

One-to-many bargaining with indissoluble agreements

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Abstract

We study a model where a central player (the principal) bargains bilaterally with each of several players (the agents) to create and share the surplus of a coalitional game. It is known that if the payments agreed with the agents are rebargained in case any bilateral negotiation breaks down, then the Shapley value is the unique efficient and individual rational outcome consistent with bilateral Nash bargaining. Here we show that when the agreed payments cannot be rebargained, i.e., they are indissoluble, that outcome is also unique but coincides instead with the Nucleolus of an associated bankruptcy problem. We provide a strategic foundation for this outcome. Then we study how the ability to rebargain affects the principal's payoff according to the properties of the surplus function. We find, for example, that indissoluble agreements hurt the principal when agents are complements and they benefit him when they are substitutes.

1 Introduction

In many situations, a central player (the principal) bargains bilaterally with each of the remaining players (the agents) to create and share a surplus which varies with the set of agents reaching an agreement—a structure that allows for “partial” cooperation if an agreement is not reached with every agent. Examples of these situations include a firm bargaining over wages with each of several workers, a drug developer acquiring rights to use multiple patents, a consumer buying different products from multiple sellers and Coase's classic case of a railroad acquiring plots of land from farmers.

Stole and Zwiebel (1996) studied how the surplus is shared in these situations when the payment agreed with each agent divides evenly the gains from a bilateral agreement between the principal and that agent relative to their respective disagreement payoffs—as in bilateral Nash bargaining. To determine those disagreement payoffs they assumed that if any bilateral agreement is not reached then the agreements with all remaining agents must be bargained anew—a situation where agreements are *non-binding*. In that case they showed that the Shapley value is the unique *stable outcome*, i.e., the unique individual rational and efficient payoff vector consistent with bilateral Nash bargaining. That outcome is also a subgame perfect equilibrium of an alternating-offers bargaining game where all bilateral agreements must be bargained anew once any negotiation breaks down.

However it is not always simple or feasible to bargain agreements anew. Rebargaining may, for example, involve significant legal costs or there may be some time during which agreements simply cannot be terminated. A natural question that follows is whether the ability to rebargain those agreements hurts or benefits the principal. The answer to this question would, for example, help

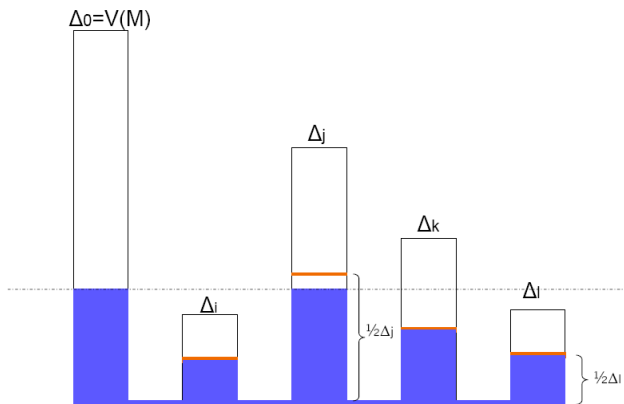
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us understand why in some situations the principal prefers a succession of short-term bilateral agreements, which usually rollover but could be bargained anew if a bilateral negotiation breaks down, and in other situations he prefers to bargain long-term agreements.

To answer this question we study a situation where the agreed payments are *indissoluble*. In this case we determine the disagreement payoffs assuming that, if any bilateral agreement is not reached, the payments agreed with all remaining agents cannot be rebargained but instead must be paid in full from the surplus of partial cooperation. Thus, the principal is the residual claimant of that surplus. This captures the fact that in practice, if a firm is unable to honour its agreements, a court will typically make use of the available surplus to pay those obligations.

We show that in this situation a *stable outcome* also exists and is unique, but it now coincides with the Nucleolus of an associated bankruptcy problem where each player’s marginal contribution to the total surplus forms a claim.¹ Each agent is then paid the minimum of half his marginal contribution and what the principal receives—and the latter is determined endogenously.

This stable outcome can easily be visualized by a system of communicating vessels where each player i is represented by a vessel with a height equal to his marginal contribution (Δ_i) (see picture below). The vessel of each agent is sealed at half its height while the principal’s (player 0) vessel is left unsealed. Once we introduce in the system an amount of liquid equal to the total cooperative surplus $v(M)$, it becomes distributed in the vessels according to the unique stable outcome with indissoluble agreements.



Moreover we show that, when this stable outcome lies in the strict Core, it is also a subgame perfect equilibrium of an alternating-offers bargaining game where the payments once agreed cannot be rebargained. This and its computational simplicity should make it an interesting benchmark in applications.

Using these results we show that whether rebargaining will hurt or benefit the principal depends on the properties of the surplus function. In general rebargaining benefits the principal if marginal contributions are “large” and it hurts him if they are “small”.

Marginal contributions are large, for example, in convex or supermodular games, introduced by Shapley (1971), and they are small in big-boss games, introduced by Muto et al. (1988)—which includes those games which are concave or submodular with respect to agent inclusion. In these particular cases the intuition is clear. In a convex game the agents are complements to each other, so if an agent is removed from the game the marginal contributions of all remaining agents are reduced. Therefore when agreements are non-binding, if any bilateral negotiation fails, all agreements will be rebargained taking this into account and the remaining agents will then accept lower payments. If instead the agreements are indissoluble then the remaining agents should still

¹Bankruptcy problems were formalized by Aumann and Maschler (1985).

be paid according to the original agreements—which are reached presuming that both the surplus and marginal contributions are large—, and thus the surplus reduction will be mostly at the principal’s expense. Therefore when a game is convex the principal’s payoff will be lower when agreements are indissoluble rather than non-binding. When a game is concave with respect to agent inclusion the agents are instead substitutes, so if an agent is removed from the game the marginal contributions of all remaining agents will increase. The opposite argument and result will then hold.

