Benchmarks in Aggregate Household Portfolios*

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Abstract

Reference–dependent preference models assume that agents derive utility from deviations of consumption from benchmark levels, rather than from consumption levels. These references can be either backward-looking (as explicit in the Habit literature) or forward-looking (as implicitly suggested by Prospect Theory). For both cases, we specify and estimate a fully structural multi-variate Brownian system in optimal consumption, portfolio and wealth using aggregate household financial and real estate wealth data. Our results reveal that references are (i) strongly relevant, (ii) state-dependent, and (iii) that the data is more consistent with the backward- than the forward-looking reference model.

JEL classification: G11, G12.

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1 Introduction

1.1 Motivation and outline

Introspection and conventional wisdom suggests that agents gauge benefits and costs of potential actions relative to subjective yardsticks. This has long been recognized by proponents of Reference Dependent (RD) utility as an alternative to standard Neoclassical theory in representing preferences over both deterministic and stochastic outcomes. RD preferences differ from conventional frameworks by assuming that agents are concerned about deviations of consumption from a benchmark (reference) level, rather than about consumption levels only; consumption above benchmark is positively valued, consumption below it causes discomfort. There is by now a large and growing set of evidence in favor of RD utility as better representation of agents' preferences over both risk-less and stochastic outcomes.[1] A second important and recurrent finding is that the reference is not constant, but adapts to the current state of the world in which the agent must make a decision. For example, the experimental evidence points to agents adjusting benchmarks rapidly following a change in assets (Munro and Sugden, 2003, p. 408).

Although researchers agree that references are important and that they are state-dependent, they disagree as to what actually constitutes a benchmark.[2] Broadly speaking, two alternative views can be found in the literature, with references being either: (i) backward-looking, customary consumption or (ii) forward-looking, expected consumption.

The Habit literature considers that the backward-looking benchmark is a cumulated habit stock acquired from past consumption experiences. An agent is concerned about deviations of current consumption with what he has become accustomed to consume in the past. Lagged consumption could either be an agent's own (internal habit), or that of his reference group with which he compares himself (external habit). Consumption that took place in a distant past is discounted more heavily, and the discount rate is a constant deep parameter.

Prospect and Regret theory instead relate references to the value of current assets.[3] If markets are complete, total wealth is equal to the expected discounted stream of fu-

[1]Experimental evidence points to agents behaving in manners which are inconsistent with standard Neoclassical theory. They resent losses from giving up an acquired good much more than they appreciate the gain from its acquisition (Endowment Effect); they excessively wish to maintain current status (Status Quo Bias); they prefer straight improvements of their status to potential tradeoffs; they give excessive weight to a same difference between two options when viewed as difference between two disadvantages rather than between two advantages (Disadvantage Bias). These can be explained when Loss Aversion and Diminishing Sensitivity are appended to RD preferences (see Kahneman and Tversky, 1979; Tversky and Kahneman, 1991, 1992; Bateman et al., 1997; Sugden, 2003, for discussions).

[2]More secondary sources of disagreement concern functional forms for deviations over benchmarks (e.g. ratios vs differences) or utility functionals over these deviations (e.g. Loss Aversion or strict concavity).

[3]For example, Kahneman and Tversky (1979) write that:

> "Gains and Losses, of course, are defined relative to some neutral reference point. The reference point usually corresponds to the current asset position . . . , the location of the reference point, and the consequent coding of outcomes as gains or losses can be affected by the formulation of the offered prospects, and by the expectations of the decision maker." [p. 274], and

ture consumption, with discount factor a function of the state-price density. Therefore, Prospect theory implicitly assumes that the agent's references are forward-looking and measure what the agent expects to consume in the future, with more distant future consumption discounted more heavily. In that respect, forward-looking benchmarks capture both internal (through the agent's choice of future consumption paths) and external (through the market discount factor) elements.

In light of these elements, this paper asks the following questions: (i) do references play any role in explaining aggregate household consumption and portfolio decisions, and if they do, (ii) are references constant or state-dependent, and (iii) is the aggregate portfolio data more consistent with a backward-looking/Habit or with a forward-looking/Prospect interpretation? Using evidence from aggregate household asset holdings data, we find unambiguous answers to these questions: references are crucial, state-dependent and determined by Habit more than by wealth.

The application of RD preferences to dynamic financial decisions is warranted by the close link between benchmarks and risk aversion. First, relative risk aversion (RRA) is inversely related to the distance between consumption and its benchmark (surplus consumption). Fluctuations in that distance therefore lead to fluctuations in RRA, contrary to the standard VNM/iso-elastic paradigm in which RRA is fixed. These movements in RRA have implications for asset holdings and could explain the strong cyclical fluctuations in observed portfolio shares (see Figures 1 and 2). In particular, recessions are associated with shifts away from more risky assets (such as stocks) in favor of relatively safer ones (cash and homes). One possible explanation is that the investment set is not constant, and that recessions are associated with falling risk premia and/or increasing quantities of relevant risk. Another is that recessions coincide with increases in risk aversion. Under RD preferences, these movements can be ascribed to pro-cyclical surpluses and cannot be explained by standard frameworks with iso-elastic utility.

Second, as is well known from state-dependent preferences, in addition to consumption risk, the RD agent is concerned with covariances between returns and any additional state variable that determines the benchmark. Under Habit, the state is (lagged) consumption, such that the agent is ultimately only concerned with consumption risk. Under Prospect/Regret, the state is (contemporary) wealth; the agent is concerned with both consumption and total wealth (i.e. market) risks.[4] Stocks, home returns and mortgages are much more correlated with wealth than with consumption; short and long-term bonds are less so (see Table 3). These differences in covariances could explain differences in levels of individual asset holdings (see Table 1).

This paper analyzes empirically the implications of backward- and forward-looking references for dynamic asset allocation. Contrary to the bulk of the preference–based re-

> "Strictly speaking, value should be treated as a function of two arguments: the asset position that serves as a reference point, and the magnitude of the change (positive or negative) from that reference point."[p. 277]

As Köszegi and Rabin (2005) point out, expectations- or endowment-based interpretations of references are identical if agents reasonably expect to own their endowment.

[4]The same result obtains under Non-Expected utility. For example, compare equations (13) in in Bakshi and Chen (1996) and (21) in in Duffie and Epstein (1992). See also Smith (2001) for further discussions of the strong parallels between Non-Expected utility and utility featuring direct preferences over wealth.

search on asset market dynamics, we focus on empirical *portfolio*, rather than on *pricing* implications. Closed-form expressions for optimal portfolio and consumption imply much more theoretical restrictions than Euler-based returns' applications. These strong restrictions are useful in pinning down the additional preference parameters for which we have scant prior information. Following a discussion of the relevant literature (Section 1.2), we characterize and contrast the closed-form expressions for optimal value functions, consumption and portfolio allocations in Section 2 for both reference models. We highlight the strong parallels, yet important differences between the two sets of predictions.

Next, we substitute the optimal rules back in the budget contraint to derive the closed–form expressions for instantaneous changes in consumption, asset holding values, and wealth. This multi–variate Brownian motion constitutes the fully structural econometric model which we estimate. In Section 3, we resort to a change–in–variables approach to address potential discretization bias of estimating continuous–time models with discretely–sampled data. We also incorporate the returns process with the optimal quantities process in a single–step Maximum Likelihood estimation to correct inference. We finally discuss how we formally test the non-nested wealth-dependent and Habit-dependent reference models, and compute test statistics on the derived series (references, surplus, and risk aversion).

The estimation results are presented in Section 4. We first present results using only financial assets in Section 4.1. Overall, we find that all preference parameters have the correct sign, are significant, and satisfy all the relevant theoretical restrictions. We find strong statistical support for the hypotheses that references are (i) relevant, and (ii) state-dependent in characterizing aggregate household portfolios. In particular, references are increasing in wealth and in past cumulated consumption. However, we find (iii) that the backward-looking, Habit model unambiguously performs better than the forward-looking, wealth-determined model in explaining the aggregate household portfolio and consumption data. We verify robustness by incorporating real estate in Section 4.2. The results remain qualitatively the same, except for (iv) a marked increase in risk aversion. A further robustness check is performed in Section 4.3 when we allow for time-varying conditional risk premia without qualitative changes in our inference results. Section 5 concludes by reviewing and discussing the main findings.

1.2 Relevant literature

This paper is related to the literature allowing an explicit role for wealth in preferences. Part of this literature restricts preferences to be over specific elements of wealth such as financial wealth (Barberis et al., 2001), or tangible wealth, such as durables (Grossman and Larocque, 1990; Detemple and Giannikos, 1996; Aït-Sahalia et al., 2004; Yogo, 2005b), or housing (Flavin and Yamashita, 2002; Chetty and Szeidl, 2004; Piazzesi et al., 2006). However, references lose their forward-looking consumption interpretation when restricted to depend only on specific components of net worth. We consequently follow Bakshi and Chen (1996); Smith (2001); Gong and Zou (2002); Kandel and Kuznitz (2004) in making utility a function of *total* wealth instead. We consider below the implications on our results of restricting total wealth to alternative definitions. In particular, consistent with Flavin and Yamashita (2002); Chetty and Szeidl (2004), we find that incorporating residential wealth results in higher implied risk aversion.

Other papers instead consider a positive direct role of total wealth in the determination of preferences. Bakshi and Chen (1996), Gong and Zou (2002), as well as Smith (2001) analyze a direct preference for total wealth which they ascribe to a "capitalistic spirit" advocated by Veblen (1899). Similarly, Kandel and Kuznitz (2004) emphasize the positive forward-looking interpretation of direct utility over wealth as preference for expected discounted stream of consumption. These papers study the implications for optimal portfolio with constant (Bakshi and Chen, 1996) or time-varying (Kandel and Kuznitz, 2004) investment set. Although our application theoretically allows for both reference-dependence and preference-for-status arguments, our estimates clearly point to a references-based role of wealth in the utility function. Furthermore, the empirical implementation focuses on returns-based Euler equations. Rather than the fully structural MLE implementation that we resort to, Bakshi and Chen (1996) do not exploit the distributional assumptions in resorting to Hansen and Jagannathan (1991) bounds and GMM estimation of a discretized version of their model. Kandel and Kuznitz (2004) consider a calibration exercise and do not estimate their model.

Our paper is also related to the empirical Habit literature (e.g. Heaton, 1995; Campbell and Cochrane, 1999a; Li, 2001, 2005; Tallarini and Zhang, 2005, among others). Following recent evidence in favor of internal as opposed to external habit (Chen and Ludvigson, 2004; Grishchenko, 2005), we focus on the Constantinides (1990) model. We differ in estimating the closed-form solution to the Habit model over aggregate data on consumption, portfolio and wealth, instead of the usual Euler equation estimation of equilibrium returns. Moreover, our specification does not nest both backward- and forward-looking references, but considers each one in turn; a formal modified Likelihood Ratio test of the two non-nested hypotheses is subsequently performed.

Finally, our paper is indirectly related to portfolio analysis under Prospect Theory. Berkelaar et al. (2004) as well as Gomes (2005) analyze the portfolio implications of wealth-dependent references with loss aversion. In their study of the dynamic updating of the reference, both papers let the benchmark be a function of total wealth, as well as of the risk-free rate. Similar to us, they consider closed-form solution to optimal portfolio in a continuous-time framework with a constant investment set. Contrary to us however, they do not analyze preferences for intermediate consumption, but focus instead on a setup where utility is defined over terminal wealth only. In addition, they allow for kinks at references through loss aversion, whereas we restrict consumption to be above references exclusively. Finally, the empirical investigation in Berkelaar et al. (2004) abandons the continuous trading framework, whereas we maintain a one-to-one relation between the structural and econometric model.

2 Model

This section outlines the model. We subsequently characterize optimal consumption and asset holdings. Finally, we obtain the closed-form expressions for the differential equations governing consumption, asset values and wealth by substituting optimal consumption and portfolio in the budget constraint.

2.1 Economic environment and RD preferences

In order to emphasize the role of alternative preference specifications, we consider a complete-markets and representative-agent framework similar that studied by Merton (1971) or by Breeden (1979). The stochastic environment is characterized by continuous information with filtration on $\boldsymbol{Z}_t \in \mathbb{R}^n$, a standard Brownian motion. The investment set consists of n risky securities and one risk-less asset. Denote by $\boldsymbol{\mu}_p \in \mathbb{R}^n$ and by $\boldsymbol{\sigma}_p \in \mathbb{R}^{n \times n}$ the constant drift and diffusion parameters for the risky returns, and by $r \in \mathbb{R}$ the short rate.

The representative agent's objective is to select consumption $C_t \in \mathbb{R}^+$ and portfolio weights $\boldsymbol{v}_t \in \mathbb{R}^n$ so as to solve:

$$J_0^i = \max_{\{C_t, \boldsymbol{v}_t\}_t} \mathrm{E}_0 \int_0^\infty \exp(-\rho t) U(C_t, X_t^i) dt, \quad \rho > 0 \tag{2.1}$$

subject to

$$dW_t = \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r] W_t - C_t\} dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \tag{2.2}$$

where J_0^i is a value function, E_0 is a conditional expectations operator, ρ is a subjective discount rate, X_t^i is a reference level which will be characterized further below for models $i = W, H$, and W_t is the agent's (total) wealth. The agent's instantaneous utility $U_t^i = U(C_t, X_t^i)$ is characterized by:

$$U_t^i = \frac{(C_t - X_t^i)^{1-\gamma}}{1-\gamma}, \quad \gamma \geq 0, \tag{2.3}$$

where γ is a curvature parameter that, under the special case of reference independence, i.e. $X_t^i = 0, \forall t$, captures relative risk aversion. Otherwise, consumption risk aversion $RR_{c,t}$ is time–varying and given by:

$$RR_{c,t}^i = \frac{\gamma}{S_t^i}, \quad S_t^i \equiv \frac{C_t - X_t^i}{C_t} \tag{2.4}$$

where S_t^i is the surplus consumption ratio; a decrease in C_t and/or increase in X_t^i leads to a reduction in surplus consumption and a corresponding increase in risk aversion. In the spirit of Constantinides (1990), policies $\{C_t, \boldsymbol{v}_t\}_{t=0}^\infty$ that solve problem (2.1)-(2.3) are considered *admissible* if they satisfy: (i) $W_t \geq 0$, (ii) $C_t \geq 0$, (iii) $C_t \geq X_t^i$, and (iv) $\int_0^t C_s ds < \infty, \forall t$ finite.[5]

Following the RD literature, X_t^i can be interpreted as a reference (benchmark) consumption level. Preferences (2.1), (2.3) state that an agent evaluates the desirability of a consumption stream from the deviations in differences over and above the benchmark levels at each periods of time. From standard practices in the Habit literature, since γ is non–negative but otherwise unrestricted, the restriction $C_t \geq X_t^i$ is necessary for utility to be well defined. Prospect Theory with Loss Aversion may allow for kinks at references $C_t = X_t$ and convexities below references $C_t < X_t^i$. In our setup, the restriction $C_t \geq X_t^i$

[5] In addition, Constantinides (1990) imposes a no-borrowing constraint $\mathbf{0} \leq \boldsymbol{v}_t \leq \mathbf{1}$. As we consider leverage below in our treatment of real estate, this condition is not imposed here.

can be interpreted as indicating that the agent is very averse to loss.[6] Alternative specifications consider other functional forms for $U_t = U(C_t, X_t^i)$, such as CES, or ratios of levels to references (Abel, 1990; Bakshi and Chen, 1996; Garcia et al., 2005). We resort to the simpler differences–*cum*–power specification to emphasize parallels with the Habit literature (Sundaresan, 1989; Constantinides, 1990; Heaton, 1995; Campbell and Cochrane, 1999a).

2.2 Forward- vs backward-looking references

As mentioned earlier, references could reflect either expected (forward-looking) or customary (backward-looking) consumption. One forward-looking specification for references is implicitly suggested by Prospect Theory and relates references to the current value of assets (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991), implying that $X_t^i = X(W_t)$. We follow this suggestion by specifying an affine reference function:

$$X_t^W = \eta_0 + \eta_w W_t. \tag{2.5}$$

We will subsequently refer to X_t^W in (2.5) as Wealth–determined References (WDR). A similar linear effect of wealth on references is used in Prospect Theory settings by Berkelaar et al. (2004) and by Gomes (2005).[7]

The parameter η_0 captures the time- and state-independent part of references, while η_w measures the sensitivity of references to contemporaneous wealth. The WDR model nests both CRRA ($\eta_0, \eta_w = 0$), and HARA ($\eta_0 = 0$) utility functions. Lemma 1 in Appendix A.1 shows that conditions sufficient to guarantee that the admissible set is non-empty are:

$$\eta_w \geq 0, \tag{2.6a}$$

$$\frac{-\eta_0}{\eta_w} \geq 0, \tag{2.6b}$$

$$W_0 \geq \frac{-\eta_0}{\eta_w}, \tag{2.6c}$$

In particular, these conditions ensure that the risk-less policy given by $C_t = X_t^W + rW_t$, and $\boldsymbol{v}_t = \mathbf{0}, \forall t$ satisfies admissibility as defined in Section 2.1.

