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Particles and Laws of Nature in Classical and Quantum Physics

Hubert Mario

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UNIL | Université de Lausanne
Section de philosophie

Particles and Laws of Nature in Classical and Quantum Physics

Thèse de Doctorat

présentée à la

Faculté de lettres
de l'Université de Lausanne

pour l'obtention du grade de
Docteur ès lettres

par

Mario Hubert

Directeur de thèse
Prof. Dr. Michael Esfeld

Jury
Prof. Dr. Detlef Dürr
Prof. Dr. Federico Laudisa

Lausanne
Novembre 2016

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Faculté des lettres

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Particles and Laws of Nature in Classical and Quantum Physics

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Lausanne, le 18 novembre 2016

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Section de philosophie

Particles and Laws of Nature in Classical and Quantum Physics

Mario Hubert

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University of Lausanne
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Doctoral dissertation for the degree "Docteur ès Lettres"

Supervisor: Prof. Dr. Michael Esfeld
External referees: Prof. D. Dürr and Prof. F. Laudisa

Contents

Executive Summary

How does the world look on the fundamental level? This question has intrigued physicists and philosophers alike. Most physicists think that the basic objects that constitute the world are particles or fields, and that it is the task of physics to find laws for the behavior of such objects. Philosophers have different categories; many of them think that the world is made of substances and properties, and they debate the status of the laws of nature. Until recently, these two camps worked rather independently of each other. In this dissertation, I combine both disciplines; in particular, I relate the debate about the ontology of quantum mechanics with current results in metaphysics and show that many arguments can be applied to classical physics as well.

What is not yet available is a comprehensive analysis of the ontological status of the most important physical entities, namely, particles, fields, forces, mass, charge, and the wave-function. I have two objectives in filling this gap. First, I argue that the existence of all these entities depends on the status of laws of nature. Second, I show that physics and metaphysics can renounce the concept of properties. In my analysis, I refer to classic texts by Isaac Newton, Albert Einstein, and John Bell, as well as to recent developments in philosophy of physics.

In Chapter 1, I introduce John Bell's notion of local beables and contrast it with the now popular *primitive ontology*. I show that both are indifferent to the status of properties, and so I introduce *primitive stuff ontology*, which consists of local objects that don't bear intrinsic properties. The chapter concludes with a critique of popular versions of ontic structural realism.

In Chapter 2, I discuss in detail the most important metaphysical theories on the status of laws of nature, namely, Humeanism, primitivism about laws, and dispositionalism. Here I develop a novel account of Humeanism that no longer has local qualities as part of its mosaic. I show that the status of properties has been hitherto completely ignored in primitivist frameworks. I then argue that ontic structural realism applied to laws of nature is best understood as relational dispositionalism.

In Chapter 3, I introduce Newtonian mechanics from an ontological point of view. I then discuss whether forces and mass exist.

Chapter 4 deals with classical electrodynamics. Here I emphasize that this theory suffers from a flaw that is as severe as the quantum measurement problem, namely, the *self-interaction problem*. The only way to avoid this pitfall is to develop a new theory of electromagnetic fields. Three strategies are presented: the Wheeler–Feynman theory, the Born–Infeld theory, and the Bopp–Podolsky theory. Moreover, I show that the famous argument by Mathias Frisch on the inconsistency of electrodynamics is an immediate consequence of the self-interaction problem. I discuss the debate between Adolf Grünbaum and John Earman about backward causation and conclude (with Grünbaum) that there is indeed no backward causation in the Maxwell–Lorentz theory. I then briefly sketch Dirac's idea of mass renormalization and end with two sections on the ontology and usefulness of fields in general.

In Chapter 5, I introduce de Broglie–Bohm quantum theory. I contrast de Broglie's first-order formulation with Bohm's second-order formulation and show that with respect to explanatory value the first-order formulation doesn't lose against the second-order formulation. Then, I argue that the problem of indistinguishable particles, which is to my mind not solvable in theories that lack a primitive stuff ontology, finds a straightforward solution in the de Broglie–Bohm theory.

Chapter 6 discusses the status of the wave-function relative to the status of laws of nature. I point out that in the nomological view the universal wave-function is not a law, nor is the effective wave-function quasi-nomological. I then present a new Humean interpretation of the wave-function and contrast this with the ideas of David Albert and Barry Loewer. After this, I offer several ways of construing the wave-function as representing dispositions. This chapter also sketches a new way to interpret the wave-function as a *multi-field*.

This dissertation points out three things. First, a primitive stuff ontology is a necessary ingredient for all fundamental physical theories, and the ontology of physics is not easily read off from mathematical formalism. Second, Humeanism and primitivism about laws can dispense with properties. Third, if properties play any role at all, they must be dispositional.

Several future projects could follow from this dissertation. First, it offers the first steps in developing a new kind of metaphysics: one that only contains primitive stuff and primitive laws. Second, it sketches a new interpretation of the wave-function as a multi-field. Third, provided that there is a consistent field theory, the status of fields within a primitive stuff ontology is particularly interesting.

Reflection on philosophical problems has convinced me that a much larger number than I used to think, or than is generally thought, are connected with the principles of symbolism, that is to say, with the relation between what means and what is meant. In dealing with highly abstract matters it is much easier to grasp the symbols (usually words) than it is to grasp what they stand for. The result of this is that almost all thinking that purports to be philosophical or logical consists in attributing to the world properties of language.

— Bertrand Russell

Preface

Why philosophy of physics? I didn't know that this branch of research existed until 2011, when I met Michael Esfeld. I only knew that I was frustrated about how physics was taught in universities. We learnt an awful lot of mathematics: calculus, linear algebra, ordinary differential equations, partial differential equations, differential geometry, tensor calculus, functional analysis. I had had the sense that mathematics was a tool for doing physics, but it turned out that the physical problems we needed to solve were almost always mathematical problems.

In fact, I intended to study mathematics in order to understand physics. I wasn't primarily interested in using Green's functions to solve linear differential equations or Sobolev inequalities to find the lower bounds of complicated integrals. These were just tools, for I thought physics was ultimately about the world. While solving (or rather while trying solve) the exercises we were given, I hoped to someday tackle deep physical questions:

- What is space and time?
- What are particles?
- What is mass?
- What are fields?
- What are forces?
- What is the wave-function?

It was only when I met Michael Esfeld that I realized there were people working full-time on these rather conceptual questions: philosophers of physics. I don't blame my physics and maths professors for teaching the way they teach, but I do think that they should have confessed that they don't have time for these kinds of problems. Physics is now far too complicated, and academic life obliges physicists to publish technical papers.

Just as you get referred to a specialist when a doctor has reached her capabilities and can no longer treat you, physics and maths professors should refer students that are interested in conceptual riddles.

And no, philosophers aren't failed physicists or unskilled mathematicians; they work on different aspects of the same scientific theories. There are experts, like Christiano Ronaldo, who score goals, but there are also experts, like Philip Lahm, whose job it is to establish a good defense. A physicist is like Ronaldo: she forms new theories, invents new experimental set-ups, and wins the Nobel prize. A philosopher is rather like Lahm: she organizes arguments, questions the words of physicists, and reflects on old questions. Unfortunately, physicists often forget that we all play for the same team: the exploration of nature.

I learnt that the philosophical tradition had a lot to contribute to current philosophy of physics. Philosophers have worked on orthogonal questions about nature for millennia:

- What is a law of nature?
- What are properties?
- Are there modal connections?
- Is there a fundamental level in the world?
- What is causation?

I'm so glad to have had the opportunity to write a thesis on exactly these pivotal physical and metaphysical questions. In working on these topics I really began to see and appreciate that the best physicists can be excellent philosophers, too. Isaac Newton, Albert Einstein, and John Stuart Bell are still tremendously influential for physics and philosophy alike.

Acknowledgements

I am lucky to have met two of the most brilliant contemporary philosophers of physics. One is Michael Esfeld; the other is Tim Maudlin. Both have been my mentors. Michael encouraged me to follow my passion by offering me a PhD position at the University of Lausanne in Switzerland. He placed so much hope in me that I was able to co-author a paper with him on the ontology of Bohmian mechanics before I got my degree in mathematics. Michael has been so generous with his time, and he was always supportive when I raised certain issues. Needless to say, I learnt a great deal from him about metaphysics. Briefly, he was the best supervisor I could wish for.

I met Tim in 2012, when I presented parts of the paper I co-authored with Michael. I remember the discussion I had with Tim after my presentation, and I was flabbergasted by his critical mind. Whenever I met him at conferences we spent hours talking. I'm the student, and he is the teacher. At some point I decided to read every paper and every book that Tim has written. I haven't succeeded yet, but I'm pretty close! In 2015, I visited Tim for an entire semester at NYU, and I thank him for his hospitality. I also thank his wife Vishnya for her hospitality, the many discussions I had with her, and the best waffles in the world!

My third mentor is Detlef Dürr. Detlef reignited my passion for physics after I lost it in the myriad of lectures on mathematical physics, and he reminded me that physics is always about nature. He has always been a source of inspiration and novelty! All my physical knowledge comes from Detlef.

I also thank Dustin Lazarovici for his support and friendship. He took the time to read the entire manuscript and revisions of the manuscript. His comments made the thesis much better. Whenever I write something it goes first to Dustin, and he always finds ways to improve the text.

I wish to thank my friend Davide Romano. We spent hours after lunch or during coffee (excuse me, Italian mocca) talking about philosophy and physics. He gave me such valuable comments on my thesis. I'm always happy when we reunite.

I would also like to thank Charles Sebens for his critical comments on my chapter on electrodynamics. Chip was in my working group at the summer school in Saig 2013. We both still profit immensely from this intense week.

I need to thank my friend Pascal Ströing, who showed me analytical philosophy. If I remember correctly, it was around 2009, when he gave me a book on philosophy of mind. It was then that I realized that philosophy can be much more than just the history of philosophy.

I wish to thank Nicole Standen-Mills for her wonderful work in proofreading the entire manuscript.

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I wouldn't have been in this position if my parents, Helana and Hans-Peter, hadn't supported me with their unconditional love. Mom and Dad, thank you for believing in me! And thank you for giving me all the freedom to follow my path! I also thank my siblings, Florian, Isabelle, and Patrik. They showed me that there is more to life than just work.

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Introduction: Physics Needs An Ontology

Cars, airplanes, telephones, computers, microwaves, MRI, GPS—they have all revolutionized our everyday life since the last century. They have created the prosperity of the western world. Just imagine how you would go to work, how you would contact your friends, how you would prepare your food without these wonderful technologies. This is the success of physics!

And the costs of new technologies have increased so much that a single country can no longer handle them. So the development of new technologies is now a matter of international agreements and cooperation. The richest countries in the world agreed to spend billions of dollars on the Large Hadron Collider (LHC) in Switzerland. Another several billion are yearly spent on the international space station program or the European ESA, plus all the other national space agencies, such as NASA (USA), FKA (Russia), or CNSA (China). Clearly, governments and other funding institutions hope for the development of new gadgets that can facilitate our life even more—or that at least increase our GDP.

Definitely, without physics we wouldn't have built the Ottomotor; nor would we have landed on the moon. But is the goal of physics really to provide new technologies? Do physicists go into their lab in hope of developing the means for the transportation or communication of the 21st century? Maybe, but not solely.

There is something deeper that encouraged geniuses such as Newton, Maxwell, Einstein, or Schrödinger to devote their entire life to it, something that reaches beyond our well-being: a never-ending curiosity about how the world works.

It's by no means obvious that matter consists of electrons, protons, and neutrons. Nor is it obvious that planetary motions are caused by forces, that matter curves space-time, or that light can be a wave or a bunch of particles. For we human beings perceive the world completely differently: we see tables and chairs and small dots in the sky. From the impressions we get from our senses we form a pre-theoretic everyday image of the world (Maudlin, 2015, p. 349).

By doing experiments we try to go beyond our senses. We try to measure electrons in particle accelerators, or we try to find pulsars with radio telescopes. But what do we actually measure? What do we do in an experiment? First and foremost, experiments deliver *data*. Data are patterns on a detector or statistics on a piece of paper. And this is what we actually *see* from experiments.

The crucial task of physics is to explain these data. There are two main approaches. Most physicists are satisfied if they cook up some recipes that allow them to predict and explain the data. Physics is about experiments, and the experiment is the *only* standard by which to measure the success of a theory. A good example is textbook quantum mechanics. Its tremendous empirical success is beyond any doubt. Yet it has strange consequences about how the structure of the world. Complementarity, naive realism about operators, the role of consciousness, the measurement problem... we can endure all this because quantum mechanics explains empirical data.

Luckily, there are others—by far the great minority. They have endeavored to revise quantum mechanics. They correctly pointed out that textbook quantum mechanics lacks an ontology. Quantum mechanics stares at us in cluelessness about the nature of the electron, proton, and neutron. What is the electron? Is it a particle? Or a wave? A matter-wave? The wave-particle dualism is a desperate move to bring light into the darkness, since it's totally obscure under which conditions the electron will show wave-like behavior or particle-like behavior.

It's therefore not enough to provide us with certain algorithms to account for experimental data. In the end, physics must come up with an ontology, that is, with some well-defined objects in our world—no wave-particle dualism, no complementarity principle, no mysterious observers on whose behavior the fate of innocent cats depend. An ontology can introduce particles, fields, or even strings. It's not important what the objects are, but there has to be something in space and time—something that builds up measurement devices and that generates all physical phenomena.

To my mind, the de Broglie–Bohm pilot-wave theory is the best non-relativistic quantum theory that we have. For the simple introduction of particles on continuous paths, let us understand how the world behaves from the microscopic level of electrons, protons, and neutrons to the macroscopic one of tables, chairs, and galaxies. Did I mention that there is no longer a measurement problem?

I don't really grasp why physicists refuse to appreciate ontology, although the old masters, like Newton and Einstein, told us a lot about the basic constituents of the world. Maybe it's the influence of quantum mechanics, because students are still instructed to forget everything about classical mechanics in their first quantum mechanics class. The quantum world is just too weird. Or maybe it's because Niels Bohr is widely regarded as having won out over Einstein in the battle for the right interpretation of quantum mechanics. I don't know.

Mathematical and physical problems have become so difficult that physicists no longer have time to ponder philosophical or conceptual questions. Reflection on these issues has been left to philosophers. And with the complexity of the physical theories the complexity of the conceptual problems have increased as well. But philosophy has its own history, a history that is sometimes indifferent to the results of physics. So philosophers have developed their own way of uncovering the ontology of the world by the mere act of hard thinking. Since the time of Aristotle this discipline has been named *metaphysics*.

But Tim Maudlin reminds us that metaphysics ignorant of physics is empty:

> Metaphysics is ontology. Ontology is the most generic study of what exists.

> Evidence for what exists, at least in the physical world, is provided solely by empirical research. Hence the proper object of most metaphysics is the careful analysis of our best scientific theories (and especially of fundamental physical theories) with the goal of determining what they imply about the constitution of the physical world. (2007a, p. 104)

Physics starts with an ontology. It postulates the existence of the basic entities of matter. But matter cannot simply exist. It has to exist somewhere—somewhere in a three-dimensional Euclidean space, or a four-dimensional Minkowski, or an eleven-dimensional space with seven compactified dimensions.

The way to the manifest image—the image of our everyday observations—then, is long and dirty. If physics contains only space and matter, we couldn't explain how matter behaves. This is the job of the laws of nature: they tell us how matter moves. Particles, fields, and strings have to move in some way to form atoms; these form molecules; these form macromolecules; these form organic or inorganic structures. The organic structures are the basis for all living beings making up cells, organs, and organisms, which finally can read the measurement outcomes of a particle accelerator. Of course, physics cannot explain all the tiny details from the microscopic to the macroscopic level, but it has to make the transition from the small to the big objects plausible.

Physics is composed of two parts: an ontology and the laws of nature. Physics has to say what the ontology looks like. What is the structure of space-time? What is matter? And physics has to say what the laws of nature are. Metaphysics complements physics. It asks whether there is more in the ontology or whether particles have intrinsic properties. What are properties in the first place? And metaphysics questions the status of laws of nature. Do laws really exist or do they just summarize what matter is doing?

These are the basic questions I pose in this thesis. And I pose these questions to three physical theories: Newtonian mechanics, classical electrodynamics, and the de Broglie–Bohm pilot-wave theory. Let's start!

Part I.

The Metaphysics of Physics

1. The Ontology of Fundamental Physics

1.1. The Legacy of John Bell

1.1.1. What Is a Measurement?

The Copenhagen interpretation of quantum mechanics postulates a division between observers and the physical systems to be observed. The act of observation changes the state of the system. The act of observation creates the position of the electron. The act of observation kills the cat. Observations and measurements are indeed peculiar processes in quantum mechanics, but what was once believed to be a revolution in science turned out to be a malicious hoax and a source of confusion.

To John S. Bell's great merit, he pointed out this artificial division:

> The usual approach, centred on the notion of 'observable', divides the world somehow into parts: 'system' and 'apparatus'. The 'apparatus' interacts from time to time with the 'system', 'measuring''observables'. During 'measurement' the linear Schrödinger evolution is suspended, and an ill-defined 'wavefunction collapse' takes over. There is nothing in the mathematics to tell what is 'system' and what is 'apparatus', nothing to tell which natural processes have the special status of 'measurements'. Discretion and good taste, born of experience, allow us to use quantum theory with marvelous success, despite the ambiguity of the concepts named above in quotation marks. But it seems clear that in a serious fundamental formulation such concepts must be excluded. (Bell, 1987, p. 174)

The notions that Bell puts in scare quotes are obscure and ill-defined in textbook accounts of quantum mechanics unless they are explained by a physical ontology. Observables, system, apparatus, and measurement have no clear meaning if they are taken to be primitive concepts. Instead, the system and the apparatus are built of the very same basic entities, which can be particles, fields, or strings, etc. And the behavior of these objects determines the behavior of the system and the apparatus.

The special role of measurement is ensured by one of the axioms of quantum mechanics, which states that whenever a system is measured its wave-function instantaneously changes into one of the eigenfunctions of a certain mathematical operator. For physicists, operators represent observables whose value is measured by an apparatus. It is not clear, however, what observables are supposed to be in the ontology.

The collapse of the wave-function, however, is a mathematically precise concept resulting in the eigenfunction of a certain operator. But it remains unclear under which precise physical circumstances a collapse can occur. To say that there is a collapse whenever an observer observes a proton or whenever a measurement apparatus measures an electron doesn't explain anything. We go round in circles. Quantum mechanics just doesn't tell us what makes up for the difference between a measurement apparatus and the system that is supposed to be measured.

These ontological difficulties notwithstanding, quantum mechanics has been applied with tremendous success. Physicists therefore think that either the ontology of a theory is completely irrelevant or there is no ontology at all, such that a theory is only about measurement results.

> It would seem that the theory is exclusively concerned with 'results of measurement' and has nothing to say about anything else. When the 'system' in question is the whole world where is the 'measurer' to be found? Inside, rather than outside, presumably. What exactly qualifies some subsystems to play this role? Was the world wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer—with a Ph.D.? If the theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time more or less everywhere? Is there ever then a moment when there is no jumping and the Schrödinger equation applies? (Bell, 1987, p. 117)

Here Bell makes the point that there is no physical feature that distinguishes a measurement process from a general physical interaction between two systems. So "'measurement-like' processes are going on more or less all the time more or less everywhere" because physical interactions are going on more or less all the time more or less everywhere. Nevertheless, we *call* certain physical interactions "measurements", and we do so because measurements tell us something about the world. But the kind of interaction that occurs between an apparatus and a quantum system is the same as that between two quantum systems, namely the one explained by the theory.

If we want to measure the momentum of an electron, we build a device whose behavior lets us deduce the momentum of the particle. The way the device is to be built is inspired by the theory, since only the theory can tell us what and how we're able to measure. At the end of the measurement, we can read a certain number on a screen and with the help of some calculations we come to know the momentum.

Since a measurement is nothing but the interaction between two physical systems the apparatus will change the state of the system. This is the very nature of an interaction. But physicists often claim that this is a unique feature of quantum mechanics: the act of measurement kills the cat. And they claim that the situation is different in classical mechanics. For a classical measurement doesn't change the state of the measured system, it rather reveals pre-existing quantities.

But this explanation is wrong: it doesn't respect how classical physics describes interactions (Bohm and Hiley, 1993, p. 108). Let's imagine two systems, A and B. Both systems are composed of particles whose behavior is governed by the gravitational and electromagnetic force. So A feels the force of B, and B feels the force of A. The interaction changes the positions of the particles of *both* systems. Nothing prevents us from choosing A as our measurement device. Therefore, the measurement of B changes the configuration of A. So the act of measurement does change the state of systems in classical physics, too.

The Difference between Classical and Quantum Measurements

But why do we have the impression that the measurements of classical systems are different from the measurements of quantum systems? There are two reasons for this. One is the size of the system we measure and the sensitivity of the measurement apparatus. If the measured system is big enough, the gravitational and electromagnetic forces disturb the system just a little. So if we want to measure the velocity of a car, the only things that we need are a clock and light. We can prepare the starting and end position with a photoelectric sensor, which is coupled to a clock such that the car will cross both sensors at a certain time. The car is so big that the light beams don't disturb the motion of the car in any significant way, and therefore we have the impression that we have found the pre-determined value of the velocity of the car. But the light beam does indeed changes the velocity of the car – though so little that we cannot measure it.

Second, for any classical physical quantity there is a fact about the value of this quantity before any measurement is performed, and it is even possible to retrieve this "undisturbed" value. Here is an example. We want to measure the temperature of a liquid by means of a thermometer. Before the measurement the liquid had temperature T_l and the thermometer T_{th}. When we bring the thermometer into thermal contact with the liquid they will reach a state of thermal equilibrium with the same temperature T after some time. But the thermometer only gives us the temperature T of the liquid *after* the measurement, and not T_l. How do we get the temperature of the liquid before the measurement? In principle, this is easily done. If we knew certain parameters of the liquid and the thermometer, thermodynamics gives us the temperature of the liquid before the measurement. So even if the measurement disturbs the thermodynamic state of the liquid, we can recover its initial state.

This is different in quantum mechanics. In the de Broglie–Bohm theory this is particularly evident if we consider an electron in a superposed spin state. Since spin is contextual we cannot say that the particle before measurement has spin-up or spin-down. The way we measure the electron determines its spin state. One measurement may show that the electron has spin-up, while a different measurement may show spin-down, even if the electron has the very same initial position.

1.1.2. Local Beables

How can we redefine measurements in quantum mechanics? The first step is to clarify what can exist in the world:

> In particular we will exclude the notion of 'observable' in favour of that of '*be*able'. The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on 'observation'. Indeed observation and observers must be made out of beables. (1987, p. 174)

The beables constitute the ontology of a physical theory; these are the entities that are supposed to exist in the world independently of any observation. In particular, the beables do not depend on measurements. As Bell has emphasized, measurement apparatus, observers, and all physical objects, from a grasshopper to the moon, are composed of beables.

He distinguishes one crucial subclass of beables:

> We will be particularly concerned with *local* beables, those which (unlike for example the total energy) can be assigned to some bounded space-time region. For example, in Maxwell's theory the beables local to a given region are just the fields $\boldsymbol{E}$ and $\boldsymbol{H}$, in that region, and all functionals thereof. [...] Of course we may be obliged to develop theories in which there *are* no strictly local beables. That possibility will not be considered here. (Bell, 1987, p. 53)

The electromagnetic field is a local beable of classical electrodynamics. It's a beable because it exists independently of the sources, and it's local because it's defined on space-time. So classical electrodynamics proposes two kinds of local beables: fields and particles. Another candidate for a local beable is Newton's gravitational field. This is introduced as another physical entity in space-time apart from particles.

Bell's definition of local beables as those entities "which [...] can be assigned to some bounded space-time region" might be a little unfortunate because it inclines us to think that the entire beable must be restricted to a bounded region of space-time in order to be a local one. Then the gravitational field would not be local, for it spreads out throughout the universe. And the electromagnetic field would not be a local beable, either, as it propagates infinitely far with the speed of light.

So what Bell really meant by a local beable was something different. Let's chop up space-time into many small regions that cover it completely. There are no restrictions on the form or sizes of the regions. Some regions may even overlap. If a beable takes values in at least one of the regions, and if by addition of the values of all the regions we can recover the entire beable, then this beable is local. With these precisifications, Newton's gravitational field and the electromagnetic field are indeed local beables.

The Wave-Function Is Not a Local Beable

The wave-function, however, isn't a local beable. In general, one cannot recover the entire wave-function from its values on bounded space-time regions. And that is because of entanglement. With two particles in a product state $\psi = |\phi\rangle_1 |\chi\rangle_2$, , for any covering of bounded regions in space-time we can retrieve ψ because both ϕ and χ are defined in three-dimensional space – we can uniquely retrieve ϕ and χ even if they overlap.

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This is no longer possible if two particles are in an entangled state $\tilde{\psi} = |\phi\rangle_1 |\chi\rangle_2 + |\chi\rangle_1 |\phi\rangle_2$ (modulo normalization). In fact, $\tilde{\psi}$ lives on configuration space and is not an entity on space-time. Although its summands $|\phi\rangle_1 |\chi\rangle_2$ and $|\chi\rangle_1 |\phi\rangle_2$ can be retrieved from their values in physical space, the superposition only makes sense on configuration space. We cannot say that there is a superposition of each component since $|\phi\rangle_1 |\chi\rangle_2 + |\chi\rangle_1 |\phi\rangle_2 \neq |\psi + \phi\rangle_1 |\phi + \psi\rangle_2$. As entanglement is a ubiquitous phenomenon in quantum mechanics, the wave-function per se cannot be local.

Local Beables and Measurements

When studying quantum mechanics, John Bell identifies two major problems that every physical theory faces. On the one hand, there is an ontological problem: What are the basic entities a physical theory is about? On the other hand, there is an epistemic problem: How does the theory make empirical contact with the real world? He considers local beables to be a solution to both problems, and I have already pointed out how to answer them. Regarding the first one, a physical theory has to state what exists. Among beables, local ones exist in space-time. In particular, matter is composed of local beables, but other entities that we do not regard as matter can be local beables, too, like the electromagnetic field.

Bell's answer to the second epistemic question also answers the status of measurement devices.

> The concept of 'observable' lends itself to very precise *mathematics* when identified with 'self-adjoint operator'. But physically, it is a rather woolly concept. It is not easy to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in 'classical terms', because they are there. The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments. 'Observables' must be

> *made*, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables. (1987, p. 52)

A measurement apparatus is build out of local beables like any physical body, and the behavior of the local beables is covered by the physical theory. What we see after a measurement process is the final configuration of local beables of the measurement device, and this configuration with the help of the physical theory allows us to make empirical contact with the world. Hence, the requirement to have an ontology is not only motivated by the personal metaphysical need to have a picture of the world, but also by physics itself in order to make the transition from the theoretic to the empiric. Bell describes this transition again in the following famous passage:

> The name is deliberately modelled on 'the algebra of local observables'. The terminology, *be*-able as against *observ*-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms'. It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk. (Bell, 1987, p. 52)

Local beables aren't only the fundamental constituents of matter; they also account for what we perceive and describe in experiments. The "classical term" Bell refers to above doesn't mean that we have to describe an experimental set-up with classical mechanics; rather, "classical terms" is used synonymous with "everyday language".

Bell uses the term "beable" for many kinds of entities: particles, fields, energy, wave-functions. We can, within reason, doubt whether we should have all these entities in the ontology. Indeed, energy shouldn't have beable status. The issue of particles, fields, and wave-functions is more complicated, and the goal of this thesis is to illuminate their ontological status.

1.2. Primitive Ontology

1.2.1. Different Ontologies

The *ontology of the world* comprises all the entities that exist. It's a fact that there are flowers and pigs but not angels or unicorns. So flowers and pigs are part of the ontology of the world, whereas angels and unicorns are excluded. When we talk about the ontology of the world, we presuppose an objective reality out there, which is independent of our perception. The world is totally indifferent to how we think about or perceive it. It is, however, utterly astonishing that we have the capacity to dig into the physical secrets of the world way beyond what we can directly see, hear, feel, smell, or taste.

There is a distinguished subset of the world's ontology. The *fundamental or basic ontology* includes all the entities that are the building blocks of the ontology as a whole. This definition presupposes that the world is not infinitely divisible, that there is some end if we zoom in to look at the structure of the world, and that there is on one level a set of primitive entities that make up all things and beings in the world.

One way to get to know more about the ontology of the world is to form physical theories. Physics is ultimately concerned with the behavior of matter. And in order to account for that behavior a physical theory poses its own ontology of what is matter and what entities have to exist such that matter behaves in the way that it behaves. Ideally, the *ontology of physics* will match the ontology of the world so that physics will come to an end and we will have a complete theory of everything. In reality, physics strives to match the world's ontology and develops further theories to offer better and better explanations and predictions.

It is possible, however, that the ontology of physics is still a proper subset of the ontology of the world, even in the ideal case where we have found the true physical theory. For example, God may exist, or it might turn out that our mind is an entity on its own, somehow detached from the brain. For minds and Gods are not part of any physical theory—even quantum mechanics has to make do without minds. A future physical theory of everything may be capable of explaining what the mind and consciousness are. Current physical theories that explicitly mention the mind introduce it as a primitive entity, however. In the many-minds interpretation (Albert and Loewer, 1988) or the Copenhagen interpretation of quantum mechanics, the mind is presupposed to explain physics.

1.2.2. What Is a Primitive Ontology?

Not all entities of the fundamental physical ontology are directly involved in observable phenomena. Absolute space is clearly part of the fundamental ontology of Newtonian mechanics, but in observations, like the motions of planets, absolute space cannot be directly experienced or measured. Particles are different from absolute space because they constitute the motion of the planets. For that reason, particles have a distinguished role because they constitute observable objects. Entities, like particles, are elements of the *primitive ontology*.

This notion was introduced by Dürr, Goldstein, and Zanghì:

> What we regard as the obvious choice of primitive ontology—the basic kinds of entities that are to be the building blocks of everything else [...]—should by now be clear: Particles, described by their positions in space, changing with time—some of which, owing to the dynamical laws governing their evolution, perhaps combine to form the familiar macroscopic objects of daily experience. (Dürr et al., 1992, p. 850)

This passage somehow misrepresents the true meaning of the primitive ontology because it seems to be rather a description of a fundamental ontology. But it's evident from the use of this notion in the literature that it bears an epistemic component. The primitive ontology selects those entities of the fundamental ontology that havea distinguished role in accounting for observable behavior (Maudlin, 2015). Therefore, the primitive ontology is defined as located in three-dimensional space or four-dimensional space-time (see, for instance, Allori, 2013).

The requirement that elements of the primitive ontology are situated in three- or four-dimensional space makes it easier to account for experiments. Having the primitive ontology in some other space, like Albert's marvelous point in high-dimensional configuration space, the connection between the primitive ontology and experiments and experience is much more difficult (see Albert, 2013, and a critique thereof in Maudlin, 2013). Our manifest image one is of objects moving in three-dimensional space. So not just any kind of basic entity is a suitable candidate for a primitive ontology.

Examples of a Primitive Ontology

There are three prominent examples of a primitive ontology. The *flash ontology* (Tumulka 2006, p. 826; Bell 1987, Chap. 22) was developed for the GRW quantum theory (Ghirardi, Rimini, and Weber, 1986). In a nutshell, the idea of GRWf is to have point-size events in space-time as the basic entities of the world. Whenever there is a spontaneous localization of the wave-function in a very high-dimensional space, corresponding events come into being in space-time. There are no continuous lines of flashes; rather, the universe consists of discrete successions of events through time. Bell described this flash ontology in lucid language:

> So we can propose these events as the basis of the 'local beables' of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here). (Bell, 1987, p. 205)

Another proposal for the ontology of GRW theory has been worked out by Ghirardi, Grassi, and Benatti (1995), and is dubbed GRWm. Instead of having discrete stuff distributed in space-time, they introduce a continuous matter density spread out in three-dimensional space. The collapse of the wave-function in configuration space represents a contraction of the matter density in physical space, thereby accounting for discrete physical objects with which we are familiar.

The oldest proposal for a primitive ontology for quantum mechanics was introduced by the de Broglie–Bohm pilot-wave theory. This theory presupposes a particle ontology, in which particles move on continuous trajectories in three-dimensional space.

Are Local Beables the Elements of the Primitive Ontology?

There are subtle but decisive differences between local beables and the elements of a primitive ontology. The primitive ontology has two features that local beables don't have: it is defined in three-dimensional space, and it is primitive. John Bell never specified the space in which local beables are defined. He seems to be open to there being local beables in high-dimensional spaces. David Albert's marvelous point and the wave-function would then be local beables in configuration space, but they aren't part of a primitive ontology. Similarly, John Bell doesn't mention whether his local beables can be composed of other smaller entities. The primitive ontology, on the other hand, is defined to as a subset of the fundamental ontology.

1.2.3. What Are Particles?

Particles aren't particles. There are particles in classical mechanics, classical electrodynamics, Bohmian mechanics, quantum field theory, and special and general relativity. These particles are not all the "same." But the differences notwithstanding, all particles must fulfill the following two requirements:

1. Particles have a precise location in space and time.
2. For any particle there is a time interval in which the particle exists for any time t of that interval, and during this very interval the particle is at rest or moves on a continuous line.

The GRW flashes form a discrete primitive ontology, too. Though they meet the first condition of particles (flashes have a precise location in space and time), they violate the second: there are no continuous lines. The average time for a collapse of a one "particle" wave-function is several billion years. The apparently continuous existence through time of macroscopic objects can be accounted for by GRWf because they are assigned a wave-function on a very high-dimensional space. And this wave-function collapses very frequently in a short period of time so that there are many flashes with respect to our usual time scale. Zooming in on a physical body, we will not find any dots that exist for longer than a moment of time. It's even possible that on a single simultaneity slice there will be no object in space-time because at this moment there may be no flashes.

The Bohmian particles are particles proper. They have a precise location in space and time, and for every particle there is a time-interval in which it moves on a continuous line. In fact, this time interval is the same for all Bohmian particles: it lasts from the beginning to the end of the universe. According to Bohmian mechanics, there are a finite number of particles that constitute all objects, and this number of particles stays the same throughout time. Furthermore, the particles don't have any spatial extension; they are point-size objects. It would be possible to grant the particles some spatial extension—they could be very tiny balls, and the location in space of the particle would be the location of the center of the ball. In knowing the diameter we know which region of space is occupied by the particle. The spatial extension of particles, however, has to be somehow reflected by the physical theory. There has to be a physical difference whether or not the particles are extended in space. Since the extension of the particles doesn't make any empirical difference, it's more parsimonious to assume the particles to be point-sized objects.

The Pre-Socractics

The idea of particles is in fact very old. It was spelled out by the pre-Socratic philosophers Leucippus and Democritus in their early theory of atomism:

> [...] substances infinite in number and indestructible, and moreover without action or affection, travel scattered about in the void. When they encounter each other, collide, or become entangled, collections of them appear as water or fire, plant or man. (fragment Diels-Kranz 68 A57, quoted in Graham, 2010, p. 537)

The pre-Socratics imagined that the world was composed of infinite particles, in contrast to Bohmian or Newtonian mechanics. And their particles didn't attract or repel each other from a distance; instead, they thought, particles can only interact via action-by-contact. When two particles touch, they either collide and move apart, or they stick to each other and move together. Of course, the word "entangled" in the quote has nothing to do with quantum entanglement. In order for two particles to stick together, the pre-Socratics imposed some spatial structure on the particles. They were thought to be spatially extended, and they could have hooks and hollows such that two separate particles could join to form a kind of chain.

Particles in Newtonian Mechanics

What the pre-Socratics didn't have, though, was a physical theory. It took more than two thousand years and the genius of Isaac Newton to fill this gap. His ontology is similar to his forefathers':

> [I]t seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, [...] the Changes of corporeal Things are to be placed only in the various Separations and new Associations and Motions of these permanent Particles[.] (1952, p. 400)

For Newton, the different configurations and motions of particles also fully account for the behavior of macroscopic objects. But he developed the conceptual and mathematical means to make quantitative predictions. Like the Bohmian particles, Newtonian ones have the following features in addition to our minimal conditions:

1. They have no spatial extension.

2. Mass and charge are intrinsic properties of particles, and therefore there are certain species of particles.

3. There are a finite number of particles, and the total number of particles remains constant.

4. Every Newtonian particle forms one long continuous trajectory from the beginning to the end of the universe.

In addition to these characteristics, Newtonian particles are generally interpreted to have the capacity to generate fields. There are two kinds of fields that classical particles can create: the gravitational field and the electromagnetic field. Both fields are related to the intrinsic properties of particles. In virtue of having mass the particle generates a gravitational field, and in virtue of having charge it generates an electromagnetic field. The fields then reach out into space and influence all the other particles. Unlike the pre-Socratic particles, Newtonian particles don't have an internal structure that allows them to hook onto one another; fields are the cause of the repulsion and attraction of particles.

Point 2 is particularly interesting since Bohmian particles also have mass and charge as intrinsic properties. How do Bohmian masses relate to their classical counterparts?

Newtonian mechanics and Bohmian mechanics are *different* physical theories, and so there is no a priori reason why the notions of one theory should be compatible with the notions of the other.

But when we look at the Bohmian laws, we recognize two parameters, m and q. At this stage, we don't know their relation to mass and charge yet. So we can replace them with some arbitrary parameters, say α and β. In most cases the parameters α and β have nothing to do with mass and charge. Only in the classical limit, when the Bohmian trajectories are approximately Newtonian, does it turn out that we can identify α and β with m and q; we have to compare Bohmian laws of motion with the classical laws (Dürr and Teufel, 2009, Sec. 8.1). So if α and β are interpreted as representing intrinsic properties of particles then they are different properties to classical mass and charge. We could name them "Bohmian mass" and "Bohmian charge," which seem to resemble classical properties in the classical limit.

The Creation and Annihilation of Particles

The idea that particles subsisted through space and time once they were created is no longer valid in modern quantum field theory (QFT). There are indeed many versions of QFT that would need to be further developed to be recognized as mature physical theories, although the Standard Model is the generally agreed framework. One attempt to offer a primitive ontology for QFT presupposes a particle ontology (Bell, 2004a). These particles meet the two minimal criteria of all particles: they have a precise location in space and time, and they follow continuous trajectories.

But contrary to Newtonian and Bohmian mechanics, there isn't one long trajectory for every particle from the beginning to the end of the universe. Instead, these particles can be created and annihilated. Therefore, the trajectories can have jumps in them. Once a particle is created it exists for a certain period of time and moves along a continuous trajectory. But it can be annihilated and thus cease to exist, until it pops into existence again and continues to move along a continuous trajectory. Of course, a deeper physical and philosophical analysis of Bell-type QFT and the mechanism of particle creation and annihilation will shed light on whether a particle keeps its identity through jumps in the trajectory. It strikes me as strange that once particle P_1 is annihilated the very same particle P_1 can reappear later out of nowhere at another location. It seems more reasonable to dismiss a cross-trajectory identity for particles.

An alternative suggestion is the Dirac sea model of QFT (Dirac, 1934). Deckert et al. (2016) explain how we can use the Dirac sea for a Bohmian QFT. The idea is that there is in fact no creation and annihilation of particles. The universe has a finite amount of particles, but most particles are in the vacuum state (the Dirac sea) and cannot be observed because they don't interact with other particles. Only excited particles can interact and form all observable objects, and they follow a deterministic non-local law of motion.

1.3. Primitive Stuff Ontology

The primitive ontology doesn't distinguish between objects and their properties. This may lead to a paradox. Imagine a Newtonian and a Bohmian particle each with mass m and charge q. Obviously, the two particles move on different trajectories. But if they share the same properties, shouldn't they behave in the same way? To avoid this question I suggest separating properties from their bearers. A particle is a propertyless object. It's not defined by its mass or charge or any other kind of intrinsic property. Particles have just one feature: they have well-defined position. And particles can only do one thing: they move. Let's call an ontology of propertyless objects a *primitive stuff ontology*.

How the elements of a primitive stuff ontology move is determined by laws. Newtonian particles move according to Newtonian laws of motion; Bohmian particles, move according to Bohmian laws of motion. We don't need properties to further specify motion.

Allori et al. introduce a primitive stuff ontology for the GRWm theory:

> Moreover, the matter that we postulate in GRWm and whose density is given by the m function does not ipso facto have any such properties as mass or charge; it can only assume various levels of density. (2014, pp. 331–2)

Originally, the matter density of GRWm was called "mass" density, but this assigns an intrinsic property to the matter density for every definition. It's not obvious how this continuous matter-distribution can have the same property as Newtonian particles.

The Role of Properties

Properties may be introduced but only to reduce the laws of motion. This leads either to Humeanism or dispositionalism. For the Humean, properties are local qualities whose identity doesn't depend on the nomological role they have in our world. And laws are the best summary of the distribution of those qualities.

The dispositionalist regards properties of particles as intrinsic and dynamic-nomological. In contrast to Humean local qualities, dispositions are defined by their causal role in the world. So a Newtonian particle moves in the way it does because of its mass and charge. The laws, then, are a representation of what dispositions are doing (more details will follow in Chapter 2).

Advantages of a Primitive Stuff Ontology

The primitive-stuff ontology separates physics from metaphysics. The task of physics is to discover the primitive stuff and the appropriate laws. Metaphysics, on the other hand, is concerned with the status of the laws of nature and the meaning of properties. With the help of primitive stuff, we can compare current metaphysical theories on laws of nature and properties (see Chapter 2).

But the most important advantage of a primitive-stuff ontology lies elsewhere. The primitive stuff ontology is sufficient for a complete ontology of matter. We can dispense with properties altogether, whether intrinsic or relational. Physics is the science of matter in motion, not the science of properties. And by throwing properties out of the ontology, we don't need to tackle purely metaphysical questions about the status and meaning and consequences of properties. Throughout this thesis we will encounter these metaphysical questions, and yet appreciate that we can ignore them without losing any physical insight. It's the task of metaphysics to help us understand physics, not to make it more obscure.

1.3.1. Does Primitive Stuff Have Haecceity?

Let's look closer at particles interpreted as primitive stuff. How can we distinguish different particles if they don't have any properties? The only dividing feature of particles interpreted as primitive stuff is their location. These particles are all identical or indistinguishable as such, but they can be distributed in different positions in space.

There is a metaphysical invention intended to help keep physically indistinguishable particles in the ontology. The idea is to equip particles with a primitive identity or haecceity—a kind of metaphysical distinction between the particles. This means that there is a primitive fact about particle P, namely that it is particle P regardless of any physical features it may possess. Though two particles may be indistinguishable with respect to all intrinsic *physical* features, they are nevertheless discernible because of the primitive identity they always "carry" along with them. The major drawback of haecceity is that it's stipulated independently of any physical theory as an ad hoc metaphysical feature of particles or any other physical object. And current physical theories don't require the concept of haecceity. Haecceitism says that it's metaphysically possible that our world could have different identities with the same physics and phenomenology:

> Imagine the following alternative history of the world: Things are *qualitatively* just as they actually are. There is no difference in anything like the shape, size, or mass of objects. There is no difference in the number of entities. Even so, there is a *non-qualitative* difference and it concerns you in particular. According to this alternative history, you fail to exist. In your place, there is a distinct individual, Double. Double has all the qualitative properties, whether mental or physical, you actually have, but, despite all these similarities, you and Double are distinct individuals. So, according to this alternative history, you do not exist. (Cowling, 2015)

Although physics never asks for haecceitism there is a mathematical structure that seems to indicate a primitive identity of particles. Usually, the configuration of n particles is represented by an n-tuple $Q := (\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_n)$. Here, particle P_1 has spatial coordinate $\boldsymbol{Q}_1$; particle P_2 has spatial coordinate $\boldsymbol{Q}_2$, and so on. A permutation of positions of the first two particles is written as $Q' := (\boldsymbol{Q}_2, \boldsymbol{Q}_1, \ldots)$. Now particle P_1 is at $\boldsymbol{Q}_2$, and particle P_2 is at $\boldsymbol{Q}_1$. So particles seem to be equipped with a primitive identity, since the n-tuple distinguishes the order of the positions. But the introduction of a primitive identity of particles is unnecessary unless the motion varies under permutation of the configuration as well, that is, with Q and Q' leading to different trajectories. It may be reasonable to introduce a primitive identity of particles if the physical theories are built in such a way. But why should the trajectories change under permutation when the particles are physically indistinguishable? Is it possible that a permutation leads to new trajectories solely because of the haecceity of the particles?

No, it's not. In fact, when a physical theory doesn't distinguish between particles, the laws of motion must be permutation invariant. So any difference between the particles must be reflected by the laws of motion so that the permutation of distinguishable particles results in different trajectories. There is no physical theory that claims that there are physically absolutely identical particles that nevertheless differ because of a primitive identity. If there is a physical difference between particles then this difference can usually be explained by the particles' having different physical properties; they may have different mass or charge.

By definition, we have no empirical access to the primitive identity of particles—if it exists. We gain knowledge about the particles only through their motion in space and time. And if the particles are physically indistinguishable their motion doesn't expose to us which particle is on which trajectory. It's true that in the case of distinguishable particles, there can be a fact that particle P_1 is on trajectory $\boldsymbol{Q}_1(t)$, but this is a physical fact, and it's not due to haecceity.

Because of mass the Newtonian laws aren't permutation invariant under a change of particle position. In this theory, mass is the physical feature that distinguishes among the particles. In the case of identical classical particles, that is, in the case of $m = m_i$ for all $i \in \{1, \dots, n\}$, the permutation of particles doesn't change the trajectories, although the n-tuple has changed. Is this a case of haecceity? No, it's a case of superfluous mathematical structure. The formalism of n-tuples is familiar and simple because the n-tuples form the vector space $\mathbb{R}^n$, which we know from high school. But it bears too much mathematical structure, for it leads to a different physical state whenever particles permute.

1.3.2. The Mathematics of Primitive Stuff

Indistinguishable particles have to be mathematically represented in a different way. We have to replace the n-tuple $(\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_n)$ by the set $\{\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_n\}$ (see Goldstein et al., 2005a,b). By definition, a set has no ordering of elements, as tuples have. So $\{\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_n\}$ only states that there is one particle at $\boldsymbol{Q}_1$, another at $\boldsymbol{Q}_2$, and so on. This is exactly what we want. The set $\{\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_n\}$ doesn't tell us anything about which particle is at which location. The mathematics now contains all the information that is necessary, but no more. Therefore, the natural configuration space for indistinguishable particles is ${}^n\mathbb{R}^3$, which is the set of all sets $\{\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_n\}$ with $\boldsymbol{Q}_i \in \mathbb{R}^3$ for $i \in \{1, \dots, n\}$.

So far, we have argued that particles can be recognized as primitive stuff, and that they are the fundamental building blocks of matter. Since particles are indistinguishable, lacking a primitive identity, the mathematical formalism should represent this, too, by replacing the ordinary configuration space $\mathbb{R}^{3n}$ with the natural one ${}^n\mathbb{R}^3$. If properties are part of the metaphysics at all (see Chapter 2), their role is solely a dynamic-nomological one, that is, they constrain the temporal development of the particles. The laws then express the role of properties in generating the motion of particles.

If we interpret mass as an intrinsic property of Newtonian particles, it has two different roles. On the one hand, it has a causal-nomological role in determining the trajectories of the particles. On the other hand, it can individuate the particles in addition to the spatial location in the case of different masses. We can say that particle P_1 is on $\boldsymbol{Q}_1$ because it is the particle with mass m_1, and particle P_2 is on $\boldsymbol{Q}_2$ because it is the particle with mass m_2. This second role of mass, however, relies on the particles' having all different masses. If there are two particles with the same mass, we cannot distinguish between the particles except in terms of their location. So this role of mass should be only used as an additional feature to distinguish non-identical particles. Taking the idea of particles as primitive stuff seriously, ontologically mass doesn't equip the particles with an identity. The only thing we can say is that there is one particle at $\boldsymbol{Q}_1$ and another at $\boldsymbol{Q}_2$ represented by $\{\boldsymbol{Q}_1, \boldsymbol{Q}_2\}$. The way the two particles move shows us on which trajectory there is a particle with mass m_1 and on which trajectory there is a particle with mass m_2.

Although particles as primitive stuff don't have a primitive identity or properties that allow us to distinguish among them, it's still possible to label the particles. It is sometimes said that identical particles don't allow for labels (Ladyman and Ross, 2007, p. 136), but this is wrong. Let's say you have two identical soccer balls, one in each hand. And we play the following game: close your eyes, kick both balls onto the field, open your eyes, and tell me which ball was in which hand. That's impossible, isn't it? One way for you to win the game is to label the balls before you kick them. The label will tell you which ball was in your left hand and which in your right. Easy.

But there is another tactic available to you. Kick the ball in your left hand, turn around 180° and kick the other ball. You'll recognize which one you kicked first just from the distribution of the balls on the field. The second tactic shows that you can identify the balls without labels. Why? Because you know their trajectories. While your eyes are closed you know how the balls fly.

We can straightforwardly apply this example to particles. The location of n particles is represented by the configuration $\{\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_n\}$. Even if this configuration is permutation-invariant we *can* label the particles: the particle at $\boldsymbol{Q}_1$ may be named P_1, the particle at $\boldsymbol{Q}_2$, P_2, etc. The only feature that distinguishes the particles is their position in space, and the labels are names of the different trajectories. This is how we can label identical particles.

Newtonian and Bohmian Particles

In the previous section we compared Newtonian mechanics and Bohmian mechanics with respect to their primitive ontologies. Now we can make the analog analysis. Both theories have the very same primitive stuff ontology consisting of propertyless particles. They only differ in how the particles move. In Newtonian mechanics, the particles move according to Newton's laws of motion, while the Bohmian particles move according to the Bohmian guiding equation. There is nothing intrinsically different between a Newtonian and a Bohmian particle; both "kinds" of particles are point-size pieces of primitive stuff.

1.4. Ontic Structural Realism

Particles are familiar and intuitive. Some philosophers, however, challenge the atomistic picture, and they claim that their view is supported by our current physical theories. They want to replace objects with relations:

> Ontic Structural Realism (OSR) is the view that the world has an objective modal structure that is ontologically fundamental, in the sense of not supervening on the intrinsic properties of a set of individuals. According to OSR, even the identity and individuality of objects depends on the relational

> structure of the world. Hence, a first approximation to our metaphysics is: 'There are no things. Structure is all there is.' We of course acknowledge that special sciences are richly populated with individual objects. Thus, to accommodate their elimination from metaphysics we will owe a non-ad hoc account of the point and value of reference to and generalization over objects in sciences other than fundamental physics. We will argue that objects are pragmatic devices used by agents to orient themselves in regions of spacetime, and to construct approximate representations of the world. (Ladyman and Ross, 2007, p. 130)

The motivation for the development of structural realism originally relied on solving issues in general philosophy of science. John Worrall (1989), who was the first to introduce this new idea based on writings of Poincaré and Duhem, wanted to find a solution to the pessimistic meta-induction. According to this argument we cannot be sure of the objects that are presupposed by our current best theories because we have rejected most of the objects that were assumed to exist by former best theories in the past. For example, physicists once believed that there had to be a caloric fluid that moves from hot bodies to cool ones in order to account for thermodynamic phenomena. But Ludwig Boltzmann showed that thermodynamics can be reduced to statistical mechanics, which is founded on an atomistic picture of particles in motion. So Worrall concluded that we should be agnostic about the basic objects that exist in the world. Although objects don't survive a theory change, something else does: the structures of the theory.

What does he mean by structure? These are laws, mathematical equations, or some other (mathematical) content of the theory. So what survives the transition from one theory to another are certain distinguished parts of the old theory, which can be retrieved in the successive theory. One famous example is Fresnel's law, which can be found in one form or the other in ether theories, Maxwell's electrodynamics, or even electroweak theory. This kind of structural realism is called *epistemic structural realism* (ESR). There may be objects. We don't know whether they exist. But we can gain knowledge of certain parts of the mathematical or law-like relationships in scientific theories.

1.4.1. Ontic Structural Realism without Relata

Ladyman and Ross (2007) use these ideas in order to form an ontology for fundamental physics. They not only claim that all we should believe in about the world are structures, but also that all that exists in the world are structures. The basic entities that are the building blocks of our universe are structures, not objects. For them these structures are not situated or distributed in space-time, and so we cannot just build physical bodies out of structures, like building a table out of particles. Therefore, they have to offer a different story of how there can be objects. They describe objects as "pragmatic devices used by agents to orient themselves in regions of spacetime."

In the quote at the beginning of this section, Ladyman and Ross claimed that structures introduce an objective modality into the world. Later they say that "[f]or 'modal' read 'nomological' if you like" (2007, p. 130), which clearly emphasizes that their structures are parts of the laws of physics. But modal isn't the same as nomological. Nomological entities are modal, while not all modal entities are nomological. Take mass, for example. You can interpret mass as a dispositional property of particles. Then it is clearly modal in constraining or determining the motion of particles. As a dispositional property mass isn't nomological.

If ontic structures are modal, then what is their ontological status as modal? This is the crucial question that Ladyman and Ross disregard. They are neither Humeans nor dispositionalists because both accounts presuppose localized objects in the world. It seems to me that they are primitivists with respect to the laws of nature. But then the world would consist only of laws, without there being any objects that instantiate these laws. It's not at all clear to me how to extract objects in space and time out of nomological entities alone. Furthermore, Ladyman and Ross aren't realists about laws per se but only about parts of them, namely structures. But parts of laws cannot create physical objects, either.

Ladyman and Ross are well aware that structures alone cannot generate objects. Although these emergent objects are sometimes summarized as "nodes" of structures or "intersections of relations" (French and Ladyman, 2011, p. 26), the emergence of objects from structures is non-trivial and not straightforward, and it requires some ingredients of Daniel Dennett's theory of patterns (Ladyman and Ross, 2007, Chap. 4). In this reconstruction of objects I can't really grasp what are instances of objects. It seems that Ladyman and Ross only want to show how macroscopic objects emerge from structures, but there are physical objects on the fundamental level as well, such as particles, forces, or fields. I don't see how these physical objects can be "pragmatic devices used by agents to orient themselves in regions of spacetime."

Besides, it isn't clear what counts as the structures of the current physical theories. The structure of a theory can be only reconstructed after a theory-change as the part of the theory that survives in the new theory with some modifications. We can only show what has survived of *previous* theories; it's not possible to identify the structures that will survive in the future. Even if a theory inherits a structure from a previous one, this doesn't guarantee that the structure will be transmitted to a successive theory. Moreover, there is too much room for the interpretation of which structures actually have survived. It may be possible that we identify a structure in a new theory that surprisingly resembles another structure in an old theory, but their resemblance may just be due to notational similarities.

So OSR à la Ladyman seems to be trivial on the one hand and mysterious on the other. It's trivial because there have to be bits of a theory to be found in a successor theory. A new theory doesn't appear out of thin air. If a theory is empirically successful, and it is replaced by another theory, then it's not surprising that the new theory contains some parts of the old one.

Mysterious is the claim that we should be realists with respect to the structure without any (primitive) relata instantiating this structure. Here is an example. Ladyman and Ross (2007, p. 95) claim that there is a structural continuity between classical mechanics and quantum mechanics. Newton's second law is the famous equation $\boldsymbol{F} = m\boldsymbol{a}$, which specifies how forces influence the acceleration of particles. The Ehrenfest theorem in quantum mechanics is of a similar form to Newton's law, namely $\langle \nabla V \rangle = m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\langle \boldsymbol{x} \rangle$. It states that the expectation value of the gradient of the potential operator and the expectation value of the position operator fulfill a mathematical equation that resembles Newton's second law. So what should we be realists about according to this example? Ladyman's OSR doesn't allow us to be realists with respect to forces, potentials, or particles since these are all objects that instantiate the structure. They aren't realists with respect to expectation values. Fortunately, they don't allow us to be naive realists about operators, either.

