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The Impact of Government Subsidies in Professional Team Sports Leagues

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Abstract

This article develops a game-theoretical model to analyze the effect of subsidies on player salaries, competitive balance, club profits, and welfare. Within this model, fan demand depends on win percentage, competitive balance, and aggregate talent. The results show that if a large-market club receives a subsidy and fans have a relatively strong preference for aggregate talent, compared to competitive balance and own team winning percentage, club profits and welfare increase for both clubs. If the small-market club is subsidized, a small subsidy increases competitive balance and player salaries of both clubs.

Keywords: subsidy, team sports, competitive balance, social welfare

Introduction

Government subsidies are a common phenomenon in professional team sports. These subsidies usually take the form of advantageous property deals,[1] tax loopholes,[2] and low or zero stadium rents.[3] As a result of these subsidies, some clubs enjoy cost advantages over their competitors.

A special case of these cost advantages are income tax differences with respect to player salaries. In France, there is a uniform income tax rate. As a result, players in the first French division face an income tax rate of 45% if their salary exceeds €150,000. Monaco, on the other hand, does not impose income taxes. Accordingly, players from AS Monaco, who also compete in the first French division, do not pay any income taxes.

In the US, state and city income tax rates also differ substantially. For example, Florida and Texas do not impose state and city income taxes, whereas California charges state taxes of up to 12.3%. The former coach of the Orlando Magic, Glenn Anton "Doc" Rivers, tried to convince free agents of opposing teams to join the Magic by yelling at them during games, "We have no state and city taxes, and it's always 80 degrees" (Alm, Kaempfer, & Sennoga, 2012, p. 619).

Kopkin (2012) identifies the effect of changes in income tax rates on player transfers. He finds that an increase in the marginal income tax rate for a given team results in a decrease of the average skill of free agents who transfer to this team. Alm et al. (2012) analyze the same effect in professional baseball and show that low tax cities benefit from a "home field advantage" in the free agent market. To the best of our knowledge, these effects have never been analyzed in a game theoretical model of a professional sports league. Our model builds on former models, which have focused on competitive balance and win percentage (see, e.g., Dietl, Lang, & Werner, 2009; Vrooman, 2008; Szymanski & Kesenne, 2004). We follow Madden (2012) by including the effect of aggregate talent on demand. Unlike Madden, however, we explicitly model fan preference for aggregate talent while he uses aggregate talent primarily as an additional factor in the revenue function.

Based on our game theoretical model we show how subsidies affect player salaries, competitive balance, club profits, and social welfare. The results show that if a large-market club receives a subsidy and fans have a relatively strong preference for aggregate talent, compared to competitive balance and own team winning percentage, then club profits and welfare increase for both clubs. If the small-market club is subsidized then a small subsidy increases competitive balance and player salaries of both clubs.

Our model clearly deviates from classical gate revenue-sharing models (cf., Szymanski & Kesenne, 2004) because subsidies affect competitive balance and aggregate talent. In classical revenue-sharing models aggregate talent is not included. While in the classical gate revenue-sharing model competitive balance decreases, we show several results in the opposite direction. Additionally, several outcomes in our model result in an increasing competitive balance.

Model Setup

We model a two-club league in which both clubs participate in a noncooperative game and independently pay a certain amount for player salaries to maximize profits. Each club k {i,j} generates its own revenues according to a fan demand function that depends on the match quality (i.e., own team win percentage, competitive balance, and aggregate talent).[4]

We introduce the concept of exogenous assistance in the form of a subsidy. In the Lang et al. (2011) model, outside actors financially assist clubs, thereby influencing a club's objective function. In our model, no outside actor has the possibility of influencing a club's objective function. One example in our case is a regional government that has its own economic interests in supporting a club via tax relief but cannot directly influence the club's decision-making process.

The gross salaries (salary payments) of club k are denoted by x_k and the net-of-tax salaries s_k the players receive at club k are given by

$$s_k = (1 - \mu_k t) x_k$$

with $t \in [0,1]$ and $\mu_k \in [0,1]$. We assume $x_k > 0$. The income tax is denoted by t and the subsidy (tax relief) that club k obtains is presented by the parameter μ_k, where a higher value of μ_k denotes a lower subsidy. For notational simplicity, we substitute the term $(1-\mu_k t)$ with α_k and obtain

$$s_k = (1 - \mu_k t)x_k = \alpha_k x_k$$

with α_k $[1-t,1]$. With the new notation, a higher parameter α_k reflects a higher subsidy. In the extreme case, $\alpha_k = 1$ and club k does not have to pay any taxes so that the gross salaries the club pays corresponds to the net-of-tax salary the players receive, i.e., $s_k = x_k$. In contrast, if $\alpha_k = 1-t$, club k does not receive any subsidy and has to pay the full tax so that $s_k = (1-t)x_k$. While this setup assumes that any revenues from a subsidy are passed on to the salary of the players, a more relaxed assumption is also plausible. By assuming that both clubs i,j forward the same share of the subsidy to their players we relax the general assumption that all tax revenues are forwarded.

Next, we specify the revenue function, which depends on win percentage, competitive balance, and aggregate talent. Win percentage is most commonly represented by the contest-success function (CSF). We select Tullock's (1980) logit approach:

$$w_k(x_i, x_j) = \frac{s_k}{s_i + s_j} = \frac{\alpha_k x_k}{\alpha_i x_i + \alpha_j x_j},$$

with k $\{i,j\}$.[5] We use the following measurement for competitive balance:

$$CB(x_i, x_j) = w_i(x_i, x_j) w_j(x_i, x_j) = \frac{\alpha_i x_i \alpha_j x_j}{(\alpha_i x_i + \alpha_j x_j)^2}.$$

If both clubs have the same winning percentage ($w_i = w_j = 0.5$), competitive balance is 1/4. The opposite case, a league with one dominant club ($w_i = 1$, $w_j = 0$), has a competitive balance of 0.