Preferences (2.5) differ from standard Habit models in that references are forward- instead of backward-looking. To see this, note that under perfect markets, wealth is

[6]For example, a typical Loss Aversion specification is given by:

$$U_t = \begin{cases} \alpha C_t + (1-\alpha)\frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}, & \text{if } C_t \geq X_t; \\ \alpha C_t - \lambda(1-\alpha)\frac{|C_t - X_t|^{1-\gamma}}{1-\gamma}, & \text{if } C_t < X_t, \end{cases}$$

where $\lambda \geq 1$ captures the degree of loss aversion. The Habit convention $\alpha = 0, C_t \geq X_t$ can be thought of as a highly loss averse agent, i.e. $\lambda \gg 1$ (Yogo, 2005a, p. 9). Values of λ above 2 are typically estimated and reported in the Prospect Theory literature (Tversky and Kahneman, 1992; Benartzi and Thaler, 1995; Berkelaar et al., 2004; Yogo, 2005a).

[7]These authors append a slow–moving effect to (2.5) by considering $X_t = (1-\eta_w)R_f X_{t-1} + \eta_w W_t$, where R_f is the gross risk-free rate of return (e.g. Gomes, 2005, eq. (6)). Under wealth–independent references ($\eta_w = 0$), references grow exogenously at the risk–free rate.

simply the expected discounted value of future consumptions streams, such that references (2.5) can be written as:

$$X_t^W = \eta_0 + \eta_w \frac{1}{\pi_t} E_t \int_t^\infty \pi_s C_s ds, \tag{2.7}$$

where $\pi_t \geq 0$ is a state-price deflator. Hence, WDR (2.7) naturally incorporate the notion that expectations about future consumption streams should determine references (Kandel and Kuznitz, 2004; Köszegi and Rabin, 2005).[8] In comparison, the Habit literature assumes that references are a function of past consumption profiles (e.g. Constantinides, 1990, eq (3), p. 522):

$$X_t^H = e^{-at} X_0^H + b \int_0^t e^{a(s-t)} C_s ds, \tag{2.8}$$

The parameter a represents the rate of discounting applied to lagged consumption, while b measures the importance of habit. Internal habit assume that C_s is a lagged control of the agent; external habit does not. Subsequent analysis will refer to X_t^H in (2.8) as Habit–determined References (HDR). The Habit model nests both CRRA ($a, b, X_0^H = 0$), and HARA ($a, b = 0$) utility functions. In parallel with our previous discussion, the conditions required for the admissible set to be non-empty (Constantinides, 1990, pp. 523-524) are:

$$a, b, X_0^H \geq 0 \tag{2.9a}$$

$$W_0 - X_0^H/(r + a - b) > 0 \tag{2.9b}$$

$$r + a - b > 0 \tag{2.9c}$$

Internal habit models (2.8) entail time-non-separability in that current choices of consumption affect future marginal utility. External habits abstract from these and retain time separability. Recent evidence on asset market dynamics suggests that the internal habit specification is more consistent with observed asset and bond returns than external habit, especially when long horizons of consumption are considered (Chen and Ludvigson, 2004; Grishchenko, 2005). We consequently focus on this specification as an alternative to WDR.

Wealth–dependent references (2.7) contain both internal and external variables, as well as time- and state-non-separabilities. Current references and marginal utility are determined by both: (i) the internal future consumption choices $C_s, s > t$ and (ii) the external state price densities $\pi_s, s \geq t$. The latter has important implications for the determinants of marginal utility risk which can be decomposed into consumption and benchmark risks. To highlight the co-movements between returns and benchmarks, observe that the laws of motion for the two benchmark processes are:

$$dX_t^W = \eta_w \mu_{w,t} dt + \eta_w \boldsymbol{\sigma}'_{w,t} d\boldsymbol{Z}_t, \tag{2.10}$$

$$dX_t^H = (bC_t - aX_t^H)dt, \tag{2.11}$$

where the wealth drift $\mu_{w,t}$ and diffusion $\boldsymbol{\sigma}_{w,t}$ are implicitly defined in the budget constraint (2.2). It follows that the (conditional) benchmark risk is proportional to total

[8] Unlike Köszegi and Rabin (2005), these expectations in (2.7) are not lagged, but contemporaneous to the choice of $C_t, \boldsymbol{v}_t$. Note that these non-separabilities are fully recognized by the agent, i.e. his decisions are dynamically consistent. This can be associated with the conditions of Personal Equilibrium required by Köszegi and Rabin (2005) that the agent's expectations determining his references are consistent with his choice of controls.

wealth (i.e. market) risk for WDR, and is zero for Habit. Consequently, WDR preferences allow for an additional market beta to supplement the consumption beta in the pricing equations. To the extent that the former is empirically high and that the latter is low (see Table 3), market risk may help in replicating high observed premia on risky assets at realistic risk aversion levels. In contrast, the Habit kernel defined by (2.8) yields a single-factor pricing equation – regardless of whether habit is internal or external – in which all the marginal utility risk is captured by consumption covariances with returns, and the persistence of pricing kernels by consumption dynamics. This bi-factorial property of WDR is also shared by generalized recursive utility. Indeed, as for the case of Non–Expected Utility (Epstein and Zin, 1989; Weil, 1989), WDR preferences are non–linear in probabilities through the power transform (2.3) applied on (2.7). Alvarez and Jermann (2005) show that these state non–separabilities are helpful in magnifying the permanent components of pricing kernels so as to jointly reproduce stocks as well as short– and long–term bonds dynamics, even if the consumption process is i.i.d..

2.3 Optimal Consumption and Portfolio Rules

The solution to the agent's problem (2.1)–(2.3), with Habit references (2.8) are well known (e.g. Constantinides, 1990); the solutions with wealth-determined references (2.5) are less so, and could be obtained using standard dynamic programming methods. It turns out however that a simpler alternative is available. Both the Wealth–determined (2.5) and the Habit–determined (2.8) reference-dependent models share a linear difference operator. Schroder and Skiadas (2002) show that closed-form expressions for linear Habit models (the primal problem) are conveniently obtained by simple modifications to the standard solutions in models without habit (the dual problem). Their analysis is cast in terms of Habit–determined references, but it can be readily extended to our WDR setup (see Appendix A.2). Based on this approach and adapting the Habit result of Constantinides (1990) to our notation and multi-variate setting reveals the following:

Proposition 1 *The indirect utility $J_t = J(W_t)$, the optimal consumption C_t and the value of risky assets $\boldsymbol{V}_t \equiv \boldsymbol{v}_t W_t$ for the agent's problem* (2.1)-(2.3) *with,*

1. *Wealth-dependent references X_t^W in* (2.5) *are:*

$$J_t \propto \left(W_t - \frac{\eta_0}{r-\eta_w}\right)^{1-\gamma} \propto \left(W_t - \frac{X_t^W}{r}\right)^{1-\gamma}, \tag{2.12}$$

$$C_t - X_t^W = \left(\frac{\gamma-1}{\gamma}\right)(r-\eta_w+\rho/(\gamma-1)+0.5M/\gamma) \times \left(W_t - \frac{\eta_0}{r-\eta_w}\right), \tag{2.13}$$

$$\boldsymbol{V_t} = \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma}\left(W_t - \frac{\eta_0}{r-\eta_w}\right), \tag{2.14}$$

2. Habit-dependent references X_t^H in (2.8) are:

$$J_t \propto \left(W_t - \frac{X_t^H}{r+a-b}\right)^{1-\gamma} \tag{2.15}$$

$$C_t - X_t^H = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{r+a-b}{r+a}\right)(r+\rho/(\gamma-1)+0.5M/\gamma) \times \left(W_t - \frac{X_t^H}{r+a-b}\right) \tag{2.16}$$

$$\boldsymbol{V_t} = \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma}\left(W_t - \frac{X_t^H}{r+a-b}\right) \tag{2.17}$$

where

$$M \equiv (\boldsymbol{\mu}_p - r)'\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r) \geq 0,$$
$$\boldsymbol{\Sigma}_{ij} \equiv \mathrm{E}[\boldsymbol{\sigma}_i d\boldsymbol{Z}_t d\boldsymbol{Z}'_t \boldsymbol{\sigma}'_j]$$

Proof.

1. WDR: Appendix A.2;

2. HDR: Constantinides (1990), Appendix A, proof of Theorem 1, pp. 536-539.

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The closed-form expressions obtained under Wealth-dependent and under Habit-dependent references in Proposition 1 share striking similarities yet display important differences. First, the value function J_t in (2.12) under WDR is unconditionally hyperbolic, with wealth benchmark $X_{w,t}^W \equiv \eta_0/(r-\eta_w)$; the one in (2.15) under Habit is conditionally (upon a realization of the consumption reference X_t^H) hyperbolic, with wealth benchmark $X_{w,t}^H \equiv X_t^H/(r+a-b)$. Under WDR, the agent selects optimal consumption and portfolio such that the risk-less annuity on wealth rW_t is always larger than the consumption benchmark $X_t^W = \eta_0 + \eta_w W_t$. Under Habit, this condition becomes $(r+a-b)W_t \geq X_t^H$, where the subjective risk-less annuity increases in the discount rate applied on past consumption and decreases in the sensitivity of habit to lagged consumption. Both RD models are therefore consistent with the portfolio-insurer strategy of ensuring that the risk-less return on wealth is sufficient to cover the benchmark on optimal wealth (Leland, 1980; Basak, 1985; Benninga and Blume, 1985; Berkelaar et al., 2004; Gomes, 2005).

Second, under both reference models, the optimal surplus consumption $C_t - X_t^i$ and the optimal risky portfolio value $\boldsymbol{V}_t$ are proportional to surplus wealth $W_t - X_{w,t}^i, i = W, H$. For $\gamma > 1$, low surplus wealth implies low surplus consumption (i.e. high savings) to increase wealth above its reference and more conservative portfolios to minimize downside risk of not beating the wealth benchmark. The wealth dependence parameter $\eta_w > 0$ has two effects: it affects the wealth benchmark (and thus indirect utility risk aversion) and the elasticity of surplus consumption to surplus wealth. For $\eta_0 < 0$, a positive η_w decreases the wealth benchmark; the opposite occurs for $\eta_0 > 0$.[9] In both instances,

[9]The empirical results reported below are consistent with a negative η_0.

$\eta_w > 0$ decreases the sensitivity of surplus consumption to surplus wealth. Note that the elasticity of the risky portfolio is independent of both η_0, η_w. Conversely $a - b > 0$ for the Habit model implies a lower wealth benchmark, and higher surplus consumption elasticity and no impact on that of the risky portfolio.

Clearly linearity in wealth for the optimal rules under both Wealth- and Habit-determined references implies that the change in consumption and portfolio will also be linear in the changes in wealth. Furthermore, we can substitute the solutions for both models in the budget constraint (2.2) to characterize dW_t. This exercise reveals the closed-form expression for instantaneous changes as follows:

Corollary 1 *The instantaneous changes in consumption, the value invested in assets, and wealth, with*

1. *Wealth-dependent references X_t^W in (2.5) are:*

$$\begin{aligned} dC_t &= \left\{ \left(\frac{\gamma - 1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5M/\gamma) + \frac{\eta_w}{\gamma} \right\} dW_t, \quad (2.18) \\ &= c_w^W dW_t; \\ d\boldsymbol{V}_t &= \left\{ \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} \right\} dW_t, \quad (2.19) \\ &= \boldsymbol{v}_w^W dW_t; \\ dW_t = dW_{s,t}^W &= \frac{W_{s,t}^W}{\gamma} \left\{ \left[\left(\frac{\gamma + 1}{\gamma} \right) 0.5M + r - \eta_w - \rho \right] dt \right. \\ &\quad \left. + (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} \boldsymbol{\sigma}_p d\boldsymbol{Z}_t \right\}, \quad (2.20) \\ &= [\mu_0^W + \mu_w^W W_t] dt + [\sigma_0^W + \sigma_w^W W_t] dZ_t; \end{aligned}$$

where $W_{s,t}^W \equiv W_t - \eta_0/(r - \eta_w)$ is surplus wealth for the Wealth-dependent reference model.

2. *Habit-dependent references X_t^H in (2.8) are:*

$$\begin{aligned} dC_{s,t}^H &= \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{r + a - b}{r + a} \right) (r + \rho/(\gamma - 1) + 0.5M/\gamma) \right\} dW_{s,t}^H \quad (2.21) \\ &= c_w^H dW_{s,t}^H; \\ d\boldsymbol{V_t} &= \left\{ \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} \right\} dW_{s,t}^H \quad (2.22) \\ &= \boldsymbol{v}_w^H dW_{s,t}^H; \\ dW_{s,t}^H &= \frac{W_{s,t}^H}{\gamma} \left\{ \left[\left(\frac{\gamma + 1}{\gamma} \right) 0.5M + r - \rho \right] dt + (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} \boldsymbol{\sigma}_p d\boldsymbol{Z}_t \right\}, \quad (2.23) \\ &= \mu_w^H W_{s,t}^H dt + \sigma_w^H W_{s,t}^H dZ_t; \end{aligned}$$

where $C_{s,t}^H \equiv C_t - X_t^H$ is surplus consumption, and $W_{s,t}^H \equiv W_t - X_t^H/(r + a - b)$ is surplus wealth for the Habit-dependent reference model.

Having derived the closed-form expressions for the laws of motion of the endogenous variables, we now turn to the estimation of the structural parameters based on these processes.

3 Estimation

3.1 Econometric Model

Estimation focuses on the multivariate Brownian motion given by Corollary 1. This approach is motivated by two concerns. First, estimating optimal allocations imposes considerably more theoretical restrictions that are related to the deep parameters on the joint first and second moments. With respect to preference parameters, standard analyses of returns treat the equilibrium quantities in the pricing kernels as exogenous; the theoretical restrictions are imposed on the prices of risk exclusively, with conditional second moments left unrestricted. In comparison, the allocations analysis produces theoretical restrictions on both first and second moments of changes in consumption, asset holdings and wealth (through the restrictions on $c_w, \boldsymbol{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$), while returns are treated as exogenous. In the absence of strong prior information, these additional restrictions will be useful in identifying the preference parameters of interest. Second, empirical studies of aggregate optimal consumption and asset holdings are much less frequent than asset pricing studies. We believe that focusing on quantities rather than on returns thus provides another perspective that complements existing results.[10]

Transformation One major problem in estimating (2.18)–(2.20) is that there exists no closed-form transition density for multi-variate Brownian motions with affine drifts and diffusions. Indeed, analytical expressions for the likelihood function exist only for a limited class of Itô processes (Melino, 1996). Unfortunately, our multi-variate process does not belong to this class. Alternative solutions include discrete (Euler) approximations, and simulating the continuous-time paths between the discretely-sampled data, either through classical (Durham and Gallant, 2002) or through Bayesian (Eraker, 2001) approaches.

Our solution to this problem is different and considerably simpler to implement. It is based on a homoscedasticity-inducing transformation for general Brownian motions. It will be shown that this approach also stationarizes the drift term. Consequently, a standard discretized approximation is appropriate, efficient, and unbiased. In particular, a straightforward application of Itô's lemma reveals the following.

Corollary 2 *For the optimal changes in Corollary 1 with:*

1. *Wealth-dependent references* X_t^W *in* (2.5)*: Let* $Y_t \in \{C_t, \boldsymbol{V}_t\}$ *be defined as follows:*

$$Y_t = y_0^W + y_w^W W_t, \tag{3.1}$$

$$dW_t = (\mu_0^W + \mu_w^W W_t)dt + (\sigma_0^W + \sigma_w^W W_t)dZ_t, \tag{3.2}$$

[10] See also Lo and Wang (2001) for arguments in favor of using the informational content of quantities more thoroughly.

where $\boldsymbol{y}^W, \boldsymbol{\mu}^W, \boldsymbol{\sigma}^W$ *are constants defined in* (2.13) *and* (2.14), *and in* (2.20), *and consider the following transformation:*

$$\tilde{Y}_t = \frac{\log[y_w^W \sigma_0^W + \sigma_w^W (Y_t - y_0^W)]}{\sigma_w^W}, \tag{3.3}$$

Then, $\tilde{Y}_t$ *has constant drift and diffusion given by:*

$$d\tilde{Y}_t = \left[\frac{\mu_w^W}{\sigma_w^W} - 0.5\sigma_w^W \right] dt + dZ_t. \tag{3.4}$$

2. *Habit-dependent references* X_t^H *in* (2.8)*: Let* $Y_{s,t}^H \in \{C_{s,t}^H, \boldsymbol{V}_t\}$ *be defined as follows:*

$$Y_{s,t}^H = y_w^H W_{s,t}^H, \tag{3.5}$$

$$dW_{s,t}^H = \mu_w^H W_{s,t}^H dt + \sigma_w^H W_{s,t}^H dZ_t, \tag{3.6}$$

where $\boldsymbol{y}^H, \boldsymbol{\mu}^H, \boldsymbol{\sigma}^H$ *are constants defined in* (2.16) *and* (2.17), *and in* (2.23), *and consider the following transformation:*

$$\tilde{Y}_{s,t}^H = \frac{\log[Y_{s,t}^H]}{\sigma_w^H}, \tag{3.7}$$

Then, $\tilde{Y}_{s,t}^H$ *has constant drift and diffusion given by:*

$$d\tilde{Y}_{s,t}^H = \left[\frac{\mu_w^H}{\sigma_w^H} - 0.5\sigma_w^H \right] dt + dZ_t. \tag{3.8}$$

Proof.

