Real Options and Technology Choice under Bertrand Competition

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Abstract

We study the investment decision problem of a duopoly with price competition on a market of finite size driven by stochastic shocks on profit. Each player has the choice between two technologies: a large unit and a small one. We prove that different equilibria may exist depending on the parameters' values: simultaneous investment equilibrium in the small unit or two preemption equilibria (preemption with the large or the small unit). Whatever the cost advantage of the small capacity technology, there is always a range of the profits for which the outcome consists in an asymmetric investment. More surprisingly, uncertainty has almost no effect on the type of equilibrium that emerge. We also prove that for some values of the demand, the preemption equilibrium in the large unit is more efficient than the joint adoption equilibrium.

1 Introduction

When a firm contemplates the possibility to undertake an investment, it has to consider all the technologies that are available on the market. Not only is the choice of the production process very important, but also the production capacity that has a direct effect on the total profit of the firm. Indeed, even if an investor alone preferred to invest in a large unit, the presence of competitors could make her invest in her least preferred technology, namely, the small unit in order to be able to sell a lower quantity but at a higher price. This effect could arise for any kind of investment, also in the R&D sector. As different authors already argued (Kulatilaka and Perotti [20] or Grenadier [13] for instance) an investment in R&D expenditures can be seen as a growth option on a future opportunity. Payoffs coming from research efforts may be very high, but may also be significantly reduced by the presence of other participants: competition can be very fierce. This is what happens in the pharmaceutical industry and more precisely in the development and production of vaccines where the number of firms is quite restricted: there are three or four major firms. Moreover, in this branch, the high production costs and the difficulties to obtain a permit to market a vaccine represent barriers to entry so that any investment can be considered as irreversible. Therefore the correct calibration of the capacity choice is a key element of such an investment decision.

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This paper proposes to study the investment decision of a firm in a competitive framework when it has the choice between different technologies to produce the same output. We consider a duopoly model with price competition on a market driven by stochastic shocks on profit. Each firm has the choice between two capacity sizes to undertake an investment.[1] Focusing on Markov Perfect Equilibria, we prove that different equilibria may emerge according to the parameter values. There are different types of preemption equilibria (the first mover choosing the small or the large capacity) and a simultaneous investment equilibrium. With this paper, we aim at studying the tension between two main effects: on the one hand, the choice for each firm would induce the participants to delay investment in order to choose the technology that is shown to be the most profitable; on the other hand, the presence of competitors make every firm reluctant to wait. Without the threat of preemption, the first effect dominates and the leader still takes the time to ensure an investment in the most profitable technology. With the threat of preemption, we prove that the second effect always dominates the first one: firms do not delay investment anymore. On the contrary, we prove that there is always a preemption incentive.

Our analysis provides different innovative insights on timing of new technology adoption under uncertainty in a competitive framework. It proposes a complete comparative static analysis with respect to the cost advantage of the small capacity technology and to uncertainty of the shock on profit. The first feature to point out is the heterogeneity in the outcomes of the investment strategy for both participants. Although firms are ex-ante identical, the outcome is heterogenous is most cases whatever the cost advantage of the small capacity technology. This result meets the conclusion of Besanko and Doraszelski [3] who study a capacity accumulation game in an oligopolistic industry. They conclude that under Cournot competition, the industry is composed of equal-sized firms. On the contrary under Bertrand competition, they claim that the industry tends to an asymmetric structure. They link their results with several empirical studies that our theoretical analysis could reinforce. This heterogeneity in the outcome can be explained with the strategic advantage of the first mover firm who invests in the large unit. In doing so, she delays the entry of her opponent who will invest later in the small capacity technology. Therefore, she takes advantage of the high profit flow as long as she is alone. This effect of course diminishes as the cost advantage of the small unit increases.

Unlike the majority of models dealing with strategic investment that highlight a strategic advantage for the second mover (see Hoppe [15] or Mason and Weeds [23] for instance), in our setting the leader still has a strategic advantage meaning that the outcome of the game consists in a preemption equilibrium (the first investor choosing the small or the large unit) for most of the parameters values. This is in part due to the setting we chose in which the second mover does not have the choice: it is always more profitable to invest in the small capacity technology. However, the fear of preemption is so high that the first mover may invest in her least preferred technology, meaning that if she were not threatened by the preemption from the other participant, she would invest in the other technology. As we already mentioned, we also propose a comparative static analysis with respect to uncertainty. We conclude that if uncertainty plays a role on the level of the thresholds that trigger investment, it has no effect on the type of equilibrium that might emerge. This is the cost advantage of one technology relative to the other that is the key factor. This conclusion completes the one of Mason and Weeds [23].

In this work, we also propose a welfare analysis that leads to results in contradiction with the existing literature. Indeed, since the pioneer work of Fudenberg and Tirole [9], it has mostly being the case that a simultaneous investment equilibrium Pareto dominates a preemption equilibrium. In our model, for all parameters values, there is a region on which two equilibria coexist: a mixed strategy equilibrium that consists in a preemption equilibrium in the large

[1] This extreme choice between only two capacity sizes makes the model much easier. We are thus able to solve it completely.

capacity technology and a simultaneous investment in the small capacity technology. A welfare analysis shows that it is very likely that the preemption equilibrium is more efficient. From the firms' perspective, the preemption equilibrium is less interesting since their expected profit is much smaller than in the simultaneous investment case. However, the consumer surplus is much higher in the case of the preemption equilibrium since prices are much lower when the large capacity has been installed. We thus show that the total surplus may be continuous or discontinuous as the value of the shock on profit increases. Locally, the expected surplus may not be increasing in the state variable, and waiting may not always be more efficient.

This work comes within the scope of timing game of entry that has been first developed by Reinganum [27] and Fudenberg and Tirole [9]. Reinganum studies the diffusion of a new technology in a duopoly when the players have precommitment to adoption dates and she shows that the outcome is a "diffusion" equilibrium (firms adopt a different dates). Fudenberg and Tirole model the adoption of a new technology as a timing game and show that two equilibria could emerge: a preemption equilibrium in which the payoffs of the two players are equalized and a joint adoption equilibrium, the latter being Pareto dominant. The introduction of an "option value" by Arrow and Fisher [1] and Henry [14] has considerably influenced the understanding of investment under uncertainty. Hoppe [16] proposes a very interesting review of literature on the timing of new technology adoption. A whole series of paper have introduced competition in the pioneer models, reducing the option value to almost zero. For instance Kulatilaka and Perotti [20] propose a model close to ours but without technology choice. They show that strategic investment with strong preemptive effect and high uncertainty tend to favor investment. Grenadier [13] describes a Cournot Nash equilibrium when each firm faces a sequence of investment opportunities. He derives a symmetric equilibrium and finds that with competition, the traditional NPV rule becomes approximatively correct. His work has triggered off many reactions. Extensions have been proposed by Back [2] in the case of Bertrand competition or Novy-Marx [25]. Many papers dealing with timing game of entry also introduced uncertainty in the initial framework of Fudenberg and Tirole [9]. For instance Hoppe [15] highlights the strategic advantage of the second mover in introducing uncertainty on the profitability of the technology. The second mover has a strategic advantage since she learns the quality of the technology thanks to the first investor's action. Boyer at al. [5] propose a model where firms face sequential investment under Bertrand competition and prove that the equilibrium results in a joint investment. Weeds [32] considers a model of R&D competition with two uncertainty sources: a technological uncertainty and an economic uncertainty on the patent influence the investment decision. The resulting equilibrium is either sequential or simultaneous, the latter being the most efficient. Mason and Weeds [23] study the effect of uncertainty in the investment decision of a monopoly and show how greater uncertainty may hasten investment. Our model can be distinguished from these ones since we focus on a choice between asymmetric investments in a competitive setting. Moreover, we propose a detailed analysis for different values of the uncertainty of the shock on profit and of the cost advantage of the small capacity technology.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the case of a natural leader and a natural follower. Section 4 is devoted to the situation where the constraint on the entry's order of the players is relaxed: preemption is thus possible. In section 5, we derive the welfare analysis and section 6 concludes. The appendix contains lengthier proofs.

2 The model

Time is continuous and indexed by $t \geq 0$. We consider a duopoly model where each firm has the choice between two technologies to undertake a unique investment. The two technologies differ in their production capacity. Both firms are risk neutral and discount future revenues and costs at a constant risk-free rate r. Variable costs are normalized to zero. Investment is irreversible and takes place in a lumpy way. The characteristics of the technologies are the following:

- technology 1 has a capacity of 1 and its sunk cost is equal to I_1,
- technology 2 has a capacity of 2 and its sunk cost is equal to I_2.

We suppose that the profit of each firm is subject to random shocks X_t described by a geometric Brownian motion

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t \tag{1}$$

where $X_0 = x$, μ and σ are positive constants and $\{W_t\}_{t\geq 0}$ is a standard Brownian motion. We denote $\Pi(n_i, n_j, X_t)$ the instantaneous expected profit flow of firm i when it holds n_i unit whereas its rival holds n_j units, and the shock equals X_t:

$$\Pi(n_i, n_j, X_t) = X_t \pi(n_i, n_j), \tag{2}$$

where $\pi(n_i, n_j)$ is the deterministic part of the profit of firm i.

At any date t, the demand side of the market is described by a linear demand function:

$$Q(P_t) = 3 - P_t, \tag{3}$$

where P_t is the market price.

Concerning competition, within each instant $[t, t + dt)$, the timing of the game is the following:

(i) first, each firm decides whether to invest or not, and if yes chooses the technology to invest in, given the realization of X_t;

(ii) next, each firm quotes a price given its investment and that of its rival;

(iii) last, consumers choose from which firm to purchase. Production and transfers take place.

Nash equilibrium in pure strategy does not always exist in such capacity constrained price competition game. It may be thus necessary to focus on mixed strategy equilibria. The following lemma gives the instantaneous profit function $\pi(n_i, n_j)$ in each state of the world of this Bertrand Edgeworth competition game.[2]

Lemma 1 *In the different states of the world, the instantaneous expected profits are equal to:*

- $\pi(1, 0) = 2$, $\pi(2, 0) = 9/4$, *and* $\pi(0, n_j) = 0, \forall n_j \in \{0, 1, 2\}$
- $\pi(2, 1) = 1$, $\pi(1, 2) = 1/2$, $\pi(1, 1) = 1$, *and* $\pi(2, 2) = 1/4$.

Pure strategy equilibrium does not exist if the state of the world is $(1, 2)$ or $(2, 2)$. In these two cases, expected profits are computed. Firms do not produce at full capacity except in the states $(1, 0)$ and $(1, 1)$. With this Lemma, we see that competition is quite fierce: if too many capacity units are installed, the expected profit may brutally decrease. We focus on Markov Perfect Equilibria (MPE) in which firms' investment and pricing decisions depend only on the current value of the shock x and the firms' capital stock measured in units of capacity (n_i, n_j).

[2] Kreps and Scheinkman [18] clearly explain how to compute the expected profit in each state of the world.

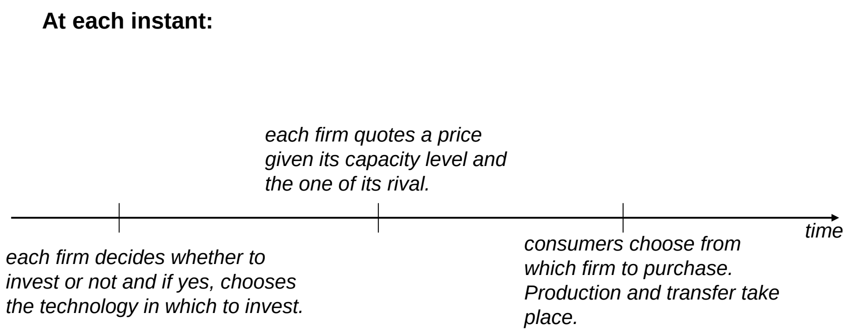

Figure 1: Timing.

