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THREE ESSAYSIN THEORETICAL CORPORATE FINANCE

Munier Jules

UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT DE FINANCE

THREE ESSAYS IN THEORETICAL CORPORATE FINANCE

THÈSE DE DOCTORAT

présentée à la

Faculté des Hautes Etudes Commerciales
de l'Université de Lausanne

pour l'obtention du grade de
Docteur ès Sciences Economiques, mention « Finance »

par

Jules MUNIER

Directeur de thèse
Prof. Norman Schürhoff

Jury

Prof. Jean-Philippe Bonardi, Président
Prof. Theodosios Dimopoulos, expert interne
Prof. Philip Valta, expert externe

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IMPRIMATUR

Sans se prononcer sur les opinions de l'auteur, la Faculté des Hautes Etudes Commerciales de l'Université de Lausanne autorise l'impression de la thèse de Monsieur Jules MUNIER, licencié en Gestion d'Entreprise de l'Université de Genève et titulaire d'un master en Finance de l'Université de Genève, en vue de l'obtention du grade de docteur ès Sciences Economiques, mention "Finance".

La thèse est intitulée:

THREE ESSAYS IN THEORETICAL CORPORATE FINANCE

Lausanne, le 20 décembre 2016

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LAUSANNE
soutenance publique, le 11 janvier 2017

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Doctorate in Finance

I hereby certify that I have examined the doctoral thesis of

Jules MUNIER

and have found it to meet the requirements for a doctoral thesis.
All revisions that I or committee members
made during the doctoral colloquium
have been addressed to my entire satisfaction.

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All revisions that I or committee members
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Executive Summary

The thesis is articulated around three theoretical corporate finance research articles. All articles model intertwined and important corporate decisions and have in common the modeling of cash policy: Each article presents a new cash model.

In the first article, I examine managers' decision to pay dividends with a dynamic cash model. The model departs from the Modigliani and Miller world according to two dimensions: Agency costs and financing frictions.

In this paper, I show that managers with more valuable outside options pay more dividends, but the outside option makes extremely costly for shareholders to optimally compensate managers. The model is solved by computing the optimal firms' ownership which should be granted to managers and by computing the optimal payout and liquidation policy decided by managers.

In the second article, I examine the impact of managerial optimism on corporate policies and agency costs within a model of dynamic corporate investment for a financially constrained firm. An optimistic manager is defined as a manager who over-evaluates the profitability of the firm assets. The main feature of the model is to allow joint investment and saving policies and the model is solved numerically.

In addition to the common sense that an optimistic manager should build an empire by investing more in average and keeping more cash under control, I find the following primary results: when the firm is highly financially constrained, an optimistic manager under-invests and can oversell the physical assets. The investment sensitivity to cash is lower. The agency costs sensitivity to cash is higher and this reinforces the need to monitor the manager.

In the third article, I examine the effects of credit rationing on corporate cash holdings by modeling the precautionary demand for cash. In the model firms can pledge part of the future cash-flows to creditors when current cash-flows are insufficient to finance investment. The discrete time model with three periods is solved with closed form solutions.

I show that the cash-flow sensitivity of cash and the investment to cash sensitivity are inappropriate indicators of financing constraint. By contrast, I show that the variation of cash holdings is monotonically decreasing with the degree of the financing constraint. An empirical study with a large sample of manufacturing firms over the 1971 to 2011 period confirms this result.

The challenge in theoretical corporate finance is to set up models that permit to highlight unexplored problems and trade-offs or to solve well known puzzles. Each model proposed in this thesis has for objective to solve a puzzle or to highlight new important trade-offs. In the first paper I highlight a new trade-off between managers outside option and payout policy. In the second paper I highlight that optimistic managers can under invest and in the third I show that a financially constrained firms can optimally reduce liquidities even with future profitable investments.

In general it is challenging to propose models rich enough to get non trivial, but simple enough solutions. A good model prediction should be testable empirically and easy to communicate. I hope that the directions of research proposed and the results found in this thesis could be useful for empirical researchers. It was an important objective.

The context of this research is rather simple. Over the past two decades, dynamic corporate finance models have taken an important part of the literature in financial economics. In particular cash models have been extensively used to provide insights and quantitative guidance for investment, financing or risk management decisions under uncertainty. Cash models are particularly useful to model intertwined decisions, because the cash is an asset used as a buffer for a financially constrained firm. This permits to analyze both cash-inflows and outflows in a natural way.

The models presented in this thesis belong to this agenda.

The next step and important perspective would be to use the trade-offs identified to set up structural estimations. Recursive models have been extensively used to perform structural estimations so far and could be maybe adapted from the theoretical models presented in this thesis. However it is not clear whether fix points techniques could be used to solve them.

The obvious improvement could be due to significantly more powerfully computers. It would permit the calibration of recursive models with more state variables.

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Managerial Compensation, Outside Option and Dividend Policy *

Jules Munier†

January 9, 2017

Abstract
I examine managers' dividend policy with a dynamic cash model. The model departs from the M&M world according to two dimensions: Financing frictions and agency costs. In the model external financing is costly and there is a conflict of interest between managers and shareholders. The conflict comes from managers' outside option. Managers have the option to leave the firm. If they decide to leave they get a fixed known compensation. This option makes managers less risk averse than shareholders and distorts the payout policy. Shareholders want to give incentives to managers to pay dividends and to mitigate the conflict of interest. The model is solved for the optimal static compensation contract which gives incentive to the managers to pay dividends and mitigate the conflict. The contract is defined as optimal if it maximizes shareholders wealth.

Keywords: Dividend; Dynamics; Cash; Managers; Liquidation

JEL Classification Number(s): G31, G35

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1 Introduction

The contribution of this essay is to better understand the impact of managers outside option on payout decisions and shareholders wealth.

1.1 Motivation

Since the beginning of the 90's there are yearly US national debates about the level of CEOs compensation: Political figures or union leaders denounce executive salaries observed in US listed firms. Debates are supported by national surveys which report that most Americans disagree with the level of top-executive compensations (e.g. D. Larcker, N. Donatiello and B. Tayan 2016).

Economic researchers would argue that it is not "how much you pay, but how" (M. C. Jensen and K. J. Murphy 2010) which should matter. In particular it is not difficult to observe how US CEOs are compensated thanks to regulatory filings[1], but difficult to determine whether board decisions about CEOs compensation maximize shareholders wealth or firm value.

In this paper I analyze theoretically the effects of managers outside options on payout policies and shareholders wealth. I define an outside option as the present value of all expected future compensations which managers may get if they decide to leave the firm. Eckbo, Thorburn and Wang (2016) study empirically CEOs career and human changes around Chapter 11 bankruptcy fillings. They find that one third of the incumbent CEOs maintain executive employment with a median compensation change of zero. The fact that managers may not systematically suffer after firms bankruptcy or liquidation may create a conflict of interests with shareholders.

The theoretical literature has been largely silent regarding the impact of this specific conflict on payout policy, which should permit to set up a new framework and find insightful results.

1.2 Model Framework and Main Results

The model is built up on standard dynamic cash models (Jeanblanc Picqué and Shiryaev 1995, Radner and Shepp 1996) with financing constraints. Standard dynamic cash models predict that managers optimally accumulate cash and decide to pay dividends only when an optimal cash holding boundary is reached, i.e. managers accumulate cash for avoiding an inefficient closure as long as the marginal value of cash is high enough.

I add one friction to this framework. I assume that managers may leave the firm at any time with a known fixed compensation unrelated to firms performance. If managers may not be dismissed after deviating from first best payout policy, it creates the need for shareholders to provide managers enough incentives to run firms and to pay dividends.

The optimal incentive is trivially a percentage of firms ownership which permits to remunerate managers if and only if they decide to pay dividends. How-

[1] Form DEF 14A filed with the Securities and Exchange Commission summarizes highest executives compensations.

ever the optimal ownership is not trivial to compute as managers liquidation and payout policy both depend on the ownership granted to the manager.

I find that shareholders must reward managers with optimally about 7% of the firm in order to give them an optimal incentive to pay dividends. It is significantly higher than the average dollar gain in firms value of about 0.1% computed in Jensen (1990).

I find that managers pay significantly less dividends than the first best solution when they have no outside option. This is not surprising as managers outside option makes them less risk averse. The corresponding inefficiency costs are equal to about 5% of the value of the firm when managers have no outside option. It means that the outside options destroy roughly 5% of firm value by giving managers the incentive to pay dividends earlier.

I find that shareholders wealth is increasing and managers wealth decreasing with firms profitability and that managers wealth is increasing and shareholders wealth decreasing with the volatility of cash inflows.

1.3 State of Research

"*The harder we look at the dividend picture, the more it seems like a puzzle, with pieces that just don't fit together*" (Black 1976). After about forty years of research, the payout policy remains one of the main puzzles in corporate finance. The payout policy puzzle is of particular interest as long as it matters for investors. I argue that this is a reasonable assumption for public firms considering the persistent and significant effect of dividend announcements on stock prices or for example the recent Apple suit for hoarding too much cash.

The payout puzzle agenda dates empirically from the Lintner's adjustment model (1956) and theoretically from the Modigliani and Miller (1958) irrelevance world. Lintner's findings seem to hold for a wide set of firms and more recent time periods (Fama and Babiak 1968, Brav and al. 2005).

Theoretically, negating the seminal Modigliani and Miller (1958) theorem, we know that payout policy matters for maximizing firm value. Since then three main theories have been proposed for explaining payout policies. Each theory is based on a small set of hypotheses departing from the M&M set of assumptions. Assuming incomplete contracting and conflict of interests between the manager and the investors leads to the Jensen's agency theory (1976). Assuming asymmetry of information leads to the Akerlof's adverse selection theory (1970) and has led to the Bhattacharya (1979), John and Williams (1985) and Miller and Rock (1985) seminal signaling models. Finally assuming heterogeneous agents leads to the clientele theory where Allen, Bernardo and Welch (2002) is an important recent contribution.

This essay is closely related to the Jensen's agency theory as a conflict of interests is highlighted and determines the payout policy decided by managers. However despite considerable research on payout policy in general, the literature has been largely silent on the role of outside options on payout policy in particular.

The structure of the paper is the following. I set up the model in section 2 and solve it in section 3. I propose a numerical analysis in section 4 a sensitivity analysis in section 5 and conclude in section 6.

2 Model Setup

I set up a simple dynamic cash model where I depart from the M&M world according to two dimensions: Agency costs and financing frictions. The model is built on standard cash models and is solved as a singular stochastic control problem, where the value function of the manager is found in closed form and the optimal controls are found numerically. The objective is to understand how shareholders can provide to managers optimal incentives to pay dividends when managers have an outside option. Thereafter in this section I begin describing the firm's financing technology, then managers' compensation technology and eventually the optimization program.

2.1 Financing Technology

I assume the existence of a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ and an economy with two players: Managers and shareholders. When managers decide to run firms, firms' activities generate the stochastic adapted cash inventory process $W = (W_t)_{t\geq 0}$ with the following dynamics:

$$dW_t \quad = \mu dt - dD_t + \sigma dZ_t, \tag{1}$$

where μ represents the constant drift of the cash process, Z_t is a Wiener process and $\sigma > 0$ a constant volatility parameter.

Managers control the cash inventory process W_t through payment of dividends dD_t, with D the cumulative dividend process (arbitrary non anticipating positive process). The shareholders cannot decide the dividend policy instead of managers.

Firms are financially constrained and can use only internal cash-holdings to avoid an inefficient closure. I denote the liquidation time τ. Firms can be liquidated in two cases:

First, I suppose that managers are not able to run firms when the cash-inventory is too low. I set this lower boundary equal to 0. The liquidation in this case is exogenous.

Second, I also assume that managers may decide to leave firms at any time. The liquidation in this case is endogenous.

If managers leave firms they cannot be replaced and firms must be liquidated. In both cases when managers leave they get B where $B > 0$ is the benefit of leaving the firm. Shareholders in turn get $(1-\beta)W_\tau$ where βW_τ are liquidation costs. I assume that managers cannot leave firms with any percentage of the cash proceeds. When managers decide to leave they only get the outside option B[2].

[2] This assumption can be relaxed without changing the main results of the model. Without this assumption managers

The risk adjusted discount rate is equal to the constant $0 < \lambda < \infty$. I also suppose that the cash inventory does not generate profits. A standard assumption is that cash-holdings generate returns below the risk adjusted discount rate[3]. A low internal rate of return creates the realistic incentive to pay dividends. By setting this rate to 0 I avoid useless parameters. I do not consider the role of taxes and the dividend payment is cost free[4].

2.2 Compensation Technology

There is a conflict of interest between shareholders and managers because liquidation is profitable for managers.

Shareholders in order to mitigate this conflict can compensate the manager with a percentage of equity shares $\alpha \in [0, 1[$.

The ownership α is contracted when the firm is created and cannot be renegotiated later.

Finally, if there is no conflict of interest i.e. when $B = 0$, I suppose that managers will maximize shareholders wealth (even with $\alpha = 0$).

2.3 Agents Optimality

- Managers maximize the expected present value of their current and future compensations M_t with respect to an admissible dividend barrier policy $D(\overline{W})$ and an optimal leaving time τ given a compensation contract α. The leaving time of the manager corresponds to the liquidation time by assumption. By denoting the optimum $\mathcal{M}_t$, managers optimization program follows:

$$\begin{aligned} \mathcal{M}(W_t) &= \sup_{\{\overline{W}, \tau\}} M(\overline{W}, \tau, W_t), \\ M(\overline{W}, \tau, W_t) &= \mathbb{E}[\textstyle\int_{t\wedge\tau}^{\tau} e^{-\lambda s} \alpha dD_s + e^{-\lambda\tau} B], \end{aligned} \quad (2)$$

  with $t \wedge \tau = \min\{t, \tau\}$ and where $\tau = \inf\{t : W_t \leq 0 \vee \mathcal{M}(W_t) \leq B\}$.

- Shareholders maximize the expected present value of their current and future dividends S_t plus the expected present value of the proceed upon liquidation. Shareholders decide the compensation contract α given managers expected dividend barrier $\overline{W}$ and leaving time τ. By denoting the optimum $\mathcal{S}_t$, shareholders optimization program follows:

$$\begin{aligned} \mathcal{S}(W_t) &= \sup_{\alpha} S(\alpha, W_t), \\ S(\alpha, W_t) &= \mathbb{E}[\textstyle\int_{t\wedge\tau}^{\tau} e^{-\lambda s}(1-\alpha) dD_s + e^{-\lambda\tau}(1-\beta) W_\tau]. \end{aligned} \quad (3)$$

  Please note that both τ and the dividend process D_t depend on the compensation control α.

could try to avoid liquidation costs by paying an extra dividend and depleting cash-reserves, before liquidating the firm

[3] In particular researchers use agency motives e.g. Kim, Mauer and Sherman 1998 or Riddick and Whited 2009 for justifications

[4] The impact of taxes on dividends policies has been widely studied e.g. by Miller and Scholes (1978), John and Williams (1985) or Allen, Bernardo and Welch (2002)

3 Model Solution

The optimization program is Markovian. For each contract α we can compute in closed form $\mathcal{M}(W_t, \alpha)$ and find the optimal contract which maximizes shareholders wealth $S(W_t)$. I start the analysis by presenting the first best solution without conflict of interest when there are only financing frictions.

3.1 First Best Solution

I denote the first best solution with the upper-script ^{FB}. Without conflict of interest managers want to maximize shareholders wealth. The optimization program becomes:

$$\begin{aligned} \mathcal{S}^{FB}(W_t) &= \sup_{\overline{W}} S(W_t), \\ S^{FB}(W_t) &= \mathbb{E}[\textstyle\int_{t\wedge\tau}^{\tau} e^{-\lambda s} dD_s], \end{aligned} \tag{4}$$

and the solution can be found in closed form[5]:

$$S^{FB}(W) \quad = A_1 e^{\zeta_+ W} + A_2 e^{\zeta_- W}, \tag{5}$$

where $\zeta_{+,-}$ are the roots solving $\frac{1}{2}\sigma^2\zeta^2 + \mu\zeta - \lambda = 0$:

$$\zeta_{+,-} = \frac{-\mu \pm \sqrt{\mu^2 + 2\lambda\sigma^2}}{\sigma^2}. \tag{6}$$

and A_1^{FB}, A_2^{FB} are

$$\begin{aligned} A_1^{FB} &= -A_2^{FB} \\ A_2^{FB} &= \frac{\mu}{\lambda} \frac{1}{e^{\zeta_- \overline{W}^{FB}} - e^{\zeta_+ \overline{W}^{FB}}} \end{aligned} \tag{7}$$

with

$$\overline{W}^{FB} = \frac{1}{\zeta_+ - \zeta_-} \ln\left(\frac{\zeta_-^2}{\zeta_+^2}\right) \tag{8}$$

3.2 Managers Value Function

Given a contract α, the managers compensation value function satisfies the following HJB (Hamilton Jacobi Bellman) equation:

$$\lambda M \quad = \mu M_W + \frac{1}{2}\sigma^2 M_{WW}, \tag{9}$$

The homogeneous ordinary equation of degree two is satisfied with the function:

$$M(W) \quad = A_1 e^{\zeta_+ W} + A_2 e^{\zeta_- W}, \tag{10}$$

Where $\zeta_{+,-}$ are the roots solving $\frac{1}{2}\sigma^2\zeta^2 + \mu\zeta - \lambda = 0$:

$$\zeta_{(+,-)} = \frac{-\mu(\pm)\sqrt{\mu^2 + 2\lambda\sigma^2}}{\sigma^2}. \tag{11}$$

[5] e.g. Décamps, Mariotti, Rochet and Villeneuve (2011).

and A_1, A_2, $\underline{W}$ and $\overline{W}$ are constants which can be determined with boundary conditions:

$$\begin{array}{rll} \text{M.C.} & M(\underline{W}) & = B \\ \text{M.C.} & M_W(\overline{W}) & = \alpha \\ \text{F.O.C.} & M_W(\underline{W}) & = 0, \text{(end.l.)} \\ \text{F.O.C.} & M_{WW}(\overline{W}) & = 0 \end{array} \tag{12}$$

where $\underline{W}$ is the liquidation boundary, $\overline{W}$ the payout boundary and en.l. stands for endogenous liquidation[6]. We find:

$$\begin{aligned} A_1 &= \frac{B - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}, \\ A_2 &= \alpha \Delta(\underline{W}) - B\zeta_+ \Delta(\overline{W}), \\ \Delta(W) &= \frac{e^{\zeta_+ W}}{\zeta_- e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}} \end{aligned} \tag{13}$$

where the boundaries $\underline{W}$ and $\overline{W}$ can be found numerically subject to the conditions $\underline{W} \geq 0$ and $\overline{W} > \underline{W}$.

3.3 Shareholders Value Function

Given a payout policy $\delta_t \in D(\overline{W})$ shareholders value function satisfies the following HJB (Hamilton Jacobi Bellman) equation:

$$\lambda S = \mu S_W + \frac{1}{2}\sigma^2 S_{WW}, \tag{14}$$

The homogeneous ordinary equation of degree two is satisfied with the function:

$$S(W) = A_1^S e^{\zeta_+ W} + A_2^S e^{\zeta_- W}, \tag{15}$$

Where $\zeta_{+,-}$ are the roots solving $\frac{1}{2}\sigma^2\zeta^2 + \mu\zeta - \lambda = 0$:

$$\zeta_{(+,-)} = \frac{-\mu(\pm)\sqrt{\mu^2 + 2\lambda\sigma^2}}{\sigma^2}. \tag{16}$$

and A_1^S, A_2^S are constants which can be determined with boundary conditions:

$$\begin{array}{rll} \text{M.C.} & S(\underline{W}) & = (1-\beta)\underline{W} \\ \text{M.C.} & S_W(\overline{W}) & = 1 - \alpha \end{array} \tag{17}$$

where $\underline{W}$ is the liquidation boundary and $\overline{W}$ the payout boundary decided by managers. We find:

$$\begin{aligned} A_1^S &= \frac{(1-\beta)\underline{W} - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}, \\ A_2^S &= (1-\alpha)\Delta(\underline{W}) - \zeta_+(1-\beta)\underline{W}\Delta(\overline{W}), \end{aligned} \tag{18}$$

where both boundaries $\underline{W}$ and $\overline{W}$ are decided by managers and found with conditions under (12) given a contract α. Both boundaries are function of the ownership parameter α, which can be found by maximizing $S(W)$ subject to the

[6]If the liquidation is exogenous i.e. when $\underline{W} = 0$ then the F.O.C. may be not satisfied.

condition $\alpha \in [0, 1]$. All details about computations and proofs can be found in appendix A.

lemma 1: *The managers want to liquidate the firm immediately (endogenous liquidation) or as late as possible (exogenous liquidation)*

The proof of **lemma 1** can be found in appendix B. Lemma 1 highlights that the managers have no incentives to liquidate the firms if they decide to manage the firms. Because shareholders have no incentives to propose a contract if managers immediately liquidate firms, we can simplify the model if the following condition is satisfied:

$$M(W_0) > B \tag{19}$$

In this case the liquidation boundary is exogenous and will be equal to 0.

The next section is dedicated to the numerical analysis of the only interesting case, when the managers accept to run the firms.

4 Quantitative Analysis

In this section, I present the benchmark calibration of the model and analyse the corresponding solution. A sensitivity analysis is presented in the next section.

4.1 Model Calibration

There are 5 parameters: μ, σ, λ, B and β.

The mean of the cash-inflow process μ is equal to 15%. It is in line with the empirical compound annual growth rate $CAGR$ of 10.13% for US firms over the last 5 years computed by A. Damodaran[7] and the 19% productivity rate used by Bolton, Chen and Wang (2011). The volatility σ is set to 20% in line with the 35% volatility found by A. Damodaran and the 9% volatility used in Bolton, Chen and Wang (2011).

The risk neutral discount rate λ is 5% as in Bolton, Chen and Wang (2011), the outside option of the manager B is set to 0.1 unit and the liquidation costs parameter β is set to 20%. It is difficult to obtain precise empirical estimates of these parameters.

The initial amount of cash-holdings is set to 0.5 unit.

All the parameters can be found in table 1.

4.2 First Best Solution

I start presenting the first best solution when there is no conflict of interest, i.e. when the manager has no outside option. The first best solution can be found in black on the top-left graphic in figure 1.

[7] $http://pages.stern.nyu.edu/~adamodar/New_Home_page/datacurrent.html$

Table 1: **Baseline Parameters**

Parameters	Values and Description	
μ	15%	Cash Process Trend
σ	20%	Cash Process Volatility
λ	5%	Risk Neutral Discount Rate
B	0.1	Managers Outside Option
β	20%	Liquidation Costs
W_0	0.5	Cash-Holdings at Time 0

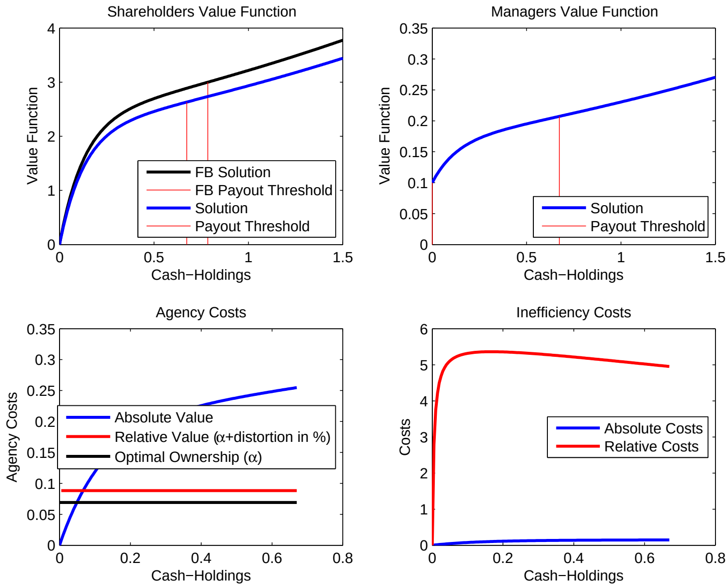

Figure 1: **Benchmark Result**, figure 1 summarizes the results of the baseline model.

In the first best case the manager wants to maximize shareholders wealth. The manager accumulates cash for avoiding liquidation and payout dividends only if the marginal value of cash is below 1. The marginal value of cash is decreasing with the cash inventory. When the marginal value of cash becomes lower than 1 it becomes optimal to pay dividends instead of saving more cash.

The liquidation threshold is 0 by assumption. The optimal dividend barrier is equal to 0.7856.

4.3 Managers Payout and Liquidation Policy

I assume now that the manager has an outside option. The outside option generates a conflict of interests, which can be mitigated if shareholders decide to give part of the ownership to the manager. When the manager becomes a shareholder it gives him incentives to pay dividends. The results are presented in figure 1.

On the top-left graphic I plot the baseline model solution with the first best solution. The liquidation barrier is unchanged as expected, however the payout barrier becomes 0.675. The manager decides to pay significantly earlier dividends in comparison to the first best case. This result is intuitive as the manager can benefit from liquidation and owns a relatively small percentage of the firm. i.e. the manager has less incentives to accumulate cash.

On the top-right graphic, we find the manager's value function. The manager's value function is only equal to 0.1 at the liquidation threshold. This result is intuitive, because the value of the option to pay dividends can only be positive.

4.4 Shareholders Agency Costs and Ownership Policy

On the bottom-left graphic of figure 1, we find the agency costs in absolute value and as a percentage of the first best solution. The black line corresponds to the optimal ownership of the manager. The red line is equal to the percentage of the firm which is really lost by the shareholders. The shareholders lose more than the optimal ownership α, because there is a distortion about the payout policy. The red line can be seen as the slice of the cake that must be optimally cut by the shareholders in order to mitigate optimally the conflict of interests.

The optimal ownership α is equal to 7% (the black line). Please note that it represents the total compensation of the manager.

The total loss in percentage of the first best solution is equal to 9% (the red line).

4.5 Inefficiency Costs

Finally, on the bottom-right graphic of figure 1, we find the inefficiency costs. The inefficiency costs are computed as $S^{FB} + B - \mathcal{M} - \mathcal{S}$ where I denote S^{FB} the first best value function, $\mathcal{M}$ the value function of the managers and $\mathcal{S}$ the value function of the shareholders. The relative costs are computed as a percentage of the total wealth without frictions, which is assumed to be $S^{FB} + B$. The relative costs are equal to 6% when the firm is close to liquidation and they drop to 5% when managers decide to pay dividends.

I propose now to perform a sensitivity analysis in order to test the robustness of the results and the role of the main parameters. The initial reserve of cash in the next section is set to 0.5 correspondingly to the benchmark case. The liquidation is always exogenous and happens when the firms run out of cash-holdings ($W = 0$).

5 Sensitivity Analysis

I start by discussing the impact of a change of the drift parameter μ on the results. Consecutively I will follow the order of presentation of the parameters used in table 1. The constant parameters have the same values as the ones in the baseline model analysis.

The sensitivity analysis is not performed with the parameter β, because a modification of the liquidation costs does not change the results if the value function at contract initiation ($t = 0$) is higher than the initial amount of cash-holdings, i.e. when $\mathcal{S}(W_0) > W_0$. If the shareholders decide to invest and if the

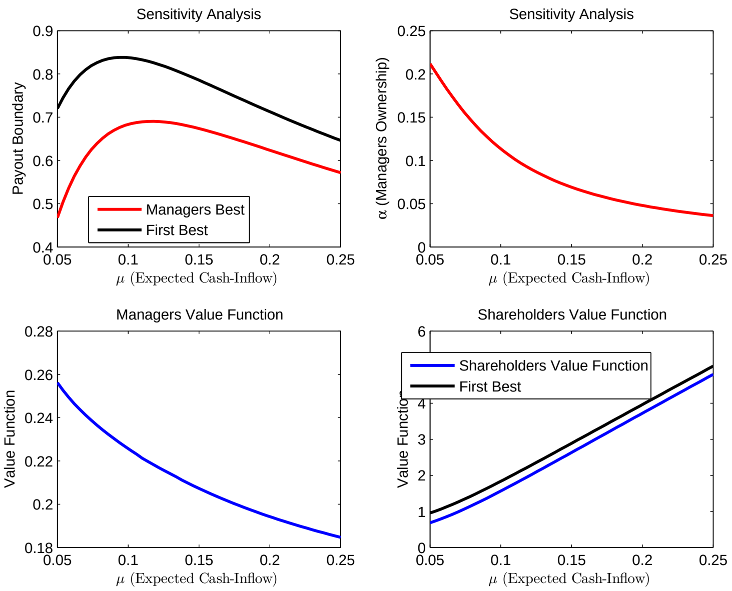

Figure 2: **Sensitivity Analysis with μ**, The figure plots the sensitivity analysis of the controls $\overline{W}$ (payout boundary) and α (managers ownership), and of the value functions M and S with respect to a change of the parameter μ (cash-inflow trend).

managers accept to run the firm, then the liquidation always happens when the firms run out of cash. When the firms run out of cash, the liquidation proceeds and the liquidation costs are equal to 0, because the liquidation costs are supposed to be a percentage of the cash-proceeds upon liquidation[8].

