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Opacity in Financial Markets*

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Abstract

This paper studies the implications of opacity in financial markets for investor behavior, asset prices, and welfare. Transparent funds (e.g., mutual funds) and opaque funds (e.g., hedge funds) trade transparent assets (e.g., plain-vanilla products) and opaque assets (e.g., structured products). Investors observe neither opaque funds' portfolios nor opaque assets' payoffs. Consistent with empirical observations, an "opacity price premium" arises: opaque assets trade at a premium over transparent ones despite identical payoffs. This accompanies endogenous market segmentation: transparent (opaque) funds trade only transparent (opaque) assets. The opacity price premium incentivizes financial engineers to render transparent assets opaque deliberately. (*JEL* D80, G10, G23)

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Opacity is central to modern finance. Opaque investment companies, such as hedge funds—typically with secretive investment strategies and undisclosed holdings—have rapidly grown in size and seem to have played a major role in the markets.[1] Moreover, the importance of opaque and complex financial assets, such as sophisticated structured products—whose payoff information is incomprehensible and/or inaccessible to most retail investors—was highlighted during the 2007–2009 financial crisis. Despite its importance, however, the implications of opacity in financial markets are not fully understood. How does opacity affect investor behavior, asset prices, and welfare?

In terms of asset prices, an intriguing empirical fact is that opaque and complex assets have been traded at a premium, rather than at a discount. Coval, Jurek, and Stafford (2009) find that senior collateralized debt obligation tranches were significantly overpriced before 2007. Henderson and Pearson (2011) report that a retail structured equity product's price is almost 8% greater than its fair value. Célérier and Vallée (2013) find that structured products are traded at a premium in Europe; the more complex a product, the more pronounced its overpricing.[2] These observations are puzzling: they appear to be inconsistent with standard asset-pricing models with rational agents. Such models might predict that investors unable to comprehend the nature of an asset would require a discount on the price, rather than pay a premium. Why do opaque assets trade at a premium? More fundamentally, why do opaque assets emerge in the first place?

To answer these questions, this paper develops a fully rational, dynamic asset-market equilibrium model with portfolio delegation. More specifically, I consider a discrete time model with infinite horizon. Its baseline version has one risky asset and one riskless asset. The risky asset's periodic payoff is the sum of a persistent component and a transitory component, both stochastic and unobservable. Over time, agents learn about the persistent component, based on the payoff history. There is a continuum of investment funds, each with one fund manager and a number of investors. The investors can invest directly in the riskless asset. However, investing in the risky asset requires that they give capital to the manager, who forms a portfolio consisting of the risky and riskless assets. The manager earns a management fee proportional to the assets under management.

The model features two types of opacity in financial markets. First, the funds are opaque in that investors cannot observe the funds' portfolios and the fund managers

[1] According to Hedge Fund Research, in 1990 there were only 610 hedge funds managing \$39 billion of assets globally, but in 2013 there are more than 10,000 funds with \$2,400 billion of assets.

[2] See also Jarrow and O'Hara (1989) and Rogalski and Seward (1991), who find overpricing of Primes and Scores and foreign currency exchange warrants, respectively.

cannot commit to their portfolio choices. An example is a hedge fund adopting a flexible trading strategy that is not communicated to investors. Second, the risky asset is opaque in that investors cannot observe its payoffs. An example is a sophisticated derivative whose payoff information is, for nonprofessional investors, unavailable or prohibitively costly. Another manifestation of opacity is the complex nature of an asset that makes it difficult to comprehend its payoffs. For instance, understanding the payoff of a structured product may require wading through its prospectus and disclosure documents, which are hundreds of pages long and filled with technical jargon. The volume of information and extent of technical difficulty make the asset's payoffs effectively unobservable to investors.[3]

An important consequence of these two layers of opacity is that, although investors can (obviously) observe the total return from a fund, they cannot see the composition of that return. This is the source of an agency problem. Investors try to infer (i.e., back out) the opaque asset's unobservable payoff from the observed fund return, learn about the persistent component of that asset, and allocate capital on that basis. Yet because the fund manager controls the portfolio that determines the fund return, that manager can potentially manipulate investor learning through his portfolio choice. More specifically, the manager can boost the expected fund return by (secretly) levering up and overinvesting in the opaque asset, in an attempt to inflate investors' estimates of the persistent component of the asset's payoff and hence their assessments of the fund's future prospects. Investors are thus led to allocate more capital to the fund, which yields more fees. That is, the manager is motivated by an implicit incentive that is akin to an endogenous "career concern."

The model yields a number of results. An increase in fund manager career concerns leads to a higher price of the opaque asset, higher fund leverage, reduced fund performance, smaller fund size, and less social welfare. (Career concerns themselves are endogenous: they increase with the volatility of the persistent component of the opaque asset's payoff and decrease with the volatility of the transitory component of the payoff.) A key mechanism for the results is "signal jamming." Managers who have greater career concerns are more inclined to lever up secretly in an effort to inflate investor expectations about their funds' future performance, thereby attracting more capital and thus more fees. Investors understand the managers' desire to fool them and hence are not fooled in equilibrium; nevertheless, the managers still lever up because otherwise their funds' future prospects would be underestimated by the investors who believe that the managers do lever up

[3]Brunnermeier and Oehmke (2009) refer to this as the information overload problem.

secretly. Overinvestment drives up the asset's price, resulting in lower expected returns for both the asset and the funds. This discourages investors from investing in the funds, so the funds shrink. In terms of utility, investors are unaffected, but managers are worse off: they attract less capital and thus earn lower fees as they fail to commit to not fool investors using opacity.

In contrast, if funds are transparent and portfolios are observable (e.g., mutual funds), then the equilibrium does not depend on whether or not the risky asset is opaque. Indeed, regardless of managers' actions, investors always correctly back out the asset's payoff from the funds' observed portfolios and returns, and therefore there is no scope for the managers to manipulate investor learning. Therefore, the managers' career concerns are also irrelevant.

Next, I extend the model to accommodate both transparent and opaque funds, as well as transparent and opaque assets with identical payoffs. Each fund can choose one of the two risky assets to which to invest. In equilibrium, the opaque asset trades at a premium over the transparent asset, consistent with the empirical facts already noted. The result holds even though it is common knowledge that these assets yield identical payoffs and all funds can purchase whichever asset they wish. The price gap between these assets—the "opacity price premium"—increases with the extent of opaque fund managers' career concerns. Accompanied by the opacity price premium, the market is endogenously segmented: transparent funds trade only transparent assets, and opaque funds trade only opaque assets. This is consistent with the real-world observation that mutual funds tend to focus on traditional asset classes, whereas hedge funds often trade opaque, complex financial instruments. The reason behind this segmentation is as follows. Because opaque fund managers cannot commit to their portfolio choices, a moral hazard problem prevents them from buying the transparent asset credibly; however, they can purchase the opaque asset credibly, being motivated by their desire to inflate the investors' expected fund assessment. In contrast, transparent fund managers simply buy the cheaper transparent asset because the asset's opacity is irrelevant for them.

Last, the fundamental question of why opaque assets emerge in the first place is addressed. In this model, they arise naturally from the demand of opaque fund managers. To study the supply of opaque assets, I introduce "financial engineers"—that is, agents who can make a transparent asset opaque and vice versa. In effect, the engineers act as arbitrageurs who exploit the opacity price premium: as long as the premium is positive, they buy transparent assets, make them opaque, and sell them at a profit. In equilibrium then the engineers serve to eliminate the premium. A novel insight is that opacity is self-feeding in financial markets: given the opacity price premium, financial engineers exploit it by supplying opaque assets, which in turn are a source of agency problems in portfolio delegation, resulting in the opacity price premium.

This paper is related to the theoretical literature on complexity and obfuscation in financial markets. Arora et al. (2009) show that the computational complexity of financial derivatives may amplify adverse selection between buyers and sellers. I also show that failing to understand financial instruments exacerbates asymmetric information problems. In the work of Carlin (2009), oligopolistic firms add complexity to the price structures of their products as a means of increasing their market power. Carlin and Manso (2011) argue that, to preserve their information rents, financial institutions alter their retail product offerings and in this way "interfere" with the learning of unsophisticated investors. As in those papers, opacity in this paper is a strategic tool for exploiting less-informed agents. Yet in contrast to those papers, our main focus is on the equilibrium price of opaque financial assets in a competitive market.[4]

A growing body of theoretical literature discusses the equilibrium implications of delegated portfolio management (Allen and Gorton 1993; Shleifer and Vishny 1997; Vayanos 2004; Cuoco and Kaniel 2011; He and Krishnamurthy 2012, 2013; Malliaris and Yan 2012; Vayanos and Woolley 2013). Kaniel and Kondor (2013) study, as do I in this paper, the equilibrium asset prices and trading strategies of fund managers concerned with fund flows. The flow-performance relationship is exogenous in their paper, but in this paper, it stems endogenously from learning. The work of Berk and Green (2004) is also related, in that fund flows endogenously stem from learning. Also, fund's asset size in this paper is determined in a similar fashion to their model, with the assumption that there are decreasing returns to scale in each fund's return. However, their model does not include asset prices and does not explicitly address the manager-investor relationship.

Finally, this paper is also related to the literature on career concerns and asset prices (Dasgupta and Prat 2008; Guerrieri and Kondor 2012; Dasgupta, Prat, and Verardo 2011). In these papers, fund managers attempt to influence investor evaluation of their ability. This paper is methodologically similar in that managers seek to influence investor expectations about their funds' future prospects.

[4]See also Caballero and Simsek (2010), who model complexity as banks' limited knowledge of the interlinkages between banks in a financial network.

1 Model

Time t is discrete and runs from zero to infinity. There is a single risky asset and a riskless asset. The riskless asset has an infinitely elastic supply at an exogenous rate of return $r > 0$ and is freely accessible to all agents. There are two classes of competitive agents—fund managers and investors—and it is only through fund managers that investors can access the risky asset. The risky asset is opaque in that its payoffs are unobservable to the investors. The managers' portfolio choices, the investors' capital investments in the funds, and the risky asset's price are determined in equilibrium.

1.1 Risky asset

1.1.1 Payoff

In period $t = 1, 2, ...$, the risky asset yields a stochastic per-share payoff of $\delta_t = \bar{\delta}_t + u_t$. Here, the persistent component $\bar{\delta}_t$ evolves according to $\bar{\delta}_t = \bar{\delta}_{t-1} + v_t$, where the noise v_t is i.i.d. across time and is normally distributed with mean 0 and variance $1/\eta_v$. The initial value $\bar{\delta}_0$ is drawn by nature from a normal distribution with mean $\hat{\delta}_0$ and variance $1/\eta_0$. The transitory component u_t is i.i.d. across time and is normally distributed with mean 0 and variance $1/\eta_u$. Nobody in this economy can observe $\bar{\delta}_t$, v_t, or u_t. The payoff history up to period t is denoted by $\mathcal{H}_t \equiv (\delta_1, ..., \delta_t)$.

1.1.2 Price and return

The asset is traded in the market at a publicly observable market-clearing price, P_t. The asset's supply, $S > 0$, is constant over time. I denote by $R_{t+1} \equiv \delta_{t+1} + P_{t+1} - (1+r)P_t$ the excess return on the risky asset per share (i.e., the dollar-value excess return multiplied by P_t). The expected excess return conditional on $\mathcal{H}_t$ is $\hat{R}_{t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{H}_t]$.

1.1.3 Asset's opacity

In Sections 1, 2, and 3, the risky asset is assumed to be opaque in the sense that the payoff realization δ_t is not directly observable to the investors.[5] The managers can observe δ_t directly. An alternative interpretation of δ_t's unobservability is that the asset is "complex" in that the investors are unable to understand its payoffs. As explained earlier, such complexity makes the asset's payoff effectively unobservable. An example of the

[5]It is not important that only the opaque asset is available for investment. Sections 4 and 5 analyze the cases in which the transparent asset (with observable δ_t) coexists with the opaque asset.

opaque asset is derivative-based securities, such as a structured product. Anecdotal evidence mentioned in the popular press is that it is virtually impossible for nonprofessional investors to track and/or comprehend such a security's payoffs because they typically "feature payouts linked underlying assets such as a narrow or proprietary index or some other obscure benchmark (*Financial Times*, September 29, 2013)."[6]

1.2 Portfolio management

There is a measure-1 continuum of investment funds, each indexed by $i \in [0,1]$. Fund i consists of an infinitely lived fund manager (manager i) and a number of risk-neutral investors from overlapping generations, who each live for two periods (investors i). Investors i can invest capital in fund i, in which manager i allocates the capital between the risky and the riskless assets. I assume that each fund is "captive" in the sense that investors i can neither invest in nor observe any activities in the other funds. This assumption simplifies the investor's inference problem, while preserving each manager's competitive behavior in the asset market.

1.2.1 Capital investment

In period t, a large number of investors i of generation t arrive at each fund i. Each investor is endowed with a certain amount of dollars and decides to invest them in the fund and the riskless asset. Let $X_{i,t}$ denote the fund's assets under management, that is, the total amount of capital that investors i invest in the fund in period t. I refer to $X_{i,t}$ as the size of fund i.

1.2.2 Portfolio choice

On behalf of investors i, manager i forms a portfolio. For each dollar of investor capital, the manager buys $\theta_{i,t} \in [0, \infty)$ shares of the risky asset and invests $(1 - P_t\theta_{i,t})$ dollars in the riskless asset.[7] That is, the manager purchases $\theta_{i,t}X_{i,t}$ shares of the risky asset in total. I refer to $\theta_{i,t}$ as the manager's portfolio.

[6] Given the assumption that P_t is observable to investors, a suitable example of opaque assets can be derivatives traded on exchanges (as opposed to over the counter). Their transaction prices are readily available at the exchanges' Web sites (e.g., Eurex Group), but their payoff information is difficult to track for most unprofessional investors. Also, even complicated structured products are traded on secondary markets and the price data are available at trading platforms' Web sites (e.g., Interactive Investor), whereas their periodic payoffs are hard to track for most retail investors.

[7] If $1 > P_t\theta_{i,t}$, then the manager buys the riskless asset; if $1 < P_t\theta_{i,t}$, then he sells the asset (i.e., borrows funds) to finance purchasing the risky asset.

1.2.3 Fund return

Following Berk and Green (2004), I assume that each fund must expend resources that are increasing and convex in the fund size $X_{i,t}$. For tractability, the cost is specified as $cX_{i,t}^2$ with $c > 0$.[8] This management cost can be viewed (as in Berk and Green 2004) as the trading costs associated with liquidity or price impacts in (unmodeled) thin financial markets. The fund's total payout to investors in period $t+1$, denoted $Y_{i,t+1}$, consists of the proceeds from the portfolio minus the management cost: $Y_{i,t+1} \equiv R_{t+1}\theta_{i,t}X_{i,t} + (1 + r)X_{i,t} - cX_{i,t}^2$. I refer to $Y_{i,t+1}$ as the fund return. Convexity of the management cost leads to decreasing returns to scale in fund return, which has been documented in the empirical literature.[9] Each investor i can observe the entire history of both $X_{i,t}$ and $Y_{i,t}$.

1.2.4 Manager's objective

The manager derives a benefit from the size $X_{i,t}$ of the fund managed. I specify the benefit as $\phi X_{i,t}$ with $\phi > 0$. This benefit can be interpreted as a fixed-percentage management fee or as utility resulting from a manager's "empire building" motive. The interpretation as a fee is plausible because the compensation contracts in most investment funds include a fixed percentage of assets under management (Deli 2002). Therefore, in what follows I refer to this benefit as the fee.[10] To prevent the manager from choosing an infinitely large $\theta_{i,t}$, I assume that he incurs a nonpecuniary cost $\kappa\theta_{i,t}^2/2$, where $\kappa > 0$, when choosing $\theta_{i,t}$. This cost can be interpreted as a reduced form of the manager's risk aversion. Indeed, in an alternative setting in which (1) the manager has a mean-variance objective function and (2) the fund's management cost, c, is stochastic, this quadratic cost would exactly represent his risk aversion (see Appendix A for details). Instead of deriving the risk-aversion formulation from first principles, I use this reduced-form specification to preserve tractability. The manager's utility in period t is the difference between the fee and his personal cost of choosing $\theta_{i,t}$. Hence, manager i's problem in period t, denoted by $\mathcal{P}_{i,t}^M$, is

[8] The quadratic form is tractable because, as shown in what follows, it will make $X_{i,t}$ linear in the investors' expected excess return on the risky asset. As in other standard signal jamming models, preserving a linear structure is critical for the model's solvability.

[9] For empirical evidence of decreasing returns to scale, see Chen et al. (2004) and Edelen, Evans, and Kadlec (2007) for mutual funds and Agarwal, Daniel, and Naik (2004) for hedge funds. Given these pieces of evidence, it is now standard to assume decreasing returns to scale in theoretical models. Besides Berk and Green (2004), see also Pastor and Stambaugh (2012) and Savov (2013).

[10] It is also possible to assume that each investor actually pays a monetary fee with a proportional rate $\phi \in (0, 1)$ to the manager. Although closed-form solutions exist for the model in that setting and the economic insights are very similar, the solutions are less tractable computationally.

choosing $\theta_{i,t} \in [0, \infty)$ to maximize

$$\mathrm{E}\left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\phi X_{i,t+\tau} - \frac{\kappa}{2}\theta_{i,t+\tau}^{2}\right) \middle| \mathcal{F}_{i,t}^{M}\right], \tag{1.1}$$

where $\beta \in (0,1)$ is a discount factor and $\mathcal{F}_{i,t}^{M} = \{Y_{i,\tau}, X_{i,\tau}, \theta_{i,\tau}, P_{\tau}, \delta_{\tau} : \tau \leq t\}$ is the manager's information set in period t.

1.2.5 Fund's opacity

I compare the case of transparent funds (Section 2), where investors can directly observe $\theta_{i,t}$, with the case of opaque funds (Section 3), where investors cannot observe $\theta_{i,t}$ and the manager cannot commit to the choice of $\theta_{i,t}$. An example of a transparent fund is a mutual fund whose portfolio holdings are disclosed and whose trading strategy is well understood by investors. An example of an opaque fund is a hedge fund that adopts a flexible trading strategy that is not communicated to investors.

1.3 Learning

Because $\bar{\delta}_t$ is unobservable, agents try to learn it using a Kalman filter. The learning processes of managers and investors can be different because of a difference in their ability to observe the asset's periodic payoff δ_t. These are detailed as follows.