But if relations as such exist, then it's mysterious what these relations are supposed to be from an ontological point of view. Ladyman and Ross claim a structure is transferred from classical mechanics to quantum mechanics. But they actually posit only a formal similarity between Newton's second law and the Ehrenfest theorem. What is the structure? What is actually transferred? For sure, the potential isn't transferred; forces aren't transferred; particles aren't transferred.

What remains is the purely formal relation $\circ = m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\circ$, where the placeholder $\circ$ indicates the spots in which we can insert some mathematical objects. But if we don't know the mathematical objects the relation as such has no physical content.

Newton's second law precisely describes the motion of a particle that is influenced by forces. Ehrenfest's theorem, on the other hand, describes the "trajectory" of the position operator, and therefore this theorem is a statement about the statistical behavior of many particles. So it's the mathematical representation of physical objects, like particles and forces, that give an equation its physical meaning. I don't see how a mathematical structure without relata can have physical meaning.

To recap, the version of OSR that I have discussed so far claims the following. All there is to the world are structures. The structures are nomological as part of the physical laws. The laws, as well as the structures, are primitive. But what do the laws of physics refer to? What does the structures refer to? What is happening in space and time? According to French, Ladyman, and Ross there is no primitive ontology; there are no local beables. The world on a fundamental level only consists of nomological entities, namely, structures that are defined as some mathematical aspects of the laws of physics. This shows that OSR is only a partial realism, because "it leaves open how the structure in question is implemented, instantiated or realized" (Esfeld, 2013, p. 20). In short, we have laws but no entities that can obey these laws.

Furthermore, French, Ladyman, and Ross also claim that their interpretation of ontic structural realism is influenced and defended by textbook quantum mechanics, in particular, by quantum statistics. Their ontology, however, doesn't help to understand what is situated in space-time according to textbook quantum mechanics. We know the quantum laws and mathematical relationships of the quantum objects, but apart from that we don't know what kind of objects there are in space and time – in fact, there are no objects on the fundamental level. Nor does their metaphysics help to solve the measurement problem or explain non-locality.

1.4.2. Ontic Structural Realism with Objects

Let's look at an alternative version of OSR. citetEsfeld:2004aa,Esfeld:2009aa suggests that OSR requires objects as the relata of structures, so that the objects instantiate those relations. What Esfeld means by structure is fundamentally different from the meaning posited by Ladyman.[1] In Esfeld's view, there are objects situated in three-dimensional space or four-dimensional space-time; they can be particles, flashes, matter density, or anything else that is primitive stuff. And these local objects instantiate certain dynamical relations that constrain their temporal development.

Esfeld criticizes Ladyman and Ross in not spelling out how their structures introduce an objective modality. Though he also interprets structures as introducing objective modality, his structures' modality is well spelled out: the structures are modal by constraining the temporal development of the objects instantiating these structures. And the physical theory at hand determines what the objects and the structures are supposed to be.

Applied to Newtonian mechanics, particles instantiate dynamical relations that are represented by Newton's second law (see Hubert, 2016, Sec. 3). In the end, these relations coincide with plain Newtonian forces. The forces are taken as primitives, and their role is to constrain the motion of particles. Newton's laws then supervene on this dynamical role of forces.

We can categorize the myriad of proposals for a fundamental ontology of the world by differentiating with respect to their commitment to the status of objects and relations (see also Stachel, 2006; Esfeld and Lam, 2011):

1. Certain objects are ontological primitives. All other objects and all relations are determined by these primitive objects.

2. Certain objects and certain relations among these objects are ontological primitives.

3. Certain relations are primitive. All objects are determined by these relations.

4. There are only relations and no objects.

The first point isn't a version of OSR. In this ontology, there are only localized objects, and the objects usually have intrinsic properties, which have two roles. On the one hand, the properties determine the behavior of the objects. On the other

[1] Ladyman and Ross (2007) are indeed ambiguous on what they mean by structures. On the one hand, parts of mathematical equations represent structures as already discussed. On the other hand, they accept space-time relations as ontic structures, too (p. 144), which is more in Esfeld's spirit.

hand, the properties equip every object with an intrinsic identity. The objects differ through having different intrinsic properties. Leibniz adheres to this view in his theory of monads. For him, the world is composed of non-extended thinking entities (monads) that perceive the world from their unique point of view or mirror the entire universe under their own aspect. Monads are the substances of the world, and everything else, including all relations, supervenes on them.

The second option allows non-supervenient relations in the ontology. For example, spatiotemporal relations among particles are candidates for non-supervenient relations. David Lewis (1986) famously supported this position in his version of Humeanism by presupposing a network of spatiotemporal relations that hold the world together. And at points of this network are objects instantiating local qualities. The spatiotemporal relations aren't responsible for the identity of objects, which is instead established by the intrinsic properties every object possesses.

The third position does grant the relation primary status and lets the objects emerge from them. This is in accordance with the view of Ladyman and Ross.

The most radical version of OSR states that the ontology of physics consists only of structures. This eliminative position gets rid of all the objects as part of the ontology; famously, French (2010, 2014) defends this view. He argues that quantum mechanics underdetermines whether quantum particles are individuated objects. On the one hand, he tries to show that quantum statistics prohibits us from recognizing quantum particles as individuals (French and Redhead, 1988). On the other hand, Muller and Saunders (2008) claim that quantum mechanics is committed to individuated objects by means of the principle of the identity of indiscernibles. In conclusion, there is no physical fact regarding whether quantum particles are individuated objects. And as there is no fact, French goes on, there are no objects.

Furthermore, French considers the symmetries of quantum mechanics represented by group theory to be the basic structures of the world. He is explicit in doing so:

> [T]he structure of the world is *presented* to us in the theoretical context under consideration by means of the relevant laws and symmetries, as informed group-theoretically. (French, 2014, p. vii)
>
> '[W]hat is structure?' It is the laws and symmetries of our theories of contemporary physics, appropriately metaphysically understood via notions of dependence and taken as appropriately modally informed. (French, 2014, p. ix)

But what is the meaning of symmetries in quantum mechanics without reference to something in space-time? Aren't there objects in space and time whose behavior is captured by whatever fancy mathematical formalism humankind can invent?

Symmetries in (non-relativistic) quantum mechanics are mathematical relations of operators—think of the rotation groups $SO(3)$ and $SU(2)$. But operators aren't fundamental objects, because they and their symmetries are ultimately derived from the wave-function and Schrödinger's equation. Operators are used to summarize measurement outcomes in a concise way. Therefore, they are not suitable foundations for an ontology of physics.[2]

I prefer Esfeld's proposal for OSR. There are objects, and there are non-supervenient relations among the objects. Objects are the stuff of which all physical objects are composed. Relations are modal by constraining the behavior of objects. Applying this to an atomist's ontology, particles are objects that stand in certain dynamical relations. Every particle is related to the other particles in the universe, and relations constrain the temporal development of the particles such that they trace out certain trajectories fixed by the physical theory.

1.4.3. Ontic Structural Realism of Space-Time

So far we have used OSR to ground the laws of motion in the dynamical relations among particles. But we can go one step further in applying OSR to space itself (Esfeld et al., 2015a,b). Then we have two kinds of structures among the particles: there is a non-modal spatiotemporal structure and a modal dynamical one.[3] This is as far as OSR can ultimately get.

Esfeld and his co-authors offer three arguments for why we should prefer relational space over absolute space:

1. The problem of the Leibniz shifts is solved;
2. Absolute space exists where there are no particles; and
3. A metaphysical distinction between points of space and points of matter is unclear.

In the famous Leibniz-Clarke debate (Leibniz and Clarke, 2000), Leibniz argues that absolute space allows for states of affairs that make no observable difference. The center of mass of the universe might be at a different location in absolute space, and we wouldn't be able to distinguish the two cases by physical experiments. We can even imagine that all objects in the universe have an additional velocity $\boldsymbol{v}$, and still there is no observational difference. According to the principle of the identity of indiscernibles, Leibniz concludes that universes that differ from ours by a translational or a kinematic shift are actually the same.

A second reason to abandon absolute space is that absolute space exists even in locations where there is no matter. The primary role of absolute space is to provide an arena for the motion of particles. But in doing so it presupposes too much mathematical and physical structure that is supposed to exist even in areas where there has never been a particle.

[2] In fact, French includes in his OSR the symmetries of QFT, which are different from the symmetries in non-relativistic quantum mechanics. Nevertheless, I don't regard them as fundamental either.

[3] In general relativity, you could argue that the spatiotemporal relations are modal, too.

Third, if there is absolute space, it's not clear what the difference is between a point of space and a point of matter, namely, a particle. Neither kind of point has any features by which to distinguish it, since they are both taken to be primitive. A particle can occupy a point in space, and, whenever there is empty space, space-time points are empty. But this manner of speaking doesn't clarify what is metaphysically different between a space-time point and a particle. Therefore, we should get rid of absolute space.

So here are the ingredients of Esfeld's ultimate version of OSR. First and foremost, there are objects. On the fundamental level, objects are interpreted as primitive stuff with no intrinsic properties at all. In addition to the primitive stuff, there are two non-supervenient structures instantiated by objects. There is a non-modal spatiotemporal structure that constitute space or space-time, and there is a modal dynamical structure that constrains the temporal development of objects. A physical theory then fills in the gaps by providing the details for both structures, since at this stage the structures are mere placeholders for physics. In fact, only Barbour's Machian theory of Newtonian mechanics poses a relational space-time from the outset (Barbour and Bertotti, 1982; Barbour, 2000); all other physical theories, including general relativity, have up to now needed some objective space-time structure independent of physical objects.[4]

Therefore, Esfeld has two possibilities. One is to interpret existing theories as presupposing a relational space-time, their standard interpretation as substantivalist theories notwithstanding. For instance, Huggett (2006) does this for Newtonian mechanics. He argues that a Humean can get rid of absolute space and time; what remains then are particles standing in spatiotemporal relations. Another strategy for Esfeld is to rewrite substantivalist theories as relational theories. He does exactly that in Esfeld et al. (2015a) for classical mechanics and Bohmian mechanics.

The situation is different for dynamical structures because all theories formulate laws to account for the temporal behavior of objects and laws can be interpreted to introduce an objective dynamical structure among objects. The task of laws is hence to describe the influence of the structure on the temporal development of objects.

Physics doesn't postulate a dynamical structure per se. Relations may come in the guise of forces, fields, or wave-functions. These are the modal entities that constrain the motion of particles. In a second step, one can analyze the ontological status of forces, fields, or wave-functions. So when we speak of dynamical relations we mean that there is some influence or connection between the objects, and the physical theory gives us the entities that make up these relations. Normally, physicists don't speak of dynamical relations; they speak of interactions. The notion of interaction, however, seems to be more restrictive than the notion of relation. Forces and fields are interactions between particles. Forces without fields are di-

[4] As always, everything is debatable. Although Barbour was inspired by Leibniz and Mach he is a substantivalist with respect to shape space, which he calls Platonia. Similarly, there are relationalist interpretations of general relativity, triggered by the hole argument.

rect interactions or actions at a distance, while forces in conjunction with fields are usually seen as mediated interaction. The wave-function, however, isn't interpreted as an interaction between particles, but can be construed as a dynamical structure (see Esfeld et al., 2015b).

Summary

I've argued against Ladyman's version of OSR because it's metaphysically mysterious and physically ill spelled out. He and Ross introduced OSR as "the view that the world has an objective modal structure that is ontologically fundamental, in the sense of not supervening on the intrinsic properties of a set of individuals." This coincides with Esfeld's proposal of a dynamical structure. They go on to say that "[a]ccording to OSR, even the identity and individuality of objects depends on the relational structure of the world." This is again true for Esfeld's version. But now they introduce a twist: "Hence, a first approximation to our metaphysics is: 'There are no things. Structure is all there is.' " This doesn't follow from the previous two statements. Their first approximation turns out to be the entire ontology: all that exists are relations without relata.

2. The Status of Laws of Nature

2.1. Humean Supervenience

Humean supervenience can be traced to David Hume's theory of causation. His *Treatise of Human Nature* from 1740 is the locus classicus:

> [W]hen we talk of any being [...] as endowed with a power or force, proportioned to any effect; when we speak of a necessary connection betwixt objects, and suppose, that this connection depends upon an efficacy or energy, with which any of these objects are endowed; in all these expressions, *so applied*, we have really no distinct meaning, and make use only of common words, without any clear and determinate ideas. But as it is more probable, that these expressions do here lose their true meaning by being wrong applied, than that they never have any meaning; it will be proper to bestow another consideration on this subject, to see if possible we can discover the nature and origin of those ideas, we annex to them.
>
> Suppose two objects to be presented to us, of which the one is the cause and the other the effect; it is plain, that from the simple consideration of one, or both these objects we never shall perceive the tie, by which they are united, or be able certainly to pronounce, that there is a connection betwixt them. It is not, therefore, from any one instance, that we arrive at the idea of cause and effect, of a necessary connection of power, of force, of energy, and of efficacy. Did we never see any but particular conjunctions of objects, entirely different from each other, we should never be able to form any such ideas.
>
> But again; suppose we observe several instances, in which the same objects are always conjoined together, we immediately conceive a connection betwixt them, and begin to draw an inference from one to another. This multiplicity of resembling instances, therefore, constitutes the very essence of power or connection, and is the source, from which the idea of it arises. The appearance of a cause always conveys the mind, by a customary transition, to the idea of the effect. (Hume, 1960, pp. 162–163, modernized spelling)

Let's use an example to illustrate what Hume means. When you flip the light switch in your bathroom the light turns on (provided that there are no defects). Then, of course, flipping the light switch is the cause of the light bulb on the ceiling's turning on. There is no doubt about the cause and the effect. But why is there no doubt? Can we observe the causal connection between the switch and the light?

Hume denies this. What we observe are constant conjunctions of flipping the switch and the burning of light. And because we have seen this happen many times as the same conjunction of events, our mind expects the effect after seeing the cause. But Hume goes even one step further. Not only can we not observe any causal connection, there isn't any such thing in the first place. For him, it's metaphysically possible for the light bulb to remain off when we flip the switch even if it bears no physical defect.

Later, in *An Enquiry of Human Understanding*, David Hume elaborates on his notion of causation:

> [W]e may define a cause to be *an object, followed by another, and where all the objects, similar to the first, are followed by objects similar to the second.* Or in other words, *where, if the first object had not been, the second never had existed.* The appearance of a cause always conveys the mind, by a customary transition to the idea of the effect. Of this also we have experience. We may, therefore, suitably to this experience, form another definition of cause; and call it, *an object followed by another, and whose appearance always conveys the thought to that other.* (Hume, 2007, p. 56)

In fact, Hume mixes up two different conceptions of causation. The first characterization of causation is very similar to that in the *Treatise*, which is known as the *regulatory theory* of causation: causation is nothing but constant conjunction of events such that similar causes precede similar effects. So flipping the light switch is the cause of the light's turning off. But there are other regularities, other constant conjunctions, that don't instantiate causal relations. For example, many vending machines have problems recognizing coins. Sometimes you need throw the coin in several times before it is accepted. But people often rub their coin on the surface of the vending machine. Voila, the coin is swallowed. But in fact, the rubbing itself doesn't have any effect. Another drawback of the regularity account is that it cannot be applied to events that aren't repeatable, like the Big Bang, although we would say that the Big Bang is the cause of the existence of our universe.

The second definition offered by Hume describes causation with the help of counterfactuals: if C had not occurred, E would not have occurred—when both C and E actually did occur. It seems that Hume confused this description of causation with the regularity account. In fact, he didn't analyze causation in terms of counterfactuals. Since counterfactuals were regarded as obscure by empiricists after Hume, a detailed analysis of causation in terms of counterfactuals was avoided until the 20$^{\text{th}}$ century (for instance Lyon, 1967; Mackie, 1980). In the end, it turned out that causation understood with the help of counterfactuals is much more satisfying than the regularity account.

But finding a more satisfactory understanding of causation with the help of counterfactuals wasn't easy. For the truth of Hume's counterfactual is neither necessary nor sufficient for causation (see Maudlin, 2007a, Ch. 5). It's not necessary because a different cause might have occurred, leading to the same effect. It's not sufficient because there are examples that show that counterfactuals go both ways, while causation only goes one way.

Certainly, the assassination of J.F. Kennedy on November 22, 1963 was the cause of his no longer being president in December 1963. So the counterfactual "if J.F. Kennedy had not been assassinated on November 22, 1963, he would have still been president in December 196" is true, too. But if we turn this sentence around, it's still true: if J.F. Kennedy had still been president in December 1963, he would not have been assassinated in November 1963. And clearly we don't associate causation with the latter counterfactual.

2.1.1. Modern Humean Metaphysics

Taking up Hume's idea that there are no modal connections in our world, David Lewis then developed an entire metaphysics:

> Humean supervenience is named in honor of the greater denier of necessary connections. It is the doctrine that all there is to the world is a vast mosaic of local matters of fact, just one little thing and then another. (But it is no part of the thesis that local matters of fact are mental.) We have geometry: a system of external relations of spatio-temporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. All else supervenes on that. (1986, pp. ix–x)

Lewis' ontology consists in the contingent distribution of local matters of particular facts: the Humean mosaic. There is a net of spatiotemporal points that are only connected by external metrical relations, and, at those points, certain qualities can be instantiated that make up all physical objects, such as particles and fields. Given some initial distribution of these qualities, there is nothing in the ontology that constrains the future development of the mosaic. Obviously our world contains regularities, but this is just a contingent fact in this framework, because Lewis's ontology lacks objective modality. If our world were Humean, it would be metaphysically possible that the regularities we have seen so far could change from one moment to the next.

Everything else with which we are familiar from the practice of science, like the laws of nature or chance, just supervenes on the behavior of the Humean mosaic:

> The question turns on an underlying metaphysical issue. A broadly Humean doctrine (something I would very much like to believe if at all possible) holds that all the facts there are about the world are particular, or combinations thereof. This need not be taken as a doctrine of analyzability, since some combinations of particular facts cannot be captured in any finite way. It might better be taken as a doctrine of supervenience: if two worlds match perfectly in all matters of particular fact, they match in all other ways too—in modal properties, laws, causal connections, chances, ... (Lewis, 1986, p. 111)

In order to avoid giving an enormously long list of particular facts describing the regularities of our world, Lewis puts forward his Humean best-system account of laws of nature:

> Take all deductive systems whose theorems are true. Some are simpler, better systematized than others. Some are stronger, more informative, than others. These virtues compete: an uninformative system can be very simple, an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law iff it is a theorem of the best system. (1994, p. 478)

According to this proposal, the laws of nature are the theorems of the best deductive system that combines or balances simplicity and strength in describing the temporal development of local matters of particular facts throughout space and time. A long list of these facts would be highly informative but very complex, whereas a single law of nature would be very simple but probably not contain enough information. So the best system comprises a certain finite number of laws of nature as theorems, which offer the perfect compromise between being simple and being informative (Loewer, 1996).

A problem for the Humean is that the laws of nature can only be written down at the end of the universe when the whole pattern of its history is on the table. For the behavior of the mosaic so far doesn't tell us anything about its behavior in the future. So the laws that we infer right now probably aren't the true laws of nature because we have no idea how the mosaic will behave in the future. We might find the best system for our world at this moment in time, but due to changes in the regularities we will probably have to revise our best system. And this is also against scientific practice. Physicists presuppose that there are necessary connections out there, and they discover the related laws. If the laws have to be revised then this is surely not because the behavior of the world has changed. Physicists rather change their laws because they have found new phenomena that can't be explained by their old theories.

One could argue that once the Humean laws are found they will no be longer unique, since they rely on some kind of equilibrium between simplicity and strength, which seem to be anthropocentric notions. Some people may agree that X is simple; others would say that X is highly complicated. Some people may agree that statement Y bears a great deal of information; others would say that Y is informationally weak. David Lewis was aware of this line of reasoning and attacked it:

> The worst problem about the best-system analysis is that when we ask where the standards of simplicity and strength and balance come from, the answer may seem to be that they come from us. Now, some ratbag idealist might say that if we don't like the misfortunes that the laws of nature visit upon

> us, we can change the laws—in fact, we can make them always have been different—just by change the way we think! [...] It would be very bad if my analysis endorsed such lunacy. I used to think rigidification came to the rescue: in talking about what the laws would be if we changed our thinking, we use not our hypothetical new standards of simplicity and strength and balance, but rather our actual and present standards. But now I think that is cosmetic remedy only. It doesn't make the problem go away, it only makes it harder to state.
>
> The real answer lies elsewhere: if nature is kind to us, the problem needn't arise. I suppose our standards of simplicity and strength and balance are only partly a matter of psychology. It's not because of how we happen to think that a linear function is simpler than a quartic or step function; it's not because of how we happen to think that a shorter alternation of prenex quantifiers is simpler than a longer one; and so on. Maybe some of the exchange rates between aspects of simplicity, etc., are a psychological matter, but not just anything goes. If nature is kind, the best system will be robustly best—so far ahead of its rivals that it will come out first under any standards of simplicity and strength and balance. We have no guarantee that nature is kind in this way, but no evidence that it isn't. It's a reasonable hope. Perhaps we presuppose it in our thinking about law. I can admit that *if* nature were unkind, and *if* disagreeing rival systems were running neck-and-neck, than lawhood might be a psychologial matter, and that would be very peculiar. I can even concede that in that case the theorems of the barley-best system would not very well deserve the name laws. But I'd balme the trouble on unkind nature, not on the analysis; and I suggest we not cross these bridges unless we come to them. (1994, p. 479)

So why does Lewis think that the Humean laws are unique? Why are simplicity and strength objective features? For Lewis the final answer lies in the "kindness of nature." Our world is built in such a way that the standards of simplicity and strength are objective and not merely a matter of human psychology such that everyone would agree on the laws of nature once the best system were found. But Lewis also considers how scientists find and discuss laws. Once proposition X is declared to be a law, all scientists agree that X is a law, and there is no debate about changing the law to make it more simple or more informative. Actually, one argument against Bohmian mechanics put forward by physicists is that the Bohmian laws are much more complicated but no more informative than the laws of textbook quantum mechanics. However, this argument is off-target, since Bohmian mechanics solves the measurement problem. Furthermore, there is no simpler physical theory than Bohmian mechanics that introduces particles on continuous trajectories due to the mathematical structure of the guidance equation.

So David Lewis rather hopes that simplicity and strength are objective features of the Humean laws, which render laws of nature unique. Everything has to supervene on the Humean mosaic such that the fact that simplicity and strength are objective has to supervene on the mosaic, too. Does this mean that the pattern of local matters of particular facts in our world is such that the laws are unique? Why not?

From a Humean perspective, we live in a very special world in which we find the same regularities over and over again, although there are no modal connections that constrain the behavior of the mosaic. So it's possible that because of the pattern of our world we can agree on what is simple and what is informative in a unique way.

2.1.2. Quiddity and Humility

Let's move forward. Qualities, which are instantiated in the Humean mosaic, are by definition independent of their causal–nomological role. So a quality q1 in one Humean world can be the very same quality q_1 in another possible world, with a completely different causal–nomological role. In other words, qualities are born with a primitive identity, dubbed *quiddity*. And we don't have epistemic access to this quiddity because all that we can find out about a quality is its causal–nomological role in our world, which is neither sufficient nor necessary to identify this quality. This epistemic limit is called *humility*, and is used as one of the main arguments against Humean metaphysics (Black, 2000, Sec. III). Lewis (2009) bites the bullet of quidditism and humility and tries to show that humility applies to at least some—if not all—fundamental properties. This is a peculiar move as Humeanism aims to establish a metaphysics that is close to science and, in particular, close to physics.

2.1.3. Quantum Mechanics against Humeanism

There aren't only purely philosophical arguments against Humeanism, like the problem of quiddity; there are also arguments from physics that aim to show that Humean supervenience cannot be right. For this purpose, Tim Maudlin (2007a, Ch. 2) uses quantum mechanics (see also Maudlin, 1998). He summarizes that Lewis's Humeanism relies on two doctrines:

> Doctrine 1 (Separability): The complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each spacetime point (or each pointlike object) and the spatio-temporal relations between those points.
>
> Doctrine 2 (Physical Statism): All facts about a world, including modal and nomological facts, are determined by its total physical state. (2007a, p. 51)

Separability states that all fundamental physical properties are intrinsic properties, and the only non-supervenient relations among objects are spatiotemporal relations. The world is then composed like a digital screen that is built out of a myriad of small pixels, one next to the other, where the state of one pixel can be changed without disturbing the states of other pixels.

Physical Statism says that all non-physical entities, like laws of nature, supervene on the physical state of the universe. This doctrine emphasizes that even the modal facts of the world are determined by the distribution of the local matters of particular facts. If two worlds fully agree in terms of the history of the Humean mosaic, then these worlds agree on all other facts, too: the two worlds are indistinguishable in all respects. Humeanism contains another doctrine: laws don't determine the behavior of the mosaic.

> Non-circularity condition: The intrinsic physical state of the world can be specified without mentioning the laws (or chances, or possibilities) that obtain in the world. (Maudlin, 2007a, p. 52)

This condition is the basis for the problem of quiddity, since physical properties, like mass and charge, can be distributed throughout the world without obeying the laws of classical physics. They aren't defined by the causal-nomological role they play in the world.

According to Maudlin, entangled quantum systems pose a threat for Humeanism. The most prominent such state is the singlet state of two spin-$\frac{1}{2}$ particles:

$$|0,0\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 \Big), \tag{2.1}$$

where $|\uparrow_z\rangle$ represents spin up in the z-direction, and $|\downarrow_z\rangle$, spin down in the z-direction. This state is the superposition of two states, namely, $|\uparrow_z\rangle |\downarrow_z\rangle$ and $|\downarrow_z\rangle |\uparrow_z\rangle$. The former state assigns particle 1 spin up and particle 2 spin down, while the latter assigns particle 1 spin down and particle 2 spin up.

So if the two-particle system were in the state $|\uparrow_z\rangle |\downarrow_z\rangle$, we could assign each particle its own state independently of the other one. But you can read directly from (2.1) that particle 1 isn't in a state that is independent of the other particle. It says that if we measure spin up for particle 1, then particle 2 is in a spin down state, and if we measure spin down for particle 1, then particle 2 is in a spin up state—and analogously, if we first measure particle 2. The singlet state doesn't assign any particle a local state; it's a global state that doesn't supervene on the local properties of the individual parts. Therefore, quantum mechanics refutes Humean supervenience.

Maudlin extends this example to strengthen his conclusion that there are quantum mechanical states that don't supervene on the local properties of the individual system. We can prepare the two spin-$\frac{1}{2}$ particles not only in the singlet state but also in the m=0 triplet state: [1]

$$|1,0\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 + |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 \Big),$$

Let's compare the two states $|0,0\rangle$ and $|1,0\rangle$. Both states lead to the same experimental results for a measurement in the z-direction: if one particle is spin up the other is spin down, and vice versa. Furthermore, if the system is in the singlet

[1] The total spin of a system consisting of two spin-$\frac{1}{2}$ particles can be either 0 or 1. The singlet state is the unique state of the spin-0 case, while there are three states for the spin-1 case. The other spin-1 states are the $m = 1$ state $|1,1\rangle = |\uparrow_z\rangle |\uparrow_z\rangle$ and $m = -1$ state $|1,-1\rangle = |\downarrow_z\rangle |\downarrow_z\rangle$.

state and we repeatedly measure the spin of particle 1, in fifty percent of cases we get spin up and in fifty percent spin down—the same for the spin-measurement of the other particle. And if the system is in the $m = 0$ triplet state, we get the very same result for the repeated spin-measurement in the z-direction of either particle: fifty percent up and fifty percent down.

Quantum mechanics lets us ascribe a density matrix to either particle, which gives us the correct experimental result if we measure the spin of one particle many times. So all the particles in the singlet and in the $m = 0$ triplet state have the very same density matrix. Can't the density matrix be the right intrinsic state on which the entangled state supervenes? This would mean that the two entangled states are identical, that is, $|0, 0\rangle = |1, 0\rangle$.

The innocent "+"-sign makes the difference. We can rewrite both states in another basis corresponding to the spin in x-direction. We then get:

$$\begin{aligned} |0,0\rangle &= \frac{1}{\sqrt{2}}\Big(|\uparrow_x\rangle_1 |\downarrow_x\rangle_2 - |\uparrow_x\rangle_1 |\downarrow_x\rangle_2 \Big), \\ |1,0\rangle &= \frac{1}{\sqrt{2}}\Big(|\uparrow_x\rangle_1 |\uparrow_x\rangle_2 - |\downarrow_x\rangle_1 |\downarrow_x\rangle_2 \Big). \end{aligned}$$

In the singlet state the spins in the x-direction are also anti-parallel, while the x-spins in the triplet state are parallel. The measurement of spin in x-direction shows us the difference between $|0, 0\rangle$ and $|1, 0\rangle$. But these correlations get lost in the density matrix formalism. Since quantum mechanics doesn't allow us to reduce the entangled states to the states of individual particles, entangled states don't supervene on the local properties of their constituents. Consequently, the Humean mosaic is not sufficient for an ontology of quantum physics.

2.1.4. Super–Humeanism to the Rescue

What can we say about Maudlin's argument? Does quantum mechanics refute Humeanism? I think that if we interpret quantum mechanics as a complete theory Maudlin's argument is waterproof, and we have to give up Humeanism. In this case, physics doesn't allow us to reduce the wave function to local qualities because there are no local features in the physics in the first place. But Miller (2014) shows that we can avoid Maudlin's radical conclusion by sticking to Bohmian mechanics instead of textbook quantum mechanics (see also Esfeld et al., 2014, 2015b). She not only considers a different quantum theory to Maudlin, but also needs a different version of Humanism too. While Maudlin sticks to traditional Humean supervenience à la David Lewis, Miller bases her argument on a Humean theory posited by Ned Hall (2009, Sec. 5.2).

Although he bites the bullet, the biggest problem for Lewis is quiddity. Therefore, Hall proposes that we should interpret properties similarly to chance and probabilities. For a Humean, chance enters the mosaic via the best system. Consequently, the world is non-deterministic if the best system says so, and the laws are such that they allow for probabilistic predictions.

Now, we can interpret properties in the very same way as Humeans interpret probabilities. Properties are introduced by the best system. They aren't part of the mosaic as entities in space and time. Rather, they are used for the best description of what's going on in the world:

> What would make it the case that there *are* masses and charges is just that there is a candidate system that says so, and that, partly *by* saying so, manages to achieve an optimal combination of simplicity and informativeness (informativeness, remember, only with respect to particle positions). So in our particle world, all they really are are facts about positions of particles at all times; but if we pretend that in addition, each particle is characterized by an unchanging value of two magnitudes (one with real values, the other with nonnegative real values), then we can write down very simple equations that encapsulate quite a lot of information about the particle motions. The final step is to let the facts about particle motions that make it the case that these equations achieve such an optimal balance of simplicity and informativeness constitute the truth-makers for claims about particle mass and charge, so that those claims can now be understood as literally correct. What results, again, is a philosophical position about mass and charge that is exactly analogous to the position the BSA already takes about objective chance. Mass, charge, and chance are all, in a certain specific sense, *manufactured* magnitudes. (Hall, 2009, pp. 27–8)

What does remain in the Humean ontology if we remove all the local qualities? Not much, as Lewis builds his entire metaphysics on the distribution of local properties. Hall replaces Lewis's qualities with particles, regarded as primitive stuff. No quiddity, no humility. Consequences that had to be swallowed by Humeans. All properties, like mass and charge, are part of the laws of motion and so are determined by the motion of the particles. Let's call Hall's version *Super-Humeanism*.

An electron, for example, no longer carries categorical properties. It has no properties; it just moves. If the Super-Humean then still says that the electron has a mass of $9.1 \cdot 10^{-31}$ kg and a charge of $1.6 \cdot 10^{-19}$ C, she just means that in the description of the motion of the electron the laws of motion mention quantities assigned to the particle so that the laws match the correct behavior.

Miller uses Super-Humeanism to counter Tim Maudlin (for further details see Chapter 6). She applies her ideas to to Bohmian mechanics and therefore gives an example of a quantum theory that is compatible with a certain version of Humeanism. Miller can do so because Bohmian mechanics introduces particles, which constitute the Super-Humean mosaic. And she interprets the wave function as part of the best system. The purpose of the wave function is solely to provide for a simple and informative description for the motion of the particles. And the ontological status of the wave function doesn't differ from the status of proper ties and probabilities. Everything still supervenes on the positions of the particles throughout history.

Too Parsimonious?

One can in fact criticize Humeanism on a point that Humeans regard as one of its greatest virtues: parsimony. This metaphysics is unsatisfactory because there are no facts of the matter as to *why* we see all the regularities in our world. Even the laws of nature as part of the best system cannot explain *why* the particles follow a Newtonian trajectory, for example; the particles just move as they do as a contingent fact. All a Humean can do is to give a good description or summary of the regularities. This is the price for not having modal connections. So why not have them in the ontology? The alternatives are primitivism about laws and dispositionalism.

2.2. Primitivism about Laws

2.2.1. What is Primitivism?

Primitivism about laws introduces laws into the ontology as independent non-supervenient entities. One famous adherent to this interpretation is Tim Maudlin:

> To the ontological question of what makes a regularity into a law of nature I answer that lawhood is a primitive status. Nothing further, neither relations among universals nor role in a theory, promotes a regularity into a law. [...] My analysis of laws is no analysis at all. Rather I suggest we accept laws as fundamental entities in our ontology. Or, speaking at the conceptual level, the notion of a law cannot be reduced to other more primitive notions. The only hope of justifying this approach is to show that having accepted laws as building blocks we can explain how our beliefs about laws determine our beliefs in other domains. Such results come in profusion. (2007a, pp. 17–8)

Maudlin starts with the central question, "What makes a regularity into a law of nature?" Let's say you have bought stocks of Apple. You are very proud of having invested in this corporation, but you are cautious, too. So you check your account every month. And you realize that the value of your portfolio has increased every time. After 36 months of increase you are pretty confident that Apple's stocks will keep growing. But you would never say that it's a law of nature that Apple's value must grow. For whatever reason, the stock price may fall in the future.

Yet you would reason completely differently when it comes to the behavior of baseballs. When you throw a baseball it falls to the ground. You throw it a hundred times, and still the ball finally falls on the ground. You have seen people throwing baseballs, and there too they fall back on the ground. You don't have a trace of doubt that the next time you throw the ball it will trace out a parabola and land on the ground as you have seen it do many times before.

Now what's the difference between the motion of baseballs and the behavior of stocks? Why do baseballs always behave the same way, while stocks might change their habitual pattern? A primitivist about laws replies that baseballs obey

Newton's laws. Newton's laws are part of the fundamental ontology of our world. They aren't determined by the motion of the objects; they aren't part of the best system. We can imagine a world that contains baseballs, but they move on different trajectories because there are different laws of motion.

Physicists and philosophers sometimes say that laws "govern" or "guide" physical objects. Though this may reflect their intuition, this manner of speaking has to be taken metaphorically (see also Loewer, 1996, p. 119):

> My own proposal is simple: laws of nature ought to be accepted as ontologically primitive. We may use metaphors to fire the imagination: among the regularities of temporal evolution, some, such as perhaps that described by Schrödinger's equation, govern or determine or generate the evolution. But these metaphors are not offered as analyses. In fact it is relatively clear what is asserted when a functional relation is said to be a law. Laws are the patterns that nature respects; to say what is physically possible is to say what the constraint of those patterns allows.
>
> Taking laws as primitive may appear to be simple surrender in the face of a philosophical puzzle. But every account must have primitives. The account must be judged on the clarity of the inferences that the primitives warrant and on the degree of systematization they reveal among our pre-analytic inferences. Laws are preferable in point of familiarity to such primitives as necessitation relations among universals. And if we begin by postulating that at each time and place the temporal evolution of the world is governed by certain principles our convictions about possibilities, counterfactuals, and explanations can be regimented and explained. (Maudlin, 2007a, p. 15)

Physical objects behave according to laws. But laws don't reach out to objects and direct them in a certain way. This is all that Maudlin says in the first paragraph. If we say that laws govern or generate motion, this doesn't explain anything:"These metaphors are not offered as analyses."

In the second paragraph we can find the advantages of taking laws as primitive: laws are familiar from the sciences; primitivism is parsimonious; we don't need to introduce further questionable philosophical concepts like necessitation relations among universals. Furthermore, Maudlin (Ch. 5 2007a) shows how to give an account of counterfactuals and causality in terms of primitive laws.

2.2.2. Primary and Secondary Ontology

Similar to Ladyman and Ross (2007), Maudlin introduces laws as primitive entities, but in contrast to their view his laws refer to objects in space and time. Maudlin (2013) describes his entire ontology of physics in the following way. First, every physical theory starts with a set of data that it intends to explain. The data maybe the positions of certain objects, the observable behavior of light, or some change in temperature of a liquid. Maudlin calls this data Primary Observables.

In a second step, a physical theory tries to explain Primary Observables. It does so by introducing some basic objects whose behavior accounts for the data. Dürr, Goldstein, and Zanghì call these objects the primitive ontology, while Maudlin calls them the *Primary Ontology*.

Primary Observables may be the phenomena involved in a quantum double slit experiment. After many runs of the experiment, we see an interference pattern on the screen. The Primary Ontologies of Bohmian mechanics, GRWm, and GRWf account for this phenomenon: in Bohmian mechanics, the particles move in non-classical trajectories; the matter density goes through both slits and contracts to one point on the screen; and in GRWf the distribution of flashes matches the pattern on the screen although nothing goes through the slits. So in all cases, the interference pattern is determined by the behavior of the Primary Ontology.

The Primary Ontology is necessary but not sufficient to explain the Primary Observables. We need more. Our Primary Observables may be apples falling to the ground. For the description of these phenomena, Newtonian mechanics postulates a Primary Ontology of particles. The apple is composed of particles, and what we observe is explained by the motion of these particles. But the explanation doesn't end here – the theory introduces further entities that are supposed to explain why an apple moves the way it does. The apple is in the gravitational field of the earth, and because the apple has a certain mass the field at the location of the apple produces a force directed towards the center of the earth. Therefore, it accelerates until it reaches the ground. Forces, fields, and mass influence the Primary Ontology and provide for a complete explanation of the motion of the apple. So these entities are part of the ontology of Newtonian mechanics as well: they constitute the *Secondary Ontology*.[2]

Epistemic reasons also play a role in the definition of the Primary and Secondary Ontology. The Primary Ontology explains Primary Observables, and the Secondary Ontology explains the behavior of the Primary Ontology. There is some freedom in finding a proper Secondary Ontology. Once we have a Primary Ontology of particles, there can be several Secondary Ontologies. For example, the fall of the apple can also be explained without forces or fields. General relativity accounts for the correct motion of the apple in this manner: it moves on a geodesic until it reaches the ground. Instead of forces and fields, space-time itself constitutes the Secondary Ontology, and its curvature determines the trajectories.

The Primary Ontology of Bohmian mechanics are particles, too, while the Secondary Ontology consists of the quantum state represented by the wave-function. The wave function guides the particles in such a way that they trace out the appropriate trajectories to account for all non-relativistic quantum phenomena.

Now we can put together Maudlin's ontology with his idea of laws of nature. His fundamental ontology consists of two distinguished subclasses: the Primary and Secondary Ontology. The behavior of the elements of both sets is governed by the laws of nature. The laws supervene neither on the Primary nor on the Secondary

[2]Forces, fields, and masses at least are primitive variables in Newtonian mechanics. Whether they are real, and therefore part of the (secondary) ontology, is another issue. For Maudlin they all exist. Furthermore, the distinction between Primary Ontology and Secondary Ontology coincides with the standard distinction between a primitive ontology and its dynamics.

Ontology, so that it is metaphysically possible to keep both ontologies unchanged while introducing different laws of nature. In sum, Maudlin's fundamental ontology consists of the Primary Ontology, the Secondary Ontology, the laws of nature, and space-time.

Making it more Parsimonious

Maudlin doesn't explicitly introduce properties into his ontology. So his Primary Ontology can be interpreted as consisting of propertyless entities. His Secondary Ontology comprises all the dynamical entities that constrain the behavior of the Primary Ontology. It's possible to make Maudlin's fundamental ontology even more parsimonious. Though the elements of the Secondary Ontology are postulated by physics, we can be anti-realists with respect to them: we can interpret forces, fields, and wave functions as purely nomological (see Chapters 3, 4, and 6). Having done so the most parsimonious fundamental ontology for a primitivist consists of primitive stuff, laws, and space-time.

2.3. Dispositionalism

This strategy keeps necessary connections while reducing laws of nature. It does so by adding properties to the primitive stuff ontology. These properties are different from the Humean ones because they are defined by their effects on objects; they are called *dispositions* or *powers* (for instance, Bird, 2007). Laws of nature then express how dispositions constrain the behavior of objects. They aren't further entities in the ontology, as in Maudlin's case, and nor do they supervene on the temporal development of objects. Rather, laws supervene on dispositions by expressing their effects.

Applied to a particle ontology, the particles have certain intrinsic properties that constrain their motion. The standard example of a physical disposition is Newtonian mass. The mass of a particle not only constrains its own motion but also the motion of all the other particles. And the effects of mass are expressed in detail by Newton's laws of motion.

2.3.1. The Problem of Action-at-a-Distance

If we want to predict the motion of a classical particle P_1, it does not suffice to know the initial position and velocity of this particle. Its motion also depends on the distribution of all the other particles in the universe. So the mass m_1 cannot "on its own" generate the motion of P_1; instead, there has to be a "connection" with all the other particles. Saying that laws establish this connection doesn't help since they supervene on dispositions. It doesn't follow from the definition of intrinsic properties that they can influence each other over spatial distances.

A primitivist about laws has no such problem since the laws of nature constitute modal connections. And in order to mediate between particles and laws, Maudlin

(2013) introduces a Secondary Ontology, which contains forces, fields, and wave functions. The Humean has also an answer: there are no modal connections between particles. All regularities are contingent.

The dispositionalist can bite the bullet: it's simply a primitive fact that masses influence each other over arbitrary distances. But then properties are no longer important for the motion of the particles because the law determines the effects of the distant action. What is the purpose of properties when they don't explain how they can influence one another?

2.3.2. Fields

Fields may help the dispositionalist because, in virtue of its mass, a particle generates a field. And all particles are affected by this field. So even if there is a particle that is very far away from all the other particles in the universe, it can be influenced by them through the net field at its position.

Fields in the Primary Ontology

The ontological status of fields is debatable. They can be part of the Primary Ontology or Secondary Ontology. In the former case, fields are primitive stuff in addition to the particles. This dualism might be bothering because there doesn't seem to be an ontological difference between a point particle and a point of the field, "A point is a point." And it might not be parsimonious to have two kinds of stuff in the ontology.

I don't see any problems with this dualism since you can stipulate that there is a primitive difference between point particles and points of fields. If that's an instance of haecceitism, so be it. And the physical theories easily distinguish between fields and particles. Therefore, there is neither a metaphysical nor a methodological ambiguity if we have fields be part of the primitive stuff ontology.

Fields in the Secondary Ontology

If fields are part of the Secondary Ontology they are construed as dynamical entities that constrain the motion of particles. This is a better interpretation since the effects of fields are accounted for by the behavior of matter. For example, we can detect the earth's magnetic field by means of a compass. But what we actually see is the behavior of the needle always pointing in the same direction; we never observe the field itself. Tim Maudlin also regards the electromagnetic field as part of the Secondary Ontology. And he denies that there is a suitable philosophical category for fields:

> Yet in the end, the question of the nature of the lines of force was settled (at least temporarily) by inventing a whole new species of physical entity: the field. A field is not a collection of particles, does not depend on the presence of polarized molecules, and does not require a mechanical ether. But what,

> we might hear a nineteenth-century philosopher or physicist ask, is it? What category of being does it belong in?
>
> By now, we have become so accustomed to the notion of a field that we are likely to be frustrated and answer roughly. There is no necessity that an electromagnetic field should fit neatly into any preexisting ontological scheme: there are more things in heaven and Earth, Horatio, than are dreamt of in your philosophy. One should stop trying to assimilate the field to some familiar concept, and rather admit it as a fundamental new sort of physical entity. (Maudlin, 2013, pp. 127–8)

Physics tells us that fields can exist in empty space; no medium has to carry a field. So fields could be interpreted as properties of space-time points. Imagine two particles P_1 and P_2 with masses m_1 and m_2 respectively. Because of its mass, P_1 generates a field, that is to say, the particle changes the intrinsic properties of space-time points. The second particle moves according to the field value at its position; for example, the field could be directed in the x-direction and is $10\frac{N}{kg}$ in strength. If $m_2 = 1kg$ then P_2 would experience an acceleration of $10\frac{m}{s^2}$ in the x-direction. This acceleration is generated by the intrinsic properties of the particles and of space.

But we instantly recognize that interpreting the field as a distribution of local properties in space leads to another dualism. There are dispositions of particles, and there are dispositions of space-time. Mass as an intrinsic property of particles affects their motion, while the field-value of a point in space doesn't affect space itself. Instead, the field only acts on particles.

Lawrence Sklar (1977) criticized this application of properties to space-time. Some philosophers wanted to get rid of objects by "replacing objects by the region of space-time they occupy" and claiming that space-time has some ominous "objectifying" feature (p. 166). Sklar calls this reduction of objects to properties of space-time a "linguistic trick", and I consider it a linguistic trick, too, to say that fields can be reduced to properties of space-time that have no effect on space-time itself. The situation would be different if fields changed the structure of space-time, as in general relativity. In that case, the property has some effect both on space-time and the motion of objects.

It's also a linguistic trick when fields spread triggering conditions for the manifestation of mass all over space-time. We might speak this way in describing the motion of particles: given the value of fields at the position of the particle, the particle moves in a certain manner. But this way of speaking doesn't elucidate the metaphysical status of fields. At least, this leaves obscure what fields are if they exist as triggering conditions on space-time. I think fields as distributions of triggering conditions have to be interpreted nomologically. They appear in the formulation of the laws to account for the motion of particles. But then action-at-a-distance remains, for the dispositionalist.

2.3.3. Ontic Structural Realism as Relational Dispositionalism

What else can we do to avoid action-at-a-distance? The problem is that intrinsic dispositions entail a metaphysical action-at-a-distance. And since dispositionalism holds to necessary connections, this position falls back to primitivism about laws. Fields may help to mediate the action between particles. But there is another viable option. Instead of intrinsic disposition, we can assign *relational* dispositions to particles, as Esfeld (2009) proposed in his suggestion regarding ontic structural realism. Then by definition there is a dynamical non-reducible connection between particles that reach over space. This would get rid of action-at-a-distance. As Esfeld points out,

> there is no action at a distance [. . .], simply because a modal structure instantiated by the configuration of matter is another conception of the determination of the temporal development of physical objects than direct interaction among the parts of that configuration. (2014, p. 10)

In fact, there are two versions of action-at-a-distance: there is *metaphysical* action-at-a-distance, and there is *physical* action-at-a-distance. A primitive ontology of particles with intrinsic properties leads to metaphysical action-at-a-distance if intrinsic properties influence each other over spatial distances. A metaphysical solution would be dynamical relations instead of intrinsic properties.

Yet OSR doesn't rule out physical action-at-a-distance. In Newtonian mechanics, we can interpret forces as instantiating a dynamical structure (see details in Chapter 3). Newton's laws then represent the effects of this structure. Because this structure is by definition relational there is no longer a problem of metaphysical action-at-a-distance.

But Newton's gravitational theory continues to be a physical action-at-a-distance theory because no physical entity in space and time mediates the action. It seems that only fields or the exchange of other particles would count as a physical non-action-at-a-distance mechanism. The action would be still mediated instantaneously, since no philosophical interpretation can make Newton's gravitation retarded. To have a finite propagation in time we need a new physics, like general relativity.

2.3.4. 'Ways Substances Are' and Powerful Qualities

Now let's delve into another version of dispositionalism:

> Substances are property bearers; properties are ways substances are. If there are substances, there are properties; if there are properties, there are substances. Every substance is some way or other, every property is a way some substance is. Substance and property are complementary categories of being. (Heil, 2012, p. 12)

Heil introduces a substance-property ontology. All material objects are composed of substances. And the substances aren't primitive stuff: they have by definition a certain way of being. His description of properties as "ways substances are" is non-standard, and I don't clearly understand what he means by it. It seems that he wants to reduce the notion of property to the substances. Usually we introduce substances and properties as primitive notions and say that substances have certain properties. But this is not Heil's aim. He tries instead to derive what properties are from the substances themselves. We see this move in the next quote, where he applies his ideas to physics.

> Substances are not bare, featureless entities to which properties attach themselves as limpets attach themselves to rocks at the seashore. Every substance is *itself* some way or other, indeed many ways. These ways are its properties. For a substance to possess a property is for it, the substance, to be a particular way. Properties—ways—do not make up a substance, they are not parts of substances. The charge, spin, and mass of an electron are not parts or constituents of the electron. As far as we know, electrons have no parts. Electrons might have spatial or temporal parts, but that is another matter, one I shall take up in due course. An electron's charge, spin, and mass are ways the electron is. (2012, p. 15)

Heil dissociates himself from a bundle theory of properties. This theory is a monistic theory, where there are only properties on the basic level and objects are derived from the properties. A certain bundle of properties constitutes an object. For example, an electron is interpreted as a bundle of mass and charge. To my mind, the bundle theory of properties rests on a misuse of language because properties are by definition properties of something. It's implausible that mass and charge exist in the void and that they can form particles.

Neither does it help to say that mass and charge are properties instantiated by a point in space. First, mass and charge don't affect space itself. Second, this would lead to super-substantivalism: all objects are properties of space. I see no good reason to reduce objects, like electrons or tables, to space itself—there may be reasons in general relativity, but that is beyond the scope of this thesis.

Heil dissociates himself from a dualistic ontology too, in that both substances and properties are primitives. For him "[s]ubstances are not bare, featureless entities to which properties attach themselves." But there seems to be only a linguistic difference in his description of properties as the ways substances are from the standard one as being intrinsic to the substance, since in the end Heil needs to introduce both substances and properties as primitives into his ontology.

Nevertheless, he somehow wants to derive properties from substances. But I see neither a motivation nor an advantage to this. I think everything is already included when substance X is said to have intrinsic property Y. Why should we try to reduce the properties to "ways substances are?" I would regard intrinsic properties as ways substances are that determine the ways substances behave without using Heil's purported reduction.

What does Heil say about the causal role of properties? Are they categorical or dispositional?

> The moment you divorce qualities and powers, you have little use for qualities (at least until you start worrying about consciousness). I have suggested that there are good reasons to think that qualities are required for the individuation of powers by reference to what those powers are powers *for*. Given the reciprocity of manifestations this, perhaps surprisingly, encourages the view that powers and qualities are not merely contingently associated. But if powers and qualities are associated of necessity, what is the nature of this necessity, what is its basis? Is it simply a 'brute' necessary correlation? That would be hard to swallow. It would, in addition, keep qualities out of the causal picture. Qualities would be necessary, but epiphenomenal, accompaniments of powers. This, I have suggested, is a source of unwelcome difficulties and, indeed, implausibilities. You can accommodate the necessity and resolve the difficulties by *identifying* powers and qualities, turning properties into *powerful qualities*.
>
> Philosophers are apt to regard such a view as contentious in the extreme, but I would wager that non-philosophers accept it as too obvious to bear mention. Things do what they do because they are as they are, and ways things *are* are qualities. When you cite qualities in causal explanations, when you say that the bull charged because the matador's cape was red, you are not citing features of objects you take to be correlated with their powers. The cape's redness, you think, sparked the bull's anger: in virtue of being red, the cape has the power to attract the attention of aggressive bulls. (Heil, 2012, p. 80)

According to Heil, properties are both qualities and powers. Because an object has a certain quality it exerts a certain power. As well as the bull example, Heil also mentions a tomato to illustrate his idea. A tomato is round, and therefore it can roll down a slope. In other words, one quality of a tomato is its roundness, and because of its roundness it can roll: "it is in virtue of possessing this quality that the tomato would roll" (2012, p. 61). The redness of the cape has certain effects on the bull, and the roundness of the tomato has some effects on itself. But these are effects either due to geometrical features or the atomic constellation of the physical body. The quality of redness or the quality of roundness can be reduced to the geometry of the tomato or the way the molecules of the cape are arranged. They are not fundamental non-reducible qualities of the objects.

This is different for mass. If it is a property of particles it cannot be reduced to their geometric or atomic features. So what is the qualitative nature of mass supposed to be? Physics doesn't tell us. Physics only describes the effects of mass. Heil simply declares that the qualitative part of any property exists; he insists that mass has a qualitative aspect without explaining what it is. Furthermore, the qualitative basis of properties inherits the problem of humility, which we encountered in Lewis's Humeanism.

Summary

The primitive stuff ontology evolved from the primitive ontology of quantum mechanics. The advantage is its lack of properties; it only introduces bare objects. It emphasizes that what we actually measure in experiments are the positions of objects. We don't measure properties. The primitive ontology also emphasizes that the positions of objects play the primary role in measurements, but it doesn't take this idea to its purest form.

Furthermore, with the introduction of a primitive stuff ontology we can dissect and compare the fundamental ontologies of the current metaphysical theories:

- Lewis's Humeanism:

  fundamental ontology = local qualities + spatiotemporal relations.

- Neo-Humeanism:

  fundamental ontology = primitive stuff + spatiotemporal relations.

- Primitivism about laws:

  fundamental ontology = primitive stuff + laws of nature + space-time.

- Dispositionalism:

  fundamental ontology = primitive stuff + intrinsic properties + space-time.

- Ontic structural realism:

  fundamental ontology = primitive stuff + dynamical relations + space-time.

Part II.

The Classical Universe

3. Newtonian Mechanics

3.1. What Is Matter?

Newton was committed to absolute space. In his seminal work *Philosophiae Naturalis Principia Mathematica* (*English:* The Mathematical Principles of Natural Philosophy; first edition published 1687, third edition published 1726), he explains what he means by this:

> Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, [. . .]. (Newton, 1999, pp. 408–9)

We can imagine absolute space as the container in which physics takes place. If we were to eliminate all material objects from the universe absolute space would still remain as a genuine entity. When Newton described it as "immovable" he might have been referring to two separate characteristics. First, absolute space is indifferent to the motion of objects; that is, space doesn't affect the motion of particles, nor do particles have any influence on space itself. Second, every point in space has an intrinsic primitive identity that persists through time. Hence, for every object there is an absolute fact about where it is located.

Newtonian Particles For Newton, the fundamental constituents of matter are particles:

[I]t seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced

Newtonian Particles

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> [I]t seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary

> Power being able to divide what God himself made one in the first Creation. While the Particles continue entire, they may compose Bodies of one and the same Nature and Texture in all Ages: But should they wear away, or break in pieces, the Nature of Things depending on them, would be changed. Water and Earth, composed of old worn Particles and Fragments of Particles, wouldn't be of the same Nature and Texture now, with Water and Earth composed of entire Particles in the Beginning. And therefore, that Nature may be lasting, the Changes of corporeal Things are to be placed only in the various Separations and new Associations and Motions of these permanent Particles; compound Bodies being apt to break, not in the midst of solid Particles, but where those Particles are laid together, and only touch in a few Points. (1952, p. 400)

In this passage from the *Opticks* (published in 1704), Newton clearly describes an ontology that coincides with the primitive ontology of particles. He considers particles to be the fundamental entities that form all the physical objects in the world, where the behavior of macroscopic bodies can be reduced to the motion of these particles. This shows that Newton wanted a physical theory that didn't only explain and predict the observable behavior of physical objects but that would give us a picture of how the world is from the ground up. Newtonian mechanics was not built to be merely an effective theory; it was supposed to be a fundamental theory of nature.