The cases where shareholders do not invest and managers do not run the firms are not interesting and not presented.

5.1 Parameter μ

In figure 2 we can find the sensitivity analysis made with the parameter μ.

On the top-left graphic we find the change of the payout boundary decided by managers in the first best case and with the conflict of interest. The payout threshold when there is a conflict of interest is significantly below the payout threshold in the first base case. The payout threshold is about 35% lower when firms profitability is low (5%) and about 10% lower when firms profitability is high (25%).

We see that both the first best and the second best payout thresholds are quasi-concave functions of the firms profitability parameter μ.

On the top-right graphic we find the optimal ownership-compensation granted to the manager α. The ownership is sharply decreasing with the profitability parameter μ. Shareholders investing in high profitable projects all other things being equal need to pay optimally much less the managers than shareholders investing in low profitable projects.

This result is also illustrated on the bottom-left graphic as we observe a

[8]It is possible to solve the model with absolute liquidation costs, but the analysis is much more complex. In this case shareholders get the incentive to liquidate firms before managers.

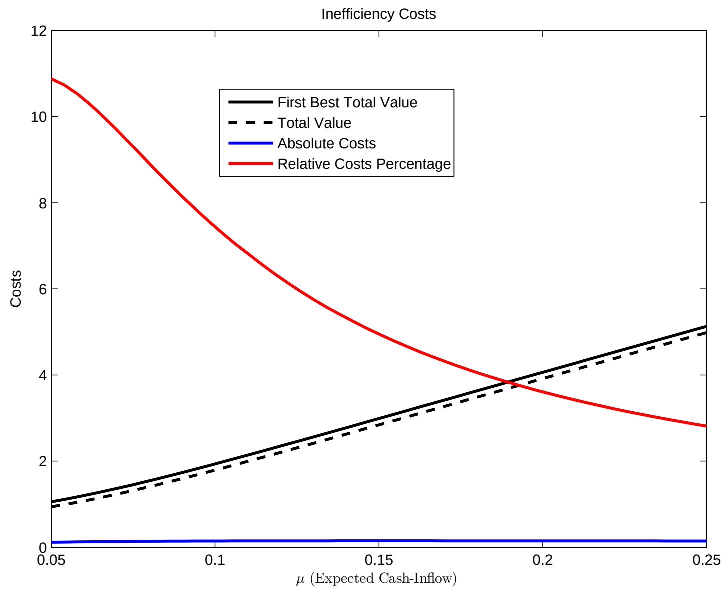

Figure 3: **Sensitivity Analysis of Inefficiency Costs with μ (cash-inflow trend)**, The figure plots the inefficiency costs ($S^{FB} + B - \mathcal{M} - \mathcal{S}$) in absolute value and as a percentage of the first base case total value ($S^{FB} + B$), the total value in the first best case and in the case with the outside option friction ($\mathcal{M} + \mathcal{S}$). All in function of the parameter μ.

sharp decrease of the managers value function (the compensation package) with respect to the profitability parameter. Managers need to be compensated less if firms are more profitable. This result is not intuitive as we could expect that managers also benefit from an increase of firms profitability.

On the bottom-right graphic we see that the shareholders value function is sharply increasing with firms profitability. It means that shareholders get a significantly larger piece of the cake when firms are more profitable.

In figure 3 we find the inefficiency costs.

The costs are slightly increasing with the profitability of the firms because the distortion of the payout policy is more pronounced when firms are more profitable. However we observe a significant decrease of the inefficiency costs as a percentage of the first best case. In other words the size of the cake is increasing much quicker than the inefficiency costs with respect to firms profitability.

5.2 Parameter σ

In figure 4 we can find the sensitivity analysis made with the parameter σ.

On the top-left graphic we find the change of the payout boundary decided by managers in the first best case and with the conflict of interests. The payout threshold when there is a conflict of interest is significantly below the payout threshold in the first base case. The payout threshold is about 15% lower when the volatility is low (10%) and about 25% lower when the volatility is high (30%).

We see that both the first best and the second best payout thresholds are increasing with the volatility parameter σ.

On the top-right graphic we find the optimal ownership-compensation granted

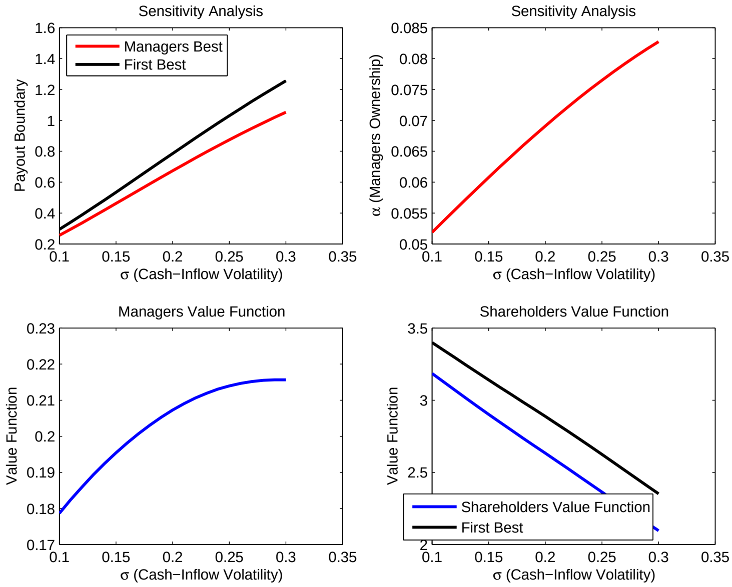

Figure 4: **Sensitivity Analysis with σ**, The figure plots the sensitivity analysis of the controls $\overline{W}$ (payout policy) and α (managers ownership), and of the value functions M and S with respect to a change of the parameter σ (volatility of the cash-inflow).

to the manager α. The ownership is sharply increasing with the volatility. Shareholders investing in risky firms, all other things being equal, need to pay optimally more the managers than shareholders investing in less risky firms.

This result is also illustrated on the bottom-left graphic as we observe a sharp increase of the managers value function with respect to the volatility. Managers need to be compensated more if the firms are more risky and the value of the compensation package is increasing with the volatility. This result is not intuitive as we could expect that managers would not benefit from an increase of firms volatility.

On the bottom-right graphic we see that the shareholders value function is sharply decreasing with firms volatility. Contrary to the increasing profitability case, shareholders get a significantly smaller piece of the cake when firms are more risky.

In figure 5 we find the inefficiency costs.

The costs are slightly increasing with the volatility of the firms because the distortion of the payout policy is more pronounced when firms cash-inflows are more volatile. Moreover we observe a significant increase of the inefficiency costs as a percentage of the first best case. The total value is decreasing and the inefficiency costs are increasing with respect to firms cash-inflows volatility.

5.3 Parameter λ

In figure 6 we can find the sensitivity analysis made with the parameter λ.

On the top-left graphic we find the change of the payout boundary decided by managers in the first best case and with the conflict of interests. The payout threshold when there is a conflict of interest is significantly below the payout

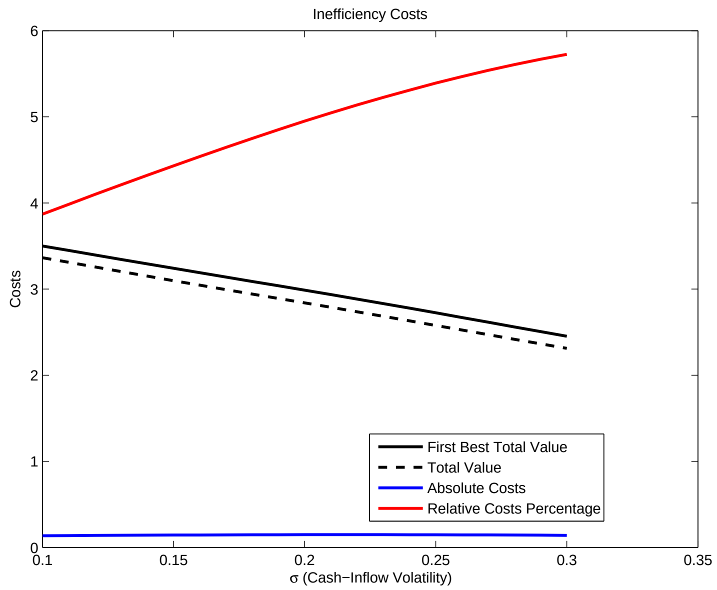

Figure 5: **Sensitivity Analysis of Inefficiency Costs with σ (cash-inflow volatility)**, The figure plots the inefficiency costs ($S^{FB} + B - \mathcal{M} - \mathcal{S}$) in absolute value and as a percentage of the first base case total value ($S^{FB} + B$), the total value in the first best case and in the case with the outside option friction ($\mathcal{M} + \mathcal{S}$). All in function of the parameter σ.

threshold in the first base case. The payout threshold is about 15% lower when the discount rate is small (1%) or high (9%).

We see that both the first best and the second best payout thresholds are decreasing with the discount rate parameter λ.

On the top-right graphic we find the optimal ownership-compensation granted to the manager α. The ownership is sharply increasing with the discount rate. All other things being equal, when the discount rate is high the shareholders need to pay optimally more the managers than in the case where the discount rate is low.

However, on the bottom-left graphic we see that the value of the compensation package is decreasing for low discount rates and increasing with high discount rates. The relationship is not monotonic.

On the bottom-right graphic we see that the shareholders value function is sharply decreasing with the discount rate, both in the first base case and the outside option friction case.

In figure 7 we find the inefficiency costs.

The costs are slightly increasing with the discount rate of the firms because the distortion of the payout policy is more pronounced when the discount rate is higher. Moreover we observe a significant increase of the inefficiency costs as percentage of the first best case. The total value is sharply decreasing and the inefficiency costs are increasing with respect to the discount rate.

5.4 Parameter B

In figure 8 we can find the sensitivity analysis made with the parameter B.

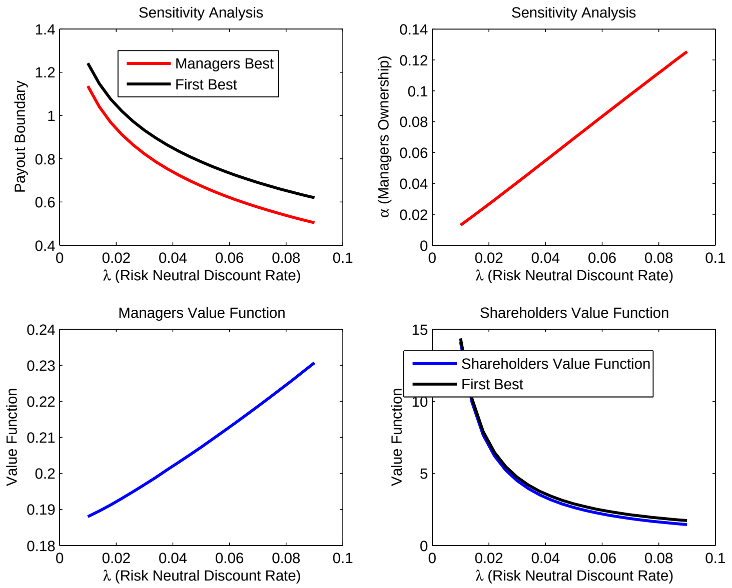

Figure 6: **Sensitivity Analysis with λ**, The figure plots the sensitivity analysis of the controls $\overline{W}$ (payout boundary) and α (managers ownership), and of the value functions M and S with respect to a change of the parameter λ (risk neutral discount rate).

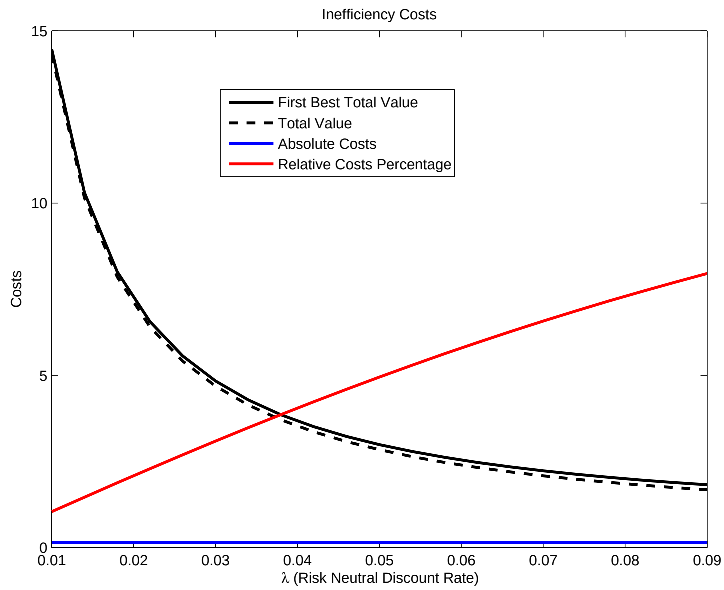

Figure 7: **Sensitivity Analysis of Inefficiency Costs with λ (risk neutral discount rate)**, The figure plots the inefficiency costs ($S^{FB}+B-\mathcal{M}-\mathcal{S}$) in absolute value and as a percentage of the first base case total value ($S^{FB}+B$), the total value in the first best case and with the outside option friction ($\mathcal{M}+\mathcal{S}$). All in function of the parameter λ.

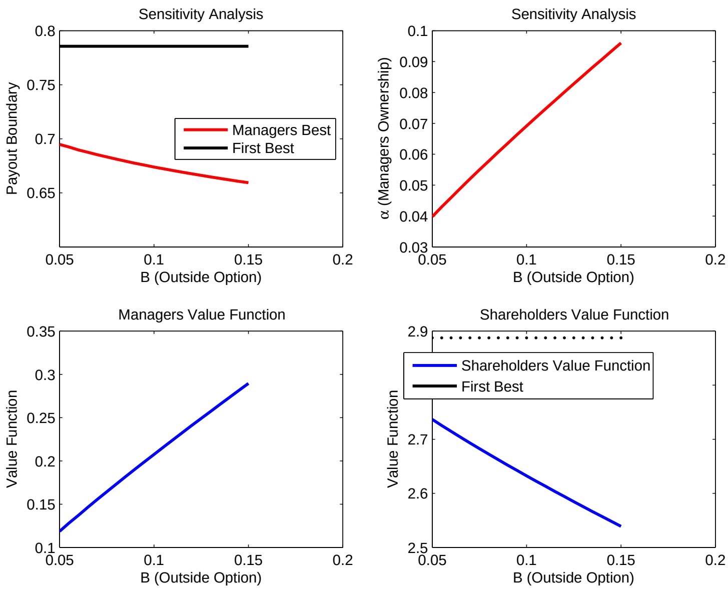

Figure 8: **Sensitivity Analysis with B**, The figure plots the sensitivity analysis of the controls $\overline{W}$ (payout policy) and α (managers ownership), and of the value functions M and S with respect to a change of the parameter B (outside option if the manager).

On the top-left graphic we find the change of the payout boundary decided by managers in the first best case and with the conflict of interests. The payout threshold when there is a conflict of interest is significantly below the payout threshold in the first base case. In the first base case the payout boundary is independent of the outside option as we assume that there is no conflict of interests. The payout threshold is about 10% lower when the outside option value is low (0.05) and about 15% lower when the outside option value is high (0.15)).

We see that the second best payout threshold is decreasing with the outside option of the manager B.

On the top-right graphic we find the optimal ownership-compensation granted to the manager α. The ownership is sharply increasing with the outside option. All other things being equal, when the outside option is more valuable the shareholders need to pay optimally more the managers than in the case where the outside option is less valuable.

Moreover on the bottom-left graphic we see that the value of the compensation package is sharply increasing with the outside option of the managers. The relationship is monotonic.

On the bottom-right graphic we observe that the shareholders value function is instead sharply decreasing with the outside option of the manager in the friction case. In the first base case, as explained before, shareholders value function is independent of managers outside option.

In figure 9 we find the inefficiency costs.

The costs are increasing with the outside option of the managers because the distortion of the payout policy is more pronounced when the outside option is more valuable. Moreover we observe a significant increase of the inefficiency

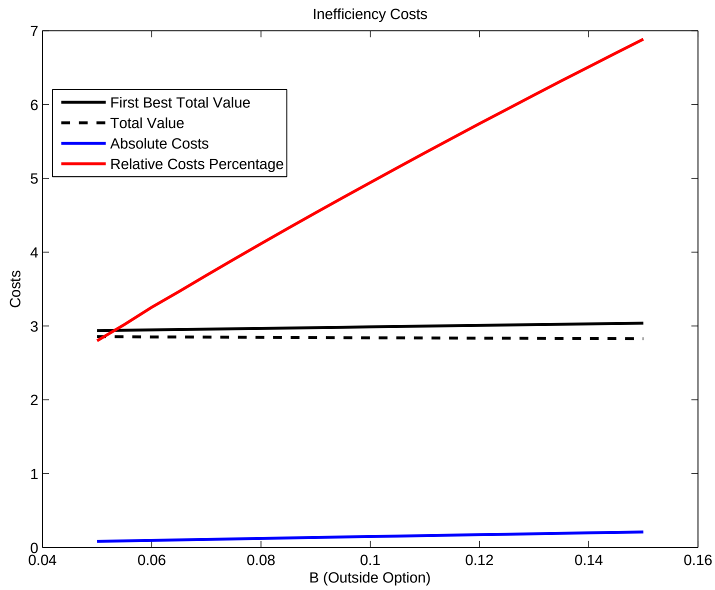

Figure 9: **Sensitivity Analysis of Inefficiency Costs with B (outside option of the manager)**, The figure plots the inefficiency costs ($S^{FB} + B - \mathcal{M} - \mathcal{S}$) in absolute value and as a percentage of the first base case total value ($S^{FB} + B$), the total value in the first best case and in the case with the outside option friction ($\mathcal{M} + \mathcal{S}$). All in function of the parameter B.

costs as a percentage of the first best case. The total value is slightly decreasing in the outside option friction case and the inefficiency costs are increasing with respect to the outside option.

5.5 Summary

You can find a summary of the sensitivity analysis in **Table** 2. The mains results are:

1. Only shareholders benefit from an increase of firms profitability.
2. The inefficiency costs are decreasing with firms profitability and the discount rate.
3. The loss for shareholders is more than compensated for managers when the risk neutral discount rate λ is high. Therefore managers benefit from high discount rates.
4. Firms with high cash-inflows volatility should pay less dividends but compensate more managers all things being equal than firms with high profitability.

6 Conclusion

I study the impact of managerial outside option on payout policy and shareholders wealth.

Table 2: **Summary for the sensitivity analysis**
P.B. stands for payout boundary. $\mathcal{S}$ is shareholders value function, $\mathcal{M}$ is managers value function and I.C. stands for inefficiency costs.

Param.	Dir.	**P. B.**	Dir.	$\mathcal{S}$	Dir.	$\mathcal{M}$	Dir.	**I. C.**	Dir.
μ	$\nearrow$	$\overline{W}$	$\nearrow\searrow$	$\mathcal{S}$	$\nearrow$	$\mathcal{M}$	$\searrow$	Costs	$\searrow$
σ	$\nearrow$	$\overline{W}$	$\nearrow$	$\mathcal{S}$	$\searrow$	$\mathcal{M}$	$\nearrow$	Costs	$\nearrow$
λ	$\nearrow$	$\overline{W}$	$\searrow$	$\mathcal{S}$	$\searrow$	$\mathcal{M}$	$\nearrow$	Costs	$\searrow$
B	$\nearrow$	$\overline{W}$	$\searrow$	$\mathcal{S}$	$\searrow$	$\mathcal{M}$	$\nearrow$	Costs	$\nearrow$

Corporate bankruptcy may impose personal economic costs on top executives. However top executives may still benefit from outside options which make them less risk averse than shareholders.

It is not clear how costly this risk making incentive is for shareholders.

I find that a significant ownership of about 7% must be granted to managers in order to maximize shareholders wealth. This percentage is much higher than the one granted to a median CFO operating in a listed company.

Another key result is that shareholders should benefit from an increase of firms profitability by getting more dividends and paying less managers. The opposite result is found for firms volatility. An increase of cash-inflows volatility, all other things being equal, reduces dividends paid and increases managers compensation at optimum.

The essay focuses mainly on payout policy: An investment model could be a further extension.

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Appendix

A. Model Solution with Details

Managers

The optimization program is:

$$\begin{aligned}\mathcal{M}(W_t) &= \sup_{\{\overline{W},\tau\}} M(\overline{W},\tau,W_t),\\ M(\overline{W},\tau,W_t) &= \mathbb{E}[\textstyle\int_{t\wedge\tau}^{\tau} e^{-\lambda s}\alpha dD_s + e^{-\lambda\tau}B],\end{aligned}$$

where α is given. If we assume that the value function is smooth enough ($\mathcal{C}^2$), by applying Ito's lemma managers value function has for dynamics:

$$\sup_{\{dD_t\}} (\alpha - M_W)dD_t + (M_W\mu + \tfrac{1}{2}\sigma^2 M_{WW})dt + M_W\sigma dZ_t$$

Then using the standard principle of optimality we obtain the following HJB equation for $M(W)$:

$$\lambda M(W) = \sup_{\{dD\}} (\alpha - M_W)dD + M_W\mu + \tfrac{1}{2}\sigma^2 M_{WW}$$

Therefore dD must be as high as possible whenever $M_W < \alpha$. It is trivial to show that it leads to a unique payout barrier policy if $M(W)$ is concave. Assuming for the moment that $M(W)$ is concave then a unique closed form solution can be found for $M(W)$ with the following boundary matching conditions.

$$\begin{aligned}M(\underline{W}) &= B,\\ M_W(\overline{W}) &= \alpha\end{aligned}$$

The first condition is trivial by no arbitrage given that managers will never liquidate the firm as long as $M(W) > B$.

The second condition comes from the differentiation of the optimality condition at payout boundary $M(W) - M(\overline{W}) \geq \alpha(W - \overline{W})$ with $\overline{W} \leq W$.

Given the HJB equation a natural candidate for the solution is:

$$M(W) = A_1 e^{\zeta_+ W} + A_2 e^{\zeta_- W},$$

Where $\zeta_{+,-}$ are the roots solving $\frac{1}{2}\sigma^2\zeta^2 + \mu\zeta - \lambda = 0$:

$$\zeta_{(+,-)} = \tfrac{-\mu(\pm)\sqrt{\mu^2+2\lambda\sigma^2}}{\sigma^2}.$$

and A_1, A_2 can be found with both previous conditions:

$$
\begin{aligned}
M(\underline{W}) &= B = A_1 e^{\zeta_+ \underline{W}} + A_2 e^{\zeta_- \underline{W}}, \\
\Rightarrow A_1 &= \tfrac{B - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}, \\
M_W(\overline{W}) &= \alpha = \zeta_+ A_1 e^{\zeta_+ \overline{W}} + \zeta_- A_2 e^{\zeta_- \overline{W}}, \\
\Rightarrow \alpha &= \zeta_+ [\tfrac{B - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}] e^{\zeta_+ \overline{W}} + \zeta_- A_2 e^{\zeta_- \overline{W}}, \\
\Rightarrow \alpha &= \zeta_+ \tfrac{B e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}} + A_2 (\zeta_- e^{\zeta_- \overline{W}} - \tfrac{\zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}}), \\
\Rightarrow \alpha &= \zeta_+ \tfrac{B e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}} + A_2 (\tfrac{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}}), \\
\Rightarrow A_2 &= \alpha \tfrac{e^{\zeta_+ \underline{W}}}{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}} - B \zeta_+ \tfrac{e^{\zeta_+ \overline{W}}}{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} e^{\zeta_+ \overline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}, \\
\Rightarrow A_2 &= \alpha \Delta(\underline{W}) - B \zeta_+ \Delta(\overline{W}), \\
\Delta(W) &= \tfrac{e^{\zeta_+ W}}{\zeta_- e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}.
\end{aligned}
$$

Then the optimal liquidation and payout boundary can be found numerically with the following first order conditions (e.g. see Dumas 1991):

$$
\begin{aligned}
M_W(\underline{W}) &= 0, \\
M_{WW}(\overline{W}) &= 0.
\end{aligned}
$$

In particular I use the "trust-region-reflective" algorithm in matlab as the gradient of the value function can be computed in closed form:

$$
\begin{aligned}
A^1_{\underline{W}} &= \tfrac{-(A^2_{\underline{W}} e^{\zeta_- \underline{W}} + A^2 \zeta_- e^{\zeta_- \underline{W}}) e^{\zeta_+ \underline{W}}}{e^{2\zeta_+ \underline{W}}} - \tfrac{\zeta_+ (B - A^2 e^{\zeta_- \underline{W}}) e^{\zeta_+ \underline{W}}}{e^{2\zeta_+ \underline{W}}}, \\
A^2_{\underline{W}} &= \alpha \tfrac{\zeta_+ e^{\zeta_+ \overline{W}} \underline{\Delta} - e^{\zeta_+ \underline{W}} \Delta_1}{\underline{\Delta}^2} + \tfrac{B \zeta_+ e^{\zeta_+ \overline{W}} \Delta_1}{\underline{\Delta}^2} \\
\underline{\Delta} &= \zeta_- e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}, \\
\Delta_1 &= \zeta_- \zeta_+ (e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}), \\
A^1_{\overline{W}} &= -A^2_{\overline{W}} \tfrac{e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}} \\
A^2_{\overline{W}} &= -\alpha \tfrac{e^{\zeta_+ \underline{W}} \Delta_2}{\underline{\Delta}^2} - B \tfrac{\zeta_+^2 e^{\zeta_+ \overline{W}} \underline{\Delta} - \zeta_+ e^{\zeta_+ \overline{W}} \Delta_2}{\underline{\Delta}^2}, \\
\Delta_2 &= \zeta_-^2 e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - \zeta_+^2 e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}, \\
\text{grad} : M(W) &= [A^1_{\underline{W}} e^{\zeta_+ W} + A^2_{\underline{W}} e^{\zeta_- W}, A^1_{\overline{W}} e^{\zeta_+ W} + A^2_{\overline{W}} e^{\zeta_- W}].
\end{aligned}
$$

Shareholders

The optimization program is:

$$
\begin{aligned}
\mathcal{S}(W_t) &= \sup_{\alpha} S(\alpha, W_t), \\
S(\alpha, W_t) &= \mathbb{E}[\textstyle\int_{t \wedge \tau}^{\tau} e^{-\lambda s} (1-\alpha) dD_s + e^{-\lambda \tau} (1-\beta) W_\tau].
\end{aligned}
\tag{20}
$$

where both τ and the cumulative dividend process D_t depend on the compensation control α. If we assume that the value function is enough smooth, by applying Ito's lemma shareholders value function has for dynamics:

$$\sup_{\{\alpha\}} [(1-\alpha) - S_W]dD_t + (S_W\mu + \tfrac{1}{2}\sigma^2 S_{WW})dt + S_W\sigma dZ_t$$

Then using the standard principle of optimality we obtain the following HJB equation for $S(W)$:

$$\lambda S(W) = \sup_{\{\alpha\}} [(1-\alpha) - S_W]dD + S_W\mu + \tfrac{1}{2}\sigma^2 S_{WW}$$

where dD depends on α too (the solution is non trivial). Assuming for the moment that $M(W)$ is concave then a unique closed form solution can be found for $S(W)$ with the following boundary matching conditions.

$$\begin{aligned} S(\underline{W}) &= (1-\beta)\underline{W}, \\ S_W(\overline{W}) &= 1-\alpha \end{aligned}$$

The first condition is given by assumption.

