Required Capital for Long-Run Risks*

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Abstract

One of the objectives of the recent microprudential regulation is to separate the computation of required capital for short-run and long-run risks. This paper provides a coherent framework to define, compute, and update these components. The approach is developed in greater details in the context of the transition to low-carbon economies. A numerical example is given.

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1 Introduction

The finance industry is under mounting pressure to take into account long-term risks. In particular, Article 2.1c of the 2015 Paris Agreement calls for "making finance flows consistent with a pathway towards low greenhouse gas emissions and climate-resilient development" (United Nations, 2015). Accordingly, an essential objective of the recent prudential regulation is to separate the computation of required capital for short-run and long-run risks (see, e.g. European Banking Authority, 2019a,b).[1] The notion of long-run risks encompasses several situations: long-term holding of illiquid assets, large-scale restructuration of production processes (for instance due to new rules on carbon emissions), risks associated with depollution costs when dismantling a nuclear power plants in coming decades, or risks faced by pension funds that have granted capital-guaranteed plans. This paper provides a coherent framework to define, compute, and update the required capital associated with such long-run risks.

1.1 Required Capital

The standard approach to compute required capital in Basel 3 is based on a computation of a Value-at-Risk (VaR) at a short horizon—typically one year—and a regulatory critical level (see Appendix A.1 for a discussion of the VaR). This approach might be directly extended to a longer horizon, but this extension feature two major limitations.[2] The first is that while those long-term risks to be covered by the required capital can be particularly large,[3] the standard prudential approach—based for instance on short term VaR—posits that it has to be covered at all dates before this risk is realized. This implies a large cost of required capital, which may discourage some investments needed to reduce long-term risks. As Campiglio (2016) put it: "imposing liquidity requirements would likely produce a reallocation of investments towards liquid shorter-term assets, while low-carbon initiatives (for instance) require long-term credit." The second reason is that long-run risks are usually not traded on financial markets, not well measured, and therefore largely unhedgeable. They have been widely neglected in the past, and are not reflected in historical data. Therefore, the conditional distributions of these risks—distributions that are required in the standard VaR approach—are difficult to approximate.

[1] A capital requirement, or regulatory capital, is the reserve amount that a financial institution holds as required by its financial regulator.

[2] See e.g. Dietz et al. (2016) for a climate Value-at-Risk based on an Integrated Assessment Model (IAM).

[3] For instance, dismantling French nuclear power plants would cost around €100bn.

This paper introduces operational definitions of required capital that are appropriate for long-term risk, defined as a risk that potentially materializes at a given large maturity. For prudential supervision, the computation is performed at the "individual" level (corporation, bank, or contract in the context of pension saving), not at the macro level. The main idea is to define a progressive profile, or target, of required capital up to this maturity, and to avoid asking for a perfect hedge at all intermediate dates. This profile involves four ingredients: (i) the updating frequency, (ii) a regulatory discount rate, (iii) a design for the evolution of the reserves, and (iv) a benchmark valuation for the underlying risk. This benchmark is not necessarily market-based.

1.2 Risks and Regulations

As far as climate-related risks are concerned, one can distinguish between physical and transition risks. Physical risks cover the economic costs and financial losses resulting from the increasing severity and frequency of extreme climate change-related weather events, as well as longer term progressive shifts of the climate; transition risks relate to the process of adjustment towards a low-carbon economy (Network for Greening the Financial System, 2019). Physical risks can, in turn, be categorised as acute risks, which are related to extreme weather events, or chronic risks associated with gradual shifts in climate. The drivers of transition risks relate to societal changes arising from a transition to a low-carbon economy; they originate for instance from (i) changes in public sector policies, (ii) innovations and changes in the affordability of existing green technologies, or (iii) investor and consumer sentiment towards a greener environment (Basel Committee on Banking Supervision, 2021, Section 2.3). In particular, for some corporations, the costs of capital and funding may increase as investors and rating agencies progressively include climate-related or Environmental, Social, and Governance (ESG) factors in their investment and rating decisions. In this context, environmental authorities, such as the European Environment Agency, are expected to become increasingly important through the provision of new types of corporate balance sheets, including, for example, information on greenhouse gas (GHG) emissions. This new information may for instance be used to calculate green ratings by public and private entities, and incorporated in standard credit-risk analysis (see, e.g., Carbone et al., 2021). The examples we propose below (in Section 3) highlight the importance of such data to feed long-run credit-risk analyses.

For financial supervisors, physical and transition climate risks are considered at the level of banks and insurance companies, and are the basis of stress tests with scenarios produced by the

Network for Greening the Financial System (NGFS).[4] In the case of transition risk, an example is the ECB 2022 "climate risk stress test" currently conducted in the euro area (European Central Bank, 2022). For the moment, this test is a learning exercise for banks and supervisors alike. The results will feed into the Supervisory Review and Evaluation Process (SREP) from a qualitative point of view only. The European Central Bank (2022) has however mentioned that this stress test could indirectly impact capital requirements through the Supervisory Risk and Evaluation Process (SREP) scores. The SREP aims at determining Pillar-2 requirements, which apply in addition to, and covers risks which are underestimated or not covered by, the minimum capital requirement (known as Pillar 1).

Even if this supervision concerns banks and insurance companies, each of these financial entities has to analyze the details of granted loans and firm values at the firm (micro) level, taking into account the possible dependence between the transition risk of the firms. That is why we focus, in this paper, on the micro-level of firms, not on a meso or macro level.

1.3 Outline of the paper

The design of the required capital is introduced in Section 2. We first consider the case of deterministic loss or profit at maturity and discuss the reserve evolution design as well as the discounting. Then, we extend the analysis to a stochastic asset value and highlight the difference between short-run run and long-run required capital. In Section 3, we explain how to treat jointly short- and long-run risk factors. We examine in particular the long-run transition risks to low-carbon economies (focusing on production processes). Illustrations are provided in Section 4. Section 5 concludes. The discussion is completed by appendices: The first appendix reviews the standard Value-at-Risk approach (Appendix A.1); the second appendix (A.2) describes the production and profit functions when carbon is taken into account; the third derives the long-run approximation of the distribution of cumulated profit (Appendix A.3).

2 The Design of Required Capital

Let us denote by t the first date at which the potential long-run risk is considered, and by T the maturity, i.e. the final date at which the risk is completely realized. The maturity T is fixed in the

[4] Hansen (2022) and Bolton et al. (2020) highlight the challenges posed by the introduction of climate-change scenarios in stress tests.

analysis. The intermediate dates are denoted by $t+h, h=0,\dots,H$, where $H=T-t$ is the initial time-to-maturity. This initial time-to-maturity is large, typically between 10 and 50 years. This horizon is beyond the traditional horizons of most economic actors—including central banks and financial supervisors, that are bound by their mandates (Carney, 2015).[5] This imposes a cost on future generations that the current generation may not endure. In this section, we consider that there is a potential loss at maturity, denoted $X_{t+H}=X_T$, and that a sequence of Required Capital Calls RCC_{t+h} ($h=1,\dots,H$) should hedge the loss. In Section 3, we consider a case where X_{t+H} is a gain, more precisely a cumulated profit. This case could be covered by keeping the definition of X_{t+H} as a loss, the gain being $-X_{t+H}$, and $-RCC_{t+h}$, $h=1,\dots,H$, being investment possibilities. Alternatively, we can also define X_{t+H} as a gain and, in this case, RCC_{t+h} would directly reflect investment possibilities. The latter sign convention will be used in Subsection 3.1 and Section 4.

2.1 Deterministic Loss at Maturity

Let us first consider a deterministic amount X_{t+H}. Depending on the case under consideration, this amount admits different interpretations: a reimbursement *in fine* for a pension saving scheme with guaranteed capital; a cumulated sum of losses in the context of portfolios of contracts with different maturity dates;[6] cumulated flows of expenses for the dismantling of nuclear power plants; cumulated flows of profits (or losses), as will be the case in our carbon transition example (developed in Appendix A.2).

This final amount X_{t+H} is constituted progressively, by cumulating a sequence of "regular payments", or Required Capital Call: RCC_{t+h}, $h=1,\dots,H$.

Without discounting, several types of profile of RCC_{t+h} can be considered. Let us focus on exponential profiles:

$$RCC_{t+h}=\mu(t,H)\delta^{h-1},$$

where δ is positive, and $\mu(t,H)$ is a factor that will be discussed below. The monthly payments are constant if $\delta=1$, increasing (respectively, decreasing), if $\delta>1$ (resp. $\delta<1$). The limiting case $\delta=0$ (resp. $\delta=\infty$) corresponds to a total protection requested as soon as of date $t+1$, the required capital being constant and fixed at X_{t+H} (resp. to zero protection up to $t+H-1$,

[5] See also Goodhart (2010), Campiglio et al. (2018), Bolton et al. (2020) for discussions of the proactivity and mandates of such institutions.

[6] In that case, if some maturities are smaller than H, then assumptions have to be made regarding the reinvestment of those bonds redeeming before $t+H$. Consider $k<H$. An approach consists in proceeding as if the payoffs of date $t+k$ are reinvested in risk-free bonds of maturity $H-k$.

and a complete protection at $t+H$). An increasing profile implies less effort of reimbursment (or reserve constitution) in the short run. The importance of such profile is easily understood for the application to pension saving with guaranteed capital paid at retirement age: the fund receives regular payments by the individual and can invest them in more or less risky assets. By fixing $\delta > 1$, the supervision is imposing a minimal proportion invested in a (weakly remunerated) riskfree asset, and this proportion increases when getting closer to the maturity date. This prudential instrument is in fact, if not in name, a form of credit guidance to monitor the credit allocation between short and long-run risks (see Bezemer et al., 2018, for a discussion of credit-guidance policies). Largely abandoned in the 1980's with the argument that they can distort the efficient allocation of resources, they have been put in place to develop priority sectors and help innovation. Relatedly, a growing literature investigates how monetary policy, through the choice of eligible collateral and/or asset purchase programs (Quantitative Easing) can affect the relative costs of green/brown investments (see, e.g. Papoutsi et al., 2020).

When X_{t+H} is a gain, a decreasing profile of RCC_{t+h} ($\delta < 1$) would allow for high investments or credits in the first periods.

