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ARTICLE

Acquisition of new arithmetic skills based on prior arithmetic skills: A cross-sectional study in primary school from grade 2 to grade 5

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Abstract

Background: In several countries, children's math skills have been declining at an alarming rate in recent years and decades, and one of the explanations for this alarming situation is that children have difficulties in establishing the relations between arithmetical operations.

Aim: In order to address this question, our goal was to determine the predictive power of previously taught operations on newly taught ones above general cognitive skills and basic numerical skills.

Samples: More than one hundred children in each school level from Grades 2 to 5 from various socio-cultural environments ($N = 435$, 229 girls) were tested.

Methods: Children were assessed on their abilities to solve the four basic arithmetic operations. They were also tested on their general cognitive abilities, including working memory, executive functions (i.e., inhibition and flexibility), visual attention and language. Finally, their basic numerical skills were measured through a matching task between symbolic and nonsymbolic numerosity representations. Additions and subtractions were presented to children from Grade 2, multiplications from Grade 3 and divisions from Grade 4.

Results and Conclusions: We show that addition predicts subtraction and multiplication performance in all grades. Moreover, multiplication predicts division performance in both Grades 4 and 5. Finally, addition predicts division in Grade 4 but not in Grade 5 and subtraction and division are not related whatever the school grade. These results are examined considering the existing literature, and their implications in terms of instruction are discussed.

KEYWORDS
arithmetic, numerical cognition, operations

INTRODUCTION

In several countries, such as the United States, Australia or France, children's math skills have been declining at an alarming rate in recent years and decades (OCDE, 2019). Numerous explanations have been provided, noticeably low investment in teachers' training (e.g., Luft & Cox, 2001) or teachers' dislike for mathematics (e.g., Ruffell et al., 1998). More specific explanations are also given and one of them is that children have lost the "sense" of arithmetical operations (Villani et al., 2018). In some school system such as the French one, this could be because the 4 operations are introduced one after the other in the curriculum, whereas a simultaneous introduction in Grade 1 could allow children to better understand the relation between them (Villani et al., 2018). The aim of the present paper is to address this question and to determine the predictive power of previously taught operations on the acquisition of new ones. This will be done using regression models in each of the Grades from 2 to 5. Because general cognitive skills can be responsible for the dependence between academic skills (e.g., Tikhomirova et al., 2020), we neutralized working memory capacities, executive function skills (i.e., a composite score of inhibition and flexibility), visual attention and language abilities in our analyses. Performance on arithmetical tasks can also be related to more basic skills in numeracy (Sasanguie et al., 2012) and this is the reason it was also considered in our model.

The approach that we adopted was inspired by Geary et al. (2017) who sought to determine the variables influencing performance in mathematics. The authors followed 167 children from the first year of primary school to the third year of secondary school. Children were subjected each year to two series of tests assessing their general cognitive abilities and their specific numerical skills. The results showed that the effect of general cognitive abilities, particularly that of working memory, was strong in the very early stages of schooling, then lessened and stabilized. In contrast, mathematical performance in a given school year became the best predictor of mathematical performance in the following years. Amongst mathematical skills, knowledge of numbers and arithmetic skills were the best predictors at all school levels, followed by fraction processing in older pupils. A similar approach was also used by Lin (2021) in the domain of arithmetic word problems. The author showed that language comprehension, working memory, attention, mathematics vocabulary and mathematics computation were unique predictors of word-problem solving in elementary school children.

As already stated, we adopted the same approach as Geary et al. (2017) or Lin (2021) in a cross-sectional design involving children from Grades 2 to 5 and applied it to mental arithmetic and, more precisely, addition and subtraction in the early school years, then multiplication and later division from the middle of primary school. Our goal was to determine the role of prior arithmetic skills on children's acquisition of new arithmetical skills once the effects of general cognitive abilities and basic numerical skills were neutralized.

Continuities in arithmetical operation learning

Arithmetic skills develop throughout elementary school and beyond. This development depends on several variables. First, cognitive maturation leads to an increase in general abilities such as attention, memory and language, allowing children to process more and more complex operations. Second, pupils formally learn arithmetic principles and procedures at school. They start with addition and subtraction, followed by multiplication and lastly division. Numerous studies have been devoted to mental arithmetic, but they rarely involve the four operations altogether and are rarely conducted over more than 2 years. For example, Xu et al. (2021) examined the development of addition, subtraction and multiplication in a longitudinal study involving children in Grades 2 and Grade 3. Other studies have assessed the four arithmetic operations together, but their goal was not to examine the relation between operations (e.g., Martens et al., 2011; Zhao et al., 2014).

