This is the peer reviewed version of the following article: Montez J. V. (2015). Controlling opportunism in vertical contracting when production precedes sales. *The RAND Journal of Economics*, 46(3), 650-670., which has been published in final form at 10.1111/1756-2171.12101. This article may be used for non-commercial purposes in accordance with Wiley Terms and Conditions for Self-Archiving.

# Controlling opportunism in vertical contracting when production precedes sales

João Montez<sup>\*</sup>

#### Abstract

In a make-to-stock vertical contracting setting with private contracts, we show that when retailers do not observe each other's stocks before choosing their prices then in any contract equilibrium the opportunism problem is extreme: a monopolist producer makes no profit. Market-wide price floors and RPM can under some conditions restore monopoly power. However other tools which do not fall under antitrust scrutiny and require only bilateral private contracts already allow the producer to exercise monopoly power in contract equilibria, such as bilateral contracts with buybacks—buybacks are a price paid by the producer to the retailer for each unit of unsold stock. We conclude that a more lenient policy towards RPM is therefore unlikely to have a significant effect on a producer's ability to control opportunism.

<sup>\*</sup>London Business School, Sussex Place Regent's Park, London NW1 4SA UK. E-mail: jmontez@london.edu. I am particularly grateful to the editor Mark Armstrong and two helpful referees, in particular for suggestions of additional policies with similar effects to buybacks. I also thank Matthew Bennett, David Myatt and Christian Wey, Jean-Pierre Benoit, Hans-Theo Normann, Patrick Rey and Mike Whinston, and participants at several seminars and conferences.

### 1 Introduction

Over recent years there has been great interest from economists and lawyers about the appropriate treatment under competition law of price floors and Resale Price Maintenance (RPM). In the US this interest was created by the Supreme Court's ruling in 2007 on the Leegin case based on reasonableness, which overruled the century-old precedent of Dr. Miles that had made RPM per se illegal.<sup>1</sup> Yet the per se illegal status of RPM remains in states' laws and there exists considerable uncertainty about its future status at the federal level as well.

In the EU the interest was motivated by the Commission's review of its guidelines on vertical agreements. Both the Commission's most recent Vertical Restraints Block Exemption Regulation and its Guidelines on Vertical Restraints unambiguously characterize RPM arrangements as hard-core restrictions.<sup>2</sup>

One of the main objections that has been raised against a more lenient policy towards price floors and RPM is that they may be used to control producer opportunism and thus help to restore monopoly power.<sup>3</sup> The main objective of this paper is to formally investigate how a more lenient policy towards price floors and RPM affects the extent to which a producer can control opportunism in industries where bilateral contracts are private and distribution is made on a make-to-stock basis, i.e. stocks are first produced and then distributed to retailers who make them readily available to consumers—e.g. foods, stationary, and apparel.

We first show that, when retailers do not observe each other's stocks before choosing their prices, in any contract equilibrium where the retailer and the producer agree on a quantity and transfer, the effect of opportunism is drastic: profits are driven down to zero. This is a new result, since previous studies have focused on situations where retailers

<sup>&</sup>lt;sup>1</sup>Leegin Creative Leather Prods., Inc. v. PSKS, Inc., 551 U.S. 877 (2007), overruling Dr. Miles Med. Co. v. John D. Park & Sons, 220 U.S. (1911).

 $<sup>^{2}</sup>$ See, in particular, the reference to "fixed or minimum sale price," at Article 4(a) of the VRBER. See further Guidelines on Vertical Restraints, at paragraph 48 and 223.

<sup>&</sup>lt;sup>3</sup>See for example "Hardcore restrictions under the Block Exemption Regulation on vertical agreements: an economic view" by the European Commission's advisory group EAGCP (2009). The proceedings from OCDE's Roundtable on Resale Price Maintenance 2008 suggest that this concern is more prevalent in Europe than in the US, possibly due to the different mandates of the competition authorities.

observe each other's stock. Unlike here, in those situations it is possible to use the stock choices as a precommitment to soften competition and partially protect industry margins.

We also find that industry-wide price floors and RPM can under some conditions be used to sustain the monopoly outcome in a contract equilibrium. Yet, as discussed above, both tolls have been illegal or seen with considerable suspicion by antitrust authorities. It would then not be surprising to find that producers have been using alternative ways to control opportunism. If this is the case, we cannot conclude that a more lenient policy towards RPM would have a significant effect on reducing the extent of producer opportunism.

Indeed we show that other instruments, which at present do not attract antitrust scrutiny but are widely used in practice, can be as effective as RPM in eliminating opportunism. Such tools rely only on bilateral contracts, which are easier to agree on and to enforce than market-wide price controls.

In particular we focus on buybacks, a standard practice where the producer receives unsold goods from its retailers and pays them an agreed price for each returned unit. It will however become clear from the intuition outlined below that other policies can be strategically equivalent to buybacks and therefore have similar effects. This is for example the case of properly specified contracts with per unit royalties or consignment agreements where the producer delegates the pricing decisions to retailers—where the retailer and producer agree on a quantity to be delivered, retailers are charged one price per unit sold and a lower price per unit not sold, and the difference between the two is equal to our return price.<sup>4</sup>

We conclude from this that, in the absence of RPM or market-wide price floors, exercising monopoly power when contracts are private is likely to be less of a challenge than previously thought. Therefore we expect that a more lenient policy towards RPM is unlikely to have a significant effect on opportunism.

<sup>&</sup>lt;sup>4</sup>Consignment is an arrangement in which a producer sends the stock to a retailer, retaining its ownership and only receiving a payment once the sale is done. Typically the producer also retains the controls of prices and it has therefore been used as a sham to conceal RPM. Yet such contractual arrangements do not immunize the parties from the price-fixing prohibitions of the Sherman Act—a little known fact among economists is that Dr. Miles landmark case concerned consignment and not RPM per se (see e.g. Peritz, 2007).

The current understanding of RPM has been largely influenced by what is now known as the Chicago Critique. It challenged a then long held view that an upstream monopolist would use vertical restraints to leverage its market power downstream. The argument was that there is a single monopoly rent in a vertical chain and it can be extracted with nonlinear contracts. It favoured instead a view where vertical restraints serve efficiency motives (e.g. Spengler, 1950, Bork, 1954 and Mathewson and Winter, 1984). For example, price floors can encourage retailers to offer valuable advice which might not be offered otherwise because of free-riding among retailers (e.g. Telser, 1960 and Mathewson and Winter, 1984).

Several authors have since argued that if the monopolist producer cannot commit to a set of public contracts (private contracting) then even with non-linear contracts it may fail to extract the monopoly rent (e.g. Hart and Tirole, 1990, O'Brien and Shaffer, 1992, McAfee and Schwartz, 1994, Rey and Verge, 2004 and, for a recent survey, Rey and Tirole, 2007). The reason is that the studied set of contracts that coordinate the vertical chain leave the producer an incentive to offer better terms to some retailers as it fails to internalize the effect this has on the profits of the other retailers. Moreover such contracting externalities are thought to be more severe when the number of retailers is large, predicting that in those situations a producer's equilibrium profit should even be negligible—what is known as "competitive convergence" (Segal and Whinston, 2003).

Direct price controls can help the producer restore its monopoly power. Indeed, if retailers were to purchase from the producer only once a consumer places an order with them—i.e. make-to-order—the opportunism problem can be solved directly with a marketwide price floor, or indirectly (by eliminating the rent-shifting incentives) with contracts consisting of a price ceiling at the monopoly price level and wholesale prices that squeeze the retail margins (O'Brien and Shaffer, 1992). Yet squeezing retail margins is ineffective if some investment from the retailers is required (Gabrielson and Johansen, 2013).

Moreover in practice most consumer goods are sold on a make-to-stock basis. We show that in these situations vertical price controls can also help to solve the opportunism problem (but may not be sufficient). It has been suggested in a survey, but not yet formally studied, that in this case a "market-wide resale price maintenance, in the form of a price floor, together with a return option would obviously solve the commitment problem" (Rey and Tirole, 2007). For some specifications of demand rationing rules we find that marketwide RPM is sufficient, while for other specifications it is not. Yet in general combining a market-wide price floor with a return option is not optimal since it can leave retailers with a double marginalization incentive—i.e. induce retailers to set their prices above the monopoly level.

The main message of this paper is that, instead of using a market-wide price floor or RPM, in make-to-stock settings the producer can use simple bilateral contracts with buybacks (complemented with a bilateral price ceiling only if needed) to eliminate opportunism and captures the monopoly profit in a contract equilibrium.

Importantly, since in practice price controls can be hard to monitor and enforce, we also show that frequently private contracts with buybacks alone are sufficient to restore monopoly power. In our model this is the case if demand is sufficiently elastic at the monopoly price—i.e., the cost of production is sufficiently high—or if the number of retailers is sufficiently high. In general this is the case if the monopoly percentage mark-up does not exceed 100%, and, more specifically, with efficient rationing when the monopoly percentage mark-up does not exceed the number of retailers—if the monopoly margin is less than 200% with two retailers and less than 500% with five retailers. In practice such large margins would be uncommon.

The latter insight has important implications to antitrust since it challenges the widely held notion of "competitive convergence". Contrary to previous belief, we find that if the bilateral contract space is sufficiently rich then it is in those situations where the retail sector is sufficiently competitive that the producer can more easily capture the monopoly rent in the absence of vertical price controls—what could be termed "monopoly convergence".

The intuition behind this result is the following. Consider a market where an upstream monopolist, producing at a constant marginal cost c, offers individual contracts specifying a fee, a quantity and a buyback price r to each of n retailers—perceived as perfect substitutes by consumers and with no distribution costs. Each retailer can then accept or reject its contract and set its retail price. Let  $p^m$ ,  $q^m$  and  $\pi^m$  denote respectively the monopoly price, quantity and revenue.

