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Three Essays in Dynamic Corporate Finance

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UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT DE FINANCE

THREE ESSAYS IN DYNAMIC CORPORATE FINANCE

THÈSE DE DOCTORAT

présentée à la

Faculté des Hautes Études Commerciales
de l'Université de Lausanne

pour l'obtention du grade de
Docteur ès Sciences Économiques, mention « Finance »

par

Jakub HAJDA

Directeur de thèse
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I M P R I M A T U R

Sans se prononcer sur les opinions de l'auteur, la Faculté des Hautes Etudes Commerciales de l'Université de Lausanne autorise l'impression de la thèse de Monsieur Jakub HAJDA, titulaire d'un bachelor en méthodes quantitatives en économie et systèmes d'information de l'École des Hautes Etudes Commerciales de Varsovie (Pologne), ainsi que d'un master en finance quantitative et science actuarielle de l'Université de Tilbourg (Pays-Bas), en vue de l'obtention du grade de docteur ès Sciences économiques, mention « finance ».

La thèse est intitulée :

THREE ESSAYS IN DYNAMIC CORPORATE FINANCE

Lausanne, le 26 mai 2020

Le doyen

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and have found it to meet the requirements for a doctoral thesis.
All revisions that I or committee members
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Prof. Laurent FRÉSARD
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This is what I see in my dreams about final exams:
two monkeys, chained to the floor, sit on the windowsill,
the sky behind them flutters,
the sea is taking its bath.

The exam is History of Mankind.
I stammer and hedge.

One monkey stares and listens with mocking disdain,
the other seems to be dreaming away —
but when it's clear I don't know what to say
he prompts me with a gentle
clinking of his chain.

— Wisława Szymborska, *Brueghel's Two Monkeys*

To my mother.

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Pursuing Ph.D. has surely been a very rewarding *and* an extremely difficult experience. I am thus immensely grateful to everyone who helped me along the way.

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I owe enormous gratitude to my parents, sisters, grandmothers, cousins, and especially to my late aunt Gabriela. Your love and support throughout the years allowed me to end up where I am now. Lastly, special thanks goes to Laurent Kilchoer for bearing with me during all ups and downs. Your support and time spent together means a lot to me. I could not have done it without you.

Lausanne, May 2020 J. H.

Summary

This thesis consists of three essays. Jointly, the essays highlight that analyzing different features of firms' operating environment allows us to better understand how capital allocation and financing decisions are made.

In Chapter 1, I quantify the extent to which product market strategy influences investment and financing decisions. As products' revenue constitutes firms' cash flows, I argue that we can better understand corporate valuation and policies when accounting for product life cycle, which implies a negative relationship between product revenue and age. I quantify the importance of product life cycle by analyzing the data through the lens of a dynamic model of investment, financing, and product portfolio decisions. I document that product-level forces are economically large and influence firms' policies. In particular, I show that capital investment and product introductions act as complements and product life cycle makes firms adopt conservative financing policy. Product life cycle effects are stronger among firms having smaller product portfolios, competing more intensely and supplying less unique products.

In Chapter 2, together with Thomas Geelen and Erwan Morellec we study how debt financing affects innovation, as recent empirical studies show that innovative firms heavily rely on debt financing. Using a dynamic model of R&D and financing decisions, we show that debt fosters innovation and growth at the aggregate level. This is the result of two opposing forces. First, debt hampers innovation by incumbents, as indebted firms invest less than they would in absence of debt due to agency frictions. Second, debt incentivizes entry as it increases the surplus from entering the industry, which stimulates innovation and growth. We also demonstrate that debt financing has large effects on firm turnover and industry structure.

In Chapter 3, I examine how the nature of cash flow risk affects firms' capital structure decisions. To do so, I develop a dynamic capital structure model in which the firm's cash flow consists of persistent and transitory parts. This distinction allows us to differentiate between shocks that affect long-run prospects of firms (e.g. changes to technology) and those that subside over time (e.g. natural disasters). I document that in this setting firms with similar observable risk can adopt different debt policies depending on risk composition. Using the model, I provide rationale as to why the observable dispersion in cash flow persistence is low, which is at odds with the large degree of heterogeneity in other firm characteristics.

Résumé

Cette thèse se compose de trois chapitres. Ensemble, les essais soulignent que l'analyse des différentes caractéristiques de l'environnement opérationnel des entreprises permet de mieux comprendre comment sont prises les décisions d'allocation de capital et de financement.

Dans le chapitre 1, je quantifie la mesure dans laquelle la stratégie du marché des produits influence les décisions d'investissement et de financement des entreprises. Comme les revenus des produits constituent les revenus des entreprises, je soutiens que nous pouvons mieux comprendre la valeur de l'entreprise et ses politiques lorsque nous prenons en compte le cycle de vie du produit, ce qui implique une relation négative entre le revenu du produit et son âge. Je quantifie l'importance du cycle de vie du produit en analysant les données en utilisant un modèle dynamique dans lequel l'entreprise fait des décisions d'investissement, de financement et de portefeuille de produits. Je montre que les forces économiques au niveau des produits sont importantes et influencent les politiques des entreprises. En particulier, je montre que l'investissement en capital et l'introduction de produits se complètent et que le cycle de vie du produit incite les entreprises à adopter une politique de financement prudente. Les effets du cycle de vie du produit sont plus forts parmi les entreprises ayant des portefeuilles de produits plus petits, celles qui se livrent une concurrence plus intense et celles qui fournissent des produits moins uniques.

Dans le chapitre 2, avec Thomas Geelen et Erwan Morellec, nous étudions comment la dette affecte l'innovation, car des études empiriques récentes montrent que les entreprises innovantes dépendent fortement du financement par emprunt. En utilisant un modèle dynamique de la R&D et des décisions de financement, nous montrons que la dette favorise l'innovation et la croissance au niveau agrégé. Ceci est le résultat de deux forces opposées. Premièrement, la dette entrave l'innovation des entreprises en place, car les entreprises endettées investissent moins qu'elles ne le feraient en l'absence de dette en raison du problème du principal-agent. Deuxièmement, la dette incite à l'entrée car elle augmente la valeur de l'industrie, ce qui intensifie la concurrence et stimule l'innovation des entreprises entrantes. Nous démontrons également que le financement par emprunt a des effets importants sur le taux de rotation des entreprises et la structure de l'industrie.

Dans le chapitre 3, j'évalue comment la nature du risque de cash flow affecte les décisions des entreprises en matière de structure du capital. Pour ce faire, je développe un modèle dynamique de la structure du capital dans lequel le cash flow est constitué de parties persistantes et transitoires. Cette distinction permet de différencier les chocs qui affectent les perspectives à long terme des entreprises (par exemple les changements technologiques) de ceux qui s'atténuent avec le temps (par exemple les catastrophes naturelles). Je constate que dans ce contexte, les entreprises présentant un risque observable similaire peuvent faire des décisions différentes en matière de dette selon la composition du risque. À l'aide du modèle, j'explique pourquoi la dispersion observable de la persistance des cash flows est faible, ce qui est en contradiction avec le degré élevé d'hétérogénéité des autres caractéristiques des entreprises.

Contents

Introduction

Recent advances in computational power and the availability of vast volumes of high quality data allow us to venture into novel and previously unexplored areas of corporate finance. In my thesis, I combine a data-driven approach with formal theoretical models of firms to better understand how firms make financing and investment decisions. I do so by considering dimensions that have not previously been considered, such as firms' product portfolio characteristics, cash flow risk composition and Schumpeterian competition. The thesis consists of three essays in which I try to shed light on the drivers of firms' observed behavior.

In the first chapter titled "*Product Market Strategy and Corporate Policies*", I study how product market strategy influences corporate investment and financing policy. This is an important question, given that product dynamics appear to vary much more than labor markets fluctuations or establishment entry and exit (Broda and Weinstein, 2010), and they microfound firms' cash flow dynamics that affect their investment and financing decisions. The large extent of product creation and destruction can be attributed to product life cycle, as firms' revenue growth crucially depends on either developing current product lines or introducing novel products (e.g. Levitt, 1965). In the essay, I study how the product life cycle channel influences corporate policies and I quantify its importance.

To do so, I use detailed product-level data of public US manufacturers of consumer goods. To analyze the empirical implications of product life cycle, I focus on firms' product portfolio age, measured by the share of products that exceed half of their lifespan. Product portfolio age is related to product life cycle because of the negative relationship between product-specific revenue and age (e.g. Argente, Lee, and Moreira, 2019). An important result of the essay is to show that this relationship aggregates to product portfolio level, because product portfolio age is negatively associated with firms' profitability. In addition, I document that the product life cycle channel results in a negative relationship between the market-to-book ratio and product portfolio age, which implies that managing product portfolios has direct implications for firm value. As such, product decisions of value-maximizing firms should be reflected in their investment and financing choices: empirically, both net leverage and capital investment are also negatively related to product portfolio age.

To quantify the importance of the product life cycle channel, I develop and estimate a dynamic model of investment, financing and product decisions. In the model, each product follows a life-cycle pattern: new products provide higher revenue and are expected to last longer than old ones. The fact that the firm can adjust the product portfolio's composition has direct implications for cash flow dynamics, and thus connects the firm's real, financial, and product decisions. The estimated model predicts that capital investment and product introductions are complements rather than substitutes, indicating that firms will expand their product lines while also investing in production capacity. I show that product life cycle also induces stronger precautionary savings motives, as older products can become obsolete, leaving the firm with a revenue gap that may have to be covered using costly external financing. Importantly, this channel is absent in standard dynamic models of the firm that do not account for product dynamics.

The estimates from the model suggest that product life cycle effects are sizeable, as they imply that firms behave as if old products only generated about a half of new products' revenue. I also provide evidence that product-level forces are stronger within firms whose products have higher sensitivity to product life cycle, that supply fewer products and that compete more intensely. Moreover, the cross-sectional evidence implies that product introduction costs are higher for firms that supply more durable and more unique products. Furthermore, I document that the effect of product dynamics is important, as they account for substantial part of variation in investment and leverage in the model. To gauge the influence of product characteristics on firm value, I provide counterfactual evidence showing that alleviating product life cycle effects, for example by introducing products that are less influenced by life cycle effects, can significantly increase firm value. Thus, the model provides empirical support for the notion that product-level economic forces are important in shaping corporate policies.

All in all, the results highlight the fact that firms' internal product setting, which can be difficult to observe in the data or may be concealed as a firm fixed effect, matters for firm value, and that the effects of product characteristics are large.

In the second chapter titled "*Debt, Innovation, and Growth*", together with Thomas Geelen from Copenhagen Business School and Erwan Morellec from École Polytechnique Fédérale de Lausanne, we develop a Schumpeterian growth model to study how debt financing affects innovation and growth.

A key result of the essay is to demonstrate that debt financing fosters innovation and creative destruction at the aggregate level. This is the outcome of two opposing forces. First, innovation and investment by incumbents are negatively associated with debt due to debt overhang. In particular, we show that the effect of debt on innovation is sizeable and larger for firms with fewer products, and those that face larger financing frictions. Second, while debt hampers innovation and investment by incumbents, it also increases the value of incumbents and leads to a higher rate of creative destruction, which increases the entry rate. We demonstrate that the latter effect dominates at the aggregate level, implying that introducing debt financing in our Schumpeterian growth model increases innovation and creative destruction and fosters growth.

We also illustrate how conclusions reached in the single-firm model, when ignoring equilibrium feedback effects, can be fundamentally altered, or even reversed, when the rate of creative destruction is taken into acocunt. Consider for example the effects of innovation costs on equilibrium quantities. Increasing innovation costs leads to a drop in the level of innovation and in the value of future innovations. This reduces the cost of debt and leads firms to increase financial leverage. These effects are much stronger in a single-firm model that does not incorporate the industry wide response. Indeed, an effect that is absent when ignoring industry dynamics is that the drop in innovation quantity and the increase in leverage feedback into the equilibrium rate of creative destruction. As shown in the essay, the effect on innovation is generally first order, leading to a negative relation between innovation costs and the rate of creative destruction. This decrease in the rate of creative destruction spurs innovation, partly offsetting the higher innovation costs. Lastly, these mechanisms translate to a lower turnover rate as innovation costs increase, because the decrease in the rate of creative destruction compensates for the lower levels of innovation. By contrast, in the single-firm model in which industry feedback effects are ignored, the sharp increase in leverage due to increasing innovation costs leads to an increase in the turnover rate.

We document that there is significant interaction between leverage and innovation. Notably, high levels of debt lead to less innovation by incumbents due to debt overhang. Moreover, R&D policies as well as the industry rate of creative destruction feed back in capital structure decisions. Crucially, firms' R&D policy affect the rate of creative destruction and the probability of default. As a result, it plays a key role in determining firms' capital structure choices. Our model thus features a rich interaction between R&D, investment, and financing decisions and predicts substantial intra-industry variation in leverage and innovation, consistent with the empirical evidence (see e.g. MacKay and Phillips, 2005 and Kogan, Papanikolaou, Seru, and Stoffman (2017)).

In the third chapter titled "*Fundamental Risk and Capital Structure*", I investigate how the nature of risk faces by firms affects their financing decisions. To this end, I develop a dynamic capital structure in which the firm's fundamental risk, captured by its cash flow process, consists of transitory and persistent parts with markedly different dynamics. The separation into transitory and persistent shocks represents the fact that some firms may experience frequent but transient cash flow shocks that influence their long-run decisions in a limited way, while others could only face infrequent disturbances, but with permanent impact on cash flows.

Unlike standard dynamic capital structure models in which the firm is exposed to a single transitory shock, this essay can rationalize the mismatch in the risk-leverage relationship by relating the observed dispersion to differences in risk composition. This happens because firms issue less debt when their cash flows are more persistent to preserve debt capacity needed to fund investment. In particular, the decomposition of fundamental volatility allows to obtain different optimal leverage ratios for firms with the same level of total volatility. Similarly, the model generates firms with high profit persistence even when the composition of their fundamental persistence, which also affects leverage choice, differs. Thus, the model provides rationale as to why the observable dispersion in cash flow persistence is low, which is at odds with the large degree of heterogeneity in other firm characteristics, as well as why persistence and leverage are weakly related in the data.

1 Product Market Strategy and Corporate Policies[1]

1.1 Introduction

Firms use products to translate their ideas into profits. Product introductions alter firms' product portfolios, which, in turn, influence their cash flows. As such, product dynamics and cash flow dynamics are closely related. Empirical evidence suggests that within-firm product creation and destruction is substantial. For example, firms in the consumer goods sector introduce or withdraw on average 10.8% of products in their portfolios every year (Argente, Lee, and Moreira, 2018).[2] The large extent of product creation and destruction can be attributed to product life cycle, as firms' revenue growth crucially depends on either developing current product lines or introducing novel products (e.g. Levitt, 1965, Argente, Lee, and Moreira, 2019). Thus, the product-level variation is bound to influence cash flow dynamics and to impact firms' policies. Moreover, as firms choose not only their product portfolio but also the way in which it is financed and implemented in their real activities, product dynamics must be related to investment and financing decisions.

The importance of product dynamics induced by product life cycle raises a number of novel questions for financial economists. First, how does product life cycle influence corporate policies? Second, to what extent do firms' product portfolio choices affect their investment and financing decisions? Third, which product characteristics are vital in determining firms' exposure to product life cycle? Finally, how quantitatively important are product dynamics for corporate policies?

In this paper, I demonstrate both empirically and quantitatively that corporate valuations and policies are better understood when taking into account the characteristics of products, which microfound firms' cash flows. First, I describe the empirical relation between product portfolio age and firms' investment and financing decisions. Second, I develop and estimate

[1] Researcher(s) own analyses calculated (or derived) based in part on data from The Nielsen Company (US), LLC and marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the Nielsen data are those of the researcher(s) and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

[2] Product dynamics contribute much more to macroeconomic fluctuations than the effects of labor markets or establishment entry and exit (Broda and Weinstein, 2010).

a dynamic model to understand and quantify the influence of the economic mechanisms underpinning the empirical results. By doing so, I document that product life cycle has significant and economically meaningful implications for corporate policies.

To analyze the empirical implications of product life cycle, I focus on firms' product portfolio age, measured by the share of products that exceed half of their lifespan. Product portfolio age is related to product life cycle because of the negative relationship between product-specific revenue and age (e.g. Argente, Lee, and Moreira, 2019). An important result of the paper is to show that this relationship aggregates to product portfolio level, because product portfolio age is negatively associated with firms' profitability. Crucially, the effect of product portfolio age on cash flow is markedly different from that of *firm* age. In addition, I document that the product life cycle channel results in a negative relationship between the market-to-book ratio and product portfolio age, which implies that managing product portfolios has direct implications for firm value. As such, product decisions of value-maximizing firms should be reflected in their investment and financing choices: empirically, both net leverage and capital investment are also negatively related to product portfolio age.[3]

I rationalize these empirical patterns by developing and estimating a dynamic model of the firm which makes investment, financing and product decisions. In the model, the firm combines capital and products to generate revenue. It finances its activities with current cash flow, net debt subject to a collateral constraint and costly external equity. Consistent with the product life cycle channel, each product follows a life-cycle pattern: new products provide higher revenue than old ones and are expected to last longer, because old products can exit. When deciding on introducing a new product to its portfolio, the firm trades off the benefits, associated with higher and more durable revenue of a younger product portfolio, versus a fixed introduction cost. The fact that the firm can adjust its product portfolio has direct implications for cash flow dynamics, and thus connects the firm's real, financial, and product decisions.[4]

The model provides economic rationale to the empirical stylized facts. First, it shows that capital investment and product introductions are complements rather than substitutes, meaning that the firm expands its product lines while also investing in production capacity. The firm increases capital investment when introducing new products, because a higher level and durability of revenues associated with a younger product portfolio increases its incentives to invest in physical capital. However, the firm tends to invest less as its product portfolio ages, because its revenues decline and become more risky as they are expected to diminish quicker. Thus, the model rationalizes the negative relationship between investment and product portfolio age observed in the data.

Second, the model documents that product life cycle induces stronger precautionary

[3] Using a text-based measure of *firm* product life cycle, Hoberg and Maksimovic (2019) document that firms in the late stage of their product life cycle have a higher investment-q sensitivity. While I do not focus on this empirical relationship, my results, obtained using product-level data, suggest that investment policy of firms with older product portfolios is more sensitive to Tobin's q.

[4] In related research, Livdan and Nezlobin (2017) argue that controlling for the vintage composition of capital stock can help explain firms' investment decisions, as the age of capital affects its profitability. In this paper, a product's age affects its revenue as well. However, product introductions are different from capital investment.

savings motives. In particular, when the product portfolio ages, the firm has higher incentives to preserve its debt capacity. This happens because the firm wants to avoid issuing costly external financing to fill the revenue gap created by old products becoming obsolete. The firm, however, also tends to increase its leverage when introducing new products, as they are predominantly financed with debt. As such, the model sheds light on the economic mechanism driving the negative empirical relationship between leverage and product portfolio age. Notably, these effects are absent in standard dynamic model of the firm that do not account for product portfolio structure.

To quantify the importance of product life cycle on corporate policies, I estimate the structural parameters of the model by matching a set of model-implied moments to their empirical counterparts. Crucially, the estimation procedure relies on using the product portfolio data, as firms' product portfolio structure is indicative of the importance of the product life cycle channel. Thus, the observable product portfolio characteristics help identify the two key parameters governing the firm's product decisions: the old product revenue discount and the product introduction cost. I find that the estimated model quantitatively matches key features of the data, particularly different moments of product portfolio age. Moreover, the estimates suggest that the product life cycle channel is quantitatively important, with each old product providing only 52.8% of a new product's revenue and the cost of introducing each new products being equal to 0.75% of assets, that is $7.64m for a typical sample firm. Both estimates are significant and substantial in magnitude, suggesting that product-level economic forces are sizeable.

To understand which features of the data help explain the exposure to the product life cycle channel, I study the cross-sectional implications of the model. I do so by estimating the model on subsamples of firms varying along key characteristics. By doing so, I examine whether the model successfully captures differences across product dimensions that might not be directly represented in product data that is aggregated to the firm level. I also investigate how the magnitude of the product life cycle channel changes along dimensions not explicitly captured by the model.

First, I demonstrate that firms whose products are more sensitive to life cycle effects are also more exposed to the product life cycle channel. These firms have a larger estimated old product revenue discount, invest more in physical capital and adopt lower leverage. Hence, the results are in line with the model's prediction that stronger product life cycle effects induce higher precautionary savings incentives, and that firm complement product introductions with capital investment. The results from this sample split also serve as a 'sanity check' for the model setup, as they indicate that the model can rationalize discrepancies across firms with markedly different product characteristics, despite using data aggregated to firm-level.

Second, I analyze whether a number of vital product characteristics help explain the magnitude of the product life cycle channel. In particular, I document that firms with smaller product portfolios are more exposed to the product life cycle channel, as they face more pronounced old product revenue discount and higher product introduction costs. As their cash flows are effectively riskier, these firms adopt lower leverage ratios. At the same time, firms supplying many products tend to have younger product portfolios, which results in higher, but less volatile profits, highlighting that firms may also use their product lines as means of revenue diversification.

I also show that competition strengthens the product life cycle channel. Specifically, firms operating in more competitive environment are more sensitive to product-level economic forces, as their products become obsolete faster. This leads to a higher rate of product introductions, which translates into their product portfolio structure and feeds back to investment and financing decisions.

Furthermore, I show that firms supplying more durable products can benefit from their products for a longer period of time but also face higher product introduction costs. This is a result of two opposing forces: on the one hand, these firms are more exposed to product life cycle, in that their products lose a larger chunk of their revenue when ageing. On the other hand, more durable products also tend to last longer, the effect of which dominates.

Finally, I demonstrate that firms with more unique products have higher product introduction costs, but are less exposed to product life cycle. Given that I use cost of sales as proxy for product uniqueness, this result means that firms may try to influence their exposure to product life cycle by managing costs such as advertising. I provide evidence that firms do so because their products also have lower durability, which incentivizes them to prolong the products' life cycle.

Overall, the cross-sectional results highlight that the estimated model provides insights concerning how the different dimensions of firms' product characteristics affect corporate policies. The empirical evidence shows that both between- and within-firm product market forces are an important determinant of investment and financing decisions.

The last contribution of the paper is to provide evidence that the product life cycle channel has quantitatively important implications for corporate policies. First, by means of variance decomposition, I show that product dynamics explain as much as 20% of the variation in leverage and investment in the model. Second, counterfactual experiments related to the severity of life cycle effects suggest that eliminating the revenue gap between new and old product increases firm value by 4.48%. Similarly, lowering the product introduction costs by 50% results in a 7.85% increase in firm value, indicating that costs related to product introduction are economically significant. Hence, the counterfactual experiments imply that managing the life cycle of products, by means of introduction cost or sensitivity to ageing, yields material benefits to firms. Third, I demonstrate that product characteristics largely influence the precautionary savings incentives of the firm. More severe product life cycle effects result in stronger precautionary savings motives, as the firm can lose a large fraction of revenue when its products age. Similarly, less frequent product introductions lower the firm's incentives to preserve debt capacity, because product introductions require less financing. These effects are large: for example, when eliminating the product life cycle channel, the firm would essentially double its leverage ratio.

All in all, the results further highlight the fact that firms' internal product setting, which can be difficult to observe in the data or may be concealed as a firm fixed effect, matters for firm value, and that the effects of product characteristics are large.

1.1.1 Related literature

This paper contributes to several strands of literature. First, the paper adds to the literature that uses dynamic models to quantitatively explain corporate investment and financing policies. Recent examples include Gomes (2001), Hennessy and Whited (2005, 2007), DeAngelo, DeAngelo, and Whited (2011), Nikolov and Whited (2014), or Nikolov, Schmid, and Steri (2018). I contribute to this literature by explicitly considering firms' product portfolio decisions. In particular, I show that product dynamics influence firms' cash flow dynamics and matter quantitatively for firms' investment and financing decisions.

To this end, the paper is also related to the growing literature on the relationship between corporate strategy and corporate policies, e.g. Titman (1984), Hellmann and Puri (2000), Parsons and Titman (2008), Gourio and Rudanko (2014), Clayton (2017), D'Acunto, Liu, Pflueger, and Weber (2018) or Hoberg and Maksimovic (2019). I differ from this literature by focusing explicitly on firms' product market strategy and showing that firms' product portfolio characteristics matter for cash flow dynamics and corporate policies. In that respect, this paper is most closely related to Hoberg and Maksimovic (2019), who infer firms' life cycle stage from its product life cycle and study its implications for investment, and D'Acunto, Liu, Pflueger, and Weber (2018), who show that pricing policy, i.e. one of the dimensions of product market strategy, affects how firms make capital structure decisions.

The paper also adds to the literature on how product market characteristics affect corporate financing policy, e.g. Spence (1985), Maksimovic (1988), Phillips (1995), Chevalier (1995a,b), Kovenock and Phillips (1995, 1997), MacKay (2003), Frésard (2010), and Valta (2012). In contrast to these papers, I focus on *within-firm* product market characteristics, that is the product market strategy, rather than *between-firm* effects such as competition and I argue that internal product market setting is an important determinant of corporate investment and financing policy.

Finally, the paper is related to the literature on multiple-product firms such as Broda and Weinstein (2010), Bernard, Redding, and Schott (2010), Hottman, Redding, and Weinstein (2016), Argente, Hanley, Baslandze, and Moreira (2019) and Argente, Lee, and Moreira (2018, 2019), who study the reasons why firms choose to supply multiple goods. In contrast to these papers, I analyze the corporate finance implications of product portfolio choice.

1.2 Data and Stylized Facts

In this section, I analyze the empirical relation between product life cycle and corporate policies. I focus on product portfolio age as the measure of firms' exposure to product life cycle, which by itself implies a negative relationship between product-specific revenue and product age (e.g. Levitt, 1965 or more recently Argente, Lee, and Moreira, 2019). I show that the product-level life cycle effects naturally translate to the product portfolio level, resulting in a negative relationship between product portfolio age and profitability. I document that both corporate investment and financing policy are negatively related to product portfolio age when controlling for other firm characteristics, indicating that product life cycle constitutes an important and novel source of variation in corporate policies.

1.2.1 Data sources

I use the data from AC Nielsen Homescan to reconstruct firms' product portfolios. The dataset contains information on prices and quantities of nondurable consumer goods sold in the US over the period of 2004 to 2018. The product data is comprehensive, covering about 66% of CPI expenditures (Broda and Weinstein, 2010), and detailed, as it contains vastly more information about products than other datasets such as BLS. I merge the AC Nielsen data with the accounting data of US public firms from quarterly Compustat. Appendix A.I provides a detailed description of the data as well as of the merging procedure. Appendix A.II contains the definitions of variables used throughout the paper.

Defining a product

I focus on the UPC-level definition of a product, as it allows to investigate the life cycle of each individual product and to construct a precise measure of product portfolio age.[5]

Table 1.1 presents the summary statistics of several product characteristics of firms in three different samples: the Nielsen Homescan, all matched public firms, and the final sample of firms used in the paper.[6] Firms vary substantially in the number of supplied products, with an average public firm supplying roughly 11 times more products and operating in 3 times more markets than an average firm in the sample. Their average net product entry and net product creation rates, however, are lower than those of private firms, given the size of their product portfolios. Table 1.1 also documents that while public firms supply ≈ 17% of all products in the market, their sales of these products constitute ≈ 48% of total product revenue. This result highlights that analyzing product market strategies of public firms remains of great importance, even though there are relatively fewer public than private firms.[7]

1.2.2 Product portfolio age

To measure product portfolio age, I follow Melser and Syed (2015) and Argente, Lee, and Moreira (2018, 2019). I define the proxy as the weighted share of *old* products, whose age exceeds half of their lifespan, in the firm's product portfolio, where the weights correspond to

[5]In principle, the data used allows for many definitions of a product. For example, one could use a very wide notion of a product that is often implicitly assumed by researchers, namely that firms supply a representative good. While in many cases reasonable, this approach would neglect the product-level dynamics that I study in this paper. The stylized facts are qualitatively robust to employing a coarser definition, e.g. one that associates brands in a given consumer good category with a product ('brand-modules'). In Appendix A.I I revisit the issue of product definition in more detail.