Each of the four operations mobilizes three types of knowledge. Declarative knowledge corresponds to chunks of specific content, such as numerical facts (e.g., $2 \times 3 = 6$ or $6+6 = 12$), which are stored in memory networks of associations between operands and answers (Ashcraft, 1992). Procedural knowledge is general, abstract, modular, relatively immune to interference and activated by specific goals (Anderson, 1993). For example, the process of decomposition into tens in order to add up numbers (e.g., $46+23$ is $40+20$ and $6+3 = 60+9 = 69$) is general because it can be applied to a large number of additions, is abstract because it contains variables that are instantiated by the values supplied by the operands, is activated by a specific goal (i.e., solving the addition) and is modular because it is independent from other procedures and is therefore relatively immune to interference (Roussel et al., 2002). Finally, conceptual knowledge refers to general properties of the operations (Crooks & Alibali, 2014) and "reflects the understanding of why a procedure works" (Scheibling-Sève et al., 2020, p. 294). For example, solving 7×6 by retrieving the results of 6×7 requires the conceptual knowledge of commutative properties (Baroody, 1999).

Learning arithmetic involves the progressive mastering of these three types of knowledge, which, through practice, conduct to mutual enrichment. For example, declarative knowledge of arithmetic facts could be created by repeated application of counting procedures to specific problems (e.g., Logan & Klapp, 1991). Still, the concomitant and mutual progression of declarative, procedural and conceptual knowledge as well as their relations is not yet well understood. It is nevertheless possible to evaluate their respective contribution to the acquisition of new arithmetic knowledge by examining the contribution of a specific arithmetic operation to the performance of operations subsequently learnt. The results of earlier work show that acquired mathematical skills at any given point during schooling constitute the best predictors of subsequent acquired learning and progress (Geary et al., 2017). This statement should generalize to the evolution of performance in arithmetical operations but, as already stated, doubts are expressed nowadays on children's abilities to articulate their knowledge and construct the sense of these operations in light of one another (Villani et al., 2018). An investigation of these questions is therefore needed.

Concerning addition, skill acquisition is initially based on counting (e.g., Bagnoud et al., 2021; Groen & Parkman, 1972). More precisely, from the age of 3 to 4 years, children are able to determine the cardinal of small quantities by subitizing (Benoit et al., 2004) and by counting one by one (Fuson, 1988). They therefore use declarative knowledge, such as a still limited verbal chain (Van Rinsveld et al., 2020), procedural knowledge, such as object pointing (Camos et al., 1999), and conceptual knowledge related to counting principles (Briars & Siegler, 1984; Gelman & Gallistel, 1978). Children grasp very soon the meaning of addition and subtraction as corresponding to increase and decrease of quantities, although in a restricted range of problem situations (i.e., change problems; Riley et al., 1983). Performance in addition and subtraction do not initially differ and remain strongly correlated throughout schooling (about $r = .80$ according to Dowker, 1998; see also Xu et al. (2021) for a study in Grade 2 and 3). As attested by rare longitudinal studies, addition and subtraction procedures evolve in parallel with age and experience (Artemenko et al., 2018; Carpenter et al., 1998). An important achievement occurs at the age of around 5 to 7 years when children understand the commutativity principle of addition and the inverse relationship between addition and subtraction (Bryant et al., 1999). From that moment onwards, they can rely on additions to solve subtractions, for example using $3+4 = 7$ to solve $7–3 = 4$. Therefore, our first hypothesis (H1) is that, from Grade 2, subtraction performance will heavily depend on addition performance. Stated more operationally, performance on subtraction should be predicted by performance on addition, even after general cognitive abilities and basic numerical skills are entered in the model. This hypothesis is relevant in the educational context in which our research took place, that is in France before 2020. At that time, subtraction was formally introduced in Grade 2 (MENJ, 2015) but situations where some objects are removed or lost in contrast to situations where objects are added or earned had been already presented in kindergarten (EDUSCOL, 2020).