Suppose first that r = 0 and the producer offers each retailer the quantity  $q^m/n$  for a fee equal to  $\pi^m/n$ . If offers were final, i.e. if the producer could commit to those contracts,

each retailer should accept its contract and set its retail price at the monopoly price level the producer would then extract the full monopoly rent. However in that case the producer can benefit from secretly offering a slightly higher quantity to retailer *i*—which *i* can sell at a profit by setting its price just below  $p^m$ —at the expense of the sales and profits of the other retailers. Anticipating this scope for opportunistic behavior, if contracts are private the retailers should reject the initial contracts—and the producer would then fail to extract the monopoly rent.

Suppose now that the producer adds a buyback price  $r = p^m$  to the initial contracts. In this case the producer and retailer *i* can no longer (jointly) benefit from overselling because it is now too costly to leave the remaining retailers with unsold stock. However, if retailer *i* holds  $q^m/n$ , and the remaining retailers are selling  $(n-1)q^m/n$  at  $p^m$ , *i* becomes a monopolist on the residual demand curve with an effective marginal cost of  $r = p^m$ . This creates a double marginalization incentive since retailer *i*'s optimal price will now exceed  $p^m$ —and the unsold portion of *i*'s stock needs to be reimbursed by the producer. For *i* to set its retail price at  $p^m$  the buyback price needs to be below some level  $\bar{r} < p^m$ , which can open again the door to opportunism—since the producer may then want to sell an additional unit to a retailer *j* if  $c + r < p^m$ , i.e. its total marginal cost is less than what *j* can get by selling an additional unit.

The producer can control this problem in two ways. One, is to set  $r = \overline{r}$  if that level is sufficiently high to remove the producer's incentive to oversell but still sufficiently low to avoid double marginalization. This is the case if the elasticity of the residual demand at the monopoly price is not too low—or, equivalently, if the cost of production is not too low. A second alternative is to set  $r \in [p^m - c, p^m]$  and prevent the double marginalization directly with an individual price ceiling equal to  $p^m$ . Unlike in a make-to-order setting, notice that the latter does not require eliminating the retailers' quasi-rents, which may prove essential to provide retailers with incentives.

This paper presents the first formal study of the role of buybacks in the context of producer opportunism. Our main contribution to that literature (as discussed above) is to show that in make-to-stock settings buybacks can be an extremely powerful tool to eliminate producer opportunism, in particular when complemented with price ceilings. Our paper therefore contributes to the literature on vertical restraints, and more specifically to the ongoing antitrust debate on the legal status of price floors and RPM.

There is in addition a large literature in economics and marketing on the use of buybacks by an upstream monopolist. That literature has two standard features: i) the producer commits to a set of public contracts and ii) there is demand uncertainty. Our paper differs from the previous literature on buybacks as it does not share either of these features. First, the problem of producer opportunism we study here is present precisely because the producer cannot commit to the contracts it offers to retailers, i.e., contracts are not public. Second, buybacks play a central role in our model even when there is no demand uncertainty (although we also show that they remain useful in controlling opportunism in the presence of uncertainty).

In that literature returns can transfer risk from a retailer to the producer (e.g. Kandel, 1996, Marvel and Peck, 1995). With a retail monopoly, a return policy and a linear wholesale price can also coordinate a supply chain with RPM (Pasternack, 1985) or improve profits over outright sales in the absence of price restraints (Marvel and Peck, 1995).

When retailing is perfectly competitive, buybacks can lead to optimal levels of inventory and price dispersion (Marvel and Wang, 2007). They may also be used as a substitute for a price floor and, by inhibiting the price cutting that would otherwise occur when demand is low, lead retailers to hold larger inventories (Deneckere, Marvel and Peck, 1996). This logic also extends to a retail oligopoly (Butz, 1997), and complementing two-part tariffs with buybacks can coordinate a supply chain under certain conditions—for example when there is substantial uncertainty over the level of demand (Narayanan et al., 2005) or if in addition there is uncertainty over consumers' preferences for differentiated retailers (Krishnan and Winter, 2007).

A general theme of the latter work is that with public contracting the producer commits to buy the unsold stock to create a de facto price floor, which corrects the retailers' incentives (horizontal pricing externality) and eventually encourages retailers to increase their stocks in equilibrium. In the present model with private contracts, the buyback price is instead used to correct the producer's incentives (to correct a contracting externality and substitute commitment) with the objective of reducing the equilibrium stocks levels.

There is also some work in the context of asymmetric information on demand. For example, returns may be used by the producer to signal to retailers private information on demand (Kandel, 1996), or they can be used to elicit private information on demand from retailers and provide them with incentives to acquire such information (Arya and Mittendorf, 2004 and Taylor and Xiao, 2009).

The rest of the paper is organized as follows. In section 2 we present the model and benchmarks. In section 3 we study the case of private contracting with buyback contracts alone and in section 4 we study their interaction with vertical price restraints. We conclude in section 5. All proofs can be found in the appendix.

#### 2 Model

An upstream monopoly producer, u, can produce any quantity at a constant marginal cost c > 0. It sells to a set of undifferentiated retailers,  $N = \{1, ..., n\}$  with  $n \ge 2$ , who then sell in the downstream market at no additional cost. The set of all players is  $M = N \cup \{u\}$ .

The downstream market is characterized by demand D(p), with D'(.) < 0 and an (absolute value) elasticity  $\varepsilon(p)$ . Demand is well-behaved, in the sense that profits are strictly concave in the relevant ranges, and so the industry profit  $\pi(p) = D(p)(p-c)$  is maximized with the monopoly price  $p^m$  satisfying the Lerner index, i.e.  $p^m - c = p^m / \varepsilon(p^m)$ , and selling  $q^m = D(p^m)$ .

A bilateral *buyback contract*  $k_i$  is a transfer  $t_i$ , a quantity  $x_i$  and a buyback price  $r_i$  paid by the producer to the retailer for each unit of unsold stock. Like in Hart and Tirole (1990), we do not need to impose particular restrictions on these contracts beyond the fact that each contract cannot condition on the contracts signed with the other retailers—in general  $k_i$  can be thought as the contract chosen by i from a general menu of contracts.<sup>5</sup>

The following timing captures a *make-to-stock* interaction typical in retailing:<sup>6</sup>

**Stage 1.** The producer offers a contract  $k_i$  to each retailer *i*.

**Stage 2.** If retailer *i* accepts contract  $k_i$ , it receives  $x_i$ , pays  $t_i$  and decides its price  $p_i$ .

<sup>&</sup>lt;sup>5</sup>The monopoly outcome can of course be achieved with contracts that condition directly on the total joint stock, with large penalties if it exceeds the monopoly quantity. Like previous authors, we view such contracts as hard to enforce and likely to violate competition laws.

<sup>&</sup>lt;sup>6</sup>Make-to-stock is more efficient than make-to-order when shipping costs are high or consumers are sufficiently impatient relative to production and delivery time—e.g. most goods sold in supermarkets.

**Stage 3.** Consumers observe prices and make purchases. The producer pays  $r_i$  to retailer i per unit of its unsold stock.

The outcome of the game depends crucially on whether retailers observe their rivals' contracts before deciding to accept or reject contracts and set prices. We say that *contracts are public* if K is observed in stage 1 and that *contracts are private* if they remain unobservable throughout the game.

A pure strategy for the producer is a  $n \times 3$  matrix K, where each row is a contract  $k_i = (t_i, x_i, r_i)$ , with  $t_i \ge 0$ ,  $x_i \ge 0$  and  $r_i \ge 0$ . T, X and R denote the respective columns of K. A pure strategy for retailer i is a set of acceptable contracts and a pricing rule. In the case of public contracts it is  $w_i(K) = (a_i, p_i)$  and in the case of private contracts  $w_i(k_i) = (a_i, p_i)$ , with  $a_i \in \{0, 1\}$  and where 0 denotes a rejection, 1 an acceptance, and  $p_i \ge 0$  the retail price. W is the matrix with rows  $w_i$  and A and P are the respective columns of W.

Note that retailers are capacity constrained by their stocks when they choose prices. Therefore the quantity consumers purchase from retailer i,  $z_i$ , must be no more than the least of his stock  $x_i$  or the demand directed to i. To specify the demand directed at each retailer we consider a family of demand rationing rules introduced by Davidson and Deneckere (1986). We denote the residual market demand at price p by

$$\gamma(X, W(K), \lambda, p) = D(p) - \theta(W, \lambda, p) \sum_{j \in J} a_j x_j$$

where J is the set of retailers that sell a positive amount by charging a price strictly below  $p, p_J$  is the highest price charged by a retailer in J, and

$$\theta(W, \lambda, p) = \lambda + (1 - \lambda)D(p)/D(p_J)$$
 with  $\lambda \in [0, 1]$ .

The extreme cases,  $\lambda = 1$  and  $\lambda = 0$ , correspond respectively to the two most common modeling rules of efficient and random rationing.<sup>7</sup> If the market demand results from the summation of individual heterogenous unit demands of a continuum of consumers,

<sup>&</sup>lt;sup>7</sup>Efficient rationing was first used by Levitan and Shubik (1972), and has since appeared, amongst others, in Kreps and Scheinkman (1983), Osborne and Pitchik (1986) and Deneckere and Kovenock (1992). Random rationing was used for example by Beckmann (1965), Allen and Hellwig (1986) and Davidson and Deneckere (1986). A similar specification can be found in Tasnadi (1999).

the parameter  $\lambda$  can then be interpreted as a measure of correlation between consumers' valuations and the time at which consumers learn about prices and make purchase decisions.