The way Newton defines particles is slightly different from how I have done so, because he assigns intrinsic properties to them, such as solidness, hardness, or impenetrability. In fact, these are dynamical features, which can be deduced from the *motion* of particles. You don't need to introduce them by definition. Instead, I defined particles as primitive stuff. And it's a primitive fact of space that one point can be either occupied or empty, so that two different particles cannot share the same location. The laws of motion must necessarily respect this constraint.

It's historically interesting to note that Newton really took his particles very seriously. In the composition of macroscopic objects, they can form "bigger" particles that are held together by some unknown "attractions":

> Now the smallest Particles of Matter may cohere by the strongest Attractions, and compose bigger Particles of weaker Virtue; and many of these may cohere and compose bigger Particles whose Virtue is still weaker, and so on for divers Successions, until the Progression end in the biggest Particles on which the Operations in Chymistry, and the Colours of natural Bodies depend, and which by cohering compose Bodies of a sensible Magnitude. (1952, p. 394)

In essence, "bigger" particles are composed of basic particles like molecules, which are composed of atoms. Newton even gave some concrete example of bigger particles:

> And so if any one should suppose that *Aether* (like our Air) may contain Particles which endeavour to recede from one another (for I don't know what

> this *Aether* is) and that its Particles are exceedingly smaller than those of Air, or even than those of Light [...]. (1952, p. 352)

> So that a Particle of Salt may be compared to a Chaos; being dense, hard, dry, and earthy in the Center; and rare, soft, moist, and watry in the Circumference. (1952, p. 386)

These quotes show that for Newton, all physical objects are indeed made of particles; even light is a conglomeration of corpuscles. They also show that Newton thought physics to be the most basic scientific theory, and that there was a continuity between physics and chemistry such that physical processes underlie all chemical processes.

Mass and Quantity of Matter

I have so far ignored one crucial feature of particles that Newton mentions in the quote from the *Opticks*: particles have mass. Mass is so central a notion to Newtonian physics that the very first definition in the *Principia* is devoted to it:

> **Definition 1**
> *Quantity of matter is a measure of matter that arises from its density and volume jointly.*
> If the density of air is doubled in a space that is also doubled, there is four times as much air, and there is six times as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction, and also for all bodies that are condensed in various ways by any causes whatsoever. [...] Furthermore, I mean this quantity whenever I use the term "body" or "mass" in the following pages. It can always be known from a body's weight, for—by making very accurate experiments with pendulum—I have found it to be proportional to the weight, as will be shown below. (Newton, 1999, pp. 403–4)

In the beginning, Newton isn't concerned about mass. He wants to define what is meant by "quantity of matter." As he defines it, the quantity of matter of a physical object is composed of the density of the object and its total volume. Then in the explanation to definition 1, we notice that the quantity of matter is the *product* of density and volume. So there are two ways to increase the amount of matter contained in a certain spatial region. Either one enlarges the volume, and because there is more space there can be more matter located in that space, or, by "squeezing" matter together and increasing its density, one manages to have more matter in a given volume, too.

A definition usually explains one notion with the help of other notions and concepts that have been previously defined or that are so clear to the reader that they can be taken for granted. As this is the first definition of the *Principia*, Newton should use notions to define the "quantity of matter" whose meaning is universally agreed upon. Volume of space poses no problems. But what about density?

Newton doesn't explain the meaning of density. Clearly, this notion depends on what matter is. Newton thought that matter consisted of particles, but he doesn't explain the density of particles. It seems that Newton presupposed that the reader would know what density is.

The situation becomes more puzzling when Newton introduces "mass" in the explanation to definition 1. First, mass coincides with quantity of matter; it's just a different word. Second, mass is also used in the laws of motion of the particles,and so it is a dynamical property. A priori, it isn't at all clear how the dynamics of particles is connected to the quantity of matter. Third, as physics evolves, density is standardly defined as mass over volume, and therefore it seems that definition 1 is circular. This bothered Ernst Mach, who repeatedly criticized Newton in his *Science of Mechanics*:

> With regard to the concept of "mass," it's to be observed that the formulation of Newton, which defines mass to be the quantity of matter of a body as measured by the product of its volume and density, is unfortunate. As we can only define density as the mass of unit of volume, the circle is manifest. (1919, p. 194)

Mach emphasizes that the identification of quantity of matter with the dynamical property mass is inconsistent:

> Definition 1 is, as has already been set forth a pseudo-definition. The concept of mass isn't made clearer by describing mass as the product of the volume into density as density itself denotes simply the mass of unit volume. The true definition of mass can be deduced only from the dynamical relations of bodies. (1919, p. 241)

Newton grants that we have epistemic access to the mass of an object by, for example, weighing that object. But he seems to regard the essence of mass to lie in the amount of stuff contained in a certain region. Mach correctly points out that mass can only be consistently defined as a dynamical property.

What is the true meaning of the quantity of matter, then? Having a primitive-stuff ontology of particles, the quantity of matter can be defined as the number of particles in a certain volume. So, given two macroscopic bodies B_1 and B_2, B_1 contains more matter than B_2 if and only if B_1 is composed of more particles than B_2. Obviously, mass is then no longer a measure of the quantity of matter because the total mass doesn't give us a hint of the number of particles. The mass of a single particle m can exceed the joint mass of two other particles $m_1 + m_2$.

Can we also make sense of the quantity of matter of a *single* particle? Can particle P_1 contain more matter than particle P_2? Doing so would mean that particles are no longer primitive entities. Matter—whatever it may be—would be the fundamental stuff in the ontology. And having differences of stuff at single points is very hard to make comprehensible. I defined particles as occupied points of space-time; either the point is occupied or it's not. It doesn't make sense to say that there is "more" particle here and "less" particle there.

The density is the number of particles per volume; hence, the density is a measure of how far apart the particles are on average. Density, as it's usually defined—as *mass* per volume—, doesn't say anything about the quantity of matter; rather, it's a measure of the dynamical properties of macroscopic objects, for which it's more convenient to give the density instead of the entire mass. For example, the mathematical treatment of fluids uses mass densities, since fluids change their shape.

Restating Mach, "The true definition of mass can be deduced only from the dynamical relations of bodies." Before examining the true dynamical role of mass in detail, we first have to prepare the field by discussing the laws of motion for classical particles.

3.2. The Laws of Motion

3.2.1. The First Law

Three laws of motion are at the center of Newton's physics. Let us start with the first one.

> **Law 1**
> *Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it's compelled to change its state by forces impressed.* (Newton, 1999, p. 416)

This law singles out a distinguished kind of motion, namely *inertial motion*. A body is in inertial motion either if it rests or if it moves in a straight line at constant speed. "Rest", "moving", and "uniform" are uniquely defined relative to absolute space and time.

The first law states that under certain circumstances a body deviates from inertial motion. Whenever a body isn't in inertial motion there have to be forces acting on this body, and if there are no forces the body will continue to move in inertial motion. This is in accordance with the way Newton defines the role of *impressed forces*:

> **Definition 4**
> *Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward.*
>
> This force consists solely in the action and doesn't remain in a body after the action has ceased. For a body perseveres in any new state solely by the force of inertia. (Newton, 1999, p. 405)

Newton's first law, as well as his fourth definition, has been the subject of much discussion and misunderstanding. Physics textbooks introduce the first law—if it's introduced in the first place—without a definition of what is meant by impressed forces. Therefore, they take the first law to be a *definition* of forces as anything that prevents the body from inertial motion. Although this is a consistent definition or interpretation of forces, it isn't the content of Newton's first law. It's a definition and not a law. Newton's first law is a law about the *motion* of particles, and it shows how *every* body will behave if there are no forces acting on it. The notion of force in the first law begs for an explanation, and this explanation is provided by definition 4. This definition confirms that there is nothing more to forces than one would expect from the first law, that is to say that they are those entities that change inertial motion. And there are no other entities that can do this.

Is the First Law a Tautology?

One might now claim that given definition 4 the first law turns out to be a tautology. Why? Because if a force is defined as that entity that changes inertial motion, it follows that whenever there are no forces on the body it has to stay in inertial motion. And therefore Newton's first law has no physical content at all, given definition 4. It was none other than Mach who pointed this out.

> We readily perceive that Law I [...is] contained in the definitions of force that precede. According to the latter, without force there is no acceleration, consequently only rest or uniform motion in a straight line. [...] It would have been enough to say that the definitions premised were not arbitrary mathematical ones, but correspond to properties of bodies experimentally given. (Mach, 1919, p. 242)

Nevertheless, Mach is mistaken: we cannot deduce physical behavior or laws from definitions. Given definition 4, it seems that a force-free body must rest or move in a straight line, but a physical body might in fact behave in a completely different way. The laws could be different. Just observing things in our daily life we would rather come up with a law that says that ordinary motion is deceleration, and there has to be something that keeps a body in inertial motion. We could keep definition 4, but it would be inconsistent with our laws of motion. So the alleged tautology confirms that the definition of forces is consistent with the dynamical laws. And Mach somehow anticipates this interpretation when he says that "the definitions premised were not arbitrary mathematical ones, but correspond to properties of bodies experimentally given."

Inertia and Inherent Forces

Newton defines impressed forces in contrast to what he calls *inherent forces*:

> **Definition 3**
> *Inherent force of matter is the power of resisting by which every body, so far as it's able, perseveres in its state either of resting or of moving uniformly straight forward.*
>
> This force is always proportional to the body and doesn't differ in any way from the inertia of the mass except in the manner in which it's conceived. Because of the inertia of matter, every body is only with difficulty put out

> of its state either of resting or of moving. Consequently, inherent force may also be called by the very significant name of force of inertia. Moreover, a body exerts this force only during a change of its sate, caused by another force impressed upon it, [. . .]. (Newton, 1999, p. 404)

For Newton inherent force is another name for the inertia of matter, whose effects are described in his first law. The choice to call it a force may result from the following scenario: when body B_1 accelerates another body B_2 (by pushing or pulling, for example), then B_1 feels a force that is exerted upon it by B_2, and the more mass B_2 has the more difficult it is to change its state of motion. In other words, every body strives to stay in inertial motion, and mass is a measure of how strongly it endeavors to be in this state. Not only is it a measure for staying in inertial motion, but it's the cause of remaining in inertial motion when there are no external forces. Nowadays inertia is no longer understood as a force. Rather, it's seen as a property, whose effect is described by the first law.

3.2.2. The Second Law

Newton's second law supplements the first law and shows how forces quantitatively act on bodies:

> **Law 2**
> *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.* (Newton, 1999, p. 416)

The second law needs the first one, since the first speaks of the *state* of inertial motion, while the second describes the *change* of inertial motion, which is parallel and proportional to the impressed force. With the help of this law we can apply Newton's physics to real-world cases and explain the temporal behavior of matter.

Three Quantities of Forces

Newton talks about motive force in his law because he introduces three quantities of force (see AppendixA): absolute quantity, accelerative quantity, and motive quantity. The absolute quantity of a force is the strength of its source. For example, the absolute quantity of the gravitational force is gravitational mass. The accelerative quantity of force is a measure of the force for producing a certain acceleration. Newton seems to refer here to a field spread in space that accelerates an object due to its mass. The acceleration of an object, though, is independent of its mass. The motive force, however, depends on the mass, and it's what we nowadays just call the force exerted on the object. Therefore, Newton refers to the *motive* force. We could perfectly understand the second law by leaving out the attribute motiv (Stein, 1970, p. 266).

Mach's Critique of the Second Law

The second law wasn't saved from Mach's critical eye, either. Here is the entire quote, without ellipses, that I have already presented:

> We readily perceive that Laws I and II are contained in the definitions of force that precede. According to the latter, without force there is no acceleration, consequently only rest or uniform motion in a straight line. Furthermore, it's wholly unnecessary tautology, after having established acceleration as the measure of force, to say again that change of motion is proportional to the force. It would have been enough to say that the definitions premised were not arbitrary mathematical ones, but correspond to properties of bodies experimentally given. (Mach, 1919, p. 242)

So according to Mach, definitions 3 and 4 don't only contain the first law, but also comprise the content (of the first part) of the second law. But still there is a difference between a tautology and consistency between laws and definitions. And even if the laws of motion restate parts of definitions, this doesn't make them wrong.

Modern Formulation

In order to make precise calculations, modern physics formulates the second law as a differential equation. Consider N-particles with positions $\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N \in \mathbb{R}^3$ that lie on the trajectories $\boldsymbol{q}_1(t), \ldots, \boldsymbol{q}_N(t)$, where $t \in \mathbb{R}$ is the parameter of time. The equation of motion for the i-th particle then is

$$\boldsymbol{F}_i\left(\boldsymbol{q}_1(t), \ldots, \boldsymbol{q}_N(t), \dot{\boldsymbol{q}}_1(t), \ldots, \dot{\boldsymbol{q}}_N(t), t\right) = m_i \ddot{\boldsymbol{q}}_i(t) \tag{3.1}$$

with the i-th particle $\boldsymbol{F}_i$, the velocity $\dot{\boldsymbol{q}}_i$ and acceleration $\ddot{\boldsymbol{q}}_i$ of the i-th particle, and N constants of proportionality $m_1, \ldots, m_N$, which happen to be the masses of the particles, respectively.

The apparent mathematical role of mass in (3.1) is to be a constant of proportionality when connecting the force on a body with its acceleration. But why is mass the correct or appropriate constant of proportionality? It isn't explicitly mentioned in Newton's original formulation of the second law. It's hidden in what Newton meant by motion. Nowadays physicists associate motion only with velocity, but for Newton motion is what we now call momentum, as one can see in his second definition.

> **Definition 2**
> *Quantity of motion is a measure of motion that arises from the velocity and the quantity of matter jointly.*
> The motion of a whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity there is twice as much motion, and if it has twice the velocity there is four times as much motion. (Newton, 1999, p. 404)

Clearly, the above differential equation shows precisely how Newton formulated his second law. And it includes the information of his first law, too: the absence of forces results in inertial motion. So both the first and the second law are summarized in one law of motion, namely, the above differential equation (3.1).

Superposition of Forces

In application you often find several component forces acting on a body such that the left hand-sight of (3.1) is the net force on that body which results in the acceleration. To cope with these cases Newton deduced a corollary from his laws:

> **Corollary 1**
> *A body acted on by [two] forces acting jointly describes the diagonal of a parallelogram in the same time in which it would describe the sides if the forces were acting separately.* (Newton, 1999, p. 417)

It strikes us as a little peculiar how one can prove the additivity of forces from the three axioms of motion; therefore we cite Newton's original proof.

> Let a body in a given time, by force M alone impressed in A, be carried with uniform motion from A to B, and, by force N alone impressed in the same place, be carried from A to C; then complete the parallelogram $ABDC$, and by both forces the body will be carried in the same time along the diagonal from A to D. For, since force N acts along the line AC parallel to BD, this force, by law 2, will make no change at all in the velocity toward the line BD in the same time whether force N is impressed or not, and so at the end of that time will be found somewhere on the line BD. By the same argument, at the end of the same time it will be found somewhere on the line CD, and accordingly it's necessarily found at the intersection D of both lines. And, by law 1, it will go with [uniform] rectilinear motion from A to D. (Newton, 1999, pp. 471–8)

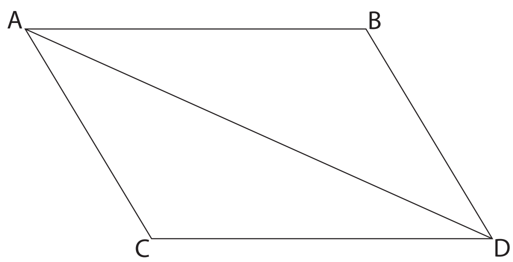

Figure 3.1.: Parrallelogram of forces

First, we notice that the forces in this proof act during a very short period of time. The body is accelerated and then moves uniformly in a straight line. Newton introduces two forces: M acts along AB, and N along AC. So if only one force were impressed on the body would it move either from A to B during Δt, or from A to C during Δt. Now comes the crucial case. If both forces act on the body, they won't influence the motion induced by the other force. This is due to the second law: a change in motion only takes place along the straight line in which that force is impressed. The second law is to be understood such that a force doesn't influence any acceleration in any other direction. So by adding up the paths that the body would follow if it were accelerated by either force, it has to move along the diagonal of a parallelogram. This proof is a nice example of how Newton uses both laws in his reasoning, the first for the description of inertial motion and the second for acceleration.

As the modern formulation of Newton's laws of motion show, forces are mathematically described as vectors. And the superposition of forces is part of the formulation of (3.1). So if forces $\boldsymbol{F}_1, \ldots, \boldsymbol{F}_r$ act on a body at the same time the net or resulting force is $\boldsymbol{F}_{net} = \sum_{k=1}^{r} \boldsymbol{F}_k$, and the second law in full generalization is

$$\boldsymbol{F}_{net} = m\boldsymbol{a}. \tag{3.2}$$

Newton did not have the mathematics of vector calculus; therefore he had to prove the superpositions of forces as a corollary to his first and second laws. But from a modern point of view the superposition of forces is included in the mathematics of vectors. Newton didn't know about vector calculus, and the proof of this corollary shows that he thought net forces to be non-existent. He considers the two forces acting on the body as two different entities that cause two different motions of the body. And since one force doesn't affect the influence of the other force, there will be a resulting motion on the diagonal of the parallelogram. It was important for Newton that there has to be a source for all the forces acting on a body. But for the net force there is no source; it's the sum of the component forces. Only component forces exist.[1]

3.2.3. The Third Law

From our own experiences, we know that when we push a body we feel a force from this body as well. Newton generalized this phenomenon in his third law.

> **Law 3**
> *To any action there is always an opposite and equal reaction; in other words the actions of two bodies upon each other are always equal and always opposite in direction.*
>
> What ever presses or draws something else is pressed or drawn just as much by it. [...] If some body impinging upon another body changes the motion of that body in any way by its own force, then, by the force of the

[1]For a modern debate on the reality of component forces see Lange (2009); Massin (2009, 2016); Wilson (2009).

> other body [. . .], it also will in turn undergo the same change in its own motion in the opposite direction. By means of these actions, equal changes occur in the motions, not in the velocities—that is, of course, if the bodies aren't impeded by anything else. (Newton, 1999, p. 417)

The third law is one of Newton's original discoveries (preliminary versions of the first and second law has already been discussed before Newton, for instance, by Descartes), and Mach (1919, p. 198) appreciated it as "[p]erhaps the most important achievement of Newton with respect to the principles.".

Is the Third Law Independent of the First Law?

As Newton discusses in the Scholium following the laws of motion (1999, pp. 427–8), there would be spontaneous acceleration in case the third law were not valid:

> I demonstrate the third law of motion for attractions briefly as follows. Suppose that between any two bodies A and B that attract each other any obstacle is interposed so as to impede their coming together. If one body A is more attracted toward the other body B than that other body B is attracted toward the first body A, then the obstacle will be more strongly pressed by body A than by body B and accordingly won't remain in equilibrium. The stronger pressure will prevail and will make the system of the two bodies and the obstacle move straight forward in the direction from A toward B and, in empty space, go on indefinitely with a motion that is always accelerated, which is absurd and contrary to the first law of motion. For according to the first law, the system will have to persevere in its state of resting or of moving uniformly straight forward, and accordingly the bodies will urge the obstacle equally and on that account will be equally attracted to each other. (Newton, 1999, pp. 427–8)

So if the third law doesn't hold then the two bodies A and B don't attract each other by the same forces. Then there would be a net force accelerating the whole system. According to Newton, the acceleration of the system violates the first law. I think Newton is mistaken here. The first law says that forces and acceleration come in pairs: whenever there is acceleration there are forces acting on the body and vice versa. And this is the case here.

Newton's thought experiment would indeed contradict the first law only if impressed forces are *external* forces. The net force from A to B doesn't originate from outside the system, but it leads to accelerations. Newton's definition of impressed forces doesn't explicitly identify impressed forces with external ones. Literally, he wrote (see page 53), "[i]mpressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward." It isn't clear what "exerted on a body" is supposed to mean here. It can mean "exerted from outside", which would identify impressed forces with external forces, or it refer to the net force. It seems, though, that the way Newton applies his laws presupposes that impressed forces are external forces.

The modern mathematical formulation of the first and second laws (3.2) clearly considers the force resulting in acceleration as the net force acting on the body. Applying this law to the system consisting of the bodies A, B, and the opposing body, there is no contradiction. As long as the system accelerates in the direction of the net force and fulfills the equation there isn't anything wrong with it. Spontaneous acceleration is then acceleration when the net force is zero; this would contradict (3.2).

Conservation of Energy and Momentum

Newton's third law is needed for the conservation of momentum and energy. We could have a mechanics that violates the third law while keeping the first two, but then the system wouldn't obey the conservation of momentum and energy. What happens in the above thought experiment is the acceleration of the center of mass of the system. If the third law were valid the motion of the bodies wouldn't change the motion of the center of mass, which would remain in inertial motion as long as there was no intervention from outside.

3.2.4. The Law of Gravitation

Although Newton knew about electric and magnetic forces, he only gave a quantitative description of gravitation:

> **Proposition 7**
> *Gravity exists in all bodies universally and is proportional to the quantity of matter in each.*
> We have already proved that all planets are heavy [or gravitate] toward one another and also that the gravity toward any one planet, taken by itself, is inversely as the square of the distance of places from the center of the planet. And it follows [...] that the gravity toward all the planets is proportional to the matter in them. (Newton, 1999, p. 810)

What he put in words in this proposition is nothing but the *universal law of gravitation*:

$$\boldsymbol{F}_i(\boldsymbol{q}_1, \dots, \boldsymbol{q}_N) = \sum_{j \neq i} \mathrm{G}\, m_i m_j \frac{\boldsymbol{q}_i - \boldsymbol{q}_j}{\left\| \boldsymbol{q}_i - \boldsymbol{q}_j \right\|^3} \tag{3.3}$$

with the gravitational constant G. The law is time-independent, only depending only on the positions of the particles. The magnitude of gravity is directly proportional to the masses of particles and inversely proportional to the square of the distance between them.

Hence, merely taking gravitation into account Newton's second law (3.2) changes to

$$\boldsymbol{F}_i(\boldsymbol{q}_1(t), \dots, \boldsymbol{q}_N(t)) = m_i \ddot{\boldsymbol{q}}_i(t),$$

where $\boldsymbol{F}_i$ is the gravitational force (3.3). $\boldsymbol{F}_i$ no longer depends on the velocities of the particles.

The gravitational force (3.3) doesn't appear out of thin air. Indeed, (3.3) is a solution of the *Poisson equation*

$$\nabla \cdot \boldsymbol{F}_i = -4\pi \mathrm{G} m_i \rho, \tag{3.4}$$

where ρ is the mass density at one point of physical space (see, for instance, Anderson, 1967, p. 118). Therefore, Newton's theory of gravity consists of two sets of laws for the description of particles, which interact only through gravity, namely, (3.1) and (3.4).

3.3. Do Forces Exist?

Newtonian mechanics is a theory of forces. Forces determine the motion of objects. All three laws of motion mention different aspects of forces. The first law says what an object does without forces acting on it: it keeps moving in a straight line with constant velocity. The second law says how forces change motion: the object will accelerate in the direction of the force. The third law says how forces on two interacting bodies are related: to every force there is a force on another body that is parallel but in the opposite direction.

If forces play this prominent role in the basic laws, they must exist, mustn't they? Doesn't the first law say that forces *cause* acceleration? How can they do so if they don't exist? In fact, *force* is a theoretical term since we cannot directly observe forces as we can directly observe tables, chairs, and trees. And as with all theoretical terms, it needs arguments and close scrutiny before we can decide whether it corresponds to something that exists.

It doesn't suffice to say that forces exist. We need to clarify their ontological status. Even if we conclude that forces don't exist, notwithstanding their usefulness, we still need to explain what they are.

3.3.1. Why Forces Don't Exist

There are three standard arguments against the existence of forces:

1. The effects of forces are causally overdetermined,
2. The existence of forces lead to a vicious regress, and
3. Forces are redundant when it comes to changes in motion.

Let's treat them one after the other.

Causal Overdetermination

This argument against the reality of forces is based on three assumptions (Wilson, 2007, p. 198):

1. Changes in motion are caused by forces.

2. Every such change in motion is caused by an entity necessitating the force.

3. Forces and their necessitating entities are distinct.

It's then concluded that every change in motion is causally overdetermined by forces and their necessitating entities.

Figure 3.2 visualizes the above argument. Double arrows represent causal relations, and the normal arrow, a necessitating relation (which might be causal). Assumption 1 then justifies the double arrow from force to acceleration. The second proposition gives rise to the other arrows. X is an arbitrary necessitating entity for forces, which at the same time causes acceleration—I'll say in a minute what X may stand for. The third proposition then just emphasizes that $X \neq force$.

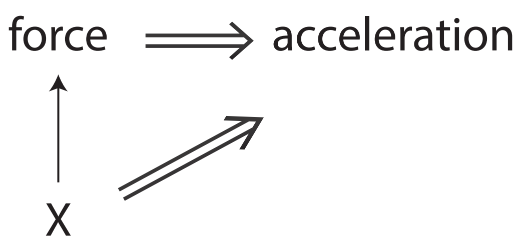

Figure 3.2.: Causal overdetermination

In order to rescue acceleration from being causally overdetermined, at least one of the three propositions needs to be rejected. Wilson argues that we can reject either the first or the third proposition. Both strategies result in abandoning forces. And so she presents her own idea that aims at keeping all three propositions without this leading to causal overdetermination. I'll present the details of Wilson's idea later in this section.

I agree with Wilson that the negation of the first proposition will deny the existence of forces—which is a viable option. Rejecting the third proposition is impossible because forces are always distinct from their necessitating entities. But we may deny the second proposition and keep the other two. Then forces exist and cause acceleration; the necessitating entities, however, lose their causal power. Let's delve into the details.

Imagine you are about to hit a golf ball with a golf club. When hitting the ball, the golf club causes the acceleration. And according to the argument of causal overdetermination the golf club necessitates a force on the ball, which causes the acceleration as well.

In my opinion, causation is a complex notion that bears heavy baggage from its use in our daily lives. It's a concept emphasizing that if one event, called the cause, hadn't occurred, then another event, called the effect, wouldn't have occurred. Furthermore, we're often not only interested in direct causes but in discovering a causal chain. For instance, a hooligan in court cannot insist that it wasn't him swinging the baseball bat that caused the basal skull fracture; instead it was the force induced on the skull that caused it. I plead not guilty. But there is a causal chain starting from the swinging of the baseball bat to the skull fracture. And that is what the judges are eager to uncover. Convicted on all charges. Similarly, there is a causal chain from the hitting of the golf club to the force and then to the acceleration of the ball.

In my opinion, causation is a complex notion that bears all its heavy baggage from the use in our daily life. It's a concept emphasizing that if the event, called cause, hadn't occurred the event, called effect, wouldn't have occurred. Furthermore, we're often not only interested in direct causes but in discovering a causal chain. For instance, a hooligan in court cannot insist that it wasn't him swinging the baseball bat, which caused the basal skull fracture; it was instead the force induced on the skull. I plead not guilty. But there is a causal chain starting from the swinging of the baseball bat to the skull fracture. And that is what the judges are eager to uncover. Convicted on all charges. Similarly, there is a causal chain from the hit of the golf club to the force and then to the acceleration of the ball.

On the fundamental level, there are other entities than golf clubs and baseball bats that necessitate forces. In Newtonian mechanics there is position, mass, and the gravitational field—in electrodynamics we have in addition charge, velocity, and the electromagnetic field.

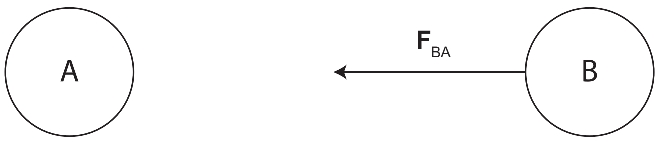

Figure 3.3.: Gravitational force on B

Here is an example. Let's have two balls, A and B, which interact at a distance via gravitational or electromagnetic forces (see Fig. 3.3). A exerts a force $\boldsymbol{F}_{BA}$ on B so that it accelerates to the right. According to proposition 2 of the argument for causal overdetermination, mass m_A causes a change in motion of B and necessitates the force $\boldsymbol{F}_{BA}$. But what is this necessitating relation? Is it a causal relation? If it were, then by denying the second proposition we can solve the causal overdetermination: mass causes forces, and forces cause acceleration. We just have to deny that mass itself can directly cause a change in motion (see Fig. 3.4).

In fact, we cannot read off causal relations from Newton's second law by just looking at the equation itself. The goal of classical physics is to explain and predict changes in motion. Newton's second law tells us that we can do so by specifying

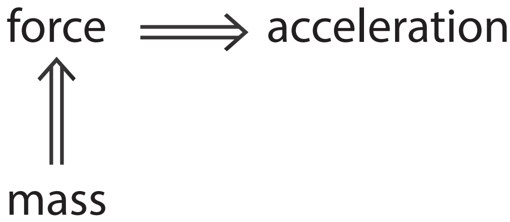

Figure 3.4.: Causal chain from mass to acceleration

forces. Newton's law then postulates a functional relationship between forces and acceleration: given the forces, it yields the acceleration. But the acceleration depends on the (inertial) mass: increasing the mass while keeping the same force will result in a smaller acceleration.[2] The mathematical structure doesn't force a causal reading on us. This has to be introduced by hand or by an interpretation.

As soon as mass causes acceleration we're in deep metaphysical water, because we tacitly regard mass as a dispositional property. Only if a property is dispositional does it have the causal power to generate an effect. And then acceleration would be causally overdetermined because mass, as well as the forces, cause the change in motion. There are now two ways out. Either mass is not a disposition or forces don't cause acceleration—this will eliminate either of the double arrows in Fig. 3.2. If we stick to mass as a disposition, forces can be construed as an epistemic tool for calculating the effects of masses. Gravity, for example, only depends on the masses and positions.

But there is still a gap between the mass m_A of A and the force $\boldsymbol{F}_{BA}$ on B (see Fig. 3.3). If m_A is an intrinsic dispositional property of A, how can it affect the motion of B over arbitrary distances? Being intrinsic to A, it's mysterious how the mass can reach out to B. The theory of dispositions just takes this for granted. There is no gap in the *explanation* of the phenomena since it's the law that explains how A affects B. Still the ontology doesn't contain all ingredients to reflect the explanatory power of the law.

Therefore, dispositionalism must introduce further entities that mediate the action from A to B. If it doesn't, dispositionalism will grant too much power to the laws, and so will fall back to primitivism about laws. It will rest in a state of primitivism in disguise: allowing for causal powers that don't have enough power to induce changes in the ontology (see page 78 for an elaboration of this argument).

Newtonian mechanics offers such a mediating entity, namely the gravitational field.[3] Then the mass of A changes the gravitational field in its vicinity, and these

[2]This is not possible if we consider the above system consisting of the balls A and B as if they interact via gravitation. For an increase in the inertial mass m_B yields an increase in the gravitational force $\boldsymbol{F}_{BA}$ in such a way that the acceleration of B won't change. But this doesn't change my argument on causation.

[3]The gravitational field, like the electromagnetic field, contains mathematical singularities that

changes propagate throughout space (with infinite speed). Since B has mass m_B the field at its position generates a force, which then causes an acceleration. So masses cause changes in the field, which causes a force, which causes acceleration (see Fig. 3.5). This is a complete causal change that avoids causal overdetermination and respects mass as a causal power. The same story can be carried over to electrodynamics. In this case it's the charge that causes changes in the field, which propagate with the speed of light.[4]

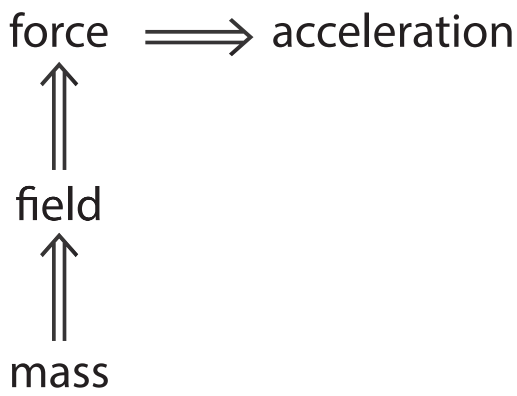

Figure 3.5.: How mass causes acceleration

Wilson doesn't go this way. She suggests that we can avoid the conclusion of causal overdetermination by rejecting either proposition 1 or proposition 3. If we reject proposition 1, that is, if forces don't cause acceleration, then either forces exist and aren't causally efficacious or forces don't exist in the first place. But being real without having a causal role doesn't make sense for forces, according to Wilson (2007, p. 198). Hence, a rejection of the first proposition leads to a rejection of the existence of forces.

Rejecting proposition 3 means that forces are no longer regarded as distinct from their necessitating entities. Then a force doesn't cause acceleration as a force proper but as a necessitating entity. In other words, forces are eliminated by being ontologically reduced to the necessitating entities. And if forces don't exist, there is no longer causal overdetermination (Wilson, 2007, p. 198). I think this move is doomed to fail. Forces are neither objects, nor properties, nor events. By definition forces differ from necessitating entities. Therefore, we cannot reject proposition 3.

Wilson doesn't question the validity of the second proposition. But its rejection would easily rescue us from causal overdetermination. For the direct cause of acceleration would be forces and no longer an entity necessitating the force. And

would prevent it from being part of the ontology. But fields are the only entities that physics offers as mediators. It's still a matter of debate whether there can be a well-defined field theory (see Ch. 4).

[4] Another option for the dispositionalist is to dismiss intrinsic properties by introducing an ontic structure. By definition the structure is relational; so there is no longer the problem of how particles can influence each other at a distance. I'll elaborate on this idea in section 3.4.4

this is surprisingly close to how physicists think: the distribution of masses produce a certain field, which generates forces, and forces cause acceleration. But Wilson has even another solution to the problem of causal overdetermination. She wants to keep all three propositions and avoid the conclusion. How this is done I'll present in Sec. 3.3.2.

The Vicious Regress

Another argument using causation shows that the standard interpretation of forces leads to a *vicious regress* (Bigelow et al., 1988, p. 621). As we have said, forces F are usually interpreted as mediators between causes C and effects E, that is, $C \Rightarrow F \Rightarrow E$. But what is supposed to mediate between C and F and between F and E? So instead of closing the gap between the cause and effect, the forces open another one, ad infinitum.

This vicious regress strikes me as a pseudo-problem. Look at Fig. 3.5. There are no causal gaps: mass causes a field, which causes a force, which causes acceleration. This is all that physics gives us. It's only by writing this causal chain in an abstract way, like $C \Rightarrow F \Rightarrow E$, that you might wonder what mediates between C and F and between F and E. But once you know what these placeholders stand for, the vicious regress disappears into infinity.

The Redundancy Argument

From Newton's formulation of the first and second laws, as well as from the mathematical equation $\boldsymbol{F} = m\boldsymbol{a}$, we see that forces explain acceleration. Once we know the forces we can calculate the acceleration. But once we know the forces they seem to disappear. Plugging in the forces in Hooke's law

$$m\ddot{\boldsymbol{q}} = -k\boldsymbol{q},$$

where k is a constant characterizing the strength of the spring, or in the law of gravitation

$$\ddot{\boldsymbol{q}}_i = \sum_{j \neq i} \mathrm{G}\, m_j \frac{\boldsymbol{q}_j - \boldsymbol{q}_i}{\left\|\boldsymbol{q}_j - \boldsymbol{q}_i\right\|^3}$$

forces are nowhere to be found in the mathematics. Therefore, it's argued that forces are just middle terms, and since they drop out in the final formulation of the second law, they don't exist.

Here is Max Jammer, who seems to have been the first to formulate the redundancy argument:

> [T]o show or to predict that a certain body A moves on a certain trajectory B, when surrounded by a given constellation of bodies C, D,... which may be gravitating, electrically charged, magnetized, and so forth, we introduce the middle term "force" and state the two "premises": (1) The constellation C, D,... gives rise to a force F; (2) The force F (according to the laws of

> motion) makes the body A move in the trajectory B. In our final conclusion, "Body A, surrounded by C, D,... under the given circumstances, moves along trajectory B," the middle term "force" again drops out. (Jammer, 1957, p. 244)

I don't think that forces don't exist just because they drop out as middle terms. The force needs to be specified as something that is not a force. So Newton's gravitational force is a function of position and masses. We can apply the same argument to electrodynamics, and we'll never conclude that the electromagnetic field doesn't exist. The electromagnetic field is a function of position and time. In the final step when plugging in the Lorentz force law the field vanishes. And in the static case, the Coulomb force and Coulomb field are almost identical to the gravitational force and the gravitational field. Still, we'll never conclude that the electric field doesn't exist.

Pace Wilson (2007, p. 192), Jammer doesn't give this argument in order to show that forces are redundant in the sense of not existing. In fact, he wants to point to the methodological role of forces: they are needed to abstract from the specific circumstances from which they evolve from. He clarifies his intention in the paragraph just before the above quote:

> The main advantage, however, of the concept of force—and this brings us to the status of our concept in present-day physics—is that it enables us to discuss the general laws of motion irrespective of the particular physical situation with which these motions are associated. The concept of force in contemporary physics plays the role of a methodological intermediate comparable to the so-called middle term in the traditional syllogism. In order to show that "Socrates is mortal," we introduce the middle term "man" and state the two premises: (1) All men are mortal; (2) Socrates is a man. In our final conclusion, "Socrates is mortal," the middle term "man" drops out. (Jammer, 1957, pp. 243–4)

As Jammer doesn't and cannot infer that men don't exist because "man" is a middle term, he doesn't and cannot infer that forces don't exist just because "forces" are middle terms. It's like saying that velocity and time don't exist because they are middle terms in $s = vt$. Once we have specified v and t, only the distance s remains. In fact, Jammer is explicit about his intention: he argues that forces play the role of middle terms, thereby unifying different phenomena. He doesn't use his argument to show that forces are redundant and don't exist.

3.3.2. Why Forces Exist

Forces as Primitives

For Mach (1919, p. 243) Newton's second law (3.1) is a definition not a law of nature. Forces don't exist because we cannot observe them and because they are nothing but mass times acceleration. For Maudlin (2013), however, forces are real and primitive.

According to his interpretation, there are forces and fields in the ontology of classical physics. And they cannot be reduced. Therefore, they have found their own ontological categories: fields form the category *field*, and forces form the category *force*. Then forces would mediate the action from fields to the physical body. If there were no fields, a force would directly mediate the action from one body to another at a distance. But as long as there is no physical theory that postulates forces or fields as part of the dynamics we cannot just interpret them as non-existent, he argues. Maudlin (2007c, p. 129) argues that Newton's second law (3.1) is a law proper. Let us assume that equation (3.1) is a definition, and image two bodies B_1 and B_2 with unknown masses m_1 and m_2 and two springs S_1 and S_2. The two bodies move frictionlessly on some surface.

First, we attach body B_1 to spring S_1, and measure the acceleration a_{11} of B_1 induced by the first spring. Since we don't know m_1 we just declare it to be 1. From (3.1) we can define the force $F_{11} := m_1 a_{11}$. From the acceleration a_{12} of B_2 by S_1 we can calculate the mass m_2 from $F_{11} = F_{12} := m_2 a_{12}$. Now we can use the second spring. Having measured the acceleration induced by the second spring on B_1, we get the following force: $F_{21} := m_1 a_{21}$.

So far we have used the second law just for the calibration of physical quantities. We defined forces and the mass m_2 with the help of this equation. We are now in a position to make a genuine *empirical* prediction that can be tested: the acceleration of B_2 induced by S_2 should fulfill $m_1 a_{21} = m_2 a_{22}$. This doesn't have to be the case. By definition $F_{22} := m_2 a_{22}$. Since we assume that the springs induce the same forces on all bodies $F_{21} = F_{22} := m_2 a_{22}$. We can measure a_{22}, and we have already fixed m_2 and F_{21} in previous experiments. What can now happen and is not logically excluded is that our definition $F_{22} := m_2 a_{22}$ is inconsistent with the experiment. It's an empirical question whether $F_{21} = m_2 a_{22}$ or $F_{21} \neq m_2 a_{22}$. But it finally turns out that in fact $F_{21} = m_2 a_{22}$.

The Aspect View

Wilson responds to causal overdetermination by using an argumentative scheme from the philosophy of mind in order to interpret forces as real entities while keeping all three premises (see page 62) and avoiding concluding that forces are causally overdetermined. In brief, the goal of the non-reductive physicalist approach to mental causation is to keep the mental and physical states ontologically distinct without causal overdetermination. The solution is said to be the *proper subset condition*: the causal powers of a mental state are a proper subset of the causal powers of the corresponding mental state. Thus, the mental state is distinct from the brain state because they have different causal powers (for this we need the "proper"), and the existence of the mental state doesn't lead to causal overdetermination because the causal powers of the mental state are contained in the causal powers of the brain state (for this we need just a subset of causal powers).

Wilson (2007, Sec. 5.1) applies this scheme to forces by adjusting the proper subset condition in the following way: the causal powers of any given force are a proper subset of the causal powers of the entity necessitating the force. Then "forces are aspects of the non-force entities necessitating them, whose causal powers satisfy the proper subset condition" (2007, p. 200).

I don't think that Wilson's interpretation of forces is tenable. In philosophy of mind, the proper subset condition doesn't clarify the ontological status of the mental state. It just says that the brain has more causal power than its corresponding brain state. Similarly, if we interpret forces as aspects of non-force entities, we still don't know the ontological status of forces, either. Nothing is said about the ontology.

But granting her interpretation, should we consider forces to be aspects of mass? Or aspects of particles? Or aspects of fields?

Forces as Causal Relations

Bigelow et al. (1988)respond to the vicious regress argument. Their way out is to grant forces a different ontological category to cause or effect: forces are a special kind of causal relation. On the fundamental level, a force relates the field, for instance, the gravitational or electromagnetic field, to the change of motion of a particle. The change of motion of a particle is the effect caused by a field at the particle's position. The role of forces is to mediate the action of a field to the particle by means of a causal relation. Therefore, forces are ontologically different from fields and the change of motion of particles.

Interpreting forces in this way doesn't lead to a vicious regress. And it emphasizes the unifying role of forces. A change of motion of particles can be caused under various circumstances: gravitation, electromagnetic interaction, direct contact, etc. The great merit of the existence of forces is that they unify all these phenomena, for we can describe all situation with forces alone without going into the details of how forces are generated.

Forces as Symmetrical Relations

Olivier Massin (2009) interprets forces as symmetrical relations. It's all about Newton's third laws, which states that forces always come in pairs. Consider two bodies A and B. Whenever body A exerts a force $\boldsymbol{F}_{BA}$ on B, then there is a force $\boldsymbol{F}_{AB}$ on A exerted by B, and vice versa. Because $\boldsymbol{F}_{AB} = -\boldsymbol{F}_{BA}$ according to the third law, the forces are generally conceived as *asymmetric* relations.

Nonetheless, Massin argues that forces are symmetric entities:

> The only thing that distinguishes the two forces of an action reaction pair is their arrow, that is, their sense. They share all their other properties: they always come together, they are determinates which fall under the same determinable, they relate the same entities, they have the same line of action (or orientation), the same magnitude and the same spatial location. Therefore,

> to ask whether we should read the Third Law literally amounts to asking: ontologically, should we take the sense of forces seriously? (2009, p. 575)

Massin is right that the forces in an action reaction pair point in opposite directions, but this isn't the only thing that distinguishes the forces as he claims. The two forces act on two *different* bodies (see Fig. 3.6). The force $\boldsymbol{F}_{BA}$ acts on B and $\boldsymbol{F}_{AB}$ acts on A.

Figure 3.6.: Anti-symmetric forces

In order to defend his claim, Massin presents three arguments claiming that forces are symmetrical relations. First, there is no way we can distinguish between action and reaction. In Fig. 3.6, there is no way to declare $\boldsymbol{F}_{AB}$ the action and $\boldsymbol{F}_{BA}$ the reaction. Thas is right.

Second, Fig. 3.6 requires four entities: the two bodies, A and B, and two forces, $\boldsymbol{F}_{AB}$ and $\boldsymbol{F}_{BA}$. Massin argues, that there are only three entities with symmetrical forces, namely the two bodies and the (symmetrical) force between them. By Ockham's razor the latter should be preferred to the former. The physical situation described by Newton's third law is rather depicted in Fig. 3.7: a symmetric force between A and B. It isn't clear, however, why there is a reduction of entities when calling the forces symmetrical. I think this is just a different description of the same physical situation that there are two bodies and two forces.

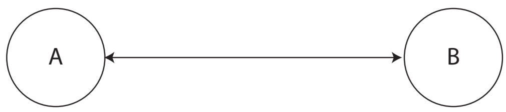

Figure 3.7.: Symmetric forces

$\boldsymbol{F}_{AB}$ is a vector representing the force exerted by B on A. If we ignore fields, $\boldsymbol{F}_{AB}$ is a relation between the bodies A and B, which shows how B affects A. Either the vector faces towards B or in the opposite direction as in Fig. 3.6. Let's say that this vector is $(-1, 0)$ then by the third law $\boldsymbol{F}_{BA} = (1, 0)$. Strictly speaking, this is not correct. The two vectors have different locations: $(-1, 0)$ is located at A, and $(1, 0)$ is located at B. And this needs to be represented by the mathematics. Hence, $\boldsymbol{F}_{AB} = (-1, 0)_A$ and $\boldsymbol{F}_{BA} = (1, 0)_B$, where the index represents the origin of the vector. And then $\boldsymbol{F}_{AB} \neq \boldsymbol{F}_{BA}$. What the third law says is that the *orientation of the vectors* is parallel and the *sense of the vectors* is opposite.

What Massin wants to do is to combine $\boldsymbol{F}_{AB}$ and $\boldsymbol{F}_{BA}$ into a *symmetric* super-relation. This relation would be $\tilde{F}(A, B) := \{\boldsymbol{F}_{AB}, \boldsymbol{F}_{BA}\}$. By definition it is symmetric, that is, $\tilde{F}(A, B) = \tilde{F}(B, A)$. But I don't see how this will help us in reducing the number of objects in the ontology. $\tilde{F}(A, B)$ still depends on the forces. Furthermore, this super-relation is by no means depicted by Fig. 3.7, since the vectors remain in different locations.

The third argument given by Massin uses an analogy with distance. It's possible to introduce a distance vector between two bodies A and B, so that the distance between A and B is represented by $\boldsymbol{D}_{AB}$, and the distance between B and A is represented by $\boldsymbol{D}_{BA}$, such that they fulfill the same equation as the forces in Newton's third law $\boldsymbol{D}_{AB} = -\boldsymbol{D}_{BA}$. This is too much of a mathematical structure to describe the distance between A and B, because all that is really necessary for the distance is captured in the magnitude of the distance vectors. Massin concludes:

> If true, given the strong analogy between the distance's law and Newton's Third Law, there is no reason to consider the asymmetry of force-vectors, but not the asymmetry of distance vectors, to be of ontological importance. Likewise, the asymmetry of forces is only a feature of their vectorial representations, but not of forces themselves. Forces are symmetrical relations, which may be referred to through asymmetrical representations, namely vectors. (Massin, 2009, p. 576)

A comparison of forces with distance doesn't show the similarities between these entities but rather points to their crucial differences. Representing distance with a vector does indeed introduce more mathematical structure than necessary. But forces aren't distances. The orientation and sense of a force are crucial features that cannot be argued away because they determine the direction of acceleration. They are by no means epiphenomena originating from a vectorial representation. In the case of distance, we could represent it as a metric $d(\cdot, \cdot)$ that covers everything that we expect from a distance relation but no more, and this $d(\cdot, \cdot)$ isn't a vector: concretely, $d(\boldsymbol{X}, \boldsymbol{Y})$ is a scalar, characterized by a single positive number

I agree that a vector representation for the distance between two objects bears more mathematical structure than needed to capture what is meant by a distance. I disagree with Massin that this is the case with forces. The sense of a force is not an epiphenomenon of its vectorial representation, for it has a dynamical consequence due to Newton's second law: the acceleration is in the direction of the force. The accelerations induced by forces $(1, 0)$ and $(-1, 0)$ on an object with mass m are of the same magnitude; they are parallel, but lead to motion in opposite directions.

Later, Massin identifies why the forces are generally considered to be asymmetrical entities instead of symmetrical.

> If Newtonian component forces are symmetrical relations, how is it that Newton and many of his followers appear to conceive them as asymmetrical? One first reason may be that they tend to confuse the property of the representations of forces (vectors) with the property of forces themselves. The second possible reason for the common belief that forces are asymmetrical relations is that Newton, like many of his followers, conceived of forces in terms of

> muscular effort. But effort implies an asymmetric relation between the active term (the voluntary subject) and the passive one (the resisting object). That is, the presence of some conation (intention, volition, trying . . .) introduces an asymmetry between the exerter and the exerted upon. (Massin, 2009, p. 578)

I don't agree that Newton confused properties of mathematical representations of forces and properties of forces themselves. First, Newton didn't know about vector calculus. Second, he was careful in explaining the role of forces and how to apply them. All that we can know about forces is their impact in the world, and for their mathematical description we need a magnitude, direction, and a sense. And there is nothing superfluous to be found in his exposition of forces, since all features of forces (magnitude, orientation, and sense) are reflected by the acceleration. Third, Newton was aware of the distinction between representation and ontology in general since he clearly states even in the title of the *Principia* that he is concerned with the *mathematical* principles of natural philosophy. And throughout his work he emphasizes this intention—most famously in his *hypotheses non fingo*. It's true that Newton may have speculated about different mechanisms for forces, but these are ideas that he shared in private conversations without building a physical theory of them.

What is really a problem for Massin's interpretation of symmetry is that the third law isn't universally fulfilled: in electrodynamics the third law is no longer valid. There are two standard examples that illustrate this phenomenon. First, consider two charged particles that are moving along two rectangular lines. Because of the cross-product in the Lorentz-force law the two forces exerted on each other aren't collinear, and the Lorentz forces result in the acceleration of the center of mass of the two bodies without the presence of external forces. Second, because the electromagnetic interaction is retarded the motion of one particle doesn't instantaneously affect the motion of the other particle. In this case, Newton's third law breaks down, too (see Chapter 4). Although Massin (2009, p. 581) is aware of this counterargument, he doesn't really offer us a clear way out.

3.4. The Metaphysics of Mass and Forces

We have seen that the authors I have presented on the existence of forces follow a top- down approach: they start with Newtonian mechanics and try to elucidate the ontology of forces. Instead, I want to follow a bottom-up approach. I start with the metaphysical theories of Chapter 2 and embed Newtonian mechanics in each theory. This will clarify not only the ontological status of forces but also of mass.

3.4.1. Humeanism

There are two versions of Humean supervenience (see Section 2.1): Lewis's version, and there is Hall's Super-Humean version. They agree on the status of forces in

Newtonian mechanics but disagree on the status of mass.

Standard Humeanism

Lewis's mosaic consists of a distribution of pure qualities. And it's straightforward to identify these qualities with Newtonian mass. Then all there is to the Humean world is the distribution of masses as pure qualities standing in spatiotemporal relations. Of course, mass as a categorical property inherits all the disadvantages of quidditism: there is, for example, a possible world in which mass behaves as what we would call charge, although it's the very same mass. The mass in the other world just has a different causal-nomological role. Masses (in our world) evolve according to Newton's laws, which are interpreted as the most efficient description of the motion of particles, balancing simplicity and strength.

Forces have a different ontological status to masses. Forces aren't—and cannot be—part of the ontology of a Humean world. Instead, forces supervene on the spatiotemporal distribution of masses over all history. They are part of the *formulation* of laws, and, therefore, they are nomological. Nevertheless, a proposition like "the strength of the gravitational force on a table is 100N" can be true in a Humean world. The truthmaker for this proposition is the motion of the masses given the laws of nature – this doesn't commit the Humean to letting forces be part of the mosaic.

We can introduce the gravitational or electromagnetic field into Humean ontology. The mosaic would gain further intrinsic qualities that instantiate the values of the field. As I discussed in section 3.3.2, I prefer to regard forces as causal relations between values of fields and the acceleration of masses. Pace Massin (2009, p. 560), forces can be reduced to the spatiotemporal relations of masses because they are part of the best system. Forces aren't primitive relations like entanglement is a primitive relation in orthodox quantum mechanics. Therefore, orthodox quantum mechanics isn't compatible with Humeanism (sec. 2.1), while forces pose no threat.

Super-Humeansim

Ned Hall (2009, Sec. 5.2) updated Lewis's version of Humean supervenience by changing the basic entities in the Humean mosaic. He proposes that the mosaic should consist of point-sized, propertyless particles, instead of pure qualities. The particles are primitive stuff, and all the non-modal facts of the world are the positions of these particles. Everything else, including the laws of nature, supervenes on the spatiotemporal distribution of particles. So even mass has to supervene on the contingent motion of particles. It's therefore a nomological entity by being part of the best system; it's no longer an intrinsic property. Yet while we can assign a mass m_1 to a particle P_1, this doesn't mean that masses exist as further entities that influence the motion of particles.

In this super-Humean ontology, forces keep their nomological status as in Lewis's version. Forces are now on a par with mass; they are only part of the Humean laws.

A proposition that "this table is attracted by the earth with $10N$" is true in virtue of the temporal development of the primitive particles composing this table. The particles behave as if they had mass and as if there were forces acting on them.

3.4.2. Primitivism about Laws

Primitivism about laws presupposes that laws are primitive and non-reducible. The central question is whether the primitive stuff and laws of nature exhaust the complete fundamental ontology. I will distinguish between different versions of primitivism. They will all agree on the primitive stuff ontology and the status of laws, but they will disagree on the status of forces and mass.

Version 1: Mass and Forces Exist

Following Maudlin (2013), Newtonian mechanics introduces particles as the Primary Ontology and mass and forces as the Secondary Ontology (see Section 2.2). The elements of the Primary Ontology are postulated to exist in space and time, and the role of the Secondary Ontology is to constrain the behavior of the Primary Ontology without making further ontological commitments. So one may interpret the elements of the Secondary Ontology as existing in addition to the Primary Ontology. Now, forces and mass may be regarded as real entities on a par with particles. Forces found their own metaphysical category, the category *force*, because, according to Maudlin, forces are novel entities of modern physics. Particles, then, are the carrier of mass such that in virtue of having mass the particles exert certain forces upon each other.

Is mass a categorical or a dispositional property if laws are primitive? Having mass as a disposition would prevent laws from being primitive, since dispositions possess all causal powers. If laws are primitive they cause the acceleration of particles, and they don't supervene on the properties that particles have. So mass must be categorical, if we want to retain intrinsic properties with primitive laws. And as in Humeanism, categorical properties are doomed to have quiddity. Like David Lewis, we may wish to bite this bullet.

