Analyzing the effect of low interest rates on the surplus participation of life insurance policies with different annual interest rate guarantees

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Abstract

We analyze the effects of a prevailing low interest rates regime on investment decisions of insurance companies and on the risk/return profile of participating life insurance policies with different contractually guaranteed minimum annual return. Our analysis is based on German legislation and a stylized insurance company with two cohorts of insured persons having different minimal return guarantees. Our findings shed some light on the non-trivial interrelation between profit distribution, minimum guarantees, and resulting profitability for the different cohorts. Moreover, we investigate the complex role of the risk reserve that allows insurance companies to redistribute profits in time and, less obviously, also between the cohorts.

Owning a participating (or with-profit) life insurance contract provides one with a contractually guaranteed minimum annual rate of return. This minimal rate of return is regulated by law and settled when the contract is signed; it is furthermore fixed until the contract expires. Heuristically speaking, this corresponds to a Cliquet-type option[2] on the return of the managed funds of the insured

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[2] A lot of attention has been drawn to the pricing of (individual) life insurance contracts with this cliquet-type options, see, for example, Bryis and de Varenne [1997], Grosen and Jørgensen [2000], Grosen and Jørgensen [2002], Bacinello [2003], Kling and Ruß [2004], Bauer et al. [2005], Barbarin and Devolder [2005], Bohnert and Gatzert [2012], and Goecke [2013].

persons. The long maturity makes this option at the same time valuable and very difficult to hedge (see, e.g., Barbarin and Devolder [2005], Graf et al. [2011]).

For many years, the minimum return rate guarantee was far below that of risk free interest rates. Thus, the risk arising from a guaranteed return in those contracts seemed negligible. Moreover, in such a situation differences in return guarantees between customers have also been negligible given the way the returns have been distributed. The sovereign debt crisis has, however, ultimately amplified problems arising from this constellation, with hardly any risk-free investments remaining and the few remaining ones paying less interest than some of the existing minimum guarantees. Insurance companies are thus struggling to earn sufficient return on the managed funds – at least without increasing the riskiness of their investments. In contrast to equity- or unit-linked policies, participating life insurance contracts reduce the risk of their policyholders by time diversification and risk sharing between generations (see, e.g., Døskeland and Nordahl [2008]). This risk-shift is implemented via a risk reserve or buffer: In good years, some fraction of the surplus is added to this reserve, whereas in bad years the reserve is used to cover at least the guarantees of the policyholders. To achieve a risk-sharing between policyholders, it is necessary that one joint portfolio (in the following called reference portfolio) is used to manage the funds of all clients – irrespectively of their minimum guarantee.[3] From a mean-variance perspective this sounds inequitable, since the portfolio strategy (and thus risk) within the joint portfolio is the same, but the returns are not distributed pro rata. On the first view, this appears to be a disadvantage of insured persons with a comparably small minimum guarantee. However, a fair risk/return analysis is very difficult, since the distribution mechanism of returns from the reference portfolio is complex and not very transparent – a fact that "*undoubtedly explains why so few articles dealt with this issue*" (Grosen and Jørgensen [2002]). Within some regulatory (country-specific) bounds, an insurance company has many freedoms in designing a surplus participation scheme, i.e. via accounting rules or reserve policies. This surplus participation scheme has a significant influence on the risk and net present value of an individual life insurance policy (see, e.g., Kling and Ruß [2004], Bohnert and Gatzert [2012]).

While some authors examine the (inter-generational) risk-sharing between policyholders of participating life insurance contracts, the effect of the surplus distribution scheme to contracts with different minimum guarantees managed via the same reference portfolio has so far not been studied in a satisfying way. In this paper, we analyze the effect of the surplus distribution scheme on the risk/return profile of policyholders with different guarantees. We concentrate on the financial features of the contract, i.e. we do not cover pure actuarial risk elements, like mortality risk or any type of administrative costs. The setup we refer to corresponds to German legislation. However, similar results can

[3] In Germany this is regulated via the minimum funding ordinance (Mindestzuführungsverordnung), see `http://www.gesetze-im-internet.de/mindzv/index.html`.

be obtained in many comparable insurance markets (see also Kling and Ruß [2004]).

With our investigation we also take up a problem that has already been hinted at by Grosen and Jørgensen [2002][4]. The findings of our investigations are threefold: First of all, we show that – under certain model assumptions – it is optimal for an insurance company to increase the riskiness in asset allocation in a prevailing low-interest rates regime (see also Graf et al. [2011]). Second, in the setup of a stylized insurance company, we compare the risk/return profile of (separately-managed) unit-linked insurance contracts with different guarantees to risk/return profiles of (jointly-managed) participating life insurance contracts that imply risk-sharing between policyholders of different guarantees. Third, our investigation reveals that the implementation of a risk reserve – has – for cohorts with different minimum guarantees – a notifiable effect on the risk/return profile of life insurance contracts. In this view, it is not justified to analyze the risk/return profile of individual contracts without taking into account strategic decision of the insurance company and the interdependence of policies within an insurance portfolio. Thus, in particular the argument of an obviously unfair treatment from a mean-variance point of view loses (at least some of) its validity.

Many aspects of the actual mechanisms inside an insurance company are difficult – if not impossible – to model, since they involve management decisions that can hardly be translated into simple algorithms. Thus, we simplified reality to the core of the present problem. Hence, the obtained results should rather be interpreted on a qualitative level than on a strict quantitative one.

The paper is structured as follows: In Section 1, the financial model for the reference portfolio and a surplus participation scheme that is common in the German insurance industry are introduced. In Section 2, we analyze the historic evolution and resulting surplus participation scheme in a historic reference portfolio. This provides us with the intuition necessary for the estimation of the model parameters in a simulation study in Section 3. We analyze the risk/return profile of unit-linked and participating life insurance contracts and then examine the effect of different reserve policies in a setting with two insured populations with different guaranteed minimum rates of return. Finally, Section 4 concludes.

1 Financial model and surplus participation scheme

The insurance company: Our stylized insurance company has a portfolio of existing life insurance contracts of K cohorts with annually guaranteed minimum returns, denoted $g^{(i)}$, $i = 1, 2, \ldots, K$.

[4] Grosen and Jørgensen [2002] state that *"many insurers now face claims from distinct groups of liability holders distinguished by different guaranteed interest rates in their policies. This raises the problem of how to avoid inequitable treatment of different classes of policyholders within the same fund. Some companies [...] have tremendous concerns over the definition of a correct and fair distribution policy [...]"*.

The relative weights of cohort i (in terms of share of ownership of the total capital) change over time and are denoted by $w^{(i)}(t)$, where t denotes the respective point in time. For the application we have in mind it suffices to discretize time in full years ($\triangle t = 1$) and we index the time grid by $t_j,\ j \in \mathbb{N}_0$. In year t_j the insurance company thus has to pay out at least the guaranteed return $g(t_{j-1}) = \sum_{i=1}^{K} w^{(i)}(t_{j-1})\, g^{(i)}$ on the total portfolio.