[6]In particular, I remove a number of firms that have been matched but are nonetheless unlikely to be affected by the product channel, as they are not exposed to selling own products. An example of such firm is Amazon, which oftentimes sell own products in retail stores, but these do not constitute the main source of revenues for Amazon. Appendix A.I provides all the details about data processing.

[7]Moreover, while public firms operate in roughly 7.4 markets at once, their average market share in these markets is fairly low, 1.4% on average, which reinforces the notion that nondurable consumer good market in the US is fairly competitive. This is no longer true, however, when looking at particular markets, in which as little as 5 firms often enjoy a combined market share of roughly 60%.

	NH	Public	Sample
Average # of UPCs	63.8	707.5	441.1
Average # of markets	2.4	7.4	6.9
Average market share (all markets)	0.4%	1.4%	2.3%
Average net product entry	0.7%	0.4%	0.2%
Average net product creation	0.5%	0.2%	0.1%
Share of aggregate revenue	100%	47.9%	22.5%
Share of all UPCs	100%	16.7%	7.3%
# establishments	37492	1376	403
# public firms		720	108

Table 1.1 – **Comparison of product characteristics in the Nielsen Homescan (NH) sample, the sample of public firms and the sample of firms used in the paper.** UPCs and markets constitute different levels of product aggregation. Market shares were computed at the market level. Net product entry is the difference between the share of entering products in a firm's product portfolio and the share of exiting products. Net product creation is the difference between the share of the revenues of the entering products in a firm's product revenue and the share of the revenue of the exiting products. Share of aggregate revenue (all UPCs) is the portion of aggregate product revenue (aggregate number of UPCs) that can be attributed to each subsample. Establishments are firms identified in the NH. Appendix A.II provides a more detailed description of all variables. All variables are winsorized at 2.5% and 97.5% percentile.

product-specific revenue:

$$\text{age}_{it} = \frac{\text{weighted \# of products with age exceeding 50\% of lifespan}(it)}{\text{total \# of products}(it)}. \tag{1.1}$$

As such, this measure captures the effective age of firm's i product portfolio in quarter t. I measure product portfolio age this way, as it allows me to directly link the data to the model.[8]

Panel A of Table 1.2 indicates that the average firm in the sample has 44.5% of old products. Product portfolio age varies substantially as its standard deviation is 0.33 and a further variance decomposition into within- and between-firm effects suggests that as much as 79% of the variance can be attributed to within-firm variation.[9]

The large variation in product portfolio age is also noticeable in Figure 1.1, which shows that the distribution of product portfolio age is spread out. In particular, there are many

[8] Using alternative breakpoints or an unweighted measure of product portfolio age generates qualitatively and quantitatively similar results. In Appendix A.II I document that these different measures produce qualitatively similar relationships with corporate policies. For example, Argente, Lee, and Moreira (2019) also document the decline in product-specific revenue can start as early as at the end of the first year for products lasting at least 4 years and that it varies with product duration. As such, using half of the lifespan is more conservative.

[9] The value is slightly higher if products are defined at the brand-module rather than UPC level (0.519), and is slightly higher when products are not weighted by their revenues (0.512). Taking a higher threshold for the proxy results in a smaller share of older products (0.315 for 75% threshold and 0.202 for 90% threshold), but the variation remains fairly high.

Panel A: summary statistics										
	age	size	adjustments	market-to-book	investment	book lev.	cash	firm size	profitability	firm age
mean	0.445	62.822	0.002	1.882	0.021	0.265	0.090	7.601	0.040	-0.067
median	0.405	37.726	0.000	1.609	0.017	0.272	0.056	7.874	0.038	-0.036
std. dev.	0.324	60.306	0.071	1.110	0.016	0.169	0.095	2.141	0.024	0.076
N	2366	2366	2366	2366	2366	2366	2366	2366	2316	2360

Panel B: pairwise correlations										
	age	size	adjustments	market-to-book	investment	book lev.	cash	firm size	profitability	firm age
age	1									
size	-0.063	1								
adjustments	-0.022	-0.028	1							
market-to-book	0.057	0.074	0.061	1						
investment	-0.072	-0.081	0.043	0.215	1					
book leverage	0.065	0.287	-0.062	-0.146	-0.067	1				
cash	0.013	-0.281	0.045	0.404	0.146	-0.506	1			
firm size	0.010	0.430	-0.0161	-0.008	-0.063	0.364	-0.210	1		
profitability	-0.037	0.120	0.068	0.611	0.107	-0.132	0.300	0.111	1	
firm age	0.030	0.131	-0.019	0.035	-0.123	-0.105	-0.011	0.191	0.118	1

Table 1.2 – **Summary statistics of product portfolio and firm characteristics.** Age is the share of old products in product portfolio, that is those exceeding half of their lifespan. Size is the number of products in product portfolio. Adjustments is the net product entry, that is the difference between the share of entering products in a firm's product portfolio and the share of exiting products. Market-to-book is the market value of equity plus book value of debt over total assets. Investment is capital investment over gross plant, property and equipment. Book leverage is book debt over total assets. Cash is cash and short-term investments over total assets. Firm size is the log of deflated total assets. Firm age is defined as in Loderer, Stulz, and Waelchli (2017). Profitability is operating profits over total assets. Appendix A.II provides a more detailed description of all variables. All variables are winsorized at 2.5% and 97.5% percentile.

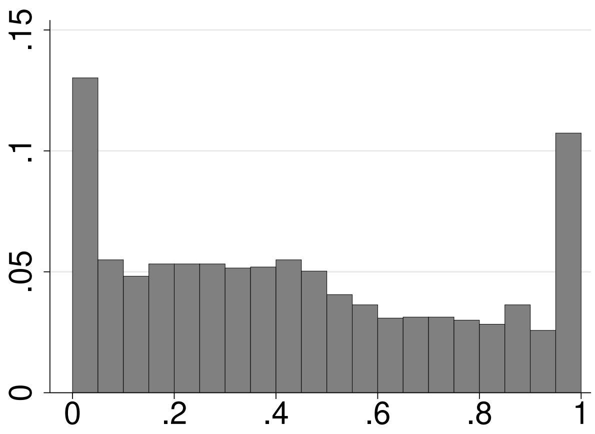


Figure 1.1 – **Histogram of product portfolio age.** Appendix A.II provides a more detailed description of all variables. All variables are winsorized at 2.5% and 97.5% percentile.

firms with only new and only old products. Finally, product portfolio age and firm age are only weakly positively related, with the correlation coefficient of 0.03. This suggests that product portfolio age can provide additional information above and beyond standard firm characteristics such as firm age. This result is expected given e.g. the evidence of Hoberg and Maksimovic (2019), who document that firms transition from being an 'old' product life cycle firm to 'new.'

Other product portfolio characteristics

While product portfolio age is of key importance for measuring the effects of product life cycle, there also exist other important product portfolio characteristics that may interact with product life cycle as well. Table 1.2 provides additional information about two such characteristics: product portfolio size and adjustments.

First, a comparison between Tables 1.1 and 1.2 suggests that the average *effective* number of products (58) is much lower than the raw one (441).[10]. This implies that not only do public firms differ in the number of products they supply, but also that the majority of firms' revenues can be attributed to a small number of products, supporting the notion that product revenues are fairly concentrated. Moreover, product portfolio size and product portfolio age are negatively correlated, implying that firms with older product portfolios have on average fewer products.

Second, I report that the average net product entry amounts to 0.26% each quarter. This corresponds to an average sample firm introducing 3.5 products each quarter, which increase its retail sales by roughly 1.2%, suggesting that within-firm product-level dynamics have important implications for cash flow dynamics. Moreover, the firm-specific average net product creation is more than 3 times lower than the aggregate one reported in Table 1.1, implying that a vast majority of product creation and destruction takes place in private firms.

[10]The effective number of products equals the inverse of their product revenue concentration measured using the normalized HHI of each firm's product revenue, see Appendix A.II for details.

Finally, product portfolio adjustments are negatively related to product portfolio age. This means that, intuitively, firms with older product portfolios also introduce new products less often.

In Appendix A.II I provide further empirical evidence about the relationship between these product portfolio characteristics and corporate policies.

1.2.3 Product life cycle matters for profitability

An important result of the paper is to show that the notion of product life cycle of Levitt (1965) and Abernathy and Utterback (1978) can be generalized to the firm level. The top left graph in Figure 1.2 documents that firms with older product portfolios have lower profitability. This means that individual-product life cycle and product portfolio age are closely related. As such, the proxy passes the natural 'sanity check' of matching the findings of Argente, Lee, and Moreira (2019) using firm- rather than product-level data. Therefore, the product life cycle provides a natural channel through which product-level economic forces interact with corporate policies.

The fact that product portfolios consist of individual products, together with product life cycle, implies that firms' cash flow should be directly affected by both product portfolio age and product introductions. I show in two ways that this is indeed the case.

First, the top right graph in Figure 1.2 shows that product portfolio age is tightly related to product sales growth, which declines as product portfolio ages. The economic significance is substantial: product revenues of firms with younger product portfolios grow by about 1.6% annually, largely due to new product introductions. On the other hand, revenue growth of firms with old product portfolios is close to zero. These numbers are consistent with the observed decline in profitability. The bottom left graph documents that firms with youngest and oldest product portfolios have on average higher cash flow volatility. This result stem from the fact that older products carry higher risk, due to both the decline in revenue as well as the chance of becoming obsolete. Finally, the cost of sales, presented in the bottom right graph, is a u-shaped function of product portfolio age. Provided that this variable proxies for firms' marketing expenses, the shape is intuitive: firms with younger products have to devote more resources to introducing new products and advertising. In the same manner, firms with older products may try to prolong the lifespan of their products by devoting more resources to marketing, or to increasing their R&D expenses, which are also contained in this measure (Peters and Taylor, 2017).

Second, if the influence of product dynamics on corporate policies operates through firms' cash flow dynamics, the effect of product introductions on cash flow should be substantial. To investigate this claim, I check how firms' profitability and sales change in quarters when firms introduce new products. Table 1.3 shows that when taking into account firm and time fixed effects, the average log sales are 4.2% higher in quarters when the number of products increases, while the average profitability is 4.0% higher and the average log product sales are 12.5% higher. These effects are even stronger when conditioning on the number of products introduced, e.g. average log sales are 5.9% rather than 4.2% higher when firms introduce more products than a typical firm in a given quarter. Finally, the effects of product introduction

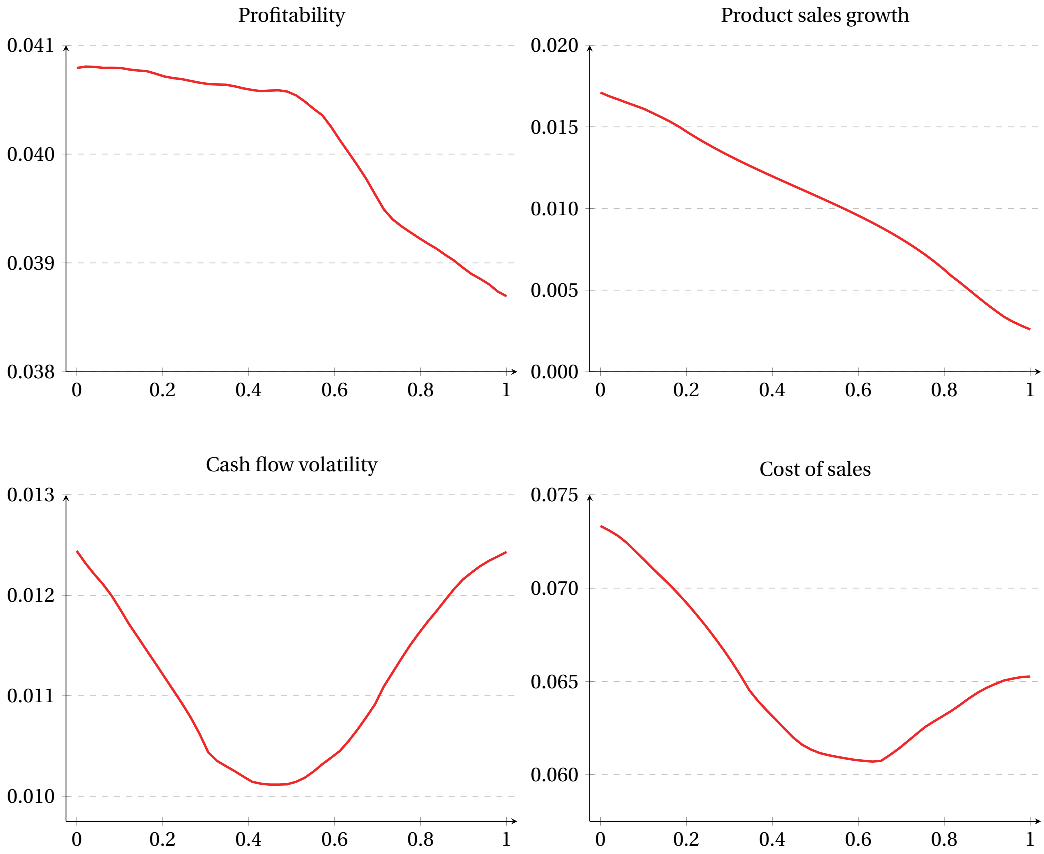


Figure 1.2 – **Product portfolio age and firm characteristics.** The figure shows how profitability, product revenue growth, cash flow volatility, and cost of sales change with product portfolio age. The solid lines are obtained from local polynomial regressions of each variable on the product portfolio age proxy using an Epanechnikov kernel function with a rule-of-thumb bandwidth estimator and local-mean smoothing. All variables are winsorized at 2.5% and 97.5% percentile. Appendix A.II provides a description of all variables.

on cash flow are persistent, but tend to become weaker over time. For example, a product introduction is associated with 3.3% higher average log sales 8 quarters after the introduction took place, thus 21% lower than the contemporaneous effect.

1.2.4 Product portfolio age and corporate policies

Having documented that product portfolio portfolio age is negatively associated with firms' profitability, the natural question that arises is whether product life cycle also has implications for firm value. In other words, do value-maximizing firms care about managing product portfolio age?

To answer this question, the top graph of Figure 1.3 presents the relationship between product portfolio age and the *unexplained* part of the market-to-book ratio. The figure documents that firm value declines with the share of old products, except for firms with

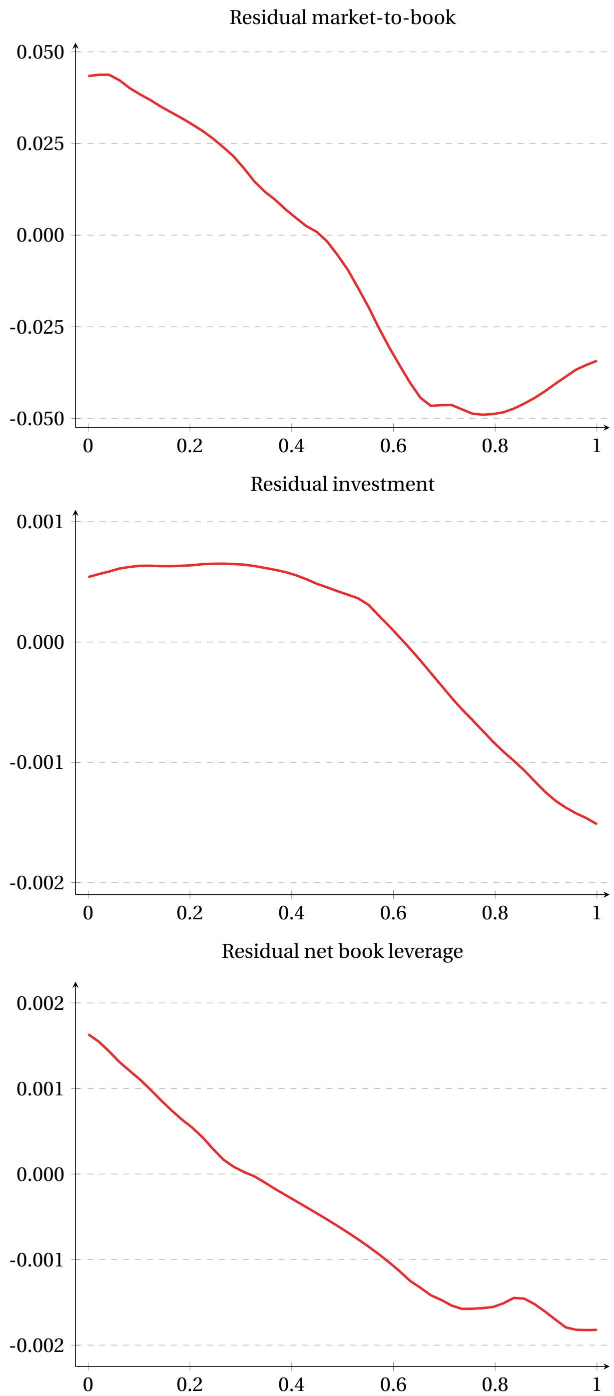


Figure 1.3 – **The relationship between product portfolio age and firms' unexplained market-to-book, investment and leverage.** The solid lines are obtained from local polynomial regressions of each variable on the product portfolio age proxy using an Epanechnikov kernel function with a rule-of-thumb bandwidth estimator and local-mean smoothing. The controls used to compute the predicted values include profitability, investment, leverage, size, cash flow volatility for market-to-book, profitability, size, cash flow volatility, market-to-book and tangibility for leverage, and size, cash flow, and market-to-book for investment. All models control for firm and time fixed effects. All variables are winsorized at 2.5% and 97.5% percentile. Appendix A.II provides a description of all variables.

Log sales			
	t	$t+4$	$t+8$
dummy	4.2%	3.7%	3.3%
below median$^+$	2.6%	2.6%	2.6%
above median$^+$	5.9%	5.1%	4.0%

Log product sales			
	t	$t+4$	$t+8$
dummy	12.5%	11.3%	10.3%
below median$^+$	7.6%	5.8%	4.5%
above median$^+$	18.0%	17.4%	16.5%

Profitability			
	t	$t+4$	$t+8$
dummy	4.0%	2.5%	3.8%
below median$^+$	2.7%	0.9%	3.3%
above median$^+$	5.5%	5.2%	3.4%

Table 1.3 – **The effect of product introductions on cash flow.** The table contains the change in average profitability, log sales, and log product sales when introducing new products today relative to no product introductions, while controlling for firm and time fixed effects. Product introductions are measured by increases in the number of UPC codes. The 'dummy$^+$' row treats all product introductions equally. The below/ above median$^+$ rows split product introductions into two equally-sized groups. All variables are winsorized at 2.5% and 97.5% percentile. Appendix A.II provides a description of all variables.

very old product portfolio.[11] In other words, product portfolio age has direct implications for firm value. The fact that the relationship survives when taking into account other firm characteristics (as in Loderer, Stulz, and Waelchli, 2017) again highlights the incremental information conveyed by product portfolio characteristics.[12]

As product life cycle influences the behavior of value-maximizing firms, it should also have an effect on firms' investment and financing policy. The middle and bottom graphs in Figure 1.3 indicate that both residualized investment and net book leverage tend to decline with product portfolio age. Importantly, the fact that the lines are not flat indicates that product portfolio age provides economically significant additional explanatory power in standard

[11] This finding is largely explained by risk. For example, Figure 1.2 indicates cash flow volatility is a u-shaped function of product portfolio age.

[12] Figure A.2 in Appendix A.II presents the raw relationships between product portfolio age, firm value, investment and net leverage.

leverage or investment regressions. For example, the within-R^2 of the leverage regression, with specification identical to the one employed in Figure 1.4, increases from 4.9% to 5.5%, that is by roughly 11%. For investment regression it increases by 6%.

The empirical relationships are also intuitive. The decline in the investment rate is consistent with the notion that product and capital investment are complements, that will be later formally confirmed by the model. The rationale behind the decline in net leverage is twofold. First, when firms' product portfolios age, that is when they do not replace their ageing products by new ones, firms are at risk of abruptly losing their revenues and prefer to pursue a more conservative financing policy in case their revenues vanish. Second, it also makes firms more risky. These effects result in a substitution between debt and cash financing: cash holdings increase and debt issuance decreases and hence net leverage declines.[13]

One remaining question to address is whether the magnitude of the reduced-form relationships in Figure 1.3 is economically meaningful. To show that the effects of product portfolio age are indeed significant, I compare how other firm characteristics, that are considered standard determinants of investment and capital structure, fare in explaining residualized investment or leverage, when controlling for all other variables.For each policy, I focus on two such characteristics: size and market-to-book for investment as well as profitability and tangibility for leverage. The results are presented in Figure 1.4. The graphs show that each variable correlates with the corresponding policy in an intuitive way, e.g. investment and market-to-book are positively related, while profitability is negatively related to leverage. More importantly, the graphs suggest that the economic magnitude of product portfolio age is larger than that of size and comparable to those of market-to-book for investment, while at least comparable to that of tangibility, and slightly smaller than that of profitability for leverage. Therefore, the results again reinforce the notion that product portfolio age constitutes an important and novel source of variation in corporate policies.

In summary, the stylized facts presented in this section showcase a non-trivial relationship between product life cycle, as captured by product portfolio age, and corporate policies. However, the presented empirical evidence makes it difficult to make statements regarding the quantitative importance of product characteristics. Isolating product-level forces is challenging in a reduced-form setting because financial data is essentially observed at the firm- rather than product-level. As such, in the remainder of the paper I examine the quantitative implication of product life cycle for financing and investment through the lens of a structural model. The structural approach allows to investigate the importance of frictions driving product portfolio adjustments and how they translate to variation in corporate policies.

1.3 Model

In this section, I develop a discrete-time dynamic model in which a firm makes optimal financing, investment, and product portfolio decisions.

[13] In Figure A.1 in Appendix A.II I investigate the robustness of these results by using other definitions of product portfolio age using the investment policy as an example. The results imply that all different measures produce qualitatively similar relationship between product portfolio age and investment.

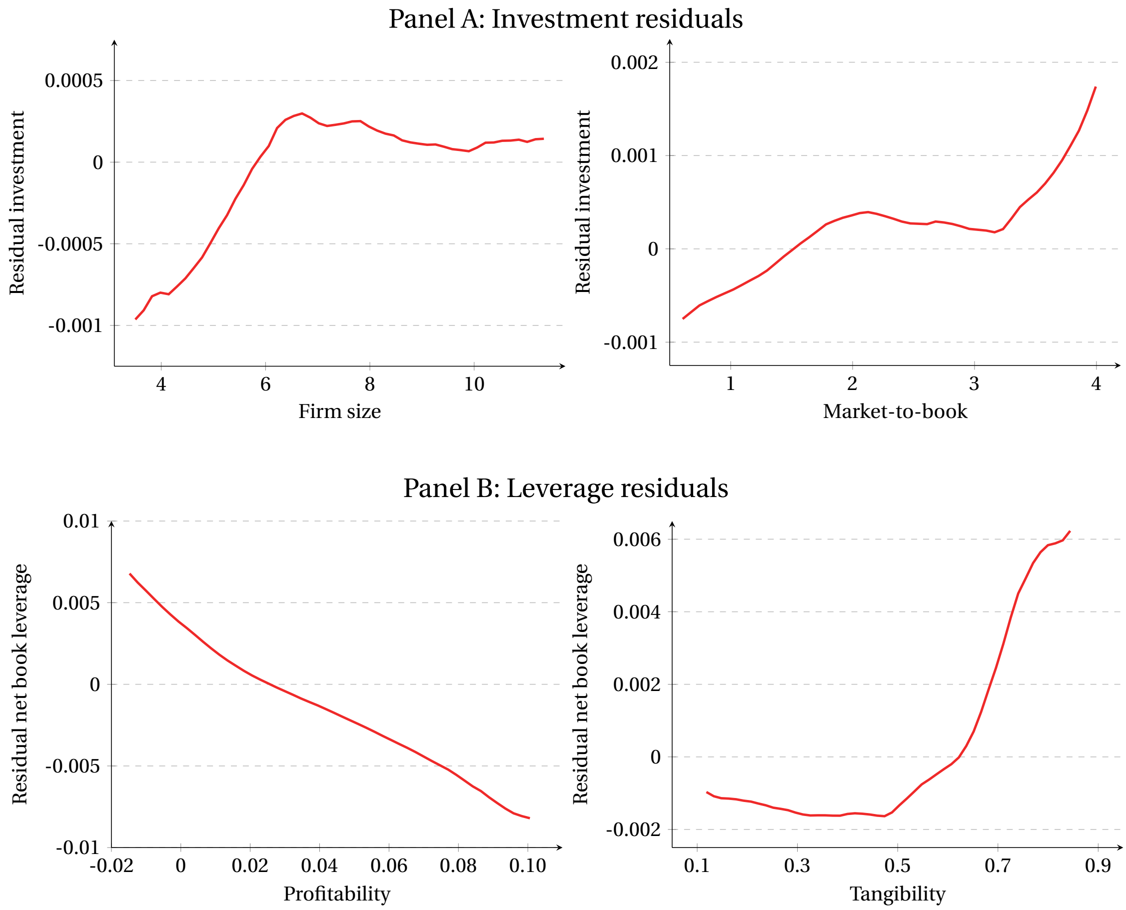


Figure 1.4 – **Assessing the significance of product portfolio age for corporate policies.** All graphs are obtained from local polynomial regressions of the residuals from an investment (or leverage) regression on a given variable, using an Epanechnikov kernel function with a rule-of-thumb bandwidth estimator and local-mean smoothing. The controls used to compute the investment residuals include size, cash flow, and market-to-book. The controls used to compute the leverage residuals include profitability, size, cash flow volatility, market-to-book, share of old products, and tangibility for profitability and profitability, size, cash flow volatility, market-to-book, and share of old products for tangibility. All regression models control for firm and time fixed effects. All variables are winsorized at 2.5% and 97.5% percentile. Appendix A.II provides a description of all variables.

1.3.1 Technology

The risk-neutral firm is governed by managers whose incentives are fully aligned with shareholders and who discount cash flows at the rate r. The firm produces homogeneous output, which can be structured into many different products, using a decreasing returns-to-scale technology. For example, one could think of the firm producing the same kind of product but marketing it to different market niches or tastes by exploiting differentiation, i.e. altering its branding, appearance, prices. The products are thus ex ante identical, but each product follows a life cycle pattern, which is the key feature of the model. Hence, the products are different ex post to the extent that they are in a different stage of their life cycle.[14] The product life cycle implies that old products contribute *less* to the firm's revenue than new products, in line with empirical evidence of Argente, Lee, and Moreira (2019). Given capital stock K and profitability shock Z, the firm generates revenue equal to

$$ZK^{\theta} \times (1-\phi(1-\xi)), \tag{1.2}$$

where ϕ is the share of old products in the firm's product portfolio and $\xi \in [0,1]$ is the old product-specific revenue discount; these are discussed in detail in the following section. Note that the model specification implies that the firm's maximum capacity is ZK^{θ}. Moreover, absent product life cycle (i.e. when $\xi = 1$), the model would collapse to the standard neoclassical benchmark.[15] The profitability shock Z follows an AR(1) process in logs,

$$\log(Z') = \rho \log(Z) + \sigma\varepsilon',\ \varepsilon' \sim N(0,1). \tag{1.3}$$

Given gross investment I, the firm's next-period physical capital stock evolves according to $K' = I + (1-\delta)K$ with capital depreciation rate $\delta \in [0,1]$. Depreciation expense is tax deductible. When the firm adjusts its capital stock, it incurs capital adjustment costs that are convex and defined as

$$\Psi(K,K') = \psi\left[K' - (1-\delta)K\right]^2 / 2K. \tag{1.4}$$

1.3.2 Product dynamics

In the model, each product follows a life-cycle pattern and can be in one of four states: 'introduction,' 'new,' 'old,' and 'exit.' New and old products are different, as each old product provides only $100 \times \xi$% of the revenue of a new product, consistent with product life cycle. A product that exits contributes nothing to the firm's revenue. The graphical illustration of an individual product's life cycle is presented in Figure 1.5.