The second condition comes from the differentiation of the optimality condition at payout boundary $S(W) - S(\overline{W}) \geq (1-\alpha)(W - \overline{W})$ with $\overline{W} \leq W$.

Given the HJB equation a natural candidate for the solution is:

$$S(W) \;\; = A_1 e^{\zeta_+ W} + A_2 e^{\zeta_- W},$$

Where $\zeta_{+,-}$ are the roots solving $\frac{1}{2}\sigma^2\zeta^2 + \mu\zeta - \lambda = 0$ (the same roots are in managers value function):

$$\zeta_{(+,-)} = \frac{-\mu(\pm)\sqrt{\mu^2 + 2\lambda\sigma^2}}{\sigma^2}.$$

and A_1, A_2 can be found with both previous conditions:

$$\begin{aligned}
S(\underline{W}) &= (1-\beta)\underline{W} = A_1 e^{\zeta_+ \underline{W}} + A_2 e^{\zeta_- \underline{W}}, \\
\Rightarrow A_1 &= \frac{(1-\beta)\underline{W} - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}, \\
S_W(\overline{W}) &= (1-\alpha) = \zeta_+ A_1 e^{\zeta_+ \overline{W}} + \zeta_- A_2 e^{\zeta_- \overline{W}}, \\
\Rightarrow (1-\alpha) &= \zeta_+ [\frac{(1-\beta)\underline{W} - A_2 e^{\zeta_- \underline{W}}}{e^{\zeta_+ \underline{W}}}] e^{\zeta_+ \overline{W}} + \zeta_- A_2 e^{\zeta_- \overline{W}}, \\
\Rightarrow (1-\alpha) &= \zeta_+ \frac{(1-\beta)\underline{W} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}} + A_2(\zeta_- e^{\zeta_- \overline{W}} - \frac{\zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}}), \\
\Rightarrow (1-\alpha) &= \zeta_+ \frac{(1-\beta)\underline{W} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}} + A_2(\frac{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}{e^{\zeta_+ \underline{W}}}), \\
\Rightarrow A_2 &= (1-\alpha)\frac{e^{\zeta_+ \underline{W}}}{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}} - (1-\beta)\underline{W}\zeta_+ \frac{e^{\zeta_+ \overline{W}}}{\zeta_- e^{\zeta_- \overline{W}} e^{\zeta_+ \underline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}, \\
\Rightarrow A_2 &= (1-\alpha)\Delta(\underline{W}) - (1-\beta)\underline{W}\zeta_+ \Delta(\overline{W}), \\
\Delta(W) &= \frac{e^{\zeta_+ W}}{\zeta_- e^{\zeta_+ \underline{W}} e^{\zeta_- \overline{W}} - \zeta_+ e^{\zeta_- \underline{W}} e^{\zeta_+ \overline{W}}}. \\
A_2 &= (1-\alpha)\Delta(\underline{W}) - \zeta_+ (1-\beta)\underline{W}\Delta(\overline{W}),
\end{aligned}$$

where both boundaries $\underline{W}$ and $\overline{W}$ are decided by managers and found with conditions under (12) given a contract α. Both boundaries are function of the ownership parameter α which can be found by maximizing $S(W)$ subject to the condition $\alpha \in [0,1]$. In particular I use the "sequential quadratic programming" algorithm in matlab as the value function is assumed to be twice continuously differentiable and the gradient cannot be computed in closed form.

B. Concavity of the Value Function

The managers value function is assumed concave in Appendix A. I first prove lemma 1 then use lemma 1 to show that the value function is concave.

lemma 1: *The managers want to liquidate the firm immediately (endogenous liquidation) or as late as possible (exogenous liquidation)*

At project initiation $t = 0$ managers decide to run the firm if the following condition is satisfied:

$$M(W_0) > B,$$

Because $M(0) = B$ by assumption, if managers decide to run the firm and if the value function $M(W)$ is strictly monotone increasing, then $M(W)$ along the optimal path can never fall below B. In this case the manager has no incentives to liquidate the firm early. It remains only to prove that the value function is strictly monotone increasing with respect to W. In the cash accumulation region $0 < W < \overline{W}$ the value function has for dynamics

$$\lambda M(W) = \mu M_W + \tfrac{1}{2}\sigma^2 M_{WW}$$

as long as $M_W > \alpha$. In the dividend paying region $W > \overline{W}$ the dynamics is:

$$\lambda M(W) = \sup_{\{dD\}} (\alpha - M_W)dD + \mu M_W + \tfrac{1}{2}\sigma^2 M_{WW}$$

with $M_W = \alpha$ and $M_{WW} = 0$ which leads to $M(W) = \frac{\alpha\mu}{\lambda} + \alpha(W - \overline{W})$. Therefore the value function is strictly monotone increasing along the optimal path with $\alpha > 0$ if the manager decides to run the firm.

Using **lemma 1** it becomes trivial to show that the payout boundary is unique, then we can prove that the value function is concave.

If the managers decide to run the firm, the value function may be simplified:

$$\begin{aligned}
A_1 &= B - A_2 \\
A_2 &= \alpha\frac{1}{\zeta_- e^{\zeta_-\overline{W}} - \zeta_+ e^{\zeta_-\overline{W}}} - B\zeta_+\frac{e^{\zeta_+\overline{W}}}{\zeta_- e^{\zeta_-\overline{W}} - \zeta_+ e^{\zeta_+\overline{W}}}, \\
\Rightarrow A_2 &= \alpha\Delta - B\zeta_+ e^{\zeta_+\overline{W}}\Delta, \\
\Delta &= \frac{1}{\zeta_- e^{\zeta_-\overline{W}} - \zeta_+ e^{\zeta_+\overline{W}}}, \\
\Rightarrow M(W) &= (\alpha\Delta + B(1 - \zeta_+ e^{\zeta_+\overline{W}}\Delta))e^{\zeta_+ W} + (\alpha\Delta - B\zeta_+ e^{\zeta_+\overline{W}}\Delta)e^{\zeta_- W}
\end{aligned}$$

and the dynamics around the unique payout boundary is:

$$\lambda M(W) = \mu M_W + \tfrac{1}{2}\sigma^2 M_{WW}$$

If we differentiate one more time we find at the payout boundary:

$$\begin{aligned}
\lambda\alpha &= \tfrac{1}{2}\sigma^2 M_{WWW}, \\
\Rightarrow M_{WWW} &= \tfrac{2\lambda\alpha}{\sigma^2} > 0
\end{aligned}$$

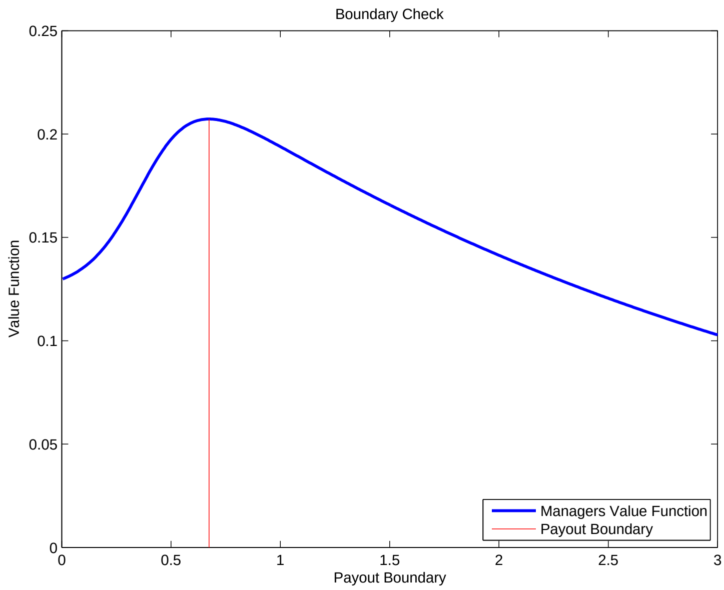

Figure 10: **Boundary Check**, The figure plots $M_{\overline{W}}(W = \overline{W}) = 0$

as $M_{WW}(\overline{W}) = 0$ and $M(\overline{W}) = \alpha$, $M_{WW}(W) < 0, \forall W < \overline{W}$ else we can find a counter example e.g. see Décamps, Mariotti, Rochet and Villeneuve (2011) p.28-p.29.

C. Payout Boundary Check

I check here that the payout boundary of the baseline model satisfies the FOC condition $M_{\overline{W}}(W = \overline{W}) = 0$. The result can be found in figure 10. The FOC is satisfied.

Managerial Optimism and Corporate Policies*

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Abstract

I examine the impact of managerial optimism on corporate policies and agency costs within a model of dynamic corporate investment for a financially constrained firm. The main feature of the model is to allow joint investment and saving policies. In addition to the common sense that an optimistic manager should build an empire by investing more in average and keeping more cash under control, I find the following primary results: 1) when the firm is highly financially constrained, an optimistic manager under-invests and can oversell the physical assets. 2) The investment sensitivity to cash is lower. 3) The agency costs sensitivity to cash is higher and this reinforces the need to monitor the manager.

Keywords: Agency conflicts; Managerial optimism; Empire building; Investment; Cash management

JEL Classification Numbers: G02; G31; G32; G34; G35

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1 Introduction

1.1 Overview

The contribution of this essay is to better understand the impact of managerial optimism on important corporate decisions.

An important theme in corporate finance is that incentive conflicts within the firm lead to distortions in corporate policy choices and to lower firm performance. With a model of dynamic corporate investments and financing for a financially constrained firm I am able to examine jointly the optimal financing, payout, cash-inventory and investment policies decided by the manager. Considering intertwined decisions permits to find new and empirically testable explanations of managers behavior and to better understand how the interests of the managers could be aligned with the ones of the shareholders at the lowest cost. In the model I assume that the manager is optimistic and that this bias of perception generates a conflict of interests between the shareholders and the manager. I analyze how the manager's optimism affects the main corporate decisions made and I evaluate how costly are these decisions for the shareholders. In particular, I obtain the following results. The optimistic manager has an incentive to build an empire: he wants to invest more in average, to keep more cash under management and to reduce payouts. However when the manager is highly financially constrained he can underinvest, oversell the physical assets, and the investment sensitivity can be lower. Moreover the costs for the shareholders of the manager's self interested decisions are mainly generated when the manager is highly financially constrained and this result supports the view that the manager should be monitored especially during periods of trouble.

Modern research has concentrated on conflicts of interests since the seminal agency theory proposed by Jensen and Meckling (1976). However, one important missing point in the neoclassical theory of the firm is the absence of managerial attitudes in creating corporate policies. In reality, the manager does not make decisions for maximizing the investors wealth, because he does not share the same interests. After three decades of empirical and theoretical research we are now able to better understand the main managerial incentives assuming that the manager is rational. Nevertheless can we reasonably assume that the manager is always rational? I suppose that this is not the case and I propose a new framework to understand managers' behavior.

1.2 Contribution

I start by presenting the model framework then I explain the main results obtained.

I develop a model in the spirit of Bolton, Chang, Wang, henceforth BCW (2011) dynamic model. In the model the manager is optimistic. I define a manager as optimistic when he overestimates the probability of good firm performance and underestimates the probability of bad firm performance like in Heaton (2002). For avoiding to embed other conflicts of interests, I suppose the manager to be remunerated like shareholders and therefore his wealth depends only on the firm value he perceives. The manager continuously decides the optimal level of investment, cash-inventory, payout, external funds and whether the assets should be liquidated. The decisions of the manager depend on two state variables, the level of the cash-inventory and physical assets. The cash can be used for investing or paying dividends and can be stored for avoiding inefficient closure and external financing costs. The incremental productivity of the physical capital has for dynamics an arithmetic Brownian motion and the cost paid for changing the level of capital is a convex function of the amount sold or invested. Hence both assets (physical and cash) are not perfect substitutes. As a starting point I assume also that the manager is not constrained when taking decisions. I relax this assumption later.

Beyond the conflict of interests between the manager and the shareholders, three frictions are assumed. Firstly, the investment is not perfectly reversible. This generates an incentive to trade-off the profitability of physical and cash assets. The physical assets cannot be used directly for avoiding inefficient closure and should be sold for avoiding bankruptcy, whenever the level of cash is too low. I assume that the bankruptcy costs represent a fixed fraction of the value of the assets during liquidation. Secondly, the firm is financially constrained, because any external funding is costly. Therefore there is a precautionary motive for increasing the cash-inventory and avoiding an inefficient closure. The firm falls into bankruptcy when the manager is forced to liquidate all the assets. Finally the cash inventory generates a return below the risk free rate and this creates an incentive to payout dividends whenever the level of cash becomes to high.

This framework corresponds to the BCW (2011) dynamic model assumptions where I embed the hypothesis that the manager is optimistic. Assuming manager's irrationality within a dynamic model, where the manager is able to save and to invest cash at the same time, I am able to find new theoretical mechanisms.

The first result I obtain is that manager's optimism generates an empire building incentive. I define the empire building desire as the incentive to increase the size or the influence of the firm and I find that an optimistic manager wants to invest more, to keep more cash and to reduce the payouts. This clearly corresponds to a situation where the manager wants to increase the size of the firm. These results are intuitive considering that the manager has an incentive to grow the size of the firm, whenever he overevaluates the profitability of the assets. Empire building is well known since Baumol (1959) and also been documented by Donaldson (1984) or Jensen (1986) among others. However it appears interesting to see that an empire building situation arises without assuming private monetary benefits or without considering the role of non-monetary form of remunerations like power or prestige as in Williamson (1963,1974).

A second result concerns investment distortions. This result highlights the key theoretical contribution of the paper. I find that the manager has an incentive to overinvest when the financing constraints are not too important and an incentive to under-invest or to sell the assets when he is more constrained. Considering an average sample or firms we can reasonably assume that an optimistic manager will overinvest in average. BCW by simulating the cash-capital ratio find that the cash inventory is equal to about 16% of the value of the physical assets. In the model I observe that the manager begins to underinvest only when the cash-ratio is equal to or below about 6%. However, as the frontier between a constrained firm and an unconstrained firm is difficult to determine, it seems difficult to anticipate whether an optimistic manager wants to overinvest or to underinvest in average for a sample of financially constrained firms. This result clearly expands on Malmendier and Tate (2005) insight. For example they do not model the role of cash-inventory in corporate decisions and argue that the use of internal funds could permit to achieve first-best investment level. I find that the internal funds do not permit to align the interests of the agents. I observe that there is only one level of cash inventory where interests are aligned and it does not correspond necessarily to the one that maximizes shareholders wealth. Moreover Malmendier and Tate find that the manager wants to curtail investments when he needs external funds and I find that the manager can cut investments before being in a distressed situation that requires external funds. More than highlighting the non intuitive result that an optimistic manager can underinvest during periods of trouble, I emphasize that a manager will not necessarily underinvest because he thinks that the stocks are underevaluated. Instead I observe that an optimistic manager by overevaluating the profitability of the physical assets will also overevaluate the value of the cash-inventory. Because the value of cash increases with the degree of the financing constraint, when the manager is highly constrained there is a point where he prefers to underinvest in order to increase the cash-inventory for avoiding an inefficient closure.

The third result I obtain concerns the investment sensitivity of cash. Even if the optimism of the manager increases the investment sensitivity of cash in average, the sensitivity is not necessarily higher when cash permits to reduce significantly the probability of early liquidation or external financing costs. During periods of trouble I find that the optimistic manager can prefer to accumulate cash and to keep investment at a low level. This result permits to understand why firms with more cash invest more in average and why at the same time investment sensitivity of cash or of cash-flows can be a bad proxy of the financing constraints degree. Again this result contrasts with past findings as Malmendier and Tate find that the investment sensitivity is higher when the firm is financially constrained. Instead I find like Fazzari, Hubbard and Petersen (1988) or Kaplan and Zingales (1997, 2000) that financially distressed firms are likely to have lower investment-cash flow sensitivities than less financially constrained firms. Moreover as argued by Kaplan and Zingales I observe that the relation between the degree of the financing constraints and the investment sensitivity should be not monotonic and that we cannot argue that firms with more cash holdings are more financially constrained.

Finally, I find that the agency costs are a concave increasing function of the cash inventory. The costs are generated mainly when the manager is highly financially constrained, when there is less cash inside the firm and when the value of cash is high. Consecutively, monitoring the manager appears significantly more important during periods of trouble than during periods of opulence where the manager is naturally more entrenched. The intuition of this result is the following: when the manager is slightly or not financially constrained he has an incentive to overinvest. Therefore the manager can decide to reduce payouts and to increase cash under management. This decision generates a loss for the shareholders that mainly resides in the impossibility to invest the liquidity captured in more profitable projects. However this cost can be seen as extremely low in comparison to the costs supported when the manager underinvests. During periods of trouble shareholders consider that the value of cash is significantly lower and would like to take more risks by investing more.

1.3 State or Research

The results I find are difficult to align with the ones obtained by assuming manager's rationality. Notwithstanding, this work is related to the modern dynamic corporate policies research with agency costs. Two important starting references are Mello, Parsons (1992) and Leland (1998). Given the number of subsequent studies, it appears useless to present an exhaustive list. Instead, we can mention that, recently, Morellec, Nikolov and Schuerhoff (2012) show that agency costs have significantly better explanatory power than financing costs for explaining leverage decisions observed and that the paper which is most closely related to this research is DeMarzo, Fishman, He and Wang (2012). They use a similar BCW framework integrating agency frictions, where the manager equilibrium effort determines the volatility of the financial slack. They look for an optimal contract that specifies for the agent the payout policy, investment policy and termination date, all as functions of the firm's profit history. My objective is precisely to avoid an optimal contracting approach, because appropriate incentives are particularly difficult to identify as reported by Stein (2003), when the manager is supposed to be irrational.

Assuming manager's irrationality, it appears rational to choose one of the main and significant bias of perception that has been emphasized by the behavioral corporate finance literature: the optimism. The experimental research in psychology leads to the conclusion that people are optimistic about outcomes that they believe they can control (Weinstein 1980). A result consistent with the studies of Larwood and Whittaker (1977), Ben-David, Graham and Harvey (2007) or March and Shapira (1987), where managers underplay inherent uncertainty when they believe that they have large control over the firm's performance and with the experimentation of Camerer and Lovallo (1999) where subjects display optimism when making entry decisions. Interestingly, despite the difficulty to measure manager's optimism, empirical and theoretical studies may support the behavioral bias for providing valuable corporate investment, mergers and acquisitions or financing explanations. In the following, I present an overview of the main results so far.

Empirically, Merrow, Phillips and Myers (1981) and Statman and Tyebjee (1985) observe that there is a strong optimism bias in project costs or sales forecasts. Cooper, Woo and Dunkelberg (1988) find that entrepreneurs overestimate the chance of success of the projects. In a Graham's survey during the internet bubble (1999), almost two-thirds of CFOs state their stock is undervalued while only three percent think it is overvalued. Malmendier and Tate (2005, 2008) identifies that the sensitivity of investment to cash flow is higher for the more optimistic CEOs and that optimistic CEOs complete more mergers and especially diversifying mergers.

Theoretically, Roll (1986) argues that managerial hubris or overconfidence about synergies valuation can explain overpayment by acquiring firms in takeovers. This should be seen as a form of overinvestment, but most importantly Roll is able to interpret the evidence on merger announcement effects and the lack of evidence of fundamental value creation through mergers surveyed by Jensen and Ruback (1983) and more recently by Andrade, Mitchell, and Stafford (2001) or Moeller, Schlingemann and Stulz (2004)[9]. Moreover he provides a framework for the studies of Malmendier and Tate (2005, 2008). Heaton (2002) finds that managerial optimism leads to an underinvestment-overinvestment trade-off without supposing asymmetric information or rational agency costs. Heaton models that an optimistic manager has an incentive to under-issue equities, because he thinks that the investors underevaluate the stocks price. Consecutively the manager prefers internal funds for financing investment consistently with the pecking order theory of Myers and Majluf (1984). At the same time Heaton argues that an optimistic manager would like to overinvest, because he overevaluates the benefits of the investments. This view corresponds to the free cash-flow theory of Jensen (1986) as the free cash-flows are used inefficiently by the manager.

These results clearly indicate that manager's optimism is a significant behavioral bias permitting to explain many of the corporate decisions observed[10]. Interestingly, the theoretical study I propose departs from previous analyses about manager's optimism in many directions and permits to provide a rich set of new theoretical predictions.

I proceed as follows. Section 2 sets up the model and the main hypotheses. Section 3 presents the model solution. Section 4 proposes a quantitative analysis. Section 5 analyses the specific problem of investment spending and section 6 concludes.

2 Model Setup

I begin describing the firm's production technology, then I present the financing hypotheses and finally state agents optimality. The model is built according to

[9] The current version of the literature points out that mergers overall for the society create value.

[10] See Baker, Ruback, Wurgler (2004) for an exhaustive survey.

the framework proposed by BCW. I use similar notations for helping comparisons.

2.1 Investment Technology

The firm needs physical assets for producing. I denote by K the capital stock and by I the gross investment. For representing the dynamic, the variables are indexed by time. I denote time by t, $t \geq 0$. Investment opportunities are constant over time and the price of capital is normalized to unity i.e. without depreciation $dK_t = I_t dt$. I assume a depreciation rate $\delta \geq 0$. The dynamics of the physical capital K becomes:

$$dK_t \quad = (I_t - \delta K_t)dt \tag{1}$$

Physical assets permit to generate cash-flows. I denote the productivity of the assets by A. Moreover I assume market completeness and that there is no scale economy. The productivity of the assets under the risk neutral measure evolves according to:

$$dA_t \quad = \mu dt + \sigma dZ_t \tag{2}$$

Z being a standard Brownian motion, $\mu \geq 0$ and $\sigma > 0$ the constant parameters of the risk-adjusted dynamics of the productivity. This setup is identical to the one proposed by BCW 2011 and makes references to the "AK" technology in the macroeconomic literature (see Cox, Ingersoll and Ross 1985).

I assume that the cash generated by the assets in place can be saved or invested. Internal financing setup will be presented in the next section. The investment is costly. I denote $G(I,K)$ the investment costs function, $G(I,K) = \frac{\theta I^2}{2K}$, with $\theta > 0$ the constant parameter measuring the degree of the adjustment costs. For constant ratio $\frac{I}{K}$, the investment costs are homogeneous of degree one in K. This assumption is standard in neoclassical investment literature (Hayashi 1982). Remark that $G(I,K) = \frac{\theta i^2}{2} K \equiv g(i)K$, with $i = \frac{I}{K}$ the investment ratio. The functional form chosen for $g(i)$ is not important as long as $g(i)$ is convex and the homogeneity assumption for $G(I,K)$ is kept.

Grouping these early pieces of information we can compute revenues from investment and production. I denote Y these operating cash-flows:

$$dY_t \quad = K_t dA_t - I_t dt - G(I_t, K_t)dt \tag{3}$$

With this continuous time setting it is convenient to represent operating cash-flows post investment. Finally, I assume for the production technology that the assets in place have a liquidation value L_t at time t. $L_t = lK_t$, with $l > 0$ a constant parameter. In case of bankruptcy, I consider that the assets in place are immediately liquidated: the bankruptcy value of the assets in place depends on the book value and not on the going concern value. I denote the bankruptcy time by τ, $\tau \geq t$.

2.2 Financing Technology

The manager can finance investment using external or internal funds. I first define external financing. I denote H_t the cumulative external financing amount at time t and X_t the cumulative costs associated for raising these funds. I assume that managers do not have access to debt financing[11]. We may interpret dH_t as seasoned equity offerings. I denote $dH_t = mK_t dt$, with $m > 0$ the proportion of existing capital issued as equity. Each time the manager decides to issue equity a cost X_t is incurred. I assume that the cost is proportional to the amount issued, $dX_t = \gamma dH_t$ with $\gamma > 0$ a constant parameter. Considering this financing friction, the Modigliani and Miller theorem (1958) does not hold.

I define W_t the cash inventory at time t. Without considering other agents like lenders it appears that an early liquidation is not optimal. The manager will liquidate or use external financing only when the firm runs out of cash. I assume that the cash inventory permits to generate earnings. However the rate of return under the risk neutral measure is below the risk free rate. This assumption can be justified by agency costs like it is standard in models with cash (Kim, Mauer and Sherman 1998 or Riddick and Whited 2009) or assuming that the taxes at firm level are higher than the ones supported by the shareholders for receiving dividends. This low rate of return increases the incentive to payout dividends to the shareholders, when the firm does not face sufficiently profitable investment opportunities. In particular, I assume that the cash kept inside the firm grows at a rate equal to $r - \lambda$ with $\lambda \in]0, r[$ a constant parameter.

Finally, I denote by U_t the firm's cumulative payout to shareholders at time t. For simplifying computations, distributing cash to shareholders is assumed to be cost free. Then the cash inventory evolves according to:

$$dW_t \quad = dY_t + dH_t - dX_t - dU_t + (r - \lambda)W_t dt \tag{4}$$

where the manager determines endogenously dH_t, dU_t and I_t. The optimal amount of savings dW_t is found by the accounting identity and is equal to the operating cash-flows plus the external cash financing and interests income minus the dividends and costs of issuing.

2.3 Agents Optimality

I consider two types of agents: the manager and the shareholder. I state first firm's optimality from the point of view of the shareholders.

2.3.1 Shareholders

I call the control solutions that maximize firm value from the perspective of the shareholders "first best". Indicating the optimum with an over-line I denote

[11] Extending the model for considering debt or credit line financing is possible but only with additional technical and strong assumptions. For example, it requires to add boundary conditions that will be satisfied only for a very small set of parameters.

"First Best" firm value $\overline{p}^{FB}$. The 'first best' solution is reached when the manager chooses dH_t, dU_t, I_t and τ for maximizing the expected present value of the sum of the payments made to the shareholders:

$$\overline{p}^{FB}(K_t, W_t) = \max_{\{dH_t, dU_t, I_t, \tau\}} \mathbb{E}[\int_t^{\tau} e^{-rs}(dU_s - dH_s - dX_s) + e^{-r\tau}(lK_\tau + W_\tau)] \quad (5)$$

K_t and W_t are the state variables and the expectation is computed under the risk-neutral probability. Usual regularity conditions are assumed to ensure that the problem is well posed.

2.3.2 Managers

The controls are not necessarily chosen optimally by managers if we assume that managers are optimistic and over-evaluate the productivity of assets. This is the key assumption of this essay. The optimism of managers creates a conflict of interests between managers and shareholders which may be costly for shareholders if managers are free to decide corporate policies[12]. Therefore I assume that managers are essential for firms and cannot be replaced and I model manager's optimism with three additional assumptions:

Firstly, I assume that the capital productivity is higher from managers point of view and state that the productivity parameter μ is increased for managers by κ, $\kappa > 0$[13]. The dynamics of the productivity for the manager becomes:

$$dA_t^m \quad = (\mu + \kappa)dt + \sigma dZ_t$$

Secondly, I consider also that the manager's optimism could lead to an overestimation of the profitability of the cash-inventory. For modeling this bias I set that the rate of return anticipated by the manager for the cash-inventory is higher. Returns are increased by an amount α, $\alpha \in]0, r - \lambda[$ and the dynamics of the cash-inventory internal returns becomes $(r + \alpha - \lambda)W_t dt$.

Finally, I assume that the manager is also optimistic regarding the amount of equity he may get for each equity offering. The manager does not anticipate any equity dilution for each equity issuance.