Random or proportional rationing ( $\lambda = 0$ ) captures a situation in which the order of purchases is random, and in this case a residual demand corresponds to a clockwise rotation of the original inverse demand around the choke price. *Efficient* or parallel rationing ( $\lambda = 1$ ) captures a situation in which there is perfect correlation between consumers' valuations and the order in which they make purchase decisions, and in this case a residual demand corresponds to a parallel inward shift of the original inverse demand. For  $\lambda \in (0, 1)$  the high valuation consumers are more likely to purchase before low valuation consumer and there is both a shift and a rotation.<sup>8</sup> We expect  $\lambda > 0$  since consumers with higher valuations have a higher incentive to learn about prices and purchase earlier than consumers with lower valuations.<sup>9</sup>

To derive the demand directed to each retailer *i*, denoted by  $\beta_i(X, W(K), \lambda)$ , we should yet consider that either retailer *i* is the only retailer charging some price  $p_i$ , in which case he receives all the residual demand  $\gamma$  at that price, or if there are several retailers charging the same price then that residual demand  $\gamma$  needs to be shared. There are at least two common ways to do that sharing: one is what we refer to as "same price fair-share" rationing (e.g. Davidson and Deneckere, 1986 and Kreps and Scheinkman, 1983), the second one is what we refer to as "same price stock-share" rationing (e.g. Rey and Tirole, 2007). We will be agnostic on this particular choice as it does not have implications to our main results. It will have an effect when the producer is allowed to use a market-wide price floor. Given the notational complexity we delay the formal description of these options until we discuss price floors in section 4.

Letting  $x^+ = \max\{0, x\}$ , we have, for either same price sharing rule, that the quantity purchased from retailer *i* is the lowest of his stock and the residual demand directed at him

 $<sup>{}^{8}\</sup>lambda$  can also capture the efficiency of a resale market where consumers can trade among themselves. In particular, if there is a perfectly efficient secondary market the equilibrium demand directed at a higher priced firm is captured by efficient rationing independently of the order of purchase decisions (Perry, 1984).

<sup>&</sup>lt;sup>9</sup>Because a consumer with a higher valuation gains more when moving from a situation where no stock is available to a situation where some stock is still available, and it also gains more from a reduction in the price at which some stock remains available.

(conditional on the remaining prices and stocks) which is given by

$$z_i(X, W(K), \lambda) = \min \left\{ a_i x_i, \beta_i(X, W(K), \lambda) \right\}.$$

The producer and retailer *i*'s profits are respectively  $\pi_u(K, W(K), \lambda)$  and  $\pi_i(K, W(K), \lambda)$ . In the remainder of the paper we drop the notational dependence of  $\pi_u$ ,  $\pi_i$  and  $z_i$  when this causes no ambiguity. The profits are then

$$\pi_u = \sum_{i \in N} a_i \left[ t_i - cx_i - r_i (x_i - z_i) \right]$$
(1)

and

$$\pi_i = a_i [p_i z_i + r_i (x_i - z_i) - t_i].$$

 $S_i$  is the set of pure strategies of player *i* and, with  $S = (S_i)_{i \in M}$ ,  $G = (M, S, (\pi_i)_{i \in M})$  denotes the present game.

Note that once a retailer accepts a contract, the buyback price  $r_i$  becomes retailer *i*'s effective marginal cost up to  $x_i$ , i.e. the opportunity cost of a unit of stock, since

$$\pi_i = (p_i - r_i)z_i + (r_i x_i - t_i)$$

and only the first term varies with  $p_i$ . As we will show later, for this reason a too high buyback price creates incentives for double marginalization.

This observation also tells us that in theory other distributions policies can be strategically equivalent to buybacks. This requires that a quantity of  $x_i$  is initially delivered, that retailers pay a large price per unit sold and a lower price per unit not sold, and that retailers remain free to set prices. Two such policies are consignment and per unit royalties. In consignment the price paid per unit sold would be equal to  $r_i$ , 0 per unit not sold, and fixed fee  $t_i - r_i q_i$  paid up-front. In the case of royalties a price of  $t_i/x_i$  per unit sold,  $t_i/x_i - r_i$  per unit not sold, and no fixed fee.

In practice important differences can persist between these different tools. For example, with consignment and royalties the producer delays a significant part of the payment until stocks are sold and therefore, to avoid cash flow problems, he should have enough liquidity in hand as he may need to wait for extended periods until a payment is made—while with buybacks the payment is made on delivery. Also with consignment the producer retains the ownership of the stocks until they are sold. Therefore he carries risks which he cannot

control, such as damage and consumer abuse which stock merchandise is generally subject to—this is not the case with buybacks since ownership is transferred to the retailer.

#### 2.1 Solution concept and benchmarks

When contracts are public we can use the concept of subgame perfect equilibrium. It is well known that with non-linear contracts the producer can extract the full monopoly profit despite the externalities present here (see e.g. Segal, 1999). The producer chooses a set of contracts K that satisfies the retailers' participation and incentive constraints—since retailers' non-cooperative pricing decisions should form a Nash equilibrium in the capacity constrained pricing game. It is stated without proof that when contracts are observable the producer can extract the monopoly profit in a subgame perfect equilibrium with contracts that satisfy

$$\sum_{i \in N} x_i = q^m, t_i = x_i p^m \text{ and } r_i = 0.$$

So in the case of observable contracts the producer has no use for buybacks.

Characterizing the outcome of the private contracting game is more problematic. Such an incomplete information game naturally admits a multitude of perfect Bayesian equilibria because of the latitude given to players in revising their off-the-equilibrium path beliefs.<sup>10</sup> Several authors have studied this problem by requiring strategies to be renegotiationproof—either directly or indirectly (e.g. Hart and Tirole , 1990, O'Brien and Shaffer, 1992, and McAfee and Schwartz, 1994). Here we avoid the complex issue of specifying off-the-equilibrium beliefs by adopting the simpler concept of contract equilibrium from O'Brien and Shaffer (1992), first introduced by Cremer and Riordan (1987):<sup>11</sup>

**Definition 1.** A pure strategy contract equilibrium (with private contracts) is a matrix of contracts  $K^*$ , and Nash equilibrium responses induced by these contracts,  $W^*$ , such that

<sup>&</sup>lt;sup>10</sup>For example, the vertical integrated outcome can be implemented as a perfect Bayesian equilibrium if retailers conjecture that the equilibrium contracts are the ones from the game with public contracts and respond to any deviation with extreme pessimistic beliefs about the remaining contracts—and therefore reject any deviating contract. As previously argued in the literature, such conjectures seem implausible (see e.g. Hart and Tirole, 1990).

<sup>&</sup>lt;sup>11</sup>We come back to this issue in more detail at the end of the next section.

for all  $i \in N$ ,  $k_i^*$  induces retailer *i* to maximize the bilateral profits  $\pi_u + \pi_i$  of the supplier and retailer *i*, taking  $(K_{-i}^*, W_{-i}^*)$  as given.

Here a contract equilibrium requires that on equilibrium each contract is optimal for any pair formed by the producer and a retailer, holding the contracts of the other retailers and their responses fixed—because they cannot react to deviations they do not observe. In addition it also requires that retailers' price choices are consistent with Nash. A contract equilibrium is symmetric if there exists an equilibrium with a contract k = (t, x, r) that is offered to all retailers, who accept it and set the same retail price.

Our first step is to study contract equilibria in the absence of buybacks, i.e. when  $r_i = 0$  for all  $i \in N$ . The result is novel and the outcome we find more drastic than in Hart and Tirole (1990): when stocks remain unobserved, two retailers are sufficient to drive industry profits to zero.

**Proposition 1.** In the absence of buybacks, there exist contract equilibria (with private contracts). They involve mixed strategies and each player makes zero profit.

The result follows from the next argument. In the absence of buybacks, for any set of proposed contracts the producer's payoff is

$$\sum_{i \in N} a_i \left[ t_i - c x_i \right].$$

Like Hart and Tirole (1990) noted, from the producer's perspective each retailer then forms an independent market. So in any contract equilibrium the producer sells each retailer i a quantity  $x_i$  that retailer i would himself pick if it was one of n vertically integrated firms with cost c, and i sets  $p_i$  accordingly. For this reason, to a contract equilibrium of our game there is a Nash equilibrium of an associated game where n firms, each producing at cost c, choose their prices and quantities simultaneously. In every equilibrium of that game competition eliminates the industry profits (Gertner, 1985), since in a situation where retailers are unable to observe other retailers' stocks before choosing prices there is no capacity precommitment that could be used to alleviate price competition. So here the producer and retailers also make zero profits—unlike in Hart and Tirole's (1990) model where retailers observe each other's quantities or stocks before choosing prices.<sup>12</sup>

<sup>&</sup>lt;sup>12</sup>The equilibrium contracts are of the form  $k_i = (cx_i, x_i, 0)$ , but X is a random vector. In equilibrium

In some situations it may be realistic to assume that capacity choices are observed before prices are chosen, like in Kreps and Scheinkmans' (1983) analysis of a situation where manufacturers make long-term capacity choices, such as the size of a plant—by their nature these investments can be either immediately observed or are likely to be learned by competitors over time.

However in retailing stocks are hard to observe, requires private warehouse information and—given the transient nature of stocks—historical data would not provide an exact estimate of current stocks levels. Thus it seems more realistic to assume, as we do here, that each retailer cannot observe the stocks and buyback prices of all other retailers before choosing its price.<sup>13</sup>

To finalize the section we comment on the use of contract equilibrium in the present context. As mentioned above, we adopted this solution concept to obtain what we think are reasonable predictions of a game that admits a multitude of PBE because of the latitude it gives to players in revising their off-the-equilibrium path beliefs. While the concept of contract equilibrium focuses on bilateral deviations only, a PBE must in addition resist multilateral deviations.

Several authors have studied PBE with a basic belief system, passive beliefs, in which each retailer following any deviation by the producer believes that what it does not observe remains unchanged. As Rey and Verge (2004) noted, a PBE with passive beliefs is always a contract equilibrium, but the reverse is not true. However a contract equilibrium that is not a PBE under passive beliefs may still be a PBE with a different set of beliefs.