The insurance company manages the investments of the policyholders in a joint reference portfolio that is kept apart from the other assets of the company. In year t_j this reference portfolio generates a return $m(t_j)$. This return is then distributed among the policyholders. In the literature, the return $m(t_j)$ is often modeled as a Brownian motion with drift (see, e.g., Grosen and Jørgensen [2000], Grosen and Jørgensen [2002], Bacinello [2003], Bauer et al. [2005], and Bohnert and Gatzert [2012]). This, however, implies that returns in consecutive years are independent, neglecting the effect of, for example, persistent low interest rate periods. For this reason, we prefer a Vasicek-model (see Vasicek [1977]) for the evolution of bond returns and interest rates (see also Barbarin and Devolder [2005], Graf et al. [2011]).

The capital market: We allow investments in three stylized types of assets: (a) a bond with return r that is considered (almost) risk free – serving as main investment vehicle, (b) a more volatile bond index with return b, and (c) a stock index whose return is denoted μ. The dynamics of the returns are modeled as

$$r_{t_j} = r_{t_{j-1}}\, e^{-\kappa_1 \triangle t} + \theta_1\big(1 - e^{-\kappa_1 \triangle t}\big) + \sigma_1 \sqrt{\frac{1 - e^{-2\kappa_1 \triangle t}}{2\kappa_1}}\, \epsilon_1\,, \tag{1}$$

$$b_{t_j} = b_{t_{j-1}}\, e^{-\kappa_2 \triangle t} + \theta_2\big(1 - e^{-\kappa_2 \triangle t}\big) + \sigma_2 \sqrt{\frac{1 - e^{-2\kappa_2 \triangle t}}{2\kappa_2}}\, \epsilon_2\,, \tag{2}$$

$$\mu_{t_j} = \theta_3 \triangle t + \sigma_3\, \epsilon_3\,, \tag{3}$$

where κ_1, κ_2, σ_1, σ_2, and σ_3 are positive constants, θ_1, θ_2, $\theta_3 \in \mathbb{R}$, and ϵ_1, ϵ_2, ϵ_3 are standard normal innovations with correlation matrix Σ. The returns described by (1) and (2) fluctuate around the long term means θ_1 and θ_2. The mean-reversion speed parameters κ_1 and κ_2 allow to steer the dependence between returns of consecutive years and thus increase the likelihood of persistent low interest rate periods.

There are – of course – more sophisticated financial market models available in the literature. For our purposes, however, the described model is sufficient: The generation of the return $m(t_j)$ of the reference portfolio[5] is a much more complex procedure than simply an investment in the financial

[5] In practice, insurance companies earn additional money by the difference between true and best-estimate actuarial

markets – extreme movements can, for example, be reduced by exploiting accounting rules.

Within this (simplified) capital market, the insurance company now has to decide on the most efficient investment decision to fulfill the interest rate guarantees.

Optimal investment strategy: We denote the investment strategy of the fund manager at time t_{j-1} by $\boldsymbol{\pi}_{t_{j-1}} = (\pi^{(1)}_{t_{j-1}}, \pi^{(2)}_{t_{j-1}}, \pi^{(3)}_{t_{j-1}})$, these must be read as weights (summing to one) of the three investment possibilities. In reality, it is realistic to assume that short selling is prohibited and especially the risky investments are subject to certain upper bounds $0 \leq \pi^{(i)}_{t_{j-1}} \leq b_i$. Fixing the investment decision over $[t_{j-1}, t_j]$ yields the return $m(t_j) := \pi^{(1)}_{t_{j-1}} r_{t_j} + \pi^{(2)}_{t_{j-1}} b_{t_j} + \pi^{(3)}_{t_{j-1}} \mu_{t_j}$. Recall that the contractual obligation on the average minimum return to be paid out is denoted as $g(t_{j-1})$. In the literature, two common risk measures are used as an objective function for an insurance company: The shortfall probability, i.e. the probability that the returns on the reference portfolio are not sufficient to cover the guarantees $g(t_{j-1})$ in a one-year period and, alternatively, the expected shortfall, i.e. the expected equity capital needed to fulfill the contractual obligations due to insufficient returns in year t_j. If shortfall risk is used, the optimal investment decision is in some cases a 100 % stock investment which obviously sets "*wrong incentives*" (see Graf et al. [2011]) and does not correspond to current insurance practice with stock investments of about 5 %. That is why, we decided to use the minimization of the expected shortfall as objective function for our investment problem. The corresponding optimization problem has the following form:

$$\begin{cases} \min_{\boldsymbol{\pi}_{t_{j-1}}} \mathbb{E}\left[\left(g(t_{j-1}) - m(t_j)\right) \mathbb{1}_{\{m(t_j) \leq g(t_{j-1})\}}\right] \\ 0 \leq \pi^{(i)}_{t_{j-1}} \leq b_i \\ \pi^{(1)}_{t_{j-1}} + \pi^{(2)}_{t_{j-1}} + \pi^{(3)}_{t_{j-1}} = 1 \end{cases} \tag{4}$$

Having specified the financial market model and the optimal investment problem, we now have a model for the yearly return $m(t_j)$ of the reference portfolio. The remaining question is how this return is distributed to the different insurance policies.

Distribution of returns: First of all, we assume that the fraction $\delta \leq 10\,\%$ is considered corporate profit.[6] The remaining share $1-\delta$ of $m(t_j)$ is distributed among the policy holders – but not necessarily immediately: If the returns are sufficient to cover all guarantees, a part $d(t_j) \leq (1-\delta)m(t_j)$ is distributed now, the remainder is stored in the buffer or risk reserve B. It is a management decision

assumptions, i.e. in their assumptions on administrative costs or mortality. This income is, however, rather stable over time and is thus – for simplicity – neglected in this analysis. Instead, we concentrate on the financial risks.

[6] In Germany, at least 90 % of returns have to be distributed to the policyholders (compare the minimum funding ordinance (Mindestzuführungsverordnung)).

of the insurance company to tactically set $d(t_j)$, at least up to some legal bounds (for more details, see Section 3.4, Grosen and Jørgensen [2000], Grosen and Jørgensen [2002], Bauer et al. [2005], and Graf et al. [2011]). An important effect of the risk reserve is that it allows to smoothen unequal returns over time.

The returns are now distributed among the policyholders with different minimum guarantee. Here, the distribution procedure is assumed to follow German legislation as in Kling and Ruß [2004]:

(1) The return on the risk reserve is identical to the return in the asset portfolio, i.e. $m(t_j)$.

(2) If $\max_{i\in\mathbb{N}} g^{(i)} \leq d(t_j)$, then all insurance holders receive the same return $d(t_j)$ on their accumulated capital.

(3) In case $g(t_j) \leq d(t_j) < \max_{i\in\mathbb{N}} g^{(i)}$, all contractual obligations are settled, i.e. all minimum guarantees are fulfilled. The remaining returns are then used to lift contracts with lower relative surplus distributions on the highest positive level. Of course, the structure implied by the different guarantees is then – at least partly – preserved as no customer with a lower guarantee gets strictly more than one with a higher guarantee.