In order to get the amount X_{t+H} at maturity, we need $RC_{t+H} = X_{t+H}$. Therefore, since

$$RC_{t+H} = \sum_{h=1}^{H} RCC_{t+h} = \mu(t,H)\frac{1-\delta^H}{1-\delta},$$

we get :

$$\mu(t,H) = X_{t+H}\frac{1-\delta}{1-\delta^H} \quad \text{for} \quad \delta \neq 1, \quad \text{and} \quad \mu(t,H) = X_{t+H}/H, \quad \text{if} \quad \delta = 1. \tag{2.1}$$

In the standard supervision, the reserves of corporations or banks under supervision are deposits that private banks and insurance companies hold with the central bank. They are weakly remunerated; in the Eurosystem for instance, minimum reserves are remunerated, but excess reserves are not.[7] For expository purpose we assume below no remuneration. Therefore the total required capital at $t+h$ is:

$$RC_{t+h} = \sum_{k=1}^{h} RCC_{t+k} = X_{t+H}\frac{1-\delta^h}{1-\delta^H}, \tag{2.2}$$

[7] See, e.g., https://www.oenb.at/en/Monetary-Policy/Monetary-Policy-Implementation/minimum-reserves/remuneration-of-minimum-reserves.html#:~:text=Excess%20reserves%2C%20i.e.%20holdings%20exceeding,whichever%20is%20the%20lower%20rate.

and the remaining balance is:

$$B_{t+h} = X_{t+H} - RC_{t+h}. \tag{2.3}$$

It is easily checked that these formulas can be rewritten as updating formulas.

Lemma 1. *Under deterministic loss, and without discounting, the exponential profile is such that:*

$$\begin{aligned} RCC_{t+h} = RC_{t+h} - RC_{t+h-1} &= \frac{1-\delta}{1-\delta^{H-h+1}}(X_{t+H} - RC_{t+h-1}), \quad h = 1, \dots, H, \\ &= \frac{1-\delta}{1-\delta^{H-h+1}} B_{t+h-1}, \end{aligned}$$

with $RC_t = 0$ and B_{t+h-1} is given by (2.3).

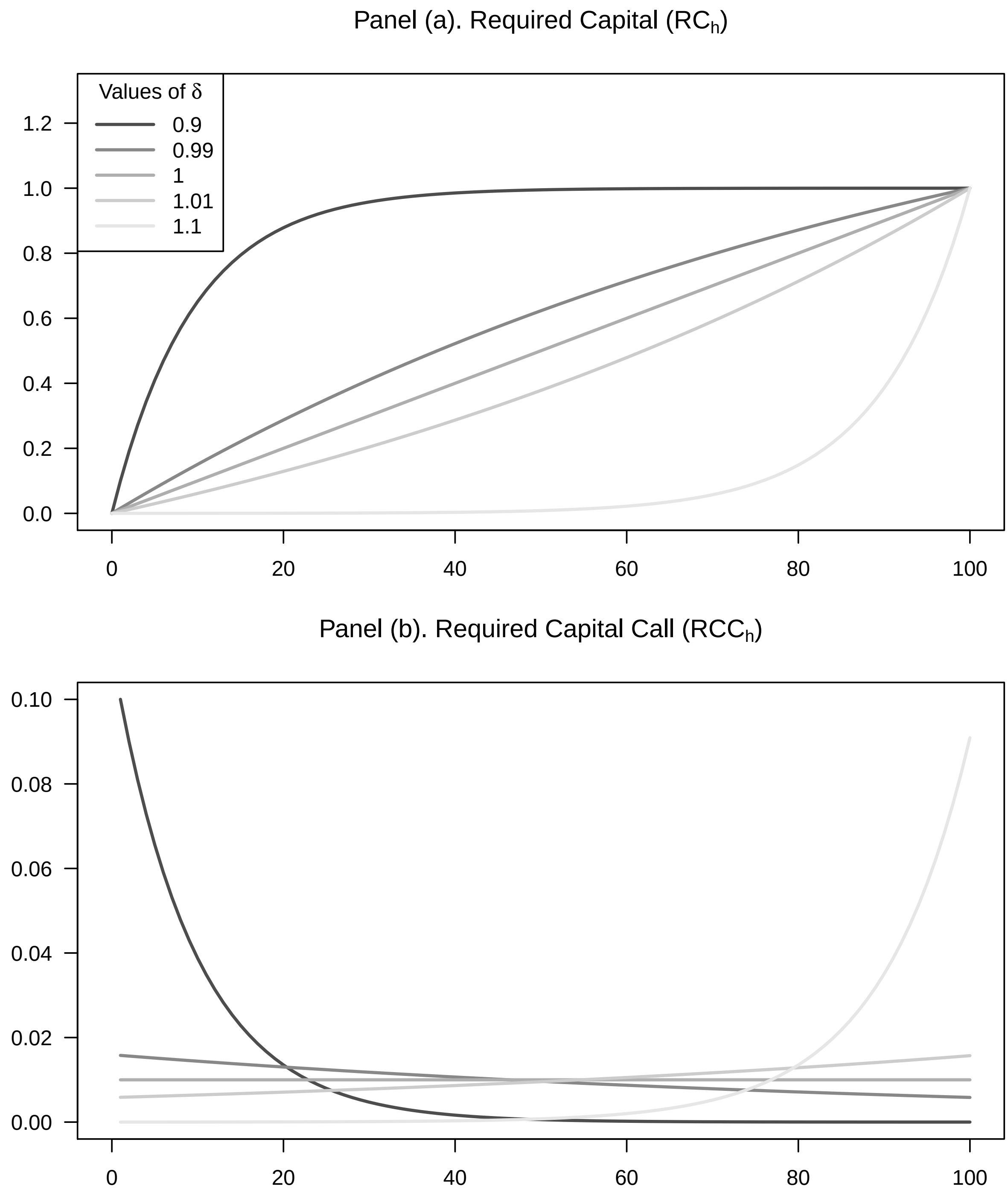
Lemma 1 shows that the required capital call at date $t+h$ is proportional to the remaining balance with a proportionality coefficient depending on the residual maturity $H-h$ and on the rate δ.

Possible evolutions of RC_{t+h} and RCC_{t+h} are illustrated in Figure 1, for different values of δ. Panel b shows that the RCC_{t+h} are monotonously decreasing (resp. increasing) when $\delta < 1$ (resp. $\delta > 1$). The evolution is linear when $\delta = 1$.

The adaptive formula in Lemma 1 is valid when there is no discounting. Let us now explain how to relax this assumption. Let us consider an initial credit amount $\overline{X}_t$ and a regulatory rate, such that $X_{t+H} = \overline{X}_t(1+r)^H$. This regulatory rate is a supervisory instrument; it is not equal in general to the (long-run) market rate. The formula in Lemma 1 is easily adjusted by considering the "discounted" loss:

Definition 1. *Under deterministic loss and regulatory discounting, the exponentially weighted profile is such that:*

$$\begin{aligned} RCC_{t+h} = RC_{t+h} - RC_{t+h-1} &= \frac{1-\delta}{1-\delta^{H-h+1}}\left[\frac{X_{t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right], \quad \textit{for } \delta \neq 1, \\ &= \frac{1}{H-h+1}\left[\frac{X_{t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right], \quad \textit{if } \delta = 1, \end{aligned} \tag{2.4}$$



Notes: This figure illustrates the updating formula given by Lemma 1. The initial date t is set to 0, and H is set to 100. The loss at maturity (X_H) is taken equal to 1. Panels (a) and (b) respectively display RC_h and RCC_h.

Figure 1: Influence of δ on the evolutions of RC_h and RCC_h

with $RC_t = 0$.

Formula (2.4) considers an objective $X_{t+H}/(1+r)^{H-h}$ at date $t+h$ and the residual exponential profiles associated with this objective. At date $t+h+1$, the objective is updated. Equation (2.4) is easily solved recursively. Figure 2 illustrates the effects of discounting.

2.2 Stochastic Loss at Maturity

The updating formula (2.4) is the basis of an extension when the loss at maturity (X_{t+H}) is stochastic. This is for instance the case when X_{t+H} corresponds to the cost of restructuration, of depollution, or to a short-sell investment in an illiquid stock (without dividends).

At each date $t+h$, we need a "valuation" $X^*_{t+h,t+H}$ of X_{t+H}.

Definition 2. *Under stochastic loss, regulatory discounting and sequence of valuations* $X^*_{t+h,t+H}$, $h = 1,\dots,H$, *with* $X^*_{t+H,t+H} = X_{t+H}$, *the profile of required capital is such that:*

$$RCC_{t+h} = RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}}\left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right], \tag{2.5}$$

with $RC_t = 0$.

This profile is perfectly defined once the supervisor has selected: (i) the regulatory (long-run) discount rate r; (ii) the rate δ of the exponential design of RCC; (iii) the sequence of benchmark "valuations" $X^*_{t+h,t+H}$, $h = 1,\dots,H$, of the risk at maturity.

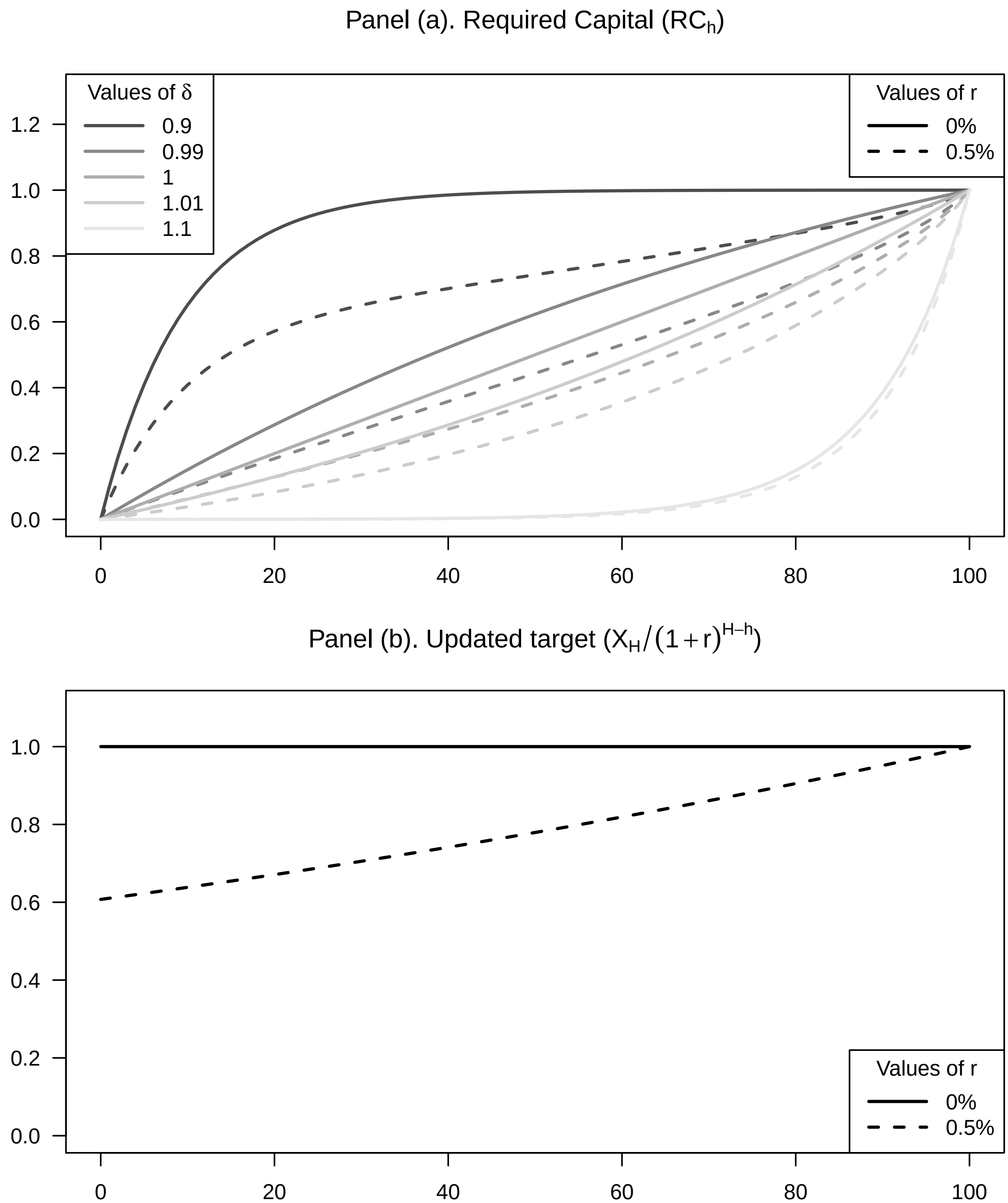
It can be noted that the updating formula (2.5) does not ensure an increasing required capital. If necessary, we obtain an increasing pattern of RC_{t+h} by transforming it into:

$$RCC_{t+h} = RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}}\left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right]^+, \quad h = 1,\dots,H-1, \tag{2.6}$$

with $RC_t = 0$, and employing the notation $X^+ = \max(X,0)$. This modification is in line with the standard formula for margin call in the definition of futures.[8]

To make the RC profiles (2.5 or 2.6) operational, the sequence of benchmark valuations has to be specified. Different valuation schemes can be selected depending on the type of long-run risk:

[8]Note that the RCC's are not margin calls, since, the required capital RC_{t+h} is not sufficient to ensure a total protection against a complete default on X_{t+H} at an intermediate date $t+h$.



Notes: Panel (a) illustrates the influence of discounting (see Definition 1) on RC_h. The initial date t is set to 0, and H is set to 100. The loss at maturity (X_H) is taken equal to 1. Panel (b) displays the updated target, that is $X_H/(1+r)^{H-h}$.

Figure 2: Discounting of the target and required capital profiles

(i) **Mean-variance scheme**. We set:

$$X^*_{t+h,t+H} = \mathbb{E}_{t+h}X_{t+H} + \frac{A}{2}\mathbb{V}ar_{t+h}X_{t+H}, \tag{2.7}$$

where $\mathbb{E}_t(X_{t+H})$ (resp. $\mathbb{V}ar_t(X_{t+H})$) is an expected loss (resp. a loss variance) conditional on the information available on date t, and A is the (absolute) risk aversion of the supervisor. This scheme is appropriate for the example of depollution cost, for which the costs and their uncertainties have to be regularly updated. They can for instance decrease if new depollution techniques are introduced, or they can increase if more severe constraints on the depollution level are imposed.

(ii) **Certainty equivalent scheme**. This approach demands a predictive distribution of the loss and a utility function $\mathscr{U}$. In this context:

$$X^*_{t+h,t+H} = \mathscr{U}^{-1}[\mathbb{E}_{t+h}\mathscr{U}(X_{t+H})]. \tag{2.8}$$

For a CARA utility function, that is $-\exp(AX)$ when X is a loss, we get:

$$X^*_{t+h,t+H} = \frac{1}{A}\log \mathbb{E}_{t+h}\exp(AX_{t+H}), \tag{2.9}$$

where A (> 0) is the absolute risk aversion of the supervisor.

(iii) **Risk-Neutral scheme**. The valuation is defined as:

$$X^*_{t+h,t+H} = \mathbb{E}^Q_{t+h}(X_{t+H}), \tag{2.10}$$

where Q is a distribution adjusted for risk, or risk-neutral distribution (under a zero riskfree rate). This valuation is the standard pricing formula when the “asset” corresponding to the loss (equal to X_{t+H} on date $t+H$) can be regularly traded between t and $t+H$ on competitive and liquid markets. This is not the case for the long-run risks we are interested in. Indeed, this approach implies, in particular, the linearity of the valuation formula, that is:

$$\mathbb{E}^Q_{t+h}(\lambda X_{t+H}) = \lambda \mathbb{E}^Q_{t+h}(X_{t+H}), \quad \text{for any } \lambda > 0.$$

Consider nuclear depollution costs. Is the cost of depollution for ten nuclear power plants equal to ten times the cost of depollution of a single plant? Likely not, since marginal costs may be decreasing (because of economies of scale) or, symmetrically, increasing if there is some rationing in the number of specialized workers available at the same time. The same remark applies if X_{t+H} represents the payment of a large short sell in an illiquid financial asset. The liquidation of this asset will likely be accompanied by penalties.

2.3 Short-run and Long-Run Required Capital

Let us illustrate the joint concepts of short-run and long-run required capitals in the context of a pension fund. We focus on a stylized situation where the fund is based on a set of contracts with the same maturity, denoted by T. Between t and $T = t+H$, the fund receives regular premiums from contractors and uses them to invest in two types of assets: illiquid and more liquid ones. The latter ones can be used to feed the RC account.

Figure 3 represents the balance sheets of this closed fund at date $t+h$, before and after the capital calls. We distinguish between frozen (illiquid) and unfrozen (liquid) components of the balance sheets. The frozen components are: (a) the guarantees (X_{t+H}) appearing on the liability side, and (b) the future premiums $\bar{A}_{t+h}$ and the RC at the beginning of period $t+h$, i.e., before the call and the premium payment, on the asset side. The unfrozen components are denoted by a_{t+h} and l_{t+h} on the asset and liability sides, respectively. For instance, l_{t+h} includes the management costs, such as salaries, whereas a_{t+h} includes financial asset holdings in more or less liquid assets. These are simplified balance sheets, assuming that there is no death of contractors before $t+H$, and that the individual contracts are not partly sold on a secondary market (as in the case of insurance linked security, ILS).[9]

In this special application, the amount of premiums payed in cash at $t+h$ is $\bar{A}_{t+h-1} - \bar{A}_{t+h}$. At the beginning of the period, and if $\delta > 1$, these premiums can likely be sufficient to cover the capital calls. But this is no longer the case close to maturity, when some financial assets in a_{t+h} have to be sold to satisfy the capital requirement.

At date $t+h$, the sum of the unfrozen value $W_{t+h} = a_{t+h} - l_{t+h}$ and of the short-run RC has to be sufficiently large to satisfy the next call for long-run risk. Here, we follow the standard definition of supervisory RC and we define the short-run RC required capital as a conditional quantile of the short-run result of the firm. Formally, the short-run level-α required capital at $t+h-1$, denoted

[9] We do not discount X_{t+H} and the future sequence of premiums. This is usually done by actuarial techniques, using a contractual interest rate.

At $t+h$ before the call and the premium payment

Asset	Liability
$\bar{A}_{t+h-1}$	X_{t+H}
RC_{t+h-1}	
	l_{t+h}
a_{t+h}	
A_{t+h}	L_{t+h}

At $t+h$ after the call and the premium payment

Asset	Liability
$\bar{A}_{t+h}$	X_{t+H}
RC_{t+h}	
rc_{t+h}	l_{t+h}
a^*_{t+h}	
A_{t+h}	L_{t+h}

Figure 3: Balance Sheets at $t+h$, before and after the call and the premium payment. We distinguish between frozen (illiquid) and unfrozen (liquid) components. The frozen components are: (a) the guarantees (X_{t+H}) appearing on the liability side, and (b) the future premiums $\bar{A}_{t+h}$ and the RC at the beginning of period $t+h$, i.e. before the call and the premium payment, on the asset side. The unfrozen components are denoted by a_{t+h} and l_{t+h} on the asset and liability sides, respectively.

by $rc_{t+h-1}(\alpha)$, is defined as the level-α conditional VaR at horizon 1 (see Appendix A.1). That is:

$$\mathbb{P}_{t+h-1}[W_{t+h} + rc_{t+h-1}(\alpha) > RCC_{t+h}] = 1 - \alpha, \tag{2.11}$$

where $RCC_{t+h} = RC_{t+h} - RC_{t+h-1}$ is the long-run call defined in previous subsections, and $\mathbb{P}_{t+h-1}$ denotes the probability conditional on the information available at date $t+h-1$. Then, the total RC at date $t+h$ is the sum of the short-run RC (i.e. rc_{t+h}), and the long-run RC (i.e. RC_{t+h}), whereas the total call is: $RC_{t+h} - RC_{t+h-1} + rc_{t+h} = RCC_{t+h} + rc_{t+h}$.

As usual in micro-supervision, the behaviour of the firm—here the pension fund—is crystallized at date t for computing the reserves. Then the supervision does not account for the optimal strategy of the fund to pay its employees, or the retirees (see, e.g., Bogentoft et al., 2001, for Asset/Liabitily management of a pension fund).

The definitions of the short-run required capital $rc_{t+h-1}(\alpha)$ and of the unfrozen net asset can be easily extended to other types of firms.

3 Long-Run Transition to Low-Carbon Economies

Let us now discuss how to treat the lines of the balance sheet depending jointly of short-run and long-run risk factors in a nonlinear way. We consider the case where there is a long-run transition between two situations. A typical example is the transition between the current economy and a low-carbon one, this transition entailing a modification of the production process of the firms.

3.1 Short-Run and Long-Run Factors

The portfolios of banks and insurance companies include assets whose values are driven by both short-run and long-run factors. To distinguish them, we can consider that short-run factors are highly volatile and mean reverting, possibly including jumps, both without effects in the long run. By contrast, whereas changes in the long-run factors are almost invisible in the short run, these factors affect macroeconomic and finance variables in a highly-persistent way. For instance consequences of a low-carbon policy are not visible at the daily frequency, but can become visible at a yearly, or lower, frequency. Indeed the supervision policy is largely driven by carbon prices, that evolve rather smoothly to avoid systemic failure of firms. These carbon prices affect corporations' results. They can increase the production costs and/or diminish the demand of the products. As a consequence, this may increase the probability of default and then diminish the value of loans and stocks (see Boermans and Galema, 2017; Devulder and Lisack, 2020). At the limit, a technology with a less damaging impact on the environment can replace a technology that is more damaging, making the latter obsolete.