1.3.1 Managers' learning

The managers' learning is standard: in period t, they estimate $\bar{\delta}_t$ based on the payoff history $\mathcal{H}_t$ that they observe directly. Let $\hat{\delta}_t \equiv \mathrm{E}[\bar{\delta}_t|\mathcal{H}_t]$ denote their period-t estimate of $\bar{\delta}_t$.[11] By Kalman filtering, if they observe δ_t, they update $\hat{\delta}_t$ as

$$\hat{\delta}_t = \lambda_t \hat{\delta}_{t-1} + (1-\lambda_t)\delta_t \tag{1.2}$$

with the updating factor $\lambda_t \in (0,1)$ evolving deterministically as

$$\lambda_{t+1} = \frac{1}{2 + \frac{\eta_u}{\eta_v} - \lambda_t}, \tag{1.3}$$

[11]Note that $\hat{\delta}_t$ is the estimate not only of $\bar{\delta}_t$ but also of its future values $\bar{\delta}_{t+\tau}$ ($\tau = 1, 2, ...$), future payoffs $\delta_{t+\tau}$ ($\tau = 1, 2, ...$), and future values of $\hat{\delta}$ itself (i.e., $\hat{\delta}_t$ is a martingale). That is, $\mathrm{E}[\bar{\delta}_{t+\tau}|\mathcal{H}_t] = \mathrm{E}[\delta_{t+\tau}|\mathcal{H}_t] = \mathrm{E}[\hat{\delta}_{t+\tau}|\mathcal{H}_t] = \hat{\delta}_t$ holds for $\tau = 1, 2, ...$

as shown in Appendix B. It is clear from (1.3) that, as $t \to \infty$, λ_t converges to a constant

$$\lambda \equiv 1 + \frac{1}{2}\left(\frac{\eta_u}{\eta_v}\right) - \left(\frac{1}{4}\left(\frac{\eta_u}{\eta_v}\right)^2 + \left(\frac{\eta_u}{\eta_v}\right)\right)^{1/2}. \tag{1.4}$$

Note that $\mathrm{d}\lambda/\mathrm{d}\eta_u < 0$, $\mathrm{d}\lambda/\mathrm{d}\eta_v > 0$, and $0 < \lambda < 1$. Given $\lambda < 1$, (1.2) implies that the true value of $\bar{\delta}_t$ is never learned, even when $t \to \infty$. This is because $\bar{\delta}_t$ itself is stochastically time varying (i.e., $\eta_v < \infty$).

1.3.2 Investors' learning

Unlike the managers, the investors cannot observe δ_t directly. Hence, they estimate $\bar{\delta}_t$ based on the value of δ_t that they infer from the available information. There is a subtlety here. As shown later, the investors will correctly infer δ_t on the equilibrium path, both in the transparent fund case (Section 2) and the opaque fund case (Section 3). So their learning on the equilibrium path is described by (1.2) as well. However, in the opaque fund case, the investors' inference of δ_t may be incorrect off the equilibrium path because the managers can manipulate their inference by secretly deviating from their equilibrium strategy (i.e., choosing $\theta_{i,t}$ that is not anticipated by the investors) in an effort to attract more investor capital and thereby increase fees. The investors are not fooled on the equilibrium path because they are rational; however, importantly, the fact that they are potentially fooled off the equilibrium path does affect the agents' actions and the prices on the equilibrium path. Therefore, it is critical (especially in the analysis of the opaque fund case) to distinguish between the true value of δ_t and the value of δ_t inferred by the investors. Let $\mathcal{H}^I_{i,t} \equiv (\delta^I_{i,1}, ..., \delta^I_{i,t})$ denote the payoff history that investors i infer, where $\delta^I_{i,t}$ is the value of δ_t they infer. Their period-t estimate of $\bar{\delta}_t$ is $\hat{\delta}^I_{i,t} \equiv \mathrm{E}[\bar{\delta}_t|\mathcal{F}^I_{i,t}]$, where $\mathcal{F}^I_{i,t}$ is the information set of investors i in period t (defined later). Similarly, their conditional expectation of the excess return is $\hat{R}^I_{i,t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{F}^I_{i,t}]$. By Kalman filtering, if they infer that the value of δ_t is $\delta^I_{i,t}$, they update $\hat{\delta}^I_{i,t}$ as

$$\hat{\delta}^I_{i,t} = \lambda_t \hat{\delta}^I_{i,t-1} + (1 - \lambda_t)\delta^I_{i,t}, \tag{1.5}$$

where $\hat{\delta}^I_{i,0} = \hat{\delta}_0$ is exogenously given and the updating factor λ_t is the same as that of (1.2) (see Appendix B). Clearly, if $\mathcal{H}^I_{i,t} = \mathcal{H}_t$, which holds on the equilibrium path, then (1.5) is identical to (1.2).

1.4 Definition of equilibrium

The equilibrium consists of the price function $P_t(\hat{\delta}_t)$, the fund size $X_{i,t}$, and the manager's portfolio $\theta_{i,t}$ for $i \in [0,1]$ such that, for all t, the following statements hold.

1. Given $P_t(\hat{\delta}_t)$ and the others' actions, each investor optimally allocates her endowment between the fund and the riskless asset.

2. Given $P_t(\hat{\delta}_t)$ and the others' actions, manager i solves $\mathcal{P}^M_{i,t}$.

3. The risky asset's market clears:

$$\int_0^1 \theta_{i,t} X_{i,t} \, \mathrm{d}i = S. \tag{1.6}$$

4. Every agent has correct beliefs about the other agents' actions.

5. Each agent updates the estimate of $\bar{\delta}_t$ via Kalman filtering.

2 Transparent Funds

As a benchmark, I start by characterizing the equilibrium for the case of transparent funds; that is, $\theta_{i,t}$ is directly observable to investors i when they make investment decisions. The risky asset is opaque: δ_t is unobservable to the investors. The information set of investors i in period t, which includes variables that are directly observable to them, is $\mathcal{F}^I_{i,t} = \{Y_{i,\tau}, X_{i,\tau}, \theta_{i,\tau}, P_\tau : \tau \leq t\}$.

It is important in the transparent fund case that all investors always correctly infer δ_t, despite it not being directly observable, whether on or off the equilibrium path. The reason is that, in period t, investors i can back out δ_t from the observed $Y_{i,t}$, $X_{i,t-1}$, P_t, P_{t-1}, and $\theta_{i,t-1}$. Thus, effectively, δ_t can be included in the investors' information set: $\mathcal{F}^I_{i,t} = \{Y_{i,\tau}, X_{i,\tau}, \theta_{i,\tau}, P_\tau : \tau \leq t\} = \{Y_{i,\tau}, X_{i,\tau}, \theta_{i,\tau}, P_\tau, \delta_\tau : \tau \leq t\}$. That is, in the transparent fund case, the investors and the managers have the same information ($\mathcal{F}^I_{i,t} = \mathcal{F}^M_{i,t}$). Therefore, all agents have the same estimate ($\hat{\delta}^I_{i,t} = \hat{\delta}_t$) and the same expected excess return ($\hat{R}^I_{i,t+1} = \hat{R}_{t+1}$) for all t.

I conjecture and later verify that there exists a publicly known constant $\gamma > 0$ such that the price function is

$$P(\hat{\delta}_t) = \frac{\hat{\delta}_t}{r} - \gamma. \tag{2.1}$$

The intuition for this conjecture is simple. The first term on the the right-hand side (RHS) is the present value of the expected future payoffs, discounted at the riskless rate; the second term is (as discussed later) the "cost premium"—that is, the return that compensates investors for the costs they incur.

2.1 Investors' optimizations

The fund size $X_{i,t}$ is determined by the investors' optimal decisions, in a fashion similar to Berk and Green (2004). First, define the fund return per dollar of capital as $y_{i,t+1} \equiv Y_{i,t+1}/X_{i,t} = R_{t+1}\theta_{i,t} + (1+r) - cX_{i,t}$. In period t, observing $\theta_{i,t}$, investors i increase their capital investment in the fund as long as the expectation of $y_{i,t+1}$ is greater than the riskless asset's return. Because $y_{i,t+1}$ exhibits decreasing returns to scale, $X_{i,t}$ is determined by the investors' indifference condition:

$$\mathrm{E}[y_{i,t+1} \big| \mathcal{F}^I_{i,t}] = 1 + r. \tag{2.2}$$

The left-hand side (LHS) of (2.2) is the expected return from investing in the fund, and the RHS is from investing in the riskless asset. Solving (2.2) for $X_{i,t}$ yields the period-t size of fund i as a function of the expected excess return $\hat{R}_{t+1}$ and the manager's portfolio $\theta_{i,t}$:

$$X_{i,t} = X(\hat{R}_{t+1}, \theta_{i,t}) \equiv \frac{1}{c}\hat{R}_{t+1}\theta_{i,t}. \tag{2.3}$$

Intuitively, more capital is invested in the fund when the expected excess return on the fund portfolio ($\hat{R}_{t+1}\theta_{i,t}$) is higher or the management cost c is lower.

2.2 Manager's optimization

A transparent fund manager's problem is simple. Because the investors infer $\mathcal{H}_t$ regardless of his actions, there is no scope for the manager to influence the decisions of future investors. Therefore, in effect, his problem is static: in each period t, he chooses $\theta_{i,t}$ to maximize his period-t utility, taking into account that $X_{i,t}$ responds to $\theta_{i,t}$ according to (2.3). That is, $\mathcal{P}^M_{i,t}$ reduces to choosing $\theta_{i,t} \in [0, \infty)$ to maximize

$$\phi X(\hat{R}_{t+1}, \theta_{i,t}) - \frac{\kappa}{2}\theta^2_{i,t}. \tag{2.4}$$

So given $\hat{R}_{t+1}$, the manager's optimal portfolio is, for all i,

$$\theta_{i,t} = \theta(\hat{R}_{t+1}) \equiv \frac{\phi}{c\kappa}\hat{R}_{t+1}. \tag{2.5}$$

Not surprisingly, the manager allocates more capital to the risky asset when the fee rate and the expected excess return are higher or the costs of investment are lower.

2.3 Equilibrium

The risky asset's equilibrium expected excess return $\hat{R}_{t+1}$ is pinned down by plugging the agents' optimal policies, (2.3) and (2.5), into the market-clearing condition (1.6). For all t,

$$\hat{R}_{t+1} = \hat{R} \equiv cS^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3}. \tag{2.6}$$

This is increasing in the fund's management cost c and in the manager's cost-benefit ratio κ/ϕ, because agents require a positive expected excess return on the risky asset as compensation for such costs.

Once $\hat{R}$ is identified, the equilibrium price and the agents' actions are readily determined. Conjecture (2.1) implies that P_t can be written as $P_t = (\hat{\delta}_t/r) - (\gamma + \hat{R}_{t+1})/(1+r)$.[12] This, together with (2.6), implies that conjecture (2.1) is correct if and only if $\gamma = (\gamma + \hat{R})/(1+r)$ or $\gamma = \hat{R}/r$. That is, the equilibrium price is equal to the present value of the expected future payoffs $(\hat{\delta}_t/r)$ minus the present value of the future expected excess returns $(\hat{R}/r)$. The agents' equilibrium actions are determined by plugging (2.6) into (2.3) and (2.5).

Proposition 2.1 summarizes the stationary equilibrium in the long run (i.e., $t \to \infty$), where all variables are constant, except for P_t, which depends on $\hat{\delta}_t$. For notational clarity, in the transparent fund case I append an asterisk to the equilibrium values of endogenous variables and omit the time indices for variables that are time invariant.

Proposition 2.1. Suppose the risky asset's payoff δ_t is unobservable, but the manager's portfolio $\theta_{i,t}$ is directly observable to the investors. A symmetric stationary equilibrium exists. In this equilibrium,

1. all investors infer δ_t correctly, and all agents update the estimate of $\bar{\delta}_t$ by (1.2) with $\lambda_t = \lambda$,

[12] Using (2.1), we obtain $R_{t+1} = \delta_{t+1} + \left(\frac{\hat{\delta}_{t+1}}{r} - \gamma\right) - (1+r)P_t$. Because $\mathrm{E}[\delta_{t+1}|\mathcal{H}_t] = \mathrm{E}[\hat{\delta}_{t+1}|\mathcal{H}_t] = \hat{\delta}_t$, we have $\hat{R}_{t+1} = \left(\frac{1+r}{r}\right)\hat{\delta}_t - \gamma - (1+r)P_t$. Rearranging it yields the result.

2. the risky asset's price is $P^*(\hat{\delta}_t) = (\hat{\delta}_t - \hat{R}^*)/r$, where $\hat{R}^* = cS^{1/3} (\kappa/\phi)^{2/3}$,

3. each fund's size is $X^* = S^{2/3} (\kappa/\phi)^{1/3}$, and

4. each manager buys $\theta^* = (\phi S/\kappa)^{1/3}$ shares of the risky asset per dollar of investor capital.

Because $\lambda < 1$, the estimate $\hat{\delta}_t$ is stochastically time varying even in the long run. The reason is that $\bar{\delta}_t$ is itself stochastic (i.e., $\eta_v < \infty$), and so the agents never learn the true value of $\bar{\delta}_t$. The price P_t is stochastic as it reflects $\hat{\delta}_t$. The fund size X^* is increasing in the manager's cost-benefit ratio κ/ϕ because it is positively related to $\hat{R}^*$. The fund's management cost c does not affect X^* because investors are compensated for a higher c exactly by a higher $\hat{R}^*$. The manager's portfolio θ^* is, as expected, negatively affected by his cost-benefit ratio.

Corollary 2.1. In the stationary equilibrium of the transparent fund case,

1. the expected return on each investor's investment is $1 + r$, and

2. each manager's one-period utility is $U^{M*} \equiv \phi X^* - \kappa\theta^{*2}/2 = S^{2/3}\kappa^{1/3}\phi^{2/3}/2$.

The expected return for the investors is equalized to the riskless return $1 + r$ by their indifference condition (2.2).[13] The manager's utility is increasing in the fee ϕ as expected. It is also increasing in his personal nonpecuniary cost κ for the following reason. On the one hand, a large κ is costly for the manager because the nonpecuniary cost $\kappa\theta_{i,t}^2/2$ is large. On the other hand, however, a large κ means that the asset's expected excess return $\hat{R}^*$ is high in equilibrium. This high $\hat{R}^*$ encourages investors to allocate a large amount of capital X^* to the fund, resulting in a large fee revenue ϕX^* for the manager. The second positive effect outweighs the first negative effect, and therefore the net impact of κ on U^{M*} is positive.

[13]This result means that, despite the positive expected excess return $\hat{R}$ on the risky asset, the expected excess return on the fund (from the investors' perspective) is zero in equilibrium. This is because decreasing returns to scale drive down the after-cost expected fund return to the riskless return $1 + r$. In relation to this point, it is worth comparing this model with Berk and Green's (2004) model. Berk and Green (2004) focus only on fund's return that investors receive, omitting individual asset's market equilibrium. They argue that each fund manager's skill could potentially generate excess return (alpha) on the fund, but it eventually disappears in equilibrium because of decreasing returns to scale. In contrast, my model concerns both the fund's and the individual asset's equilibrium returns. As shown in (2.6), the presence of investment cost (i.e., the manager's risk aversion κ) leads to excess return $\hat{R}$ on the asset, remaining positive in equilibrium, whereas excess return on the fund disappears in equilibrium because of decreasing returns to scale.

Last, an obvious, but important, implication of the transparent fund case is presented in the following corollary.

Corollary 2.2. In the transparent fund case, the risky asset's opacity is irrelevant: that is, whether or not δ_t is observable to the investors, the equilibrium outcome would be identical.

The intuition for Corollary 2.2 is that, as long as the manager's portfolio $\theta_{i,t}$ is observable, every investor can infer the unobservable payoff history $\mathcal{H}_t$ correctly, both on and off the equilibrium path, just as if it could be observed directly. As discussed in the next section, this will not be the case if the fund's portfolio is opaque. The idea of Corollary 2.2 will play a key role in the models of Sections 4 and 5, where the transparent and opaque funds and assets coexist.

3 Opaque Funds

This section characterizes the equilibrium for the case of opaque funds; that is, investors cannot directly observe $\theta_{i,t}$ and the manager cannot commit to his choice of $\theta_{i,t}$. The risky asset is opaque: δ_t is unobservable to the investors. Thus, the information set of investors i in period t is $\mathcal{F}^I_{i,t} = \{Y_{i,\tau}, X_{i,\tau}, P_\tau : \tau \leq t\}$.

Because the investors cannot observe δ_t, they try to infer it. However, unlike the transparent fund case, the investors cannot back out δ_t immediately from the observed variables because here both δ_t and $\theta_{i,t}$ are unobservable. Thus, the investors must infer δ_t based on their beliefs about $\theta_{i,t}$ (instead of the actual $\theta_{i,t}$ chosen). Although investors do infer δ_t correctly on the equilibrium path, their inference may be incorrect off the equilibrium path because the manager can secretly deviate from his equilibrium strategy (i.e., choose $\theta_{i,t}$ that is not anticipated by investors) in an effort to manipulate the investor inference of δ_t and attract more investor capital and thus more fees. Investors are not fooled on the equilibrium path because they are rational; however, the fact that they can be fooled off the equilibrium path does affect the agents' actions and the prices on the equilibrium path. Therefore, it is critical in this section (especially when solving the manager's problem in Section 3.3) to study investor beliefs and actions on off-the-equilibrium paths, where the investors may be fooled by the manager. For this purpose, it is important to distinguish between the variables related to the investors' beliefs $(\delta^I_{i,t}, \mathcal{H}^I_{i,t}, \hat{\delta}^I_{i,t}, \hat{R}^I_{i,t+1})$ and those of the managers $(\delta_t, \mathcal{H}_t, \hat{\delta}_t, \hat{R}_{t+1})$.

3.1 Conjectures and out-of-equilibrium beliefs

I propose (and later verify) the following two conjectures.

1. There exists $\{\gamma_t\}_{t=0}^{\infty} > 0$, which is nonstochastic and publicly known, such that the price function is
$$P_t(\hat{\delta}_t^I) = \frac{\hat{\delta}_t^I}{r} - \gamma_t \quad \text{where } \hat{\delta}_t^I \equiv \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i. \tag{3.1}$$

2. There exists $\{\theta_t^{**}\}_{t=0}^{\infty} > 0$, which is nonstochastic and publicly known, such that, for all $i \in [0,1]$, manager i optimally plays
$$\theta_{i,t} = \theta_t^{**} \tag{3.2}$$
on the equilibrium path and also on off-the-equilibrium paths, where $\hat{\delta}_{i,t}^I \neq \hat{\delta}_t$.

The intuition for conjecture (3.1) is simple. The first term, $\hat{\delta}_t^I/r$, is the present value of the average of all the investors' expected future payoffs discounted at the riskless rate. The second term, γ_t, is the cost premium; unlike the transparent fund case, this premium is now (deterministically) time varying. Note that every agent knows the form of price function (3.1), and therefore can infer the investors' average estimate $\hat{\delta}_t^I$ by observing P_t. Conjecture (3.2) states that the manager will optimally buy a deterministic number of shares of the risky asset per dollar of investor capital, regardless of his own past actions $(\theta_{i,0}, ..., \theta_{i,t-1})$.