1. WDR: Appendix A.
2. HDR: By Itô's lemma.

∎

The transformation (3.3) requires that its first derivative with respect to the Itô process Y_t is the inverse of the diffusion. It is usually introduced in order to stationarize the diffusion (Shoji and Ozaki, 1998; Aït-Sahalia, 2002; Durham and Gallant, 2002). In our case, both drift and diffusion are affine and have intercept and slope coefficients that are closely inter-related. Consequently the theoretical restrictions implied by the model are such that the transformation *also* stationarizes the drift term. This is fortunate since the resulting transformed model is easily estimated by maximum likelihood. In particular, the discretization of (3.4) and (3.8):

$$\Delta\tilde{Y}_t^i = \left[\frac{\mu_w^i}{\sigma_w^i} - 0.5\sigma_w^i \right] + \epsilon_t \tag{3.9}$$

where ϵ_t is a standard Gaussian term, can be consistently and efficiently estimated by MLE (e.g. Gourieroux and Jasiak, 2001, pp. 287–288).

Unfortunately, such a transformation does not exist for the Habit reference process X_t^H. Consequently we follow the standard approaches of discretizing the habit process. Two alternatives are available for this purpose. First, we can discretize the integral in (2.8) to obtain:

$$X_t^H = e^{-at} X_0^H + b \sum_{s=0}^{t-1} e^{a(s-t)} C_s.$$

An alternative is to discretize the o.d.e. (2.11) to obtain:

$$\Delta X_t^H = bC_{t-1} - aX_{t-1}.$$

These two possibilities imply the following alternatives for the difference equation:

$$X_t^H = e^{-a}[X_{t-1} + bC_{t-1}] \quad \text{or,} \tag{3.10a}$$
$$X_t^H = (1-a)X_{t-1} + bC_{t-1}, \quad \text{for } t \geq 1 \tag{3.10b}$$

where, for both cases, the initial observation X_0^H is treated as a parameter and is estimated jointly with the other preference parameters a, b, γ. Heaton (1995); Li (2001, 2005) implicitly consider variants of (3.10a) by making current habit a weighted average of lagged habit and lagged consumption, multiplied by the loading b. Grishchenko (2005) instead estimates (3.10b). In the absence of a clear consensus, we estimate both, and test which of the two habit models performs best in reproducing the data using non-nested hypotheses tests discussed below. As will become clear shortly, the inference results reported below are robust to the choice of discretized process. However, the point estimates are not.

Likelihood function The optimal rules in Proposition 1 and Corollary 1 take the moments of the returns' distribution $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$, as well as the risk-free rate r as given. These moments could be estimated in an external round, using a two-step method, and substituted back into the optimal rules to obtain the predicted rules. Instead, we perform a single-step procedure and incorporate the mean and covariance matrix of the risky returns into the calculation of the likelihood function. This approach has the advantage of factoring in the parametric uncertainty concerning $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ into the calculation of the standard errors of the deep parameters.[11] Specifically, denote by $\tilde{\boldsymbol{Y}}_t \equiv [\tilde{C}_t, \tilde{\boldsymbol{V}}_t, \tilde{W}_t]'$ the $n+2$ vector of transformed variables, and by $\boldsymbol{p}_t$ the cumulated log (cum-dividend) price process. The model to be estimated is the following:

$$\begin{pmatrix} \Delta \tilde{\boldsymbol{Y}}_t^i \\ \Delta \boldsymbol{p}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_y^i \\ \boldsymbol{\mu}_p \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_y^i \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \quad \begin{pmatrix} \boldsymbol{\epsilon}_y^i \\ \boldsymbol{\epsilon}_p \end{pmatrix} \sim \text{N.I.D.} \left(\begin{pmatrix} \boldsymbol{0}_y \\ \boldsymbol{0}_p \end{pmatrix}, \begin{pmatrix} \boldsymbol{I}_y & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{pp} \end{pmatrix} \right), \tag{3.11}$$

where $\boldsymbol{\mu}_y^i$ is given by Corollary 1, and $\boldsymbol{I}_y$ is an $n+2$ identity matrix.

First, in accordance with the maintained assumption of the model, all the innovations are Gaussian. Second, as mentioned earlier, the transformation in Corollary 2 implies that the quantities innovations are standardized white noise. Third, consistent with the

[11] Following standard practices, the risk-free rate r is calibrated to its mean value.

model, the covariance matrix is block diagonal, i.e. we impose the absence of cross-correlations between innovations in quantities and returns. Any potential covariance between the two is fully taken into account in the closed-form solutions; allowing for additional correlations is not theoretically justified.

With these elements in mind, the contributions to the likelihood function (with constant term omitted) are given by:

$$f_t^i = -0.5 \log[\det(\boldsymbol{\Sigma})] + \log[\det(\boldsymbol{K}_t^i)] - 0.5 \boldsymbol{\epsilon}_t^{i\prime} \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}_t^i \tag{3.12}$$

where $\boldsymbol{\Sigma}$ is defined implicitly in (3.11), while $\boldsymbol{K}_t^i$ is a Jacobian correction term associated with the transformations (3.3) and (3.7).[12] The parameter vector is then $\boldsymbol{\theta}^W \equiv \{\gamma, \rho, \eta_0, \eta_w, \boldsymbol{\mu}_p, \boldsymbol{Q}_{pp}\}$ for the WDR model and $\boldsymbol{\theta}^H \equiv \{\gamma, \rho, a, b, X_0, \boldsymbol{\mu}_p, \boldsymbol{Q}_{pp}\}$ for the HDR model, where $\boldsymbol{Q}_{pp} \equiv \mathrm{Chol}(\boldsymbol{\Sigma}_{pp})$ is the n-dimensional triangular Cholesky root of the returns covariance matrix.

Hypothesis tests We consider two benchmarks in assessing the performance of the WDR and Habit models. As mentioned earlier, CRRA utility is obtained by imposing that $\eta_0, \eta_w = 0$ for the WDR model and $a, b, X_0 = 0$ for HDR, whereas HARA utility imposes $\eta_w = 0$ for the WDR model and $a, b = 0$ for HDR. To the extent that it has been studied extensively in asset pricing models, CRRA utility constitutes a natural benchmark. HARA utility, although less popular, has the advantage of optimal rules which are not proportional to wealth, and are thus able to capture time variation in consumption and portfolio shares of wealth highlighted in Figures 1, and 2.

The WDR and Habit models however are strictly non-nested, as one cannot be expressed as a theoretical restriction of the other. We therefore resort to a modification to the Likelihood Ratio test suggested by Vuong (1989) for non-nested model selection tests. Given a set of dependent variables $\boldsymbol{Y}_t$, of explanatory variables $\boldsymbol{Z}_t$ two non-nested models $\boldsymbol{\theta}^i, \boldsymbol{\theta}^j$, with corresponding contributions to the likelihood $f(\boldsymbol{Y}_t \mid \boldsymbol{Z}_t, \boldsymbol{\theta}^i)$ and $f(\boldsymbol{Y}_t \mid \boldsymbol{Z}_t, \boldsymbol{\theta}^j)$, the appropriate test statistic and asymptotic distributions are:

$$n^{-1/2} LR_n(\hat{\boldsymbol{\theta}}^i, \hat{\boldsymbol{\theta}}^j)/\hat{\omega} \xrightarrow{d} N(0,1) \tag{3.13}$$

$$n^{1/2}\hat{\omega} \equiv \hat{\boldsymbol{m}}'\hat{\boldsymbol{m}} \tag{3.14}$$

$$\boldsymbol{m} \equiv \{\log[f(\boldsymbol{Y}_t \mid \boldsymbol{Z}_t, \hat{\boldsymbol{\theta}}^i)/f(\boldsymbol{Y}_t \mid \boldsymbol{Z}_t, \hat{\boldsymbol{\theta}}^j)]\}_{t=1}^n, \quad \hat{\boldsymbol{m}} = \boldsymbol{m} - \mathrm{E}(\boldsymbol{m}). \tag{3.15}$$

(Vuong, 1989, Theorem 5.1, p. 318).[13] A significant positive (negative) test statistic indicates that the i (j) model performs better in reproducing the data. We also use the Vuong (1989) test to compare the two habit discretization models (3.10a), and (3.10b).

Finally, we compute descriptive statistics for the derived series of interest, namely, the reference levels for consumption X_t^i, and at the optimum $X_{w,t}^i$; the corresponding surplus

[12] Specifically:

$$\boldsymbol{K}_t^i \equiv \mathrm{Diag}([K_{c,t}^i, \boldsymbol{K}_{v,t}^i, K_{w,t}^i, 1, \dots 1])$$
$$K_{y,t}^W = 1/[y_w^W \sigma_0^W + \sigma_w^W (Y_t - y_0^W)]$$
$$K_{y,t}^H = 1/[\sigma_w^H Y_t^H].$$

[13] We use a correction for the different number of parameters in the WDR and Habit models.

levels $S_t^i = (C_t - X_t^i)/C_t$ and $S_{w,t}^i = (W_t - X_{w,t}^i)/W_t$; and the consumption $RR_{c,t}^i = \gamma/S_t^i$ and indirect utility $RR_{w,t}^i = \gamma/S_{w,t}^i$ risk aversion. These statistics are evaluated at the point estimates and fully incorporate the parametric uncertainty.[14]

Estimation details The econometric model (3.11) to be estimated is highly non-linear in both parameters, and in variables (because of the transformation). Consequently, we experimented with numerous identification strategies to ensure convergence, and global optimization of the likelihood function. Following standard practices, the short rate r was calibrated to the mean real quarterly return on T-Bills. Also, the subjective discount rate ρ was calibrated to a realistic annual rate of 3.5%. As for estimation, the strategy we used was based on a sequential estimation procedure.

In a first step, we estimated the most restricted CRRA model ($\eta_0 = \eta_w = 0$, or $a = b = X_0 = 0$). We used the unconditional means of returns to generate the starting values for the returns process. Then, we performed a grid search over the starting value for remaining preference parameter γ to control for potential multiplicity of optimum. To facilitate convergence, we used a sequential combination of Simplex and Newton-based algorithms in which outputs were used as starting values until final convergence was obtained.

In a second step, we used these estimated results as starting values for the HARA model ($\eta_0 \neq 0, \eta_w = 0$, or $a = b = 0, X_0 \neq 0$) and performed a grid search for the starting value of η_0, using the one which yielded the highest likelihood at the optimum. Finally, we used the HARA results as starting values in the WDR model ($\eta_0, \eta_w \neq 0$) and for the HDR model ($a, b, X_0 \neq 0$), again performing a grid search over potential starting values for η_w or for a, b, and resorting to successive use of different algorithms to ensure global optimization.

To summarize, our estimation strategy imposes a one–to–one mapping between the continuous–time theoretical model in Corollary 1 and its fully–structural, discrete–time econometric analog (3.11); there is *no* auxiliary hypothesis appended between the two. These strong restrictions are necessary to help identify deep preference parameters $\gamma, \eta_0, \eta_w, a, b, X_0$ for which we have no strong priors. Moreover, our transformation avoids the usual pitfalls of estimating a continuous–time model using discretely–sampled data. Also, the single–step estimation approach that we use ensures that the inference tests are adequately computed. Finally, we perform rigorous testing of the preference models, including a test of the non-nested WDR and HDR hypotheses, and compute moments for the derived series of interest that incorporate the parametric uncertainty.

3.2 Data

Our data set consists of post-war U.S. quarterly observations on aggregate consumption, asset holdings and corresponding returns indices. The time period covered ranges from 1963:Q2 to 2005:Q3, for a total of 170 observations. All quantities are expressed in real,

[14]Specifically, we used $\sqrt{\hat{\boldsymbol{g}}'[\hat{\boldsymbol{G}}\hat{\boldsymbol{\Xi}}\hat{\boldsymbol{G}}']\hat{\boldsymbol{g}}}$, where $H_0 : \hat{\boldsymbol{g}} = \boldsymbol{g}(\hat{\boldsymbol{\theta}}) = 0$ is the relevant moment restriction, $\hat{\boldsymbol{G}} = \boldsymbol{G}(\hat{\boldsymbol{\theta}})$ is the matrix of derivatives with respect to parameter set $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ evaluated at the MLE estimates, and $\hat{\boldsymbol{\Xi}} = \boldsymbol{\Xi}(\hat{\boldsymbol{\theta}})$ is the VCV matrix of the parameters. The δ method is used to compute numerical derivatives.

per-capita terms, where the aggregate price index is taken to be the implicit GDP deflator (base year: 2000). Similarly, all returns are converted in real terms by subtracting the inflation index.

Consumption The consumption series is the aggregate expenditure on Non-Durables and Services. The source of the data is the Bureau of Economic Analysis NIPA series. This series has been used in most asset pricing studies.

Assets The aggregate portfolio holdings are defined as follows:

$$\begin{aligned} \boldsymbol{V}_t &= [V_{0,t}, V_{1,t}, V_{2,t}, V_{3,t}, V_{4,t}] \\ &= [\text{Deposits, Bonds, Stocks, Home, Mortgages}]. \end{aligned}$$

Each asset holdings are obtained from the Flow of Funds Accounts made available by the Board of Governors of the Federal Reserve (Table L.100). They correspond to the level values of asset holdings by households and non-profit organizations (see also Lettau and Ludvigson, 2003; Poterba, 2000, for discussions). We consider two definition of wealth: the narrower Financial wealth, used in most empirical asset pricing and portfolio applications, and the broader Financial + Real estate wealth. More precisely the wealth and corresponding individual assets (notation, mnemonic) are:

1. Financial wealth only: Deposits + Bonds + Stocks ($W_{fin,t} = V_{0,t} + V_{1,t} + V_{2,t}$), where:
    (a) Deposits ($V_{0,t}$, FL15400005): Includes foreign, checkable, time, savings deposits and money market fund shares.
    (b) Bonds ($V_{1,t}$, FL153061005): U.S. government securities (Treasury and Agency).
    (c) Stocks ($V_{2,t}$, FL153064105): Corporate equities directly held by households, does not include indirectly held mutual funds.
2. Financial and real estate wealth: Financial wealth + Home + Mortgages ($W_t = W_{fin,t} + V_{3,t} + V_{4,t}$), where:
    (a) Home ($V_{3,t}$, FL155035015): Real estate corresponding to all owner–occupied housing, whether primary or second homes, plus vacant land.
    (b) Mortgage ($V_{4,t}$, FL153165105) Value of mortgage liabilities faced by households, including home equity lines of credit.

Deposits are thus taken to represent the risk-free asset, whereas long-term government bonds, corporate equity, home equity, and mortgages are taken to be the risky assets. A main advantage of defining wealth in this way (instead of modeling one of its component as an unobservable latent variable) is that it is observable and the definition provides more structure on the econometric model since one of the theoretical asset holding is defined residually.[15]

[15] For example, (2.14) reveals that, for the risk-free asset:

$$V_{0,t} = v_{00} + v_{w,0} W_t, \quad v_{00} = -\sum_{i=1}^{n} v_{i0}, \quad v_{w0} = 1 - \sum_{i=1}^{n} v_{iw}.$$

The choice of specific portfolio holdings is dictated by a number of practical elements. First, these financial assets correspond to some of the largest asset holdings for U.S. households, and their returns have been studied extensively in the asset pricing literature, thus providing useful benchmarks for our analysis. Moreover, we also incorporate the empirically important real estate assets so as to encompass tangible assets in total wealth. Real estate has been shown to result in higher estimates of risk aversion. It will be of interest to verify whether or not this is the case in Section 4.2 when using this broader definition of total wealth. Second and related, these assets have corresponding returns series that are required to evaluate the distributional parameters $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ used to compute the theoretical asset holdings. Other portfolio holdings such as pension and life insurance reserves are also important in relative size. However, no clear returns indices are available for these assets.[16]

Table 1 reports the sample moments for the consumption and asset holdings in percentage of financial wealth and in parentheses of financial + real estate wealth. Those series are plotted in Figure 1, and 2. A first observation is that the shares of wealth allocated to consumption, deposits and stocks are roughly of the same order of magnitude, whereas bonds on the other hand represent a much lower share of wealth and are smoother. Finally, incorporating real estate reveals that net home equity is large, and appears to have a smoothing influence on consumption and portfolio shares.

Returns We follow Campbell et al. (2003) in constructing the returns series that correspond to our assets definition. The return on cash is taken to be the real return on 3-months Treasury Bills. The return on bonds is proxied by the real return on 5-years T-Bills. The stock returns are evaluated as the value-weighted returns on the S&P–500 index. In order to compute the return on homes, we used the capital gains based on median sales prices for new single–family houses (source CITIBASE).[17] Finally, the mortgage rate was proxied by the effective interest rate on conventional, closed home mortgage loans (source CITIBASE). Again, the inflation series is computed from the GDP deflator.

Table 2 presents sample moments of the real returns. These series have been widely discussed in the asset pricing literature, so we only briefly outline their main features. First, we observe that all risky assets warrant a positive premia. Equity returns however are clearly larger, and much more volatile. The return on home equity is lower, but

Our definition abstracts from other elements, such as durables, mutual funds, and human wealth. Unfortunately, real returns indices on both durable goods and mutual funds are difficult to evaluate, and these assets were omitted from our selected holdings series $\boldsymbol{V}_t$. Moreover, human wealth is not observable, whether in levels or in rates of returns and thus also eliminated.

[16]For example, pension reserves are typically invested differently by fund managers whether they are defined benefit or defined contribution. Finding a unique pricing index for this series in the absence of detailed information on the funds' composition is impractical.