Observational implications of these results are, for example, that a drug developer acquiring rights to use multiple complementary patents would prefer to bargain short-term licensing agreements to protect itself from a sort of hold-up arising in situations with complementarities, while a firm hiring from a pool of substitute workers would prefer to bargain long-term employment contracts. Or yet that a producer selling to multiple buyers (with decreasing marginal utilities) would prefer short-term supply agreements, which are expected to rollover but could be rebargained sooner rather than later, if he has economies of scale but long-term supply agreements if he has diseconomies of scale—in the former case buyers are complements to each other and in the latter case they are substitutes.

This paper contributes to a large literature on coalitional bargaining.² However we focus only on the specific situation of one-to-many bargaining, which has been studied in a number of papers but with distinct considerations from ours. For example, Jun (1989), Horn and Wolinsky (1988), Inderst and Wey (2007), Atakan (2008) and Dobbelaere and Lutgens (2011) were interested in the payoff effects of agents merging. Hendon and Tranaes (1991), Cai (2000, 2003), Menezes and Pitchford (2004) and Chowdhury and Sengupta (2012) focused on the hold-out problem and bargaining delays. Noe and Wang (2004), Marx and Shaffer (2007) and Krastvea and Yildirim (2012) studied the optimal sequence of negotiations.

A notable exception that looks at a similar question in the context of intrafirm bargaining is the work of Westermarck (2003). He studied a non-cooperative model where agreements are indissoluble, and characterized the solution when the firm’s production function has decreasing returns to labour: each worker then receives half his marginal contribution.³ In that case indissoluble agreements reduce the total wage bill relative to non-binding agreements. Inderst and Wey (2007) also consider a solution where each agent is paid half his marginal contribution, but they remark that in general the solution violates the individual rationality of the principal—for example in the presence of complementarities.

The present model solves that issue while delivering a simple solution for general surplus functions. In addition we show that the principal’s preference for indissoluble or non-binding agreements is crucially determined by the agents’ productive characteristics. For example, in the case of intrafirm bargaining indissoluble agreements reduce the wage bill when workers are substitutes—Westermarck’s (2003) decreasing returns—but they increase the total wage bill if workers are instead complements—the increasing returns case. Sufficient conditions also allow us to examine other situations where complementarity and substitution of workers are simultaneously present.

Horn and Wolinsky (1988) and Cai (2003) studied non-cooperative bargaining games with agreements similar to ours—the former between a firm and two workers and the latter with multiple but indispensable agents. While in their work bilateral negotiations are sequential, here

²Well known examples include Gul (1989), Hart and Mas-Colell (1996) and Serrano (1995). The last one mixes non-cooperative and cooperative approaches to implement the Nucleolus of subadditive surplus sharing games (the games considered here are on the other hand superadditive). There, the largest creditor makes a proposal about how to split the estate and those creditors who accept the proposal receive their payments, while those who reject appeal to an arbitrator who shares the remaining estate using the contested garment rule.

³In his model, players are induced to reach an agreement by impatience, while in both Stole and Zwiebel’s (1996) and the present model the players are induced to reach an agreement by the fear that negotiations may breakdown.

they take place simultaneously.⁴ Still, as theirs and our efficient solutions coincide in their more specific settings, we view the present work as a generalization of their games to multiple agents and general surplus functions.

The bargaining outcome we find here is efficient also in games with strong complementarities. The hold-out literature mentioned above has found that in those situations the last agent to sign a contract may be able to extract a large share of the surplus, which in turn can lead to delays and even a complete breakdown of negotiations. In those models bargaining is typically sequential (while here it is simultaneous) and payments are made immediately (while here they are made from the cooperative surplus). The present work then also helps us better understand how those two features can lead to hold-out instances.

We characterize the outcome with indissoluble agreements in section 2, we study how these change the sharing of the surplus in section 3 and we conclude in section 4.

2 Bargaining with indissoluble agreements

We consider a transferable utility game in coalitional form (M, v) . Player 0 is the principal and players in $N = \{1, \dots, n\}$ are agents, so $M = 0 \cup N$ is the set of players. These labels are used to reflect our focus on situations in which one player, the principal, bargains bilaterally with all remaining players, the agents, but these do not bargain with each other.

$v : 2^M \rightarrow R$ is a zero-normalized characteristic function denoting the surplus each subset of players $S \subseteq M$ can create by cooperation—so $v(\emptyset) = v(i) = 0$. The marginal contribution of player i to S and the joint marginal contribution of a subset S to M are respectively⁵

$$\Delta_i v(S) = v(S \cup i) - v(S \setminus i) \text{ and } \Delta_S v(M) = v(M) - v(M \setminus S).$$

$v(M) > 0$ and v is superadditive, i.e., $\Delta_i v(S) \geq 0$ for all $i \in M$ and $S \subseteq M$. Given the principal's special role, $v(S) = 0$ if $0 \notin S$ and if the principal cooperates only with those agents in S then a surplus $v(S \cup 0)$ is created.

2.1 The stable bargaining outcome

The bilateral bargaining problem between each pair $\{0, i\}$, with $i \in N$, is described by the possible payoffs ϕ_0 and ϕ_i in case of agreement and the disagreement payoffs d_0^i and 0 for the principal and agent i respectively. The gain from a bilateral agreement is $(\phi_i + \phi_0 - d_0^i)$. Since utility is transferable, bilateral Nash bargaining awards agent i half the gains from bilateral agreement and the principal the other half plus his disagreement payoff, i.e.,

$$\phi_i = \frac{1}{2}(\phi_i + \phi_0 - d_0^i) \text{ and } \phi_0 = \phi_i + d_0^i \tag{1}$$

Solutions satisfying this condition are called standard for two player games.