It will be shown later that all investors correctly infer $\mathcal{H}_t$ on the equilibrium path, and therefore have the same estimate of $\bar{\delta}_t$. Thus, on the equilibrium path, each investor observes P_t and confirms that her estimate based on her inferred payoff history $\mathcal{H}_{i,t}^I$ is the same as the other investors' average estimate $\hat{\delta}_t^I$ (revealed by P_t). In other words, $\mathrm{E}[\bar{\delta}_t|\mathcal{H}_{i,t}^I] = \hat{\delta}_t^I$ for all i and t on the equilibrium path. However, on off-the-equilibrium paths for which some agents deviate from their equilibrium strategies, some investors may observe P_t, learn $\hat{\delta}_t^I$ from it, and realize that their estimate $\mathrm{E}[\bar{\delta}_t|\mathcal{H}_{i,t}^I]$ does not match $\hat{\delta}_t^I$. For such cases, I specify the following out-of-equilibrium belief of investors:

$$\text{If } \mathrm{E}[\bar{\delta}_t|\mathcal{H}_{i,t}^I] \neq \hat{\delta}_t^I \text{ then } \hat{\delta}_{i,t}^I = \mathrm{E}[\bar{\delta}_t|\mathcal{H}_{i,t}^I]. \tag{3.3}$$

This states that an investor whose estimate (based solely on her inferred payoff history $\mathcal{H}_{i,t}^I$) disagrees with the other investors' average estimate $\hat{\delta}_t^I$ believes that her own estimate is more accurate. Note that such a belief makes sense: in this model, investors have no reason to revise their estimate in favor of the observed price because that price need not convey information that is superior to theirs.[14]

3.2 Investors' optimizations

As in the transparent fund case, the investors' indifference condition (2.2) pins down the fund size:

$$X_{i,t} = X(\hat{R}^I_{i,t+1}, \theta^{**}_t) = \frac{1}{c}\hat{R}^I_{i,t+1}\theta^{**}_t. \tag{3.4}$$

Importantly, (3.4) differs from its transparent fund counterpart (2.3) in two ways. First, $X_{i,t}$ depends on the expected excess return from investors i's point of view, $\hat{R}^I_{i,t+1}$, which is not necessarily equal to $\hat{R}_{t+1}$ on some off-the-equilibrium paths. On the equilibrium path, of course, $\hat{R}^I_{i,t+1} = \hat{R}_{t+1}$ for all i and t because all the investors infer $\mathcal{H}_t$ correctly. Second, $X_{i,t}$ depends on the investors' belief (θ^{**}_t) about the manager's action, and not on the action itself ($\theta_{i,t}$), which they cannot observe. This is the source of the manager's moral hazard, which will play a central role in the following analyses.

3.3 Manager's optimization

The manager's problem drastically differs from that of the transparent fund case. Recall that the fund size $X_{i,t}$ is determined by the investors' belief that manager i chooses θ^{**}_t, regardless of the $\theta_{i,t}$ actually chosen. So if the manager were to maximize only his period-t utility, an extreme moral hazard would occur: for any belief θ^{**}_t of investors, the manager would just choose $\theta_{i,t} = 0$ because $\theta_{i,t} > 0$ would not affect $X_{i,t}$ and yet is costly. However, in the dynamic setting considered here, the manager will choose $\theta_{i,t} > 0$ in equilibrium because he is motivated by a (endogenous) career concern: although his choice of $\theta_{i,t}$ does not affect the current fund size, it can potentially affect the future fund size through influencing the investor beliefs in future periods. In sum, the main difference in the manager's problem in the two cases is that the manager of a transparent fund chooses his current portfolio $\theta_{i,t}$ by weighing its cost $\kappa\theta^2_{i,t}/2$ against the benefit from influencing the

[14] In asymmetric information models à la Grossman and Stiglitz (1980) or differential information models à la Grossman (1976), investors should revise their estimates in favor of the price because it reflects superior information. In the model proposed here, however, no investor has information superior to the other funds' investors and no one has private information that would be collectively useful. On the equilibrium path, the price just confirms each investor's estimate. Also note that investors i have no reason to believe that discrepancy in the estimates is caused by manager i's deviation; after all, it could be other funds' managers or investors who have deviated.

current fund size $X_{i,t}$, whereas the manager of an opaque fund weighs such costs against the benefit from influencing the future fund sizes $X_{i,t+\tau}$, $\tau = 1, 2, ...$

Now, let us verify that it is indeed optimal for manager i to play $(\theta_0^{**}, \theta_1^{**}, ...) > 0$ deterministically. To do so, first we need to consider what would happen if he deviated from his equilibrium play and instead chose a sequence of portfolios $(\theta_{i,0}, \theta_{i,1}, ...) \neq (\theta_0^{**}, \theta_1^{**}, ...)$ even as investors i still believe that he chooses $(\theta_0^{**}, \theta_1^{**}, ...)$. Such a deviation does not affect the asset's prices because each manager has measure zero.

I first determine the effect of a sequence of deviations $(\theta_{i,0}, \theta_{i,1}, ...)$ on the payoff $\delta^I_{i,t+1}$ as inferred by investors i. The value of $\delta^I_{i,t+1}$ solves

$$\left(\delta^I_{i,t+1} + P_{t+1} - (1+r)P_t\right)\theta_t^{**} X_{i,t} = \left(\delta_{t+1} + P_{t+1} - (1+r)P_t\right)\theta_{i,t} X_{i,t}. \tag{3.5}$$

The RHS is the excess return on the fund's portfolio, $R_{t+1}\theta_{i,t}X_{i,t}$, whose value is known to investors i who observe $Y_{i,t+1}$. This value depends on the manager's actual choice, $\theta_{i,t}$, and the true payoff, δ_{t+1}. The LHS is the decomposition of that excess return as (incorrectly) inferred by investors i. It depends on their incorrect belief about the manager's action, θ_t^{**}, and implies an erroneous inferred payoff, $\delta^I_{i,t+1}$. Clearly, the source of the investors' error is their inability to observe two variables: δ_{t+1} and $\theta_{i,t}$. Solving (3.5) for $\delta^I_{i,t+1}$, we have

$$\delta^I_{i,t+1} = \delta_{t+1} + \left(\frac{\theta_{i,t} - \theta_t^{**}}{\theta_t^{**}}\right) R_{t+1}. \tag{3.6}$$

Hence, if the manager plays $\theta_{i,t} > \theta_t^{**}$ and if $R_{t+1} > 0$, then the investors will overshoot their inference, that is, $\delta^I_{i,t+1} > \delta_{t+1}$.

Given $\delta^I_{i,t+1} > \delta_{t+1}$, the investors' estimate of the persistent component of the asset's payoff will be biased upward in future periods, that is, $\hat{\delta}^I_{i,t+\tau} > \hat{\delta}_{t+\tau}$ for $\tau = 1, 2, ...$ (see Appendix C).[15] Then their expectation about the asset's excess return is also inflated in the future, that is, $\hat{R}^I_{i,t+\tau+1} > \hat{R}_{t+\tau+1}$ for $\tau = 1, 2, ...$ (see Appendix C).[16] Thus, by (3.4), the future fund size $X_{i,t+\tau}$ for $\tau = 1, 2, ...$ is larger than that on the equilibrium path. That is, the manager can potentially influence $X_{i,t+\tau}$ for $\tau = 1, 2, ...$ through his choice of $\theta_{i,t}$. Of course, he takes this fact into account when choosing $\theta_{i,t}$. It can be shown that

[15]A key for this overshoot of $\hat{\delta}^I_{i,t+\tau}$ to occur is the out-of-equilibrium belief (3.3), which leads the investors to stick by their own estimate when it disagrees with the other investors' average estimate $\hat{\delta}^I_{t+\tau}$ that is revealed by $P_{t+\tau}$. Note that, on this off-the-equilibrium path, the investors are aware that their estimate $\hat{\delta}^I_{i,t+\tau}$ is higher than $\hat{\delta}^I_{t+\tau}$ but are unaware that it is too high: they (incorrectly) believe that $\hat{\delta}^I_{i,t+\tau}$ is more accurate than $\hat{\delta}^I_{t+\tau}$.

[16]Intuitively, in period $t + \tau$, the investors' high estimate $\hat{\delta}^I_{i,t+\tau}$ leads them to expect both $\delta_{t+\tau+1}$ and $P_{t+\tau+1}$ to be high; given the observed price $P_{t+\tau}$, they expect that $R_{t+\tau+1}$ will also be high.

his maximization problem in period t (both on and off the equilibrium path) is written as follows.

Lemma 3.1. Manager i chooses $\theta_{i,t} \in [0,\infty)$ to maximize

$$-\frac{\kappa}{2}\theta_{i,t}^2 + \frac{\phi}{c}\left(\frac{\theta_{i,t} - \theta_t^{**}}{\theta_t^{**}}\right)\Omega_t \hat{R}_{t+1} \tag{3.7}$$

$$\text{where} \quad \Omega_t \equiv (1-\lambda_{t+1})\sum_{\tau=1}^{\infty}\beta^\tau \theta_{t+\tau}^{**}\left(1 + \frac{1-\lambda_{t+\tau+1}}{r}\right)\left(\prod_{\nu=t+2}^{t+\tau}\lambda_\nu\right).$$

The first term of (3.7) is the manager's personal cost of choosing $\theta_{i,t}$; the second term corresponds to his expected gain from influencing the fund size in future periods. On the equilibrium path, the second term of (3.7) is zero because $\theta_{i,t} = \theta_t^{**}$. Nonetheless, this term still affects the manager's equilibrium action because the marginal effect of $\theta_{i,t}$ on that term is nonzero even on the equilibrium path. The variable Ω_t measures the sensitivity of the manager's expected future gain to an increase in $\theta_{i,t}$. Observe that Ω_t is deterministic and publicly known.

Maximizing (3.7) given $\hat{R}_{t+1}$ and θ_t^{**}, the manager's optimal choice of $\theta_{i,t}$ is

$$\theta_{i,t} = \frac{1}{\theta_t^{**}}\frac{\phi}{c\kappa}\Omega_t \hat{R}_{t+1}. \tag{3.8}$$

As will be shown in Section 3.4, $\hat{R}_{t+1}$ is a deterministic variable. So conjecture (3.2) is correct (and the investors' beliefs are consistent) if $\theta_{i,t} = \theta_t^{**}$ holds in (3.8)—that is, if

$$\theta_t^{**} = \left(\frac{\phi}{c\kappa}\Omega_t \hat{R}_{t+1}\right)^{1/2}. \tag{3.9}$$

The values of θ_t^{**} and $\hat{R}_{t+1}$ will be obtained explicitly once the market-clearing condition is imposed and another relation between θ_t^{**} and $\hat{R}_{t+1}$ is identified (in Section 3.4).

3.4 Equilibrium

The market-clearing condition (1.6) determines the asset's price and the agents' actions. Plugging the agents' optimal policies (3.4) and (3.8) into (1.6), and noting that $\hat{R}_{i,t+1}^I = \hat{R}_{t+1}$ holds for all i in equilibrium, $\hat{R}_{t+1}$ is obtained as

$$\hat{R}_{t+1} = c\left(\frac{\kappa S}{\phi \Omega_t}\right)^{1/2}. \tag{3.10}$$

It is intuitive that $\hat{R}_{t+1}$ is decreasing in Ω_t. For a given $\hat{R}_{t+1}$, a rise in Ω induces the manager to increase $\theta_{i,t}$. This increase then leads to a higher $X_{i,t}$ because, holding $\hat{R}_{t+1}$ constant, a rise in $\theta_{i,t}$ increases the expected value of $y_{i,t+1}$. These higher $\theta_{i,t}$ and $X_{i,t}$ lead to a higher aggregate demand for the risky asset. Thus, $\hat{R}_{t+1}$ decreases (i.e., P_t increases) to clear the market.

Now we can obtain the price P_t. Conjecture (3.1) implies that P_t is written as $P_t = (\hat{\delta}_t^I/r) - (\gamma_{t+1} + \hat{R}_{t+1})/(1+r)$.[17] This implies that conjecture (3.1) is correct if and only if $\gamma_t = (\gamma_{t+1} + \hat{R}_{t+1})/(1+r)$ or

$$\gamma_t = \sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau+1} \hat{R}_{t+\tau+1}. \tag{3.11}$$

That is, again, the price equals the present value of the expected future payoffs $(\hat{\delta}_t^I/r)$ minus the present value of the future expected excess returns.

Note that $\hat{R}_{t+1}$ and γ_t are not entirely solved in (3.10) and (3.11), respectively, in that their RHS depend on Ω_t, which in turn still depends on endogenous variables $\theta_{t+1}^{**}, \theta_{t+2}^{**}, \ldots$. However, in the long-run stationary equilibrium, where $t \to \infty$, these variables are solved in closed form. To clarify matters, I append two asterisks to the equilibrium values of endogenous variables in the opaque fund case and omit the time indexes for time-invariant variables. It can be shown (see Appendix D) that the stationary values of Ω_t and θ_t^{**} satisfy

$$\Omega^{**} = \xi \theta^{**}, \tag{3.12}$$

$$\text{where} \quad \xi \equiv \left(1 + \frac{1-\lambda}{r} \right) \frac{\beta(1-\lambda)}{1-\beta\lambda} > 0. \tag{3.13}$$

The composite parameter ξ plays a central role in the following analyses. It measures the degree of the manager's career concern in the stationary equilibrium. When ξ is higher, the expected marginal gain to the manager from investing in the risky asset is higher, and thus his desire to "fool" future investors by increasing $\theta_{i,t}$ is stronger. Note that ξ is increasing in β and η_u but is decreasing in η_v and r. Intuitively, the manager's career

[17] Using price function (3.1), $R_{t+1} = \delta_{t+1} + \left(\frac{1}{r} \int_0^1 \hat{\delta}_{i,t+1}^I \, \mathrm{d}i - \gamma_{t+1} \right) - (1+r)P_t = \delta_{t+1} + \frac{1}{r} \int_0^1 \left(\lambda_{t+1} \hat{\delta}_{i,t}^I + (1-\lambda_{t+1})\delta_{t+1} \right) \mathrm{d}i - \gamma_{t+1} - (1+r)P_t = \left(1 + \frac{1-\lambda_{t+1}}{r} \right) \delta_{t+1} + \frac{\lambda_{t+1}}{r} \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i - \gamma_{t+1} - (1+r)P_t$. Thus, $\hat{R}_{i,t+1}^I = \left(1 + \frac{1-\lambda_{t+1}}{r} \right) \hat{\delta}_{i,t}^I + \frac{\lambda_{t+1}}{r} \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i - \gamma_{t+1} - (1+r)P_t$. Integrating over i and noting that $\hat{R}_{i,t+1}^I = \hat{R}_{t+1}$ for all i in equilibrium, we have $\hat{R}_{t+1} = \left(1 + \frac{1-\lambda_{t+1}}{r} \right) \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i + \frac{\lambda_{t+1}}{r} \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i - \gamma_{t+1} - (1+r)P_t = \left(\frac{1+r}{r} \right) \hat{\delta}_t^I - \gamma_{t+1} - (1+r)P_t$. Rearranging it, the result holds.

concern is stronger if his interest in the future is high (high β) and when the investors' estimate $\hat{\delta}_t$ is inaccurate and thus susceptible to manipulation (low λ, which stems from high η_u or low η_v). The constant ξ may take any positive value because $\xi \to 0$ when $\beta \to 0$, but $\xi \to \infty$ when $r \to 0$.[18]

Using (3.12), we can rewrite the stationary versions of (3.9) and (3.10) as a system of equations with two unknowns, $\hat{R}^{**}$ and θ^{**}, yielding simple closed-form solutions (see Proposition 3.1). Then X^{**} and γ^{**} are obtained from (3.4) and (3.11).

Proposition 3.1 summarizes the long-run stationary equilibrium. Note that $\hat{\delta}_t^I = \hat{\delta}_t$ for all t on the equilibrium path because all investors correctly infer δ_t.

Proposition 3.1. Suppose the risky asset's payoff δ_t and the manager's portfolio $\theta_{i,t}$ are both unobservable to the investors. A symmetric stationary equilibrium exists. On the equilibrium path,

1. all investors infer δ_t correctly, and all agents update the estimate of $\bar{\delta}_t$ by (1.2) with $\lambda_t = \lambda$,
2. the risky asset's price is $P^{**}(\hat{\delta}_t) = (\hat{\delta}_t - \hat{R}^{**})/r$, where $\hat{R}^{**} = \xi^{-2/3}\hat{R}^*$,
3. each fund's size is $X^{**} = \xi^{-1/3}X^*$, and
4. each manager buys $\theta^{**} = \xi^{1/3}\theta^*$ shares of the risky asset per dollar of investor capital.

The equilibrium is supported by the investors' out-of-equilibrium belief (3.3).

Clearly, the key difference from the transparent fund case (Proposition 2.1) is the presence of the manager's career concern ξ. Indeed, all the stationary equilibrium variables of the opaque fund case coincide with those of the transparent fund case if $\xi = 1$. This fact facilitates comparing the equilibrium values of the transparent and opaque fund cases. If $\xi > 1$, then the risky asset is more expensive, the fund size is smaller, and the fund's portfolio is riskier in the opaque fund case than in the transparent fund case.

The effects of ξ on the equilibrium are summarized in Corollary 3.1. Recall that ξ is an endogenous composite parameter that increases with β and η_u and decreases with r and η_v.

[18] That ξ can exceed unity is a major difference from Holmström's (1999) result. The counterpart of ξ in his paper is the LHS of Equation (22), which is never greater than one. That threshold is crucial for his main result because it ensures that the labor supply is never greater than the efficient level in the stationary state. The reason why $\xi > 1$ may occur in the model developed here is that the risky asset not only yields δ_t but also can be sold at price P_t. This resale of the asset creates an additional component (captured by the term $(1 - \lambda)/r$ in the definition of ξ in (3.13)) and allows ξ to be large.

Corollary 3.1. The manager's career concern ξ affects the price and the agents' actions: $\mathrm{d}P^{**}/\mathrm{d}\xi > 0$, $\mathrm{d}\hat{R}^{**}/\mathrm{d}\xi < 0$, $\mathrm{d}X^{**}/\mathrm{d}\xi < 0$, and $\mathrm{d}\theta^{**}/\mathrm{d}\xi > 0$.

The intuition is as follows. For a given $\hat{R}$, a rise in ξ induces the manager to (secretly) lever up and increase investment in the risky asset in an effort to boost next-period fund return and thus inflate investor expectations about future fund returns (i.e., $\partial\theta/\partial\xi > 0$ given $\hat{R}$). The investors are rational and are not fooled on the equilibrium path, but the manager levers up nonetheless. Indeed, as in Holmström (1999) or Stein (1989), given the investors' beliefs that the manager will try to fool them, it is optimal for him to do so for fear of being underestimated. This increase in θ then induces the investors to invest more capital in the fund for a given $\hat{R}$ (i.e., $\partial X/\partial\xi > 0$ given $\hat{R}$) because, ceteris paribus, a higher θ implies a higher expected fund return. Higher θ and X result in a higher aggregate demand, θX, for a given $\hat{R}$. In equilibrium, that demand is settled by a higher market-clearing price and hence a lower expected return (i.e., $\mathrm{d}P^{**}/\mathrm{d}\xi > 0$ and $\mathrm{d}\hat{R}^{**}/\mathrm{d}\xi < 0$). Taking this price adjustment into account, the manager's portfolio is still riskier than the original level ($\mathrm{d}\theta^{**}/\mathrm{d}\xi > 0$); however, the negative effect of ξ on X^{**} through $\hat{R}^{**}$ dominates its positive effect through θ^{**}, and so the net effect is negative ($\mathrm{d}X^{**}/\mathrm{d}\xi < 0$).

