degryse and Nguyen (2007).

[8]To simulate two correlated random variables from an exponential distribution, I first simulate two independent variables from the distribution, and then I apply a transformation to one of the generated arrays using an R package *mc2d*. The resulting two vectors of simulated variables have a specified Pearson correlation ρ.

in and *out* degrees of each node, two assumptions used in the positive section are mutually exclusive: that the interbank liability take a given fraction of the bank's balance sheet (35%), and that all banks are of the same size. Consider a network in which a highly connected bank j has *in* degree of 10, which means that the bank lent money to 10 banks. Next, consider that these 10 banks have only one counterparty, the bank j. The assumption that all banks are of the same size and that interbank liabilities represent 35 of bank's balance sheet means that bank j has more interbank assets than the presumed size of its balance sheet.[9] To overcome the issue, I perform the following modification: after the network is simulated and the value of each interbank connection is established in the same way as for a network with Poisson distribution, I find banks whose interbank assets exceed 90% of the balance sheet.[10] For these banks I increase the size of the loans to the real economy, A^{Loans} by an exogenously given amount h_i. To preserve the balance sheet identity I also increase the amount of bank's deposits. As a result of this transformation, banks with the highest number of connections have a larger size, but for these banks the total value of interbank liabilities is not any longer 35% of the assets, but less. This transformation allows to solve the issue with highly connected banks, to capture the observed feature that more connected banks tend to be larger (Li and Schuerhoff (2017)), and by not increasing the value of the interbank liabilities of the highly connected banks it does not create the problem for other banks that the total interbank assets exceed the size of the balance sheet. When a bank is hit by an exogenous shock, its assets are reduced by the an exogenously given amount z_i, the same for all banks hit by the shock. This means that more centrally connected banks which are larger and which are hit by the shock experience a reduction in the asset value by the same absolute amount as smaller banks. This assumption keeps the size of the shock constant and as a result it allows a comparison of the simulation results not only for the core-periphery network, but also a comparison of the core-periphery network with the one with binomial degree distribution. The parameters used for the model simulation are presented in Appendix .11. The simulation results, presented in the figures 3.5 and 3.6 show the results similar to those of the network with binomial degree distribution: a higher connected network is able to better support the asset price and as a result is more resilient to the propagation of shocks. The frequency of systemic collapse for the core-periphery network is presented in the Appendix .12 and its results are similar to those of a binomial network.

[9]Under the normalisation that bank's balance sheet is 100, the interbank liabilities of bank j are $10 \times 35 > 100$.

[10]The value of each bank's interbank liabilities cannot exceed 35% by construction.

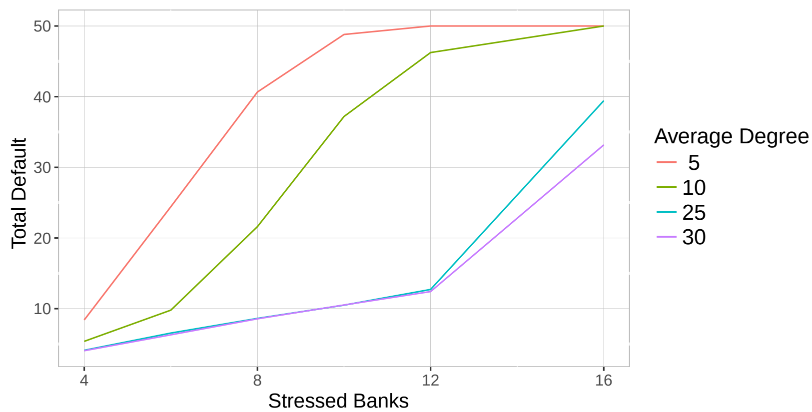

Figure 3.5: Simulation results for core-periphery network, total number of defaults.

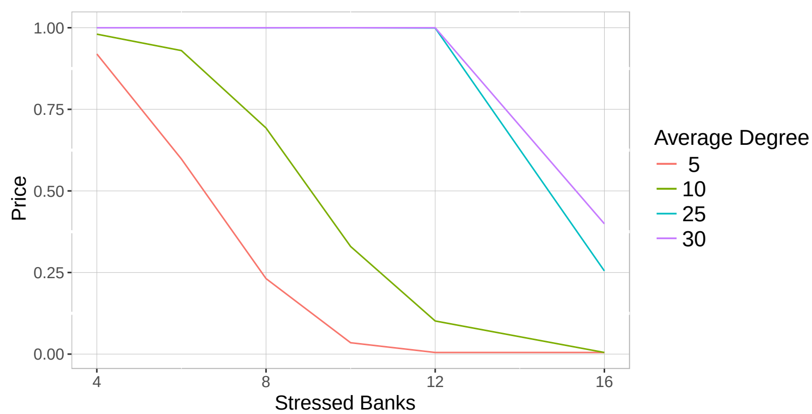

Figure 3.6: Simulation results for core-periphery network, total number of defaults.

It is of interest to compare the resilience of a network with binomial degree distribution to that of the network with exponential degree distribution. While there is evidence that networks with exponential degree distribution are more resilient than networks with the binomial degree distribution[11], it is of interest however to compare the resilience of both types of networks once prices are endogenised. To this end, I perform the simulation of a network with a binomial degree distribution, but unlike in section 3.2, I use the same assumptions (see appendix .11) as for the simulation of the core periphery network.

As presented in the figures 3.7 and 3.8, that depict the asset price in networks with exponential and binomial degree distribution, unconcentrated network with a binomial distribution is better able to support the asset prices in the system, both in cases when the networks are poorly and densely connected.[12] Figure 3.7 shows that a poorly connected non-concentrated network is able to sustain a larger shock size until it collapses than a concentrated network with exponential degree distribution. At the same time a network with binomial degree is able to support to asset price for larger values of shock, thus exhibiting a lower rate of network collapse than a concentrated network with exponential distribution. Similar results hold for

[11]See Gai and Kapadia (2011), Albert et at. (2000)

[12]Average degree for the two types of networks are 5 and 30 respectively

the frequency of systemic collapse: it is higher in a core-periphery structure. A higher frequency of systemic collapse can be explained by the fact that if a highly connected bank is hit by a shock, it renders the entire system more fragile, thus constituting a weak point in the system. At the same time, given that I assume that a system is hit by the same level of shock regardless of the size of the balance sheet, the effect of a weak point in a concentrated network is mitigated.