3.1.1 Factor Decomposition

Let us now consider an entity (corporation, bank) with a result, or profit, P_{t+h} at date $t+h$. Let us assume that this result depends on a short-run factor and a long-run one, these factors being respectively denoted by $y_{s,t+h}$ and $y_{l,t+h}$:

$$P_{t+h} = g(y_{s,t+h}, y_{l,t+h}), \tag{3.1}$$

where g may be nonlinear. The cumulated result on the period is:

$$CP_{t+H} = \sum_{h=1}^{H} P_{t+h} = \sum_{h=1}^{H} g(y_{s,t+h}, y_{l,t+h}). \tag{3.2}$$

We expect the short-run factor to be highly volatile, but with a weak serial correlation, and the long-run factor to vary gradually, without significant visible impact in the short run. As usual the factors are defined up to an invertible transform. More precisely, let us introduce two invertible functions a_s and a_l, that are such that $y_{s,t} = a_s(z_{s,t})$ and $y_{l,t} = a_l(z_{l,t})$. Equation (3.1) becomes:

$$P_{t+h} = g[a_s(z_{s,t+h}), a_l(z_{l,t+h}] \equiv g^*(z_{s,t+h}, z_{l,t+h}).$$

When there is a gradual evolution around an "equilibrium" value y_l^*, the formula (3.2) can be replaced with its first-order expansion with respect to y_l around y_l^*:

$$CP_{t+H} \simeq \sum_{h=1}^{H} g(y_{s,t+h}; y_l^*) + \sum_{h=1}^{H} (y_{l,t+h} - y_l^*) \frac{\partial g}{\partial y_l}(y_{s,t+h}; y_l^*). \tag{3.3}$$

3.1.2 Dynamics of the Long-Run Factor

Let us now consider dynamic assumptions on the factors and the effect of these factors on he sequence of cumulated profits. We assume that the short-run and long-run factors are independent. Process $(y_{s,t})$ is assumed to be strictly stationary. This process can be a linear or a nonlinear process, and is not necessarily Gaussian. The other process (y_{lt}) is modelled as an Ultra Long-Run (ULR) component, i.e. a process that is closed to a unit root stationary process. Loosely speaking, this long-run process entails trajectories that are almost constant in time at a stochastic level (i.e. a singular process). That is, the trajectories are smooth with local trends effects, while staying stationary over long period (see, e.g., Gouriéroux and Jasiak, 2020, for a general definition of an ULR process). For sake of simplicity, we base the ULR component on an Ornstein-Uhlenbeck (OU) process on which we apply a time deformation. Specifically, we assume:[10,11]

$$\begin{aligned} y_{l,t} &= \tilde{y}(t/H) && (3.4) \\ \text{where } d\tilde{y}(\tau) &= -k\tilde{y}(\tau)d\tau + \sqrt{2k}\, dW(\tau), \quad k > 0, && (3.5) \end{aligned}$$

The drift and volatility function of the OU diffusion are constrained to get a stationary distribution of the ULR component that is independent of H. The ULR component satisfies a discretized stationary Gaussian autoregressive process of order 1:

$$y_{l,t} = \rho_H y_{l,t-1} + \sqrt{1-\rho_H^2}\, \varepsilon_{l,t}, \tag{3.6}$$

where $(\varepsilon_{l,t})$ is a Gaussian standard noise, $\rho_H = \exp(-k/H) < 1$, and the stationary distribution of $y_{l,t}$ is $\mathcal{N}(0,1)$. When H is large (tends to infinity), ρ_H tends to 1 (close to unit root), the volatility tends to zero (small variance) and the trajectory of the ULR component is close to $y_{l,t} = y_{l,t-1}, \forall t$.

[10] In fact, the ULR process would have to be indexed also by H, as $y_{l,H,t} = \tilde{y}(t/H)$, leading to a triangular array modelling and to a stationary close-to-unit root model.

[11] The standard approach used in microprudential modeling is the value-of-the-firm model of Vasicek, which leads to probit/logit models for probabilities of default and allow for default correlations. Choosing an ULR Ornstein-Uhlenbeck process can is the natural step to extend this standard approach to short/long-run risks.

Since the process of profit is a nonlinear transform of $y_{s,t}, y_{l,t}$ (equation 3.1), it is generally not Gaussian—even if $(y_{s,t})$ and $(y_{l,t})$ are—and it admits a complicated nonlinear dynamics.

Let us stress that the assumption of a latent Ornstein-Uhlenbeck process is not very restrictive. Indeed, since the long-run factor is defined up to an invertible transform, the OU diffusion equation can be replaced by another diffusion equation obtained by applying Ito's formula, leading to other types of ULR processes.

3.1.3 Long-Run Approximation

The approach above is usually applied to different corporations $i = 1, \ldots, n$, say. In that context, equation (3.1) becomes:

$$P_{i,t+h} = g_i(y_{i,s,t+h}, y_{l,t+h}), \quad i = 1, \ldots, n,$$

where the $y_{i,s}$, $i = 1, \ldots, n$, are idiosyncratic factors, y_l a common long-run factor, and g_i is corporate-specific. The common (or systematic) factor creates the dependence between individual corporate risks and individual exposures in the balance sheets of banks and insurance companies.

In the long-run microprudential framework (see Subsection 1.2), both the number of firms n and the horizon H are large. It is always possible to evaluate jointly the individual exposures at the different horizons by intensive simulations (bottom-up approach). However these simulations can be facilitated by using asymptotic approximations of the distribution of aggregate loss exposure for either n large, or H large. More precisely, the long-run supervisory formula for the aggregate loss at time t and horizon H is:

$$L_{t,H} = \sum_{i=1}^{n} E_{i,t,H}\, ELGD_{i,t,H} Z_{i,t,H},$$

where $E_{i,t,H}$ is the individual exposure, $ELGD_{i,t,H}$ is the individual expected loss-given-default and $Z_{i,t,H}$ is the default indicator, defined by:

$$Z_{i,t,H} = \begin{cases} 1 & \text{if } \sum_{h=1}^{H} g_i[y_{i,s,t+h}, y_{l,t+h}] < 0, \\ 0 & \text{otherwise.} \end{cases}$$

It differs from the standard formula for aggregate loss since default is only possible at the long-run horizon H, based on the cumulated profit.

For large n, the distribution of the aggregate loss can by obtained by applying the granularity theory (see Gouriéroux and Gagliardini, 2022), which consists in extending Vasicek's single-risk-factor approach (Vasicek, 1991). For large H, another approximation can be introduced in models with short-run and long-run components.

This large-H approximation is given below for a given firm. For notational simplicity, the firm subscript is omitted. Let us denote by f the stationary density of the short-run factor $y_{s,t}$, and by G the function defined by:

$$G(y_l) = \int g(y_s, y_l) f(y_s) dy_s. \tag{3.7}$$

Then, for large H, we approximately have (Gouriéroux et al., 2022):

$$\frac{1}{H} CP_{t+H} \simeq \int_0^1 G(\tilde{y}_u) du \equiv P^*. \tag{3.8}$$

We see, from formulas (3.7) or (3.8), that, for large H, CP_{t+H} is stochastic (and hence P^* is stochastic) through the long-run component $\tilde{y}$ only. One can then apply the approach of Section 2 with $X_{t+H} = -CP_{t+H} \simeq -H\ P^*$ (or $X_{t+H} = CP_{t+H} \simeq H\ P^*$ by changing the sign convention). This provides a sequence of approximated required capital RCC^*_{t+h} (with $RCC^*_{t+h} \approx RCC_{t+h}$), for $h = 1, \ldots, H$. And, at each date $t+h-1$, the approximated short-run required capital rc^* can be fixed as in (2.11) by:

$$\mathbb{P}_{t+h-1}[g(y_{s,t+h}, y_{l,t+h-1}) + rc^*_{t+h-1}(\alpha) > RCC^*_{t+h}] = 1 - \alpha.$$

An example of application of this long-run approximation is given in Section 4.

When the ULR component is close to a given value y_l^*, say, then the H-asymptotic approximation can be replaced by:

$$\frac{1}{H} CP_{t+H} \simeq \mathbb{E}[g(y_{s,t}; y_l^*)] + \mathbb{E}\left[\frac{\partial g}{\partial y_l}(y_{s.t}; y_l^*)\right] \left(\int_0^1 \tilde{y}(u) du - y_l^*\right) \equiv P^{**}. \tag{3.9}$$

When the firm index is reintroduced, this approach accounts for joint defaults of firms that may result from adverse trajectories of the long-run systematic factors. However, this model does not account for the potential chains of failures—or cascades of defaults—that may arise from additional contagion effects. While this aspect is beyond the scope of this paper, a promising line of research would consist in combining long-run factors and network analysis (see, e.g., Darolles and Gouriéroux, 2015, Chapter 3, for common frailty vs contagion in dynamic models, Battiston et al., 2017 for a network-based approach, without long-run factor, and Cahen-Fourot et al., 2020, for input/output networks).

3.2 Structural Model for Low-Carbon Transition

In the present subsection, we provide interpretations of g and of the underlying factors in the context of the transition to low-carbon economies. Appendix A.2 provides additional details.

3.2.1 The Profit in the Context of Low-Carbon Transition

Consider the case where the firm's production function has to account for carbon input and output on top of the standard ones. Moreover, the firm has to adjust its technology to the change of environment, especially to the evolution of carbon prices. As developed in Appendix A.2, the profit $\hat{\hat{P}}$ is a nonlinear function of current and past carbon prices, as a result of optimal choice of technology and input allocations. More precisely, it can be written as:

$$\hat{\hat{P}} = \hat{P}[y, p, \pi, \pi_c; \hat{\theta}(y, p, \pi, \pi_c, q)],$$

where $\hat{\theta}$ is the optimal technology, y is the output of non carbon good, π is its price, vector p gathers the prices of non carbon inputs, π_c and q are the prices of carbon as output and input, respectively. All these variables are supposed to be observed.