(4) In the critical case $d(t_j) < g(t_j)$, all contractual obligations must still be fulfilled, even though the return of this year is not sufficient to do so. To make up the difference, the risk reserve $B(t)$ is used and if this buffer is still not sufficient, the insurance company has to provide the difference. Table 1 schematically shows how the reference portfolio is subdivided into policy reserves $P_i(t)$, $i = 1, 2, \ldots, K$ that belong to the policyholders and a common risk reserve $B(t)$.

<table>
<tr><td rowspan="2">reference portfolio $A(t)$</td><td>policy reserve $P_1(t)$
policy reserve $P_2(t)$
...
policy reserve $P_K(t)$</td></tr>
<tr><td>risk reserve $B(t)$</td></tr>
</table>

Table 1: Reference portfolio of a fictitious insurance company consisting of K cohorts with annually guaranteed minimum return.

At this point we observe that contracts with different annual minimal guarantees can not be considered independently: in both scenarios (3) and (4) it is the case that the returns from the joint portfolio are not distributed pro rata. One might say that policyholders with a small guarantee subsidize the ones with higher guarantee (see also Kling and Ruß [2004]). Apart from that there is a second – less obvious – reason why the contracts interact: the optimal investment decision depends on the average minimum guarantee $g(t_{j-1})$ that needs to be earned in year t_j – a fact that also leads to some interdependence between different policies. This is made precise in the subsequent paragraph.

2 Reconsidering the recent past

	10y German gov. bond	REX index	DAX index
mean realized returns	4.25 %	5.41 %	11.0 %
emp. standard deviations	0.010	0.052	0.218

Table 2: Realized yearly mean returns and standard deviations for the asset classes German government bonds (10 years), the REX index, and the DAX index (period 1994–2014).

Our journey through the recent past starts in 1994 and ends 2014. The three assets in our financial model are represented by 10-year German government bonds to reflect an almost risk-free investment with return r, the German REX index as a slightly more risky alternative with return b, and the German stock index DAX to represent a typical stock investment with return μ.[7] The first two moments of the empirical yearly returns for the period 1994 until 2014 are given in Table 2. In the right-hand columns of Table 3 we have computed returns of three portfolios that differ in the weighting of stocks, for which we assumed 10 %, 5 %, and 0 %. In a few cases, the resulting portfolio return falls below 3.5 % – which is the annual minimal guarantee for contracts settled 01.01.1994. This happened especially during the years 2001, 2002, and 2008 (caused by decreasing stock prices) and 2011–2013 (the current low interest rate period). Besides the individual moments we have computed the empirical correlation matrix of the returns and found

$$\hat{\Sigma} = \begin{pmatrix} 1.00 & 0.34 & -0.38 \\ 0.34 & 1.00 & -0.35 \\ -0.38 & -0.35 & 1.00 \end{pmatrix} . \tag{5}$$

2.1 A stylized insurance portfolio with two contracts

We construct a fictitious portfolio consisting of only two policies: with-profit insurance of a 50-year old female (born 1964) and a 30-year old female (born 1984). These contracts are chosen to represent

[7] Historic daily data was obtained from Reuters (ticker names: DE10YT=RR, GREXP, and GDAXI).

Year y	10y govern– ment bonds r	REX index b	DAX index μ	portfolio return with weights $\boldsymbol{\pi} = \dots$ $(0.65, 0.25, 0.10)$	$(0.70, 0.25, 0.05)$	$(0.75, 0.25, 0.00)$
1994	6.72 %	-2.51 %	-7.17 %	**3.02 %**	3.72 %	4.41 %
1995	6.89 %	16.69 %	22.20 %	10.87 %	10.11 %	9.34 %
1996	6.22 %	7.54 %	22.88 %	8.22 %	7.38 %	6.55 %
1997	5.68 %	6.56 %	46.37 %	9.97 %	7.94 %	5.90 %
1998	4.64 %	11.24 %	16.15 %	7.44 %	6.86 %	6.29 %
1999	4.43 %	-1.94 %	32.47 %	5.64 %	4.24 %	**2.84 %**
2000	5.29 %	6.86 %	-0.59 %	5.09 %	5.38 %	5.68 %
2001	4.81 %	5.62 %	-24.83 %	**2.05 %**	3.53 %	5.02 %
2002	4.83 %	9.02 %	-46.20 %	**0.77 %**	**3.32 %**	5.87 %
2003	4.07 %	4.09 %	47.70 %	8.44 %	6.26 %	4.08 %
2004	4.12 %	6.70 %	4.84 %	4.84 %	4.80 %	4.76 %
2005	3.41 %	4.08 %	33.36 %	6.57 %	5.07 %	3.58 %
2006	3.74 %	0.27 %	19.65 %	4.46 %	3.67 %	2.87 %
2007	4.20 %	2.51 %	0.92 %	**3.45 %**	3.61 %	3.78 %
2008	4.10 %	10.15 %	-36.68 %	**1.53 %**	3.57 %	5.61 %
2009	3.22 %	4.92 %	29.28 %	6.25 %	4.95 %	3.65 %
2010	2.77 %	4.01 %	26.19 %	5.42 %	4.25 %	**3.08 %**
2011	2.74 %	8.29 %	-8.74 %	**2.98 %**	3.56 %	4.13 %
2012	1.56 %	4.64 %	20.39 %	4.21 %	**3.27 %**	**2.33 %**
2013	1.57 %	-0.49 %	20.88 %	**2.99 %**	**2.02 %**	**1.06 %**

Table 3: Yearly returns of the three asset classes (left) and resulting portfolio returns given the constant strategy $\boldsymbol{\pi} = (\pi^{(1)}, \pi^{(2)}, \pi^{(3)})$ (right) for three sets of weights. Whenever the portfolio return fails to exceed the guarantee of an insurance contract settled at 01.01.1994, that is 3.5 %, the respective portfolio returns are emphasized in bold.

cohorts of insured persons with strongly different minimum return guarantee. Both invest at their 30th birthday the lump sum of 10 000 € in a single premium contract. In case of death,[8] the accumulated value of this insurance is paid out. The minimum guarantees, when the two contracts have been settled, were $g^{(1)} = 3.5\,\%$ (for 01.01.1994) and $g^{(2)} = 1.75\,\%$ (for 01.01.2014). The resulting weighted average $g(t_{j-1}) = \sum_{i=1}^{2} w^{(i)}(t_{j-1})\, g^{(i)}$ is the relevant return for the fund manager to be earned in year $[t_{j-1}, t_j]$. For the moment, we assume that the returns are completely distributed to the two contracts, i.e. $d(t_j) = m(t_j)$ and $B(t) \equiv 0$.