First, these results support the regulatory policy of increased capital and regulatory control of large systemically important financial institutions, as those are hit by a shock and are unable to sustain it, the shock wave will be much larger. Second, a policy of price support is of higher importance for core-periphery than for non-concentrated networks, as the former network is more prone to system-wide collapse.

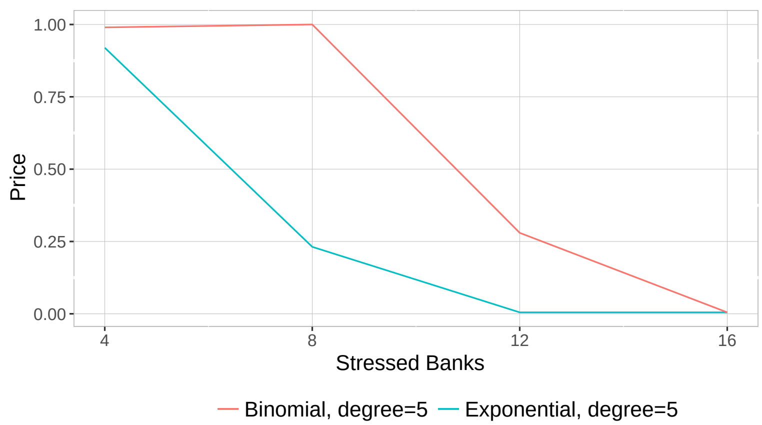

Figure 3.7: Comparison of the equilibrium price for network with exponential and binomial distribution, when the average degree is 5.

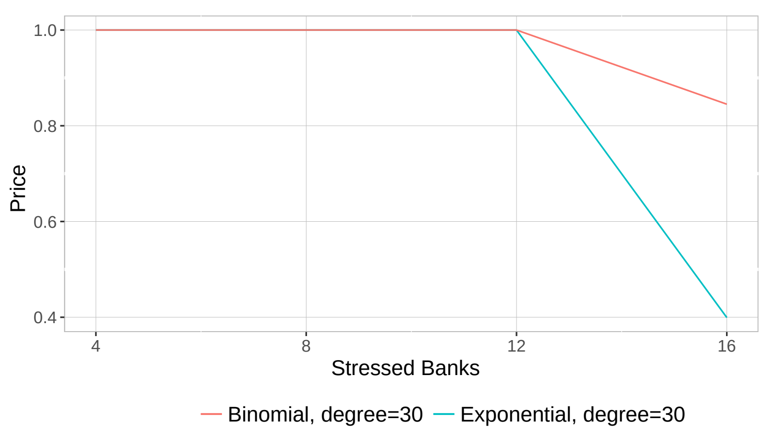

Figure 3.8: Comparison of the equilibrium price for network with exponential and binomial distribution, when the average degree is 30.

3.4 Conclusion

The paper aims to analyse the resilience of a financial network with two sources of network propagation: a network of interbank exposures and a fire sale liquidation. To this end I develop a framework in which banks borrow from each other thus creating a network of interbank liabilities. Banks also finance loans to real economy. When some banks are hit by a shock, they are unable to reimburse their interbank liabilities, which can create a default cascade. When banks are unable to reimburse their debt, they have to sell the loans they issued to the real economy in the centralised market. Other banks from the system are the natural buyers of these loans and if they have enough cash, they will acquire these loans. If the ability of the natural buyers to acquire these loans is limited by the losses banks incur through interbank liability network, the loans are sold below their economic value at fire sale prices.

The model yields the following results. I find that a more connected network is able to withstand a higher level of shock as opposed to poorly connected network and it is able to better support the asset price. Higher asset price enables banks to reimburse a higher fraction of their interbank liabilities which in turn helps support the asset price in the system. This creates a mutually enforcing mechanism: higher price leads to higher recovery value in default, which in turn leads to higher market asset price.

The comparison of concentrated networks (exponentially distributed network) versus non-concentrated networks (binomial degree distribution) demonstrates that non-concentrated networks are more prone to systemic collapse, and are less able to sustain the asset price in the market. The results suggest that a sound financial position of banks which are centrally located in the network is critical for the stability of such networks and thus support the recent regulatory policy to increase regulatory requirements for the large systemically important financial institutions.

Appendix

.1 Optimisation problem of non-securitising bank

Solution of the optimisation problem of a non-securitising bank that issues secured on balance sheet debt.

At $t = 1$ a bank already holds loans from the previous period with face value R_0, that will be reimbursed with some probability p_0 determined by the effort a bank put at $t = 0$. On the liability side, it holds deposits k, unsecured debt with face value D_{sec}, plus the deposits in the amount $(1 - g)$ that it receives at $t = 1$.

The manager of a bank maximises the expected shareholder value: it optimally chooses p_1 that weighs the benefit of putting more effort (making a loan safer) against the costs incurred from additional screening. The value function of a such bank is composed of the expected return on the loans, minus the debt claims, minus the cost of loan screening and monitoring. I assume that if both loans default, shareholders get nothing. The realisations of the payoff R_0 and R_1 are i.i.d, the expected shareholder value at $t = 1$ writes as follows:

$$\begin{aligned} V_{t=1}^{NoSec} =& max_{p_1} \Big(p_1 p_0 \left(R_0 + R_1 - k - D_{sec} - (1-g)\right) \\ & + p_1(1-p_0) max\left(0, nR_0 + R_1 - k - D_{sec} - (1-g)\right) \\ & + p_0(1-p_1) max\left(0, nR_1 + R_0 - k - D_{sec} - (1-g)\right) \\ & - (\gamma p_0)^2 - (\gamma p_1)^2 - 2g \Big) \end{aligned} \tag{7}$$

where the index $NoSec$ stands for "non-securitising bank". The limited liability insures that in case of default of one loan or of both, the payoff of shareholders is non-negative, hence the term $max\left(0, nR_0 + R_1 - k - D_{sec} - (1-g)\right)$. $2g$ is the total amount of equity that bank shareholders put over two periods, $1 + k$ is the total amount of deposits that a bank collected over two periods, γp_0^2 is the cost of screening per unit of loans. $V_{t=1}^{NoSec}$ must be non-negative, to make sure that shareholders break through.