It is interesting to see the impact of career concerns on social welfare. Corollary 3.2 shows that the effect is a negative one.

Corollary 3.2. In the stationary equilibrium of the opaque fund case,

1. the expected return on each investor's investment is $1 + r$, whereas each manager's one-period utility is $U^{M**} \equiv \phi X^{**} - \kappa\theta^{**2}/2 = ((2-\xi)/\xi^{1/3})U^{M*}$,

2. the manager's career concern does not affect the investors' expected return but makes the manager worse off, that is, $\mathrm{d}U^{M**}/\mathrm{d}\xi < 0$, and

3. the allocation is Pareto inferior (superior) to that of the transparent fund case if $\xi > 1$ ($\xi < 1$).

The reason for $\mathrm{d}U^{M**}/\mathrm{d}\xi < 0$ is that $\mathrm{d}X^{**}/\mathrm{d}\xi < 0$ and $\mathrm{d}\theta^{**}/\mathrm{d}\xi > 0$, as shown in Corollary 3.1. Intuitively, the manager is worse off because he attracts lower capital (and thus lower fees) as he cannot commit to not fool the investors by using opacity. Because each investor's expected return is $1 + r$ in both transparent and opaque fund cases, comparing the social welfare that results from these cases amounts to comparing U^{M*} and U^{M**}. It is clear that $U^{M*} \gtreqless U^{M**}$ if and only if $\xi \gtreqless 1$. It may be surprising that the transparent fund case is Pareto inferior when $\xi < 1$, because the transparent fund managers could choose the opaque fund managers' superior solution θ^{**} while the investors are directly observing the choices. This does not occur in equilibrium because the asset's price adjusts so that the action is no longer optimal for the managers (who are price takers). Indeed, if in the transparent fund case all managers happened to choose θ^{**} when $\xi < 1$, the resulting $\hat{R}$ would be so high that it would not be individually optimal for each manager to stick with such a low θ^{**}.

Corollary 3.3. In the opaque fund case, if the risky asset were transparent (i.e., if δ_t were observable to the investors), then no manager would buy that asset (i.e., $\theta_{i,t} = 0$ $\forall i$), and hence no equilibrium would exist under the conjectures specified in Section 3.1.

This result starkly differs from its counterpart in the transparent fund case (Corollary 2.2) due to a fundamental difference in the nature of maximization problems of transparent and opaque fund managers. The transparent one does not have any career concern: he chooses portfolio to maximize (in effect) his within-period utility, taking into account the current investors' best responses to his observable portfolio choice (see (2.4)). In contrast, the opaque one is solely motivated by a career concern: he chooses portfolio in an attempt to influence the future investors' decisions, knowing that his unobservable portfolio choice does not affect the current investors' decisions (see (3.7)). If the risky asset were transparent, the investors would observe $\mathcal{H}_t$ directly irrespective of the manager's actions, and hence the opaque fund manager's career concern would be eliminated. Then what remains would be the manager's pure moral hazard: for any asset prices and for any belief $\theta_t^{**} \geq 0$ of the investors, the manager would optimally choose $\theta_{i,t} = 0$ because choosing $\theta_{i,t} > 0$ is costly for him and yet does not affect the investors' capital provision (and therefore his fees) at all. In other words, if the asset were transparent, the second term of the manager's objective function (3.7) would disappear and only the cost $-\kappa\theta_{i,t}^2/2$ in the first term would remain; hence, his optimal choice would be $\theta_{i,t} = 0$. So the only possible situation in which the investors have a consistent belief would be that the manager chooses $\theta_{i,t} = 0$ and the investors believe $\theta_t^{**} = 0$. However, $\theta_{i,t} = 0$ would not clear the risky asset's market, and thus there would be no such an equilibrium. If the opaque fund manager could promise credibly to buy the transparent asset, he would do so because it would attract more investor capital; however, a lack of commitment prevents him from doing so. Corollaries 3.3 and 2.2 will be useful when understanding the working of the models of Sections 4 and 5, where the transparent and opaque funds and assets coexist.

4 Opacity Price Premium: Coexistence of Transparency and Opacity

So far I have focused on models with extreme assumptions: there is only a single, opaque risky asset, and the funds are either all transparent or all opaque. Do the insights generated by these models carry through in a more realistic setting in which various degrees of opacity coexist? In this section, I present a model in which both transparent and opaque funds, as well as transparent and opaque assets with identical payoffs, coexist. In equilibrium, an "opacity price premium" arises: the opaque asset trades at a premium over the transparent one, even though it is common knowledge that their payoffs are identical and both assets are accessible to all funds. The opacity price premium is accompanied by endogenous market segmentation: transparent funds purchase only transparent assets, whereas opaque funds purchase only opaque assets.

4.1 Setup

The model setup used here is much like that used in Section 1. Unless otherwise noted, the same assumptions prevail (see Appendix E for the details).

4.1.1 Risky assets

There are two risky assets, a and b, referred to (respectively) as the transparent and opaque assets. The supply of asset a is πS and that of asset b is $(1-\pi)S$, where $\pi \in (0,1)$ is taken to be exogenous in this section. In focusing on the role of opacity, I assume that these two assets are identical in terms of their payoffs: in period t, they yield the same payoff δ_t from the same distribution specified in Section 1. It is common knowledge that these assets yield identical payoffs. Asset a's payoff δ_t is directly observable only to the agents in the funds that bought that asset in period $t-1$.[19] Asset b's payoff δ_t is never directly observable to the investors. Obviously, if a fund purchases asset a, then the investors in that fund will learn asset b's δ_t as well. Asset $k \in \{a, b\}$ is traded in the market at a publicly observable price P_t^k, and its excess return is denoted by $R_{t+1}^k \equiv \delta_{t+1}^k + P_{t+1}^k - (1+r)P_t^k$. The expected excess return conditional on the true payoff history $\mathcal{H}_t = (\delta_1, ..., \delta_t)$ is $\hat{R}_{t+1}^k \equiv \mathrm{E}[R_{t+1}^k|\mathcal{H}_t]$.

[19] A story behind this assumption is as follows. There are potentially a large number of transparent assets that are ex ante distinguishable to the managers, but not to the investors. Hence, only if the manager picks one particular asset from them the investors can track that asset's periodic payoff.

4.1.2 Funds

There is a measure-1 continuum of funds. An exogenous proportion $\alpha \in (0,1)$ of them are funds A, indexed by $i \in [0,\alpha)$, and the rest are funds B, indexed by $i \in [\alpha, 1]$. Funds A and B are referred to as the transparent and opaque funds, respectively. Note that the funds' types are not the managers' choices but their inherent characteristics.[20] Fund $j \in \{A, B\}$ with index i ("fund j-i") consists of a manager ("manager j-i") and a large number of investors ("investors j-i"). The size of fund j-i in period t is $X_{i,t}^{j}$, which gives the manager a fee of $\phi X_{i,t}^{j}$ but requires the fund to expend the management cost $cX_{i,t}^{j2}$.

4.1.3 Manager's decisions

In each period, each manager makes two decisions. First, he chooses the type of risky asset to hold in his fund portfolio. Second, he decides how much of that asset to purchase. These are detailed as follows.

- Decision 1: asset class. I assume that each manager can form a portfolio consisting of only one risky asset (i.e., either a or b) and the riskless asset. In each period t, before $X_{i,t}^{j}$ is determined, each manager chooses the type $k \in \{a, b\}$ of the risky asset he holds in his period-t portfolio. This choice is directly observable to the investors, whether the fund is transparent or opaque.[21] The choice of k is interpreted as a fund's choice of the asset classes it invests in. Choosing $k = a$ can be viewed as committing to focus on traditional, "plain vanilla" asset classes, whereas choosing $k = b$ can be interpreted as advertising that it will invest in more sophisticated, complex financial instruments.

- Decision 2: portfolio. After choosing $k \in \{a, b\}$, manager j-i forms a portfolio: for each dollar of investor capital, he purchases $\theta_{i,t}^{jk} \in [0, \infty)$ shares of asset k and $\left(1 - P_t^k \theta_{i,t}^{jk}\right)$ units of the riskless asset. In fund A-i, the manager's portfolio $\theta_{i,t}^{Ak}$ is directly observable to investors A-i in period t, irrespective of k. In fund B-i, in

[20] In Section 5, I discuss the managers' endogenous choice of fund types. See footnote 27 and the corresponding discussion.

[21] Even if we allow each manager to choose to make this choice observable or unobservable, every manager chooses to make it observable. A transparent fund manager is indifferent between observable and unobservable choices because his portfolio is always observable to the investors. An opaque fund manager strictly prefers to make it observable: if he instead chose to make it unobservable, then the investors would not invest capital in the fund, anticipating rationally a moral hazard situation in which the manager chooses $k = a$ and then "shirks," that is, invests no capital in asset a to avoid incurring the personal cost of portfolio formation.

contrast, the portfolio $\theta_{i,t}^{Bk}$ is unobservable to investors B-i regardless of k, and the manager cannot commit to his portfolio choice. As in Section 1, manager j-i incurs a personal cost of portfolio formation: $\kappa\theta_{i,t}^{jk2}/2$.

Note that, in fund B-i, the chosen asset class $k \in \{a, b\}$ is observable, whereas the number of shares of that asset purchased (i.e., $\theta_{i,t}^{Bk} \in [0, \infty)$) is unobservable to the investors regardless of k.

4.2 Equilibrium

Proposition 4.1 summarizes a long-run stationary equilibrium of this economy.

Proposition 4.1. Suppose there are both transparent and opaque assets, as well as transparent and opaque funds. Assume the opaque fund managers' career concerns satisfy $\xi \in (\xi_\ell, 2)$, where $\xi_\ell \equiv (\alpha/(1-\alpha))^{1/2}((1-\pi)/\pi)^{1/2}$. Then a stationary equilibrium exists. On the equilibrium path,

1. all investors infer δ_t correctly, and all agents update the estimate of $\bar{\delta}_t$ by (1.2) with $\lambda_t = \lambda$,

2. (Opacity price premium.) the prices of the transparent and opaque assets are, respectively, $P^{a*}(\hat{\delta}_t) = (\hat{\delta}_t - \hat{R}^{a*})/r$ with $\hat{R}^{a*} = c(\pi S/\alpha)^{1/3}(\kappa/\phi)^{2/3}$ and $P^{b*}(\hat{\delta}_t) = (\hat{\delta}_t - \hat{R}^{b*})/r$ with $\hat{R}^{b*} = \xi_\ell^{2/3}\xi^{-2/3}\hat{R}^{a*}$, where $P^{a*}(\hat{\delta}_t) < P^{b*}(\hat{\delta}_t)$,

3. the sizes of the transparent and opaque funds are, respectively, $X^{A*} = (\pi S/\alpha)^{2/3}(\kappa/\phi)^{1/3}$ and $X^{B*} = \xi_\ell^{4/3}\xi^{-1/3}X^{A*}$,

4. (Market segmentation.) every transparent fund manager buys $\theta^{Aa*} = (\pi S\phi/(\alpha\kappa))^{1/3}$ shares of the transparent asset per investor capital, and every opaque fund manager buys $\theta^{Bb*} = \xi_\ell^{2/3}\xi^{1/3}\theta^{Aa*}$ shares of the opaque asset per investor capital, and

5. the expected return on each investor's investment is $1 + r$, whereas the one-period utilities of the transparent and opaque fund managers are, respectively, $U^{MA*} = (\kappa^{1/3}/2)(\phi\pi S/\alpha)^{2/3}$ and $U^{MB*} = \xi_\ell^{4/3}\xi^{-1/3}(2-\xi)U^{MA*}$.

The equilibrium is supported by the investors' out-of-equilibrium belief (E.8) specified in Appendix E.2 (which is the obvious counterpart of (3.3)).

The second part of Proposition 4.1 states that the opaque asset trades at a higher price than does the transparent one in equilibrium, even though it is common knowledge that these assets yield identical payoffs and each manager is allowed to buy whichever risky asset he wishes. This result provides a theoretical support for the empirical findings that opaque, complex financial instruments have been traded at a premium (Coval, Jurek, and Stafford 2009; Henderson and Pearson 2011; Célérier and Vallée 2013). The price gap between the two asset types—the opacity price premium—is increasing in the extent of the opaque fund managers' career concerns, ξ. Importantly, this price gap accompanies endogenous market segmentation: the transparent asset is purchased only by transparent funds, whereas the opaque asset is purchased only by opaque funds. This segmentation is consistent with the real-world observation that mutual funds with disclosed positions tend to focus on traditional assets, whereas hedge funds with secretive positions often trade opaque assets, such as complex derivatives. Indeed, from a sample of over 5,000 hedge funds, Chen (2011) documents that 71% of them trade derivative securities, whereas Koski and Pontiff (1999) find that only 21% of mutual funds in their sample use derivatives. Moreover, as Chen (2011) points out, the result in Koski and Pontiff's work (1999) implies that many mutual funds do not use derivatives even if they are permitted to do so, because nearly two-thirds of mutual funds are actually allowed to invest in derivatives (Deli and Varma 2002). This is also consistent with the model's result that funds A do not buy asset b even if they could.

The economic intuition for the market segmentation (and the associated price gap) is obtained by revisiting Corollaries 2.2 and 3.3. For a transparent fund manager, the risky asset's opacity is irrelevant because his fund's investors infer the opaque asset's payoff δ_t correctly, regardless of whether it is on or off the equilibrium path, and thus there is no scope for the manager to influence the investor assessment of his fund's future prospect by investing in the opaque asset (Corollary 2.2). Therefore, given that the opaque asset is more expensive, he simply buys the cheaper transparent asset to attract investor capital.

In contrast, an opaque fund manager does not buy the transparent asset, even if it is cheaper, due to a moral hazard problem and a lack of commitment (Corollary 3.3). Let us see more carefully why opaque funds buy only the opaque asset despite the higher price. First, consider what would happen if manager B-i chose $k = a$ in period t. In this case, investors B-i would directly observe the true value of δ_{t+1} in period $t + 1$, regardless of the $\theta_{i,t}^{Ba}$ chosen. Hence, the manager would not be able to influence the future investors' capital provisions, $X_{i,t+\tau}^{B}$ for $\tau = 1, 2, ...,$ by choosing $\theta_{i,t}^{Ba} > 0$. Furthermore, because investors B-i make decisions based on their beliefs about the manager's (unobservable) choice of $\theta_{i,t}^{Ba}$, not on the $\theta_{i,t}^{Ba}$ actually chosen, the manager would not be able to affect the current investors' capital $X_{i,t}^{B}$ by choosing $\theta_{i,t}^{Ba} > 0$ either. Then because choosing $\theta_{i,t}^{Ba} > 0$ has no impact on the investors' capital provisions but is yet personally costly for him, it would be optimal for the manager to choose $\theta_{i,t}^{Ba} = 0$. Clearly, this would give the investors a negative rate of return. So, given that the manager chose $k = a$, the investors would not invest any capital in the fund in that period (i.e., $X_{i,t}^{B} = 0$), anticipating that the manager will choose $\theta_{i,t}^{Ba} = 0$. Because $X_{i,t}^{B} = 0$ yields no fees, the manager's period-t utility when choosing $k = a$ would be zero. Of course, if the manager could make the investors believe that $\theta_{i,t}^{Ba} > 0$, then $X_{i,t}^{B} > 0$ would be provided; however, a lack of commitment prevents him from doing so.[22] Next, consider what happens if the manager chooses $k = b$. In this case, being motivated by a career concern, he can credibly purchase the opaque asset (as in the model of Section 3), and the investors rationally anticipate that the manager will indeed buy a positive number of shares of that asset (i.e., $\theta_{i,t}^{Bb} = \theta^{Bb*} > 0$). Thus, the investors will provide a positive amount of capital to the fund (i.e., $X_{i,t}^{B} > 0$), which yields a positive fee $\phi X_{i,t}^{B}$. As shown in the fifth part of Proposition 4.1, this fee leads the manager to attain a positive utility level, even after the cost (i.e., $U^{MB*} > 0$). Therefore, the manager is better off purchasing the opaque asset than purchasing the transparent one, even if it is more expensive.

Note that, for this equilibrium to exist, the degree of each opaque fund manager's career concern needs to be "moderate." On the one hand, ξ must be small enough ($\xi < 2$) to make the opaque fund managers willing to invest in the opaque asset rather than the transparent asset (i.e., $U^{MB*} > 0$). But, on the other hand, it must be large enough ($\xi > \xi_\ell$) to make these managers drive up the opaque asset's price to the level at which the transparent fund managers invest only in the transparent asset (i.e., $P^{a*} < P^{b*}$).

Corollary 4.1 offers testable implications on the opacity price premium that are drawn from Proposition 4.1.

Corollary 4.1. The overpricing of opaque assets is positively related to

1. the proportion of opaque funds: $\mathrm{d}(P^{b*} - P^{a*})/\mathrm{d}(1-\alpha) > 0$, and

2. the volatility of the persistent component $\bar{\delta}_t$ of the risky assets' periodic payoff: $\mathrm{d}(P^{b*} - P^{a*})/\mathrm{d}(\eta_v^{-1}) > 0$.

[22] For $\theta_{i,t}^{Ba} = 0$ to hold, the assumption that investors are short lived plays a role. If they were infinitely lived and sufficiently patient, they could commit not to invest in the fund in the future if the fund return reveals ex post that the manager chose $\theta_{i,t}^{Ba} = 0$, and thereby the manager would be incentivized to choose $\theta_{i,t}^{Ba} > 0$. I thank an anonymous referee for pointing this out.

The first part of Corollary 4.1 indicates that the rapid growth in the hedge fund industry in the past few decades may have driven the overpricing of opaque assets from the demand side. Indeed, a large proportion $(1-\alpha)$ of opaque funds would be associated with a large aggregate demand for the opaque asset and thus a high price of that asset, leading to a large opacity price premium. The second part of the corollary predicts that the overpricing is pronounced when financial assets with highly unpredictable payoffs prevail in the market. This makes sense too. When the volatility η_v^{-1} of the periodic payoff's persistent component is high, the investors' estimate $\hat{\delta}_t$ tends to be highly inaccurate and thus susceptible to manipulation; therefore, each opaque fund manager has a strong desire to influence their estimate, leading to a large career concern ξ. This large ξ induces the manager to lever up and invest aggressively in the opaque asset, driving up its price.