year y	portfolio return using $\boldsymbol{\pi} = (0.70, 0.25, 0.05)$	mort. rate ${}_1q_y$	$P_1(t)$ (no mortality)	$P_1(t)$ (with mortality)
			10 000.00 €	10 000.00 €
1994	3.72 %	0.090 %	10 350.00 €	10 340.65 €
1995	10.11 %	0.093 %	11 291.58 €	11 270.90 €
1996	7.38 %	0.104 %	12 041.83 €	12 007.31 €
1997	7.94 %	0.106 %	12 902.10 €	12 851.47 €
1998	6.86 %	0.116 %	13 699.19 €	13 629.67 €
1999	4.24 %	0.130 %	14 222.20 €	14 131.69 €
2000	5.38 %	0.130 %	14 911.41 €	14 797.21 €
2001	3.53 %	0.151 %	15 433.31 €	15 291.93 €
2002	**3.32 %**	0.165 %	15 973.47 €	15 801.10 €
2003	6.26 %	0.174 %	16 873.29 €	16 662.14 €
2004	4.80 %	0.195 %	17 602.29 €	17 348.18 €
2005	5.07 %	0.215 %	18 406.14 €	18 101.44 €
2006	3.67 %	0.231 %	19 050.35 €	18 691.80 €
2007	3.61 %	0.266 %	19 717.11 €	19 294.63 €
2008	3.57 %	0.255 %	20 407.21 €	19 918.99 €
2009	4.95 %	0.297 %	21 316.07 €	20 744.27 €
2010	4.25 %	0.318 %	22 131.56 €	21 469.35 €
2011	3.56 %	0.347 %	22 906.16 €	22 143.71 €
2012	**3.27 %**	0.364 %	23 707.88 €	22 835.42 €
2013	**2.02 %**	0.404 %	24 537.66 €	23 539.08 €

Table 4: Portfolio return if the share $\boldsymbol{\pi} = (0.70, 0.25, 0.05)$ is invested into the three assets (left, see also Table 3). Mortality rates ${}_1q_y$ of females born 1964 (second column). The right part of the table presents the evolution of the policy reserve with and without mortality risk. All presented numbers refer to the end of the given year. The initial lump sum on 01.01.1994 is 10 000 €, the contractual guaranteed interest rate is 3.5 %, the insurance company receives a share of $\delta = 10\,\%$ from the realized returns on the reference portfolio.

2.2 Portfolio returns for the accumulation period 1994–2014

For the today 50-year old female, we compute the returns and accumulated capital of a portfolio with weights $\boldsymbol{\pi} = (0.70, 0.25, 0.05)$ and report them in Table 4 for the period from 1994 until 2014. This accumulated portfolio value is computed twice: with and without considering mortality. We observe that in 2002, 2012, and 2013 the returns are insufficient to cover the minimum guarantee of 3.5 %. This deficit must be closed by the accumulated risk reserve, or – if the risk reserve is insufficient – by equity capital of the insurance company. Important for the subsequent analysis is a realistic

[8] Mortality tables are, for example, available from the German government agency *Statistisches Bundesamt*, see `https://www.destatis.de`.

assumption on today's value of the contract that was settled 20 years ago, since this influences the relative weights when a second contract is added.

3 Modeling the future

The future development is analyzed for the period from 2014 until 2043. On 01.01.2014, a second female invests a lump sum of $P_1(t_0) = 10\,000$ € in a with-profit insurance single premium policy, she enjoys a minimal guarantee of $g^{(2)} = 1.75\,\%$. In the mean time, the portfolio value of the first contract has accumulated to 23 539.08 €, see Table 4. The weighted portfolio guarantee thus decreases from $g^{(1)} = 3.5\,\%$ to $g(t_0) = \sum_{i=1}^{2} w^{(i)}\, g^{(i)} = 2.98\,\%$. We now use our stochastic model (1), (2), and (3) of the financial market to simulate the development of the three assets from $t_0 := 2014$ until $t_{30} := 2044$. The base case set of parameters is chosen as

$$\kappa_1 = \kappa_2 = 1, r_0 = 1.5\,\%, \theta_1 = 3.0\,\%, \sigma_1 = 0.005, b_0 = 3.0\,\%, \theta_2 = 4.0\,\%, \sigma_2 = 0.075, \theta_3 = 7.5\%, \sigma_3 = 0.250,$$

and the correlation matrix is the historical one, i.e. $\hat{\Sigma}$ from (5).

In contrast to the available literature, we want to take into account that the insurance company changes its investment decision (i.e. increases its riskiness) if (risk-less) interest rates are not sufficient to cover the guarantees. Therefore, we determine at the beginning of each year optimal portfolio weights according to optimization problem (4). Using our base case parameter set, Figure 1 presents optimal portfolio weights depending on the average portfolio guarantee $g(t_j)$. We observe that the latter significantly influences the investment decision. If the difference between interest rates observed on the market and interest guarantees decreases, the insurance company increases the riskiness of its investment decision.

To quantify risk and return of the insurance policy, we simulate the returns of our reference portfolio in the 30–year period from 2014 until 2043. In each simulation run, we determine the internal rate of return of the different insurance policies. This internal rate of return $R_I^{(i)}$ of cohort i, $i = 1, 2, \dots, K$, is defined as the geometric mean of the returns of the reference portfolio – adjusted for mortality, i.e.

$$R_I^{(i)} := \sqrt[30]{\frac{P_i(t_{30})}{P_i(t_0) \cdot \prod_{j=2014}^{2043} (1 - {}_1q_j)}}\,.$$

From all simulation runs, we can then determine an empirical distribution of this internal rate of return. Due to our model assumptions, this distribution is approximately normal. From this, we

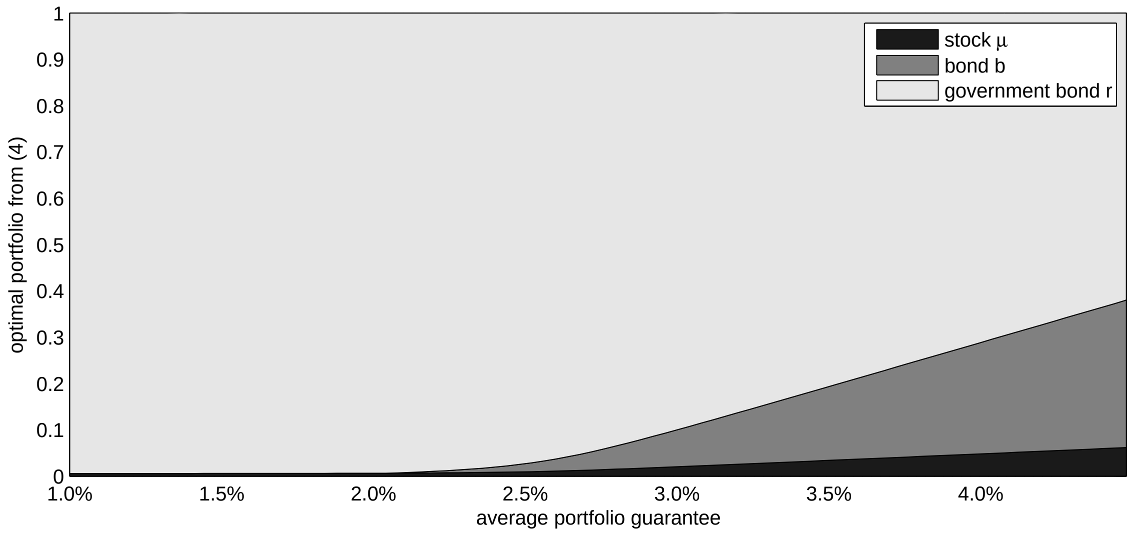

Figure 1: Optimal portfolio weights $\boldsymbol{\pi} = (\pi^{(1)}, \pi^{(2)}, \pi^{(3)})$ resulting from optimization problem (4); depending on the average guarantee. We use our base case parameter set with slowly rising interest rates.

quantify our risk-return profile via the empirical mean of the internal rate of return $\mu_I^{(i)}$ and its standard deviation $\sigma_I^{(i)}$. We now examine the effect of different interest rate regimes (see Section 3.1) and contract type (unit-linked vs. participating life insurance contract, see Section 3.2) on the risk/return profile.