Fans value not only competitive balance but also aggregate playing talent within the league.

$$AT(x_i, x_j) = \alpha_i x_i + \alpha_j x_j.$$

Without aggregate talent, supporters are unable to distinguish between teams in a high or low league (assuming that win percentages are the same). By including $\sigma > 0$ to measure the relative importance of aggregate talent, the quality function of club k is

$$q_k(x_i, x_j) = w_k(x_i, x_j) + CB(x_i, x_j) + \frac{1}{\sigma} AT(x_i, x_j).$$

We assume that every supporter, denoted by v, has a preference for a game's quality, denoted by θ. For simplicity, we assume that these preferences are uniformly distributed in [0,1]; that is, the measure of potential fans is 1.[6] The payoff for a supporter is described as the utility a supporter derives from attending a game, $\theta_v q_k$, minus the cost a supporter has to pay for it, p_k. We take for granted that a consumer's payoff cannot be negative, $\max\{\theta_v q_k - p_k, 0\}$. The consumer who is indifferent with respect to attending a game (or, similarly, paying a television fee to watch the game) is described by $\theta_v = \frac{p_k}{q_k}$. Thus, the number of supporters who are willing to attend a game at price p_k is expressed by $1 - \theta_v = \frac{q_k - p_k}{q_k}$.

By assuming that each club has a market size or drawing potential given by $m_k > 0$, the aggregate demand function for club *k* is denoted by

$$d_k(m_k, p_k, q_k) = m_k \frac{q_k - p_k}{q_k} = m_k(1 - \frac{p_k}{q_k}).$$

Thus, the club's revenue function is

$$R_k = p_k \cdot d_k(m_k, p_k, q_k).$$

The optimal choice for a club to maximize earnings is to set $p_k = \frac{q_k}{2}$, which results in the following revenue function:

$$R_k = \frac{m_k}{4} q_k = \frac{m_k}{4}(w_k(x_i, x_j) + CB(x_i, x_j) + \frac{1}{\sigma} AT(x_i, x_j)).$$

Our revenue functions differ from the revenue functions of other papers (i.e., Szymanski, 2003; Vrooman, 2007, 2008) because our revenue function depends on consumer preferences for aggregate talent. The revenue functions allow us to measure social welfare consisting of club profits, consumer surplus, supporter surplus, and player salaries. Profit π_k for club *k* is given by revenues R_k minus gross salaries (salary payments) x_k.

$$\pi_k = \frac{m_k}{4}(w_k(x_i, x_j) + CB(x_i, x_j) + \frac{1}{\sigma} AT(x_i, x_j)) - x_k.$$

Given that the maximal price consumers are willing to spend is $p_k = q_k$ and that in equilibrium, consumers have to pay a price of $p_k = \frac{q_k}{2}$, we receive the following aggregate consumer surplus (CS):

$$\begin{aligned} CS &= CS_i + CS_j \text{ with } CS_k = \int_{\frac{q_k}{2}}^{q_k} m \frac{q_k - p_k}{q_k} dp_k = \frac{m}{8} q_k, \ k \in \{i, j\} \text{ so that} \\ CS &= \frac{1}{8}(mq_i + q_j). \end{aligned}$$

Net-of-tax player salary is given by $s_k = (1 - \mu_k t)x_k = \alpha_k x_k$ and aggregate player salary by $PS = s_i + s_j$. We calculate aggregate club profits in the same way: $\pi_i + \pi_j$. The social planner receives $(1 - \alpha_i)x_i$ and $(1 - \alpha_j)x_j$ as taxes. With all four components social welfare is[7]

$$W(x_i, x_j) = \pi_i + \pi_j + CS + \alpha_i x_i + \alpha_j x_j + (1 - \alpha_i)x_i + (1 - \alpha_j)x_j,$$

With $\pi_i = R_i - x_i$ social welfare reduces to

$$W(x_i, x_j) = R_i + R_j + CS.$$

We have thus defined all our main variables: competitive balance, aggregate talent, club profit, consumer surplus, and social welfare. In the next section we calculate the equilibrium outcomes.

Equilibrium Outcomes

For notational simplicity, we normalize m_j=1 and write m_i=m. To maximize profits, each club chooses the optimal salary payment x_k^*. Thus, the clubs' optimization problems are $max_{x_i \geq 0}\ \pi_i$ and $max_{x_j \geq 0}\ \pi_j$ with the corresponding first-order conditions

$$\frac{\partial \pi_i}{\partial x_i} = 0.25 \frac{-4\sigma(x_i\alpha_i + x_j\alpha_j)^3 + m\alpha_i((x_i\alpha_i + x_j\alpha_j)^3 + 2\sigma\alpha_j^2 x_j^2)}{\sigma(x_j\alpha_j + x_i\alpha_i)^3} = 0,$$
$$\frac{\partial \pi_j}{\partial x_j} = \frac{\alpha_j(x_i\alpha_i + x_j\alpha_j)^3 + 2\sigma(x_i^2\alpha_i^2\alpha_j - 2(x_i\alpha_i + x_j\alpha_j)^3)}{\sigma(x_j\alpha_j + x_i\alpha_i)} = 0,$$

and second-order conditions[8]

$$\frac{\partial \pi_i^2}{\partial x_i^2} = -\frac{3mx_j^2\alpha_i^2\alpha_j^2}{2(x_i\alpha_i + x_j\alpha_j)^4} < 0 \text{ and } \frac{\partial \pi_j^2}{\partial xj^2} = -\frac{3x_i^2\alpha_i^2\alpha_j^2}{2(x_i\alpha_i + x_j\alpha_j)^4} < 0.$$