Proposition 4.1 focuses on an equilibrium in which opaque funds invest aggressively in the risky asset compared with transparent funds. For this result, the assumption that ξ is relatively large ($\xi > \xi_\ell$) is critical. Then what happens if $\xi \le \xi_\ell$ instead?

Proposition 4.2. Suppose there are both transparent and opaque assets, as well as transparent and opaque funds. Assume the opaque fund managers' career concerns satisfy $\xi \le \xi_\ell$. Then a stationary equilibrium exists. On the equilibrium path,

1. all investors infer δ_t correctly, and all agents update the estimate of $\bar{\delta}_t$ by (1.2) with $\lambda_t = \lambda$,
2. the prices of the transparent and opaque assets are equal: $P^{a*}(\hat{\delta}_t) = P^{b*}(\hat{\delta}_t) = (\hat{\delta}_t - \hat{R}^*)/r$, where $\hat{R}^* = c\,(\kappa/\phi)^{2/3}\left(S/((1-\alpha)\xi^2+\alpha)\right)^{1/3}$,
3. the sizes of the transparent and opaque funds are, respectively,
   $X^{A*} = (\kappa/\phi)^{1/3}\left(S/((1-\alpha)\xi^2+\alpha)\right)^{2/3}$ and $X^{B*} = \xi X^{A*}$,
4. a fraction $\omega^* = \pi\left(1+\xi^2(1-\alpha)/\alpha\right) \in (0,1]$ of transparent fund managers buy $\theta^{A*} = \left(S\phi/(\kappa\left((1-\alpha)\xi^2+\alpha\right))\right)^{1/3}$ shares of the transparent asset per investor capital, and the rest of them buy θ^{A*} shares of the opaque asset per investor capital. Every opaque fund manager buys $\theta^{Bb*} = \xi\theta^{A*}$ shares of the opaque asset per investor capital, and
5. the expected return on each investor's investment is $1+r$, while the one-period utilities of the transparent and opaque fund managers are, respectively, $U^{MA*} = (\kappa^{1/3}/2)\left(S\phi/((1-\alpha)\xi^2+\alpha)\right)^{2/3}$ and $U^{MB*} = \xi(2-\xi)U^{MA*}$, where $U^{MA*} \ge U^{MB*}$.

The equilibrium is supported by the investors' out-of-equilibrium belief (E.8) specified in Appendix E.2.

Proposition 4.2 states that if $\xi \leq \xi_\ell$, then the opacity price premium vanishes, that is, $P^{a*} = P^{b*}$. Moreover, market segmentation does not occur: although all opaque funds buy the opaque asset, only a fraction $\omega^* \in (0, 1]$ of transparent funds buy the transparent asset, and the rest of them buy the opaque asset.[23] This result underscores that the opacity price premium arises only if the opaque fund manager's career concern is strong enough. The result's intuition is as follows. If all funds A purchased asset a and all funds B purchased asset b (as in Proposition 4.1), then the demand of funds B for asset b is so weak (because ξ is small) that $P^a > P^b$ would hold. But $P^a > P^b$ could not be sustained in equilibrium because fund-A managers—for whom asset's opacity is irrelevant—flock to the cheaper asset b until $P^a = P^b$ holds. This equality pins down ω^*. Fund-B managers buy only asset b because the commitment problem prevents them from buying asset a.

4.3 Graphical analysis

Although the model's solutions are all expressed in closed form, a graphical representation helps us grasp the big picture of the equilibrium. Figure 1 plots the model's key endogenous variables versus the opaque fund manager's career concern ξ (which itself is an endogenous composite parameter defined in (3.13)). The parameter values used in the figure are $\alpha = 0.44$, $\pi = 0.92$, $c = 1$, $\hat{\delta}_t = 4$, $S = 1$, $\kappa = 0.1$, $\phi = 0.02$, and $r = 0.04$. In choosing the values of α and π, I consider mutual funds as an example of transparent funds; hedge funds as an example of opaque funds; stocks as an example of transparent assets; and structured products as an example of opaque assets. According to Investment Company Institute, there were 7,555 mutual funds in 2010, whereas there were around 9,500 hedge funds in 2010, according to TheCityUK. Thus, I set $\alpha = \frac{7{,}555}{7{,}555+9{,}500} = 0.44$. As for π, because of the limited availability of data, I use the relative market size of stocks and structured products in Europe as a proxy. The World Bank reports that the market capitalization of the companies listed on the stock exchanges in the European Union in 2010 is EUR 7,985 billion. According to Célérier and Vallée (2013), the total value of outstanding structured tranche products in Europe as of December 2010 is EUR 704 billion. So, I set $\pi = \frac{7{,}985}{7{,}985+704} = 0.92$.

[23] Here, note that none of the funds buys both assets a and b because, by assumption of Decision 1, each fund is allowed to buy only one risky asset. That is, there is a measure $\omega^*\alpha$ of transparent funds buying only asset a and $(1 - \omega^*)\alpha$ of them buying only asset b.

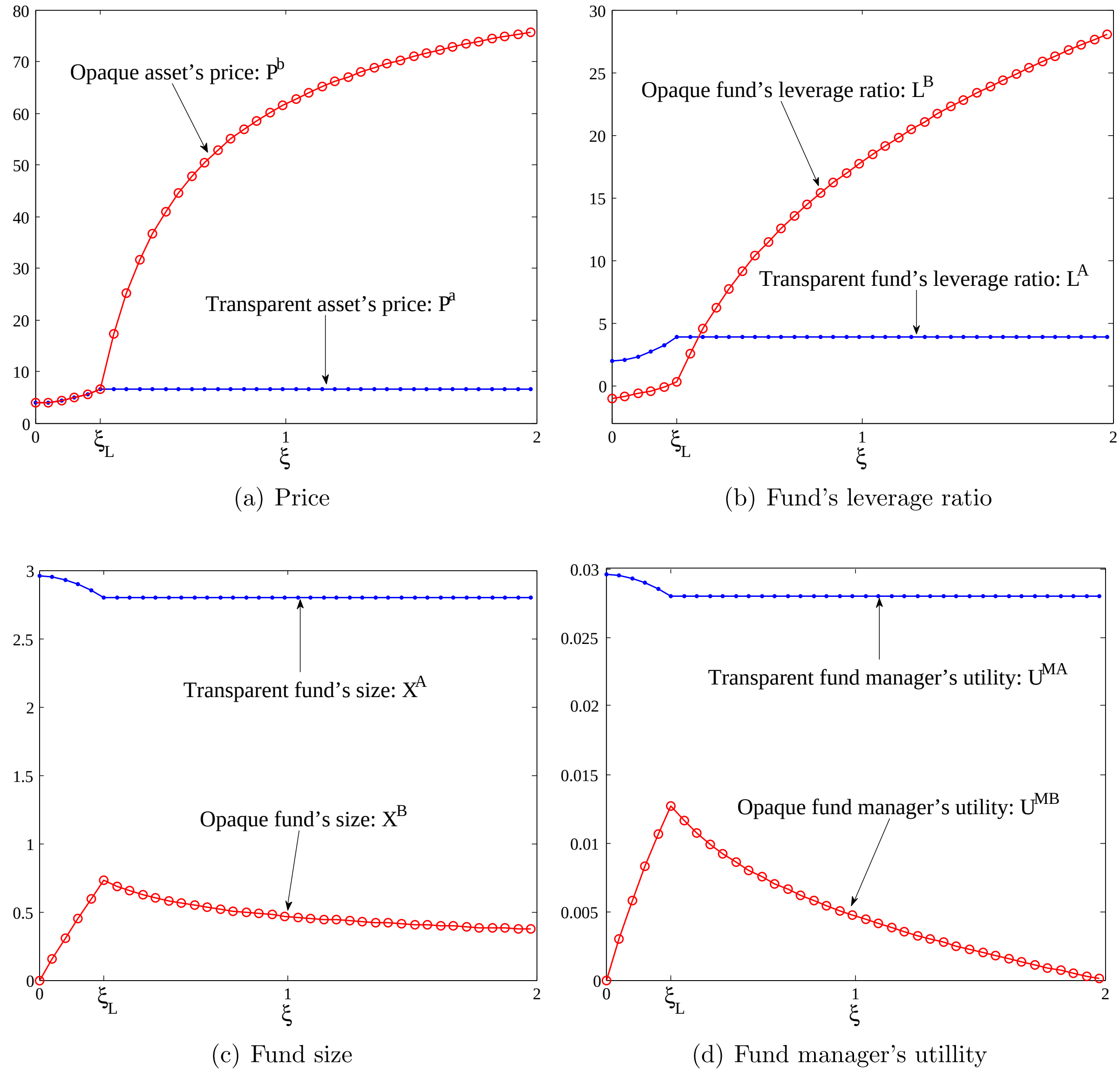


Figure 1: Fixed supply model. The graphs plot the key endogenous variables versus the opaque fund manager's career concern, ξ. Note that ξ is an endogenous composite parameter that increases with β and η_u and decreases with r and η_v. The parameter values used in the graphs are $\pi = 0.92$, $\alpha = 0.44$, $c = 1$, $\hat{\delta}_t = 4$, $S = 1$, $\kappa = 0.1$, $\phi = 0.02$, and $r = 0.04$.

Figure 1(a) plots the assets' prices. For $\xi \leq \xi_\ell$, P^a and P^b are equal. P^b increases with ξ because the opaque funds' demand increases with ξ. P^a also increases with ξ due to the increased demand pressure from the transparent funds that flee from the opaque asset and flock to the transparent asset (i.e., ω increases with ξ). For $\xi > \xi_\ell$, the market is totally segmented: only funds A buy asset a and only funds B buy asset b. So only P^b is increasing in ξ due to the opaque funds' demand, and therefore the opacity price premium increases with ξ.

Figure 1(b) plots the fund's leverage ratio, defined as the amount of borrowing ($P\theta X - X$) divided by capital X; that is, $L = (P\theta X - X)/X = P\theta - 1$. The opaque fund's leverage ratio L^B rises sharply with ξ because (1) the opaque asset's price P^b rises and (2) the manager purchases a larger number of shares of the asset per investor capital (i.e., θ^{Bb} increases).

Figure 1(c) shows $X^B < X^A$ for all ξ, consistent with the empirical observation that the asset size of each hedge fund tends to be much smaller than that of mutual fund.[24] To see the intuition, recall that the fund size is increasing in the expected return on the fund portfolio, $\hat{R}\theta$ (see (2.3) and (3.4)). Because the opaque funds' return is lower than the transparent funds' (because they purchase the expensive opaque asset), they attract fewer investor capital. For $\xi \leq \xi_\ell$, X^A decreases with ξ because the transparent fund's return decreases as $\hat{R}^a$ and θ^{Aa} both decrease. In contrast, X^B increases with ξ for $\xi \leq \xi_\ell$ because the opaque fund manager increases θ^{Bb} so much that its positive effect on X^B outweighs the negative effect of a decrease in $\hat{R}^b$ on X^B. For $\xi > \xi_\ell$, however, X^B decreases with ξ because $\hat{R}^b$ decreases so much (due to the sharp rise in P^b) that it exceeds the positive effect of a rise in θ^{Bb}. Figure 1(d) shows that the manager's one-period utility follows the similar pattern as Figure 1(c) because her fee revenue is proportional to the fund size.

5 Endogenous Supply of Opaque Assets

Why do opaque assets emerge in the first place? To address this question, I make one modification to the model of Section 4: the opaque asset is endogenously supplied by a new set of competitive agents called financial engineers. That is, π is endogenously determined in this section. The engineers not only trade the two risky assets but also

[24]The total assets of mutual funds in 2010 are USD 11,832 billion, according to Investment Company Institute, whereas those of hedge funds are USD 1,920 billion, according to TheCityUK. Thus, dividing them by the number of funds, each mutual fund's average size is USD 1.56 billion, whereas that of the hedge fund is only USD 0.19 billion.

(costlessly) transform transparent assets into opaque assets and vice versa. I assume there are $S > 0$ shares of the transparent asset and none of the opaque asset in the beginning of period 0; afterward, the engineers competitively trade and transform the assets.

The engineers effectively act as arbitrageurs who exploit the opacity price premium: as long as the opaque asset's price is higher the engineers will buy the transparent asset, make it opaque, and sell (i.e., supply) it at a profit. Because the prices of transparent and opaque assets are decreasing in their supplies (see Proposition 4.1), the engineers should continue transforming the assets until the price gap is closed.[25] Thus, the equality $P^{a*} = P^{b*}$ pins down the stationary level of π.

Proposition 5.1 summarizes the long-run stationary equilibrium of this economy.

Proposition 5.1. Suppose transparent and opaque funds coexist and financial engineers supply the opaque asset. Assume the opaque fund managers' career concerns satisfy $\xi < 2$. Then a stationary equilibrium exists. On the equilibrium path,

1. all investors infer δ_t correctly, and all agents update the estimate of $\bar{\delta}_t$ by (1.2) with $\lambda_t = \lambda$,
2. the number of shares of the opaque asset supplied by the engineers is $(1 - \pi^*)S$ and that of the transparent asset is $\pi^* S$, where $\pi^* = \alpha/((1 - \alpha)\xi^2 + \alpha)$,
3. the prices of the transparent and opaque assets are equal: $P^{a*}(\hat{\bar{\delta}}_t) = P^{b*}(\hat{\bar{\delta}}_t) = (\hat{\bar{\delta}}_t - \hat{R}^*)/r$, where $\hat{R}^* = c\,(\kappa/\phi)^{2/3}\,(S/((1 - \alpha)\xi^2 + \alpha))^{1/3}$,
4. the sizes of the transparent and opaque funds are, respectively, $X^{A*} = (\kappa/\phi)^{1/3}\,(S/((1 - \alpha)\xi^2 + \alpha))^{2/3}$ and $X^{B*} = \xi X^{A*}$,
5. every transparent fund manager buys $\theta^{Aa*} = (S\phi/(\kappa\,((1 - \alpha)\xi^2 + \alpha)))^{1/3}$ shares of the transparent asset per investor capital, and every opaque fund manager buys $\theta^{Bb*} = \xi\theta^{Aa*}$ shares of the opaque asset per investor capital, and
6. the expected return on each investor's investment is $1 + r$, whereas the one-period utilities of the transparent and opaque fund managers are, respectively, $U^{MA*} = (\kappa^{1/3}/2)\,(S\phi/((1 - \alpha)\xi^2 + \alpha))^{2/3}$ and $U^{MB*} = \xi(2 - \xi)U^{MA*}$, where $U^{MA*} \geq U^{MB*}$.

The equilibrium is supported by the investors' out-of-equilibrium belief (E.8) specified in Appendix E.2.

[25] Here, the price gap is closed completely because the engineers' asset transformation is costless. If instead it is costly, then the gap would remain positive, reflecting the transformation cost. I thank an anonymous referee for pointing this out.

An insight drawn from this model is that opacity is self-feeding in financial markets. Given the opacity price premium, the engineers try to exploit it by supplying the opaque asset, but that asset becomes a source of agency problems in portfolio delegation, resulting in the opacity price premium.

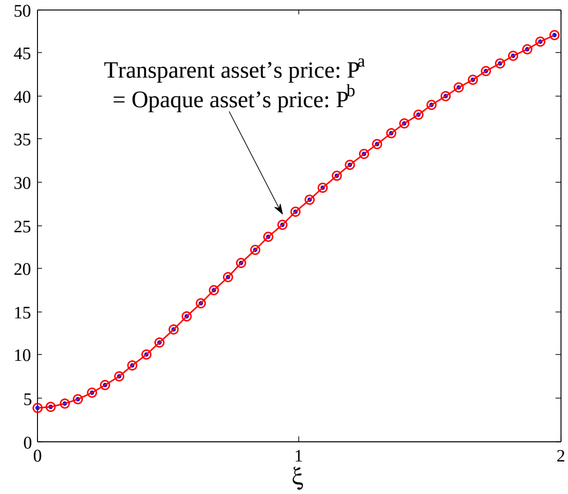
It is worth noting that the expressions of P, $\hat{R}$, X, θ, and U^M in Proposition 5.1 are identical to those of Proposition 4.2. This makes sense. In Proposition 4.2, the fraction ω of transparent funds that buy the transparent asset adjusts in equilibrium so that $P^a = P^b$ holds. In Proposition 5.1, the supply πS of the transparent asset adjusts until $P^a = P^b$ holds. Although the mechanisms are different, these cases achieve the same price level, leading to the same equilibrium actions and utility levels.

Figure 2 plots the key endogenous variables versus ξ for the same parameter values as Figure 1. Unlike Figure 1, there are no kinks on the graphs because the financial engineers exploit the opacity price premium and "smooth out" the potential regime change that would have occurred at a certain threshold level of ξ (i.e., ξ_ℓ in Section 4). Because of the similarity between Propositions 4.2 and 5.1, Figure 2 can be viewed as the low-ξ part of Figure 1 extended to the entire range of ξ. Thus, the economic intuitions for Figure 2 are similar to those for the low-ξ part of Figure 1 discussed in Section 4.3.

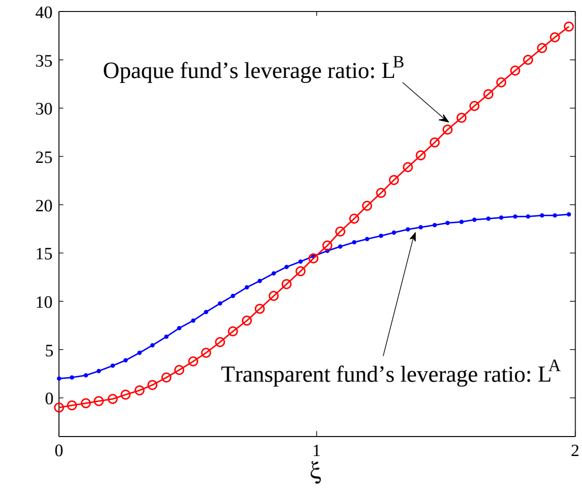
Proposition 5.1 implies that it is only in the knife-edge case $\xi = 1$ that the agents' actions and utilities are identical between transparent and opaque funds. Regarding the actions, $X^{B*} \gtreqless X^{A*}$ and $\theta^{Bb*} \gtreqless \theta^{Aa*}$ if $\xi \gtreqless 1$. As for the managers' utilities, interestingly, $U^{MA*} > U^{MB*}$, as long as $\xi \neq 1$, regardless of whether $\xi > 1$ or $\xi < 1$.[26] This means that if the managers were allowed to choose the type of funds (i.e., if α were endogeneized), then all of them would choose to be transparent (i.e., $\alpha = 1$). This somewhat unappealing feature could be eliminated if we assume that transparency comes at a cost: if disclosing $\theta_{i,t}$ to investors requires a periodic cost $q > 0$, then an interior solution of α could be obtained from the managers' indifference condition.[27] The disclosure cost q can be interpreted as representing hedge funds' benefits of being opaque—which they would lose if

[26] For this result, convexity of the manager's personal cost plays a role. If $\xi > 1$, then $X^{A*} < X^{B*}$ and $\theta^{Aa*} < \theta^{Bb*}$. Thus, clearly $\phi X^{A*} < \phi X^{B*}$. Nevertheless, $U^{MA*} > U^{MB*}$ holds because $\frac{1}{2}\kappa\theta^{Aa*2}$ is much lower than $\frac{1}{2}\kappa\theta^{Bb*2}$ because of convexity. On the other, if $\xi < 1$, then $\phi X^{A*} > \phi X^{B*}$. Because $\frac{1}{2}\kappa\theta^{Aa*2}$ is not so higher than $\frac{1}{2}\kappa\theta^{Bb*2}$ because of convexity, $U^{MA*} > U^{MB*}$ holds again.