[17]In the absence of rents data at a quarterly frequency that correspond to our Household and Nonprofit sector home equity holdings, we follow Flavin and Yamashita (2002) in constructing home equity returns based on capital gains only. Available NIPA rent measures, such as those used by Piazzesi et al. (2006) correspond to rents paid to *all* home-owning sectors, including non-personal ones, and therefore overestimate the actual rent received by households. Unlike Flavin and Yamashita (2002), but consistent with our perfect market hypothesis, we do not incorporate fiscal distortions, such as the deductability of interest payments on mortgages, in our analysis. Incorporating these distortions was found to have only a marginal effect on our results.

the volatility is similar to other capital-gains-based estimates of home returns.[18] In comparison, the return on medium–term government bonds are slightly lower, though much less volatile. Note in Table 3 that the correlation of returns with consumption is lower than the one with wealth.

4 Results

This section presents the estimation results for the CRRA, HARA, WDR and HDR models in Corollary 1. We start by discussing the estimated parameters and hypotheses tests, followed by the derived series of interest. We first present in Section 4.1 the estimation results when assets and wealth are restricted to the narrower financial-assets definition. We then extend the assets and wealth definition to incorporate real estate in Section 4.2 to verify the impact on our results. Finally, in Section 4.3, we again check robustness by allowing for time-varying risk premia.

4.1 Financial wealth only

4.1.1 Estimation results

Parameter estimates Table 4 presents the estimated parameters for the CRRA (column 1), HARA (column 2), WDR (column 3) Habit_a (column 4) and Habit_b (column 5) models, with associated t-statistic in parentheses. Panel A reports the preference parameters, while the drift and diffusion parameters for the risky returns process are presented in panels B and C. Overall, all the preference parameters have the correct sign and are statistically significant.

First, for the CRRA model, the curvature parameter γ is associated with relative risk aversion. The level is within the range usually considered as reasonable (e.g. between 0 and 10, Mehra and Prescott, 1985). Second, for HARA preferences, the curvature is virtually unchanged, while the constant references η_0 is positive. Third, turning to the WDR model, the curvature is larger. The parameter η_0 is negative, and significant; the wealth sensitivity parameter η_w is positive, and significant. From Table 5, panel A.1, the estimates η_0, η_w satisfy all the theoretical restrictions (2.6a), (2.6b), and (2.6c). The positive and significant value for η_w confirms the experimental finding of Prospect theory that for forward-looking references, higher wealth increases the consumption benchmark.

Fourth, compared to CRRA, HARA, and WDR, we witness a lower curvature index for the two Habit models, Habit_a, and particularly for Habit_b. The initial value of the habit process X_0^H, the discount parameter a and the sensitivity parameter b are all positive and significant, thus satisfying the Habit restriction (2.9a). From panel B.1 in Table 5, the estimated Habit parameters also satisfy the theoretical restrictions (2.9b) and (2.9c).

[18]Using PSID data, Flavin and Yamashita (2002), Tab. 1.A, p. 350, find a mean return of 0.0656, with a standard deviation of 0.1424 for the period 1968-1992. The higher mean return can be explained by the inclusion of the risk-free rate, and tax deduction in their returns calculation (11), p. 348, and the fact that they use individual, rather than aggregate housing returns.

Compared with Habit_a, we witness an increase in X_0^H, a, b under Habit_b. The parameter a is larger than one, indicating mean reversion in the habit process (3.10b).[19]

In order to contrast our estimated parameters X_0^H, a, b, γ with other estimates in the empirical literature, a convenient basis of comparison is to compute steady-state habit-to-consumption ratios X^H/C and risk aversion levels $\gamma/(1 - X^H/C)$. Panel A in Table 6 reveals that, compared to other internal habit estimates, our steady-state habit-to-consumption is lower, especially for Habit_a. These differences can be related to differences in our empirical strategy: (i) we estimate, instead of calibrate a continuous-time model, (ii) we do not impose ad-hoc restriction of duration of habit, but instead (iv) we estimate the initial value of the habit process X_0^H, (iv) we use a closed-form, fully structural econometric model and (v) we estimate over quantity, instead of returns' data. Note finally that our steady-state risk aversion estimates are nonetheless very similar to those in the literature.

Finally, the other drift and diffusion parameters in panels B, C are mainly instrumental to our analysis and are not discussed in details. It should be noted that, as could be expected since returns are exogenous, they are reasonably robust to the choice of preference model.

Inference Table 7 presents the inference results for the various model selection tests. First, we find that the null of CRRA preferences is strongly rejected against any alternative. This results points to strong statistical evidence in favor of the relevance of references in characterizing aggregate US household portfolio and consumption decisions.

Second, we find strong statistical support for the hypothesis that these references are not constant, but are state-dependent. The null of HARA is rejected against either the WDR and the two Habit models. Hence our results reveal that the references used by households are state-contingent, whether the state is measured by contemporary wealth, or lagged consumption.

Third, we clearly reject the null that the WDR and two Habit models perform equally well in reproducing the data. The test results unambiguously indicate that the aggregate household portfolio and consumption data is more consistent with backward-looking, Habit-determined benchmarks than forward-looking, wealth-determined references. Fourth, we also reject the null that the two Habit models Habit_a and Habit_b perform equally well; Habit_b clearly dominates Habit_a in reproducing the data.

4.1.2 Derived series

Table 8 presents summary statistics for the derived reference (panel A), surplus (panel B) and risk aversion (panel C) levels. The first four columns are the statistics for the within-period utility function $U(C_t, X_t^i)$ in (2.3) with references (2.5), for WDR and (2.8) for Habit; the last four columns are those for the indirect utility function $J(W_t^i)$ in Proposition 1. All statistics are evaluated at the point estimates for the parameters presented in Table 4, with associated t-statistics in parentheses. The latter fully accounts for parametric uncertainty at the MLE estimates. Overall, these statistics indicate that for the three RD models, references, surpluses and risk aversion are all positive and

[19] Larger estimates of mean reversion can be associated with the discretization bias from using discrete Euler approximations such as Habit_b (e.g. Phillips and Yu, 2005, Tab. 1, p. 340).

significant throughout the sample. This implies that both within-period U_t and indirect utility J_t are well-defined and satisfy the relevant theoretical restrictions of monotonicity and concavity. We plot the derived reference, surplus and risk aversion series in Figures 3, 4, and 5.

First, in panel A of Figure 3 the reference consumption levels are identically zero for CRRA and virtually so for HARA. Habit produces the largest consumption references. Note that mean reversion identified earlier from the estimated $a > 1$ is quite moderate. The consumption references for WDR are noticeably smaller and subject to more short-term, and pro-cyclical fluctuations. This is unsurprising since wealth was found to be more volatile and covariant with returns than consumption (see Table 3). The wealth references plotted in panel B are constant, and indistinguishable for WDR and HARA, while they are increasing for Habit.

Second, in Figure 4, panel A, the Habit_a and especially Habit_b consumption surpluses are clearly lower, and hover closely around their respective steady-state value. The consumption surplus ratio under WDR is more volatile, counter-cyclical and stationary. This suggests that wealth-determined references fall more than actual consumption in recessions. The surplus wealth ratios in panel B reveal very similar pro-cyclical properties and levels under WDR and Habit. However, since references are constant at the optimum under WDR, the surplus exhibits a clear positive trend, whereas that of Habit is stationary.

Third, the implied relative risk aversion indices are plotted in Figure 5. The estimates for consumption risk aversion in panel A are within admissible range for all models, except for WDR and Habit_b. The high risk aversion for WDR is a result of the lower surplus and higher curvature; that of Habit_b can be explained by its much lower surplus offsetting the lower curvature index. All consumption risk aversion series appear stationary. In panel B, we find that both Habit specifications yield lower levels of risk aversion at the optimum, and WDR the highest. Clear counter-cyclical patterns can be found for HARA, WDR and Habit. Moreover, since the wealth references are constant for HARA and WDR, risk aversion exhibits a clear downward trend for these models, while it is stationary under Habit. Finally, it should be noted that all RD models yield moderate risk aversion volatility. Indeed, the coefficient of variation varies between [0.06%, 2.28%] for $RRA_{c,t}$ and [4.17%, 8.89%] for $RRA_{w,t}$. These numbers pale in comparison with those found in asset shares [16.05%, 31.62%] or asset returns [53.36%, 485.39%].

To summarize, our results obtained over financial portfolio data confirm the ones obtained under estimation of pricing kernels. Representative–agent models with frictionless markets require high levels of risk aversion to reconcile high observed premia with the relatively low observed risky financial asset holdings. Introducing reference-dependent preferences does not result in significantly lower risk aversion at the optimum. However, high risk aversion comes about for different reasons: high surplus and curvature for WDR, low surplus and curvature for Habit. A second salient difference between WDR and Habit is that the latter generates stationary risk aversion both for consumption and at the optimum; the former does so only for consumption. Finally, a common feature to all the RD models is the strong counter-cyclical patterns for optimal risk aversion that they generate.

4.2 Incorporating real estate

Our previous results were obtained using financial wealth only. To verify their robustness to the definition of wealth, we now augment the set of assets to incorporate real estate. We include home equity ($V_{3,t}$) and mortgages ($V_{4,t}$), and modify the wealth variable accordingly ($W_t = W_{fin,t} + V_{3,t} + V_{4,t}$).

Overall, the parameter estimates of Table 9 confirm our earlier findings; again, all parameters have the correct sign and are statistically significant, while the theoretical restrictions for both the WDR and the Habit models are satisfied (see panel B of Table 5). Compared to our earlier results, we witness (i) a strong increase in the curvature parameter γ for all models, (ii) a decrease in the wealth sensitivity η_w for WDR, (iii) an increase in the initial value for the habit process X_0^H, and (iv) a decrease in both the habit sensitivity parameter b and in the habit discount rate a. The end result is an increase in steady-state habit-to-consumption ratios (panel B, Table 6) which are now closer to estimates found elsewhere.

The results of the model selection tests in Table 10 confirm our earlier findings that the nulls of no or constant references are both strongly rejected against the WDR and both Habit alternatives. When tested against one another, we again reach a clear conclusion that the Habit model is better at explaining the data than the WDR model. Finally, as before, the discretized Habit_b in (3.10b) outperforms Habit_a in (3.10a).

The descriptive statistics for derived references, surplus and risk aversion series are presented in Table 11. Once again, the derived series are all significant and positive for the RD models, thereby indicating that the relevant monotonicity and concavity conditions are satisfied. The derived references, surplus and risk aversion series are plotted in Figures 6, 7, and 8. The results reveal that the Habit surpluses are now much lower compared to the others. This coupled with the increase in curvature leads to large increase in risk aversion for all models, and for Habit in particular. Indeed, panel B, Table 6 shows that the internal Habit risk aversion are now of levels found in the external Habit literature. Clear counter-cyclical risk aversion at the optimum are again apparent in panel B of Figure 8. Finally, volatility again remains moderate for both consumption risk aversion [0.06%, 1.13%], and optimal risk aversion [3.30%, 5.75%].

4.3 Time-varying risk aversion, investment set, or both?

This paper has highlighted the important time variation in empirical portfolio shares. This variation can be explained by two elements: (i) time-varying risk aversion, and/or (ii) time-varying investment set. By abstracting from the second explanation, we have found strong empirical support for the first one, with counter-cyclical risk aversion induced by non-zero, and state-dependent benchmarks.

However, one might reasonably object that these results depend crucially on our assumption of constant investment set. Since we impose constant mean excess returns, short rates and covariances, the empirical results may put excessive weight on counter-cyclical risk aversion in order to artificially generate counter-cyclical movements in risky portfolio shares, that could otherwise be explained by cyclical fluctuations in conditional moments of returns. Put differently, lower risky portfolio shares in recessions could be explained by factors unrelated to higher risk aversion, such as higher risk or short rate, or lower premia.

In order to assess the relevance of this objection, we therefore modify the model to allow for some time variation in the investment set (TVIS). A major obstacle is that closed-form solutions for consumption and portfolio shares with TVIS are notoriously difficult to attain, even in simplified settings. This is even the more so in our particular framework that allows for time- and state-non-separabilities.[20] Nonetheless, we can still check the robustness of our results if we are willing to accept a few simplifying assumptions.

Denote by $\boldsymbol{\chi}_t \in \mathbb{R}^k$ the set of macro state factors determining the conditional drift $\boldsymbol{\mu}_{p,t} = \boldsymbol{\mu}(\boldsymbol{\chi}_t)$ and diffusion $\boldsymbol{\sigma}_{p,t} = \boldsymbol{\sigma}(\boldsymbol{\chi}_t)$ of the risky returns process. A standard argument establishes that the optimal portfolio will be composed of the (mean-variance) efficient portfolio plus the dynamic hedging portfolio:

$$\boldsymbol{V}_t = \frac{-J_{w,t}}{J_{ww,t}} \boldsymbol{\Sigma}_{pp,t}^{-1}(\boldsymbol{\mu}_{p,t} - r_t) - \boldsymbol{\Sigma}_{pp,t}^{-1} \boldsymbol{\Sigma}_{p\chi,t} \frac{-\boldsymbol{J}_{w\chi,t}}{J_{ww,t}}$$

where $J_{w,t} = U_c(C_t, X_t)$ from first-order conditions. We can simplify this expression by assuming that:

(A1) the states can be replicated by factor-mimicking portfolios;

(A2) these replicating portfolios are the assets under consideration, i.e. $\boldsymbol{p}_t = \boldsymbol{\chi}_t$, where $\boldsymbol{p}_t$ is the cumulated cum-dividend log price process.

The first assumption is routinely adopted in APT settings with excess returns on market base portfolios capturing the prices of state risk (e.g Lehmann and Modest, 1988). The second assumption is reasonable, considering that some of our asset returns are the same broad-based indices that are typically introduced in empirical factorial analysis of returns (e.g. Chen et al., 1986; Fama and French, 1993).

Under these two assumptions, the optimal portfolios simplify to:

$$\boldsymbol{V}_t = \frac{-J_{w,t}}{J_{ww,t}} \left(\boldsymbol{\Sigma}_{pp,t}^{-1}(\boldsymbol{\mu}_{p,t} - r_t) + \frac{-\boldsymbol{J}_{wp,t}}{J_{w,t}} \right).$$

Two observations stem from this analysis (i) the hedging portfolio is independent of state risk and (ii) the mean-variance portfolio with time-varying Sharpe ratios will dominate portfolio allocations if the second term in the parentheses is small. An empirical argument shows that this is the case for our data set. Indeed, for our utility function (2.3), we have:

$$\boldsymbol{J}_{wp,t}^i = U_{cc,t} \boldsymbol{C}_{p,t} + U_{cx,t} \boldsymbol{X}_{p,t}^i$$

A regression of consumption on a constant, wealth, time trend and our portfolios' cumulated excess returns yielded insignificant parameters for 3 out of 4 cases, indicating that $\boldsymbol{C}_{p,t}$ is negligible; similar results were obtained for the WDR model in which $X_t^W = W_t$

[20]For example, Campbell and Viceira (1999); Campbell et al. (2003) need to resort to numerical approaches to solve consumption and portfolio with Epstein and Zin (1991) preferences and a simple factorial model of returns. Berkelaar et al. (2004), and Gomes (2005) simplify the dynamic problem with loss-averse investors by abstracting from utility over intermediate consumption.

and wealth was regressed on those same cumulated returns, indicating that $\boldsymbol{X}^W_{p,t}$ is negligible as well.[21] Importantly, this implies that our previous solutions can be adapted to incorporate the time-varying elements of the distribution without reference to the complexities induced by the dynamic hedging portfolios.

We operationalize this TVIS framework by assuming constant short rate r and diffusions $\boldsymbol{\sigma}$, but allowing for time-varying mean excess returns. Specifically, denote by $\boldsymbol{p}^e_t \equiv \int_0^t \boldsymbol{r}^e_s ds$ the cumulated excess returns process, we assume that:

$$d\boldsymbol{p}^e_t = [\boldsymbol{A}\boldsymbol{p}^e_t + \boldsymbol{B}]dt + \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \tag{4.1}$$

where $\boldsymbol{A} \in \mathbb{R}^n \times \mathbb{R}^n$ and where $\boldsymbol{B} \in \mathbb{R}^n$. Clearly, our previous constant investment set can be obtained as $\boldsymbol{A} = 0$, and $\boldsymbol{B} = \boldsymbol{\mu}_p - r$.

The arithmetic Brownian motion (4.1) has two important properties. First, it is sufficiently flexible to accommodate a wide array of returns' dynamics, and second, it is associated with closed-form expressions for the transitional densities in discrete-time. This implies that this model can also be estimated again taking into full consideration the discretization bias. Indeed, it can be shown that the discrete-time equivalent of (4.1) satisfies the following VARMA(1,1) representation:

$$\boldsymbol{p}^e_t = \boldsymbol{\lambda}\boldsymbol{p}^e_{t-1} + \boldsymbol{g} + \boldsymbol{\eta}_t \tag{4.2}$$

where

$$\boldsymbol{\lambda} \equiv \exp(\boldsymbol{A}), \quad \boldsymbol{g} \equiv \boldsymbol{B}(\boldsymbol{\lambda} - \boldsymbol{I})\boldsymbol{A}^{-1}$$
$$\mathrm{E}(\boldsymbol{\eta}_t\boldsymbol{\eta}_s) = \begin{cases} \boldsymbol{\Omega}_0, & s = t \\ \boldsymbol{\Omega}_1, & s = t-1 \\ \boldsymbol{0} & \text{otherwise} \end{cases}$$

where the $\boldsymbol{\Omega}_i$ are complex non-linear functions of $\boldsymbol{A}, \boldsymbol{\Sigma}_{pp}$ (see Grossman et al., 1987; Gordon and St-Amour, 2004, for details).

