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Economics Letters 87 (2005) 347–353
economics letters
www.elsevier.com/locate/econbase

Fertility, volatility, and growth

Aude Pommeret[a], William T. Smith[b,*]

[a]*University of Lausanne, Switzerland*
[b]*University of Memphis, Department of Economics, Fogelman College of Business and Economics, Memphis, TN 38152, USA*

Received 1 June 2004; received in revised form 12 July 2004; accepted 20 January 2005
Available online 22 April 2005

Abstract

Empirically, growth rates are negatively correlated with birth rates; they are also correlated with production risk. We argue that these stylized facts are related and arise jointly from the decision of how many children to have in a risky environment.

Keywords: Fertility; Stochastic growth; Volatility

JEL classification: D80; J13; O41

1. Introduction

Why do poor countries get trapped in a vicious circle of high birth rates and low growth rates? This paper will develop a unified theory of how production volatility affects the growth rate of an economy by altering both saving decisions and the decision to have children. The model may be particularly relevant in some developing countries, where production volatilities are much greater than in developed economies (Turnovsky and Chattopadhyay, 2003).

In developing such a theory, we are motivated by two "stylized facts" about long-term growth that have emerged from cross-sectional empirical studies. First, countries with low birth rates tend to grow faster than countries with high birth rates (Yip and Zhang, 1996). Second, in a cross-section, growth

* Corresponding author. Tel.: +1 901 678 3675.
E-mail address: wtsmith@memphis.edu (W.T. Smith).

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doi:10.1016/j.econlet.2005.01.002

rates are correlated with the volatilities of GDP growth. Although the sign of the correlation is disputed, there seems to be a consensus that risk matters for growth (Turnovsky and Chattopadhyay, 2003).

We contend that these two observations are inextricably linked to the decision of how many children to have in a risky economic environment. To develop this proposition, we integrate two previously distinct strands of growth theory, the literature on endogenous fertility and the recent literature on stochastic growth.

There is a rich literature that explains the simultaneous determination of fertility and growth rates, see the survey in Barro and Sala-i-Martin (1999). In these models, people derive pleasure from having children, but raising children entails costs, in terms of output or foregone time. So far, this literature has not allowed for the possibility that uncertainty may affect fertility choices. Nevertheless, it is now admitted that uncertainty "matters" for growth even in the long run because it alters saving behavior and affects portfolio choices between alternative capital investments. This is one of the key insights of the modern literature on stochastic growth (see Turnovsky (2000) for example). However, this literature ignores fertility decisions.

In this paper we link these literatures by incorporating fertility choice into a stochastic growth model. We construct a stochastic version of the model by Yip and Zhang (1996): Raising a child requires the diversion of time and effort away from the production of goods and services. This reduces growth, as in the nonstochastic model of Yip and Zhang (1996). However, it also reduces the variance of production, much as changes in employment alter it in models of stochastic growth with wage income (Turnovsky, 2003). Since having more children reduces both the mean and the variance of production, these in turn alter saving decisions. The saving and fertility decisions interact simultaneously to determine the growth rate.

2. Technology and preferences

Consider an economy populated by a large number of identical households. Time is continuous. Each family is endowed with a fixed amount of time in each period—normalized to unity—which can be spent either in production or in child-rearing activities. Following Yip and Zhang (1996) let $\phi(n)$ be the amount of time required for child rearing if the birth rate is n where $\phi'>0$. Without much loss in generality, we set $\phi(n)=n$ to simplify exposition.

The production function (in per capita terms) is

$$dy = A(1-n)^{1-\alpha}k[dt + \sigma dz] \tag{1}$$

where dz is the increment to a standard–normal Wiener process and time indices have been deleted for simplicity. Output is random, with a mean of $A(1-n)^{1-\alpha}k$ and a variance of $A^2(1-n)^{2(1-\alpha)}\sigma^2k^2$. If $\alpha>0$ and $\sigma^2=0$ this reduces to the non-stochastic production function in Yip and Zhang (1996). If $\sigma^2>0$ but $\alpha=0$, then labor supply disappears and we have the stochastic production function now standard in the stochastic growth literature. If $\sigma^2=0$ and $\alpha=0$, we would recover the simple linear, nonstochastic (a–k) technology. Notice that an increase in the birth rate reduces the variance of output.