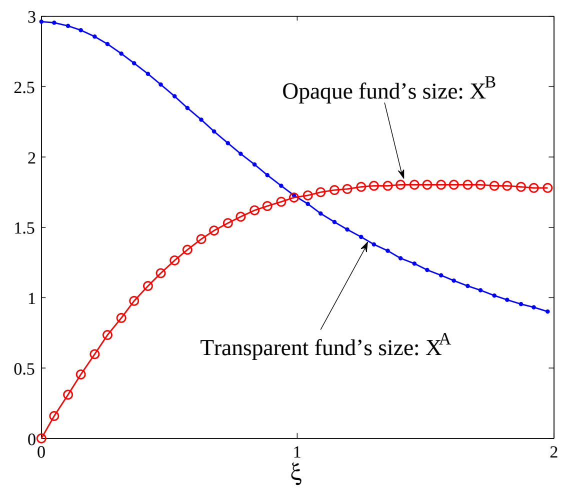
[27] Suppose that each manager is allowed to choose the fund's type and that each transparent fund manager needs to pay a periodic disclosure cost $q > 0$. In equilibrium, all managers should be indifferent between the two fund types, that is, $U^{MA*} - q = U^{MB*}$ must hold. Solving this condition for α, we obtain its equilibrium value: $\alpha^* = \frac{1}{1-\xi^2}\left(S\phi\left(\frac{(1-\xi)^2\kappa^{1/3}}{2q}\right)^{3/2} - \xi^2\right)$. Choosing the parameter values properly, we can ensure that $\alpha^* \in (0, 1)$.



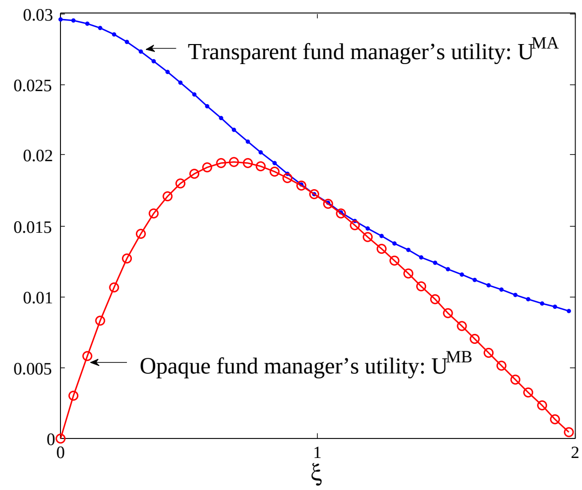
(a) Price



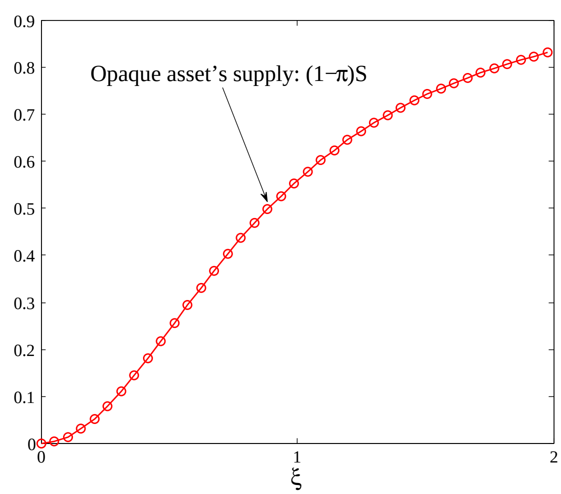
(b) Fund's leverage ratio



(c) Fund size



(d) Fund manager's utillity



(e) Opaque asset's supply

Figure 2: Endogenous supply model. The parameter values used in the graphs are $\alpha = 0.44$, $c = 1$, $\hat{\delta}_t = 4$, $S = 1$, $\kappa = 0.1$, $\phi = 0.02$, and $r = 0.04$.

becoming transparent—that are not captured in the model. For example, disclosure of hedge funds' positions could expose these funds to front running and predatory trading, and also "could allow others to infer their trading strategies and information, inducing mimicking trade which could erode the profitability of strategies (Easley, O'Hara, and Yang 2012)."[28] Indeed, Aragon, Hertzel, and Shi (2013) find empirical evidence that confidentiality of hedge funds' positions allows them to earn positive and significant abnormal returns.

Last, the next corollary offers testable implications on the supply of opaque assets.

Corollary 5.1. The supply of opaque asset is positively related to

1. the proportion of opaque funds: $\mathrm{d}(1-\pi^*)/\mathrm{d}(1-\alpha) > 0$, and

2. the volatility of the persistent component $\bar{\delta}_t$ of the underlying asset's periodic payoff: $\mathrm{d}(1-\pi^*)/\mathrm{d}(\eta_v^{-1}) > 0$.

The intuition for Corollary 5.1 is closely related to that for Corollary 4.1. A large proportion $(1-\alpha)$ of opaque funds and/or a large volatility η_v^{-1} of the persistent component of the underlying asset's payoff would be associated with a large opacity price premium (absent engineers), which requires a large number of shares of the opaque asset to be eliminated. Note that the opaque asset's price P^{b*} increases with ξ, even though the supply $(1-\pi^*)S$ increases with ξ, because the aggregate demand $(1-\alpha)\theta^{Bb*}X^{B*}$ is more increasing with ξ. Similarly for the transparent asset, P^{a*} increases with ξ, whereas the aggregate demand $\alpha\theta^{Aa*}X^{A*}$ decreases with ξ because the supply π^*S is more decreasing with ξ.

6 Conclusion

This paper proposes a dynamic equilibrium model of delegated portfolio management to study the implications of opacity in financial markets for investor behavior, asset prices, and welfare. Opacity creates agency problems in portfolio delegation, leading to excessively high fund leverage and lower welfare. An opaque asset's price is driven up as the fund managers' career concerns become stronger, since opacity potentially allows them to collect higher fees by manipulating investor assessments of their funds' future prospects. This is so even though investors are not fooled in equilibrium. When opacity and transparency coexist, opaque assets trade at a premium over transparent assets despite it

[28] I thank an anonymous referee for suggesting these interpretations of the disclosure cost.

is common knowledge that the respective asset types yield identical payoffs; this is the opacity price premium. This price gap is accompanied by endogenous market segmentation: transparent funds trade only transparent assets, and opaque funds trade only opaque assets. The opacity price premium is exploited by financial engineers, capable of transforming transparent assets into opaque ones. In that sense, opacity is self-feeding: engineers exploit the opacity price premium by supplying opaque assets, which in turn are the source of agency problems (in portfolio delegation) that result in a premium for opacity.

A limitation of this paper is that it focuses on competitive equilibria of liquid financial markets while in reality a significant proportion of complex assets are traded in over-the-counter markets (where asset prices are not made public) or thin markets (where investors have price impact). It will be fruitful for future research to explore the implications of opacity based on a search-based model of OTC markets (e.g., Duffie, Gârleanu, and Pedersen 2005) or a double-auction model of thin markets (e.g., Kyle 1989).

In this paper, opacity arises for its own sake—that is, it is created by engineers for the sole purpose of obfuscating the asset's true payoffs. This extreme way of modeling is useful for understanding one aspect of opacity. Naturally it is also important to consider other, more positive views on opaque assets. For instance, opacity may be a minor side effect associated with sophisticated financial techniques to achieve better risk sharing. Future research exploring this possible trade-off would be both interesting and important.

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Appendix

A Fund Manager's Nonpecuniary Cost as a Reduced Form of Risk Aversion

This section shows that the manager's nonpecuniary cost $\kappa\theta_{i,t}^2/2$ can be interpreted as a reduced form of his risk aversion because it endogenously arises in a slightly modified model setting.

Consider a setting with the following two modifications.

1. Fund i's management cost is $c_{i,t}X_{i,t}^2$, where $c_{i,t} \in (0, \infty)$ is stochastic, i.i.d. across funds and also across time. The realization of $c_{i,t}$ is after $\theta_{i,t}$ is chosen by manager i but before $X_{i,t}$ is determined by investors i. (The timing assumption that $X_{i,t}$ is determined after $\theta_{i,t}$ is chosen would not affect any results of the model in the main text.) For notational simplicity, define two constants: $c \equiv \mathrm{E}\left[c_{i,t}^{-1}\right]^{-1}$ and $\sigma_c^2 \equiv \mathrm{Var}\left[c_{i,t}^{-1}\right]^{-1}$. Note that, because there is a continuum of funds,

$$\int_0^1 \frac{1}{c_{i,t}}\mathrm{d}i = \mathrm{E}\left[\frac{1}{c_{i,t}}\right] = \frac{1}{c} \tag{A.1}$$

   holds by the law of large numbers. This relation plays an important role later in this section.

2. The manager's per-period objective is a mean-variance function of his fee revenue:

$$\mathrm{E}[\phi X_{i,t}] - \frac{\nu}{2}\mathrm{Var}[\phi X_{i,t}], \tag{A.2}$$

   where $\nu > 0$ represents the degree of his risk aversion.

Now, conjecture (and later verify) that $\hat{R}_{t+1}$ is a positive constant $\hat{R}$ in equilibrium. Then, as in (2.3), $X_{i,t}$ is determined as

$$X_{i,t} = \frac{1}{c_{i,t}}\hat{R}\theta_{i,t}. \tag{A.3}$$

So, by (A.2) and (A.3), the manager's period-t objective is rewritten as

$$\mathrm{E}\left[\phi X_{i,t}\right] - \frac{\nu}{2}\mathrm{Var}\left[\phi \frac{1}{c_{i,t}}\hat{R}\theta_{i,t}\right] = \mathrm{E}\left[\phi X_{i,t}\right] - \frac{1}{2}\left(\frac{\nu\phi^2\hat{R}^2}{\sigma_c^2}\right)\theta_{i,t}^2 = \mathrm{E}\left[\phi X_{i,t}\right] - \frac{\kappa}{2}\theta_{i,t}^2, \tag{A.4}$$

where $\kappa \equiv \nu\phi^2\hat{R}^2/\sigma_c^2$. Thus, (1.1) can be viewed as a reduced form of this risk-aversion formulation with the constant $\nu\phi^2\hat{R}^2/\sigma_c^2$ replaced by κ.

Last, I verify that $\hat{R}_{t+1}$ is indeed nonstochastic in equilibrium. Using (A.3) again, the manager's objective (A.4) is further rewritten as

$$\mathrm{E}\left[\phi \frac{1}{c_{i,t}}\hat{R}\theta_{i,t}\right] - \frac{\kappa}{2}\theta_{i,t}^2 = \frac{\phi}{c}\hat{R}\theta_{i,t} - \frac{\kappa}{2}\theta_{i,t}^2. \tag{A.5}$$

The manager's choice of $\theta_{i,t}$ that maximizes (A.5) is, for all i and t,

$$\theta_{i,t} = \frac{\phi}{c\kappa}\hat{R}. \tag{A.6}$$

Using (A.3) and (A.6), the risky asset's market clearing condition is

$$\int_0^1 \theta_{i,t} X_{i,t} \mathrm{d}i = S \iff \int_0^1 \frac{\phi}{c\kappa}\hat{R}\frac{1}{c_{i,t}}\hat{R}\frac{\phi}{c\kappa}\hat{R}\mathrm{d}i = S \iff \frac{\phi^2}{c^2\kappa^2}\hat{R}^3 \int_0^1 \frac{1}{c_{i,t}}\mathrm{d}i = S$$

$$\iff \frac{\phi^2}{c^3\kappa^2}\hat{R}^3 = S, \tag{A.7}$$

where the last line follows from (A.1). From (A.7), it is verified that $\hat{R}_{t+1}$ is indeed nonstochastic:

$$\hat{R} = cS^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3}. \tag{A.8}$$

Observe that the expression of (A.8) is identical to (2.6).

B Derivation of (1.3)

Let $\eta_t^- \equiv 1/\mathrm{Var}(\bar{\delta}_t|\mathcal{H}_{t-1})$ be the precision of the managers' estimate of $\bar{\delta}_t$ before observing δ_t, and $\eta_t^+ \equiv 1/\mathrm{Var}(\bar{\delta}_t|\mathcal{H}_t)$ be the precision after observing δ_t. By standard Kalman filtering, a new observation of δ_t will update the estimate of $\bar{\delta}_t$ as $\hat{\delta}_t = \lambda_t\hat{\delta}_{t-1} + (1-\lambda_t)\delta_t$ with $\lambda_t \equiv \eta_t^-/\eta_t^+$, where

$$\eta_t^+ = \eta_t^- + \eta_u. \tag{B.1}$$

Take $\mathrm{Var}(\cdot|\mathcal{H}_t)$ to $\bar{\delta}_{t+1} = \bar{\delta}_t + v_{t+1}$. By normality and independence,

$$\frac{1}{\eta_{t+1}^-} = \frac{1}{\eta_t^+} + \frac{1}{\eta_v} \iff \eta_{t+1}^- = \frac{\eta_t^+\eta_v}{\eta_t^+ + \eta_v} \iff \eta_{t+1}^- = \frac{(\eta_t^- + \eta_u)\eta_v}{\eta_t^- + \eta_u + \eta_v}. \tag{B.2}$$

Meanwhile, from the definition of λ_t it follows that

$$\lambda_t = \frac{\eta_t^-}{\eta_t^- + \eta_u} \iff \eta_t^- = \eta_u\frac{\lambda_t}{1-\lambda_t}. \tag{B.3}$$

Plugging (B.3) into (B.2) and then rearranging yields (1.3):

$$\eta_u\frac{\lambda_{t+1}}{1-\lambda_{t+1}} = \frac{\left(\eta_u\frac{\lambda_t}{1-\lambda_t} + \eta_u\right)\eta_v}{\eta_u\frac{\lambda_t}{1-\lambda_t} + \eta_u + \eta_v} \iff \lambda_{t+1} = \frac{1}{2 + \frac{\eta_u}{\eta_v} - \lambda_t}.$$

The updating factor λ_t is the same for (1.2) and (1.5). This is shown as follows. Let $\eta_{i,t}^{I-} \equiv 1/\mathrm{Var}(\bar{\delta}_t|\mathcal{H}_{i,t-1}^I)$ be the precision of the investors i's estimate of $\bar{\delta}_t$ before inferring δ_t (i.e., before having the value of $\delta_{i,t}^I$), and $\eta_{i,t}^{I+} \equiv 1/\mathrm{Var}(\bar{\delta}_t|\mathcal{H}_{i,t}^I)$ be the precision after inferring δ_t. As in (B.1), the relation between $\eta_{i,t}^{I+}$ and

$\eta^{I-}_{i,t}$ is

$$\eta^{I+}_{i,t} = \eta^{I-}_{i,t} + \eta_u. \tag{B.4}$$

Because $\mathcal{H}_0 = \mathcal{H}^I_{i,0}$ (i.e., they are empty sets in period 0), $\eta^{I-}_{i,0} = \eta^-_0$ for all i. Given this, (B.1) and (B.4) imply that $\eta^{I+}_{i,t} = \eta^+_t$ and $\eta^{I-}_{i,t} = \eta^-_t$ for all i and t. Thus, we have $\eta^{I-}_{i,t}/\eta^{I+}_{i,t} = \eta^-_t/\eta^+_t = \lambda_t$, as required.

C Proof of Lemma 3.1

First, I determine the size of fund i on an off-the-equilibrium path where manager i is deviating from the equilibrium strategy.

Lemma C.1. If manager i plays $(\theta_{i,0}, \theta_{i,1}, ...)$ when investors i believe that he plays $(\theta^{**}_0, \theta^{**}_1, ...)$, then the size of fund i in period $t = 1, 2, ...$ is $X_{i,t} = X_t + X^+_{i,t}$, where

$$X_t \equiv \frac{1}{c}\hat{R}_{t+1}\theta^{**}_t \tag{C.1}$$

$$\text{and} \quad X^+_{i,t} \equiv \frac{\theta^{**}_t}{c}\left(1 + \frac{1-\lambda_{t+1}}{r}\right)\sum_{s=1}^{t}\left(\prod_{\nu=s+1}^{t}\lambda_\nu\right)(1-\lambda_s)\left(\frac{\theta_{i,s-1}-\theta^{**}_{s-1}}{\theta^{**}_{s-1}}\right)R_s.^{29} \tag{C.2}$$

Proof of Lemma C.1: To prove Lemma C.1, I prove the following two claims.

Claim 1: If investors i believe that the payoff history up to period t is $\mathcal{H}^I_{i,t} = (\delta^I_{i,1}, ..., \delta^I_{i,t})$, then the estimate of investors i in an arbitrary period t is

$$\hat{\delta}^I_{i,t} = \hat{\delta}_t + \sum_{s=1}^{t}\left(\prod_{\nu=s+1}^{t}\lambda_\nu\right)(1-\lambda_s)(\delta^I_{i,s} - \delta_s).$$

Proof of Claim 1: For $t = 0$, we have $\hat{\delta}^I_{i,0} = \hat{\delta}_0$ (exogenous). For $t = 1$, using the updating rule (1.5),

$$\begin{aligned}\hat{\delta}^I_{i,1} =& \lambda_1\hat{\delta}_0 + (1-\lambda_1)\delta^I_{i,1}\\ =& \underbrace{\lambda_1\hat{\delta}_0 + (1-\lambda_1)\delta_1}_{\hat{\delta}_1} + \underbrace{(1-\lambda_1)(\delta^I_{i,1} - \delta_1)}_{\text{misperception}}\\ =& \hat{\delta}_1 + (1-\lambda_1)(\delta^I_{i,1} - \delta_1).\end{aligned}$$

For $t = 2$,

$$\begin{aligned}\hat{\delta}^I_{i,2} =& \lambda_2\hat{\delta}^I_{i,1} + (1-\lambda_2)\delta^I_{i,2}\\ =& \lambda_2\left(\hat{\delta}_1 + (1-\lambda_1)(\delta^I_{i,1} - \delta_1)\right) + (1-\lambda_2)\delta_2 + (1-\lambda_2)(\delta^I_{i,2} - \delta_2)\\ =& \underbrace{\lambda_2\hat{\delta}_1 + (1-\lambda_2)\delta_2}_{\hat{\delta}_2} + \underbrace{\lambda_2(1-\lambda_1)(\delta^I_{i,1} - \delta_1) + (1-\lambda_2)(\delta^I_{i,2} - \delta_2)}_{\text{misperception}}\\ =& \hat{\delta}_2 + \lambda_2(1-\lambda_1)(\delta^I_{i,1} - \delta_1) + (1-\lambda_2)(\delta^I_{i,2} - \delta_2).\end{aligned}$$

[29] In (C.2), I abuse notation and set $\prod_{\nu=t+1}^{t}\lambda_\nu \equiv 1$.