3 Asymmetric players: a natural leader and a natural follower

As a benchmark, we focus on the case of a natural leader (L) and a natural follower (F). The natural leader invests first and the natural follower may enter the market only once the first investment occurred. Moreover, according to Lemma 1, once the leader has made her capacity choice, the follower does not have the choice any more. She always invests in technology 1: indeed, $\pi(1,1) = \pi(2,1)$ and $\pi(1,2) > \pi(2,2)$. In the next subsections, we solve this problem by backward induction, focusing first on the behavior of the follower.

3.1 Follower's strategy

The follower's strategy depends on the leader's choice.

3.1.1 The leader has invested in technology 1

We are here in the classic setting of optimal stopping time models. Let us first compute the expected net discounted profit of the follower when she is going to invest in technology 1, given that the leader has invested in technology 1 and that the shock equals x

$$\Phi_{11}^F(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}\Pi(1,1,X_t)\,dt | X_0 = x\right] - I_1 = \frac{\Pi(1,1,x)}{r-\mu} - I_1 = \frac{x}{r-\mu} - I_1. \quad (4)$$

The option value created by such a possible investment is

$$V_{11}^F(x) = \sup_\tau \mathbb{E}\left[e^{-r\tau}\Phi_{11}^F(X_\tau) | X_0 = x\right]. \quad (5)$$

This problem can be easily solved.[3] The solution is

$$V_{11}^{F}(x)=\begin{cases}\left(\frac{x}{x_{11}^{F*}}\right)^{\beta}\left(\frac{x_{11}^{F*}}{(r-\mu)}-I_1\right) & \text{if } x\leq x_{11}^{F*},\\ \frac{x}{(r-\mu)}-I_1 & \text{if } x>x_{11}^{F*},\end{cases}$$

where

$$x_{11}^{F*}=\frac{\beta}{\beta-1}(r-\mu)I_1, \tag{6}$$

and β is the positive root of the second order equation

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r=0. \tag{7}$$

The investment strategy of the follower is to invest in technology 1 as soon as x crosses the threshold x_{11}^{F*}. If $x<x_{11}^{F*}$, she prefers to wait and see the evolution of demand. As $\beta>1$, $x_{11}^{F*}>(r-\mu)I_1$. Indeed the follower values the information she can collect on the demand and prefers to delay investment: this is a classic result of the real option theory.

3.1.2 The leader has invested in technology 2

The problem is very similar in the case where the leader has invested in technology 2. We thus give directly the expression for the profit function and the option value.

$$\Phi_{12}^{F}(x)=\mathbb{E}\left[\int_0^{+\infty}e^{-rt}\Pi(1,2,X_t)\,dt|X_0=x\right]-I_1=\frac{x}{2(r-\mu)}-I_1. \tag{8}$$

$$V_{12}^{F}(x)=\begin{cases}\left(\frac{x}{x_{12}^{F*}}\right)^{\beta}\left(\frac{x_{12}^{F*}}{2(r-\mu)}-I_1\right) & \text{if } x\leq x_{12}^{F*},\\ \frac{x}{2(r-\mu)}-I_1 & \text{if } x>x_{12}^{F*},\end{cases}$$

where

$$x_{12}^{F*}=\frac{\beta}{\beta-1}2(r-\mu)I_1. \tag{9}$$

Here, investment in technology 1 is triggered as soon as x reaches the threshold x_{12}^{F*}. Note that $x_{12}^{F*}=2x_{11}^{F*}$. The follower invests earlier if the leader has invested in technology 1 than if she has invested in technology 2. Indeed, when the leader invests in technology 2, the follower's net profit is twice less than if the leader had invested in technology 1. $V_{11}^{F}(x)$ is greater than $V_{12}^{F}(x)$ whatever the demand value x. The follower prefers that the leader invests in technology 1. We now turn to the analysis of the leader's strategy.

3.2 Leader's investment decision: two auxiliary problems

The leader has the choice between the two technologies. In a first step, we consider two auxiliary problems when the leader does not have this choice.

[3] See for example Dixit and Pindyck, Chapter V [7].

3.2.1 Leader's investment decision in technology 1

The leader's profit flow depends on whether or not the follower has already invested. Indeed, her instantaneous profit flow is modified by the entry of the follower. The expected net discounted profit of the leader equals

$$\Phi_1^L(x) = \mathbb{E}\left[\int_0^{\tau_{11}^{F*}} e^{-rt} 2X_t dt + \int_{\tau_{11}^{F*}}^{+\infty} e^{-rt} X_t dt | X_0 = x\right] - I_1, \tag{10}$$

where $\tau_{11}^{F*} = \inf\left\{t | X_t = x_{11}^{F*}\right\}$. If $x \geq x_{11}^{F*}$, there is a simultaneous investment by the two players and the expected net discounted profit of the leader is equal to

$$\Phi_1^L(x) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt} X_t dt | X_0 = x\right] - I_1. \tag{11}$$

We obtain that

$$\Phi_1^L(p) = \begin{cases} \frac{2x}{r-\mu} - \frac{x_{11}^{F*}}{(r-\mu)}\left(\frac{x}{x_{11}^{F*}}\right)^\beta - I_1 & \text{if } x \leq x_{11}^{F*}, \\ \frac{x}{r-\mu} - I_1 & \text{if } x > x_{11}^{F*}. \end{cases}$$

If $x \leq x_{11}^{F*}$, when the leader invests, she is alone on the market. The term $-\frac{x_{121}^{F*}}{r-\mu}\left(\frac{x}{x_{11}^{F*}}\right)^\beta$ represents the loss in the leader's profit induced by the potential entry of the follower. On the contrary, if $x \geq x_{11}^{F*}$, both the leader and the follower enter at the same time.

As usual, the option value of the investment in technology 1 has the following expression

$$V_1^L(x) = \sup_\tau \mathbb{E}\left[e^{-r\tau} \Phi_1^L(X_\tau) | X_0 = x\right], \tag{12}$$

and is analytically equal to

$$V_1^L(x) = \begin{cases} \left(\frac{x}{x_1^{L*}}\right)^\beta \Phi_1^L\left(x_1^{L*}\right) & \text{if } x \leq x_1^{L*}, \\ \Phi_1^L(x) & \text{if } x > x_1^{L*}, \end{cases}$$

where

$$x_1^{L*} = \frac{\beta}{\beta - 1} \frac{r-\mu}{2} I_1. \tag{13}$$

3.2.2 Leader's investment decision in technology 2

If the leader invests in technology 2, the expected payoff and the option value equal

$$\Phi_2^L(x) = \begin{cases} \frac{9x}{4(r-\mu)} - \frac{5x_{12}^{F*}}{4(r-\mu)}\left(\frac{x}{x_{12}^{F*}}\right)^\beta - I_2 & \text{if } x \leq x_{12}^{F*}, \\ \frac{x}{r-\mu} - I_2 & \text{if } x > x_{12}^{F*}. \end{cases}$$

$$V_2^L(x) = \begin{cases} \left(\frac{x}{x_2^{L*}}\right)^\beta \Phi_2^L\left(x_2^{L*}\right) & \text{if } x \leq x_2^{L*}, \\ \Phi_2^L(x) & \text{if } x > x_2^{L*}, \end{cases}$$

where

$$x_2^{L*} = \frac{\beta}{\beta - 1} \frac{4}{9}(r-\mu) I_2. \tag{14}$$

Note that $x_2^* > x_1^*$. What happens in the case where the leader has indeed the choice between the two technologies?

3.3 Leader's investment decision when she has the choice

If the leader has the choice between the two technologies, the stopping time problem she faces is:

$$V^L(x) = \sup_\tau \mathbb{E}\left[e^{-r\tau} \max\left(\Phi_1^L(X_\tau), \Phi_2^L(X_\tau)\right) | X_0 = x\right]. \tag{15}$$

Indeed, while the investment has not been undertaken, the leader still has the choice between the two technologies. This kind of problems has been studied by Décamps et al. [6] for instance and we remind their main results. First of all, as we do not want one technology to dominate strictly the other, we put some restrictions on the parameters' values.

Lemma 2 *Under the assumption*

$$A1: I_2 < \left(1 + \frac{5\beta}{4(\beta-1)}\left(1 - \frac{1}{2^{\beta-1}}\right)\right) I_1, \tag{16}$$

there are two thresholds $\widetilde{x}$ *and* $\widetilde{\widetilde{x}}$*, such that*

- $\forall x \in [0, \widetilde{x}[$, $\Phi_1^L(x) > \Phi_2^L(x)$,
- $\forall x \in \left]\widetilde{x}, \widetilde{\widetilde{x}}\right[$, $\Phi_1^L(x) < \Phi_2^L(x)$,
- $\forall x \in \left]\widetilde{\widetilde{x}}, +\infty\right[$, $\Phi_1^L(x) > \Phi_2^L(x)$.

Proof: We study $\Phi_1^L(x) - \Phi_2^L(x)$. This function is differentiable everywhere except for $x = x_{11}^{F*}$ and $x = x_{12}^{F*}$. We want to determine the parameters' values such that this function is positive, then negative to finally end up positive. This comes down to finding a condition under which the minimum of this function is strictly negative.

We first study this function on $\left[0, x_{11}^{F*}\right]$.

$$\Phi_1^L(x) - \Phi_2^L(x) = \frac{x_{11}^{F*}}{r-\mu}\left(\frac{x}{x_{11}^{F*}}\right)^\beta \left(\frac{5}{2^{\beta+1}} - 1\right) - \frac{x}{4(r-\mu)} + I_2 - I_1$$

This function is decreasing and attains its minimum at $x = \left(\frac{1}{4\beta}\frac{2^{\beta+1}}{5-2^{\beta+1}}\right)^{\frac{1}{\beta-1}} x_{11}^{F*}$. As $\left(\frac{1}{4\beta}\frac{2^{\beta+1}}{5-2^{\beta+1}}\right)^{\frac{1}{\beta-1}} > 1 \forall x \in \left[0, x_{11}^{F*}\right]$,$\Phi_1^L(x) - \Phi_2^L(x)$ is strictly decreasing on this interval.

We then study this function on $\left[x_{11}^{F*}, x_{12}^{F*}\right]$.

$$\Phi_1^L(x) - \Phi_2^L(x) = \frac{5x_{12}^{F*}}{4(r-\mu)}\left(\frac{x}{x_{12}^{F*}}\right)^\beta - \frac{5x}{4(r-\mu)} + I_2 - I_1.$$

This function is either increasing or decreasing and then increasing on $\left[x_{11}^{F*}, x_{12}^{F*}\right]$, depending in the ranking of $\left(\frac{1}{2}\right)^{\frac{1}{\beta-1}}$ relative to $\frac{1}{2}$.

Then, it follows that in order this function to have two zeros, it is necessary and sufficient that $\Phi_1^L\left(x_{11}^{F*}\right) - \Phi_2^L\left(x_{11}^{F*}\right) < 0$ what comes down to $I_2 < \left(1 + \frac{5\beta}{4(\beta-1)}\left(1 - \frac{1}{2^{beta-1}}\right)\right) I_1$.

Note that we have proven that $\widetilde{x} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$. □

Note that by proving this Lemma, we have also proved that $\widetilde{x} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$.

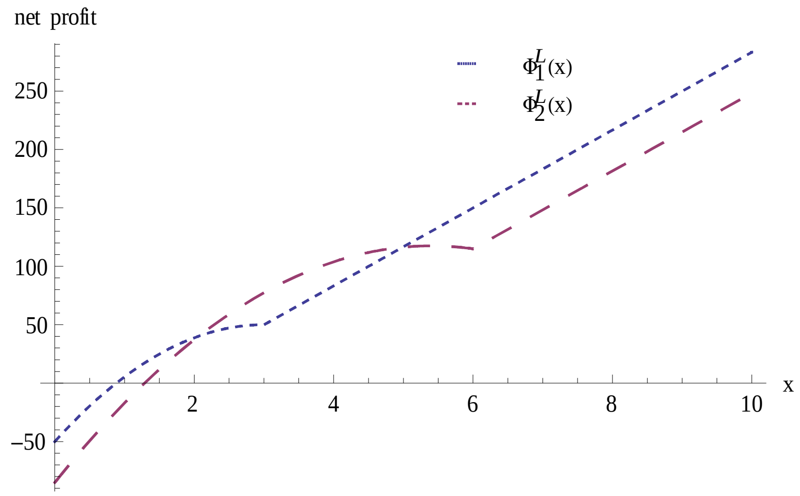

Figure 2: Net profits $\Phi_1^L(x)$ and $\Phi_2^L(x)$.