The contract equilibria derived in Proposition 1 above are PBE with passive beliefs. However those derived below in the analysis with buybacks in Proposition 2 may not be since, by removing the buybacks from the contracts, the producer can find profitable retailer *i* chooses a price above *c* anticipating that with some probability it may not sell all its stock  $x_i$  but it has no incentive to change its price given its stock—and its expectation about rivals' stocks and prices, which is correct in equilibrium.

<sup>13</sup>Assuming that stocks are observable also makes the treatment of buybacks more complex since then we would have a model of capacity constrained price competition with heterogenous retailing costs (or here buyback prices), and which for that exact reason have been rarely used—for an exception with just two retailers and efficient rationing see Deneckere and Kovenock (1994). multilateral deviations.

In our view this is reveals a severe limitation of passive beliefs in the presence of buybacks. As we will see, in the proposed equilibria, buybacks have no actual cost to the producer and thus would send a costless "signal" to the retailers that the producer is not behaving opportunistically. Therefore the absence of buybacks should not be meet passively by retailers, but instead trigger extreme caution. Unfortunately, given the dimensionality of the contract space, we were unable to formalized this idea in the context of PBE.

#### **3** Private contracting with buybacks

When the retailer complements the contracts with buybacks we find:

**Proposition 2**. The producer can extract the monopoly profit in a contract equilibrium with private buyback contracts if the monopoly percentage markup is not too large, *i.e.* 

$$\frac{p^m - c}{c} \le 1 + \lambda(n - 1),\tag{2}$$

or equivalently the demand elasticity at the monopoly price is not too low, i.e.

$$\varepsilon(p^m) \ge 1 + \frac{1}{1 + \lambda(n-1)}.$$

The result follows essentially from two constraints. Consider a symmetric contract equilibrium where each retailer is offered a contract  $k = (t^*, x^*, r)$ . The first constraint deals with *opportunism*, i.e. the producer's temptation to sell a retailer *i* more than  $x^* = q^m/n$ while expecting *i* to sell all its units by setting its price just below  $p^m$ . If the producer sells an additional unit to *i* (which costs *c* to produce), retailer *i* will find it optimal to just undercut its rivals (so the total quantity remains unchanged and therefore the other retailers end up with an unsold unit in total, which costs *r* to compensate). To prevent such deviations the total cost of selling one additional unit to *i*, c + r, must then be higher than the marginal benefit *i* gets with that additional unit by setting its price arbitrarily close to  $p^m$ , i.e.

$$p^m \le c + r$$

A second one deals with *double-marginalization*. When the remaining retailers are selling  $(n-1)q^m/n$  at a price  $p^m$ , retailer *i* is a monopolist on the residual demand curve with an effective marginal cost of *r*. If *r* is too close to  $p^m$  the retailer *i* finds it optimal to set its price above  $p^m$  and sell only a fraction of its stock. This incentive creates an upper bound on *r* which is

$$r \le \overline{r}(n,\lambda) \equiv p^m (1 - \frac{1}{\varepsilon(p^m)(1 + \lambda(n-1))}).$$
(3)

These two constraints push r in opposite directions. By replacing  $\overline{r}(n, \lambda)$  in the first constraint we obtain a single condition that determines the limit of buybacks in controlling opportunism in a symmetric contract equilibrium—which is simplified to (2) with the Lerner index.<sup>14</sup>

Notice that  $\overline{r}(n, \lambda)$  increases with  $\lambda$ , and therefore condition (2) becomes less stringent as  $\lambda$  increases. The reason is that if some retailer *i* sets  $p_i$  above  $p^m$  its residual demand is a sample of the original demand, but this sample becomes more biased towards low valuation consumers as  $\lambda$  increases. Therefore as  $\lambda$  increases the residual demand of a retailer with a higher price becomes more elastic (relative to the original demand), which relaxes the constraint associated to double marginalization and allows *r* to be increased to correct the opportunism incentive.

This result suggests that even with private contracts there are many situations where bilateral buyback contracts (and by extension consignment and royalties) allow the producer to retain its monopoly power. In the limiting and least favorable case of random rationing ( $\lambda = 0$ ) it requires that  $p^m \leq 2c$ , or equivalently  $\varepsilon(p^m) \geq 2$ . Empirical estimates suggests that this is the case of many branded consumer goods, such as breakfast cereals, beer and sodas (see e.g. Nevo, 2001, Pinkse and Slade, 2004, and Dhar et al., 2005). In the most favorable case of efficient rationing ( $\lambda = 1$ ) the producer can extract the monopoly rent with buyback contracts alone if the number of retailers is larger than the monopoly percentage markup, i.e.

$$\frac{p^m - c}{c} \le n.$$

<sup>&</sup>lt;sup>14</sup>There is an additional constraint that is trivially satisfied: a retailer and the producer should not find it optimal to agree to sell less than  $q^m/n$  at a price above  $p^m$ .

This is less than 200% with two retailers and less than 500% with five. In practice margins of this magnitude are uncommon.

It is also interesting to note that if the conditions of Proposition 2 are verified, and if demand is log-concave, then a similar result holds if there is a probability z that demand vanishes (i.e. is equal to zero).<sup>15</sup> The reason is that the monopoly price is then higher than in the case without uncertainty. Since a log-concave demand is more elastic at higher prices, so the incentive for double marginalization decreases and the return price can be further increased (relative to a situation without uncertainty). Further, the opportunism constraint (no bilateral deviation) is then  $(1-z)p^m \leq c + (1-z)\overline{r}$ , and it is satisfied under the conditions of Proposition 2 if  $p^m - \overline{r}$  decreases with the increase in  $p^m$  ( $\overline{r}$  also depends on  $p^m$ ). It can be shown that this is always the case when demand is log-concave.

A similar logic also explains why (2) is relaxed as n increases. For  $\lambda > 0$ , when the remaining retailers are selling  $(n-1)q^m/n$  at  $p^m$ , if i sets  $p_i$  above  $p^m$  its residual demand is biased towards low valuation buyers and it is therefore more elastic than the original demand. Moreover that elasticity increases as the share of the monopoly output sold by the remaining retailers increases. So double marginalization is easier to control when n is large, meaning that the return price can then be increased to eliminate opportunism.

We view this corollary as an important result from an antitrust perspective, since it questions "competitive convergence", the widely held notion that when the producer cannot commit to its contracts then the total industry profit becomes arbitrarily small when the number of retailers is large (e.g. Segal and Whinston, 2003). To the contrary, this result suggests that if the bilateral contracting space is realistically enlarged then it is precisely in situations with many retailers that the producer should be able to obtain the monopoly rent with private bilateral contracts only. We have what may be called "monopoly convergence":

**Corollary 1.** For all  $\lambda > 0$  and c > 0 there exists a finite  $\underline{n}(c, \lambda)$  such that the producer extracts the monopoly profit with private buyback contracts in a contract equilibrium if  $n \ge \underline{n}(c, \lambda)$ . Moreover the critical  $\underline{n}(c, \lambda)$  decreases with  $\lambda$  and c.

To summarize, the analysis of this section suggests that buyback contracts alone (and  $^{15}$ Log-concavity of demand, i.e.  $p/\varepsilon(p)$  decreasing, is sufficient to ensure that the monopoly problem can be solved with first order conditions and is for example verified if demand is concave.

by extension consignment and royalties contracts) are more likely to be sufficient to control producer opportunism in make-to-stock markets with the following characteristics: market demand is relatively elastic, the marginal cost of production is not too low, consumers can trade in secondary markets and the number of retailers is high.

What happens when the conditions of Proposition 2 are not satisfied? We may have thought, because producer opportunism cannot be completely eliminated, that the equilibrium price would always be below the monopoly level. However this is not always the case. In appendix B we show that, if the market demand is log-concave and the monopoly outcome cannot be sustained in a contract equilibrium, then the market price in symmetric pure strategy contract equilibria exceeds the monopoly level.

The reason is that when demand is log-concave the elasticity of the retailers' residual demand is high when the market price is also high, and vice versa. So the incentive for double marginalization is low when the market price is high, and high when that price is low. If the market equilibrium price is high, the producer can offer a high buyback price to control opportunism without inducing double marginalization, thereby sustaining that high price as an equilibrium. This may not work for prices below the monopoly level since in those cases the incentive to price above the equilibrium level can be too high—and hence such a price will not be sustained in a symmetric equilibrium.

### 4 Buybacks and vertical price restraints

RPM is an agreement between the producer and the retailers that limits the prices the retailers can charge. With *strict RPM* the market price is fixed. We can also have a *price floor* (an agreement in which the retailer promises not to sell the product for less than a set minimum price), or a *price ceiling* (an agreement in which a retailer promises not to sell the product for more than a set maximum price). These agreements are also classified according to the range of applicability and may be *individual*, i.e. each agreement only limits the price charged by an individual retailer, or *market-wide*, i.e. applying simultaneously to all retailers.

On the issue of price ceilings there is considerable agreement in antitrust law that they

should be legal because they help with double marginalization. For that reason price ceilings fall in the EU Commission block exemption for vertical agreements and are considered under a rule of reason in the United States since 1997.<sup>16</sup>

If we extend the producer's strategy space to allow for private contracts of the form  $\hat{k}_i = (t_i, x_i, r_i, \overline{p}_i)$ , where  $\overline{p}_i$  is a individual price ceiling, we find that buybacks can be powerful in controlling opportunism:

**Proposition 3.** Private buyback contracts with a buyback price  $r_i \in [p^m - c, p^m]$  and an individual price ceiling  $\overline{p}_i = p^m$  (and by extension with strict individual RPM) are sufficient for the producer to extract the monopoly profit in a contract equilibrium.

This Proposition can be related to the result that in a make-to-order setting opportunism can be eliminated with contracts where both the wholesale price and the price ceiling are set at the monopoly price to completely eliminate retailers' quasi-rents (O'Brien and Shaffer, 1992). Note however that in our make-to-stock setting we can have a gap of c between the price ceiling and the return price, and therefore retailer quasi-rents do not have to be eliminated.