Learning new operations results in new conceptual and declarative knowledge and new procedures. There are both continuities and discontinuities between additions and subtractions on the one hand and multiplications and divisions on the other hand but generally, reference to additions and subtractions constitutes a basis to learn conceptual, declarative and procedural aspects of multiplication and division (Cooney et al., 1988; Lemaire & Siegler, 1995). Although repeated addition is introduced in classrooms as a procedure that can be used to solve multiplication problems (e.g., 5×4 can be solved by performing $5+5+5+5$), teaching a conception of multiplication as a repeated addition is not necessarily the best way to develop a deep understanding of the multiplication concept in children (Park & Nunes, 2001). Indeed, Grade 2 children better grasp this concept when they are taught the scheme of correspondence or, in other words, the fact that a multiplication is an invariant relation of correspondence between two quantities. More precisely, children perform better when they have been trained with word problems such as "Yesterday, Tom ate 2 fruits at each of the 3 meals. How many fruits did he eat yesterday?" (i.e., scheme of correspondence) than with problems such as "Yesterday, Tom ate 2 fruits during breakfast, 2 fruits during the lunch and 2 fruits during diner. How many fruits did he eat yesterday?" (i.e., repeated addition). This conception of multiplication as a scheme of correspondence allows children to understand that it is possible to multiply 4.3 by 2.1, for example, which would not make sense in a repeated addition conception (Larsson et al., 2017). For Piaget (1965) or Steffe (1988, 1992), overcoming the addition scheme addition to reach a higher level of abstraction is necessary to master multiplication. Clark and Kamii (1996) showed that some children in Grade 2 already master such multiplicative thinking, but they also show that this ability develops slowly. Even if children must construct their representation of multiplication out of addition, it remains that when whole numbers are used in the text of a problem, repeated additions can be used as a resolution procedure.

However, once the multiplication scheme is acquired by children, multiplication tables are often learnt by heart in classrooms (Geary, 1994), and, by Grade 4, retrieval of the answers from memory is the dominant strategy (Cooney et al., 1988). Memorization of new associations between operands and results can create the emergence of interference, some specific to multiplications (Barrouillet et al., 1997; De Visscher & Noël, 2014) and some others related to prior associations, particularly with addition facts (Lemaire et al., 1994; Miller & Paredes, 1990). Therefore, negative side effects can arise from conflicts between previous addition facts in memory and newly acquired associations. Still, either negative or positive, these effects would reflect the impact of operations previously learned on more recently acquired ones. Therefore, our second hypothesis (H2) is that, in Grade 3, multiplication performance will depend on addition performance but that this relation will disappear in Grades 4 or 5. This is because, as just stated, retrieval, which is disconnected from addition procedures (e.g., Mathieu et al., 2016), becomes the dominant strategy over development. Moreover, and as also explained above, as children grow older, they depart progressively from the addition scheme to understand multiplicative structures (Clark & Kamii, 1996). These hypotheses make sense in the educational context in which the study was conducted. At that time in France, multiplication was introduced only at the end of Grade 2 and initially and uniquely presented as a shortcut for repeated additions. It was only later that multiplication was presented as a combination between variables. At a procedural level, multiplication tables are taught through rote learning in French schools and are expected to have been automatized by the end of Grade 3 up to the 9 times table (MENJ, 2015).

Compared with other operations, studies related specifically to division processing are the least advanced. This operation is taught at a later stage during schooling and is practised less frequently than other operations. During Grade 4 children rely heavily on iterated addition (e.g., 20/5 is $5+5+5+5$) and sometimes, but rarely, on repeated subtractions (20/5 is 20–5 -5 -5 -5) (Mulligan & Mitchelmore, 1997). During Grade 5, children move to the use of multiplication (48/6 is 6 x? = 48) but still rely infrequently on direct retrieval. In fact, the percentages of retrieval do not increase with age (Robinson, Arbuthnott, et al., 2006). At a more conceptual level, children have difficulties in understanding the relations between division and multiplication (e.g., Robinson, Arbuthnott, et al., 2006; Robinson & LeFevre, 2012; Robinson, Ninowski, & Gray, 2006). More precisely, still 80% of children in Grade 8 do not apply their knowledge that multiplying is the inverse of dividing when they solve problems involving several operations (Dubé & Robinson, 2018). In fact, this difficulty in grasping the relation between division and multiplication is also observable in adults who have a better comprehension of the relation between addition and subtraction (Robinson & Ninowski, 2003). As a consequence, performance in division could benefit less, or at least could take longer than other operations to benefit, from the mastery of previously learnt operations. At the same time, Parmar (2003) notes that other operations can be used as a basis for learning division. Indeed, repeated subtraction may form the basis for understanding the quotitive schema associated with division (i.e., how many groups of × objects can be formed from a specific amount?) (Fischbein et al., 1985). Therefore, it is possible that addition, subtraction and multiplication performances are related to division because they can be used as procedures to solve them. Nevertheless, we have seen that subtraction is rarely used by children (e.g., Mulligan & Mitchelmore, 1997). Therefore, its relation to division could be inexistent in both Grades 4 and 5 (H3). In contrast, addition is the dominant strategy to solve division in Grade 4 and the relation between these two operations could therefore be limited to this grade (H4). Indeed, in Grade 5, resort to the inverse multiplication becomes the dominant strategy for division and our sixth hypothesis (H5) is that the relation between these two operations could be limited to this grade. Theses hypotheses are based on previous literature showing that children struggle in establishing the relation between multiplication and division, despite the facts that in France, in which the research took place, division is sometimes presented as soon as Grade 2 in sharing situations or situations in which the number of times a number is comprised in a larger number has to be determined. Division is then more formally introduced in Grade 4 (MENJ, 2015).