From the first-order conditions, we derive the equilibrium salary payments

$$x_i^* = \frac{\alpha_i\alpha_j m^2\sigma(\alpha_j - 4\sigma)}{2(3m\alpha_i\alpha_j\sigma(\alpha_j - 4\sigma) - \alpha_j\sigma\beta + m\alpha_i\beta(\alpha_j - 3\sigma) - m^2\alpha_i^2(\alpha_j - 4\sigma)(\alpha j - \sigma)},$$
$$x_j^* = \frac{m\alpha_i(\alpha_j(m\alpha_i - 4\sigma)(\sigma(3m\alpha_i + \alpha_j) - m\alpha_i\alpha_j) + \beta(3\alpha_j\sigma + m\alpha_i(\sigma - \alpha_j))}{8\sigma^2(m\alpha_i - \alpha_j)^3},$$

with $\beta = (m\alpha_i\alpha_j(m\alpha_i - 4\sigma)(\alpha_j - 4\sigma))^{1/2}$. To ensure non-negative equilibrium salary payments, we assume from now on that the fan preference for aggregate talent is sufficiently large, with $\sigma < \sigma^* = \min\{\frac{\alpha_i m}{4}, \frac{\alpha_j}{4}\}$.

Next, we derive the equilibrium win percentages:

$$w_i^* = \frac{1}{1 + \frac{\alpha_j(4\sigma - m\alpha_i)}{(m\alpha_i\alpha_j(4\sigma - m\alpha_i)(\alpha_j - 4\sigma))^{1/2}}} \text{ and } w_j^* = \frac{1}{1 - \frac{m\alpha_i(\alpha_j - 4\sigma)}{(m\alpha_i\alpha_j(m\alpha_i - 4\sigma)(\alpha_j - 4\sigma))^{1/2}}}.$$

Club profits in equilibrium are

$$\pi_i^* = \frac{m^2(4\alpha_j\beta\sigma + m^2\alpha_i^2(6\alpha_j\sigma + 8\sigma^2 - 3\alpha_j^2) + m\alpha_i\alpha_j(2\sigma(5\alpha_j - 12\sigma) - 3\beta}{32\sigma^2(\alpha_j - m\alpha_i)^2},$$
$$\pi_j^* = \frac{2m\alpha_i\alpha_j\sigma(5m\alpha_i + 3\alpha_j) + 8\alpha_j\sigma^2(\alpha_j - 3m\alpha_i) - 3m^2\alpha_i^2\alpha_j^2 - 3m\alpha_i\alpha_j\beta + 4m\alpha_i\sigma\beta}{32\sigma^2(\alpha_j - m\alpha_i)^2}.$$

Consumer surplus in equilibrium is given by

$$CS_i^* = \frac{m(4\alpha_j\sigma\beta + m^2\alpha_i^2(6\alpha_j\sigma + 8\sigma^2 - 3\alpha_j^2)}{64\sigma^2(\alpha_j - m\alpha_i)^2} + \frac{m\alpha_i\alpha_j(2\sigma(5\alpha_j - 12\sigma) - 3\beta)}{64\sigma^2(\alpha_j - m\alpha_i)^2},$$
$$CS_j^* = \frac{2m\alpha_i\alpha_j\sigma(5m\alpha_i + 3\alpha_j) + 8\alpha_j\sigma^2(\alpha_j - 3m\alpha_i)}{64\sigma^2(\alpha_j - m\alpha_i)^2} + \frac{-3m^2\alpha_i^2\alpha_j^2 - 3m\alpha_i\alpha_j\beta + 4m\alpha_i\sigma\beta}{64\sigma^2(\alpha_j - m\alpha_i)^2}.$$

Consequently, social welfare in equilibrium is

$$\begin{aligned} W^* &= \frac{3m\alpha_i\alpha_j(3m\alpha_i(5 + m + m^2) + \alpha_j(9 + 5m(1 + 2m))}{64\sigma^2(\alpha_j - m\alpha_i)^2} \\ &+ \frac{8\sigma^2(m^3\alpha_i^2(1 + 2m) - 3m\alpha_i\alpha_j(3 + m + m^2) + 3\alpha_j^2) - 3m^2\alpha_i^2\alpha_j^2(3 + m + 2m^2)}{64\sigma^2(\alpha_j - m\alpha_i)^2} \\ &+ \frac{m\beta(9\alpha_i\alpha_j - 3m\alpha_i\alpha_j - 6m^2\alpha_i\alpha_j + 12\alpha_i\sigma + 4\alpha_j\sigma + 8m\alpha_j\sigma))}{64\sigma^2(\alpha_j - m\alpha_i)^2}. \end{aligned}$$

In the next section we examine the effect of a subsidy on the equilibrium outcomes.[9]

Effects of Subsidies on Equilibrium Outcomes

In this section we discuss the implications of our results step by step. First, we explore the consequences of a subsidy $\alpha_i \in (\alpha_j, \alpha_i^*)$ for club *i* on its salary payments.[10]

Proposition 1. *Suppose club i receives a subsidy. Increasing the subsidy always increases the salary payments of club i, while it increases the salary payments of club j if and only if the subsidy is not too large. Otherwise, a subsidy for club i reduces salary payments of club j. Formally,* $\frac{\partial x_i}{\partial \alpha_i} > 0 \ \forall \alpha_i \in (\alpha_j, 1)$ *and* $\frac{\partial xj}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^*)$ $with$ $\alpha_i^* = \frac{16\alpha_j\sigma}{m(3\alpha_j+4\sigma)}$.

Proof. See Appendix.

To explain Proposition 1, we use Figure 1, which displays subsidies on the x-axis and salary payments on the y-axis. The dotted line denotes the large club. The solid line denotes the small club. Panel A shows the case when a large club receives a subsidy. Panel B shows the case when a small club receives a subsidy.