Continuing this way, for an arbitrary t, we have

$$\begin{aligned}\hat{\delta}^I_{i,t} &= \hat{\delta}_t + \lambda_t\lambda_{t-1}\cdots\lambda_2(1-\lambda_1)(\delta^I_{i,1}-\delta_1)\\ &\quad + \lambda_t\lambda_{t-1}\cdots\lambda_3(1-\lambda_2)(\delta^I_{i,2}-\delta_2)\\ &\quad \vdots\\ &\quad + \lambda_t(1-\lambda_{t-1})(\delta^I_{i,t-1}-\delta_{t-1})\\ &\quad + (1-\lambda_t)(\delta^I_{i,t}-\delta_t)\\ &= \hat{\delta}_t + \sum_{s=1}^{t}\left(\prod_{\nu=s+1}^{t}\lambda_\nu\right)(1-\lambda_s)(\delta^I_{i,s}-\delta_s),\end{aligned}$$

where (abusing notation somewhat) I set $\prod_{\nu=t+1}^{t}\lambda_\nu \equiv 1$. (End of proof of Claim 1.)

Claim 2: Investors i's expected excess return on the risky asset is

$$\hat{R}^I_{i,t+1} = \hat{R}_{t+1} + \left(1+\frac{1-\lambda_{t+1}}{r}\right)(\hat{\delta}^I_{i,t}-\hat{\delta}_t). \tag{C.3}$$

Proof of Claim 2: Using (3.1) and (1.5), the risky asset's excess return is written as

$$\begin{aligned}R_{t+1} \equiv&\delta_{t+1} + P_{t+1} - (1+r)P_t\\ =&\delta_{t+1} + \frac{1}{r}\int_0^1 \hat{\delta}^I_{i,t+1}\,\mathrm{d}i - \gamma_{t+1} - (1+r)P_t\\ =&\delta_{t+1} + \frac{1}{r}\int_0^1\left(\lambda_{t+1}\hat{\delta}^I_{i,t} + (1-\lambda_{t+1})\delta^I_{i,t+1}\right)\mathrm{d}i - \gamma_{t+1} - (1+r)P_t\\ =&\delta_{t+1} + \frac{1-\lambda_{t+1}}{r}\int_0^1 \delta^I_{i,t+1}\,\mathrm{d}i + \frac{\lambda_{t+1}}{r}\hat{\delta}^I_t - \gamma_{t+1} - (1+r)P_t.\end{aligned}$$

Thus, the expected excess return conditional on the true payoff history $\mathcal{H}_t$ is

$$\hat{R}_{t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{H}_t] = \left(1+\frac{1-\lambda_{t+1}}{r}\right)\hat{\delta}_t + \frac{\lambda_{t+1}}{r}\hat{\delta}^I_t - \gamma_{t+1} - (1+r)P_t. \tag{C.4}$$

Similarly, investors i's expected excess return (conditional on their inferred history $\mathcal{H}^I_{i,t}$) is

$$\hat{R}^I_{i,t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{F}^I_{i,t}] = \left(1+\frac{1-\lambda_{t+1}}{r}\right)\hat{\delta}^I_{i,t} + \frac{\lambda_{t+1}}{r}\hat{\delta}^I_t - \gamma_{t+1} - (1+r)P_t. \tag{C.5}$$

From (C.4) and (C.5) we obtain

$$\hat{R}^I_{i,t+1} = \hat{R}_{t+1} + \left(1+\frac{1-\lambda_{t+1}}{r}\right)(\hat{\delta}^I_{i,t}-\hat{\delta}_t),$$

as required. (End of proof of Claim 2.)

Claim 2 shows that an overestimation of $\bar{\delta}_t$ (i.e., $\hat{\delta}^I_{i,t} > \hat{\delta}_t$) leads to an overestimation of the excess return (i.e., $\hat{R}^I_{i,t+1} > \hat{R}_{t+1}$). Now Claims 1 and 2 can be used to rearrange the fund size (3.4) as follows:

$$\begin{aligned}
X_{i,t} =& \frac{1}{c}\hat{R}^I_{i,t+1}\theta^{**}_t \\
=& \frac{1}{c}\hat{R}_{t+1}\theta^{**}_t + \frac{\theta^{**}_t}{c}(\hat{R}^I_{i,t+1} - \hat{R}_{t+1}) \\
=& \frac{1}{c}\hat{R}_{t+1}\theta^{**}_t + \frac{\theta^{**}_t}{c}\left(1 + \frac{1-\lambda_{t+1}}{r}\right)(\hat{\delta}^I_{i,t} - \hat{\delta}_t) \\
=& \frac{1}{c}\hat{R}_{t+1}\theta^{**}_t + \frac{\theta^{**}_t}{c}\left(1 + \frac{1-\lambda_{t+1}}{r}\right)\sum_{s=1}^{t}\left(\prod_{\nu=s+1}^{t}\lambda_\nu\right)(1-\lambda_s)(\delta^I_{i,s} - \delta_s).
\end{aligned} \tag{C.6}$$

Substituting (3.6) into (C.6) then yields

$$\begin{aligned}
X_{i,t} =& \frac{1}{c}\hat{R}_{t+1}\theta^{**}_t + \frac{\theta^{**}_t}{c}\left(1 + \frac{1-\lambda_{t+1}}{r}\right)\sum_{s=1}^{t}\left(\prod_{\nu=s+1}^{t}\lambda_\nu\right)(1-\lambda_s)\left(\frac{\theta_{i,s-1} - \theta_{s-1}}{\theta_{s-1}}\right)R_s \\
=& X_t + X^+_{i,t},
\end{aligned}$$

as we wanted. (End of proof of Lemma C.1.)

Lemma C.1 shows that the fund size $X_{i,t}$ is the sum of two terms. The first, (C.1), is the size on the equilibrium path. The second, (C.2), is an additional component on the off-the-equilibrium paths where the manager is deviating. (This term is zero on the equilibrium path because $\theta_{i,s-1} = \theta^{**}_{s-1}$ holds for all s.) From the manager's perspective, the equilibrium fund size (C.1) is independent of his actions and is thus uncontrollable. Hence, the impact of his actions $(\theta_{i,0}, ..., \theta_{i,t-1})$ on the current fund size $X_{i,t}$ is summarized in the off-equilibrium component (C.2). Given that $R_s > 0$ for $s \leq t$, the investors invest additional $X^+_{i,t} > 0$ dollars of capital in the fund in period t after the manager's deviation $\theta_{i,s-1} > \theta^{**}_{s-1}$ in period $s-1$. As is clear from (3.4), this additional capital investment is caused by an overshoot in the investors' expectation of excess return (i.e., $\hat{R}^I_{i,t+1} > \hat{R}_{t+1}$). The key for this to occur is the investors' out-of-equilibrium belief (3.3), which leads the investors to stick by their own estimate of $\bar{\delta}_t$ when it disagrees with the all investors' average estimate $\hat{\delta}^I_t$ implied by the price. Note that, on this off-the-equilibrium path, investors i are aware that $\hat{\delta}^I_{i,t}$ is higher than $\hat{\delta}^I_t$, but are unaware that it is too high: they (incorrectly) believe that $\hat{\delta}^I_{i,t}$ is more accurate than $\hat{\delta}^I_t$. As shown in (C.3), this overestimation of $\bar{\delta}_t$ leads to an overshoot in the expectation of R_{t+1}. Intuitively, the investors' high estimate of $\bar{\delta}_t$ leads them to expect both δ_{t+1} and P_{t+1} to be high; given the observed current price P_t, these estimates lead them to expect that R_{t+1} will also be high.

Now I prove Lemma 3.1. To simplify the manager's period-t objective, note the following points.

- The fee generated by the current fund size, $\phi X_{i,t}$, can be omitted from the original objective function (1.1) because, from (3.4), $X_{i,t}$ is independent of the manager's actual choice of $\theta_{i,t}$.
- By Lemma C.1, the future fund size $X_{i,t+\tau}$ ($\tau = 1, 2, ...$) is linear in $X_{t+\tau}$ and $X^+_{i,t+\tau}$. Because (1.1) is linear in $X_{i,t+\tau}$, it follows that (1.1) is linear in $X_{t+\tau}$ and $X^+_{i,t+\tau}$. This implies that $X_{t+\tau}$ can be omitted from (1.1) because the manager cannot influence $X_{t+\tau}$ by his choice of $\theta_{i,t}$. That is, only $X^+_{i,t+\tau}$ is relevant for his choice of $\theta_{i,t}$.
- Conjecture (3.2)—which is verified later—implies that the manager's current action ($\theta_{i,t}$) does not

affect his own future actions $(\theta_{i,t+1}, \theta_{i,t+2}, ...)$, both on and off the equilibrium path. Thus, his personal costs of forming future portfolios can be omitted from (1.1).

Taking these points into account, the manager's period-t maximization problem reduces to

$$\max_{\theta_{i,t}\in[0,\infty)} -\frac{\kappa}{2}\theta_{i,t}^2 + \mathrm{E}\left[\sum_{\tau=1}^{\infty}\beta^\tau \phi X_{i,t+\tau}^+ \middle| \mathcal{F}_{i,t}^M\right],$$

$$\text{where} \quad X_{i,t+\tau}^+ = \frac{\theta_{t+\tau}^{**}}{c}\left(1+\frac{1-\lambda_{t+\tau+1}}{r}\right)\sum_{s=1}^{t+\tau}\left(\prod_{\nu=s+1}^{t+\tau}\lambda_\nu\right)(1-\lambda_s)\left(\frac{\theta_{i,s-1}-\theta_{s-1}^{**}}{\theta_{s-1}^{**}}\right)R_s. \tag{C.7}$$

Because $X_{i,t+\tau}^+$ is a linear function of $(\theta_{i,0}, ..., \theta_{i,t+\tau-1})$, the marginal effect of the manager's current action $\theta_{i,t}$ on $X_{i,t+\tau}^+$ is independent of his actions in other periods, $(\theta_{i,0}, ..., \theta_{i,t-1}, \theta_{i,t+1}, ..., \theta_{i,t+\tau-1})$. Hence, in $X_{i,t+\tau}^+$ given by (C.7), only the term corresponding to $s = t+1$ is relevant for the problem. Thus, an equivalent problem is

$$\max_{\theta_{i,t}\in[0,\infty)} -\frac{\kappa}{2}\theta_{i,t}^2 + \mathrm{E}\left[\sum_{\tau=1}^{\infty}\beta^\tau\phi\frac{\theta_{t+\tau}^{**}}{c}\left(1+\frac{1-\lambda_{t+\tau+1}}{r}\right)\left(\prod_{\nu=t+2}^{t+\tau}\lambda_\nu\right)(1-\lambda_{t+1})\left(\frac{\theta_{i,t}-\theta_t^{**}}{\theta_t^{**}}\right)R_{t+1}\middle|\mathcal{F}_{i,t}^M\right].$$

This is rewritten as

$$\max_{\theta_{i,t}\in[0,\infty)} -\frac{\kappa}{2}\theta_{i,t}^2 + \frac{\phi}{c}\left(\frac{\theta_{i,t}-\theta_t^{**}}{\theta_t^{**}}\right)\Omega_t\hat{R}_{t+1},$$

$$\text{where} \quad \Omega_t \equiv (1-\lambda_{t+1})\sum_{\tau=1}^{\infty}\beta^\tau\theta_{t+\tau}^{**}\left(1+\frac{1-\lambda_{t+\tau+1}}{r}\right)\left(\prod_{\nu=t+2}^{t+\tau}\lambda_\nu\right). \quad \square$$

D Derivation of (3.12)

By the definition of Ω_t,

$$\Omega_t = (1-\lambda_{t+1})\sum_{\tau=1}^{\infty}\beta^\tau\left(\prod_{\nu=t+2}^{t+\tau}\lambda_\nu\right)M_{t+\tau}, \quad \text{where } M_{t+\tau} \equiv \theta_{t+\tau}^{**}\left(1+\frac{1-\lambda_{t+\tau+1}}{r}\right).$$

Then, noting that $\prod_{\nu=t+2}^{t+1}\lambda_\nu \equiv 1$, we have

$$\frac{\Omega_t}{1-\lambda_{t+1}} = \beta M_{t+1} + \beta^2\lambda_{t+2}M_{t+2} + \beta^3\lambda_{t+2}\lambda_{t+3}M_{t+3} + \beta^4\lambda_{t+2}\lambda_{t+3}\lambda_{t+4}M_{t+4} + \cdots,$$

$$\frac{\Omega_{t+1}}{1-\lambda_{t+2}} = \beta M_{t+2} + \beta^2\lambda_{t+3}M_{t+3} + \beta^3\lambda_{t+3}\lambda_{t+4}M_{t+4} + \cdots.$$

From these two equations it follows that

$$\frac{\Omega_t}{1-\lambda_{t+1}} = \beta M_{t+1} + \beta\lambda_{t+2}\frac{\Omega_{t+1}}{1-\lambda_{t+2}}.$$

In the stationary state when $t \to \infty$, this relation becomes

$$\frac{\Omega^*}{1-\lambda} = \beta\theta^* \left(1 + \frac{1-\lambda}{r}\right) + \beta\lambda\frac{\Omega^*}{1-\lambda}.$$

Rearranging, we have

$$\Omega^* = \xi\theta^*, \quad \text{where} \quad \xi \equiv \left(1 + \frac{1-\lambda}{r}\right)\frac{\beta(1-\lambda)}{1-\beta\lambda}.$$

E Proof of Proposition 4.1

E.1 Setup

First, I present some model details that are omitted in the main text (because of their obvious analogy to those in Section 1).

Manager j-i's maximization problem in period t, denoted $\mathcal{P}^{Mj}_{i,t}$, is choosing $k \in \{a, b\}$ and $\theta^{jk}_{i,t} \in [0, \infty)$ to maximize

$$\mathrm{E}\left[\sum_{\tau=0}^{\infty} \beta^\tau \left(\phi X^j_{i,t+\tau} - \frac{\kappa}{2}\theta^{jk2}_{i,t+\tau}\right) \middle| \mathcal{F}^{Mj}_{i,t}\right],$$

where $\mathcal{F}^{Mj}_{i,t} = \left\{Y^j_{i,\tau}, X^j_{i,\tau}, \theta^{jk}_{i,\tau}, P^a_\tau, P^b_\tau, \delta_\tau : \tau \le t\right\}$ is the manager's information set in period t. Let $\mathcal{I}^{jk}_t$ be the set of index i such that manager j-i chooses asset class k in period t.

The investors' information sets, which include variables that are directly observable to them, are as follows.

$$\text{Investors } A\text{-}i\text{:} \quad \mathcal{F}^{IA}_{i,t} = \left\{\delta_\tau \text{ iff manager } A\text{-}i \text{ chose } k = a \text{ in period } \tau - 1, Y^A_{i,\tau}, X^A_{i,\tau}, \theta^{Ak}_{i,\tau}, P^k_\tau : \tau \le t\right\}.$$

$$\text{Investors } B\text{-}i\text{:} \quad \mathcal{F}^{IB}_{i,t} = \left\{\delta_\tau \text{ iff manager } B\text{-}i \text{ chose } k = a \text{ in period } \tau - 1, Y^B_{i,\tau}, X^B_{i,\tau}, P^k_\tau : \tau \le t\right\}.$$

Only for investors A-i, we can alternatively use $\mathcal{F}^{IA}_{i,t} = \left\{\delta_\tau, Y^A_{i,\tau}, X^A_{i,\tau}, \theta^{Ak}_{i,\tau}, P^k_\tau : \tau \le t\right\}$ because they can always infer δ_t correctly regardless of the manager's actions.

The equilibrium consists of the price function $P^k_t(\hat{\delta}_t)$, the fund size $X^j_{i,t}$, and the manager's portfolio $\theta^{jk}_{i,t}$ for $i \in [0, 1]$, $j = A, B$, and $k = a, b$ such that, for all t, the following statements hold.

1. Given $P^k_t(\hat{\delta}_t)$ and the others' actions, each investor optimally allocates her endowment between the fund and the riskless asset.
2. Given $P^k_t(\hat{\delta}_t)$ and the others' actions, manager j-i solves $\mathcal{P}^{Mj}_{i,t}$.
3. The risky assets' markets clear.

$$\text{Asset } a\text{:} \quad \int_{i\in\mathcal{I}^{Aa}_t} \theta^{Aa}_{i,t} X^A_{i,t}\,\mathrm{d}i + \int_{i\in\mathcal{I}^{Ba}_t} \theta^{Ba}_{i,t} X^B_{i,t}\,\mathrm{d}i = \pi S. \tag{E.1}$$

$$\text{Asset } b\text{:} \quad \int_{i\in\mathcal{I}^{Ab}_t} \theta^{Ab}_{i,t} X^A_{i,t}\,\mathrm{d}i + \int_{i\in\mathcal{I}^{Bb}_t} \theta^{Bb}_{i,t} X^B_{i,t}\,\mathrm{d}i = (1-\pi) S. \tag{E.2}$$

4. Every agent has correct beliefs about the other agents' actions.
5. Each agent updates the estimate of $\bar{\delta}_t$ via Kalman filtering.

Denote by $\mathcal{H}^{Ij}_{i,t} \equiv (\delta^{Ij}_{i,1}, ..., \delta^{Ij}_{i,t})$ the payoff history believed by investors j-i, where $\delta^{Ij}_{i,t}$ is the value of δ_t that they believe. These investors' estimate of $\bar{\delta}_t$ is $\hat{\delta}^{Ij}_{i,t} \equiv \mathrm{E}[\bar{\delta}_t|\mathcal{F}^{Ij}_{i,t}]$, and their conditional expectation of the excess return on asset $k \in \{a, b\}$ is $\hat{R}^{Ijk}_{i,t+1} \equiv \mathrm{E}[R^k_{t+1}|\mathcal{F}^{Ij}_{i,t}]$.

E.2 Conjectures and out-of-equilibrium beliefs

I look for an equilibrium with the following properties.