Now I can formalize manager's optimality. The manager chooses the corporate policies in order to maximize the firm value he perceives. I denote p_m the firm value from managers point of view and I state:

$$\overline{p_m}(K_t, W_t) = \max_{\{dH_t^m, dU_t^m, I_t^m, \tau^m\}} \mathbb{E}^m[\int_t^{\tau^m} e^{-rs}(dU_s^m - dH_s^m - dX_s^m) + e^{-r\tau^m}(lK_{\tau^m} + W_{\tau^m})] \quad (6)$$

Please remark that the manager faces an optimization problem similar to the shareholders one, but the dynamics of the value maximizing controls are different. I use the upper-script m for indicating that the processes have changed, when the manager decides the corporate policies, in distinction with the first

[12] I remove the assumption that managers are free to decide payout policies in section (4.3).

[13] In a model extension I also suppose in section 5 that the manager may overestimate the benefit associated with the investment spending. The value of the investment spending for the manager in comparison to the shareholder is multiplied by an amount $1 + \iota$, $\iota > 0$. In this case the value of the assets acquired is increased without even changing the profitability.

best case.

In the next section I present the solution of the model.

3 Model Solution

The manager's optimization problem can be solved by deriving the firm value dynamics and by finding appropriate boundaries. No closed form solutions can be found for the firm valuation. The dynamics of the firm value is a non linear homogeneous Partial Differential Equation (PDE) of degree two. The PDE is parabolic and could be transformed into a form analogous to the heat equation by a change of variables. However, considering both state variables K and W, the optimal corporate policies are difficult to interpret.

A key result of the model is that firm value is homogeneous with respect to the physical capital stock, $p(K,W) = Kp^{\sim}(w)$ [14], $\forall K \in \mathbb{R}_+^*$, with $w = \frac{W}{K}$. The model setting is exploited to transform the PDE into an Ordinary Differential Equation (ODE). The ODE can be easily solved numerically and most importantly it permits to characterize and to interpret the different corporate policies in a simple way as a function of the unique state variable w.

Similarly to BCW 2011, I propose a benchmark solution in which the Modigliani and Miller theorem holds. I pursue by presenting the model solution dynamics for the internal financing region. I identify the boundaries corresponding to external financing and payout policies and I conclude by analyzing firm value for shareholders.

3.1 A Benchmark

An interesting benchmark can be obtained when there are no financing frictions and when the Modigliani and Miller theorem holds. In this case there are no incentives for keeping cash. The cash inventory, if any, grows at a rate equal to the risk free rate and external financing are free of costs. Without financing frictions the marginal value of cash is always equal to one, $p_W = 1$ or equivalently $p_w = 1$. Considering there are also no conflicts of interests, the Hamilton-Jacobi-Bellman (HJB) equation is the following:

$$r\overline{p}(K, W=0) \quad = \max_{I} (I - \delta K)\overline{p}_K + \mu K - I - G(I,K) \tag{7}$$

This equation is easy to interpret. The $\overline{p}_K$ term represents the effect of net investment on firm value and the last term represents the cash-flows generated by the investment policy. Exploiting the homogeneity assumption, the equation can be rewritten in terms of the investment ratio $i = \frac{I}{K}$:

$$r\overline{p}(K, W=0) \quad = \max_{i} K[(i-\delta)\overline{p}_K + \mu - i - g(i)] \tag{8}$$

Now the optimal investment ratio policy i^* can be computed in closed form (details in appendix **A**). First we observe that the benchmark Tobin's q, $\overline{p}_K$

[14] Latter, I drop the superscript ($\sim$) for simplifying the notations.

equals $1 + \theta i^*$. Then plugging this solution inside the ODE permits to find i^*:

$$i^* \quad = r + \delta - \sqrt{(r + \delta)^2 - 2\frac{\mu - (r + \delta)}{\theta}} \tag{9}$$

Obviously i^* is constant, an investment situation corresponds to the case $\mu > r + \delta$. In this case net investments permit to generate higher expected returns than the risk neutral rate. Moreover, remark that if the productivity of the assets is too low for financing optimal investments, external financing resources should be used.

Alternatively a disinvestment situation corresponds to the case $\mu < r + \delta$ where the investments are not profitable enough.

The optimal investment ratio permits to compute the benchmark firm value $\overline{p}(w)$. This benchmark allows comparison with the financially constrained case that I want to analyze. I present now the solutions for the financially constrained firm when the main corporate policies are determined by the manager.

3.2 Internal Financing Region

For solving the model, the key state variable is the cash ratio w. Depending on the cash-ratio level the manager can decide to obtain external funds, to accumulate cash or to pay out dividends. I call internal financing region the range of cash ratio values for which the manager wants to finance investment with internal funds and to accumulate cash. Clearly the boundaries of this region depend on the marginal value of cash $p_w(w)$. When the marginal value of the cash ratio is sufficiently low the manager decides to stop accumulating cash and to pay dividends. When the marginal value of the cash ratio is sufficiently high the manager decides to use external funds for financing investment even if these funds are more costly than the internal funds.

I first present the dynamic of the firm value in the internal financing region. From the point of view of the manager the firm value $p_m(K, W)$ satisfies the following HJB equation:

$$rp_m \quad = (I - \delta K)p_K + [(r + \alpha - \lambda)W + (\mu + \kappa)K - I - G(I, K)]p_W + \frac{\sigma^2 K^2}{2} p_{WW} \tag{10}$$

Using the homogeneity assumption we know that $P_K(K, W) = p(w) - wp_w(w)$, $p_W(K, W) = p_w(w)$ and $p_{WW} = \frac{p_{ww}}{K}$. Substituting these terms into (10) it permits to transform the PDE into an ODE:

$$rp(w) \quad = (i - \delta)(p(w) - wp_w(w)) + [(r + \alpha - \lambda)w + \mu + \kappa - i - g(i)]p_w(w) + \frac{\sigma^2}{2} p_{ww}(w) \tag{11}$$

Please note that i is function of the new state variable w and I have dropped the manager index m for simplifying the presentation. The ODE can be easily interpreted.

On the right hand side of the equality, the left term still represents the effect of net investment on firm value. The marginal q, $q = \frac{d(P(K,W) - W)}{dK}$ (in the benchmark case p_K), has been replaced by $p(w) - wp_w(w)$. Because the firm is now financially constrained, the spending for changing the physical assets generate a cost by reducing the internal funds available. A reduction of the cash inventory increases the expected external financing costs or bankruptcy costs.

The central term represents the effect of the expected savings on firm value. In comparison to the BCW framework the value of the expected savings perceived by the manager is higher: the manager has an incentive to grow the assets in place. Consecutively. the cash generated by the cash inventory or the physical capital, from the point of view of the manager, is more valuable.

Finally the right side term is related to the quadratic variation of the Brownian motion and represents the effect of the volatility of the productivity on firm value.

Now I assume that the manager wants to maximize $\overline{p}_m$, the perceived firm value. Remember that the manager can determine in the internal financing region the investment ratio i. Using a first order condition we can find the following relation for the optimal control:

$$i(w) \quad = \frac{1}{\theta}\left(\frac{p(w)}{p_w(w)} - w - 1\right) \tag{12}$$

Consecutively the dynamic of the firm value for the manager and the shareholders can be identified by plugging the investment ratio obtained. Even if the investment ratio is not controlled by the shareholders, it does not represent an additional state variable. The impact of the modification of the investment ratio is captured by the change of the state variable w. In other words, the investment ratio becomes an additional exogenous parameter for the shareholders optimization problem and the fundamental HJB equation can be defined as:

$$r\overline{p}(w) \quad = (i_m - \delta)(\overline{p}(w) - w\overline{p}_w(w)) + [(r-\lambda)w + \mu - i_m - g(i_m)]\overline{p}_w(w) + \frac{\sigma^2}{2}\overline{p}_{ww}(w) \tag{13}$$

with $i_m = \frac{1}{\theta}\left(\frac{pm(w)}{pm_w(w)} - w - 1\right)$ the investment ratio decided by the manager. I propose now to determine the boundaries for solving the optimization problem numerically.

3.3 Boundaries: External Financing or Liquidation

External use of funds and liquidation decisions are competing together. There are no reasons to liquidate the firm before it runs out of cash. When there is no more cash, external funds are preferable if and only if the liquidations costs $(1 - l)$ are too high.

Intuitively it appears optimal to use external funds when the marginal value of cash is sufficiently high. In this case even if the external financing is costly it permits to increase firm value. The optimal liquidation boundary $w = 0$ is easy to identify whereas the external financing boundary is less intuitive.

Internal and external funds are not competing together as both can be used as investment financing. Consecutively external funds should be used when the marginal value of the cash inventory is higher than the marginal costs $1+\gamma$ borne to obtain the additional financing. However as it will be clear in the quantitative analysis, the marginal value of cash $p_w(w)$ is a monotone decreasing function of the cash ratio in the internal financing region. In this region an increase of the cash ratio helps to relax the financing constraint and we find $p_{ww}(w) < 0$. At the same time the manager **anticipates** that he can obtain external funds even if there are no more cash savings. Therefore, the marginal value of cash cannot be higher than the marginal cost of the last dollar obtained, when the firm runs out of cash. As the marginal value of cash is a monotone decreasing function of the cash ratio, an external financing is not optimal as long as the cash ratio is positive, $w > 0$. It means that for both external and liquidation policies the optimal cash ratio boundary is identical. This result provides also a different explanation of the pecking order between internal and external funds than the one proposed by Heaton (2002). Explicitly, I do not consider the fact that the investors will pay issued stocks at a lower price than the one expected by the manager. Nevertheless the manager still decides to use in priority internal funds, because he thinks that internal funds are a cheaper way to finance investment.

It appears also interesting to mention that a fixed cost assumption for external financing will not change this result. Assuming that the manager gears a fixed cost for obtaining the external financing increases the "tightness" of the financing constraint. Consecutively, the marginal value of cash is higher at the external boundary. Therefore liquidation and external financing boundaries will be still equal as long as we find $p_{ww} < 0$ in the internal financing region.

We can now formalize the conditions at the boundary cash ratio $w = 0$. For the liquidation boundary we can impose the following matching condition:

$$p(0) \quad = l \tag{14}$$

Concerning the external financing boundary, it is not necessary to assume that the firm value is continuous before and after equity issuance. Without considering the possibility of negative cash-inventory[15], the boundary $w = 0$ corresponds to the boundary of the set of the admissible values for w. Then, the only relevant condition for the lower boundary is the following smooth pasting condition:

$$p_w(m) \quad = 1 + \gamma, \tag{15}$$

where m represents the optimal fraction of the physical capital K issued. Comparing the value of the firm $p(0)$ or the marginal value of cash $p_w(0)$ for both policies at the boundary threshold permits to determine the optimal corporate decision. Most important the optimal lower boundary $w = 0$ is identical for both the manager and the shareholders, however the optimal fraction of capital m can differ. The amount of equity issued will be not optimal from the point of view of the shareholders as long as the marginal value of cash will be differently evaluated by the manager and the shareholders. I present now the methodology for identifying the optimal payout threshold.

[15]BCW 2011 consider the possibility of negative cash-inventory, when a credit line is available.

3.4 Boundaries: Payout

The optimization of the flow of the dividends is a well known topic since the seminal contributions of Picqué and Shiryaev (1995), and Radner and Shepp (1996). The marginal value of cash for the manager is greater or equal to one, $p_w(w) \geq 1$ in the cash accumulation region because the manager is financially constrained. Each time the manager is able to invest i^* and to save an optimal amount of cash I denote $\overline{w}$, the remaining cash should be paid out to the shareholders. Assuming that the total reserve of cash is equal to $w > \overline{w}$ we can formulate the following value matching equation:

$$p(w) \quad = p(\overline{w}) + (w - \overline{w}) \tag{16}$$

Computing the derivatives with respect to w on both sides we obtain the following smooth pasting and super contact (Dumas (1991)) conditions:

$$\begin{aligned} p_w(\overline{w}) &= 1 \\ p_{ww}(\overline{w}) &= 0 \end{aligned} \tag{17}$$

Plugging these conditions inside the manager firm value dynamic permits (details can be found in appendix B) to characterize the relationship between the firm value and the payout boundary at optimum $\overline{w}$:

$$\overline{p_m}(\overline{w}) \quad = \overline{w} + 1 + \theta(r+\delta) - \sqrt{\frac{\overline{w}^2}{2} + \overline{w}(1 + \theta(r + \alpha - \lambda + \delta)) + \frac{2\theta(\mu+\alpha)+1}{2}} \tag{18}$$

Using this equation, a numerical solution can be obtained for $\overline{w}$.

3.5 Shareholders Valuation and Agency Costs

The agency costs (AC) are equal to the firm's value for shareholders when the manager is not optimistic (First Best) minus the firm's value for shareholders when the manager is optimistic: $AC = \overline{p}^{FB}(w) - \overline{p}(w)$. I have identified the corporate decisions made by the managers yet. However, if I want to evaluate the agency costs I need to approximate the dynamic of the shareholders firm value when the manager decides the value of the controls, i.e. corporate decisions. A problem arises because we cannot assume that smooth pasting conditions are respected when the manager is free to make decisions. Consecutively there is an infinite number of solutions for the second order problem encountered.

The simplest solution is to guess the marginal value of cash at the upper boundary. The guess can be precise. The manager is not financially constrained when he decides to pay dividends $p_m(\overline{w}) = 1$. Recall that the manager has an incentive to pay dividends later. This can be verified by computing firm value with or without conflict of interests at the payout boundary. The firm value at the payout boundary without conflict of interests satisfies the following equation:

$$\overline{p}^{FB}(\overline{w}) \quad = \overline{w} + 1 + \theta(r+\delta) - \sqrt{\frac{\overline{w}^2}{2} + \overline{w}(1 + \theta(r - \lambda + \delta)) + \frac{2\theta\mu+1}{2}} \tag{19}$$

Comparing the last equation with the equation 18, $\forall \overline{w}$, we find that the firm is more valuable for the manager, $\overline{p}_m(\overline{w}) > \overline{p}^{FB}(\overline{w})$. It means that a payout region for the manager corresponds always to a payout region for the shareholders. Obviously this result comes from the structural setting I have used for considering manager's optimism desire and is not surprising. Denoting the first best payout threshold for the shareholders $\overline{w}^{FB}$, we always have $\overline{w} > \overline{w}^{FB}$. Moreover, even if the marginal value of cash can be below one, $p(w) < 1$ when $w \in]\overline{w}; \overline{w}^{FB}[$, at the optimum $\overline{w}$ we can approximate the Neumann condition: $p_w(\overline{w}) \lessapprox 1$. The only agency cost for the shareholders is induced by the low return of the cash-inventory. This cost should be low and the marginal value of cash should be close to one even if below. This hypothesis permits to reduce the number of admissible solutions and to approximate shareholders firm value for all payout boundaries decided by the manager.

With this additional condition I am able to compute sufficiently precisely the dynamic of the firm value for the shareholders. Interestingly we can also consider that the manager tries to respect the Neumann conditions $p_w(\overline{w}) = 1$ and $p_{ww}(\overline{w}) = 0$, corresponding to the first best case at the payout boundary. In other words that the manager is free only for selecting investments and not for determining dividends. This case is analyzed in section 3.3.

4 Quantitative Analysis

I define the parameters for the baseline analysis. I state first the investment parameters then the financing parameters and conclude with the parameters capturing, in a reduced form approach, the optimism of the manager. In general the values of the parameters are in line with the BCW (2011) analysis.

The mean of the productivity shock μ is equal to 17% and the volatility σ is set to 9%. These values correspond to the empirical results obtained by Eberly, Rebelo and Vincent (2009) for large US firms. The depreciation rate δ is set to 10.5%. All the preceding estimates are annualized rates. The adjustment cost parameter is θ and equal 1.5 in line with Whited (1992). The liquidation value of the assets in place l is set to 0.9 according to the Hennessy and Whited (2007) results. Assuming no financing frictions BCW reports a Tobin's q equal to 1.23 and a corresponding optimal investment ratio i equal to 15.1%. I find comparable results assuming managerial optimism.

Concerning the financing parameters, the risk free rate $r = 6\%$. I assume that the cost carrying cash $\lambda = 2\%$ and the marginal cost of external financing $\gamma = 6\%$ in line with Altinkilic and Hansen (2000). Differently from BCW I do not assume fixed costs, because they do not change the results that I emphasize.

Finally, the manager's bias of perception for the productivity of the assets is $\kappa \in [0.25\%, 1\%]$ and the additional benefit rate for the cash under management is $\alpha \in [0, 0.75\%]$. The analysis concerning the investment spending will be made in section 4. Actually there are no empirical studies permitting to justify the value of these parameters. The objective of this paper is to assess the impact of manager's optimism on the main corporate policies. This could also lead to an

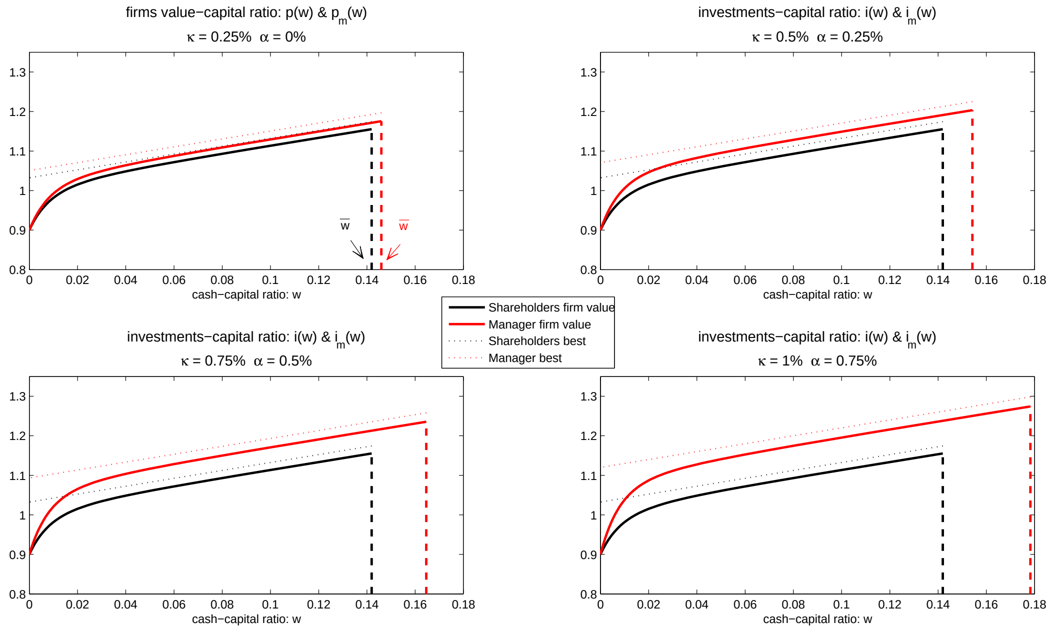

Figure 1: **Liquidation Case, Payout Analysis**, this figure plots the evolution of the firm value $p_m(w)$ and payout boundary $\overline{w}$ for different combinations of the parameters κ and α capturing the manager's optimism.

improvement of calibration for future empirical studies.

4.1 The Effect of Manager's Optimism on Corporate Policies

I begin with the liquidation case for presenting the impact of manager's optimism on payout and investment decisions.

In this subsection I compare corporate decisions with or without the manager bias of perception. The analysis for each figure is made with the same methodology. I begin with general comments, then I analyze the consequences of the conflict of interests on the corporate policies. One of the main contribution of this essay is probably that the dynamics of the control for the baseline model is different than in Bolton Chang Wang. The calibration of the baseline model is therefore different, because I add two parameters capturing the optimism of the manager.

Figure 1 plots the evolution of the firm value $p_m(w)$ and payout boundary $\overline{w}$ in the cash accumulation region for different sets of the parameters. The parameters capturing the manager's optimism κ and α are chosen increasing. I use three benchmarks for comparisons: best case firm values without financing frictions for both shareholders and manager and the firm value with a rationale manager (when $\kappa = \alpha = 0$). The firm values for the shareholders are identical in each plot.

We remark that firm values are strictly concave[16], $p_{ww}, p_{mww} < 0$ for both manager and shareholders. Interestingly, the firm values are close to be linear with respect to the cash-capital ratio w before the optimal boundary thresholds. This means that the impact of the financing constraint on the different corporate policies could appear not significant, even if the manager has not saved the optimal cash inventory $\overline{w}$ and is financially constrained. However, without financing frictions, firms value is linear and higher everywhere. The spread between the best case and the corresponding constrained case in the linear region ($p_{ww} \sim 0$) is mainly the consequence of the low rate of returns for the cash inventory. Remember that with $\lambda > 0$ keeping cash instead of paying dividends generates a cost in comparison to the best case. In the strongly concave region $p_{ww} << 0$ the financing constraints are particularly costly for both shareholders and manager. In this region, in addition to the cost induced by a low rate of return for the cash balance, the expected bankruptcy costs are also significantly higher.

Considering only the impact of manager's optimism on corporate policies we can formulate two comments. Firstly the firm value perceived by the manager is always higher than the one perceived by the shareholders. This result is not surprising as the assets of the firm are assumed to be more valuable for the manager. Corresponding to this result, we find that the manager firm value is increasing with both parameters κ and α. Secondly, the payout boundary is always higher for the manager and increasing with the parameters too. This result is also not difficult to understand. The cash inventory is assumed to be more valuable for the manager. Therefore the manager in comparison to the shareholders prefers to accumulate more cash before paying dividends. The optimism generates an empire building.

Figure 2 plots the marginal value of cash for the same sets of parameters. We remark that the marginal value of cash is a monotone decreasing convex function of the cash-ratio for both the manager and the shareholders. The marginal value of cash is high when the firm runs out of cash. Then, increasing more and more the cash ratio increases less and less the firm value. According to the previous results, the marginal value of cash is close to one, $p_w(w) \sim 1$, before the optimal payout boundary. We can also mention that the marginal value of cash at the boundary $w = 0$, $p_w(0)$ is higher than the marginal cost of external financing $1 + \gamma$. This implies that external financing should help to relax the financing constraint and a preferable solution.

The manager's optimism has for impact to increase the marginal value of cash for each cash ratio in the accumulation region. The cash appears two times more valuable for the manager for the case $\kappa = 1\%$, $\alpha = 0.75\%$, when the firm is close to be liquidated. It emphasizes the necessity the manager has for keeping the firm alive.

Figure 3 plots the optimal investment ratio i^* and highlights one of the key non intuitive results found. The optimal investment is a monotone increasing function of the cash-ratio. However the function is convex near the liquidation

[16] Strict concavity is assumed for using super contact condition (Dumas 1991).

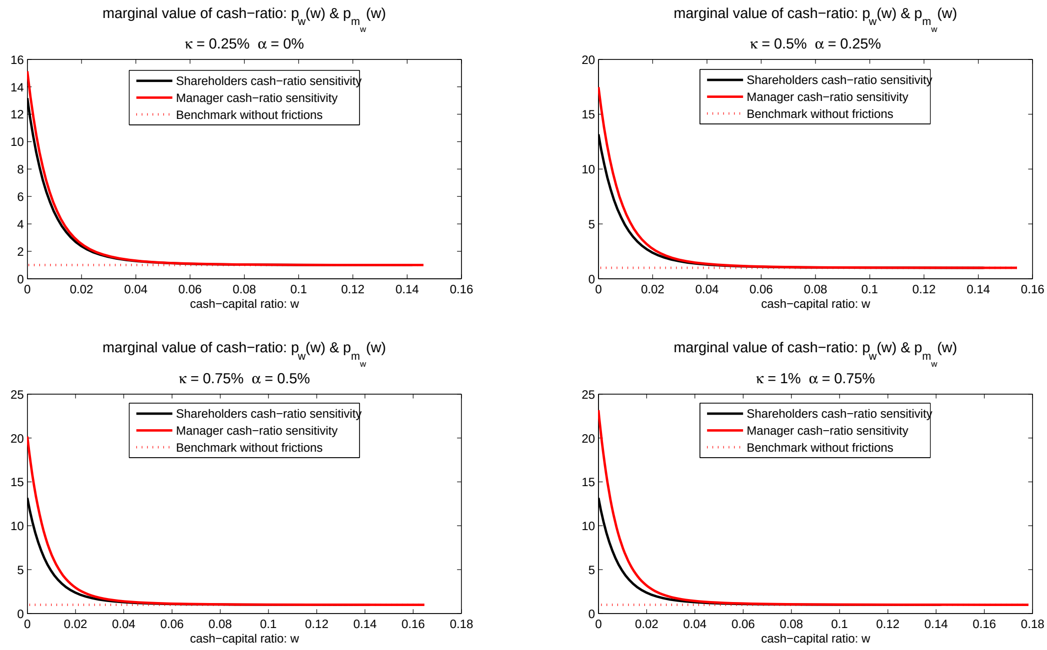

Figure 2: **Liquidation Case**, this figure plots the evolution of the marginal value of cash $p_{m_w}(w)$ and payout boundary $\overline{w}$ for different combinations of the parameters κ and α capturing the manager's optimism.

and concave near the payout boundary. This result can be explained by the convexity of the investment adjustment costs.

Without financing constraints the optimal investment ratio i^* is fixed. We remark that the manager can decide to invest more than the shareholders best case when he is sufficiently optimistic, when the value of the parameters κ and α is sufficiently high. Moreover, as a consequence of the manager's optimism, the manager invests not enough when the firm is highly financially constrained and too much when the cash ratio is high. This result can be observed also on the top left plot when the manager overevaluates only the profitability of the physical assets. An increase of the profitability of the physical assets has for consequence to increase also the profitability of the more liquid assets. When the firm is close to bankruptcy the sensitivity to the profitability of the marginal value of cash $\frac{\partial p_w(w)}{\partial \mu}$ can be higher than the sensitivity of the marginal value of investment $\frac{\partial p_{i^*}(w)}{\partial \mu}$. Therefore an increase of the profitability perceived by the manager can lead to an underinvestment. However, we can also expect that an optimistic manager invests more in average, because the firms should be closer to the payout threshold in average as reported by BCW 2011. Interestingly the investment sensitivity of cash is positive everywhere (the slope of the investment function), but the investment sensitivity can be lower when the firm is highly constrained. The investment sensitivity is not a monotonic function of the cash ratio and assuming that the manager is optimistic should not necessarily lead to an increase of the investment sensitivity for a sample of constrained

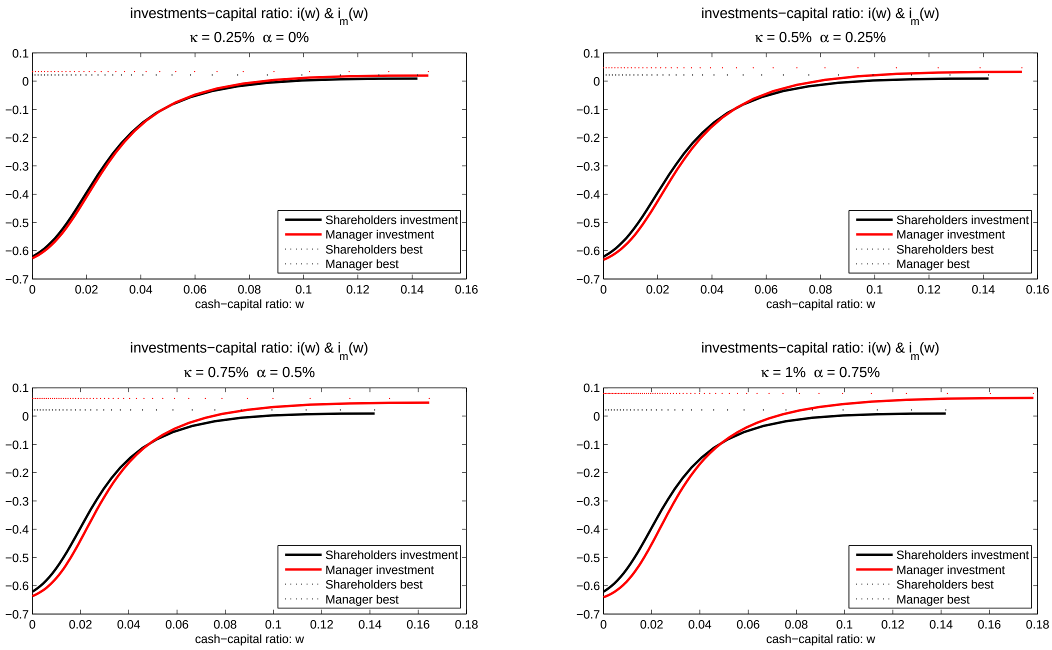

Figure 3: **Liquidation Case, Investment Analysis**, this figure plots the evolution of the optimal investment ratio $i^*(w)$ for different combinations of the parameters κ and α capturing the manager's optimism.

firms.