In light of the tremendous non-linearities involved, we used a two-step estimation procedure applied to the Financial assets subset. Specifically, we started with the (exogenous) cumulated price process (4.2) to obtain the predicted excess returns $\boldsymbol{\mu}_{p,t} - r = [\boldsymbol{A}\boldsymbol{p}^e_t + \boldsymbol{B}]$ and the covariance matrix $\boldsymbol{\Sigma}_{pp} = \boldsymbol{\sigma}_p\boldsymbol{\sigma}'_p dt$. These were then substituted in the closed-form solutions in Corollary 2 in the second round of estimation to obtain the deep preference parameters. All the test statistics in Tables 12 and 13 are corrected to take into account the parametric uncertainty in the first round of estimations.

Overall, the estimation results confirm the robustness of our earlier findings to allowing for time variation in the investment set. As before, we find that both the null of no, and that of constant benchmarks are strongly rejected against the WDR, and Habit alternatives. Moreover, the backward-looking benchmark perspective of the Habit model dominates the forward-looking one of WDR when the discretized habit process is taken to be (3.10b), although by a much narrower margin. Interestingly, the curvature index is

[21] Since the Habit benchmark process X^H_t is unobservable and is generated from the estimated parameters, and since the latter are obtained under the constant investment set assumption, an evaluation of the empirical relevance of $\boldsymbol{X}^H_{p,t}$ cannot be performed.

now centered around one for the CRRA, HARA and WDR models. This is unsurprising given that our TVIS model heavily relies on the mean-variance portfolio, as would be the case under log-utility. Unfortunately, the curvature is negative and significant for the Habit models, thereby violating the concavity requirement, so that some caution must be used in interpreting these results.

As for the question which, of time-varying risk aversion or expected returns is responsible for explaining time variation in aggregate portfolios, the answer has to be: both of them are. The parameters of the $\boldsymbol{A}$ matrix in (4.1) are almost all significant, indicating that the hypothesis of constant investment set is rejected. Moreover, our maintained findings of significant and state-dependent benchmarks add support to the time variation in risk aversion.

5 Summary and discussion

5.1 Summary

A main feature of Reference Dependent preferences is that agents gauge possible outcomes using deviations from benchmark consumption levels. These benchmarks can be characterized as either anticipated (i.e. forward-looking) or customary (i.e. backward-looking) consumption. Prospect Theory assumes the former and associates references with assets being held, whereas Habit models relate references to past consumption profiles. These benchmarks have important implications for asset selection in that agents are more risk averse when consumption is close to reference. Fluctuations in references thus generate movements in risk aversion beyond those associated with fluctuations in consumption. These could explain the asset reallocations observed over the business cycle.

This paper adresses three questions. First we ask whether references play any role in explaining aggregate US household decisions. Second, assuming they do, we ask whether or not references are state-dependent. Finally, we ask whether the data is more consistent with a forward- or a backward-looking interpretation of references. References are of course inherently subjective and unobservable. Consequently, to study these issues, we depart from standard empirical applications in focusing on optimal decision rules instead of returns-based Euler equation applications. We construct a fully structural continuous-time econometric model from the closed-form expressions for instantaneous changes in consumption, asset holdings and wealth. We estimate this model for both the wealth-dependent and the habit-dependent reference specification using aggregate household asset holdings data on financial and real estate wealth. Compared to standard returns-based approaches, our portfolio- and consumption-based approach imposes much more cross-equations restrictions from which the underlying benchmark processes can be identified.

Our main results can be summarized as follows. First, we find strong evidence in favor of references. Second, these references are found to be state-dependent; they are increasing in wealth and in habit. Third, the model-selection tests unambiguously favor the backward-looking, Habit-determined over the forward-looking, wealth-determined reference model. Fourth, we find that all our inference results are robust to the choice of assets, but that incorporating real estate yields even higher estimates for risk aversion.

Finally, allowing for time-varying conditional risk premia does not qualitatively affect our main findings.

5.2 Discussion

The fact that references are important and state-dependent is intuitively appealing. If a consumption benchmark is to retain any economic meaning, then it should be allowed to grow at a similar rate than consumption. Both the WDR and Habit models allow for this possibility, whereas CRRA and HARA do not. Our estimates point to a clear rejection of the latter two models. Consumption benchmarks are determined by wealth and by lagged consumption in such a way as to increase at the same rate as contemporary consumption.

Our findings for the WDR model are consistent with known experimental evidence. Prospect Theory has long recognized the very strong role of references, and the fact that agents adjust benchmarks rapidly following a change in assets. This evidence is consistent with our finding of a strong positive wealth elasticity of references in the WDR model.

Nonetheless, the Habit specification has a further advantage compared to WDR in that the wealth references at the optimum are also state-dependent, whereas those of WDR are not. This could explain why we find that the Habit model outperforms the WDR. The fact that references roughly grow at the same speed as consumption and wealth implies that surpluses are stationary, both for within-period utility, and at the optimum (Figures 4, and 7). Consequently, so is risk aversion (Figures 5, and 8). This result is intuitively comforting. It would be difficult to rationalize secular declines in risk aversion without similar movements in risk premia, events which we simply do not observe.

Does this mean that the forward-looking interpretation of references is inconsistent with the data? Not necessarily. This interpretation relies heavily on the perfect market hypothesis. It could very well be that agents indeed use expected future consumption as benchmarks, but that imperfect markets mean that this value is not fully captured by available wealth measures. Allowing for non-observable human wealth or market frictions could alter the results. Moreover, alternative functional forms for the wealth-determined reference process could lead to a state-dependent wealth benchmark at the optimum. We leave both issues on the research agenda.

All the RD models (HARA, WDR, and Habit) estimated in this study are consistent with strong counter-cyclical risk aversion at the optimum. This confirms similar findings in the returns-based empirical literature, whether the state-of-the-world variable is specified (as in this study), or not (Gordon and St-Amour, 2000, 2004). It is unsurprising given the pro-cyclical movements in risky asset shares identified in Figures 1 and 2. When the investment set is fixed, such movements can *only* be ascribed to changing attitudes toward risk. The better performance of RD models is largely reflective of this property. Interestingly, time-varying risk aversion *also* remains prevalent when we allow for time variation in predicted premia (Section 4.3). Whether or not strong counter-cyclical risk aversion is maintained when a wider state set or more complex hedging strategies are allowed will require the use of numerical methods, such as those of Campbell et al. (2003) that are beyond the scope of this study.

Our finding that risk aversion increases when real estate is introduced has been documented elsewhere (e.g. Grossman and Larocque, 1990; Heaton and Lucas, 2000; Flavin and Yamashita, 2002; Chetty and Szeidl, 2004; Chetty, 2004). It can be understood as a dual of the main pricing anomalies. For instance, the equity premium puzzle states that the observed equity premium is too high relative to its theoretical counterpart. In portfolio terms, this translates in observed portfolio shares that are too low compared to those predicted by the model. When a narrow definition of wealth is used, the denominator W_t in the definition of portfolio shares $v_{i,t} \equiv V_{i,t}/W_t$ is too low. Consequently, the observed shares are artificially higher, and the model is able to reproduce them at more realistic risk aversion levels. When a broader definition of wealth is used, empirical shares fall mechanically and are tantamount to increasing risk aversion.

Another, more intuitive interpretation is also suggested by the real estate literature. Buying a house involves large, usually incompressible mortgage payments, rather than state-contingent ones. This forces the agent into more conservative portfolio positions (Chetty and Szeidl, 2004). These market imperfections are not taken into account in the model. Consequently, the observed portfolio mix is reproducible through large risk aversion. Along these lines, another interpretation is suggested by the strong reduction in the elasticity of inter-temporal substitution (EIS) when real estate is introduced. As Constantinides (1990) shows, the Habit EIS is simply the inverse of the consumption relative risk aversion. For Habit_b, the mean EIS thus falls almost 80%, from 0.0674 to 0.0139. Since the correlation between short- and mortgage rates is large (85%, see Table 2), an increase in the short rate is associated with an increase in mortgage rates. As mortgage positions are important (see Table 1) and largely incompressible, this severely reduces the household's capacity to curb consumption and increase savings.

Finally, the fact that risk aversion remains excessive when allowing for references is at the same time both comforting and depressing. All models other than CRRA predicted high, and counter-cyclical risk aversion at the optimum, confirming findings based on returns data. This could be interpreted as a salient, model-, and data-free feature of asset markets. Yet, the fact that these models are unable to address the main anomaly of excessive risk aversion is disappointing, although perhaps predictable. The single source of risk in the Habit model ultimately remains consumption; this risk, no matter *how* it is determined, cannot justify the high premia (or low asset holdings) unless an excessive price of risk is used. As wealth co-varies more than consumption with returns, incorporating wealth-determined references is a right element, but goes in the wrong direction. Declines in references in recessions have a pacifying influence on marginal utility risk. Again a high price of risk needs to be obtained somehow.

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A Proofs

A.1 Non-empty admissible set

Lemma 1 *The conditions* (2.6a), (2.6b), *and* (2.6c), *are sufficient for the admissible set to be non-empty.*

Proof. In the spirit of Constantinides (1990), consider the no-risky investment policy $\boldsymbol{v}_t = \mathbf{0}$ where surplus consumption is equal to the risk-less return on wealth:

$$C_t - X_t^W = rW_t \tag{A.1}$$

with $X_t^W = \eta_0 + \eta_w W_t$. We show that (2.6a), and (2.6c) are sufficient to ensure that this policy satisfies the conditions for admissibility.

(i) $W_t \geq 0$**:** Substituting (A.1) in the budget constraint (2.2) yields the non-homogenous differential equation:

$$dW_t/dt + \eta_w W_t = -\eta_0 \tag{A.2}$$

with solution

$$W_t = \frac{-\eta_0}{\eta_w} + \left(W_0 - \frac{-\eta_0}{\eta_w}\right) e^{-\eta_w t}. \tag{A.3}$$

(2.6b) ensures that the intercept is positive; (2.6c) ensures that the slope is positive, such that $W_t \geq 0, \forall t$.

(ii) $C_t \geq 0$**:** Substituting solution (A.3) in policy (A.1) yields:

$$C_t = r\left(\frac{-\eta_0}{\eta_w}\right) + (r + \eta_w)\left(W_0 - \frac{-\eta_0}{\eta_w}\right) e^{-\eta_w t}. \tag{A.4}$$

Since $r \geq 0$, (2.6b) ensures that the intercept is positive; (2.6a) and (2.6c) jointly ensure that the slope is positive, such that $C_t \geq 0, \forall t$.

(iii) $C_t \geq X_t^W$**:** Follows directly from (i) and (A.1) since $r \geq 0$.

(iv) $\int_0^t C_s ds < \infty$**:** Using (A.4) yields:

$$\int_0^t C_s ds = r\left(\frac{-\eta_0}{\eta_w}\right) t + (r + \eta_w)\left(W_0 - \frac{-\eta_0}{\eta_w}\right) [e^{-\eta_w t} - 1] \tag{A.5}$$

which is finite for all finite t, since $\eta_w \geq 0$ from restriction (2.6a).

∎

A.2 Proposition 1

Proof. First, by appropriately redefining the agent's problem, expressions for the dual consumption, dual short rate and dual risk premia can be obtained. Second, these expressions are then substituted back into the known solutions to the dual problem. Third, the solutions to the primal problem are obtained by adding in the wealth-in-the-utility term to the second-step solutions.

In what follows let Y_t refer to a variable in the primal problem and let $\hat{Y}_t$ refer to its dual problem counterpart. We start by defining the dual variables as follows:

$$\hat{C}_t \equiv C_t - \eta_w W_t, \tag{A.6}$$

$$\hat{U}_t \equiv \frac{(\hat{C}_t - \eta_0)^{1-\gamma}}{1-\gamma} = U_t. \tag{A.7}$$

Next, replace for C_t in budget constraint (2.2) by using (A.6) to obtain:

$$\begin{aligned} dW_t &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r]W_t - \hat{C}_t - \eta_w W_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + (r - \eta_w)]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + \hat{r}]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \end{aligned} \tag{A.8}$$

Observe that wealth, portfolio, and the risk premia $(\boldsymbol{\mu}_p - r)$ remain unchanged, whereas the short rate is replaced by $\hat{r} \equiv r - \eta_w$, and consumption is replaced by $\hat{C}_t$ as given in (A.6).[22] Moreover, dual utility (A.7) suppresses any explicit dependence on wealth, and is simply a HARA over dual consumption $\hat{C}_t$. Under the iso-morphism result of Schroder and Skiadas (2002), we can:

1. use the known solutions of Merton for the dual problem $(\hat{C}_t, \hat{\boldsymbol{v}}_t)$ as functions of $W_t, \hat{r}, \boldsymbol{\mu}_p - r$,
2. correct the short rate in these solutions using $\hat{r} = r - \eta_w$,
3. get back the expression for C_t by inverting (A.6); the expression for $\boldsymbol{v}_t$ is the same as that for $\hat{\boldsymbol{v}}_t$.

Following this iso-morphism approach yields the solutions in Proposition 1. It can be shown that these solutions correspond exactly to those obtained using the more traditional dynamic programming approach. ∎

A.3 Corollary 2

Proof. First, (3.1) and (3.2) reveal that:

$$dY_t = [y_w \mu_0 + \mu_w (Y_t - y_0)]dt + [y_w \sigma_0 + \sigma_w (Y_t - y_0)]dZ_t \tag{A.9}$$

$$= \mu(Y_t)dt + \sigma(Y_t)dZ_t. \tag{A.10}$$

[22] This suggests that if π_t is the primal state-price density, then $\hat{\pi}_t \equiv e^{\eta_w t}\pi_t$ is its dual counterpart, since a standard no-arbitrage argument establishes that:

$$\hat{r} = -\mu_{\hat{\pi}}/\hat{\pi}_t = r - \eta_w$$
$$\hat{\boldsymbol{\mu}}_{\boldsymbol{p}} - \hat{r} = -(1/\hat{\pi}_t)\, \boldsymbol{\sigma}_{\boldsymbol{p}} \boldsymbol{\sigma}_{\hat{\boldsymbol{\pi}}}' = \boldsymbol{\mu}_{\boldsymbol{p}} - r,$$

Next, by Itô's lemma, we have for $\tilde{Y}_t = \tilde{Y}(Y_t)$:

$$d\tilde{Y}_{j,t} = \left[\mu(Y_t)\tilde{Y}'(Y_t) + 0.5\sigma(Y_t)^2\tilde{Y}''(Y_t)\right] dt + \sigma(Y_t)\tilde{Y}'(Y_t)dZ_t \tag{A.11}$$

Observe that $\mu_0/\mu_w = \sigma_0/\sigma_w$ to substitute in (A.11) to obtain (3.4). ∎

B Tables

Table 1: Descriptive statistics: Consumption, portfolio and wealth

Variable	Series	min	max	mean	std
		A. Levels ($)			
C_t	consumption	8,002	23,350	14,486	4,298
$V_{0,t}$	cash	7,186	17,815	12,825	2,813
$V_{1,t}$	bonds	1,003	3,630	1,849	715
$V_{2,t}$	stock	5,169	34,990	13,062	6,548
$V_{3,t}$	home	12,640	57,261	26,289	10,829
$V_{4,t}$	mortgage	-24,557	-4,270	-9,920	5,128
$W_{fin,t}$	fin. wealth only	17,630	52,651	27,736	8,190
W_t	fin. + real estate wealth	27,904	74,768	44,105	13,011
		B. Shares of fin. wealth only			
$C_t/W_{fin,t}$		35.8%	66.6%	52.7%	8.8%
$V_{0,t}/W_{fin,t}$		27.9%	66.0%	48.1%	10.8%
$V_{1,t}/W_{fin,t}$		2.9%	12.0%	6.7%	1.8%
$V_{2,t}/W_{fin,t}$		27.4%	66.5%	45.2%	10.8%
		C. Shares of fin. + real estate wealth			
C_t/W_t		26.6%	38.9%	32.9%	3.0%
$V_{0,t}/W_t$		19.7%	38.3%	29.9%	4.8%
$V_{1,t}/W_t$		1.7%	7.5%	4.3%	1.2%
$V_{2,t}/W_t$		14.9%	46.9%	29.1%	9.2%
$V_{3,t}/W_t$		39.9%	81.6%	58.0%	10.0%
$V_{4,t}/W_t$		-35.0%	-14.1%	-21.3%	5.1%

Note: Financial wealth only is $W_{fin,t} = \sum_{i=0}^{2} V_{i,t}$; financial + real estate wealth is $W_t = \sum_{i=0}^{4} V_{i,t}$. Sample period is 1963:Q1–2005:Q3 (170 quarterly observations). Levels are in real, (year 2000) per-capita $.