Let us now consider progressive changes of carbon prices π_{ct} and q_t, the ouput level and other prices being crystallized. We have:

$$\hat{\hat{P}}_t \quad = \quad \hat{P}\left[y, p, \pi, \pi_{ct}, q_t; \hat{\theta}(y, p, \pi; \pi_{ct}, q_t)\right]. \tag{3.10}$$

In this model, carbon prices—or log-carbon prices—can be used as underlying structural factors. Moreover, if carbon prices are close to (target) prices denoted by π_c^* and q^*, a Taylor expansion leads to:

$$\hat{\hat{P}}_t \simeq \hat{\hat{P}}^* + \frac{\partial \hat{P}^*}{\partial \pi_c}(\pi_{ct} - \pi_c^*) + \frac{\partial \hat{P}^*}{\partial q}(q_t - q^*) + \frac{\partial \hat{P}^*}{\partial \theta}\left[\frac{\partial \hat{\theta}^*}{\partial \pi_c}(\pi_{ct} - \pi_c^*) + \frac{\partial \hat{\theta}^*}{\partial q}(q_t - q^*)\right], \tag{3.11}$$

where the superscript $*$ means that the derivatives are taken at y, p, π, π_c^*, q^*. Therefore, locally, we can apply (3.11) for any date $t+h$, and also to the cumulated (optimal) profit $CP_{t+H} = \sum_{h=1}^{H} \hat{\hat{P}}_{t+h}$. We get:

$$CP_{t+H} = H\hat{\hat{P}}^* \quad + \quad \left(\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial \pi_c}\right)\sum_{h=1}^{H}(\pi_{c,t+h} - \pi_c^*)$$
$$+ \quad \left(\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial q}\right)\sum_{h=1}^{H}(q_{t+h} - q^*). \tag{3.12}$$

We can use this expansion to translate the low-carbon transition risk due to the changes in prices into prudential risks, i.e., we can measure the long-run sustainability of this business model.

3.2.2 Carbon Prices Dynamics

As mentioned earlier we will consider a progressive change of carbon prices, following Carney (2016)'s recommendation: "a too rapid movement towards a low-carbon economy could materially damage financial stability, [. . .] destabilize markets, crystallize losses." More precisely, to evaluate the long-run uncertainty on the cumulated profit, we introduce ULR dynamics for prices π_{ct} and q_t. The evolution of these prices may for instance reflect the introduction of carbon taxes (see, e.g., Nordhaus, 2014; Gollier, 2018; Bolton et al., 2020, for related estimates of the social cost of carbon), or new rules for eligible collateral that would penalize brown assets (see, e.g., Matikainen et al., 2017; Papoutsi et al., 2020). These ULR dynamics should be consistent with positive prices.

Let us consider the expansion (3.12). We get:

$$\frac{1}{H}CP_{t+H} = \hat{\hat{P}}^* + B\frac{1}{H}\left[\sum_{h=1}^{H}(\pi_{c,t+h} - \pi_c^*), \sum_{h=1}^{H}(q_{t+h} - q^*)\right]', \tag{3.13}$$

where B is the row vector of multipliers, that is:

$$B = \left[\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial \pi_c}, \quad \frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial q}\right].$$

If the sequence of prices is a ultra long-run process:

$$(\pi_{c,t+h}, q_{t+h})' = \tilde{Y}(h/H), \tag{3.14}$$

where $(\tilde{Y}(\tau))$ is a bivariate diffusion process with positive components. Let us now discuss the possible dynamics for $\tilde{Y}$ to account for the positivity and nonstarionarity of prices.

The difference with Subsection 3.1 is that these smooth dynamics have to be nonstationary in order to account for the transition between the two economies.[12] This can be modeled in different ways, for instance by considering positive combinations of processes satisfying stochastic-logistic-transition dynamics:

$$d\tilde{y}(\tau) = k\tilde{y}(\tau)\{c - \tilde{y}(\tau)\}d\tau + \eta dW(\tau), \quad k > 0,\, c > 0,\, \eta > 0.$$

This model ensures a price evolution around the deterministic logistic function, obtained for $\eta = 0$, i.e.:

$$d\tilde{y}(\tau) = k\tilde{y}(\tau)\{c - \tilde{y}(\tau)\}d\tau \Leftrightarrow \log \frac{\tilde{y}(\tau)}{c - \tilde{y}(\tau)} = \frac{k}{c}\tau \Leftrightarrow \tilde{y}(\tau) = \frac{c}{1 + \exp(-k\tau/c)}.$$

This standard stochastic logistic model does not strictly ensures positive prices when $\eta > 0$, but is easily modified to get the positivity.[13]

If the prices follow a logistic transition from initial zero prices (the carbon was not priced at the initial date), price changes are small at the beginning and at the end of the transition, but reach a maximum in between. The transition speed directly impacts this maximum. This can be the moment of the largest required changes to which corporations will not adjust and then will default. Using the vocabulary of integrated assessment models, this constitutes a form of "tipping point" in which the change can be irreversible for the firm (see, e.g., Lenton et al., 2019, for a discussion of tipping points).

3.2.3 Long-Run Approximation

Under (3.14), we get the long-run approximation (for large H):

$$\frac{1}{H} CP_{t+H} \simeq \hat{\hat{P}}^* + B \left[\int_0^1 \tilde{Y}(u)du - \begin{pmatrix} \pi_c^* \\ q^* \end{pmatrix} \right]. \tag{3.15}$$

As in Subsection 3.1.3, formula (3.15) can be used to analyze the uncertainty on the cumulated profit at any intermediate date, then to propose a required capital profile, or to evaluate the probability of default. For instance, considering that a default takes place when CP_{t+H} is negative, the

[12] This nonstationarity is sometimes called "non equilibrium model" in the literature (see, e.g., Bolton et al., 2020, p.44).

[13] For instance by considering $\tilde{y}(\tau) = \frac{c}{1+\exp(-kz(c)/c)}$ where process $z(c)$ satisfies $dz(c) = dc + \eta dW(c)$.

probability of default of a given firm, evaluated at a date $t+h$, is:

$$\begin{aligned} PD_{t+h} &= \mathbb{P}_{t+h}[CP_{t+H} < 0] \\ &\simeq \mathbb{P}_{t+h}\left[\hat{\hat{P}}^* + B\left[\int_0^1 \tilde{Y}(u)du - \begin{pmatrix} \pi_c^* \\ q^* \end{pmatrix}\right] < 0\right]. \end{aligned}$$

If $(t+h)/H \simeq h/H \simeq \gamma$ for large H, we get:

$$PD_{t+h} \simeq \mathbb{P}\left[\hat{\hat{P}}^* + B\left[\int_0^\gamma \tilde{Y}(u)du - \begin{pmatrix} \pi_c^* \\ q^* \end{pmatrix}\right] + B\int_\gamma^1 \tilde{Y}(u)du < 0 \middle| \underline{\tilde{Y}(\gamma)}\right],$$

where $\underline{\tilde{Y}(\gamma)} = (\underline{\pi_{c,t+h}}, \underline{q_{t+h}})'$ is the information on prices available at date $t+h$.

3.3 The Individual Information and the Control Parameters

The planned policy of the microprudential approach described in Subsections 3.1 and 3.2 requires both information on the new balance sheets and the values of several parameters, including in particular the maximal term, the regulatory rate, and the profile δ for computing the *RCC*. In the framework of low-carbon transition, partial answers have been given by the supervisors.

3.3.1 Information

The present example highlights the importance of updating the knowledge of carbon inputs and outputs involved in the production process, which, itself, requires a complete and transparent accounting framework.[14] This is currently under discussion in Europe, in the context of the climate-related prudential supervision (European Banking Authority, 2019a, 2020) and in that of the 2022 ECB Climate Risk Stress Test (CST). This need has also been stressed in the context of the recent French pilot exercise (see Allen et al., 2020; Authority of Prudential Control and Resolution, 2021).

The implementation of such a supervision suppose data, standardizations and taxonomies. For the transition to a low-carbon economy, the standardizations concern the precise definitions of homogenous classes of goods (inputs and outputs), of the GHG, of the production functions, of the

[14] See Klaassen and Stoll (2021) for the importance of harmonizing firm-level carbon accounting.

reporting frequency, of the industrial sectors,[15] of the control of the reporting quality, and of the horizon of analysis.

3.3.2 Control Parameters

The computation of required capital is based on different parameters such as the maximal term, the rate δ of the exponential profile, the "contractual" discount rate r, and on the criterion to compute valuations $X^*_{t+h,t+H}$. In the context of the transition to a low-carbon economy, the choice of these control parameters and valuation criterion may impact credit costs, credit granting, or credit allocation between firms. Therefore, the calibration of these parameters, which is beyond the scope of the present paper, is critical. We acknowledge that this calibration is very peculiar given the long horizons that are considered (difficulties associated with long-run stress tests and scenarios are highlighted by Hansen, 2022).

Regarding the contractual rate r, some guidance may be found in the literature that studies the discounting of very long-term payoffs relating to environmental projects (see, e.g., Giglio et al., 2021; Bauer and Rudebusch, 2022). It should be acknowledged, however, that the long-standing debate regarding the proper discounting of climate-related costs, initiated by two prominent names of the field—namely Nicholas Stern and William Nordhaus—has not yet been concluded (see, e.g., Dasgupta, 2008; Weisbach and Sunstein, 2009; Arrow et al., 2014; DeCanio et al., 2022).

3.3.3 Micro-Supervision vs Macro-Supervision

Important questions of coherency between the analysis of microprudential supervision and dynamic macro-models introduced for macro predictions have also to be solved. Let us give an example of such questions: "prices" π_c are an instrument of economic policy, that can be used at the individual level of the firm. During the transition, the costs of carbon emissions may differ across industrial sectors, reflecting different degrees of vulnerability to—or effect on—carbon (and carbon footprints, see Appendix A.2). The supervision has to be "proportionated, tailored for different business models around the sector, recognizing that the zero failure is neither desirable, nor realistic" (Carney, 2015). This is not compatible with standard macro-models, where "the car-

[15] A dozen of nomenclatures are currently used but not yet harmonized. They are not exempt from conflicting views. They include the International Standard Industrial Classification of all Economic Activities (ISIC) from the United Nations, the Nomenclature des Activités Commerciales et Économiques (NACE) for the European Union, the Harmonized System (HS) by the World Customs Organization, the Global Industry Classification Standard (GICS) created by Standard and Poor's and Barra, the Industry Classification System (ICS) created by Dow Jones and FTSE.

bon prices should be equalized in every sector and country" [Nordhaus (2019), p. 2002, see also Weitzman (2014)]. Indeed standard macro-models rarely account for firm heterogeneity.[16]

4 Implementation

Let us now illustrate how the approach can be implemented. We consider the framework of Subsection 3.1 which mixes short-run and long-run factors.