A product that is introduced immediately becomes new, which corresponds to t_n in Figure 1.5. Every period, a new product can transition to being an old product with probability $p_{n\to o}$, which happens at time t_o in Figure 1.5, or remains new with probability $p_{n\to n} \equiv 1 - p_{n\to o}$. Similarly, every period an old product can either remain old with probability $q_{o\to o}$, or exits with probability $q_{o\to e} \equiv 1 - q_{o\to o}$, which happens at time t_e in Figure 1.5. A product that exits remains in that state forever. The product life cycle of a single product can thus be

[14] It should be noted further that the model does not distinguish between vertical and horizontal differentiation explicitly, but is more consistent with the latter, given that all product varieties are priced in the same way and only differ in their features.

[15] Note that the firm's revenue is the sum of the revenue generated by new and old products, i.e. $ZK^{\theta} \times (1-\phi(1-\xi)) = (1-\phi)ZK^{\theta} + \xi\phi ZK^{\theta}$. Here the implicit assumption is that it is not the number of products per se that matters for the firm's revenue, but rather its product portfolio structure.

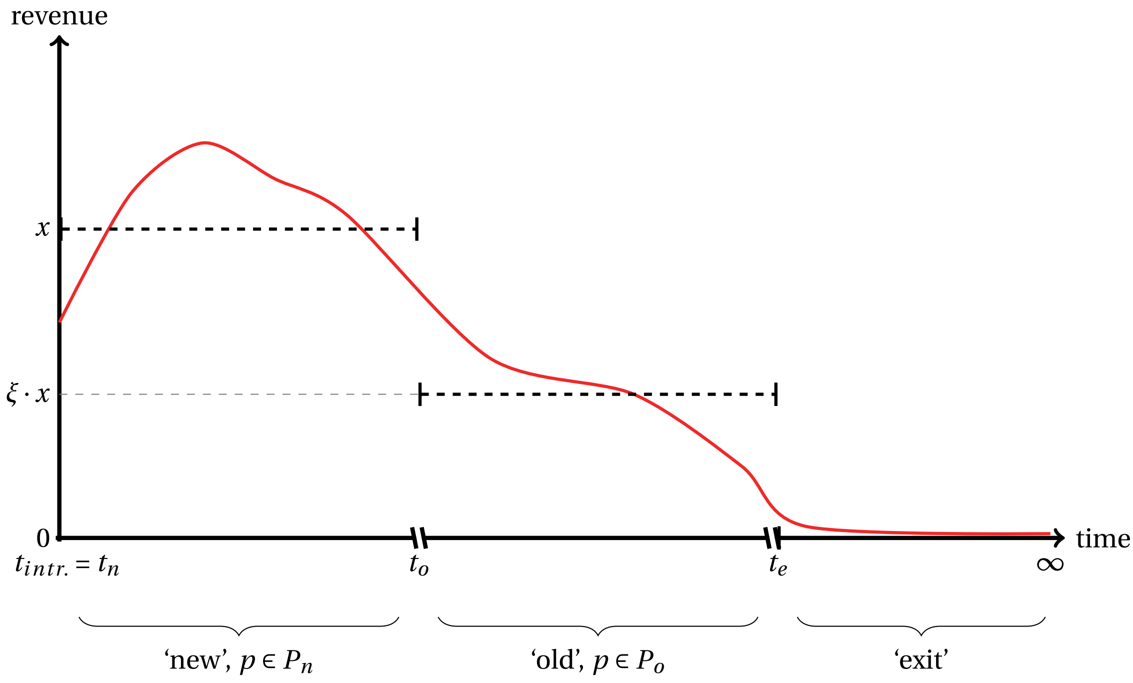


Figure 1.5 – **Graphical representation of each product's evolution in the model.**

characterized by a transition matrix

$$
\begin{array}{c} \\ \text{intr}_{t+1} \\ \text{new}_{t+1} \\ \text{old}_{t+1} \\ \text{exit}_{t+1} \end{array}
\begin{array}{c}
\begin{array}{cccc} \text{intr}_t & \text{new}_t & \text{old}_t & \text{exit}_t \end{array} \\
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & p_{n \to n} & 0 & 0 \\ 0 & p_{n \to o} & q_{o \to o} & 0 \\ 0 & 0 & q_{o \to e} & 1 \end{bmatrix}
\end{array}
$$

At the beginning of each period, the firm owns P_n new products and P_o old products, and decides whether to introduce Δ_P new products. It does so by trading off the benefits of a younger product portfolio, that is higher current revenue and higher durability of revenue, versus product introduction costs equal to $\eta K \cdot \Delta_P$. The product introduction costs capture the fact that introducing new products is costly, as it requires the firm to conduct market research, repurpose its production technology, or hire workers to market the products. Thus, the stock of new products P_n can change in two ways: the firm can introduce more products or existing new products can become old. The stock of old products P_o changes due to the ageing of new products and because old products can exit. As such, the transition probability for the firm's end-of-period product portfolio state $\Phi \equiv (P_n, P_o)$ (also called the product portfolio structure) can be expressed by a transition matrix T_Φ, which contains the probability that the firm's products transition to the state $\Phi' = (P'_n, P'_o)$ conditional on being in the state $\Phi = (P_n, P_o)$. The construction of the transition matrix T_{Φ} is described in detail in Appendix A.III.[16]

Given the structure of product dynamics in the model, we can compute the share of old products in the firm's product portfolio as:

$$\phi \equiv \phi(\Delta_P, \Phi) = \frac{P_o}{P_n + \Delta_P + P_o}, \tag{1.5}$$

which is tightly linked to the empirical proxy for product portfolio age developed in Section 2. Furthermore, the transition matrix allows to infer the expected lifetime of each product,

$$m^{(\text{intr.,exit})} = \frac{1}{1 - p_{n\to n}} + \frac{1}{1 - q_{o\to o}}. \tag{1.6}$$

Formally, Equation (1.6) is the expected hitting time of state 'exit' of a product starting at state 'introduction' and it implies that each product is expected to remain 'new' for $1/(1-p_{n\to n})$ periods and 'old' for $1/(1-q_{o\to o})$ periods. Given that we can observe the left-hand side of Equation (1.6) in the data, and given the break point assumption used to create the measure of product portfolio age, the model can be tightly linked to the data using this definition of product portfolio age.

1.3.3 Financing frictions

The firm's financing choices consist of internal funds (cash and current profits), risk-free debt, and costly external equity. Since in the model it is never optimal for the firm to hold both debt and cash at the same time, I define the stock of net debt D as the difference between the stock of debt and the stock of cash.

Debt takes the form of a riskless perpetual bond incurring taxable interest at a rate $r(1-\tau)$. As in Hennessy and Whited (2005) and DeAngelo, DeAngelo, and Whited (2011), the stock of debt is subject to a collateral constraint proportional to the depreciated value of capital

$$D \leq \omega(1-\delta)K, \tag{1.7}$$

where ω is the collateral constraint parameter such that $\omega \in [0,1]$. Alternatively, the firm may choose to hoard liquid assets to save on the costs of external equity issuance or to avoid depleting its debt capacity. However, the interest the firm earns on its cash balance is equal to $r(1-\tau)$, meaning that liquid assets earn a lower rate of return than the risk-free rate.

The cost of raising external equity is modeled in reduced form, similar to Hennessy and Whited (2005, 2007)

$$\Lambda(E(\cdot)) = \lambda E(\cdot)\mathbb{1}_{\{E(\cdot)<0\}}, \tag{1.8}$$

[16] In the model, the firm does not have the possibility to *remove* a product from its portfolio, meaning that product exit is purely stochastic. This modelling choice captures the notion of product exit being driven by exogenous customer demand forces: the firm would withdraw the product when it contributes nothing to revenue. Allowing the firm to retire a product early would require incorporating a more granular product state, as otherwise firms could artificially increase their product portfolio age by retiring old products rather than introducing new ones.

where E is the firm's cash flow, implying that the firm has to bear a proportional equity financing cost λ if it issues external equity.

1.3.4 The firm's cash flow

This setup implies the firm's cash flows E, which is a function of $(\Delta_P, K, K', D, D', \Phi, Z)$, consists of operating, investment, and financing cash flow

$$\begin{aligned} E(\cdot) = & \underbrace{(1-\tau)[ZK^{\theta} \times (1-\phi(1-\xi)) - \eta K \cdot \Delta_P]}_{\text{after-tax operating profit}} + \underbrace{\tau\delta K}_{\substack{\text{depreciation} \\ \text{tax credit}}} \\ & - \underbrace{I}_{\text{investment}} - \underbrace{\psi I^2/2K}_{\substack{\text{capital} \\ \text{adjustment cost}}} \\ & + \underbrace{D' - [1 + r(1-\tau)]D.}_{\substack{\text{net debt issuance} \\ \text{less interest expense}}} \end{aligned} \tag{1.9}$$

This formulation implies that the firm issues external equity if its cash flow is negative or pays out a dividend otherwise.[17]

1.3.5 Recursive formulation

The firm's problem is to maximize the present value of its future cash flows by choosing the investment, debt and product policies, subject to the external equity issuance cost $\Lambda(\cdot)$ and the collateral constraint. The Bellman equation for the problem is

$$\begin{aligned} V(K, D, \Phi, Z) = & \max_{\Delta_P, K', D'} \left\{ E(\cdot) + \Lambda(E(\cdot)) + \beta \mathbb{E}\left[V(K', D', \Phi', Z') \right] \right\}, \\ & \text{s.t. } D \leq \omega(1-\delta)K. \end{aligned} \tag{1.10}$$

The model is solved numerically using value function iteration. It should be noted that we only have to keep track of two out of four possible product states, given that entering products are translated into new products and exiting products produce revenue of zero. The grid for the productivity shock Z and transition matrix T_Z, are created following Tauchen (1986). The grid for capital is formed around the approximated steady-state capital. The grid for debt is formed such that its upper end point is equal to the upper end of the grid for capital, while the lower end is half of the upper end, with a reversed sign.

1.3.6 Optimal policies

In this section, I analyze the optimal product, investment and financing policies implied by the model. I derive the first-order conditions and investigate how product portfolio decisions interact with the firm's choice of investment and debt. I focus on highlighting insights that are inherently different from those stemming from standard dynamic models of the firm.

[17] I assume that the product introduction costs are considered as part of operating expenses, so that they can be deduced from taxes. Hence, firm's operating profits can be consequently interpreted as gross profits minus operating expenses.

Product portfolio

To understand how firms optimally adjust their product portfolios, I derive the approximate first-order condition for product choice Δ_P, assuming for simplicity that the firm does not issue equity:[18]

$$\underbrace{\eta K}_{\substack{\text{product}\\\text{introduction}\\\text{cost}}} \approx \underbrace{ZK^{\theta}(1-\xi)\frac{\Delta(\phi)}{\Delta(\Delta_P)}}_{\substack{\text{profit increase}\\\text{today}}} + \underbrace{\beta\mathbb{E}\left[\frac{\Delta(V(K',D',\Phi'(\Delta_P),Z'))}{\Delta(\Delta_P)}\right]}_{\substack{\text{profit increase}\\\text{tomorrow}}}. \tag{1.11}$$

Firms will introduce new products as long as the marginal cost on the left-hand side of Equation (1.11) is smaller than the marginal benefit of introducing a new product on the right-hand side of Equation (1.11). The marginal cost consists of a product introduction cost. The marginal benefit depends on the old-product specific revenue discount ξ. Furthermore, the marginal benefit also changes with product portfolio structure Φ and the profitability shock Z. For example, when the profitability shock is more persistent (higher ρ), the firm has more incentives to introduce new products to reap the benefits associated with the profitability shock whose effects last longer. Finally, the marginal benefit of a new product today also contains the expected marginal change in firm value, because today's product portfolio adjustment affects its potential future evolution. Thus, Equation (1.11) shows that investment and debt decisions of the firm indirectly affect how it chooses its product portfolio structure.[19]

Investment

Equation (1.11) shows that the firm's product portfolio adjustment is intertwined with other corporate policies through the effect on the expected marginal change in firm value. To see the exact link between investment and product decisions, I derive the investment Euler equation, which sets the discounted expected return on capital investment equal to the value of a dollar payout today:[20]

$$1 = \beta\mathbb{E}\left[\frac{(1+\Lambda'(E(\cdot))}{(1+\Lambda(E(\cdot))}\left(\frac{MB_i}{MC_i} + \frac{MB_i^{\Phi}(K',Z',\Delta_P',\Phi')}{MC_i}\right)\right], \tag{1.12}$$

where

$$MB_i^{\Phi}(K',Z',\Delta_P',\Phi') = -\theta(1-\tau)(1-\xi)\phi' K'^{\theta-1}Z' - \eta\Delta_P'. \tag{1.13}$$

Equation (1.12) shows that the return on capital investment consists of two parts. The first part, common to e.g. the neoclassical investment model, is the ratio of the marginal benefit of investment MB_i, which comprises the marginal increase in output, the value of additional

[18] In Equation 1.11, $\Delta(\cdot)$ indicates the discrete derivative, defined as $\Delta(f(n)) = f(n+1) - f(n)$.

[19] More specifically, Equation (1.11) shows that the next period stock of new products P_n' and old products P_o' both depend on how many new products were introduced in the current period, as it affects the transition matrix T_{Φ}. Thus, $\partial V'/\partial\Delta_P$ is a non-trivial quantity that depends on $\partial P_n/\partial\Delta_P$ and $\partial P_o/\partial\Delta_P$.

[20] Details of the computation are provided in Appendix A.III.

depreciated capital, and lower adjustment costs in the future, to the marginal cost MC_i, equal to a dollar spent on investment and the corresponding investment adjustment costs. The second part is the ratio of the marginal benefit of investment due to the product portfolio structure, captured by $MB_i^{\Phi}(\cdot)$, to the marginal cost.

Product and investment policies are related, because older product portfolio negatively affects the firm's revenue, resulting in a lower marginal benefit of investment. Introducing more new products increases the marginal benefit of investment, because lower revenue discount associated with younger product portfolios and higher durability of revenue increase the firm's incentives to invest in physical capital. Intuitively, the firm can now benefit from its physical capital for a longer period of time. This suggests that product introductions and capital investment act as complements. Finally, a direct computation shows that $\partial MB_i^{\Phi}(\cdot)/\partial\phi' < 0$, documenting that the model is able to reconcile the stylized fact that product portfolio age and investment are negatively related. Overall, the Euler equation shows that incentive to invest in physical capital can vary with the firm's product portfolio structure.

Net debt

To examine how financing and product decisions are interrelated, I combine the first-order condition for the debt choice D' and the corresponding envelope condition, which yields

$$1 = \beta\mathbb{E}\left[\frac{(1+\Lambda'(E(\cdot)))}{(1+\Lambda(E(\cdot)))}\left(1 + r(1-\tau) + \mu'\right)\right], \qquad (1.14)$$

where μ is the Lagrange multiplier associated with the collateral constraint. The right-hand side of Equation (1.14) is the expected discounted value of debt, which is equal to the interest payments less the tax shield and the shadow value of relaxing the constraint on issuing debt. The Lagrange multiplier μ indicates that debt is more valuable when the collateral constraint is expected to bind, highlighting that the firm may have incentives to preserve its debt capacity today to avoid reaching the collateral constraint tomorrow and having to issue costly external equity. This result, standard in dynamic investment models such as e.g. Gamba and Triantis (2008) or DeAngelo, DeAngelo, and Whited (2011), shows that debt capacity has value as it grants the firm more financial flexibility. One implication of this notion is the fact that financial, investment and product policies will be intertwined: if the firm is more likely to introduce new products tomorrow, it will follow a conservative debt policy today.

Equation (1.14) suggests that the model can reconcile the stylized facts: absent positive product introduction opportunities, the firm will preserve its debt capacity, resulting in a negative relationship between product portfolio age and leverage. Thus, even though product choice does not *directly* affect the firm's debt policy, it has an indirect effect, because it affects the firm value as well as the probability that the firm has to incur the equity issuance cost $\Lambda(\cdot)$.[21]

[21] While the model puts emphasis on the fact that product introduction decisions affect firms' financing decisions only through the 'quantitative rationing' effects of the collateral constraint, the negative association between leverage and product portfolio age is also consistent with firms issuing debt for tax reasons. Indeed, since younger product portfolios are associated with higher profits, their incentives to shield these profits from taxation are also

1.4 Estimation and Identification

I structurally estimate the model to examine the quantitative implications of product decisions on corporate policies. In this section, I describe the estimation procedure, discuss the identification strategy, present the baseline results and the cross-sectional implications of the model.

1.4.1 Estimation

Throughout the paper, I set the tax rate τ to 20% as an approximation of the corporate tax rate relative to personal taxes. While the majority of the structural parameters of the model are estimated using simulated method of moments (SMM), several parameters are estimated separately. The risk-free interest rate r is estimated at 1.4%, which is the average 3 month T-bill rate over the sample period. I also estimate separately the probability of a 'new' product remaining 'new' $p_{n\to n}$ and the probability of an 'old' product exiting $p_{o\to e}$. These probabilities can be inferred directly using the expected lifetime of a product implied by the model, shown in Equation (1.6), and the definition of the empirical proxy. In particular, in the data a product is considered 'old' if it exceeds half of its lifetime. This means that each product spends half of its lifespan being 'new' and the other half being 'old.' In terms of the model, this implies that

$$m^{(\text{intr.,exit})} = \frac{1}{2}\frac{1}{1-p_{n\to n}} + \frac{1}{2}\frac{1}{1-q_{o\to o}}, \quad \frac{1}{1-p_{n\to n}} = \frac{1}{1-q_{o\to o}}. \tag{1.15}$$

In the data, the average lifespan of a product (weighted by revenue) is 15.94 quarters. This implies that $p_{n\to n} = 0.8746$ and $q_{o\to e} = 1 - q_{o\to o} = 0.1254$. Finally, I directly estimate the proportional external equity financing cost by regressing issuance proceeds on the underwriting fees, which implies a value of 0.0223.[22]

I estimate the remaining 8 parameters $(\theta, \sigma, \rho, \delta, \psi, \omega, \eta, \xi)$ using SMM, where θ is the production function curvature, σ is the standard deviation and ρ the autocorrelation of the profitability process; δ is the physical capital depreciation rate; ψ is the capital adjustment cost parameter; ω is the parameter governing the collateral constraint; η is the product introduction cost and ξ is the old-product specific revenue discount. To do so, I first solve the model numerically, given the parameters, and generate simulated data from the model. Then, I compute a set of moments of interest using both the simulated and actual data. The SMM estimation procedure determines the parameter values that minimize the weighted distance between the model-implied moments and their empirical counterparts. Appendix A.IV provides further details on the estimation procedure.

It is important to note that the fact that the sample of firms in the data is fairly homogeneous speaks in favor of using SMM, because SMM estimates the parameters of an average firm, the concept of which is more appropriately defined in subsamples of similar firms.

higher and thus they will issue more debt. This channel is also present in the model.

[22] By doing so, I only control for direct costs of equity issuance, as in e.g. Warusawitharana and Whited (2016) or Michaels, Page, and Whited (2018).

1.4.2 Identification

Before proceeding with estimation, I discuss the identification of the structural parameters. SMM estimators are identified when the selected empirical moments equal the simulated moments if and only if the structural parameters are at their true value. A sufficient condition for this is a one-to-one mapping between a subset of structural parameters and the selected moments, that is the moments have to vary when the structural parameters vary. Because the firm's investment, financing, and payout decisions are intertwined, all of the moments are to some extent sensitive to all the parameters. However, some relationships are strongly monotonic in the underlying parameters and as such more informative of the relationship, thus useful for identifying the corresponding parameter. For example, the mean and variance of operating profits are informative of μ and σ while ρ is easily identified from the serial correlation of operating profits, which is estimated using the technique of Han and Phillips (2010).

I select 12 moments related to firms' operating profits, investment, net leverage and product portfolio characteristics. I do not choose the moments arbitrarily but rather include a wide selection of moments to understand which features of the data the model can and cannot explain. Therefore, I examine all means, variances and serial correlations of all main variables of interest that can be computed in the model. Notably, in the estimation procedure I refrain from using moments related to the size of the product portfolio (i.e. the number of products), given that the model is unlikely to match the data on this margin, as firms introduce products for variety of reasons that are not captured by this model (see e.g. Hottman, Redding, and Weinstein, 2016). Instead, I focus primarily on the product portfolio age and product portfolio adjustments, which, as I argue, help identify parameters related to the product space characteristics.

The remaining parameters are identified as follows. The physical capital depreciation rate δ is strongly linked to the mean of investment. The capital adjustment cost parameter ψ is identified by the variance and autocorrelation of investment, as higher adjustment costs result in the firm smoothing its investment. The collateral constraint parameter ω is identified by the mean of net leverage. The product introduction cost η is identified by the variance and autocorrelation of old product share, as higher cost results in more lumpy product introduction policy. The old-product specific discount ξ, on the other hand, is tightly linked to the mean of old product share, as it determines the trade off the firm faces when deciding on product introductions today.

1.4.3 Estimation results

I summarize the results of the structural estimation in Table 1.4. Panel A contains simulated and actual moments. Panel B reports the structural parameter estimates and their standard errors.

The estimated model fits the data fairly well on financial, real and product dimensions, which is justified by the low values of t-statistics in Panel A testing the difference between the model- and data-implied moments. The only exceptions are the mean and serial correlation of net leverage and the variances of investment and product portfolio age. Nevertheless, even if the difference between simulated and actual moments is statistically significant, the economic difference is negligible, especially for the variances and autocorrelations.

Panel A: Moments

	Simulated	Actual	t-stat
Mean operating profits	0.0390	0.0401	0.5259
Variance of operating profits	0.0010	0.0009	-0.3700
Serial correlation of operating profits	0.2401	0.2093	-0.2749
Mean investment	0.0204	0.0212	0.5619
Variance of investment	0.0004	0.0006	2.1910
Serial correlation of investment	0.1740	0.1742	0.0024
Mean net leverage	0.2428	0.1716	-2.7116
Variance of net leverage	0.0087	0.0093	0.4914
Serial correlation of net leverage	0.6424	0.7848	3.0527
Mean old product share	0.4301	0.4444	1.3965
Variance of old product share	0.0823	0.0946	2.4429
Serial correlation of old product share	0.4113	0.4392	0.3108

Panel B: Parameters

Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.6593	0.3457	0.3392	0.0810	0.8786	0.3591	0.0075	0.5282
Std. error	(0.0354)	(0.0284)	(0.0453)	(0.0038)	(0.2613)	(0.0332)	(0.0017)	(0.0518)

Table 1.4 – **Structural estimates and model-implied moments.** The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the the estimated parameters and their standard errors. Standard errors are clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix A.IV provides the details about the estimation procedure.

Panel B documents that all model parameters are economically meaningful and statistically significant. It is worth noting that the structural parameters have been estimated precisely, as their standard errors are low, indicating that the model is well identified.[23]

The estimate of the product introduction cost η is equal to 0.75%, which implies that a typical sample firm behaves as if it had to incur a cost of approximately \$7.64m when introducing a new product. While this cost appears substantial, it is required to square the fact that firms do not continuously adjust product portfolios in the data, as the distribution of product entry is fairly lumpy (see also Figure A.2 in Appendix A.II). Moreover, the estimated cost is fixed and as such can be interpreted as if it comprised both the direct costs of introduction

[23]In Appendix A.IV I show that the model parameters are locally identified by the underlying moments by computing the diagnostic measure of Andrews, Gentzkow, and Shapiro (2017).

(such as marketing, R&D expenditures, etc.) as well as indirect ones, such as the present value of costs related to supplying the product.

The old-product specific discount ξ is estimated at 0.5282, meaning that firms act as if each old product in their portfolio only contributed 52.82% of a new product's revenue. As such, the discount is fairly large, which is consistent with the notion of product life cycle. In particular, the fact that the estimate of ξ is strictly larger than 0 implies that product life cycle effects are present and important. On the other hand, the fact that it is not equal to 1 suggests firms still benefit from old products, which is indeed the case as their profitability does not drop to 0, as per Figure 1.2. To gauge whether the magnitude of the estimate is sensible, I consider a back-of-the-envelope calculation and compute the average revenue of 'old' and 'new' products in the data. The obtained value of 59.1% suggests that the estimated value of ξ is in a reasonable range. Barring potential measurement error, the fact that it is lower than its data 'counterpart' could be explained by the fact that firms in the model do not withdraw products by themselves, which makes the old products relatively 'worse' compared to the new ones.

The structural estimates of the remaining parameters are in range of those in extant studies of firms' financing and investment policy. For example, the standard deviation of the profitability shock σ and the collateral constraint parameter ω that determines the firm's debt capacity are close to the ones obtained in Nikolov, Schmid, and Steri (2018) and the persistence of the profitability process ρ is similar to the one reported in Warusawitharana and Whited (2016) for the food manufacturing industry, which comprise the majority of the sample firms. The only parameter that may seem on the higher end of the range compared to the existing literature is the convex investment adjustment cost ψ, estimated at 0.8786, which results in a fairly sticky investment policy.[24]

1.4.4 Cross-sectional implications of the model

The results discussed until now show that the model is able to jointly explain the corporate investment, financing and product portfolio policies of an average sample firm. In this section, I provide further empirical evidence of the importance of the product life cycle channel by estimating the model on subsamples of firms that vary along key firm characteristics. In particular, I focus on two specific sets of sample splits. First, I investigate whether the estimated model can reconcile differences between firms varying in their products' sensitivity to product life cycle. This analysis serves as a 'sanity check' whether the product life cycle effects in the model correspond to the ones observed in the data, despite using firm-level rather than product-level data in the estimation procedure. Second, I analyze whether other important product market characteristics matter for the product life cycle channel. To this end, I focus on sample splits based on the size of product portfolio, the degree of product market competition, the durability of the products and product uniqueness. This exercise, in turn, provides further insight as to how the economic forces behind product life cycle affect corporate policies of firms differing in dimensions not explicitly captured in the model. It also

[24] In a different model, Warusawitharana and Whited (2016) also obtain a much higher investment adjustment cost for the food manufacturing industry.

shows which features of the data explain the magnitude of the product introduction cost.