Operationalization of the present research

Then, with instruction and practice, conceptual, declarative and procedural knowledge play a growing role on the acquisition of arithmetic skills (Geary et al., 2017). However, the acquisition, memorization and implementation of knowledge and procedures might depend in turn on the cognitive abilities that control their activation, use and checking (Archambeau & Gevers, 2018; Geary, 2011). This can be especially true for division for which, as just stated, conceptual understanding could be particularly disconnected from the knowledge of other operations (e.g., Dubé & Robinson, 2018).

To investigate this matter, we used a cross-sectional approach and examined pupils' performance on different arithmetic operations adapted to their levels of schooling. This set of data was subjected to regression analyses in which 3 categories of variables were successively introduced: (1) general cognitive abilities (i.e., working memory, executive functions, visual attention and language) (2) basic numerical skills and (3) previously learnt arithmetic operations. Our main goal was to assess the specific weight of this last variable on children's performance for each arithmetic operation after the role of the other variables had been taken into account.

As just stated, four general cognitive abilities were entered in the models. We entered a measure of visual attention because performing calculations first requires the encoding of the problem operands and arithmetic signs (Thevenot et al., 2011; Thevenot & Barrouillet, 2006, 2010). Children who are able to deliberately focus their attention and to resist distraction are more efficient during this phase (Ortega et al., 2020). More generally, attentive children are more successful than other children in processing arithmetical operations (Aunola et al., 2004; Commodari & Di Blasi, 2014; Geary, 2013). This is the reason why we also included two measures of executive functions that were combined, one related to inhibition and the other to flexibility. A measure of working memory was also included because working memory resources are mobilized to manage the calculation implementation process (Brysbaert, 2018). Working memory integrates the outcomes of the encoding phase and the outcomes of the activation of declarative and procedural knowledge in long-term memory. The management cost of such integration depends on pupils' level of mastery related to this knowledge. For example, early during schooling, small additions impose a minimal processing demand on the cognitive system because they rely on automatic processing corresponding either to memory retrieval or fast counting (e.g., Ashcraft, 1992; Ashcraft & Battaglia, 1978; Fayol & Thevenot, 2012; Thevenot et al., 2016; Thevenot & Barrouillet, 2020). In contrast, division requires a substantial level of cognitive control because it involves to-and-fro between multiplication, addition and subtraction processing. Inhibition of interferences and updating of intermediate results are also needed to solve division (Raghubar et al., 2010; Swanson, 2011,) and this is the reason why we entered a measure of executive functions in the models. Finally, language skills play an important role in the development of arithmetical competencies (Brysbaert, 2018), and a measure combining lexical skills and written language was also entered in the first step of the models. To sum up, working memory capacities, executive function skills, visual attention and language abilities were independently entered in Step 1 of our hierarchical regression models.

In order to assess the specific role of arithmetical skills in the acquisition of more and more complex operations, it was important to ensure that the contribution of arithmetic was indeed specific and not due to numerical skills in general. This is the reason why we measured children's basic numerical skills through a classical matching task between symbolic and nonsymbolic numerosity representations (Billard et al., 2021; Geary et al., 2009).

To test the 5 hypotheses formulated, we asked children in each school level from Grades 2 to 5 to solve arithmetic operations. Additions and subtractions were presented to children from Grade 2, multiplications from Grade 3 and divisions from Grade 4.