To determine the disagreement payoff d_0^i we assume that i) each bilateral agreement cannot be rebargained and ii) the principal is the residual claimant of the cooperative surplus. In this case, if the principal does not reach an agreement with an agent i , the equilibrium agreements with all other agents remain valid. Moreover those agents should be paid in full from $v(M \setminus i)$, unless $v(M \setminus i)$ is insufficient to satisfy all payments and then the surplus gets shared only among the agents—the exact way in which this is done is however not essential.

⁴This approach keeps our model simple and tractable—for example, in Horn and Wolinsky (1988) characterizing the bargaining outcome with two agents is an exercise of 15 pages. Also, we see no reason to restrict negotiations to be sequential since, for example, offers may be exchanged by mail or email.

⁵We use shorthand notation by writing $S \cup i$ for $S \cup \{i\}$ and $S \setminus i$ for $S \setminus \{i\}$. $|S|$ denotes the cardinality of S .

Formally, $\phi_{-i} = (\phi_j)_{j \in N \setminus i}$ denotes the vector of payments agreed with the agents in $N \setminus i$, V the set of all functions, u_j the payoff of player j and $x_+ = \max\{0, x\}$. The gains from cooperation in the absence of agent i are shared according to any allocation rule $u : \phi_{-i} \times V \rightarrow R^n$ satisfying

$$\sum_{M \setminus i} u_j = v(M \setminus i), u_0 = \left[v(M \setminus i) - \sum_{N \setminus i} \phi_j \right]_+ \text{ and } \phi_j \geq u_j \geq 0 \text{ for all } j \in N \setminus i. \quad (2)$$

The principal's disagreement payoff when bargaining with agent i is then

$$d_0^i = \left[v(M \setminus i) - \sum_{N \setminus i} \phi_j \right]_+. \quad (3)$$

Let $\phi_M = (\phi_i)_{i \in M}$ denote a generic payoff vector.

Definition 1. *A stable bargaining outcome with indissoluble agreements is a payoff vector ϕ_M that is i) individually rational, ii) efficient and iii) consistent with bilateral Nash bargaining, i.e.,*

i) $\phi_i \geq 0$ for all $i \in M$, ii) $\sum_M \phi_i = v(M)$ and iii) satisfies (1) for each pair $\{0, i\}$ with $i \in N$,

when the disagreement payoffs d_0^i are given by (3).

Stole and Zwiebel (1996) showed that when the agreed payments are instead non-binding the Shapley value of (M, v) is the unique payoff vector that satisfies these same three properties.⁶ Here we obtain the following sharing of the surplus:

Proposition 1. *There exists a unique stable bargaining outcome with indissoluble agreements, denoted by ϕ_M^B , where*

$$\phi_i^B = \min \left\{ \phi_0^B, \frac{\Delta_i v(M)}{2} \right\} \text{ for all } i \in N \text{ and } \phi_0^B = v(M) - \sum_N \phi_i^B.$$

ϕ_M^B coincides with the Nucleolus of a bankruptcy problem associated to (M, v) , where the estate is $v(M)$ and each player's claim is his marginal contribution $\Delta_i v(M)$.

With indissoluble agreements each agent is then paid the minimum of half his marginal contribution and what the principal receives—with the latter being determined endogenously. This outcome is easy to compute and the connected vessels metaphor presented in the introduction, based on Kaminski's (2000) ingenious hydraulic interpretations, allows us to visualize it in a simple way.

In a one-to-many bargaining situation, the equivalence above implies that bilateral bargaining over indissoluble payments results in a sharing of $v(M)$ which minimizes (among all subset of players) the difference between the sum the players in each subset S are awarded and the sum those players would get if instead they received the remainder of $v(M)$ after paying each of the remaining players his marginal contribution.

⁶These correspond respectively to their definitions 3, 1 and 2. The only difference thus lies in the way disagreement points were obtained—in their work they are obtained recursively, by considering the stable outcome of the subgame without agent i , and so forth.

2.2 The extensive-form game

We now show that ϕ_M^B can also be the outcome of a strategic game. We study a one-to-many extension of a standard bilateral bargaining protocol with an exogenous risk of breakdown—where the principal starts by making a proposal to each agent and subsequent proposals are made in alternating order (as in Binmore et al., 1986). These protocols coincide if there is a single agent.

Bargaining takes place in periods $k = 0, 1, 2, \dots$. Players are risk neutral. There is no discounting but in every period there is a constant probability $(1 - \delta) \in (0, 1)$ that the game will end before reaching the next period, so it ends at a random date \bar{k} . In line with the literature we are interested in the limiting case where $\delta \rightarrow 1$.

Let N_k denote the set of agents without an agreement in period k —for example $N_0 = N$. In every period $k = 2h$, with $h = 0, 1, 2, \dots$, the principal proposes simultaneously to every agent $i \in N_k$ to pay that agent $\phi_i \geq 0$ for i 's cooperation at \bar{k} . Each agent $i \in N_k$ then simultaneously accepts or rejects ϕ_i —when indifferent, all players accept. A proposal that is accepted by i at k becomes a bilateral agreement ϕ_i^* between the pair $\{0, i\}$. A bilateral agreement once reached cannot be rebargained in the future. Therefore, if the pair $\{0, i\}$ reaches an agreement at k , we have that $i \notin N_{k'}$ for all $k' > k$. If agent i rejects the proposal made to him at k then $i \in N_{k+1}$ and i will make a proposal to the principal in period $k + 1$.

In every period $k = 2h + 1$, with $h = 0, 1, 2, \dots$, each agent $i \in N_k$ makes a simultaneous proposal to the principal to cooperate at \bar{k} and be paid $\phi_i \geq 0$. The principal then decides which of those proposals to accept or reject. Again, a proposal that is accepted by 0 at k becomes a bilateral agreement ϕ_i^* between the pair $\{0, i\}$, while if the principal rejects the proposal made by i at k then $i \in N_{k+1}$ and the principal will make a proposal to i in the next period.