Version 2: Mass and Forces Don't Exist

You may dismiss mass as a categorical property and regard it as a parameter in the law of motion. Then particles really remain primitive stuff. Since we have only one universe and a constant number of particles, according to Newtonian mechanics, interpreting mass as a parameter seems to be slightly inappropriate. Rather, masses are better construed as constants of nature. Supposing that the universe consists of N particles, Newton's laws of motion contain N constants $m_1, \ldots, m_N$. These constants no longer represent intrinsic properties; instead, the status of mass is nomological.

The interpretation of the masses as constants of nature is more feasible if all masses coincide. Consider a Newtonian universe containing N identical particles, that is, particles with identical mass m. Then Newton's laws would contain only two constants: the mass m and the gravitational constant G. It's common to call the gravitational constant a *constant of nature*. But having a universe of identical particles and primitive laws, there isn't anything special about m with respect to G. Saying that m is the mass *of the particles* would just be a manner of speaking.

When applying Newtonian mechanics to subsystems it may be reasonable to call mass a parameter. A subsystem of the universe that contains M particles ($M < N$) is described by M parameters $m_{i_1}, \ldots, m_{i_M}$. Here the masses are often said to be parameters that have to be adjusted to the underlying subsystem. What is really meant is that we consider a subset of particles of all the particles in the universe, and the laws of motion for the subsystem include $m_{i_1}, \ldots, m_{i_M}$ as constants fitting the correct motion of the particles. Actually, we don't fine-tune the masses to fit the subsystem as if they were arbitrary parameters. What we really do is to extract M appropriate constants out of the entire set of N constants of nature $m_1, \ldots, m_N$.

It's possible to reduce the fundamental ontology even further so that forces, too, are no longer part of the fundamental ontology. So both forces and mass are interpreted as nomological entities. Particles move as they do *because* there are Newton's laws. In particular, they do *not* move because they carry mass or because there are forces.

If mass ceases to be an intrinsic property of particles, then how can particles "know" how to move? If particle P_1 has mass m_1 it moves in a certain way because of its mass m_1. Once we change its mass to $\tilde{m}_1$ it changes its motion accordingly. But if P_1 doesn't carry mass then it seems mysterious why a change in m_1 changes its motion.

We can introduce mass as a constant of nature, but still this constant is different from the gravitational constant G or Planck's constant $\hbar$. If we change G or $\hbar$ we change the dynamics of all the particles. But if we change just m_1 (and keep the forces fixed), it's the particle with mass m_1 and only this particle that changes its motion. Or if we have a system of particles that interact just by gravitation, and again we change the mass m_1 of particle P_1, the motion of all the particles except for P_1 will change – remember that m_1 cancels out in Newton's second law. In both cases, you can identify the particle by changing m_1. Therefore, it seems that mass is somehow attached to particles.

There is another difference between mass and constants, like G, $\hbar$, or Boltzmann's constant k_B. The gravitational constant, Planck's constant, and Boltzmann's constant are dimensional scale factors that relate two different theories (Dürr and Teufel, 2009, Sec. 4.1.3 and 8.1). They appear once a theory is reduced to another theory. G relates the units of classical mechanics with general relativity; $\hbar$, the units of classical mechanics with quantum mechanics; and k_B, the units of thermodynamics with statistical mechanics.

Primitive Laws Have to be Permutation-Invariant

The mathematical representation of a configuration as an n-tuple allows us to make two kinds of permutations: we can permute masses, or we can permute particles. When permuting particles, we permute the masses as well, but we can permute the masses without permuting the particles. So if the laws aren't permutation-invariant, both a permutation of particles and a mere permutation of mass yield a new physical situation. The mathematical structure suggests that the mass is somehow attached to the particles.

With permutation-invariant laws and configurations represented as sets instead of n-tuples, a permutation doesn't result in a new physical state. A permutation of particles doesn't change the motion of any particle. And a permutation of the masses is no longer possible. What we can do, however, is increase or decrease the value of mass, say of m_1. Then, as above, we distinguish one particle or one trajectory that "carries" this mass. But this mass isn't attached to the particles because every particle in the same position will behave the same way. So an ontologically parsimonious account of primitivism about laws must be committed to permutation-invariant laws. Otherwise, the mathematics suggests that particles carry categorical properties.

3.4.3. Dispositionalism

Dispositionalism introduces modal connections into the ontology. In contrast to primitivism, the laws of nature don't have this capacity qua laws; instead, the elements of the primitive-stuff ontology have certain dispositional properties or powers, and the laws express the action of dispositions. In Newtonian mechanics, mass can be interpreted as a disposition.

But mass doesn't give an intrinsic identity to the particles. As I pointed out in section 1.3, every particle is discernible due to its location in space or, more generally, by its metrical relations relative to the other particles. The role of mass is solely a dynamical one.

Moreover, it's essential for mass (as a disposition) to have the same causal- nomological role in all possible worlds; it's not a categorical property, and consequently it doesn't bear the problems of quiddity. This causal-nomological role is represented by the laws. Metaphorically, Newton's laws are grounded or anchored in the ontology by the intrinsic mass of particles interpreted as a causal power. Our epistemic access to mass as a disposition is through observation of what it does in the world, namely, enacting change in motion.

A crucial feature of dispositions is their need for certain triggering conditions in order to be manifested. Zooming into Newton's second law (3.2), we see the following: the manifestation of the mass m_j of particle P_j at time t is its acceleration $\boldsymbol{a}_j(t)$ given the positions and velocities of all the particles (including P_j) at time t. So the positions and velocities of *all* the particles are the triggering conditions for the manifestation of mass. In the case of gravitation, the mass m_j cancels out on both sides of (3.2), and we deduce that the acceleration $\boldsymbol{a}_j(t)$ doesn't depend on m_j. Yet, $\boldsymbol{a}_j(t)$ is the manifestation of mass of the j-th particle, although it's independent of the precise value of m_j.

Having mass as a disposition, it seems that there is no room in the ontology for further entities that may influence motion. So Newtonian forces must be interpreted as non-existent. Rather, they are nomological in the formulation of the laws of motion; they only express the influence of the masses on particles.

The Mathematical Structure of Causal Relations

Blondeau and Ghins (2012) offer a general scheme for all laws of motion in physics from quantum mechanics to classical mechanics, to thermodynamics. They postulate that all the relevant differential equations are of the following form:

$$\frac{\partial x}{\partial t} = C_1(t) + \ldots + C_n(t). \tag{3.5}$$

On the left side of the equation are the effects, and on the right side one finds all the causes. The variable x stands for the quantity in whose temporal development one is interested: position, momentum, forces, wave functions, entropy, temperature, etc. What is important is the structure of the right side: $C_1(t), \ldots, C_n(t)$ are the causes, and they are summed up. By setting $C_i = \boldsymbol{F}_i$ this scheme reflects the standard view that (component) forces are the cause of acceleration (Blondeau and Ghins, 2012, p. 385).

I don't think that scheme (3.5) is universal. Causes and effects must be part of the ontology—an exception is Humeanism, where causation is introduced by means of counterfactuals. For our dispositionalist, who grants only particles and mass in the primitive ontology, forces don't exist and the scheme (3.5) cannot be applied. Instead, the distribution of dispositions is the causes of the change in motion, and a general causal scheme for the laws of motion of classical mechanics rather has to look like this:

$$\frac{\mathrm{d}^\alpha \boldsymbol{x}}{\mathrm{d}t^\alpha}(t) = f\left(t, \frac{\mathrm{d}^{\alpha-1}\boldsymbol{x}(t)}{\mathrm{d}t^{\alpha-1}}, \ldots, \frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}, \boldsymbol{x}(t); D_1(\boldsymbol{x}(t), t), \ldots, D_r(\boldsymbol{x}(t), t)\right),$$

where $D_i(\boldsymbol{x}(t), t)$ is the disposition of the ith particle, which can a priori depend on time and position, and f is a function that expresses the functional relationship among the disposition, position, velocity, and higher-order derivatives. The function f can also depend on time. Applying this to Newtonian gravity in the case of N particles, we get, by setting $D_i(\boldsymbol{x}(t), t) = m_i$ and $\boldsymbol{x} = \boldsymbol{Q} = (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N)$,

$$\frac{\mathrm{d}^2 \boldsymbol{Q}}{\mathrm{d}t^2}(t) = f\left(\boldsymbol{Q}; m_1, \ldots, m_N\right) = \sum_{j \neq i}^{N} \mathrm{G}\, m_j \frac{\boldsymbol{q}_j - \boldsymbol{q}_i}{\left\|\boldsymbol{q}_j - \boldsymbol{q}_i\right\|^3}.$$

What Is Wrong with Intrinsic Dispositions?

If the ontology of a dispositionalist only consists of particles and intrinsic properties, there is a problem regarding how particles can influence each other over spatial distances. If we want to predict the motion of particle P_1, it doesn't suffice to know the initial position and velocity of this particle; its motion also depends on the distribution of all the other particles in the universe. So the mass m_1 cannot "on its own" generate the motion of P_1; instead, there has to be a "connection" with the other particles.

How is this "connection" established? In a dispositionalist ontology there is nothing but particles and intrinsic properties. It doesn't follow from the definition of intrinsic properties that they can influence each other over spatial distances. If particle P_1 has a certain intrinsic property, then this property can only change the behavior of P_1. One might say that the laws tell us how the intrinsic properties influence one another. That's right, but this explanation grants laws too much power. The laws supervene on the dispositions, and if we say that intrinsic dispositions can influence each other over spatial distances, then we commit ourselves to laws as further entities in our ontology—something that a dispositionalist is eager to avoid.

The Humean doesn't face this problem. For her intrinsic properties are categorical, and laws summarize the motion of the particles throughout history. Humean laws don't add anything to the ontology that isn't already found in the mosaic.

One option for the dispositionalist is to bite the bullet: it's simply a primitive fact that masses influence each other over arbitrary distances—some kind of metaphysical action-at-a-distance. But then I fear that properties are no longer important because the law determines the effects of the distant action. Hence dispositionalism only differs from primitivism about laws in terms of the language it uses. I therefore think that a dispositionalism which maintains an ontology solely of particles and intrinsic properties, fails to metaphysically ground Newtonian mechanics.

A solution for the dispositionalist may be to introduce fields and forces. These may help the dispositionalist because, in virtue of its mass, a particle generates a field, which generates a force. So even if there is a particle that is far away from other particles, it can still be influenced by them through the net force generated by the net field.

Another potential escape for the dispositionalist would be to introduce physical modal relations (that is, an ontic structure) instead or on top of the intrinsic properties. I see two problems with this idea: either these relations reduce to forces or fields, or they coincide with the laws of nature themselves. Let's analyze this option in more detail.

3.4.4. Ontic Structural Realism

OSR as introduced and defended by Esfeld (2004, 2009) is a general scheme that is well suited to providing for the dynamics in a primitive-stuff ontology. In this scheme, different physical theories agree on the primitive stuff, whereas they differ in the ontic structure. I will show that in Newtonian mechanics the dynamical structure isn't unique; it depends on whether fields exist. The ontology of fields will also make a difference to how we should interpret mass.

OSR without Fields

The most parsimonious ontology for Newtonian mechanics consists of particles in motion: no mass, no forces, no fields. In a non-Humean metaphysics, however, there have to be modal connections. Apart from laws and intrinsic properties, we can impose dynamical relations on particles that generate change in motion.

Here we could stop insisting that the relations are primitive metaphysical relations that account for the correct motion of particles irrespective of the laws. And we don't spell out what the relations are or what instantiates the relations. This would turn OSR into a purely metaphysical idea, something akin to humility but with respect to the structure.

But we can go beyond this. Newtonian mechanics gives us a candidate for on ontic dynamical structure: component forces. For gravity they look like

$$\boldsymbol{F}_{ij} = \mathrm{G}\, m_i m_j \frac{\boldsymbol{q}_j - \boldsymbol{q}_i}{\left\|\boldsymbol{q}_j - \boldsymbol{q}_i\right\|^3},$$

which is the force that particle P_j exerts on particle P_i.

The component forces are dynamical bipartite particle–particle relations; that is, they dynamically relate the change of motion of particle P_i to the change of motion of P_j. We can even summarize all the forces into one "universal" force $\boldsymbol{F} = (\boldsymbol{F}_1, \ldots, \boldsymbol{F}_N)$, where $\boldsymbol{F}_i$ are net forces. This is a concise way of writing down the entire structure. The net forces aren't anything in addition to the component forces; they are just a name for the resulting action.

Let me briefly mention the features of this ontic structure. First, it instantiates a dynamical non-locality: the acceleration of one particle is determined by the position of all the other particles on a simultaneity slice irrespective of the spatial distances. But the structure $\boldsymbol{F}$ is itself regarded as a *local* object, because it's determined by the values it takes in 3-dimensional physical space.

Second, the influence of the gravitational structure decreases with distance. The more the particles move apart, the less they contribute to the acceleration of the other particle.

Third, the gravitational structure always consists of bipartite particle–particle relations. The whole structure can be decomposed into relations of two particles, which physicists would call *direct interactions*. Although this is action-at-a-distance because there are no entities continuously transmitting the action, these direct interactions respect the structure of Galilean space-time since they relate only particles on a simultaneity slice. Furthermore, the action-at-a-distance of OSR is not as problematic as that accepted by the standard dispositionalist. The latter only allows the intrinsic properties to be dynamically efficacious, and so masses must influences each other at a distance. It's a problem for the dispositionalist to justify how intrinsic masses can do this; it's not enough to refer to the laws of motion because they only express the effects of the disposition. OSR, on the other hand, introduces a net of dynamical relations as part of the fundamental ontology. So the change in motion of one particle still influences the motion of all the other particles at the same time, but there is no longer a metaphysical action-at-a-distance.

Mass as a Coupling Constant

Interpreting the component forces as a dynamical structure, mass cannot be an intrinsic property of particles. Mass would either be categorical or dispositional. Categorical properties are to be avoided. Whether mass can be a disposition depends on the ontology; more precisely, it depends on the status of fields. Let's first say that there are no fields. If mass were a disposition it would be a disposition that generates force or acceleration. In neither case is this needed, as the structure is sufficient to ground the dynamics in the ontology. So like a primitivist about laws, it's better to regard mass as a parameter or constant. The role of mass is rather to "tune the intensity" of the structure: the higher the masses the more the particles attract each other or the more they resist the impact of forces. In order to emphasize these two roles, namely, being a parameter and regulating the strength of the structure, mass is more suitably called a *coupling constant*.

OSR with Fields

Forces instantiate an ontic structure. Since the role of forces depends on the existence of fields (see Sec. 3.3) we need to slightly adjust the structure. With fields in our ontology, forces relate the value of the field to the acceleration of particles. The ontic structure ceases to be a bipartite particle–particle relation; it becomes a bipartite field–acceleration relation.

This application of OSR to classical mechanics is close to the interpretation of forces as causal relations given by Bigelow et al. (1988). They interpret forces as relations between the particle and the (gravitational) field. As they respond to the vicious regress argument, they want to elucidate the status of force as the mediator between causes and effects: $C \Rightarrow F \Rightarrow E$. Bigelow et al. associate C with the change in the field and with E the acceleration. They interpret the force F as a causal relation between C and E.

Mass as a Dispostion

If fields exist then mass is rather to be interpreted as a disposition: a disposition to generate fields. Physics doesn't introduce any abstract relations, like forces between fields and acceleration, that mediate between the mass and the generation of fields. Therefore, we need to presuppose that it's a primitive capacity of mass to produce a field. But this doesn't exhaust the role of mass. It also influences how big the force is in generating acceleration. So it's still a coupling constant in the ontic structure between fields and acceleration, a coupling constant that has a dispositional aspect.

4. Classical Electrodynamics

Well, it seemed to me quite evident that the idea that a particle acts on itself, that the electrical force acts on the same particle that generates it, is not a necessary one—it is a sort of a silly one, as a matter of fact. And so I suggested to myself that electrons cannot act on themselves, they can only act on other electrons. That means there is no field at all. You see, if all charges contribute to making a single common field, and if that common field acts back on all the charges, then each charge must act back on itself. Well, that was where the mistake was, there was no field. It was just that when you shook one charge, another would shake later.

— Richard Feynman (1966, p. 699)

4.1. Particles and Fields

The Maxwell–Lorentz theory of electromagnetism has a wide scope of application. It can be used for phenomena on the macroscopic level, like the behavior of electromagnetic waves in different media or even the heating of a frying pan through induction. This theory may be also applied to the behavior of single charged particles. The latter situation allows us to analyze this theory as a candidate for a fundamental theory of the world, as we have done with Newtonian mechanics, and we can work out what the world would be like if this theory were to introduce the true laws of physics governing all matter in our universe.

The Maxwell–Lorentz theory introduces two kinds of fundamental entities situated in space-time. First, there are particles. And these have two properties, namely, mass and charge. Charge comes in two varieties: positive and negative. And in virtue of having charge the particles generate an electromagnetic field, the second kind of basic entity.

Since there are two fundamental kinds of entities, namely fields and particles, the laws should show us how the motion of particles influences the behavior of fields, and how the behavior of fields changes the motion of particles. The former case is accounted for by the *Maxwell equations*:

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c}\boldsymbol{j}, \qquad \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}; \tag{4.1}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi\varrho, \qquad \nabla \cdot \boldsymbol{B} = 0. \tag{4.2}$$

The Maxwell equations tell us that there are two species of fields, namely, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$. According to equations (4.2), also known as the *constraint equations*, the sources of the electric field lie in the charge density ϱ, while the magnetic field has no sources or sinks. Instead, magnetic fields are only generated by the motion of charges resulting from a current density $\boldsymbol{j}$, and therefore there is no magnetic field produced by static charges. Equations (4.1), conventionally dubbed the *evolution equations*, point to a correlation between the change of the electric field and the change of the magnetic field and vice versa. They are the origin of the wave behavior of fields and, in particular, of radiation effects, which will play a central role in my philosophical analysis.

The Maxwell equations are non-homogeneous linear equations; so they can be globally solved given certain initial conditions (Spohn, 2004, section 2.1). It turns out that the values of the fields on a Cauchy hypersurface are sufficient to deliver enough data for a solution. Given the field values on a space-like hypersurface at a time $t = t_0$, namely,

$$\boldsymbol{E}(t_0, \boldsymbol{x}), \quad \boldsymbol{B}(t_0, \boldsymbol{x})$$

together with the boundary conditions

$$\nabla \cdot \boldsymbol{E}(t_0, \boldsymbol{x}) = \varrho, \quad \nabla \cdot \boldsymbol{B}(t_0, \boldsymbol{x}) = 0,$$

the Maxwell equations can be thus solved for all times t. And they instantly lead to a law of conservation. Since the charge density ϱ and the current density $\boldsymbol{j}$ fulfill the continuity equation

$$\partial_t \varrho(t, \boldsymbol{x}) + \nabla \cdot \boldsymbol{j}(t, \boldsymbol{x}) = 0, \tag{4.3}$$

the change of the charge density in a given volume is only due to charges moving out of or into the volume. In other words, charges are locally conserved so that they cannot appear or disappear without following a continuous path. This leads to a global law of conservation: the number of charges in the universe remains constant.

One basic ingredient is still missing. Given fields, the theory must state how particles are moving. The *Lorentz force law* takes care of this:

$$\frac{\mathrm{d}\boldsymbol{P}(t)}{\mathrm{d}t} = e\Big(\boldsymbol{E}(t, \boldsymbol{q}(t)) + \dot{\boldsymbol{q}} \times \boldsymbol{B}(t, \boldsymbol{q}(t))\Big). \tag{4.4}$$

This is a relativistic generalization of Newton's second law (3.2). The change of the relativistic momentum of a particle with elementary charge e is caused by the electromagnetic forces acting upon it. The Lorentz force law has two important features. First, magnetic fields aren't only generated by moving charges, but only moving charges can be influenced by the magnetic force because of the velocity of the particle in the cross-product on the right side of (4.4). Second, experiments confirm that charges are quantized by appearing only in certain "packets". The smallest such packet is the elementary charge e carried by an electron.

So far, this has been a standard exposition of the Maxwell–Lorentz theory that you can find in every physics textbooks. Some philosophy is still missing. The primitive-stuff ontology of electrodynamics consists of point-size particles and the electric and magnetic fields. These are the basic objects or substances of which our world is composed. The Maxwell equations and the Lorentz force law contain mass and charge. Whereas the notion of mass is taken from Newtonian mechanics, charge is newly introduced. And depending on the metaphysics of laws of nature, the status of mass and charge changes: they may be intrinsic properties of the particles or just mathematical parameters (see Chapters 2 and 3). The Maxwell equations and the Lorentz force law show that all these primitive entities influence each other: the fields change the motion of particles, and the motion of particles changes the fields. According to the equations (4.1), the fields don't only influence the motion of particles; they mutually influence their temporal development as well.

The Covariant Formulation

The idea is to dispense with the electric and magnetic fields as two independent fields in space-time and unite them into a single electromagnetic field F. This field manifests itself in a certain combination of electric and magnetic fields in every inertial reference frame.

The field F is mathematically a tensor field on space-time. Given the components of the electric and magnetic field in a reference frame, the field tensor F has the following component representation:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}.$$

The crucial property of the tensor field F is its Lorentz-invariance in contrast to the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields. This makes it suitable to be the real entity on space-time as it doesn't change relative to the Lorentz frames, in the same way that the relativistic interval doesn't change under Lorentz transformation.

In covariant notation, the Maxwell equations (4.1) and (4.2) are written as

$$\partial_\mu F^{\mu\nu} = -\frac{4\pi}{c} j^\mu \tag{4.5}$$

$$\partial_\varrho F_{\nu\sigma} + \partial_\nu F_{\sigma\varrho} + \partial_\sigma F_{\varrho\nu} = 0, \tag{4.6}$$

with $j = (c\varrho, \boldsymbol{j})$ and all tensors depending on $x = (ct, \boldsymbol{x})$.

Sometimes it's more convenient to use potentials instead of fields in calculations. There are infinitely many scalar fields ϕ and vector fields $\boldsymbol{A}$ that generate the same electric and magnetic fields if they fulfill the following relations:

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{A}, \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

In covariant form the above equations are subsumed into one equation for the field tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

with $A^\mu = (\phi, \boldsymbol{A})$. With the help of potentials, the first Maxwell equation (4.5) can be written as

$$\Box A^\nu = -\frac{4\pi}{c} j^\nu$$

with the d'Alembert operator $\Box = \partial^\nu \partial_\nu$.

4.2. Infinity: The Threat of a Moving Particle

Maxwell–Lorentz electrodynamics is an extremely successful physical theory. It's thus surprising that this theory isn't able to deal with the most basic scenario: the motion of a single charged particle. The equations of motion don't deliver any exact solution for this. Let's see how this can be and what we can do about it

The charge density of a single charge is $\varrho(t, \boldsymbol{x}) = e\delta(\boldsymbol{x} - \boldsymbol{q}(t))$—the δ-function indicates that the charge is only concentrated on its trajectory $\boldsymbol{q}(t)$. The corresponding density current is the charge density multiplied by the velocity of the charge, namely, $\boldsymbol{j}(t, \boldsymbol{x}) = \varrho(t, \boldsymbol{x})\boldsymbol{v}(t) = e\delta(\boldsymbol{x} - \boldsymbol{q}(t))\boldsymbol{v}(t)$. Therefore, the Maxwell equations are as follows:

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c} e\delta(\boldsymbol{x} - \boldsymbol{q}(t))\boldsymbol{v}(t), \qquad \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}; \tag{4.7}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi e\delta(\boldsymbol{x} - \boldsymbol{q}(t)), \qquad \nabla \cdot \boldsymbol{B} = 0. \tag{4.8}$$

Calculating the fields is standard (Spohn, 2004, section 2.1). The electric field is given by

$$\boldsymbol{E}(t, \boldsymbol{x}) = \frac{e}{4\pi}\left(\frac{(1 - \boldsymbol{v}^2)(\hat{\boldsymbol{n}} - \boldsymbol{v})}{(1 - \boldsymbol{v}\cdot\hat{\boldsymbol{n}})^3 |\boldsymbol{x} - \boldsymbol{q}|^2} + \frac{\hat{\boldsymbol{n}} \times ((\hat{\boldsymbol{n}} - \boldsymbol{v}) \times \dot{\boldsymbol{v}})}{(1 - \boldsymbol{v}\cdot\hat{\boldsymbol{n}})^3 |\boldsymbol{x} - \boldsymbol{q}|}\right), \tag{4.9}$$

and the magnetic field can be calculated as

$$\boldsymbol{B}(t, \boldsymbol{x}) = \hat{\boldsymbol{n}} \times \boldsymbol{E}(t, \boldsymbol{x}). \tag{4.10}$$

The right side of (4.9) has to be evaluated at the retarded time t_{ret}, which is implicitly defined by

$$t_{ret} = t - |x - \boldsymbol{q}(t_{ret})|.$$

It is the time at which the backwards light-cone with apex at $\boldsymbol{x}$ crosses the world-line of the particle (see Figure 4.1). Equation (4.9) states that the value of the electric field at position $\boldsymbol{x}$ at time t only depends on the position and the velocity of the moving particle at the retarded time t_{ret}, as $\boldsymbol{q}$, $\dot{\boldsymbol{v}}$, and $\hat{\boldsymbol{n}}$ has to be evaluated at t_{ret}. The vector $\hat{\boldsymbol{n}}$ is a unit vector derived from the position of the particle, namely,

$$\hat{\boldsymbol{n}} := \frac{\boldsymbol{x} - \boldsymbol{q}(t_{ret})}{|\boldsymbol{x} - \boldsymbol{q}(t_{ret})|}.$$

So the vector $\hat{\boldsymbol{n}}$ at $\boldsymbol{x}$ is parallel to the backwards light-cone pointing away from the trajectory $\boldsymbol{q}(t)$.

The retarded fields (4.9) and (4.10) aren't the only solutions to the Maxwell equations. In particular, they allow for advanced solutions as well. The advanced fields differ from the retarded ones only in the time at which they are evaluated; instead of the retarded time t_{ret}, they are evaluated at the advanced time t_{adv} given by

$$t_{adv} = t + |x - \boldsymbol{q}(t_{adv})|.$$

The advanced time lies on the forward light-cone with apex $\boldsymbol{x}$ intersecting the world-line of the particle (see Figure 4.1). This is the reason why the advanced solutions are abandoned in the application of the Lorentz-Maxwell electrodynamics. It suggests that the field value at $\boldsymbol{x}$ at time t depends on the future location of the particle as if the electromagnetic fields travelled from the future to the past. This phenomenon isn't confirmed by experiments, so that only the retarded fields have physical significance.

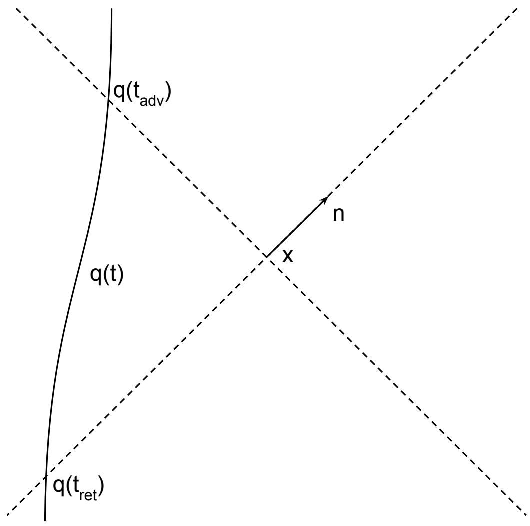

Figure 4.1.: Retarded and advanced times

In order to calculate the Liénard–Wiechert fields (4.9) and (4.10) we have to presuppose the trajectory of the particle, which must be derived from the Lorentz force law (4.4). So if we want to get the exact trajectory of the particle we have to couple the Maxwell equations with the Lorentz force law by inserting the self-fields in the law of motion. This poses huge difficulties. For the fields have to be evaluated at the particle's position, and this leads to infinities as the denominators of (4.9) become zero. Briefly, the Maxwell–Lorentz theory breaks down in the most basic case of a moving charged particle.

How can it be that the theory is still successfully applied? In physics textbooks, it seems that one can easily calculate the motion of a particle. But the motion is only under the influence of *external* fields generated by other particles; the self-fields are always ignored. So what physicists are doing in these cases is using test charges to avoid the infinities. A test charge is a hypothetical particle whose features don't perturb the physical system. If physicists claim to have calculated the motion of a particle P they treat this particle as a test particle without explicitly mentioning so. Though P has mass and charge, its own field is simply neglected. So it moves in the field of the other particles without disturbing their fields. In this case, the mathematics is perfectly fine. But once you treat the whole story by considering the self-field, the theory is of no use.

Because we need the equations later on, I'll summarize the physics of a single charge in covariant notation. The four-current of a moving single charge is then given by

$$j^{\nu}(x) = ec \int_{-\infty}^{\infty} \delta(x - z(\tau)) \dot{z}^{\nu}(\tau) \, \mathrm{d}\tau$$

with $z(\tau)$ the four-position, $\dot{z}(\tau)$ the four-velocity of the particles parametrized by its proper time τ. The Maxwell equation (4.5) is solved by the Liénard-Wiechert potentials

$$A^{\nu}_{ret}(x) = e \frac{\dot{z}^{\mu}(\tau_{ret})}{(x_{\sigma} - z_{\sigma}(\tau_{ret}))\, \dot{z}^{\sigma}(\tau_{ret})}, \tag{4.11}$$

which is the retarded potential corresponding to the retarded field of (4.9), and

$$A^{\nu}_{adv}(x) = e \frac{\dot{z}^{\mu}(\tau_{adv})}{(x_{\sigma} - z_{\sigma}(\tau_{adv}))\, \dot{z}^{\sigma}(\tau_{adv})}, \tag{4.12}$$

being the advanced potential.

In order to calculate the trajectory of the particle in its own field we need to solve the Lorentz force law

$$\frac{\mathrm{d}p^{\mu}(\tau)}{\mathrm{d}\tau} = m\ddot{z}^{\mu}(\tau) = \frac{e}{c} F^{\mu\nu}(z(\tau)) \dot{z}_{\nu}(\tau)$$

by using the Liénard-Wiecher potentials. So we need to couple the Maxwell equations with the equation of motion for the particle yielding the following system of differential equations:

$$\begin{aligned} \partial_{\mu} F^{\mu\nu}(x) &= -4\pi e \int_{-\infty}^{\infty} \delta(x - z(\tau)) \dot{z}^{\nu}(\tau) \, \mathrm{d}\tau \\ m\ddot{z}^{\mu}(\tau) &= \frac{e}{c} F^{\mu\nu}(z(\tau)) \dot{z}_{\nu}(\tau). \end{aligned}$$

The Lorentz force law needs to be evaluated at the position of the particle, but in plugging the particle's position into the Liénard-Wiechert potentials, the potentials become infinite. Consequently, the motion of the particle in its own field cannot be calculated.

4.2.1. How to Tame Infinity?

What can be done about this situation? Maxwell–Lorentz electrodynamics simply breaks down for the motion of a single charged particle. And if a theory breaks down one has to find another one, a successor without infinities. The self-interaction problem has been known since Maxwell, and many physicists have worked on it. Most physicists hoped that quantum mechanics would solve everything. But it did not. The ultra-violet divergence in QFT notwithstanding, the infinite self-energies in classical electrodynamics has been almost forgotten.

In physics textbooks, one can still find analogies between classical electrodynamics and quantum mechanics that are, to put it mildly, question-begging. One example can be found in the otherwise very meticulous book by Orhan Barut:

> The field [...] is infinite at the point $x = z(s)$. Physically, we know that it represents the effect of the near field of the particle. In quantum mechanical language the particle carries with itself a cloud of photons which are continuously emitted and absorbed. It is, therefore, to be expected that this force [i.e., the self-force] contributes to the inertia or mass of the particle. (Barut, 1980, p. 190)

First, it isn't clear what "represents the effect of the near field of the particle." Is it the field itself, the force, or the infinity? Second, as the equations of motions break down there is no effect of the near field; the theory doesn't say how the particle is affected by its own field. Third, the "cloud of photons" of quantum mechanics by no means lets us expect that the self-force, which is infinite, contributes to the mass of the particle. Rather, we would expect an impact on the particles' trajectory from radiation and the associated loss of energy. Infinities and quantum mechanics do not help. But some physicists and mathematicians have tried to develop new classical theories that avoid the self-interaction problem.

The Wheeler–Feynman Theory

Wheeler and Feynman (1945, 1949) got rid of fields, and so there is no longer a problem of interaction between fields and particles. And the equation of motion for a single point particle is well defined. Apart from the metaphysical problems of an action-at-a-distance theory (see, for example, Pietsch, 2010, sections 5 and 6), the price, however, is that we have to take the advanced fields seriously. Some regard this as indicating backwards causation. At least the advanced fields cannot be dismissed as unphysical because they play a major role in the equations of motion. Whether this is a case of backward causation is another issue.

Furthermore, the mathematics of the Wheeler–Feynman theory is extremely difficult, and many important problems are still open (Bauer, 1997; Deckert, 2009). For now, physicists have not found a physical system that allows for an initial value formulation, and it isn't clear yet whether there could be an initial value problem for certain systems in the first place (Bauer et al., 2013). Initial-value problems are the sort of problems physicists and mathematicians have grown up with, and they have developed many effective tools to solve such problems. But as long as the differential equations of the Wheeler–Feynman theory cannot be rearranged as allowing certain system to be formulated as initial-value problems, the proper tools have yet to be invented.

The Born–Infeld Theory

Another potential successor to Maxwell–Lorentz electrodynamics is the Born–Infeld theory (Born and Infeld, 1934). This is a field theory proper. The idea is to change the Maxwell equations to non-linear equations. Born and Infeld propose a non-linear relation between the electric field $\boldsymbol{E}$ and the electric displacement field $\boldsymbol{D}$ and between the magnetic field $\boldsymbol{B}$ and the magnetic induction field H—in the Maxwell–Lorentz theory the relations between these fields in a vacuum are trivial, that is, $\boldsymbol{E} = \boldsymbol{D}$ and $\boldsymbol{B} = \boldsymbol{H}$.

The Born–Infeld self-field of a static particle is bounded

$$\boldsymbol{E} = \frac{q}{4\pi\sqrt{r_0^4 + r^4}}\boldsymbol{e}_r,$$

with $r_0^2 = \frac{q}{4\pi b}$ (see Perlick, 2015, p. 530). The constant b, called *Born's field strength*, is a new constant of nature in the Born-Infeld theory. We immediately see that $|\boldsymbol{E}| \to b$ if $r \to 0$; that is, the absolute value of the electric field is finite (see Fig. 4.2).

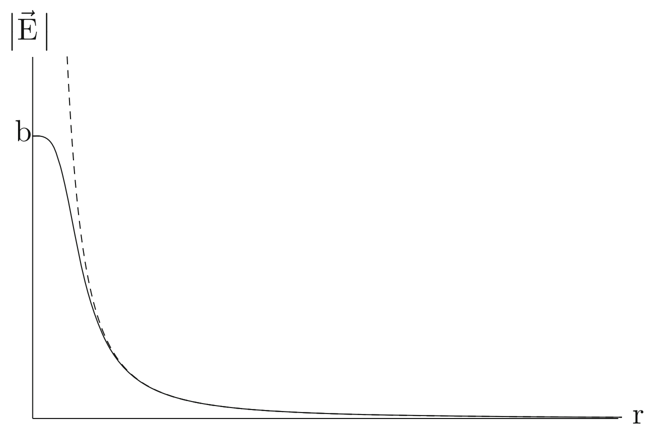

Figure 4.2.: Static Born–Infeld field compared to the (dashed) Coulomb field (picture taken Perlick, 2015, p. 531)

The dynamical case is very hard and poses many obstacles (Kiessling, 2012). Since the Born–Infeld equations are non-linear there are no standard methods for solving these equations. There are no solutions for the field of a moving particle analogous to the Liénard–Wiechert fields. Even qualitative results are difficult to deliver. No one knows whether the self-force and the self-energy are finite.

The Bopp–Podolsky Theory

In the 1940s, Bopp (1940), Podolsky (1942), and Landé and Thomas (1941) independently developed another field theory aimed at taming the infinities, which is now called the Bopp–Podolsky theory. This theory is linear but of higher order than the Maxwell equations. The Bopp–Podolsky analogue of the Coulomb field reads

$$\boldsymbol{E} = \frac{q}{4\pi r^2}\left(1 - \left(\frac{r}{l} + 1\right)\mathrm{e}^{-\frac{r}{l}}\right)\boldsymbol{e}_r,$$

with l being a new constant of nature. The modulus of the electric field is depicted in Fig. 4.3. When $r \to 0$ then $|\boldsymbol{E}| \to \frac{q}{8\pi l^2}$.

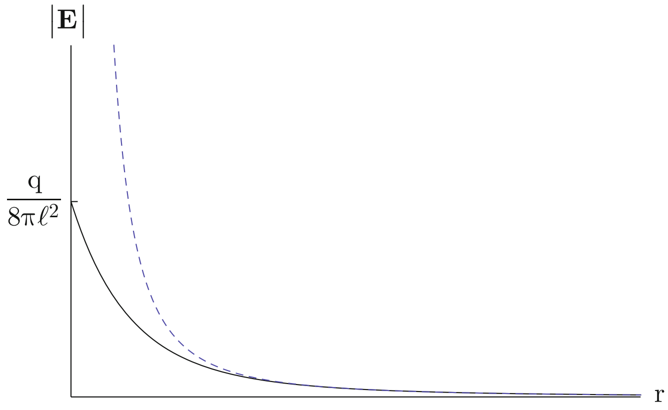

Figure 4.3.: Static Bopp–Podolsky field compared to the (dashed) Coulomb field (picture taken from Perlick, 2015, p. 531)

Unlike the Born–Infeld theory, an analogue of the Liénard–Wiechert potential has been calculated:

$$A(x) = \left(\int_{-\infty}^{\tau} \frac{J_1\left(\frac{s(x,\tau')}{l}\right)}{l_s(x,\tau')}\dot{z}^a(\tau')\,\mathrm{d}\tau'\right)\eta_{ab}\,\mathrm{d}x^b, \tag{4.13}$$

with J_1 as the Bessel function of the first kind. We don't need to explain all the details of this potential, but one feature is important. If x is in the future light cone of $z(\tau)$, the field value at x depends on the entire trajectory $z(t)$ from $t = -\infty$ to $t = \tau$. We also encounter this in the Wheeler–Feynman theory, where there are no fields.

Workarounds within the Maxwell–Lorentz Theory

The Maxwell–Lorentz theory needs to be replaced by another theory. Nevertheless, there have been many attempts to derive or justify an equation of motion for single charged particles *within* this theory. There are three major strategies. One is based on an energy-balance argument and leads to the *Abraham–Lorentz equation* (sections 4.3 and 4.4). Another is based on Dirac's idea of mass renormalization and leads to the *Lorentz–Dirac equation* (section 4.5). Both equations pose a dilemma: either the particles accelerate infinitely (runaway behavior), or the particles accelerate before the action of a force (pre-acceleration). Furthermore, the Lorentz–Dirac equation introduces an electromagnetic mass in addition to the familiar mechanical mass.

Inflating the Particles

But some diagnose the problem as lying in the size of the particles themselves. As they are point particles, it isn't surprising that there are singularities at their position. The *Abraham model* and the *Lorentz model* introduce particles that have a non-zero finite diameter. The Abraham model is a non-relativistic model of a spherical charged particle (Abraham, 1908, p. 130). Abraham introduced a rigid charge distribution ϕ, which is independent of the motion of the particle, and the particle is spherical in all inertial systems. The Lorentz model, on the other hand, is relativistic (Lorentz, 1916). Here, Lorentz introduces a "flexible" charge distribution that undergoes a physical Lorentz–Fitzgerald contraction during acceleration. As the names already suggest, neither attempt is a fully worked-out theory; they are actually models for one electron derived from the Maxwell equations. And it seems that extended particles are ruled out by experiments since electrons would have to be too large.[1]

4.2.2. Worse than Inconsistency

The equations of motion for point-size particles that are derived from the Maxwell equations are simply ill-defined. The Maxwell–Lorentz theory isn't capable of solving the simplest physical system consisting of just one charged particle. That's a big problem that begs for a genuine solution. Frisch (2005, Ch. 2) purports to show that there is another troublemaker lying at the heart of electrodynamics: an inconsistency with the principle of energy conservation.

This inconsistency arises because physicists regard only the Lorentz forces that act on a particle as generated solely by external fields. On the other hand, the theory predicts that particles radiate, and there is only energy conservation if the radiating energy is respected in the energy balance. The problem now is that if we calculate the work done by the external Lorentz forces on the particle using Newton's second law, there are no terms representing the radiated energy. So the standard way to derive a law for the conservation of energy fails to give the correct relation between energies. Therefore, classical electrodynamics is inconsistent with the principle of energy conservation.

These are the four assumptions that Frisch (2005, p. 33) puts forward to derive the inconsistency:

[1] I thank Detlef Dürr for pointing this out to me.

(i) There are discrete, finitely charged accelerating particles.

(ii) Charged particles function as sources of electromagnetic fields in accordance with the Maxwell equations.

(iii) Charged particles obey Newton's second law (and thus, in the absence of non-electromagnetic forces, their motion is governed by the Lorentz force law);

(iv) Energy is conserved in particle-field interactions, where the energy of the electromagnetic field and the energy flow are defined in the standard way.

Let's look in more detail at how Frisch argues that these assumptions lead to an inconsistency.Imagine one particle that feels the force $\boldsymbol{F}_{ext}$ from external fields so that it moves from its initial position A to its final position B within the time interval $[t_A, t_B]$. According to point (iii), the force does the work

$$W = \int_A^B \boldsymbol{F}_{ext} \cdot \mathrm{d}l = \int_{t_A}^{t_B} \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \cdot \boldsymbol{v} \, \mathrm{d}t = E_{kin}(t_B) - E_{kin}(t_A). \tag{4.14}$$

So it turns out that the work done by the force is the difference between the initial and final kinetic energy of the particle.

But if we consider that the particle radiates when it moves from A to B, the work is partially transformed into radiation. So from (iv) we get

$$W = E_{kin}(t_B) + E_{rad} - E_{kin}(t_A). \tag{4.15}$$

Since we get from (i) and (ii) that

$$E_{rad} > 0,$$

the two equations (4.14) and (4.15) are inconsistent.

I think that Frisch's argument is at best misleading.[2] It appears to be well-structured with a clear and concise conclusion, but as always the devil is in the details. In the end, Frisch has put up a smokescreen because he obfuscates the real problem. Let's get rid of the smoke.

First, the justification that the radiation energy is bigger than zero doesn't depend, as Frisch suggests in (i), on the size of particles (Muller, 2007, p. 256). Whether the particles are point-like or extended doesn't make a difference to the Maxwell equations with respect to radiation. If a charged object, be it a point or a ball, accelerates, it radiates.

Moreover, in equation (4.15), Frisch only considers the radiation energy, but for a rigorous energy balance he would have to respect the entire self-field of the particle. The radiating field is just part of the self-field; the near field, the field that is always attached to the particle irrespective of its state of motion, is still ignored.

[2] In fact, Frisch's inconsistency argument has attracted a series of critical replies, such as Belot (2007), Muller (2007), Vickers (2008), and Zuchowski (2013).

Following his general derivation of the inconsistency as I've just shown, Frisch makes the calculation by inserting the actual electromagnetic fields in equations (4.14) and (4.15). Equation (4.14) needs to be replaced by Poynting's theorem, the electromagnetic law of energy conservation—see equation (4.31) later in this chapter. The inconsistency again arises when we look deeper into the system. Poynting's theorem ignores the self-field, and by inserting the self-field, Poynting's theorem is inconsistent with its adjusted cousin that respects the self-field. So Frisch says that in this case there is an inconsistency because of the self-field, while in the general case above there is an inconsistency because of the radiation field. It seems that the way he applies his own argumentative scheme is inconsistent.

But more importantly, when Frisch talks of inconsistency, what does he really mean? Is inconsistency the same as contradiction? At first sight, it seems so, as equation (4.14) obviously contradicts (4.15). A physical theory that leads to contradiction is doomed to death. If we require anything from a scientific theory, it mustn't be contradictory. Have physicists used classical electrodynamics while remaining oblivious to a contradiction within its basic equations? The solution is pretty easy: equations (4.14) and (4.15) are derived from different assumptions. When deriving (4.14), we only count the external fields, while in (4.15) we respect the radiation field as well. And it's not surprising that different assumptions lead to different consequences. Hence, the two equations don't contradict each other; they just model two different physical situations. Still, they are inconsistent according to Frisch. But I don't see how they can be inconsistent without being contradictory.

The true problem, the genuine root of Frisch's argument, is the self-interaction problem. And Frisch seems to be aware of this when he concludes:

> As this discussion suggests, the inconsistency is most plausibly seen as arising from the fact that the Lorentz force equation of motion ignores any effect that the self-field of a charge has on its motion. The standard scheme treats charged particles as sources of fields and as being affected by fields—yet not by the total field, which includes a contribution from the charge itself, but only by the field external to the charge. This treatment is inconsistent with energy conservation. Intuitively, if the charge radiates energy, then this should have an effect on its motion, and thus a radiation term representing a force due to the charge's self-field should be part of the equation of motion. (Frisch, 2005, p. 35)

But still Frisch regards the inconsistency with energy conservation as more important than the self-interaction problem, since his subsequent discussions on how to rescue the Maxwell–Lorentz theory are centered around the inconsistency. In his reply to his critics (see Frisch, 2008), he goes back on his claim that the inconsistency is a problem of the theory itself. Rather, his argument aims to show that inconsistent theories can be empirically successful, and philosophers should think about their idea of perfectly consistent theories.

But in order to claim that an empirically successful theory doesn't need to match the high standards of ideal theories imagined by philosophers, one can argue without the detour into Frisch's inconsistency argument. Quantum mechanics is a good example. One can see its empirical success by using a computer or an LED lamp. Yet quantum mechanics is plagued by the measurement problem. And the Maxwell–Lorentz theory is plagued by the self-interaction problem, its indisputable usefulness for physicists and engineers notwithstanding.

What should physicists do? Throw away quantum mechanics? Throw away the Maxwell–Lorentz theory? Physicists are pragmatic: postulate a collapse postulate and ignore the self-fields in order to do physics. And this approach has its merits. But it has its downsides, too. Physicists tend to forget the deep fundamental problems with their own theories. The fundamental problem of classical electrodynamics is *not* its inconsistency with the principle of energy conservation. Indeed, there is no inconsistency. In application, physicists need to ignore the self-fields, otherwise the theory will be silent about any empirical consequences. So the problem of electrodynamics is much more subtle and much more severe. The truth is that the Maxwell–Lorentz theory is much more than inconsistent: it's not well defined.

4.3. Runaway Behavior

It's possible to model the motion of a single point charge within the Maxwell–Lorentz theory. We know from experiments that charged objects are more difficult to accelerate than uncharged ones, that is to say that the same external force acting on a charged object leads to a decreased acceleration compared to an uncharged object (other things being equal). And from the Liénard-Wiechert potentials, as well as from experiments, we know that accelerating charges radiate.

This inspired Abraham and Lorentz to search for an equation of motion for a single charged particle (see Jackson, 1999, section 16.2; Barut, 1980, pp. 184–5). In doing so, they introduced a new kind of force $\boldsymbol{F}_{rad}$, the *radiation reaction force*, which is supposed to act on an accelerating charged particle apart from the external forces $\boldsymbol{F}_{ext}$; it's a kind of back-reaction from the radiating field to the particle. Newton's equation of motion for a particle with mass m changes to

$$m\ddot{\boldsymbol{x}} = \boldsymbol{F}_{ext} + \boldsymbol{F}_{rad}. \tag{4.16}$$

In order to get a mathematical expression for $\boldsymbol{F}_{rad}$, Abraham and Lorentz argued that the work done by $\boldsymbol{F}_{rad}$ during a certain time interval $[t_1, t_2]$ equals the radiated energy E_{rad}, that is,

$$\int_{t_1}^{t_2} \boldsymbol{F}_{rad} \cdot \boldsymbol{v} \, \mathrm{d}t = E_{rad}.$$

E_{rad} can be calculated with the help of the *Larmor power formula*

$$P(t) = \frac{2}{3}\frac{\mathrm{e}^2}{c^3} \left|\dot{\boldsymbol{v}}\right|^2 .$$

This formula represents the total instantaneous energy radiated by a slowly moving particle, and it's derived directly from the Liénard-Wiechert potentials. Now we get

$$\int_{t_1}^{t_2} \left(\boldsymbol{F}_{rad} - \frac{2}{3}\frac{\mathrm{e}^2}{c^3}\ddot{\boldsymbol{v}} \right) \cdot \boldsymbol{v}\,\mathrm{d}t = 0.$$

A possible solution for the radiation reaction force is the obvious

$$\boldsymbol{F}_{rad} = \frac{2}{3}\frac{\mathrm{e}^2}{c^3}\ddot{\boldsymbol{v}}.$$

Having thus an expression for $\boldsymbol{F}_{rad}$, Newton's equation (4.16) turns into the *Abraham–Lorentz equation*

$$m\,(\ddot{\boldsymbol{x}} - \tau\dddot{\boldsymbol{x}}) = \boldsymbol{F}_{ext}, \tag{4.17}$$

where $\tau := \frac{2}{3}\frac{\mathrm{e}^2}{mc^3}$ has the dimension of time.[3]

The remarkable feature of this equation is the third derivative in the position coordinate. In contrast to an ordinary Newtonian equation, the initial acceleration $\boldsymbol{a}_0$ is now part of the initial conditions and can be freely chosen. The values of the initial position and velocity are no longer sufficient in for yielding unique trajectories.

But this isn't the most important consequence of the Abraham–Lorentz equation. It gets much more exciting when we consider its solutions (Plass, 1961). The general solution to (4.17) in one dimension is

$$a(t) = \mathrm{e}^{t/\tau}\left(a_0 - \frac{1}{\tau m}\int_0^t \mathrm{e}^{-t'/\tau} F_{ext}(t')\,\mathrm{d}t' \right), \tag{4.18}$$

if we choose the initial time as zero without any loss of generality (Plass, 1961, p. 40).

If we assume the external forces to be zero, there are two solutions to the Abraham–Lorentz equation (4.17). One solutions is $a_1(t) = 0$, which says that the particle will remain in inertial motion. Another solution, the *runaway solution*, is $a_2(t) = a_0\mathrm{e}^{t/\tau}$ $(a_0 \neq 0)$. Given the initial acceleration a_0, the acceleration $a_2(t)$ is unbounded when t increases. This is a non-Newtonian trajectory.

4.3.1. Infinite Energies and Singularities

You might reply, "Runaway solutions are physically possible because of the infinite self-energy of charged particles." The infinite self-energy is nothing but an inexhaustible source of energy. And as long as the particle has access to this unlimited source it can use this form of energy for its never-ending acceleration.

[3] Instead of the Larmor formula, we could have used a relativistic formula for the radiating energy. This would then yield the Lorentz–Dirac equation (Barut, 1980, p. 185).

Let's see whether this line of reasoning is sound. First and foremost, we need to distinguish between singularities and infinite energies. Singularities arise whenever we plug a forbidden number into a mathematical term. Look at the function $\frac{1}{x}$. It's well-defined for all real numbers except 0 since $\frac{1}{0}$ is mathematically meaningless. This is analogous to the self-interaction problem. The Liénard-Wiecher potentials (4.11) and (4.12), as well as the corresponding fields (4.9) and (4.10), are singular on the particle's trajectory. The fields and the potentials simply have no value on the trajectory. And so the Maxwell–Lorentz theory renders no equation of motion for a single particle in its own field.

Independently of the self-interaction problem, we can calculate the energy

$$W(R) = \int_{K_R} \frac{1}{2} \boldsymbol{E} \cdot \boldsymbol{E} \, \mathrm{d}^3 \boldsymbol{x} \tag{4.19}$$

of the self-field within a ball K_R with radius R. In the Maxwell–Lorentz theory this quantity is infinite for any $R > 0$, and the particle is said to have *infinite self-energy*.

What is the relation between the infinite self-energy and the singularity of the fields? There are four logical possibilities:

1. Infinite field energy and singularity,
2. Finite field energy and singularity,
3. Infinite field energy and no singularity,
4. Finite field energy and no singularity.

The Liénard–Wiechert fields have singularity at the position of the source, and the integral (4.19) is infinite. Clearly, the infinite field energy is a result of the singularity. But there may be "well behaved" singularities that yield a finite integral, for example, if the potential goes like $-\ln x$. An infinite field energy without a singularity can only arise if we take the integral (4.19) over the entire space, that is, for $R \to \infty$. The field density can decrease very slowly so that the integral is infinite.

The Born–Infeld theory and the Bopp–Podolsky theory have no singularities and yield finite self–energies—remember that in the Born–Infeld theory we only have results for the static case. The fields decrease so rapidly that the energy integral is finite over the entire space. This is as well-behaved a field as one can get. But it's still an open issue whether the Bopp–Podolsky theory contains runaway solutions (Perlick, 2015, p. 540).

It might sound paradoxical, but we can have runaway solutions even if the fields have finite energies.Coleman (1961, pp. 33) also argues that it's wrong to assume that a runaway particle gets its kinetic energy from its self-energy. For there are mathematical models of charged particles that allow runaway solutions to have *finite* self-energy. Although Coleman's models fall within quantum field theory

(Norton and Watson, 1959), this doesn't weaken the argument: runaway behavior is possible with a finite self-energy.

4.3.2. The Meaning of Energy–Momentum Conservation

Another reaction to runaway solutions might be that these solutions should be dismissed because they violate energy-momentum conservation. Since the particle accelerates without upper bounds it conserves neither energy nor momentum. The "violation" of conservation of momentum is obvious. Due to its change in velocity, the particle's momentum changes. But this is nothing serious. A moving car changes its momentum all the time. What about conservation of energy? The kinetic energy of the particle isn't conserved for the simple reason that its momentum isn't either.

So which momentum and which energy are allegedly not conserved? That the kinetic energy and the momentum of the particle isn't conserved is trivial and offers no argument against the physical possibility of runaway behavior. Unless there is a precise physical quantity to be identified with energy or momentum, the argument of violation of energy-momentum conservation loses its power because it's too general.

Theorists often ignore the fact that all conservation laws are derived from the equations of motions—as pointed out by Grünbaum (1976, pp. 174–5). Laws of conservations are never axioms or independent claims supplementing the equations of motion. They are theorems of the laws. Therefore, it's impossible to derive a law of conservation from the Abraham–Lorentz equation (4.17) that forbids its own solutions. In particular, it's impossible to have a law of conservation of energy and momentum that would be violated by the runaway solutions.

Moreover, conservation laws aren't universally valid; they are derived from the laws of motion *by fulfilling certain assumptions*. Unless a physical system fulfills the very same assumptions, it doesn't have to obey these conservation laws. So given certain laws of motion, only a subset of solutions under certain circumstances can fulfill a law of conservation.

Energy-Momentum Conservation in Newtonian Mechanics

Let's have a look at Newtonian mechanics to illustrate these two issues: laws of conservation are derived from the laws of motion, and they aren't universally applicable. The trivial case is a particle in inertial motion. Since the velocity is constant for all times the kinetic energy, as well as the momentum, is constant. Therefore, its kinetic energy and momentum are conserved. But if this particle moves in a force field, its velocity will certainly change. So its kinetic energy and its momentum aren't conserved quantities because the initial energy and momentum differ from the final energy and momentum, that is to say, $\frac{1}{2}m\boldsymbol{v}(t_1) \neq \frac{1}{2}m\boldsymbol{v}(t_0)$ and $m\boldsymbol{v}(t_1) \neq m\boldsymbol{v}(t_0)$.