METHOD

Participants

Our research included 435 children attending school in classrooms from Grades 2 to 5 (105 to 111 pupils per school level) in a range of public and private schools all over the Paris region in France. Written informed consent to participate was obtained from all the parents or legal tutors of the children involved. All procedures performed in this study have been conducted in compliance with the recommendations of the 1964 Helsinki declaration and its later amendments or comparable ethical standards. Because only behavioural data were collected in a nonvulnerable population of children, the official approval of a committee of ethics was not required. Our research protocol was however accepted by inspectors of the French National Education.

Table 1 shows the characteristics of the sample: gender, parents' socio-professional category (SPC, following the recommendation of French national statistics office, the higher socio-professional category between the two parents was retained to classify children in one of three SPC categories).

Material and procedure

The protocol was established on the basis of the BMT-i (Modulable Battery of Computerized Tests), which assesses cognitive skills (Billard et al., 2021). Eight speech therapists and three neuropsychologists were in charge of the testing after having been trained to the test administration. The tests were

TABLE 1 Socio-demographic characteristics of the sample.

Grade	2	3	4	5
Sample size (*N*)	109	111	110	105
Girls (*N* and %)	56 (51%)	53 (48%)	62 (56%)	58 (55%)
SPC (%)				
1 = low	12%	34%	9%	16%
2 = middle	16%	18%	27%	25%
3 = high	72%	48%	64%	59%

administered in one or two 45-minute sessions, at least 15 days apart, during school hours. Children were first tested on language skills, then on numerical skills, then on working memory and finally on executive functions and visual attention.

The tests were administered using a secure website using a Surface Pro3 tablet operating under Windows 8. The items needed to be read out loud were recorded in advance and read out by the software. Responses were either recorded automatically or noted by the examiner.

Tasks

Numerical skills

Arithmetic

Mental arithmetic fluency was evaluated using an adaptation of the Dutch Tempo-Test-Rekenen test (TTR) (de Vos, 1992) in which children had to solve a maximum of 4 series of operations. Each of the series corresponded to 40 problems related to a specific arithmetic operation (i.e., addition, subtraction, multiplication and division). At all school level, children had to solve additions and subtractions, multiplications were added from Grade 3 and divisions were added from Grade 4 upwards. For each series, children were given 1 minute to solve as many operations as possible. A 30-second pause was set up between series. Children's scores were calculated for each operation and corresponded to the number of correct answers out of 40.

Basic numerical skills

This task included in the BMT-i (Billard et al., 2021) assesses children's ability to match nonsymbolic numerosities (i.e., dots) and numerical symbolic representations (i.e., Arabic digits). This type of tasks is especially well suited for our purpose because it allows the detection of children with mathematics difficulties or disabilities and therefore assesses the core knowledge of basic numerical representations (Geary et al., 2009). Children were presented with 66 pairs of stimuli in rapid succession on a computer screen and had to decide whether two stimuli of a pair represented the same numerosity. The pairs always contained a set of dots and an Arabic digit ranging from 1 to 9, which were presented sequentially either with the sets of dots or the Arabic digit presented first. The distance between the two quantities was controlled and varied from 0 to 2. The rate of correct answers in the task was calculated for each child.

General cognitive abilities

General cognitive abilities were assessed through a battery of six subtests conceived to assess working memory, executive functions (i.e., two subtests), visual attention and language (i.e., two subtests) (Iannuzzi et al., 2019).

Working memory

In the working memory span test, children had to repeat series of 3 to 7 digits presented verbally through the software at a fixed pace of one digit per second. Children had to repeat the numbers in the reverse order (i.e., backward span), and a score was calculated by considering the maximum number of digits correctly recalled.

Executive functions

Executive functions were assessed through a flexibility and an inhibition tasks. The inhibition task was a simple "go/no-go"-type task. The child had to touch a circle as quickly as possible every time the word "circle" was uttered by the software and had to stay still when another word was uttered. In the flexibility task, the child had to shift between two types of instructions. The first instruction was to press a triangle when the word triangle was uttered by the software. By contrast, following the second instruction, the child had to press a circle when the word square was uttered. Children's executive function scores were calculated by adding the number of correct responses across the two subtests.

Visual attention

Sustained visual attention was assessed using an adaptation of the Conners test (Conners et al., 2011). During 15 minutes, children were presented with coloured circles and black circles. Following a go-no-go task methodology, children had to react as quickly as possible to coloured circles and not to more seldom black circles randomly intertwined. The ability to sustain attention was measured by the percentages of missed targets.

Language

Written language Reading speed and accuracy were assessed through the processing of a text tailored to children's school level. We calculated the number of words read correctly in one minute (NWRC/min), and this score was combined with the measures collected in the following language test.