The first order condition gives the following optimal level of effort of a bank at $t = 1$.

Non-securitising bank at $t = 0$

At $t = 0$ a bank knows what p_1 will be, as a function of the probability p_0 and the promised return R_1. This investment will take place only with a probability $\frac{1}{2}$. At $t = 0$ banks maximise the expected shareholder value, accounting for the possibility that at $t = 1$ there might be no investment opportunities.

$$V_{t=0}^{NoSec} = max_{p_0}\left(\frac{1}{2}\Big(p_0(R_0-k-D_{sec})-(\gamma p_0)^2-g\Big)+\frac{1}{2}V_{t=1}^{NoSec}(p_0)\right) \quad (8)$$

where the expression $\frac{1}{2}\Big(p_0(R_0-k-D_{sec})-(\gamma p_0)^2-g\Big)$ is the expected payoff to shareholders if a bank invests only at $t=0$, and $V_{t=1}^{NoSec}$ is the payoff if a bank also invests at $t=1$, defined in the equation 7.

In the presence of moral hazard the pricing of the secured debt with present value $1-k-g$ is done in the following way. First banks make a take it or leave offer to the market to buy a debt with the face value $\bar{D}_{sec}$, and present value d_{sec}, where $d_{sec}=1-k-g$. Markets observe the offer as well as the level of deposits k and the promised payoff on the investment R_0, and they correctly infer the effort that banks will choose to put into loan screening. This level of effort also determines the probability with which the debt holders will be reimbursed If the offer $d_{sec}, \bar{D}_{sec}$ is such that prospective buyers of the debt break even in expectation, they will buy the debt. If not, the offer is rejected and a bank is left without additional funding. Banks anticipate this, and they will offer the pair (d_0, D_{sec}) such that the debt is fairly priced. After the debt $\bar{D}_{sec}$ is raised, banks choose the optimal level of effort. This ensures that banks do not have incentives to shirk, and the secured debt is fairly priced.

From the technical perspective this means that banks maximise the shareholder value w.r.t. p_0 for a given value of D_{sec}. p_0 and D_{sec} are jointly determined by the following system of equations:

$$\begin{cases} \frac{\partial V_{t=0}(R_1,R_0,D_{sec})}{\partial p_0}=0 \\ 1-k-g=E(D_{sec}) \end{cases}$$

$E(D_{sec})$ is determined by the prospective buyers of the secured debt who take into account the probability with which they will be reimbursed in full, and the amount they receive if a bank goes bankrupt. Denote a as the fraction of the loan that secures the claim D_{sec}. The face value of debt D_{sec} with the present value equal to $1-k-g$ must satisfy the following equation:

$$\begin{aligned} 1-k-g=&\frac{1}{2}(p_0D_{sec}+(1-p_0)(min(D_{sec},nR_0a+min(D_{sec}-nR_0a,max((1-a)nR_0-k,0))))) \\ &+\frac{1}{2}(p_0D_{sec}+(1-p_0)(p_1min(D_{sec},nR_0a+min(D_{sec}-nR_0a,max((1-a)nR_0+R_1-(1-g)-k,0))) \\ &+(1-p-1)min(D_{sec},nR_0a+min(D_{sec}-nR_0a,max((1-a)nR-0+nR-1-(1-g)-k,0))))) \end{aligned} \quad (9)$$

The first line in 1.13 accounts for the expected payoff in no investment is done at $t=1$, and the rest is the expected payoff if a bank finances the project at $t=1$, and all funding comes from deposits. I solve the model numerically.

.2 Objective function of a securitising bank

The objective function of a securitising bank under the conjecture that $p_s > p_h$

When a bank faces an investment opportunity at $t = 1$, it already has loans in place, some of which are financed through securitisation, and some are financed with the available on-balance sheet funds. At $t = 1$ a bank chooses the optimal level of effort as a function of R_1, and of the optimal level of effort taken at $t = 0$.

The realisations of R_1 and R_0 are i.i.d. The expected equity value of a securitising bank at $t = 1$ under the conjecture that $p_{s,0} \leq p_{h,0}$ writes as follows:

$$
\begin{aligned}
V_{t=1}^{Sec,Ps>Ph} = max_{p_1} \Big(& p_h \Big(p_1 \left(R_0 + R_1 - (1-g) - k - D_{0,s} \right) \\
& + (1-p_1) max \left(0, nR_1 + R_0 - (1-g) - k - D_{0,s} \right) \Big) \\
& + (p_s - p_h) \Big(p_1 max \Big(0, R_0 n(k+g)(1-a_0) + R_0((k+g)a_0 + 1 - k - g) \\
& + R_1 - (1-g) - k - D_{0,s} \Big) \\
& + (1-p_1) max \Big(0, nR_1 + R_0 n(k+g)(1-a_0) + R_0((k+g)a_0 + 1 - k - g) \\
& - (1-g) - k - D_{0,s} \Big) \Big) \\
& + (1-p_s) p_1 \Big(R_0 n (1-a_0)(k+g) + R_1 - (1-g) - k \Big) \\
& - p_s^2 \left((k+g)a_0 + d_{0,sec} \right) - p_h^2 (k+g)(1-a_0) - p_1^2 - 2g \Big)
\end{aligned}
\tag{10}
$$

where p_s and p_h are the success probabilities of a securitised and retained portfolio, determined at $t = 0$ by the effort of a bank. k is the amount of deposits a bank had at $t = 0$, $(1 - g)$ is the quantity of deposits a bank receives at $t = 1$. Note the state-contingent nature of the off-balance sheet debt: in the states of the world $(1 - p_s)$, when a securitised portfolio of loans does not pay off, a bank does not reimburse the holders of the ABS. In these states of the world, a bank looses the fraction $a_0(k + g)$ of the defaulted assets, which is seized by the ABS holders, but they do not have to share with them the profit from a new investment opportunity R_1. $d_{0,s} = 1 - k - g$ is the present value of a securitised debt with the face value D_s,defined as

$$D_{0,s} = -\frac{1}{p_s}((1 - (1 - a)(g + k))n(1 - p_s)R_0 - 1 + g + k)$$

The optimal level of effort is determined by the first order condition: $\frac{\partial V_{t=1}^{Sec}}{\partial p_1} = 0$.