On Figure 3.3, we represent the net profit functions in both cases, $\Phi_1^L(x)$ and $\Phi_2^L(x)$, when $r = 0.05$, $\mu = 0.02$, $\sigma = 0.1$, $I_1 = 50$ and $I_2 = 85$.

Assumption $A1$ implies that technology 2 should not be too expensive. Indeed, in the opposite case, it would not be worth investing in it, and technology 1 would always be the preferred technology. Before going further with the analysis of the leader's behavior, a second restriction has to be put on the parameters' values, indeed that at the indifference point, $\widetilde{x}$, the two technologies are already profitable. If this were not the case, then it would never be worthwhile to invest in technology for low levels of x. This restriction is summarized in Assumption $A2$.

$$A2 : \Phi_1^L(\widetilde{x}) = \Phi_2^L(\widetilde{x}) > 0. \tag{17}$$

It is shown in the Appendix that this assumption is not satisfied if I_2/I_1 is too low. This is quite intuitive. If the cost advantage of technology 1 is small, there is no reason to invest in technology 1 when x is small. Any investor would be better off by investing in technology 2. In the Appendix, Assumption $A2$ is expressed with the help of the parameters of the model so that Assumption $A2$ can be represented as a function of β and I_2/I_1.

Let us introduce the three sets

$$E^L = \left\{x \geq 0 | V^L(x) = \max\left(\Phi_1^L(x), \Phi_2^L(x)\right)\right\},$$

$$E_1^L = \left\{x \geq 0 | V^L(x) = \Phi_1^L(x)\right\}, \text{ and } E_2^L = \left\{x \geq 0 | V^L(x) = \Phi_2^L(x)\right\}.$$

E^L is the "exercise region" (also called "investment region") of the leader when she can choose between the two technologies, E_1^L (resp. E_2^L) is the exercise region of the leader when technology

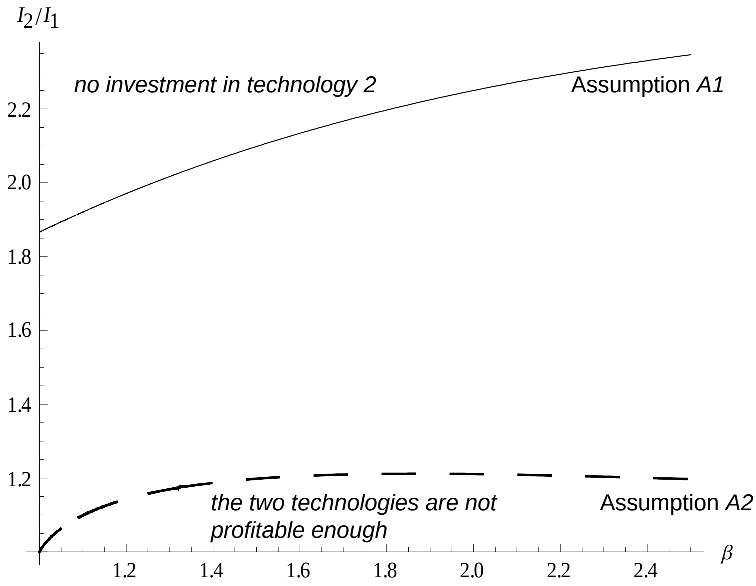

Figure 3: Assumptions $A1$ and $A2$.

1 (resp. 2) is the preferred one. According to Décamps et al. [6], we know E^L is the disjoint union of the two exercise sets E_1^L and E_2^L. They show the following theorem that we apply to our setting: *The indifference points* $\widetilde{x}$ *and* $\widetilde{\widetilde{x}}$ *do not belong to the exercise region* E^L.[4] Therefore, E_1^L can also be decomposed into two disjoint sets:

$$E_1^L = \underline{E}_1^L \cup \overline{E}_1^L,$$

where $\underline{E}_1^L = \left\{0 \leq x < \widetilde{x} | V^L(x) = \Phi_1^L(x)\right\}$ and $\overline{E}_1^L = \left\{x > \widetilde{\widetilde{x}} | V^L(x) = \Phi_1^L(x)\right\}$. Concerning the leader's exercise region in technology 2, we have that $E_2^L \subset \left]\widetilde{x}, \widetilde{\widetilde{x}}\right[$. We will see in the next subsections that depending on the values taken by the parameters of the model, the exercise region for investment in technology 1, $\underline{E}_1^L$, does not always exist (is empty). Lemma 3 gives a first result on the exercise region $\overline{E}_1^L$.

Lemma 3 *The exercise region* $\overline{E}_1^L$ *is never empty.*

Proof: Suppose that $\overline{E}_1^L = \emptyset$. Then, $\forall x > \widetilde{\widetilde{x}}, V(x) > \Phi_1^L(x) > \Phi_2^L(x)$. at the same time, $\forall x \geq x_1^{L*}, V_1^L(x) = \Phi_1^L(x)$. But, when $x \to +\infty, V^L(x) = V_1^L(x)$. This leads to $\lim_{x\to+\infty} \Phi_1^L(x) > \lim_{x\to+\infty} \Phi_1^L(x)$, a contradiction. □

When demand is high enough, whatever the parameters' values, the leader is going to invest in technology 1 and $\overline{E}_1^L$ is never empty. Let us now focus on the other exercise region in technology 1, $\underline{E}_1^L$.

[4]See proposition 2.2 p.431 [6].

3.3.1 The case of two exercise regions: $\underline{E}_1^L$ is empty

In this part, we study the case where $\underline{E}_1^L$ is empty. Let us consider

$$A3 : \Phi_1^L\left(x_1^{L*}\right) < V_2^L\left(x_1^{L*}\right). \tag{18}$$

Assumption $A3$ implies that, at the threshold x_1^{L*}, the option value of investing in technology 2 is strictly greater than the expected discounted net profit of investing in technology 1. The leader prefers not to invest immediately in technology 1 and to keep alive the option to invest in technology 2.

Lemma 4 *We have the following result concerning the shape of* $\underline{E}_1^L$.

- *If* $9I_1 > 8I_2$, $\underline{E}_1^L = \emptyset$.
- *If* $9I_1 < 8I_2$, $\underline{E}_1^L = \emptyset$ *if and only if*

$$\left(\frac{9I_1}{8I_2}\right)^\beta \left(\frac{I_2}{I_1} - \frac{5}{2}\beta\left(\frac{2I_2}{9I_1}\right)^\beta\right) - 1 + \beta\left(\frac{1}{2}\right)^\beta > 0.$$

The condition given in this lemma is satisfied if and only if the ratio I_2/I_1 is small enough. Indeed, when technology 2 is relatively cheap, the leader does not want to waste her opportunity of receiving high revenues and thus she refrains from investing in technology 1 for low values of x. This condition is also satisfied if β is quite high (meaning that σ is quite low, r and μ being constant). Indeed in this case, the probability that x takes high values quite in a near future is low high, and the leader takes advantage of the time she is alone on the market.

Under Assumption $A3$ and according to Décamps et al. [6], we know that the leader's option value is equal to:

$$V^L(p) = \begin{cases} B_2 x^\beta & \text{if } x \leq x_2^{L*}, \\ \Phi_2^L(x) & \text{if } x_2^{L*} < x \leq x_3, \\ Ax^\alpha + Bx^\beta & \text{if } x_3 < x \leq x_4, \\ \Phi_1^L(x) & \text{if } x > x_4, \end{cases}$$

where

$$B_2 = \left(\frac{1}{x_2^{L*}}\right)^\beta \Phi_2^L\left(x_2^{L*}\right),$$

and x_3, x_4, A and B can be numerically obtained thanks to the value matching and smooth pasting conditions at x_3 and x_4.

The leader invests in technology 2 if $x \in \left[x_2^{L*}, x_3\right]$, he invests in technology 1 if $x \geq x_4$. The indifference point $\widetilde{\widetilde{x}}$ does not belong to any exercise region as we already mentioned: $\widetilde{\widetilde{x}} \in [x_3, x_4]$. Between these two exercise regions $\left[x_2^{L*}, x_3\right]$ and $[x_4, +\infty[$, the leader faces an inaction region in which she prefers to wait rather than to invest in one of the two technologies.

In the case of a unique inaction region, two equilibria may arise: a sequential equilibrium (the leader invests in technology 2 and the follower invests in technology 1 at x_{12}^{F*}) when $x \in \left[x_2^{L*}, x_3\right]$ and a simultaneous investment in technology 1 equilibrium $\forall x \geq x_4$.

3.3.2 The case of three exercise regions: $\underline{E}_1^L$ is not empty

To ensure the existence of $\underline{E}_1^L$, we need to assume that

$$A4 : \Phi_1^L\left(x_1^{L*}\right) \geq V_2^L\left(x_1^{L*}\right). \tag{19}$$

Lemma 5 *Under Assumption A4,* $\underline{E}_1^L$ *and* E_2^L *are not empty.*

This implies that at x_1^{L*}, the leader prefers to invest immediately in technology 1 rather than to wait to invest in technology 2. This is a sufficient condition for $\underline{E}_1^L$ not to be empty. Assumption $A4$ is the contrary of Assumption $A3$, and then we know according to Lemma 4 that it is satisfied when when I_2/I_1 is high enough. Indeed, in this case the leader prefers to earn profit quite early in the time by investing in technology 1 because she knows that demand has to be quite high in order an investment in technology 2 to be profitable. In this case, the leader's option value is a succession of three disjoint exercise regions: $\underline{E}_1^L$, E_2^L and $\overline{E}_1^L$.

$$V^L(x) = \begin{cases} B_1 x^\beta & \text{if } x \leq x_1^{L*}, \\ \Phi_1^L(x) & \text{if } x_1^{L*} \leq x \leq \widehat{x}_2, \\ \underline{A}x^\alpha + \underline{B}x^\beta & \text{if } \widehat{x}_2 \leq x \leq \widehat{x}_3, \\ \Phi_2^L(x) & \text{if } \widehat{x}_3 \leq x \leq \widehat{x}_4, \\ \overline{A}x^\alpha + \overline{B}x^\beta & \text{if } \widehat{x}_4 \leq x \leq \widehat{x}_5, \\ \Phi_1^L(x) & \text{if } x \geq \widehat{x}_5, \end{cases}$$

where

$$B_1 = \left(\frac{1}{x_1^{L*}}\right)^\beta \Phi_1^L\left(px_1^{L*}\right),$$

and $\widehat{x}_2$, $\widehat{x}_3$, $\widehat{x}_4$, $\widehat{x}_5$, $\underline{A}$, $\underline{B}$, $\overline{A}$ and $\overline{B}$ can be numerically obtained thanks to the value matching and smooth pasting conditions at $\widehat{x}_2$, $\widehat{x}_3$, $\widehat{x}_4$ and $\widehat{x}_5$.

In this case with three exercise regions, when $x \in \left[x_1^{L*}, \widehat{x}_2\right]$, the equilibrium is a sequential investment in technology 1 for the two players, when $x \in [\widehat{x}_3, \widehat{x}_4,]$, the equilibrium is a sequential investment in technology 2 for the leader and then 1 for the follower, and when $x \in [\widehat{x}_5, +\infty[$. Let us now introduce the possibility for any player to preempt her rival.