3.1 Analysis of different interest rate scenarios

A crucial input for our simulation is the long term mean of the returns of the two bond investments. These might be interpreted as some level of interest rates to which the current (low interest rates) have a tendency to return to. We consider the following scenarios:

- **Scenario 1**: Persistent period of low interest rates ($\theta_1 = 1.5\,\%$, $\theta_2 = 3.0\,\%$).
- **(Base case) Scenario 2**: Moderate rise of interest rates ($\theta_1 = 3.0\,\%$, $\theta_2 = 4.0\,\%$).
- **Scenario 3**: Gradual return to high interest rates ($\theta_1 = 4.0\,\%$, $\theta_2 = 5.0\,\%$).

3.2 Unit-linked vs. participating life insurance contracts

In our analysis we want to distinguish unit-linked insurance contracts (separately managed policies that do not shift risks between policyholders) and participating life insurance contracts (policies managed in a joint reference portfolio and sharing risk by a joint risk reserve). We consider the following three portfolios:

- **Portfolio 1**: Participating life insurance portfolio of two contracts initiated on 01.01.1994 and 01.01.2014.
- **Portfolio 2**: Unit-linked life insurance contract initiated on 01.01.1994.
- **Portfolio 3**: Unit-linked life insurance contract initiated on 01.01.2014.

This comparison allows us to examine how (strong) the different insurance policies interact with each other. Their interdependence results from both the surplus distribution scheme and the different investment decisions depending on the average guaranteed interest in the insurance portfolio. Figure 2 presents the average investment decision in Scenario 1 depending on the average annually guaranteed interest rate $g(t_j)$ in the portfolio.

3.3 Default of the insurance company

In the literature, default of an insurance company is triggered as soon as its risk reserves or equity fall below a certain threshold (see, for example, Grosen and Jørgensen [2000], Barbarin and Devolder [2005], Graf et al. [2011], Barbarin and Devolder [2005], Bohnert and Gatzert [2012]). In practice, the event of a default of an insurance company, however, can be avoided by recapitalization and political or regulatory interventions. Tax-payers money might be used to safe the insurance company in order to keep the pension benefits above a certain level; regulators can decide to cut the level of interest rate guarantees. That is why, we decided against such a default trigger. Instead, we keep track of a fictitious equity account that increases each year by the amount the equity holders have to provide in order to fulfill the interest rate guarantees. This equity account increases by income sources that are neglected in the current study (i.e. by the difference between true and best-estimate actuarial assumptions for administrative costs or mortality) and by the share $\delta \leq 10\%$ of returns that is immediately considered corporate profit. To get some intuition on the likelihood of a default of the insurance company, we monitor the average ratio κ of the fictitious equity account at t_{30} and the total policy reserves $P(t_{30})$, i.e.

$$\kappa := \frac{\mathbb{E}\big[E(t_{30})\big]}{\mathbb{E}\big[P(t_{30})\big]} . \tag{6}$$

In our simulation study, the investment decisions in Scenario 1 now lead to a distribution of the internal rate of return in the period 2014 until 2043. Figure 3 displays the distribution of the internal rate of return in the case of a jointly managed participating life insurance portfolio (above) and of