Securitising bank at $t = 0$, $p_s > p_h$

The optimisation problem of a securitising bank at $t = 0$ is has two components: the shareholder value if there is no investment at $t = 1$, and the shareholder value if a bank will invest at $t = 1$:

$$V_0^{sec} = \frac{1}{2} V^{Sec,Ps>Ph} + \frac{1}{2} V_{t=1}^{Sec,Ps>Ph}$$

where $V^{Sec,Ps>Ph}$ is the expected equity value if a bank does not invest at $t = 1$, which writes as:

$$
\begin{aligned}
V^{Sec,Ps>Ph} =& p_h(R_0 - k - D_{0,s}) + (p_s - p_h)(R_0 n(k+g)(1-a_0) + R_0((k+g)a_0 + 1 - k - g) \\
& - k - D_{0,s}) - p_s^2\left((k+g)a_0 + d_{0,sec}\right) - p_h^2(k+g)(1-a_0) - g
\end{aligned}
$$

The optimal level effort for a securitised portfolio and non-securitised portfolio are defined by the first order condition:

$$
\begin{cases}
\frac{\partial V_0^{sec}}{\partial p_s} = 0 \\
\frac{\partial V_0^{sec}}{\partial p_h} = 0
\end{cases}
$$

Note that p_s is taken for a given level of D_s, and therefore when taking the derivative of V_0^{sec} w.r.t. p_s, D_s is taken as constant. The functional form of D_s is plugged in the first order condition after the derivative is taken. The optimal level of securitisation a_0 takes into account the impact on all the variables that it will have. I solve the model numerically: for given values (R_0, n, k, g) I find the optimal p_s and p_h for all a_0 defined on a grid, and then take a_0 that gives the highest value. I do not find parameter values for which an interior solution for this case would exist.

.3 Default threshold

A bank defaults to maximize the expected payoff to shareholders. This means that a bank will default for the asset values for which shareholders get nothing in solvency. The payoff to shareholders in solvency is:

$$Payoff^{SH}_{SOLV} = V(H_1 + 1) - (H_1 + D)$$

I consider only risky level of total debt, hence the payoff to shareholders in default is zero. Therefore, the default threshold for banks is defined by the following expression:

$$V(H_1 + 1) - (H_1 + D) \leq 0$$

$$\hat{V}_{def} = \frac{H_1 + D}{H_1 + 1}$$

When no fire sale liquidation occurs, bank's default threshold is just its total debt divided by the total number of assets:

$$\hat{V}^{NoFS}_{def} = \frac{H_0 + D}{H_0 + 1}$$

However, when banks have to deleverage, those who were close to the threshold $\frac{H_0+D}{H_0+1}$ will be forced into bankruptcy, because they number of their assets will decrease:

$$V(H_1 + 1) - (H_1 + D) = V(H_0 + 1 - m) - (H_0 + D - m * P_1) \leq 0$$

$$\hat{V}^{FS}_{def} = \frac{H_0 + D - mP}{H_0 + 1 - m}$$

Thus, the fire sale liquidation increases a banks default rate, all other things being equal.

.4 Solution of the benchmark case with no fire sale externality

.4.1 Solution of the model

I start the solution by finding the price that would clear the market. The supply of assets is composed entirely from the amount of assets of bankrupt firms:

$$Supply = \frac{1}{2}\int_0^{V_{def}} (H+1)dV = (H+1)\frac{V_{def}}{2} = (H+1)\frac{D+H}{2(H+1)} = \frac{D+H}{2}$$

The demand for the assets is a sum of demands (integral as this is a continuous case) across all banks who have an added value from acquiring the assets:

$$Demand = \frac{1}{2}\int_{\frac{P}{n}}^{2} \frac{(H+1)V-(D+H)}{P}dV = -\frac{(2n-P)(2(D-1)n-(H+1)P)}{4n^2P}$$

Therefore, equilibrium price P_{eq} is such that must equalizes supply and demand:

$$\frac{(2n-P)(2(D-1)n-(H+1)P)}{4n^2P} = \frac{D+H}{2} \tag{11}$$

Next, firms will be conjecturing this level of price and will take the optimal level of debt H and D as a function of price P that will later clear the market as in equation 11.

The expected firm value is the following:

$$\begin{aligned}
&V_0\\
&= \frac{1}{2}\bigg(\int_{\hat{V}_{def}}^{2} VdV + \int_0^{\hat{V}_{def}} PdV + \int_{\frac{P}{n})}^{2} \frac{V(H+1)-(H+D)}{P}(nV-P)dV\bigg) - s(1-D_0) =\\
&= \frac{1}{12}\Big(-\frac{(P-2n)^2(n(3d-H-4)-(H+1)P)}{n^2P} + \frac{6s((H+1)P(D+H)-(D-2)D-H(H+2)-2)}{H+1}\\
&+ 6P(D+H) - \frac{3(D-H-2)(D+3H+2)}{H+1}\Big)
\end{aligned}$$

The first order condition for D is:

$$\frac{\partial V_0}{\partial D} = -((4(1+H)n^2+(1+H)P^2-2nP(2-D+H+P+HP+(2-2D+P+HP)s))/(4(1+H)nP))$$

$$\Rightarrow D_{opt} = -\frac{4(H+1)n^2-2nP(s(HP+P+2)+HP+H+P+2)+(H+1)P^2}{2nP(2s+1)}$$

The FOC for H gives the following roots for H:

$$H = -1 \pm \frac{\sqrt{3}(D-1)^2n^2P(2s+1)}{\sqrt{-(D-1)^2n^2P(2s+1)\left(4n^3+3n^2P(2P(s+1)-2s+3)-3nP^2+P^3\right)}}$$

However, it is straightforward to show in *Mathematica* that the discriminant $-(D-1)^2n^2P(2s+1)\left(4n^3+3n^2P(2P(s+1)-2s+3)-3nP^2+P^3\right)$ is never positive for any values of the parameters allowed in this model. This means that there are no real roots of the derivative which means that V_0 is either increasing or decreasing in H.