Lexical skills Two tests were used to assess children's lexical knowledge depending on school level, one for Grades 2 to 4 and one for Grade 5. Lexical production was assessed through the naming of 40 pictures. Lexical comprehension was assessed through the selection of the picture corresponding to the spoken word uttered by the software amongst 5 pictures. A set of 32 words was used for Grades 2 to 4, and a different set of 33 words was used for Grade 5.
The score corresponding to the language variable combined the previous NWRC/min score and the scores obtained from children in the lexical production and comprehension tests.

RESULTS

Descriptive analyses

Table 2 presents the mean performance (and standard deviations) for each of the variables that we studied across grades. The coefficients of asymmetry and flattening did not reveal any violation of the normality of their distribution (Kline, 1998). The reliability of the scores for each of the operations, measured using Cronbach's alpha, was very good (alpha between .82 and .92).

TABLE 2 Children's performance in the different tasks.

	2nd	3rd	4th	5th
Grades	***M* (*SD*)**	***M* (*SD*)**	***M* (*SD*)**	***M* (*SD*)**
Mental arithmetic				
Addition	11.88 (3.02)	13.66 (3.55)	16.56 (3.98)	18.95 (4.08)
Subtraction	9.23 (4.46)	11.46 (4.33)	15.07 (5.30)	17.87 (4.53)
Multiplication	--	10.26 (3.32)	14.19 (4.27)	16.64 (4.12)
Division	--	--	5.38 (4.05)	8.09 (4.66)
General cognitive abilities				
Working memory	1.53 (1.17)	1.82 (1.39)	2.27 (1.24)	2.30 (1.29)
Executive functions	7.52 (2.65)	8.48 (2.27)	9.56 (1.99)	9.67 (2.24)
Visual attention	.34 (.08)	.36 (.06)	.38 (.04)	.39 (.05)
Language	33.63 (8.83)	41.07 (9.11)	47.94 (9.76)	60.27 (10.39)
Basic numerical skills	.58 (.11)	.63 (.13)	.70 (.10)	.72 (.09)

Hierarchical regression models

To determine the main predictors of success in mental arithmetic amongst the variables we studied, our data were processed through hierarchical regression analyses. In successive steps, we introduced our different sets of variables.

Step 1: General cognitive abilities were introduced (i.e., working memory, executive functions, visual attention and language): Model 1.

Step 2: Basic numerical skills (matching between nonsymbolic and symbolic numerosity representations) were introduced in addition to general cognitive abilities: Model 2.

Step 3: The variables concerning mental arithmetic were introduced at this stage in addition to general cognitive abilities and basic numerical skills. We explored first the contribution of performance in addition to the explanation of performance in subtraction; then, the contribution of addition and subtraction to multiplication; lastly, the contribution of addition, subtraction and multiplication to division: Model 3.

Ultimately, 9 models of three-stage hierarchical regression were tested (i.e., contribution of addition on subtraction in Grades 2, 3, 4 and 5; contribution of addition and subtraction on multiplication in Grades 3, 4 and 5 and contribution of addition, subtraction and multiplication on division in Grades 4 and 5) (Tables 3 to 5). Table 3 reports the results obtained for subtraction. As it can be seen, our first hypothesis (H1) that addition will explain subtraction performance in all grades (i.e., from Grade 2 to Grade 5) after cognitive and basic numerical skills are entered in the regression analyses was confirmed (β = .585; .522; .682 and .630 for Model 3 in Grades 2, 3, 4 and 5, respectively).

Table 4 reports the results for multiplication. Contrary to our second hypothesis (H2), addition predicted multiplication in all grades (β = .339; .277; .525 for Model 3 in Grades 3, 4 and 5, respectively) and not only in Grades 3 or 4.

Finally, Table 5 reports the results for division. As expected (H3), above general cognitive and basic numerical skills, subtraction did not explain division performance neither in Grade 4 nor Grade 5 (β = .137 and .194 for Model 3 in Grades 4 and 5, respectively). H4 was also confirmed because addition predicted division only in Grade 4 (β = .490 and .166 for Model 3 in Grades 4 and 5, respectively). However, contrary to H5 according to which multiplication will predict division only in Grade 5, we can see that multiplication predicted division in both Grades 4 and 5 (β = .353 and .235 for Model 3 in Grades 4 and 5, respectively).