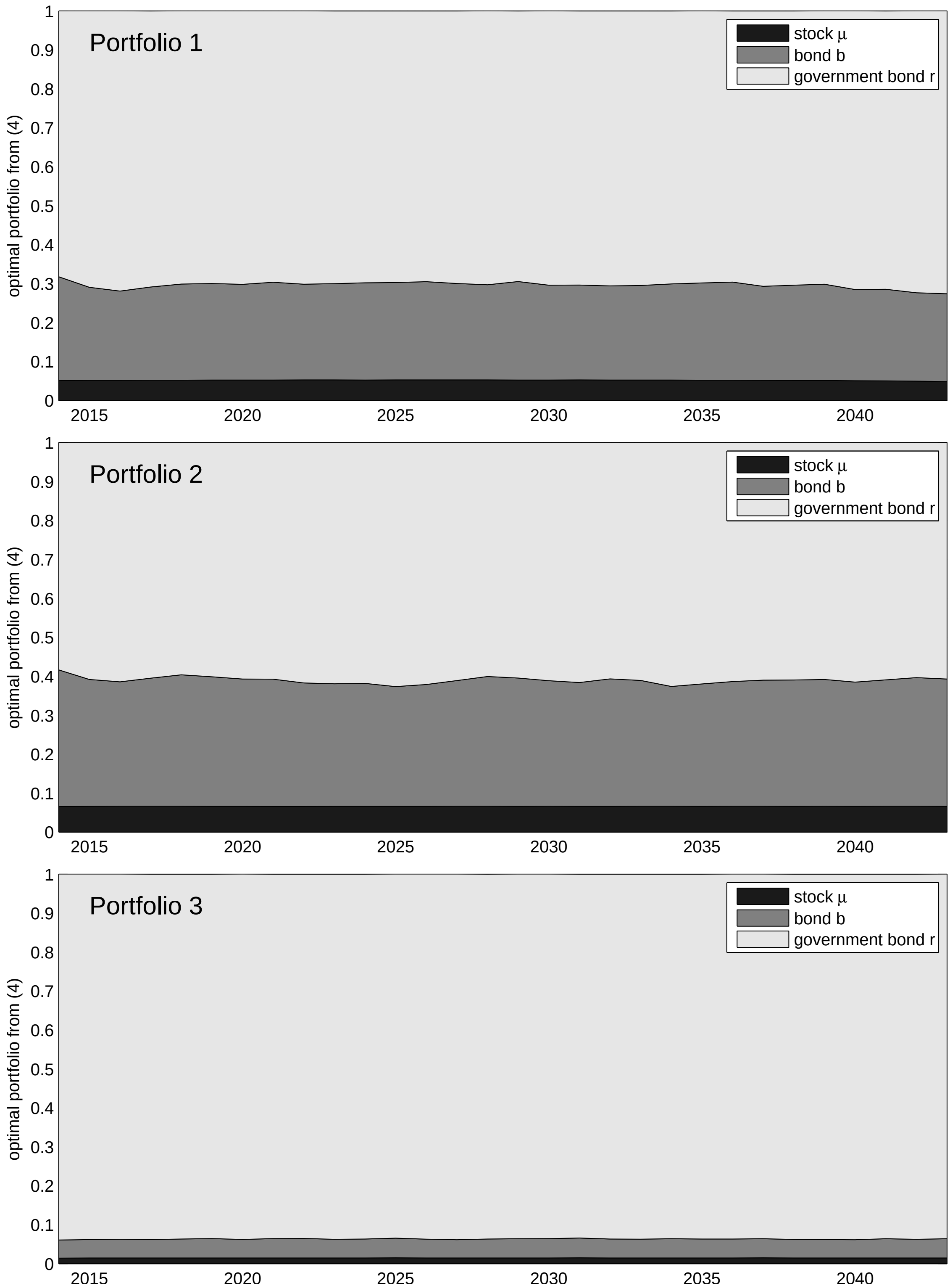

Figure 2: Scenario 1: Average investment decision following the optimization problem (4) in a participating life insurance portfolio (above) of both contracts. This is compared to (separately managed) unit-linked life insurance contracts with a guaranteed interest of 3.5 % (middle, contract initiation 01.01.1994), respectively 1.75 % (below, contract initiation 01.01.2014).

(separately managed) unit-linked life insurance contracts (below). One observes that – surprisingly – the contract with the lower guarantee of 1.75 % profits from the jointly managed insurance portfolio – at least in terms of the average rate of return. In a separately managed portfolio, however, this contract would profit from a significantly lower investment risk.

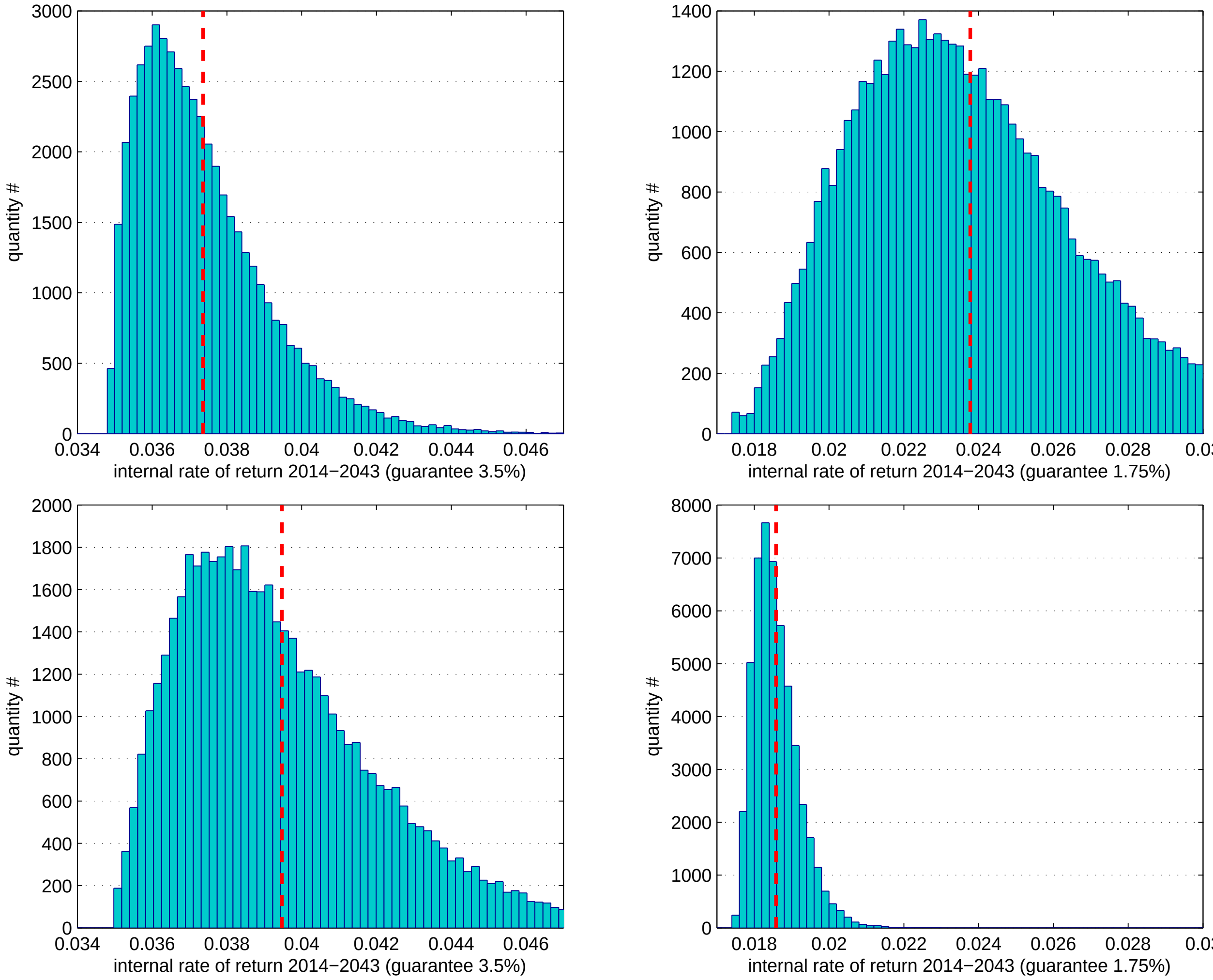

Figure 3: Scenario 1: Distribution of the internal rate of return $R_I^{(i)}$ for contracts with an annually guaranteed interest of 3.5 % (left) and 1.75 % (right) in the period 2014 until 2043. Both policies are managed in a joint reference portfolio (above), respectively in separate portfolios (below).

If we approximately assume a normal distribution of the internal rate of return, we can compress the distributions in Figure 3 to the risk/return profile $(\mu_I^{(i)}, \sigma_I^{(i)})$ in the three portfolios. Table 5 presents the results and compares them to the fictitious portfolio that also invests according to the optimization rules (4) but does not guarantee any minimum interest rate. In Scenario 1, the insurance company has to add equity in order to ensure its interest rate guarantees.