Sensitivity to product life cycle

Panel A: Moments

	Low sensitivity to PLC		High sensitivity to PLC	
	Simulated	Actual	Simulated	Actual
Mean operating profits	0.0395	0.0428	0.0328	0.0376
Variance of operating profits	0.0008	0.0008	0.0009	0.0011
Serial correlation of operating profits	0.1543	0.1455	0.2067	0.3123
Mean investment	0.0178	0.0190	0.0183	0.0240
Variance of investment	0.0002	0.0003	0.0006	0.0012
Serial correlation of investment	0.0653	-0.0030	0.2805	0.3189
Mean net leverage	0.2096	0.1863	0.1848	0.1565
Variance of net leverage	0.0070	0.0073	0.0039	0.0112
Serial correlation of net leverage	0.6546	0.8056	0.6958	0.7673
Mean old product share	0.4250	0.4248	0.4349	0.4647
Variance of old product share	0.0839	0.0860	0.0842	0.1034
Serial correlation of old product share	0.3832	0.4420	0.4141	0.4367

Panel B: Parameters

Low sensitivity to product life cycle								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.5641	0.3253	0.2167	0.0707	0.5657	0.3140	0.0066	0.6145
Std. error	(0.0469)	(0.0305)	(0.0259)	(0.0033)	(0.1147)	(0.0308)	(0.0005)	(0.0213)
High sensitivity to product life cycle								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.7002	0.3951	0.3197	0.0719	0.4775	0.3173	0.0079	0.5021
Std. error	(0.0547)	(0.0570)	(0.0550)	(0.0077)	(0.3556)	(0.0313)	(0.0016)	(0.1031)

Table 1.5 – **Structural estimates and model-implied moments: firms whose products have low and high sensitivity to product life cycle.** This table reports the estimation results for subsamples of firms more and less exposed to product life cycle, classified using the firm-specific regression coefficient of product-level revenue on age, while controlling for product-level and cohort-level fixed effects. The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the estimated parameters and their standard errors, clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix A.IV provides the details about the estimation procedure.

To analyze whether the model can reflect differences across firms whose products vary in their sensitivity to product life cycle, for each firm in the sample I estimate a product-level regression of the form

$$\log(\text{rev})_{it} = \alpha + \beta \times \log(\text{age})_{it} + \eta_i + \gamma_c + \varepsilon_{it}, \tag{1.16}$$

where i and t indicate the product and the quarter, and c indicates the product's corresponding cohort.[25] I then split the firms into two groups based on the estimates of β. The firms with the below-median (above-median) sensitivity of product-specific revenue to product age β should be less (more) exposed to product life cycle effects, for example due to the fact that their products are more (less) durable or are less (more) susceptible to ageing.

Table 1.5 presents the estimation results for the two subsamples. Panel A documents that the model-implied and data moments are relatively close, implying that the model captures well the policies of firms in both subsamples. The results also suggest that firms whose products are less exposed to product life cycle have on average higher profitability and older product portfolios. The fact that these firms also adopt higher net leverage speaks to the importance of the precautionary savings motive of product life cycle, that should be less pronounced when firms are less exposed to product life cycle. On the other hand, firms with higher sensitivity to product life cycle invest more on average, which is related to the fact that they tend to introduce more products and complement it with capital investment. This is also true in the data, as the net product entry rate of these firms is approximately 3 times higher as compared to the one of firms with low product life cycle sensitivity (0.4% vs 1.2% per year, on average). The fact that this moment is not used in the estimation procedure serves as a test for the external validity of the subsample analysis.

The parameter estimates in Panel B indicate that products of firms with high product life cycle sensitivity lose about 49.79% of revenue when they become old, as compared to 38.55% for products of firms with low product life cycle sensitivity. This result shows that the model successfully captures the intuition underlying the relationship between product revenue and age, and as such the product-level information is not lost when aggregating product-level data to firm-level. The fact that the average net leverage across the two subsamples is different while the estimate of the collateral constraint parameter ω is nearly the same further reinforces the importance of product dynamics on firms' precautionary savings incentives. Finally, firms with higher product life cycle sensitivity are also more sensitive to firm-wide productivity shocks, as the estimate of production function curvature θ is higher for these firms, as is the standard deviation of the profitability shock. This finding suggests that there can be some differences in the underlying economic environment across the two subsamples of firms, for example they may supply products in industries that may be subject to different kinds of customer demand dynamics.

Product characteristics

I now turn to investigating the differences in estimates along product dimensions not explicitly captured by the model. I focus on four sample splits, based on the size of product portfolio, the

[25] I control for the cohort-specific fixed effects using the Deaton (1997) adjustment as in Argente, Lee, and Moreira (2019).

degree of product market competition, the durability of the products and product uniqueness. Table 1.6 contains the parameter estimates for each subsample. To keep the presentation of the results concise, the corresponding data- and model-implied moments are relegated to Appendix A.IV.

Product portfolio size

Panel A of Table 1.6 shows the estimation results for firms with small and large product portfolios, that is those with below- and above-median effective number of products, respectively. The parameter estimates indicate that firms with small product portfolios face a higher costs of introducing new products and a more pronounced old-product specific revenue discount. These results reinforce the notion that for managing product portfolios is even more important for small firms, as they are more exposed to product life cycle. Additionally, it is interesting to note that for firms with small product portfolios the fraction of capital that can be collateralized ω is much lower than for firms with large product portfolios. This result has two explanations. First, since the correlation between product portfolio size and firms size is positive, a part of the result is simply due to small firms having less capital (in terms of total assets) that can be pledged as collateral, consistent with e.g. Nikolov, Schmid, and Steri (2018). The fact that the estimate of ω varies substantially across the two samples, however, suggests that the number of products also plays a critical role, which can be consistent with firms behaving as if their intangible assets (e.g. patents or trademarks) can be pledged as collateral as well (see e.g. Mann, 2018, Suh, 2019 or Xu, 2019). Finally, the size of product portfolio also appears to serve as a way for firms to diversify their revenues, as firms with large product portfolios have lower estimated variance of profitability shock and higher estimated profitability shock autocorrelation.

Product market competition

Panel B of Table 1.6 presents the estimates from two subsamples differing in the degree of product market competition.[26] Investigating this dimension of the data is important for two reasons. First, the degree of competition could affect the trade-offs determining product life cycle, for example firms operating in more competitive markets could be forced to introduce more new products to be able to keep up with their competitors or gain market share. Second, the empirical literature on product markets has largely focused on this dimension of the data, that is on how the between-firm effects affect firms' investment and financing policy. It is therefore instructive to examine how *within*-firm product market forces, such as product life cycle, are related to corporate policies. Importantly, the measure of competition adopted in this paper is better suited to characterize the competitive environment faced by firms as it incorporates complete information about the product markets they operate in.

The results in Panel B suggest that each old product of firms operating in more concentrated

[26] This is done by first computing the HHI of each 'market' in which firms operate, which are defined by product groups (see Appendix A.I), and then computing the firm-specific exposure to the markets, by computing the average HHI weighted by the firm's share of sales in a given market. In particular, the HHI of each market is computed using all available data on private and public firms. More details about how the competition proxy is computed are provided in Appendix A.II.

Panel A: Product portfolio size								
	θ	σ	ρ	δ	ψ	ω	η	ξ
Smaller	0.7062 (0.0383)	0.3294 (0.0452)	0.1070 (0.0196)	0.1041 (0.0086)	0.5509 (0.1087)	0.2385 (0.0428)	0.0096 (0.0017)	0.4334 (0.0547)
Larger	0.6636 (0.0520)	0.2610 (0.0237)	0.2799 (0.0620)	0.0773 (0.0028)	0.5967 (0.1996)	0.3043 (0.0291)	0.0053 (0.0007)	0.6141 (0.0213)

Panel B: Product market competition								
	θ	σ	ρ	δ	ψ	ω	η	ξ
Competitive	0.7457 (0.0452)	0.3558 (0.0404)	0.3802 (0.0997)	0.0810 (0.0056)	0.6455 (0.1711)	0.3429 (0.0397)	0.0064 (0.0010)	0.4484 (0.0535)
Concentrated	0.5873 (0.0347)	0.1878 (0.0237)	0.2813 (0.0614)	0.0865 (0.0056)	0.5315 (0.0953)	0.2720 (0.0269)	0.0063 (0.0012)	0.6310 (0.0575)

Panel C: Product durability								
	θ	σ	ρ	δ	ψ	ω	η	ξ
Lower	0.6473 (0.0429)	0.2238 (0.0423)	0.2905 (0.0846)	0.0904 (0.0031)	0.4003 (0.1271)	0.3757 (0.0368)	0.0067 (0.0011)	0.6004 (0.0705)
Higher	0.7113 (0.0434)	0.2299 (0.0373)	0.3270 (0.1130)	0.0926 (0.0069)	0.6709 (0.0990)	0.3104 (0.0367)	0.0091 (0.0010)	0.4208 (0.0277)

Panel D: Cost of sales								
	θ	σ	ρ	δ	ψ	ω	η	ξ
Lower	0.7203 (0.0743)	0.1283 (0.0556)	0.1897 (0.0356)	0.0708 (0.0056)	0.8094 (0.2971)	0.4248 (0.0355)	0.0080 (0.0012)	0.4114 (0.0588)
Higher	0.6050 (0.0390)	0.3756 (0.0334)	0.1294 (0.0542)	0.0822 (0.0086)	0.3190 (0.2499)	0.3330 (0.0334)	0.0087 (0.0026)	0.5586 (0.1030)

Table 1.6 – **Cross-sectional evidence from sample splits.** This table reports the estimation results for subsamples of firms varying according to specific firm characteristics. Panel A presents the results for firms with with small and large product portfolios, classified using the median breakpoint of the effective number of products. Panel B presents the results for firms exposed to more and less competitive product markets, computed using the exposure of each firm's sales to the HHI of each market. Panel C presents the results for firms supplying more and less durable products, computed using the products' average calendar age at exit. Panel D presents the results for firms with higher and lower selling-related expenses, computed using Compustat item `xsga` scaled by total assets. The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. The table reports the estimated parameters, standard errors clustered at firm-level are in parentheses. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix A.IV provides the details about the estimation procedure.

markets provides about 63% of a new product's revenue; compared to 43% for firms subject to more competitive pressure. Firms in less competitive product markets also face lower costs of introducing new products. Given that higher competition may result in products become obsolete quicker, that is ξ being lower, the results show that product life cycle is related to product market competition, highlighting that both between- and within-firm product market forces play an important role in shaping corporate policies. The product market competition dimension also appears to affect the extent of product life cycle more product portfolio size does, given that the difference in ξ across the two samples is larger than for firms differing in the size of product portfolios. Finally, model- and data-implied moments in Table A.6 in Appendix A.IV suggest that firms operating in less competitive product markets have higher operating profits, consistent with less intensive competition. Importantly, while these firms have similar level of old product share to firms operating in competitive environment, the effect on profitability is mitigated as these firms are less exposed to product life cycle channel.

Product durability

In the model, each product is ex ante homogeneous and is expected to last for the same amount of time. However, in reality firm can influence the expected durability of their products, e.g. by expanding to more lasting product categories or investing in consumer retention. To this end, I investigate how product durability interacts with the product-level forces in the model. Panel C of Table 1.6 presents the estimation results from two subsamples split according to the expected durability of firms' products, measured by the products' average calendar age at exit.

The estimates suggest that firms with higher product durability face higher product introduction costs (0.913% vs. 0.667%) *and* are more exposed to product life cycle, as their old products provide only 42% of their revenue, as compared to 60% for firms that introduce less durable products. While the first result is intuitive, the second one might appear contrary to the notion of product durability. However, one important thing to note is that product durability should, by definition, greatly affect the product transition probabilities. This is indeed the case, as the implied $p_{n \to n}$ changes from 81.74% to 90.55% across the two subsamples, which suggests that products of firms with higher product durability are expected to last twice as long than those of firms with lower product durability. This means that the net effect of having product durability can still be positive, despite products of these firms losing a larger chunk of revenue when they age. Finally, it is interesting to note that other parameter estimates remain similar for both subsamples, indicating that product durability remains a fairly distinctive product feature that does may reveal itself in other firm characteristics.

Product uniqueness – cost of sales

The last sample split that I investigate is related to cost of sales, which captures all costs not directly related to goods sold, such as advertisement or corporate expenses (Compustat item `xsga`). This sample split is important, for several reasons. First, it speaks to the notion of selling more or less specialized products, thus following different strategies (e.g. product uniqueness of Titman and Wessels, 1988). Second, since cost of sales contains marketing expenses, it should be related to product life cycle, to the extent that firms try to influence the lifetime of their products by e.g. advertising. Finally, it measures a cost of selling the product, which should be closely related to the product introduction cost η, and as such the sample split serves as a sanity check of whether the model can correctly recover this feature of the data. To this end, I split the firms into two samples based on their cost of sales as a fraction of total assets and re-estimate the model.

The results of this exercise are presented in Panel D of Table 1.6 and indicate that firms with higher cost of sales have higher product introduction costs (0.797% vs. 0.872%), which is intuitive. Their products are, however, less exposed to product life cycle, as new products only lose 44% of their revenue when becoming old, as compared to 59% for firms with lower cost of sales. Interestingly, higher cost of sales do not translate to higher durability. In fact, firms with higher cost of sales have *lower* average lifespan of their products (85.98% vs. 88.78%), indicating that lower product durability could be the underlying reason why firms spend more on selling-related activities. It also highlights that product durability is markedly different from product uniqueness, plausibly proxied by cost of sales: more specialized firms are not necessarily those that also supply more durable products.

1.5 Analysis and Counterfactuals

In this section, I study the implcations of product-level economic forces for corporate policies. First, I analyze the numerical policy functions implied by the model. Second, I consider a number of counterfactuals to better understand how product market strategy interacts with corporate policies and how quantitatively important its effects are. Finally, I analyze the role of product cannibalization in shaping corporate policies.

1.5.1 Numerical policy functions

To examine the implications of the estimated parameters for the firm's optimal policies, I compute the numerical policy functions $\{I/K, D/K, \Delta_P\} = h(K, D, \Phi, Z)$ for investment rate I/K, net leverage D/K, and product introductions Δ_P. In the discussion that follows, I focus on two sets of policy functions. First, I fix K and D at their average values in the simulated sample and set $Z = 1$ as I want to focus on the economic forces driven by product portfolio setting. Panel A of Figure 1.7 plots the policy functions $\{I/K, D/K, \Delta_P\} = \tilde{h}(P_o|P_n^i)$ for a firm with a low and high number of new products, i.e. $i \in \{l, h\}$. Second, in Panel B of Figure 1.7 I fix K and D at their average values in the simulated sample and plot the policy functions for the profitability shocks Z: $\{I/K, D/K, \Delta_P\} = \tilde{h}(Z|\Phi^i)$, while varying Φ from a low to a high value, i.e. $i \in \{l, h\}$.

The numerical policy functions in Panel A of Figure 1.7 show how the firm optimally responds to changing the product portfolio structure. In particular, the left graph in Panel A shows that the policy function for product introductions Δ_P can be characterized by an inaction region due to a fixed cost of product introduction. That is, the firm only starts introducing new products once its current stock of old products is sufficiently large. The threshold at which it happens depends on the stock of new products, as firms with more new products are less exposed to product life cycle than firms with less new products.

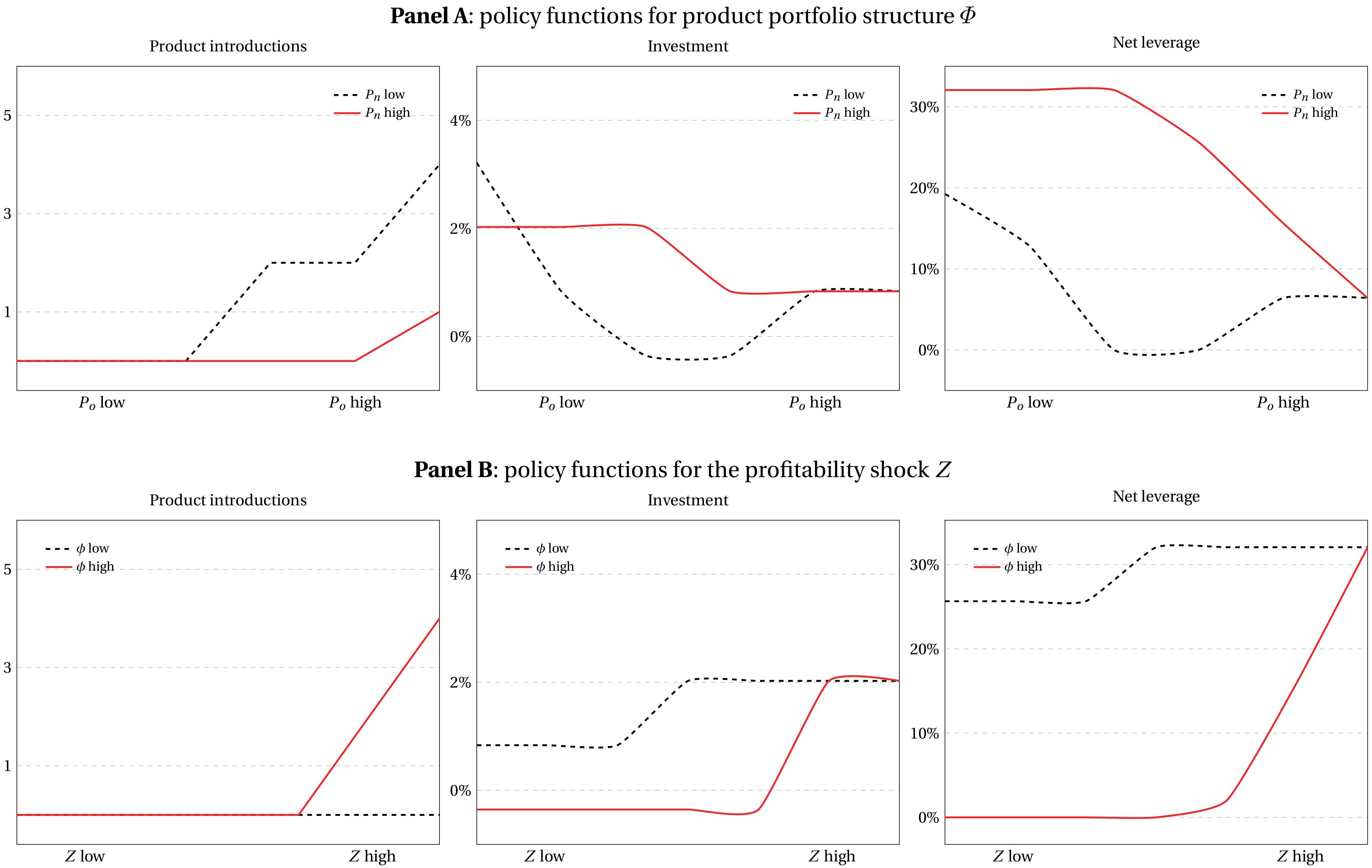


Figure 1.7 – **Numerical policy functions for the product portfolio structure Φ and the profitability shock Z.** In Panel A, the policy functions are computed for the baseline parameter estimates and average values of capital, debt, and profitability shock. In Panel B, the policy functions are computed for the baseline parameter estimates and average values of capital and debt.

The policy function for product introductions Δ_P has natural implications for investment and financing policy. The middle graph in Panel A illustrates that investment decreases in the number of old products, consistent with the investment Euler equation. The Euler equation also reveals the intuition behind the spike in investment that is visible for a large number of old products, which coincides with the firm introducing new products. That is, if the product introduction cost is sufficiently small, the firm's marginal benefit of investment increases in Δ_P. In other words, capital and product investment are *complements*: the firm wants to invest more in physical capital as its product portfolio becomes younger to benefit from higher and more durable operating revenue.

The right graph in Panel A documents the precautionary savings motive induced by product dynamics, as it indicates that the firm's leverage policy depends on its product portfolio structure. Importantly, the firms appears to finance product introductions to a large extent with debt, given how the increase in the policy function coincides with Δ_P. It should also be mentioned that when the share of old products in the portfolio is low, leverage tends to decrease with P_o. This happens largely because the firm has higher precautionary savings incentives and thus values preserving debt capacity more, because it would have to tap external financing when the old products exit and its revenue drops. Thus, given the costly nature of external equity it is optimal for the firm to act conservatively and adopt lower leverage. In the counterfactual experiments below, I show that this effect largely depends on the product-level characteristics. Finally, the firm also adopts lower leverage as it benefits less from tax shields due to lower operating income.

Panel B of Figure 1.7 documents how the firm optimally responds to profitability shock when varying its product portfolio structure. The left graph in Panel B suggests that the firm with a high share of old products may choose not to introduce net products when it experiences a low realization of Z, because introducing new products is costly. The result in the middle graph in Panel B is fairly standard in the dynamic investment models, as investment increases with Z, but it also confirms the intuition conveyed in Panel A of Figure 1.7 that the firm invests less when it has an older product portfolio. That is, the product dimension changes the firm's sensitivity of investment to the profitability shock.

The right graph in Panel B illustrates that the firm's choice of leverage varies differently with Z, depending on its product portfolio structure. When hit by a low shock realization, the firms tend to disinvest and use the proceeds to pay down debt, resulting in lower leverage. Similarly, when the realization of the shock is high, the firms prefer to preserve their debt capacity to fund future profitable investment opportunities, and thus adopt lower leverage ratios. The only exception is the firm with a low share of old products, which also issues debt to fund investment for a very high shock realization. It is also worth noting that for high Z realizations, the firm with a high share of old products focuses on introducing new products, that is renewing its product portfolio, rather than investing in capital. This coincides with no apparent spike in leverage, unlike in Figure 1.7, because now the firm finances introducing new products internally, following a high realization of Z.

To conclude the discussion of the policy functions, I investigate whether the dynamics induced by the product-level economic forces are economically important. To this end, I perform a variance decomposition of the firm's investment and leverage policy. The results highlight that product dynamics account for roughly 20% of the total investment and leverage variance, independently of how they are measured in the model (by share of old products or stock of new and old products separately).[27] Moreover, most of the variation is due to the dynamics of the stock of old products, which is intuitive given their importance in the product life cycle channel. Overall, the results suggest that product dynamics can contribute substantially to the observed variation in corporate policies.

1.5.2 Counterfactuals

I now turn to investigating the quantitative importance of product dynamics for the firm's corporate policies by means of several counterfactual exercises. First, I consider how product-level economic forces affect firm value, firms' precautionary savings incentives and the relationship between product characteristics and corporate policies. Second, I examine the impact of changing parameter values related to the product market dimension on investment and leverage policies. I do so by varying the old-product specific discount ξ and the probability of product exit $q_{o\to e}$ that govern the expected benefit per product and expected product lifetime.

Firm value implications

I first investigate how important product market characteristics are in shaping firm value. In Table 1.7, I consider the effects of changing the cost of introducing new products η, the old product-specific discount ξ and the individual-product transition probabilities $p_{n\to o}$ and $q_{o\to e}$ from their baseline estimates.

In the first panel of Table 1.7, I conduct counterfactual experiments related to the severity of product life cycle. When setting $\xi = 0$, product life cycle is very severe, as old products generate zero revenue. In contrast, $\xi = 1$ implies that new and old products contribute the same amount to the firm's cash flow. Comparing the baseline results and those for $\xi = 0$ indicates that firm value is 3.55% higher due to the fact that each of their old products generates 58% of a new product's revenue rather than 0%. However, the firms still lose from the product life cycle effects, as changing ξ from its baseline estimate of $\xi = 0.5282$ to $\xi = 1$ would increase firm value by 4.48%. All in all, this evidence suggests that introducing products that age slower over their life cycle can bring material benefits to the firm.

It is also interesting to investigate how the correlations between corporate policies and product portfolio age vary in these different cases from their baseline value. Not surprisingly, making the distinction between old and new products irrelevant by setting $\xi = 1$ dampens the correlations to essentially 0, as product portfolio age loses any impact. In the second case, the relationships remain the same or become stronger. This shows that the channel between product portfolio structure and corporate policies described in the paper is sensitive to product characteristics.

[27] To get these numbers, I compute the Type III partial sum of squares for each variable in the model and then compute its share in the total variance. That is, the number represents the % of total variance explained by the variable.

	%Δ firm value	mtb	Corr(·, age) inv	lev	%Δ debt capacity
Baseline	0.00%	-0.208	-0.088	-0.060	0.00%
Old product generates no revenue $\xi = 0$	-3.55%	-0.045	-0.044	-0.201	117.54%
Old and new products are the same $\xi = 1$	4.48%	0.000	0.000	-0.009	-61.41%
50% lower product introduction cost η	9.14%	-0.113	-0.090	0.000	11.38%
50% higher product introduction cost η	-5.72%	-0.231	0.004	-0.171	-43.59%
New product immediately becomes old $p_{n\to o} = 1$	-10.71%	-0.203	0.081	0.090	-59.40%
Old product immediately exits $q_{o\to e} = 1$	-3.03%	0.009	-0.019	-0.154	-62.73%

Table 1.7 – **Counterfactual experiments.** The table reports the outcomes of alternative model parametrizations, resulting from varying the product introduction cost η (first panel), the old-product specific revenue discount ξ (second panel), and the individual-product transition probabilities p and q (third panel). The first column reports the % change in the average market-to-book relative to baseline estimation results, the three following columns the correlation between product portfolio age and market-to-book, investment, and leverage, respectively, and the last column the % change in the average debt capacity (measured as the difference between maximum debt capacity ωK and current net debt, both as a fraction of capital) relative to baseline estimation results.

The second panel of Table 1.7 indicates that lower values of η result in higher firm value, as product introductions become cheaper, and thus firms have more flexibility in adjusting their product portfolio. The effects are quantitatively large as well: increasing η by 50% results in 5.72% lower firm value. Increasing the product introduction cost also changes the correlations between corporate policies and product portfolio age, because the firm's product policy becomes more lumpy and product introductions are less frequent.

Finally, I analyze how changing product durability affects firm value. The results in the third panel of Table 1.7 show that changing the probability of a new product becoming old to $p_{n\to o} = 1$ lowers firm value by 10.71%, and is much lower than increasing the probability of an old product exiting to $q_{o\to e} = 1$ that results in 3.03% lower firm value. However, these can be reconciled by the fact that the estimated old-product specific discount implies that old products are approximately two times worse than new ones in terms of their contribution to the firm's revenues. Overall, the results indicate that managing product durability can also be beneficial for firms.

Quantifying the effects on precautionary savings incentives

As argued before, the product life cycle effects induce particular precautionary savings incentives for firms. In the counterfactual exercise, I examine how the magnitude of these incentives is affected by product-level economic forces. The last column of Table 1.7 presents the percentage change in firms' debt capacity, measured as the difference between the maximum debt capacity ωK and net debt D (both scaled by capital K), relative to the values implied by the estimated model.

The results suggest that product characteristics can largely magnify firms' incentives to preserve debt capacity. For example, changing product introduction costs affects how often firms' decide to introduce new products. Less frequent product introductions lower their incentives to preserve debt capacity, because they require less funding for product introductions. The effects of varying the exposure to the product life cycle channel, by altering the old-product specific discount, are even stronger: more severe product life cycle effects result in stronger precautionary savings motives. This happens because the firm can lose a larger fraction of revenue due to product exit, which makes it value spare debt capacity more. This is the reason why removing the difference between new and old products results in the firm preserving its debt capacity less, as then the influence of the product life cycle channel is non-existent. This finding also highlights that the financing behavior induced by the product life cycle channel would be absent in standard dynamic models of the firm that do not account for product-level dynamics (e.g. the AK framework). In other words, product life cycle magnifies the precautionary savings effects.

Comparative statics: product portfolio characteristics

I now consider a different type of a counterfactual exercise by examining the effect of changing product-level characteristics on average firm policies. Figure 1.8 presents the resulting comparative statics for the old product-specific revenue discount ξ in Panel A and for the probability of old product exit $q_{o\to e}$ in Panel B, which govern the firm's exposure to the product life cycle channel and the durability of each product, respectively. In each panel, I examine the effect of a 20% upward or downward change in each parameter on average investment, net leverage, and product portfolio age. To construct these figures, I solve a model in which the values of each parameter deviate from its baseline estimated value, and simulate using the resulting optimal policies.