DISCUSSION

In this study, we aimed at determining the impact of previously taught operations on performance in subtraction, multiplication and division in children from Grade 2 to Grade 5 beyond general cognitive abilities and basic numerical skills. We formulated 5 hypotheses concerning the possible relations between operations. In accordance with our first hypothesis, we showed that addition predicts subtraction performance in all school grades. This confirms previous observations that addition and subtraction performances remain strongly correlated throughout schooling (Dowker, 1998; Xu et al., 2021). This result strengthens the legitimacy of a pedagogical approach introducing addition and subtraction at the same time in the first year of formal schooling (Villani et al., 2018).

Our second hypothesis that addition will predict multiplication performance only in early grades was not confirmed because the relation was observed in all grades (i.e., 3, 4 and 5). Therefore, it is possible that children do not depart as we should expect from the addition schema when they consolidate their conception of multiplication. This interpretation relates to Post et al.'s (1985) observations that children tend to inadequately extend their knowledge of addition when they encounter situations requiring multiplicative thinking, such as fraction problems (Tobias & Andreasen, 2013). This result supports Park and Nunes' (2001) proposition that addition should be introduced at school in relation to multiplication only as a mean to solve the problems and not as a scheme to help children understanding the multiplication concept. Alternatively, the result that addition predicts multiplication performance in all grades could be

TABLE 3 Hierarchical regression models for mental subtraction in Grades 2 to 5.

	Grade 2			Grade 3			Grade 4			Grade 5		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Independent variables	β	β	β	β	β	β	β	β	β	β	β	β
Step 1												
Working memory	.077	.074	.055	.175*	.162*	.162*	.124	.115	.015	.050	.001	−.002
Executive functions	.118	.091	.007	.114	.099	.077	.081	.073	.063	.036	−.033	−.057
Visual attention	.005	−.007	.004	.219*	.207*	.096	.206*	.132	−.003	.141	.090	.038
Language	.291**	.296**	.169*	.183*	.181	.103	.216*	.153	−.005	.230*	.239*	.065
R^2	.14			.17			.15			.11		
$F(4, 103) = 3.981, p < .001$				$F(4, 110) = 5.518, p < .001$			$F(4, 104) = 4.329, p < .001$			$F(4, 98) = 2.764, p < .001$		
Step 2												
Basic numerical skills		.082	−.044		.095	−.011		.267**	.151*		.233*	.028
ΔR^2		.01			.01			.06			.04	
R^2		.15			.18			.21			.15	
$\Delta F(5, 103) = 3.297, p < .001$				$\Delta F(5, 110) = 4.628, p < .001$			$\Delta F(5, 104) = 5.169, p < .001$			$\Delta F(5, 98) = 3.193, p < .001$		
Step 3												
Mental addition			.585**			.522**			.682**			.630**
ΔR^2			.28			23			.36			.29
R^2			.43			.41			.57			.44
$\Delta F(6, 103) = 12.165, p < .001$				$\Delta F(6, 110) = 12.085, p < .001$			$\Delta F(6, 104) = 21.819, p < .001$			$\Delta F(6, 98) = 12.233, p < .001$		

*, $p < .01$; **, $p < .001$.

TABLE 4 Hierarchical regression models for mental multiplication in Grades 3 to 5.

	Grade 3			Grade 4			Grade 5		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Independent variables	β	β	β	β	β	β	β	β	β
Step 1									
Working memory	.040	.003	−.044	.135	.130	.039	.125	.094	.092
Executive functions	.062	.021	−.022	.093	.089	.053	.087	.044	.025
Visual attention	.137	.104	−.028	.166	.129	.017	.159	.128	.084
Language	.255**	.250**	.148	.150	.118	−.013	.273**	.279**	.131
R^2	.12			.10			.19		
$F(4, 110) = 3.551, p < .001$				$F(4, 104) = 2.832, p < .001$			$F(4, 98) = 5.384, p < .001$		
Step 2									
Basic numerical skills		.261**	.165*		.133	−.030		.144	−.029
ΔR^2		.06			.02			.01	
R^2		.18			.12			.20	
$\Delta F(5, 110) = 4.631, p < .001$				$\Delta F(5, 104) = 2.612, p < .001$			$\Delta F(5, 98) = 4.713, p < .001$		
Step 3									
Mental addition			.339**			.277*			.525**
Mental subtraction			.288**			.435**			.010
ΔR^2			.25			.34			.21
R^2			.43			.46			.41
$\Delta F(7, 110) = 11.194, p < .001$				$\Delta F(7, 104) = 11.573, p < .001$			$\Delta F(8, 98) = 9.169, p < .001$		

*, $p < .01$; **, $p < .001$.

due to the use of backup strategies relying on addition. Indeed, it has been shown that even adults' resort to decomposition strategies such as $7 \times 6 = (6 \times 6) + 6$ when retrieval fails (LeFevre et al., 1996).