The total payments agreed with the principal, prior to and at k , plus the proposals made by him at k must not exceed $v(M)$. When the game ends at \bar{k} cooperation between the principal and those agents with an agreement, i.e., the set $N \setminus N_{\bar{k}}$, takes place and a surplus of $v(M \setminus N_{\bar{k}})$ is created. Those agents without an agreement are removed from the game and receive their zero standalone payoffs. Each agent $i \in N \setminus N_{\bar{k}}$ is paid ϕ_i^* from the cooperative surplus $v(M \setminus N_{\bar{k}})$, unless $v(M \setminus N_{\bar{k}})$ is lower than the sum of the agreed payments and then it is shared proportionally only among those agents. The payoffs are then

$$u_0 = \left[v(M \setminus N_{\bar{k}}) - \sum_{N \setminus N_{\bar{k}}} \phi_i^* \right]_+, \quad u_i = \phi_i^* \min \{1, \lambda\} \text{ if } i \notin N_{\bar{k}} \text{ and } 0 \text{ if } i \in N_{\bar{k}},$$

where λ satisfies $\lambda \sum_{N \setminus N_{\bar{k}}} \phi_i^* = v(M \setminus N_{\bar{k}})$.

Proposals and agreements are private, i.e., each proposal ϕ_i and its acceptance is only observed by the pair $\{0, i\}$. Therefore the principal observes the entire history of the game but each agent i observes only the proposals ϕ_i and their acceptance or rejection. Each agent's strategy must then depend on his beliefs about the agreements reached by the principal with the remaining agents.

In a Perfect Bayesian Equilibrium (PBE) agents can hold arbitrary out-of-equilibrium beliefs, which gives rise to a multitude of equilibria. We make a usual restriction to “passive beliefs”, a belief system where after observing an unexpected action each agent still believes the other agents reach the equilibrium agreements.⁷ There is *immediate agreement* if and only if $N_1 = \{\emptyset\}$.⁸

Proposition 2. *When ϕ_M^B lies in the strict Core of (M, v) , for any δ sufficiently close to 1, there exists a unique PBE outcome with passive beliefs and immediate agreement. It converges to ϕ_M^B as $\delta \rightarrow 1$.*

⁷Passive beliefs were introduced in vertical contracting (e.g. Hart and Tirole, 1990; McAfee and Schwartz, 1994), and have since been widely used in both theory and applications.

⁸Immediate agreement is equivalent to efficiency when the marginal contribution of each agent is strictly positive.

The role of the Core in this result can be understood as follows.⁹ The stability approach considered the optimality of each agreement assuming the agreements with all remaining agents are always reached. In the present non-cooperative game it must be the case that the principal does not want to reach an agreement with some—but not all—agents, and so there must not exist an $S \subset M$ such that the principal could increase his payoff by reaching the equilibrium agreements with only each agent $j \in S$, i.e., we must not have that for some $S \subset M$

$$\phi_0^B = v(M) - \sum_N \phi_j^B < v(S) - \sum_{S \setminus \emptyset} \phi_j^B \Leftrightarrow \Delta_{N \setminus S} v(M) < \sum_{N \setminus S} \phi_j^B$$

or, equivalently, the total payment to be made to any subset of agents must not exceed their joint marginal contribution. This is the case if ϕ_M^B lies in the strict Core.

This model differs in two important aspects from the non-cooperative alternating proposals bargaining model of Stole and Zwiebel (1996). In their model the principal meets agents sequentially. Agent i is removed from the game if, along that sequence, bilateral negotiations break down while the principal is bargaining with that agent. Importantly, in that case all previously reached agreements become void and a similar bargaining (sub)game starts anew with all agents remaining in the game—and so forth until agreements can be reached with all agents present at the start of a particular (sub)game.

Their result, that the Shapley is a SPE outcome, does not rely on that particular sequential specification. The same result is obtained in a variant of our game where bilateral negotiations are simultaneous.¹⁰ The fundamental difference between these two is that in our variant a bilateral agreement once reached cannot be rebargained, while in their case all agreements are void and must be rebargained in case a single bilateral negotiation breaks down.

3 Indissoluble vs non-binding agreements

A natural question is whether indissoluble agreements hurt or benefit the principal, relative to non-binding, in an otherwise similar bargaining situation. Here we provide sufficient conditions on the surplus functions that help answer this question.

We say that marginal contributions are “large” when all marginal contributions are high or, if there are agents with a sufficiently low marginal contribution, the marginal contribution of each of those agents is larger than their respective inframarginal contributions. Marginal contributions are “small” when the sum of the joint marginal contribution of any subset S of agents with that sum of its complement $N \setminus S$ is larger than the sum of all agents’ individual marginal contributions. Formally,

Definition 2. *Marginal contributions are large if and only if $\Delta_i v(M) \geq \Delta_i v(S)$ for all $S \subset M$*

$$\text{for each } i \in N \text{ such that } \frac{1}{2} \sum_{j \in M} \min \{ \Delta_i v(M), \Delta_j v(M) \} < v(M). \quad (4)$$

Marginal contributions are small if and only if

$$\Delta_S v(M) + \Delta_{N \setminus S} v(M) \geq \sum_N \Delta_i v(M) \text{ for all } S \subseteq N. \quad (5)$$

⁹The strict Core is the set of efficient payoff vectors ϕ_M such that there exists no $S \subseteq M$ where $v(S) > \sum_S \phi_i$.

¹⁰In that variant, if by the end date \bar{k} a bilateral agreement has not been reached with each agent $i \in N$, i.e., $N_{\bar{k}} \neq \{\emptyset\}$, then a (sub)game must be restarted anew with only those agents who have reached an agreement before \bar{k} , i.e., with each $i \notin N_{\bar{k}}$, and so forth until agreements can be reached with each of the agents present at the start of a particular (sub)game.

In particular a game has large marginal contributions if the marginal contribution of each agent is larger than each of his inframarginal contributions. A game has small marginal contributions if the inframarginal contributions of each agent to subsets that include the principal always exceeds the marginal contribution of that agent.