Table 2: Descriptive statistics: Returns

Variable	Series	min	max	mean	std
		In real, annual rates			
$r_{0,t}$	30-d T-Bills	-4.50%	7.39%	1.80%	2.27%
$r_{1,t}$	5-y T-Bills	-4.23%	9.38%	3.02%	2.45%
$r_{2,t}$	S&P-500	-70.20%	137.02%	10.89%	30.47%
$r_{3,t}$	$\%\Delta(P_{home,t})$	-35.81%	55.29%	3.49%	16.94%
$r_{4,t}$	Mortg. rate	-2.72%	10.60%	4.61%	2.46%

Note: Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

Table 3: Correlation Consumption, Wealth, Returns

A. Financial assets and wealth only

	$\Delta\log(W_{fin,t})$	$r_{0,t}$	$r_{1,t}$	$r_{2,t}$
$\Delta\log(C_t)$	0.185	0.175	0.203	0.163
$\Delta\log(W_{fin,t})$		0.096	0.147	0.849
$r_{0,t}$			0.917	0.093
$r_{1,t}$				0.146

B. Financial and real estate assets and wealth

	$\Delta\log(W_t)$	$r_{0,t}$	$r_{1,t}$	$r_{2,t}$	$r_{3,t}$	$r_{4,t}$
$\Delta\log(C_t)$	0.198	0.175	0.203	0.163	0.038	0.192
$\Delta\log(W_t)$		0.086	0.139	0.809	0.145	0.194
$r_{0,t}$			0.917	0.093	-0.166	0.851
$r_{1,t}$				0.146	-0.103	0.959
$r_{2,t}$					0.154	0.226
$r_{3,t}$						-0.060

Note: Assets are cash (0), bonds (1), stocks (2), home (3) and mortgages (4). Financial wealth is cash + bonds + stocks. Financial and real estate wealth is financial wealth + real estate - mortgages. See Tables 1 and 2 for further description of series. Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

Table 4: Estimation results: Financial wealth only

Model	CRRA	HARA	WDR	Habit_a	Habit_b
			A. Preference parameters		
γ	8.3096	8.4449	10.2691	7.0100	6.5525
	(8.3353)	(6.5439)	(5.1076)	(6.3087)	(6.5226)
η_0		28.3000	-384.9929		
		(53.0600)	(-5.6715)		
η_w			0.0647		
			(5.9144)		
X_0^H				1889.4971	4368.3082
				(4.1539)	(28.7881)
b				0.7552	1.0589
				(4.6696)	(25.7733)
a				1.3983	1.8891
				(7.7903)	(49.9839)
			B. Drifts of risky returns process		
μ_1	0.0065	0.0067	0.0074	0.0064	0.0064
	(25.1760)	(20.1926)	(20.7452)	(20.7262)	(21.4101)
μ_2	0.0170	0.0178	0.0182	0.0168	0.0167
	(9.8923)	(19.1923)	(2.3097)	(20.2153)	(15.4796)
			C. Diffusions of risky process		
$Q_{1,1}$	0.0062	0.0061	0.0060	0.0062	0.0062
	(16.9519)	(17.3530)	(12.0772)	(17.1418)	(17.1048)
$Q_{1,2}$	0.0100	0.0099	0.0107	0.0099	0.0099
	(1.7415)	(1.7420)	(1.8826)	(1.7462)	(1.7437)
$Q_{2,2}$	0.0728	0.0728	0.0728	0.0728	0.0728
	(18.2942)	(18.3122)	(18.2605)	(18.3027)	(18.3002)
LLF	-5674.7728	-5560.9877	-5558.8910	-4801.0109	-4743.1844
nb. obs.	1190	1190	1190	1190	1190

Note: Estimation results (t-stat) for model (3.11) where assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Fixed parameter $\rho = (1 + 0.035)^{1/4} - 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations). Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 5: Theoretical restrictions tests

Model/restriction	A. Fin. wealth only		B. Fin. + real est. wealth	
1. WDR:				
$\eta_w \geq 0$	0.0647		0.0257	
	(5.9144)		(5.3718)	
$-(\eta_0/\eta_w) \geq 0$	5,947.1		5,106.3	
	(57.1486)		(209.3904)	
$W_0 - (-\eta_0/\eta_w) \geq 0$	14,612.3		23,823.5	
	(140.4162)		(976.9079)	
2. Habit:	$\mathrm{Habit_a}$	$\mathrm{Habit_b}$	$\mathrm{Habit_a}$	$\mathrm{Habit_b}$
$W_0 - X_0/(r + a - b) \geq 0$	17,641.2	15,325.6	22,454.3	19,670.3
	(21.0944)	(38.7955)	(19.8420)	(32.4883)
$r + a - b \geq 0$	0.6475	0.8346	0.4211)	0.5734
	(10.7536)	(22.6443)	(12.5193)	(21.3347)

Note: Theoretical restrictions (t-stat) sufficient to guarantee that the admissible set is non-empty. For WDR, conditions given by (2.6a), (2.6b), (2.6c); for Habit, by (2.9b), (2.9c) (Constantinides, 1990, p. 523). $\mathrm{Habit_a}$ is for discretized habit process (3.10a); $\mathrm{Habit_b}$ is for discretized habit process (3.10b).

Table 6: Comparisons Habit steady-state references, consumption risk aversion

author	Steady-state X/C	γ	$\gamma/(1-X/C)$
Internal habits			
Constantinides (1990), Tab. 1, p. 532	0.79-0.82	2.2	10.48-12.22
Heaton (1995), Tab. VI, p. 702	0.71	2.44	8.41
Grishchenko (2005), Tab. 3, p. 38	0.40-0.82	3.96-8.46	12.58-47.02
St-Amour (2006)	A. Financial wealth only		
Habit_a	0.2478	7.0100	9.3188
	(2.2419)	(6.3087)	(8.5874)
Habit_b	0.5605	6.5525	14.9096
	(18.9511)	(6.5226)	(6.6764)
	B. Financial + real estate wealth		
Habit_b	0.3411	24.3036	36.8853
	(2.7897)	(2.6660)	(2.8114)
Habit_b	0.6790	23.2913	72.5603
	(17.3105)	(2.5212)	(2.5490)
External habits			
Campbell and Cochrane (1999b), Tab. 1, p. 218	0.94	2.0	33.33
Tallarini and Zhang (2005), Tab. 3, p. 25	0.95	6.27	125.40

Note: Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 7: Model selection LR tests: Financial wealth only

	Alternative H_1:			
Null H_0:	HARA	WDR	Habit_a	Habit_b
CRRA	227.5702	231.7636	1747.5239	1863.1769
	[0]	[0]	[0]	[0]
HARA		4.1934	1519.9536	1635.6067
		[0.0406]	[0]	[0]
WDR			-9.7640^{*}	-9.9882^{*}
			[0]	[0]
Habit_a				-10.6594^{*}
				[0]

Note: Likelihood Ratio tests [p-value] of null vs alternative hypotheses, where assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. $*$: modified LR test for non-nested hypotheses following (Vuong, 1989, Thm. 5.1, p. 318), test statistic asymptotically distributed as $N(0,1)$. A significant positive (negative) test statistic indicates that the null (alternative) model performs better in reproducing the data. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 8: Derived series: Financial wealth only

model	min.	max.	mean	std.	min.	max.	mean	std.
A. Reference levels								
	A.1 Consumption X_t^i				A.2 Wealth $X_{w,t}^i$			
CRRA	0	0	0	0	0	0	0	0
HARA	28.300	28.300	28.300	0	6395.284	6395.284	6395.284	0
	(53.060)	(53.060)	(53.060)		(53.060)	(53.060)	(53.060)	
WDR	756.273	3023.429	1410.555	530.167	6383.478	6383.478	6383.478	0
	(6.077)	(5.952)	(5.998)	(5.914)	(52.832)	(52.832)	(52.832)	
Habit_a	1889.497	5718.046	3558.655	1057.402	2918.173	8831.050	5496.050	1633.070
	(4.154)	(12.532)	(12.525)	(12.510)	(3.489)	(5.788)	(5.786)	(5.786)
Habit_b	4368.308	13006.950	8092.284	2402.319	5233.797	15584.005	9695.601	2878.289
	(28.788)	(31.191)	(31.200)	(31.200)	(13.249)	(13.486)	(13.480)	(13.482)
B. Surplus								
	B.1 Consumption $S_t^i = (C_t - X_t^i)/C_t$				B.2 Wealth $S_{w,t}^i = (W_t - X_{w,t}^i)/W_t$			
CRRA	1	1	1	0	1	1	1	0
HARA	0.997	0.999	0.998	0.001	0.637	0.879	0.752	0.062
	(14949.082)	(43726.207)	(24813.111)	(53.060)	(93.208)	(383.772)	(160.837)	(53.060)
WDR	0.852	0.934	0.902	0.020	0.638	0.879	0.752	0.062
	(34.300)	(85.146)	(55.505)	(5.940)	(93.076)	(382.925)	(160.538)	(52.832)
Habit_a	0.750	0.764	0.754	0.002	0.745	0.864	0.800	0.034
	(37.468)	(13.437)	(38.460)	(0.721)	(16.863)	(36.715)	(23.130)	(5.770)
Habit_b	0.430	0.454	0.441	0.004	0.550	0.757	0.647	0.059
	(23.421)	(23.944)	(24.654)	(8.924)	(16.557)	(42.075)	(24.712)	(13.488)
C. Relative risk aversion								
	C.1 Consumption $RR_{c,t}^i = \gamma/S_{c,t}^i$				C.2 Wealth $RR_{w,t}^i = \gamma/S_{w,t}^i$			
CRRA	8.310	8.310	8.310	0	8.310	8.310	8.310	0
	(8.335)	(8.335)	(8.335)		(8.335)	(8.335)	(8.335)	
HARA	8.455	8.475	8.463	0.005	9.613	13.252	11.306	0.918
	(6.544)	(6.544)	(6.544)	(6.516)	(6.534)	(6.516)	(6.523)	(6.552)
WDR	11.000	12.051	11.386	0.259	11.686	16.098	13.739	1.112
	(5.762)	(7.546)	(6.259)	(2.619)	(5.068)	(4.945)	(5.008)	(4.635)
Habit_a	9.177	9.348	9.292	0.019	8.116	9.412	8.779	0.366
	(6.922)	(6.367)	(6.362)	(0.743)	(6.370)	(6.715)	(6.507)	(5.091)
Habit_b	14.431	15.235	14.846	0.128	8.653	11.923	10.210	0.908
	(6.677)	(6.685)	(6.664)	(88.311)	(6.550)	(6.933)	(6.686)	(21.453)

Note: Derived references, surplus, and risk aversion (t-stat) for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. In-sample moments (t-statistics) evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 4. The t-statistic incorporates parametric uncertainty (see footnote 14 for details). Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 9: Estimation results: Financial and real estate wealth

Model	CRRA	HARA	WDR	Habit_{a}	Habit_{b}
			A. Preference parameters		
γ	27.4235	27.6717	25.7202	24.3036	23.2913
	(5.0921)	(7.9935)	(32.4089)	(2.6660)	(2.5212)
η_0		27.3508	-131.2752		
		(44.9911)	(-4.8050)		
η_w			0.0257		
			(5.3718)		
X_0^H				2726.7883	5309.1264
				(7.9007)	(45.3101)
b				0.6617	1.2035
				(5.7755)	(28.8938)
a				1.0784	1.7725
				(8.6650)	(38.5413)
			B. Drifts of risky returns process		
μ_1	0.0069	0.0069	0.0069	0.0068	0.0068
	(8.8238)	(9.3790)	(14.9514)	(5.8136)	(5.6207)
μ_2	0.0222	0.0226	0.0222	0.0220	0.0219
	(8.9944)	(7.6794)	(6.3392)	(10.2368)	(9.0787)
μ_3	0.0118	0.0119	0.0116	0.0117	0.0117
	(4.9843)	(4.8872)	(4.8029)	(5.0618)	(5.0887)
μ_4	0.0090	0.0092	0.0091	0.0089	0.0089
	(9.6367)	(10.4434)	(16.5955)	(6.2160)	(5.9899)
			C. Diffusions of risky returns process		
$Q_{1,1}$	0.0061	0.0061	0.0061	0.0061	0.00061
	(14.5567)	(14.3219)	(14.4268)	(15.1111)	(15.2305)
$Q_{1,2}$	0.0047	0.0045	0.0052	0.0047	0.0043
	(0.6916)	(0.6456)	(0.7596)	(0.7418)	(0.6813)
$Q_{1,3}$	-0.0130	-0.0133	-0.0121	-0.0130	-0.0129
	(-3.4391)	(-3.4422)	(-3.2271)	(-3.6703)	(-3.7137)
$Q_{1,4}$	0.0067	0.0067	0.0066	0.0067	0.0067
	(12.7551)	(12.6258)	(12.7745)	(12.9879)	(13.0352)
$Q_{2,2}$	0.0720	0.0720	0.0719	0.0720	0.0720
	(18.4030)	(18.3867)	(18.4079)	(18.4316)	(18.4153)
$Q_{2,3}$	0.0043	0.0043	0.0046	0.0043	0.0043
	(1.1979)	(1.1817)	(1.2807)	(1.2157)	(1.2114)
$Q_{2,4}$	0.0002	0.0002	0.0002	0.0002	0.0002
	(0.5706)	(0.5392)	(0.5893)	(0.6262)	(0.6147)
$Q_{3,3}$	0.0408	0.0409	0.0407	0.0408	0.0408
	(18.5632)	(18.2123)	(17.8581)	(20.3205)	(20.6098)
$Q_{3,4}$	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005
	(-2.1037)	(-2.6366)	(-13.2811)	(-1.2126)	(-1.1494)
$Q_{4,4}$	0.0031	0.0030	0.0031	0.0031	0.0031
	(44.8955)	(54.2553)	(19.1934)	(21.2237)	(21.0309)
-LLF	-7208.3901	-7103.6437	-7100.0121	-6280.8784	-6198.7259
nb. obs.	1870	1870	1870	1870	1870

Note: Estimation results (t-stat) for model (3.11), where assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. Fixed parameter $\rho = (1+0.035)^{1/4} - 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations). Habit_{a} is for discretized habit process (3.10a); Habit_{b} is for discretized habit process (3.10b).

Table 10: Model selection LR tests: Financial and real estate wealth

	Alternative H_1:			
Null H_0:	HARA	WDR	Habit_a	Habit_b
CRRA	209.4927	216.7561	1855.0235	2025.3283
	[0]	[0]	[0]	[0]
HARA		7.2634	1645.5308	1815.8356
		[0.0070]	[0]	[0]
WDR			-10.6903*	-8.5213*
			[0]	[0]
Habit_a				−8.8755*
				[0]

Note: Likelihood Ratio tests [p-value] of null vs alternative hypotheses, where assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. $*$: modified LR test for non-nested hypotheses following (Vuong, 1989, Thm. 5.1, p. 318), test statistic asymptotically distributed as $N(0, 1)$. A significant positive (negative) test statistic indicates that the null (alternative) model performs better in reproducing the data. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 11: Derived series: Financial and real estate wealth

model	min.	max.	mean	std.	min.	max.	mean	std.
				Data				
	Consumption				Wealth			
	8002	23350	14486	4298	27904	74768	44105	13011
				A. Reference levels				
	A.1 Consumption X_t^i				A.2 Wealth $X_{w,t}^i$			
CRRA	0	0	0	0	0	0	0	0
HARA	27.351	27.351	27.351	0	6180.769	6180.769	6180.769	0
	(44.991)	(44.991)	(44.991)		(44.991)	(44.991)	(44.991)	
WDR	586.086	1790.892	1002.605	334.491	6168.011	6168.011	6168.011	0
	(5.528)	(5.421)	(5.462)	(5.372)	(60.381)	(60.381)	(60.381)	
Habit_a	2726.788	7861.978	4894.831	1452.647	6475.485	18670.362	11624.081	3449.698
	(7.901)	(19.699)	(19.683)	(19.650)	(5.722)	(7.655)	(7.652)	(7.650)
Habit_b	5309.126	15764.345	9800.859	2909.305	9259.465	27494.052	17093.340	5074.019
	(45.310)	(49.769)	(49.674)	(49.677)	(15.293)	(15.445)	(15.469)	(15.469)
				B. Surplus				
	B.1 Consumption $S_t^i = (C_t - X_t^i)/C_t$				B.2 Wealth $S_{w,t}^i = (W_t - X_{w,t}^i)/W_t$			
CRRA	1	1	1	0	1	1	1	0
HARA	0.997	0.999	0.998	0.001	0.778	0.917	0.848	0.040
	(13117.270)	(38365.135)	(21771.565)	(44.991)	(158.127)	(499.262)	(251.765)	(44.991)
WDR	0.913	0.946	0.931	0.007	0.779	0.918	0.849	0.040
	(56.785)	(96.956)	(73.831)	(5.383)	(212.781)	(671.557)	(338.711)	(60.381)
Habit_a	0.656	0.668	0.662	0.002	0.682	0.786	0.736	0.025
	(37.362)	(39.688)	(38.565)	(5.909)	(16.363)	(28.221)	(21.352)	(7.569)
Habit_b	0.312	0.336	0.323	0.004	0.537	0.683	0.612	0.036
	(22.437)	(22.978)	(23.750)	(16.086)	(17.851)	(33.426)	(24.408)	(15.473)
				C. Relative risk aversion				
	C.1 Consumption $RR_{c,t}^i = \gamma/S_{c,t}^i$				C.2 Wealth $RR_{w,t}^i = \gamma/S_{w,t}^i$			
CRRA	27.423	27.423	27.423	0	27.423	27.423	27.423	0
	(5.092)	(5.092)	(5.092)		(5.092)	(5.092)	(5.092)	
HARA	27.704	27.767	27.729	0.017	30.165	35.545	32.689	1.541
	(7.993)	(7.993)	(7.993)	(8.133)	(7.995)	(8.006)	(7.999)	(8.251)
WDR	27.187	28.176	27.627	0.202	28.033	33.019	30.372	1.429
	(28.017)	(24.768)	(26.542)	(4.435)	(32.936)	(34.470)	(33.601)	(216.233)
Habit_a	36.402	37.064	36.706	0.109	30.905	35.654	33.049	1.091
	(2.670)	(2.671)	(2.671)	(3.092)	(2.676)	(2.698)	(2.684)	(3.300)
Habit_b	69.219	74.641	72.016	0.840	34.104	43.409	38.179	2.196
	(2.533)	(2.534)	(2.532)	(2.644)	(2.526)	(2.543)	(2.532)	(2.700)