4 Symmetric players: when preemption is allowed

When no constraint holds on the order players enter the market, each player may want to preempt her rival in order to take advantage of high profit flows the time she is alone on the market. Three kinds of equilibria may emerge:

- an equilibrium where the first mover invests in technology 1,
- an equilibrium where the first mover invests in technology 2,
- an equilibrium where the two players have an incentive to simultaneously invest in technology 1.

If the two players only focus on technology 1, until $x < x_{11}^{F*}$, each of them wants to be the first to move in order to take advantage of the time period she is alone on the market. In fact there is a gain in being the first mover as soon as $\Phi_1^L(x) \geq V_{11}^F(x)$. Thus the threshold $\overline{x}_1$ such that $\Phi_1^L(\overline{x}_1) = V_{11}^L(\overline{x}_1)$ is decisive. In fact, as soon as $\Phi_1^L(x) \geq V_{11}^L(x)$, or equivalently $x \geq \overline{x}_1$, there is the risk of being preempted.

As in the previous case, the threshold $\overline{x}_2$ such that $\Phi_2^L(\overline{x}_2) = V_{12}^F(\overline{x}_2)$ plays a key role in the other preemption equilibrium. Indeed, as soon as $x \geq \overline{x}_2$ or equivalently $\Phi_2^l(x) \geq V_{12}^F(x)$, there is a gain in being the first who moves and invests in technology 2.

In these two mixed strategy equilibria, one should check that no other player has an incentive to deviate and play an other strategy (investing in technology 2 for the first case, and in technology 1 for the second case).

This means that there are 4 types of equilibria (one equilibrium in pure strategy and three in mixed strategies) for which we carefully describe the possible outcomes:[5]

- ***preemption equilibrium with technology 1,*** $\mathbf{PR_1}$: this equilibrium may happen if $x \in \left[\overline{x}_1, x_{11}^{F*}\right]$ and if no player has an incentive to deviate. The two possible outcomes of this mixed strategy equilibrium are:

  (i) one player immediately invests in technology 1, the other waits and optimally responds to this action by investing in technology 1 when x crosses the threshold x_{11}^{F*},

  (ii) the two players immediately invest in technology 1.

- ***preemption equilibrium with technology 2,*** $\mathbf{PR_2}$: this equilibrium may happen if $x \in \left[\overline{x}_2, x_{12}^{F*}\right]$ and if no player has an incentive to deviate. It is also necessary that $x \in \left[\widetilde{x}, \widetilde{\widetilde{x}}\right]$. Indeed the first mover must have an incentive to invest in technology 2. The two possible outcomes of this mixed strategy equilibrium are:

  (i) one player immediately invests in technology 2, the other waits and optimally responds to this action by investing in technology 1 when x crosses the threshold x_{12}^{F*},

  (ii) the two players simultaneously invest in technology 2.

- ***preemption equilibrium with both technologies 1 and 2,*** $\mathbf{PR_{12}}$: this equilibrium happens when, either in PR_1 or in PR_2, a player has an incentive to deviate and invests in the other technology. In this equilibrium, outcomes are different whether x is less than x_{11}^{F*} or not. If $x < x_{11}^{F*}$, the outcomes of this mixed strategy equilibrium, denoted $\mathbf{PR_{12}^{I}}$, are:

  (i) one player immediately invests in technology 1, the other waits and optimally responds to this action by investing in technology 1 when x crosses the threshold x_{11}^{F*},

  (ii) one player immediately invests in technology 2, the other waits and optimally responds to this action by investing in technology 1 when x crosses the threshold x_{12}^{F*},

  (iii) the two players immediately invest in technology 1,

  (iv) the two players immediately invest in technology 2,

  (v) the two players immediately invest one in technology 1, the other in technology 2.

  If $x \geq x_{11}^{F*}$, the outcomes of this mixed strategy equilibrium, denoted $\mathbf{PR_{12}^{II}}$, are:

  (i) one player immediately invests in technology 2, the other waits and optimally responds to this action by investing in technology 1 when x crosses the threshold x_{12}^{F*},

  (ii) the two players immediately invest in technology 1,

  (iii) the two players immediately invest in technology 2,

  (iv) the two players immediately invest one in technology 1, the other in technology 2.

- ***simultaneous equilibrium in technology 1,*** **SI**: this equilibrium may happen if $x \geq x_{11}^{F*}$. This is a pure strategy equilibrium in which each player immediately invests in technology 1.

[5] The intensities with which all the equilibria are played and the condition under which players do not have interest to deviate are in the Appendix.

Note that in all the three preemption equilibria, if x is lower than the threshold that triggers the equilibrium ($\overline{x}_2$ or $\overline{x}_2$), when investment will be triggered (at $\overline{x}_2$ or $\overline{x}_2$ depending in the type of equilibrium), the two players have the same payoff. There will be no mistake in the investment at this point. However, if x is higher than $\overline{x}_2$ or $\overline{x}_2$ at the beginning of the game, then a mistake may happen since the leader is better off than the follower.

To derive the equilibrium, it is thus fundamental to know exactly how the different thresholds are ranked. Therefore let us introduce

$$E_{12} = \{(\beta, I_2/I_1)\,; \overline{x}_1 < \overline{x}_2\}\,, E_{1I} = \{(\beta, I_2/I_1)\,; \overline{x}_1 < \widetilde{x}\}\,, \text{ and } E_{2I} = \{(\beta, I_2/I_1)\,; \overline{x}_2 < \widetilde{x}\}\,. \tag{20}$$

Lemma 6 *We have the following result:*

$$E_{12} \subset E_{1I} \subset E_{2I} \tag{21}$$

The different thresholds are thus ranked according to four possible outcomes:

- $\widetilde{x} < \overline{x}_2 < \overline{x}_1 < x_1^{L*} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$ *(case I),*
- $\overline{x}_2 < \widetilde{x} < \overline{x}_1 < x_1^{L*} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$ *(case II),*
- $\overline{x}_2 < \overline{x}_1 < \widetilde{x} < x_1^{L*} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$ *(case III),*
- $\overline{x}_2 < \overline{x}_1 < \widetilde{x} < x_1^{L*} < x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$ *(case IV).*

Proof: The ranking $x_{11}^{F*} < \widetilde{\widetilde{x}} < x_{12}^{F*}$ has been proved in Lemma 2. And we already noted that $x_1^{L*} < x_{11}^{F*}$.

We first suppose that $\exists\,(\beta, I_2/I_1) \in \overline{E}_{1T} \cap E_{12} = \emptyset$. In this case, there is $x \in [\overline{x}_1, \overline{x}_2]$ such that

$$\begin{aligned} V_{12}^F(x) &> \Phi_2^L(x) \\ &> \Phi_1^L(x) \\ &> V_{11}^F(x)\,, \end{aligned}$$

where the first inequality holds since $x > \widetilde{x}$, the second since $x > \overline{x}_1$ and the third since $x < \overline{x}_2$. But $V_{11}^F(x) > V_{12}^F(x)\,\forall x$. This leads to a contradiction implying that $E_{12} \subset E_{1I}$.

It is straightforward to extend this reasoning to obtain that $E_{12} \subset E_{2I}$ and $E_{1I} \subset E_{2I}$. Therefore, $E_{12} \subset E_{1I} \subset E_{2I}$. □

This result means that if $\exists\,(\beta, I_2/I_1) \in E_{1I}$ (meaning that $\max[\overline{x}_1, \overline{x}_2] < \widetilde{x},$), then there is a preemption motive to invest in technology 1 but not in technology 2 $\forall x \in [\overline{x}_1, \widetilde{x}]$. On the contrary, if $\exists\,(\beta, I_2/I_1) \in \overline{E}_{1T}$ (meaning that $\max[\overline{x}_2, \widetilde{x}] < \overline{x}_1$), then there is a preemption motive to invest in technology 2 but not in technology 1 $\forall x \in [\overline{x}_2, \overline{x}_1]$. Lemma 6 suggests that E_{1I} corresponds to situations where technology 2 is quite expensive relative to technology 1 and $\overline{E}_{1T}$ corresponds to situations where technology 2 is quite cheap. The distinction between E_{2I} and E_{1I} and between E_{12} and $\overline{E}_{12}$ is less important and specifies more exactly the starting points of these regions where the preemption motive exists. The following lemma helps us in determining the influence of the cost advantage I_2/I_1.

Lemma 7 *For a given β,*

(i) suppose that I_2/I_1 is such that $\overline{x}_1 = \overline{x}_2$. If I_2 increases, then $\overline{x}_1 < \overline{x}_2$,

(ii) suppose that I_2/I_1 is such that $\overline{x}_1 = \widetilde{x}$. If I_2 increases, then $\overline{x}_1 < \widetilde{x}$,

(iii) suppose that I_2/I_1 is such that $\overline{x}_2 = \widetilde{x}$. If I_2 increases, then $\overline{x}_2 < \widetilde{x}$.

Proof: Recall that $\widetilde{x}$ satisfies

$$\frac{\beta}{\beta-1}\frac{5-2^{\beta+1}}{2^{\beta+1}}\left(\frac{x}{x_{11}^{F*}}\right)^{\beta} - \frac{\beta}{4(\beta-1)\frac{x}{x_{11}^{F*}}} + k - 1 = 0, \tag{22}$$

positive before and negative after. $\overline{x}_1$ satisfies

$$\frac{2\beta}{\beta-1}\frac{x}{x_{11}^{F*}} - \frac{\beta+1}{\beta-1}\left(\frac{x}{x_{11}^{F*}}\right)^{\beta} - 1 = 0, \tag{23}$$

negative before and positive after. $\overline{x}_2$ satisfies

$$\frac{9\beta}{4(\beta-1)}\frac{x}{x_{11}^{F*}} - \frac{5\beta+2}{2^{\beta+1}(\beta-1)}\left(\frac{x}{x_{11}^{F*}}\right)^{\beta} - k = 0, \tag{24}$$

negative before and positive after.

$\overline{x}_1$ does not depend on I_2 and when I_2 increases, $\widetilde{x}$ increases. Therefore if $(\beta, k) \in E_{12}$, then $(\beta, k') \in E_{12}$ with $k' > k$. Similarly, as $\overline{x}_2$ increases when I_2 increases, if $(\beta, k) \in E_{2I}$, then $(\beta, k') \in E_{2I}$ with $k' > k$. And finally, because of the properties of E_{12}, E_{1I} and E_{2I} if $(\beta, k) \in E_{1I}$, then $(\beta, k') \in E_{1I}$ with $k' > k$. □

This Lemma has direct consequences:

- (i) implies that when the cost advantage of technology 1 increases (meaning that technology 2 becomes more expensive), the threshold that triggers PR_1 moves away from the one that triggers PR_2. This means that the preemption incentive with technology 1 increases as I_2/I_1 increases.

- (ii) tells a very similar story: the preemption incentive with technology 1 increases as I_2/I_1 increases. Here the comparison concerns the indifference point $\widetilde{x}$. Whereas the preemption incentive is triggered in the zone where technology 2 is the preferred technology of any first mover, if I_2/I_1 increases, it is triggered earlier (for $x < \widetilde{x}$).

- (iii) implies that when technology 2 becomes more expensive relative to technology 1, the point up to which technology 2 is the preferred technology for a first mover ($\widetilde{x}$) becomes greater than the threshold that triggers PR_2 ($\overline{x}_2$). This means that as technology 2 becomes more costly, the preemption incentive exists in a region for x on which nobody is ready to invest in technology 2. Thus there is a region where the dominance of one technology over the other is more importance than the preemption incentive. On $[\overline{x}_2, \widetilde{x}]$, competition with technology 2 is meaningless since technology 2 is totally dominated by technology 1.