4.1 A Multistep Approach

The approach follows the steps below:

1. Define the number(s) of short-run and long-run factors and the (nonlinear) function g, in (3.1).

2. Define the horizon $T = t + H$.

3. Specify the distribution of the short-run component(s), and the dynamics of the long-run component(s).

4. Deduce, by simulation, or by applying a first-order expansion, the conditional distribution of CP_{t+H} at date $t+h$. In simple models (see Subsection 4.2), this conditional distribution is known under closed form. Otherwise, it is obtained by simulation.

5. Fix the valuations $X^*_{t+h,t+H}$ from this conditional distribution, for instance by a mean-variance scheme (see other possibilities in Subsection 2.2).

6. Fix the control parameters δ, r defining the exponential profile and the supervisory discount rate, respectively.

7. Compute the sequences RCC and RC by formula (2.5).

8. Compute the short-run required capital rc using (2.11).

[16]See Nordhaus (2014) for a description of the DICE (Dynamic Integrated Climate-Economy) model, and Keen (2019) for a critical view.

4.2 Stochastic Volatility Model with ULR Volatility

Let us first consider a profit with a long-run linear evolution and a stochastic volatility defined by:

$$P_t \;=\; g(y_{s,t}, y_{l,t}) = a + by_{l,t} + \sqrt{y_{l,t}}\, y_{st},$$

where y_{st} is a strong white noise, $y_{l,t} = \tilde{y}(t/T)$ an ULR stochastic process based on a diffusion $\tilde{y}$. The modelling above includes a risk premium with an effect of the long-run volatility on the conditional mean. By applying (3.9)-(3.8) and noting that $\mathbb{E}(y_{s,t}) = 0$, we get:

$$\frac{1}{H} CP_{t+H} \simeq a + b \int_0^1 \tilde{y}(u)du = P^*. \tag{4.1}$$

In the long run, the effect of $\sqrt{y_{l,t}} y_{s,t}$ can be neglected, but the randomness associated with the long-run risk premium has to be taken into account.

Next we apply the mean-variance scheme to fix the intermediate valuations (see Section 2.2). Since $y_{l,t}$ is a volatility, we posit a dynamics of $y_{l,t}$ based on a continuous time CIR process (Cox et al., 1985).[17] Specifically, we take $y_{l,t} = \tilde{y}(t/H)$, where the continuous-time dynamics of $\tilde{y}(\tau)$ is defined by the diffusion equation:

$$d\tilde{y}(\tau) = K\{\theta - \tilde{y}(\tau)\}d\tau + \eta\, \tilde{y}^{1/2}(\tau) d\tilde{W}(\tau), \tag{4.2}$$

where K, θ, η are parameters, with $K > 0$, $\theta > 0$, $\eta > 0$, and where the Feller condition holds ($2K\theta > \eta^2$), and $\tilde{W}$ is a Brownian motion.

Since the CIR process is an affine process, the conditional log-Laplace transform [or cumulant generating function] of its future cumulated values is a linear affine function of its current value (see Appendix A.3). Then by considering the second-order Taylor expansion of this log-Laplace transform, we deduce that the associated conditional mean and variance are linear affine as well. Thus we have:

$$\mathbb{E}\left[\int_\gamma^1 \tilde{y}(u)du | \underline{\tilde{y}(\gamma)}\right] \;=\; m_1(1-\gamma)\, \tilde{y}(\gamma) + m_0(1-\gamma), \tag{4.3}$$

$$\mathbb{V}ar\left[\int_\gamma^1 \tilde{y}(u)du | \underline{\tilde{y}(\gamma)}\right] \;=\; \sigma_1(1-\gamma)\, \tilde{y}(\gamma) + \sigma_0(1-\gamma), \tag{4.4}$$

[17] An alternative would be to assume that $\log \tilde{y}$ follows an Ornstein-Uhlenbeck process.

where m_1, m_0, σ_1, and σ_0 are functions of the time-to-maturity and of the parameters characterizing the dynamics (4.2) of the CIR process (see Appendix A.3 for details).

When H is large:

$$\begin{aligned}\frac{1}{H}CP_{t+H} &\simeq a+b\int_0^1 \tilde{y}(u)du \\ &= \frac{H-h}{H}a+\frac{1}{H}CP_{t+h}+b\int_{h/H}^1 \tilde{y}(u)du.\end{aligned}$$

This implies, for the mean-variance scheme:[18]

$$\begin{aligned}X^*_{t+h,t+H} &= \mathbb{E}_{t+h}(CP_{t+H})-\frac{A}{2}\mathbb{V}ar_{t+h}(CP_{t+H}) \\ &\simeq (H-h)a+CP_{t+h}+ \\ &\quad bH\left(m_0\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_0\left(1-\frac{h}{H}\right)\right)+ \\ &\quad bH\left(m_1\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_1\left(1-\frac{h}{H}\right)\right)y_{l,t+h}. \end{aligned} \tag{4.5}$$

Hence, $X^*_{t+h,t+H}$ can be approximated by an affine combination of the current cumulated profit (CP_{t+h} as of date $t+h$) and of the current value of the long-run component, with coefficients depending on the time-to-maturity.

Next, we can apply (2.5), the recursive formula defining the RCC. For instance, with $\delta=1$ and $r=0$, we get:

$$\begin{aligned}RCC_{t+h} &= X^*_{t+h,t+H}/(H-h)\simeq RCC^*_{t+h}= \\ &= a+\frac{CP_{t+h}}{H-h}+\frac{bH}{H-h}\left[m_0\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_0\left(1-\frac{h}{H}\right)\right] \\ &\quad +\frac{bH}{H-h}\left[m_1\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_1\left(1-\frac{h}{H}\right)\right]y_{l,t+h}.\end{aligned} \tag{4.6}$$

For large h, and using the ULR property of component $y_{l,t}$, we obtain:

$$\begin{aligned}RCC^*_{t+h+1} &\simeq RCC^*_{t+h}, \\ P_{t+h+1} &\simeq a+by_{l,t+h}+\sqrt{y_{l,t+h}}\,y_{s,t+h+1},\end{aligned}$$

and following the approach proposed in (2.11), the short-run required capital $rc_{t+h}(\alpha)$ is approxi-

[18] The minus sign in front of A results from the fact that X is, here, a gain.

mately given by:

$$\mathbb{P}_{t+h}\left[a+by_{l,t+h}+\sqrt{y_{l,t+h}}\,y_{s,t+h+1}+rc^*_{t+h}(\alpha)>RCC^*_{t+h}\right]=1-\alpha,$$

or

$$rc^*_{t+h}(\alpha)=RCC^*_{t+h}-a-by_{l,t+h}-q(\alpha)\sqrt{y_{l,t+h}}, \tag{4.7}$$

where $q(\alpha)$ is the α-quantile of the distribution of the short-run component.

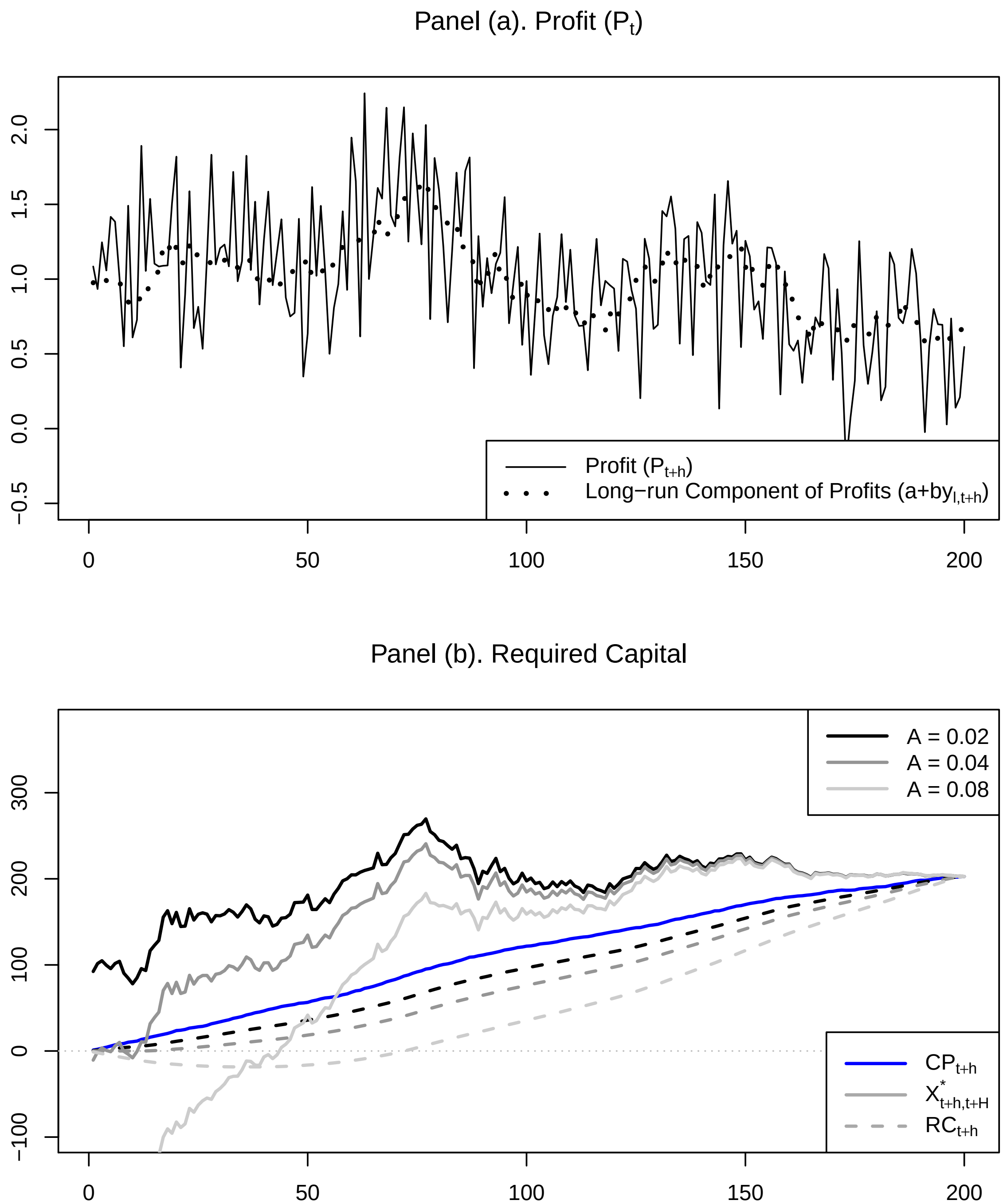
This approach requires the knowledge of the distribution of the short-run component, of the parameters of the underlying CIR process, and an approximation of the ULR component. These estimation and filtering issues are out of the scope of the present paper.