I turn now to the external financing case. As understood, allowing external financing increases firm value by relaxing the financing constraint. This result is true as long as the costs borne for levering the funds are not too high. Because the external financing case can be seen as a particular example of the liquidation case, I present only the results on firm valuation and the external amount of funds issued by the manager. The manager empire desire has the same impact on the payout and investment policies than in the liquidation case.

Figure 4 plots the evolution of the firm value $p_m(w)$ and indicates the cash-capital ratio m issued by the manager. The firm value appears to be almost a linear function of the cash-ratio. Even if the possibility to use external funds avoids to bear bankruptcy costs, the firm value is still below the best case without financing frictions. There are still issuing costs and most importantly the cash inventory generates returns below the minimal rate required by the investors. We remark also that the optimal payout boundary is lower in comparison to the liquidation case: as the value of cash is reduced the optimal payout boundary decreases.

Because the manager knows that he can use external funds if the cash-inventory falls to zero, this reduces the value of cash. Consecutively, the manager can always sell the physical assets in order to increase the cash ratio and is no more constrained by an hypothetical bankruptcy. Indeed, as long as the manager does not want to invest, he has no reason to obtain external funds. Instead, the manager prefers to sell the appropriate fraction of the assets in

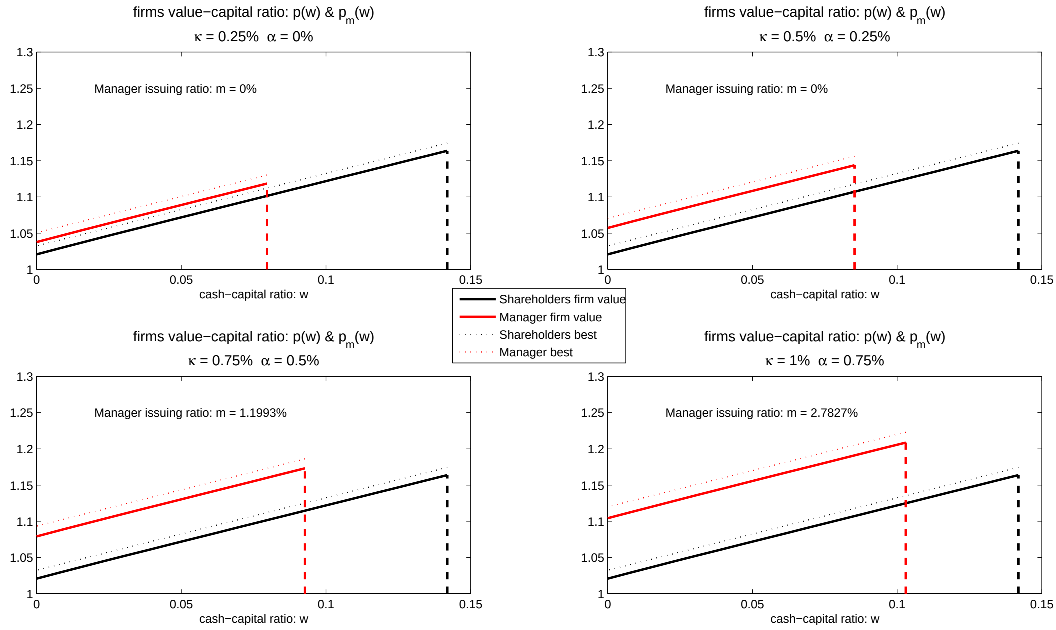

Figure 4: **External Financing Case, Payout and External Financing Analysis**, this figure plots the evolution of the firm value $p_m(w)$ and indicates the cash-capital ratio m issued by the manager for different combinations of the parameters κ and α capturing the manager's optimism.

place for obtaining new liquidity. When the manager wants to invest and the firm runs out of cash, he decides to obtain liquidity from new investors. In this case the external cash-capital ratio m is positive. Moreover it appears that the cash-capital ratio grows with the value of cash and as understood the value of cash increases with the manager's optimism. Therefore the manager has an incentive to issue more funds in comparison to the shareholders' optimum. This result is in line with the issuance distortion measured in Warusawitharana and Withed (2014): they find that the consequence of overevaluated shares is a high equity issuance and that managers issue much more equities when firms are financially constrained. However we cannot argue that the manager will effectively decide to issue more equities as we have not considered that investors will accept to pay less for stocks than the value evaluated by the manager. Here we can only emphasize that manager's optimism generates an incentive which mitigates the result obtained by Heaton (2002) to issue less equities.

4.2 Agency Costs

In the previous section I have compared firm values using different degrees of manager's optimism. The degree of manager's optimism has been captured in a reduced form approach with the parameters α and κ. I try now to compute firm value for the shareholders with and without the conflict of interests identified. The objective is to assess agency costs. By imposing the corporate decisions decided by the manager inside the shareholder's optimization problem I am

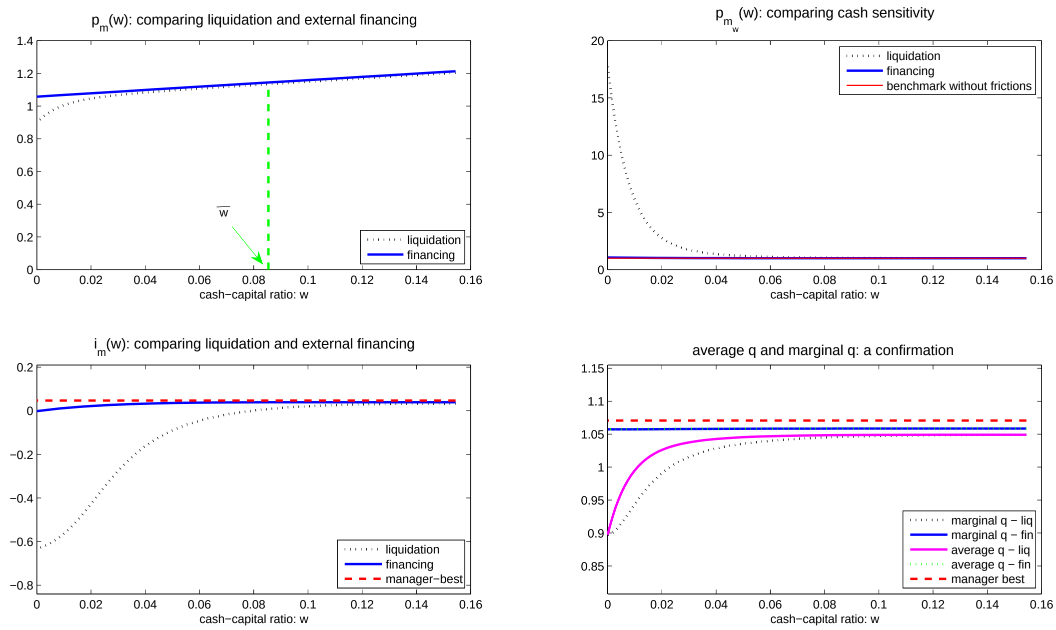

Figure 5: **Liquidation and External Financing Case**, this figure plots the corporate policies decided by the manager, Tobin's q and the average q for both liquidation and external financing cases with parameters $\alpha = 0.25\%$ and $\kappa = 0.5\%$.

able to evaluate the firm value for the shareholders. Then I can compare this result with the benchmark case when the manager is rational and maximizes shareholders wealth.

I begin by summarizing the different optimal corporate policies for both liquidation and external financing case from the point of view of the manager and by presenting the computations of marginal and average q. I draw the evaluation of the firm value for the shareholders with and without manager's optimism. The agency costs can be computed as the difference between shareholders' firm values. For simplifying the analysis I treat only the liquidation case when I evaluate agency costs, because I have found similar results with the external financing assumption. I decide to set $\alpha = 0.25\%$ and $\kappa = 0.5\%$ for the analysis in this section. Similar results are obtained for lower values of the parameters. However, for high value of the parameters the ODE of the shareholders is no more solvable. In this case the investment policy of the manager is not feasible.

Figure 5 plots the corporate policies decided by the manager, Tobin's q and the average q for both liquidation and external financing cases. The results confirm the previous analysis made and are in line with BCW 2011. The financing constraint is relaxed when external funds are not too costly. This reduces the marginal value of cash and permits to invest more. The firm value is still a concave function of the cash-capital ratio in the cash accumulation region. Computing marginal q, $q = \frac{d(P(K,W)-W)}{dK}$ and average q^{av}, $q^{av} = \frac{P(K,W)-W}{K}$ we find that $q \simeq q^{av}$ when the firm is slightly financially constrained and $q^{av} > q$ when the firm is more constrained. Because the spread $q^{av} - q = (p_w(w) - 1)w$,

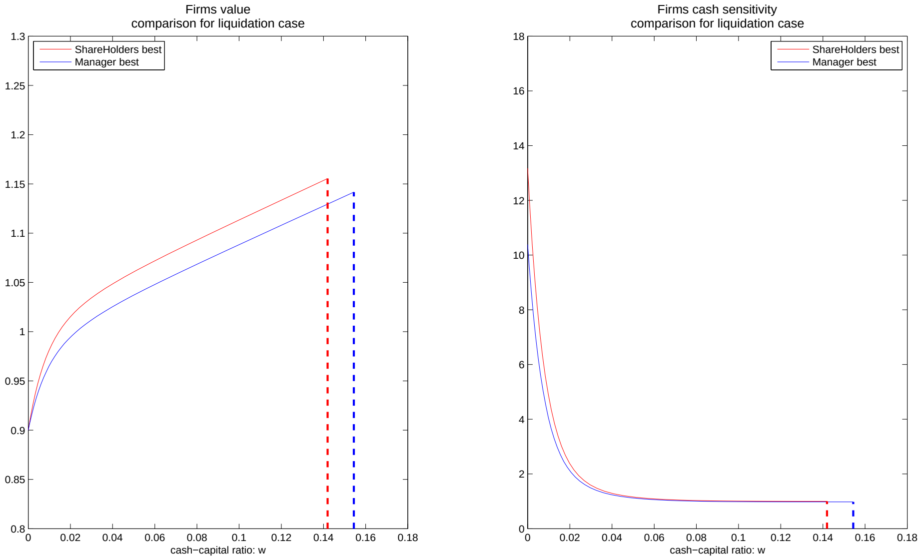

Figure 6: **Liquidation Case**, this figure plots the value of the firm and the marginal value of cash for the shareholders with and without conflict of interests with parameters $\alpha = \mathbf{0.25\%}$ and $\kappa = \mathbf{0.5\%}$.

we find $q^{av} \geq q$. Moreover with the setting I use for capturing the manager's optimism we have seen that $p_{mw}(w) \geq p_w(w)$. Therefore the optimism of the manager increases the spread between both measures of investment. The average q becomes a poor proxy of marginal q when the firm is financially constrained **and/or** when the manager is strongly optimistic.

I evaluate now the agency costs. In **Figure** 6, I plot shareholders' firm value with and without the conflict of interests. The costs are significant, they represent about 2.5% of the firm value at the payout boundary. However even if the cost is a monotonic non decreasing function of the cash-ratio, agency costs are generated mainly during periods of trouble. When the firm is close to default the marginal value of cash is high. The manager sells a dramatical fraction of the assets in place in order to avoid bankruptcy (or external financing costs). This emphasizes the importance to monitor the manager when the firm runs out of cash. When the firm has enough liquidity even if the manager wants to over-invest this does not generate significant agency costs.

4.3 The Case of a Constrained Manager

In the last subsection, the manager was free to determine the different corporate policies. However, as argued, the manager can be constrained and not allowed to decide freely the optimal corporate policies. In particular, we can imagine a situation where the board of directors is not as optimistic as the manager and decides the amount and frequency of payouts to shareholders. In this case the manager could be constrained by selecting payout policies according to conditions of optimality. The free corporate policy for the manager becoming the investment policy. If so we can expect the agency costs to be significantly

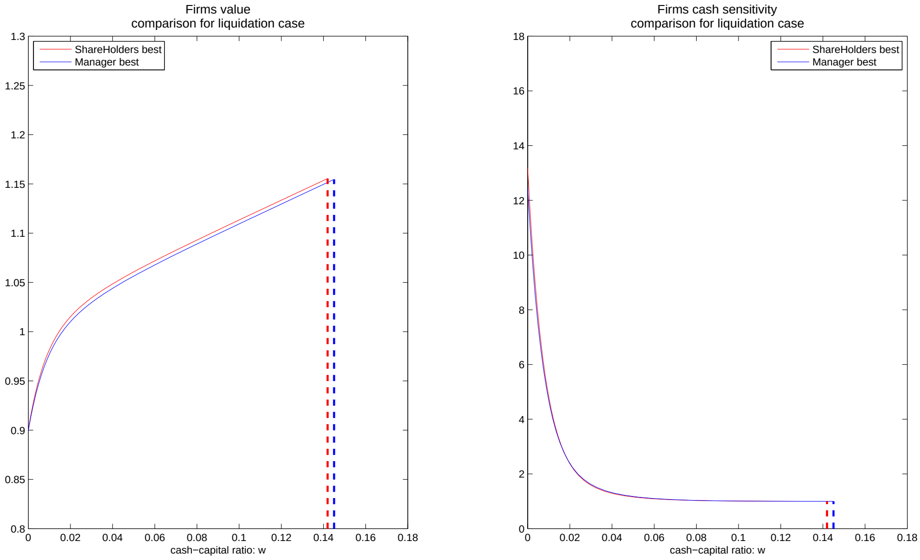

Figure 7: **Liquidation Case**, this figure plots the value of the firm and the marginal value of cash for the shareholders with and without conflict of interests with parameters $\alpha = \mathbf{0.25\%}$ and $\kappa = \mathbf{0.5\%}$.

lower.

In this section, I test this intuition. I assume that the board of directors determines the payout policy that maximizes shareholders wealth taking the investment policy of the manager as given. I plot the results in **Figure** 7. It appears that the difference is too small for distinguishing a significant spread between the firm values. Trivially assuming we can monitor the manager without costs leads to a reduction of the agency costs. The costs become, in average, equal to 0.1% of the value of the assets in place. The difference is small but still not negligible. Remark also that the payout boundary is still higher to the shareholders optimum. The investment decisions of the manager affects the payout decisions made by the board of directors, even if we assume that the board is independent and tries to maximize shareholders wealth. Finally, agency costs look like a concave monotone function of the cash-ratio and the need for monitoring is still more pronounced when the firm is more financially constrained. However, it appears particularly interesting to see that imposing a dividend policy permits to increase significantly the firm value for the shareholders.

Technical comments here can be of importance for the reader. Computing exactly the agency costs requires to interpolate firm value for a finite set of the state variable w. Indeed firm value with or without conflict of interests have not been computed for identical cash-capital ratios yet. The draw is made by interpolating firm value in-between the state variable points with a polynomial of order 5. An approximation of the firm values using the same values for the state variable w is possible. However the magnitude of the agency costs was not really significant and the precision of the approximation seems difficult to estimate. Instead I propose another direction of research. So far I have assumed that the manager was irrational being optimistic. A simple and interesting case can be proposed. Supposing that the manager has different information about the profitability of the assets in place, it can be that the high estimation of the assets profitability is justified by a higher evaluation of the volatility of the productivity. I consider now this special case.

4.4 The Case of a Rational Manager

In practice the manager can smooth the results published in order to mislead investors. For example we can assume that the manager manipulates the results for improving a statistic like the Sharpe-ratio. With this assumption it could be that the investors under-evaluate the volatility and the profitability of the assets. In this case the manager is no more optimistic and irrational, but is simply better informed. Intuitively we can expect that the spread between firm values is now higher because the manager has another incentive to deviate from the shareholders' optimal corporate policies.

For testing the hypothesis, I assume now that the manager evaluates differently the productivity shock. I still assume that the manager is not free when selecting the payout policy and I treat only the liquidation case[17]. I denote the productivity shock perceived by the manager σ_m and I set $\sigma_m = 18\%$. This changes the dynamic of the firm value for the manager and for the shareholders even if the shareholders still evaluate that the productivity shock is equal to 9%. Intuitively, the manager perceiving a higher risk should invest less, because this increases liquidations costs and so the value of cash. Referring to the incentives I have highlighted about investment policy in **figure 3**, we can formulate the following assumption: when the manager evaluates sigma higher than the shareholders, $\sigma_m > \sigma$, this should increase the agency costs especially when the firm runs out of cash. Remember, when the firm runs out of cash, the manager has an incentive to under-invest and when the cash-capital ratio is high, the manager has an incentive to over-invest. Like this reducing the investment magnifies the under-investment problem but mitigates the over-investment.

Figure 8 plots the value of the firm and the marginal value of cash for the shareholders with and without conflict of interests when the risk perceived by the manager is higher, $\sigma_m = 18\%$. As expected, the agency costs are increasing until the cash-capital reaches a threshold (about 2.5% in the plot) and decrease until the optimal shareholders' payout boundary. In comparison to the previous case the agency costs are higher for all the values of the state variable. However, the marginal value of cash is now higher without conflict of interests only for low values of the cash-capital ratio. When the manager overinvests, the value of cash is higher as the shareholders would have preferred to accumulate more cash and to invest less. Alternatively, when the manager perceives that the investment is more risky he has another incentive to underinvest. In this case the increase of the cash inventory can help to mitigate the underinvestment

[17]I find similar results when the manager is free to select all the corporate policies.

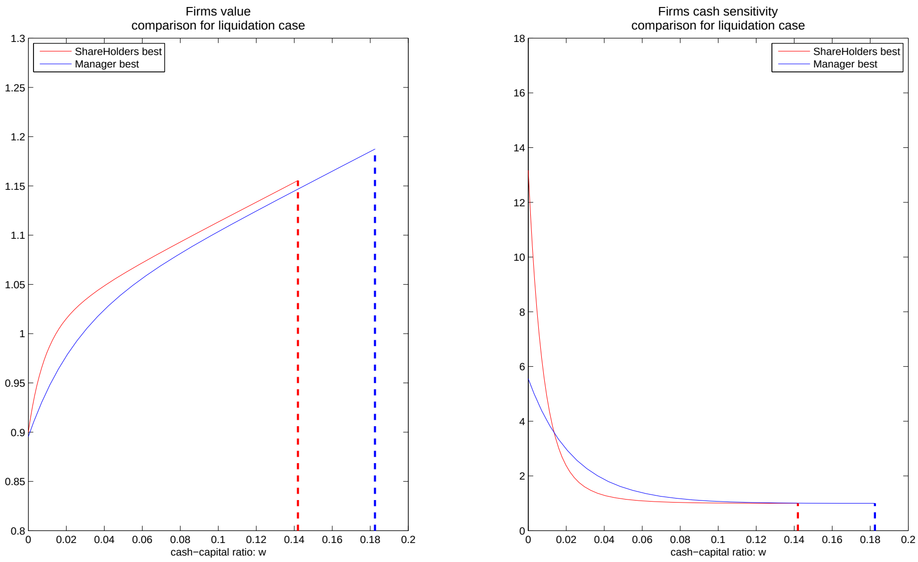

Figure 8: **Liquidation Case**, this figure plots the value of the firm and the marginal value of cash for the shareholders with and without conflict of interests with parameters $\alpha = 0.25\%$, $\kappa = 0.5\%$, $\sigma = 9\%$ and $\sigma_m = 18\%$.

problem.

These results reinforce the need to monitor the manager when the firm runs out of cash. Moreover this indicates that natural mechanisms exist reducing the incentive for optimal contracting solutions when the firm is less financially constrained. On one side we have seen that the manager has little flexibility for deviating from the optimal corporate policies when the payout policies are imposed by an independent board and on the other side that agency costs are naturally low when the size of the firm is sufficiently important.

5 Investment Spendings

In the last section we have considered that an optimistic manager has an incentive to grow the assets size of the balance sheet. A reduced form approach exploiting the framework by BCW (2011) has been proposed. In this section I consider another incentive to increase the investment spending. Imagine that the manager overestimates the value of the spending and not the productivity of the assets. If the manager is irrational and optimistic it could be he considers that the assets acquired are more valuable even if he agrees with the shareholders about the productivity. I capture this hypothesis assuming that the manager investment spending are worth $(1+\iota)i$ instead of i from the point of view of the manager. Intuitively we can anticipate that the manager will still over-invest. However, the objective is to show that the effect on the different corporate policies of this hypothesis is ambiguous.

Firstly we can easily compute the new optimal investment ratio i^*:

$$i^* = \frac{1}{\theta}((1+\iota)(\frac{p(w)}{p_w(w)} - w) - 1) \tag{20}$$

Even if the private benefits generate an incentive to increase the spending i, we do not know ex ante the effect on the marginal value of cash $p_w(w)$. This second effect could reduce the optimal amount invested. Clearly the marginal value of cash can increase or decrease depending on the magnitude of the financing constraint and the choice of the payout boundary.

Secondly it appears impossible to identify if the payout boundary will be higher or lower. Using the new optimal investment obtained we find the following relation at $\overline{w}$:

$$\overline{p} \quad = \overline{w} - \frac{1+\theta(r+\delta+\sqrt{(r+\delta)^2+\iota^2+\frac{2}{\theta}(r+\delta+\iota^2(\kappa+\mu)-(\kappa+\mu)+\overline{w}(\lambda-\alpha+\iota^2(\alpha-\lambda)))})}{\iota^2-1} \tag{21}$$

As the derivative $\overline{p}_\iota$ can be positive or negative, it appears impossible to generalize the results obtained. It means that we can not predict the impact of the manager empire desire on the different corporate policies, when we assume that the manager can earn private benefits by increasing the investment spending. The intuition of the result is the following: when the manager thinks that the investment is more profitable this increases the value of the firm he perceives and this for all values of the state variable w. Consecutively the manager does not need to invest more. The amount invested depends on the marginal value of cash and we cannot generalize how an increase of the value of the investments will affect the value of cash.

6 Conclusion

I use a dynamic model of corporate investment and financing with financing constraints for analyzing the impact of manager's optimism on main corporate policies and agency costs. I propose an extension of the framework proposed by Bolton, Chen, Wang (2011) where the dynamics of the controls are distorted by the optimism of the manager. It permits to find a broad set of theoretical but also testable predictions.

Notably, I find that the internal funds do not permit to align the interests of the manager and the shareholders. However this paper shows that there exists a natural convergence of interests between the agents. The corporate decisions made by the manager generate significantly higher agency costs when the firm is more financially constrained. The agency costs appear to be a concave and not necessarily monotone function of the cash-ratio and the manager has an incentive to increase the cash-ratio until an optimal trigger is reached. Therefore a reinforcement of the manager's monitoring during periods of trouble appears suitable.

I also find that an increase of the profitability of the physical assets has for consequence to increase the marginal value of cash. This key result is driven by the overall increase of the firm value after an increase of the physical assets profitability, which in turn increases the precautionary motive to accumulate liquidity for avoiding inefficient closure. Hence an optimistic manager may have an incentive to underinvest when the firm is close to bankruptcy and more financially constrained.

This work opens many research directions. Proposing a different setting of manager compensations or allowing the shareholders to identify the suboptimal behavior of the manager could be of particular interest. For example a signaling game could be introduced where the shareholders are able, even if intermittently, to observe the decisions of the manager.

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Appendix

A. Benchmarks investment ratio policy and firm value

I present the computations for obtaining the benchmarks investment ratio and firm value. Without financing friction, recall that the ODE for firm valuation in terms of the optimal investment ratio is the following:

$$\begin{aligned} rp(K, W=0) &= \max_i K[(i-\delta)p_K + \mu - i - g(i(w))] \Rightarrow \\ rp(w=0) &= \max_i (i-\delta)p_K + \mu - i - g(i) \end{aligned}$$

Differentiating firm value with respect to i, we obtain the following first order condition:

$$\begin{aligned} 0 &= p_K - 1 - \theta i^* \Rightarrow \\ p_K &= 1 + \theta i^* \end{aligned}$$

Using the homogeneity assumption $p_K(K, W) = p(w) - wp_w(w)$ and assuming no financing frictions $p_K(K, W = 0) = p(w = 0)$. Replacing p_K by $p(w = 0)$ and $p(w = 0)$ by $1 + \theta i^*$ inside the ODE we find the following quadratic relation for i^* at optimum:

$$\begin{aligned} r(1+\theta i^*) &= (i^*-\delta)(1+\theta i^*) + \mu - i^* - g(i(w)) && \Rightarrow \\ 0 &= i^* - \delta - \delta\theta i^* + \theta {i^*}^2 + \mu - i^* - \theta \frac{{i^*}^2}{2} - r - r\theta i^* && \Rightarrow \\ 0 &= \mu - \delta - r + i^*(-\delta\theta - r\theta) + {i^*}^2 \frac{\theta}{2} && \Rightarrow \\ i^* &= \frac{\theta(r+\delta) - \sqrt{\theta^2(r+\delta)^2 - 2\theta(\mu - r - \delta)}}{\theta} && \Rightarrow \\ i^* &= r + \delta - \sqrt{(r+\delta)^2 - 2\frac{\mu-(r+\delta)}{\theta}} \end{aligned}$$

Then firm value can be eventually found using the previous relationship obtained:

$$p(w=0) = 1 + \theta i^* = 1 + \theta(r + \delta - \sqrt{(r+\delta)^2 - 2\frac{\mu-(r+\delta)}{\theta}}) \tag{22}$$

B. Payout boundary and firm value

I present the computations for obtaining the value of the firm at the payout boundary $\overline{w}$. The firm value, perceived by the manager, before hitting the payout boundary satisfies the following HJB equation:

$$rp(w) = (i-\delta)(p(w) - wp_w(w)) + [(r + \alpha - \lambda)w + \mu + \kappa - i - g(i)]p_w(w) + \frac{\sigma^2}{2} p_{ww}(w)$$

Therefore we can obtain an analytic expression for the optimal control i^* using a first order condition:

$$i^* = \frac{1}{\theta}(\frac{p}{pw} - w - 1)$$

At the optimal boundary threshold, knowing that both the smooth pasting condition $p_w(\overline{w}) = 1$ and super contact conditions $p_{ww}(\overline{w}) = 0$ should be satisfied, and plugging the value of the optimal control inside the ODE we find:

$$
\begin{aligned}
r\overline{p} &= \left(\frac{1}{\theta}(\overline{p}-\overline{w}-1)-\delta\right)(\overline{p}-\overline{w})+(r+\alpha-\lambda)\overline{w}+\mu+\kappa-\frac{1}{\theta}(\overline{p}-\overline{w}-1)-\frac{(\overline{p}-\overline{w}-1)^2}{2\theta} \quad \Rightarrow \\
\overline{p} &= \overline{w}+1+\theta(r+\delta)-\sqrt{\frac{\overline{w}^2}{2}+\overline{w}(1+\theta(r+\alpha-\lambda+\delta))+\frac{2\theta(\mu+\alpha)+1}{2}}
\end{aligned}
$$

Credit Rationing and Cash Holdings*

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Abstract

I examine the effects of credit rationing on corporate cash holdings by modelling the precautionary demand for cash. In the model, firms can pledge part of the future cash-flows to creditors when current cash-flows are insufficient to finance investment. I show that the cash-flow sensitivity of cash and the investment to cash sensitivity are inappropriate indicators of financial constraint. By contrast, I show that the variation of cash holdings is monotonically decreasing with the degree of the financing constraint. An empirical study with a large sample of manufacturing firms over the 1971 to 2011 period supports these results.

Keywords: Financing frictions; Cash holdings; Identification; Test;

JEL Classification Numbers: G32; G35; G39

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1 Introduction

The contribution of this essay is to provide a new theory about the determination of precautionary cash-holdings in financially constrained firms.