Higher values of ξ imply that each product provides the firm with more benefits over the same expected lifetime. As such, the firm has more incentives to introduce new products and its product portfolio age declines, see the leftmost graph of Panel A of Figure 1.8. This, in turn, results in the firm substituting capital for product investment, as the latter becomes relatively cheaper for the same level of total output and average investment decreases with ξ. As for leverage, there are three main reasons why it increases in ξ. First, the firm finances product introductions by issuing debt, especially when it has an old product portfolio. Second, smaller old product discount ξ results in higher profits, which incentivizes the firm to issue debt to benefit from tax shields. Finally, higher ξ means that the firm is less exposed to the product life cycle channel and thus having many old products is less risky for the firm, as they differ less from new products, which means that the firm values preserving debt capacity less. These channels are consistent with the policy functions in Figure 1.7.

Panel B of Figure 1.8 presents the effects of changing the probability that an old product exits $q_{o\to e}$. Essentially, this parameter determines the expected longevity of the firm's products. When the probability is lower, the firm needs to introduce less new products to achieve the same level of product portfolio longevity firm, as the existing products are expected to survive for a longer period of time. This results in a higher average product portfolio age, see the rightmost graph in Panel B. Thus, when the firm's products become less durable, it invests more in physical capital to make up for the revenue lost due to shorter lifetime of its products. However, the effect is quantitatively smaller than in case of ξ. The firm also adopts lower leverage to ensure that it has enough debt capacity to fund investment and introduce new products using debt rather than resorting to costly external financing.

All in all, the results of the comparative statics further reinforce the notion that product portfolio characteristics play a major role in shaping corporate policies.

1.5.3 The effects of cannibalization

Arguably, the product setting in the model is silent on many aspects of real-world product portfolio management, for example the fact that introducing new products usually results in negative externalities for firms' existing product lines, which is known as 'cannibalization.' Thus, one could argue that these effects could play a major role in shaping firms' product market strategies. To study whether this is indeed the case, I examine how quantitatively important the effects of cannibalization are on financing and investment.

To this end, I extend the model by explicitly allowing for a dependence between the number of introduced products and the probability of a existing products becoming old $p_{n\to o}$. In the

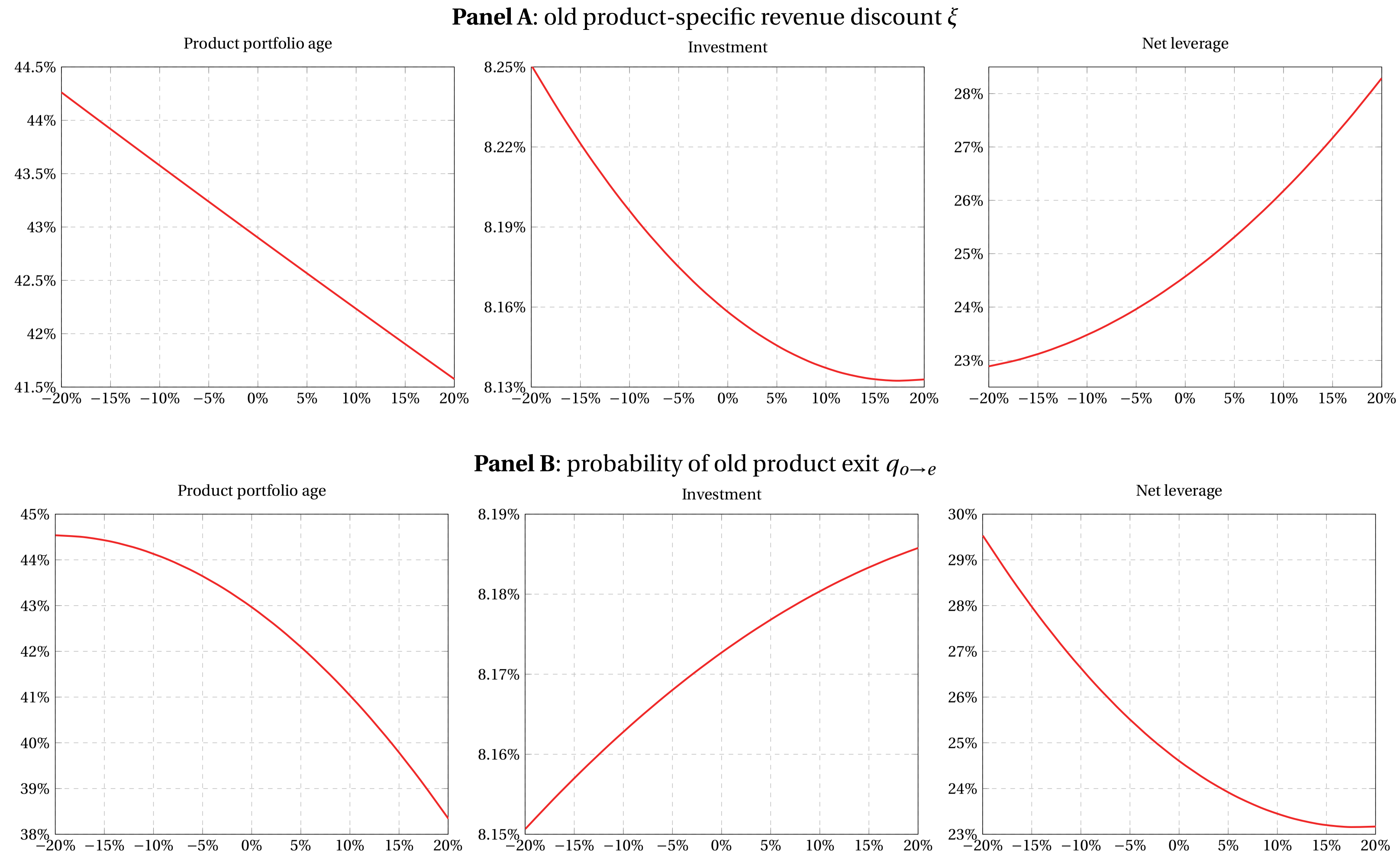


Figure 1.8 – **Comparative statics of product-related parameters.** Each point on the curve corresponds to the value of a given moment from a counterfactual experiment, starting from the baseline estimates of structural parameters and changing only the respective parameter by 20% up and 20% down. Each curve is a polynomial interpolation of moments from a discrete set of counterfactual experiments.

extended model, it is parametrized as

$$\tilde{p}_{n\to o} = p_{n\to o} + \sum_{p=1}^{\Delta_P} \epsilon^p, \tag{1.17}$$

where Δ_P is the number of products introduced by the firm and ϵ can be considered as a parameter related to the firm's elasticity of substitution between existing and new products' revenues. Since in the model old products generate lower revenue than new ones, I effectively assume that product cannibalization acts through ageing the firm's products, which is reasonable given that, as argued by Argente, Lee, and Moreira (2019), product life cycle is largely due to changes in customer preferences. In the exercise to follow, I assume that ϵ varies from 0 (the baseline case) to 0.0627, that is a half of $p_{n\to o}$. This implies that the product-specific revenue is expected to lower by 21.9% when introducing one new product, as compared to the "no cannibalization" benchmark.[28]

Figure 1.9 presents the effects of the cannibalization parameter ϵ on the firm's profitability, product portfolio age, net product entry and net leverage. The figures indicate that controlling for potential cannibalization has intuitive implications for corporate policies: average profitability decreases, as new product now have shorter lifespan, which translates to higher product portfolio age, and much higher new product entry, that nearly doubles. Finally, leverage increases for two reasons. First, as firms finance product introductions by issuing debt, they also adopt higher leverage. Second, higher cannibalization rate results in a more stable product portfolio structure, and thus profitability, as both variances decrease. This lowers the firm's precautionary savings incentives and thus results in higher leverage.

Overall, the effect of cannibalization can magnify the effects of product life cycle, but does not appear to alter the main mechanisms through which product dynamics interact with corporate policies.

1.6 Conclusion

In this paper, I demonstrate that product life cycle has important implications for corporate policies by developing and estimating a dynamic model of product portfolio decisions. In line with the product life cycle channel, new products are more profitable, and are expected to last longer than old ones. Thus, when deciding whether the introduce new products, the firm trades off the benefits of a younger product portfolio versus product introduction costs.

I embed the product life cycle channel into a flexible model of financing and investment that can be taken to the data by means of structural estimation. The firm's product introduction decisions have direct implications for cash flow dynamics. As a result, investment, financing and product decisions are intertwined at firm level. In particular, the model implies that product introductions and capital investment act as complements and that product life cycle

[28]The IO and marketing literature do not specify a clear-cut candidate for the value of this parameter. For example, Hottman, Redding, and Weinstein (2016) estimate the product elasticity of substitution due to *price* increases, which implies a cannibalization rate of 0.5 for the median firm in their sample. This means that about half of the sales of a new product introduced by a firm comes from the sales of existing products and half from the new ones. That is, in this analysis I assume a less pronounced effect.

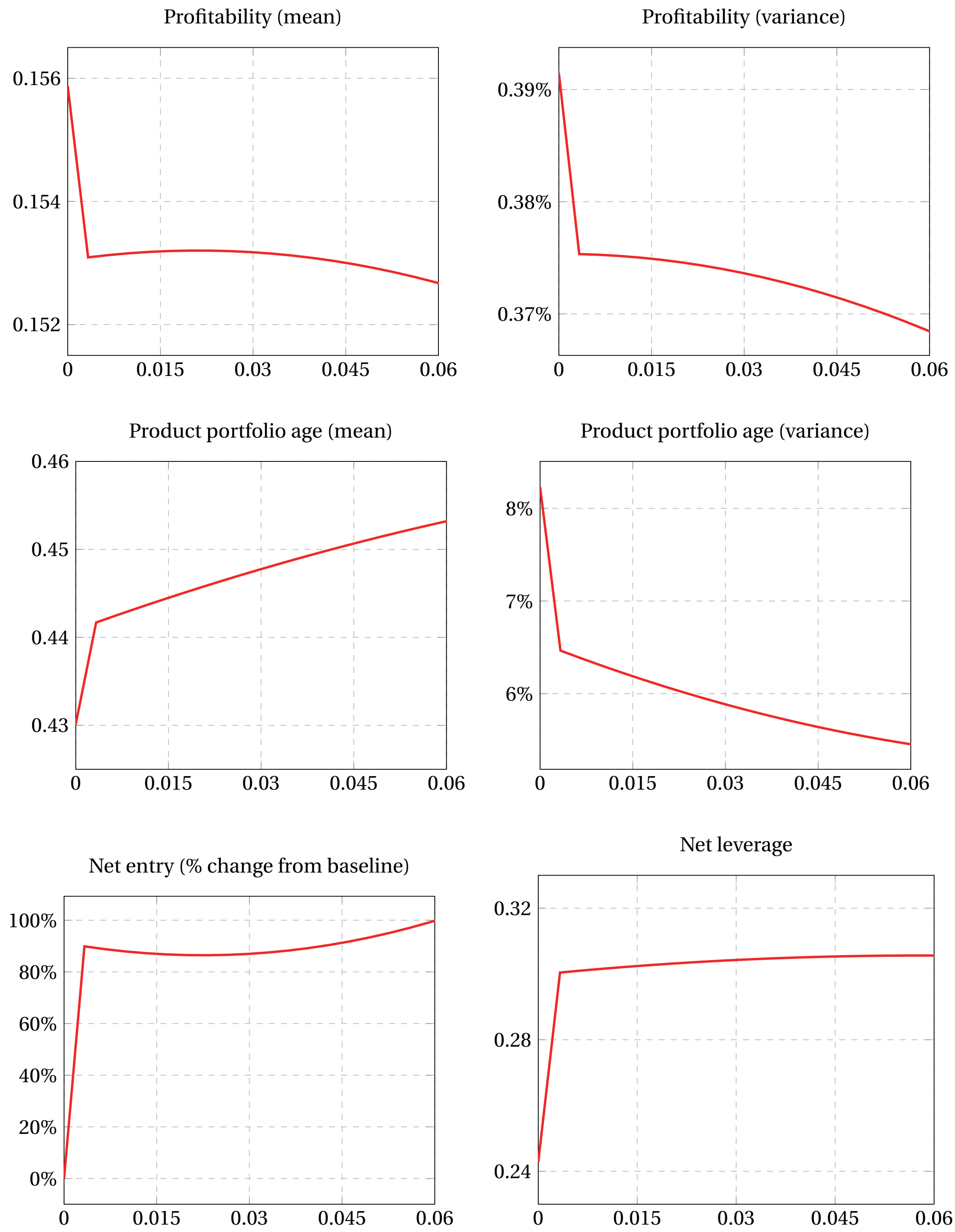


Figure 1.9 – **The effects of cannibalization.** Comparative statics of the cannibalization parameter ϵ. Each point on the curve corresponds to the value of a given moment from a counterfactual experiment, in which the baseline estimates of structural parameters are retained while changing only the cannibalization parameter ϵ. Except for the point corresponding to $\epsilon = 0$, each curve is a polynomial interpolation of moments from a discrete set of counterfactual experiments.

induces stronger precautionary savings motives for firms. These predictions are in line with empirical stylized facts about product portfolio age, investment and leverage.

Using detailed data on firms' product portfolios, I structurally estimate the model to quantify the effects of the product life cycle channel. By doing so, the paper delivers three novel results. First, the structural estimates imply that the product-level forces are quantitatively important, as products lose 48% of their revenue when they become old and firms spend $7.64m per product introduction. Second, the estimated model provides important cross-sectional predictions, as the estimates suggest that firms supplying fewer products, competing more intensely, and supplying products more sensitive to ageing are also more exposed to the product life cycle channel. Third, by means of counterfactual experiments I find that product life cycle substantially affects firm value as well as corporate investment and financing decisions. Overall, the data suggests that managing the life cycle of products, by means of introduction cost or sensitivity to ageing, yields material benefits to firms.

2 Debt, Innovation, and Growth[1]

2.1 Introduction

Over the last few decades, the US economy has become innovation driven. Public firms now spend twice as much on research and development than on capital expenditures, and fixed assets have fallen from 34% to less than 20% of total assets between 1975 and 2016 (see for example Corrado and Hulten, 2010 or Doidge, Kahle, Karolyi, and Stulz, 2018). Creative destruction has been a driving force of this transition to a knowledge-based economy. A good example of this phenomenon is the swift rise to power of Apple and Samsung in the mobile phone industry, replacing Nokia as the market leader. This example of creative destruction was driven by the innovative success of Apple and Samsung, even though all three firms devoted large amounts of resources to R&D.[2]

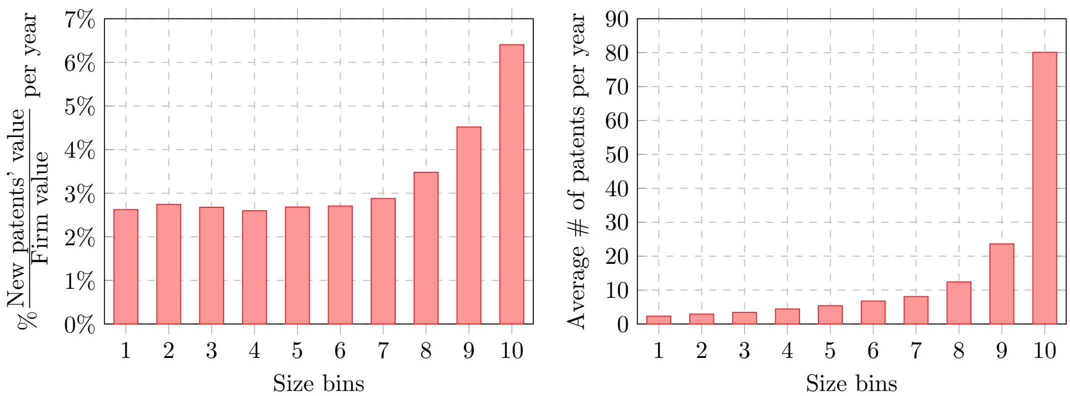


Figure 2.1 – **Innovation quality and intensity.** The innovation data is based on Kogan, Papanikolaou, Seru, and Stoffman (2017) and the firm size data is from Compustat. The averages are conditional on issuing a new patent.

As shown in Figure 2.1, large firms play an important role for aggregate levels of innovation. Decades of empirical research have shown that debt is a key source of financing for these

[1]The material in this chapter represents joint work with Thomas Geelen from Copenhagen Business School and Erwan Morellec from EPFL.

[2]See https://thenextweb.com/plugged/2019/03/29/24-years-global-phone-sales-graph-visualization/ for an impressive visualization of this change in market share.

firms; see e.g. Graham, Leary, and Roberts (2015). In addition, even though debt is widely cast as an unlikely way to fund young and risky ventures, recent empirical studies show that small and young firms also heavily rely on debt financing. For example, Robb and Robinson (2014) find that formal debt financing (business bank loans, credit lines, and owner-backed bank loans) provides about 40% of firms' initial startup capital. The reliance on formal credit channels holds true even for the smallest firms at the earliest stages of founding. Looking only at those firms that access equity sources, such as venture capital or angel financing, the average firm still has around 25% of its capital structure in the form of debt. A recent study by Hochberg, Serrano, and Ziedonis (2018) further documents a widespread use of loans to finance technology startups, even in early stages of development. Relatedly, Davis, Morse, and Wang (2018) find that venture debt is often a complement to equity financing, with over 40% of all financing rounds including some amount of debt.[3]

Given the change to an innovation-based economy and the heavy reliance of innovative firms on debt financing, a number of questions naturally arise. First, how does debt financing influence innovation at the firm level? Second, how do innovation and creative destruction in turn feed back into firms' financing policies? Third, what are the implications of debt financing in innovative firms for aggregate levels of innovation and growth?

This paper attempts to answer these questions by developing a Schumpeterian growth model in which firms' innovation and financing policies are endogenously determined. In this model, each incumbent has a portfolio of products and invests in R&D. Innovations by firms improve the quality of the goods produced. Firms therefore expand into new product lines when R&D is successful, which allows them to profit from their own innovations. But these profits are compromised when competitors develop better products. The force of creative destruction therefore affects firms R&D policies, as each product remains profitable until it is overtaken by another firm's innovation.

In line with Figure 2.1, firms decide on both R&D intensity, that is the rate at which they generate new innovations, and R&D quality, that is the expected number of products that each innovation creates. Shareholders' choice of R&D therefore determines firms' cash flow dynamics, which feeds back into their financing decisions. In the model, R&D and financing policies maximize shareholder wealth. As a result, financing choices reflect conflicts of interest between shareholders and debtholders, on top of the standard trade-off between the tax advantage of debt, the costs of issuing securities, and default costs.[4]

After solving for individual R&D, investment, and financing choices, we embed the single-firm model into an industry equilibrium in which the rate of creative destruction is endogenously determined. We derive a steady state equilibrium in which new product lines replace

[3] While start-ups cannot typically obtain debt financing from traditional banks, major U.S. banking institutions, public firms, and private firms specialize in providing loans to the very start-ups that banks turn away. In related research, Mann (2018) shows that patents are pledged as collateral to raise significant debt financing, and that the pledgeability of patents contributes to the financing of innovation. Suh (2019) finds that firm ownership of patents increases firms' total debt-to-assets ratio by 18%. Xu (2019) shows that firms use trademarks as collateral for debt financing.

[4] A simplified version of this model has been shown to capture the main stylized facts about corporate leverage (Strebulaev, 2007, Morellec, Nikolov, and Schürhoff, 2012, and Danis, Rettl, and Whited, 2014).

existing ones and entrants replace incumbents that default and exit the industry. Firms in this equilibrium exhibit a wide variation in leverage, size, and innovation rates. Furthermore, all industry-wide equilibrium variables are constant over time, although individual firms continue innovating, investing, and adjusting their capital structure.

In this equilibrium, capital structure and R&D influence each other through three main channels. First, R&D policy influences firms' risk profile and the aggregate level of creative destruction, which in turn affects their capital structure decisions. Second, levered firms are subject to debt overhang, which alters their incentives to innovate and the level of competition. Third, debt financing changes the surplus from entering the industry, which again influences the aggregate level of creative destruction and competition.

Starting with firm-level policies, we find that there is significant interaction between leverage and innovation. Notably, high levels of debt lead to less innovation by incumbents due to debt overhang, in that shareholders endogenously cut R&D and investment when their benefits mostly accrue to debtholders by rendering debt less risky. We find that the effect of debt on innovation by incumbents is sizeable, larger for firms with fewer products, and present both when firms follow a static debt policy or can dynamically adjust their debt levels. We also show that R&D policies and the industry rate of creative destruction play a key role in determining financing choices by affecting cash flow risk and the probability of default. Our model predicts substantial intra-industry variation in leverage and innovation, in line with the evidence in MacKay and Phillips (2005) and Kogan, Papanikolaou, Seru, and Stoffman (2017). It also shows that debt financing has large effects on firm turnover and industry structure.

Underinvestment by incumbents suggests that debt may hamper innovation and growth. A striking result of the paper is to demonstrate that debt financing does in fact lead to more investment and growth at the aggregate level. This is the outcome of two opposing forces. First, as discussed above, innovation by incumbents is negatively associated with debt. Second, debt increases the value of incumbents (by providing a tax shield) and the surplus from entering the industry, leading to a higher entry rate and to greater innovation by entrants. We demonstrate that the latter effect always dominates in our endogenous growth model, so that debt financing fosters creative destruction and growth.

Importantly, the economic mechanism underlying this result suggests that measuring the effects of debt on innovation and growth using shock-based causal inference can potentially be problematic. Indeed, according to our model, any exogenous policy shock that would make debt more valuable—e.g., a decrease in debt issuance costs due to a change in regulation—would lead to higher leverage ratios and to lower innovation rates for incumbents. This could lead to a negative relation between innovation rates of firms and debt levels in the data. Yet, at the aggregate level, cheaper debt would foster entry and spur innovation and growth. Shock responses would therefore not recover theory-implied causal effects as they would not capture the influence of debt financing on entry.

Remarkably, our result that debt fosters creative destruction and growth does not hinge upon the specific trade-off we use to determine firms' financing decisions. This result would also hold for example if debt reduced the cost of informational asymmetries between insiders and outsiders (Myers and Majluf, 1984) or the cost of free cash flow and managerial flexibility (Jensen, 1986), as in both cases debt financing would increase the surplus of entrants (thereby stimulating entry), and reduce innovation by and facilitate exit of incumbents.[5]

Lastly, this result is consistent with the evidence in Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following US banking deregulation. Their empirical analysis shows that US banking reforms, that made bank debt widely available and cheaper by increasing competition, brought growth in both entrepreneurship and business closures (see also Amore, Schneider, and Žaldokas, 2013 and Chava, Oettl, Subramanian, and Subramanian, 2013).

We also illustrate how the conclusions reached in the single-firm model, when ignoring equilibrium feedback effects, can be fundamentally altered, or even reversed, when the rate of creative destruction is endogenized in industry equilibrium.

Consider for example the effects of innovation costs on equilibrium quantities. Increasing innovation costs leads to a drop in the level of innovation and in the value of future innovations. This reduces the cost of debt (overhang) and leads firms to increase financial leverage. These effects are stronger in a single-firm model that does not incorporate the industry-wide response. Indeed, an effect that is absent when ignoring industry dynamics is that the drop in innovation quantity and the increase in leverage feed back into the equilibrium rate of creative destruction. As shown in the paper, the effect on innovation is generally first order, leading to a negative relation between innovation costs and the rate of creative destruction. This decrease in the rate of creative destruction—and the corresponding increase in the expected life of product lines—spurs innovation, partly offsetting the higher innovation costs. Lastly, in industry equilibrium these mechanisms translate to a lower turnover rate as innovation costs increase. By contrast, when industry feedback effects are ignored, the sharp increase in leverage due to increasing innovation costs increases the turnover rate.

2.1.1 Related literature

Our article contributes to several strands of the literature. First, we contribute to the literature studying innovation in Schumpeterian growth models. Schumpeterian growth theory has been widely used in the literature on innovation and industry structure and evolution; see for example Klette and Kortum (2004), Lentz and Mortensen (2008), Aghion, Akcigit, and Howitt (2014), Akcigit and Kerr (2018), and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). However, to the best of our knowledge, this literature has not studied the effects of debt financing on innovation, Schumpeterian competition, and industry dynamics. This is relatively surprising given that innovative firms heavily rely on debt financing. Our paper fills this gap by extending the model proposed by Klette and Kortum (2004) to incorporate debt financing. In our model, firms hold debt and default, which influences their R&D policies and the industry level of

[5]We do not use either a mechanism based on financing constraints because these models generally require debt to be fully collateralized and lead to the counterfactual prediction that all debt is risk-free. The turmoil in corporate debt markets that took place in March 2020 shows this is clearly not the case. In addition, the empirical studies cited above show that debt financing is used by innovative firms that have access to and use both debt and equity financing.

creative destruction.[6]

Second, our paper relates to the literature on dynamic capital structure choice initiated by Fischer, Heinkel, and Zechner (1989) and Leland (1994). Models in this literature generally maintain the Modigliani and Miller (1958) assumption that investment and financing decisions are independent by assuming that the assets of the firm are exogenously given. This allows them to focus solely on the liability side of the balance sheet (see for example Fan and Sundaresan, 2000, Duffie and Lando, 2001, Hackbarth, Miao, and Morellec, 2006, Gorbenko and Strebulaev, 2010, Glover, 2016, or DeMarzo and He, 2018). Our paper advances this literature by endogenizing not only firms' capital structure choices but also their investment policy. In line with the evidence in Chava and Roberts (2008), Giroud, Mueller, Stomper, and Westerkamp (2012), and Favara, Morellec, Schroth, and Valta (2017), we find that debt financing has a negative effect on innovation and investment at the firm level, due to debt overhang (Myers, 1977). The distortions in investment due to debt financing are large and imply important feedback effects of (endogenous) investment on capital structure choice. A key contribution with respect to this literature is that we embed the individual firm choices into a Schumpeterian industry equilibrium. We show that while debt leads to underinvestment by incumbents, it increases creative destruction and growth by stimulating entry. This result goes against standard economic intuition.

Third, our paper relates to the literature on debt in industry equilibrium. In a closely related paper, Miao (2005) builds a competitive equilibrium model in which firms face idiosyncratic technology shocks and can issue debt at the time of entry before observing their profitability. In this model, all firms have the same debt level. However, the model has heterogeneity in firm size because firms are allowed to invest after entry. An important assumption in Miao (2005) is that there are no costs of adjusting capital. As a result, there is no debt overhang in the sense of Myers (1977) because the absence of adjustment costs or frictions make investment independent of financing (Manso, 2008).[7] By contrast, firms have different (endogenous) debt levels in our model and can adjust capital structure after entry as profitability evolves. In addition, investment and financing decisions interact, leading to debt overhang and underinvestment by incumbents.[8] Other important contributions to this literature include Fries, Miller, and Perraudin (1997) and Zhdanov (2007), which respectively study static and dynamic capital structure choices in the Leahy (1993) model. In these models, incumbent firms are exposed to a single industry shock. They all have the same assets and the same debt level and there is no investment.

Lastly, our paper relates to the literature initiated by Mello and Parsons (1992) and Parrino and Weisbach (1999) on the effects of debt financing on corporate investment in dynamic

[6] Another departure from Klette and Kortum (2004) is that we introduce heterogeneity in the quality of innovations, which is key to match the patterns in Figure 2.1.

[7] In Miao (2005), firms underinvest in that levered firms exit the industry at a higher rate than unlevered firms would. This feature is also present in our model.