Denote by ϕ_M^A the Shapley value of (M, v) , which Stole and Zwiebel (1996) found to be the unique stable bargaining outcome when the agreed payments are non-binding. The inability to rebargain the agreements has the following effect on the principal's payoff:

Proposition 3. *Indissoluble agreements hurt the principal (relative to non-binding) if marginal contributions are large, i.e., $\phi_0^B \leq \phi_0^A$, while they benefit the principal if marginal contributions are small, i.e., $\phi_0^B \geq \phi_0^A$.*

We now apply this result to particular well known classes. The second-order difference operator

$$\Delta_{ij}^2 v(S) = \Delta_j [\Delta_i v(S)] = [v(S \cup i \cup j) - v(S \setminus i \cup j)] - [v(S \setminus j \cup i) - v(S \setminus i \setminus j)]$$

expresses player i 's effect on player j 's marginal contribution to S . It captures the complementarity of the two players in S (see e.g. Segal, 2003). Players i and j are complements in S if $\Delta_{ij}^2 v(S) \geq 0$ and substitutes if $\Delta_{ij}^2 v(S) \leq 0$. In any game $\Delta_{i0}^2 v(S) \geq 0$ for all $i \in N$ and S , so the principal is always a complement.

Convex games, introduced by Shapley (1971), capture situations with increasing returns. It is well known that a game is convex or supermodular if and only if $\Delta_{ij}^2 v(S) \geq 0$ for all $i, j \in M$ and all $S \subseteq M$.

Big-boss games, introduced by Muto et al. (1988), capture situations with an indispensable player where the remaining non-indispensable players can increase their influence by forming a group. Player i is indispensable if and only if for all $S \subset M$ we have $v(S) = 0$ if $i \notin S$. Since the principal is an indispensable player, it follows from their definition that our setting is a big-boss game if and only if

$$\sum_S \Delta_i v(M) \leq \Delta_S v(M) \text{ for all } S \subseteq N.$$

The subclass of *total big boss games*, introduced by Voorneveld et al. (2002), requires the stronger union property

$$\sum_S \Delta_i v(T \cup 0) \leq \Delta_S v(T \cup 0) \text{ for all } S \subset T \subseteq N.$$

This condition is equivalent to $\Delta_{ij}^2 v(S \cup 0) \leq 0$ for all $i, j \in N$ and $S \subseteq N$. So, in our setting, total big boss games coincide with concave or submodular games with respect to agent inclusion—where agents are substitutes to each other.

Definition 3. *In one-to-many bargaining, agents are complements if and only if $\Delta_{ij}^2 v(S) \geq 0$ for all $i, j \in N$ and $S \subseteq M$, or equivalently if and only if (M, v) is convex. Agents are substitutes if and only if $\Delta_{ij}^2 v(S) \leq 0$ for all $i, j \in N$ and $S \subseteq M$, or equivalently if and only if (M, v) is a total big boss game.*

In convex games the marginal contributions are large and in big-boss games they are small. The following result has the simple intuition explained in the introduction.

Corollary 1. *If agents are complements, i.e., if (M, v) is convex, then indissoluble agreements hurt the principal (relative to non-binding). If (M, v) is a big-boss game then indissoluble agreements benefit the principal—and more specifically if agents are substitutes then they also hurt each agent, i.e., $\phi_i^B \leq \phi_i^A$ for all $i \in N$.*

In two situations $\phi^B = \phi^A$. The first situation are games that are simultaneously convex and concave with respect to agent inclusion, corresponding to n independent bilateral bargaining games. Each agent's payoff is then the standard Nash bargaining solution of his bilateral game. The second situation is when only cooperation by all players has a positive worth, so each player is indispensable, corresponding to a pure bargaining game with $n + 1$ players. Each player then gets an equal share of the total surplus, also the standard Nash bargaining solution of that game.

3.1 An application to intrafirm bargaining

In Stole and Zwiebel's (1996) intrafirm bargaining setting the principal is a producer hiring labour from workers with an inelastic unit supply of labour—the outside wage can be zero-normalized without loss of generality. If workers are homogenous then $v(S)$ depends only on the number of workers but not their identities and thus $v(S) = f(s)$, where $s = |S \setminus 0| + |S \cap 0|$ and $f(0) = 0$.

Westermarck (2003) used a non-cooperative game to study the effect of indissoluble agreements in the total wage bill when f is concave and he found that they lead to lower wages than non-binding ones. Do however indissoluble agreements always benefit the firm?

Our results show that this is not the case, since the effect of indissoluble agreements on the total wage bill crucially depends on the production technology. For example, they lead to a higher wage bill if f is instead convex—when f is concave the workers are substitutes while if f is convex the workers are complements.

The sufficient conditions also allow to compare situations that present simultaneously complementarity and substitution. Suppose for example that f is s-shaped (i.e., first convex and then concave) and that a worker's marginal productivity does not exceed the average worker productivity (i.e., $f(n) - f(n - 1) \leq f(n)/n$). In that case indissoluble agreements decrease the total wage bill since marginal contributions are small.¹¹

Another textbook example is a firm with a *Leontief production* function where workers of two disjoint subsets are needed in fixed proportions—say, workers in subset W are white-collar, in B are blue-collar, and $N = B \cup W$. If a unit of output is worth 1 then

$$v(S) = \begin{cases} \min(|W \cap S|, |B \cap S|) & \text{if } 0 \in S \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

In this case there is *same-side* substitution and *cross-side* complementarity, i.e., blue-collar workers are substitutes to each other but complementary to white-collar workers—and vice versa.¹² The total wage bill is higher with indissoluble wage agreements if $|B| = |W|$ and lower if $|B| \neq |W|$, since marginal contributions are then respectively large and small.

4 Conclusion

We studied a one-to-many bargaining situation where the agreed payments are indissoluble, i.e., once reached they cannot rebargained. In that case both an axiomatic and a non-cooperative approach yield the same simple solution.