If we take the forces into account we get for the difference in kinetic energy

$$\frac{1}{2}m\boldsymbol{v}^2(t_1) - \frac{1}{2}m\boldsymbol{v}^2(t_0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_1} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{x}.$$

So the difference in kinetic energy is traced back to the work done by the force field. And if we further assume that the force is conservative, that is, $\boldsymbol{F} = \nabla U$, then

$$\frac{1}{2}m\boldsymbol{v}^2(t_1) + U(\boldsymbol{x}_1) = \frac{1}{2}m\boldsymbol{v}^2(t_0) + U(\boldsymbol{x}_0).$$

Now we have a conserved quantity: the total energy, being the sum of the kinetic and the potential energy ($E_{tot} = E_{kin} + E_{pot}$), is constant for all trajectories.

This relation is derived from Newton's second law; it isn't a separate axiom in addition to the laws of motion. And the forces have to be conservative so that the total energy E_{tot} is conserved. Furthermore, the total energy in the form $E_{kin} + E_{pot}$ can only be *defined* if the forces are conservative. There is no potential for non-conservative forces. This shows that this law of conservation is only valid after putting further constraints on the physical system.

The momentum of a single particle isn't a conserved quantity, as the kinetic energy isn't a conserved quantity either. In order to get a law of conservation for the momentum, we have to enlarge the system of one particle to comprise all the other particles generating the force $\boldsymbol{F}$. I would like illustrate this using the two-body problem. This is the only problem that can be solved from the ground up by solving Newton's second law; there are no exact solutions for systems consisting of three or more particles. The equations of motion for a two-particle system are

$$\boldsymbol{F}_{12} = m_1\ddot{\boldsymbol{x}}_1,$$
$$\boldsymbol{F}_{21} = m_2\ddot{\boldsymbol{x}}_2.$$

$\boldsymbol{F}_{12}$ is the force exerted on particle P_1 by P_2, and $\boldsymbol{F}_{21}$ is the force exerted on P_2 by P_1.

Because of Newton's third law, the two particles are exerting forces against each other that only differ in terms of their direction, namely, $\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$. So we get

$$0 = \boldsymbol{F}_{12} + \boldsymbol{F}_{21} = m_1\ddot{\boldsymbol{x}}_1 + m_2\ddot{\boldsymbol{x}}_2 = (m_1 + m_2)\ddot{\boldsymbol{R}},$$

where $\boldsymbol{R} = \frac{m_1\boldsymbol{x}1 + m_2\boldsymbol{x}_2}{m_1 + m_2}$ is the vector of the center of mass. The equation states that the center of mass is in inertial motion, and it follows by integration that the sum of the initial total momentum equals the sum of the final momenta:

$$m_1\boldsymbol{v}_1(t) + m_2\boldsymbol{v}_2(t) = m_1\boldsymbol{v}_1(t_0) + m_2\boldsymbol{v}_2(t_0).$$

While the momentum for each particle changes, the total momentum of the whole system remains constant.

Besides, conservation of total momentum depends on the validity of Newton's third law. If the forces did not obey this axiom, the integration would yield

$$m_1\boldsymbol{v}_1(t) + m_2\boldsymbol{v}_2(t) = m_1\boldsymbol{v}_1(t_0) + m_2\boldsymbol{v}_2(t_0) + \int\limits_{\boldsymbol{x}(t_0)}^{\boldsymbol{x}(t_1)} (\boldsymbol{F}_{12} + \boldsymbol{F}_{21})\ \mathrm{d}\boldsymbol{x},$$

and the total momentum would no longer be a conserved quantity.

How do we get conservation of energy for the system of two particles? The first step is to subtract both equations of motion, yielding

$$\frac{m_1 m_2}{m_1 + m_2}(\ddot{\boldsymbol{x}}_1 - \ddot{\boldsymbol{x}}_2) = \boldsymbol{F}_{12}.$$

Now we are in a similar scenario to the one-particle case. If the force is conservative we get conservation of energy; otherwise, there will be a term left indicating how the system is losing or gaining energy from the outside.[4]

The Purpose of Energy-Momentum Conservation

We have finally seen that laws of conservation aren't a priori true. They are theorems of the laws of motion. But what is their meaning, purpose, or significance? Sometimes I have the impression that laws of conservations have to be fulfilled and defended by any means, as if it were a law of nature that every system under every circumstance has to conserve energy and momentum. If a system violates energy-momentum conservation it's often denounced either as bizarre or as unimportant. Something must be strange about this system.

But nothing is strange if a system doesn't conserve energy or momentum or any other mathematical quantity. And nothing is special if a system does. Energy and momentum conservation are consequences of the equations of motion. They don't add anything new that isn't included in the laws. Of course, the same holds true for all the other familiar candidates for conserved quantities, like angular momentum, the local charge, or probability.

Nonetheless, kinetic energy, potential energy, linear momentum, and angular momentum seem to be distinguished from other conserved quantities since they appear in all kinds of physical theories from quantum mechanics to cosmology. There seems to be something ontological about the conservation of total energy, as if energy were a kind of substance in or a property of a physical system. The language of physics supports this view: "energy is radiating from the particle to the environment," or

[4] One word on collisions of balls. To ensure the validity of energy conservation in a collision, the balls have to be elastic, that is to say that they don't convert energy into heat during a collision. If this is fulfilled the sum of the initial kinetic energies equals the sum of the final kinetic energies. That there is conservation of energy during collisions is, however, reasoning from induction or an assumption we make on the behavior of the system. There is no exact solution for this case. The conservation of momentum is induced from its validity in the two-particle case.

"an electron can only absorb certain energy quanta." These ways of speaking may be underpinned by historical reasons and support intuition, but they are misleading when it comes to what happens in the ontology. The primitive-stuff ontology of particles and fields comprises all the objects in the ontology. Adding the masses and charges as properties of the particles completes the ontological picture of classical electrodynamics if one is a dispositionalist—the analysis of Chapter 2 can easily be applied to electrodynamics.

As the mathematical definitions of all kinds of energies and momenta show, they ultimately depend on the position and motion of the particles. This is obvious for the kinetic energy and linear and angular momentum. The same holds true also for the potential energy. If we write the potential energy for a particle P as $U(\boldsymbol{x})$, the potential energy doesn't only depend on the position $\boldsymbol{x}$ of P, but also on the other particles in the universe that exert a force on P. Remember that the potential energy can only be defined for conservative forces, and whether the forces are conservative depends on the behavior of the other particles as well. Philosophically speaking, energy and momenta supervene on the motion of the particles. Depending on the particles' behavior, energy and momentum may remain constant throughout the trajectory.

There is another argument favoring the view that energy and momentum are distinguished. The definitions change from physical theory to physical theory, and only the notions remain. For instance, the kinetic energy of a quantum particle is different from the kinetic energy of a classical particle. The kinetic energy in classical mechanics has to be defined differently to its quantum correlate because the laws of motions are different. Therefore, it isn't the case that the mathematical formula of kinetic energy refers to some stuff or additional property of particles or system of particles. And the search for laws of conservation doesn't lie in the description of a further property of a physical system or of additional stuff apart from the particles and fields; the motivation for defining energy and momentum and for the search for laws of conservation lie elsewhere.

Indeed, the laws of conservation are indispensable for many *practical* purposes. One such purpose is that they are a means for checking consistency. Let's assume we have calculated a trajectory and we know that this trajectory has to fulfill energy conservation. We can then plug it in our law of conservation in order to check whether we have made a mistake in our calculations. Or given an arbitrary trajectory, we can instantly dismiss it as false or unphysical if it does or does not fulfill the laws of conservation. In these cases the conservation of a certain quantity turns out to be very useful.

And by far the most important use of conservation laws is that they help us in solving the equations of motion in the first place. In Newtonian mechanics, only the two-body problem is solvable directly from Newton's laws. But with the help of conserved quantities the number of systems that can be solved greatly increases. And physicists try to make calculations as simple as possible. Since solving the equations of motion by hand is either extremely difficult or even impossible, they have to discover further ways that lead to results. And many systems, like colliding particles, can be solved only with the help of energy and momentum conservation.

A Conserved Quantity in the Abraham–Lorentz Equation

Now we can go back to the runaway solutions of the Abraham–Lorentz equation. The runaway solutions are genuine solutions of this equation, and they don't violate any energy-momentum conservation derived from this very equation. As we can now appreciate, the laws of motion cannot contain solutions that violate a law of conservation by fulfilling the very same assumptions that lead to the law of conservation itself. Coleman (1970, p. 294) shows that the total (relativistic) energy of runaway solutions to the Abraham–Lorentz equation is conserved once you take the fields into account. It's given by

$$E = \frac{1}{2}\left(m_0 \dot{\boldsymbol{x}}^2 + \int \left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right) \mathrm{d}^3\boldsymbol{x}\right). \tag{4.20}$$

The mass m_0 is the bare mass instead of the physically measurable mass m, and it's negative. It's derived from a mass renormalization procedure—which I introduces in section 4.5. This means that the left side of the sum decreases, while the right side increases, when the particle accelerates. The decrease of the kinetic energy and the increase of the field energy is such that the total energy E doesn't change.

Change in Energy without External Forces

Another argument against runaway solutions says that "[r]unaway solutions are unphysical because a particle cannot gain energy without external forces." For example, Plass uses this line of argument:

> [T]he solutions of the equations of motion with radiative reaction for a particular force always contains terms which require that the acceleration of the particle must eventually increase exponentially with time. These solutions have been called "self-accelerated," "run-away," and "nonphysical". The particle doesn't obtain its added energy from any physical force which acts upon it. Clearly, these are absurd solutions when applied to our real physical world. (1961, p. 38)

His description is ambiguous in the penultimate sentence. On the one hand, Plass refers to energy, which has to be added from the outside. This reminds me of an a priori validity of energy conservation, and as I have shown above a physical system doesn't have to obey energy conservation. On the other hand, he emphasizes that external forces must provide the change in energy. But why are external forces necessary? Couldn't an infinite reservoir of self-energy in principle account for runaway behavior without the use of external forces?

The possibility of the particle's accessing an infinite amount of self-energy has to be explicitly excluded. And it has to be excluded that a particle can change its energy out of nowhere or that it can change its motion spontaneously, disregarding the motion of the other particles. The latter is an instance of the principle of sufficient reason, which is an essential principle in physics.[5] Only if all these requirements are spelled out is Plass's conclusion sound.

If we rephrase his central sentence and dispense with the word "energy", then his statement is that the particle changes its state of motion without the influence of external forces. Doesn't this remind us of Newton's first law? So runaway solutions do violate Newton's law of inertia. And this may be a good reason to expel them from physically possible solutions, if we stick to the irrevocable truth of Newton's axiom. But Newton's axioms aren't written in stone. Their validity depends on theoretical arguments, as well as on experiments. Newton's first law doesn't only define the (causal) role of forces, but presupposes their very existence. There are other theories, like Bohmian mechanics, that don't rely on Newton's first axiom, or on forces in general.

Empirical Adequacy

All the aforementioned arguments against the physical possibility of runaway behavior have been theoretical. They refer to energy-momentum conservation or Newton's laws. But runaway solutions may be dismissed because they don't reflect physical behavior: "Runaway solutions are unphysical because they aren't observed in nature." Nothing can be said against empirical inadequacy. The solutions may even fulfill all nice features, such as energy-momentum conservation, but if there is no physical system that behaves accordingly, then these solutions are indeed unphysical.

4.4. Pre-Acceleration and Backward Causation

How can we get rid of the runaway solutions? The general solution (4.18) to the Abraham–Lorentz equation comprises more solutions than just the runaway ones. We don't want the acceleration of the particle to rise to infinity as time passes. Mathematically, this *asymptotic condition* is written as

$$\lim_{t \to \infty} |\boldsymbol{a}(t)| < \infty. \tag{4.21}$$

Applying this to the general solution (4.18) yields that the term in brackets has to go (sufficiently fast) to zero; otherwise, the exponential function will take over and lead to infinity again. This constrains the possible initial accelerations, so that there is exactly one initial acceleration that doesn't lead to runaway solutions:

$$\tilde{a}_0 = \frac{1}{\tau m} \int_0^\infty \mathrm{e}^{-t'/\tau} F_{ext}(t')\, \mathrm{d}t'.$$

[5]The principle of sufficient reason should not be confused with determinism. Standard quantum mechanics does obey the principle of sufficient reason, but it's an indeterministic theory. The measurement of identical systems illustrates the difference.

Plugging this into the general solution (4.18), we get the non-runaway solutions[6]

$$\tilde{a}(t) = \frac{\mathrm{e}^{t/\tau}}{\tau m} \int_t^\infty \mathrm{e}^{-t'/\tau} F_{ext}(t')\,\mathrm{d}t'. \tag{4.22}$$

What's remarkable about this equation is that the acceleration at time t is determined by all future forces exerted on the particle. Earman (1976) claims that this is a clear case of backward causation. In his discussion he refers to the relativistic Lorentz–Dirac equation, but his line of reasoning applies without constraint to the non-relativistic Abraham–Lorentz equation as well.

> The Lorentz–Dirac equation of motion for classical relativistic charged particles is a third-order (in time) differential equation. When certain asymptotic conditions are imposed, the equation is converted into a second-order integro-differential equation according to which the acceleration of a charged particle at proper time [t] is equal to the integral over the "effective force" acting for all times [$t' > t$]. As a result, the integro-differential equation predicts "pre-acceleration" effects; e.g., if a sharp impulsive force acts on the particle at time [t], then the particle will begin to accelerate before [t]. For the moment, I will assume that this result constitutes prima facie evidence that if the equation in question were a true law of nature, then force would have backward causal effects. (Earman, 1976, p. 13)

Adolf Grünbaum, Earman's harshest opponent on this matter, tried to prove him wrong in two papers (Grünbaum, 1976; Grünbaum and Janis, 1977). Grünbaum was convinced that electrodynamics doesn't yield retrocausation. Let's discuss their disagreement.

4.4.1. Pre-Acceleration as Truncated Runaway

Earman's argument above contains three steps. First, he notices that the acceleration $\tilde{a}(t)$ is mathematically determined by forces exerted on all future times $t' > t$ in the integral. Second, this obviously leads to *pre-acceleration*: the particle starts to change its motion before external forces act on the particle. Third, Earman claims that the pre-acceleration is a case of *retrocausation*: future forces cause the change of motion of the particle at an earlier time, although they aren't yet present.

I cannot disagree with the first point. It's a mathematical fact that according to equation (4.22) the acceleration at time t is mathematically determined by the forces at $t' > t$. The second point isn't controversial, either. The solutions to the Abraham–Lorentz equation just state that there is acceleration before the imposition of forces. There is even acceleration without any forces at all, as seen by the run-away solutions.

[6]They are often confused with integro-differential equations (see, for example, Grünbaum and Janis, 1977, pp. 478 and 481). It's true, however, that in the relativistic case the same procedure starting from the Lorentz–Dirac equation does lead to integro-differential equations. I deal with this equation in section 4.5.

It's Earman's third step that is contentious because there he mentions causation. The problem, however, is that the motion of the particles isn't only governed by equation (4.22), but also by the Abraham–Lorentz equation, which describes how forces at time t influence the acceleration at t. But Earman derives his conclusion just from the functional dependencies of (4.22). Is his line of reasoning valid in deducing causal relations from functional relations and in disregarding the Abraham–Lorentz equation? Are future forces indeed the cause for the acceleration at t?

Let's first discuss a toy model before we delve into the details of Grünbaum's attack on Earman (see Plass, 1961, section III; the paper contains many more examples). The most instructive case is when the external forces act constantly during a finite time interval, that is,

$$F_{ext}(t) := \begin{cases} 0, & t < 0, \\ mk, & 0 \leq t \leq t_0, \\ 0, & t_0 < t, \end{cases}$$

where m is the mass of the particle and k is an arbitrary constant. The general solution to the Abraham–Lorentz equation is

$$a(t) = \begin{cases} (k + A)\, \mathrm{e}^{t/\tau}, & t < 0, \\ k + A\mathrm{e}^{t/\tau}, & 0 \leq t \leq t_0, \\ \left(k\mathrm{e}^{-t_0/\tau} + A\right) \mathrm{e}^{t/\tau}, & t_0 < t. \end{cases}$$

The constant A can be freely chosen. But if we want to avoid pre-acceleration then we must choose $A = -k$, and the solutions becomes

$$a_\infty(t) = \begin{cases} 0, & t < 0, \\ k\left(1 - \mathrm{e}^{t/\tau}\right), & 0 \leq t \leq t_0, \\ k\left(\mathrm{e}^{-t_0/\tau} - 1\right) \mathrm{e}^{t/\tau}, & t_0 < t. \end{cases}$$

This ultimately leads to a runaway behavior of the particle because of the exponential function for $t_0 < t$.

The non-runaway solutions require $A = -k\mathrm{e}^{t_0/\tau}$:

$$\tilde{a}(t) = \begin{cases} k\left(1 - \mathrm{e}^{t_0/\tau}\right) \mathrm{e}^{t/\tau}, & t < 0, \\ k\left(1 - \mathrm{e}^{t\,-\,t_0/\tau}\right), & 0 \leq t \leq t_0, \\ 0, & t_0 < t. \end{cases}$$

Now we see that the particle starts moving before the forces act at $t = 0$. Consequently, we cannot get rid of both runaway and pre-acceleration: either there is runaway behavior, or there is pre-acceleration.

It's illuminating to zoom in on how the general solution to this toy model is found in the first place. First, three different versions of the Abraham–Lorentz equation are independently solved corresponding to the behavior of the external forces F_{ext} during $t < 0$, $0 \leq t \leq t_0$, and $t_0 < t$. Second, the continuity of the acceleration constrains the behavior at $t = 0$ and $t = t_0$, where three different parts have to be stuck together. This then leads to the general solution $a(t)$, as shown above.

It's remarkable that the solution for $t < 0$ doesn't depend on future forces. If $F_{ext}(t) = 0$ for *all* times t, there would still be the same exponential acceleration. So the counterfactual, "If there were no forces, there won't be pre-acceleration," is wrong if we hold to the asymptotic condition (4.21). The external force doesn't make the particle move before it's effective; the particle starts moving for $t < 0$ even if no external forces are exerted on it. How can the force cause pre-acceleration when the counterfactual case isn't true?

The case $t < 0$ in our toy model is nothing but the runaway solution that is cut-off after $t = 0$ when the force is applied. The role of the force isn't to generate motion during $t < 0$. In solving the equation of motion, we see that the motion of the particle in this time interval is independent of the external forces. Rather, the forces *prevent* the particle from accelerating infinitely—this is a consequence of the continuity condition.[7]

Is pre-acceleration observed in nature?

> I believe that backward causation is a conceptual possibility and that the question of whether backward causation exists in nature is a question which must be settled not by armchair philosophers but by natural philosophers. A possible mechanism for backward causation is contained in the Dirac-Plass theory of classical relativistic electrodynamics. According to this theory, an impulsive force causes (I believe that 'causes' is the right word) a particle to accelerate before the pulse arrives. For an electron, the preacceleration effect is on the order of 10^{-23} seconds, and, therefore, it's unlikely that it could be detected by any classical apparatus; so even if it exists in nature, preacceleration may not give rise to any recognizable future analogues of traces. But still the point remains that a coherent mechanism for the production of traces of the future is at hand. (Earman, 1974, p. 41)

The Abraham–Lorentz equation isn't an exact equation. So it isn't a severe problem when it predicts phenomena that cannot be experimentally tested. Even if the pre-acceleration effect is too small to be observed, a discussion of this phenomenon may be applied to a possible exact theory predicting some kind of pre-acceleration.

A Common Fault

Before analyzing Earman's claim in detail, I would like to mention an interesting point made by the physicist David Griffiths regarding the Abraham–Lorentz

[7] There is an interesting feature that isn't related to pre-acceleration: external forces in the Abraham–Lorentz equation behave differently from forces in Newton's second law. The force doesn't always increase the acceleration in the same direction. You can see this in the runaway solution in our one-dimensional toy model. The external force is positive (indicating a force from left to right), but the acceleration *decreases* until in the end it vanishes.

equation:

> These difficulties [namely, pre-accleration] persist in the relativistic version of the Abraham–Lorentz equation, which can be derived by starting with Liénard's formula instead of Larmor's [. . .]. Perhaps they are telling us that there can be no such thing as a point charge in classical electrodynamics, or maybe they presage the onset of quantum mechanics. (Griffiths, 1999, p. 467)

I would like to say it straight: pre-acceleration has nothing to do with the particles being point-sized. The problem with point charges appears much earlier in the Maxwell equations. The infinite self-energies can be interpreted as a problem of point-sized charges, instead of a problem of fields, so that one strategy to avoid the infinities is to introduce extended particles.

The idea that pre-acceleration foresees quantum mechanics is more than far-fetched. This is one of many examples of physicists hoping that quantum mechanics can solve the deepest issues in classical physics. In quantum field theory, the infinities remain.

4.4.2. The Analogy with Newtonian Velocities

According to the standard interpretation of Newtonian mechanics, forces cause acceleration. Since only future forces determine the acceleration in equation (4.22), these forces retrocause their effects. This is Earman's argument in a nutshell. More precisely, the conjunction of all external forces $F_{ext}(t')$ on the interval $[t, \infty)$ uniquely determines the acceleration at t according to (4.22), while the acceleration at time t doesn't uniquely determine the external forces—the integral may have the same value for different forces. This asymmetry between the forces and the acceleration supports the standard interpretation of forces as being the cause of acceleration. As the forces lie in the future, this is backward causation.

Grünbaum (1976) argues against backward causation by working out an analogy with Newtonian mechanics. In doing so, he in fact uses two kinds of arguments. First, there is a formal mathematical similarity between Newtonian mechanics and the Abraham–Lorentz theory. Since there is no backward causation in Newtonian mechanics, it has to be excluded from electrodynamics as well.

Second, the asymptotic condition (4.21) isn't a law. Grünbaum interprets it as a "de facto" boundary condition, that is, a *contingent* boundary condition imposed on the Abraham–Lorentz equation. It's contingent because not all solutions to the Abraham–Lorentz equation vanish at infinity; the sole purpose of the asymptotic condition is to exclude runaway solutions. Being contingent, it cannot yield causal relations that aren't included by the Abraham–Lorentz equation itself. For Grünbaum, the Abraham–Lorentz equation is the source of causal relation, as he construes it as a law of motion for single charged particles.

Let's delve into the details of his arguments. In one dimension, Newton's second law is

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F(t),$$

and the general solution for the velocity is given by

$$v(t) = v(t_0) + \frac{1}{m}\int_{t_0}^{t} F(t')\,\mathrm{d}t'. \tag{4.23}$$

The above formula bears the following important characteristics:

(a) The velocity $v(t)$ can be zero even if the forces $F(t)$ are non-zero for the whole interval $[t_0, t]$. This can happen if the initial velocity is minus the right summand.

(b) The velocity $v(t)$ can be non-zero even if the forces $F(t)$ are zero for the whole interval $[t_0, t]$. This can happen if $v(t_0) \neq 0$.

These two consequences of (4.23) show that a non-zero integral over the forces is neither necessary nor sufficient for a non-zero velocity. So just considering equation (4.23) we cannot deduce any causal relation between forces and acceleration. The mathematics abstains from giving us a clue.

Nevertheless, physicists and philosophers generally agree that the forces at a time t cause the acceleration at t in Newtonian mechanics. Newton's first law strong supports this view. But Newton's laws in the form of a differential equation are indeed ambiguous about the causal relations between forces and acceleration because acceleration mathematically determines the forces and vice versa. As I discussed in section 3.4, Blondeau and Ghins (2012) show, however, that in a differential equation the derivative is always the effect and the rest are the causes, thus again establishing that forces are the causes of acceleration. Although the general solution (4.23) doesn't fix the causal relationship between forces and accelerations, forces can be construed as the causes of acceleration if we include Newton's law.

To make the argument more vivid, we can impose a boundary condition on the Newtonian solutions (4.23). We demand that the velocity be zero if time goes to infinity, that is, $v(t) \to 0$ if $t \to 0$. Applying this to the initial condition $v(t_0)$, we get

$$v(t) = -\frac{1}{m}\int_{t}^{\infty} F(t')\,\mathrm{d}t'. \tag{4.24}$$

No one would ever refer to this equation in order to defend backward causation in Newtonian mechanics. It is true that the forces on $[t, \infty)$ determine the velocity at t, but the causal role of forces is explained by Newton's second law. The forces at t cause the acceleration at t. Choosing special initial conditions, the velocities in Newtonian mechanics depend on the future forces. But this is only a mathematical dependency. In no way does equation (4.24) prove the existence of backward causation in Newtonian mechanics.

The pre-acceleration solutions (4.22) of the Abraham–Lorentz equation are mathematically almost identical to equation (4.24). So why should the forces retrocause the acceleration in electrodynamics but not in Newtonian mechanics?

Grünbaum (1976) argues that owing to the mathematical similarities the solutions (4.22) mustn't be interpreted causally, since the Newtonian solutions aren't causally interpreted, either. He adds that the mathematical form of the solutions of differential equations aren't suitable for deriving causal relations. In particular, the pre-acceleration solutions rely on the asymptotic condition (4.21), which is contingent, like any other boundary or initial condition. The correct sources for a causal analysis are the laws of motion.

In Newtonian mechanics, forces cause acceleration. This is supported by the fact that the forces at time t uniquely determine the acceleration at t. But in the Abraham–Lorentz equation the forces only determine $m\,(\ddot{\boldsymbol{x}} - \tau\dddot{\boldsymbol{x}})$. Therefore, Grünbaum goes on, it's wrong to regard the forces as the causes of acceleration. Hence, there is no backward causation in classical electrodynamics.

4.4.3. Differential Equations and their Solutions

Grünbaum and Janis (1977) don't agree, either, with the conclusion drawn by Earman (1974, 1976) that pre-acceleration solutions (4.22) instantiate retro-causation. They try to disprove him by showing that the forces don't uniquely determine the acceleration. This cannot be deduced solely from the mathematical structure of the pre-acceleration solutions; they will need to include the Abraham–Lorentz equation. They stage their argument in five steps.

First, they point out that differentiating the pre-acceleration solution (4.22) yields the Abraham–Lorentz equation (4.17). Therefore, the pre-acceleration solution *entails* the Abraham–Lorentz equation.

Second, the crucial consequence of the first step is the following: if the Abraham–Lorentz equation entails some proposition u, this very proposition can also be deduced from the pre-acceleration solution (Grünbaum and Janis, 1977, p. 479), because the pre-acceleration solutions mathematically entail the Abraham–Lorentz equation. The proposition that is central to their argument is "acceleration determines forces." So if we can deduce from the Abraham–Lorentz equation that acceleration determines forces, then the pre-acceleration solutions also entail that acceleration determines forces.

Third, assume that the acceleration $\dot{v}(t)$ is given for all times t in a finite time interval $[t_0, t_1]$. Then the Abraham–Lorentz equation states that the forces during the time interval are uniquely determined by the acceleration. And because of the previous two steps, the pre-acceleration solutions uniquely determine the forces, too, given the acceleration in this interval.

But if the *forces* are given during $[t_0, t_1]$ the pre-acceleration solutions don't determine the acceleration, because they need the forces on the infinite interval $[t_0, \infty)$. Similarly, the Abraham–Lorentz equation doesn't determine the acceleration, either. If the external forces are given, the Abraham–Lorentz equation rather determines $m\,(\ddot{\boldsymbol{x}} - \tau\dddot{\boldsymbol{x}})$. Therefore, accelerations determine forces for a given *finite* time interval, while the forces don't determine the acceleration.

Furthermore, if the forces are given on $[t_0, \infty)$, they uniquely determine the acceleration in the pre-acceleration solutions in contrast to the Abraham–Lorentz equation. And if the *accelerations* are given on this infinite interval, then the forces are uniquely determined in the Abraham–Lorentz equation—as with the finite interval. Since the pre-acceleration solutions entail the Abraham–Lorentz equation, these solutions uniquely determine the forces, too, according to the first and second steps. On the infinite interval $[t_0, \infty)$, accelerations determine forces, and forces determine acceleration.

Fourth, let's look at the limiting case where the time interval reduces to one point. According to the pre-acceleration solutions, the acceleration $a(t_0)$ at an instant t_0 doesn't determine the forces for any time $t \geq t_0$. And the forces at one moment don't determine the acceleration for any time $t \geq t_0$ either.

Fifth, Earman's argument rests solely on an analysis of the pre-acceleration solutions. Just considering this equation without the Abraham–Lorentz equation, the values for the forces during $t \in [t_0, \infty)$ seem to uniquely determine the acceleration at t_0, while the acceleration at t_0 doesn't determine any value for future forces. Earman regards this asymmetry between the forces and acceleration as an instance of backward causation: the forces cause the acceleration $a(t_0)$, although they lie in the future of $a(t_0)$. Grünbaum and Janis counter that they show in their third step that the forces don't determine the acceleration for any finite subinterval of $[t_0, \infty)$. Moreover, they explain in their fourth step that forces at one moment t don't determine the acceleration either. Hence, there is no backward causation.

Do Solutions Entail their Equation?

The crucial ingredients for their argument lie in the first two steps; the rest is a consequence thereof. In the first step, they claim that the pre-acceleration solutions *entail* the Abraham–Lorentz equation because the latter follows from a simple differentiation of the former. So the relation between these two equations is supposed to be the same as the relation between the functions x^2 and $2x$: one is the derivative of the other.

There is a subtle but crucial difference. The Abraham–Lorentz equation (4.17) is a differential equation. There is a mathematical space of functions assigned to it that represents all solutions—and we know that all solutions are of the form (4.18). So a differential equation is nothing but a shorthand for all the functions that fulfill this equation. In order to pick one solution we have to specify a set of initial conditions. The initial conditions for the Abraham–Lorentz equation yielding a unique solution are the position, velocity, and acceleration at time t_0. If we choose the initial acceleration, regardless of the position and velocity, as given by the asymptotic condition (4.21), then we get our well-known pre-acceleration solutions (4.22). In other words, the Abraham–Lorentz equation entails the pre-acceleration solutions if we require the asymptotic condition.

By construction, the pre-acceleration solutions must fulfill the Abraham–Lorentz equation. Indeed Grünbaum and Janis base their argument on the observation that whatever we can deduce from a differential equation we can deduce from a special solution as well. When we deduce features from the differential equations we actually deduce features of the space of solution to this differential equation.

The Moral of the Argument

In contrast to Earman, Grünbaum and Janis trace back where the pre-acceleration solutions come from. They take the Abraham–Lorentz equation as the basic law of motion and argued that forces there don't uniquely determine the accelerations. Furthermore, they emphasize that only if the forces are given on the infinite interval $[t_0, \infty)$ do they uniquely determine the acceleration at t_0 according to the pre-acceleration solution, while the Abraham–Lorentz equation uniquely determines the forces whenever the accelerations are given.

Grünbaum and Janis successfully counter Earman by carefully examining the functional relationships between force and accelerations. Earman (1976, p. 19) refers only to the mathematical dependencies of the pre-acceleration solutions. Grünbaum and Janis, on the other hand, disprove him with the help of the Abraham–Lorentz equation interpreted as a basic law of motion; the solutions aren't sufficient. By no means do they aim to argue that the accelerations cause the forces; their goal is simply to show that Earman could not read off backward causation from the mathematical form of solutions of differential equations.

4.5. Electromagnetic Mass

In his seminal paper, Paul Dirac (1938) derives the Lorentz–Dirac equation, a relativistic equation of motion for a single charged particle. A decisive step in his derivation is his method of mass renormalization. I follow Barut (1980, Ch. V) in his presentation of Dirac's ideas. In Appendix B, I discuss what Dirac really did in detail.

4.5.1. Mass Renormalization

Let's start with the retarded Liénard-Wiechert field (4.9), which consists of a near field F^{near} and a radiation field F^{rad}—the near field is like $\frac{1}{x}$, and the radiation field like $\frac{1}{x^2}$:

$$F^{\text{ret}}_{\mu\nu} = \frac{1}{2}F^{\text{rad}}_{\mu\nu} + \frac{1}{2}F^{\text{near}}_{\mu\nu}.$$

According to this decomposition, there are three kinds of forces acting on the particle. First, there is the force K^{rad}_{self}, which is the back-reaction of the radiation. Second, there is the force originating from the particle's near field K^{∞}_{self}. Third, there may be external forces K^{ext} originating from other particles. Hence, the equation of motion reads

$$mc\ddot{z}_\mu = K^{\text{rad}}_{\text{self}\,\mu} + K^{\infty}_{\text{self}\,\mu} + K^{\text{ext}}_\mu. \tag{4.25}$$

The radiating field F^{rad} poses no problems since it's bounded at the particle's position, and its force on the particle is given by

$$K^{\text{rad}}_{\text{self}\,\mu} = \frac{2}{3}\frac{q^2}{4\pi}\left(\dddot{z}_\mu + \dot{z}_\mu \ddot{z}^2\right). \tag{4.26}$$

Dirac (1938, Appendix) derived this expression from the Liénard-Wiechert fields (4.9) and (4.10) by means of a Taylor expansion and further approximations (see also Barut, 1980, section 5.C).

What is problematic, though, is the near field F^{near}, as well as its associated self-force $K^{\infty}_{\text{self}\,\mu}$; neither are defined at the particle's position (Barut, 1980, p. 190). To circumvent this problem, we can use a *mass renormalization* procedure. We stipulate that the self-force has to be finite and has to mimic Newton's second law by introducing a new mass δm—we can justify this by making a clever Taylor expansion (Barut, 1980, Ch. V, section 5D). Then the self-force has the form:

$$K^{\infty}_{\text{self}\,\mu} = -\delta m c\ddot{z}_\mu. \tag{4.27}$$

All in all, the equation of motion (4.25) becomes

$$(m + \delta m)\, c\ddot{z}_\mu = \frac{2}{3}\frac{q^2}{4\pi}\left(\dddot{z}_\mu + \dot{z}_\mu \ddot{z}^2\right) + K^{\text{ext}}_\mu, \tag{4.28}$$

and Dirac concludes that the *Lorentz–Dirac equation*

$$m_{\text{exp}} c\ddot{z}_\mu = \frac{2}{3}\frac{q^2}{4\pi}\left(\dddot{z}_\mu + \dot{z}_\mu \ddot{z}^2\right) + K^{\text{ext}}_\mu \tag{4.29}$$

is the correct equation of motion for a single charged particle, where $m_{\text{exp}} = m + \delta m$ is the experimentally verified mass of the particle.

Runaway and Pre-Acceleration

Like the Abraham–Lorentz equation, the Lorentz–Dirac equation immediately yields runaway solutions. With the help of the asymptotic condition (4.21), we can transform this equation into an integro-differential equation whose solutions no longer reflect runaway behavior:

$$\ddot{z}_\mu(s) = \frac{\mathrm{e}^{s/\tau}}{\tau}\int_s^\infty \mathrm{e}^{-s'/\tau}\left(K_\mu(s') + \tau\ddot{z}^2(s)\dot{z}_\mu(s')\right)\mathrm{d}s'. \tag{4.30}$$

As we have seen with the Abraham–Lorentz equation, once we have eliminated the runaway solutions, the Lorentz–Dirac equation presents pre-acceleration solutions—the entre discussion of section 4.3 and 4.4 can be immediately applied.

4.5.2. A New Kind of Mass?

Now let's zoom into the transition from (4.28) to the Lorentz–Dirac equation (4.29), focusing on what happens to the masses. According to special relativity, every particle has an invariant rest mass m (Adler, 1987; Okun, 1989). The rest mass as the analogue of Newtonian mass in classical mechanics is a dynamical quantity that determines how particles move. And classical electrodynamics introduces charge as another dynamical quantity. Mass renormalization postulates the quantity δm as a third dynamical quantity, which has the dimension of a mass. What is the meaning of δm? And does it affect the meaning or numerical value of m and q?

Beware that equation (4.27) is indeed mathematically false because the left side is infinite, while the right side is finite. Nonetheless, it's needed in order to yield an equation of motion. According to this relation, the near field can be interpreted as contributing to the inertia of the particle. So apart from its rest mass m, a particle also bears an electromagnetic mass δm. And the experimentally detected mass of particles m_{exp} actually consists of a mechanical and an electromagnetic component such that $m + \delta m = m_{\text{exp}}$.

Disregarding the derivation of the Lorentz–Dirac equation and taking it rather as a fundamental equation of motion, there are two further options for interpreting the masses.

Zero Electromagnetic Mass

One is to set $\delta m = 0$, so that the experimentally verified mass coincides with the Newtonian one, that is, $m_{\text{exp}} = m$. Setting $\delta m = 0$ even for charged particles and claiming that the entire mass is m isn't convincing. The mass renormalization procedure rests on the hypothesis that δm is finite but non-zero. If the electromagnetic mass were zero then equation (4.27) would be even more of a cheat than it already is: the infinite self-force would not affect the particle in any way.

Zero Rest Mass

Another radical way of interpreting the electromagnetic mass is to regard it as the sole mass of a particle (Barut, 1980, pp. 199–200); that is to say, the entire mass is of electromagnetic origin, and $m_{\text{exp}} = \delta m$. This interpretation became popular in the beginning of the 20th century (see Muller, 2007, p. 268). At this this time, physicists believed that matter was composed solely of charged particles. The electron was discovered by J. J. Thomson in 1897, and the proton was discovered by Rutherford around 1911. So there were sensible experimental grounds for believing that mass was of electromagnetic origin.

But in 1932 the neutron was discovered by James Chadwick, and Dirac (1938, p. 148) admitted that the discovery of the neutron was a major threat for the electromagnetic interpretation of mass—the theory of quarks, which says that electrically charged quarks built the neutron, didn't exist when Dirac wrote his article in 1938. For the mass of neutrons cannot arise from their self-field because there is no self-field. And this would lead to a strange dichotomy too: electrically charged particles get their mass from their self-field, while neutral particles get their mass from elsewhere. And this "elsewhere" is totally obscure, as there is no physical mechanism that assigns neutral particles their masses.

But what is the correct option? Is there electromagnetic mass in addition to the mechanical rest mass? Or is all mass even electrodynamic? One might say that the Lorentz–Dirac equation is derived from Taylor approximations and other mathematical and physical idealizations; that it relies on mass renormalization; that it doesn't spell out what happens to the near fields; that it's an effective equation; that it's not a fundamental law of motion. All this skepticism stops the Lorentz–Dirac equation from making any ontological conclusions. Being an effective description, the Lorentz–Dirac equation doesn't add anything to our metaphysical picture. The electrodynamic mass δm is just a useful hypothesis in deriving the equation. It originates from the mass renormalization procedure in getting rid of the infinities by stipulation. And in the end we only need to calculate with m_{exp}. Whether this mass has different components isn't only irrelevant—there is no ontological division in the first place.

But we can take a different stance. What would the world be like if the Lorentz–Dirac equation were a fundamental equation of motion? First and foremost, we would have to draw our conclusions from equation (4.29) and the consequences thereof; how we got to the equation in the first place isn't relevant. The Lorentz–Dirac equation assigns to every particle a mass m_{exp} and a charge q, in case it's charged. Due to the first summand of the right side in (4.29), a moving charged particle radiates electromagnetic waves. If a particle is electrically neutral the Lorentz–Dirac equation can still be applied because it reduces to the relativistic version of Newton's second law if you grant that the external forces are of non-electromagnetic origin.

Imagine that we have an electrically neutral particle of mass m. But once we charge it the mass has to increase to match m_{exp}, owing to the dynamical effects of the near field. This is unproblematic because mass is neither stuff nor an intrinsic property that sets up the identity of the particle. If mass were stuff then it would be utterly mysterious how this stuff can increase by just changing charge. And if the particle's identity were grounded in its mass then it had better be constant. As I argued in Chapter 3, mass has to be interpreted as a dynamical property, that is, as a property whose sole role is to generate the motion of a particle. Therefore, a change in mass due to charging or uncharging is in accordance with this interpretation of mass: altering the charge alters the dynamics.

4.6. The Ontology of Fields

The self-interaction problem is the Achilles' heel of Maxwell–Lorentz electrodynamics. And there is no general remedy. Depending on how you diagnose the source of the problem, there are different ideas about how to cope with it. Some regard the size of the particles as the root of all evil. If the particles were bigger, with a non-zero extension, the field would not become infinite. The Abraham model and the Lorentz model are two prominent ideas for how to treat extended particles in the Maxwell–Lorentz theory. The Lorentz model is relativistic, allowing the particle to change its shape due to a Lorentz–Fitzgerald contraction, while the Abraham model is non-relativistic because the shape and the size of the particle don't change during acceleration.

The electromagnetic field itself was found guilty of self-interaction by Wheeler and Feynman. They therefore dismissed the fields altogether in their own theory and kept point-size particles. There cannot be a self-interaction problem in an action-at-a-distance theory. All interactions between the particles take place on the light-cones without any mediating entity.

In the Born–Infeld theory and the Bopp–Podolsky theory, on the other hand, the field is acquitted of any responsibility for the infinities. For them, the problem lies in how the fields are implemented by the Maxwell equations. So they proposed adjusting the field equations to tame the infinities.

There are other problems within the Maxwell–Lorentz theory in which the fields are involved. Still, field theories may have certain advantages over action-at-a-distance theories. Let's discuss the advantages and drawbacks of fields.

4.6.1. What is a Field?

What is the electromagnetic field in the first place? In the Maxwell–Lorentz theory the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are introduced by the Maxwell equations (4.1) and (4.2)—in the covariant formulation, there is just an electromagnetic tensor field F described by the equations (4.5) and (4.6). So physics defines fields either as vector-valued functions or tensors on space-time. But the mathematical description by itself isn't enough to elucidate the ontological status of the field. This is similar to the controversy over the status of the wave-function (see Chapter 6). The wave-function is mathematically defined as a complex-valued function on configuration space, but there has been ongoing debate about its ontological meaning since the inception of quantum mechanics.

There are three possible modes of existence for the electromagnetic field (Lange, 2002): property, stuff, or mathematical device. David Lewis (1986, pp. ix–x) regarded a field as a property or a continuous distribution of properties throughout space-time. But his whole metaphysics leads to quiddity and humility, which is unacceptable. If the electromagnetic field is construed as a distribution of dispositional properties on space-time, it is, however, problematic to see how space-time can have properties that don't affect its structure (see section 2.3).

If fields are interpreted as stuff, then they are ontologically on a par with particles. And this is what the Maxwell–Lorentz theory at first sight proposes. The Maxwell equations are the fundamental equations for the temporal development of the fields, while the Lorentz force law is the fundamental equation of motion for the particles. As there is radiation and the possibility of further free fields that aren't generated by particles, we can straightforwardly interpret the fields as independent stuff in addition to the particles. Then the primitive stuff ontology consists of particles and fields.

It's crucial for the primitive stuff ontology that the fundamental entities don't have intrinsic properties that distinguish one entity from the other. The particles are nothing but a *discrete* distribution of points throughout space, and an electromagnetic field is mathematically just a *continuous* distribution of points throughout space. But what distinguishes a point that refers to a particle from a point that refers to a field-value? Obviously, the two points move in a different manner: one behaves according to the Maxwell equations, while the other behaves according to the Lorentz force law. Nonetheless, being members of the primitive stuff ontology the points are intrinsically the same.

If the elements of the primitive stuff ontology have properties according to a dispositional account of laws of nature, the properties are only dynamical ones. Their role is solely to generate the temporal development; they don't contribute to the identity of the entities. Rather, this identity stems from spatiotemporal relations. And here lies the problem. If the points are only distinguished by the spatiotemporal relations, then given two points one cannot distinguish whether the two points are two particles or two points of the field, or whether they consist of one particle and one point of the field. Therefore, in a primitive stuff ontology, where the elements are distinguished by their spatiotemporal relations alone, it seems that there is only room for one kind of stuff: either there are particles, or there are fields. Furthermore, whenever there are two kinds of basic entities in the ontology, it has to be clear how one kind influences the other. A prominent case is the mind–body problem. If you adhere to a Cartesian dualism claiming that the mind is ontologically independent from the body or the brain, then you must explain in what way the mind is able to influence the body, and vice versa. Descartes thought that the mind met the brain in the pineal gland situated between the cerebral hemispheres. This simple explanation has turned out to be wrong, and the mind–body problem is still unsolved for a dualist. But if you're a monist, thinking that the mind is nothing but a certain configuration of the brain, then there is no longer a conceptual problem about how the mind can affect the brain.

In the Maxwell–Lorentz theory, the interaction between fields and particles yields the self-interaction problem. The interaction between the two kinds of substances always leads to the question of how the two substances influence one another. But is the infinite self-energy of a charged particle solely a mathematical–physical problem, or does it arise from an ontological dualism? The history of philosophy has shown us that every dualism bears an "interaction" problem when it comes to how one kind can affect the other. So we expect some problems in the interaction between fields and particles too (see also Feynman, 1966).

But it would exceed the power of philosophy to solve the self-interaction problem from dualism alone. That the existence of fields and particles must lead to ill-defined equations of motion is rather a mathematical–physical problem. Not every kind of dualism leads to insurmountable physical obstacles. Bohmian mechanics, for instance, introduces particles and the wave-function. Notwithstanding that the wave-function needs a philosophical explanation, the Bohmian theory is perfectly mathematically consistent. In this theory, there is no self-interaction problem since the particles don't affect the guiding field, which is generated by the wave-function.

The Born–Infeld theory and the Bopp–Podolsky theory are also counterexamples to the idea that the dualism between fields and point-size particles has to lead to a self-interaction problem. While the Born–Infeld theory gives us results for static sources, the Bopp–Podolsky theory shows us that even for dynamic systems particles and fields can peacefully coexist.

If fields were to be interpreted as further stuff in the ontology, a dispositionalist would have to specify the dynamical properties of them. In classical electromagnetism the particles have mass and charge. But what are the dispositions of the electromagnetic field? Does it have intrinsic properties, like the particles? The Maxwell–Lorentz theory doesn't propose anything that could be a disposition of the fields. That the fields are disposed to behave according to the Maxwell equations is a tautology. Similarly, particles are disposed to move according to the Lorentz force law, and the sun is disposed to shine.

Fields don't have a dispositional property similar to mass or charge, which might indicate that they play a different role to particles. The particles are there to form material objects. Fields, in contrast, are there to affect the motion of particles. In the end, the theory is about the behavior of matter that is nothing but the motion of particles. So while particles are ontological entities, the electromagnetic field is better seen as dynamical.

Interpreting particles as ontologically primary, because they compose material objects, and the electromagnetic field as secondary, because it only influences the behavior of the particles (see Maudlin's distinction between primary and secondary Ontology in section 2.2), the Maxwell–Lorentz theory should be introduced in a slightly different order. One needs to start with the Lorentz force law, which ultimately determines the motion of particles. And in its mathematical formulation it uses fields as dynamical entities. The standard exposition of the Maxwell–Lorentz equation is a little misleading because it introduces the field as the primary object. Fields may be of primary concern for mathematical and physical problems, but from an ontological point of view they are part of the secondary ontology, while the particles are part of the primary ontology.

So it's best for a dispositionalist to interpret the field as a mathematical device rather than as additional stuff in the ontology. Mass and charge are the dispositional properties. The electromagnetic field is a mathematical representation of the properties of the particles. And the manifestation of the dispositions is the motion of the particles.

In Hall's Super-Humeanism too the electromagnetic field can be straightforwardly interpreted as a mathematical device. Similar to the interpretation of the wave-function by Esfeld et al. (2014), the electromagnetic field can be part of the best system. A Humean can do so because she can easily put all dynamical entities, all entities of the secondary ontology, into the best system.

Maudlin (2013), however, argues that fields establish their own ontological category, thereby granting the metaphysical possibility for and consistency of the simultaneous existence of fields and particles in general. In his version of primitivism about laws, there is the field, in addition to the particle as part of the primary ontology of the Maxwell–Lorentz theory. And he argues that an electromagnetic field, like forces or the wave-function, could not be found or invented through purely philosophical reasoning. Consequently, the classic ontological categories set up by the Ancient Greeks aren't appropriate for modern physics. Consequently, no metaphysical problem arises from a dualism of particles and fields. The self-interaction problem is a purely physical issue.

4.6.2. Is the Electromagnetic Field Stuff?

Let's see whether the interpretation of the electric and magnetic fields as two separate entities ontologically on a par with the particles is tenable. The Maxwell equations as such don't give us any further information to clarify this issue; we therefore need to examine specific examples.

Imagine a wire, as in figure 4.4.[8] In the rest frame of the wire, there are negatively charged particles moving in the upper part of the wire on a straight line from right to left with velocity $\boldsymbol{v}$. In the lower part, there are exactly the same number of positively charged particles moving on a straight line from left to right with velocity $-\boldsymbol{v}$. The motion of the negatively charged particles cause a current $-\lambda$, while the positively charged particles generate the current $+\lambda$. In a distance s from the center of the wire, there is a particle with charge q moving from left to right with velocity $\boldsymbol{u}$.

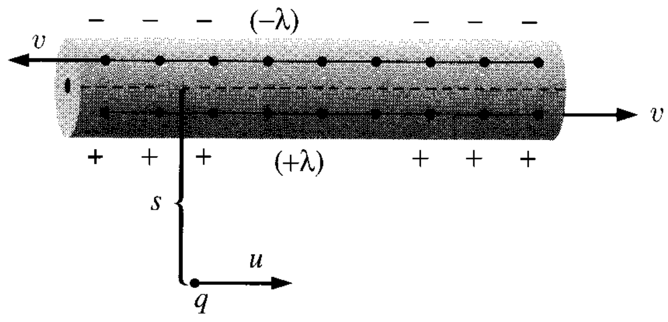

Figure 4.4.: Rest frame of wire

Assuming the radius of the wire is very small compared to the distance s, there is no electrical net force on the particle q. The change of motion of the particle is only due to its motion in the *magnetic* field generated by the currents in the wire.

Imagine now the very same situation but described from the rest frame of the particle as depicted in figure 4.5. Because of the Lorentz contraction, the distance

[8]This example and Figures 4.4 and 4.5 are taken from Griffiths (1999, pp. 522–5).

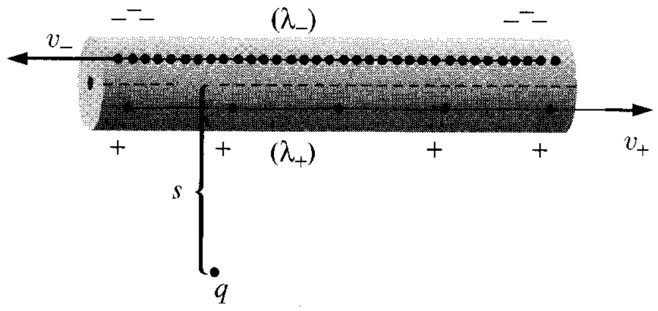

Figure 4.5.: Rest frame of charge q

between the negatively charged particles decreases, while the distance between the positively charged particles increases. Moreover, the velocities of both currents are no longer equal; the velocity of the negatively charged particles $\boldsymbol{v}_-$ is bigger than the velocity $\boldsymbol{v}_+$ of the other particles in the wire. Because the particle q is no longer in motion in its own rest frame, its change in motion cannot be accounted for by any magnetic field acting on it. Nevertheless, it has to experience an acceleration either towards the wire or in the opposite direction (depending on its charge) that is independent of any inertial reference frame. Consequently, an *electric* field has to be changing the state of the particle. A net electric field is indeed generated by both currents in the wire because the charges inside no longer equal out because of Lorentz contraction.

Acceleration is independent of inertial reference frames, and the acceleration of the particle in our example has to be explained in both of the frames we introduced. It turns out that the fields causing this acceleration changes with the reference frame. The electric and magnetic fields aren't invariant entities relative to inertial reference frames. Therefore, they cannot be stuff in the world. More precisely, the electric field cannot be stuff, for example, electric stuff, and the magnetic field cannot be magnetic stuff. This would imply the invariance of these entities under a change of inertial reference frames—it's not a logical consequence of the definition that stuff needs to be invariant under coordinate transformation, but my philosophical intuition tells me that this is a reasonable requirement.

Another example that shows that the electric and magnetic field aren't invariant stuff in space-time is the following. Imagine a metallic rod moving uniformly inside a homogeneous magnetic field crossing its field lines perpendicularly, as indicated in figure 4.6. Because of the motion of the rod, the free electrons inside experience a Lorentz force $\boldsymbol{F}_m$ directed downwards so that the rod comes to be negatively charged at the lower end b and positively charged at the upper end a. The Lorentz force includes only terms of the magnetic field here, since there is no electric field in this situation.

But in the rest frame of the rod the magnetic field is moving, and the free electrons begin to accelerate towards the lower end b, too. This time, however, the motion of the electrons isn't due to the magnetic field but due to the forces caused

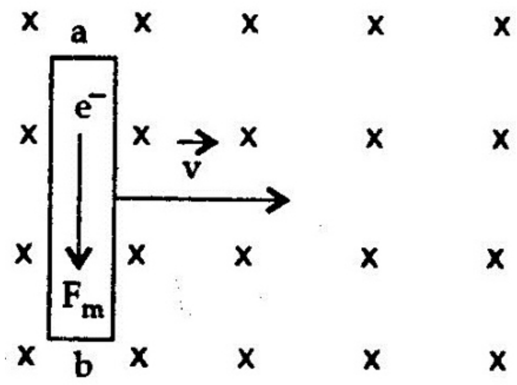

Figure 4.6.: Rod moving in magnetic field

by an *electric* field generated by the moving magnetic field by means of induction. A change of reference, of course, yields the same distribution of charges inside the rod. The behavior of the particles doesn't change, but the fields do.

4.7. Why Fields?

4.7.1. Locality

Action-at-a-distance is bad; locality is good. This has been a ubiquitous mantra since Newton stated his reservations about his own theory:

> That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else by which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers. (reprinted in Cohen, 1958, pp. 302–3)

Action-at-a-distance means that one physical object can affect another over space and time without another entity mediating the action. The action can be instantaneous, or it can be retarded. Retarded action-at-a-distance is generally regarded as the more troublesome version of action-at-a-distance because there is a time difference between cause and effect in the lack of an entity transmitting the effect through space. A field theory postulates a field as the mediating entity between cause and effect. Depending on the field theory the effect can be instantaneously transmitted, or it can take a certain amount of time.

Newton himself was concerned about his theory being an instantaneous action-at-a-distance theory. He couldn't do anything against the action being *instantaneous* because this is just what his laws of motion say. What he was concerned about, rather, was the lack of a mediating entity in his theory of gravitation. He personally thought about discrete particle-like transmitters, as well as of continuous immaterial entities. Nothing succeeded, so his ideas were only ever expressed in private letters (Stein, 1970).

Nowadays, we can postulate a gravitational field, which is the entity that transmits the action of one particle to another. In this way, Newton's gravitational theory becomes a field theory. Nevertheless, this theory of gravitation bears an element of non-locality: it's *dynamically* non-local because of the instantaneous action of the field. The motion of one particle influences the motion of all the other particles that are on the same simultaneity slice. And this instantaneous effect continues in both the field theory version and the action-at-a-distance version.

In another sense, Newton's gravitational field is local (see also Section 1.1). Though dynamically non-local, it's *ontologically* local.[9] If we were to chop up space into arbitrarily bounded chunks, the sum of the field values in every bounded region of space gives us the whole gravitational field (modulo the field values on overlapping regions). So in Bell's words, the gravitational field is a local beable. With respect to this terminology, the electromagnetic field of the Maxwell–Lorentz theory is as local as we could wish for: it's ontologically and dynamically local.