1. Asset a has a higher expected excess return than asset b; that is, for all t,
$$\hat{R}^a_{t+1} > \hat{R}^b_{t+1}. \tag{E.3}$$

2. Asset a is purchased only by funds A, and asset b is purchased only by funds B. That is, for all t,
$$\mathcal{I}^{Aa}_t = [0, \alpha) \text{ and } \mathcal{I}^{Bb}_t = [\alpha, 1]. \tag{E.4}$$

3. There exists a publicly known constant $\gamma^a > 0$ such that asset a's price function is
$$P^a\big(\hat{\delta}^{IA}_t\big) = \frac{\hat{\delta}^{IA}_t}{r} - \gamma^a \quad \text{where } \hat{\delta}^{IA}_t \equiv \frac{1}{\alpha}\int_0^\alpha \hat{\delta}^{IA}_{i,t}\,\mathrm{d}i. \tag{E.5}$$

4. There exists $\{\gamma^b_t\}^\infty_{t=0} > 0$, which is nonstochastic and publicly known, such that asset b's price function is
$$P^b_t\big(\hat{\delta}^{IB}_t\big) = \frac{\hat{\delta}^{IB}_t}{r} - \gamma^b_t \quad \text{where } \hat{\delta}^{IB}_t \equiv \frac{1}{1-\alpha}\int_\alpha^1 \hat{\delta}^{IB}_{i,t}\,\mathrm{d}i. \tag{E.6}$$

5. There exists $\{\theta^{Bb*}_t\}^\infty_{t=0} > 0$, which is nonstochastic and publicly known, such that, for all $i \in \mathcal{I}^{Bb}_t$, manager B-i optimally plays
$$\theta^{Bb}_{i,t} = \theta^{Bb*}_t \tag{E.7}$$
on the equilibrium path and also on off-the-equilibrium paths such that $\hat{\delta}^{IB}_{i,t} \neq \hat{\delta}_t$.

I specify the following out-of-equilibrium belief of the opaque funds' investors, which is the obvious counterpart of (3.3) in Section 3.

$$\text{If } \mathrm{E}[\bar{\delta}_t|\mathcal{H}^{IB}_{i,t}] \neq \hat{\delta}^{IB}_t \text{ then } \hat{\delta}^{IB}_{i,t} = \mathrm{E}[\bar{\delta}_t|\mathcal{H}^{IB}_{i,t}]. \tag{E.8}$$

E.3 Optimizations: Transparent funds

First, for a given $k \in \{a, b\}$ chosen by manager A-i, let us consider the decisions of investors A-i. As in Section 2.1, $X^A_{i,t}$ is determined by the investors' indifference condition: $\mathrm{E}[y^A_{i,t+1}\big|\mathcal{F}^{IA}_{i,t}] = 1 + r$, where $y^A_{i,t+1} = R^k_{t+1}\theta^{Ak}_{i,t} + (1 + r) - cX^A_{i,t}$ is the fund return per dollar of capital. Solving this condition for $X^A_{i,t}$ yields
$$X^A_{i,t} = \frac{1}{c}\hat{R}^k_{t+1}\theta^{Ak}_{i,t}. \tag{E.9}$$

Observe that the investors' expected excess return $\hat{R}^{IAk}_{i,t+1}$ is replaced with $\hat{R}^k_{t+1}$ in (E.9) because they always correctly infer $\mathcal{H}_t$.

Second, consider manager A-i's problem. For a given $k \in \{a, b\}$, what is his optimal choice of $\theta^{Ak}_{i,t}$? Because there is no scope for manipulating investor learning regardless of k, his problem is, in effect, simply choosing $\theta^{Ak}_{i,t}$ to maximize his period-t utility, taking into account that $X^A_{i,t}$ responds to $\theta^{Ak}_{i,t}$ according to (E.9). His optimal choice is

$$\theta^{Ak}_{i,t} = \frac{\phi}{c\kappa}\hat{R}^k_{t+1}. \tag{E.10}$$

From (E.9) and (E.10), the manager's maximized period-t utility given $\hat{R}^k_{t+1}$ is

$$\phi X^A_{i,t} - \frac{\kappa}{2}\theta^{Ak2}_{i,t} = \frac{\phi^2}{2c^2\kappa}\hat{R}^{k2}_{t+1}, \tag{E.11}$$

which is increasing in $\hat{R}^k_{t+1}$. Therefore, given that $\hat{R}^a_{t+1} > \hat{R}^b_{t+1}$ (by conjecture (E.3)), it is optimal for the manager to choose $k = a$.

E.4 Optimizations: Opaque funds

According to conjecture (E.4), it is optimal for each fund-B manager to choose $k = b$ in every period. In this section, I take it as given and characterize the opaque-fund agents' optimal decisions. Later, in Section E.6, I will verify that it is indeed optimal for the manager to choose $k = b$.

Using conjecture (E.7), the investors' indifference condition implies that the size of fund B-i is

$$X^B_{i,t} = \frac{1}{c}\hat{R}^{IBb}_{i,t+1}\theta^{Bb*}_t. \tag{E.12}$$

This size depends on the investors' belief (θ^{Bb*}_t) about the manager's unobservable choice ($\theta^{Bb}_{i,t}$). Note also that it depends on the investors' expected excess return on asset b, $\hat{R}^{IBb}_{i,t+1}$, which need not be equal to $\hat{R}^b_{t+1}$ off the equilibrium path.

Following steps similar to those in Section 3.3, the manager's problem is reduced to the following.

Lemma E.1. Suppose that manager B-i chooses $k = b$ in period t. Then, he chooses $\theta^{Bb}_{i,t} \in [0, \infty)$ to maximize

$$-\frac{\kappa}{2}\theta^{Bb2}_{i,t} + \frac{\phi}{c}\left(\frac{\theta^{Bb}_{i,t} - \theta^{Bb*}_t}{\theta^{Bb*}_t}\right)\Omega_t\hat{R}^b_{t+1}, \tag{E.13}$$

$$\text{where} \quad \Omega_t \equiv (1 - \lambda_{t+1})\sum_{\tau=1}^{\infty}\beta^\tau\theta^{Bb*}_{t+\tau}\left(1 + \frac{1 - \lambda_{t+\tau+1}}{r}\right)\left(\prod_{\nu=t+2}^{t+\tau}\lambda_\nu\right).$$

Proof of Lemma E.1: First, consider what happens when manager B-i deviates from his equilibrium play $\left(\theta^{Bb*}_0, \theta^{Bb*}_1, ...\right)$ and chooses an arbitrary sequence $\left(\theta^{Bk}_{i,0}, \theta^{Bk}_{i,1}, ...\right)$. Note that here the manager is allowed to choose $k = a$ when deviating. If the manager chooses $k = a$ in period t, then the investors will directly observe δ_{t+1}. However, if the manager chooses $k = b$ in period t, then, as discussed in Section 3.3, the investors' inference about δ_{t+1} can be misled. That is,

$$\text{if } \ i \in \mathcal{I}_t^{Ba}, \quad \text{then} \quad \delta_{i,t+1}^{IB} = \delta_{t+1};$$
$$\text{if } \ i \in \mathcal{I}_t^{Bb}, \quad \text{then} \quad \delta_{i,t+1}^{IB} = \delta_{t+1} + \left(\frac{\theta_{i,t}^{Bb} - \theta_t^{Bb*}}{\theta_t^{Bb*}} \right) R_{t+1}^b.$$

Then, following steps similar to those in the proof of Lemma C.1, the size of fund B-i in period $t = 1, 2, ...$ is written as

$$X_{i,t}^B = X_t^B + X_{i,t}^{B+}, \tag{E.14}$$

where

$$X_t^B \equiv \frac{1}{c} \hat{R}_{t+1}^b \theta_t^{Bb*}$$

and

$$X_{i,t}^{B+} \equiv \frac{\theta_t^{Bb*}}{c} \left(1 + \frac{1 - \lambda_{t+1}}{r} \right) \sum_{s=1}^{t} \left(\prod_{\nu=s+1}^{t} \lambda_\nu \right) (1 - \lambda_s) \mathbf{1}_{\left(i \in \mathcal{I}_{s-1}^{Bb}\right)} \left(\frac{\theta_{i,s-1}^{Bb} - \theta_{s-1}^{Bb*}}{\theta_{s-1}^{Bb*}} \right) R_s^b,$$

where $\mathbf{1}_{(Z)}$ is an indicator function that takes 1 if Z is true and 0 otherwise. The indicator function is needed because the manager's deviation in period $s-1$ influences $X_{i,t}^{B+}$ only if he chooses $k = b$ in that period.

Note that, from (E.14) and (E.4), the future fund size $X_{i,t+\tau}^B$ ($\tau = 1, 2, ...$) is a linear function of $(\theta_{i,0}^{Bk}, ..., \theta_{i,t+\tau-1}^{Bk})$. It follows that the marginal effect of the manager's current action $\theta_{i,t}^{Bb}$ on $X_{i,t+\tau}^B$ is independent of his actions in other periods, $(\theta_{i,0}^{Bk}, ..., \theta_{i,t-1}^{Bk}, \theta_{i,t+1}^{Bk}, ..., \theta_{i,t+\tau-1}^{Bk})$. So, by the same logic as in the proof of Lemma 3.1, the manager's objective function is simplified to (E.13). (End of proof of Lemma E.1.)

The first term of (E.13) is the manager's personal cost of choosing the portfolio, and the second term corresponds to his expected gain from influencing future investor estimates. The optimal choice of $\theta_{i,t}^{Bb}$ (given $\hat{R}_{t+1}^b$ and θ_t^{Bb*}) is

$$\theta_{i,t}^{Bb} = \frac{1}{\theta_t^{Bb*}} \frac{\phi}{c\kappa} \Omega_t \hat{R}_{t+1}^b. \tag{E.15}$$

It is shown in Section E.5 that $\hat{R}_{t+1}^b$ is nonstochastic. Hence, conjecture (E.7) is correct (and the investors' beliefs are consistent) if $\theta_{i,t}^{Bb} = \theta_t^{Bb*}$ in (E.15)—that is, if

$$\theta_t^{Bb*} = \left(\frac{\phi}{c\kappa} \Omega_t \hat{R}_{t+1}^b \right)^{1/2}. \tag{E.16}$$

The values of θ_t^{Bb*} and $\hat{R}_{t+1}^b$ will be determined explicitly after the market-clearing condition is imposed and another relation between θ_t^{Bb*} and $\hat{R}_{t+1}^b$ is identified (in Section E.5).

E.5 Equilibrium

According to conjecture (E.4) (which is verified later), asset a is purchased only by funds A and asset b only by funds B. Plugging the fund-A agents' optimal policies, (E.9) and (E.10), into asset a's market-clearing condition (E.1) pins down the expected excess return on asset a: for all t,

$$\hat{R}^a_{t+1} = \hat{R}^a \equiv c\left(\frac{\pi S}{\alpha}\right)^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3}. \tag{E.17}$$

Substituting (E.17) into (E.9) and (E.10) determines the fund-A agents' equilibrium actions. On the other, plugging the fund-B agents' optimal policies (E.12) and (E.15) into asset b's market-clearing condition (E.2), and noting that $\hat{R}^{IBb}_{i,t+1} = \hat{R}^b_{t+1}$ holds for all i in equilibrium, we obtain the expected excess return on asset b as

$$\hat{R}^b_{t+1} = c\left(\frac{(1-\pi)S\kappa}{(1-\alpha)\phi\Omega_t}\right)^{1/2}. \tag{E.18}$$

Now the price P^k_t for $k = a, b$ can be determined. Following the same steps as in Sections 2.3 and 3.4, conjectures (E.5) and (E.6) are correct if and only if

$$\gamma^a = \frac{\hat{R}^a}{r} = \frac{c}{r}\left(\frac{\pi S}{\alpha}\right)^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3} \quad \text{and} \quad \gamma^b_t = \sum_{\tau=0}^{\infty}\left(\frac{1}{1+r}\right)^{\tau+1}\hat{R}^b_{t+\tau+1}.$$

As in Section 3.4, it is readily shown that $\Omega^* = \xi\theta^{Bb*}$ in the stationary state when $t \to \infty$, where ξ is given by (3.13). Using this relation, the stationary versions of (E.16) and (E.18) can be written as a system of equations with two unknowns: $\hat{R}^{b*}$ and θ^{Bb*}. The solutions are

$$\hat{R}^{b*} = c\left(\frac{(1-\pi)S}{1-\alpha}\right)^{1/3}\left(\frac{\kappa}{\phi\xi}\right)^{2/3} \tag{E.19}$$

and

$$\theta^{Bb*} = \left(\frac{(1-\pi)S\phi\xi}{(1-\alpha)\kappa}\right)^{1/3}. \tag{E.20}$$

From (E.17) and (E.19), conjecture (E.3) is correct (i.e., $\hat{R}^a > \hat{R}^{b*}$ holds) if and only if

$$\xi > \xi_\ell \equiv \left(\frac{\alpha}{1-\alpha}\frac{1-\pi}{\pi}\right)^{1/2}. \tag{E.21}$$

In other words, for this equilibrium to exist, the career concerns of fund-B managers need to be strong enough to make them drive up asset b's price to the level at which the fund-A managers invest only in asset a. The parametric assumption $\xi \in (\xi_\ell, 2)$ ensures (E.21).

Substituting (E.17) and (E.19) into the agents' optimal policies determines their equilibrium actions:

$$X^{A*} = \left(\frac{\pi S}{\alpha}\right)^{2/3}\left(\frac{\kappa}{\phi}\right)^{1/3} \quad \text{and} \quad X^{B*} = \xi_\ell^{4/3}\xi^{-1/3}X^{A*};$$

$$\theta^{Aa*} = \left(\frac{\pi S\phi}{\alpha\kappa}\right)^{1/3} \quad \text{and} \quad \theta^{Bb*} = \xi_\ell^{2/3}\xi^{1/3}\theta^{Aa*}.$$

From these, the managers' one-period utilities are determined as follows:

$$U^{MA*} = \phi X^{A*} - \frac{\kappa}{2}\theta^{Aa*2} = \frac{\kappa^{1/3}}{2}\left(\frac{\phi\pi S}{\alpha}\right)^{2/3}$$

and

$$U^{MB*} = \phi X^{B*} - \frac{\kappa}{2}\theta^{Bb*2} = \xi_\ell^{4/3}\xi^{-1/3}(2-\xi)U^{MA*}. \tag{E.22}$$

Note that each fund-B manager attains a positive utility level (i.e., $U^{MB*} > 0$) because $\xi < 2$ by assumption. This fact ensures that fund-B managers optimally choose $k = b$, as formally shown in Section E.6.

E.6 Verification of optimality

Last, we need to verify that it is indeed optimal for each fund-B manager to choose $k = b$ in every period. In this section, I show that the manager's one-period utility level would be zero if he deviated from the equilibrium strategy and chose $k = a$ in an arbitrary period t. With this result, we can verify that $k = b$ is indeed optimal for him because we already know from (E.22) that $k = b$ allows him to attain a positive utility $U^{MB*} > 0$.

Suppose that manager B-i chooses $k = a$ in period t. First, consider the decisions of investors B-i. Because they cannot observe the manager's portfolio choice $\theta_{i,t}^{Ba}$, they make decisions based on their belief about $\theta_{i,t}^{Ba}$, not on the actual $\theta_{i,t}^{Ba}$. Let $\tilde{\theta}_{i,t}^{Ba}$ denote the investors' belief about $\theta_{i,t}^{Ba}$. Then the investors' indifference condition implies that the size of fund B-i in period t is

$$X_{i,t}^{B} = \frac{1}{c}\hat{R}_{i,t+1}^{IBa}\tilde{\theta}_{i,t}^{Ba}. \tag{E.23}$$

Second, consider the manager's portfolio choice. Because the investors will observe the true value of δ_{t+1} directly, there is no scope for the manager to influence investors' future estimates by choosing $\theta_{i,t}^{Ba}$. So, in effect, the manager's problem is static: he chooses $\theta_{i,t}^{Ba} \in [0, \infty)$ to maximize his period-t utility

$$\phi X_{i,t}^{B} - \frac{\kappa}{2}\theta_{i,t}^{Ba2}, \tag{E.24}$$

taking into account that $X_{i,t}^{B}$ is determined according to (E.23). However, because the first term of (E.24) is independent of the manager's actual choice of $\theta_{i,t}^{Ba}$ and thus it is out of his control, effectively the manager's problem is choosing $\theta_{i,t}^{Ba}$ to minimize the cost $\kappa\theta_{i,t}^{Ba2}/2$ on the second term. Clearly, then, for any belief $\tilde{\theta}_{i,t}^{Ba}$ of the investors, the manager's optimal choice is $\theta_{i,t}^{Ba} = 0$.

The investors should rationally anticipate that the manager will choose $\theta_{i,t}^{Ba} = 0$. So their belief is $\tilde{\theta}_{i,t}^{Ba} = 0$. Then (E.23) implies that the investors do not provide any capital in the fund, that is, $X_{i,t}^{B} = 0$. Thus, given $X_{i,t}^{B} = 0$ and $\theta_{i,t}^{Ba} = 0$, the manager's maximized period-t utility is 0, as we wanted.

F Proof of Proposition 4.2

Let $\omega \in (0,1)$ be a (endogenous) fraction of funds A that purchase asset a; the rest of them purchase asset b. As in the case of $\xi > \xi_\ell$, all funds B buy asset b because a commitment problem prevents them from buying asset a. In equilibrium, the market clearing conditions imply the following.

$$\text{Asset } a\text{:} \quad \omega\alpha\theta_t^{Aa}X_t^A = \pi S. \tag{F.1}$$

$$\text{Asset } b\text{:} \quad (1-\omega)\alpha\theta_t^{Ab}X_t^A + (1-\alpha)\theta_t^{Bb}X_t^B = (1-\pi)S. \tag{F.2}$$

Plugging the agents' optimal policies (Eqs (E.9), (E.10), (E.12), and (E.15)) into (F.1) and (F.2) and using the relation $\Omega^* = \xi\theta^{Bb*}$, the assets' expected excess returns in the long-run stationary equilibrium (given ω) are obtained as

$$\hat{R}^{a*} = c\left(\frac{\pi S}{\omega\alpha}\right)^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3} \tag{F.3}$$

and

$$\hat{R}^{b*} = c\left(\frac{(1-\pi)S}{(1-\omega)\alpha + (1-\alpha)\xi^2}\right)^{1/3}\left(\frac{\kappa}{\phi}\right)^{2/3}. \tag{F.4}$$

From (E.11), each fund-A manager's maximized utility is increasing in $\hat{R}^{k*}$, $k \in \{a, b\}$. Thus, she is indifferent between $k = a$ and $k = b$ if and only if $\hat{R}^{a*} = \hat{R}^{b*}$. This equality pins down the equilibrium value of ω, denoted by ω^*. Note that $\omega^* \leq 1$ because $\omega^* = 1$ if $\xi = \xi_\ell$, $\mathrm{d}\omega^*/\mathrm{d}\xi > 0$, and $\xi \leq \xi_\ell$. Plugging ω^* back into (F.3) and (F.4) yields the value of $\hat{R}^* = \hat{R}^{a*} = \hat{R}^{b*}$. Plugging $\hat{R}^*$ into the agents' optimal policies yields their equilibrium actions. Note that all fund-A managers choose the same level of leverage irrespective of whether $k = a$ or $k = b$, that is, $\theta^{A*} = \theta^{Aa*} = \theta^{Ab*}$. □