A club's profit depends on its win percentage, competitive balance, and aggregate talent. When a club receives a subsidy, it will always invest it. If the club that does not receive a subsidy is small, it mainly benefits from aggregate talent. As competitive balance decreases the small club cannot profit from it. If the club that does not receive a subsidy is large, it can benefit from both aggregate talent and competitive balance.

On the left side of Figure 1, both clubs invest in salary until the subsidy reaches a limit. Once the subsidy reaches a limit only the large club invests in salary. The small club decreases investment in salary.

As shown on the right side of Figure 1, the salary investment of the large club that does not receive the subsidy first increase and then decrease. The salary investment for the small club increases. In both cases (left and right side in Figure 1) the salary payments for the subsidy-receiving club always increase.

The following proposition analyzes the effect of subsidies on win percentage.

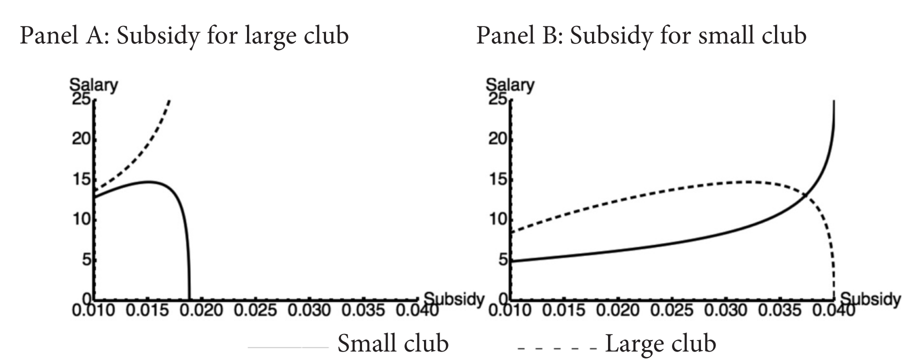


Figure 1. Salary payments

Proposition 2. *Suppose club i receives a subsidy. Increasing the subsidy always increases the win percentage of club i and decreases the win percentage of club j. Formally,* $\frac{\partial w_i}{\partial \alpha_i} > 0$ *and* $\frac{\partial w_j}{\partial \alpha_i} < 0 \ \forall \alpha_i \in (\alpha_j, 1)$.

Proof. See Appendix.

The proposition shows that a higher subsidy has an unambiguous effect on win percentages. Thus, even if club *j* increases salary payments as a result of a subsidy lower than the subsidy of club *i*, this increase is overcompensated for by an increase in the salary payments of club *i*.

Next, we examine the effect of subsidies on competitive balance.

Proposition 3. *Suppose club i receives a subsidy. If club i is the large club, increasing the subsidy always decreases competitive balance. If club i is the small club, increasing the subsidy increases competitive balance if and only if the subsidy is not too large. Otherwise, a subsidy for the small club decreases competitive balance.*

Formally, $\frac{\partial CB}{\partial \alpha_i} < 0 \ \forall \alpha_i \in (\alpha_j, 1) \quad for\ m > 1\ and\ \frac{\partial CB}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^{CB})$

$$with\ \alpha_i^{CB} = \frac{\alpha_j}{m}\ for\ m < 1.$$

Proof. See Appendix.

To explain Proposition 3, we use Figure 2, which displays competitive balance.

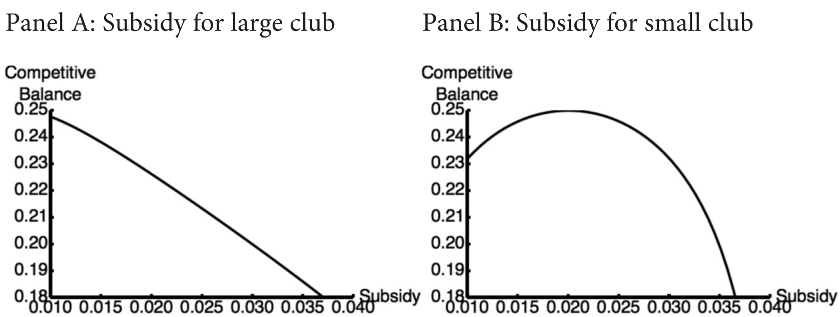


Figure 2. Competitive balance

The effect of a subsidy on competitive balance depends on which club receives the subsidy. If one club dominates the league, competitive balance is comparably low. When a (small or large) club benefits from a subsidy, the club's salary payments will increase as the club has more funds available. If the large club receives a subsidy, competitive balance will decrease as the large club's salary payments further increase in relation to the salary payment of the small club. The opposite holds true for the small club: An increasing subsidy leads to higher salary payments of the small club, resulting in a more balanced league until a maximum. After the maximum, competitive balance decreases. Figure 2 shows this case on the right-hand side. Additionally, a relatively high preference for aggregate talent leads to lower (higher) competitive balance if the large (small) market club receives a subsidy.

We derive the following numerical result for the effect of a subsidy on aggregate talent:

Result 1. *Suppose club i receives a subsidy. Increasing the subsidy always increases aggregate talent in the league if and only if the subsidy is not too large. Otherwise a subsidy for club i reduces aggregate talent. Formally,* $\frac{\partial AT}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^{AT})$.
Proof. See Appendix.

If club *i* receives a subsidy, it will increase its salary payments. If the subsidy does not exceed a certain threshold (see previous result), club *j* will have positive salary payments as well. Club *j* also benefits from the subsidy through the increase in aggregate talent. However, if the subsidy is too large, club *j* will considerably decrease its salary payments. The additional salary payments of club *i* will then be lower than the decrease in salary payments of club *j*. Therefore, aggregate talent decreases. In the next result, we analyze how a subsidy affects club profits.