We contrasted this situation with a situation where those agreements are instead non-binding, i.e., can be rebargained. We showed that the preference of the central player (the principal) for indissoluble or non-binding agreements will be determined by the productive characteristics of the

¹¹Since the joint marginal contribution of a subset of workers of size s first increases and then decreases with s , we have that for any $s \leq n$ the average joint contribution is smaller than the marginal contribution, i.e., $f(n) - f(n - 1) \leq (f(n) - f(n - s))/s$, and therefore in those games marginal contributions are small.

¹²Then $\Delta_{ij}v(S \cup 0) \in \{0, 1\}$ if $i \in B$ and $j \notin W$ while $\Delta_{ij}v(S \cup 0) \in \{-1, 0\}$ otherwise.

other players (the agents). In general rebargaining benefits the principal if the agents' marginal contributions are "large" and it hurts him if they are "small".

The present approach also helps to understand how trust in legal institutions can affect non-contractible investments by changing players' expectations over bargaining outcomes. Hart and Moore (1990) showed that giving additional assets to an agent always increases both his payoff and ex-ante investment incentives when assets are complements and players receive their Shapley value, which in a one-to-many situation is consistent with bargaining with non-binding bilateral agreements. However, if in that same situation the players instead expect the agreements to be indissoluble, then asset ownership may actually discourage ex-ante investment.¹³

To see this, consider a situation, based on the introductory example of Hart and Moore (1990), where a tycoon (the principal) needs to bargain for his holidays with a boat owner, a skipper and a chef (the agents). Suppose that cooperation of all players is worth 100, cooperation of all players except the chef is worth 80 and any other alternative is worth zero. So the tycoon, the boat and the skipper are indispensable but the chef is not. Moreover suppose the chef can make a non-contractable ex-ante investment to acquire a relationship specific skill so that the cooperation of all players is worth 124 instead of 100. It follows from our work that the chef's marginal incentive to invest is 6 if agreements are non-binding and it is 12 if they are indissoluble.

Consider now a situation where instead the chef also owns the boat, so he becomes an indispensable player. The chef's marginal incentive to invest is then 8 with both non-binding and indissoluble agreements. Therefore the ownership of the boat, an indispensable asset, makes the chef more likely to invest if the agreements are non-binding ($8 > 6$), but less likely to invest if they are indissoluble ($8 < 12$).

In general, when agreements are indissoluble an agent with a small marginal contribution has a strong incentive to increase the total surplus because he receives half of his marginal contribution, giving him a fifty percent return on investment. Owning an indispensable asset increases the share that agent receives of the total surplus, but typically this also shrinks the share he gets of his marginal contribution—and it is the latter and not the former that determines investment incentives. Ultimately bargaining leverage depends on the nexus of agreements, and it would be interesting to study further how the nature of agreements affects ex-ante investments.

¹³The present rationale is distinct from Chiu (1998) and de Meza and Lockwood (1998), who find that asset ownership can discourage investment in a two-player setting when assets provide benefits even if an agreement is not reached (as inside options)—which is not the case in our setting.

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Appendix

Proof of Proposition 1. In step 1 we characterize the unique stable outcome and in step 2 we obtain the Nucleolus equivalence.

Step 1. In the bilateral bargaining problem with agent i , for any allocation rule u satisfying (2), we have

$$\phi_0^B = v(M) - \sum_N \phi_i^B \text{ and } d_0^i = \left[v(M \setminus i) - \sum_{N \setminus i} \phi_j^B \right]_+.$$

Thus the gains from a bilateral agreement between 0 and i do not exceed i 's marginal contribution

$$(\phi_i + \phi_0 - d_0^i) \leq \Delta_i v(M).$$

Since $d_0^i \geq 0$, to be consistent with bilateral Nash bargaining we must have that

$$\phi_0^B = \phi_i^B = \left[\frac{v(M) - \sum_{N \setminus i} \phi_j^B}{2} \right]_+ \text{ if } d_0^i = 0$$

while

$$\phi_i^B = \frac{\Delta_i v(M)}{2} < \phi_0^B = v(M) - \sum_N \phi_i^B \text{ if } d_0^i > 0.$$

Thus

$$\phi_i^B = \min \left\{ \phi_0^B, \frac{\Delta_i v(M)}{2} \right\} \text{ for all } i \in N. \quad (7)$$

It follows that ϕ_M^B is unique. To see this, suppose otherwise, that there were two distinct vectors ϕ_M^B and $\phi_M^{B'}$. By Pareto optimality $\sum_M \phi_i^{B'} = \sum_M \phi_i^B = v(M)$ and we could then find at least one agent i such that

$$\phi_0^B \geq \phi_0^{B'} \text{ and } \phi_i^B < \phi_i^{B'},$$

contradicting (7). Its existence is insured by construction: just note from (7) that ϕ_M^B divides equally each dollar of the surplus by all players until half the lowest marginal contribution is reached. At that moment those agents receiving half their marginal contributions are removed and the next dollar is divided equally among the remaining players. It proceeds in this way, removing those agents who reach half their marginal contributions, until the total surplus is distributed or, if each agent can get half of his marginal contribution the remainder goes to the principal.

Step 2. The payoffs defined in (7) can be rewritten as

$$\phi_i^B = \begin{cases} \min \left\{ \lambda, \frac{\Delta_i v(M)}{2} \right\} & \text{if } \sum_M \frac{\Delta_i v(M)}{2} \geq v(M) \\ \Delta_i v(M) - \min \left\{ \lambda, \frac{\Delta_i v(M)}{2} \right\} & \text{if } \sum_M \frac{\Delta_i v(M)}{2} < v(M) \end{cases} \quad (8)$$

where λ is such that $\lambda \sum_M \phi_i^B = v(M)$. Associate to (M, v) a bankruptcy problem where the amount to be divided (the estate) is $v(M)$ and each player's claim is his marginal contribution $\Delta_i v(M)$. Then (8) satisfies the Talmud rule of that problem and therefore it follows from Aumann and Maschler (1985) that ϕ_M^B coincides with the Nucleolus of the bankruptcy problem we associated to (M, v) .