The per capita capital stock accumulates according to

$$dk = dy - nkdt - cdt. \tag{2}$$

The family has an infinite planning horizon. We follow Becker and Barro (1988) and Yip and Zhang (1996) in assuming that it derives utility from both consumption and the birth rate[1]. Using time-separable utility, as in Yip and Zhang (1996), would confound preferences for substitution over time with risk aversion and with ordinal preferences between c and n. In order to disentangle these different aspects of preferences we employ a form of generalized isoelastic (GIE) utility:

$$(1-\gamma)U(t) = \left\{ \left[cn^{\theta}\right]^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\rho dt}[(1-\gamma)E_t U(t+dt)]^{\frac{1\varepsilon-1}{1-\gamma\varepsilon}} \right\}^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}}. \tag{3}$$

$\theta \geq 0$ governs the taste for having children, $\varepsilon > 0$ is the intertemporal elasticity of substitution, and $\gamma > 0$ is the coefficient of relative risk aversion. This nests several familiar cases. If $\theta = 0$, Eq. (1) reduces to the GIE preferences, defined over consumption alone, proposed by Svensson (1989). If in addition $\gamma = 1/\varepsilon$ we recover conventional time-separable, isoelastic preferences. If $\gamma = 1/\varepsilon$ and $\theta \geq 0$ we have time-separable preferences over c and n. An important benchmark case is $\gamma = 1$ and $\varepsilon = 1$; this captures time-separable, logarithmic intertemporal and risk preferences.

The household maximizes lifetime utility (Eq. (3)) given the production function (Eq. (1)) and the resource constraint (Eq. (2)).

3. Equilibrium

The solution to this problem satisfies the following first-order conditions[2]:

$$c = n^{\theta(\varepsilon-1)} Bk \tag{4}$$

$$\theta n^{\theta(\varepsilon-1)-1} B\left(n, \sigma^2\right) = (1-\alpha)A(1-n)^{-\alpha} + 1 - \gamma A^2 (1-\alpha)(1-n)^{1-2\alpha} \sigma^2 \tag{5}$$

where

$$n^{\theta(\varepsilon-1)} B\left(n, \sigma^2\right) = \varepsilon\rho + (1-\varepsilon) \underbrace{\left[A(1-n)^{1-\alpha} - n - \frac{1}{2}\gamma A^2 (1-n)^{2(1-\alpha)} 2 \right]}_{\text{certainty equivalent rate of return}}. \tag{6}$$

Eq. (4) is the consumption function. The marginal propensity to consume is a linear function of the certainty equivalent rate of return to capital. As shown by Weil (1990), the intertemporal elasticity of substitution governs the sign of the effect of risk σ^2 on consumption: an increase in risk (given a birth rate n) reduces the certainty equivalent rate of return, which in turn may increase or decrease consumption depending upon whether $\varepsilon < 1$ or $\varepsilon > 1$. If there are log preferences towards intertemporal substitution ($\varepsilon = 1$), risk has no effect on consumption. Notice that the birth rate alters consumption by changing the certainty equivalent rate of return.

[1] See Becker and Barro (1988) and Barro and Sala-i-Martin (1999) for a discussion of these preferences and for a justification of both the continuous-time formulation and the assumption of infinite horizons. As noted by Yip and Zhang (1996), the model can be reformulated as a continuous-time model with overlapping generations, such as Blanchard (1985), without changing its conclusions.

[2] The derivation, along with all other mathematical details, is relegated to an appendix, available on request.

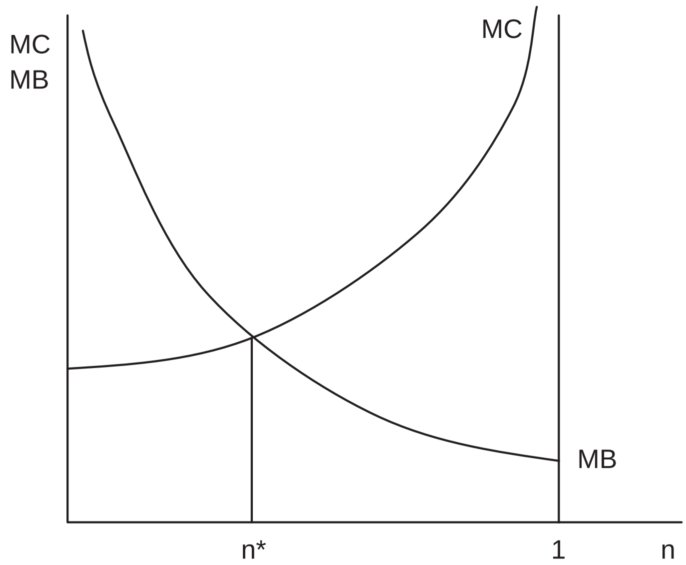

Fig. 1. $\varepsilon<1$ and $\gamma>1$.