[8] In related research, Malamud and Zucchi (2019) develop a model of cash holdings, innovation, and growth in the presence of Schumpeterian competition. Firms are all equity financed in their model. Maksimovic and Titman (1991) develop a three-period model in which investment decisions reflect debt choices in industry equilibrium. They do not study entry and exit decisions, which are central to our analysis.

models of the firm. Our study departs from prior work by endogenizing capital structure choices and by embedding the single-firm model in an industry equilibrium in which the rate of creative destruction and the persistence of firm cash flows are endogenous.[9] This literature generally emphasizes the negative effects of debt on investment. By contrast, we show that in aggregate debt fosters investment and growth by stimulating entry.

This article is organized as follows. Section 2.2 describes individual firm choices and then embeds the single-firm model into an industry equilibrium. Section 2.3 analyzes the model implications. Section 2.4 closes the model in general equilibrium. Section 2.5 concludes.

2.2 Model

We present the model in steps, starting with the investment and financing decisions of an individual firm. We then embed the single-firm model into an industry equilibrium.

2.2.1 Assumptions

Throughout the paper, time is continuous and shareholders and creditors are risk neutral and discount cash flows at a constant rate $r > 0$. The economy consists of a unit mass of differentiated goods that are produced by incumbent firms.

A firm is defined by the portfolio of goods it produces. The discrete number of different products supplied by any given firm at time $t \geq 0$, denoted by P_t, is defined on the integers and is bounded from above by $\bar{p}$. As a result of competition between firms, each good is produced by a single firm and yields a profit flow of one. The profit flow of the firm evolves through time as a birth-death process that reflects product creation and destruction.

To increase the number of goods it produces, a firm invests in innovative effort, i.e. spends resources on R&D. A firm's R&D choice is two-dimensional. Each instant, it chooses both the frequency of arrival of new innovations $\lambda_t \in [0, \bar{\lambda}]$ and the quality of new innovations $\theta_t \in [0, 1]$. The arrival intensity λ_t determines the Poisson rate at which innovations arrive. Conditional on an innovation, the number of new product lines generated is given by

$$X_t = \min\left(Y_t, \bar{p} - P_{t^-}\right) \; with \; Y_t \sim Bin(n, \theta), \tag{2.1}$$

where $n < \bar{p}$ is an exogenous upper bound on the number of new product lines that can be developed following an innovation and $Bin(n,\theta)$ is the binomial distribution. This specification implies that the expected number of new product lines is approximately $n\theta$. Therefore, a higher quality θ leads to a higher expected number of new product lines. Bounding the number of new product lines X_t from above by $\bar{p} - P_{t^-}$ ensures that P_t never exceeds $\bar{p}$. These assumptions imply that the total number of product lines the firm has developed up to time t,

[9] In related research, Kurtzman and Zeke (2018) quantify the aggregate implications of debt overhang on firms' innovation activity and macroeconomic outcomes. In their model, innovations only temporarily boost productivity while the persistence of innovations is endogenous in our model and reflects firms' individual R&D decisions and the industry rate of creative destruction. This allows us to study the implications of debt financing on macroeconomic growth. Another important difference is that creative destruction by competitors influences firms' cash flow risk in our framework, which is a first-order determinant of their financing and investment decisions. Lastly, our model considers dynamic capital structure choice.

denoted by I_t, evolves as

$$dI_t = X_t dN_t^I, \tag{2.2}$$

where dN_t^I is a Poisson process with intensity λ_t.

A firm's existing product lines can become obsolete because some other firm innovates on a good it is currently producing. In this case, the incumbent producer loses the good from its portfolio due to creative destruction. Since any firm is infinitesimal, we can ignore the possibility that it innovates on a good it is currently producing. Because of creative destruction, each product becomes obsolete at an exponentially distributed time with intensity f. We call f the rate of creative destruction, that each firm takes as given. Subsection 2.2.3 embeds the single-firm model into an industry equilibrium and endogenizes the rate f of creative destruction. The total number O_t of product lines lost by the firm up to time $t \geq 0$ because of creative destruction evolves as

$$dO_t = dN_t^O, \tag{2.3}$$

where dN_t^O is a Poisson process with intensity fP_{t^-}. The total number product lines in a firm's portfolio P_t is therefore given by

$$P_t = I_t - O_t. \tag{2.4}$$

A firm with zero product lines exits the economy at time $\tau_0 \equiv \inf\{t > 0 : P_t = 0\}$.

A firm performing R&D with intensity and quality (λ_t, θ_t) incurs flow costs $q(P_t, \lambda_t, \theta_t)$. To make sure that shareholders are better off with more product lines, we impose that the R&D cost function does not increase too fast in the number of product lines in that

$$q(p+1, \lambda, \theta) - q(p, \lambda, \theta) < 1. \tag{2.5}$$

An incumbent firm's operating profit is the profit that comes from the operation of the product lines minus the endogenous costs of performing R&D:

$$P_t - q(P_t, \lambda_t, \theta_t). \tag{2.6}$$

Profits are taxed at the constant rate $\pi > 0$. As a result, firms have an incentive to issue debt to reduce corporate taxes.[10] To stay in a simple time-homogeneous setting, we follow the literature (e.g. Leland, 1994, Duffie and Lando, 2001, and Manso, 2008) and consider debt contracts that are characterized by a perpetual flow of coupon payments c. The firm incurs a proportional cost ξ when issuing debt. Because of creative destruction, firms may default on their debt obligations. Default risk leads to endogenous distortions in the firm's R&D decisions when close to distress, reflecting debt overhang. An additional cost of debt is that default leads

[10] The main benefit of debt in our model is that it provides tax savings thereby raising the value of incumbents and, therefore, the surplus from entering the industry. We could similarly assume that firms obtain better financing terms with debt for example due to lower sensitivity to informational asymmetries.

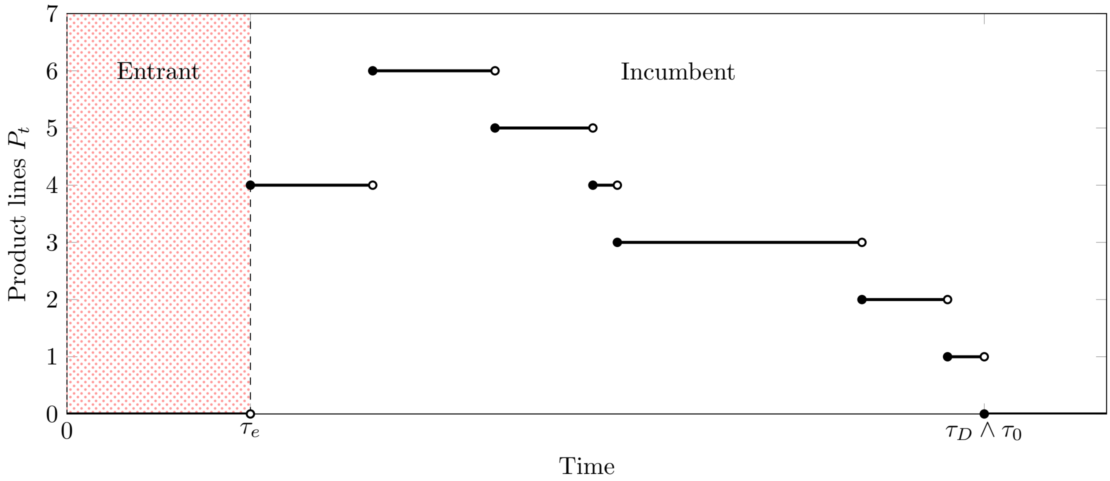


Figure 2.2 – **Life-cycle of a firm.** The firm starts as an entrant and becomes an incumbent (with 4 product lines) at τ_e. The number of product lines then evolve over time and the firm defaults at $\tau_D \wedge \tau_0$ where $x \wedge y = \inf\{x, y\}$.

to exit and liquidation with a fraction $\alpha > 0$ of assets in place being lost as a frictional cost. When choosing the amount of debt, shareholders balance the tax benefits of debt against its costs. Internet Appendix A allows firms to dynamically optimize their capital structure.

As in Klette and Kortum (2004), a mass of entrants invests in R&D to become producers upon a successful innovation. An entrant that generates an innovation becomes an incumbent. Similarly to an incumbent, the entrant chooses its R&D intensity λ_t and quality θ_t. The entrant has R&D cost function $q_e(\lambda, \theta)$. Because an entrant has no product lines before becoming an incumbent, it has (optimally) no debt and its optimal innovation strategy is time-homogenous: $\lambda_t = \lambda_e$ and $\theta_t = \theta_e$. As soon as an entrant has an innovative breakthrough and knows how many product lines this breakthrough generates, it has the possibility to issue debt. The cost of becoming an entrant is denoted by $H > 0$. We consider that entry costs are tax deductible, e.g. through depreciation. This assumption ensures that taxes have no bearing on innovation when firms are unlevered so that our results are directly comparable to those in prior work.

Figure 2.2 illustrates the life cycle of a firm. An entrepreneur first pays the entry cost and becomes an all-equity financed entrant, which incurs R&D expenses until it innovates for the first time. At τ_e, the entrant experiences a breakthrough resulting in new product lines, decides how much debt to issue, and becomes an incumbent. Once the firm becomes an incumbent, it generates profits from its portfolio of products and continues to make R&D decisions, which influences the intensity at which new innovations arrive as well as their quality. This process continues until the firm exits at time τ_D in case of default or at time τ_0 in case it loses all of its product lines to competitors.

2.2.2 Optimal financing and investment

We start by analyzing the case in which debt policy is static (as in e.g. Leland, 1994, Duffie and Lando, 2001, Manso, 2008, or Antill and Grenadier, 2019) and solve the model recursively, starting with the value of levered equity for a given financing policy. Since each good generates the same flow of profits, we only need to keep track of the number of goods it produces and the coupon when describing the state of the firm.

After debt has been issued, shareholders maximize equity value by choosing the firm's default and R&D policy. As a result, equity value for a given coupon c satisfies

$$E(p,c) = \sup_{\{\lambda_t,\theta_t\}_{t\geq 0},\tau_D} \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0} e^{-rt}(1-\pi)\left(P_t - c - q(P_t,\lambda_t,\theta_t)\right) dt \right], \tag{2.7}$$

where $\mathbb{E}_p[\cdot] = \mathbb{E}_0[\cdot|P_0 = p]$, τ_0 is the first time the firm has zero product lines, and $x \wedge y = \inf\{x, y\}$. As shown by (2.7), shareholders receive the after-tax profits from P_t product lines minus the coupon payments c and R&D expenses $q(P_t,\lambda_t,\theta_t)$ until they decide to default or the firm exists with zero products. They select the R&D strategy $\{\lambda_t,\theta_t\}_{t\geq 0}$ and default time τ_D to maximize the equity value. The presence of debt as well as the rate of creative destruction alter shareholders' incentives to invest in R&D or to continue operations.

From equation (2.7), it follows that equity value solves the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} rE(p,c) = \sup \Big\{ 0, (1-\pi)(p-c) + &\underbrace{fp\left(E(p-1,c) - E(p,c)\right)}_{\textit{Creative destruction}} \\ &+ \underbrace{\sup_{\lambda,\theta}\left\{\lambda\left(\mathbb{E}^\theta\left[E(\min\{p+x,\bar{p}\},c)\right] - E(p,c)\right) - (1-\pi)q(p,\lambda,\theta)\right\}}_{\textit{R\&D}} \Big\}, \end{aligned} \tag{2.8}$$

where $\mathbb{E}^\theta$ takes the expectation over $x \sim Bin(n,\theta)$ and $E(0,c) = 0$. We then have the following result.

Theorem 1 (Equity Value)**.** *A unique solution to the equity value* (2.7) *exists. Equity value is non-decreasing in p and therefore the optimal default strategy is a barrier strategy* $\tau_D = \inf\{t > 0|P_t \leq p_D\}$. *If the optimal level of R&D is interior* $((\lambda,\theta) \in (0,\bar{\lambda}) \times (0,1))$, *it solves*

$$\mathbb{E}^\theta\left[E(\min\{p+x,\bar{p}\},c)\right] - E(p,c) = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial \lambda}, \tag{2.9}$$

$$\lambda\frac{\partial \mathbb{E}^\theta\left[E(\min\{p+x,\bar{p}\},c)\right]}{\partial\theta} = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial\theta}. \tag{2.10}$$

The optimal R&D strategy, if interior, equates the marginal benefits and the marginal costs of R&D.[11] The marginal cost depends on the R&D cost function $q(p,\lambda,\theta)$. If an innovation

[11] If there exists a λ^* such that for any $\lambda > \lambda^*$ and $\theta \in [0,1]$

$$\frac{\partial q(p,\lambda,\theta)}{\partial\lambda} \geq \frac{\theta n}{r} \tag{2.11}$$

then in equilibrium $\lambda < \lambda^*$ and imposing the bound on λ becomes void.

arrives, the expected increase in equity value is

$$\underbrace{\mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c)\right]}_{\textit{Post innovation}} - \underbrace{E(p,c),}_{\textit{Pre innovation}} \tag{2.12}$$

which is the marginal gain from increasing the arrival rate of innovations λ. Similarly, higher R&D quality θ increases the expected number of new product lines when an innovation arrives. The marginal increase in equity value from higher R&D quality θ is

$$\lambda \frac{\partial \mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c)\right]}{\partial \theta}. \tag{2.13}$$

The presence of debt in the firm's capital structure implies that shareholders do not fully capture the benefits of investment in that cash flows to shareholders are truncated at $\tau_D \wedge \tau_0$. This in turn implies that the level of R&D that maximizes shareholder value is lower in a levered firm, notably when close to distress (see Section 2.3).

We also perform a comparative statics analysis with respect to the model's parameters:

Proposition 1 (Equity Value: Comparative statics)**.** *If $E(p,c) > 0$, equity value is decreasing in the tax rate π, the coupon c, the rate of creative destruction f, and the cost $q(p,\lambda,\theta)$ of performing R&D.*

An increase in these parameters decreases the cash flows to shareholders or the expected lifetime of the firm and, therefore, reduces equity value.

Given the rate of creative destruction f and shareholders' optimal R&D $\{\lambda_t,\theta_t\}_{t\geq 0}$ and default τ_D policies, the debt value $D(p,c)$ is the discounted value of the coupon payments until the time of default plus the liquidation value of assets in default. That is, we have

$$D(p,c) = \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0} e^{-rt} c\,dt + e^{-r(\tau_D\wedge\tau_0)}(1-\alpha)\frac{(1-\pi)P_{\tau_D\wedge\tau_0}}{r+f}\right]. \tag{2.14}$$

Finally, we can also determine the value of an entrant given the rate of creative destruction f. Let τ_e be the time at which the entrant has a breakthrough and can develop its first product lines, which happens with intensity λ_e. The entrant's shareholders choose the R&D intensity and quality that maximize their equity value, which consists of the proceeds once there is a breakthrough minus the tax-deductible R&D costs. That is, we have

$$E^e(f) = \sup_{\lambda_e,\theta_e} \mathbb{E}_0\left[e^{-r\tau_e}V(f,\theta_e) - \int_0^{\tau_e} e^{-rt}(1-\pi)q_e(\lambda_e,\theta_e)\,dt\right] \tag{2.15}$$

$$= \sup_{\lambda_e,\theta_e}\left(\frac{\lambda_e V(f,\theta_e) - (1-\pi)q_e(\lambda_e,\theta_e)}{r+\lambda_e}\right), \tag{2.16}$$

where

$$V(f,\theta_e) = \mathbb{E}^{\theta_e}\left[\sup_{c\geq 0}\left\{E(p_0,c) + (1-\xi)\,D(p_0,c)\right\}\right], \tag{2.17}$$

with $p_0 = \min(x, \bar{p})$ and $x \sim Bin(n, \theta_e)$. As shown by equation (2.17), shareholders select the coupon that maximizes the value of their claim once they know how many product lines their innovative breakthrough generates. Because the debt choice is affected by the number of product lines, the heterogeneity in entrants' R&D outcomes naturally leads to cross-sectional variation in the amount of debt issued, even in the static debt model.

Lastly, an entrepreneur pays the cost $H(1-\pi) > 0$ upon becoming an entrant. The free entry condition then implies that

$$E^e(f) \leq H(1-\pi), \tag{2.18}$$

which becomes an equality when there is a positive mass of entrants. In equilibrium, competition implies that the value of becoming an entrant can never exceed the cost of entry.

2.2.3 Industry equilibrium

This section incorporates the individual-firm decisions into a Schumpeterian industry equilibrium. We look for a Markovian steady state industry equilibrium in which the number of firms and product lines is constant over time. In this industry equilibrium, both incumbents and entrants maximize their equity value. That is, incumbents optimally choose their R&D and default decisions and entrants optimally choose their R&D and capital structure decisions. Given that we look for a Markovian steady state equilibrium, incumbents' optimal policies are a function of the number of product lines they own and the coupon payment on their debt, which is a function of the number of product lines at entry p_0. Entrants' optimal policies are time-homogenous. Finally, the free entry condition ensures that new entrants continue to enter as long as entry is profitable.

Definition 1 (Industry Equilibrium)**.** *The parameters and policies*

$$\Psi^* = \{f^*, c^*(p_0), \lambda^*(p|p_0), \theta^*(p|p_0), p_D^*(p_0), \lambda_e^*, \theta_e^*\} \tag{2.19}$$

are an industry equilibrium if:

1. ***Incumbents:*** *Given the rate of creative destruction f^* and coupon payment $c^*(p_0)$, incumbents level of R&D $(\lambda^*(p|p_0), \theta^*(p|p_0))$ and default decision $p_D^*(p_0)$ maximize shareholder value.*
2. ***Entrants:*** *Given the rate of creative destruction f^*, entrants level of R&D $(\lambda_e^*, \theta_e^*)$ and capital structure upon becoming an incumbent $c^*(p_0)$ maximize shareholder value.*
3. ***Entry:*** *The free entry condition holds:*

$$E^e(f^*) \leq H(1-\pi), \tag{2.20}$$

and the inequality binds when there is creative destruction $f^* > 0$.

Figure 2.3 shows an industry equilibrium in which new product lines replace existing ones and entrants replace incumbents that default and exit the industry. The size of the circles

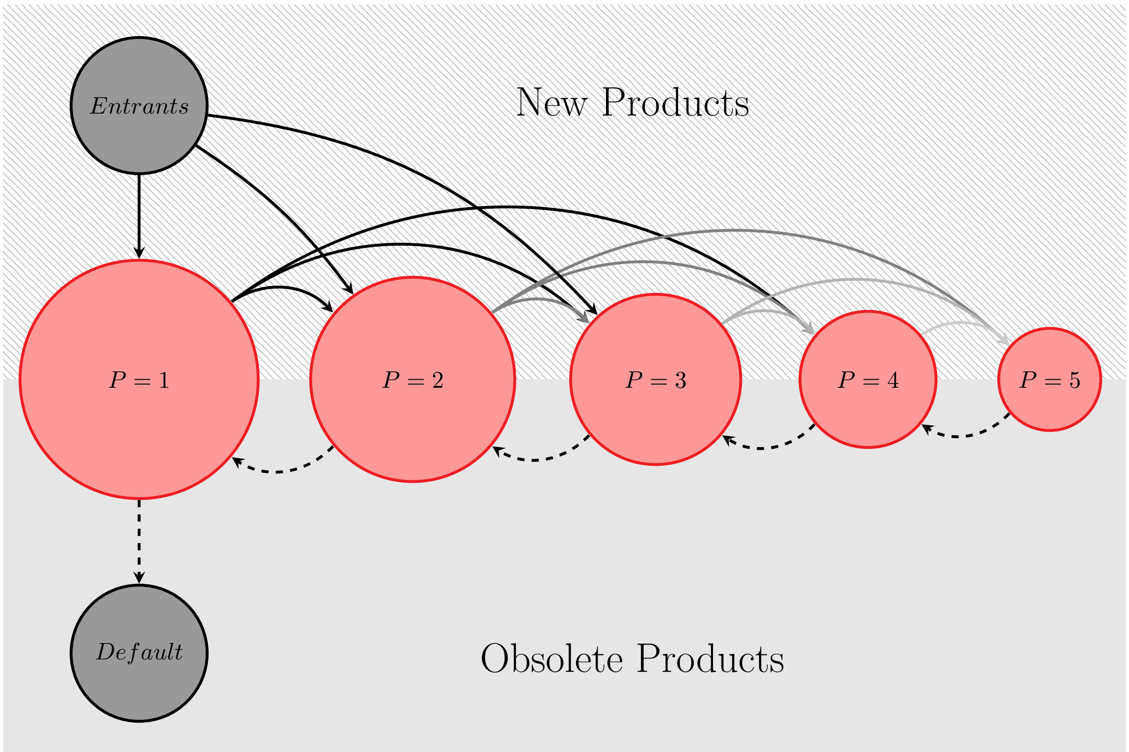


Figure 2.3 – **Steady state equilibrium.** This figure gives an example of a steady state distribution in which there is entry. Appendix B.III derives the steady state firm size distribution.

indicates the mass of firms of each type. In a steady state equilibrium, the size of these circles is constant over time. Incumbents can move up due to innovations, which generate new product lines, and move down due to creative destruction. Because an innovation can generate more than one product line and the number of product lines generated is random, there are multiple upward flows. In this equilibrium, firms exit when they have zero product lines and therefore there is a positive mass of entrants. All industry-wide variables are constant over time, even though individual firms can create new product lines, can lose product lines to competitors, and can even exit. Debt financing affects industry structure and dynamics by changing firms' R&D policies and the rate of creative destruction.

In industry equilibrium, creative destruction arises because of innovations by incumbents and entrants. The level of innovation by incumbents depends on the mass of incumbents in the economy M_i and the flow of innovations they generate per firm $\mathbb{E}[\lambda X] \approx \mathbb{E}[\lambda n\theta]$,[12] where an innovation generates X new product lines and the expectation is taken over the steady state distribution of incumbent firms. The level of innovation by entrants depends on the mass of entrants M_e and the average number of innovations per entrant $\lambda_e n\theta_e$. Altogether,

[12]The equality is approximate since a firm can never have more than $\bar{p}$ product lines.

this implies that

$$f^* = f^i + f^e = \underbrace{M_i \mathbb{E}[\lambda X]}_{Incumbents} + \underbrace{M_e \lambda_e n \theta_e}_{Entrants}, \tag{2.21}$$

where we denote by f^i and f^e the respective contributions of incumbents and entrants to the rate of creative destruction. As this equation makes clear, innovation and investment are driven by both incumbents and entrants in industry equilibrium.

The following theorem establishes equilibrium existence:

Theorem 2 (Equilibrium Existence)**.** *If Assumption 1 in Appendix B.II holds then there exists an industry equilibrium Ψ^*.*

Under additional conditions, we can establish that all equilibria have the same rate of creative destruction f^*:

Proposition 2 (Uniqueness of the Rate of Creative Destruction)**.** *If the debt value is strictly decreasing in the rate of creative destruction f, then all equilibria have the same rate of creative destruction f^*.*

The condition that debt value is strictly decreasing in f ensures, when combined with Proposition 1, that firm value is strictly decreasing in f so that a higher rate of creative destruction makes the firm worse off. Therefore, there can only exist one level of creative destruction for which the free entry condition binds.

In the model, firms that innovate choose to issue debt because debt financing increases firm value (i.e. issuing some debt is positive NPV). By increasing the value of incumbents, debt increases the surplus from entering the industry. This leads to an increase in the entry rate and in innovation, thereby increasing the rate of creative destruction. Notably, we have the following result:

Proposition 3 (Debt Financing and Creative Destruction)**.** *Let $f^*_{No\ Debt}$ be the equilibrium rate of creative destruction in case firms are restricted to have no debt. Then there exists an industry equilibrium with a rate of creative destruction*

$$f^* \geq f^*_{No\ Debt}. \tag{2.22}$$

Proposition 3 demonstrates that debt financing fosters innovation and creative destruction. As we show below, this is the outcome of two opposing forces. First, debt hampers innovation by incumbents due to debt overhang. Second, debt increases the surplus from entering the industry, thereby stimulating entry. As shown by Proposition 3, the latter effect dominates in equilibrium so that debt spurs innovation and growth. Importantly, our results are consistent with the evidence in Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following US banking deregulation. Their empirical analysis shows that US banking reforms—that made bank debt widely available and cheaper by increasing competition—brought growth in both entrepreneurship and business closures.

2.2.4 Refinancing

Internet Appendix A extends the model by allowing firms to dynamically optimize their capital structure as their portfolio of products evolves. Notably, firms that perform well may releverage to exploit the tax benefits of debt. We show in this appendix that all the results derived in this section go through when we allow firms to restructure and demonstrate that there exists an industry equilibrium.

2.3 Model analysis

This section examines the implications of the model for innovation, financing policy, and industry dynamics. To do so, we calibrate the model to match the observed characteristics of innovation and capital structure policies of an average US public firm, using firms' financial data from Compustat and the data on firms' innovation activity from Kogan, Papanikolaou, Seru, and Stoffman (2017).

2.3.1 Parameter values

We first set the interest rate r at 4.2% as in Morellec, Nikolov, and Schürhoff (2012). We choose a tax rate π of 15%, consistent with the estimates of Graham (1996). The bankruptcy cost α is set to 45%, in line with the estimates of Glover (2016). The proportional cost of debt issuance ξ is set to 1.09%, consistent with the evidence on debt underwriting fees in Altinkilic and Hansen (2000). We choose a cost function separable in R&D intensity and quality, as in Akcigit and Kerr (2018). Notably, we assume that

$$q\left(p,\lambda,\theta\right) = p\left(\beta_i\left(\frac{\lambda}{p}\right)^{\frac{1}{\gamma}} + \beta_q\theta^{\frac{1}{\gamma}}\right), \tag{2.23}$$

$$q_E\left(\lambda,\theta\right) = \beta_i\lambda^{\frac{1}{\gamma}} + \beta_q\theta^{\frac{1}{\gamma}}, \tag{2.24}$$

where $\beta_q = 2\beta_i$. This specification captures the notion that investment in innovation quality is more expensive than investment in innovation intensity. To obtain the remaining parameter values, we focus on matching several key moments of interest in the data: the mean and variance of the leverage ratio, the mean of the innovation value per patent, and the turnover rate. Firms' choice of leverage is tightly linked to the parameters governing the R&D cost function β and γ. Furthermore, innovation quantity is directly linked to the maximum number of new products per innovation n. These parameters also determine the cost of performing R&D and are thus informative about the innovation value per patent. Lastly, the entry cost H pins down the turnover rate. Panel A of Table 2.1 summarizes the baseline values of the parameters.

To compute the data counterparts of the model-implied variables, we use the Kogan, Papanikolaou, Seru, and Stoffman (2017) data on patent quantity and value merged with accounting variables from Compustat. We use the sample period 1980 - 2010. Furthermore, we apply standard Compustat filters and remove firms with negative book equity and market-to-book larger than 15. All variables are then winsorized at 1% and 99% in each fiscal year. Panel B of Table 2.1 presents the definitions of the moments of interest in the data as well as

Panel A: Baseline parameter values		
Parameter	**Symbol**	**Value**
Max # products per firm	$\bar{p}$	25
Interest rate	r	4.2%
Tax rate	π	15%
Bankruptcy cost	α	45%
Debt issuance cost	ξ	1.09%
Max # new products per innovation	n	3
After-tax entry cost	$H(1-\pi)$	5
Innovation curvature	γ	0.345
Innovation intensity: scale	β_i	26
Innovation quality: scale	β_q	52
Panel B: Variable definitions		
Moment	**Model**	**Data**
Leverage	$\frac{D(P_t,c_t)}{D(P_t,c_t)+E(P_t,c_t)}$	$\frac{dltt_t+dlc_t}{dltt_t+dlc_t+prcc\_f_t*csho_t}$
Innovation value per patent	$\frac{E(P_t+n,c_t)-E(P_t,c_t)}{nE(P_t,c_t)}$	$\frac{tsm_t}{prcc\_f_t*csho_t*fnpats_t}$
Tax benefit	$\frac{\mathbb{E}[\pi D(P_t,c_t)]}{\mathbb{E}[V(P_t,c_t)]}$	

Table 2.1 – **Baseline parameter values and definitions of moments.**

their model counterparts. We compute the model-implied moments by simulating a balanced panel of $N = 15000$ firms over $T = 15$ years, similar to the ones observed in the data. Firms that exit are replaced with entrants to keep the panel balanced.