Proof of Proposition 2. In step 1 we look at the agents' behavior in bilateral bargaining. In step 2 we show that behavior implies that there can be at most one equilibrium outcome, which

converges to ϕ_M^B as $\delta \rightarrow 1$. In step 3 we propose a strategy profile such that, if ϕ_M^B lies in the strict Core, forms an equilibrium with the proposed outcome for any δ sufficiently close to one.

Step 1. An equilibrium has immediate agreement if and only if the principal's proposals at $k = 0$ are accepted by all agents. When agents' beliefs are passive, for any proposal made to agent i at $k = 0$, i believes that each remaining agent j has received and accepted his equilibrium proposal denoted by $\phi_j^*(\delta)$. Therefore agent i views the remainder of the game as a standard bilateral bargaining game with alternating proposals. Since the sum of the offers made and accepted cannot exceed $v(M)$, the gains from agreement in that game are perceived by i to be

$$G_i(\delta) = \left[v(M) - \sum_{N \setminus i} \phi_j^*(\delta) \right] - \left[v(M \setminus i) - \sum_{N \setminus i} \phi_j^*(\delta) \right]_+ \geq 0.$$

It is well known that in such situation, in the unique subgame perfect equilibrium of that game, agent i uses a simple cut off strategy at any date k : in every even period he accepts any proposal equal or greater than $\frac{\delta}{1+\delta}G_i(\delta)$ and in every odd period he proposes $\frac{1}{1+\delta}G_i(\delta)$ (see e.g. Binmore et al., 1986).

Step 2. It follows that in any PBE with passive beliefs and immediate agreement the principal must make at $k = 0$ to each $i \in N$ a proposal

$$\phi_i^*(\delta) = \frac{\delta}{1+\delta} \min \left\{ v(M) - \sum_{N \setminus i} \phi_j^*(\delta), \Delta_i v(M) \right\},$$

and the principal's payoff will be

$$\phi_0^*(\delta) = \left[v(M) - \sum_N \phi_i^*(\delta) \right]_+,$$

since the principal would not make a proposal lower than $\phi_i^*(\delta)$ to any $i \in N$ as that would not affect i 's acceptance decision. The solution to these m equations, $\phi_M^*(\delta)$, exists and is unique. To see this, suppose there were two solution vectors $\phi_M^*(\delta)$ and $\phi'_M(\delta)$. Then for at least one agent i we have that

$$\phi_0^*(\delta) \geq \phi'_0(\delta) \text{ and } \phi_i^*(\delta) < \phi'_i(\delta),$$

contradicting the equations above. This solution exists, as can be shown by constructing a system of connected vessels—as explained in the introduction—but here the vessels representing the agents have a *width of* δ (instead of 1) and are sealed at $1/(1+\delta)$ of their height (instead of half), while the principal's vessel remains unchanged—i.e., unsealed with unitary width and height $v(M)$. If we introduce an amount of water $v(M)$ in the system, a stable situation will be reached with the water distributed according to $\phi_M^*(\delta)$. So for each agent i there is a unique strategy consistent with PBE with passive beliefs and immediate agreement where in every even period he always accepts any proposal equal or greater than $\phi_i^*(\delta)$ and in every odd period makes a proposal of $\phi_i^*(\delta)/\delta$. Therefore the vector $\phi_M^*(\delta)$ is the unique candidate outcome of PBE with passive beliefs and immediate agreement. It follows from its construction above that $\phi_M^*(\delta)$ converges to ϕ_M^B as $\delta \rightarrow 1$.

Step 3. We now show that an equilibrium with the outcome $\phi_M^*(\delta)$ exists for δ sufficiently close to 1 if ϕ_M^B lies in the strict Core. Take a strategy profile where in any even period k the principal makes offers $\phi_i^*(\delta)$ to each agent without a contract and each agent in N_k accepts any proposal equal or greater than $\phi_i^*(\delta)$. Moreover in every even period each agent without a contract offers $\phi_i^*(\delta)/\delta$ and the principal accepts the set of offers that maximizes his expected payoff. The principal's optimal deviation should maximize his profit subject to the agents' strategies. Notice

however that at $k = 0$ it is never optimal for the principal to deviate and instead accept offers in a future odd period because if an agreement is supposed to be reached with a subset of agents at some odd period k it would have been optimal for the principal to reach them at the even period $k - 1$ while making lower payments, thus increasing his expected payoff. But then we can also rule out any deviations by the principal that involve some finite delay in reaching an agreement, since starting with the latest set of agreements to be reached in an even period in a potential deviation, if those agreements were supposed to be reached at some even period k then it would have been at least as good to reach them at $k - 2$ given that the same payments need to be made but avoiding the expected loss from a potential breakdown. It remains to consider the possibility of perpetual disagreement with some subset of players. Suppose that ϕ_M^B lies in the strict Core, then for all $S \subseteq N$ we have

$$\Delta_S v(M) < \sum_S \phi_i^B.$$

and, by the continuity of $\phi_M^*(\delta)$, for any δ sufficiently close to 1 we have that

$$\Delta_S v(M) \leq \sum_S \phi_i^*(\delta).$$

So, when ϕ_M^B lies in the strict Core, for any δ sufficiently close to 1 immediate agreement is optimal for the principal since

$$\phi_0^*(\delta) = v(M) - \sum_N \phi_i^*(\delta) \geq v(M/S) - \sum_{N/S} \phi_i^*(\delta) \text{ for all } S \subseteq N$$

and all players accept when indifferent. Since $\phi_M^*(\delta)$ changes continuously with δ , if ϕ_M^B lies in the strict Core then for δ sufficiently close to 1 the candidate vector $\phi^B(\delta)$ is the unique PBE outcome with passive beliefs and immediate agreement.