Next note that V_0 is discontinuous in H in the point $H = -1$. It is continous over the interval $(-\infty, -1) \bigcup (-1, \infty)$. Therefore, it is enough to determine the sign of the derivative at one point of the interval $(-1, \infty)$, to know the sign over the entire interval. I set $H = 0$:

$$\frac{\partial V_0}{\partial H}|_{H=0} = \frac{3n^2P(2s((D-2)D+P)+(D-2)D+2(P+2))+4n^3-3nP^2+P^3}{12n^2P}$$

This derivative is always positive for the range of parameter values allowed in this model. This means that the optimal level of H_0 is the maximum that banks can take:

$$H_{opt} = \frac{1-h}{h} \tag{12}$$

This gives the following optimal level of debt:

$$D_{opt}^{NoExtern} = -\frac{-2nP(2hs+h+Ps+P+1)+4n^2+P^2}{2hnP(2s+1)} \tag{13}$$

The price is determined by the following market clearing condition:

$$\int_0^{\frac{H_{opt}+D_{opt}^{NoExt}}{H_{opt}+1}} (H_{opt}+1)dV = \int_{\frac{P}{n}}^{2} \frac{V(H_{opt}+1)-(H_{opt}+D_{opt}^{NoExt})}{P} dV \tag{14}$$

where the l.h.s. of the expression is the total supply of assets in the market, and the r.h.s. is the demand for these assets. Once this expression is simplified, it turns out that P_{equil} does not depend on H_{opt}, and consequently on h.

To solve for the equilibrium price P_{eq} and risky debt $D_{eq}^{NoExtern}$, I plug the optimal level of debt 13 and the optimal level of H_0 into the market clearing condition 11 and I find the following polynomial of third degree w.r.t. P:

$$F(P) = -4n^3(P+2)+2n^2P(P(P+4)(s+1)+4)-nP^2(P+2)(2s+3)+2P^3(s+1) \tag{15}$$

The equilibrium price P_{eq} is the root of the polynomial $F(P)$. Even though a polynomial of third degree can have up to three roots, in all the numerical parameter values that I looked at, there is only one root that satisfies the inequalities of this case.

.5 Model solution with a systemic externality, when price is high: $P \in (1, 2)$

When price is high, not only banks do not have to sell anything to meet the collateral constraint in period $t = 1$, but in addition they can borrow more against the collateral because a favourable price movement creates additional borrowing capacity for banks.

The supply and demand for the assets has the following form:

$$Supply = \frac{1}{2}\int_0^{V_{def}} (H+1)\, dV$$
$$Demand = \frac{1}{2}\int_{\frac{P}{n}}^{2} \frac{(H+1)V - (D+H) - mP}{P}\, dV$$

where m is negative, implying that a bank borrow more against collateral and can buy additional $-m$ units of assets in the secondary market, and $V_{def} = \frac{H+D}{H+1}$.

Therefore, the price has to satisfy the following market clearing condition:

$$\frac{(2n-P)(2nD+mP-1)-(H+1)P)+2(H+1)n^2PV_{def}}{4n^2P} = 0$$

The expected firm value takes the following form:

$$\begin{aligned}
&V_0^{EnhDem}\\
&= \frac{1}{2}(H+1)\Big(\int_0^{V_{def}} P\, dV + \int_{V_{def}}^{2} V\, dV\Big) - (1-D_0)s\\
&+ \frac{1}{2}\int_{\frac{P}{n}}^{2} \frac{((H+1)V-(D+H)-m*P)(nV-P)}{P}\, dV
\end{aligned}$$

The FOC gives the following optimal level of unsecured debt D:

$$D_{opt}^{EnhDem} = -\frac{4(H+1)n^2 - 2nP(s(HP+P+2)+HP+H+P+2)+(H+1)P^2}{2nP(2s+1)} \tag{16}$$

The derivative of V_0 w.r.t. H_0 has no real roots for the range of parameters $(0,1)$ and price $P \in (1,2)$. Next, the function $V_0(H_0)$ is discontinuous in H_0 only in the point $H = -1$. This means that V_0 is either monotonically increasing or decreasing over the interval $(-\infty,-1)\bigcup(-1,\infty)$. I find the sign of the derivative for $H_0 = 0$, and this will give me the its sign over the entire interval.

$$\begin{aligned}
&\frac{\partial V_0^{EnhDem}}{\partial H_0}|_{H_0=0} =\\
&\frac{1}{12n^2P}\Big(3n^2P(2s((D-2)D+P)+(D-2)D+6P+4)\\
&+\frac{3n(P-1)(P-2n)^2}{h} - 4n^3(3P-1) - 3nP^2(P+1)+P^3\Big)
\end{aligned} \tag{17}$$

$$\frac{\partial V_0^{EnhDem}}{\partial H_0}|_{H_0=0} > 0\ \forall\ \begin{cases} (h,n,s,) \in (0,1)\\ P \in (1,2)\\ D > 0 \end{cases}$$

This means that banks will take the maximum collateralized debt they can:

$$H_{opt}^{EnhDem} = \frac{1-h}{h}$$

However, when price is within the interval $(1, 2)$ and H is at its max, then the expected value of the risky unsecured debt exceeds 1: $D_0 > 1$. This cannot be done, as the number of assets that bank acquires with debt and equity is normalized to 1. This implies that the corner solution for D is s.t. $D_0 = 1$.

However, there is no price that would clear the market in this case.