Eq. (5) governs fertility choice. Having a child has a cost in terms of the lost production caused by working less, as in Yip and Zhang (1996). Now, however, having a child also reduces the variance of production. The right hand side of Eq. (5) is the net (or certainty equivalent) marginal cost of raising a child; it is the decrease in the certainty equivalent rate of return caused by increasing n. This is depicted with the positively sloped curve $\mathrm{MC}(n,\sigma^2)$ in the Fig. 1[3]. The left hand side of Eq. (5) is the marginal benefit of having kids, given by the marginal utility of having children. Notice that the marginal utility of fertility is an increasing function of consumption. This means that having kids alters marginal utility both directly, as in Yip and Zhang (1996), and indirectly by changing consumption. The negatively sloped curve $\mathrm{MB}(n,\sigma^2)$ in Fig. 1 depicts the marginal benefit[4].

The equilibrium birth rate n^* is determined where $\mathrm{MB}(n^*,\sigma^2)=\mathrm{MC}(n^*,\sigma^2)$.

4. Risk, fertility, and growth

How does risk affect the birth rate and growth?

To address this question it is useful to first consider the special case with logarithmic preferences for intertemporal substitution. In this case consumption is independent of the certainty equivalent rate of return [from Eqs. (4) and (6), $c=\rho k$], so the marginal benefit of having children is independent of risk. For a given n an increase in σ^2 unambiguously lowers the certainty equivalent rate of return. This lowers the marginal cost of having children, shifting the MC curve to the right. Therefore the birth rate will

[3] We assume that labor's share exceeds one half ($\alpha<1/2$) and that $1>\gamma A\sigma^2/2$. These inequalities guarantee that the certainty equivalent rate of return is a decreasing, concave function of n.

[4] For convenience, we depict the case where MB is negatively sloped. This is always true if $\varepsilon\leq 1$. If $\varepsilon>1$, MB may be positively sloped. Even if MB is positively sloped, the ensuing comparative statistics were still obtained as long as MB is not steeper than MC, a condition implied by the necessary second-order conditions.

increase. Intuitively, people self-insure against production risk by investing more time in the "risk-less" investment of having children. Moreover, the expected growth rate of the economy is only affected by uncertainty through the change in the birth rate: in this case, more uncertainty unambiguously reduces the growth rate.

Proposition 1. *In the case of a log utility function, an increase in risk unambiguously raises the birth rate, which in turn reduces the growth rate.*

Now consider the general model with GIE preferences. An increase in risk still reduces the marginal cost of having children, exactly as in the log case. However, by changing the certainty equivalent rate of return it also changes consumption, which alters the marginal benefit of having children. If people like to substitute over time, so that $\varepsilon > 1$, then the increase in risk increases consumption and with it the marginal benefit of having children: the MB curve shifts up. This reinforces the effect of the decrease in MC so that the birth rate unambiguously increases. This is shown in Fig. 2. However, if people do not like to substitute over time, so that $\varepsilon < 1$, the opposite occurs: consumption falls as risk increases, so MB shifts down. In this case, the effect on MB offsets the effect on MC. In general, we have

$$\frac{\partial n^*}{\partial \sigma^2} = \frac{\partial MC/\partial \sigma^2 - \partial MB/\partial \sigma^2}{\partial MB/\partial n - \partial MC/\partial n}. \tag{7}$$

It is possible to find numerical examples, when $\varepsilon < 1$ where the birth rate will decrease when risk increases.