The expressions for the boundaries $B_3(\beta, I_2/I_1)$, $B_4(\beta, I_2/I_1)$ and $B_5(\beta, I_2/I_1)$ of E_{12}, E_{1I} and E_{2I} are determined in the Appendix. They are very complicated and it is quite tricky to see how they evolve as the different parameters move. More precisely, the shape of the boundary with respect to β is very tricky. However thanks to the results of the previous two lemmas, we are already able to state the following proposition:

Proposition 1 *The boundaries of E_{12}, E_{1I} and E_{2I} satisfy:*

$$B_3(\beta, I_2/I_1) > B_4(\beta, I_2/I_1) > B_5(\beta, I_2/I_1). \tag{25}$$

Moreover for a given β as I_2/I_1 increases,

- *you cannot switch from Case II to Case I,*

- *you cannot switch from Case III to Case II,*

- *you cannot switch from Case IV to Case III.*

Proof: This proposition is a direct consequence of the previous two lemmas. □
The different regions are represented in Figure 4.

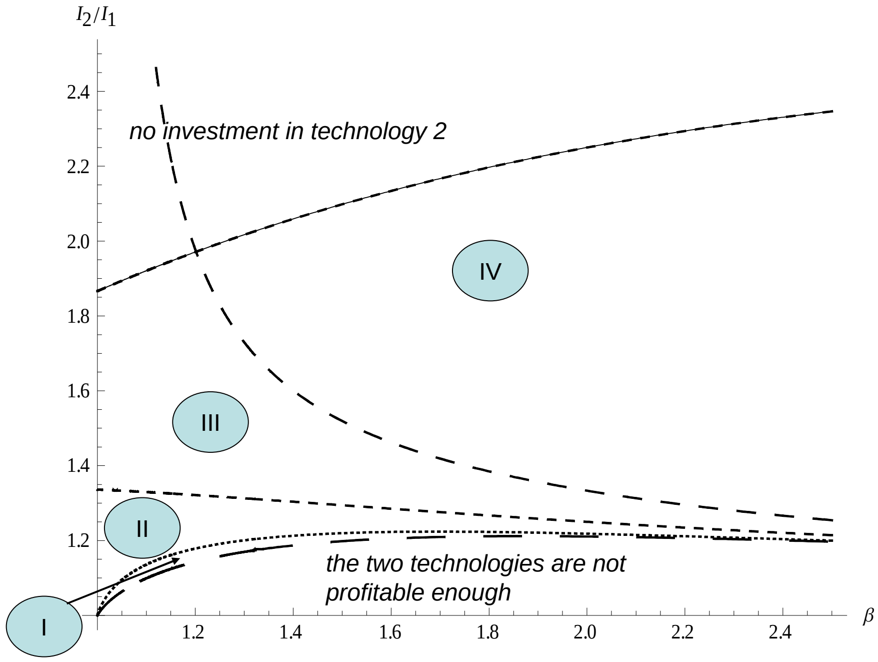

Figure 4: The different regimes.

We already mentioned that the effect of β (or uncertainty) on the shape of the different regions E_{12}, E_{1I} and E_{2I} is very difficult to obtain. However, thanks to Figure 4, we are able to have some insights. First note that β is a decreasing function of uncertain, σ^2, that varies from 1 (when σ^2 tends to $+\infty$) to r/μ (when $\sigma^2 = 0$). When there is no uncertainty, all the four boundaries get closer and the most likely region that remains corresponds to E_{12} (or Case IV). In fact as uncertainty decreases, the different regions narrow and end up disappearing except E_{12}. When uncertainty is very high, and thus β close to 1, the regions corresponding to Cases II and III are maximal.

The question is then to know which equilibrium occurs when. Next proposition determines the different equilibria that emerge.

Proposition 2 *The equilibrium for* $x \in \left[\min\left(\overline{x}_1, \max\left(\widetilde{x}, \overline{x}_2\right)\right), x_{11}^{F*}\right]$ *depends of the parameters' values and is represented in Figures 4 and 4:*

(i) Case I: as x *increases, the equilibrium is first* PR_2*, then* PR_{12}^I *and finally* PR_1*,*

(ii) Case II: as x *increases, the equilibrium is first* PR_{12}^I *and then* PR_1*,*

(iii) Cases III and IV: as x increases, the equilibrium is PR_1.

In the four cases, for $x \in \left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right[$, the equilibria are PR_{12}^{II} and SI, and for $x \in \left]\widetilde{\widetilde{x}}, +\infty\right[$, the equilibrium is SI.

Proof: See the Appendix. □

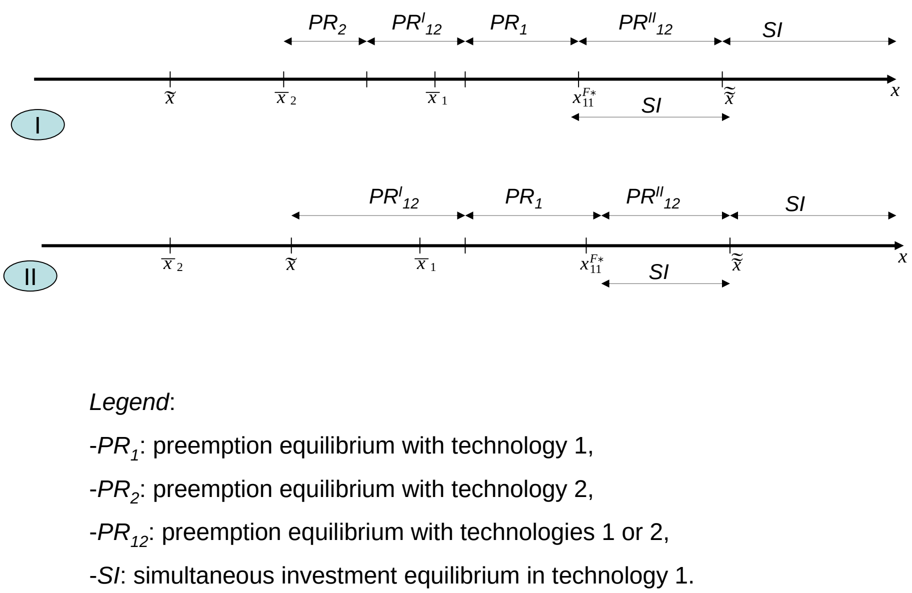

Figure 5: The different equilibria in Cases I and II.

The description of the different equilibria lead to the following comments. We consider first the case where $x < x_{11}^{F*}$ since the outcome is the same in the four cases for higher values of x.

Note that Cases I and II and Cases III and IV work together. In Cases I and II, the preemption equilibria succeed one another as the value of the shock x increases. When it is very low and technology II is very cheap (Case I), the equilibrium begins with PR_2 and then (also in Case II) switches to PR_{12}^{I} and finally then to PR_1. As x increases, technology 2 is first favored, then technology 1 comes into play and finally technology 2 is abandoned. In PR_2 that is a mixed strategy equilibrium, there always exists the risk that the two players simultaneously invest in technology 2 what leads to a disaster for the two players because of the fierce price competition. Their expected payoff in this case is indeed quite low (see Lemma 1). Therefore when x increases, the potential loss that is an increasing function of x also increases, and players have an incentive to deviate and invest in technology 1 that is safer. Therefore the equilibrium switch to PR_{12}^{I} where the two strategies, investing in technology 1 and investing ni technology 2, are played. And when x continues to increases, there is a threshold above which technology 2 is totally abandoned. With Proposition 1 and Figure 4 we know that Cases I and II occur when I_2/I_1 is quite low. In Cases III and IV, the unique equilibrium is PR_1 on $\left[\overline{x}_1, x_{11}^{F*}\right]$. Indeed, technology 2 is quite expensive relative to the gain in profit therefore it is totally abandoned. An interesting point to highlight is that uncertainty plays almost no role on the type of equilibrium

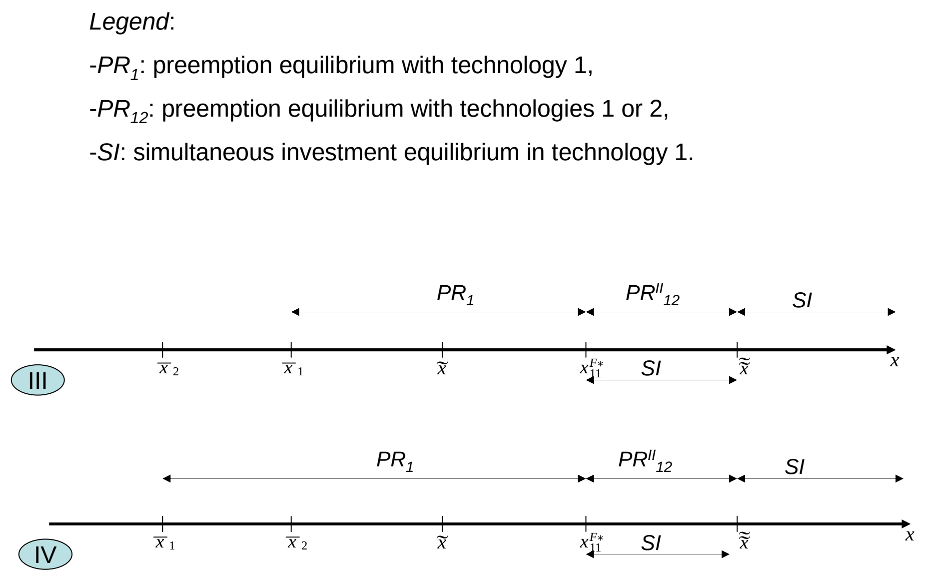

Figure 6: The different equilibria in Cases III and IV.

that arises. Indeed, we see in Figure 4 that the boundary between Cases I and II and Cases III and IV is a function that is almost independent of β and thus of uncertainty. Of course, β plays a role on the thresholds $\overline{x}_1$, $\widetilde{x}$... that trigger the different equilibria but not on the type of equilibrium itself. This implies that the cost advantage determines the technology investors chose whereas uncertainty determines the timing of investment. Going further, we can argue that uncertainty only has a minor impact on the industry structure.

When $x > x_{11}^{F*}$, the four cases lead to the same outcome:

- two equilibria for $x \in \left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$: SI and PR_{12}^{II},

- SI for $x \geq \widetilde{\widetilde{x}}$.

Lemma 8 *For a given β, the size of the interval $\left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$ decreases when I_2 increases.*

Proof: We know from Lemma 2 that $\widetilde{\widetilde{x}} \in \left[x_{11}^{F*}, x_{12}^{F*}\right]$. Therefore $\widetilde{\widetilde{x}}$ satisfies

$$\frac{2\beta}{\beta-1}\frac{x}{x_{11}^{F*}} - \frac{5}{2^{\beta+1}}\frac{\beta}{\beta-1}\left(\frac{x}{x_{11}^{F*}}\right)^{\beta} - \frac{I_2}{I_1} + 1 = 0. \tag{26}$$

As this expression is positive when $x < \widetilde{\widetilde{x}}$ and negative otherwise, it follows that $\widetilde{\widetilde{x}}$ decreases when I_2 increases. Noting that x_{11}^{F*} does not depend on I_2, the result is proved. □

As I_2/I_1 increases, the interval on which the two equilibria coexist narrows. Why do the two equilibria coexist on this interval? First as the threshold x_{11}^{F*} is crossed, the two players have an incentive to invest simultaneously in the small unit. But in the same time, the values of the shock x are still less than $\widetilde{\widetilde{x}}$ meaning that technology 2 is still the preferred choice of the first mover. Once x_{11}^{F*} is crossed, there is no choice anymore since the first and the second mover want to invest immediately in technology 1.