This difference should not be overlooked since it can have important consequences in practice, where the existence of quasi-rents is often essential to provide retailers with incentives—for example, to make sure that the goods are properly stored or services are offered. After all, if quasi-rents are eliminated, and the motive to recoup the investment is not present, it seems unlikely that a retailer would "push" the producer's stocks. Indeed, if retailers must exert some effort in the make-to-order setting of O'Brien and Shaffer (1992) then bilateral price ceilings are ineffective and the Bertrand outcome always arises in a contract equilibrium (Gabrielson and Johansen, 2013). In the present make-to-stock setting this is less of a concern since it is often possible to set a price ceiling to control double marginalization, and a buyback sufficiently high to prevent opportunism and yet sufficiently low to leave a gap between the two prices and provide such incentives.

We now turn our attention to price floors and strict RPM. Both have been seen unfavorably under competition law in most developed countries. The EU Commission has

<sup>&</sup>lt;sup>16</sup>See "Restrictions that remove the benefit of the block exemption", 4 (a) on EU Commission's regulation No 330/2010 and "State Oil Co. v. Kahn", 522 U.S. 3, 15 (1997).

generally seen them as contrary to Article 81(1) and certain prohibitions exist for example in Australia, Canada, France, and the UK. Both were also per se illegal in the United States until 2007, when the U.S. Supreme Court's decision in the Leegin case made RPM come under a rule of reason—legal uncertainty remains at the state level.

In the case where the producer can commit ex-ante to a public market-wide price floor individual price floors have no effect—the producer's strategy space is then a matrix K of private contracts and a public and enforced price floor p applying to all retailers.

As mentioned above, a formal evaluation on the usefulness of price floors in controlling opportunism requires particular attention to how residual demand is shared among retailers when two or more retailers set the same price.

With "same price fair-share" rationing each of those retailers is entitled to an equal share of the residual market demand at that price, and only if some consumers still remain unserved (due to the stock constraint of another retailer) will the remainder be shared proportionally to the stock those retailers charging that same price still hold after serving their "fair share". In Davidson and Deneckere (1986) and Kreps and Scheinkman (1983) there are only two retailers and the quantity sold by each retailer becomes

$$\min \{a_i x_i, \max \{D(p)/2, D(p) - a_j x_j\}\}.$$

Extending this notion to a setting with multiple retailers, and letting  $x^+ = \max\{0, x\}$ , we have that the quantity purchased from retailer *i* is the lowest of his stock and the demand directed at him (conditional on the remaining prices and stocks) which is given by

$$\min\left\{a_i x_i, \beta_i(X, W(K), \lambda)\right\}$$

where

$$\beta_i(X, W(K), \lambda) = \eta + \frac{(a_i x_i - \eta)^+}{\sum_{j \in L_i} (a_j x_j - \eta)^+} (|L_i| \eta - \sum_{j \in L_i} \min\{k_j, \eta\})$$

with  $L_i$  the set of retailers with a price equal to  $p_i$  and where for notational simplicity we have set  $\eta = \gamma(X, W(K), \lambda, p_i) / |L_i|$ .

Rey and Tirole (2007) argued, but did not formally prove, that an industry-wide price floor alone would not solve the commitment problem, but suggested that complementing it with a buyback option would. They had in mind that when several retailers charge the same price then they share the residual market demand proportionally to their stocks, which we call "same price stock-share".<sup>17</sup> In that case the quantity purchased from retailer i would be instead given by

$$\min\left\{a_i x_i, \widehat{\beta}_i^+(X, W(K), \lambda)\right\}$$

where

$$\widehat{\beta}_i(X, W(K), \lambda) = \frac{x_i}{\sum_{j \in L_i} a_j x_j} \left( D(p_i) - \theta(W, \lambda) \sum_{j \in J_i} a_j x_j \right).$$

We find that a industry-wide price floor alone can solve the commitment problem in the case of "same price fair-share" rationing, but in the case of "same price stock-share" it is not sufficient—even when it is complemented with a buyback option because of the potential for double marginalization (after all the two instruments are better seen as substitutes and not as complements).

**Proposition 4.** With "same price fair-share" rationing, private contracts with a public market-wide price floor (and by extension with market-wide strict RPM) are sufficient for the producer to extract the monopoly profit in a contract equilibrium, even in the absence of buybacks. With "same price stock-share" rationing buybacks together private contracts with a public market-wide price floor or RPM may not be sufficient for the producer to extract the monopoly profit in a contract equilibrium, even when buybacks are used.

While market-wide price floors could be used to control opportunism, in practice such direct price controls can be hard to monitor and enforce as they need to be public and apply to all retailers (see e.g. Alexander and Reiffen, 2005 for a discussion on the difficulties of enforcing RPM and its limitations as a strategic commitment). In our view, simple bilateral buyback contracts, perhaps with a bilaterally agreed price-ceiling, seem to be easier to agree on and to enforce.

<sup>&</sup>lt;sup>17</sup>As they explain, "suppose that c = 0, and that when both sellers charge the same price ... sellers sell an amount proportional to what they bring to the market. Let the upstream firm supply  $q^m/2$  to each downstream firm and impose price floor  $p^m$ . Then the upstream firm can supply some more units at a low incremental price to one of the sellers, thus expropriating the other seller."

Direct price controls can also be less effective than simple buyback contracts for other reasons. For example, in the presence of demand uncertainty it may be better to delegate the final price decision to retailers who can better gauge market conditions.

To make this point simply, consider a variant of our model with two retailers and where a unit mass of consumers has an inelastic demand and a willingness to pay of v uniformly distributed in  $[\underline{v}, \overline{v}]$ . At the time of contracting the realization of v is unknown, but it is learned by retailers before they choose their respective prices. In the vertical integrated outcome the producer sells to one retailer  $\alpha$  and the other  $1 - \alpha$  and each retailer sets its price equal to the realization of v provided that the expected unit margin is positive, i.e.  $c \leq \frac{v+\overline{v}}{2}$ , and otherwise produce and sell nothing.

Here double marginalization is not an issue since demand is inelastic and therefore price ceilings are not required. To explain why a market-wide price floor is ineffective in achieving the vertical integrated outcome, note that retailers must be able to charge a price equal to the realization of v, regardless of its realization. The industry-wide price ceiling should therefore not exceed  $\underline{v}$ . To curb the opportunism incentive it must also be the case that, when one retailer is buying  $\alpha$  and is expected to set his price equal to the realization of v, the other retailer and the producer have no incentive to produce and sell one more unit by undercutting him. That will only be the case if the expected bilateral surplus from that deviation is lower than c, i.e.  $c \geq \frac{v+\overline{v}}{2}$ . Clearly this condition is incompatible with the previous condition on the existence of a positive margin.

Consider now the case of a simple buyback contract. It can be shown that the optimal buyback is  $r = \underline{v}$  and both conditions of no profitable bilateral deviation and a positive margin are simultaneously satisfied if  $c \in \left[\frac{\overline{v}-\underline{v}}{2}, \frac{\underline{v}+\overline{v}}{2}\right]$ , as then

$$c+r=c+\underline{v}\geq rac{\underline{v}+\overline{v}}{2} ext{ and } c\leq rac{\underline{v}+\overline{v}}{2}.$$

So, where the other instruments have failed, buybacks in private bilateral contracts implement the vertically integrated outcome if either c or v are not too low.

## 5 Conclusion

Producer opportunism may have important effects in vertical contracting. The main message of this paper is that many common and currently legal practices—such as buybacks, consignment sales and per unit royalties with price-ceilings if needed—can be sufficient to eliminate the opportunism problem in make-to-stock situations, and even be more effective than market-wide RPM. This suggests that alternative explanations for the use price floors may be more relevant—such as reducing free-riding in services among retailers. We therefore hope these insights contribute to the important current antitrust debate on price floors and RPM.

From a managerial perspective the proposed alternatives increase the range of tools that are known to be effective in mitigating the opportunism problem. For example, it has been shown that the producer can achieve vertical control by integrating downstream (e.g. Hart and Tirole, 1990, McAfee and Schwartz, 1994, and Gans 2007). However vertical integration does not seem a realistic solution in the case of supermarkets. Buyback contracts offer an alternative that seems more practical, and also avoids costs associated with integration or potential retail disruptions.

Another proposed solution are (term-by-term) most-favoured-nation clauses, which presupposes that retailers can observe the contracts of each other before deciding on their final contract (McAfee and Schwartz, 1994, De Graba, 1996 and Marx and Shaffer, 2004). This solution may be hard to implement given that private contracting is already the initial source of the problem. On the other hand, the solution proposed in this paper requires only bilateral contracts, i.e. they do not depend on the agreements reached with the remaining retailers.

A tight capacity constraint, and more generally producing under decreasing returns to scale, can also help to control opportunism (Segal and Whinston, 2003). This effect can be reinforced with buyback contracts since in that case vertical control is even easier to sustain.

There are however circumstances that can limit the usefulness of buybacks. As mentioned in the introduction, buybacks can allow for risk-sharing between the producer and the retailers. We found that to overcome opportunism the producer may need to offer buybacks within particular bounds. Such bounds may create active constraints and lead to inefficient risk sharing. Also, return systems can be costly to administer and retailer moral hazard can be an issue.<sup>18</sup> A buyback policy can also raise additional and complex issues when the good sold by the upstream firm is an intermediate good used in production by the downstream firms, as they may distort the incentives of downstream firms to improve their production efficiency. We leave such issues to future research.

Finally, while there is both experimental and anecdotal evidence of the problem, we are not aware of direct empirical tests.<sup>19</sup> We hope that the present work contributes to determine when to expect to observe producer opportunism in the real world. Based on the present research, we think it may be less prevalent than previous theory would suggest.

 $<sup>^{18}</sup>$ As mentioned in Deneckere et al. (1997), an illustration of both aspects is provided by books and magazines, where the shipping costs are so high that often retailers only return the covers to guarantee their buyback reimbursement.