Result 2. *Suppose club i receives a subsidy. Increasing the subsidy increases the profits of club i if and only if the subsidy is not too large. The opposite is true for club: increasing the subsidy for club i increases the profits of club j if and only if the subsidy is sufficiently large. Formally,* $\frac{\partial \pi_i}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^{\pi})$ *and* $\frac{\partial \pi j}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j^{\pi}, 1)$.

Proof. See Appendix.

To explain Result 2, we use Figure 3, which displays subsidies on the x-axis and club profits on the y-axis. The dotted line denotes the large club. The solid line denotes the small club. Panel A shows the case when a large club receives a subsidy. Panel B shows the case when a small club receives a subsidy.

In panel A we see the case when the large club receives a subsidy leading to an increase in salary payments. Accordingly, this increase leads to a more unbalanced league (effect on competitive balance). Nevertheless, the small club benefits from the increase of salary payments because of an increase in aggregate talent.

In panel B we see the case when the small club receives a subsidy. The subsidy will lead to an increase in salary payments. A small subsidy leads to an increase in competitive balance while a large subsidy leads to a decrease in competitive balance. Aggregate talent increases for both a large and small subsidy and almost always has a positive effect on revenues. The small club's increase in salary payments is smaller than the decrease in the large club's salary payments. We see in panel B that a very large subsidy decreases the profit of the small club (see Appendix for proof).

Next, we examine the effect of subsidies on social welfare. We derive the following numerical finding:

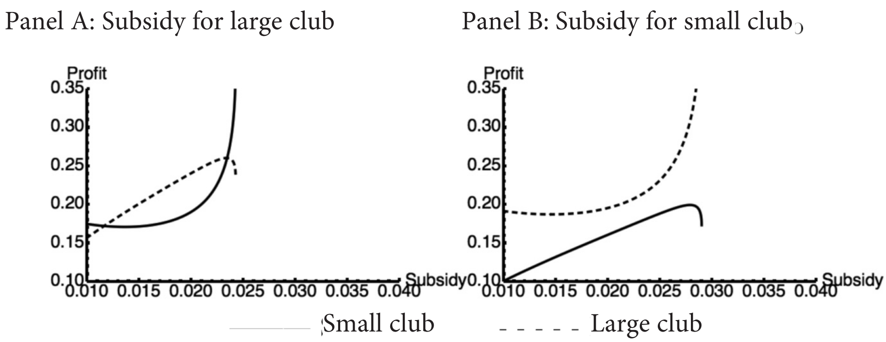


Figure 3. Club profits

Result 3. *Suppose club i receives a subsidy.*

If supporters have a high preference for aggregate talent, increasing the subsidy always increases social welfare (see left panel of Figure 4), i.e., $\frac{\partial W}{\partial \alpha_i} > 0 \ \forall \alpha_i \in (\alpha_j, 1)$ *for* $\sigma < \sigma^*$.

If supporters have a low preference for aggregate talent, increasing the subsidy increases social welfare if and only if the subsidy is not too large, i.e., $\frac{\partial W}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^W)$ *for* $\sigma > \sigma^*$.

Proof. See Appendix.

To explain Result 3, we use Figure 4, which displays subsidies on the x-axis and social welfare on the y-axis. Panel A shows the case when supporters have a high preference for aggregate talent, (thus $\frac{1}{4} \leq \sigma \leq \frac{1}{2}$). Panel B shows the case when supporters' preference for aggregate talent is comparatively small (i.e., $\frac{1}{4} < \sigma$).

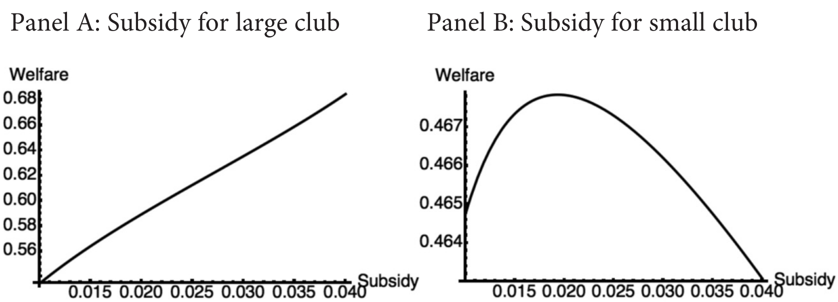


Figure 4. Social welfare

Panel A shows that aggregate talent (almost) always increases through higher subsidy (kink in aggregate talent seems not to have an effect; see Figure 5 in the Appendix). If preference for aggregate talent is sufficiently high, then this effect dominates all other effects. Panel B shows that the social planner must be more careful when setting the subsidy if supporters have a low preference for aggregate talent. This means that competitive balance is more important for the generation of revenues. In this case, social welfare first increases as the subsidy increases until a maximum level is reached. Increasing the subsidy beyond this optimum level leads to a reduction in social welfare. To clarify the results from the previous proofs all effects are summarized in Table 1.

Analysis When Winning Percentage Is Irrelevant

In this scenario we assume that the quality function of clubs depends only on competitive balance and aggregate talent. This means that the weight for winning percentage is 0. This quality function models a league that generates most of its revenues from the sale of television rights like the NFL.

Salary payments

When the large-market club receives a subsidy then salary payment decreases for the large-market club and increases for the small-market club. The intuition is that the

Table 1. Results Overview

	Subsidy for large-market, effect on		Subsidy for small-market, effect on	
	large-market	**small-market**	**large-market**	**small-market**
Salary payments	+	ambiguous	ambiguous	+
Win percentage	+	-	-	+
Competitive balance	-		ambiguous	
Aggregate talent	ambiguous		ambiguous	
Club profits	+	ambiguous	ambiguous	ambiguous
Social welfare	ambiguous		ambiguous	

revenue of both clubs depends on competitive balance. Thus, the large-market club decreases its own revenue when salary payments lead to a more unbalanced league. This is different in the case when winning percentage and competitive balance are equally weighted. Salary payments for the large-market club increase (left side of Figure 1).