.6 Model solution with a fire sale spillover: $P \in (1-h, 1)$

The market clearing condition takes the following form:

$$\begin{aligned}
Supply &= \int_0^{Vdef} (H+1)\, dV + \int_{V_{def}}^{2} m\, dV \\
Demand &= \int_{\frac{P}{n}}^{2} \frac{V(H-m+1) - (D+H-mP)}{P}\, dV
\end{aligned}$$

where m is the number of units banks will have to sell because they exceeded the margin constraint.

The expected firm value is the following:

$$\begin{aligned}
&V_0^{FS} \\
&= \frac{1}{2}(H+1)\Big(\int_0^{V_{def}} P\, dV + \int_{V_{def}}^{2} V\, dV\Big) - (1-D_0)s - \frac{1}{2}\int_{V_{def}}^{2} m(V-P)\, dV \\
&+ \frac{1}{2}\int_{\frac{P}{n}}^{2} \frac{((H+1-m)V - (D+H-mP))(nV-P)}{P}\, dV
\end{aligned}$$

The FOC gives the following optimal level of unsecured debt D:

$$D_{opt}^{FS} = \frac{(H(P-1)+P)\,(2nP(s((2h-1)P+2)+hP+2) - 4n^2 - P^2)}{2hnP^2(2s+1)}$$

The derivative of V_0^{FS} w.r.t. H_0

$$\begin{aligned}
&\frac{\partial V_0^{FS}}{\partial H_0} \\
&= 3\left(\frac{(P-1)P\left(d^2h^2(2s+1) - (h-1)(2hs+h+1)(H(P-1)+P)^2\right)}{h(H(P-1)+P)^2} + 4\right) \\
&+ \frac{(P-1)P}{hn^2} + \frac{4n(P-1)(3(h-1)P+4)}{hP^2} + \frac{3(h-1)(P-1)P}{hn}
\end{aligned} \tag{18}$$

Once D_{opt}^{FS} is plugged in the equation 18, then $\frac{\partial V_0^{FS}}{\partial H_0}$ does not depend on H_0 any more. This means that when D is at the optimal level determined by the FOC, then H_0 optimally takes either its maximum or its minimum level:

$$\begin{cases} \frac{\partial V_0^{FS}}{\partial H_0}|_{D=D_{opt}} > 0 \Rightarrow H_{opt} = \frac{1-h}{h} \\ \frac{\partial V_0^{FS}}{\partial H_0}|_{D=D_{opt}} < 0 \Rightarrow H_{opt} = \frac{P-hP}{1-P+hP} \end{cases}$$

.6.1 Market clearing for the optimal levels of debt H_0 and D

Once the optimal levels of debt H_0 and D are plugged in, I get the following market condition when $H_0 = H_{max} = \frac{P-hP}{1-P+Ph}$:

$$F(P)^{NoFS} = \frac{-4n^3(P+2) + 2n^2P\left(P^2(s+1) + 4P(s+1) + 4\right) - nP^2(P+2)(2s+3) + 2P^3(s+1)}{2n^2P^2(2s+1)((h-1)P+1)} \tag{19}$$

.7 Model solution without a fire sale spillover: $P \in (1-h, 1)$

When banks choose collateralized debt H_0 within the range $\left(0, \frac{P-hP}{1-P+hP}\right)$, again, it turns out that once D_{opt}^{NoFS} is plugged in,[13] the derivative of V_0 w.r.t. H_0 does not depend on H_0 any more. This means that H_0 is either 0 or $\frac{P-hP}{1-P+hP}$. However, the derivative is never negative, hence banks are always better off choosing

$$H_{opt}^{NoFS} = \frac{P-hP}{1-P+hP}$$

However, I show that when within a framework with a fire sale it is optimal to choose $H_0 = \frac{1-h}{h}$, then choosing $H_0 = \frac{1-h}{h}$ gives a higher expected firm value then the level $H_0 = \frac{P-hP}{1-P+hP}$. This means that *the only determining condition of the optimality of H_0 is the optimality condition of H_0 in the model with a fire sale spillover, presented in in the Appendix .6.*

.8 Analysis of the case when $\frac{P}{n} < \frac{D+H-m*P}{H+1-m}$

I analyze a possibility that the threshold above which it is optimal to acquire assets (which is P/n) is below the default threshold, $\frac{D+H-m*P}{H+1-m}$.

The expected firm value under this conjecture takes the following form:

[13] $Dopt^{NoFS}$ is determined by a FOC as in previous cases

$$
\begin{aligned}
&V_0^{FS} \\
&= \frac{1}{2}(H+1)\Big(\int_0^{V_{def}} P\,dV + \int_{V_{def}}^2 V\,dV\Big) - (1-D_0)s - \frac{1}{2}\int_{V_{def}}^2 m(V-P)\,dV \\
&+ \frac{1}{2}\int_{\frac{D+H-m*P}{H+1-m}}^2 \frac{((H+1-m)V-(D+H-mP))(nV-P)}{P}\,dV
\end{aligned}
$$

Given that the FOC gives a system of equations which is not tractable analytically, I search for solutions on the grid. I find that there is a solution for the following values of the parameters:

$$
\begin{cases}
h \in (3/10, 1) \\
n \in (0.765, 1) \\
s \in (0.775, 1)
\end{cases}
$$

.9 Frequency of systemic collapse when a given number of banks is hit by an exogenous shock.