<sup>&</sup>lt;sup>19</sup>See for example Martin et al. (2001) and "The supply of groceries in the UK market investigation" by UK's Competition Commission (2008).

## References

Alexander, C. and Reiffen, D. (2005), "Vertical Contracts as Strategic Commitments: How are they Enforced?", Journal of Economics and Management Strategy, 4: 623-649.

Allen, B. and Hellwig, M. (1993), "Bertrand-Edgeworth duopoly with proportional residual demand", International Economic Review, 34: 39-60.

Arya, A. and Mittendorf, B. (2004) "Using Return Policies to Elicit Retailer Information", RAND Journal of Economics, 35: 617-630.

Bandyopadhyay, S. and Paul, A. (2010), "Equilibrium Returns Policies in the Presence of Supplier Competition", Marketing Science, 29: 846-857.

Beckmann, M. (1965), "Edgeworth-Bertrand duopoly revisited" in R. Henn, ed., Operations Research Verfahren III, Meisenhein: Verlag Anton Hain.

Bork, R. "Vertical Integration and the Sherman Act; the Legal History of an Economic Misconception", University of Chicago Law Review, 22: 157-201.

Butz, D. (1997), "Vertical Price Controls with Uncertain Demand", Journal of Law and Economics, 40: 433-460.

Crémer, J. and Riordan, M. (1987), "On Governing Multilateral Transactions with Bilateral Contracts", RAND Journal of Economics , 18: 436-451.

Davidson, C. and Deneckere, R. (1986), "Long-run competition in capacity, short-run competition in price, and the Cournot model", RAND Journal of Economics, 17: 404-415.

DeGraba, P. (1996), "Most-Favored-Customer Clauses and Multilateral Contracting: When Nondiscrimination Implies Uniformity", Journal of Economics & Management Strategy, 5: 565-579.

Deneckere, R., Marvel, H. and Peck, J. (1996), "Demand Uncertainty, Inventories and Resale Price Maintenance", Quarterly Journal of Economics, 111: 885-913. Deneckere, R., Marvel, H. and Peck, J. (1997), "Demand Uncertainty and Price Maintenance: Markdowns as Destructive Competition", American Economic Review, 87: 619-641.

Deneckere, R. and Kovenock, D. (1992), "Price Leadership", Review of Economic Studies, 59: 143-162.

Dhar, T, Chavas, J.P., Cotterill, R. and Gould, B. (2005) "An Econometric Analysis of Brand-Level Strategic Pricing Between Coca-Cola Company and PepsiCo", Journal of Economics & Management Strategy, 14: 905-931.

Gabrielsen, T. and Johansen, B. (2013) "The opportunism problem revisited: the case of retailer sales effort", Working paper University of Bergen.

Gans, J. (2007) "Vertical Contracting when Competition for Orders Precedes Procurement", Journal of Industrial Economics, 55: 325-346.

Gertner, R. (1985), "Simultaneous Move Price-Quantity Games and Non-Market Clearing Equilibrium", Mimeo, Massachusetts Institute of Technology.

Hart, O. and Tirole, J. (1990), "Vertical Integration and Market Foreclosure", Brookings Papers on Economic Activity (Microeconomics), 1990: 205-285.

Kandel, E. (1996), "The Right to Return", Journal of Law and Economics, 39: 329-356.

Kreps, D. and Scheinkman, J. (1983), "Precommitment and Bertrand competition yield Cournot outcomes", Bell Journal of Economics, 14: 326-337.

Krishnan, H. and Winter, R. (2007), "Vertical Control of Price and Inventory", American Economic Review, 97: 1840 - 1857.

Levitan, R. and Shubik, M. (1972), "Price Duopoly and Capacity Constraints", International Economic Review, 13: 111-22.

Marvel, H., and Peck, J. (1995), "Demand Uncertainty and Returns Policy", International Economic Review, 36: 691-714.

Marvel, H., and Wang, H. (2007), "Inventories, Manufacturer Returns Policies, and Equilibrium Price Dispersion under Demand Uncertainty", Journal of Economics & Management Strategy, 16: 1031-1051.

Marx, L. and Shaffer, G. (2004), "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity: Comment", American Economic Review, 94: 796–801.

McAfee, P. and Schwartz, M. (1994), "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity", American Economic Review, 84: 210–230.

Martin, S., Normann, H.T., and Snyder C., (2001), 'Vertical Foreclosure in Experimental Markets', RAND Journal of Economics, 32: 667-685.

Mathewson, G. and Winter, R. (1984) "An Economic Theory of Vertical Restraints" RAND Journal of Economics , 15: 27-38

Narayanan, V., Raman, A. and Singh, J. (2005) "Agency Costs in a Supply Chain with Demand Uncertainty and Price Competition", Management Science, 51: 120.

Nevo, A., (2001) "Measuring Market Power in the Ready-to-Eat Cereal Industry" Econometrica, 69: 307-42.

O'Brien, D. and Shaffer, G (1992), "Vertical Control with Bilateral Contracts", RAND Journal of Economics, 23: 299-308.

Osborne, M. and Pitchik, C. (1986), "Price competition in a capacity-constrained duopoly", Journal of Economic Theory, 38: 238-260.

Pasternack, B. (1985), "Optimal Pricing and Return Policies for Perishable Commodities", Marketing Science, 4: 166-176.

Peritz, R. (2007), "Nervine' and Knavery: The Life and Times of Dr. Miles Medical Company" in Antitrust Stories, ed Daniel A. Crane and Eleanor M. Fox, Foundation Press.

Perry, M. (1984), "Sustainable Positive Profit Multiple-Price Strategies in Contestable Markets", Journal of Economic Theory, 32: 246-265. Pinkse, J. and Slade M. (2004), "Mergers, brand competition, and the price of a pint", European Economic Review, 48: 617-643.

Rey, P. and Tirole, J. (2007), "A primer on Foreclosure", in Mark Amrstrong and Rob Porter eds, Handbook of Industrial Organization, vol. III.

Rey, P. and Verge, T. (2004), "Bilateral Control with Vertical Contracts", RAND Journal of Economics, 35: 728-746.

Segal, I. (1999), "Contracting with Externalities", Quarterly Journal of Economics, 114: 337-388.

Segal, I. and Whinston, M. (2003), "Robust Predictions for Bilateral Contracting with Externalities", Econometrica, 71: 757-791.

Spengler, J. (1950), "Vertical Integration and Antitrust Policy", Journal of Political Economy, 58: 347-352

Taylor, T. and Xiao, W. (2009), "Incentives for Retailer Forecasting: Rebates vs. Returns", Management Science, 55:1654-1669

Tasnadi, A. (1999), "A two-stage Bertrand–Edgeworth game", Economics Letters, 65: 353–358.

Telser, L. (1960), "Why Should Manufacturers Want Fair Trade?", Journal of Law and Economics,3: 86-105.

# Appendix A

**Proof of Proposition 1.** In step we 1 show that when R = 0 there is no contract equilibrium in pure strategies. In step 2 we extend the game and the concept of contract equilibrium to mixed strategies. In step 3 we show that any contract equilibrium in mixed strategies must have a corresponding Nash equilibrium of an auxiliary game—which was studied by Gertner (1985). In step 4 we derive the implications to the players' strategies and profits.

**Step 1.** In a pure strategy contract equilibrium retailers make zero profits (since the producer can otherwise increase the fixed fee) and  $z_i^* = x_i^*$  for all i (otherwise the producer could deviate to a lower  $x_i$  and save on the production cost). When R = 0 the producer's net benefit of contracting with any retailer i,  $(p_i^* - c)x_i^*$ , must be non-negative and the same for all  $i \in N$  (suppose not, that  $(p_i^* - c)x_i^* > (p_j^* - c)x_j^*$  for some i and j, then there exists  $\varepsilon > 0$  such that the producer benefits from offering j a deviating contract  $k'_{j} = ((p_{i}^{*} - 2\varepsilon)x_{i}^{*}, x_{i}^{*}, 0)$ , which retailer j would accept as it can always undercut retailer i by  $\varepsilon$  and make a net profit  $\varepsilon x_i^*$  instead of zero). Moreover  $(p_i^* - c)x_i^* > 0$  for at least one i (since in an equilibrium one retailer i must sell  $z_i^* > 0$  and therefore is a monopolist on its residual demand curve, and therefore the bilateral benefit can be increased by raising the price and decreasing  $x_i^*$ ). It follows, since the net benefit of contracting with each retailer must be the same, that both  $(p_j^* - c)$  and  $x_j^*$  must be strictly positive for all  $j \in N$ . But if  $(p_i - c) > 0$  for all *i* then there exists  $\varepsilon > 0$  such that the producer benefits from offering some retailer j, which in equilibrium charges the lowest price, a deviating contract  $k'_j = ((p_j^* - 2\varepsilon)D(p_j^*), D(p_j^*), 0)$  with  $(p_j^* - 2\varepsilon) > c$ , which retailer j would accept as it can always undercut  $p_j^*$  by  $\varepsilon$  and make a net profit  $\varepsilon D(p_j^*)$  instead of zero. But then  $z_i^* \neq x_i^*$ for all  $i \neq j$ , which is a contradiction. This implies that there is no equilibrium in pure strategies.

Step 2. We therefore introduce the mixed strategy extension of G. Let  $\Delta(S_i)$  be the set of probability measures over the subsets of  $S_i$  and  $\sigma_i \in \Delta(S_i)$  denote a mixed strategy of player i. Extend the profits to this space by defining  $\sigma = (\sigma_i)_{i \in M}$  and having  $\pi_i(\sigma) = \int_S \pi_i(x) d\sigma$ , and we obtain the mixed extension  $\overline{G} = (M, \Delta(S_i)_{i \in M}, (\pi_i)_{i \in M})$ . A mixed strategy contract equilibrium (with unobservable contracts) is a probability distribution over all possible contracts and Nash equilibrium responses induced by that distribution such that, for all  $i \in N$ , each contract realization  $k_i$  induces retailer i to maximize  $\pi_u + \pi_i$  while taking the remainder of  $\sigma$  as given.