We notice that the pre-quantum theories may already be non-local, but this non-locality is always a dynamical non-locality. The great innovation of quantum mechanics was the wave-function as an ontologically non-local object. This non-local object leads to a new kind of dynamical non-locality unknown to classical physics. Tim Maudlin calls it *EPR-non-locality.*[10]

> A physical theory is *EPR-local* iff according to the theory procedures carried out in one region don't immediately disturb the physical state of systems in sufficiently distant regions in any significant way. (2014, p. 8)

EPR-locality is the condition that Einstein et al. (1935) used to argue that quantum mechanics is incomplete. This condition is natural to adopt because it had been valid throughout classical physics.. It seems only natural to adopt this condition since it had been valid throughout classical physics. The Maxwell–Lorentz theory is EPR-local because one system cannot "immediately disturb the physical state" of another system because of the finite speed of light. Newton's gravitational theory is EPR-local, too, because gravity decreases with the square of the distance. So it doesn't affect another system in any significant way if the two systems are sufficiently far away. For example, the gravitational effects of the moon don't disturb me writing on a computer in any significant way, while they are responsible for the tides. So in EPR-local theories two systems can always be isolated by increasing their spatial distance; the systems then evolve independently from each other. This is no longer possible for entangled quantum systems.

[9] I wish to thank Tim Maudlin for pointing out the distinction between ontological and dynamical locality in a private conversation. In fact, ontological locality coincides with separability.

[10] There are many more locality conditions mentioned in the literature. Some of them are discussed by Frisch (2005, Ch. 4).

Beware that EPR-locality is a feature of a physical theory; it isn't a feature of the physical objects postulated. Independently of the ontological status of the gravitational and electromagnetic field, Newton's theory and the Maxwell–Lorentz theory are both EPR-local. All (single-world) quantum theories, however, are EPR-non-local theories—they'd better be, lest they violate Bell's theorem.

4.7.2. Energy and Momentum

Newton's third law is no longer valid for electrodynamics. There are two mechanisms by which Newton's third law may fail. One is due to the finite speed of the electromagnetic interaction (Lange, 2002, Ch. 5). If there are two particles a certain distance apart whose forces on each other initially obey Newton's third law, then the forces no longer cancel out whenever one particle starts to move. The motion of the particle changes the electromagnetic field, but the effect can only travel with the speed of light. So as long as the force on the other particle doesn't change due to a change in the field, the forces aren't even collinear.

Newton's third law also fails because of the cross-product in the Lorentz force law (Griffiths, 1999, section 8.2.1). Imagine two positively charged particles as indicated in Figure 4.7 (which is taken from Griffiths' book). Particle q_1 moves on the x-axis with velocity $\boldsymbol{v}_1$, and particle q_2, on the y-axis with velocity $\boldsymbol{v}_2$.

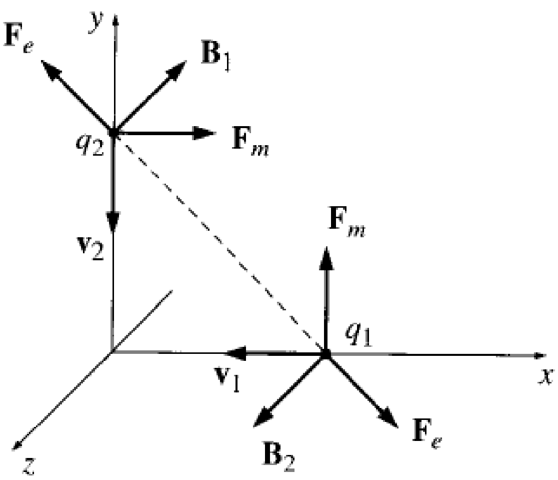

Figure 4.7.: Violation of Newton's third law

The magnetic field $\boldsymbol{B}_1$ on q_2, which is generated by q_1, is parallel to the z-axis pointing in its negative direction; similarly, the magnetic field $\boldsymbol{B}_2$ points to the positive direction of the z-axis. Although the electric force $\boldsymbol{F}_e$ fulfills Newton's third law, the magnetic forces $\boldsymbol{F}_m$ are rectilinear according to the right-hand rule. Disregarding the electromagnetic field, therefore, violates Newton's third law and also the conservation of momentum.[11].

[11]Breitenberger (1968), however, shows that the generalized forces in the Lagrange formalism

Fields, however, can restore the conservation of energy and momentum. The most general result, disregarding self-fields, is Poynting's theorem (Jackson, 1999, section 6.7):

$$\dot{E}_m + \dot{E}_f = \oint_S \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{A}, \tag{4.31}$$
$$\dot{P}_m + \dot{P}_f = \oint_S \boldsymbol{T} \cdot \mathrm{d}\boldsymbol{A}.$$

Here, the dot stands for the derivative with respect to time. E_m and P_m are the mechanical energy and the mechanical momentum of particles within a volume V. E_f and P_f are the field energy and the field momentum within V. The right side of the equations symbolizes the energy and momentum crossing the surface S—$\boldsymbol{S}$ is the energy flux density (the *Poynting vector*), and $\boldsymbol{T}$ is the momentum flux density (the *Maxwell stress tensor*). Poynting's theorem then states: energy and momentum change inside a volume V if and only if energy and momentum cross the surface S of V (see Fig. 4.8).

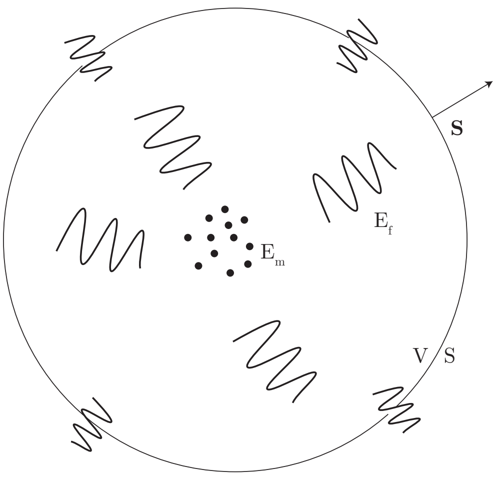

Figure 4.8.: The Poynting Theorem

4.7.3. Cauchy Data

Newton's equations of motion have a familiar mathematical structure: once we fix the positions and velocities of the particles at a certain time t_0 the laws of motion generate all future and past trajectories. These are the familiar initial value problems or *Cauchy problems*. For this setup, mathematicians and physicists have developed a rich arsenal of tools that can be applied to ordinary and partial differential equations.

fulfill Newton's third law and that the canonical momentum is conserved.

The Maxwell–Lorentz theory also allows us to formulate initial value problems, but only if we include field variables. The positions and velocities of particles, as well as the values of the electromagnetic field on a (sufficiently smooth) space-like hyperplane, are the *Cauchy data* that uniquely determine the future of the system. Without the field values, we would not have a Cauchy problem, and the familiar mathematical tools could not be applied. In this case, the entire past of the system had to be taken into account in order to figure out the future evolution. This is impossible for all practical purposes.

Initial Values are not Arbitrary

There is a crucial difference between Newton's gravitational field and the Maxwell–Lorentz field. Whether we interpret Newton's theory of gravitation as a field theory or as an action-at-a-distance theory, the Cauchy data always consists of the positions and velocities of the particles on a simultaneity slice. The gravitational field doesn't give any input that isn't included in the initial condition of particles. For the gravitational field is always attached to the particles and transmits the effects with infinite speed.

The electromagnetic field encodes more information than is comprised in the initial conditions of particles. But we need to be careful in setting the initial conditions. Deckert and Hartenstein (2016) prove that the initial value problem isn't well-posed in the Maxwell–Lorentz theory. The idea is that on a Cauchy surface the initial values for the particles and fields at time t_0 cannot be chosen arbitrarily because the history of the particles before t_0 determines the fields at t_0—Maudlin (2007a, p. 130) also mentions that boundary conditions cannot be freely chosen, but he doesn't make the calculations. Whenever the field values are chosen disregarding the history of the particles discontinuities or even singularities can arise.

Let's look at a simple example. Consider a particle at rest (see Fig. 4.9). If the field at t_0 is the electrostatic Coulomb field, the particle stays at rest and the initial value problem is well-posed. But if we change the field on the Cauchy surface irrespective of the behavior of the particles before t_0, the new field might cause a kink in the trajectory (see Fig. 4.10). The kink arises from a discontinuity in the velocity because the acceleration is infinite at t_0.

Deckert and Hartenstein conclude that in order to calculate the entire trajectory $q(t)$ one needs to have a certain piece of the trajectory as one's initial data. This then leads to delay differential equations one already encounters in the Wheeler–Feynman theory.

The Maxwell–Lorentz Theory as an Action-at-a-Distance Theory

Although the Maxwell–Lorentz theory is formulated as a field theory, it has been suggested that it would be better regarded as an action-at-a-distance theory. One example is Richard Feynman in his *Lectures on Physics* (Feynman et al., 1977, p.

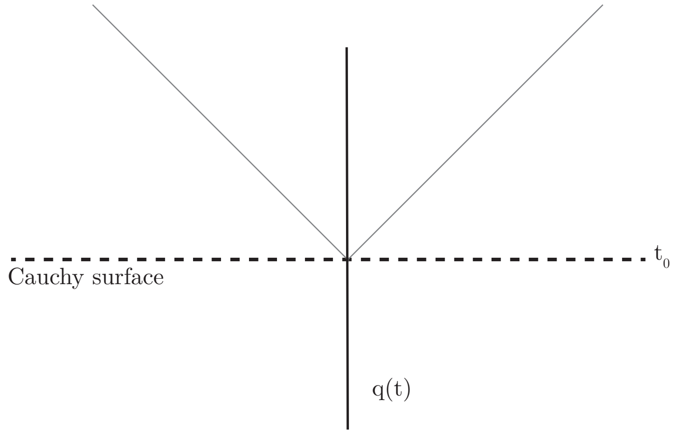

Figure 4.9.: Resting particle and Coulomb field as initial field values

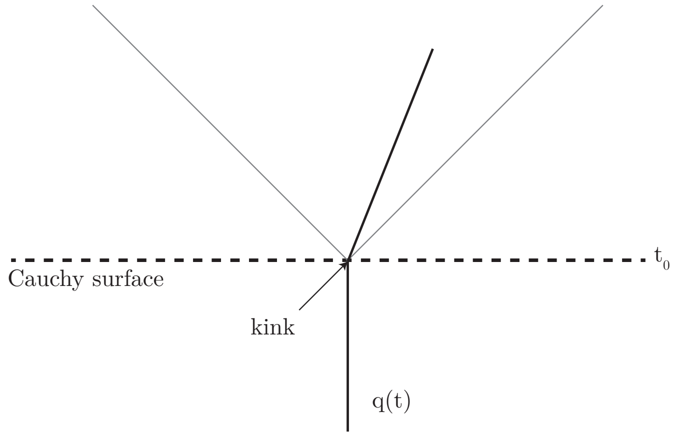

Figure 4.10.: A change in the field values on the Cauchy surface leads to a kink on the trajectory

21-1). He proposes taking the Liénard-Wiechert fields (4.9) and (4.10) and plugging them into the Lorentz force law (4.4). The result is the force generated by a charged particle on an arbitrary test particle while no longer explicitly mentioning fields. By superposing the forces of all the sources we can get the force on a particle generated by an arbitrary charged object (Griffiths, 1999, p. 439). A similar strategy using the retarded electromagnetic field tensor has been suggested by Mundy (1989).

Both strategies are a re-interpretation of Maxwell–Lorentz electrodynamics rather than new action-at-a-distance formulations thereof. Both still use the same standard formalism. But what is the advantage? These theories put us in a very strange situation. For such a theory itself lets us solve systems as Cauchy problems. But if it's interpreted as an action-at-a-distance theory we're strictly speaking not allowed to use the field values. The field has very useful practical advantages, but it's somehow not there. It would be insincere to call these interpretations genuine action-at-a-distance theories. And more importantly, neither solve the self-interaction problem, either. A self-interaction problem in an action-at-distance-theory—what a grotesque situation!

The Interpretation of Fields

There might be a dilemma: on the one hand, we can be anti-realist with respect to fields, but on the other hand we cannot interpret the Maxwell–Lorentz theory as an action-at-a-distance theory. Why? Because there is a difference between interpreting the ontological status of the field in a field theory and having an action-at-a-distance theory. In the former case, there are several ways to embed the theory in a metaphysical framework:you can be a realist or an anti-realist with respect to the field. In the latter case, you have to be an anti-realist. It isn't possible to interpret the field as part of the ontology if the theory itself is an action-at-a-distance theory.

The Wheeler–Feynman theory is an action-at-a-distance theory. Because it doesn't contain fields, there are unfortunately no Cauchy problems – actually, it isn't yet clear whether special systems exist that may allow for this. All the interactions take place on the light-cones. And to calculate the motion of a particle you have to know its entire past, as well as everything that happens on the forward and backward light-cones. The proper mathematical tools have yet to be developed for solving such problems. As a purely mathematical tool, however, the Wheeler–Feynman theory uses the electromagnetic field tensor. Although this tensor is part of the theory, we cannot interpret the theory as postulating fields. And there is a philosophical problem, too. Usually, metaphysical theories on the status of laws of nature, namely, Humeanism, primitivism, and dispositionalism, are interchangeable. Given an arbitrary fundamental physical theory it used to be possible to interpret it in a Humean framework, in a primitivist framework, and in a dispositionalist framework. But the Wheeler–Feynman theory seems to be at odds with a dispositional interpretation. Dispositionalism seems to presuppose objective simultaneity slices or the possibility of Cauchy hypersurfaces because the dispositions at one time or at one hypersurface determines motion for all future times. This is no longer appropriate in an action-at-a-distance theory that is formulated on Minkowski space-time. A detailed analysis of the relation between dispositionalism and relativistic theories is a task for future research.

There might be a dilemma: on the one hand, we can be anti-realist with respect to fields, but on the other hand we cannot interpret the Maxwell–Lorentz theory as an action-at-a-distance theory. Why? Because there is a difference between interpreting the ontological status of the field in a field theory and having an action-at-a-distance theory. In the former case, there are several ways to embed the theory in metaphysical framework; you can be a realist or an anti-realist with respect to the field. In the latter case, you have to be an anti-realist. It isn't possible to interpret the field as part of the ontology, while the theory itself is an action-at-a-distance theory.

In general, though, it seems that physics and philosophy agree that laws of nature must have the standard form of initial conditions plus time evolution. If it turns out that there is no initial value problem in the Wheeler–Feynman theory, then we need to revise our conception of laws of nature. It was Alfred North Whitehead (1925) who questioned whether physics should be formulated as successions of instantaneous states. Instead, he proposed an ontology of processes. It's beyond the scope of this thesis to examine Whitehead's ideas in detail, but they are definitely worth exploring in future research.

Part III.

The Quantum Universe of Particles

5. What Is the De Broglie–Bohm Theory?

But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing 'impossibility' proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as 'metaphysical' and 'ideological'? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?

— John Bell (2004c, p. 160)

5.1. Definite Trajectories

Quantum mechanics has serious physical and philosophical problems; they have been known since its inception in the 1920s. One is the notorious measurement problem (see Maudlin, 1995), brilliantly illustrated by Schrödinger (1935) in his famous cat paradox. The other is the lack of an ontology. Are quantum particles really particles, or are they waves? And if there is this duality, when is a particle a particle, and when is it a wave? In search of an ontology and in trying to clarifying what quantum mechanics tells us about the fundamental structure of our world, the big names, such as Bohr, Heisenberg, Schrödinger, Einstein, and Pauli, tried to elucidate the meaning of this theory. But discussions about *realism* or Bohr's *complementary principle* rather seem to be desperate attempts to make sense of a badly constructed physical theory.

Therefore, it's hard to understand the reaction of physicists (and some philosophers too) to the possibility of having quantum particles on definite trajectories. What if we could remold quantum mechanics to have point particles that move on plain continuous trajectories? Of course, the trajectories aren't the familiar Newtonian ones, but that's exactly the point: to introduce particles that move in such a way that they give rise to the same empirical predictions as quantum mechanics. Every physical and philosophical riddle about quantum mechanics can be solved through an analysis of these very trajectories. Nothing else has been done for a long time with classical mechanics.

You might guess that such a theory would have been welcomed by the scientific community. Nothing could be further from the truth. When de de Broglie (1928) presented his ideas about a theory of this kind at the 5th Solvay Conference in 1927, he faced harsh criticism and was discouraged (see Bacciagaluppi and Valentini, 2009). After this, de Broglie dropped his project.

5.1.1. The Second-Order Theory

It was 25 years later that David Bohm (1952a,b) dared to search for trajectories in quantum mechanics, unaware of the accomplishments of his French colleague. As in standard quantum mechanics, Bohm took the Schrödinger equation as the fundamental equation for the evolution of the wave-function:

$$i\hbar\frac{\partial\psi}{\partial t} = -\left(\frac{\hbar}{2m}\right)\nabla^2\psi + V(x)\psi. \tag{5.1}$$

Bohm's crucial move was to rewrite the wave-function in polar form:

$$\psi = R\exp\left(\frac{iS}{\hbar}\right). \tag{5.2}$$

In so doing, he derived a differential equation for the phase, which resembles a Hamilton–Jacobi equation:

$$\frac{\partial S}{\partial t} = -\left(\frac{(\nabla S)^2}{2m} + V(x) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right).$$

This equation differs from the classical Hamilton–Jacobi equation only in the last term, which is missing in the classical case. Bohm interpreted it as an additional potential, the notorious *quantum potential*:

$$U(x) := -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}.$$

Having developed the mathematical similarity with classical mechanics, the whole formalism thereof was now at Bohm's disposal. In particular, the particles have to follow a revised Newtonian equation

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\nabla\left(V(x) + U(x)\right), \tag{5.3}$$

in which the total potential is adjusted by the quantum potential U.

In the Hamilton–Jacobi theory, the trajectories aren't usually orthogonal to the wave-fronts. But if they are orthogonal at one time t_0 they are orthogonal forever. The orthogonality of trajectories was crucial for Bohm, because the trajectories should be consistent with the continuity equation and he wanted his theory to match the predictions of ordinary quantum mechanics. Therefore, Bohm (1952a, p. 170) postulated that the initial velocity of particles needed to fulfill

$$\boldsymbol{v}(\boldsymbol{x}) = \frac{\nabla S(\boldsymbol{x})}{m}. \tag{5.4}$$

Bohm believed that his second-order theory would be a more fundamental theory than ordinary quantum mechanics. On atomic scales the trajectories would be orthogonal to the wave–fronts, so his theory matches the standard quantum predictions, but on the subatomic scale (5.4) is violated, leading to new phenomena.

Like de Broglie, Bohm was attacked by his peers (see Myrvold, 2003), but his theory didn't fade into oblivion. On the contrary, Bohm's theory has been exciting philosophers and physicists alike for around two decades—although only a tiny minority of physicists and philosophers of physics really defend it (for instance, Belousek, 2000; Holland, 1993).

5.1.2. The First-Order Theory

John S. Bell has been the most prominent advocate of Bohm's quantum theory. In fact, he supported Bohm's theory in a slightly different version, a version that has been further developed by the mathematical physicists Detlef Dürr, Sheldon Goldstein, and Nino Zanghì (DGZ). This theory is (slightly inappropriately) called *Bohmian mechanics*. As in Bohm's second-order theory, here one basic equation is Schrödinger's equation. The law of motion for the particles, dubbed the *guiding equation*, is a different one. What Bohm used as a boundary condition for particle motion on the atomic level, namely equation (5.4), is here raised to the law of motion – a first-order differential equation. In a more prolific notation the guiding equation is

$$\frac{\mathrm{d}Q_k}{\mathrm{d}t} = \boldsymbol{v}_k^{\psi}(Q_1, \dots, Q_N), \tag{5.5}$$

where the N-tuple $(Q_1(t), \dots, Q_N(t))$ represents the trajectories of particles, $\psi(q_1, \dots, q_n)$ is the wave-function, and $\boldsymbol{v}_k^{\psi}$ is the k-th component of the vector field $\boldsymbol{v}^{\Psi} = \frac{\nabla S(\boldsymbol{x})}{m}$ generated by the wave-function.

Every physical prediction of this theory can be derived from to these two fundamental equations. If we know the positions of the particles at one time t, the *guiding equation* (5.5) gives us their velocities at t and hence the entire trajectories. It's quite remarkable that we only need the positions as initial conditions. Contrary to Newton's second law, which is a second-order equation and hence requires positions and momenta as initial conditions, the law of motion of Bohmian mechanics is first-order. Its mathematical structure is the simplest possible for a differential equation: ask for the trajectories by specifying a vector field $\boldsymbol{v}^{\Psi}$. Newton's law is in this sense more complicated because in order to get the trajectories we have to take a detour over the accelerations by specifying the forces.

What de Broglie Really Did

Louis de Broglies work is generally underestimated. But he is indeed the genuine inventor of Bohmian mechanics. Bacciagaluppi and Valentini (2009, Ch. 2) discovered that de Broglie had at the 5th Solvay conference presented at a theory much more developed than generally recognized. There he showed the audience the Schrödinger equation (5.1) and the first-order guiding equation (5.5) for a many-particle system.

While he borrowed Schrödinger's equation from its inventor, he reached the guiding equation all by himself *some years before* in his PhD thesis. De Broglie was motivated by physical problems: he wanted to explain discrete atomic energy levels and the diffraction of photons. He reasoned that one needs to find a new dynamics that violates Newton's first law because photons are diffracted by an object without touching the object itself. According to Newton's first law, the light beam must follow a straight line. In the quest for his new law of motion, de Broglie was inspired by Newton's idea of a particle theory of light, and combined ideas from wave optics and geometrical optics. He eventually used two action principles: Fermat's principle (an action principle for light-waves) and Maupertuis' principle (an action principle for particles), to derive the guiding equation.

Nowadays, the de Broglie–Bohm theory is justified because it solves the measurement problem and provides an ontology for quantum mechanics. But this wasn't de Broglie's concern! He wanted to explain empirical phenomena. That his theory has these other (crucial) features was only later recognized and appreciated. As a matter of historical fact, de Broglie developed his theory years before quantum mechanics was born.

The Guiding Equation from Schrödinger's Equation

There are many ways to derive the first-order guiding equation:

1. De Broglie combined Fermat's principle with Maupertuis' principle.
2. Dürr et al. (1992, section 3) suggested deriving the guiding equation from symmetry principles.
3. One can integrate Bohm's second-order equation.

But there is a simple derivation that only uses Schrödinger's equation. Indeed, the de Broglie–Bohm theory is already contained in standard quantum mechanics (see also Sakurai and Napolitano, 1994, pp. 101–2).

From Schrödinger's equation one can derive the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = 0,$$

where $\rho = |\psi|^2$ and $\boldsymbol{j} = (\frac{\hbar}{m})\mathrm{Im}\,(\psi^* \nabla \psi)$ is the quantum flux. From the polar form of the wave-function (5.2), we immediately get that

$$\boldsymbol{j} = \frac{\rho \nabla S}{m}.$$

Since the standard continuity equation from fluid dynamics reads

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

we get the velocity for particles following the quantum flux as

$$\boldsymbol{v} = \frac{\nabla S}{m},$$

which is the guiding equation.

5.1.3. Two Kinds of Wave-Functions

The Universal Wave-Function

Up to now, I have talked about *the* wave-function. But Bohmian mechanics distinguishes two kinds of wave-function: the universal wave-function Ψ and the effective wave-function ψ. This distinction stems from two different modes of application. On the one hand, we can apply this theory to certain systems in a laboratory, like the hydrogen atom or silver atoms emitted by an oven. In these cases we always use the effective wave-function. On the other hand, we can apply Bohmian mechanics to the biggest system at our disposal, namely, the universe. The wave-function assigned to the entire universe is the universal wave-function, which generates the universal vector field $\boldsymbol{v}^{\Psi}$.

The problem is that we don't know the universal wave-function. And no one knows whether the universal wave-function would obey Schrödinger's equation in the first place. We can only guess what the universal wave-function might look like. For example, it may be time-dependent, or it may be time-independent. It's often claimed that the wave-function must change in time because a dynamic universe cannot have evolved from a static wave-function. But for change the universal wave-function doesn't have to change; it's sufficient for the particles to keep moving. The Bohmian laws allow the existence of non-trivial particle trajectories that are guided by a static universal wave-function. We'll encounter more arguments in section 6.1 for a time-independent universal wave-function.

The Effective Wave-Function

There is both good news and bad news. The bad news is that if Bohmian mechanics only consisted of the universal wave-function and the guiding equation it wouldn't be possible to make calculations. First, no one knows the universal wave-function.

Second, even if we knew the universal wave-function the computational power that we would need to get trajectories would exceed all current supercomputers taken together.

The good new is, though, that from the universal wave-function we can derive another kind of wave-function that is essential for empirical predictions. Let's say we would like to predict the motion of an electron. Its position in three-dimensional space is $\boldsymbol{Q}_1(t)$; the positions of all the other particles are $Y(t) := (\boldsymbol{Q}_2, \ldots, \boldsymbol{Q}_n)$. We can now define from the universal wave-function Ψ another wave-function:

$$\psi_t(x) := \Psi(x, Y(t)). \tag{5.6}$$

This is the *conditional wave-function* of the electron; it depends on time because of the motion of the environment $Y(t)$.

Although we can now assign a wave-function to an electron—and more generally, to any physical subsystem via (5.6)—, we cannot use this wave-function because we not only need to know the universal wave-function, we also need the position of all the particles in the environment. In other words, if we want to predict the motion of an electron we need to find the universal wave-function and the motion of 10^{80} particles. Impossible.

Still, we can define another wave-function. For what we would like to describe is the behavior of subsystems that are in some (yet to be specified) way isolated from the environment. The definition of the conditional wave-function is too broad; it allows us to assign wave-functions to systems that aren't sufficiently isolated.

What does isolated mean here? The most straightforward isolated subsystem ψ is found in a product state:

$$\Psi = \psi(x)\Phi(y),$$

where Ψ is as always the universal wave-function, and Φ is the wave-function of the environment. The product state is a very special quantum state since the behavior of the y-system doesn't influence the behavior of the x-system. Hence the guiding equation gets decoupled into two equations, where the velocity of the x-system only depends on the positions of x and the velocity of the y-system only depends on the positions of y. But a subsystem that forms a product state with the environment is too strict in application and highly unlikely, for there will be interactions between both systems that result in an entangled state. Consequently, the universal wave-function is probably in a superposition.

It turns out that we can get a reasonable wave-function for a subsystem if the universal wave-function is in the superposition of a product state and some remaining wave-function $\Psi^\perp$ (Dürr et al., 2013, Sec. 2.5):

$$\Psi(x, y) = \psi(x)\Phi(y) + \Psi^\perp(x, y). \tag{5.7}$$

In order to assign the wave-function $\psi(x)$ to the x-system two conditions must be met. First, the positions of the environment need to be in the support of Φ, that is, $Y(t) \in \operatorname{supp}\Phi$. Second, $\Phi(y)$ and $\Psi^\perp(x, y)$ need to have a *macroscopically disjoint*

y-support. That is to say, their y-supports are so far apart in configuration space that there is a macroscopic difference whether $Y(t) \in \operatorname{supp} \Phi$ or $Y(t) \in \operatorname{supp} \Psi^{\perp}$.

As soon as all these requirements are satisfied $\psi(x)$ is called the *effective wave-function* of the x-system. We can define for every subsystem a conditional wave-function, but only under the above conditions is there an effective one. If the effective wave-function exists it coincides with the conditional wave-function.

The definition of the effective wave-function is more complicated than the definition of the conditional wave-function. And both definitions presuppose the universal wave-function, which is impossible to know. So why have we introduced the effective wave-function in the first place?

There is a shortcut to reach the effective wave-function, a shortcut where we don't need to know the universal wave-function. It goes by the name of *Schrödinger's equation* (5.1). Given the quantum Hamiltonian of the subsystem—physicists know very well how to built Hamiltonians—Schrödinger's equation gives us the effective wave-function ψ. But why does the effective wave-function follow Schrödinger's equation in the first place? The universal wave-function by definition fulfills Schrödinger's equation—even if it were time-independent it could fulfill Schrödinger's equation. If the effective wave-function exists, we can for all practical purposes neglect $\Psi^{\perp}$ in (5.7), and so ψ (as well as Φ) fulfills its own Schrödinger evolution once the interaction potential between system and environment becomes negligible (Dürr and Teufel, 2009, pp. 216–7).

Textbook quantum mechanics doesn't make this Bohmian distinction between universal and effective wave-function. Indeed a wave-function for the entire universe would collapse all the time, and so it doesn't play a distinctive role as in the de Broglie–Bohm theory. Nevertheless, in experiments physicists manipulate the wave-functions of more or less closed systems. And in textbooks the systems introduced, like the hydrogen atom or the potential well, don't interact with the environment. So the effective wave-function makes precise what physicists are already familiar with and thereby embeds quantum mechanics into a quantum theory of particles.

5.1.4. First-Order or Second-Order: Which is Better Physics?

Initial Conditions and Quantum Equilibrium

The first and second-order theories aren't equivalent. They differ in their physical predictions, in the way they explain phenomena, and in their ontology. Bohm's equation of motion for particles is (5.3). In order for it to yield trajectories, we need to specify both the positions and velocities. While the guiding equation (5.5) is first-order, we only need to specify positions.

In order to match the predictions of ordinary quantum mechanics, which fit extremely well with experiments, Bohm had to demand that the velocities of particles equal the gradient of the phase of the wave-function (see equation 5.4). This boundary condition can only be justified by empirical adequacy. In the first-order version this condition is nothing but the guiding equation (5.5). In this case, it has the status of a law of nature.

Bohm has a different law of motion, and he needs to accept (5.4) as a very special boundary condition to make his theory empirically successful. He clarified that equation (5.4) is the proper boundary condition on atomic scales because it's in accordance with quantum mechanics and experiments. On shorter length scales, however, "an arbitrary relation between [$\boldsymbol{v}$] and ∇S" is possible (Bohm, 1952a, p. 171). It seems that Bohm took these alternative trajectories very seriously, since this would amount to new empirical predictions (see also Valentini, 1991a,b). I'm not aware of any empirical results that require a violation of the boundary condition (5.4), and Dürr et al. (1992) confirm that there is no need to analyze *quantum non-equilibrium*, since our universe is already in quantum equilibrium.

Difference in Ontology and Explanatory Value?

There are many arguments regarding which theory to prefer. Some rely on the ontological or explanatory differences instead of the mathematical or physical difference that I've just mentioned.. Belousek (2003) argues that the second-order theory is explanatorily more powerful than the first-order theory. For him the first-order theory only represents particle trajectories in a mathematical–formal way:

> But [. . .] this 'guidance' is merely a mathematical-formal definition of the particle velocity in terms of the phase of the quantum state ψ and can in no sense be understood in physical terms unless ψ itself is endowed with some direct physical significance; that is, insofar as the quantum state ψ is merely a mathematical abstraction existing in a (fictitious) configuration space, as DGZ (and de Broglie) insist that it is, one can in no sense say that it guides particle motions in a physically significant way. On the DGZ view, then, the guidance equation allows for only the prediction of particle trajectories. And while correct numerical prediction via mathematical deduction is constitutive of a good physical explanation, it is not by itself exhaustive thereof, for equations are themselves 'causes' (in some sense) of only their mathematical-logical consequences and not of the phenomena they predict. (Belousek, 2003, p. 136)

For Belousek, the second-order theory is superior to the first-order theory because it introduces forces and potentials that explain the motion of particles. The first-order theory lacks these concepts; it describes motion by directly specifying velocities. This specification of velocities is said to be of mathematical-formal character. Belousek is correct in emphasizing that particles in physical space cannot really be guided by the wave-function, which is itself defined on configuration space – only Albert's marvelous point is truly guided by the wave-function (Albert, 2015, Ch. 6, for more on the marvelous point). But this is not what DGZ have in mind when they talk about guidance. Guidance is just a metaphorical picture for guiding our intuition. Moreover, it's not a special feature of the first-order theory that the wave-function exists "in a (fictitious) configuration space." This is just a mathematical truism in all quantum theories and is the source of quantum non-locality.

The first sentence of Belousek's paragraph can be equally formed in terms of the second-order theory:

> But the second-order equation is merely a mathematical-formal definition of the particle acceleration in terms of the amplitude of the quantum state ψ, as well as the classical potential, and can in no sense be understood in physical terms unless ψ itself is endowed with some direct physical significance.

Unless the wave-function is assigned to single particles it cannot be interpreted as a field in physical space, and therefore the quantum potential isn't a potential in physical space, either. So due to the quantum potential the velocity of one particle depends on the position of all particles. This is also true for the classical potential, but the classical potential, in contrast to the quantum potential, is determined by direct interaction between two particles in *physical* space. The relation between particles in the quantum potential takes place on the level of the wave-function, and that happens in configuration space.

Belousek falls for the illusion that the quantum potential can "physically" explain what is going on in 3-D space, because all the examples that he gives—the double-slit, tunneling, and the stability of atoms—are systems that only need single-particle wave-functions. Instead of explaining the trajectories in the double-slit experiment as due to changes in the quantum potential, one could equally explain them by means of changes in the values of the wave-function in physical space.

I don't think that the quantum potential provides a physical explanation of trajectories which the wave-function itself cannot render. But forces seem to be different. First, forces are familiar from classical mechanics. Once forces are specified acceleration is specified. And having forces on the quantum level lets us recover more of the classical ontology in the classical limit. In the classical limit of the first-order theory forces don't emerge. The task is "just" to recover classical trajectories within an epsilon. The ontological continuity from the quantum to the classical level may be an advantage of forces.

Second, forces are ontologically local objects since they are defined on three-dimensional space. So the direct cause for the motion of the particles can be given in local terms. As in classical mechanics, it's possible that ontologically local objects could generate dynamically non-local effects; non-local correlations, like in the EPR-experiment, remain of course in the second-order theory. But this gain in local explanations shouldn't fool one into thinking that quantum non-locality can be entirely reduced to local interactions. In the background, there is still the wave-function sitting on configuration space. Although forces mediate non-local correlations on $3D$ space, they are themselves generated by a non-local beable. And there is still a gap in physical space from one force to the other, a gap in the sense of action-at-a-distance—in classical electrodynamics, for example, this gap is filled by the field.

To my mind, Belousek is both too strict with and mistaken about the first-order theory. The mistake lies in the last sentence of the quoted paragraph: "equations are themselves 'causes' (in some sense) of only their mathematical–logical consequences and not of the phenomena they predict." This statement isn't valid for primitivism about laws—and DGZ regard laws as primitive entities. It's perfectly legitimate and consistent to interpret certain equations as being part of the fundamental ontology.

Belousek is too strict when he says that the guiding equation is only a mathematical–formal description of trajectories of particles. Similarly, Newton's equation is also a mathematical–formal description of trajectories, and the Maxwell equations are a mathematical–formal description of the behavior of electromagnetic fields. Modern physics is founded on differential equations, and to interpret them as merely mathematical–formal descriptions designates our entire physics as at least explanatorily insufficient. Physics is the science of matter in motion, and it's also the science of how to find clever differential equations.

And the Winner Is...

The first-order and second-order theories differ in terms of their mathematical–physical structure. First, they require different initial conditions. Second, the first-order theory excludes the possibility that our universe might have evolved from quantum non-equilibrium or that there is a violation of the guiding equation on the subatomic level. To my mind, this is a strength of the first-order theory. We cannot experimentally select initial positions or velocities; all we can do is manipulate a system to have a certain wave-function. The distribution of positions and velocities is always follows Born's rule. Moreover, there has up to now been no empirical confirmation of new (non-relativistic) quantum phenomena on the subatomic level. So the first-order theory is as empirically successful in our world as the second-order theory without introducing further parameters. As a matter of parsimony, this point goes to the first-order theory.

What about Belousek's argument? The second-order theory is said to explain the behavior of particles causally and more informatively because it introduces the quantum potential and forces. Since the first–oder theory lacks these tools, it merely gives a mathematical–formal explanation of trajectories. While in the first-order theory the wave-function directly determines the velocities, there is a causal chain from the wave-function to the velocities in the second-order theory: wave-function $\rightarrow$ quantum potential $\rightarrow$ forces $\rightarrow$ acceleration $\rightarrow$ velocity. According to Belousek, the second-order equation isn't a mathematical–formal description of trajectories because it postulates the quantum potential and forces as objects in the world that do change in motion.

Belousek's argument, however, doesn't hold up to scrutiny. The quantum potential is a non-local beable as it is the wave-function. So it doesn't literally guide particles in physical space as the electromagnetic field might guide charged particles. Examples of one-particle systems that Belousek uses are misleading in acknowledging the quantum potential as living on physical space.

Forces, on the other hand, even forces generated by the quantum potential, act on each single particle. They support this anachronistic picture of a push–pull physics. But must a law of motion be formulated in terms of forces? Even classical mechanics can be reformulated without forces, and the dynamics would be first-order on phase space (Hamiltonian dynamics) or on the tangent bundle of configuration space (Lagrangian dynamics).

The second-order theory might be more comprehensible in running the classical limit, and the transition might be smoother. If the quantum potential goes to zero, the trajectories are the familiar classical ones changed by the classical forces. But if we don't stick to these requirements—forces as necessary for dynamical explanations and the classical limit—the first-order theory is again the preferred version.

5.2. Indistinguishability, Identity, and Individuality

5.2.1. The Mystery of Indistinguishable Particles

Quantum mechanics is said to be a revolutionary theory. It has shown us that particles may not have positions, that observers can collapse the wave-function, that consciousness might be even more powerful than we think, that we may live in Hilbert space, that there may be infinitely many copies of us in different worlds, that physics is about our state of belief, and that quantum particles aren't individuals. It is this last point I will focus on now. In particular, it will become apparent that quantum mechanics is "revolutionary" on the individuality of particles because it's an incomplete theory. In a complete theory, like the de Broglie–Bohm theory, there would be no mystery about individuality, indistinguishability, or identity.

Let's first see what this mystery is supposed to be and where it arises. Imagine a box that is equally partitioned by a wall. The left part is filled with a gas A, argon for example, with entropy S_A, and the right part is filled with a different gas B, oxygen for example, with entropy S_B. Together the gases have entropy $S = S_A + S_B$. Once the partition is lifted, the gases will adiabatically mix, that is, without an exchange of heat with the environment, and entropy S will increase. The total increase is the sum of the entropy increase of gases A and B: $\Delta S = \Delta S_A + \Delta S_B$. But if gases A and B are identical, both gases are argon for example, the entropy S of the entire system remains constant, that is, $\Delta S = 0$. This is Gibbs' paradox (see, for instance, Schroeder, 2000, pp. 79–81).

In this case we expected, when the two identical gases were mixed, the entropy to increase as well. The mistake was to assume that the calculation of the entropy woudn't change whether we mixed two different or two identical gases. But for the mixing of argon with oxygen there are many more microstates leading to the same macrostate, because the permutation of an argon molecule with an oxygen molecule leads to a new microstate. Permuations among argon molecules don't change the microstate of argon. Therefore, if we calculate the entropy of the mixture of gases A and B, we need to divide the microstate by the permutations that don't change in case A and B are identical. The paradox is resolved because it rests on wrong expectations.

The discussion of Gibbs paradox should end here. Everything that is physically explainable is explained. Nonetheless, physicists have questioned the validity of indistinguishable particles and found justification in quantum mechanics.

> Physicists were skeptical about Gibbs' proposal to ignore all states in calculating [the entropy] that arises from a permutation of particles. Years later, he found justification in quantum theory. Identical particles, which share all their properties (mass, charge, angular momentum, magnetic moment, etc.), are indistinguishable. If they were distinguishable (by a number, a color, a position, etc.), they would no longer be identical. Atoms cannot be numbered, colored, or fixed at certain positions without changing their properties or structure. (Stierstadt, 2010, p. 70; my translation)

Indistinguishable particles are not a novelty of quantum mechanics! Identical particles are not a novelty of quantum mechanics! The Gibbs' paradox is a classical problem. Distinguishability is about our capability to discriminate particles. Identity, on the other hand, is an ontological notion about the properties of particles. We may distinguish particles even if they are identical. Two perfect black billiard balls are identical but they are distinguishable by their positions. Likewise, electrons are identical but distinguishable by their position. As I argued in section 1.3, identical particles can always be labeled because we can label their location.

If particles don't have position, if a physical theory is incomplete, we get all this fuss about the difference between identity, indistinguishability, and individuality. The argument that quantum "particles" cannot be distinguished goes as follows (Ladyman and Ross, 2007, section 3.1). Take a helium atom in the ground state; the two electrons share the same energy eigenstate and position state, and their spin state is the singlet state:

$$|0,0\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \Big).$$

The two electrons coincide in all their intrinsic properties: mass, charge, energy state, position state. And they coincide in all their relational properties: if one electron is spin-up the other is spin-down and if one electron is spin-down the other is spin-up. Therefore, Ladyman and Ross (and others) conclude that electrons are identical and indistinguishable. But can they be individuated? Are there two electrons, or is there just one? Quantum mechanics doesn't tell us. But somehow there are two electrons in the helium atom. What we need to do is to go beyond the physics of quantum mechanics. Here are two suggestions for doing so.

One can equip each electron with a primitive identity. Although electrons in the ground-state of a helium atom share all their physically intrinsic and extrinsic (relational) properties, there are two separate objects here, due to a primitive identity.

It's strange that a physical theory is incapable of telling us whether there are one or two objects without introducing a metaphysical ad-hoc haecceity.

It would be much easier and less dubious if electrons had a precise position. Then they could be distinguished by their location even if they could not be distinguished by merely looking at their intrinsic and extrinsic quantum properties. When the de Broglie–Bohm theory describes two electrons in a helium atom the electrons are always here and there, and the discussion of identity and distinguishability becomes trivial.

Now we can better understand Gibbs' paradox. Classical particles have precise positions, and so they are distinguishable. But differences in entropy are due to something deeper, something objective about particles: whether particles are *identical* or not changes the entropy of the gas.

There is still a small paradox left to be resolved: if classical particles are primitive stuff, as I've been arguing, shouldn't entropy be constant for all mixtures of gases? At first sight there is nothing, apart from position, that distinguishes particles. As a permutation of primitive stuff particles doesn't change the distribution of intrinsic properties, so the microstate shouldn't change either. This would be bad because we couldn't account for the physical truism that the mixture of oxygen with argon increases entropy.

How can we resolve this problem? There are indeed two kinds of permutation. We can permute particles, and we can permute trajectories. If we were realists with respect to properties both permutations would be the same. But since we disentangle the particles from their dynamics, we have more options. A permutation of particles switches the positions of two actual particles, and since particles have no intrinsic properties this kind of permutation doesn't change the physical state of affairs. Not so a permutation of trajectories. Here we don't touch the particles; they remain in their position. What we do, though, is permute the physical quantities in the law of motion as if we would have switched the properties of particles if they really had some (see Fig. 5.1). These permutations change trajectories. So what we need to consider in Gibbs' paradox isn't the permutation of particles, but permutations of trajectories, because these can change the physical state.

5.2.2. Bohmian Mechanics of Indistinguishable Particles

In Bohmian mechanics, identical particles are no more mysterious than non-identical particles. The theory tells us how to deal with identical particles, that is, a system of electrons, protons, or neutrons, etc. From the Bohmian formalism of identical particles, we can say that there are two kinds of particles, fermions and bosons, whose wave-function is constrained by certain symmetries. Fermion wave-functions are antisymmetric with respect to particle permutations, whereas boson wave-functions are symmetric. To get these results it is crucial that we define the configurations of identical particles on ${}^N\mathbb{R}^3$ (see section 1.3).

We can take the idea of identical particles a step further and claim that not only are particles of one species identical with each other, but particles from different

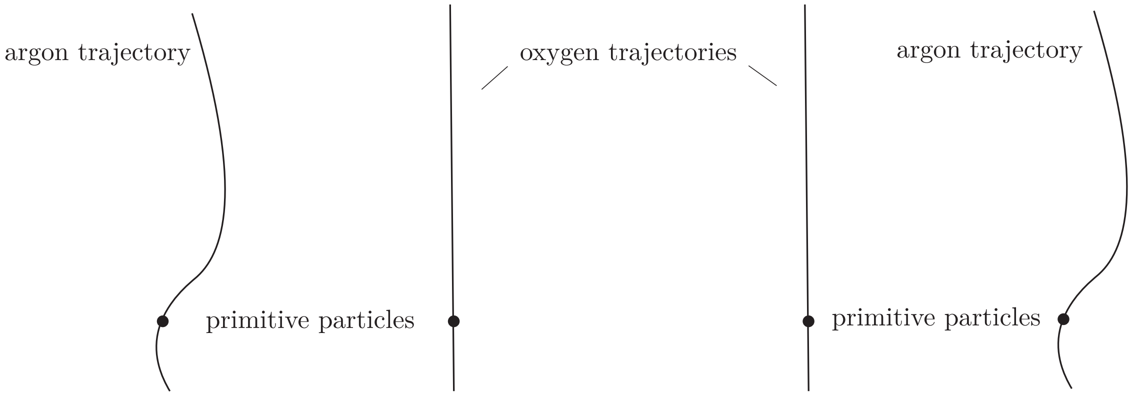

Figure 5.1.: Permutation of trajectories of an argon and an oxygen molecule without permutation of the molecules themselves. The permutation takes place on the level of physical quantities in the law of motion. The permutation may even change the shape of the trajectories (not depicted in the picture)

species are identical as well. Intrinsically nothing distinguishes an electron from a proton, for example. If we could freeze the world at one moment and zoom in on matter down to the level of particles, we would only see a distribution of identical dots, and nothing that we could do with the particles would allow us to identify which dot belonged to which species. But if we turn on time, we see that some dots move like electrons, and some dots move like protons, etc. It is motion that distinguishes particles, not intrinsic properties. This idea must also be captured in ${}^N\mathbb{R}^3$, but in contrast to the single species case we also need to change the law of motion in order to make it permutation-invariant for *non*-identical particles.

How to Deal with One Species of Particle?

Let's say we want to describe a system of electrons. How would we do this? First, these configurations need to be formalized in ${}^N\mathbb{R}^3$. Second, we need to define a wave-function on this new configuration space. Intuitively, we would think that we can only define symmetric wave-functions on ${}^N\mathbb{R}^3$, but this would mean that indistinguishable particles are always bosons. So we need to dig deeper to find the fermions and to exclude that wave-functions of indistinguishable particles can have further symmetry properties apart from being symmetric or anti-symmetric.

What we need to notice is that ${}^N\mathbb{R}^3$ is isomorphic to another space, namely the space in which we subtract the diagonal line from the standard configuration space $\mathbb{R}^{3N}$ and quotient by the permutation group S_N (see Dürr et al., 2006, for the topological features):

$$^N\mathbb{R}^3 \simeq \mathbb{R}^{3N}_{\neq}/S_N$$

We see that $\mathbb{R}^{3N}_{\neq}$ is a (periodic) covering space of $\mathbb{R}^{3N}_{\neq}/S_N$, similar to the Riemann surface of $\ln z$ as a (periodic) covering space of the complex plane. The goal is to define a Bohmian velocity field on the covering space that is projected on the configuration space of identical particles. We have to require permutations of particles to be guided by the same velocity vector. This periodicity condition for the vector field translates to a constraint on the wave-function. Together with the requirement that particles are distributed according to Born's rule, the wave-function (on the covering space) can be either symmetric or anti-symmetric (see all the missing details in Dürr and Teufel, 2009, section 8.5). So the fermion–boson distinction arises in Bohmian mechanics from a topological analysis of the configuration space ${}^N\mathbb{R}^3$.

There Are No Particle Species: Bohmian Mechanics of Primitive Stuff

In order to treat *all* particles as identical we need to go beyond the usual treatment of identical particles by making the guiding equation permutation-invariant for arbitrary particle species[1]. This was proposed by Goldstein et al. (2005a,b). To preserve equivariance, that is, the conservation of total probability by the Bohmian flow, the symmetrization has to be performed in the following way. First, Goldstein et al. re-wrote the standard guiding equation (5.5) in the form

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{j\left(Q(t)\right)}{\rho\left(Q(t)\right)}, \tag{5.8}$$

where

$$\rho = \psi^* \psi$$

is the probability density and $j = (\boldsymbol{j}_1, \ldots, \boldsymbol{j}_N)$ with

$$\boldsymbol{j}_i = \frac{\hbar}{m_i} \mathrm{Im} \psi^* \nabla_i \psi$$

the probability current corresponding to the system's wave-function ψ.

Then they independently symmetrized both the numerator and the denominator in equation (5.8) by summing over all possible permutations of particles. Hence, we get a new, permutation-invariant guiding equation, which reads[2]

$$\frac{\mathrm{d}Q_k}{\mathrm{d}t} = \frac{\sum_{\sigma \in S_N} \boldsymbol{j}_{\sigma(k)} \circ \sigma}{\sum_{\sigma \in S_N} \rho \circ \sigma} (Q(t)). \tag{5.9}$$

In this theory, which Goldstein et al. dubbed *identity-based* Bohmian mechanics (*symmetrized* Bohmian mechanics would be more appropriate), we don't a priori

[1] Most of what I'm going to discuss here has previously been published in Esfeld, Lazarovici, Lam, and Hubert (2015b, section 3).

[2] Here, the sum goes over all elements of the permutation group S_N, and

$$\sigma Q := \left(Q_{\sigma^{-1}(1)}, \ldots, Q_{\sigma^{-1}(N)}\right)$$

means that every coordinate Q_i is assigned a new index $Q_{\sigma^{-1}(i)}$, changing the order in the N-tuples.

attribute any mass to any particle. The law of motion merely determines N trajectories for N particles, and it's characteristic of this law that one of those trajectories happens to behave—at least in the relevant circumstances—like the trajectory of a particle with mass m_1, another like the trajectory of a particle with mass m_2, and so on, depending only on the (contingent) initial conditions of the system.

Let's discuss an example given in Goldstein et al. (2005a, section 3) that compares the standard formulation of Bohmian mechanics with the symmetrized version. Consider a two-particle universe consisting of an electron with mass m_e and a muon with mass m_μ. Suppose that they are in a product state $\Psi(q_1, q_2) = \phi(q_1)\chi(q_2)$. Then, the standard guiding law (5.5) leads to the following equations of motion:

$$\begin{aligned} \frac{\mathrm{d}Q_1}{\mathrm{d}t} &= \frac{\hbar}{m_e}\mathrm{Im}\frac{\nabla\phi(Q_1)}{\phi(Q_1)}, \\ \frac{\mathrm{d}Q_2}{\mathrm{d}t} &= \frac{\hbar}{m_\mu}\mathrm{Im}\frac{\nabla\chi(Q_2)}{\chi(Q_2)}. \end{aligned} \tag{5.10}$$

In contrast, the symmetrized guiding equation (5.9) reads

$$\begin{aligned} \frac{\mathrm{d}Q_1}{\mathrm{d}t} &= \frac{\frac{\hbar}{m_e}\left|\chi(Q_2)\right|^2 \mathrm{Im}\left(\phi^*(Q_1)\nabla\phi(Q_1)\right) + \frac{\hbar}{m_\mu}\left|\phi(Q_2)\right|^2 \mathrm{Im}\left(\chi^*(Q_1)\nabla\chi(Q_1)\right)}{\left|\phi(Q_1)\right|^2\left|\chi(Q_2)\right|^2 + \left|\phi(Q_2)\right|^2\left|\chi(Q_1)\right|^2} \\ \frac{\mathrm{d}Q_2}{\mathrm{d}t} &= \frac{\frac{\hbar}{m_\mu}\left|\phi(Q_1)\right|^2 \mathrm{Im}\left(\chi^*(Q_2)\nabla\chi(Q_2)\right) + \frac{\hbar}{m_e}\left|\chi(Q_1)\right|^2 \mathrm{Im}\left(\phi^*(Q_2)\nabla\phi(Q_2)\right)}{\left|\phi(Q_1)\right|^2\left|\chi(Q_2)\right|^2 + \left|\phi(Q_2)\right|^2\left|\chi(Q_1)\right|^2}. \end{aligned} \tag{5.11}$$

We see that equation (5.10) ascribes an intrinsic mass to every particle: particle 1, described by the coordinates Q_1, is the electron with mass m_e, while particle 2, described by the coordinates Q_2, is the muon with mass m_μ. In equation (5.11), by contrast, neither Q_1 nor Q_2 is designated as the position of the electron, respectively the muon. A priori, the two particles are distinguished only by the position that they occupy at time t. But if we consider a situation in which ϕ and χ have disjoint support, one of the two sums in the nominators and denominators will be zero, so that the equation of motion *effectively* reduces to equation (5.10) (possibly with the indices 1 and 2 interchanged). In other words, in situations where the two-particle wave-function is suitably decohered, one of the particles will play the role of the electron—being effectively described by equations (5.5) and (5.1) with the parameter m_e—while the other will play the role of the muon—being effectively described by equations (5.5) and (5.1) with parameter m_μ.

Which trajectory turns out to be guided by which part of the wave-function thereby depends only on the law of motion and the (contingent) initial conditions of the system, rather than on intrinsic properties of the particles. In fact, if both parts of the wave-function were brought back together and then separated again, one and the same particle could switch its role from being the electron to being the muon. Therefore, like a particle's spin, to be an electron, a muon, or a positron is nothing more and nothing less than to *move* under certain circumstances electronwise, muonwise, or positronwise. No properties in this theory define different species of particles, but only primitive stuff, following a law of motion that accounts for the phenomena conventionally attributed to a multiplicity of particle-types.

One obvious objection to symmetrized Bohmian mechanics is that the guiding law (5.9) is much more contrived and complex than that in standard Bohmian mechanics. That's correct, and we are indeed trading a sparse ontology for a more complicated mathematical formalism. But this worry can be addressed in a few ways.

First, the apparent complexity of equation (5.9) is indeed the price for expressing a law of motion for configurations in ${}^{N}\mathbb{R}^3$ instead of the coordinate space $\mathbb{R}^{3N}$. It's often true that a sparser ontology leads to a mathematically more complicated physical theory, and symmetrized Bohmian mechanics is an example.

Second, the symmetrized guiding equation (5.9) in general yields *different* trajectories to the usual one—the above example of the electron and muon is a special case in which both guiding equations coincide in the decoherence regime. In application, however, the very shape of the trajectories is irrelevant. What's important is that the theory predicts the correct statistical distributions. Although the equation of motion is different, symmetrized Bohmian mechanics manages to reproduce Born's rule. And so it's empirically equivalent to Bohmian mechanics and to standard quantum mechanics.

Conclusion

Ontology must never be read off from the mathematical formalism. Yet mathematical formalism isn't indifferent to all ontological interpretations. The mathematical form of the guiding equation suggests that particles have intrinsic properties. We can ignore this; we can ignore that the law uses n-tuples to describe configuration; we can ignore that the guiding equation (for non-identical particles) isn't permutation-invariant. These mathematical features notwithstanding, a metaphysician may still stipulate that particles have neither a primitive identity nor intrinsic properties. But that would be like interpreting electromagnetic fields in the Maxwell–Lorentz theory as non-existent: one would be disregarding crucial features of a physical theory.