5 Welfare Analysis

The welfare analysis is meaningful in the case where there are multiple equilibria that is when $x \in \left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$. The first step consists in computing the total surplus in all the states of the world. Following Kreps and Sheinkman [18], we have the following lemma:

Lemma 9 *The instantaneous expected surplus in the different states of the world is*

$S_{10} =$	$5/2$	$S_{11} =$	4
$S_{21} =$	$3/2 + 4\ln 2$ ($\simeq$4.27)	$S_{20} =$	$27/8$
$S_{22} =$	$167/64 + 383/32 \ln 2 - 189/32 \ln 3$ ($\simeq$4.42)		

We thus compare the welfare in SI and in PR_{12}^{II} when x belong to this interval. In the case of the simultaneous investment equilibrium, the surplus is easily computed since this equilibrium is a pure strategy one. Thus

$$S_{SI}(x) = \frac{S_{11}x}{r-\mu} - 2I_1. \tag{27}$$

The welfare in the case of PR_{12}^{II} is less straightforward to compute since there may be different outcomes. It is thus the expected value of the surplus in the different outcomes.[6]

The surplus is the sum of the profit of the two firms and the consumers' surplus. The expected profit of each firm in PR_{12}^{II}, equals to V_{21}^{F}, is smaller than the profit in case of a simultaneous investment in technology 1, $\frac{x}{r-\mu} - I_1$. On the contrary, for most of the parameters values, the consumers surplus is higher in the case of PR_{12}^{II} than in the case of SI. Indeed, with an investment in the large capacity technology, produced quantities are higher and prices are lower. Therefore, PR_{12}^{II} is more favorable for the consumers, whereas SI is more favorable for the firms. In the case of PR_{12}^{II}, there is a jump in the total surplus as x crosses x_{11}^{F*}, due to the discontinuity in the expected payoff of the firms ($V_{12}^{F}\left(\widetilde{\widetilde{x}}_{-}\right) < \Phi_1^{L}\left(\widetilde{\widetilde{x}}_{+}\right)$). Locally (around $\widetilde{\widetilde{x}}$), the expected surplus is thus not increasing in x and waiting may not be optimal.

The comparisons of the total surplus in the two equilibria allows to determine which effect dominates. Because of the complexity of the expression of the intensities, we compute numerically the difference $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ on the interval $\left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$ for different parameters values.

First note that for a given β and I_2/I_1, $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ is decreasing. This means that for $\underline{x} < \overline{x}$, $S_{SI}(\underline{x}) - S_{PR_{12}^{II}}(\underline{x}) > S_{SI}(\overline{x}) - S_{PR_{12}^{II}}((\overline{x})$. The efficiency of SI decreases relative to the efficiency of PR_{12}^{II}. Three cases happen:

[6] The expression of the expected surplus is

$$S_{PR_{12}^{II}}(x) = \frac{2v_2(1-v_1-v_2)}{1-(1-v_1-v_2)^2 - 2v_1(1-v_1-v_2)}\left(\frac{S_{20}x}{r-\mu} - I_2 + \left(\frac{x}{x_{12}^{F*}}\right)^{\beta}\left(\frac{(S_{21}-S_{20})x_{12}^{F*}}{r-\mu} - I_1\right)\right) + $$
$$+\frac{2v_1v_2\left(\frac{S_{21}x}{r-\mu} - I_1 - I_2\right) + v_1^2\left(\frac{S_{11}x}{r-\mu} - 2I_1\right) + v_2^2\left(\frac{S_{22}x}{r-\mu} - 2I_2\right)}{1-(1-v_1-v_2)^2 - 2v_1(1-v_1-v_2)}. \tag{28}$$

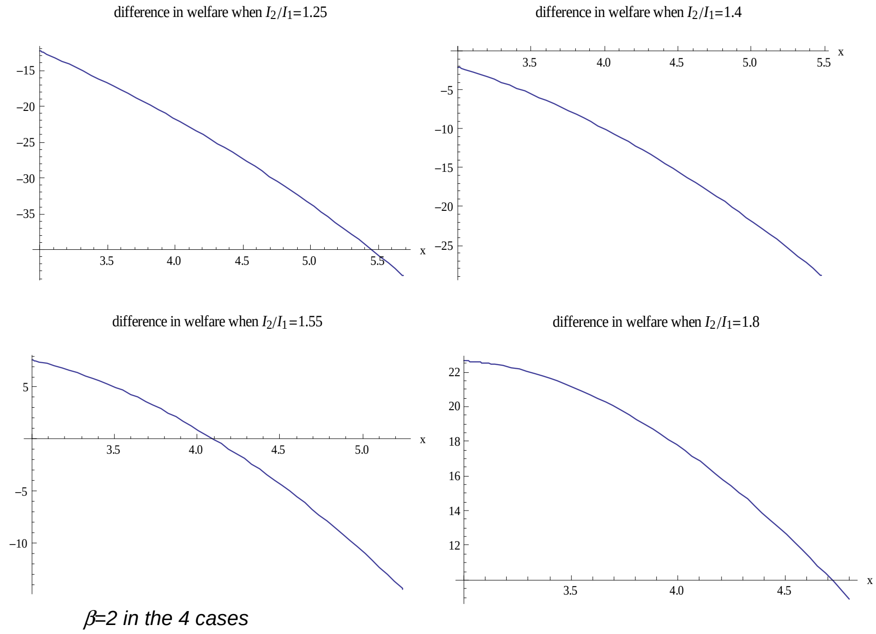

Figure 7: The difference in surplus.

- $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ negative on $\left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$. The gain in efficiency in the case of PR_{12}^{II} increases when x increases,
- $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ is positive and then negative as x increases from x_{11}^{F*} to $\widetilde{\widetilde{x}}$. SI is more efficient for low values of x and then PR_{12}^{II} is more efficient for high values,
- $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ is always positive on $\left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$. The gain in efficiency in the case of SI decreases as x increases.

In almost all the outcomes of the mixed strategy equilibrium, the instantaneous expected surplus is higher than in the case of SI. This effect is magnified as x increases, hence the shape of the difference. On Figure 5, we see that for a given β and x, $S_{SI}(x) - S_{PR_{12}^{II}}(x)$ is increasing when I_2/I_1 increases, meaning that PR_{12}^{II} becomes less efficient relative to SI as the cost advantage of technology 1 increases. This results is very intuitive, since as I_2 becomes more expensive, any investor is more reluctant to invest in it. Thus if $\beta = 2$ and $I_2/I_1 = 1.25$, PR_{12}^{II} is more efficient than SI on the whole interval. If if I_2/I_1 increases to 1.8, this is SI that is the most efficient, and if $I_2/I_1 = 1.55$, SI is first the most efficient and then PR_{12}^{II} as x increases from x_{11}^{F*} to $\widetilde{\widetilde{x}}$. On Figure 5, we allow the other parameter β to vary.

When $I_2/I_1 = 1.3$, $\forall\beta$ PR_{12}^{II} is always the most efficient equilibrium. On the contrary, when $I_2/I_1 = 1.8$, PR_{12}^{II} is the most efficient equilibrium if β is low (equal to 1.5), but when there is no uncertainty ($\beta = 2.5$), SI is first the most efficient equilibrium and then this is PR_{12}^{II}. If the cost advantage of technology 1 is low, whatever the uncertainty, PR_{12}^{II} is more efficient than SI. But if the cost advantage of technology 1 is high, then uncertainty comes into play:

- with high uncertainty, PR_{12}^{II} is more efficient,

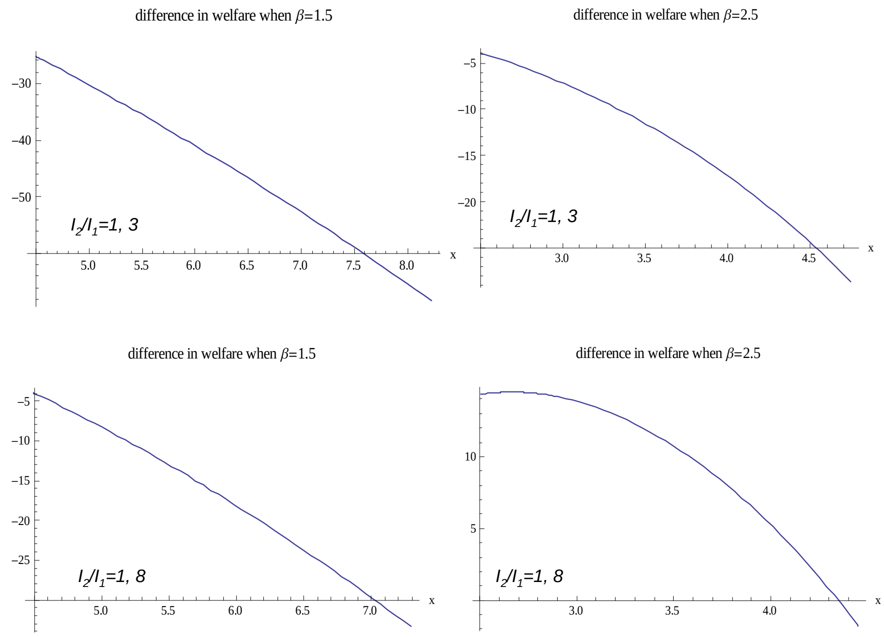

Figure 8: The difference in surplus.

- with low uncertainty, SI is more efficient.

Uncertainty increases the option values in PR_{12}^{II}, therefore its surplus increases.

6 Concluding remarks

This paper analyzes the investment strategy of a duopoly with price competition on a market of finite size with shocks on profit. Each firm has the choice between two technologies: a large unit and a small unit.

We first study the case where one firm is constrained to invest second. We find that, depending on the parameters values, there are one or two inaction region(s). In these regions, the first mover does not invest in any technology whereas without choice, she would have immediately invested in one of the two. The leader prefers to wait and see which technology turns out to be the most profitable to invest later in it. The inaction regions reveal the existence of a choice value for the leader.

When no constraint on the investment's order holds any more, the inaction regions disappear. The fear of being preempted indeed makes firms invest earlier. Depending on the parameters' values, three types of equilibria exist: a simultaneous investment equilibrium in which both firms invest in the small unit and two mixed strategy equilibria. We show that whatever the cost advantage of the small capacity technology, there is always an equilibrium with asymmetric investment. Moreover, even under fierce competition, the first mover still has a strategic advantage meaning that there is a preemption incentive. We also propose a detailed study of the role of uncertainty: it does not play any role in the type of equilibrium that emerges, but on the level of thresholds that trigger investment. Last, we lead a welfare analysis that reveals a Pareto dominance of the preemption equilibrium relative to the joint adoption equilibrium.

A Computations of the intensities in the different equilibria

A.1 PR_1

In this mixed strategy equilibrium, the player is indifferent between not moving and investing in technology 1. Therefore, denoting the expected utilities of investing $U_\emptyset$ and not investing U_1, we have:

$$U_\emptyset = V_{11}^F, \tag{29}$$

$$U_1 = (1-t)\,\Phi_1^L + t\,(\delta - I_1)\,. \tag{30}$$

Equating the two expressions implies that

$$t = \frac{\Phi_1^L - V_{11}^F}{\Phi_1^L - \delta + I_1} \tag{31}$$

that is positive and less than 1. PR_1 is an equilibrium if and only if no player has an incentive to deviate and invest in technology 2. The condition for PR_1 to be an equilibrium is thus that

$$(1-t)\,\Phi_1^L + t\,(\delta - I_1) > (1-t)\,\Phi_2^L + t\,(\delta - I_2)\,. \tag{32}$$

Therefore function f:

$$f(x) = \left(V_{11}^F(x) - \delta(x) + I_1\right)\left(\Phi_2^L(x) - \Phi_1^L(x)\right) - \left(\Phi_1^L(x) - V_{11}^F(x)\right)(I_2 - I_1) \tag{33}$$

is highly important. If it is negative, then PR_1 is an equilibrium.

A.2 PR_2

In this equilibrium, the expected utilities of waiting and investing in technology 2 are equalized:

$$U_\emptyset = V_{12}^F, \tag{34}$$

$$U_1 = (1-u_2)\,\Phi_2^L + u_2\,(\delta/2 - I_2)\,, \tag{35}$$

implying that

$$u_2 = \frac{\Phi_2 - V_{12}^F}{\Phi_2^F - \delta/4 + I_2} \tag{36}$$

that is positive and less than 1. In order PR_2 to be an equilibrium, no player should have an incentive to deviate:

$$(1-u_2)\,\Phi_2^L + u_2\,(\delta/4 - I_2) > (1-u_2)\,\Phi_1^L + u_2\,(\delta/2 - I_1)\,. \tag{37}$$

Thus, function

$$g(x) = \left(V_{12}^F(x) - \delta(x)/4 + I_2\right)\left(\Phi_2^L(x) - \Phi_1^L(x)\right) - \left(\Phi_2^L(x) - V_{12}^F(x)\right)\left(\delta(x)/4 + I_2 - I_1\right) \tag{38}$$

is highly important and PR_2 is an equilibrium if and only if $g(x) > 0$.