2.3.2 Baseline calibration and model-implied moments

We calibrate the model parameters using the static debt version of the model and report the model-implied variables in Table 2.2. The numbers in the table suggest that the model succeeds in replicating the magnitude of observed financing and innovation policies. In particular, the average (market) leverage ratio is equal to 21.47% in the static debt specification and to 28.21% in the dynamic debt specification, both of which are close to the empirical value of 22%. As we will show later on, the relatively low value of leverage in the model is the result of the endogenous rate of creative destruction that disciplines firms' financing policy and the endogenous R&D policy that feeds back in financing decisions. The model also closely matches the variance of leverage, which equals 1.8% in the data and 2.2% in the model, thus generating sizeable variation in financing policy. The average innovation quality per patent is close to the observed value of 0.5%. The model generates a turnover rate of 1.21%, which is close to the observed turnover rate of 1.1% reported by Corbae and D'Erasmo (2017).

Baseline calibration. All values are in %.					
	Leverage Mean	**Leverage Variance**	**Value p.p. Mean**	**Tax benefit**	**Turnover rate**
No debt	0.00	0.00	0.43	0.00	0.54
Static debt	21.47	2.24	0.41	3.21	1.21
Dynamic debt	28.21	2.51	0.40	4.23	0.94

Table 2.2 – **Baseline calibration of the model.**

A comparison between the static debt and no debt case indicates that debt lowers R&D investment and facilitates firm exit by increasing the turnover rate. The results also show that despite the debt induced distortions in R&D, firms benefit substantially from debt financing. The implied tax benefit of debt is around 3.21% of firm value, which is close to the estimates of Korteweg (2010) and Binsbergen, Graham, and Yang (2010).

Table 2.2 reveals that there are quantitative differences between the static and dynamic specifications. In particular, when the maximum number of new product lines that can be developed following an innovation is low, firms initially select a low debt level in the static debt model to avoid default and cannot subsequently readjust. This implies that the average firm has a lower leverage ratio in the static debt model than in the dynamic debt model, in which firms that perform well can issue additional debt. Finally, because the model with dynamic debt generates the same qualitative results as the model with static debt, we focus hereafter on analyzing the predictions of the model with static debt.

Table 2.3 shows how changes in the firm's environment affect outcome variables in the static debt case. The table illustrates that frictions (i.e. the corporate tax rate or the cost of issuing debt) and the quality of the firm's investment opportunity set have important effects on financing decisions and the industry turnover rate (Internet Appendix C reports a similar table for the dynamic debt case). The next subsection provides an in-depth analysis of the relation between debt financing, innovation, and competition.

2.3.3 Debt and innovation

As shown by Proposition 3, our model predicts that debt financing fosters innovation and creative destruction at the aggregate level by increasing the surplus from entering the industry and, therefore, the entry rate. To better understand this mechanism, we turn to analyzing the effects of debt financing on R&D investment by incumbents and entrants.

R&D investment and debt overhang

We first examine the effects of debt financing on investment by incumbents. To do so, we first show how debt affects investment in innovation intensity λ and quality θ depending on incumbent's size, as captured by the number of product lines p. Notably, Figure 2.4 shows the effects of debt overhang by plotting the difference between R&D investment in the static first-best case, in which R&D policy maximizes firm value, and the static debt case, in which

Comparative statics. All values are in %.					
	Leverage Mean	**Leverage Variance**	**Value p.p. Mean**	**Tax benefit**	**Turnover rate**
Max # new products per innovation					
$n=2$	34.42	4.82	0.30	5.16	1.01
$n=3$	21.47	2.24	0.41	3.22	1.21
$n=4$	20.35	1.79	0.47	3.05	1.52
After-tax entry cost					
$H=4$	27.38	2.24	0.37	4.11	2.87
$H=5$	21.47	2.24	0.41	3.22	1.21
$H=6$	20.99	2.22	0.39	3.15	0.71
Innovation cost scale					
$\beta=23$	17.43	1.36	0.51	2.61	1.33
$\beta=26$	21.47	2.24	0.41	3.22	1.21
$\beta=29$	27.64	1.84	0.45	4.15	1.09
Innovation cost curvature					
$\gamma=0.333$	20.91	1.94	0.44	3.14	1.41
$\gamma=0.345$	21.47	2.24	0.41	3.22	1.21
$\gamma=0.357$	27.97	3.68	0.37	4.20	1.24
Tax rate					
$\pi=0.10$	16.26	1.47	0.42	1.63	0.97
$\pi=0.15$	21.47	2.24	0.41	3.22	1.21
$\pi=0.20$	30.45	2.37	0.40	6.09	1.06
Debt issuance cost					
$\xi=0\%$	21.81	2.40	0.41	3.27	1.23
$\xi=1.09\%$	21.47	2.24	0.41	3.22	1.21
$\xi=4.36\%$	21.27	2.13	0.41	3.19	1.16

Table 2.3 – **Comparative statics of selected moments.**

R&D policy maximizes shareholder value and is therefore subject to debt overhang. The first-best case uses the optimal coupon from the static debt case.

Figure 2.4 shows that when investment decisions maximize shareholder value and firms have debt outstanding, firms not only spend less on R&D overall, but also innovate less on each margin. The effects of debt overhang are substantial in the model. Depending on firm size p and leverage, firms invest up to 23% more in innovation intensity and quality in the first-best case compared to the baseline case. This distortion, that is solely due to debt, is especially strong for small firms. These effects tend to become smaller when firm size p increases as debt becomes less risky. As a result, wealth transfers to debtholders due to new

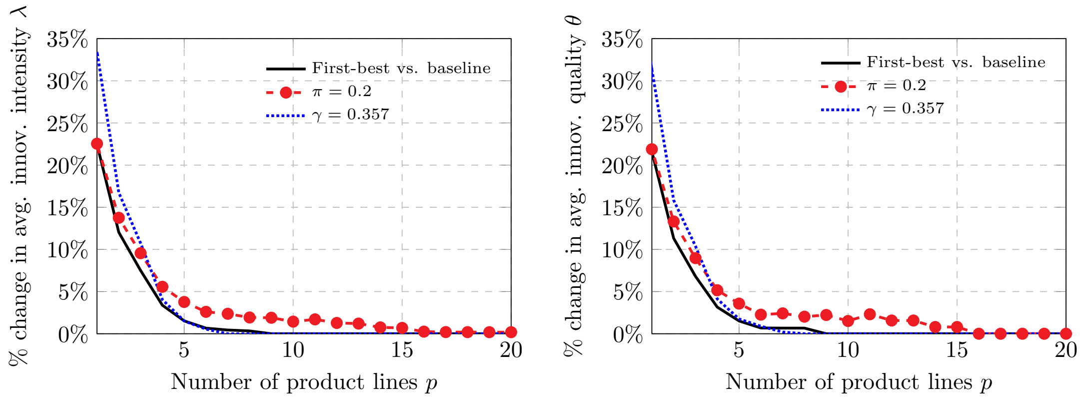


Figure 2.4 – **Debt overhang.** The figure plots the change in innovation intensity and quality by incumbents due to debt overhang as a function of firm size p.

investment are limited and so are the distortions in investment policy due to debt overhang. Figure 2.4 also demonstrates that the magnitude of these effects varies with input parameter values. Distortions in investment are greater when the tax rate is larger or when the quality of the investment opportunity set worsens, as firms adopt higher leverage ratios (see Table 2.3). Overall, the analysis indicates that debt has first-order effects on incumbents' R&D policy, notably for smaller firms close to distress.

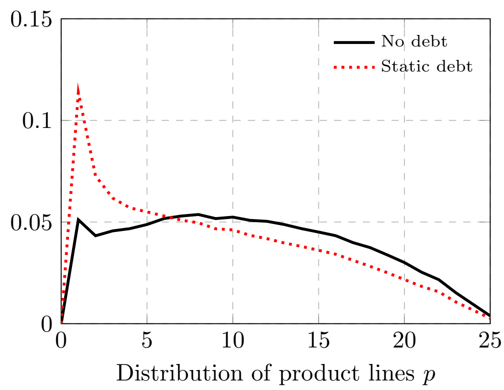


Figure 2.5 – **Distribution of the number of products p.** The figure shows the distribution of the number of products p in the no debt (solid) and static debt (dotted) cases.

Because of its effects on investment, debt financing also has important implications for the size distribution of firms. To illustrate these implications, Figure 2.5 presents this distribution for the no debt and static debt cases. The figure shows that the distribution is positively skewed when firms are allowed to issue debt. This change can be attributed to the higher entry and turnover rates and to debt overhang, which reduces incumbents' incentives to innovate and grow (see Subsection 2.3.3).

Debt and entry

To further characterize the effects of debt financing on innovation, we now examine the effects of debt financing on entry and the change in the contribution of entrants to the rate of creative destruction.[13] The left panel of Figure 2.6 plots the increase in f^e (i.e. $\frac{f^e - f^e_{No\ Debt}}{f^e_{No\ Debt}}$) due to debt financing. In our base case environment, allowing firms to issue debt increases f^e by 161%, which indicates that debt does indeed significantly foster turnover and entry. This increase in the entry rate is greater when the benefits of debt increases are reaches 270% when the tax rate increases to 20%. It is also greater when the cost of debt decreases (i.e. n decreases or γ increases). This increase in f^e due to debt financing is also illustrated by Figure 2.5, in which debt financing increases the positive skew in the size distribution.

As discussed above, a second effect of debt is that it leads to underinvestment by incumbents. To assess the relative magnitudes of these effects, we compare the change in f^e due to debt to the change in the aggregate rate of creative destruction f due to debt:

$$\frac{f^e - f^e_{No\ Debt}}{f - f_{No\ Debt}} \approx 1.43.$$

In our base case environment, about 143% of the increase in the rate of creative destruction f due to debt can be attributed to the increase in the entry rate. Underinvestment by incumbents acts as a balancing force and has a negative effect on the aggregate rate of creative destruction. That is, the net effect of debt financing on the rate of creative destruction results from two large and opposing forces, that partially offset each other in equilibrium.

Lastly, we can examine the effects of debt on firm value by computing the increase in the value of incumbents due to debt financing. The left panel of Figure 2.7 corresponds to the exercise undertaken in the empirical studies of Korteweg (2010) and Binsbergen, Graham, and Yang (2010). It shows that debt financing leads to a significant increase in incumbent firm value *when holding fixed the rate of creative destruction* (i.e applying the policy change to a single firm by assuming that $f = f^*_{No\ Debt}$). This increase is larger when firms have greater incentives to issue debt, due e.g. to a higher tax rate or to a higher cost of innovation. This increase in the value of incumbents leads to an increase in the benefits of entry and, therefore, to an increase in the entry rate and in the rate of creative destruction.

The right panel of Figure 2.7 shows the change in the value of incumbents due to debt financing when the rate of creative destruction is endogenized. In equilibrium, debt financing leads to an increase in the rate of creative destruction, which is such that the entry condition (given by equation (2.18)) binds. This leads to a dampening of the effects of debt financing on firm value. In our base case calibration, the entry condition binds for low p since the number of new products that can be developed following an innovation is low ($n = 3$).

[13] The number of product lines that entrants generate every period is given by the mass of incumbent firms times the turnover rate of incumbent firms times the expected number of product lines an innovation by an entrant generates, conditional on the entrant generating at least one product line.

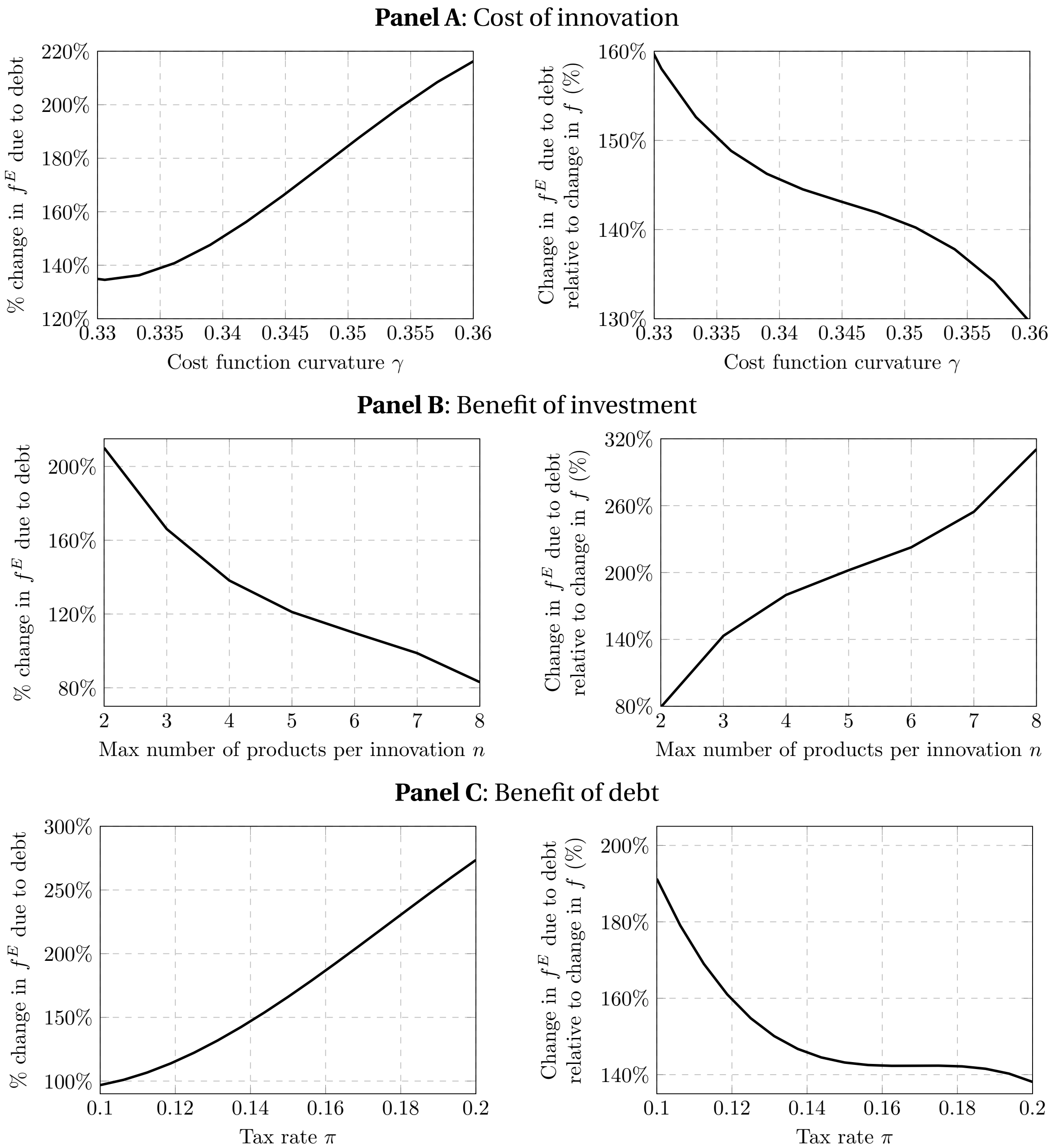


Figure 2.6 – **The effects of debt financing on innovation by entrants.** The left figures show the effects of using debt financing on innovation by entrants. The right figures show entrants' contribution to the aggregate increase in creative destruction due to debt financing. The comparative statics are smoothed using a third-order polynomial.

Financing policy and investment opportunities

Shareholders choose a leverage ratio that balances the marginal benefits and marginal costs of debt. Interest expenses on debt are tax deductible, which gives shareholders an incentive to issue debt. The presence of debt gives shareholders an option to default, which is costly. Debt also reduces the benefits of innovation to shareholders because part of the benefits of

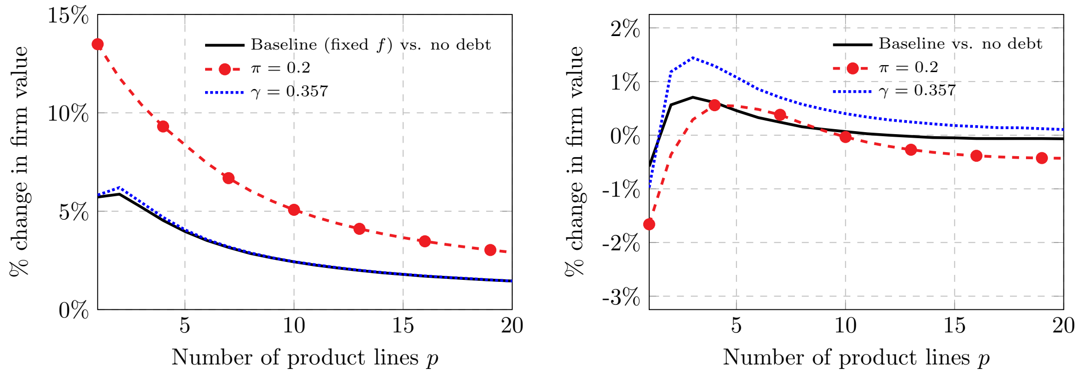


Figure 2.7 – **Net benefits of debt.** The figure plots the relative change in firm value due to debt financing with exogenous rate of creative destruction (left panel) and endogenous rate of creative destruction (right panel).

investment accrue to creditors (due to the fact that debt becomes less risky). Therefore, debt distorts innovation incentives and leads to underinvestment by incumbents. These distortions in innovation policy feed back into firms' cash flow dynamics which influences the optimal leverage choice. Investment and financing policy are therefore jointly determined.

To illustrate these mechanisms, Figure 2.8 shows how leverage is affected by several key parameters describing the quality of the firms' investment opportunities: The cost function curvature γ, the cost function level β_i, the maximum number of new products per innovation n, and the maximum number of product lines $\bar{p}$.

Figure 2.8 shows that higher costs of innovation lower individual firms' incentives to innovate, so that a smaller amount of their value comes from growth opportunities. In response, firms increase financial leverage. Figure 2.8 also shows that when each innovation has the potential of creating more product lines (as n gets larger), the potential costs of debt overhang are larger and firms issue less debt. The effect of changing $\bar{p}$ on leverage is more muted. This is due to the fact that $\bar{p}$ has been chosen large enough so that its effects on firm policies are limited. Overall, these results show that investment decisions feed back into financing choices. Our results are consistent with evidence in Smith and Watts (1992) and Barclay and Smith (1995) that firms with better growth opportunities adopt lower leverage ratios.

2.3.4 Industry equilibrium

In equilibrium, the industry rate of creative destruction and firms' capital structure decisions are jointly and endogenously determined. To better understand the underlying economic mechanism, Panel A of Figure 2.9 shows how changing the cost of innovation γ affects equilibrium quantities. The top left graph of Panel A shows that increasing the cost of innovation γ lowers firms' investment in R&D. Interestingly, when f is fixed, the drop in R&D is much stronger as it does not incorporate the feedback from the industry. Because firms face worse growth opportunities when the cost of R&D investment is high, much of their value is attributable to assets in place. As a result, they increase leverage, as shown by the top right panel of the figure. The effect is again weaker in industry equilibrium as the effects of γ on R&D

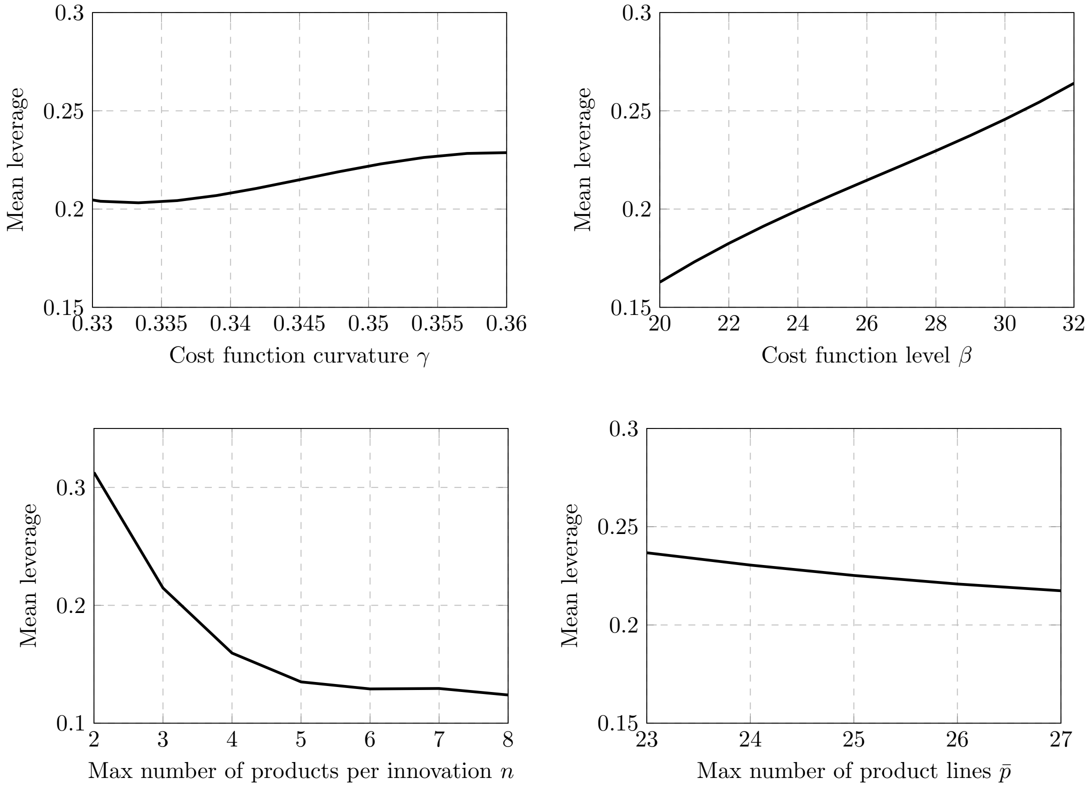
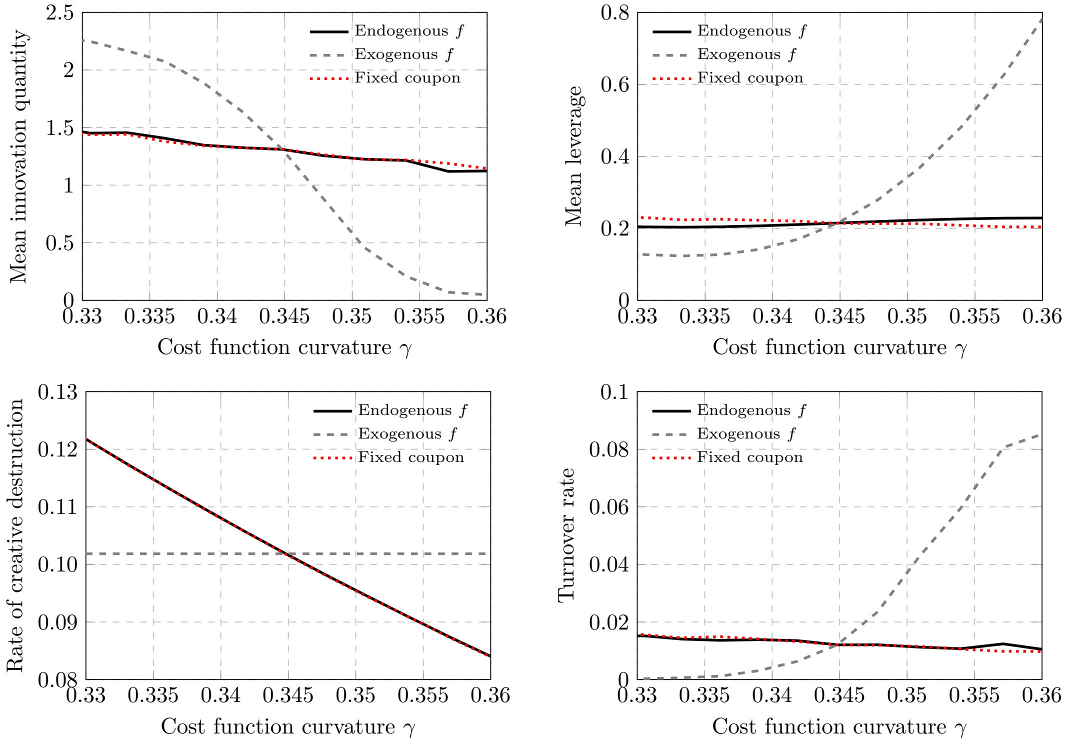


Figure 2.8 – **Investment opportunities and financing policy.** The figure shows the effects of the quality of investment opportunities on financing decisions. The comparative statics are smoothed using a third-order polynomial.

get muted. The drop in innovation quantity and the increase in leverage in turn feedback into the equilibrium rate of creative destruction, as illustrated in the bottom left panel of the figure. In equilibrium, the effect on innovation quantity is first order, leading to a negative relation between γ and f. This decrease in the rate of creative destruction—and therefore the longer expected productive life of each product line—spurs innovation, partly offsetting the higher innovation costs (top left graph). Lastly, as illustrated by the bottom right graph, these mechanisms translate to a lower turnover rate as γ increases. By contrast, in the single-firm model in which f is fixed, the sharp increase in leverage leads to a sharp increase in the turnover rate.

Panel B of Figure 2.9 shows the effects of varying the maximum number of new products per innovation on outcome variables. There again, endogenizing the rate of creative destruction has first order effects on model predictions. For instance the bottom right figure of Panel B shows that when f is endogenous, increasing n increases the turnover rate due to the large increase in the rate of creative destruction (and even more so when the coupon is fixed). By contrast, when f is exogenous, increasing n decreases the turnover rate. Again this is due to the fact that in the latter case, an increase in n only leads to a decrease in leverage. Another result illustrated by the top right figure of Panel B is that an increase in n leads to a decrease in

Panel A: Cost of innovation



Panel B: Quality of investment opportunities

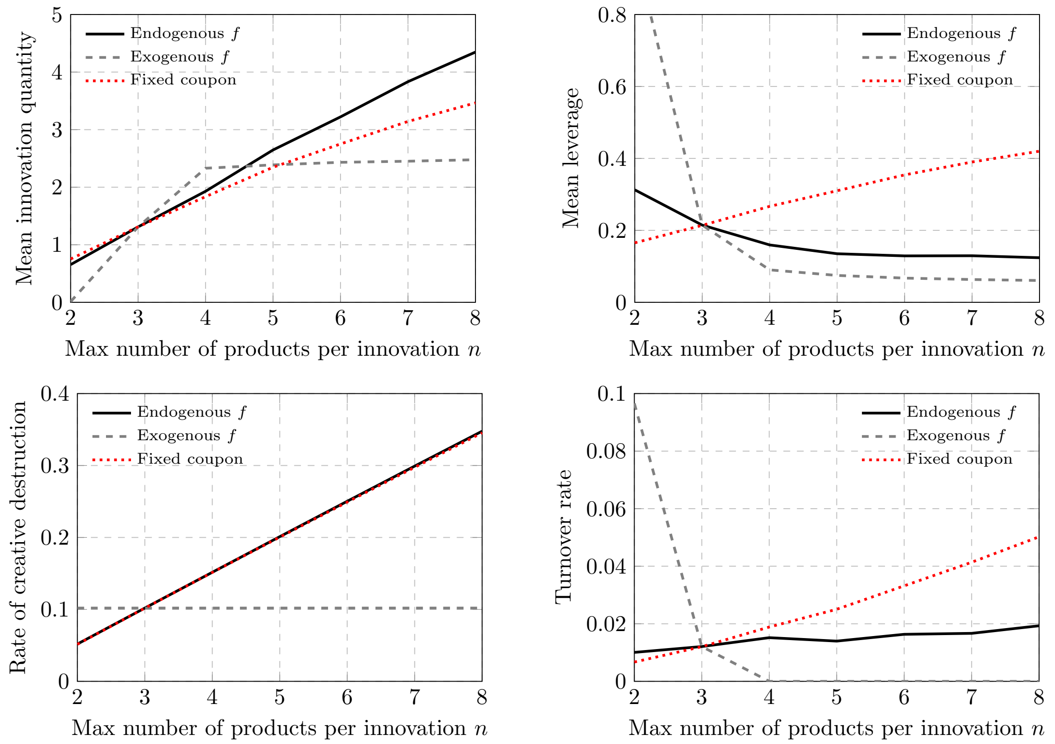


Figure 2.9 – **The effects of the endogenous rate of creative destruction.** The figure shows the effects of changing the cost of innovation and the quality of investment opportunities on outcome variables in the single-firm model and in industry equilibrium. The comparative statics for leverage are smoothed using a third-order polynomial

leverage when financing decisions are endogenous. By contrast, with a fixed coupon, leverage increases with n because the higher rate of creative destruction impairs firm value.