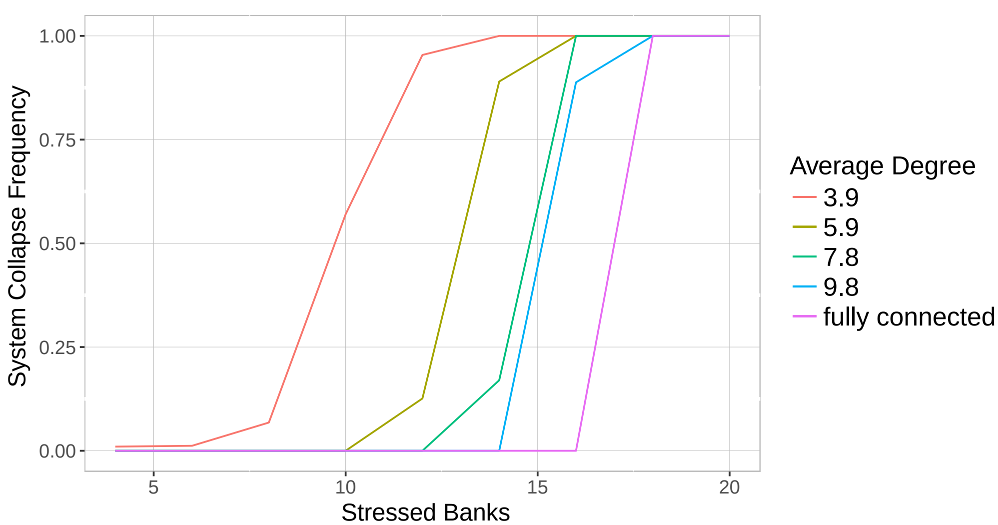

Figure 9: The frequency of systemic collapse in the numerical simulations, as a function of the number of banks that are hit by an exogenous shock.

.10 Simulation results following a systemic shock

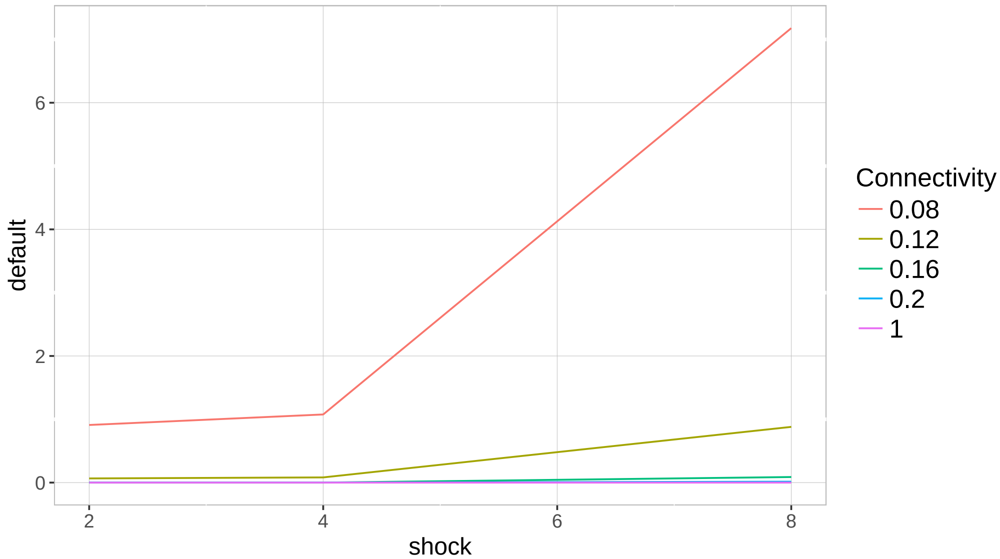

Figure 10: The average number of defaults following a systemic shock, a negative hit to the assets of all financial institutions. X-axes, *shock*, reflects a percentage of the balance sheet of all banks that was wiped out following a negative systemic shock.

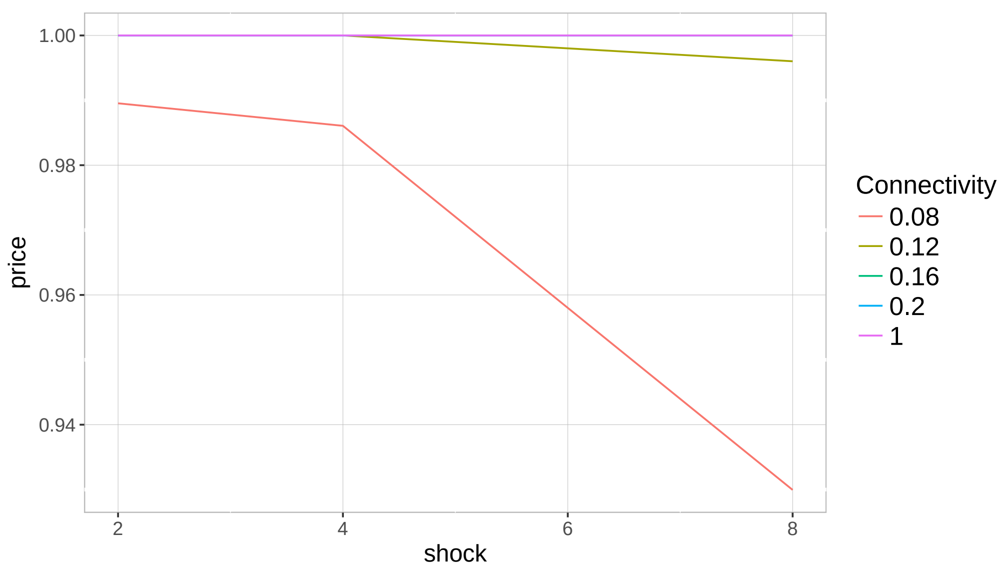

Figure 11: The average equilibrium asset price following a systemic shock, a negative hit to the assets of all financial institutions. X-axes, *shock*, reflects a percentage of the balance sheet of all banks that was wiped out following a negative systemic shock.

.11 Parameters for the simulation of core-periphery network.

Parameter	Description	Baseline calibration
N	number of banks in the network	50
C	bank's cash	2% of the total assets
A^{IB}	interbank assets	endogenously determined
A^{Loans}	loans to real economy	balancing item
D^{IB}	interbank liabilities	35% of the total liabilities
D^{dep}	deposits	61% of the total liabilities
E	bank equity	4% of the total liabilities
h_i	asset increase of core banks	30
z_i	asset decrease of shock-hit banks	60
s_{banks}	number of stressed banks	$(4, 6, 8, 10, 12, 16)$
$E(degree)$	average degree	$(5, 10, 25, 30)$

Table 3: Model parameter calibration for core-periphery network simulation.

.12 Frequency of systemic collapse in a core-periphery network.