A.3 PR_{12}

We have to distinguish whether x is greater than x_{11}^{F*} or not.

A.3.1 $x < x_{11}^{F*}$

In this equilibrium, the expected utilities of waiting, investing in technology 1 and investing in technology 2 are equalized:

$$U_\emptyset = \frac{s_1 V_{11}^F + s_2 V12^F}{s_1 + s_2}, \tag{39}$$

$$U_1 = (1 - s_1 - s_2)\,\Phi_1^L + s_1\,(\delta - I_1) + s_2\,(\delta/2 - I_1)\,, \tag{40}$$

$$U_2 = (1 - s_1 - s_2)\,\Phi_2^L + s_1\,(\delta - I_2) + s_2\,(\delta/4 - I_2)\,. \tag{41}$$

Equating the three terms allows to solve and find s_1 and s_2. Denoting

$$D = \Phi_2^L - \Phi_1^L + \delta/4 + I_2 - I_1, \tag{42}$$

$$A = -\left(\frac{\delta}{4}\right)^2 + \frac{\delta D}{4}(\delta - I_1) - \frac{\delta}{4}\left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right)\left(\frac{\delta}{4} - I_1\right), \tag{43}$$

$$\begin{aligned} B = &-\left(\Phi_2^L - \Phi_1^L\right)\frac{\delta}{4}\Phi_1^L + \left(\Phi_2^L - \Phi_1^L\right) D\left(\delta - I_1\right) - \left(\Phi_2^L - \Phi_1^L\right)\left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right)\left(\frac{\delta}{2} - I_1\right) + \\ &+\frac{\delta}{4}\left(\frac{\delta}{4} + I_2 - I_1\right)\Phi_1^L + \frac{\delta}{4}\left(\Phi_2^L - \Phi_1^L\right)\left(\frac{\delta}{2} - I_1\right) - D^2 V_{11}^F + \left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right) D V_{12}^F, \end{aligned} \tag{44}$$

$$C = \left(\Phi_2^L - \Phi_1^L\right)\left(\frac{\delta}{4} + I_2 - I_1\right)\Phi_1^L + \left(\Phi_2^L - \Phi_1^L\right)\left(\frac{\delta}{4} + I_2 - I_1\right)\Phi_1^L + \left(\Phi_2^L - \Phi_1^L\right)^2\left(\frac{\delta}{2} - I_1\right) - D\left(\Phi_2^L - \Phi_1^L\right)V_{12}^F. \tag{45}$$

s_1 is the smallest solution of the second order equation $As^2 + Bs + C = 0$, leading to

$$s_1 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \tag{46}$$

and

$$s_2 = \frac{\Phi_2^L - \Phi_1^L - s_1\left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right)}{\Phi_2^L - \Phi_1^L + \delta/4 + I_2 - I_1}. \tag{47}$$

A.3.2 $x \geq x_{11}^{F*}$

In this equilibrium, the expected utilities of waiting, investing in technology 1 and investing in technology 2 are equalized:

$$U_\emptyset = V12^F, \tag{48}$$

$$U_1 = v_1(\delta - I_1) + v_2(\delta/2 - I_1), \tag{49}$$

$$U_2 = (1 - v_1 - v_2)\Phi_2^L + v_1(\delta - I_2) + v_2(\delta/4 - I_2). \tag{50}$$

Equating the three terms allows to solve and find v_1 and v_2:

$$v_1 = \frac{\left(\Phi_2^L - V_{12}^F\right)\left(\delta/2 - I_1 - V_{12}^F\right)}{V_{12}^F 3\delta/4 - \left(\Phi_2^L - \delta/4 + I_2\right)(\delta - I_1) + \left(\Phi_2^L - \delta + I_2\right)(\delta/2 + I_1)}, \tag{51}$$

and

$$v_2 = \frac{\Phi_2^L - V_{12}^F - v_1\left(\Phi_2^L - \delta + I_2\right)}{\Phi_2^L - \delta/4 + I_2}. \tag{52}$$

B Computations of the boundaries of E_{12}, E_{1I} and E_{2I}

The boundary of $E12$ equals

$$B_3(\beta, k)\frac{2\beta}{\beta - 1}a_3(\beta, k) - \frac{\beta + 1}{\beta - 1}\left(a_3(\beta, k)\right)^\beta - 1, \tag{53}$$

where

$$a_3(\beta, k) = \frac{\beta - 1}{\beta}\frac{k - \frac{5\beta + 2}{2^{\beta+1}(\beta+1)}}{\frac{9}{4} - \frac{5\beta + 2}{2^\beta(\beta+1)}}. \tag{54}$$

The boundary of E_{1I} equals

$$B_4(\beta, k) = \frac{2\beta}{\beta - 1}a_4(\beta, k) - \frac{\beta + 1}{\beta - 1}\left(a_4(\beta, k)\right)^\beta - 1, \tag{55}$$

where

$$a_4(\beta,k) = \frac{\beta-1}{\beta}\frac{\frac{5-2^{\beta+1}}{2^{\beta+1}}\frac{\beta}{\beta+1} - (k-1)}{\frac{5-2^{\beta+1}}{2^{\beta+1}}\frac{2\beta}{\beta+1} - \frac{1}{4}}. \quad (56)$$

The boundary of E_{2I} equals

$$B_5(\beta,k) = \frac{9\beta}{4(\beta-1)}a_5(\beta,k) - \frac{5\beta+2}{2^{\beta+1}(\beta-1)}(a_5(\beta,k))^\beta - k, \quad (57)$$

where

$$a_5(\beta,k) = \frac{4(\beta-1)}{\beta}\frac{\frac{5-2^{\beta+1}}{5\beta+2}\beta k - (k-1)}{\frac{5-2^{\beta+1}}{5\beta+2}9\beta - 1}. \quad (58)$$

C Proof of Proposition 2

We first state a Lemma that allows to have preliminary results.

Lemma 10 *We have the following results concerning the equilibria:*

(i) PR_1 *and* PR_{12}^I *never coexist,*

(ii) PR_2 *and* PR_{12}^I *never coexist,*

(iii) PR_2 *and* PR_{12}^{II} *never coexist,*

(iv) as x *increases but still remains less than* x_{11}^{F*}*, the equilibrium cannot switch from* PR_1 *to* PR_{12}*,*

(v) PR_2 *is never an equilibrium in cases II and III.*
In case I, it is an equilibrium for some values of x *and as* x *increases but still remains less than* x_{11}^{F*}*, the equilibrium cannot switch from* PR_{12}^I *to* PR_2*.*
In case IV, it may be an equilibrium for some values of x *and as* x *increases but still remains less than* x_{11}^{F*}*, the equilibrium cannot switch from* PR_2 *to* PR_{12}^I*,*

(vi) when $x < x_{11}^{F*}$*,* PR_{12}^I *is never an equilibrium in cases III and IV,*

(vii) when $x \in \left[x_{11}^{F*}, \tilde{\tilde{x}}\right]$*,* PR_{12}^{II} *and SI are the two equilibria,*

(viii) when $x > \tilde{\tilde{x}}$*, SI is the unique equilibrium.*

Proof:

(i) Suppose the two equilibria exist for the same values of x. $PR12^I$ is characterized by the equality between the utility with the three strategies: $U_\emptyset = U_1 = U_2$. This leads to

$$\frac{s_1V_{11}^F + s_2V_{12}^F}{s_1+s_2} = (1-s_1-s_2)\Phi_1^L + s_1(\delta - I_1) + s_2(\delta/2 - I_1) \quad (59)$$

$$= (1-s_1-s_2)\Phi_2^L + s_1(\delta - I_2) + s_2(\delta/4 - I_2). \quad (60)$$

PR_1 is characterized by $U_\emptyset = U_1 > U_2$. This leads to

$$V_{11}^F = (1-t)\Phi_1^L + t(\delta - I_1) \quad (61)$$

$$> (1-t)\Phi_2^L + t(\delta - I_2). \quad (62)$$

Computing the difference between U_2 and U_1 in the two equilibrium and comparing the two expressions implies that

$$(t-s_1-s_2)\left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right) > s_2\delta/4. \quad (63)$$

$\left(\Phi_2^L - \Phi_1^L + I_2 - I_1\right) > 0$ and $s_2\delta/4 > 0$. Thus, $t > s_1 + s_2$.

As $V_{11}^F > V_{12}^F$, $s_1V_{11}^F + s_2V_{12}^F < (s_1+s_2)V_{11}^F$, leading to

$$(1-t)\Phi_1^L + t(\delta - I_1) > (1-s_1-s_2)\Phi_1^L + s_1(\delta - I_1) + s_2(\delta : 2 - I_1) \quad (64)$$

and thus

$$(s_1 + s_2 - t)\left(\Phi_1^L - \delta + I_1\right) > s_2\delta/2. \tag{65}$$

The left hand side is negative and the right hand side is positive. Therefore there is a contradiction, and the two equilibria never exist for the same value of the shock x.

(ii) Suppose the two equilibria exist for the same values of x. PR_{12}^I is characterized as in the previous paragraph. PR_2 is characterized by $U_\emptyset = U_2 > U_1$. This leads to

$$\begin{aligned} V_{12}^F &= (1-u_2)\,\Phi_2^L + u_2\,(\delta/4 - I_2) && (66)\\ &> (1-u_2)\,\Phi_1^L + u_2\,(\delta/2 - I_1)\,. && (67)\end{aligned}$$

The same steps as before imply that $u_2 < s_1 + s_2$. Using as before $V_{11}^F > V_{12}^F$ leads to

$$(u_2 - s_1 - s_2)\left(\delta/4 - I_2 - \Phi_2^L\right) > s_1 3\delta/4. \tag{68}$$

This leads to a contradiction since the left hand side is negative and the right hand side is positive. The two equilibria never exist for the same value of the shock x.

(iii). Suppose that the two equilibria exist for the same value of x. $PR12^{II}$ is characterized by the equality between the utility with the three strategies: $U_\emptyset = U_1 = U_2$. This leads to

$$\begin{aligned} V_{12}^F &= \frac{v_1\,(\delta - I_1) + v_2\,(\delta/2 - I_1)}{v_1 + v_2} && (69)\\ &= (1 - v_1 - v_2)\,\Phi_2^L + v_1\,(\delta - I_2) + v_2\,(\delta/4 - I_2)\,. && (70)\end{aligned}$$

PR_2 is, as before, characterized by $U_\emptyset = U_2 > U_1$:

$$\begin{aligned} V_{12}^F &= (1-u_2)\,\Phi_2^L + u_2\,(\delta/4 - I_2) && (71)\\ &> (1-u_2)\,\Phi_1^L + u_2\,(\delta/2 - I_1)\,. && (72)\end{aligned}$$

This implies that

$$\frac{v_1\,(\delta - I_1) + v_2\,(\delta/2 - I_1)}{v_1 + v_2} > (1-u_2)\,\Phi_2^L + u_2\,(\delta/4 - I_2)\,. \tag{73}$$

But this is not possible since $\Phi_1^L > \delta - I_1$ and the four quantities v_1, v_2, u_2 and $1-u_2$ are positive. There is thus a contradiction and the two equilibria PR_2 and PR_{12}^{II} never coexist.