It is common knowledge that cash holdings are one of the main preoccupations of the corporate deciders. The managers of US listed firms have decided to accumulate an important amount of cash in the years preceding the 2008 crisis. Bates, Kahle, Stulz (2009) report that the average cash ratio has more than doubled in 26 years: from 10.5% in 1980 to 23.2% in 2006. Because unused financing generates a cost of opportunity, researchers have tried to explain this phenomena.

One angle of approach is to consider the role of the financing frictions. Interestingly enough, reversing the conclusions of the famous Modigliani & Miller irrelevance theorem (1958), we know that financing frictions impact firm value.

Even if the impact of costly financing has been discussed in many dimensions, the theoretical analyses concerning the role of a limited supply are rare. We can mention exhaustively Stiglitz and Weiss (1981), Almeida, Campello, Weissbach (2002, 2004 and 2010), Han and Qiu (2007), Morellec, Hugonnier and Malamud (2011 and 2011).

Conscious of this lack in the literature, I identify and analyze a new theoretical relation between the external availability of debt funds and the level of cash-holdings. The availability of funds is not only a theoretical question. The drying up of liquidity was one of the major concerns during the last crisis. Notably, the impact of a liquidity shock on corporate decisions appeared ambiguous, the difficulty probably comes from the number of cash-holdings determinants. Kahle and Stulz (2011) reports that the firms have experienced an increase of cash to assets ratio after September 2008 in addition to a decrease of net equity issuance. This observation does not corroborate in favor of a significant reduction of credit supply. In parallel, Campello, Graham, Harvey (2011) find in a survey study that 86% of the constrained CFOs have reduced or postpone their planned investments. Therefore even though the shock on the liquidity supply has an impact on the lending capacity of the banks, it seems that the main cost of the last crisis was the decrease of the current or future investment spending by the firms.

The purpose of this paper is not to discover a preventive tool which appears to be more a regulator job but to identify a new and fundamental relationship between the degree of financing rationing and the cash-holdings policy of the US firms. In particular, I propose a model which embed in a simple way three main determinants of the cash-holdings: the risk of the cash flows, the profitability of current and/or future investment opportunities and the availability of external funds, three of the main shocks identified during the last crisis.

In order to be sure that the reader follows me, I need to make some definitions. I define that the supply is limited when the manager is unable to pledge all the expected cash-flows. I call this situation financing rationing[18]. I define

[18] My definition is not based on an equilibrium relation between supply and demand as in Stiglitz and Weiss 1981.

a constrained manager as a manager who is not able to invest the amount that maximizes shareholders wealth.

In the model, I assume that the manager can pledge only a fraction of the expected cash-flows i.e. that the supply of funds is limited. As a consequence, the manager becomes financially constrained when the internal capacity of financing is too low to reach the optimal level of investment. I analyze the impact of this constraint on corporate cash-holdings choice. This assumption constitutes the main departure with existing comparable frameworks (ACW 2004, HQ 2007). The advantage of this hypothesis is to establish a link between future investment decisions and current corporate policies.

Computing the impact of a debt supply shock on the optimal level of cash-holdings, I find that there is a negative monotonic relation between the time variation of cash holdings and the degree of the financing constraint. It means that the more the manager is constrained, the less he is able to adapt the optimal level of cash holdings. It permits to identify the time variation of cash holdings as a new theoretical proxy of the degree of the constraint. This finding is particularly interesting considering the difficulty to assess precisely the financing constraint, an unobservable variable.

The intuition of the result is the following: a constrained manager has an incentive to finance the firm early and to constitute a cash buffer. In the model, the incentive is motivated by the non zero probability to lose some external financing capacity. A reduction of the future financing capacity implies a drop in profitable investments. To avoid this situation, the manager prefers to use the external financing capacity early and to constitute a reserve of cash. The capacity to modify the level of cash-holdings depends on the external financing capacity and so on the degree of the financing constraint. The relation is monotonic because a relaxing of the constraint permits always to the manager to increase the level of cash holdings. Even if the manager will not systematically use this opportunity in practice, we can expect he will do it in average as long as there is an incentive to increase the value of the firm.

I complete this theoretical result by an analysis of robustness of the two most influential theoretical indicators of financing constraint: the investment to cash sensitivity (Fazzary, Hubbard, Petersen 1988) and the cash flow sensitivity of cash (Almeida, Campello, Weisbach 2004)[19]. I find that these indicators are not robust to the hypothesis that the manager can pledge a fraction of the expected cash-flows. Notably, it appears impossible to justify the irrelevance of the financing constraint in the ACW proposition. The relation holds even if we consider the possibility to invest in the same projects today and tomorrow. Concerning the investment to cash sensitivity, I find that we cannot assume a systematic relationship, because a shock on cash flows affects not only the current level of investment but also the future one. This argument is similar to the one proposed by Kaplan and Zingales (1997).

I check the validity of my results with an empirical part. I decide to approx-

[19]The Whited and Wu (2006) index is another important contribution. However, their framework is not theoretical. The index is calibrated from a reduced form model. The impact of the external financing constraint (the lagrange multiplicator) is evaluated by an empirical proxy.

imate the degree of the financing constraint with classical empirical proxies: payout ratio, size, Kaplan-Zingales (1997) and Whited-Wu (2006) indexes. The objective is not to identify the firms that are financially constrained. The sample is made up of a priori financially constrained firms and I want to assess the correlation between the variations of cash-holdings and the proxies of the financing constraint. Using a large sample of manufacturing firms over the period 1976-2009, I find solid evidence supporting my predictions.

In particular, I observe a significant and negative relationship between the degree of the financing constraint and the variation of cash holdings for the Whited-Wu index and the size of the firm. Interestingly, the relation becomes insignificant for all the proxies of the credit constraint when I build a sample of firms with a priori strong hedging motives. This emphasizes the difficulty to identify the determinants of cash-holdings when a shock affects both the credit supply and the risk perceived by the agents. In the model, there is an incentive to finance the firm early when the cash-flows are risky, because the constrained manager wants to maximize the financing capacity of future profitable investment opportunities. I explicitly leave aside the other cash-holdings benefits. Another key empirical result is to find that the current level of operational cash-flows appears to be a poor determinant of cash-holdings. It confirms the necessity to use directly a proxy of the financing rationing to explain cash-holdings decisions.

Throughout the paper I try to assess the robustness of both the theoretical and empirical hypotheses I make. Because the understanding of the hypotheses used is based on the role attributed to cash holdings I propose now to replace this work into a wider context.

1.1 The Determinants of Cash-Holdings

It is difficult to identify the main role of cash holdings. A reserve of cash can be constituted for many reasons. Intuitively, the objective to reduce spending today is to increase your benefits tomorrow. Your benefits can increase because of costs saving or profits increasing. This motive has been called by the literature precautionary, emphasizing the fact that future internal financing resources or needs are uncertain. The challenge for the researchers was to identify and sub-classify these sources of cost-savings or profits.

Keynes in 1936 already explains that a firm can save transaction costs by using cash to make payments without having to liquidate assets [20]. Underpinning demand models under transactions costs have been proposed by Baumol (1952) and Miller and Orr (1966). Empirically, Keynes assertion has been tested by an important number of papers, generally, using the fact that large firms could reduce the cost by economies of scale. In that research direction, Mulligan (1997) remains an important contribution.

[20]Keynes initially identified three motives to hold cash (p.108): transaction, precautionary and speculative. In fact, by splitting the transaction and the precautionary motive the intention of Keynes was explicitly to distinguish between the objective to realize current or future economies. In fine, it is only a question of classification.

Another objective of cash holdings is the possibility to increase the financing capacity of of profitable investments. Interestingly enough it is possible to establish a link between the volatility of the cash-flows and the level of cash holdings. Han and Qiu (2007) show that an increase of the anticipated volatility of the cash-flows generates for financially constrained firms an incentive to raise the reserve of cash. Differently from Han and Qiu, I also show that the uncertainty affecting the cash-flows can generate an incentive to finance the firm early without specifying the third order derivative of the production functions.

The precautionary motive of cash holdings is also to reduce the probability of bankruptcy. This is suitable assuming that bankruptcies are costly. Bankruptcies are the consequence of the uncertainty affecting the value of cash-flows or the timing the cash-flows [21]. Numerous authors have chosen to use a Leland (1994) type model to analyze the interaction between an endogenous bankruptcy threshold, the level of cash holdings and some other important corporate policies. Among others we can mention Nikolov and Morellec (2009) who try to identify the impact of competition on cash holdings and Miao (2009) who proposes an analysis of the timing of debt repurchase. A model able to embed the maturity of debt as a control variable or the randomness of the timing of the cash-flows stays an important challenge in corporate finance. The analysis in discrete time of Diamond and He (2010) should be considered as a good starting point.

Finally, a last precautionary motive has been identified by Foley, Hartzell, Titman and Twite (2007). The authors highlight the fact that US multinational firms have an incentive to hold larger amount of cash holdings in order to avoid the taxes associated with the repatriation of earnings.

In parallel to the precautionary motive, another important determinant of cash holdings is the agency motive proposed by Jensen (1986)[22]. Referring to his theory of conflict of interests between shareholders and managers (1976), Jensen discusses that entrenched managers try to increase their personal benefits generated among other by an accumulation of cash. Arguing that the conflict of interests concerns mainly the optimal size of the firm and payout to the shareholders, Jensen specifies that the incentive to retain cash comes from the fact that part of the remuneration of the managers is based on sales growth (in adequacy with Murphy 1985) and that the power of the manager is correlated with the size of the firm.

Now the reader should be conscious than a general model including all the motives to keep the cash inside the firm is impossible. When the number of trade-offs considered increases it becomes impossible to obtain a unique prediction. However, empirically, it is possible to test the importance of many determinants of cash holdings. The papers of Opler, Pinkowitz, Stulz and Williamson (1999) and Bates, Kahle and Stulz (2009) are important empirical contribu-

[21] Cash holdings are also used to permit the payments to the suppliers and to avoid some fees in the case of a payment delay.

[22] Keynes identified also a speculative motive. Assuming asymmetry of information Keynes emphasizes the benefit of keeping cash when we anticipate privately a rise of the rate of interest. This objective of cash retention is not pertinent at firm level.

tions. Opler and all find strong evidences in favor of the precautionary motive of cash holdings. Using a large sample of public US firms they observe that the main firm characteristics correlated with the level of cash-holdings are the size (negative correlation) and some proxies of the growth opportunities and risk of the activities. They argue that their "results are consistent with the view that firms hold liquid assets to ensure that they will be able to keep investing when cash flow is too low". Bates and all find also evidence for the precautionary motive of cash holdings. In addition to Opler and all, they observe that the increase of the level of cash characterized firms paying less dividends and having experienced the greater increase of idiosyncratic volatility. They find a negative correlation with inventories or capital expenditures, and a positive correlation with R&D expenditures. Similarly, they find that a residual substantial cross-sectional variation in cash holdings is not explained, but that there is no evidence concerning the agency motive.

Considering these primary empirical results, I assume that the level of cash holdings depends strongly on firms characteristics. Therefore I decide to model the precautionary motive. In particular, I assume that cash-holdings can always be used to increase the financing capacity in future profitable investments, but that the financial slack is limited. This underpinning is necessary to justify the analysis of the impact of credit rationing on the level of cash holdings.

This financing constraint framework is useful for two reasons: firstly it permits to propose a new theory to identify the firms subject to financing constraints. This objective is particularly suitable for the realization of empirical studies. Secondly, it permits to analyze the robustness of previous theoretical models. This objective is justified by the observed lack of consensus regarding the impact of financing constraint on corporate and in particular cash policies.

1.2 A Debate About Financially Constrained Firms

In their seminal paper FHP (1988) develop a theory where the manager decides the amount invested by trading-off the marginal output of the production function and the marginal dead-weight cost of external funds required. Assuming a large wedge between the internal and external costs of financing, the investment should vary with the availability of internal funds.

Empirical and theoretical criticisms exist for the FHP work. Empirically, principal reprobation can be found in Erickson and Whited (2000), Gomes (2001) and Alti (2003) where they argue that the methodology of FHP is inappropriate, as Tobin's Q does not control for investment opportunities. In addition, Kaplan and Zingales (1997) find a higher investment-cash flow sensitivity for lower constrained firms in opposite to the expected relation. Considering the popularity of the FHP indicator[23], we could argue that an empirical review is not sufficient to invalidate a theoretical framework. Nevertheless, I present the three major set of criticisms so far formulated.

Firstly, Kaplan and Zingales (1997, 2000) discuss the pervasiveness of the

[23] Hubbard (1998) provides an useful survey.

non monotonic relationship between the investment-cash flow sensitivity and the degree of financial constraint. In this sense, their argumentation is similar to the one proposed in this paper. FHP (2000) and KZ (2000) agree on the fact that the relationship is no longer monotonic, if we consider financially distressed firms. However, they are in contradiction concerning the impact of the non monotonicity. For KZ, the non monotonicity should be a crucial issue or in other words the investment-cash flow sensitivity $\frac{\partial I}{\partial W}$ (I = investment, W = cash level) theory is not sufficiently robust: the authors recommend to use a second order variation $\frac{\partial^2 I}{\partial W^2}$ arguing that the future amount invested depends on the level of cash. In addition, they mention that the level of constraints is a function of firm specific characteristics, which are not captured by the cross sectional methodology of FHP.

The second set of criticisms comes from the work of Povel and Raith (2001, 2007). They show that the relationship between investment and internal funds could be U-shaped. For sufficiently low levels of cash, they find an increase of the investment after a negative shock on cash-flows. Consecutively, no prediction could be made only by considering the investment-cash flow sensitivity.

The last set of relevant criticisms has been made by Almeida and Campello (2002). They demonstrate that the link between investment decision and financing capacity influences the change of the cash reserve. In addition, they also identify the lack of robustness of FHP results. The magnitude of the investment variation depends on the concavity of functions of both marginal external financing cost and marginal productivity.

The cash-flow sensitivity of cash is becoming an influential test to identify constrained firm [24]. Theoretical and empirical negations are rare. Empirically, Yi-Chen Lin (2007) finds a positive cash-flow sensitivity of cash for both constrained and unconstrained firms. Riddick and Whited (2009) use a different methodology and find that the sensitivities are often negative. Theoretically, ACW (2010) find that the sensitivity can be negative considering different investment liquidity. I show that a negative sensitivity can be found with a simple setting.

Grouping these considerations, a research of consensus constitutes another important justification of the framework I propose. To conclude, the choice of the model is guided by a pragmatic motivation. Models with infinite horizon are useful to analyze the joint relationship between financing and investment decisions (or timing), but generally they do not allow to consider explicitly security design (for example see Hennessy, Livdan and Miranda (2010) or Morellec and Schürhoff (2010)). It could justify the choice of a finite horizon model in discrete time. It facilitates the analysis.

The paper is organized as follows. In section 2, I present the model and the results. I propose a theoretical robustness analysis in section 3. I test my main results in section 4 and I conclude in section 5.

[24] Without the empirical validation of the authors, a non exhaustive list is Ferreira and Vilela (2004), Costa and Paz (2005), Nguyen (2006), Saddour (2006), Chang, Tan and Wong (2007), and Sufi (2007). In addition, an empirical study of the financial development determinants has been proposed by Khurana, Martin and Pereira (2006).

2 Model

2.1 Structure

I develop a framework permitting to identify the financially constrained firms. To this end, I construct a three-period model that illustrates the effect of financing constraints on firms cash policy.

There are three stages 0,1 and 2. In stage 0, a firm has a finite set of assets with value $A > 0$ and a reserve of cash equal to $C_0 \in [0; \overline{C_0}]$. C_0 represents the cash holdings plus the cash-flows at date 0. The manager can invest in a profitable project $I_0 \in [0; \overline{I}]$ and subscribe an amount of debt $D_0 \in [0; \overline{D}]$. $\overline{C_0}$, $\overline{I}$ and $\overline{D}$ are positive finite numbers. The manager can invest all the financing available in stage 0 or transfer an amount of cash in stage 1. I denote this amount C, $C = max\{C_0 + D_0 - I_0; 0\}$. C will be the amount of cash holdings in date 1.

Uncertainty is present in period 0. In period 1, there are two states of nature: H (high) and L (low). The probability to be in state H is $p \in [0; 1]$ and the probability to be in state L is $(1 - p) \in [0; 1]$. This uncertainty is at the origin of the precautionary motive to transfer cash at date 1. Assets in place generate positive and finite cash-flows $C_H > 0$ in state H and $C_L > 0$ in state L, with $C_H > C_L$. In stage 0, I denote $C_1 \in \{C_H; C_L\}$ the random cash-flows in stage 1. Moreover, the firm in period one can invest a variable amount in a new project: I_H in state H and I_L in state L. In stage 0, I denote $I_1 \in [0; \overline{I}]$ the future random investment in stage 1. This investment project constitutes the incentive to increase the financing capacity at date 1.

The maturity of the debt contract is the beginning of period 2. Uncertainty is resolved in stage 1: in stage 2, the firm remains in the same state as in stage 1 and earns an amount of cash $f(I_H) + g(I_0)$ or $f(I_L) + g(I_0)$ which is known at date 1. The function g characterizes the profitability of the first investment, the function f is state independent. The function $f, g : [0; \overline{I}] \times [0; \overline{I}] \rightarrow \Re^{+2}$ satisfy the usual Inada conditions: I assume monotone decreasing marginal returns to scale.

To conclude, the manager can use risk management to reduce the variability of the future cash-flows: with a derivative contract, it is possible to transfer cash from state H to state L. The derivative security works like an insurance, but has the same structure as a future contract. No payment is made in stage 0, and $R_1 \in [0; \overline{R}[$ is settled or received in stage 1: $R_H = -(1 - p)R$ in state H and $R_L = pR$ in state L. $R \in [0; \overline{R}[$ is chosen by the manager. The manager can hedge partially or totally the firm against the variance of the cash-flows. A full insurance situation corresponds to the special case $\overline{R} \geq \frac{C_H + C_L}{2}$. I do not make particular assumption concerning the value of $\overline{R}$. In other terms the hypothesis that the manager can hedge the firm against the variance of the cash-flows does not affect the results, but permits to propose a base case model more general. The model's time line is given in appendix B.

2.2 Main Hypotheses

- All agents are risk neutral and the risk free rate is zero.
- Expected cash flows are verifiable and partially pledgeable.

The assumption that the expected cash-flows are pledgeable seems to be realistic. The risky debt is the consequence of a partially verifiable contract. In practice, liquidity ratios like current, quick or operational cash-flows ratios are used. It permits precisely to ensure that a certain amount of cash will remain in the firm.

The recognition and analysis of the relation between future investments and corporate borrowing is not new (e.g. Myers 1977). Empirically, Bradley and Roberts (2004) demonstrate that there is a link between growth opportunities and bond covenants and that the covenant structure and the yield on corporate bonds are determined simultaneously. Roberts and Sufi (2009) show that one of the primary determinants of the credit contract renegotiation is the investment opportunity.

Theoretically, this assumption is used in many frameworks (e.g. ACW 2006 or 2010). It allows examining the relation between current financing capacity and future investment decisions.

- Financing constraint: creditors accept to finance a limited constant fraction τ, $\tau \in [0;1[$ of expected cash-flows.

Choice of the friction:

I need a financing rationing, because if the resources are unlimited the optimal corporate policy decisions are trivial: the optimal investment amount can be always achieved. Importantly, the limited financing supply is the unique friction I use. Parsimoniousness simplifies the understanding of the analysis.

Financing rationing occurs, when the manager is not able to attract the amount of funds desired. Two reasons can be identified: the financing is too costly or the supply of funds is limited. I exploit the second reason. In the model, there is no financing costs, but the investors accept to finance only a limited fraction of the expected cash-flows. The consequence of financing rationing is a capital rationing. The manager is not able to optimally invest in all the profitable projects available.

In practice, we could argue that the availability of the funds is as important as the costs supported to obtain the financing. Moreover, many works highlight that the access to the credit supply is the main determinant of the investment decisions (e.g. with a macroeconomic approach Stiglitz and Weiss (1981) or Greenwald, Stiglitz and Weiss (1984)).

Origin of the friction:

The financing constraint can be related to the inalienability of human capital or to moral hazard. Both frictions are the consequence of incomplete contracting.

Hart and Moore (1994) are the first ones to propose an analysis of the inalienability of human capital: the manager can have exclusive competences that increase the value of the project. Because it is contractually impossible to prevent the manager from leaving the firm, the manager can try to renegotiate the agreed upon payments. Assuming the manager skills are necessary to ensure the success of the project, he will obtain a fraction of the expected cash-flows.

The consequence of moral hazard in project choice has been studied by Holmstrom and Tirole (1997): supposing it is impossible to impose contractually the optimal investment, the manager has an incentive to use his freedom to select the projects where he will get some private benefits. The optimal behavior of the manager can be obtained by granting some portion of the expected cash-flows.

In both cases, the costs generated by the identified frictions need to be proportional to the expected cash-flows: if the cost produced by the friction is fixed, it can be included into the evaluation of the project. As a consequence, the investors would finance all the investment for any profitable project. It is because an increase in the financing of the project increases the wealth of the manager that the investors accept to fund only a fraction of the expected cash-flows.

Choice of the parameter τ :

The parameter τ captures the financing constraint: it represents the fraction of the expected cash-flows that the investors accept to supply. Supposing that the expected cash-flows cannot be verified, Myers and Rajan (1998) show that the availability of funds is a function of the tangibility of the assets and the legal environment. An explicit parameter to assess the liquidity of the assets could be used. I argue that it plays no role and overloads the analysis. I discuss this assumption in the next section.

In general, the parameter τ can also include some direct fees proportional to the amount of funds. Including these variable costs reduces the financing capacity and also simplifies the analysis. Nevertheless, I do not explicitly consider variable costs. More importantly, the parameter τ permits to define adequate limit cases: when $\tau = 0$, there is no external financing available. When $\tau = 1$, there is no financing constraint. It is always possible to invest the required amount of funds and the net present value of the project is always maximized. Finally, I need to assume $\tau f' \bigwedge \tau g' \in [0, 1[$ else the problem is not well defined: an increase of the investment generates a stronger increase of the financing.

2.3 Analysis

Without agency problems, the manager's objective is to maximize shareholders wealth (SW). The shareholders wealth corresponds to the sum of the expected dividends over the lifetime of the firm. Assuming that securities are fairly priced, the maximization of shareholders wealth corresponds to the maximization of the firm value. I denote P_0 the initial dividend (P for Payment) and the future random dividends are P_1 and P_2. The optimization program is:

$$\begin{aligned} \underset{C,D_0,I_0,I_1,R}{max(SW)} &= P_0 + E[P_1 + P_2] \\ s.t.: P_0 &= C_0 + D_0 - I_0 - C \geq 0 \\ P_1 &= C_1 - I_1 + C + R_1 \geq 0 \\ P_2 &= g(I_0) + f(I_1) - D_0 \\ D_0 &\leq \tau(g(I_0) + E[f(I_1)]) \end{aligned} \tag{1}$$

I explicitly admit that there is a link between future investment opportunities and the firm's ability to raise funds: $D_0 \leq \tau(g(I_0) + E[f(I_1)])$. I consider that a fraction of the expected cash-flows is pledgeable as discussed in the last sub-section. Using this main assumption we will see that the amount of cash transferred C depends on the level of the constraint.

I solve now the optimization problem. Two cases need to be analyzed depending on whether the manager is financially constrained or not. The details of the following computations are available in appendix A.

2.3.1 Unconstrained solution(s):

An unconstrained manager can maximize the value created by the projects available. The net value of the first investment equals $g(I_0) - I_0$ and the net value of the second project $f(I_1) - I_1$. I denote $I_0^{**} = g'^{-1}(1)$ the optimal first amount invested and $I_1^{**} = f'^{-1}(1)$ the optimal second amount invested. Two cases can be identified:

When the constraints are all saturated, but the manager is still able to invest at optimal levels, all the borrowing capacity should be used. Moreover, there is one unique solution for C. I denote the optimal transfer of cash C^{**}: I use the double star for the optimal unconstrained value of the variables of choice. We find $D_0^{**} = \tau(g(I_0^{**}) + E[f(I_1^{**})])$ and $C^{**} = C_0 + D_0^{**} - I_0^{**}$.

When the financing constraint is not binding, but the optimal investment thresholds are reachable, there is an infinite number of solutions. The corporate policies are undetermined.

2.3.2 Constrained solution:

Considering each stage separately, a constrained manager wants to maximize investment. In other words, each dollar available and not invested reduces shareholders wealth. Therefore, there is no reason to pay a dividend before stage 2 and all the financing capacity should be used in stage 1 (the constraints are saturated). The objective becomes to maximize the final payment P_2, but the borrowing capacity D_0 depends also on the optimal cash policy C^*. I use the single star for the optimal constrained value of the endogenous variables. The manager's problem becomes:

$$\max_C \quad g(C_0 + D_0 - C) + E[f(C + C_1 + R_1)] \tag{2}$$

I denote $D_0^* + C_0 - C^* = I_0^*$ and $C_1 + R_1^* + C^* = I_1^*$. Assuming that the firm can use hedging instrument permits to eliminate part or all of the variance of the cash flows in stage 1. Supposing hedging instruments are fairly priced, the manager's objective is to equalize the financing capacities in the states H and L. It is simply a traditional full insurance result, when the concave productivity function is state invariant (Froot, Scharfstein and Stein (1993)). Therefore, $C_H - (1-p)R^* = C_L + pR \Rightarrow R^* = C_H - C_L$. In this case, the manager's problem becomes $\max_C g(C_0 + D_0^* - C) + f(E[C_1] + C)$. The optimal cash policy C^* is determined by the following equality:

$$(f'(I_1^*) - g'(I_0^*))(1 + \frac{\tau g'(I_0^*)}{1-\tau g'(I_0^*)}) \quad = 0 \Leftrightarrow \frac{f'(I_1^*) - g'(I_0^*)}{1-\tau g'(I_0^*)} = 0 \tag{3}$$

At optimum the manager chooses $C^* = D_0^* + C_0 - I_0^*$ with $D_0^* = \tau(g(I_0^*) + f(I_1^*))$ to equalize the marginal productivities of present $g'(I_0^*)$ and future $f'(I_1^*)$ investment opportunities.

When the manager is unable to eliminate the variance of the future operational cash-flows (a reasonable assumption) the manager uses his assumption concerning the distribution of the states and the uncertainty is maintained. We find:

$$(E[f'(I_1^*)] - g'(I_0^*))(1 + \frac{\tau g'(I_0^*)}{1-\tau g'(I_0^*)}) \quad = 0 \Leftrightarrow \frac{E[f'(I_1^*)] - g'(I_0^*)}{1-\tau g'(I_0^*)} = 0 \tag{4}$$

The manager chooses $D_0^* = \tau(g(I_0^*) + E[f(I_1^*)])$ with the objective to equalize the marginal productivity of present $g'(I_0^*)$ and expected $E[f'(I_1^*)]$ investment opportunities.

Now, I am not interested by the value of the optimal cash transfer or optimal debt borrowings. I want to propose an analysis of the cash policy in order to identify the constrained firms. The optimal level of cash C^* is a function (e.g. h) of two exogenous parameters, C_0 and τ: $h : [0; \overline{C_0}] \times [0; 1[\rightarrow [0; \overline{C_0} + \overline{D_0}]$. Assuming that τ, C_0 and C_1 are independent, by characterizing the relation between the function and the parameters, I am able to suggest the following propositions.

2.3.3 Results

Proposition 1: the variation of cash holdings decreases with the degree of financing constraints.

First of all, remark that the optimal variation of cash holdings is equal to

$C^* + \triangle C_0 - C_0$ with $\triangle C_0 \leqslant 0$, I denote, the finite amount of cash-flows at date 0. Consecutively, the constraint sensitivity of the variation of the cash holdings (the constraint sensitivity) equals $\frac{\partial(C^*+\triangle C_0 - C_0)}{\partial\tau} = \frac{\partial C^*}{\partial\tau}$. Using the equality between the productivity of both investments at optimum to compute the effect of the variation of the constraint parameter τ on the optimal cash level C^*, we find:

$$\frac{\partial C^*}{\partial\tau} = \frac{g(I_0^*)+E[f(I_1^*)]}{1-\tau g'(I_0^*)} \frac{g''(I_0^*)}{g''(I_0^*)+E[f''(I_1^*)]} > 0$$

The variation of the cash holdings decreases with the degree of constraint. The relationship is monotonic. The ratio $\frac{1}{1-\tau g'(I_0^*)}$ is similar to the multiplier effect of Kiyotaki and Moore (1997). However, it is not possible to identify ex ante if the shock on the credit limit τ will be amplified or reduced.