When the small-market club receives the subsidy then salary payment decreases for the large-market club and increases for the small-market club. Thus, both clubs aim to increase competitive balance. Once the amount of salary payment is equal the opposite reaction occurs. This is, to some extent, similar to the case when aggregate talent and competitive balance are equally weighted. However, in this case salary payments for the large-market club first increase before they decrease.

Club profits

Profits always increase for the club that receives the subsidy and decrease for the club that does not receive the subsidy. When winning percentage is weighted, profits also increase for the club that does not receive the subsidy (cf., Figure 3). However, once the subsidy is too large then only one club benefits from the subsidy. This is then similar to the case of the analysis when winning percentage is not weighted.

Social welfare

When the large-market club receives a subsidy, social welfare always decreases. A larger subsidy exacerbates this tendency. Fan preference for aggregate talent has only a minor influence. However, when the small-market club receives the subsidy, social welfare increases up to a maximum before declining again. Social welfare only declines when the subsidy for the small-market club is too large. Again, this result is different from the case when winning percentage is weighted because social welfare is influenced by fan preference for aggregate talent.

Analysis When Competitive Balance Is Irrelevant

In the following scenario we assume that the quality function of clubs depends only on winning percentage and aggregate talent. This means that the weight for competitive balance is 0. Examples of a league with such characteristics are the European Champions League or Europa League. Competitive balance is not the driving force in these leagues like European soccer leagues, which means winning percentage and aggregate talent determine club profits.

Salary payments

When the large-market club receives a subsidy salary payments for both clubs increase. However, once the subsidy is too large both clubs' payments sharply decrease. Both clubs invest because their revenue depends on winning percentage and aggregate talent. This is different from the case when winning percentage and aggregate talent are weighted equally (left side of Figure 1).

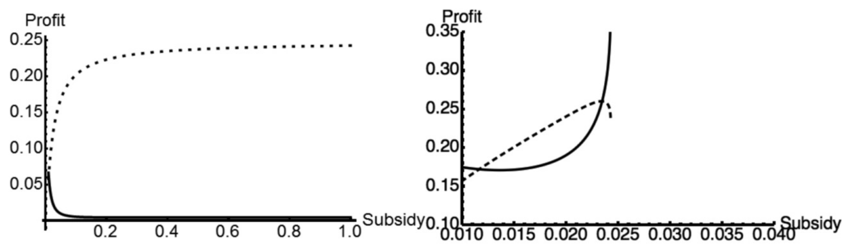
When the small-market club receives a subsidy, salary payments for both clubs increase. However, once the subsidy is too large both clubs payments sharply decrease. Again, this result is different when winning percentage is weighted (right side of Figure 1).

Club profits

When the large-market club receives a subsidy, the profits of only the large-market club are increasing (left side of Figure 5). In the case when competitive balance is weighted a subsidy for the large-market club leads, to some extent, to an increase in profits for both clubs (right side of Figure 5). This result is clearly different from the present case.

When the small-market club receives the subsidy, the profit of only the small-market club increases. This is different in the case when aggregate talent and winning percentage are equally weighted. Then the subsidy is positive for both clubs (to some extent). Even the large-market club benefits, although the small-market club receives the subsidy (right side of Figure 3).

Panel A: Subsidy for large club Panel B: Subsidy for small club



——— Small club - - - - - Large club

Figure 5. Profit when large-market club receives a subsidy

Social welfare

Social welfare decreases when the small-market club receives a small subsidy. When the subsidy is considerably large then social welfare increases. This is different in the case when competitive balance is weighted. Social welfare depends on whether fans have a high preference for aggregate talent. In this analysis fan preference for aggregate talent has only a minor influence.

When the large-market club receives a subsidy, social welfare always increases. Again, this is different for the case when winning percentage and competitive balance are equally weighted. The driving force for social welfare is fan preference for aggregate talent. When fan preference for aggregate talent is high a subsidy always increases social welfare.

Policy Implications

Four parts of our model are especially interesting for policy makers: competitive balance, aggregate talent, club profit, and social welfare.

We show that competitive balance significantly changes when one club receives a subsidy. When a large-market club receives a subsidy, competitive balance decreases. When a small-market club receives a subsidy, competitive balance first increases but decreases when the subsidy is too large. Thus, when a policy maker wants to set the socially optimal competitive balance, he/she has to limit the amount of the subsidy that a small-market club receives.

Managing the amount of a subsidy is an effective policy instrument to influence aggregate talent in a league. Increasing the amount of the subsidy increases aggregate talent until a maximum is reached. Increasing the subsidy above this talent-maximizing level decreases aggregate talent in the league. Thus, it is important for the policy maker to find the optimal amount of subsidy to maximize aggregate talent in a league.

Club profit depends on win percentage, competitive balance, and aggregate talent. We describe for policy makers two situations regarding club profits. One scenario is when fans have a low preference for aggregate talent. Aggregate club profit increases only when competitive balance increases, which means that the small-market club should be subsidized. In the second scenario, fans have a high preference for aggregate talent. Aggregate club profit increases only if the large club receives a subsidy. Thus, it is important for a policy maker to identify whether fans have a high or low preference for aggregate talent.

Aggregate club profits increase when the large-market club receives the subsidy and fans have a low preference for aggregate talent. When the small-market club receives the subsidy, aggregate club profits decrease. When fans have a high preference for aggregate talent then a subsidy for the large-market club increases aggregate profit. However, a subsidy for the small-market club first increases aggregate profit, but when the subsidy is too large, aggregate profit decreases.