**Step 3.** From (1), when R = 0 for any realization K of  $\sigma_u$  the producer profit is

$$\pi_u(K, W) = \sum_{i \in N} a_i \left[ t_i - c x_i \right],$$

and therefore the producer's benefit of contracting with each retailer is independent of the contracts it offers to the other retailers. Since the acceptance and pricing decisions of each retailer are unaffected by unobserved changes in the contracts offered to the other retailers, the stock offered to any retailer *i* should maximize the expected profit of that retailer *i*—conditional on that contract—as if *i* could himself produce at a constant marginal cost *c* and then choose its price. Therefore the distribution of *X* and *P* in any contract equilibrium of  $\overline{G}$  should be similar to the distribution of the stocks and prices chosen by the retailers in a Nash equilibrium of a game  $\overline{H}$  where retailers choose both quantities and prices simultaneously while producing at a marginal cost *c*, and for any realization *X* the fee  $t_i$  equals  $cx_i$  plus *i*'s expected profit in  $\overline{H}$  conditional on choosing  $x_i$ .

Step 4. Gertner (1985) showed that mixed strategy Nash equilibria of  $\overline{H}$  exist and in all equilibria retailers make zero profits (while he uses a specific rationing rule, it can be shown that this results also holds for all rationing rules considered in this paper). It follows that in  $\overline{G}$  the producer's payoff is also zero. Moreover for any  $\sigma_u^*$  the set of contracts K are such that  $k_i = (cx_i, x_i, 0)$  but X is random, and retailers accept the realized contracts and each chooses a price  $p_i(x_i)$  as if they had chosen  $x_i$  themselves in the game  $\overline{H}$ . Existence can be shown by construction using the equilibria identified by Gertner (1985).

**Proof of Proposition 2.** The proof proceeds in 3 steps. Step 1 ensures that contracts do not lead to double-marginalization, i.e. they induce each retailer *i* to set  $p_i = p^m$  while taking the remaining contracts as given. Step 2 ensures that the producer does not have an incentive to behave opportunistically, i.e. it is not profitable to replace a contract  $k_i^*$  with a contract in which  $x_i > x^*$ . Step 3 ensures that there is no profitable deviating contract with  $x_i < x^*$ . **Step 1.** If the monopoly outcome is to be sustained in equilibrium we must have that a set of contracts with

$$k_i^* = (t_i^*, x_i^*, r_i^*) = (\alpha_i^* p^m D(p^m), \alpha_i^* D(p^m), r_i^*)$$

is proposed by the producer, accepted by the retailers and induces the retailers to set the monopoly price—where  $\alpha_i^* \geq 0$  represents the share of the monopoly quantity sold to retailer *i* and  $\sum_n \alpha_i^* = 1$ . In that case pricing below  $p^m$  is dominated for all retailers. Once retailer *i* accepts its contract it can either get  $p^m \alpha_i^* D(p^m)$  with  $p_i = p^m$  or set  $p_i > p^m$  and get the profit made on its residual demand curve when its effective marginal cost of selling a unit is  $r_i^*$ , i.e.

$$(p_i - r_i^*)z_i(X, W, \lambda) + r_i^*\alpha_i^*D(p^m) \Leftrightarrow$$

$$(p_i - r_i^*) \max \left\{ 0, (D(p_i) - (1 - \alpha_i^*)(\lambda D(p^m) + (1 - \lambda)D(p_i))) \right\} + r_i^* \alpha_i^* D(p^m)$$
(4)

When demand is well behaved, to insure that the retailer's optimal price does not exceed  $p^m$  it is sufficient that

$$\frac{\partial(4)}{\partial p_i}\Big|_{p_i=p^m} \le 0 \Leftrightarrow \alpha_i^* \left[ D(p^m) + (p^m - r_i^*) D'(p^m) \right] + \lambda (1 - \alpha_i^*) \left[ (p^m - r_i^*) D'(p^m) \right] \le 0 \quad (5)$$

The second left-hand term of (5) is strictly negative—except for  $\lambda = 0$  where it is zero. For the first left-hand term, notice that

$$D(p^m) + (p^m - r_i^*)D'(p^m)$$

corresponds to the first-order condition of the vertical monopoly evaluated at  $p_i = p^m$  when the cost of production is r instead of c. So this term is larger, equal or smaller than zero if  $r_i$  is respectively larger, equal or smaller than c. So, for all  $\lambda \in [0, 1]$ , (5) is satisfied if  $r_i^* \leq c$ , while (5) increases with  $\alpha_i^*$  for  $c < r_i^* \leq p^m$ —obviously  $r_i^*$  should not exceed  $p^m$ . Since these constraints must be satisfied for all i, this set of constraints is then more easily satisfied if each retailer receives in equilibrium an equal share of the surplus, i.e  $\alpha_i^* = 1/n$ for all  $i \in N$ . So the contracts do not induce retailers to set prices above  $p^m$ , i.e. there is no double-marginalization, if

$$r_i^* \le p^m (1 - \frac{1}{\epsilon(p^m)(1 + \lambda(n-1))}) = \overline{r}(n,\lambda) \text{ and } \alpha_i^* = \frac{1}{n} \text{ for all } i \in N.$$

**Step 2.** We now check that the producer has no incentives to deviate to an alternative contract  $k'_i$  such that  $x'_i > x^*_i$  with some retailer *i*. Given that deviation, if retailer *i* prices above  $p^m$  it still does not sell more than  $x^*_i$ , so that deviation is unprofitable (otherwise the producer could deviate to  $x^*_i$  and save on the production cost). It may however price at  $p^m$  to sell

$$\frac{x'_i}{D(p^m) + (x'_i - x^*)} D(p^m) < x'_i \text{ or } \frac{D(p^m)}{n}$$

depending on whether we have "same price stock-share" or "same price fair-share" rationing respectively, or else reduce its price to  $p'_i < p^m$  and sell min  $\{x'_i, D(p')\}$ . But for any  $x'_i > x^*_i$ undercutting the other retailers dominates pricing at  $p^m$ , and so  $p_i < p^m$  if  $x'_i > x^*_i$ . Note that if a deviation with  $p'_i < p^m$  and  $x'_i > x^*_i$  takes place then the remaining retailers would not sell all their stock and therefore make a loss, i.e. they will be victims of producer opportunism. For such deviations to be unprofitable it must be the case that the most the producer can get from selling any additional unit to retailer *i* is less than the (average) value of buybacks that need to be paid to the remaining retailers, i.e.

$$p^m - c \le \sum_{j \in N \setminus i} \frac{r_j \alpha_j^*}{1 - \alpha_i^*}.$$
(6)

Replacing each  $r_j$  in the expression above with the bound  $\overline{r}(n, \lambda)$  and setting  $\alpha_j^* = 1/n$ , we get that all constraints associated with both double marginalization and opportunism can be simultaneously satisfied if for all  $i \in N$ 

$$\alpha_i^* = \frac{1}{n} \text{ and } p^m - c \le \overline{r}(n, \lambda) \Leftrightarrow \epsilon(p^m) \ge 1 + \frac{1}{1 + \lambda(n-1)}.$$

Using the Lerner index we find that this expression is also equivalent to (2).

Step 3. To complete the proof we should also consider deviations to some quantity  $x_i < x^*$ and a price  $p'_i > p^m$  (selling less at a lower price is then not optimal). Since profits are concave in p, when  $\alpha_i^* = 1/n$  for all  $i \in N$  such deviation is unprofitable if

$$\frac{\partial}{\partial p_i} \left[ (D(p_i) - \frac{n-1}{n} D(p^m) (\lambda + (1-\lambda) \frac{D(p_i)}{D(p^m)}))(p-c) \right] \Big|_{p_i = p^m} \le 0.$$

or equivalently

$$[D(p^m) + (p^m - c)D'(p^m)] + (p^m - c)(\lambda(n-1))D'(p^m) \le 0.$$

Note that the first left-hand element is zero (from the first order conditions of monopoly), and the latter is negative since  $D'(p^m) < 0$ . So this condition is trivially satisfied for all  $\lambda \in [0, 1]$ .

**Proof of Corollary 1.** Follows directly from (2) that for all  $\lambda \in (0, 1]$  the producer can always extract the monopoly rent with buyback contracts alone if either  $p^m \leq 2c$  or otherwise when  $p^m > 2c$  if

$$n \ge 1 + \frac{p^m - 2c}{c\lambda}.$$

So for all  $\lambda > 0$  and c > 0 there exists a finite  $\underline{n}(c, \lambda) \equiv 1 + \max\left\{0, \frac{p^m - 2c}{c\lambda}\right\}$  such that for all  $n \geq \underline{n}(c, \lambda)$  the producer extracts the monopoly profit in a contract equilibrium of G. Moreover  $n(c, \lambda)$  is decreasing in  $\lambda$  and c.

**Proof of Proposition 3.** A symmetric strategy profile with  $\hat{k}_i = (p^m \frac{D(p^m)}{n}, \frac{D(p^m)}{n}, r_i, p^m)$ , with  $r_i \in [p^m - c, p^m]$ , and  $w_i = (1, p^m)$  forms a contract equilibrium of  $\hat{G}$  because, taking the other contracts and prices as given, the bilateral benefit  $\pi_u + \pi_i$  is maximized by selling  $q^m/n$  at  $p^m$ . There is no other set of contracts that improves upon it, since the producer already obtains the monopoly profit.