This result needs to be validated empirically. It is precisely the objective of the section 4.

Proposition 2: for constrained firms, the cash-flow sensitivity of cash is positive, but the relation with the degree of constraint is not monotonic.

The difference with the ACW setting is to recognize that the future investment project can impact the current financing capacity. At this point, it is interesting to check whether their results remain valid. ACW find a cash flow sensitivity of cash equal to: $\frac{g''(I_0)}{g''(I_0^*)+f''(I_1^*)}$. Keeping their assumption that perfect hedging is possible and recognizing that the relationship between the future ($E[C_1] + C^*$) and present ($C_0 + D_0^* - C^*$) financing capacities distorts the sensitivity, the cash-flows sensitivity of cash becomes:

$$\frac{g''(I_0^*)(\frac{\partial D_0^*}{\partial C_0}+1)}{g''(I_0^*)+f''(I_1^*)}$$

An exogenous shock affecting the cash-flows changes the current and future financing capacities and modifies the optimal cash buffer C^*. We see that ACW results are only valid if we assume $\frac{\partial D_0^*}{\partial C_0} = 0$. In the model I propose it is only the case for highly constrained firms: $\tau = 0 \Rightarrow \frac{\partial D_0^*}{\partial C_0} = 0$. Computing the sensitivity of the debt financing $\frac{\partial D_0^*}{\partial C_0}$, we find:

$$\frac{\partial C^*}{\partial C_0} = \frac{\frac{g''(I_0^*)}{1-\tau g'(I_0^*)}}{g''(I_0^*)+f''(I_1^*)} > 0$$

The cash-flow sensitivity of cash remains positive. However, we see there is a link between the cash flow sensitivity and the constraint sensitivity: $\frac{\partial C^*}{\partial C_0} =$

$\frac{\partial C^*}{\partial \tau}\frac{1}{g(I_0^*)+f(I_1^*)}$. The cash flow sensitivity of cash is lower than the constraint sensitivity.

Computing the second order derivative $\frac{\partial^2 C^*}{\partial C_0 \partial \tau}$, the main drawback of the indicator is obvious. It is impossible to assess the magnitude of the financing constraint without specifying the third derivative of the production functions g''' and f'''. Moreover, the relation is not monotonic, because the third order derivative is not necessarily constant.

To summarize, with the ACW specification the degree of the constraint plays no role for a certain class of utility function (similar concavities for current and future production functions). Assuming a link between the present financing capacity and the cash holding choice, the effect of the constraint remains for all type of Cobb Douglas functions. In addition the hypothesis of perfect hedging is not necessary.

Proposition 3: the current investment to cash sensitivity is positive. The variation of investment to cash sensitivity can be positive or negative.

Using previous results we can easily compute the investment to cash sensitivity. First, we find: $\frac{\partial D_0^*}{\partial C_0} = \frac{\tau g'(I_0^*)}{1-\tau g'(I_0^*)}$. Consecutively, we obtain:

$$\frac{\partial I_0^*}{\partial C_0} = \frac{1}{1-\tau g'(I_0^*)} - \frac{\frac{g''(I_0^*)}{1-\tau g'(I_0^*)}}{g''(I_0^*)+E[f''(I_1^*)]} = \frac{1}{1-\tau g'(I_0^*)}(1 - \frac{g''(I_0^*)}{g''(I_0^*)+E[f''(I_1^*)]}) > 0$$

The relationship is monotonic as long we assume that the production functions satisfy Inada conditions. However, computing the variation of investment between stage 0 and 1, we find:

$$\frac{\partial(I_0^*-I_1^*)}{\partial C_0} = \frac{1}{1-\tau g'(I_0^*)}(1 - 2\frac{g''(I_0^*)}{g''(I_0^*)+f''(I_1^*)}) \lesseqgtr 0$$

The sensitivity can be negative (Povel and Raith (2001) find a similar result). An increase of the marginal productivity of the future investment project f' and a decrease of the concavity of the future production function f'' reduces the investment to cash sensitivity. This result is important, because it shows that an exogenous shock on the cash-flows changes not only the current level of investment, but also the future one. However the variations of the amount invested are difficult to predict, because they depend on the concavity of the production functions. This result confirms one of the main criticisms formulated for the FHP framework.

I propose now to show that the randomness of the financing capacity generates itself an incentive to finance the firm early. This result is important, because it illustrates a situation where the level of cash-holding is simply the consequence of the desire to maximize the financing capacity of a future profitable investment.

Proposition 4: the randomness of the financing capacity generates an incentive to finance the firm early.

The randomness of the cash-flows has for consequence a random financing capacity. When the cash-flows are low the creditors accept to lend a smaller amount of funds. At date 0 the creditors use an average scenario to predict the level of cash-flows at date one of the firm. Consecutively, if we assume there is some uncertainty affecting the level of cash-flows, there is always at least one state of the world where the financing capacity is lower at date 1 than at date 0.

Formally, the amount the creditors accept to lend at date 0 is equal to $\tau(g(I_0^*)+E[f(I_1^*)])$ and at date 1 is equal to $\tau(g(I_0^*)+f(I_L^*)])$. Moreover by Jensen inequality we find $E[f(I_1^*)] > f(E[I_1^*]) > f(I_L^*)$, implying $\tau(g(I_0^*)+E[f(I_1^*)]) > \tau(g(I_0^*)+f(I_L^*)])$. It means that the manager has an incentive to finance the firm early in order to save some financing capacity. Importantly, in state H the manager has always the possibility to refinance the firm. I have not treated this case to be parsimonious.

Knowing that the uncertainty of cash-flows generates an incentive to finance the firm early and consecutively to create a cash-buffer, a natural question is: can the variance of the cash-flows explain the amount observed of cash-holdings? It was precisely one of the main objective of the Han and Qiu (2007) paper. Their empirical analysis seems to confirm that the idiosyncratic volatility of the cash-flows affects the level of cash-holdings: an increase of the risk corresponding to an increase of the reserve of cash.

With the framework I propose, two important remarks can be formulated. First of all it seems not necessary, as they do, to specify the sign of the third derivative of the production function to explain the link between the variance of the cash-flows and the level of cash-holdings. However, it is true that a systematic positive relation between the level of cash and the risk does not hold if we do not specify this third derivative. It is one of the main critics we can addressed to the Han and Qiu framework, because their assumptions concerning the behaviour of the production functions are difficult to justify in a systematic way in practice.

A second remark concerns the hedging against the variance of the cash-flows. It is not necessary to assume an imperfect hedging to find a relationship between the variance of the cash-flows and the level of cash-holdings.

For example, in my model, an exogenous variation of the anticipated cash-flows can also affect the optimal level of cash holdings. Assuming a perfect hedging, the manager equalizes the financing available in all the states in stage 1. The optimal investment I_1^* is equal to:

$$\frac{2C^*+C_H+C_L+pR^*-(1-p)R^*}{2}$$

It is simply the sum of all the financing (in each state) divided by the number of states. The expected investment $E[I_1^*]$ is equal to $C^* + E[C_1]$ and depends on the spread $C_H - C_L$ [25]. Consecutively, the optimal cash transfer C^* is also a function of the anticipated spread of cash-flows. Because a change of the variance of the cash-flows could be the consequence of the change of the spread, it becomes impossible to identify if the value of C^* has changed consecutively to the anticipated variance of the cash-flows or to an exogenous change of the constraint. The conclusion is that it appears preferable to group the firms with similar anticipated cash-flows variances if we want to be sure to identify the impact of a shock on the credit rationing. However we can reverse this proposition and argue that it would be preferable to control for credit rationing shocks if we want to observe the effect of the variance of the cash-flows on the cash-holdings. Importantly, empirically, we can approximate the anticipated variance using the past variance of the cash-flows, because we can assume that the manager and the creditors use the historical variance to assess the future one.

Finally and independently of the model I propose, we can discuss the fact that a perfect hedging does not leave the incentive to finance the firm early. The manager could have access to a better financing, if he is able to provide some guarantees concerning the future of the firm. In other words, a perfect hedging at date 0 against the variation of the cash-flows at date 1 could improve the current financing conditions. Without considering this research direction, it seems obvious that the optimal cash buffer depends on future hedging needs and hedging capacities. Acharya, Almeida and Campello (2007) is a good example. The detection of a constrained firm should be made by taking into account a proxy to the hedging needs. The variance of the cash-flows is an example.

The last proposition I formulate concerns the difficulty using a shock on the cash-flows to anticipate a change of the cash-holdings. In particular, I relax the Inada conditions assuming the existence of a minimal investment threshold.

Proposition 5: the cash-flow sensitivity of cash can be negative.

This proposition is not new. ACW (2010) find also that the cash-flow sensitivity of cash can be negative. In their model, they consider investments with different recoveries. Using an investment opportunity, which delivers intermediate cash-flows, they use an alternative way to increase the financing capacity. For a constrained firms, the objective is to increase the future financing capacity, but by taking into account the costs associated with the increase of the financing. If the investments delivering an intermediate cash-flow are less costly (or permit more to increase the financing capacity, all things being equal) than the cash-holdings, the optimal reserve of cash can be reduced after an increase of the cash-flows. Nevertheless, they assume implicitly an historical suboptimal level of cash-holdings or that the cost to keep the cash and the liquidity of the investment have changed. Else, in the model they propose, the manager has no need to wait an increase of the cash-flows to reduce the optimal reserve of cash S (p.18-19). Finding that the optimal level of liquid investment increases system-

[25] we could also make the analysis using the distribution of the states.

atically, there is no reason to keep the cash into the firm. The only possibility that the optimal reserve would be reduced after an increase of the cash-flows (all things being equal) is that the additional cash obtained destroys a fraction of the value of the cash-holdings. This hypothesis is not realistic. Assuming suboptimal reserve of cash, their result can be obtained with various model specifications.

For example, it is sufficient to relax the Inada conditions and to assume there is a cost to hoard the cash. If the slope of the production function is not infinite at the origin, it could be the case that a minimal investment threshold need to be reached to obtain a positive net value.

Denoting the minimal investment thresholds $\underline{I_0} : g(\underline{I_0}) - \underline{I_0} = 0$ and $\underline{I_1} : f(\underline{I_1}) - \underline{I_1} = 0$ and assuming a highly constrained manager is not able to invest at the minimal investment threshold in stage 0 and 1, the manager has no reason to look for a particular financing. Notably, the level of the financing constraint can be now assessed by the ratio $\frac{\tau}{\underline{I_0}+\underline{I_1}}$. An increase of the sum of the minimal investment thresholds or a decrease of the parameter τ, raises the difficulty to invest in a profitable project. Moreover, if the storage of the cash is costly, the optimal solution becomes to maximize the payout in stage 0. In this case, if the historical (sub-optimal) value for C^* was positive or equal to zero, then the cash flow sensitivity of cash could be negative or null.

ACW(2004) discuss that no optimal corporate policies can be identified for an unconstrained firm, because in this case there is no systematic manager's behaviour. Wherefore, the sensitivity of cash should be undetermined. In other terms, they argue that the variation of the target cash reserve should be in average equal to zero for an unconstrained firm. If it is not possible to distinguish a constrained firm and an unconstrained firm, when the sensitivity is close to zero, the sensitivity of cash appears to be a useful measure only for significantly positive and negative sensitivities.

3 Robustness

While the main hypotheses have been already commented (subsection 2.2), I propose a discussion of the other implicit hypotheses or explicit model specifications I have made. The discussion concerns only the manager behaviour of the financially constrained firms. I do not claim to be exhaustive, the objective is to evaluate the robustness of my main results.

3.1 Assets Liquidity

I have supposed that the assets with value A, which generate the operational cash-flows, are illiquid. It means they are not used as a collateral and do not increase the financing capacity. I argue that an increase of the liquidity of the assets does not change the nature of the relation between the future investment opportunity, the financing capacity and the optimal cash buffer. Assuming that the existing assets and the future acquired assets can be used as a collateral has for consequence to increase the financing capacity today and tomorrow. Only the level of the financing capacities is modified.

Supposing that the liquidity value of the assets is equal to $q(A+I_0+I_1)$ with $q \in [0;1]$, the current financing capacity remains D_0+C_0-C. The debt capacity becomes $\tau(q(A+I_0+E[I_1])+g(I_0)+E[f(I_1)])$. Computing the optimal amount of cash-holdings, we find now at optimum $\frac{E[f']-g'}{1-\tau g'-\tau q}=0$ (details in appendix A). The manager does not take into account the liquidity of the assets and continue to equalize the current productivity of the investments with the expected one. Only the value of the financing multiplicator is changed.

However, assuming that the liquidity of the assets is different in stage 0 and 1 there is a new trade-off that the manager must face choosing between the marginal productivity of one investment and the marginal gain of liquidity of the other one. Formally, the relation characterizing the optimal level of cash holdings becomes: $E[f']-g'=\tau(q_0-q_1)$ with $q_0 \in [0;1]$ representing the liquidity of the current assets and $q_1 \in [0;1]$ the liquidity of the expected one. In that case, the solution is not necessarily unique depending on the concavity of both production functions.

3.2 Random Output

So far I have assumed that the production function at date 1 is constant. In other words that the profitability of the investment is known. Introducing randomness does not change the analysis. For example, supposing that the production function becomes a random variable generates no additional uncertainty, because the manager tries already to anticipate the level of cash-flows at date 1 to chose the appropriate amount of cash-holdings. Moreover, if we assume that the profitability of the investment is also random, the manager will simply try to equalize the expected productivity of all the accessible investments.

The key incentive to use the cash-holdings is the randomness of the financing capacity at date 1. As long as this financing capacity is random, there is at least one state of the world at date 1 where the value of the collateral is lower than the expected value at date 0.

3.3 Costs of Carry

The increase of the cash buffer C has already a cost. It reduces the financing available C_0+D_0-C for the first investment project. Adding a supplementary cost has for effect to reduce the optimal level of cash-transfer.

Assuming that no external financing is available at date 2, two extreme cash policies can be identified. When the costs of carry become sufficiently high the first policy would be to invest all the cash-flows in the first investment project and not to use a cash buffer. The second policy would be to use all the cash-flows to increase the payout to the shareholders.

However, the introduction of hoarding costs creates an interesting trade-off if we assume that the manager is able to finance the firm tomorrow with an additional amount of borrowing. In that case, the manager has no more reason to use systematically all the borrowing capacity in stage 1. Consecutively, the manager can decide to reduce the borrowing today and to borrow more tomorrow instead of using the cash reserve . A new trade off appears between the reduction of the current debt capacity and the increase of the future one. Consecutively, the amount of long term or short term borrowing should depend on the level of the constraint. Considering sufficiently high costs of carry, the precautionary motive of cash buffer becomes insufficient. For this extreme case, with simple computations we find the same optimal relationship $g'(I_0^*) = f'(I_1^*)$ and a debt sensitivity $\frac{\partial D_1}{\partial D_0}$ equal to $\frac{\tau g'(I_0^*)-1}{1-\tau g'(I_0^*)} < 0$. The sensitivity depends on the level of the constraint in a simple way. Moreover, the most constrained and unconstrained firms should be indifferent between long term and short term debt. The other firms should engage in more active maturity management. Obviously, this result is not satisfactory. In practice, the optimal cash buffer matters. Consecutively, it is more reasonable to assume that significant costs of carry generate a trade-off between the saving of the financing capacity in low cash-flows state of the world and the costs supported to increase the future financing capacity using the cash-holdings. It means that for sufficiently low level of costs of carry the relationship between the degree of the financing rationing and the variation of cash-holdings holds.

At this stage of the analysis, it is important to recall that the objective is not to take into account all the determinants of the cash-holdings. The variation of the cash buffer is undetermined if we consider to much trade-offs. I argue that the model structure I propose is able to identify one of the main determinants of the link between the level of the financing constraints and the financing capacities.

3.4 Cash-Flows

So far I have assumed that the assets in place generate the cash-flows only in stage 0 and 1. They are the consequence of ordinary activities. Even if it could be more realistic to assume that the assets generate cash-flows in each stage of the model, the operational cash-flows earned in stage 2 are useless. The uncertainty is already present via the random value of the optimal investment at date 1.

Concerning the randomness of the cash-flows at date 1, trivially, without uncertainty, the previous propositions do not hold. The optimal financing tomorrow can be financed by an increase of the cash-buffer C or by a reduction of the first debt borrowing D_0 without changing the initial amount invested I_0.

3.5 Priority Structure and Random Financing Constraint

The impact of financing constraints on priority structure is always an open question[26]. In the model I propose there is no external financing at date 1. The manager has no incentive to wait before funding the firm.

[26] We can mention the paper of Hackbarth, Mauer (2009). With a dynamic Leland type model, they show the existence of an interior optimal priority structure which eliminates the over and underinvestment problem.

We have seen that the introduction of costs of carry generates an incentive to wait before to finance the firm. Another interesting case to analyze would be to imagine that the manager anticipates a reduction of the financing constraints[27]. For a constrained manager, it becomes systematically optimal to raise funds at date 1, because it permits to increase the investments. Consecutively, because the priority structure determines at date 2 the sharing of cash-flows between the creditors, it becomes a new important variable of decisions. In particular, we should identify that the optimal priority structure depends on the profitability function of both investments. Moreover a shock on the financing constraints will affect the external financing capacity at date 0 and 1. Notably, an increase of the constraint should reduce both financing and modifies the optimal priority structure.

Now as long as we assume that the manager tries to equalize the productivity of both investments, there is no reason that a reduction of the overall financing capacity should modify only one investment. The fact I assume that the manager can pledge a fraction of all expected cash-flows has for consequence to link the financing capacity at date 0 and 1. The priority structure does not affect this setting.

4 Empirical Analysis

4.1 Methodology and Data

I test my model's main prediction about a monotonic negative relationship between the degree of the financing constraint faces by the managers and the variation of cash-holdings. The objective is not to identify the determinants of cash-holdings but a new, significant and theoretically justified relation. To do so, I propose a panel data approach estimating a fixed effect model. To test my theoretical findings, I implicitly assume that the manager is relatively myopic by considering only one future investment opportunity. Like this, the model prediction can be tested in a panel data approach taking an appropriate laps of time between the observations.

I constitute a sample of yearly data of all manufacturing firms (SICs 100 to 3999) available on WRDS-COMPUSTAT over the period 1971-2011. I keep manufacturing, mining and agricultural firms and I leave health and financial sectors. Like this, the sample permits to compare my results with the ones in ACW (2004). I cannot choose exactly the same period of time or data, because I obtain more missing values by computing the indices of constraints. I eliminate firm-years observations for which cash holdings exceeded the value of total assets, those for which the market capitalization is less than \$15 million dollars and those displaying sales growth exceeding 50%. I remove firms with small capitalization and strong growth of sales with the intention to avoid eventual mergers and acquisitions biases. My final sample consists of 21'337 rows of

[27]The optimal cash policy will be changed only if the manager expects a reduction, else no additional funds can be obtained in stage 1. In other words the incentive to maximize the financing at date 0 would be maintained.

observations (firm-years)[28].

4.2 Empirical Baseline Model

The empirical model is built to describe the role of the financing constraint and to capture the precautionary motive that I use to keep cash into the firm. I test proposition 1 first i.e. that there is a negative correlation between financing constraints and cash-holdings variations. Other incentives to save cash are considered later. The baseline model can be written as:

$$\begin{aligned}\triangle CashHoldings_{i,t} &= \alpha_0 + \alpha_i + \tau_t + \alpha_1 ConstraintIndicator_{i,t-1} + \alpha_2 Q_{i,t-1} + \\ &\quad \alpha_3 logSize_{i,t-1} + \alpha_4 EBIT_{i,t-1} + \epsilon_{i,t} \\ \triangle CashHoldings_{i,t} &= CashHoldings_{i,t} - CashHoldings_{i,t-1}\end{aligned} \tag{5}$$

where α_i captures firms fixed effect and τ_t time effect. I correct the residuals covariance matrix for heteroskedasticity and for within-period residuals correlation using the Eicker-Huber-White estimator. $CashHoldings$ is the ratio of holdings of cash and marketable securities to total assets. $ConstraintIndicator$ is one of the proxies I use to identify the degree of the constraint. Q is the market value divided by the book value of assets. This variable is used to capture the impact of future investment opportunities on cash decisions (the precautionary motive in my model). The Q is an approximation and subject to difficulty of measurement. Similarly to ACW 2004 I argue that measurement problems are unlikely to bias my inferences, because I use a financial (as opposed to a real) dependent variable. I control for size with the variable $logSize$. I use this additional control, because standard arguments concerning economies of scale in cash management and to capture the mechanical relationship between the variables as the assets size appears on both sides of the regressions. I take the log to reduce the impact of extreme variations, $logSize$ is the natural log of assets[29]. Finally, I use the $EBIT$ variable as a measure of the operational cash-flows. I define $EBIT$ as the earnings before interests and taxes divided by the book value of assets. I do not try to approximate the free cash-flows to firms. I explicitly avoid to make some assumptions regarding some extraordinary items or the depreciation policy of the firms.

4.3 Indicators of the Financing Constraints

I propose to select four of the most commonly used proxies of the financing constraints. I have no prior concerning the best indicator. I cannot select the proxies assessing the credit quality of the firm (commercial paper or bond ratings), because I need numerical values. It is my main criterion of selection.

[28] There is a small difference between the number of observations available for the different proxies of financing constraint.

[29] All the regressions are also made taking the asset size directly as a control variable. The results are very similar.

Table 1: **Dependent Variable Description**

This table displays the main information on the dependent variable.

	Statistics			
	Mean Value	Standard Deviation	Minimum	Maximum
Cash-Holdings	.209	.224	0	0.997
Data by firms	7.47	3.38	0	11

N. obs: 17'640

- I compute for each firm the Whited and Wu index (2006). To do so, I apply the following linearisation to the data:

$$\begin{aligned} WW_{i,t} &= -0.091CF_{i,t} - 0.062DIVPOS_{i,t} + 0.021TLTD_{i,t-1} \\ &-0.44LNTA_{i,t-1} + 0.102ISG_{i,t} - 0.035SG_{i,t} \end{aligned} \quad (6)$$

  with CF the cash-flows from operations, $DIVPOS$ a dummy variable equal to one if the firm pays dividends, $TLTD$ the ratio of long term debt over assets, $LNTA$ the natural log of assets, ISG the three-digit SIC industry sales growth rate and SG the firm sales growth rate. The computation of the WW index generates a loss of observations, because the industry sales are not reported as frequently as the firms level data.

- I compute for each firm their payout ratio (similar to $Dividends$ in the KZ regression). The payout ratio is defined as the sum of dividends plus stock repurchases divided by operating income. The firms with higher payout ratio should be less constrained, because they try to keep less cash into the firm.

- Finally, I use the total of assets as a proxy for firms size and an additional financing constraint indicator. We can reasonably assume that small firms are typically young, are followed by a smaller number of analysts and have more difficulties to fund profitable projects.

4.4 Results

4.4.1 Data summary

I propose to begin by describing the main characteristics of the main variables.

In **Table** 1 you find the main information concerning the dependent variable ($CashHoldings$), the cash-holdings ratio. Cash-holdings in the sample I build represents a very high proportion of the asset value, about 21% in average. The result is partially explained by a small number of firms keeping more than 50% of the assets in cash. In addition, we observe a high standard deviation, about 22.4%. Another important remark concerns the number of observations, quite important (17'640), but the number of rows of data by firm is relatively

Table 2: **Correlations Between the Financing Constraints Indicators**

This table displays the adjusted Pearson's correlation coefficients between the financing constraint indicators, the corresponding t-stats at 5% level and the number of data.

	Financing Constraints Indicators		
	WW Indicator	Payout Indicator	Size Indicator
WW Indicator	1		
	13'244		
Payout Ratio Indicator	-0.0014	1	
	(0.868)		
	13'211	21'242	
Size Indicator	$\mathbf{-0.7352}^{\dagger}$	0.0079	1
	(0.000)	(0.2519)	
	13'244	21'242	21'337

Significance: $\dagger\ p < \mathbf{0.05}$

small, about 7.5 in average. The absence of market values among others missing variables reduces the number of useful explained variables.

I present now the Pearson correlation coefficients between the proxies of the financing constraint used. This analysis is relevant to understand the link between the explanatory variables. For example, we could assume that there is a mechanical relationship between the Whited and Wu index and the level of assets, because the total of assets (the proxy of firms size) is used in the computation of the index. In particular we can expect to find a negative relation between the index and the book value of the assets, because the level of assets is negatively weighted in the index.

In **Table** 2 we observe the significant negative and anticipated correlation between the asset size variable and the WW index. In addition and interestingly, we observe a positive correlation between the index and the total asset value and between the total of asset and the payout ratio, and a negative correlation between the index and the payout ratio. However the correlations are not significantly different from 0. Overall the correlations are the ones expected. For example an increase of the payout ratio or firms size should correspond to a decrease of the financing constraint.

4.4.2 Baseline model results

I propose now the results of the baseline regression model. The expected results are a significant coefficient for at least one of the proxies of the financing constraints and a low R^2. I do not expect to identify the main determinants of the cash-holdings variation, but an alternative determinant of cash-holdings variations than the one proposed by Almeida, Campello and Weisbach (2004).

In **Table** 3 we find significant coefficients at 5% level for two of the financing constraints indicators: the WW index and firms size. Interestingly enough even if there is a mechanical relationship between the log of assets and the cash-holding ratio, the coefficient of the size is positive and exhibits the predicted effect. However the WW index coefficient has not the predicted sign. The economic magnitude of the effects is difficult to interpret, because the dependent variable is a ratio. For example if firm size in year t is one million dollars then the predicted change of the cash-holding ratio from year t to year $t+1$ is an increase of 10% ($10^6 \times 10^{-5}\%$ in this case). The payout coefficient also has the predicted sign but is not significant at the 10% level.

The Q coefficient is positive but not significant at the 10% level. The $EBIT$ variable has no explanatory power neither. Moreover, the negative sign for the coefficient associated with the $EBIT$ variable is not the one predicted by the cash-flow sensitivity of cash theory. Finally, the firms fixed effect coefficients are all significant at the 1% level (unreported). This result is not surprising considering the few number of data available by firm.

Apart from the Q and the $EBIT$ variables, the values of the coefficients are the ones expected and confirm, at least for the size of the firm, the theoretical relation established. However the R^2 [30] are small and underscore the low explanatory power of the model. This result is understandable for three reasons. There are many alternative motives to keep cash, the theoretical model I set predicts only a monotonic relationship and the removal of extreme values that are candidates for outliers is difficult.

Now it becomes interesting to analyze whether the relation between the proxies of the financing constraints and the variation of cash holdings holds for alternative model specifications.

4.5 Model Extensions

I perform two robustness tests in this section.

Firstly, I want to identify whether the relation between the financing constraint and the change of cash-holdings is robust to a sub-sample classification of firms with similar cash-flows variances. Proposition 1 assumes that the cash-flows variance is constant and that the change of cash-holdings is the consequence of a change of the financing constraint. The variance is assumed to be constant, because both the probability to reach a particular state and the corresponding cash-inflows are constants in the model. Therefore it is necessary to control whether the empirical results are robust to a sub sample filtering.

Secondly, I need to extend the baseline empirical model by considering other important variables as investments and financing capacity which are supposed to be driven only by a change of the financing constraint in the theoretical model, but could be also driven by other forces like investments profitability. This would permit to improve the fitting of the model too.

[30] The R^2 reported in the Tables are the R^2 adjusted for the number of explanatory variables.

Table 3: **Baseline Models Results**

This table displays results for OLS estimations of the baseline regression model (equation (5)). All data are from the annual WRDS-COMPUSTAT database. The sampled firms include only manufacturers (SICs 100 to 3999) and the sample period is 1971 through 2011. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the Eicker-White-Huber estimator. The associated t-statistics are reported in parentheses. The numbers are rounded at the 4^{th} decimal place.