The idea of primitive particles without particle species finds a rigorous form in symmetrized Bohmian mechanics. Here the mathematics doesn't allow us to assign primitive identities or intrinsic properties to particles. But the price we pay is a more complicated law of motion, so calculating trajectories is much more tedious and cumbersome. The precise shape of Bohmian trajectories isn't relevant for empirical predictions. What's important is the statistical distribution of particles, and the symmetrized version of Bohmian mechanics agrees with its standard version on this. For physicists symmetrized Bohmian mechanics might not be interesting, but it shows the metaphysician that some of her ideas can be woven into the formalism of physics.

6. The Status of the Wave-Function

What is the ontological status of the wave-function? This question is as old as quantum mechanics itself. Historically the debate centered on what the wave-function actually represents. Does it represent some objective facts? Certain elements of reality? Or does it rather stand for our epistemic knowledge about a physical system? These questions have seeped into the philosophical community and are still being heavily debated. Contemporary philosophers of physics merged physicists' ideas about the wave-function with their own insights on what the status of the laws of nature is (Humeanism, primitivism about laws, and dispositionalism)—but in fact, the ontological ideas of Dürr, Goldstein, and Zanghì preceded this philosophical analysis of the early 1990s.

This strategy, making the status of the wave-function dependent on the status of laws of nature, was particularly fruitful for the de Broglie–Bohm theory. The latter theory was susceptible to philosophical work because it started with a well-formulated primitive ontology. When we have a primitive ontology this makes it straightforward to talk about the fundamental level of reality, and it lets us start to discuss the relation between laws of nature and this fundamental reality. Only after we have gotten clear about the status of Schrödinger's equation and the guiding equation can we philosophize about the status of the wave-function. More precisely, we need to clarify the status of both the universal and effective wave-functions.

6.1. The Nomological View

Let's say that Schrödinger's equation and the guiding equation are primitive. What might the status of the wave-function be? It might be a physical field. In crucial features, it would differ from the electromagnetic field. First, it's defined on configuration space. Second, it would be a non-local beable on physical space.

Third, particles don't act back on the wave-function. In electromagnetism, the field influences the motion of particles, and the motion of particles changes the field. In particular, accelerated particles radiate. But the wave-function is indifferent to the behavior of particles. This might show us that it cannot refer to something physical in space-time. If it were physical it would need to interact with matter, but in the present case the "interaction" only goes one-way, namely, from the field to particles.

Dürr, Goldstein, and Zanghì therefore argue that the universal wave-function is instead a *nomological* entity (Dürr et al., 1997; Goldstein and Zanghì, 2013), that is, something law-like. More precisely, being nomological means here that there is no physical entity out there in the world that is mathematically described by the wave-function. The single role of the wave-function is to generate the vector field in the guiding equation, and hence to generate motion via the guiding equation.

6.1.1. The Universal Wave-Function is not a Law!

In their writings, Dürr, Goldstein, and Zanghì sometimes claim that the wave-function *is* the law (see, for example, Dürr et al., 1997). I don't think that this is a good way of explaining the status of the wave-function. The wave-function is a mathematical function, and the law of motion of Bohmian mechanics is the guiding equation. One can interpret the wave-function as nomological but not *as a law*.

Maudlin (2013) agrees with Dürr, Goldstein, and Zanghì that the Bohmian law of motion is primitive, while the wave-function isn't nomological. For him, the wave-function refers to a physical entity, which he calls the *quantum state*. The quantum state is a non-local beable on four-dimensional space-time and is mathematically represented by the wave-function on configuration space.

We know that the quantum state bears some unusual features: it's independent of the motion of particles, it's non-local in space-time, and we don't have direct empirical access to it. So Maudlin remarks that no philosophical category is appropriate for the quantum state:

> We do not even know the right general ontological category in which to put it. Indeed, there is no reason to believe that any theorizing or speculation on the nature of the physical world that took place before the advent of quantum theory would have hit on the right ontological category for the quantum state: because it is so hidden, there would have been nothing relevant to speculate about. Whether one finds the possibility invigorating or disheartening, the best ontological category for the quantum state might simply be the category *Quantum State*, just as the right ontological category for a classical field is *Field*, not 'stress in a medium' or 'collection of particles.' (2013, pp. 151–2)

This means that, for Maudlin, the quantum state is neither a substance nor a property. Philosophy just hasn't been able to discover or invent the appropriate categories for modern physics. In particular, the quantum state founds the category "quantum state". In sum, what distinguishes Maudlin from Dürr, Goldstein, and Zanghì is that he adds the quantum state as a novel physical and metaphysical entity to the fundamental ontology, whereas the laws remain primitive.

6.1.2. The Effective Wave-Function is not Quasi-Nomological!

What is the status of the effective wave-function if the universal one is nomological? At first sight, there seems to be an overdetermination of motion. As the universe has its own wave-function, every subsystem is guided by the velocity-field generated universal wave-function. There are certain subsystems, namely those that are sufficiently decohered from the environment, that seem in addition to be guided by an effective wave-function. Overdetermination as such isn't always a problem, but here it is, if we construe the effective wave-function as a primitive nomological entity in addition to and on a par with the universal wave-function. It suffices to have just one entity that "really" guides the particles. Ockham's razor is here well applied here.

Goldstein and Zanghì (2013) oddly claim that the status of the effective wave-function isn't interesting to analyze:

> Our point is rather that once the status of the wave function of the universe has been settled, the question about the status of ψ is rather secondary—something about which one might well feel no need to worry. (Goldstein and Zanghì, 2013, p. 276)

Nevertheless, the metaphysical status of the effective wave-function needs to be explained, just as everyone agrees that the status of the universal wave-function needs to be explained.

Notwithstanding their initial statement, Goldstein and Zanghì still try to elucidate the metaphysics of the effective wave-function:

> Be that as it may, we would like to regard ψ as quasi-nomological. We mean by this that while there are serious obstacles to regarding the wave function of a subsystem as fully nomological, ψ does have a nomological aspect in that it seems more like an entity that is relevant to the behavior of concrete physical reality (the primitive ontology) and not so much like a concrete physical reality itself. (2013, p. 276)

So the universal wave-function is fully nomological, while the effective one is not—it's *quasi-nomological*. But what does "fully" mean here? And is the effective wave-function only half nomological?

Let's try to make this more precise. The "nomological aspect" of the effective wave-function is inherited from the universal wave-function. Since the effective wave-function is derived from its universal mother, it's ontological status must be nomological; that is, the effective wave-function doesn't refer to an entity out there in the world.

For all practical purposes, we use the effective wave-function by means of Schrödinger's equation and never the universal wave-function. So in applying Bohmian mechanics, the entity that is "relevant to the behavior of concrete physical reality" is the effective wave-function. This might give us a clue about the meaning of "quasi". The universal wave-function is the primary (because primitive) nomological entity in Bohmian mechanics; the effective wave-function is nomological, too, but it's defined in terms of the universal wave-function.

The notion *quasi-nomological* is a bit unfortunate. For it suggests that the effective wave-function belongs to a novel ontological category. But there is no such category, and therefore I wouldn't dub it quasi-nomological. The effective wave-function is a device for calculation, which can be defined if certain circumstances are fulfilled. But it's always the universal wave-function that is dynamically efficacious.

6.2. Quantum Humeanism

6.2.1. The Marvelous Point

David Albert (1996) and Barry Loewer (1996) were the first to develop a Humean theory of Bohmian mechanics. For them the universal wave-function is a field on configuration space:

> The sorts of physical objects that wave functions *are*, on this way of thinking, are (plainly) *fields* – which is to say that they are the sorts of objects whose states one specifies by specifying the values of some set of numbers at every point in the space where they live, the sorts of objects whose states one specifies (in *this* case) by specifying the values of two numbers (one of which is usually referred to as an *amplitude*, and the other as a *phase*) at every point in the universe's so-called *configuration* space.
>
> The values of the amplitude and the phase are thought of (as with all fields) as intrinsic properties of the points in the configuration space with which they are associated. (Albert, 1996, p. 278)

Albert takes the $3N$-dimensional configuration space as the fundamental space of Bohmian mechanics, and every point of this very high-dimensional space is assigned a unique value of the universal wave-function. This is Albert's Humean mosaic.

Maudlin (2013) points out that Albert must accept unobservable physical facts, since any change in the wave-function induces a change in values of the intrinsic properties of configuration space. For example, we can multiply wave-functions by a global phase without changing the Bohmian velocity field. Albert, however, has to bite the bullet and say that those wave-functions represent different *physical* situations or different physical worlds.

Bohmian mechanics isn't a monist theory, in which only the universal wave-function exists. There are also particles. But in Albert's ontology particles don't live in three-dimensional space. And there isn't a multitude of them either; there is just one.

> On Bohm's theory, for example, the world will consist of exactly two physical objects. One of those is the universal wave function and the other is the universal *particle*.
>
> And the story of the world consists, in its entirety, of a continuous succession of changes of the *shape* of the former and a continuous succession of changes in the *position* of the latter.
>
> And the dynamical laws that *govern* all those changes – that is: the Schrödinger equation and the Bohmian guidance condition – are completely deterministic, and (in the high-dimensional space in which these objects live) completely *local*. (Albert, 1996, p. 278)

Here Albert claims that our world is very peculiar on the fundamental level. Our universe actually consists of one single particle in configuration space, which is a mere dot in a $3N$-dimensional space. Sheldon Goldstein (with a little grain of irony) dubbed Albert's universal particle *the marvelous point*. The familiar world that we perceive is in fact a *shadow* of the marvelous point (Albert, 2015, Ch. 6)

Life in configuration space is local. It's dynamically local because there is only one particle; there are no non-local effects on other particles in configuration space. And this world is ontologically local because the universal wave-function is defined on the very space, in which the universal particle is defined. According to Bell's definition, Albert construes the universal wave-function as a local beable *in configuration space*.

The Problem of Communication

When we have particle and fields in the same space, they are geometrically related: the value of the field in position x determines the motion of a particle at x. The situation is different if you regard configuration space and three-dimensional physical space as both real and independent.

> But this makes no sense. Think about it: what the guidance condition would have to amount to, on a picture like this, is a fundamental law of nature whereby one concrete entity (the wave function) in a $3N$- dimensional space tells a set of N concrete entities (the corpuscles) in an altogether *different* space—the three-dimensional space of our everyday physical experience—how to move. What we're *used* to doing in physics (remember) is writing down laws of the interactions of two or more concrete entities in the *same* space. And in circumstances like *that*, questions like "In what spatial direction does B move as a result of its interaction with A?" (think here, say, of collisions, and of Newtonian gravitational interactions, and of interparticle electrical forces, and so on) are invariably settled by geometrical relations between A and B *themselves*. But in the *present* case there *are* no geometrical relations between A (the wave function) and B (the corpuscles) at all! In the present case (then) there can be no idea whatever of A's affecting B by pushing or pulling or poking or prodding, either directly or indirectly, either locally or nonlocally. And this (mind you) is not merely an offence to intuition—it is a straightforward *logical* problem: lacking any geometrical relationship between A and B, there is nothing about the condition of A in *its* space that is structurally capable of *picking out* anything like a *direction*, or anything like a *particular corpuscle*, or anything *whatsoever*, in the B-space. Period. End of story. (Albert, 2015, pp. 124–5)

If configuration space and physical space are primitive spaces that exist independently of each other, then there is no link between how things happening in configuration space and what happens in three-dimensional space. How can anything that exists outside of our universe affect the motion of objects in our universe? God may be capable of doing so but not the wave-function. For Albert, this isn't due to physical or metaphysical reasons, "it is a straightforward *logical* problem."

If configuration space and three-dimensional space exist independently from each other we not only face the problem of how configuration space might affect our familiar physical space in principle but also that of fleshing out the details.

> And note that the trajectory of a world-particle like this one can patently contain *no suggestion whatever* as to whether we are dealing here with a single material particle moving freely in an N-dimensional physical space, or (say) $N/3$ distinct material particles moving freely in a *three*-dimensional physical space, or $N/2$ distinct material particles moving freely in a *two*-dimensional physical space, or N distinct material particles moving freely in a *one*-dimensional physical space. *Nothing* about a trajectory like that (to put it slightly differently) can make it natural or make it plausible or make it reasonable or make it simple or make it elegant or make it any other desirable thing to suppose that any particular *one* of those possibilities, as opposed to any one of the *others* I mentioned, or any one of the others I *didn't* mention, actually *obtains*. (Albert, 1996, p. 280)

In order to connect configuration space and physical space there has to be a mapping, an isomorphism, that maps what happens in one space to what happens in the other. A point in configuration space is written as a $3N$-tuple $(x_1, x_2, \ldots, x_{3N})$, and Albert correctly notices that there is no natural or a priori grouping of the coordinates so that (x_1, x_2, x_3) represent the position of the first particle in physical space and (x_4, x_5, x_6) represent the position of the second particle, and so on. The same configuration space might represent particles in a two-dimensional or one-dimensional space.

I question Albert's argument. He obviously takes the mathematical formalism of Bohmian mechanics at face value and tries to read off the ontology off from the mathematical representation. For mathematical representations never uniquely refer to the same physical system. Similar mathematical formalisms used in physics can even represent biological or economic behavior.

As Maudlin (2013) shows, Albert indeed used an inappropriate mathematical formalism. Since classical mechanics, configurations have been represented by n-tuples. This mathematical structure leads to relatively simple equations of motions, but creates unnecessary surplus structure if you describe indistinguishable particles. In particular, a permutation of particles leads to a different n-tuple and hence to a different mathematical representation. So Albert has to accept further unobservable facts in his metaphysics: two configurations that differ by permutations cannot be distinguished by observation although they represent different physical states of affairs (see Section 1.3 for more details).

The proper mathematical space for configurations of particles is rather ${}^N\mathbb{R}^3$. Had Albert based his interpretation of Bohmian mechanics on this space he wouldn't have wondered about the different physical situations to which an n-tuple might refer. From the elements of ${}^N\mathbb{R}^3$ alone it's set that we deal with N particles in three-dimensional space—an element is $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$, where $\boldsymbol{x}_i \in \mathbb{R}^3$ for $i \in \{1, \ldots, N\}$. Nevertheless, the problem of communication remains.

6.2.2. Super-Humeanism

There is a completely different way of constructing a Humean world of Bohmian mechanics (see Esfeld et al., 2014; Miller, 2014), a way that Albert (2015, p. 126) simply calls crazy. Let's see how crazy this strategy really is. We start by postulating a three-dimensional world, in which particles move. This space is the fundamental physical space of Bohmian mechanics. Contrary to Albert's mosaic, this mosaic consists of a primitive stuff ontology of particles. The wave-function, the guiding equation, Schrödinger's equation, and configuration space are determined by the motion of particles throughout the history of the universe. They are all part of the best system in describing the behavior of the particles.

We cannot simply derive the wave-function from the positions of particles at any time t. It is, however, possible to interpret the universal wave-function as supervening on the entire history of particles. Imagine an ideal observer who is capable of observing everything that happens in the universe from beginning to end without interfering with it. What laws will the ideal observer write down if we ask her to do so? Laws that best combine simplicity and strength will be the Bohmian ones.

But writing down the laws doesn't reify the wave-function or configuration space. Nor is there a problem regarding how the wave-function can refer to objects in ordinary space. The whole theory supervenes on the primitive stuff ontology.

Miller's Humeanism

In a footnote, Elizabeth Miller claims that she has developed a Humean theory of Bohmian mechanics that differs from the account given by Esfeld et al. (2014):

> Esfeld and his co-authors suggest that Bohmian mechanics may be combined with a Humean account of laws, but it is not clear from their discussion exactly how they expect this to go and, especially, whether they take this to be a *realist* treatment of the Bohmian pilot wave. My proposal may be read as a suggestion in a similar Humean spirit, one that fleshes out a (distinct, explicitly realist) Humean treatment of the pilot wave by way of an analogy to a Humean account of objective chance and that offers this treatment as a response to the non-separability argument. (Miller, 2014, p. 579)

I don't see the differences that Miller purports to notice. Her Humean account is exactly the same as the one I introduced above, and it's exactly the one that Esfeld et al. advocate.

Moreover, Miller distinguishes her project as a "*realist* treatment of the Bohmian pilot wave," while Esfeld et al. argue for a nomological interpretation. To my mind, this is just a matter of terminology. Some Humeans defend a realist interpretation of chance, a realist interpretation of mass, or a realist interpretation of the wave-function, although all these are always part of the best system. But claiming to be a realist about the best system doesn't make the best system real in the same way that particles of the Humean mosaic are real.

I would say that a real realist about chance is someone who regards chance as an irreducible part of nature, like someone who regards the GRW laws to be true. A real realist about mass is someone who regards mass as a categorical or dispositional property. And a real realist about the wave-function regards it as non-nomological.

Bhogal and Perry on Separability

Another treatment of Humean Bohmian mechanics has recent been put forward by Bhogal and Perry (2015). I don't intend to discuss their results regarding how to apply Humeanism to Bohmian mechanics, since their core idea matches Miller's and Esfeld's while introducing some new (and, I think, not particularly illuminating) terminology. What I would like to analyze in some detail, though, is their discussion of the principle of separability.

Remember that Maudlin (2007a, p. 51) defines separability in the following way:

> The complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each spacetime point (or each pointlike object) and the spatio-temporal relations between those points.

Maudlin aims to refute Humeanism by showing that entangled states cannot be reduced to intrinsic states of the Humean mosaic. The simplest entangled state is the singlet state. Contrary to a product state, where each particle has a well-defined quantum state, the system in a singlet state only has this state as a whole. If quantum mechanics is taken to be complete, and the wave-function refers to a physical quantum state, the argument goes, Humeanism has to add these entangled states as primitive relations. But then there are further physical facts that no longer supervene on local properties. And that is no longer a Humean world.

Now Bhogal and Perry (2015) accuse Maudlin of using too strict a notion of separability. According to them, Maudlin actually shows the incompatibility of Humeanism with *strong separability*:

> The complete physical state of any region R is determined by (supervenes on) the intrinsic physical states [of] (and relations between) R's sub-regions. (Bhogal and Perry, 2015, p. 4)

Given two (disjunct) regions R_1 and R_2, the intrinsic properties of either region determine the physical state of both regions, that is, the state of $R_1 \cup R_2$. But if $R_1 \cup R_2$ is in an entangled state, neither subregion has intrinsic properties that are independent of the other region. This is the meaning of strong separability, and this is also the meaning to which Maudlin adheres in his definition.

As Maudlin's formulation of separability is about the complete physical state of the world, it's sufficiently general to allow another interpretation. If the two particles in the singlet state are in regions R_1 and R_2 respectively, their physical state doesn't merely depend on the intrinsic properties of R_1 and R_2 but on the intrinsic properties of the entire Humean mosaic:

> A pair of particles being in the Singlet state is not determined by the intrinsic physical states of those two particles; rather, it's determined by the states of the pair together with the intrinsic physical states at other points in the mosaic. (Bhogal and Perry, 2015, p. 3)

This is exactly the same as the idea proposed by Esfeld et al. (2014) and Miller (2014), namely to apply Humeanism to Bohmian mechanics. I don't think one can properly speak about the Humean mosaic of ordinary quantum mechanics because only the wave-function on the fundamental level lacks a physical description of intrinsic properties. But the wave-function in Bohmian mechanics may be interpreted as supervening on the motion of all the particles in the universe.

The main claim of Bhogal and Perry is that they have rescued separable Humean quantum metaphysics. I don't agree. Indeed they have changed the meaning of separability so that physical states are separable states whenever they supervene on the behavior of the whole mosaic. Although I think that this is an appropriate strategy for interpreting the wave-function, I disagree with their definition of separability. For what they dub strong separability is the core idea of separability in the first place. This is not only obvious in Maudlin's paper but also when Einstein speaks about this notion in a famous letter to Max Born:

> If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following: the concepts of physics relate to a real outside world, that is, ideas are established relating to things such as bodies, fields, etc., which claim 'real existence' that is independent of the perceiving subject – ideas which, on the other hand, have been brought into as secure a relationship as possible with the sense-data. It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space'. Unless one makes this kind of assumption about the independence of the existence (the 'being-thus') of objects which are far apart from one another in space – which stems in the first place from everyday thinking – physical thinking in the familiar sense would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localizing the elementary objects on which it is based and which exist independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four-dimensional) elements of space. (Born, 1971, pp. 170–1)

The essence of separability is spelled out by Einstein: "An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space.' " So if a physical system is situated in a region that can be divided into two disjunct parts R_1 and R_2, and the subsystem in R_1 is in a physical state that is independent of the state of R_2, then the state of $R_1 \cup R_2$ is a separable state.

The singlet state, however, is a non-separable state. Ordinary quantum mechanics interpreted as an ontologically complete theory[1] is a non-separable theory. Bohmian mechanics is a non-separable theory. Separability is hence rather a feature of a physical theory than a feature of a metaphysical framework.

Somehow Albert's interpretation of Bohmian mechanics seems to be an exception. The universal wave-function interpreted as a physical field is separable since the entire wave-function is determined by its intrinsic values in each point of configuration space. Bhogal and Perry claim that Albert follows neither the principle of separability nor of strong separability; instead, they say that Albert denies separability in favor of a weakened *fundamental state separability*:

> The complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each point in the fundamental space of that theory (and on the geometric relations between points in that fundamental space). (Bhogal and Perry, 2015, p. 16)

Of course, Albert violates both Maudlin's definition of separability and Bhogal and Perry's definition of strong separability. And he does so because he takes configuration space to be the fundamental space instead of space-time. But it seems a little odd that this violation of separability just hinges on which space you take to be the most basic. The crucial idea behind separability is not to build an ontology on three-dimensional space or four-dimensional space-time. Maudlin's formulation notwithstanding, the heart of separability is that it should be always possible to assign physical states to arbitrary subregions of *your fundamental space*, whether it's low-dimensional or high-dimensional. And this is exactly what Einstein (as I understand him) wanted to point out. Therefore, the wave-function in super-Humeanism is non-separable, while it's separable in Albert's ontology.

What are Physical States?

Something important but less visible in Maudlin (2007a, Ch. 2) and Bhogal and Perry (2015) is their use of *physical states*. They seem to presuppose that the reader clearly understands what physical states are, as they nowhere attempt to define them. Physicists talk all the time about the physical states of systems, but they use the term in a very narrow sense. I'm not always sure whether this is the intended sense in the abovementioned papers; nevertheless, it's the sense that I prefer and generally use. So for physicists a physical state is the complete description of a system that is needed for the basic laws to predict the behavior of the system.

In Newtonian mechanics, for example, the physical state of a particle at time t consists of its position and velocity, that is, $(\boldsymbol{x}(t), \boldsymbol{v}(t))$. The physical state of a

[1] Maudlin (2007b) distinguishes between informationally complete and ontologically complete physical theories.

particle in Hamiltonian mechanics consists of position and momentum $(\boldsymbol{q}(t), \boldsymbol{p}(t))$. A physical state in Maxwell-Lorentz electrodynamics consists of the positions and velocities of the particle, as well as of the values of the electric and magnetic field, that is, $(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{E}, \boldsymbol{B})$. In textbook quantum mechanics the state of a system is fully described by its wave-function ψ, whereas in Bohmian mechanics the physical state also comprises the positions. So the Bohmian state is represented as $(\boldsymbol{Q}, \psi)$.

The physical state is first and foremost an abstract entity that together with the laws describes the behavior of the system (see Curiel, 2014, section 2). It's then a matter of interpretation when it comes to disentangling the ontological status of the objects that are part of the state or working out what the state refers to in the first place. Like Curiel, you can be indifferent to this ontological issue. Physical states as such don't make any ontological commitments, and they are necessary and sufficient for accurate empirical predictions. But they aren't sufficient for a complete description of empirical phenomena. To do so, we need an ontology. And as we see above, the objects that describe physical states are the notorious ones: particles, fields, and wave-functions.

6.3. The Wave-Function as a Disposition

We saw in Chapter3 that a primitive ontology consisting merely of particles and intrinsic dispositions doesn't work because it leads to metaphysical action-at-a-distance. Hence, we needed to introduce fields or relations. Now, with Bohmian mechanics, we have a theory that obliges us for *physical* reasons to introduce relational dispositions owing to quantum entanglement. The novelty of quantum entanglement is that the entangled state cannot be reduced to actual individual states of particles—Maudlin used this feature to argue against Humeanism (see section 2.1.3). Consider two electrons in the spin singlet state. Neither electron has spin-up or spin-down, but after a measurement in the same direction one electron is always spin-up and the other is always spin down. Contrary to Maudlin, who construes entanglement as established by a novel and unspecified quantum state that is mathematically represented by the wave-function, we can regard the wave-function as representing the disposition of particles to move in a certain way.

There are two ideas for a dispositional account of the wave-function. Esfeld et al. (2014) state that all particles taken together have one holistic disposition, while Suárez (2015) introduces an infinite set of intrinsic dispositional properties for each particle. After explaining these views, I will outline the bare bones of an often neglected interpretation of the wave-function, namely as a multi-field on ordinary physical space. A multi-field wave-function is as ontologically local as possible without changing the mathematics—it's an attempt to *interpret* the wave-function or what the wave-function represents as a local beable. Having the wave-function as a genuine local beable on three-dimensional space, however, requires a new physical theory, and this has been attempted by Norsen (2010).

6.3.1. A Holistic Disposition, aka an Ontic Structure

If the universal wave-function were in a product state, every particle would have its own quantum state, and the motion of every particle would be independent of the other particles. Then we could interpret quantum states of particles as local dispositions; each local disposition determines how each particle moves. Consequently, the universal wave-function, as it determines the dynamics of all the particles, would represent a set of individual dispositions: it would supervene on local properties of particles. The universal wave-function, however, is in general not a product state.

The idea of Esfeld et al. (2014) is therefore to interpret the universal wave-function as a *holistic* disposition; that is, the set of all particles has exactly *one* disposition. This holistic disposition constrains the motion of all particles so that they follow Bohmian trajectories. But we must make sure that there are no external triggering conditions for the manifestation of the holistic dispositions because there is nothing outside the universe. In the standard cases of the fragility of glass or the solubility of sugar, the triggering conditions are imposed on the system—namely, by a flying stone or water. But here the disposition is always manifested; particles have a velocity because they have position.

As the motion of each particle depends on the positions of all the other particles, this holistic disposition is relational. And so the universal wave-function instantiates an ontic structure (Esfeld et al., 2015b)—an ontic structure that dynamically relates all particles in three-dimensional space (Lam, 2015).

Advantages and Disadvantages

The universal wave-function counters two related problems. Remember Albert's problem of communication (see p. 153), namely that it would be mysterious how the wave-function could affect anything in the real world if the real world were a three-dimensional one. A dispositionalist would understand the problem of communication differently: it would be mysterious how the wave-function could affect anything in the real world if the real entity that the wave-function refers to indeed existed in configuration space. Therefore, the dispositionalist (similar to a Humean, I would say) emphasizes that the wave-function is only mathematically defined on configuration space and thus that it's a tool for calculating trajectories and statistical distribution, but (contrary to a Humean) the wave-function stands for a holistic disposition (or an ontic structure) in ordinary physical space (Esfeld et al., 2015b, section 5)—in section 2.3.3 I argued that the only intelligible version of OSR I accept coincides with relational dispositionalism.

The dispositionalist's interpretation of the wave-function is said to solve a (to my mind alleged) problem of primitive laws. If laws are abstract primitive entities, it's mysterious, according to the dispositionalist, how they can govern the behavior of real, concrete, substantive particles. It is, in particular, unfathomable how a nomological wave-function can do so. By assigning a disposition to particles, there is something in the world that changes the behavior of something in the world. By having a holistic disposition in three-dimensional space, you cannot object that it's mysterious how laws or a mathematical wave-function can guide real substantive particles. For the dynamical efficacious entity is neither the law nor the mathematical wave-function but the disposition.

In primitivism, particles move as they do because there are laws. The dispositionalist then asks how laws can effect this motion and expects an answer that goes beyond laws. Something in the world or in the particles must move particles, not abstract mathematical laws. Abstract entities influencing concrete objects in space and time sounds like a category mistake. But primitive laws govern particles because they are primitive laws. There is nothing to add or to unravel.

Let's turn to another feature of the structuralist construal of the wave-function. If the wave-function represents an ontic structure, it's said that we no longer face the miracle of particles affecting each other instantaneously over arbitrary distances. Quantum non-locality is hence not action-at-a-distance because this action is mediated by dynamical relations:

> By contrast to an account of quantum non-locality in terms of superluminal interaction [...], there is no action at a distance among anything in BM, simply because a modal structure instantiated by the configuration of matter is another conception of the determination of the temporal development of physical objects than direct interaction among the parts of that configuration. Hence, there is quantum non-locality because the temporal development of quantum systems is not determined by properties that are intrinsic to each object (as mass and charge are intrinsic properties of particles in classical mechanics and determine their temporal development), but by a structure that is instantiated by the configuration of all the quantum objects. (Esfeld, 2014, p. 10)

To my mind, this argument is a red herring. First, one needs to distinguish between a metaphysical and a physical action-at-a-distance. What a dynamical ontic structure overcomes is metaphysical action-at-a-distance, not the physical kind. For there is no physical mechanism, like the exchange of particles or the transmission of fields, that mediates between two entangled particles. For all kinds of actions-at-a-distance—for retarded, but especially for instantaneous action—one can introduce an ontic structure that "generates" the dynamics. But then one replaces one black box, non-local interactions, with another black box, namely an ontic structure.

To my mind, this argument is a red herring. First, you need to distinguish between a metaphysical and a physical action-at-a-distance. What a dynamical ontic structure overcomes is metaphysical action-at-a-distance, not a physical one. For there is no physical mechanism, like the exchange of particles or the transmission of fields, that mediates between two entangled particles. You can introduce for all kinds of actions-at-a-distance—also for retarded but especially for instantaneous action—an ontic structure that is "generating" the dynamics. But then you replace one black box, non-local interactions, with another black box, an ontic structure.

Second, an ontic structure doesn't specify how it can be causally or dynamically efficacious. The precise dynamics is always described by the law of motion. So why not take the law itself as the dynamical relation? The law is something that we need anyway, and by raising the law to be the primary dynamical entity we abstain from postulating dubious dynamical relations that are purported to be different from laws, although they don't do more than laws or better clarify any issue..

6.3.2. Myriads of Dispositions

Suárez (2015) presents another idea for a dispositional wave-function. The aim is to have a distribution of intrinsic dispositions on every point of space. The wave-function defines a velocity vector field on configuration space; that is, it assigns to every point of configuration space a velocity vector at every time t. We can now fix a point $\boldsymbol{x}_i \in \mathbb{R}^3$ and attribute a set of velocity vectors in the following way: take the universal particle in configuration space and move it along $\boldsymbol{x}_i = const$. This will give us all the possible velocity vectors at $\boldsymbol{x}_i$ at time t. Of course, we can do this for all points and receive an infinite stack of velocity vectors for every point in physical space.

So according to Suárez, a particle in an arbitrary spot has the disposition, represented by the velocity vectors, to move in all those directions indicated by these vectors (see Fig. 6.1). Of course, only one velocity per particle can be manifested

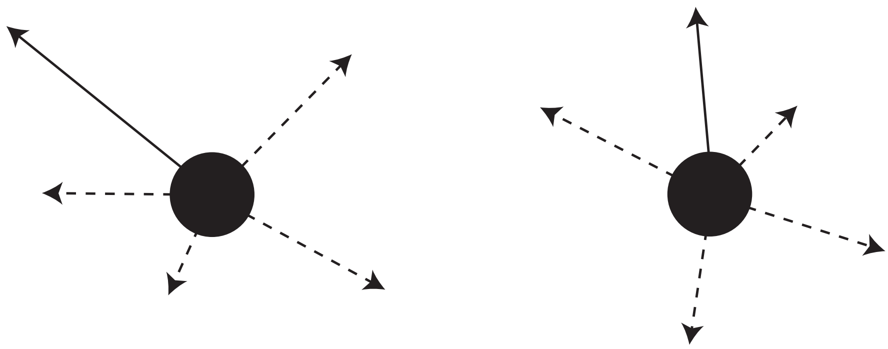

Figure 6.1.: A stack of dispositions for each particle. Drawn lines represent manifested dispositions; dashed lines represent unmanifested ones.

at a moment in time. But this manifestation depends non-locally on the position of all the other particles in the universe. So even if one particle remains in its spot (because we hold it tight), it's future motion will change once the particles in the environment have changed their position (see Fig. 6.2).

Advantages, Disadvantages, and Misconceptions

With respect to the problem of communication and primitivism about laws, the advantages of Suarez' interpretation of the wave-function are the same as Esfeld's.

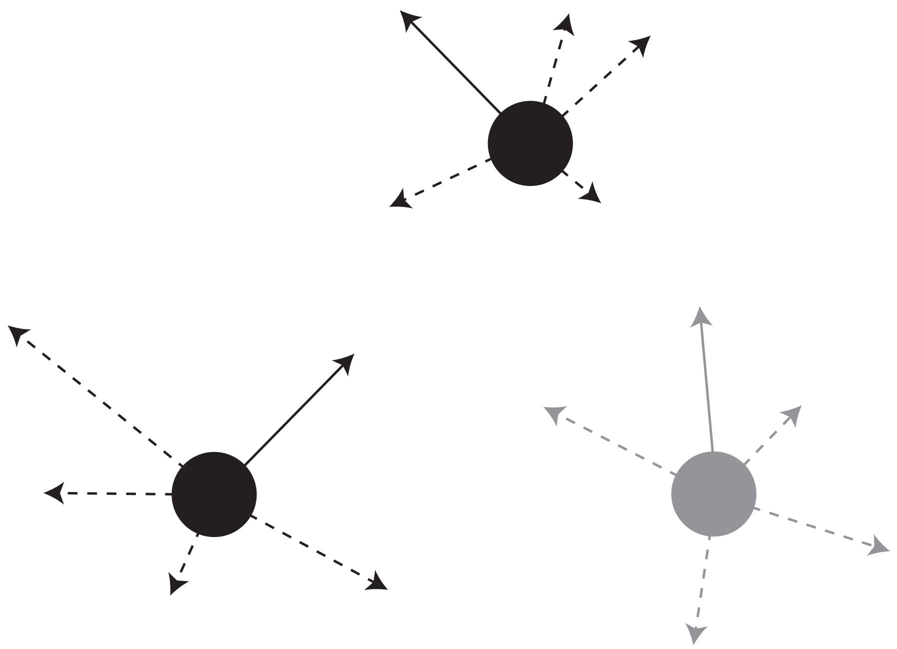

Figure 6.2.: Non-local dependency of velocity by the positions of other particles. The particle on the right from Fig. 6.1 is now in a different position. The manifested disposition of the other particles changes as well.

What Suarez doesn't have in this ontology, however, is an ontological connection among particles; Esfeld, on the other hand, introduces a holistic disposition, where due to holism all particles are by definition ontologically related. Suarez' ontology consists of a distribution of local dispositions; that is, every point in physical space is equipped with infinitely many dispositions, which are mathematically represented as velocity vectors. A particle will choose exactly one of those velocity vectors—the disposition comes to be manifested—according to the positions of the other particles at a time t. This is what the guiding equation tells us. Although this non-local connection is described by the law, which is supposed to be a mere representation of dispositions, there is nothing in Suarez' ontology that establishes this connection. So this ontology already suffers from a metaphysical action-at-a-distance, something that a dispositionalist is supposed to avoid as she defends modal connections.

Suárez and Esfeld agree on the primitive stuff ontology: the Bohmian world is three-dimensional and contains particles. They agree that particles have dispositions. And they agree that the manifestation of dispositions is the velocity of each single particle. Where they disagree—and this is subtle but crucial—is on the number of dispositions and where those dispositions are "located." But for some inexplicable reason, Suárez misrepresents Esfeld's theory:

> A comparison is instructive with the alternative account offered by [Esfeld et al. (2014)]. On their account only the universal particle in configuration space has dispositions, and the velocity field is defined over configuration

> space. The actual velocities of particles in 3D space are thought instead as the manifestation of these universal particle dispositions [(Esfeld et al., 2014, p. 785)]. I understand this to mean that there are no genuine dispositions for the individual particles taken in isolation. The only dispositions that exist are those represented by the universal wavefunction for all the particles, which concern the universal particle; these dispositions manifest themselves in the first instance in the evolution of the universal particle itself. True, these are perceived as motions of each of the individual particles in 3D space, but there are no dispositions 'residing in' physical 3d space—neither in the points of 3D space nor in the 3d particles that occupy such points.
>
> This 'Esfeld' disposition, as we may call it, is a holistic property of the universal wavefunction. So the manifestations will appear to be non-local, in the sense that it will appear as if the velocity of a particular particle over here depends upon the positions of particles elsewhere. But in reality there are no dispositions in 3D space or in the 3D particles, there is only one higher rank disposition of the universal particle, which is necessarily entangled or holistic (but not non-local since there is only one universal particle with a particular location at all times). (2015, p. 3217)

Nearly every sentence of this quote is wrong. Let me briefly correct some misconceptions:

1. In Esfeld's ontology, there is no universal particle, let alone one that has a disposition. The ontology is about particles in three-dimensional space. Suárez confuses the mathematical representation of the holistic disposition with the actual bearers of this disposition.

2. The velocity field is always defined on configuration space. That's not special to Esfeld's account. But Esfeld thinks (as all Bohmian physicists do) that this velocity field eventually describes the motion of particles in three-dimensional space.

3. The holistic disposition is not a "holistic property of the universal wavefunction." The universal wave-function rather represents a holistic disposition of all the particles taken together.

4. I don't understand what Suárez means when he says that "the manifestations will appear to be non-local." Non-locality isn't a matter of appearance. In the de Broglie–Bohm theory, non-locality is a physical fact that immediately pops out of the guiding equation once you have entangled wave-functions.

5. The non-locality—rather the *not* non-locality; is that locality?—that Suárez mentions in the last sentence is particularly confusing. Does he want to characterize the disposition or the universal particle? Both the universal wave-function and the universal particle are local beables in configuration space; this is ontological locality or separability. There is, of course, no dynamical non-locality in configuration space, because there is no other particle that can interact with the universal particle.

6.3.3. The Multi–Field Account: Two Versions

Belot (2012) gives an idiosyncratic interpretation of the wave-function first posited by Forrest (1988), which now seems to be gaining popularity. The basic idea is to regard the wave-function not as an ordinary field on configurations space, as David Albert does, but as a novel physical field: a *multi-field* in three-dimensional physical space. This would counteract the problem of communication because what the universal wave-function actually represents would be a field in the same space, in which particles move.

The basic idea is to regard the wave-function not as an ordinary field on configurations space, as David Albert does, but as a novel physical field: a *multi-field* on three-dimensional physical space. This would counteract the problem of communication because what the universal wave-function actually represents would be a field in the same space, in which particles move.

Forrest (1988, pp. 155–9) distinguished between *monowaves* (monadic waves) and *polywaves* (polyadic waves, which were dubbed *multi-fields* by Belot, 2012). Monowaves are already familiar from classical electrodynamics and hydrodynamics; these are scalar or vector-valued fields that depend on the three-dimensional position $\boldsymbol{x}$ and time t. The electric field $\boldsymbol{E}(\boldsymbol{x}, t)$ and the one-particle wave-function $\psi(\boldsymbol{x}, t)$ are examples of monowaves.

Polywaves, on the other hand, take as arguments $\boldsymbol{x}_1, \dots \boldsymbol{x}_N$ such that the many-particle wave-function $\psi(\boldsymbol{x}_1, \dots \boldsymbol{x}_N, t)$ is a polywave. While a monowave assigns a unique value to a point in space at a certain time t, a polywave relates N spatial points by assigning all these points together a unique value at a time t. These kind of waves are unknown from classical physics, and I know of no good way to depict them, but they are objects in three-dimensional space.

Only some polywaves can be decomposed into monowaves, whereas all monowave can be decomposed into their Fourier parts. In case the wave-function is in a product state $\psi_1(\boldsymbol{x}_1) \otimes \cdots \otimes \psi_N(\boldsymbol{x}_N)$, this many–particle wave-function is trivially decomposed into monowaves, where each particle receives its own wave-function.

I suspect that the multi-field account over-promises and under-delivers. It pretends to make the wave-function local, or rather, to make the entity that it represents local. Yet, it does so in an unsophisticated and non-convincing way. Forrest just draws a formal analogy with ordinary fields, and pretends to have invented or discovered a new physical object, whose only difference to standard fields is the larger argument—the multi-field depends on N points $\boldsymbol{x}_1, \dots \boldsymbol{x}_N$, whereas an ordinary field depends on $\boldsymbol{x}$. In the end, , Forrest just switches the name tags around: what was formerly called a wave-function is now dubbed a multi-field or polywave. There is no change in the mathematics, nor any alteration in the ontology.

In a way, this is similar to Maudlin's interpretation of the wave-function. For him, the wave-function represents a non-local quantum state in physical space. We cannot ask what the quantum state is or where it is localized. For the quantum state is the quantum state, a novel ontological entity, and it's represented by the wave-function on configuration space. Apart from its effects on particles, this is all that we can know about the quantum state, according to Maudlin. I find Maudlin's explanation unsatisfactory, but I prefer it over the multi-field interpretation, because the introduction of the multi-field happens only on the formal mathematical level. Maudlin, on the other hand, is explicit about his ontology, both about its virtues and about its limitations.

Therefore, I would propose an alternative reading of the multi-field, based on Suárez' ontology. The wave-function mathematically defines a velocity vector field on configuration space, that is, a distribution of field values on points of configuration space. By projecting this vector field onto three-dimensional space, we get an ontologically local field in physical space similar to the electromagnetic field. Here is how the projection looks like. The universal particle at $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ assigns to every particle at $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N$ a velocity $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_N$. We can now keep one $\boldsymbol{x}_i$ fixed and change the universal particle in all other possible directions. Then, every infinitesimal movement of the universal particle gives us a new velocity $\boldsymbol{v}_i$. By bundling all these $\boldsymbol{v}_i$ together we get a stack of possible velocities for the particle at $\boldsymbol{x}_i$.

So contrary to classical fields, the projected universal velocity field assigns a multitude of velocity vectors to each point in space. To have a well-defined dynamics, only one vector out of this multitude must be chosen to "guide" the actual particle, and this choice depends on the position of all the other particles. Besides, particles don't act back on the multi-field; the stack of field values doesn't change whatever particles do.

What's the difference between this multi-field interpretation and Suárez' dispositions? The projections of the universal vector field on three-dimensional space coincide.But Suárez presents only one option for interpreting what fields are. In section 4.6, I discussed how fields (from an ontological point of view) can be either stuff, properties, or mere mathematical devices (see Lange, 2002). By interpreting the wave-function as a distribution of dispositions, Suárez is clearly on the side of properties. The multi-field interpretation, on the other hand, is liberal to its ontological reading and shows that there is a conceptual continuity between electrodynamics and quantum mechanics. Fig. 6.1 and Fig. 6.2, therefore, in fact represent in fact a multi-field.

This latter version of the multi-field interpretation of the wave-function shows that the wave-function can be seen as a field on three-dimensional space. We don't need to go as far as David Albert (2015, p. 130) in downgrading our world as a mere shadow of what happens in configuration space. We can retain our familiar space as the fundamental space, but if we have the wave-function as a field in this space we need to generalize what we mean by a field. Once we accept that this novel multi-field is a genuine physical entity, we have an interpretation of the wave-function at hand that makes it (the wave-function) ontologically local without changing the mathematics. It's now a local beable mathematically represented in configuration space.

But if you happen to be unsatisfied, because the mathematical representation of a local object must be in physical space as well,there is a quantum theory that can help: *the theory of exclusively local beables.* Let's zoom in and look at it more closely.

6.3.4. The Wave-Function as a Local Beable

Travis Norsen (2010) proposed a Bohmian quantum theory in which there is no longer a wave-function on configuration space (see also Norsen et al., 2015). Instead, the main dynamical entity is the conditional wave-function; that is to say, every particle is guided by a conditional wave-function. The conditional wave-functions, however, don't suffice to recover all the predictions of the de Broglie–Bohm theory, and Norsen presents a nice example showing why this is so.

Imagine two particles that are about to collide. We can prepare the system in two different ways. In the first case, we start with a non-entangled wave-function (see Fig. 6.3), and in the second case, we prepare the system to be entangled (see Fig. 6.4). The initial particle positions are the same. And, more importantly, the initial conditional wave-functions of both particles are the same, too. Yet we can prepare each system in such a way that the particles move differently after collision.

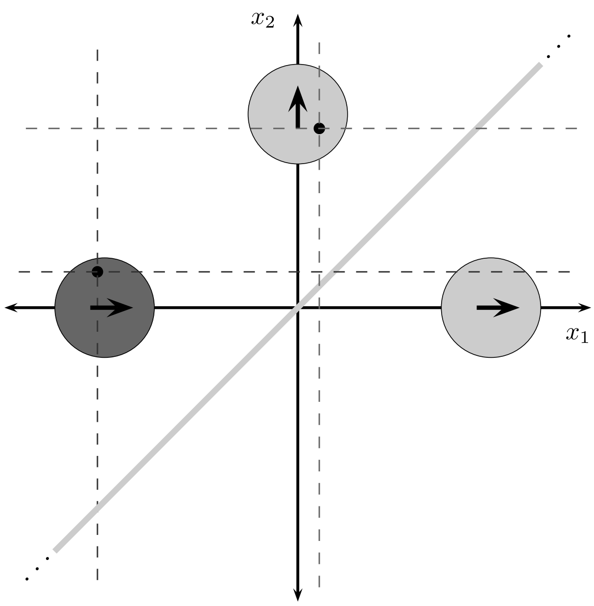

Figure 6.3.: Scattering of two non-entangled particles represented in the two-particle configuration space. A particle moves parallel to the x_1-axis approaching a particle that sits at $x_2 = 0$. The potential of the resting particle is marked as a light grey diagonal line. Their initial wave-function is marked in dark grey. After scattering, the first moving particle stops, and the other particle moves upwards. The wave-function is then in a superposition indicated by two light grey wave-functions. (Picture from Norsen, 2010, p. 1867)

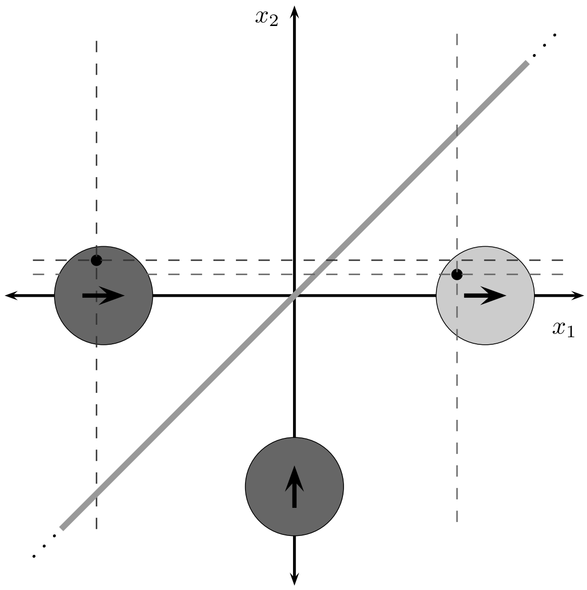

Figure 6.4.: Scattering of two entangled particles represented in the two-particle configuration space. As in Fig. 6.3, a particle approaches a resting particle from the left and collides at $x_2 = 0$. Their entangled initial wave-function is depicted in dark grey. After scattering, the resting particle starts moving to the right parallel to the x_1-axis, while the other particle stops at $x_2 = 0$. The post-scattering wave-function is drawn in light grey. (Picture from Norsen, 2010, p. 1868)

This shows that conditional wave-functions cannot do the job alone when it comes to retrieving all Bohmian trajectories. While conditional wave-functions can render the correct trajectories in the first example, they cannot do so for the entangled system. The information about entanglement gets lost in the definition of conditional wave-functions—it's the same for the reduced density matrix in an EPR experiment, where it merely gives us the statistics for one particle irrespective of what happens to the other particle.

Norsen's idea is now to add additional local fields to the conditional wave-functions. The task of these new fields is to change the conditional wave-functions of each particle in such a way that they render the correct trajectories even if the system is entangled; in fact, these fields are non-zero only if there is entanglement.

Norsen's theory of exclusively local beables makes the very same empirical predictions as the de Broglie–Bohm theory—it even predicts the very same trajectories—, but the price is a more contrived law for the evolution of all those local fields. First, it turns out that there are infinitely many such interacting fields since the evolution of the interaction fields requires further interaction fields—a never ending recursion. And it's not clear yet that one can get satisfactory results with only a finite set of these fields. Second, each conditional wave-function follows a modified Schrödinger equation, in which the other interaction fields are included. And these interaction fields themselves have their own evolution equation. This makes the theory almost useless for making calculations.

But this doesn't make the theory less interesting for metaphysics. On the contrary! First, Norsen gave us an outline of what a theory with local wave-functions might look like. Second, we can take Norsen's de Broglie–Bohm theory to investigate the ontological status of the wave-function(s), and see whether his physical theory makes a difference to the current debate. Here, I'll briefly discuss his theory in the context of dispositions.

Esfeld and Suarez deliver a dispositional interpretation that strive to have the wave-function represent something in three-dimensional space, although it is mathematically defined on configuration space. Either the wave-function represents a holistic disposition, or it represents a distribution of intrinsic dispositions in three-dimensional space.

Norsen's theory has only local beables; even the wave-function is turned into a local beable, since every particle is guided by its own quantum state. We can interpret every such (conditional) wave-function as representing primary dispositions, namely those that directly guide particles. And the interaction fields can be construed as secondary dispositions influencing the primary dispositions. By having these interaction fields in the ontology, this metaphysics of dispositions no longer suffers from a metaphysical or a physical action-at-a-distance. These shortcomings are now overcome. Remember that Suarez only has these primary dispositions in his ontology, and it's there mysterious how these dispositions are connected over arbitrary distances. But Norsen provides a *physical* mechanism in three-dimensional space showing how the position of a particle can influence the velocity of another one.

Since the wave-function is now a local beable, Norsen's theory is the first and only quantum theory that is ontologically local in physical space. Still, the motion of one particle depends on the positions of all particles. This dynamical non-locality is not something we can get rid of. The interacting fields must mediate the action with infinite speed in order to be consistent with Bell's theorem. At some point we always have to deal with quantum non-locality.

6.3.5. There are still Classical Properties, Aren't There?

As we have seen, the universal wave-function can be construed as a disposition, that is, a certain kind of property that causes motion. There are several versions on the table. The wave-function can be seen as a holistic disposition of all particles together, or it can be regarded as an ontic structure—I don't think there is a difference between these positions. Another possibility is to take the wave-function as representing a distribution of *intrinsic* dispositions in three-dimensional space.

What all these strategies have in common is that the wave-function is sufficient to generate trajectories: once the initial wave-function and the initial particle positions are given, the motion follows from the Schrödinger equation and the guiding equation. But there are still "classical" properties, such as mass and charge. What is their role in determining motion? As is generally agreed upon, in classical physics, they function as intrinsic dispositions, and it seems that in the quantum realm they are epiphenomena since they are not mentioned or used in either interpretation of the wave-function that I have presented so far. But the wave-function still might not be enough to fix the trajectories (given some initial conditions); maybe classical properties need to complement the wave-function.

It's important, though, not to be fooled by our classical intuitions. The de Broglie–Bohm theory isn't a classical theory. As I argued in section 1.2.3 the free parameters in the guiding equation can be named "mass" and "charge" once we get to the classical limit. But they are parameters in quantum law, so if they are construed as intrinsic properties they are different from their classical cousins. They may in some sense coincide within a certain regime, but due to their role in the new theory they are to be understood as novel entities.

It's not clear, however, whether Bohmian masses and Bohmian charges really reside in the particles. There are quantum phenomena that incline us to think that mass and charge are rather properties of the wave-function. If so, Bohmian masses and Bohmian charges would be second-order properties or second-order dispositions for the first-order disposition represented by the wave-function. There are even arguments aiming to show that mass and charge are in fact properties of both the wave-function and particles. Let's see how plausible all this is.

Does the Wave-Function bear Classical Properties?

The most compelling arguments that there may be some reality to the (effective) wave-function and that the (effective) wave-function carries mass and charge come from research on weak measurements (Aharonov et al., 1988) and protective measurements (Aharonov and Vaidman, 1993). Here, the measurement apparatus is very weakly coupled to the system so that a measurement doesn't change the quantum state—in standard language, weak and protective measurements don't collapse the wave-function. With the help of this idea, physicists have a new method for examining quantum systems; for example, they can now reconstruct the shape of the wave-function or measure the expectation value of an arbitrary observable by making measurements on just one quantum system.

The consequences for experimental physics aren't relevant here. What's important, though, is another experiment, in which weak interactions play a pivotal role. And this experiment can be used to show that mass and charge are properties of the wave-function.

Let's start with a preliminary set-up as in Fig. 6.5; we need this to illustrate how the trajectories of the wave-functions differ from the trajectories of particles (Bell, 2004b). As shown in the figure, the wave-functions follow what might be called classical trajectories. They cross the interference region and hit the screen. A Bohmian particle does something different. Let's say it enters the beam splitter and is guided by ψ_1. Like its guiding wave, it is reflected by the upper mirror

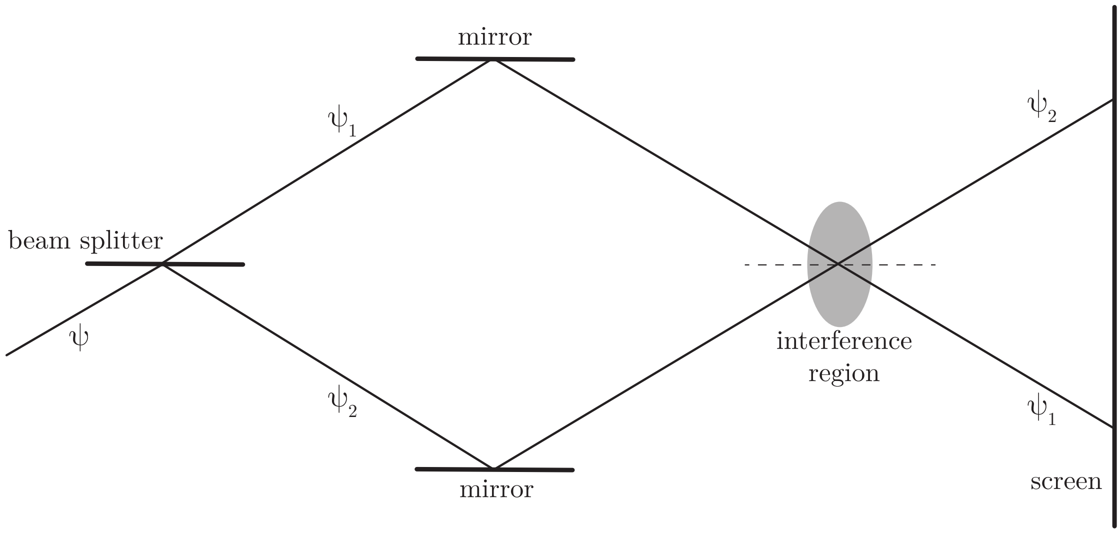

Figure 6.5.: An effective wave-function ψ enters a beam splitter and is split into two parts ψ_1 and ψ_2. One follows the upper path; the other follows the lower. After they are reflected by a mirror they interfere where both paths cross. Then ψ_1 hits the lower part of the screen, while ψ_2 hits the upper part.

and enters the interference region. And here something happens that we're not familiar with from classical physics. When the particle approaches the symmetry axis (dashed line), it bounces off and is suddenly guided by ψ_2, heading to the upper part of the screen. By symmetry a particle first guided by ψ_2 after the beam splitter will be guided by ψ_1 when it has reached the interference region.