Concerning division, we showed that it was not predicted by subtraction performance. This result confirmed our third hypothesis, which was based on previous observations that children only rarely resort to the repeated subtraction procedure to solve division problems (e.g., 24 / 6 = 24–6 = 18–6 = 12–6 = 6–6 = 0 so 4) (Robinson, Arbuthnott, et al., 2006). Indeed, in Grade 4, children preferentially use the iterated addition procedure (Mulligan & Mitchelmore, 1997). This is confirmed by our results because, in accordance with our fourth hypothesis, addition predicts division only in Grade 4 but not in Grade 5, where children preferentially solve division through retrieval of inverse multiplication facts (Mulligan & Mitchelmore, 1997). Still, and contrary to our fifth hypothesis, multiplication was a significant predictor of division in both Grades 4 and 5. This shows that conceptual understanding or procedural mastering of multiplication must be associated with division as soon as this last operation is introduced. This result strengthens the position that extra effort to link these two operations across instruction is primordial (e.g., Mulligan & Mitchelmore, 1997; Nunes & Bryant, 1996). Such multiplicative thinking is also viewed as essential for the development of concepts needed to be mastered by pupils in later grades, such as ratio, proportion, area, volume or proportions (Mulligan & Watson, 1998). Teaching the relation between multiplication and division early during school curriculum could be achieved by introducing multiplication and division instruction at the same time in classrooms (Villani et al., 2018). One efficient way to promote the understanding of this relation is the use of arrays or in other words of arrangements of objects in columns and rows (Jacob & Mulligan, 2014). This tool helps children focus their attention on three quantities that can be apprehended flexibly for the description of multiplication or division situations (i.e., 12 objects are divided into 4 lines of 3 objects and multiplying 3 objects by the number of lines give the total amount of objects).

TABLE 5 Hierarchical regression models for mental division in Grades 4 to 5.

	Grade 4			Grade 5		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Independent variables	β	β	β	β	β	β
Step 1						
Working memory	.039	.033	−.069	.059	.005	−.017
Executive functions	.208*	.203*	.174*	.146	.072	.062
Visual attention	.122	.079	−.045	.326**	.270**	.209*
Language	.257*	.220*	.086	.099	.109	−.049
R^2	.18			.20		
$F(4, 104) = 5.424, p < .001$				$F(4, 98) = 5.937, p < .001$		
Step 2						
Basic numerical skills		.154	.061		.252*	.119
ΔR^2		.03			.05	
R^2		.20			.25	
$\Delta F(5, 104) = 4.890, p < .001$				$\Delta F(5, 98) = 6.191, p < .001$		
Step 3						
Mental addition			.490**			.166
Mental subtraction			−.137			.194
Mental multiplication			.353**			.235**
ΔR^2			.35			.18
R^2			.55			.43
$\Delta F(8, 104) = 14.466, p < .001$				$\Delta F(8, 98) = 8.536, p < .001$		

*, $p < .01$; **, $p < .001$.

To sum up and conclude, we have shown here that subtraction and multiplication performance capitalize on the acquisition of addition and that division performance capitalizes on multiplication performance. We have discussed the fact that, therefore, mutual development and articulation of arithmetical concepts must be given special attention from teachers and educators in arithmetic instruction. Stated differently and in accordance with the conclusion of Xu et al. (2021), we show here that learning arithmetic is a hierarchical process. Thus, at the very least, ensuring that children master the operations that have been taught before moving to the teaching of new operations is crucial.

AUTHOR CONTRIBUTIONS

Catherine Thevenot: Conceptualization; writing – original draft; writing – review and editing. **Youssef TAZOUTI:** Data curation; formal analysis; visualization. **Catherine Billard:** Conceptualization; investigation; methodology; software. **Jasinta Dewi:** Formal analysis; visualization. **Michel Fayol:** Conceptualization; methodology; writing – original draft.

CONFLICT OF INTEREST STATEMENT

The authors report there are no competing interests to declare.

DATA AVAILABILITY STATEMENT

The data set generated and analyzed for this research can be find here in a SPSS format: https://osf.io/nw36h/?view_only=197929aac89440419beec0754952e60e.

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