(iv). We are going to prove that function f is either positive then negative or negative when x increases from $\overline{x}_1$ to x_{11}^{F*}. To do so, we have to differentiate function f three times. It is straightforward to show that the sign of the third derivative of function f is the same than the sign of

$$g^{'''}(y) = \frac{4\beta\,(2\beta+1)}{\beta - 1}\left(1 - \frac{5}{2^{\beta+1}}\right) y^{\beta+2} + (\beta+1)\left(\frac{1}{4} - \beta\left(1 - \frac{5}{2^{\beta+1}}\right)\right) y + (\beta-2)\,(\beta-1)\left(\frac{I_2}{I_1} - \frac{5}{2^{\beta+1}}\right) \tag{74}$$

where $y = x/x_{11}^{F*}$. With $\overline{\beta}$ the solution of $1 - 5/2^{\beta+1} = 0$ (it is negative if $\beta < \overline{\beta}$ and positive otherwise) and $\overline{\overline{\beta}}$ the solution of $1/4 - \beta\left(1 - 5/2^{\beta+1}\right) = 0$ (it is negative if $\beta > \overline{\overline{\beta}}$ and positive otherwise), three cases have to be distinguished (note that $\overline{\beta} < \overline{\overline{\beta}}$):

(a) if $\beta < \overline{\beta}$, then $g^{'''}$ is decreasing on $[\overline{y}_1, 1]$, first positive then negative,

(b) if $\overline{\overline{\beta}} < \beta < \overline{\beta}$, then $g^{'''}$is increasing and positive on $[\overline{y}_1, 1]$,

(c) if $\beta > \overline{\beta}$, then $g^{'''}$is increasing on $[\overline{y}_1, 1]$, first negative and then positive.

These three cases lead to the same conclusion for $g^{''}$ and thus for $f^{''}$. Either it is positive or negative and then positive as y increases from $\overline{y}_1$ to 1. Indeed $f^{''}(1) > 0$:

$$f^{''}(1) = \beta\left(1 - \frac{5}{2^{\beta+1}}\right) + \frac{\beta}{4} + (\beta-1)\,(\beta-2)\left(\frac{I_2}{I_1} - 1\right) \tag{75}$$

is positive $\forall\beta \geq 1$ and $I_2 > I_1$. As $f^{'}(1) = (I_2/I_1 - 1)(\beta - 1) > 0$, either $f^{'}$ is positive or negative and then positive as y increases from $\overline{y}_1$ to 1.

The last point is to focus on f (i.e. on the condition itself). First note that $g(1) = 0$. In order to have our result, it remains to show that $f(\overline{y}_1) < 0 \Rightarrow f'(\overline{y}_1) < 0$. $f(\overline{y}_1) < 0$ is possible if and only if $\overline{y}_1 < \widetilde{y}$.[7] It is also straightforward to show that

$$f'(\overline{y}_1) = (1 - \overline{y}_1)\left[-\frac{2\beta^2}{(1+\beta)^2}\left(1 - \frac{5}{2^{\beta+1}}\right)\overline{y}_1 + \frac{\beta(\beta-1)}{(\beta+1)^2}\left(1 - \frac{5}{2^{\beta+1}}\right) - \frac{k-1}{\beta+1}\right] + (2\overline{y}_1 - 1)(k-1). \tag{76}$$

In order to sign this expression, we solve the second order equation

$$\frac{2\beta^2}{(1+\beta)^2}\left(1 - \frac{5}{2^{\beta+1}}\right)t^2 + \left(-\frac{\beta(3\beta-1)}{(\beta+1)^2}\left(1 - \frac{5}{2^{\beta+1}}\right) + \frac{(2\beta+3)(k-1)}{\beta+1}\right)t + \frac{\beta(\beta-1)}{(\beta+1)^2}\left(1 - \frac{5}{2^{\beta+1}}\right) - \frac{(\beta+2)(k-1)}{\beta+1}, \tag{77}$$

whose roots are

$$\overline{y}_1^{\pm} = \frac{\left(\frac{\beta(3\beta-1)}{(\beta+1)^2}\left(1 - \frac{5}{2^{\beta+1}}\right) - \frac{(k-1)(2\beta+3)}{\beta+1}\right) \pm \sqrt{\Delta}}{\frac{4\beta^2}{(\beta+1)^2}\left(1 - \frac{5}{2^{\beta+1}}\right)} \tag{78}$$

where Δ is the usual determinant.

It is straightforward to see that $\overline{y}_1^{+} > 0$ whereas $\overline{y}_1^{-} < 0 \Leftrightarrow \beta < \overline{\beta}$. Tow cases have to be distinguished:

(a) when $\beta < \overline{\beta}$, the coefficient of the second order equation is negative, meaning that the expression is first increasing then decreasing.In this case, $\overline{y}_1^{+} < \overline{y}_1^{-}$ and we have to check that $\overline{y}_1 < \overline{y}_1^{+}$ knowing that Assumptions $A1$ and $A2$ are satisfied and that $\overline{y}_1 < \widetilde{y}$.

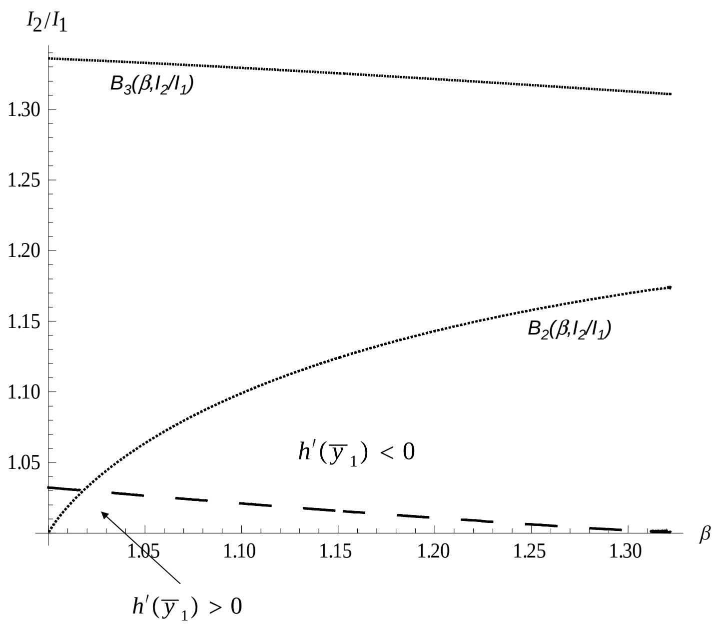

Figure 9: The different boundaries.

[7]Recall that the variable y is equal to the variable x normalized by x_{11}^{F*}.

(b) when $\beta > \overline{\beta}$, the coefficient of the second order equation is positive, meaning that the expression is first decreasing and then increasing. In this case, we have to check that $\overline{y}_1 < \overline{y}_1^-$. But in this case we can easily prove it by finding a lower bound for $\overline{y}_1^+$, indeed $\overline{y}_1^+$is the lowest when k equals 1 and in this case 1 is the same than $\overline{y}_1$. Therefore, $\overline{y}_1 < \overline{y}_1^+$ and the resulted is proved.[8]

(v) This point is very similar than the previous one, as far as we have to study the equilibrium condition under which a player trying to preempt with two units does not have an incentive to deviate and invest in the small unit:

$$h(x) = \left(V_{12}^F(x) - \delta(x)/4 + I_2\right)\left(\Phi_2^L(x) - \Phi_1^L(x)\right) - \left(\Phi_2^L(x) - V_{12}^F(x)\right)\left(\delta(x)/4 + I_2 - I_1\right). \quad (79)$$

On again we introduce the variable $y = x/x_{11}^{F*}$. By deriving three time function h, it can be shown that three cases may happen:

(a) if $\beta < \overline{\beta}$, h''' is decreasing, positive then negative,

(b) if $\overline{\beta} < \beta < \overline{\beta}+1$, h''' is increasing and always positive,

(c) if $\beta > \overline{\beta}+1$, h''' is increasing, negative then positive.

As $h''(1) > 0$, h'' is either positive or negative and then positive as y increases from $\overline{y}_2$ to 1. Moreover as $h'(\overline{y}_2) < 0$, h' is either negative or negative and then positive as y increases from $\overline{y}_2$ to 1. Proving that $h'(\overline{y}_2) < 0$ is quite tricky and necessitates different steps. First we work with another variable, indeed $z = x/x_{12}^{F*}$. Computations lead to:

$$h'(\overline{z}_2) = A(\beta,k)\,\overline{z}_2^2 + B(\beta,k)\,\overline{z}_2 + C(\beta,k), \quad (80)$$

where

$$A(\beta,k) = \frac{\beta}{(5\beta-2)(\beta-1)^2}\left(\frac{3(5-2\beta)}{2} + \frac{9\beta}{5\beta-2}\left(2^\beta - \frac{5}{2}\right)\left(-5\beta^2+10\beta+4\right)\right), \quad (81)$$

$$\begin{aligned} B(\beta,k) =\; & \frac{\beta}{(5\beta-2)^2(\beta-1)}\left(2^\beta - \frac{5}{2}\right)\left(9(5\beta-2) + 2k\left(5\beta^2 - 19\beta - 13\right)\right) + \frac{4-5k-\beta}{(\beta-1)(5\beta-2)} + \\ & + \frac{1-\beta k}{2(\beta-1)} + \frac{9(k-1)}{2}, \end{aligned} \quad (82)$$

$$C(\beta,k) = \frac{2\beta k}{5\beta-2}\left(2^\beta - \frac{5}{2}\right)\left(\frac{4k}{5\beta-2} - 1\right) + k(k-1)\left(\frac{2}{5\beta-2} - 1\right). \quad (83)$$

Coefficient $A(\beta,k)$ is negative if $\beta < \overline{\overline{\overline{\beta}}}$ or $\beta > \overline{\overline{\overline{\overline{\beta}}}}$, it is positive in the other cases. Moreover, it can be shown that the two roots of the second order equation, $\overline{y}_2^+$ and $\overline{y}_2^-$, are such that:

(a) if $\beta < \overline{\overline{\overline{\beta}}}$, $\overline{y}_2^+$ and $\overline{y}_2^-$ are negative, $h'(\overline{y}_2)$ is increasing and then decreasing, being first negative, then positive and finally negative,

(b) if $\overline{\overline{\overline{\beta}}} < \beta < \overline{\overline{\overline{\overline{\beta}}}}$, $\overline{y}_2^+$ is positive and $\overline{y}_2^-$ is negative, $h'(\overline{y}_2)$ is decreasing and then increasing, being first positive, then negative and finally positive,

(c) if $\beta > \overline{\overline{\overline{\overline{\beta}}}}$, $\overline{y}_2^+$ and $\overline{y}_2^-$ are positive, $h'(\overline{y}_2)$ is increasing and then decreasing, being first negative, then positive and finally negative.

In the three cases, $h'(\overline{y}_2)$ is negative.

The last step is to show that it is not possible to have for the same parameters' values $h(\overline{y}_2) > 0$ and $h(1) > 0$. Recall that $h(\overline{y}_2) > 0$ is equivalent to $\overline{y}_2 > \widetilde{y}$, or equivalently $\overline{x}_2 > \widetilde{x}$. TERMINER CA

(vi). Note that

$$f(\overline{y}_1) > 0 \Leftrightarrow \widetilde{y} < \overline{y}_1. \quad (84)$$

[8] In fact, we proved that when $\beta > \overline{\beta}$, $f'(\overline{y}_1)$ is negative whether $\overline{y}_1$ is greater than $\widetilde{y}$ or not.

Moreover, in (iv), we proved that function f is either positive then negative or negative when x increases from $\overline{x}_1$ to x_{11}^{F*}. Thus when $\widetilde{x} > \overline{x}_1$ (what happens in cases III and IV),f is always negative and PR_1 is always an equilibrium.

(vii). When $x \in \left[x_{11}^{F*}, \widetilde{\widetilde{x}}\right]$, PR_1 cannot be an equilibrium anymore, and PR_2 is not an equilibrium neither. Therefore the only possible equilibria are SI and PR_{12}^{II}.

(viii.) In this case, the nobody has an interest to invest in technology 2, thus the unique equilibrium that remains is SI. □

This Lemma helps us to state Proposition 2. □

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