Figure 4 displays simulated paths based on the framework described in the present subsection. Specifically, we consider the process of profits P_t described by (4.1), with $a=-3$ and $b=1$. Moreover, the specification of the CIR process (4.2) is as follows: $K=0.05$, $\eta=0.45$, and $\theta=4$. Finally, $y_{s,t}\sim i.i.d.\,\mathcal{N}(0,\sigma^2)$, with $\sigma=0.2$. The valuation X^* is based on a mean-variance scheme, and is approximated by (4.5). We consider different values of A, reflecting different degrees of absolute risk aversion. Using $\delta=1$ and $r=0$, the computation of the Required Capitals Calls is based on (4.6). We obtain lower RCs for larger values of A. The figure also illustrates the convergence of required capitals RC_{t+h} to the valuation $X^*_{t+h,t+H}$ when h goes to H.

5 Concluding Remarks

This paper introduces operational definitions of required capital that are appropriate to address long-term risks, defined as risks that potentially materialize at a given large maturity. We first consider pure long-run risks. Then we extend the approach to a joint computation of reserves for short-run and long-run risks. We discuss the example of the transition to a low-carbon economy.

Even if the focus is on microprudential supervision, the approach provides measures of long-run risks—through the long-run required capital—that can be used for other purposes. They could, in particular, serve as a basis to construct short-run and long-run ratings in the context of the transition to low-carbon economies, or labels for low-carbon responsible funds, green bonds,or sustainability-linked bonds. These applications are beyond the scope of this paper.



Notes: We consider a process of profits P_t described by (4.1) (with $a = -3$ and $b = 1$). The process followed by the long-run component $y_{l,t}$ is based on (4.2); specifically $y_{l,t} = \tilde{y}(t/H)$. The CIR process (4.2) is parameterized with $K = 0.05$, $\eta = 0.45$, and $\theta = 4$. The short-run component $y_{s,t}$ is a Gaussian white noise of standard deviation 0.2. The valuation X^* is based on a mean-variance scheme, and is approximated by (4.5). The computation Required Capital Calls is based on (4.6) (using $r = 0$ and $\delta = 1$). Parameter A is the risk aversion of the mean-variance scheme (equation 2.7).

Figure 4: Simulated required capital RC_{t+h}

A.1 The Limits of the Value-at-Risk for Long-Run Required Capital

Let us briefly review the standard approach for required capital. We keep the same notations as in the main text. That is, X_{t+H} is a (positive) loss, and $-X_{t+H}$ is a gain. The required capital at $t+h$ is chosen to bound the conditional probability of loss at $t+H$. When RC is not remunerated, we have:

$$\mathbb{P}_{t+h}[-X_{t+H}+RC_{t+h}<0]=\alpha, \tag{a.1}$$

where α is the regulatory critical level and $\mathbb{P}_{t+h}$ is the conditional probability given the information available at date $t+h$. Equation (a.1) rewrites:

$$\begin{aligned} & \mathbb{P}_{t+h}[-X_{t+H}+RC_{t+h}>0]=1-\alpha \\ \Leftrightarrow \quad & \mathbb{P}_{t+h}[X_{t+H}<RC_{t+h}]=1-\alpha \\ \Leftrightarrow \quad & RC_{t+h}=q_{t+h}(1-\alpha), \end{aligned} \tag{a.2}$$

where q_{t+h} is the conditional quantile of X_{t+H} at date $t+h$, usually called Value-at-Risk (VaR).

This VaR measure of required capital depends on the following policy instruments: (i) the horizon H; (ii) the updating frequency; (iii) the information available at date $t+h$; and (iv) the specification of the conditional distribution.

Example 1. *Arbitrage portfolio*

Denote by p_t the value of a stock with geometric returns such that:

$$y_{t+1}=\log p_{t+1}-\log p_t=\mu+\sigma u_t,$$

where the u_t's are independent identically normally distributed. This simple dynamics underlies the Black-Scholes approach.

We have:

$$\log p_{t+H}-\log p_{t+h}=\mu(H-h)+\sigma\sqrt{H-h}\,U, \quad \text{where } U\sim N(0,1).$$

Consider a portfolio invested at date t in one unit of the stock and a short sell of p_t in riskfree *asset. This is an arbitrage portfolio with zero value at date t. With zero riskfree rate, we have, at date $t+H$: $X_{t+H} = p_t - p_{t+H}$. We deduce:*

$$
\begin{aligned}
& \mathbb{P}_{t+h}(p_t - p_{t+H} < RC_{t+h}) \\
= \; & \mathbb{P}_{t+h}(p_{t+H}/p_t > 1 - RC_{t+h}/p_t) \\
= \; & \mathbb{P}_{t+h}[\log(p_{t+H}/p_t) > \log(1 - RC_{t+h}/p_t)] \\
& \textit{(note that } RC_{t+h} < p_t\textit{, since the maximum loss is } p_t\textit{).} \\
= \; & \mathbb{P}_{t+h}\left[U > \{\log[(p_t - RC_{t+h})/p_{t+h}] - \mu(H-h)\}/\sigma\sqrt{H-h}\right].
\end{aligned}
$$

For this to be equal to $1-\alpha$, we need to have:

$$
RC_{t+h} = p_t - p_{t+h}\exp\left[\mu(H-h) + q(\alpha)\sigma\sqrt{H-h}\right], \tag{a.3}
$$

where $q(.)$ is the quantile function of the standard normal distribution. In practice α is small, and $q(\alpha)$ is negative.

This example is not purely theoretical, especially when considering the balance sheet of a bank. Indeed, the updating of such balance sheet is largely due to the issuance of new credits. Newly issued credits change the balance sheet by introducing the same value of the initial balance in both asset and liability sides, implying a zero initial value of this portfolio change.

Let us now discuss some drawbacks of this standard approach when applied to long-run risk, that is, if $H-h$ is large.

First, if $H-h$ is large, we get: $RC_{t+h} \simeq -p_{t+h}\exp[\mu(H-h)]$, whenever $\mu > 0$. Thus, the pure risk σ is not taken into account in the definition of the required capital. In other words, in the model of random walk with drift for the price, the pure risk is implicitly assumed diversified in the long run. Moreover the value RC_{t+h} tends to $-\infty$, that means that the investor could profit of this diversification to invest $-RC_{t+h}$ in risky asset.[19]

Second, and symmetrically, if μ is negative, RC_{t+h} tends to p_t when $H-h$ tends to infinity. The standard VaR approach then requires a perfect hedge of the stock at any date to cover the short sell, and this RC is independent of the critical level α.

Third, let us consider how the potential RC (without taking into account the nonnegativity

[19] In fact the current supervision assumes a nonnegative required capital, and then this natural incentive to take more risk does not exist.

constraint) depends on time-to-maturity. Distinguish two cases: (i) for an adverse evolution ($\mu < 0$), RC_{t+h} is an increasing function of time-to-maturity (there is no incentive to invest in long-run risk); (ii) for a positive evolution ($\mu > 0$), RC_{t+h} can be first increasing, then decreasing. Indeed in the short run ($H - h$ small) the volatility dominates the tendency.

Fourth, this approach assumes the knowledge, or at least an approximation, of the conditional distribution of p_{t+H} (or of the returns). This is not realistic for the cases we are interested in, where the associated long-run risks are not traded on liquid organized markets.

All in all, "(whereas) existing modelling instruments allow for a good measurement of market risk, [...] over relatively small time intervals, these (VaR) approaches feature severe deficiencies if they are routinely applied to longer time periods" (Embrechts et al., 2005).

A.2 Transition to Low-Carbon Economy

This appendix introduces a structural dynamic model describing the transition to a low-carbon economy. In the model, firms are led to adjust their production process in response to changes in carbon prices, and we examine the impact of this adjustment on their probability of default. This section concerns productive corporations and does not apply to financial institutions. The main reason is the assumption of a fixed number of inputs, whereas a bank can increase the numbers and types of inputs by increasing the number and type of granted loans, say. That is, the modelling of this section is not appropriate to account for the creation of money by private banks.

This structural analysis suggests, in particular, that a microprudential supervision does not only require the knowledge of financial balance sheets at the firm level, but also technical reports for the production function, including the carbon features. This is consistent with the Greenhouse Gas Emission Reports (GHGRP),[20] which include a carbon balance and are expected to be produced every year.

A.2.1 The Standard Production Function

For the sake of simplicity, we consider a production function g with two inputs and one output:[21]

$$y = g(x_1, x_2), \tag{a.4}$$

where x_1 and x_2 are input quantities and y is the output quantity. Input prices are denoted by $p = (p_1, p_2)$, and π denotes the price of the output. Under standard assumptions, the producer maximizes her profit:

$$\max_{x_1, x_2} \pi g(x_1, x_2) - p_1 x_1 - p_2 x_2, \text{ s.t. } g(x_1, x_2) = y, \tag{a.5}$$

[20]Such GHGRP include all Greenhouse Gas, as methan (H_4), nitrous oxyde ($N_2 0$), hydrofluorocarbon (HCF), perfluorinated hydrocarbons (PFC), sulfur hexafluoride (SF_6), not only CO_2. It can be extended to other environmental aspects as air pollutants, water stress, and various wastes (see, e.g., British Columbia (BC), 2015; European Banking Authority, 2020, Annex 1). For simplicity, we consider only carbon in the present model.

[21]In practice the production process of the firm involves a large numbers of inputs and outputs, leading to a large input-output table (see e.g., Timmer, 2012, at the world level, Wilting and van Oorschot, 2017, at the country level, and Klaassen and Stoll, 2021, for the firm level). Defining precisely a standardized list of inputs/outputs is currently one of the main goals of microprudential supervision.

or, equivalently, minimizes her cost:

$$\min_{x_1,x_2} p_1x_1 + p_2x_2, \text{ s.t. } g(x_1,x_2) = y.$$

If the production function is differentiable, this leads to the first-order conditions:

$$\frac{\partial g(x_1,x_2)}{\partial x_j} = \frac{p_j}{\lambda}, \quad \text{for } j = 1,2, \tag{a.6}$$

where λ is a Lagrange multiplier in the cost minimization problem. Let us denote by $\hat{x}_1$ and $\hat{x}_2$ the solutions of (a.6). The solutions of (a.5) are:

$$\begin{cases} \hat{x}_1, \hat{x}_2 & \text{if } \pi g(\hat{x}_1,\hat{x}_2) - p_1\hat{x}_1 - p_2\hat{x}_2 > 0, \\ \text{no production,} & \text{otherwise.} \end{cases} \tag{a.7}$$

Example 2. *Strict complementary*

If $g(x_1,x_2) = \min(a_1x_1, a_2x_2)$, with $a_1 > 0$ and $a_2 > 0$, then the optimum is:

$$\begin{cases} \hat{x}_1 = y/a_1 \text{ and } \hat{x}_2 = y/a_2 & \text{if } \pi - p_1/a_1 - p_2/a_2 > 0 \\ \text{no production,} & \text{otherwise.} \end{cases}$$

Note that $c_j = 1/a_j$, $j = 1,2$, are technical coefficients usually given in input-output tables: to produce one unit of output, we need c_1 units of input 1 and c_2 units of input 2.