Scenario 1	$(\mu_I^{(1)}, \sigma_I^{(1)})$	$(\mu_I^{(2)}, \sigma_I^{(2)})$	no guarantee	κ
Portfolio 1 (joint)	$(3.74\,\%, 0.17 \cdot 10^{-2})$	$(2.38\,\%, 0.31 \cdot 10^{-2})$	$(2.21\,\%, 0.39 \cdot 10^{-2})$	29.9 %
Portfolio 2 (only 3.5 %)	$(3.95\,\%, 0.28 \cdot 10^{-2})$		$(2.47\,\%, 0.53 \cdot 10^{-2})$	42.1 %
Portfolio 3 (only 1.75 %)		$(1.86\,\%, 0.06 \cdot 10^{-2})$	$(1.56\,\%, 0.12 \cdot 10^{-2})$	3.6 %

Scenario 2	$(\mu_I^{(1)}, \sigma_I^{(1)})$	$(\mu_I^{(2)}, \sigma_I^{(2)})$	no guarantee	κ
Portfolio 1 (joint)	$(3.51\,\%, 0.01 \cdot 10^{-2})$	$(1.94\,\%, 0.11 \cdot 10^{-2})$	$(2.78\,\%, 0.09 \cdot 10^{-2})$	3.4 %
Portfolio 2 (only 3.5 %)	$(3.56\,\%, 0.05 \cdot 10^{-2})$		$(2.91\,\%, 0.15 \cdot 10^{-2})$	0.0 %
Portfolio 3 (only 1.75 %)		$(2.70\,\%, 0.08 \cdot 10^{-2})$	$(2.70\,\%, 0.08 \cdot 10^{-2})$	0.0 %

Scenario 3	$(\mu_I^{(1)}, \sigma_I^{(1)})$	$(\mu_I^{(2)}, \sigma_I^{(2)})$	no guarantee	κ
Portfolio 1 (joint)	$(3.67\,\%, 0.05 \cdot 10^{-2})$	$(3.37\,\%, 0.14 \cdot 10^{-2})$	$(3.58\,\%, 0.08 \cdot 10^{-2})$	0.0 %
Portfolio 2 (only 3.5 %)	$(3.68\,\%, 0.06 \cdot 10^{-2})$		$(3.60\,\%, 0.08 \cdot 10^{-2})$	0.0 %
Portfolio 3 (only 1.75 %)		$(3.58\,\%, 0.09 \cdot 10^{-2})$	$(3.58\,\%, 0.09 \cdot 10^{-2})$	0.0 %

Table 5: Average internal rate of return $\mu_I^{(i)}$ and its standard deviation $\sigma_I^{(i)}$ in Scenarios 1, 2, and 3. We distinguish again contracts with an annual guaranteed interest of 3.5 % ($i = 1$) and 1.75 % ($i = 2$) in the period 2014 until 2043. The column "no guarantee" compares the results to the fictitious portfolio that also invests according to the optimization rules (4) but does not guarantee any minimum interest rate. The right column presents the ratio κ that serves as an indicator for the likelihood of default of the insurance company (see Section 3.3 for details).

The risk/return profile can similarly be estimated in Scenarios 2 and 3. Table 5 presents the results in the shape of the risk/return profile $(\mu_I^{(i)}, \sigma_I^{(i)})$. In Scenario 2 the 3.5 %-contract is – in a jointly managed portfolio – still subsidized by the 1.75 %-contract. Additional equity from the insurance company to cover the guarantees is in this scenario on average not necessary. A separate management is a big advantage for the 1.75 %-contract (higher average return, reduction of investment risk).

In Scenario 3 – a gradual return to high interest rates – there is no noteworthy interdependence between the two life insurance policies. However, now both contracts – especially the 1.75 %-contract – profit from a separate management of their policies.

3.4 Smoothing the surplus distribution over time

Up to now, we assumed that there is no risk reserve, i.e. that the share $1 - \delta$ of the returns of the reference portfolio is immediately transferred to the policy reserves. Now, we examine the effect of a return smoothing algorithm. Following Grosen and Jørgensen [2000], we denote by $P(t_j) = \sum_{i=1}^{K} P_i(t_j)$ the total amount invested in policy reserves. The company has a constant target for the ratio of bonus reserves to policy reserves, i.e. the ratio $B(t)/P(t)$. This target buffer ratio is denoted by γ. We assume that if the ratio $B(t)/P(t)$ exceeds γ, a positive fraction α of the excessive risk reserve is distributed to the policyholders. Of course the insurance company hereby has to ensure that they credit at least the contractually minimum return. This leads to

$$d(t_j) = \max\left(g(t_j); \alpha\left(\frac{B(t_{j-1})}{P(t_{j-1})} - \gamma\right)\right).$$

We set $\gamma = 10\,\%$ and assume an initial reserve of $B(t_0) = 5\,000$ €. Then, we examine the effect of the surplus distribution mechanism to the risk/return profile of the policyholders. We compare the case of no risk reserve (see Section 3.2) to participation rates of $\alpha = 20\,\%$ and $\alpha = 50\,\%$. We first observe, that the return-smoothing leads to a risk-shift between insurance company and policy holders: The insolvency risk of the insurance company significantly decreases – especially in Scenario 1. Second, we also observe a risk-shift between the 1.75 %-contract and the 3.5 %-contract. While in Scenario 1, the 1.75 %-contract loses significantly by the return smoothing, it profits in the setup of rising interest rates (Scenario 3, $\alpha = 50\%$).

Portfolio 1	$(\mu_I^{(1)}, \sigma_I^{(1)})$	$(\mu_I^{(2)}, \sigma_I^{(2)})$	no guarantee	κ
Scenario 1: no reserve	$(3.74\,\%, 0.17 \cdot 10^{-2})$	$(2.38\,\%, 0.31 \cdot 10^{-2})$	$(2.21\,\%, 0.39 \cdot 10^{-2})$	29.9 %
Scenario 1, $\alpha = 20\,\%$	$(3.51\,\%, 0.04 \cdot 10^{-2})$	$(1.81\,\%, 0.13 \cdot 10^{-2})$	$(2.21\,\%, 0.39 \cdot 10^{-2})$	11.9 %
Scenario 1, $\alpha = 50\,\%$	$(3.53\,\%, 0.08 \cdot 10^{-2})$	$(1.87\,\%, 0.20 \cdot 10^{-2})$	$(2.21\,\%, 0.39 \cdot 10^{-2})$	12.7 %
Scenario 2: no reserve	$(3.51\,\%, 0.01 \cdot 10^{-2})$	$(1.94\%, 0.11 \cdot 10^{-2})$	$(2.78\,\%, 0.09 \cdot 10^{-2})$	3.4 %
Scenario 2, $\alpha = 20\,\%$	$(3.50\,\%, 0.00 \cdot 10^{-2})$	$(1.78\,\%, 0.01 \cdot 10^{-2})$	$(2.78\,\%, 0.09 \cdot 10^{-2})$	0.0 %
Scenario 2, $\alpha = 50\,\%$	$(3.50\,\%, 0.01 \cdot 10^{-2})$	$(1.87\,\%, 0.15 \cdot 10^{-2})$	$(2.78\,\%, 0.09 \cdot 10^{-2})$	0.0 %
Scenario 3: no reserve	$(3.67\,\%, 0.05 \cdot 10^{-2})$	$(3.37\%, 0.14 \cdot 10^{-2})$	$(3.58\,\%, 0.08 \cdot 10^{-2})$	0.0 %
Scenario 3, $\alpha = 20\,\%$	$(3.79\,\%, 0.06 \cdot 10^{-2})$	$(2.89\,\%, 0.15 \cdot 10^{-2})$	$(3.58\,\%, 0.08 \cdot 10^{-2})$	0.0 %
Scenario 3, $\alpha = 50\,\%$	$(3.86\,\%, 0.07 \cdot 10^{-2})$	$(3.58\,\%, 0.11 \cdot 10^{-2})$	$(3.58\,\%, 0.08 \cdot 10^{-2})$	0.0 %

Table 6: Average internal rate of return $\mu_I^{(i)}$ with standard deviation $\sigma_I^{(i)}$ in Scenario 1, 2, and 3 for different surplus distribution schemes. The column "no guarantee" compares the results to the fictitious portfolio that also invests according to the optimization rules (4) but does not guarantee any minimum interest rate. The right column presents the ratio κ, i.e. the average amount of equity relative to the total amount invested in policy reserves the insurance company has to add to ensure their contractually guaranteed interest (see Section 3.3).