Note: Derived references, surplus, and risk aversion (t-stat) for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$. Assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. In-sample moments (t-statistics) evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 9. The t-statistic incorporates parametric uncertainty (see footnote 14 for details). Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Table 12: Estimation results: Financial wealth only and Time-varying expected excess returns

Model	CRRA	HARA	WDR	Habit$_\mathrm{a}$	Habit$_\mathrm{b}$
			A. Preference parameters		
γ	0.9614	0.9681	0.9955	-0.1661	-0.1526
	(22.7544)	(23.8309)	(22.4515)	(-67.7677)	(-94.0588)
η_0		60.3675	-5901.6036		
		(779.3747)	(-32.4198)		
η_w			0.6188		
			(33.4114)		
X_0^H				4349.5545	999.9913
				(1.0223)	(3.4914)
b				0.3581	1.4741
				(4643.4627)	(307.4177)
a				0.6135	1.7626
				(7993.1912)	(327.6955
			B. Drifts of risky returns process		
$A_{1,1}$	0.0233	0.0233	0.0233	0.0233	0.0233
	(39.2204)	(38.6565)	(35.5398)	(22.4317)	(29.6386)
$A_{1,2}$	-0.0039	-0.0039	-0.0039	-0.0039	-0.0039
	(-57.0216)	(-55.6992)	(-46.5008)	(-16.2464)	(-21.3599)
$A_{2,1}$	-0.0046	-0.0046	-0.0046	-0.0046	-0.0046
	(-1.1478)	(-1.1512)	(-1.1493)	(-1.3935)	(-1.3132)
$A_{2,2}$	0.0117	0.0117	0.0117	0.0117	0.0117
	(7.7965)	(7.6751)	(8.0265)	(12.1459)	(12.4686)
B_1	0.0018	0.0018	0.0018	0.0018	0.0018
	(34.4337)	(33.4611)	(26.9853)	(15.1823)	(25.1210)
B_2	0.0034	0.0034	0.0034	0.0034	0.0034
	(4.4095)	(4.3994)	(3.3678)	(4.4731)	(5.6516)
			C. Diffusions of risky process		
$Q_{1,1}$	0.0023	0.0023	0.0023	0.0023	0.0023
	(29.6736)	(29.2640)	(19.6231)	(20.1438)	(24.8512)
$Q_{1,2}$	0.0253	0.0253	0.0253	0.0253	0.0253
	(10.9827)	(10.8031)	(6.3708)	(34.7481)	(38.7523)
$Q_{2,2}$	0.0889	0.0889	0.0889	0.0889	0.0889
	(27.7571)	(26.8925)	(18.6372)	(29.6861)	(36.5844)
LLF	-8680.1705	-8185.2641	-8025.7534	-4376.3823	-4228.8432
nb. obs.	1190	1190	1190	1190	1190

Note: Estimation results (t-stat) for portfolio model (3.11), with returns following the arithmetic Brownian motion (4.1), with disscret-time equivalent (4.2). Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Fixed parameter $\rho = (1{+}0.035)^{1/4}{-}1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \mathrm{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process. Number of observation is number for full system. Estimation period is 1963:Q1–2005:Q3 (170 quarterly observations). Habit$_\mathrm{a}$ is for discretized habit process (3.10a); Habit$_\mathrm{b}$ is for discretized habit process (3.10b).

Table 13: Model selection LR tests: Financial wealth only and Time-varying expected excess returns

	Alternative H_1:			
Null H_0:	HARA	WDR	Habit_a	Habit_b
CRRA	989.8128	1308.8343	8607.5765	8902.6547
	[0]	[0]	[0]	[0]
HARA		319.0215	7617.7637	7912.8419
		[0.0406]	[0]	[0]
WDR			-1.2424^*	-2.0099^*
			[0.1071]	[0.0222]
Habit_a				-0.16804^*
				[0.4333]

Note: Likelihood Ratio tests [p-value] of null vs alternative hypotheses, where assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. $*$: modified LR test for non-nested hypotheses following (Vuong, 1989, Thm. 5.1, p. 318), test statistic asymptotically distributed as $N(0,1)$. A significant positive (negative) test statistic indicates that the null (alternative) model performs better in reproducing the data. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

C Figures

C.1 Data

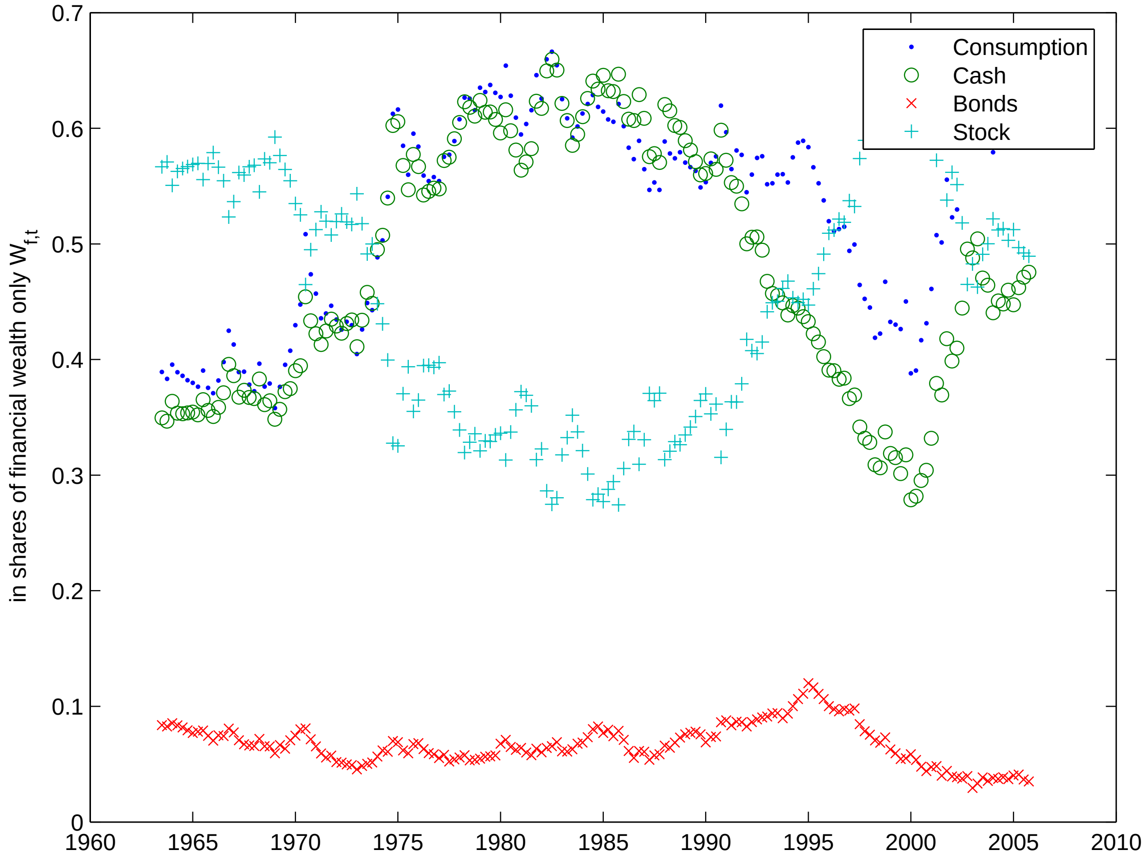


Figure 1: Consumption and Asset Shares of Financial Wealth Only

Note: Assets are cash (0), bonds (1), stocks (2), home (3) and mortgages (4). Financial wealth is cash + bonds + stocks. Financial and real estate wealth is financial wealth + real estate - mortgages. Shares are expressed in percentage of relevant wealth measure. Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

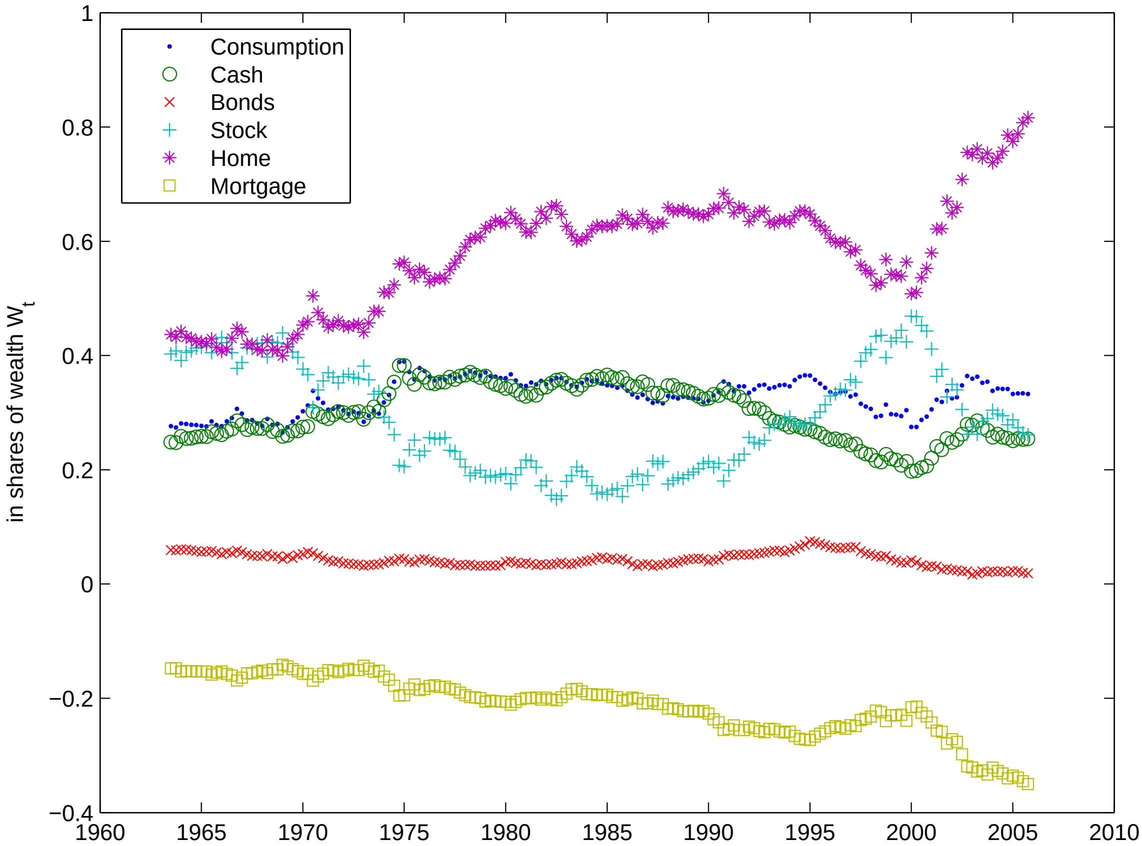
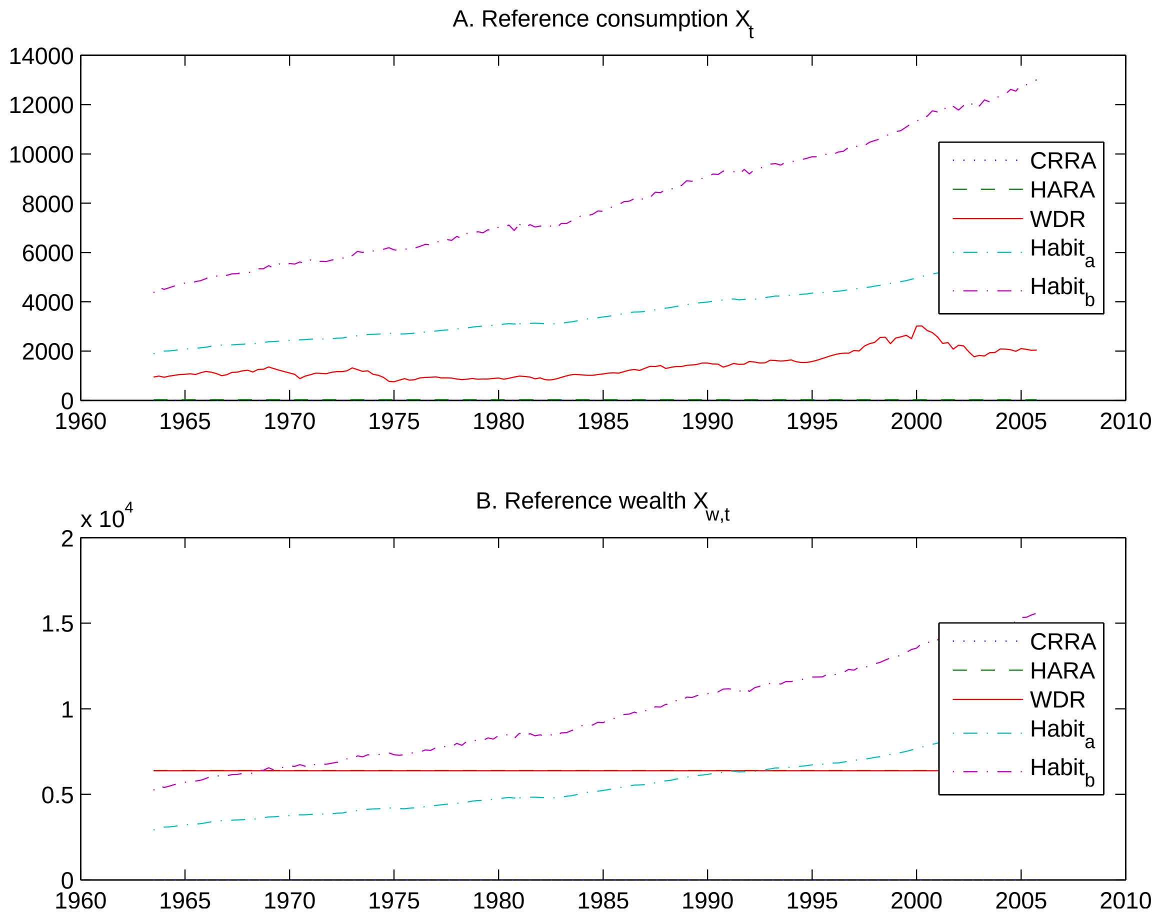


Figure 2: Consumption and Asset Shares of Financial + Real Estate Wealth

Note: Assets are cash (0), bonds (1), stocks (2), home (3) and mortgages (4). Financial wealth is cash + bonds + stocks. Financial and real estate wealth is financial wealth + real estate - mortgages. Shares are expressed in percentage of relevant wealth measure. Sample period is 1963:Q1–2005:Q3 (170 quarterly observations).