We can now slightly, but very importantly, modify the experimental set-up—first presented by Englert et al. (1992), here in a simplified version from Dewdney et al. (1993). We insert a measurement device into one of the paths before the interference region. The device is built so that it weakly interferes with the wave-function of the particle in order to leave the quantum state unchanged. The crucial step is that we can prepare the entire system in such a way that the physical state of the weak measurement device will change, that is, it will measure something, although the particle takes the other path. It seems that the measurement device measures the trajectory of the particle without the particle being actually there. On this basis, Englert et al. (1992) dubbed Bohmian trajectories, those that are predicted by the guiding equation, *surrealistic*. The true path, according to them, is a different one, namely the one we can measure.

Dürr et al. (1993) and Dewdney et al. (1993) reply that there is nothing surrealistic about Bohmian trajectories and that there is nothing surprising in the fact that the weak measurement device changes in terms of its physical state when interacting with the empty wave (see also Barrett 1999, Ch. 5, and Barrett 2000). That's just a prediction of the theory. A detailed analysis of this model by means of

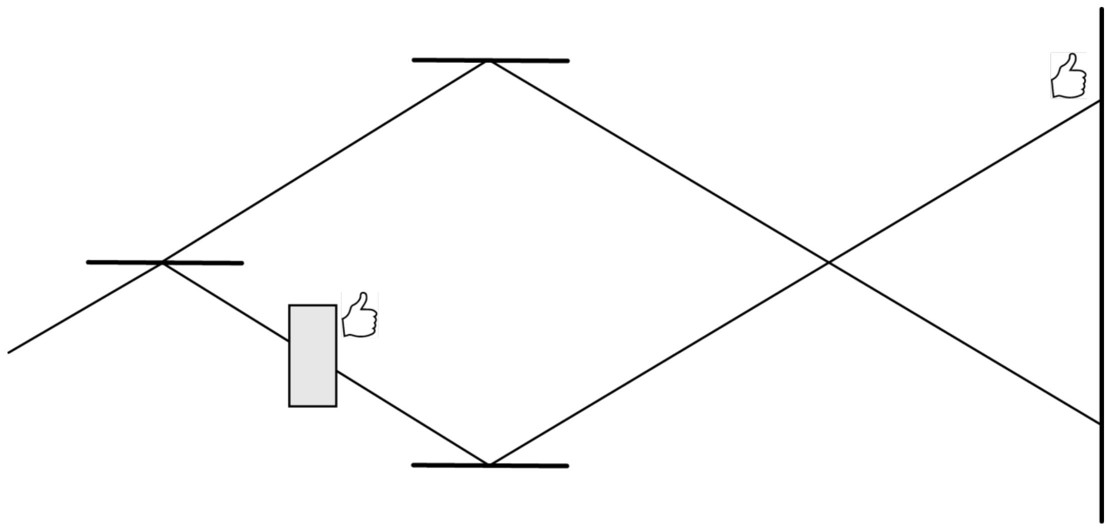

Figure 6.6.: This is the same arrangement as in Fig. 6.5 but with a weakly coupled measurement device (grey box) between the beam splitter and the lower mirror. We assume that the particle crosses the beam splitter and is guided by ψ_1. The de Broglie–Bohm theory tells us that the particle will hit the screen in the upper half. Nevertheless, the weakly coupled measurement device changes its state after the particle hits the screen. What has it measured?

Bohmian mechanics shows that under certain circumstances a measurement device measures something without there being any particle. That's a manifestation of non-locality rather than a flaw of the theory (Holland, 2014).

From a metaphysical point of view, one may reason that the wave-function itself must carry some dynamical properties in order to interact with matter. Since the particles in the weak measurement device have mass and charge—and they change their state after an interaction with the empty wave—the empty wave itself must carry mass and charge. This is one argument that Brown et al. (1996) use to assign properties to the wave-function—Brown et al. (1995) even claim that the wave-function carries gravitational mass. But for Brown et al. (1996) the wave-function cannot be the sole bearer of mass and charge; particles must have them too.

Classical Properties in the Particles and in the Wave–Function

Brown et al. (1996) use two arguments in order to show that the classical properties must be part of the wave-function and particles. Let's begin with the first one. Assume that the classical properties merely reside in the wave-function. Then particles are propertyless stuff, and there is no intrinsic feature by which to distinguish one particle from the other. This leads, Brown et al. (1996) claim, to the *problem of recognition*; that is, a wave-function cannot know how to guide a particle because it doesn't recognize which species the particle belongs to.

Imagine a wave-function that is in a superposition of a muon wave-function and an electron wave-function, and let's assume that these two wave packets are spatially separated. If the (propertyless) particle is in one of the wave packets it moves like a muon or like an electron. The problem of recognition arises if we let the two wave packets interfere: if the particle doesn't have intrinsic properties that identify it as either a muon or an electron the wave-function doesn't "know" how to guide this propertyless object. Is the particle in the interference regime a muon or an electron? Of course, if the particle were to have intrinsic properties, a specific mass and a specific charge, then the wave-function could decide how to guide it; the intrinsic properties uniquely identify the species of a particle.

According to Brown et al. (1996), there are two ways out of this. First, one may label particles; that is, "each corpuscle carried a distinct label which can be read by the associated ψ-field [...]. This first option is tantamount to accepting that the pilot wave interpretation is incomplete, since no such labels are specified in the theory" (p. 314). In my opinion, labelling particles doesn't solve the problem of recognition, because labels don't change the ontology. The wave-function doesn't care whether we label particles or not. Therefore, what Brown et al. mean to suggest, I assume, is equipping particles with a quiddity. This primitive identity can be assigned to particles without them having intrinsic properties. A particle would then be a muon or an electron because it carries with it an intrinsic primitive feature (or essence) that identifies the particle as belonging to this species irrespective of the wave-function. The way the wave-function guides a particle must of course be consistent with its quiddity; it's not possible for a muon to be guided by an electron wave-function. As Brown et al. correctly emphasize, quiddities aren't specified by the de Broglie–Bohm theory—indeed by any physical theory.[2]

A second way to solve the problem of recognition would be to distinguish particles not by their intrinsic features but by their motion, that is, by their history. An electron is an electron because it moves like an electron. And a muon is a muon because it moves like a muon. Brown et al. dismiss this idea since "[t]he second option amounts to an admission that the interpretation is *non-local in time as well as space*" (p. 314).

Introducing non-locality here is like killing a fly with a bulldozer. Usually we use non-locality in two different situations. One is an ontological reading, which coincides with the famous notion of separability, and another is a dynamical reading,

[2]There is a subtle difference between quiddity and haecceity (Wüthrich, 2009, p. 1042). Whereas quiddity is the same as type-identity, that is, a primitive feature that selects a type or kind an object belongs to, haecceity is much stronger. Haecceity is token-identity, that is, it specifies the identity of a particular object. In more metaphysical terms, quiddity is the essence of universals, and haecceity is the essence of particulars. Often any kind of primitive identity is dismissed. I think, however, that there are situations in which they are useful. It's true that we don't have empirical access to these primitive identities, but this doesn't mean we should dismiss them as occult concepts that need to be avoided at all costs, like a vampire avoiding garlic. For example, if you adhere to space-time substantivalism, and in particular to Newton's absolute space, the haecceity of each spatial point can prevent a substantival space from collapsing into one single point (Maudlin, 2012, p. 41).

when two objects non-locally interact. Neither of these cases applies to the history of particles. The trajectories are not non-separable, and nor is there a non-local dynamical influence from the past trajectory to future points on the trajectory.

Next, there are two things we need to distinguish. First, a particle's *momentary* motion specifies whether the particle is an electron or a muon provided that it's guided by an electron wave-function or a muon wave-function. Remember that it's (theoretically) possible for a particle to change its species once we bring two waves in to interfere. Second, the supposedly "non-local" aspect might be that the identity of a particle persists throughout its entire history, irrespective of the shape of the trajectory as long as it remains continuous. So a particle's history determines that at time $t = t_1$ we have the same particle—the same primitive stuff—as before, at time $t = t_0$. But that the identity of particles as grounded in their location in space is the only means we have for identity unless we introduce intrinsic properties, quiddities, or haecceities. And it's parsimonious and straightforward. You cannot disqualify the idea because it leads to an alleged non-locality.

In the light of a primitive stuff ontology, I think that the problem of recognition is a pseudo-problem. Only in the decoherence regime can we say that this particle here is an electron, while that particle over there is a muon. What determines the species of particles in a primitive stuff ontology is their motion, and their motion is determined by the wave-function. So if we happen to have an electron wave-function interfering with a muon wave-function, the particle in the interference region is neither an electron nor a muon. And the particle will still move in a unique way due to the velocity field generated by the interfering wave-functions. A wave-function will always correctly recognize the particle!

Here is the second argument from Brown et al. regarding why Bohmian particles carry intrinsic properties: mass and charge appear in the guiding equation. And they appear in the guiding equation in such a way that m_1 is assigned to particle 1, m_2 to particle 2, etc. This seems to be a pretty strong argument (the same that can be used in classical physics), unless you know that you can symmetrize the guiding equation without altering any empirical prediction (see section 5.2.2). In the symmetrized formalism, it's no longer possible to assign an intrinsic property to any particle. Only under special circumstances (the decoherence regime), can we assign particles mass and charge for practical or intuitive purposes.

One might be inclined to think that there are two versions of Bohmian mechanics, the standard version and the symmetrized version, which require two different ontologies, one with intrinsic properties and the other without. Since we cannot and should not read ontologies off of mathematical formalisms, we could have developed a primitive stuff ontology for the standard version of Bohmian mechanics. But within the symmetrized version this ontology is much more natural. In the end, we can think about symmetrized Bohmian mechanics for ontological purposes and use the standard version for making calculations.

Misapprehensions about Bohmian Particles

We're now in a position to clear up some misunderstandings about Bohmian particles. Ladyman and Ross try to downgrade the de Broglie–Bohm theory on some dubious arguments:

> Of course, there is a version of quantum theory, namely Bohm theory, according to which QM is not complete and particles do have definite trajectories at all times. However, Harvey Brown et al. (1996) argue that the 'particles' of Bohm theory are not those of classical mechanics. The dynamics of the theory are such that the properties, like mass, charge, and so on, normally associated with particles are in fact inherent in the quantum field and not in the particles. It seems that the particles only have position. We may be happy that trajectories are enough to individuate particles in Bohm theory, but what will distinguish an 'empty' trajectory from an 'occupied' one? Since none of the physical properties ascribed to the particle will actually inhere in points of the trajectory, giving content to the claim that there is actually a 'particle' there would seem to require some notion of the raw stuff of the particle; in other words haecceities seem to be needed for the individuality of particles of Bohm theory too. (2007, p. 136)

Let's unpick this paragraph. First, Ladyman and Ross misrepresent the aims of Brown et al.. Brown et al. clearly argue that mass and charge are properties of *both* particles and the wave-function. Second, the rhetorical question "What will distinguish an 'empty' trajectory from an 'occupied' one?" rests on a confusion about the meaning of experiments with empty waves, which I discussed above. The actual trajectories of Bohmian particles are those that are predicted by the guiding equation. An "unoccupied" trajectory is an oxymoron; it doesn't make sense, and it doesn't exist. The interaction of an empty wave with a measurement device or the traces of an empty wave in a bubble chamber (see Vaidman, 2005) are often misnamed "unoccupied" or "empty" trajectories—notions that expel these experimental facts as contradictions within the de Broglie–Bohm theory. But due to non-locality in Bohmian mechanics, empty waves can affect other particles. As I've repeatedly argued, if we interpret particles as raw stuff their individuality still doesn't need haecceity. So the initial appeal of the arguments by Ladyman and Ross against the de Broglie–Bohm theory vanishes into thin air.[3]

One might read the above quote as saying that Ladyman and Ross are concerned with the difference between a bare particle and a spatial point. What distinguishes a particle from a spatial point when the particle doesn't have intrinsic properties? In my opinion, this is yet another occasion for introducing benign primitive identities.

[3] People often argue against Bohmian mechanics because it's non-local and it doesn't allow for a relativistic extension. Both arguments can be successfully countered. A natural reading of Bell's theorem—and in fact Bell's own understanding of his theorem—shows that there are non-local connections in our world (see, for instance, Norsen, 2006, 2009, 2011). A relativistic Bohmian theory is possible, but it forces us to be very clear on what we mean by *relativistic* (see Maudlin, 2011; Dürr et al., 2014).

Not haecceities, since they are too strong, but quiddities. A particle has a quiddity that indicates that it's a particle and not a point in space. Two particles can be distinguished by their location in space; no haecceities are needed for that. For points of space, however, we need haecceities to distinguish that this point here is different from that point over there. But that's harmless.

The Nomological Interpretation of Classical Properties

What, now, is the true status of classical properties in the de Broglie–Bohm theory? We have discussed very strong arguments supporting the view that mass and charge reside as intrinsic properties in Bohmian particles. Yet, to every one of these arguments, even if they involve empirical phenomena, there is an equal or even more convincing counter. If mass and charge aren't intrinsic properties of particles or the wave-function, what are they?

Mass and charge have a dynamical role, because when you change them you change the shape of trajectories—on the mathematical level the change takes place in the guiding equation, as well as in the Schrödinger equation. Primitive propertyless particles can be maintained when you regard the wave-function as a disposition governing the motion of particles and the "classical properties" as *nomological*.

By interpreting properties as nomological, they are no longer properties of anything; rather, they are physical parameters in the law that fine-tune the disposition—laws in dispositionalism are but representations of the dispositions. Intrinsic properties sit on particles, and they don't change what the particles do. Whether an intrinsic property is categorical or dispositional doesn't change that the property is part of the essence of a particle. Hence, by having mass and charge as nomological "properties" the only dynamical efficacious entities in the ontology of Bohmian mechanics are dispositions represented by the universal wave-function, and in the mathematical description of these dispositions (beware: in Esfeld's proposal there is just *one* holistic disposition) we find physical parameters, namely mass and charge, which specify the dispositions.

Results and Future Research

This thesis provides the following results in metaphysics and philosophy of physics:

1. I gave a historical and conceptual analysis of John Bell's local beables and primitive ontology. I pointed out that there is a slight but crucial difference between these concepts.

2. Based on Esfeld, Lazarovici, Lam, and Hubert (2015b), I discussed and developed a *primitive stuff ontology*.

3. Also based on the above paper, I introduced and discussed a novel version of Humeanism that dispenses with local qualities.

4. I gave a new critique of Ladyman's ontic structural realism.

5. I analyzed mass and forces in classical mechanics. I embedded the status of mass in the current metaphysical debate and criticized the major contemporary interpretations of forces.

6. I explicitly mentioned and explained the self-interaction problem in classical electrodynamics, which is still ignored by physicists and philosophers alike.

7. I criticized Mathias Frisch's argument that classical electrodynamics is inconsistent.

8. I evaluated the arguments regarding whether there is backward causation in classical electrodynamics and concluded that there is not.

9. I highlighted the most important arguments regarding why electromagnetic fields may exist.

10. I gave the first philosophical discussion of the Born–Infeld theory and the Bopp–Podolsky theory.

11. I discussed recently published work by Deckert and Hartenstein (2016) showing that the initial value problem of classical electrodynamics is ill-defined unless one restricts the initial values.

12. I countered old and recent arguments claiming that the first-order formulation of the de Broglie–Bohm theory is explanatorily weaker than the second-order formulation.

13. I criticized the current discussion of identical particles and showed that all problems can be solved within the de Broglie–Bohm theory by means of a primitive stuff ontology.

14. I pointed out that the wave-function cannot be construed as a law and that the effective wave-function is not quasi-nomological.

15. I showed how the wave-function can be interpreted in the framework of super-Humeanism (see also Esfeld, Lazarovici, Lam, and Hubert, 2015b).

16. I discussed the dispositional interpretation of the wave-function (see also Esfeld, Lazarovici, Hubert, and Dürr, 2014).

17. I showed how the wave-function in the de Broglie–Bohm theory can be seen as a multi–field.

18. I made the connection between Norsen's theory of exclusively local beables and a dispositional interpretation of the wave-function.

19. Based on Esfeld, Lazarovici, Lam, and Hubert (2015b), I showed how classical properties are better regarded as nomological in the de Broglie–Bohm theory.

My thesis shows that there are a lot of old and new problems to be tackled by philosophers and physicists alike. Four future projects are of particular interest for philosophers of physics:

1. A theory of primitive stuff with primitive laws.

2. A study of the status of fields within the primitive stuff ontology.

3. A detailed philosophical analysis of the work of Deckert and Hartenstein (2016). Does their proof serve as a novel derivation of the Wheeler–Feynman theory?

4. Analysis of whether we need a new concept of laws of nature in order to understand theories like the Wheeler–Feynman theory, which aren't formulated as initial value problems. Would the ideas of Whitehead (1925) suit such a new understanding?

5. A detailed study of the multi-field interpretation of the wave-function in the de Broglie–Bohm theory. Can it be extended to other quantum theories?

A. Newton's Three Aspects of Forces

One of Newton's goals in the *Principia* was to explain the motion of planets. Therefore, he defines centripetal forces very early.

> **Definition 5**
> *Centripetal force is the force by which bodies are drawn from all sides, are impelled, or in any way tend, toward some point as to a center.*
> One force if this kind is gravity, by which bodies tend toward the center of the earth; another is magnetic force, by which iron seeks a lodestone; and yet another is that force, whatever it may be, by which the planets are continually drawn back from rectilinear motions and compelled to revolve in curved lines. (Newton, 1999, p. 405)

Then he defines three aspects of centripetal forces, but it's clear from his description that they are to be found in all forces.

> **Definition 6**
> *The absolute quantity of centripetal force is the measure of this force that is greater or less in proportion to the efficacy of the cause propagating it from a center through the surrounding regions.*
> An example is magnetic force, which is greater in one lodestone and less in another, in proportion to the bulk or potency of the lodestone. (Newton, 1999, p. 406)
>
> **Definition 7**
> *The accelerative quantity of centripetal force is the measure of this force that is proportional to the velocity which it generates in a given time.*
> One example is the potency of a lodestone, which, for a given lodestone is greater at a smaller distance and less at a greater distance. Another example is the force that produces gravity, which is greater in valleys and less on the peaks of high mountains and still less (as will be made clear below) at greater distances from the body of the earth, but which is everywhere the same at equal distances, because it equally accelerates all falling bodies (heavy or light, great or small), provided that the resistance of the air is removed. (Newton, 1999, p. 407)
>
> **Definition 8**
> *The motive quantity of centripetal force is the measure of this force that is proportional to the motion which it generates in a given time.*

An example is weight, which is greater in a larger body and less in a smaller body; and in one and the same body is greater near the earth and less out the heavens. This quantity is centripetency, or propensity toward a center, of the whole body, and (so to speak) its weight, and it ay always be known from the force opposite and equal to it, which can prevent the body from falling. (Newton, 1999, p. 407)

These quantities of forces, for the sake of brevity, may be called motive, accelerative, and absolute forces, and, for the sake of differentiation, may be referred to bodies seeking a center, to the places of the bodies, and to the center of the forces: that is, motive force my be referred to a body as an endeavor of the whole directed toward a center and compounded of the endeavors of all the parts; accelerative force, to the place of the body as a certain efficacy diffused from the center through each of the surrounding places in order to move the bodies that are in those places; and absolute force, to the center as having some cause without which the motive forces are not propagated through the surrounding regions, whether this cause is some central body (such as a lodestone in the center of a magnetic force or the earth in the center of a force that produces gravity) or whether it is some other cause which is not apparent. This concept is purely mathematical, for I am not now considering the physical causes and sites of forces.

Therefore, accelerative force is to motive force as velocity to motion. For quantity of motion arises from velocity and quantity of matter jointly, and motive force from accelerative force and quantity of matter jointly. For the sum of the actions of the accelerative force on the individual particles of body is the motive force of the whole body. As a consequence, near the surface of the earth, where the accelerative gravity, or the force that produces gravity, is the same in all bodies universally, the motive gravity, or weight, is as the body, but in an ascent to regions where the accelerative gravity becomes less, the weight will decrease proportionately and will always be as the body and the accelerative gravity jointly. Thus in regions where the accelerative gravity is half as great, a body one-half of one-third as great will have a weight four or six times less. Further, it is in this same sense that I call attractions and impulses accelerative and motive. Moreover, I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, let the ready beware of thinking that by words of this kind I am anywhere defining a species or mode of action or physical cause or reason, or that I am attributing forces in a true and physical sense to centers (which are mathematical points) if I happen to say that centers attract or that centers have forces. (Newton, 1999, pp. 407–8)

B. How Dirac Introduced Mass Renormalization

Dirac (1938) never mentioned the decomposition

$$F^{\text{ret}}_{\mu\nu} = \frac{1}{2}F^{\text{rad}}_{\mu\nu} + \frac{1}{2}F^{\text{near}}_{\mu\nu}.$$

But from the formulas he gave we can derive this equation.

The Maxwell equations don't distinguish between the retarded and advanced fields; both can be used to build up the actual field $F^{\mu\nu}_{act}$ at a position x. If we pick out the retarded fields we need to add the external fields $F^{\mu\nu}_{in}$, which Dirac calls *incident fields*:

$$F^{\mu\nu}_{\text{act}} = F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{in}}.$$

If we pick out the advanced fields we get the decomposition

$$F^{\mu\nu}_{\text{act}} = F^{\mu\nu}_{\text{adv}} + F^{\mu\nu}_{\text{out}},$$

thereby defining the field $F^{\mu\nu}_{\text{out}}$ of outgoing radiation.

It turns out that the radiation field $F^{\mu\nu}_{\text{rad}}$ has the following form

$$F^{\mu\nu}_{\text{rad}} = F^{\mu\nu}_{\text{out}} - F^{\mu\nu}_{\text{in}} = F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}}.$$

Then Dirac defines a new field

$$f^{\mu\nu} := F^{\mu\nu}_{\text{act}} - \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}\right) = \frac{1}{2}\left(F^{\mu\nu}_{\text{in}} + F^{\mu\nu}_{\text{out}}\right).$$

Now from this new field we get the decomposition of the retarded field into the radiation and near field:

$$\begin{aligned} F^{\mu\nu}_{\text{ret}} &= \frac{1}{2}\left(F^{\mu\nu}_{\text{in}} + F^{\mu\nu}_{\text{out}}\right) - F^{\mu\nu}_{\text{in}} + \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}\right) \\ &= \frac{1}{2}\underbrace{\left(F^{\mu\nu}_{\text{out}} - F^{\mu\nu}_{\text{in}}\right)}_{F^{\mu\nu}_{\text{rad}}} + \frac{1}{2}\underbrace{\left(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}\right)}_{F^{\mu\nu}_{\text{near}}}. \end{aligned}$$

Now let's follow how Dirac derived the Lorentz–Dirac equation. In order to do so, he approached the field $f^{\mu\nu}$ from two sides.

On the one hand, it follows from the definition

$$\begin{aligned} f^{\mu\nu} &= F^{\mu\nu}_{\text{act}} - \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}\right) \\ &= F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{in}} - \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} + F^{\mu\nu}_{\text{adv}}\right) \\ &= F_{\text{in}^{\mu\nu}} + \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}}\right) \\ &= F^{\mu\nu}_{\text{in}} + \frac{1}{2}F^{\mu\nu}_{\text{rad}} \end{aligned}$$

As Dirac shows in the Appendix (see also Barut, 1980, Chap. V, Sec. 5C) that

$$F^{\text{rad}}_{\mu\nu} = \frac{4\text{e}}{3}\left(\dddot{v}_\mu v_\nu - \dddot{v}_\nu v_\mu\right),$$

we get

$$f^{\nu}_{\mu} = F^{\nu}_{\text{in}\,\mu} + \frac{2\text{e}}{3}\left(\dddot{v}_\mu v^\nu - \dddot{v}^\nu v_\mu\right). \tag{B.1}$$

And now comes the mass renormalization procedure in order to get rid of f^{ν}_{μ}. The argument rests on energy–momentum conservation. We imagine the world-line of a particle to be surrounded by a thin tube, whose surface has a distance ϵ from the world-line (see Fig. B.1).

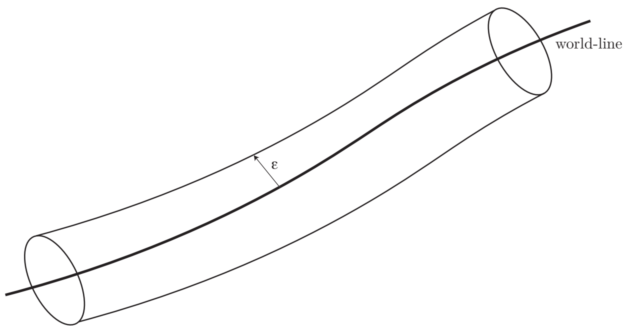

Figure B.1.: Dirac's reasoning for mass renormalization

Dirac calculated the flow of energy and momentum crossing the surface of the tube. In doing so he used the stress–energy tensor $T_{\mu\rho}$ given by

$$4\pi T_{\mu\rho} = F_{\text{act}\,\mu\nu}F^{\nu}_{\text{act}\,\rho} + \frac{1}{4}g_{\mu\rho}F_{\text{act}\,\alpha\beta}F^{\alpha\beta}_{\text{act}},$$

where $g_{\mu\rho}$ is the Minkowski metric.

Dirac very precisely describes his strategy:

We then have the information that the total flow of energy (or momentum) out from the surface of any finite length of tube must equal the difference in the energy (or momentum) residing within the tube at the two ends of this length, and must thus depend only on conditions at the two ends of this length. In mathematical language, the rate of flow of energy (or momentum) out from the surface of the tube must be a perfect differential.

It is easily seen that the information obtained in this way is independent of what shape and size we give to our tube (provided it is sufficiently small for the Taylor expansions used in the calculations to be valid). If we take two tubes surrounding the singular world-line, the divergence of the stress tensor will vanish everywhere in the region of space-time between them, since there are no singularities in this region [...]. Expressing the integral

$$\int\int\int\int \frac{\partial T_{\mu\rho}}{\partial x_\rho} \cdot \mathrm{d}x_0 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3$$

over the region of space-time between a certain length of the two tubes as a surface integral over the (three-dimensional) surface of this region, we obtain immediately that the difference in the flows of energy (or momentum) across the surfaces of the two tubes depends only on conditions at the two ends of the length considered. Thus the information provided by the conservation laws is well defined. (Dirac, 1938, p. 153)

Dirac calculated (and that's rather cumbersome) the flow of energy and momentum through the surface of the tube to be

$$\int \left(\frac{e^2}{2\epsilon} \dot{v}_\mu - e v_\nu f_\mu^\nu \right) \mathrm{d}s,$$

which is a vectorial integral over the length of the tube. Beware that this integral isn't exact; it's part of a Taylor series, where the higher order terms in ϵ were omitted.

As Dirac explained above, the integrand must be a conservative vector-field. So there is a vector B_μ such that

$$\frac{e^2}{2\epsilon} \dot{v}_\mu - e v_\nu f_\mu^\nu = \dot{B}_\mu. \tag{B.2}$$

The whole procedure of energy–momentum conservation was aimed at deriving this very equation.

Now we need to specify B_μ. Since a short calculation shows that $\boldsymbol{v} \cdot \boldsymbol{B} = 0$, the simplest choice of $\boldsymbol{B}$ would be to have it parallel to the velocity of the particle, that is,

$$B_\mu = k v_\mu,$$

with some constant k.

We then get

$$\left(\frac{e^2}{2\epsilon} - k \right) \dot{v}_\mu = e v_\nu f_\mu^\nu.$$

Dirac postulated that the term in parentheses to be a constant m, that is *independent* of ϵ, that is,

$$m := \frac{e^2}{2\epsilon} - k. \tag{B.3}$$

Here it is where Dirac stipulated a term that goes to infinity to be *finite*. This move give us the equation

$$m\dot{v}_\mu = ev_\nu f^\nu_\mu. \tag{B.4}$$

And with his former calculation (B.1), we get the Lorentz–Dirac equation

$$m\dot{v}_\mu - \frac{2}{3}e^2\ddot{v}_\mu - \frac{2}{3}e^2\dot{\boldsymbol{v}}^2 v_\mu = ev_\nu F^\nu_{\mathrm{in}\,\mu}.$$

The zeroth component of this equation,

$$m\dot{v}_0 - \frac{2}{3}e^2\ddot{v}_0 - \frac{2}{3}e^2\dot{\boldsymbol{v}}^2 v_0 = ev_\nu F^\nu_{\mathrm{in}\,0},$$

has a nice physical interpretation. The first term, $m\dot{v}_0$ is the derivative of the relativistic kinetic energy of the particle mv_0. The second term, $-\frac{2}{3}e^2\ddot{v}_0$, is the derivative of $-\frac{2}{3}e^2\dot{v}_0$, which can be interpreted as a kind of acceleration energy of the particle. Changes in the accelerative energy correspond to a reversible form of emission or absorption of field energy, which never gets far from the electron. It so describes the effects of the (now finite) near field. The third term, $-\frac{2}{3}e^2\dot{\boldsymbol{v}}^2 v_0$, is the radiation damping term describing the irreversible emission of radiation. Of course, the term on the right side of the equation represents the external fields doing work on the particle.

Let's end with how Dirac interprets mass renormalization:

> Let us see how the kinetic energy term arises. The B, introduced in [(B.2)] can be interpreted as minus the vector of energy and momentum residing within the tube at any value of the proper-time. Thus, from [(B.3)] and [(B.4)], the energy within the tube must be negative and must tend to $-\infty$ as ϵ tends to zero. This negative energy is needed to compensate for the large positive energy of the Coulomb field just outside the tube, to keep the total energy down to the value appropriate to the rest-mass m. If we want a model of the electron, we must suppose that there is an infinite negative mass at its centre such that, when subtracted from the infinite positive mass of the surrounding Coulomb field, the difference is well defined and is just equal to m. Such a model is hardly a plausible one according to current physical ideas but, as discussed in the Introduction, this is not an objection to the theory provided we have a reasonable mathematical scheme. (1938, p. 155)

Bibliography

M. Abraham. *Theorie der Elektrizität: Elektromagnetische Theorie der Strahlung.* Leipzig: B. G. Teubner, 1908.

C. G. Adler. Does mass really depend on velocity, dad? *American Journal of Physics*, 55(8):739–43, 1987.

Y. Aharonov and L. Vaidman. Measurement of the schrödinger wave of a single particle. *Physics Letters A*, 178(1–2):38–42, 1993.

Y. Aharonov, D. Z. Albert, and L. Vaidman. How the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100. *Physical Review Letters*, 60(14):1351–4, 1988.

D. Albert and B. Loewer. Interpreting the many worlds interpretation. *Synthese*, 77(2):195–213, 1988.

D. Z. Albert. Elementary quantum metaphysics. In J. T. Cushing, A. Fine, and S. Goldstein, editors, *Bohmian Mechanics and Quantum Theory: An Appraisal*, pages 277–84. Springer Netherlands, 1996.

D. Z. Albert. Wave function realism. In A. Ney and D. Z. Albert, editors, *The Wave Function: Essays On The Metaphysics of Quantum Mechanics*, chapter 1, pages 52–7. New York: Oxford University Press, 2013.

D. Z. Albert. *After Physics.* Cambridge, MA: Harvard University Press, 2015.

V. Allori. Primitive ontology and the structure of fundamental physical theories. In A. Ney and D. Z. Albert, editors, *The Wave Function: Essays on the Metaphysics of Quantum Mechanics*, pages 58–75. New York: Oxford University Press, 2013.

V. Allori, S. Goldstein, R. Tumulka, and N. Zanghì. Predictions and primitive ontology in quantum foundations: a study of examples. *The British Journal for the Philosophy of Science*, 65(2):323–52, 2014.

J. L. Anderson. *Principles of Relativity Physics.* New York: Academic Press, 1967.

G. Bacciagaluppi and A. Valentini. *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference.* Cambridge, UK: Cambridge University Press, 2009.

J. Barbour. *The End of Time.* New York: Oxford University Press, 2000.

J. Barbour and B. Bertotti. Mach's principle and the structure of dynamical theories. *Proceedings of the Royal Society A*, 382:295– 306, 1982.

J. A. Barrett. *Quantum Mechanics of Minds and Worlds*. New York: Oxford University Press, 1999.

J. A. Barrett. The persistence of memory: Surreal trajectories in bohm's theory. Extended version of the talk given at the UC Irvine Laguna Beach Physics Conference, February 2000. URL `https://arxiv.org/abs/quant-ph/0002046`.

A. O. Barut. *Electrodynamics and Classical Theory of Fields and Particles*. New York: Dover Publications, 1980.

G. Bauer. *Ein Existenzsatz für die Wheeler-Feynman Elektrodynamik*. PhD thesis, LMU Munich, Department of Mathematics, April 1997.

G. Bauer, D.-A. Deckert, and D. Dürr. On the existence of dynamics of Wheeler-Feynman electromagnetism. *Zeitschrift für angewandte Mathematik und Physik*, 64(4):1087–124, 2013.

J. S. Bell. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge, UK: Cambridge University Press, 1987.

J. S. Bell. Beables for quantum field theory. In *Speakable and Unspeakable in Quantum Mechanics*. Cambridge, UK: Cambridge University Press, 2004a.

J. S. Bell. de Broglie–Bohm, delayed-choice double–slit experiment, and density matrix. In *Speakable and Unspeakable in Quantum Mechanics*, chapter 14, pages 111–6. Cambridge, UK: Cambridge University Press, 2004b.

J. S. Bell. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge, UK: Cambridge University Press, 2nd edition, 2004c.

G. Belot. Is classical electrodynamics an inconsistent theory? *Canadian Journal of Philosophy*, 37(2):263–82, 2007.

G. Belot. Quantum states for primitive ontologists. *European Journal for Philosophy of Science*, 2(1):67–83, 2012.

D. W. Belousek. Statistics, symmetry, and (in)distinguishability in bohmian mechanics. *Foundations of Physics*, 30(1):153–64, 2000.

D. W. Belousek. Formalism, ontology and methodology in Bohmian mechanics. *Foundations of Science*, 8(2):109–72, 2003.

H. Bhogal and Z. Perry. What the humean should say about entanglement. *Noûs*, forthcoming, 2015. doi: 10.1111/nous.12095.

J. Bigelow, B. Ellis, and R. Pargetter. Forces. *Philosophy of Science*, 55(4):614–30, 1988.

A. Bird. *Nature's Metaphysics: Laws and Properties*. New York: Oxford University Press, 2007.

R. Black. Against quidditism. *Australasian Journal of Philosophy*, 78(1):87–104, 2000.

J. Blondeau and M. Ghins. Is there an intrinsic criterion for causal lawlike statements? *International Studies in the Philosophy of Science*, 26(4):381–401, 2012.

D. Bohm. A suggested interpretation of the quantum theory in terms of "hidden" variables. I. *Physical Review*, 85(2):166–79, 1952a.

D. Bohm. A suggested interpretation of the quantum theory in terms of "hidden" variables. II. *Physical Review*, 85(2):180–93, 1952b.

D. Bohm and B. J. Hiley. *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. London: Routledge, 1993.

F. Bopp. Eine lineare Theorie des Elektrons. *Annalen der Physik*, 430(5):345–84, 1940.

M. Born. *The Born-Einstein Letters*. London: Macmillan, 1971.

M. Born and L. Infeld. Foundations of the new field theory. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 144(852):425–51, 1934.

E. Breitenberger. Magnetic interactions between charged particles. *American Journal of Physics*, 36(6):505–15, 1968.

H. Brown, C. Dewdney, and G. Horton. Bohm particles and their detection in the light of neutron interferometry. *Foundations of Physics*, 25(2):329–47, 1995.

H. Brown, A. Elby, and R. Weingard. Cause and effect in the pilot-wave interpretation of quantum mechanics. In J. T. Cushing, A. Fine, and S. Goldstein, editors, *Bohmian Mechanics and Quantum Theory: An Appraisal*, pages 309–19. Springer Netherlands, 1996.

I. B. Cohen. *Isaac Newton's Papers and Letters in Natural Philosophy*. Cambridge, MA: Harvard University Press, 1958.

S. Coleman. Classical electron theory from a modern standpoint. The Rand Corporation, 1961. URL `http://www.rand.org/content/dam/rand/pubs/research_memoranda/2006/RM2820.pdf`.

S. Coleman. Acausality. In A. Zichichi, editor, *Subnuclear Phenomena Part A*, pages 282–327. New York: Academic Press, 1970.

S. Cowling. Haecceitism. The Stanford Encyclopedia of Philosophy, 2015. URL `http://plato.stanford.edu/archives/win2015/entries/haecceitism/`.

E. Curiel. Classical mechanics is Lagrangian; it is not Hamiltonian. *The British Journal for the Philosophy of Science*, 65(2):269–321, 2014.

L. de Broglie. La nouvelle dynamique des quanta. In *Electrons et photons. Rapports et discussions du cinquième Conseil de physique tenu à Bruxelles du 24 au 29 octobre 1927 sous les auspices de l'Institut international de physique Solvay*, pages 105–32. Paris: Gauthier-Villars, 1928. English translation in G. Bacciagaluppi and A. Valentini. *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*, pages 341–71. Cambridge, UK: Cambridge University Press, 2009.

D.-A. Deckert. *Electrodynamic Absorber Theory: A Mathematical Study.* PhD thesis, LMU Munich, Department of Mathematics, October 2009.

D.-A. Deckert and V. Hartenstein. On the initial value formulation of classical electrodynamics. Draft, 2016. URL `http://arxiv.org/abs/1602.04685`.

D.-A. Deckert, M. Esfeld, and A. Oldofredi. Bohmian mechanics for QFT: a persistent particle ontology in terms of the dirac sea. *Forthcoming in The British Journal for the Philosophy of Science*, 2016.

C. Dewdney, L. Hardy, and E. Squires. How late measurements of quantum trajectories can fool a detector. *Physics Letters A*, 184(1):6–11, 1993.

P. A. M. Dirac. Théorie du positron. In *Rapport du 7ème Conseil Solvay de physique: structure et propriétés des noyaux atomiques.* Paris: Gauthier-Villars, 1934. Reprinted in Julian Schwinger (ed.): Selected papers on quantum electrodynamics. New York: Dover 1958. Chapter 7, pp. 82–91.

P. A. M. Dirac. Classical theory of radiating electrons. *Proceedings of the Royal Society A, Mathematical and Physical Sciences*, 167(929):148–69, 1938.

D. Dürr and S. Teufel. *Bohmian Mechanics: The Physics and Mathematics of Quantum Theory.* Berlin: Springer, 2009.

D. Dürr, S. Goldstein, and N. Zanghì. Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics*, 67(5):843–907, 1992.

D. Dürr, W. Fusseder, S. Goldstein, and N. Zanghì. Comment on "surrealistic bohm trajectories". *Zeitschrift für Naturforschung*, 48a:1261–2, 1993.

D. Dürr, S. Goldstein, and N. Zanghì. Bohmian mechanics and the meaning of the wave function. In R. S. Cohen, M. Horne, and J. Stachel, editors, *Experimental Metaphysics: Quantum Mechanical Studies for Abner Shimony Vol. 1*, pages 25–38. Dordrecht: Kluwer, 1997.

D. Dürr, S. Goldstein, J. Taylor, R. Tumulka, and N. Zanghì. Topological factors derived from Bohmian mechanics. *Annales Henri Poincaré*, 7(4):791–807, 2006.

D. Dürr, S. Goldstein, and N. Zanghì. *Quantum Physics without Quantum Philosophy*. Heidelberg: Springer, 2013.

D. Dürr, S. Goldstein, T. Norsen, W. Struyve, and N. Zanghì. Can bohmian mechanics be made relativistic? *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, 470(2162), 2014. doi: 10.1098/rspa.2013.0699.

J. Earman. An attempt to add a little dircetion to "the problem of the direction of time". *Philosophy of Science*, 41(1):15–47, 1974.

J. Earman. Causation: A matter of life and death. *The Journal of Philosophy*, 73 (1):5–25, 1976.

A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47(10):777–80, 1935.

B.-G. Englert, M. O. Scully, G. Süssmann, and H. Walther. Surrealistic Bohm trajectories. *Zeitschrift für Naturforschung*, 47a:1175–1186, 1992.

M. Esfeld. Quantum entanglement and a metaphysics of relations. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 35(4):601–17, 2004.

M. Esfeld. The modal nature of structures in ontic structural realism. *International Studies in the Philosophy of Science*, 23(2):179–94, 2009.

M. Esfeld. Ontic structural realism and the interpretation of quantum mechanics. *European Journal for Philosophy of Science*, 3(1):19–32, 2013.

M. Esfeld. How to account for quantum non-locality: ontic structural realism and the primitive ontology of quantum physics. *Synthese*, forthcoming:1–16, 2014. doi: 10.1007/s11229-014-0549-4.

M. Esfeld and V. Lam. Ontic structural realism as a metaphysics of objects. In A. Bokulich and P. Bokulich, editors, *Scientific Structuralism*, chapter 8, pages 143–159. Dordrecht: Springer, 2011.

M. Esfeld, D. Lazarovici, M. Hubert, and D. Dürr. The ontology of Bohmian mechanics. *The British Journal for the Philosophy of Science*, 65(4):773–96, 2014.

M. Esfeld, D.-A. Deckert, and A. Oldofredi. What is matter? The fundamental ontology of atomism and structural realism. In A. Ijjas and B. Loewer, editors, *A guide to the philosophy of cosmology*. Oxford: Oxford University Press, 2015a.

M. Esfeld, D. Lazarovici, V. Lam, and M. Hubert. The physics and metaphysics of primitive stuff. *The British Journal for the Philosophy of Science*, advance access, 2015b. doi: 10.1093/bjps/axv026.

R. Feynman. The development of the space-time view of quantum electrodynamics. *Science*, 153(3737):669–708, 1966.

R. Feynman, R. B. Leighton, and M. Sands. *The Feynman Lectures on Physics Vol. 2*. Addison Wesley, 1977.

P. Forrest. *Quantum Metaphysics*. Oxford: Basil Blackwell, 1988.

S. French. The interdependence of structure, objects and dependence. *Synthese*, 175(1):89–109, 2010.

S. French. *The Structure of the World: Metaphysics and Representation*. Oxford: Oxford University Press, 2014.

S. French and J. Ladyman. In defence of ontic structural realism. In A. Bokulich and P. Bokulich, editors, *Scientific Structuralism*, chapter 2, pages 25–42. Dordrecht: Springer, 2011.

S. French and M. Redhead. Quantum physics and the identity of indiscernibles. *The British Journal for the Philosophy of Science*, 39(2):233–46, 1988.

M. Frisch. *Inconsistency, Asymmetry, and Non-Locality: A Philosophical Investigation of Classical Electrodynamics*. New York: Oxford University Press, 2005.

M. Frisch. Conceptual problems in classical electrodynamics. *Philosophy of Science*, 75(1):93–105, 2008.

G. C. Ghirardi, A. Rimini, and T. Weber. Unified dynamics for microscopic and macroscopic systems. *Physical Review D*, 34(2):470–91, 1986.

G. C. Ghirardi, R. Grassi, and F. Benatti. Describing the macroscopic world: closing the circle within the dynamical reduction program. *Foundations of Physics*, 25(1):5–38, 1995.

S. Goldstein and N. Zanghì. Reality and the role of the wave function in quantum theory. In D. Dürr, S. Goldstein, and N. Zanghì, editors, *Quantum Physics without Quantum Philosophy*, chapter 12, pages 263–78. Heidelberg: Springer, 2013.

S. Goldstein, J. Taylor, R. Tumulka, and N. Zanghì. Are all particles identical? *Journal of Physics A: Mathematical and General*, 38(7):1567–76, 2005a.

S. Goldstein, J. Taylor, R. Tumulka, and N. Zanghì. Are all particles real? *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 36(1):103–12, 2005b.

D. W. Graham, editor. *The Texts of Early Greek philosophy. The Complete Fragments and Selected Testimonies of the Major Presocratics.* Cambridge, UK: Cambridge University Press, 2010.

D. J. Griffiths. *Introduction to Electrodynamics*. Upper Saddle River, NJ: Pearson Prentice Hall, 3rd edition, 1999.

A. Grünbaum. Is preacceleration of particles in dirac's electrodynamics a case of backward causation? the myth of retrocausation in classical electrodynamics. *Philosophy of Science*, 43(2):165–201, 1976.

A. Grünbaum and A. I. Janis. Is there backward causation in classical electrodynamics? *The Journal of Philosophy*, 74(8):475–82, 1977.

N. Hall. Humean reductionism about laws of nature. Manuscript, 2009. URL `http://philpapers.org/rec/halhra`.

J. Heil. *The Universe As We Find It*. Oxford: Oxford University Press, 2012.

P. Holland. The roads not taken: empty waves, wave function collapse and protective measurement in quantum theory. In S. Gao, editor, *Protective Measurement and Quantum Reality: Towards a New Understanding of Quantum Mechanics*, chapter 11, pages 145–63. Cambridge, UK: Cambridge University Press, 2014.

P. R. Holland. *The Quantum Theory of Motion*. Cambridge, UK: Cambridge University Press, 1993.

M. Hubert. Quantity of matter or intrinsic property: Why mass cannot be both. In L. Felline, A. Ledda, F. Paoli, and E. Rossanese, editors, *New Developments in Logic and Philosophy of Science*, volume 3, pages 267–77. London: College Publications, 2016.

N. Huggett. The regulatory account of relational spacetime. *Mind*, 115(457):41–73, 2006.

D. Hume. *A Treatise of Human Nature*. Oxford: Clarendon Press, 1960. Edited, with an analytical index, by L. A. Selby-Bigge.

D. Hume. *An Enquiry Concerning Human Understanding*. New York: Oxford University Press, 2007. Edited with an Introduction and Notes by Peter Millican.

J. D. Jackson. *Classical Electrodynamics*. New York City: John Wiley & Sons, 3rd edition, 1999.

M. Jammer. *Concepts of Force: A Study in the Foundations of Dynamics.* Harvard: Harvard University Press, 1957.

M. K.-H. Kiessling. On the motion of point defects in relativistic fields. In F. Finster, O. Müller, M. Nardmann, J. Tolksdorf, and E. Zeidler, editors, *Quantum Field Theory and Gravity: Conceptual and Mathematical Advances in the Search for a Unified Framework*, pages 299–335. Basel: Springer, 2012.

J. Ladyman and D. Ross. *Every Thing Must Go: Metaphysics Naturalized.* New York: Oxford University Press, 2007.

V. Lam. Quantum structure and spacetime. In C. Wüthrich and T. Bigaj, editors, *Metaphysics in Contemporary Physics*, chapter 4, pages 81–99. Leiden: Brill, 2015.

A. Landé and L. H. Thomas. Finite self-energies in radiation theory. part ii. *Physical Review*, 60(7):514–23, 1941.

M. Lange. *An Introduction to the Philosophy of Physics: Locality, Fields, Energy, and Mass.* Oxford: Blackwell, 2002.

M. Lange. A tale of two vectors. *Dialectica*, 63(4):397–431, 2009.

G. W. Leibniz and S. Clarke. *Correspondence.* Indianapolis, IN: Hackett Publishing, 2000. Edited, with Introduction, by Roger Ariew.

D. Lewis. *Philosophical Papers Vol. 2.* New York: Oxford University Press, 1986.

D. Lewis. Humean supervenience debugged. *Mind*, 103(412):473–90, 1994.

D. Lewis. Ramseyan humility. In D. Braddon-Mitchell and R. Nola, editors, *Conceptual Analysis and Philosophical Naturalism*, pages 203–22. Cambridge, MA: MIT Press, 2009.

B. Loewer. Humean supervenience. *Philosophical Topics*, 24(1):101–27, 1996.

H. A. Lorentz. *The theory of electrons and its applications to the phenomena of light and radiant heat.* Leipzig: B. G. Teubner, 1916.

A. Lyon. Causality. *The British Journal for the Philosophy of Science*, 18(1):1–20, 1967.

E. Mach. *The Science of Mechanics: A Critical and Historical Account of Its Development.* Chicago: The Open Court Publishing Co., 4$^{\text{th}}$ edition, 1919.

J. L. Mackie. *The Cement of the Universe.* New York: Oxford University Press, 1980.

O. Massin. The metaphysics of forces. *Dialectica*, 63(4):555–89, 2009.

O. Massin. The composition of forces. *The British Journal for the Philosophy of Science*, advance access, February 2016.

T. Maudlin. Three measurement problems. *Topoi*, 14(1):7–15, 1995.

T. Maudlin. Part and whole in quantum mechanics. In E. Castellani, editor, *Interpreting Bodies*, pages 46–60. Princeton, NJ: Princeton University Press, 1998.

T. Maudlin. *The Metaphysics Within Physics*. New York: Oxford University Press, 2007a.

T. Maudlin. Completeness, supervenience and ontology. *Journal of Physics A: Mathematical and Theoretical*, 40(12):3151–3171, 2007b.

T. Maudlin. On the passing of time. In *The Metaphysics Within Physics*, chapter 4, pages 104–142. New York: Oxford University Press, 2007c.

T. Maudlin. *Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics*. Chichester, UK: Wiley-Blackwell, 3 edition, 2011.

T. Maudlin. *Philosophy of Physics: Space and Time*. Princeton, NJ: Princeton University Press, 2012.

T. Maudlin. The nature of the quantum state. In A. Ney and D. Z. Albert, editors, *The Wave Function: Essays on the Metaphysics of Quantum Mechanics*, chapter 6, pages 126–53. New York: Oxford University Press, 2013.

T. Maudlin. What Bell did. *Journal of Physics A: Mathematical and Theoretical*, 47(42), 2014.

T. Maudlin. The universal and the local in quantum theory. *Topoi*, 34(2):349–58, 2015.

E. Miller. Quantum entanglement, Bohmian mechanics, and Humean supervenience. *Australasian Journal of Philosophy*, 92(3):567–83, 2014.

F. Muller and S. Saunders. Discerning fermions. *The British Journal for the Philosophy of Science*, 59(3):499–548, 2008.

F. A. Muller. Inconsistency in classical electrodynamics? *Philosophy of Science*, 74(2):253–77, 2007.

B. Mundy. Distant action in classical electromagnetic theory. *The British Journal for the Philosophy of Science*, 40(1):39–68, 1989.

W. C. Myrvold. On some early objections to Bohm's theory. *International Studies in the Philosophy of Science*, 17(1):7–24, 2003.

I. Newton. *Opticks: or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light Treatise of the Reflections, Refractions, Inflections and Colours of Light*. New York: Dover Publications, 1952.

I. Newton. *The Principia: Mathematical Principles of Natural Philosophy*. Berkeley: University of California Press, 1999. Translated into English by I. Bernard Cohen and Anne Whitman.

T. Norsen. Bell locality and the nonlocal character of nature. *Foundations of Physics Letters*, 19(7):633–655, 2006.

T. Norsen. Local causality and completeness: Bell vs. Jarrett. *Foundations of Physics*, 39(3):273–94, 2009.

T. Norsen. The theory of (exclusively) local beables. *Foundations of Physics*, 40 (12):1858–84, 2010.

T. Norsen. J. S. Bell's concept of local causality. *American Journal of Physics*, 79 (12):1261–75, 2011.

T. Norsen, D. Marian, and X. Oriols. Can the wave function in configuration space be replaced by single-particle wave functions in physical space? *Synthese*, 192 (10):3125–51, 2015.

R. E. Norton and W. K. R. Watson. Commutation rules and spurious eigenstates in soluble field theories. *Physical Review*, 116(6):1597–603, 1959.

L. B. Okun. The concept of mass. *Physics Today*, 42(6):31–6, 1989.

V. Perlick. On the self-force in electrodynamics and implications for gravity. In D. Puetzfeld, C. Lämmerzahl, and B. Schutz, editors, *Equations of motion in relativistic gravity*, pages 523–42. Heidelberg: Springer, 2015.

W. Pietsch. On conceptual issues in classical electrodynamics: Prospects and problems of an action-at-a-distance interpretation. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 41(1): 67–77, 2010.

G. N. Plass. Classical electromagnetic equations of motion with radiative reactions. *Reviews of Modern Physics*, 33(1):37–62, 1961.

B. Podolsky. A generalized electrodynamics part I—non-quantum. *Physical Review*, 62(1-2):68–71, 1942.

J. Sakurai and J. Napolitano. *Modern Quantum Mechanics*. San Francisco: Addison Wesley, 2$^{\text{nd}}$ edition, 1994.

E. Schrödinger. Die gegenwärtige Situation in der Quantenmechanik. *Die Naturwissenschaften*, 23(48-50), 1935. English translation *The present situation in quantum mechanics*. In J. A. Wheeler and W. H. Zurek, editors. *Quantum Theory and Measurement*, section I.11, pages 152–67. Princeton Series in Physics. Princeton, NJ: Princeton University Press, 1983.

D. V. Schroeder. *An Introduction to Thermal Physcis*. San Francisco: Addison Wesley, 2000.

L. Sklar. *Space, Time, and Spacetime*. Berkeley: University of California Press, 1977.

H. Spohn. *Dynamics of Charged Particles and Their Radiation Field*. New York: Cambridge University Press, 2004.

J. Stachel. Structure, individuality and quantum gravity. In D. Rickles, S. French, and J. Saatsi, editors, *Structural Foundations of Quantum Gravity*, pages 53–82. Oxford: Clarendon Press, 2006.

H. Stein. On the notion of field in Newton, Maxwell, and beyond. In R. H. Stuewer, editor, *Historical and Philosophical Perspectives of Science*. Minneapolis: University of Minnesota Press, 1970.

K. Stierstadt. *Thermodynamik: Von der Mikrophysik zur Makrophysik*. Heidelberg: Springer, 2010.

M. Suárez. Bohmian dispositions. *Synthese*, 192(10):3203–28, 2015.

R. Tumulka. A relativistic version of the Ghirardi–Rimini–Weber model. *Journal of Statistical Physics*, 125(4):821–40, 2006.

L. Vaidman. The reality in Bohmian quantum mechanics or can you kill with an empty wave bullet? *Foundations of Physics*, 35(2):299–312, 2005.

A. Valentini. Signal locality, uncertainty, and the subquantum H-theorem I. *Physics Letters A*, 156(1-2):5–11, 1991a.

A. Valentini. Signal locality, uncertainty, and the subquantum H-theorem II. *Physics Letters A*, 158(1-2):1–8, 1991b.

P. Vickers. Frisch, Muller, and Belot on an inconsistency in classical electrodynamics. *The British Journal for the Philosophy of Science*, 59(4):767–92, 2008.

J. A. Wheeler and R. P. Feynman. Interaction with the absorber as the mechanism of radiation. *Reviews of Modern Physics*, 17(2–3):157–81, 1945.

J. A. Wheeler and R. P. Feynman. Classical electrodynamics in terms of direct interparticle action. *Reviews of Modern Physics*, 21(3):425–33, 1949.

A. N. Whitehead. *Science and the Modern World*. New York: The Macmillan Company, 1925.

J. Wilson. Newtonian forces. *The British Journal for the Philosophy of Science*, 58(2):173–205, 2007.

J. Wilson. The causal argument against component forces. *Dialectica*, 64(4):525–54, 2009.

J. Worrall. Structural realism: The best of both worlds? *Dialectica*, 43(1–2): 99–124, 1989.

C. Wüthrich. Challenging the spacetime structuralist. *Philosophy of Science*, 76 (5):1039–51, 2009.

L. Zuchowski. For electromagnetic consistency. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 44(2): 135–42, 2013.