We assume that a policy maker in the decision-making process primarily wishes to increase win percentage and tax revenues. A subsidy for a large-market share club always decreases competitive balance while a subsidy for a small-market share club has ambiguous effects. The subsidy for the small-market club results in higher salary payments, which ultimately affects the win percentage of both clubs. When a maximum is reached (i.e., a balanced league), further increasing the subsidy decreases competitive balance.

Our model provides essential policy implications regarding social welfare. While common intuition says that a subsidy is beneficial for only the subsidy-receiving club, our model shows that a different interpretation is appropriate. When supporters from both clubs have a high preference for aggregate talent, both clubs can benefit in terms of profit. However, when supporters have a low preference for aggregate talent, social welfare depends mainly on competitive balance and win percentage. Thus, when a subsidy results in a more unbalanced league, social welfare decreases.

Conclusion

The aim of this article is to develop a game-theoretic model that analyzes how subsidies influence a professional team sports league. In addition to win percentage and competitive balance, the model includes fan preference for aggregate talent. The paper examines how subsidies influence salary payments, competitive balance, club profits, and social welfare. Future research can weigh competitive balance, win percentage, and aggregate talent differently. Additionally, a similar setting in a league where clubs are win or utility maximizers may yield interesting results.

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Endnotes

[1] The European Commission states that the Spanish soccer club Real Madrid" appears to have benefitted from a very advantageous real property swap with the City of Madrid" (see http://europa.eu/rapid/press-release IP-13-1287_en.htm).

[2] See, for example, Barcelona CF, Athletic Club Bilbao. The European Commission states that they might have received "possible privileges regarding corporate taxation."

[3] One example for stadium rent is the Dutch soccer club Willem II (see http://europa.eu/rapid/press-release IP-13-192_en.htm). Clubs that might have benefited by paying no or lower rent for their training facilities are FC Den Bosch and MVV (these are also Dutch soccer clubs).

[4] Dietl and Lang (2008) use a similar approach but without the inclusion of aggregate talent. Aggregate talent is included in a different way in the models of Dietl, Lang, and Rathke (2009) and Madden (2011).

[5] Note, it is not the gross salary x_k but the net-of-tax salary s_k that determines the playing talent, and, in turn, the win percentage.

[6] For more detail, see Dietl et al. (2009), and Falconieri, Palomino, and Sakovics (2004), who use a similar approach.

[7] Note that $(1-\alpha_k)x_k = \mu_k t x_k$.

[8] Second-order conditions for a maximum are satisfied because $\frac{\partial \pi_i^2}{\partial x_i^2} < 0$ and $\frac{\partial \pi_j^2}{\partial x j^2} < 0$.

[9] A benchmark case (when the tax subsidy is symmetric) does not yield different results compared with no subsidies.

[10] An equivalent proposition can be derived if club j receives the tax break.

Appendices

Appendix A. Proof of Proposition 1

To proof the claim in Proposition 1, we proceed as follows. First, we show that the subsidy always increases the salary payments of club i if this club receives a subsidy, i.e., $\frac{\partial x_i^*}{\partial \alpha_i^*} > 0 \quad \forall \alpha_i > \alpha_j$. We compute

$$\frac{\partial x_i^*}{\partial \alpha_i} = 0 \Leftrightarrow \alpha = \{\alpha_1, \alpha_2\} \text{ with}$$

$$\alpha_{i1} = -\frac{m(\alpha_j - 2\sigma) + (m(\alpha_j - 4\sigma)(\alpha_j - \sigma))^{1/2}}{m^2}$$

$$\alpha_{i2} = \frac{-m(\alpha_j - 2\sigma) + (m(\alpha_j - 4\sigma)(\alpha_j - \sigma))^{1/2}}{m^2}$$

We have $a_i<0$ and $a_j>1$. Moreover, $\frac{\partial x_i^*}{\partial \alpha_i} > 0 \Leftrightarrow \alpha \in (\alpha_i, \alpha_j)$. Thus, we conclude that $\frac{\partial x_i^*}{\partial \alpha_i^*} > 0 \; \forall \alpha_i \in (\alpha_j, 1)$, which proves the claim.

Second, we show that a subsidy for club *i* increases the salary payments of club *j* if and only if the subsidy is not too large, i.e., $\frac{\partial x_j^*}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i < \alpha_i^* = \frac{16\alpha_j\sigma}{m(3\alpha_j + 4\sigma)}$. We compute

$$\frac{\partial x_j^*}{\partial \alpha_i} = 0 \Leftrightarrow \alpha = \alpha_i^* = \frac{16\alpha_j\sigma}{m(3\alpha_j + 4\sigma)}.$$

Moreover, we derive $\lim_{\alpha_i \to 0} \frac{\partial x_j^*}{\partial \alpha_i} = \frac{m}{2\alpha_j} > 0$ and thus $\frac{\partial x_j^*}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i < \alpha_i^*$, which proves the claim.

Appendix B. Proof of Proposition 2

Competitive balance is defined as

$$CB(x_i, x_j) = w_i(x_i, x_j)w_j(x_i, x_j) = \frac{\alpha_i x_i \alpha_j x_j}{(\alpha_i x_i + \alpha_j x_j)^2}$$

so that competitive balance in an equilibrium is

$$CB^* = \frac{1}{(1-(\frac{m\alpha_i(\alpha_j-4\sigma)}{\sqrt{m\alpha_i\alpha_j(m\alpha_j-4\sigma)(\alpha_j-4\sigma)}})(1+(\frac{\alpha_j(4\sigma-m\alpha_i)}{\sqrt{m\alpha_i\alpha_j(m\alpha_i-4\sigma)(\alpha_j-4\sigma)}})}.$$

Recall that large values of CB characterize a more balanced league. The most balanced league—a league with two equally strong clubs—has a maximum value of 0.25.