	Models(regressions)		
	WW Model	Payout Model	Size Model
	Coeff SE (cluster)	Coeff SE (cluster)	Coeff SE (cluster)
WW Index	$\mathbf{-0.0001}^{\ddagger}$ (0.0000)		
Payout Ratio		0.0001 (0.0001)	
Size			$\mathbf{0.0000}^{\dagger\dagger}$ (0.0000)
Q	0.0049 (0.0035)	0.0021 (0.0016)	0.0021 (0.0016)
LogSize	**-0.0774**‡ (0.0084)	$\mathbf{-0.0693}^{\ddagger}$ (0.0056)	$\mathbf{-0.0721}^{\ddagger}$ (0.0059)
Op. Cash (EBIT)	-0.0880 (0.0844)	-0.0078 (0.0457)	-0.0072 (0.0454)
Constant	$\mathbf{0.4717}^{\ddagger}$ (0.0534)	$\mathbf{0.4194}^{\ddagger}$ (0.0340)	$\mathbf{0.4330}^{\ddagger}$ (0.0352)
N. obs	11’350	16’168	16’213
R^2	0.065	0.042	0.043

Significance: † $p < 0.1$; †† $p < 0.05$; ‡ $p < 0.01$; (two-tail test levels)

4.5.1 Hedging needs and profitability shocks

I have discussed under proposition 4 that there is a link between the variance of the cash-flows and the incentive to finance the firm early. An increase of external financing should correspond to an increase of the cash-holding for a constrained firm.

Empirically, it is useful to describe if the changes of the cash reserves are due to an exogenous shock on the profitability of the firm or on the degree of the financing rationing. Naturally, I want to find a proxy of these cash-flows shocks. I decide to approximate the size of the shocks with the variance of the cash-flows. The firms that have higher cash-flows variances are natural candidates to be assumed more concerned with unexpected productivity variations. Moreover, the variance of the cash-flows can be also interpreted as an approximation of the hedging needs. That constitutes an advantage in comparison to the use of macroeconomic variables. Yet I have approximated the precautionary role of cash-holdings with a Q measure. It becomes interesting to use an alternative proxy of this precautionary motive. Assuming that a perfect hedging instrument is unavailable for the manager, it permits to identify if the relation between the financing constraint indicator and the variation of cash-holdings is robust and to compare the precautionary motives. Moreover, it could be the case that there is no more significant relationship if the main determinant of cash-holdings is to hedge the firm against a variation of the cash-flows.

To analyse the role of the variance of the cash-flows, I decide to divide the initial sample of firms in two sub-samples. The first sample is made up of the 20% of the firms with the higher variance of the cash-flows. Symmetrically the second sample is made up of the 20% of the lower cash-flows variance firms. This division does not capture the idiosyncratic volatility, but approximates the overall risk supported by the firms [31]. The variance is computed using the ratio of cash-flows over assets. The quantiles are selected to keep samples with substantial sizes. Note that the number of data reported for each model and groups is slightly different because of missing items.

The results of the firms with the higher variances of the cash-flows are presented in **Table** 4. The findings support the idea that the variation of cash holdings depends on the degree of the financing constraint when the hedging needs are not too important. None of the proxies of the financing constraint appears to be significant now and the explanatory power of the model is particularly low. It does not mean that the sample of high variances firms is not constrained. These results are in line with proposition 4. In the model, the variation of cash-holdings can be the consequence of a shock on the supply of funds or on the hedging needs. When the volatility of the cash-flow increases, this generates an incentive to create a reserve of cash. This incentive exists as long as a prefect hedging is not possible. When there is no uncertainty affecting the cash-flows, there is no more an incentive to finance the firm early. Therefore the variation of the supply of funds should be a better indicator of

[31] I do not make assumptions on market equilibrium or efficiency

Table 4: **Baseline Models Results for High Cash-flows Variance Firms**

This table displays results for OLS estimations of the base line regression model where I keep only the 20% of the firms with the highest variance of cash-flows (high hedging needs). All data are from the annual WRDS-COMPUSTAT database. The sampled firms include only manufacturers (SICs 100 to 3999) and the sample period is 1971 through 2011. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the Eicker-White-Huber estimator. The associated t-statistics are reported in parentheses. The numbers are rounded at the 4^{th} decimal place.

	Models(regressions)		
	WW Model	Payout Model	Size Model
	Coeff SE (cluster)	Coeff SE (cluster)	Coeff SE (cluster)
WW Index	-0.0000 (0.0000)		
Payout Ratio		-0.0002 (0.0001)	
Size			0.0000 (0.0000)
Q	0.0118 (0.0073)	**0.0119**‡ (0.0044)	**0.0124**‡ (0.0044)
LogSize	**−0.0280**‡ (0.0089)	**−0.0150**‡ (0.0057)	**−0.0170**‡ (0.0065)
Op. Cash (EBIT)	-0.0439 (0.0900)	-0.0510 (0.0412)	-0.0498 (0.0410)
Constant	**0.2392**‡ (0.0742)	**0.1279**‡ (0.0457)	**0.1414**‡ (0.0514)
N. obs	2’397	3’449	3’471
R^2	0.033	0.029	0.031

Significance: † $p < 0.1$; †† $p < 0.05$; ‡ $p < 0.01$;
(two-tail test levels)

the cash-holdings. I present the results of the firms with the lower variances of the cash-flows in **Table** 5.

I find a significant relationship at 5% between the variation of cash-holdings and the degree of the financing constraints for the WW index and the size of the firm. The magnitude of the effect is considerably higher in comparison to the baseline model. Using the same example as before if firm size is 1 million then the corresponding predicted increase of the cash-holding ratio is now 130%. Again this result is difficult economically to interpret, because I do not control for a change of the total of assets. The R^2 are also higher between 14% and 27%, when the hedging needs are low. A constrained manager can adapt the optimal amount of cash-holdings depending on the supply of funds. Even if I do not try to explain all the roles of cash-holdings, this result seems to confirm the existence of a precautionary motive for holding-cash. Trivially, the variation of cash holdings is explained by the degree of the financing constraint if and only if there is no other good reasons to keep cash within the firm. The uncertainty affecting the financing of the future investment opportunities seems to be one of these reasons.

Apart from the WW regression, hedging needs and investment opportunities seem to be linked: without future profitable investments there is no reason to hedge the firm against a variation of the cash-flows. The Q coefficients have the predicted sign in both model settings: positive when the hedging needs are high and negative when the hedging needs are low. However, the coefficients are not significant at the 10% level for the low hedging needs sample. Finally the $EBIT$ control has a negative and significant impact on the optimal level of cash-holdings. I propose now an empirical model extension in order to check whether alternative determinants of cash-holdings may improve the explanatory power of the model.

4.5.2 Alternative cash-holdings determinants

As already discussed and in order to test the robustness of proposition 1, I propose to take into account alternative and competing uses of funds. The uses of funds are different but the wide objective stays to capture the role of future investment opportunities in determining cash-holding variations. For this second robustness check, I assume now that the change in cash-holdings is also a function of capital expenditures ($Expenditures$), acquisitions ($Acquisitions$) and changes in short-term debt ($\triangle ShortDebt$). In addition of controlling for other investments I use short-term debt as a control for the following reason: I argue that we cannot directly control the working capital management or precautionary motive of the cash-holding, because the level of cash is precisely one of the main components of the working capital. Even if we could try to identify the impact of the variation of the non-cash elements, I have found a too much important number of missing data concerning the different proxies of the non-cash elements, which makes the analysis not relevant. Therefore, I have decided to keep only the impact of short term debt variations as a control for financing

Table 5: **Baseline Models Results for Low Cash-flows Variance Firms**

This table displays results for OLS estimations of the base line regression model where I keep only the 20% of the firms with the lowest variance of cash-flows (low hedging needs). All data are from the annual WRDS-COMPUSTAT database. The sampled firms include only manufacturers (SICs 100 to 3999) and the sample period is 1971 through 2011. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the Eicker-White-Huber estimator. The associated t-statistics are reported in parentheses. The numbers are rounded at the 4^{th} decimal place.

	Models(regressions)		
	WW Model	Payout Model	Size Model
	Coeff SE (cluster)	Coeff SE (cluster)	Coeff SE (cluster)
WW Index	**−0.0979**‡ (0.0345)		
Payout Ratio		0.0002 (0.0009)	
Size			**0.0013**‡ (0.0005)
Q	0.0006 (0.0004)	-0.0000 (0.0003)	-0.0001 (0.0003)
LogSize	**−0.1603**‡ (0.0293)	**−0.1383**‡ (0.0247)	**−0.2043**‡ (0.0440)
Op. Cash (EBIT)	**−0.4464**‡ (0.1140)	**−0.2504**‡ (0.0636)	**−0.2253**‡ (0.0655)
Constant	**0.5504**‡ (0.1178)	**0.4900**‡ (0.0960)	**0.6514**‡ (0.1408)
N. obs	1’869	2’879	2’884
R^2	0.272	0.146	0.153

Significance: † $p < 0.1$; †† $p < 0.05$; ‡ $p < 0.01$;
(two-tail test levels)

capacity. All the new controls are scaled by the total of assets and $\triangle$ remains a lag operator. The model becomes:

$$\begin{aligned} \triangle CashHoldings_{i,t} &= \alpha_0 + \alpha_i + \tau_t + \alpha_1 ConstraintIndicator_{i,t-1} + \alpha_2 Q_{i,t-1} \\ &+ \alpha_3 logSize_{i,t-1} + \alpha_4 EBIT_{i,t-1} + \alpha_5 Expenditures_{i,t-1} \\ &+ \alpha_6 Acquisitions_{i,t-1} + \alpha_7 \triangle ShortDebt_{i,t} + \epsilon_{i,t} \end{aligned} \tag{7}$$

where α_i captures firms fixed effect and τ_t time effect as in the baseline model. I correct the residuals covariance matrix for heteroskedasticity and for within-period residuals correlation using the Eicker-Huber-White estimator too. I report the results of the regression in **Table** 6. I do not report the constant coefficients that are systematically significant. The results are very similar to the baseline model. The coefficients of the additional controls are all significant except for the changes in short-term debt. They have the anticipated sign. This highlights the different ways to identify the determinants of cash-holdings. The link between the WW index and the variation of cash-holdings remains valid. The adjusted R^2 have increased as predicted. It is interesting to identify if the hedging motive has the same impact on the coefficients. Using the extended model with both sub-samples of firms I find similar and consistent results (unreported). A significant relationship for the indicators of financing constraints is observed only for the low hedging needs sample. At the same time the addition of the controls improves significantly the adjusted R^2.

5 Conclusion

There is a debate among researchers whether financially constrained firms should accumulate cash after a positive shock on cash-inflows. The main objective of this chapter is to propose a framework for identifying financially constrained firms. In the model I assume explicitly that one of the main role of cash-holdings is to permit to increase the financing capacity of future profitable investments. I model this precautionary motive.

Theoretically and empirically, I emphasize that there is a monotonic decreasing relation between cash-holdings and the degree of the financing constraints. In parallel, I analyze theoretically the robustness of previous influential indicators of financing constraints, the investment to cash sensitivity (FHP 1988) and the cash-flow sensitivity of cash (ACW 2004). I find that the cash-flow sensitivity of cash is positive for constrained firms even if we assume that the expected cash-flows are pledgeable, however the cash-flow sensitivity does not permit to assess the magnitude of the financing constraint. The cash-flow sensitivity of cash may be extremely small for a financially constrained firm. I find that the investment sensitivity is positive only considering current investments. In a time series approach, the relation should no longer be monotonic, because the increase of the internal financing capacity determines both current and future investments.

Table 6: **Extended Models Results**

This table displays results for OLS estimations of the extended regression model (equation(8)) where I add as controls capital expenditures (*Expenditures*), acquisitions (*Acquisitions*) and changes in short term debt (**Δ** *Short Debt*). All data are from the annual WRDS-COMPUSTAT database. The sampled firms include only manufacturers (SICs 100 to 3999) and the sample period is 1971 through 2011. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the Eicker-White-Huber estimator. The associated t-statistics are reported in parentheses. The numbers are rounded at the 4^{th} decimal place.

	Models(regressions)		
	WW Model	Payout Model	Size Model
	Coeff SE (cluster)	Coeff SE (cluster)	Coeff SE (cluster)
WW Index	**−0.0001**‡ (0.0000)		
Payout Ratio		0.0000 (0.0000)	
Size			**0.0000**†† (0.0000)
Q	0.0048 (0.0035)	0.0022 (0.0016)	0.0022 (0.0016)
LogSize	**−0.0882**‡ (0.0087)	**−0.0772**‡ (0.0057)	**−0.0796**‡ (0.0059)
Op. Cash (EBIT)	-0.0729 (0.0853)	-0.0048 (0.0370)	-0.0042 (0.0367)
Expenditures	**−0.6994**‡ (0.0724)	**−0.6519**‡ (0.0605)	**−0.6530**‡ (0.0604)
Acquisitions	**−0.3747**‡ (0.0274)	**−0.3694**‡ (0.0120)	**−0.3684**‡ (0.0196)
△ShortDebt	0.0056 (0.0568)	-0.0679 (0.0495)	-0.0670 (0.0495)
N. obs	10’895	15’404	15’448
R^2	0.107	0.080	0.081

Significance: † $p < 0.1$; †† $p < 0.05$; ‡ $p < 0.01$;
(two-tail test levels)

An empirical study with a large sample of manufacturing firms over the 1971 to 2011 period confirms my main result. There is a monotonic relationship between the supply of funds and the variation of cash-holdings. However another important determinant appears to be the risk of the cash-flows. The proxies I use to assess the financial constraints lose their explanatory power of cash-holdings variations considering a sample of firms with high hedging needs.

Based on my results, I recommend to use the variation of the cash holdings as new proxy of the financial constraints. Statistically, the relation I have identified should be sufficiently robust to other models specifications. Relying on the famous sentence of Samuelson "if only the right general direction of cause and effect can be determined, we shall have made a tremendous step forward".

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Appendix A

Section 2

Computations of the cash-flow sensitivity of cash:

The cash-flow sensitivity of cash is distorted when we admit a link between the financing capacity and the future investment opportunity. I start with the presentation of the cash-flow sensitivity of cash because he permits to understand the main difference with close frameworks (ACW 2004, HQ 2007).

Firstly, we need to determine the optimal cash policy C^* which solves $\max_C g(C_0+D_0-C)+E[f(C_1+C)]$. To simplify the presentation, I denote $g'(C_0+D_0-C)=g'$ and $f'(C_1+C)=f'$. I recall we need to assume $\tau f' \bigwedge \tau g' \in [0,1[$, else the problem is not well defined. Maximizing the expected cash-flows, the cash policy is determined by the following relations:

$$\begin{aligned} g'(\tfrac{\partial D_0}{\partial C^*}-1)+E[f'] &= 0 \\ \tfrac{\partial D_0}{\partial C^*} &= \tfrac{\tau(E[f']-g')}{1-\tau g'} \Rightarrow \\ g'(\tfrac{\tau(E[f']-g')}{1-\tau g'}-1)+E[f'] &= 0 \Rightarrow \\ (E[f']-g')(1+\tfrac{\tau g'}{1-\tau g'}) &= \tfrac{E[f']-g'}{1-\tau g'}=0 \end{aligned} \tag{8}$$

Wherefore we find: $E[f']=g'$. It is simply the standard result that the current marginal productivity should be equal at optimum to the expected one. We can recall that the manager has an incentive to fully hedge the firm against the variance of the cash-flows, because we have assumed a concave production function. This is the Froot, Scharfstein, Stein (1993) result. Trivially, we can omit the no investment solution for the analysis. We can now compute the financing sensitivities:

$$\begin{aligned} g'(1-\tfrac{\partial C^*}{\partial D_0}) &= E[f']\tfrac{\partial C^*}{\partial D_0} \Rightarrow \\ \tfrac{\partial C^*}{\partial D_0} &= \tfrac{g'}{g'+E[f']}=\tfrac{1}{2} \end{aligned} \tag{9}$$

Without surprise, the marginal increase of the cash buffer at optimum is half of the marginal increase of the financing capacity. Now, we are more interested to compute $\frac{\partial C^*}{\partial C_0}$ (the cash-flow sensitivity of cash). From that point, I denote $g''(C_0+D_0-C)=g''$ and $f''(C_1+C)=f''$:

$$\begin{aligned} \tfrac{\partial g'}{\partial C_0} &= \tfrac{\partial E[f']}{\partial C_0} \\ g''(1+\tfrac{\partial D_0}{\partial C_0}-\tfrac{\partial C^*}{\partial C_0}) &= E[f'']\tfrac{\partial C^*}{\partial C_0} \Rightarrow \\ \tfrac{\partial C^*}{\partial C_0} &= \tfrac{g''(\frac{\partial D_0}{\partial C_0}+1)}{g''+E[f'']} \end{aligned} \tag{10}$$

This intermediate result contrasts with existing theoretical frameworks. In order to identify the role of the constraint parameter I need to compute the ratios $\frac{\partial D_0^*}{\partial C_0}$. I denote $C_0 + D_0^* - C^* = I_0^*$ and $C_1 + C^* = I_1^*$:

$$\begin{aligned}
\frac{\partial D_0^*}{\partial C_0} &= \frac{\partial \tau(g(I_0^*)+E[f(I_1^*)])}{\partial C_0} = \frac{\partial \tau(g(D_0^*+C_0-C^*)+E[f(C_1+C^*)]}{\partial C_0} \\
&= \tau(g'(\frac{\partial D_0^*}{\partial C_0}+1-\frac{\partial C^*}{\partial C_0}) + E[f']\frac{\partial C^*}{\partial C_0}) = \frac{\tau(g'(1-\frac{\partial C^*}{\partial C_0})+E[f']\frac{\partial C^*}{\partial C_0})}{1-\tau g'} \\
&= \frac{\tau g'}{1-\tau g'}
\end{aligned} \tag{11}$$

The last equality is obtained using the relation between current and expected productivity for the optimal cash policy. The constraint parameter appears in the debt sensitivity. We can now characterized the cash-flow sensitivity as a function of the constraint:

$$\frac{\partial C^*}{\partial C_0} = \frac{g''(\frac{\partial D_0^*}{\partial C_0}+1)}{g''+E[f'']} = \frac{g''(\frac{\tau g'}{1-\tau g'}+1)}{g''+E[f'']} = \frac{\frac{g''}{1-\tau g'}}{g''+E[f'']} > 0 \tag{12}$$

With $\tau = 0$ (when the firm is highly constrained) and assuming that the manager can fully hedge the variation of the cash flows, we find the same sensitivity than ACW: $\frac{\partial C^*}{\partial C_0} = \frac{g''}{g''+f''}$. The sensitivity is equal to zero only when the slope of the production of the date 0 investment is null (when $g'' = 0$).

Computing the second order variation $\frac{\partial^2 C^*}{\partial C_0 \partial \tau}$, we find that the result depends on the third order derivative of both current g''' and future f''' production functions. It is clearly the main motive to assess directly the constraint sensitivity of cash.

Computations of the constraint sensitivity of cash:

$$\begin{aligned}
\frac{\partial D_0^*}{\partial \tau} &= g + E[f] + \tau(g'(\frac{\partial D_0^*}{\partial \tau} - \frac{\partial C^*}{\partial \tau}) + E[f']\frac{\partial C^*}{\partial \tau}) \\
&= \frac{g+E[f]+\tau(E[f']-g')\frac{\partial C^*}{\partial \tau}}{1-\tau g'} = \frac{g+E[f]}{1-\tau g'} \\
\\
\frac{\partial g'}{\partial \tau} &= \frac{\partial E[f']}{\partial \tau} \\
g''(\frac{\partial D_0^*}{\partial \tau} - \frac{\partial C^*}{\partial \tau}) &= E[f'']\frac{\partial C^*}{\partial \tau} \Rightarrow \\
\frac{\partial C^*}{\partial \tau} &= \frac{g''\frac{\partial D_0^*}{\partial \tau}}{g''+E[f'']} = \frac{g''}{g''+E[f'']}\frac{g+E[f]}{1-\tau g'} = \frac{\partial C^*}{\partial C_0}(g+E[f]) > 0
\end{aligned} \tag{13}$$

The second order derivative will depend on the concavity of the production function.

Computations of the investments to cash sensitivities:

The investments to cash sensitivities are:

$$\begin{aligned}\frac{\partial I_0^*}{\partial C_0} &= \frac{\partial C_0 + D_0^* - C^*}{\partial C_0} = 1 + \frac{\tau g'}{1-\tau g'} - \frac{\frac{g''}{1-\tau g'}}{g''+E[f'']} = \frac{1}{1-\tau g'} - \frac{\frac{g''}{1-\tau g'}}{g''+E[f'']} > 0 \\ \frac{\partial I_1^*}{\partial C_0} &= \frac{\partial C^*}{\partial C_0} > 0\end{aligned} \tag{14}$$

The sensitivity of the investment change is equal to:

$$\frac{\partial (I_0^* - I_1^*)}{\partial C_0} = \frac{1}{1-\tau g'} - 2\frac{\partial C^*}{\partial C_0} \lesseqgtr 0 \tag{15}$$

Even it the relation between the current investment and the cash-flows is monotonic considering Inada production functions. The variation of investments can be positive or negative. We can compute the investment to constraint sensitivities. We should find a similar result:

$$\begin{aligned}\frac{\partial I_0^*}{\partial \tau} &= \frac{\partial (C_0 + D_0^* - C^*)}{\partial \tau} = \frac{g+E[f]}{1-\tau g'}(1 - \frac{g''}{g''+E[f'']}) = (g + E[f])\frac{\partial I_0^*}{\partial C_0} > 0 \\ \frac{\partial I_1^*}{\partial \tau} &= \frac{\partial C^*}{\partial \tau} = \frac{\partial C^*}{\partial C_0}(g + E[f]) > 0\end{aligned} \tag{16}$$

The sensitivity of the investment change is equal to:

$$\frac{\partial (I_0^* - I_1^*)}{\partial \tau} = \frac{\partial I_0^*}{\partial \tau} - \frac{\partial I_1^*}{\partial \tau} = (g + E[f])\frac{\partial (I_0^* - I_1^*)}{\partial C_0} \lesseqgtr 0 \tag{17}$$

Section 3

Assets liquidity:

I propose to analyze the effect of the liquidity of the assets on the optimal cash condition. The objective stays to characterize the choice of the cash-holdings. I assume that the assets can be used as a collateral permitting to rise the external financing. The lenders accept to fund a fraction τq, $q \in [0;1]$ of the assets value. The managers objective becomes ${}^{max}_{C}\, g(C_0+D_0-C)+E[f(C_1+C)]+q(A+I_0+E[I_1])$ with $D_0 = \tau(g(C_0 + D_0 - C) + E[f(C_1 + C)] + q(A + I_0 + E[I_1])$. The optimal cash policy is determined by the following relations:

$$\begin{aligned} g'(\frac{\partial D_0}{\partial C^*} - 1) + E[f'] + q\frac{\partial D_0}{\partial C^*} &= 0 \\ \frac{\partial D_0}{\partial C^*} &= \frac{\tau(E[f'] - g')}{1-\tau g' - \tau q} \Rightarrow \\ g'(\frac{\tau(E[f'] - g')}{1-\tau g' - \tau q} - 1) + E[f'] &= 0 \Rightarrow \\ \frac{E[f'] - g' + \tau q - \tau q}{1-\tau g' - \tau q} &= 0 \end{aligned} \tag{18}$$

We need to assume $\tau(g' - q) \in [0;1[$. Confirming the intuition the liquidity of the assets scales only the financing available, but does not change the optimal relationship between the marginal productivities. However it becomes interesting to analyze the case where the liquidity of the current assets is not the same as the liquidity of the assets the manager anticipates to acquire. I assume that the liquidity of the current assets is equal to $q_0 \in [0;1]$ and the liquidity of the assets obtained at date 1 to $q_1 \in [0;1]$. Intuitively different liquidity of assets should generate a new trade-off between current and future investments.

Now the managers objective is $\max_C g(C_0 + D_0 - C) + E[f(C_1 + C)] + q_0(A + I_0) + q_1 E[I_1])$ with $D_0 = \tau(g(C_0 + D_0 - C) + E[f(C_1 + C)] + q_0(A + I_0) + q_1 E[I_1])$. Computing C^*, we find:

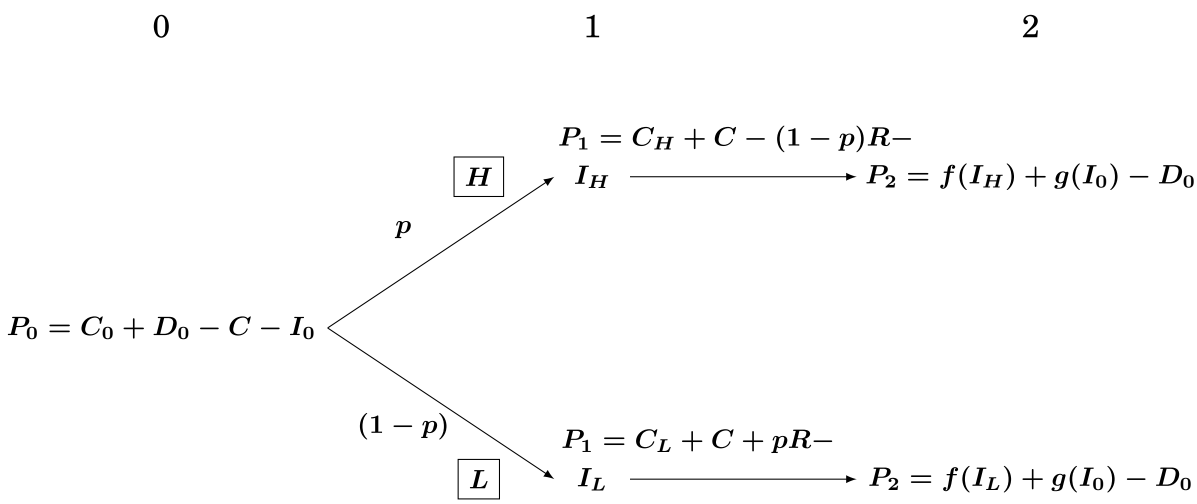
$$\begin{aligned} g'(\tfrac{\partial D_0}{\partial C^*} - 1) + E[f'] + q_0(\tfrac{\partial D_0}{\partial C^*} - 1) + q_1 &= 0 \\ \tfrac{\partial D_0}{\partial C^*} &= \tfrac{\tau(E[f'] - g' + q_1 - q_0)}{1 - \tau g' - \tau q_0} \Rightarrow \\ g'(\tfrac{\tau(E[f'] - g' + q_1 - q_0)}{1 - \tau g' - \tau q_0} - 1) + E[f'] &= 0 \Rightarrow \\ \tfrac{\tau g' E[f'] - g' + \tau q_1}{1 - \tau g' - \tau q_0} + E[f'] = 0 \Rightarrow \tfrac{E[f'] - g' + \tau(q_1 - q_0)}{1 - \tau g' - \tau q_0} &= 0 \\ \Leftrightarrow \tfrac{E[f'] - g'}{1 - \tau g' - \tau q_0} &= \tfrac{\tau(q_0 - q_1)}{1 - \tau g' - \tau q_0} \end{aligned} \tag{19}$$

We assume $\tau(g' - q_0) \in [0;1[$. Interestingly, at optimum, we see that the gain of productivity of one of the investment should be compensate by the marginal gain of liquidity with the assets of the other investments: $E[f'] - g' = \tau(q_0 - q_1)$. At the same time there is no necessary one unique solution. The optimal cash policy can not be identified without specifying the third derivative of the production functions.

Appendix B

Model Timeline

Section 2: based model (value of the assets = A , two states H and L)



legend:

- C_0 the initial Cash reserve
- C the Cash reserve at date 1
- C_H, C_L the random Cash-flows
- D for Debt borrowing
- I for Investments
- P for Payments (the dividends)
- $f; g$ the functions of profitability
- p the state probability