4 Conclusion and outlook

We summarize our findings from the above simulated evolution of our simplified insurance company:

(1) **Accepting more risky investment decisions**: We conclude that with increasing contractually guaranteed minimal annual return (weighted average over all insured policies) the insurance company will allocate the managed funds in increasingly risky asset classes, as can be seen e.g. in Figure 1. If the current low interest rate regime prevails and most insurance companies behave similarly, this might lead to a substantial relocation of capital from government bonds into riskier (or alternative) asset classes or into government bonds with longer maturity. It is to be expected that this increased risk profile might cause financial distress to insurance companies in years of adverse market conditions.

(2) **One portfolio to manage all funds**: From the perspective of a contract with small (resp. large) minimal guarantee the legal rule to manage all funds in one joint portfolio increases (resp. decreases) the riskiness and expected return of the position taken compared to the fictitious scenario that these are managed in two separate portfolios. This can be seen in Figure 2 and the derivations leading to it. Clearly, this observation is consistent with Observation (1).

    It is difficult to decide who ultimately profits from the legal situation of one joint portfolio. Obviously, whenever the yearly return is below the maximal guarantee, then the distribution of the returns is in favor of the cohorts with high guarantee. But (due to decreasing guarantee levels) these cohorts are the ones who have contributed most to existing risk reserves and these risk reserves now also yield a bigger benefit for insured persons with low guarantees.

(3) **The effect of the risk reserve**: The most obvious effect is a smoothing of realized returns in time. Accumulating some buffer in successful years allows paying out more return in adverse situations. The most obvious effect is a smoothing of the empirical variance of yearly returns. But there is a second effect. Namely, using a risk reserve mechanism makes it less likely that the return of one year is below the maximal guarantee and thus a different return is distributed to the different cohorts. In this regard, clients with small guarantee will profit more from a prudent use of the risk reserve, a price clients with a high guarantee have to pay.

    In view of Observations (2) and (3), we find it crucial to conclude that the intrinsic value of an annuity insurance contract thus should not be computed without taking into account other existing contracts and even potential new contracts in the future – since these will alter the allocation of capital and there will necessarily be some redistribution processes taking place. This redistribution depends on, among others, how the risk reserve is used.

(4) **Equity capital requirements**: It is also obvious that a prevailing low interest rate scenario might call for higher capital requirements, given the fact that the low interest rate setting forces the insurance companies to take higher risks. However, it is on the one hand beyond the scope of this simple study to quantify these requirements. On the other hand, the currently offered low guarantees might leave a potential for taking such risks in the future if the interest rates will increase.

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A Optimization

At each time point t_{j-1}, we choose an optimal investment strategy $\boldsymbol{\pi}_{t_{j-1}} = (\pi^{(1)}_{t_{j-1}}, \pi^{(2)}_{t_{j-1}}, \pi^{(3)}_{t_{j-1}})$ such that the probability of the portfolio return $m(t_j) := \pi^{(1)}_{t_j} r_{t_j} + \pi^{(2)}_{t_j} b_{t_j} + \pi^{(3)}_{t_j} \mu_{t_j}$ being less than the portfolio guarantee $g(t_{j-1})$ is minimized, i.e.

$$\begin{cases} \min_{\boldsymbol{\pi}_{t_{j-1}}} \mathbb{E}\big[\big(g(t_{j-1}) - m(t_j)\big)\, \mathbb{1}_{\{m(t_j) \leq g(t_{j-1})\}}\big] \\ 0 \leq \pi^{(i)}_{t_{j-1}} \leq b_i \\ \pi^{(1)}_{t_{j-1}} + \pi^{(2)}_{t_{j-1}} + \pi^{(3)}_{t_{j-1}} = 1 \end{cases}$$

Abbreviating

$$\boldsymbol{d} := \begin{pmatrix} r_{t_{j-1}}\, e^{-\kappa_1 \triangle t} + \theta_1\big(1 - e^{-\kappa_1 \triangle t}\big) \\ b_{t_{j-1}}\, e^{-\kappa_2 \triangle t} + \theta_2\big(1 - e^{-\kappa_2 \triangle t}\big) \\ \theta_3 \triangle t \end{pmatrix}, \quad \boldsymbol{\sigma} := \begin{pmatrix} \sigma_1 \sqrt{\frac{1-e^{-2\kappa_1 \triangle t}}{2\kappa_1}} \\ \sigma_2 \sqrt{\frac{1-e^{-2\kappa_2 \triangle t}}{2\kappa_2}} \\ \sigma_3 \end{pmatrix},$$

then $m(t_j)$ is normally distributed with mean $\boldsymbol{\pi}_{t_j}\,\boldsymbol{d}$ and variance $\boldsymbol{\pi}'_{t_j}\,(\boldsymbol{\sigma}\boldsymbol{\sigma}' \,.\, \Sigma)\,\boldsymbol{\pi}_{t_j}$ (where . denotes point-wise matrix-multiplication and $'$ transpose). The portfolio guarantee $g(t_j)$ is a constant. In the following, we write the covariance matrix as $V := \boldsymbol{\sigma}\boldsymbol{\sigma}' \,.\, \Sigma$.

If returns are normally distributed, the objective function can easily be evaluated if one recalls basic properties of the truncated normal distribution, i.e.

$$\begin{aligned}
&\mathbb{E}\big[\big(g(t_{j-1}) - m(t_j)\big)\,\mathbb{1}_{\{m(t_j)\le g(t_{j-1})\}}\big] \\
&\quad = \Phi\left(\frac{g(t_{j-1}) - \boldsymbol{\pi}'_{t_{j-1}}\,\boldsymbol{d}}{\sqrt{\boldsymbol{\pi}'_{t_{j-1}}\,V\,\boldsymbol{\pi}_{t_{j-1}}}}\right)\cdot\big(g(t_{j-1}) - \boldsymbol{\pi}'_{t_{j-1}}\,\boldsymbol{d}\big) + \sqrt{\boldsymbol{\pi}'_{t_{j-1}}\,V\,\boldsymbol{\pi}_{t_{j-1}}}\cdot\phi\left(\frac{g(t_{j-1}) - \boldsymbol{\pi}'_{t_{j-1}}\,\boldsymbol{d}}{\sqrt{\boldsymbol{\pi}'_{t_{j-1}}\,V\,\boldsymbol{\pi}_{t_{j-1}}}}\right),
\end{aligned}$$

where $\phi(\,\cdot\,)$, respectively $\Phi(\,\cdot\,)$, denote the density, respectively distribution function, of the standard normal distribution.