Example 3. *Substitutability*

Substitutable inputs can be represented by a Cobb-Douglas production function:

$$g(x_1,x_2) = Ax_1^{\alpha_1}x_2^{\alpha_2}, \quad \text{with } A > 0,\ \alpha_1 > 0,\ \alpha_2 > 0.$$

The optimum is:

$$\begin{cases} \hat{x}_1 &=& \left(\dfrac{y}{A}\right)^{\dfrac{1}{\alpha_1+\alpha_2}} \left(\dfrac{p_2}{p_1}\dfrac{\alpha_1}{\alpha_2}\right)^{\dfrac{\alpha_2}{\alpha_1+\alpha_2}}, \\ \hat{x}_2 &=& \left(\dfrac{y}{A}\right)^{\dfrac{1}{\alpha_1+\alpha_2}} \left(\dfrac{p_1}{p_2}\dfrac{\alpha_2}{\alpha_1}\right)^{\dfrac{\alpha_1}{\alpha_1+\alpha_2}}. \end{cases}$$

A.2.2 Production function that includes "carbon"

Let us now explain how carbon can be included in the production function.[22] Carbon can be taken into account as an input as well as an output. We obtain a multiple input-output production function:

$$\begin{cases} y &=& f_1(x_1,x_2;z_1), \\ z &=& f_2(x_1,x_2;z_1), \end{cases} \tag{a.8}$$

where z_1 (resp. z) is the quantity of carbon input (resp. carbon output). Let us denote by $-\pi_c$ the price of carbon emissions (the negative sign reflects negative externalities), and by q the price of carbon inputs. For instance, in a macroanalysis, f_1 could be a three-factor Cobb-Douglas function, with capital, labor, and energy as factors (see Keen et al., 2019, for the Energy-Augmented Cobb Douglas Production Function, EACDPF). In an analysis at the firm level, this EACDPF modelling has to be avoided. Indeed, if, at the origin, the carbon is a "public good" with price zero, this modelling with substitutability will lead to an infinite amount of energy to produce any y. As seen below, complementarity has to be introduced.

The profit maximization problem becomes:

$$\begin{aligned} & \max_{x_1,x_2,z_1,z} \pi y - \pi_c z - p_1 x_1 - p_2 x_2 - q z_1, \\ & \text{s.t.} \begin{cases} f_1(x_1,x_2;z_1) &=& y \\ f_2(x_1,x_2;z_1) &=& z \end{cases} \\ \Leftrightarrow \quad & \max_{x_1,x_2,z_1} \pi f_1(x_1,x_2;z_1) - \pi_c f_2(x_1,x_2,z_1) - p_1 x_1 - p_2 x_2 - q z_1 \\ & \text{s.t. } f_1(x_1,x_2;z_1) = y. \end{aligned}$$

This leads to another input allocation: $\{\hat{x}_1(y,p,\pi;\pi_c,q), \hat{x}_2(y,p,\pi;\pi_c,q), \hat{z}_1(y,p,\pi;\pi_c,q)\}$, and to a (potential) profit $P(y,p,\pi;\pi_c,q)$. There is no production if $P(y,p,\pi;\pi_c,q)$ is negative.

[22] This model can be extended to multivariate z and z_1 (see below) to include other GHG, or wastes.

Example 4. *Carbon footprint*

The regulation for low carbon has implicitly selected specific forms of production functions of the type:

$$\left\{ \begin{array}{lcl} y & = & g(x_1,x_2) + \mu \min(z_1, \gamma_1 x_1 + \gamma_2 x_2), \\ z & = & f_2(x_1,x_2;z_1), \end{array} \right. \tag{a.9}$$

where $\mu > 0$, $\gamma_1 > 0$ *and* $\gamma_2 > 0$. *At the optimum, we have:* $z_1 = \gamma_1 x_1 + \gamma_2 x_2$, *and the profit becomes:*

$$\pi g(x_1,x_2) - \pi_c f_2(x_1,x_2;\gamma_1 x_1 + \gamma_2 x_2) + [(\mu\pi - q)\gamma_1 - p_1]x_1 + [(\mu\pi - q)\gamma_2 - p_2]x_2 \tag{a.10}$$

The intuition behind these formulas is the following: the initial inputs x have not been disaggregated enough to account for the carbon used to get them. This approach tries to account for the carbon quality of each input by associating to each of them a so-called carbon-footprint coefficient: γ_1, $\gamma_2 > 0$.

For instance, the 'electricity" footprint of 'depends on its origin (nuclear plants, solar panels, or gas power plant). The "labour" footprint is not the same for a worker or a manager who regularly takes intercontinental flights. Profit formula (a.10) *shows how input prices are adjusted for their carbon component by the terms* $(\mu\pi - q)\gamma_j$. *Even if the cost of input j increases from* p_j *to* $p_j + \gamma_j q$, *the use of this input can be profitable if* $(\mu\pi - q)\gamma_j > p_j$, *even if* $\mu\pi > q$ *only.*

Practical implementation of production functions of the type (a.9) can be found, e.g., in European Banking Authority (2020); Network for Greening the Financial System (2020); Boermans and Galema (2017).

A limitation of this approach is that it necessitates the knowledge of the production function itself, and of the exposure to carbon, either direct—called Scope 1—or indirect—Scope 2 and Scope 3 (see, e.g., European Union, 2019; Klaassen and Stoll, 2021).

A.2.3 Change of technology

The previous derivation does not account for the possibility of changes in the technology to adjust for carbon valuation, that is to reshape the current productive structure. This possibility can be captured by making the technology parameter-dependent. Let us replace (a.8) with:

$$\begin{cases} y &= f_1(x_1, x_2; z_1; \theta), \\ z &= f_2(x_1, x_2; z_1; \theta). \end{cases} \tag{a.11}$$

The optimization is then performed jointly with respect to x_1, x_2, z_1 and θ. This leads to an optimal technology $\hat{\theta}(y, p, \pi; \pi_c, q)$ and an optimal profit $\hat{\hat{P}}(y, p, \pi, \pi_c, q) = \hat{P}\left[y, p, \pi, \pi_c, q; \hat{\theta}(y, p, \pi, \pi_c, q)\right]$, where, by abuse of notation, P still is the profit function (defined in Subsection A.2.2), but for a given θ.

Considering Example 4 of carbon footprint, we see that we can parameterize the technology either through function g, the footprint coefficients (γ_1, γ_2), the function f_2, or the parameter μ. Typically, the firm can change the carbon quality of the inputs—making them more costly—but could also increase its added value by employing a more efficient production process. The firm can also decide to diminish or treat its carbon emissions: this is a costly change in production function f_2, ceteris paribus, that can be compensated by decreasing direct carbon emissions z.

This production function, that includes technology changes, can be given a "portfolio" interpretation. At the optimum the producer has chosen the best combination of inputs x_1, x_2 and technology θ, i.e., the best allocations associated with the (exogenous) prices and demand level y. In this respect new prudential supervision should follow not only the final profit of the firm, but also the changes in these allocations, including changes in technology. This is in line with the recent prudential supervision for hedge funds, that follows not only the hedge funds returns, but also their portfolio allocations.

In what precedes, we assume that prices and demand are exogenous to the firms (prices evolve due to exogenous taxes, or penalties, and the firm does not account for the future evolution of demand). The analysis could be extended to account for a modification of the selected technology due to a change of demand by consumers, that become more sensitive to environmental issues, say. We consider also each firm as an autonomous entity. Thus we do not take into account the chain value and their impact on potential cascades of default. This would necessitate the knowledge of an input-output network (see Cahen-Fourot et al., 2020, 2021, for attempts in this direction). This is out of the scope of this paper.

A.3 Nonlinear Prediction Formulas for the CIR Process

Since it is an affine process, the CIR process admits an exponential affine Laplace transform, that is:

$$\begin{aligned}\psi(v;t,h) &= \log \mathbb{E}_t \left\{ \exp\left[-v \int_t^{t+h} \tilde{y}(u)du \right] \right\} \\ &= C(v;h) - D(v;h)\tilde{y}_t.\end{aligned}$$

Closed-form solutions for functions C and D can be found, e.g., in Hurd and Kuznetsov (2008). Since the Taylor expansion of a log-Laplace transform is:

$$\log \mathbb{E}[\exp(-vZ)] \simeq -v\mathbb{E}(Z) + \frac{v^2}{2}\mathbb{V}arZ,$$

we deduce, using the notations of (4.3) and (4.4), that:

$$\begin{aligned}m_0(h) &= -\frac{\partial C(0;h)}{\partial v}, \quad m_1(h) = \frac{\partial D(0;h)}{\partial v}, \\ \sigma_0(h) &= \frac{\partial^2 C(0;h)}{\partial v^2}, \quad \sigma_1(h) = -\frac{\partial^2 D(0;h)}{\partial v^2}.\end{aligned}$$

Acronyms

ACPR: Autorité de Contrôle Prudentiel et de Résolution

BC: British Columbia

BIS: Bank for International Settlements

DICE: Dynamic Integrated Model of Climate and the Economy

EACDPF: Energy-Augmented Cobb-Douglas Production Function

EBA: European Banking Authority

ES: Expected Shortfall

EU: European Union

ESG: Environmental, Social and Governance

GHG: Greenhouse Gas.

GHGRP: Greenhouse Gas Emission Report

GICS: Global Industry Classification Standard

HCF: Hydrofluorocarbon

HLCCP: High Level Commission on Carbon Prices

HS: Harmonized System

HSBC: Hong-Kong Shanghai Banking Corporation

IAM: Integrated Assessment Model

ICS: Industry Classification System

ILS: Insurance Linked Security

IPCC: Intergovernmental Panel on Climate Change

IR: Investment Responsable (fund)

ISIC: International Standard Industrial Classification

NACE: Nomenclature des Activités Commerciales et Économiques

NGFS: Network for Greening the Financial System

OECD: Organisation for Economic Co-operation and Development

PFC: Perfluorinated Hydrocarbons

RCC: Required Capital Call

SEC: Securities and Exchange Commission

SLB: Sustainability Linked Bonds

SREP: Supervisory Review and Evaluation Process

UIC: Uniform Integrable in Cesaro (sense)

VaR: Value-at-Risk

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