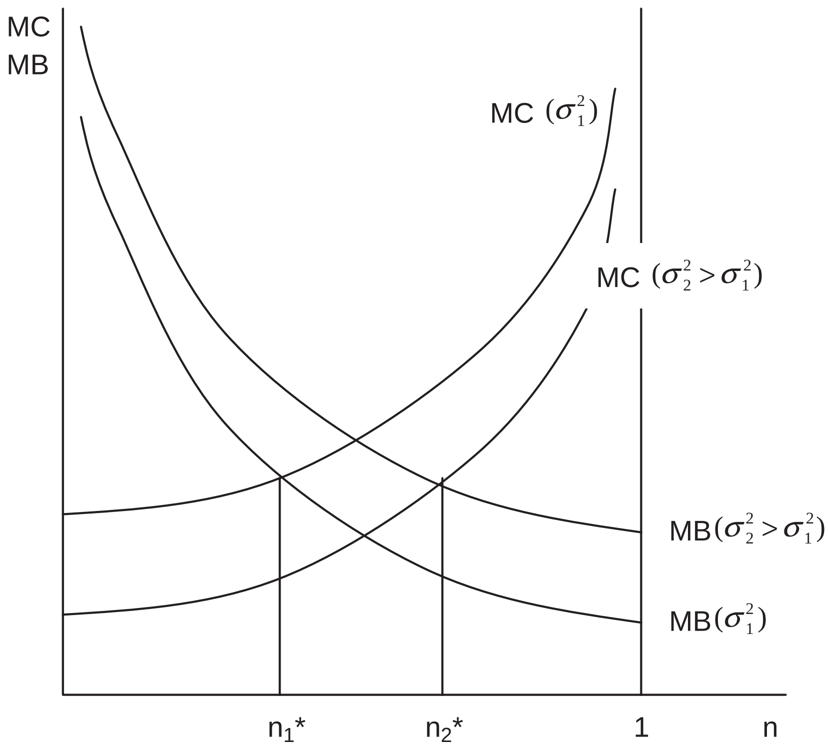

Fig. 2. $\varepsilon > 1$ and $\gamma > 1$.

What about growth? Using Eqs. (2), (5), (6) and (7) the equilibrium growth rate is

$$\frac{dk}{k}=\left\{\varepsilon\left[A\left(1-n^*\right)^{1-\alpha}-n^*-\rho\right]+(1-\varepsilon)\gamma A^2\left(1-n^*\right)^{2(1-\alpha)}\frac{\sigma^2}{2}\right\}dt+A(1-n^*)^{1-\alpha}\sigma dz. \quad (8)$$

Define the expected growth rate of capital (the expression in braces) by μ_k^*. An increase in uncertainty affects the expected growth rate directly and indirectly: the direct effect is to alter the saving decision; the indirect effect is to increase the birth rate, which changes the risk and expected return of capital, feeding back to change the saving decision. The net effect of an increase in σ^2 on μ_k^* is

$$\frac{\partial\mu^*_{(k)}}{\partial\sigma^2}=-\left\{\varepsilon\left[(1-\alpha)A\left(1-n^*\right)^{1-\alpha}+1\right]+(1-\varepsilon)\gamma A^2\left(1-n^*\right)^{2(1-\alpha)}\frac{\sigma^2}{2}\right\}\frac{\partial n^*}{\partial\sigma^2}$$
$$+(1-\varepsilon)\frac{\gamma}{2}A^2(1-n^*)^{2(1-\alpha)}. \quad (9)$$

From our previous discussion we know that if $\varepsilon\geq 1$ then $\partial n^*/\partial\sigma^2>0$. Eq. (9) then implies that if $\varepsilon\geq 1$ it will also be true that $\partial\mu_k^*/\partial\sigma^2<0$. However, if $\varepsilon<1$ then $\partial\mu_k^*/\partial\sigma^2$ cannot be signed; in particular, using plausible values for the parameters (see the Appendix) we obtain that the expected growth rate may be increased by σ^2. Therefore:

Proposition 2. *An increase in risk unambiguously raises the birth rate and reduces the growth rate only if* $\varepsilon\geq 1$. *If* $\varepsilon<1$ *it is possible for an increase in risk to reduce the birth rate and raise the growth rate.*

5. Conclusion

The premise of this paper is that birth rates and growth rates are jointly affected by production uncertainty. Our analysis points toward—and suggests a way of achieving—the integration of the literatures on stochastic growth and on fertility. The crucial parameter governing the effect of risk on fertility and growth is the intertemporal elasticity of substitution ε. If $\varepsilon\geq 1$ our model predicts that countries with more volatile GDPs should have higher birth rates and lower growth rates than those with less volatile GDPs. If $\varepsilon<1$ the converse holds: risky countries should have lower birth rates and higher growth rates than less risky countries. The conventional wisdom is that $\varepsilon<1$ [see Campbell et al., 1997, for example], so the presumption would seem to be that risk dampens fecundity and stimulates growth. However, there is some evidence in favor of $\varepsilon>1$(Attanasio and Weber, 1993; Bufman and Leiderman, 1990; Koskievic, 1999). Such evaluations may therefore not be taken as definitive. This suggests that the model proposed in this paper may provide an explanation for developing countries with risky technologies that are trapped in equilibria with high birth rates and low growth rates.

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