**Proof of Proposition 4.** If the monopoly outcome is to be sustained in equilibrium we must have that an industry-wide price floor  $p^m$  and a set of contracts with

$$k_i^* = (t_i^*, x_i^*, r_i^*) = (\alpha_i^* p^m D(p^m), \alpha_i^* D(p^m), r_i^*)$$

is proposed by the producer, accepted by the retailers and induces them to set the monopoly price—where  $\alpha_i^* \ge 0$  represents the share of the monopoly quantity sold by retailer *i* and  $\sum_N \alpha_i^* = 1$ . It must then be the case that the producer has no incentive to offer a contract  $k'_i$  where retailer *i* is given  $x'_i > x^*_i$ . If there is an industry-wide price floor at  $p^m$  retailer *i* cannot undercut its rivals, but it can set  $p_i = p^m$  and sell

$$\frac{x'_i}{D(p^m) + (x'_i - x^*)} D(p^m) < x'_i \text{ or } \frac{D(p^m)}{n}$$

depending on whether we have "same price stock-share" or "same price fair-share" rationing respectively. Therefore with "same price fair-share" rationing there is no incentive to deviate. On the other hand, with "same price stock-share" to avoid this deviation it must be the case that the net benefit made with i after paying the buybacks is less than the benefit made with the equilibrium contract, i.e.

$$p^{m} \frac{x_{i}' D(p^{m})}{(1-\alpha_{i}^{*}) D(p^{m}) + x_{i}'} - \sum_{j \in N \setminus i} r_{j}^{*} \left( \alpha_{j}^{*} - \frac{\alpha_{j}^{*} D(p^{m})}{(1-\alpha_{i}^{*}) D(p^{m}) + x_{i}'} \right) D(p^{m}) - cx_{i}' \leq (p^{m} - c)\alpha_{i}^{*} D(p^{m})$$

$$\tag{7}$$

In addition, the equilibrium contracts should not induce double marginalization which, like in Step 1 of Proposition 2, implies that for all  $i \in N$ 

$$\alpha_i^* \left[ D(p^m) + (p^m - r_i^*) D'(p^m) \right] + \lambda (1 - \alpha_i^*) \left[ (p^m - r_i^*) D'(p^m) \right] \le 0$$

or equivalently

$$r_i^* \le p^m (1 - \frac{\alpha_i^*}{\epsilon(p^m)(\alpha_i^* + \lambda(1 - \alpha_i^*))}),$$

which is strictly less than  $p^m$  for all  $\alpha_i^* > 0$ . There must also exist at least one  $i \in N$  such that there is some  $j \neq i$  with  $\alpha_j^* > 0$ . For any such *i* note that the left-hand side of (7) is differentiable, equal to the right-hand side for  $x'_i = \alpha_i^* D(p^m)$ , and if we take the derivative of the left-hand side evaluated at that point is strictly positive for *c* sufficiently close to zero. This implies that for *c* sufficiently close to zero there exists no vector  $\alpha = (\alpha_1, ..., \alpha_n)$ with  $\alpha_i \geq 0$  and  $\sum_N \alpha_i^* = 1$  that satisfies both constraints for all  $i \in N$ . This means that for sufficiently low *c* there always exists a profitable deviation, and hence in those cases there cannot be a contract equilibrium where the producer extracts the monopoly profit.

## Appendix B

The result below suggests that this may not be the case since the producer may use the buybacks to protect its profit.

**Result** If the market demand is log-concave but the monopoly outcome cannot be sustained in a contract equilibrium, then the market price in any symmetric pure strategy contract equilibria exceeds the monopoly level.

**Proof.** Let  $P(n, \lambda)$  be the set of prices that can be sustained in a symmetric contract equilibrium in pure strategies of G with n retailers, so that if  $p^* \in P(n, \lambda)$  the producer offers each retailer the contract  $k^* = (p^*D(p^*)/n, D(p^*)/n, r^*)$ , which is accepted and each retailer sets  $p_i = p^*$ . The proof proceeds in 4 steps. Step 1 is ensuring that these contract induces each retailer i to set  $p_i = p^*$  when it takes the remainder of  $S^*$  as given. The second step is to ensure that retailer i does not find it profitable to make an offer to the producer to replace its contract  $k^*$  with a contract in which  $x_i > x^*$ . The third step is ensuring that the same holds for a contract with  $x_i < x^*$ . Step 4 brings the constraints found in steps 1 to 3 together to characterize the set  $P(n, \lambda)$ .

**Step 1.** When the remaining retailers are selling an aggregate quantity of  $\frac{n-1}{n}D(p^*)$  at a price of  $p^*$ , retailer *i* does not want to set  $p_i < p^*$  since it can sell all its stock  $\frac{1}{n}D(p^*)$ for  $p^*$  and make a higher profit—provided of course that  $r^* \leq p^*$ . On the other hand if  $p_i > p^*$  it will sell less than  $x^*$  but it gets  $r^*$  on those units it does not sell and therefore its opportunity cost of selling each unit is  $r^*$ . With profits concave in  $p_i$ , to avoid double marginalization we must have

$$\frac{\partial(p_i - r^*) \left( D(p_i) - \left(\frac{n-1}{n}\right) \left(\lambda D(p^*) + (1-\lambda)D(p_i)\right) \right) + r^* \frac{D(p^*)}{n}}{\partial p_i} \bigg|_{p_i = p^*} \le 0$$

or equivalently

$$\frac{1}{n} \left[ D(p^*) + (p^* - r^*) D'(p^*) \right] + \lambda(\frac{n-1}{n}) \left[ (p^* - r^*) D'(p^*) \right] \le 0,$$

which implies that

$$r^* \le p^*(1 - \frac{1}{\epsilon(p^*)(1 + \lambda(n-1))}) \equiv r(p^*, n, \lambda).$$

**Step 2.** Consider now an alternative contract  $k'_i$  where retailer *i* is given  $x'_i > x^*_i$ . Such deviation induces *i* to set  $p_i \leq p^*$  as otherwise *i* does not sell more than  $x^*_i$ . Retailer *i* may then price at  $p^*$  to sell

$$\frac{x'_i}{D(p^*) + (x'_i - x^*)} D(p^*) < x'_i \text{ or } \frac{D(p^*)}{n}$$

depending on whether we have "same price stock-share" or "same price fair-share" rationing respectively, or reduce its price to some  $p'_i$  just below  $p^*$  and sell min  $\{x'_i, D(p'_i)\}$ , and therefore undercutting  $p^*$  dominates pricing at  $p^*$ . So deviations to  $x'_i \in (x^*_i, D(p^*)]$  are not optimal since if retailer *i* is offered a stock  $x'_i \in (x^*_i, D(p^*)]$  he will sell all its stock by undercutting  $p^*$  and the net benefit the producer can obtain from this deviation instead of offering  $k^*_i$  is

$$(p'_i - c)x'_i - r^*(x'_i - x^*) - (p^*_i - c)x^*_i,$$

which if positive implies that

$$(p'_i - c - r^*)x'_i > (p^*_i - c - r^*)x^*_i$$

and therefore can be improved by selling *i* a stock  $D(p'_i) > D(p^*)$ , which exceeds the proposed  $x'_i$  since  $p'_i < p^*$ . So we consider deviations to  $x'_i > D(p^*)$ , a situation in which the remaining retailers sell nothing and therefore the total paid in buybacks is

$$r^* \frac{n-1}{n} D(p^*).$$

To avoid such deviations it must be that

$$\max_{p_i \in [c,p^*)} \left[ (p'_i - c)D(p'_i) - r^* \frac{n-1}{n}D(p^*) \right] < (p^* - c)\frac{D(p^*)}{n}$$

which is satisfied if and only if

$$\frac{D(p^*)}{n} \left( p^* - c + r^*(n-1) \right) \ge \begin{cases} \pi(p^*) \text{ if } p^* \le p^m \\ \pi(p^m) \text{ if } p^* > p^m \end{cases}$$
(8)

**Step 3.** Finally we look at deviations to some quantity  $x_i < x^*$  with a price  $p_i > p^*$  (selling less at a lower price is obviously not optimal). Since profits are concave in p, for any such deviation to be unprofitable we must have

$$\frac{\partial}{\partial p_i} \left[ (D(p_i) - \frac{n-1}{n} D(p^*)(\lambda + (1-\lambda) \frac{D(p_i)}{D(p^*)}))(p-c) \right] \Big|_{p_i = p^*} \le 0$$

or equivalently

$$\frac{D(p^*)}{n} + (p^* - c)(\frac{1 + \lambda(n-1)}{n})D'(p^*) \le 0 \Leftrightarrow \frac{p^*}{\epsilon(p^*)} \le (1 + \lambda(n-1))(p^* - c)$$
(9)

**Step 4.** Replacing  $r(p^*, n, \lambda)$  in (8) and rearranging the elements, we have that  $p^* \in P(n, \lambda)$  if and only if (9) and

$$\frac{p^*}{\epsilon(p^*)} \le \begin{cases} c(1+\lambda(n-1)) \text{ if } p^* \le p^m \\ c(1+\lambda(n-1))(1-\frac{n}{n-1}\frac{\pi(p^m)-\pi(p^*)}{cD(p^*)}) \text{ if } p^* > p^m \end{cases}$$
(10)

are simultaneously satisfied. D is log-concave if  $p/\epsilon(p)$  is decreasing in p. It follows from (9) and (10) that for any  $p < p^m$  we have  $p \notin P(n, \lambda)$  if  $p^m \notin P(n, \lambda)$ , since the right-hand side of the inequalities are weakly increasing in  $p^*$  and the left-hand side decreasing, and so both inequalities become harder to satisfy as  $p^*$  decreases. For  $p > p^m$  the left-hand side is again decreasing in  $p^*$ , the right hand side of (9) is increasing and the right-hand side of (10) is also increasing for prices sufficiently close to  $p^m$  (to see the latter take the derivative of the right-hand side of (10) with respect to  $p^*$ , evaluate it at  $p^* = p^m$  and use the Lerner index to find that it is strictly positive). So for prices slightly above  $p^m$  both inequalities are relaxed as the price increases. It that when D is log-concave the set  $P(n, \lambda)$  can be non-empty and not include  $p^m$ , but then  $p^m < p^*$  for any  $p^* \in P(n, \lambda)$ .