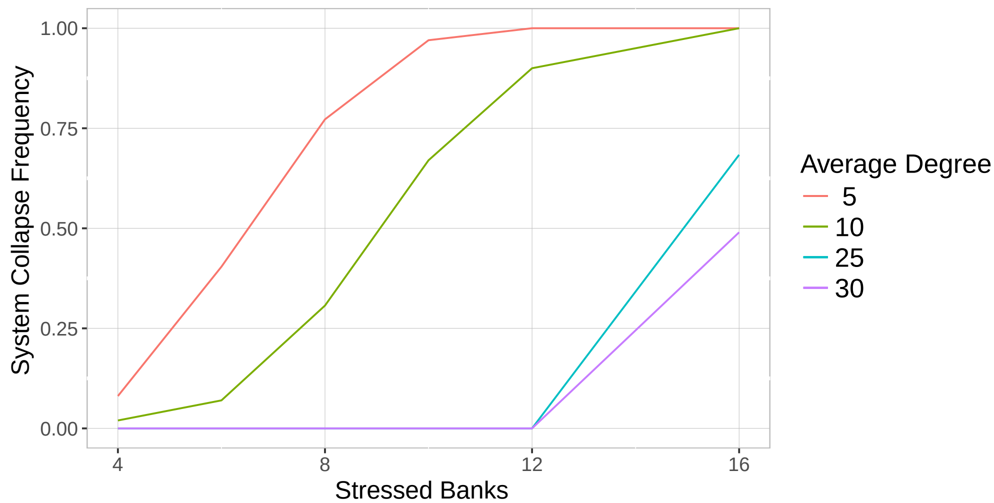

Figure 12: The frequency of systemic collapse in the numerical simulations, as a function of the number of banks that are hit by an exogenous shock. Case of a core-periphery network with exponential degree distribution.

.13 Comparison of the frequency of systemic collapse and number of defaults in networks with exponential and binomial degree distribution.

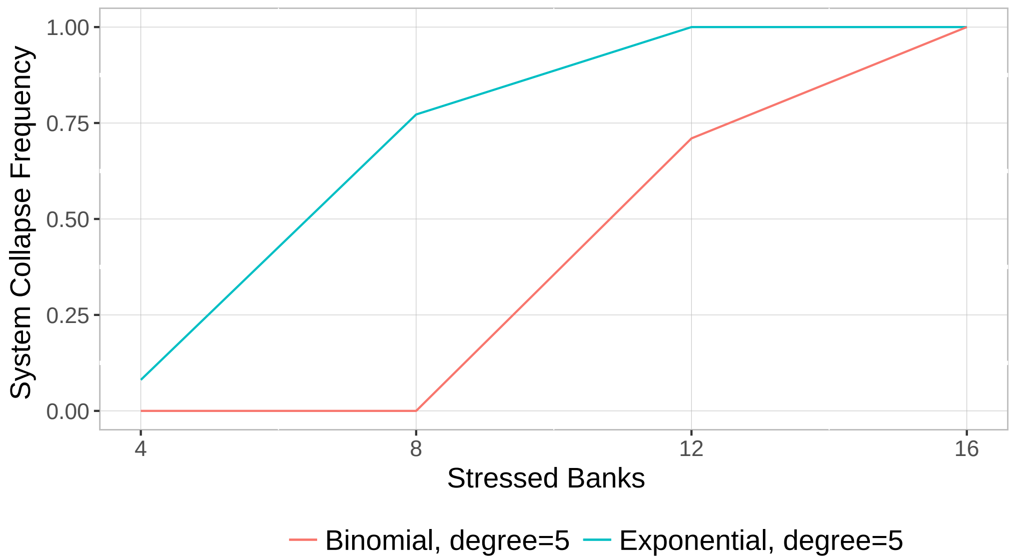

Figure 13: The frequency of systemic collapse in the numerical simulations, as a function of the number of banks that are hit by an exogenous shock, when the average connectivity is 30. Case of a core-periphery network with exponential degree distribution.

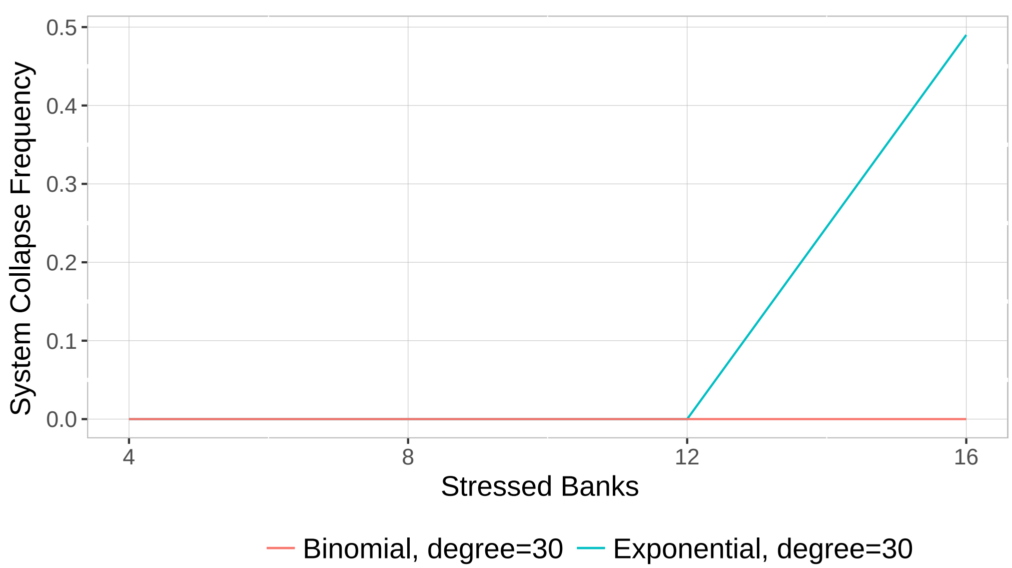

Figure 14: The frequency of systemic collapse in the numerical simulations, as a function of the number of banks that are hit by an exogenous shock, when the average connectivity is 30. Case of a core-periphery network with exponential degree distribution.

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