Next, we compute

$$\frac{\partial CB^*}{\partial \alpha_i^*} > 0 \Leftrightarrow \alpha_i < \alpha_i^{CB} = \frac{\alpha_j}{m}.$$

Note that $a_i^{CB}<a_j$ for $m>1$ and $a_i^{CB}>a_j$ for $m<1$.

(a) Suppose that club i is the large club with $m>1$ so that $a_i^{CB}<a_j$. Given that club i receives the subsidy, it holds $a_i>a_j$ and thus $a_i>a_i^{CB}$. We conclude that $\frac{\partial CB^*}{\partial \alpha_i^*} < 0$ $\forall \alpha_i \in (\alpha_j, 1),$ i.e., competitive balance in the league decreases.

(b) Suppose that club i is the small club with $m<1$ so that $a_i^{CB}>a_j$. Given that club i receives the subsidy, it holds $a_i>a_j$ and thus by increasing the subsidy the league becomes more balanced for all a_i (a_j, a_i^{CB}) and less balanced for all a_i $(a_i^{CB},1)$. Formally, $\frac{\partial CB}{\partial \alpha_i} > 0$ if $\alpha_i < \alpha_i^{CB}$ and $\frac{\partial CB}{\partial \alpha_i} < 0$ if $a_i>a_i^{CB}$. This completes the proof of Proposition 2.

Appendix C. Proof of Proposition 3

To show that a subsidy for club i always increases the win percentage of club i and decreases the win percentage of club j, we proceed as follows. We derive

$$\frac{\partial w_i^*}{\partial \alpha_i} = \frac{2\alpha_j\sigma\gamma}{\alpha_i\left(-m\alpha_i\alpha_j + 4\alpha_j\sigma + \gamma\right)^2},$$

$$\frac{\partial w_j^*}{\partial \alpha_i} = -\frac{2m^2\alpha_i\alpha_j\sigma(\alpha_j - 4\sigma)^2}{\gamma\left(-m\alpha_i(\alpha_j - 4\sigma) + \gamma\right)^2},$$

with $\gamma = \sqrt{m\alpha_i\alpha_j(m\alpha_i - 4\sigma)(\alpha_j - 4\sigma)}.$ so that $m\alpha_i\alpha_j(m\alpha_i - 4)(\alpha_j - 4) > 0.$ It is straightforward to show that $\frac{\partial w_i^*}{\partial \alpha_i} > 0$ and $\frac{\partial w_j^*}{\partial \alpha_i} < 0\ \forall \alpha_i \in (\alpha_j, 1).$

Appendix D. Proof of Result 1

To prove the claim of Result 1, we have to rely on numerical simulations. We set m=0.5; a_j=0.01; s=0.5. Solving the maximization problem $\max_{\alpha_i \in (\alpha_j,1)}(x_i^* + x_j^*)$ yields a_i^{AT}=0.038 and thus $\frac{\partial x_i^* + x_j^*}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^{AT})$. Thus, a higher subsidy for club i always increases aggregate talent in the league if and only if the subsidy is not too large. This completes the proof of Result 1.

To explain Proof of Result 1, we use Figure 6 that displays subsidies on the x-axis and aggregate talent on the y-axis.

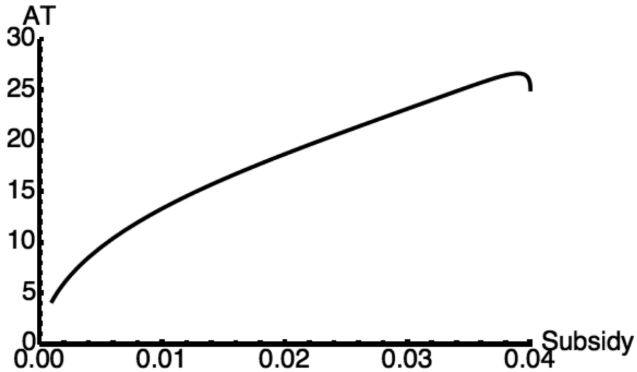

Figure 6. Aggregate talent

Appendix E. Proof of Result 2

To prove the claim of Result 2, we have to rely on numerical simulations again. We set m = 0.5; a_j = 0.01; s = 0.5: (i) Solving the maximization problem $\max_{\alpha_i \in (\alpha_j,1)} \pi_i^*$ yields $\alpha_i^\pi = 0.038$ and thus $\frac{\partial \pi_i}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^\pi)$. (ii) Solving the minimization problem $\min \pi_j^*$ yields $\alpha_j^\pi = 0.02$ and thus $\frac{\partial \pi j}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j^\pi, 1)$. Thus, part (i) shows that a higher subsidy for club i always increases the profits of club i if and only if the subsidy is not too large. Part (ii) shows that a higher subsidy for club i increases the profits of the other club j if and only if the subsidy is sufficiently large.

Appendix F. Proof of Result 3

To prove the claims of Result 3, we have to rely on numerical simulations again. We set m = 0.5; a_j = 0.01; s = 0.5. The maximization problem $\max_{\alpha_i \in (\alpha_j,1)} W^*$ has no interior solution and thus $\frac{\partial W}{\partial \alpha_i} > 0 \; \forall \alpha_i \in (\alpha_j, 1)$ for $\sigma < \sigma^*$. Thus increasing the subsidy always increases social welfare if supporters have a high preference for aggregate talent.

Second, we set m = 0.5; a_j = 0.01; s = 2. Solving the maximization problem $\max_{\alpha_i \in (\alpha_j,1)} W^*$ yields $\alpha_i^W = 0.015$ and thus $\frac{\partial W}{\partial \alpha_i} > 0 \Leftrightarrow \alpha_i \in (\alpha_j, \alpha_i^W)$ for $\sigma > \sigma^*$. Thus, if supporters have a low preference for aggregate talent, increasing the subsidy increases social welfare if and only if the subsidy is not too large.