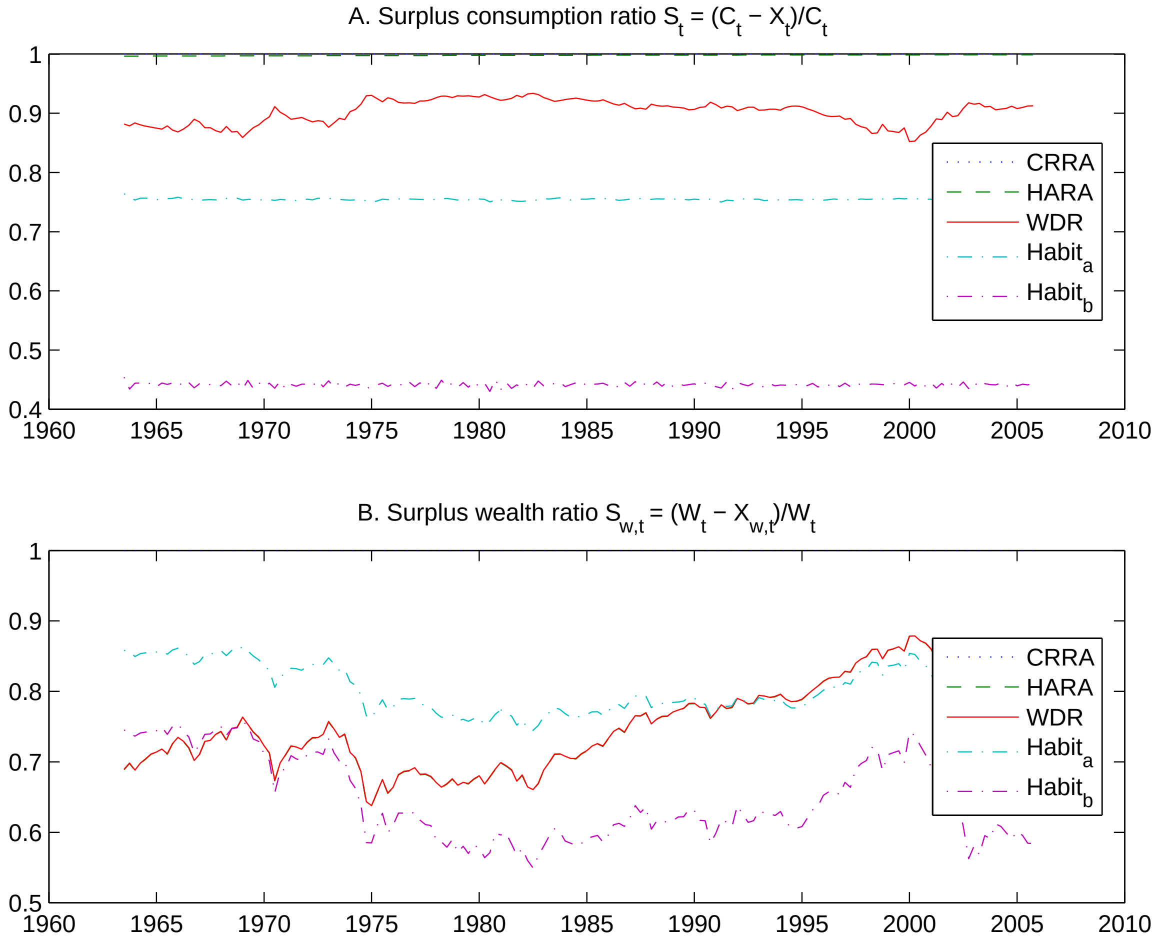
C.2 Derived series: Financial wealth only

Figure 3: Reference consumption and reference wealth levels: Financial wealth only



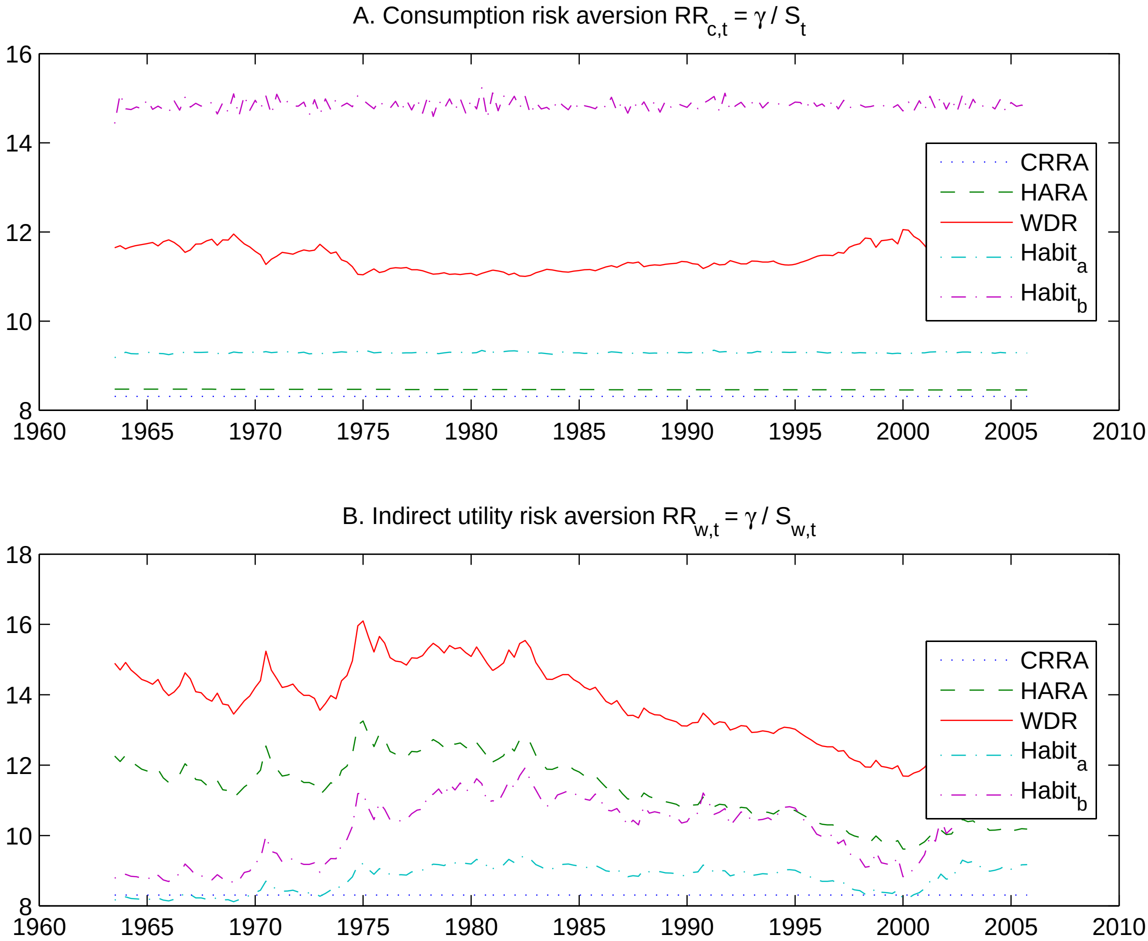
Note: Derived references X_t^i and $X_{w,t}^i$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 4. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

Figure 4: Surplus consumption and surplus wealth ratios: Financial wealth only



Note: Derived surpluses $S_t^i = 1 - X_t^i/C_t$ and $S_{w,t}^i = 1 - X_{w,t}^i/W_t$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 4. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).

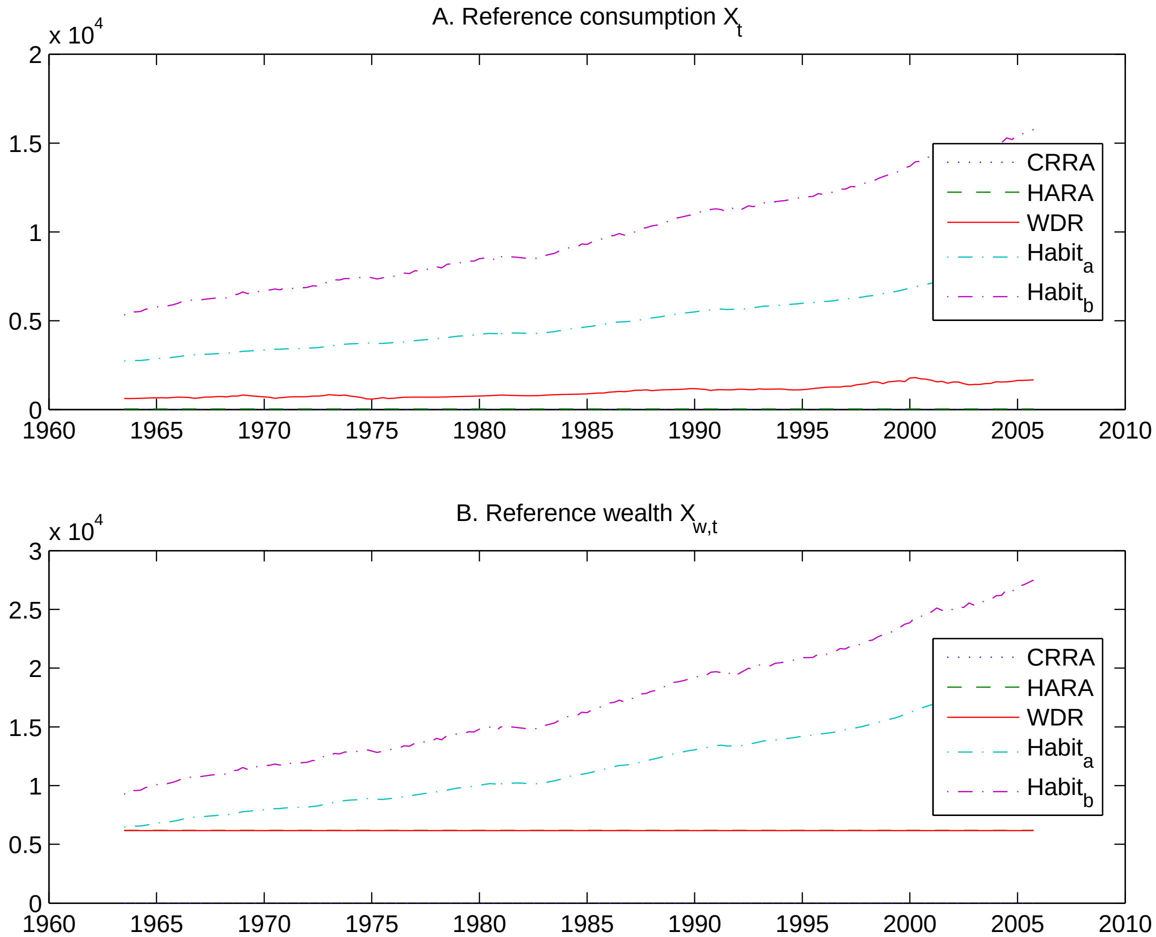
Figure 5: Risk aversion: Financial wealth only



Note: Derived risk aversion $RRA_t^i = \gamma/S_t^i$ and $RRA_{w,t}^i = \gamma/S_{w,t}^i$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 4. Habit$_a$ is for discretized habit process (3.10a); Habit$_b$ is for discretized habit process (3.10b).

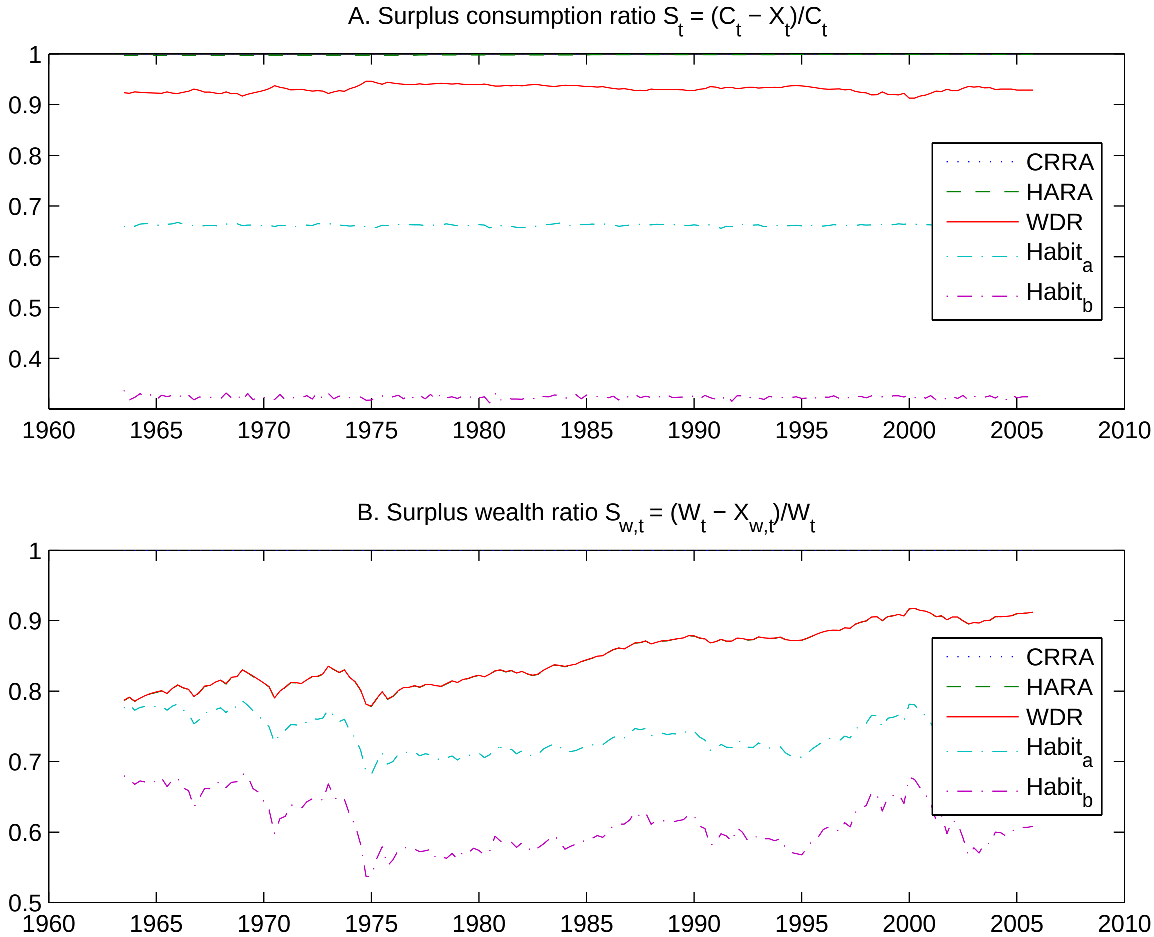
C.3 Derived series: Financial and real estate wealth

Figure 6: Reference consumption and reference wealth levels: Financial wealth only



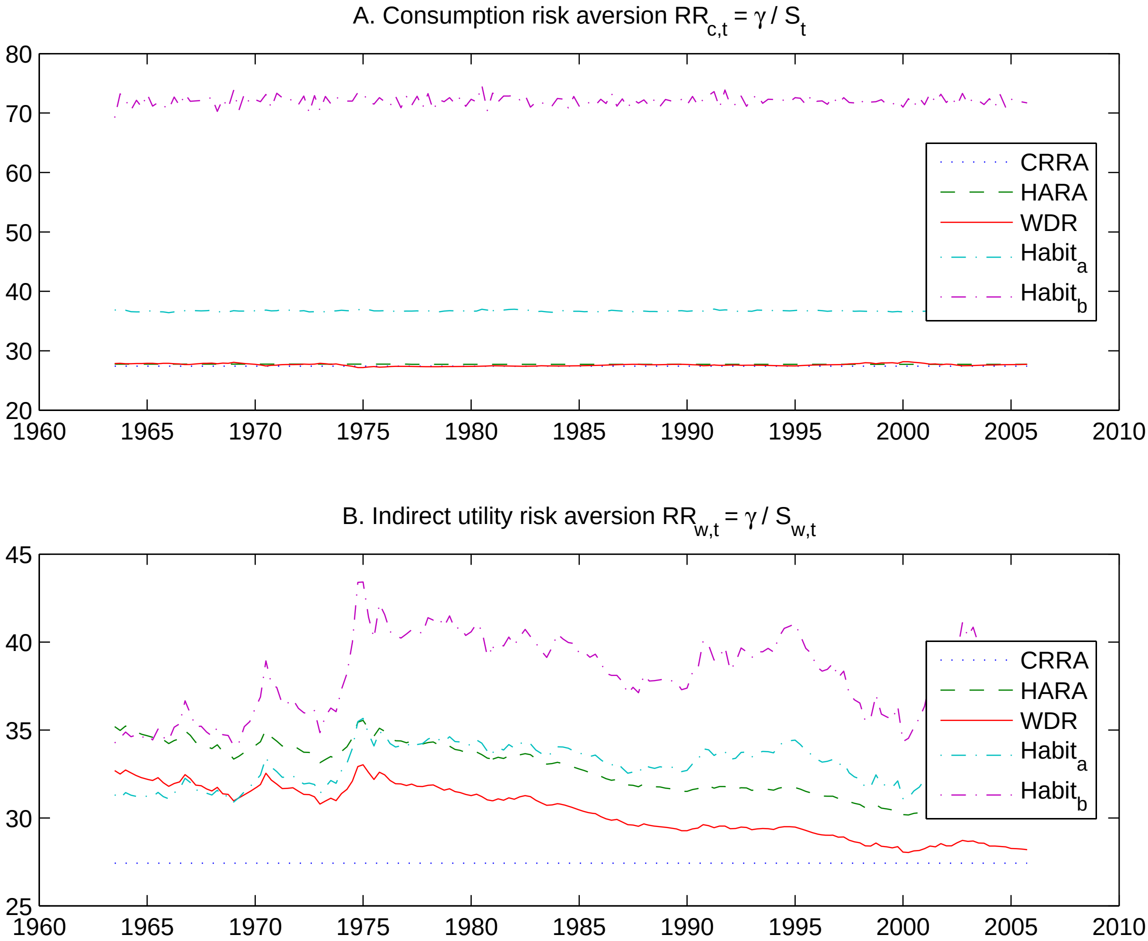
Note: Derived references X_t^i and $X_{w,t}^i$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial and real estate assets: $V_t = [CASH, BONDS, STOCK, HOME, MRTG]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 9. Habit$_a$ is for discretized habit process (3.10a); Habit$_b$ is for discretized habit process (3.10b).

Figure 7: Surplus consumption and surplus wealth ratios: Financial and real estate wealth



Note: Derived surpluses $S_t^i = 1 - X_t^i/C_t$ and $S_{w,t}^i = 1 - X_{w,t}^i/W_t$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 9. Habit$_\mathrm{a}$ is for discretized habit process (3.10a); Habit$_\mathrm{b}$ is for discretized habit process (3.10b).

Figure 8: Risk aversion: Financial and real estate wealth



Note: Derived risk aversion $RRA_t^i = \gamma / S_t^i$ and $RRA_{w,t}^i = \gamma / S_{w,t}^i$ for the within-period $U(C_t, X_t^i)$ and indirect $J(W_t^i)$ utility functions for reference models $i = W, H$, and restricted models CRRA, HARA. Assets are financial assets: $V_t = [CASH, BONDS, STOCK]$. Derived series evaluated at point estimates $\hat{\boldsymbol{\theta}}$ of parameters in Table 9. Habit_a is for discretized habit process (3.10a); Habit_b is for discretized habit process (3.10b).