Proof of Proposition 3. In step 1 we present our working definition of the Shapley value. In step 2 we study the case where marginal contributions are large, and in step 3 the case where marginal contributions are small.

Step 1. The Shapley value can be captured in the following simple way: Imagine that the players are ordered randomly and let Π denote the set of $(n + 1)!$ possible orderings of the elements of M . If a player is placed after a set of players S then he is paid $\Delta_i v(S)$. The Shapley value is simply the expectation of these payments taken over all orderings when each ordering is equally likely. Formally, let $\pi(i)$ be the rank of player i in the ordering $\pi \in \Pi$, and π^i the set of players that come before i in ordering π , i.e., $\pi^i = \{j \in M : \pi(j) < \pi(i)\}$. Then

$$\phi_i^A = \frac{1}{(n + 1)!} \sum_{\pi \in \Pi} \Delta_i v(\pi^i) \text{ for all } i \in M.$$

Note that $\phi_0^A \geq \phi_i^A$ for all $i \in N$. Also, take an ordering of players $\pi \in \Pi$ such that the principal takes the first position, i.e., $\pi(0) = 1$. For any such π we have that $v(M)$ is equal to the sum of the contribution of each agent i to the set of the proceeding players, i.e.,

$$v(M) = \sum_N \Delta_i v(\pi^i) \text{ for any } \pi \in \{\Pi : \pi(0) = 1\}.$$

Step 2. When marginal contributions are large there must be at least one $i \in M$ such that (4) is not satisfied. To see this suppose that (4) is satisfied for all $i \in M$. Since

$$v(M) = \sum_N \Delta_i v(\pi^i) \leq \sum_N \Delta_i v(M) \text{ and } \Delta_0 v(M) = v(M)$$

we have a contradiction as

$$\sum_{j \in M} \min \{ \Delta_0 v(M), \Delta_j v(M) \} \geq 2v(M).$$

Therefore when marginal contributions are large ϕ_M^B is obtained with (8). This partitions the set of players into a subset of players \widehat{S} (which may be empty) that satisfy (4) and receive half of their individual marginal contribution, and its complement $M \setminus \widehat{S}$ whose members receive an equal share of the remainder, i.e., $i \in \widehat{S}$ if and only if it satisfies (4) and

$$\phi_i^B = \frac{\Delta_i v(M)}{2} \text{ if } i \in \widehat{S} \text{ while } \phi_i^B = \frac{v(M) - \sum_{\widehat{S}} \phi_j^B}{|M| - |\widehat{S}|} = \phi_0^B \text{ if } i \in N \setminus \widehat{S}. \quad (9)$$

For each $i \in \widehat{S}$ we have that $\Delta_i v(S) \leq \Delta_i v(M)$ for all $S \subseteq M$ and therefore

$$\phi_i^A \leq \frac{\Delta_i v(M)}{2} \text{ for all } i \in N$$

since the principal is placed before agent i in only half the orderings $\pi \in \Pi$. Moreover since $\phi_0^A \geq \phi_i^A$ for all $i \in N$, we have that $\phi_0^A \geq v(M)/m$ and therefore

$$\phi_0^A \geq \frac{v(M) - \sum_S \phi_i^A}{|M| - |S|} \text{ for any } S \subseteq N. \quad (10)$$

It follows from (9) and (10) that $\phi_0^B \leq \phi_0^A$.

Step 3. When marginal contributions are small we have that

$$v(M) = \Delta_N v(M) \geq \sum_N \Delta_i v(M) \text{ and } \Delta_0 v(M) = v(M),$$

so from (8) we have that $\phi_i^B = \Delta_i v(M)/2$ for all $i \in N$ and therefore

$$\phi_0^B = v(M) - \sum_N \frac{\Delta_i v(M)}{2}.$$

Now take an ordering π such that $\pi(0) = k$, so π^0 the set of the first $k-1$ players in π . Take also the ordering π' such that for all $i \in M$ we have $\pi'(i) = n+1 - \pi(i)$, so $\pi'(0) = n+1 - k$. Note that $N \setminus \pi^0$ is now the set of the first $n-k$ players in π' , and we have

$$\Delta_0 v(\pi^0) + \Delta_0 v(\pi'^0) = 2v(M) - \Delta_{\pi^0} v(M) - \Delta_{N \setminus \pi^0} v(M).$$

With (5) we then have

$$\Delta_0 v(\pi^0) + \Delta_0 v(\pi'^0) \leq 2v(M) - \sum_N \Delta_i v(M).$$

Since the $(n+1)!$ orderings can be grouped in $(n+1)!/2$ such pairs, we have that

$$\phi_0^A = \frac{1}{(n+1)!} \sum_{\pi \in \Pi} \Delta_0 v(\pi^0) \leq v(M) - \sum_N \frac{\Delta_i v(M)}{2},$$

and therefore $\phi_0^A \leq \phi_0^B$.

Proof of Corollary 1. It is immediate to see that convex games are games where marginal contributions are large and that big boss games are games where marginal contributions are small. The case where marginal contributions of the agents are decreasing with respect to agent inclusion: For any agent $i \in N$ in half the orderings $\pi \in \Pi$ we have that $0 \notin \pi^i$ and so $\Delta_i v(\pi^i) = 0$. In the remaining half of the orderings we have that $0 \in \pi^i$, but in the present case for those π we also have that $\Delta_i v(\pi^i) \geq \Delta_i v(M)$. Therefore in this case $\phi_i^A \geq \frac{\Delta_i v(M)}{2}$ for all $i \in N$. We saw in the proof of Proposition 3 that when marginal contributions are small, and therefore when agents are substitutes, $\phi_i^B = \frac{\Delta_i v(M)}{2}$. Therefore in this case $\phi_i^A \geq \phi_i^B$ for all $i \in N$.