2.4 General equilibrium

This section closes the model in general equilibrium to endogenize the growth rate, labor demand, and the interest rate in the economy. The general equilibrium setup builds on Klette and Kortum (2004). We study a stationary equilibrium with a balanced growth path. This subsection describes the key features of the general equilibrium framework. Appendix B.IV provides a detailed and formal description.

There is a representative household with logarithmic preferences who perfectly elastically supplies labor at a fixed wage. Entrants and incumbents use labor to perform R&D and produce goods. All costs in the model come in the form of labor costs, and therefore aggregate production equals aggregate consumption.

An innovation improves a product's production technology and therefore increase aggregate production and consumption. Each firm uses one unit of labor for each product line. Given the representative agent's preferences, this setup implies that a firm's profits per product line only depend on the wage rate, which allow us to use the industry equilibrium framework we developed before. As a consequence, all the results derived in industry equilibrium still hold in general equilibrium.

This also implies that Proposition 3, which shows that creative destruction is higher in an industry equilibrium with debt, still holds true in general equilibrium. As we show in Appendix B.IV, this higher rate of creative destruction implies that the growth rate is also higher in the presence of debt. The following proposition formalizes this result.

Proposition 4 (Debt Financing and Growth)**.** *Let $g^*_{No\ Debt}$ be the equilibrium growth rate in case firms are restricted to have no debt. There exists an equilibrium with growth rate*

$$g^* \geq g^*_{No\ Debt}. \tag{2.25}$$

This result follows directly from Propostion 3 and the fact that, as we show in Appendix B.IV, the growth is proportional to the rate of creative destruction f^*. When firms are allowed to issue debt, levered incumbents face debt overhang which lowers investment. But the possibility to issue debt also increases firm value, which spurs entry and therefore innovation and growth.

2.5 Conclusion

This paper investigates the relation between debt financing, innovation, and growth in a Schumpeterian growth model in which firms' dynamic R&D and financing choices are jointly and endogenously determined. In the model, each firm's R&D policy influences its risk profile, which feeds back in its capital structure decisions. In addition, a levered firm's R&D policy can be altered by its financing decisions, due to conflicts of interest between shareholders and debtholders. As a result, financing and investment are intertwined at the firm level.

We embed the single-firm model into a Schumpeterian industry equilibrium that endogenizes the rate of creative destruction and derive a steady state equilibrium in which innovating firms introduce new products that replace existing ones, and new entrants replace exiting incumbents. In this equilibrium, firms' R&D and capital structure decisions affect the aggregate level of creative destruction, which in turn feeds back in their policy choices.

Based on the resulting equilibrium, the paper delivers several novel results. First, we show that while debt hampers innovation by incumbents due to debt overhang, it also stimulates entry, thereby fostering innovation and growth at the aggregate level. Second, we show that debt financing has large effects on firm entry, firm turnover, and industry structure and evolution. Third, we show that our model predicts substantial intra-industry variation in leverage and innovation, in line with the empirical evidence.

3 Fundamental Risk and Capital Structure

3.1 Introduction

The negative relationship between risk and leverage, illustrated in Figure 3.1, is one of the most well-established phenomena in finance. The association is robust in the data and existing dynamic models of capital structure, starting with Leland (1994), provide intuitive theoretical underpinning of how risk affects firm's debt policy. Even practitioners acknowledge that risk plays an important role in shaping firms' capital structure, as according to Graham and Harvey (2001) it constitutes the third most important factor of debt issuance decisions.

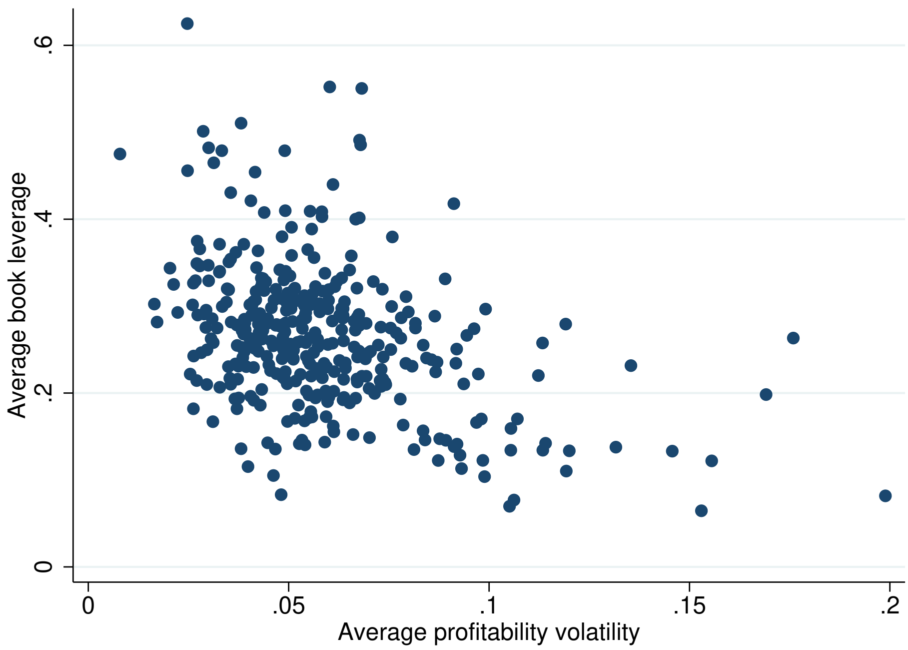


Figure 3.1 – **The risk-leverage trade-off.** The graph presents the relationship between the average annual book leverage and the average operating profitability volatility. Each point corresponds to a different 4-digit SIC industry. Definitions of variables are provided in Appendix C.II. All variables are winsorized at 1% and 99%.

Risk, however, is inherently unobservable. This is why empirical researchers have to resort to using proxies which typically focus on a single dimension of firms' fundamental risk, such as cash flow volatility. While there is little doubt that this characteristic plays an important role in determining capital structure, it may not be able to capture more in-depth features of firms' true riskiness. For example, it could express the degree of total risk in a firm's operations while missing out on other important determinants of cash flow dynamics, such as their exposure to aggregate market conditions or the structure of profits. These claims are not unfounded, for instance Schwert and Strebulaev (2014) document that asset beta provides additional explanatory power for leverage above and beyond the effect of total volatility, which shows that further dissecting firms' cash flow process could provide additional insights concerning leverage variation in the data. Figure 3.1 suggests that while the negative correlation between risk and leverage holds on average, there also exists a large degree of dispersion in the data. Firms in industries with similar risk adopt markedly different leverage and firms in industries with similar leverage vary in their riskiness. [1]

In this paper, I argue that using a more general notion of risk helps explain the dispersion in firms' capital structure policies. In particular, the nature of risk faced by firms should wield influence on its leverage policy through its effect on the firm's cash flows and investment. To make this claim, I develop a dynamic capital structure model in which the firm's cash flows can be exposed to both transitory and persistent shocks. The separation into transitory and persistent shocks represents the fact that some firms may experience frequent but transient cash flow shocks that influence their long-run decisions in a limited way, while others could only face infrequent disturbances, but with permanent impact on cash flows. To capture the distinction, the shocks are modeled using a stationary and a non-stationary process.[2] The firm's fundamental risk can then be directly linked to the composition of cash flows and described by their volatility as well as persistence.

The main implication of the model is that leverage is a decreasing function of not only total volatility, but also of persistent shock exposure for the same level of total risk. The intuition underpinning this finding results from the fact that leverage choice is closely related to the firm's investment decisions, which are, in turn, highly sensitive to persistent shock realizations. The firm wants to invest more when experiencing a persistent rather than a transitory shock, as its long-term effects on cash flows are lasting. Therefore, the firm preserves more debt capacity when investment opportunities are more persistent, as it values financial flexibility (e.g., DeAngelo, DeAngelo, and Whited, 2011). The irreversibility of the persistent shock reinforces this mechanism further. The higher the firm's exposure to persistent shocks, the more conservative leverage ratio it adopts, given that it wants to avoid at all cost the prospect of having to forgo valuable long-lasting investment opportunities if they have to be financed with costly external equity.

[1] Appendix C.II provides more empirical evidence on the relationship between risk and leverage.

[2] While I do not take a particular stance on what these shocks may represent, the literature typically associates persistent shocks with events that affect long-run prospects of the firm, such as changes to production technology, human capital, tastes. Transitory shocks, on the other hand, subside over time and can result from demand or supply shocks, regulatory changes requiring real adjustments, changes to production cost structure, machine failure or natural disasters.

The theoretical results have important implications for empirical research. For example, unlike standard dynamic capital structure models in which the firm is exposed to a single transitory shock, the model rationalizes the mismatch in the risk-leverage relationship in Figure 3.1 by relating the observed dispersion to differences in risk composition. In particular, the decomposition of fundamental volatility allows to obtain different optimal leverage ratios for firms with the same level of total volatility. Similarly, the model generates firms with high profit persistence even when the composition of their fundamental persistence, which also affects leverage choice, differs.

Furthermore, the model explains why firms tend to have persistent cash flows despite varying substantially in other dimensions. Taking the empirical evidence by face value, one could infer that profit persistence has no bearing on investment, leverage or other firm characteristics, which is strongly at odds with model evidence predicting a robust effect of changing shock persistence on these characteristics. In the model, the overall persistence of firm's cash flow process can be very high if it contains a small persistent part. Therefore, two firms with similar observable dynamic properties of cash flows may adopt markedly different leverage and investment policies, depending on the true dynamics of their profits. To this end, the model gives rationale why firms typically have highly persistent profits and why the estimates profit persistence only provide limited explanatory power for explaining variation in capital structure and other firm policies if we do not control for the overall cash flow composition.

Finally, the model shows that the firm's risk composition has a significant impact on the dispersion of leverage as well as other characteristics of capital structure such as volatility and persistence. The properties describing the overall composition of cash flow dynamics are therefore bound to provide extra explanatory power above and beyond total volatility in explaining capital structure variation in the data, not only in leverage level but also in its higher-order moments.

3.1.1 Related literature

While the empirical and theoretical literature on capital structure is vast, only a handful of studies deal with the implications of transitory and permanent shocks for corporate policies and even less consider their effects on the firm's leverage choice.[3].

Gorbenko and Strebulaev (2010) study financing policy in a model where firms can be exposed to both types of shocks and show that firms with more transitory shock exposure adopt conservative leverage policies, but the shocks interact additively. The shock separation results in an imperfectly correlated firm value and cash flow as well as between earnings and asset volatility. Décamps, Gryglewicz, Morellec, and Villeneuve (2016) extend this analysis by considering the effects of transitory and permanent shocks on investment, financing and

[3] In general, such decomposition of shocks dates back to Blundell and Preston (1998) who use the permanent income hypothesis to study consumption dynamics. More generally, models with a stationary and a non-stationary shock are popular e.g. in asset pricing, household finance or labor economics. Some (by no means exhaustive) examples include Kaltenbrunner and Lochstoer (2010), Adrian and Rosenberg (2008), Guiso, Pistaferri, and Schivardi (2005) or Gourio (2012)

liquidity policies. In their model, financing constraints increase the cash-flow sensitivity of cash and firms prefer to hoard liquid assets as their exposure to transitory shocks increases. Even though these papers do address the relationship between shock exposure and capital structure, they do not provide an explicit link with investment policy, which, as this paper shows, is the most important channel affecting the firm's leverage through its risk composition.

Other papers investigate the empirical implications of separating the shocks. Chang, Dasgupta, Wong, and Yao (2014) use macroeconometric filters to decompose firm-specific cash flow into trend and cycle components, which can be interpreted as persistent and transitory parts of the firm's cash flow. Their analysis implies that a one standard deviation shock to the persistent component of cash flow is associated with a 3.6% increase in investment rate and 2.5% decrease in book leverage, these effects are approximately 50% larger than the ones resulting from a shock to the transitory component. However, most of their analysis focuses on the investment-cash flow sensitivity and financial constraints but not the effect of shock composition. Byun, Polkovnichenko, and Rebello (2016) propose a dynamic investment model with cash in which the firm is subject to a transitory and an idiosyncratic shock and show that each has different implications for the dynamics of savings and investment. Their model contains no debt, however, and all shock processes are stationary, which is different from the setting considered in this paper. Lastly, and perhaps most importantly, this paper is closely related to the work of Gourio (2008), who structurally estimates a dynamic neoclassical model of investment with persistent and transitory shocks and shows that investment policy reacts much stronger to persistent shocks. While the model in this paper can replicate these findings, it also yields further predictions regarding the impact of shocks on capital structure.

Finally, this paper shares many features with the discrete-time neoclassical dynamic investment models of capital structure such as Hennessy and Whited (2005, 2007) or DeAngelo, DeAngelo, and Whited (2011), for example investment is endogenous and debt is risk free and subject to a collateral constraint.

3.2 Model

In this section, I develop a discrete-time dynamic model in which a firm exposed to transitory and persistent shocks makes optimal financing and investment decisions.

3.2.1 Model setup

Time is discrete and the time horizon is infinite. The firm is governed by risk-neutral firm managers who discounting cash flows at rate r. Their incentives are fully aligned with shareholders. The firm uses capital K to produce output and the per-period profit function $\Pi(K, Z)$ depends on the firm's capital K as well as profitability shock Z. The profit function is continuous, concave and satisfies the Inada conditions. The concavity of the profit function reflects the decreasing returns to scale faced by the firm. I specify that the firm's after-tax profits are equal to

$$\Pi(K, Z) = (1-\tau) Z K^{\theta}, \tag{3.1}$$

where τ is the corporate tax rate and θ is the curvature of the firm's production function. The firm's capital stock K depreciates over time and thus evolves according to

$$K' = I + (1-\delta)K \tag{3.2}$$

with depreciation rate $\delta \in (0,1)$. When adjusting the capital stock, the firm incurs capital adjustment costs, which are convex and defined as

$$\Psi(K,K') = \psi/2\,(I/K)^2\,K. \tag{3.3}$$

The firm is also able to write off a part of its tax bill due to depreciation tax credit, which amounts to $\tau\delta K$.

The profitability shock Z consists of two components Z_P and Z_T corresponding to permanent and transitory parts, respectively. The law of motion for Z is multiplicative in both shocks and given by

$$\begin{aligned} Z = Z_P \times Z_T \iff \log(Z) &= \log(Z_P) + \log(Z_T) \\ \log(Z'_P) &= \log(Z_P) + \sigma_P \varepsilon'_P \\ \log(Z'_T) &= \rho \log(Z_T) + \sigma_T \varepsilon'_T, \end{aligned} \tag{3.4}$$

where each ε'_i is iid standard normal and $\varepsilon'_T \perp \varepsilon'_P$. The shock Z_P takes values in a compact set $[\underline{Z}_P, \overline{Z}_P]$. The choice of shocks interacting multiplicatively, similar to Décamps, Gryglewicz, Morellec, and Villeneuve (2016), is motivated by their effect on firms of different sizes. Intuitively, small firms are worse off when they face additive shocks, because large firms are less sensitive to shock realizations, whereas shock effects are proportional to firm size when they are modeled multiplicatively, as in this paper.

The firm's financing choices consist of internal funds (cash and current profits), costly external equity and risk-free debt. The stock of net debt P is defined as the difference between the stock of debt (D) and the stock of cash (C). This implies that we can write $D = \max(P,0)$ and $C = -\min(P,0)$ and thus $P = D - C$. Debt takes form of a riskless perpetual bond incurring taxable interest at a rate $r(1-\tau)$. The firm may also choose to hoard liquid assets to save on the costs of external equity issuance or to avoid depleting its debt capacity. However, the interest the firm earns on its cash balance is equal to $r(1-\tau)$, meaning that liquid assets earn a lower rate of return than the risk-free rate. Finally, as in DeAngelo, DeAngelo, and Whited (2011) and Hennessy and Whited (2005), the stock of debt is subject to a collateral constraint proportional to the firm's capital stock

$$P' \leq \omega K',\ \omega \in [0,1]. \tag{3.5}$$

This setup implies the following sources and uses of funds constraint defining the firm's cash flow, which result in external equity issuance (if negative) or distributions (if positive)

$$\begin{aligned}E(K,K',P,P',Z_T,Z_P) = (1-\tau)Z_T Z_P K^{\theta} + \tau\delta K \\ -[K'-(1-\delta)K] - \psi/2\left[(K'-(1-\delta)K)/K\right]^2 K \\ +P' - [1+r(1-\tau)]P.\end{aligned} \tag{3.6}$$

External equity issuance is costly and subject to linear issuance costs, resulting from the underwriting costs or the adverse selection problem of Myers and Majluf (1984). The cost of raising external equity is modeled in reduced form as in Gomes (2001) or Hennessy and Whited (2005, 2007):

$$\Phi(E(\cdot)) = \left[\eta E(\cdot)\right]\mathbb{1}_{E(\cdot)<0}. \tag{3.7}$$

3.2.2 Solution method

The firm's problem is to maximize the present value of its future cash flows by choosing the investment and debt policies subject to equity issuance cost $\Phi(\cdot)$

$$\begin{aligned}V(K_0,P_0,Z_{0,T},Z_{0,P}) = \max_{\{K_{t+1},P_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0\Bigg[\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \Big(E(K_t,K_{t+1},P_t,P_{t+1},Z_{t+1,T},Z_{t+1,P}) \\ +\Phi(E(K_t,K_{t+1},P_t,P_{t+1},Z_{t+1,T},Z_{t+1,P}))\Big)\Bigg]\end{aligned} \tag{3.8}$$

The Bellman equation for the problem and the laws of motions of the shocks can be written as

$$\begin{aligned}V(K,P,Z_T,Z_P) = \max_{K',P'}\Big\{E(K,K',P,P',Z_T,Z_P) + \Phi(E(K,K',P,P',Z_T,Z_P)) \\ + \frac{1}{1+r}\mathbb{E}_{Z'_T,Z'_P}\left[V(K',P',Z'_T,Z'_P)\right]\Big\}, \\ \text{s.t. } P' \le \omega K', \\ K' = I + (1-\delta)K, \\ \log(Z'_P) = \log(Z_P) + \sigma_P\varepsilon'_P, \\ \log(Z'_T) = \rho\log(Z_T) + \sigma_T\varepsilon'_T.\end{aligned} \tag{3.9}$$

The numerical solution of the model is described in detail in Appendix C.I.

3.2.3 Optimal financing policy

In this subsection I provide further intuition underpinning the model by analyzing its optimality conditions. These follow closely Hennessy and Whited (2005, 2007) and DeAngelo, DeAngelo, and Whited (2011) given that the model belongs to the same class of discrete time dynamic capital structure models. For simplicity, I assume that V is differentiable. I denote the Lagrange multiplier accompanying the collateral constraint as ξ'.

The optimal financing policy, obtained by taking the first-order condition of the Bellman equation with respect to P', satisfies the following equality

$$1+\eta\mathbb{1}_{E(\cdot)<0}=\xi'-\frac{1}{1+r}\mathbb{E}_{Z'_T,Z'_P}\left[V_2(K',P',Z'_T,Z'_P)\right], \tag{3.10}$$

with $V_2(\cdot)$ being the derivative of the value function with respect to the second argument. The left-hand side contains the marginal benefit of debt financing. If the firm has a financing deficit ($E(\cdot)<0$), then an extra dollar of debt financing allows to avoid costly external equity financing today: the benefit of the extra dollar of debt is thus $1+\eta$. If the firm is running a financing surplus, then using an extra dollar of debt means that it can distribute an extra dollar to its shareholders and thus the benefit of debt is 1. To gain more intuition about the expected marginal costs of debt on the right-hand side, we may expand the first-order condition further by using the corresponding envelope condition for P.

The first-order condition can thus be expressed as

$$1+\eta\mathbb{1}_{E(\cdot)<0}=\xi'+\frac{[1+r(1-\tau)]}{1+r}\mathbb{E}_{Z'_T,Z'_P}\left[(1+\eta\mathbb{1}_{E'(\cdot)<0})\right]. \tag{3.11}$$

The right-hand side can be seen as the expected principal and interest on debt that must be repaid tomorrow. The term $\eta\mathbb{1}_{E'(\cdot)<0}$ suggests that the marginal cost of debt is higher when the firm is expected to run financing deficit next period: raising an extra dollar of debt today implies debt repayment tomorrow and therefore a higher probability of having to issue costly external equity. The presence of the Lagrange multiplier ξ' implies that the marginal cost of debt is also higher when the firm expects to exhaust its debt capacity next period: choosing a high level of debt today results in less financial flexibility in the future, thus the cost of borrowing today includes the value lost when the firm loses the option to borrow in the future. The equation also shows that the firm's financial and real policies are deeply intertwined: if any given firm's characteristic makes it invest more at optimum, it will also imply that the firm will want to preserve its debt capacity right now. This feature is particularly important given the effect of the persistent shock Z_P on investment. This channel will be thoroughly investigated in the following section.

3.3 Analysis

In this section I analyze the main implications of the model for the relationship between risk and capital structure. I focus on two characteristics of the fundamental risk: volatility and persistence. In particular, I highlight how the composition of the firm's fundamental risk affects its capital structure characteristics and provide a thorough analysis of all the parameters describing the firm's fundamental risk resulting from its cash flows. I also consider how analyzing risk composition helps explain the observed heterogeneity in corporate policies. Finally, I discuss whether model-implied moments are informative about the parameters governing the firm's risk composition.

3.3.1 Model calibration

I calibrate the model to build intuition about the interactions between the nature of risk faced by the firm and model-implied moments. As I do not want to target any particular moments and since the model belongs to the same class of models as Hennessy and Whited (2005, 2007) and DeAngelo, DeAngelo, and Whited (2011), I set the parameter values close to the estimates resulting from these papers, given that they constitute a plausible starting point. All parameter values are summarized in Table 3.1. In particular, I set the interest rate r at 0.02, the curvature of the profit function θ at 0.75, the corporate tax rate τ at the statutory rate of 35%, the depreciation rate δ at 0.15, the capital adjustment cost parameter ψ at 0.1 and the external financing cost η at 0.15. The collateral constraint parameter ω is set at 0.6, implying that firms cannot raise more than 60% of their concurrent capital value as debt. This value of the collateral constraint parameter does not appear to be restrictive given than the 95th percentile of firm-level leverage distribution is 0.65 and 0.49 for industry-level data and their net leverage counterparts are 0.59 and 0.4, respectively.

The parameters driving the shock processes used in this exercise were chosen such that the total volatility of the shocks is equal to a value from the interval 0.15–0.35, which is close to the estimates from Nikolov and Whited (2014) and between the relatively high estimates from DeAngelo, DeAngelo, and Whited (2011) and the lower estimates such as those in Hennessy and Whited (2005, 2007) or Riddick and Whited (2009). For the assumed parametrization, any level of total volatility below the lower bound of the interval produces leverage ratios equal to the collateral constraint. Persistent shock volatility varies between 0.00 and 0.05; the upper end of the interval is close to the value of 0.07 in Gourio (2008).[4] Finally, the persistence of the transitory shock varies between 0.00 and 0.80.

Interest rate	r	0.02
Corporate tax rate	τ	0.35
Production function curvature	θ	0.75
Capital depreciation rate	δ	0.15
Convex capital adjustment cost	ψ	0.10
External equity issuance cost	η	0.15
Collateral constraint	ω	0.60
Persistence of transitory shock Z_T	ρ	0.00–0.80
Total volatility	σ	0.15–0.35
Volatility of persistent shock Z_P	σ_P	0.00–0.05

Table 3.1 – **Baseline parameters used in the calibration of the model.** The persistent shock volatility is implied by the equality $\sigma = \sqrt{\sigma_T^2 + \sigma_P^2}$.

[4]As explained in Appendix C.I, it is not possible to solve the model for an arbitrary value of persistent shock volatility σ_P, as it is closely related to θ, whose higher value limits the plausible range of σ_P. Therefore I only consider 'small' values which are nevertheless consistent with extant literature and intuition concerning persistent shocks.

3.3.2 The fundamental volatility channel

In the model, fundamental risk affects leverage primarily through its effect on investment policy. One dimension of the firm's fundamental risk is its fundamental volatility. High total volatility implies that there is a higher chance that large investment is optimal, so the firm preserves its debt capacity as it places a higher value on its option to borrow to fund higher investment. On the other hand, low volatility firms have more predictable cash flows, thus they do not value preserving their debt capacity as much to address their funding needs and adopt higher leverage. The first main channel through which the nature of the firm's risk affects its capital structure is related to the composition of its fundamental volatility, which goes beyond the effect of total volatility. This is because persistent shocks reinforce the risk-leverage trade-off by increasing the size of investment outlays and making the profitability of investment more persistent. These effects result in firms placing even higher value on their ability to borrow, which further reduces their optimal leverage ratios. Importantly, firms more exposed to persistent shocks not only use less debt financing, but also more internal funds. Furthermore, as suggested by the impulse response functions, firms spread out their investment outlays over time following a positive realization of a persistent shock, which reduces their need to use external finance even further.

Indeed, higher persistent shock exposure results in a lower optimal leverage for the same level of total volatility. Figure 3.2 illustrates the negative association between persistent shock exposure and leverage for different levels of total volatility σ. It documents that the relationship between volatility composition and leverage crucially depends on total volatility. For high values of σ, the firm's exposure to persistent shock is relatively small and increasing it further has muted effect on the firm's debt policy. However, when the firm's cash flow process contains a relatively larger persistent part, then its leverage is sensitive to changing the volatility composition. Even firms with very low total volatility of $\sigma = 0.15$, which otherwise would lever up to their collateral constraint, prefer to substantially decrease their leverage ratio when increasing the importance of the persistent component in their cash flow process. This observation provides an alternative explanation for the long-standing debt conservatism puzzle, as shock decomposition is not an additional financing friction, but merely allows for a more flexible definition of firms' cash flow process.

Another implication of Figure 3.2 is that the one-to-one link between total volatility and leverage, present in standard capital structure models, is broken. In other words, while extant models are able to explain the values in the graph when $\sigma_P = 0$, they are unable to generate firms with the same optimal leverage but different total volatility or firms with distinctive debt ratios but the same total volatility. Both of these cases can be obtained in the model, which reinforces the claim that volatility composition may be able to explain a portion of the dispersion in the risk-leverage relationship illustrated in Figure 3.1.

Fundamental volatility and leverage dynamics

Volatility composition also has important implications for moments describing capital structure dynamics such as leverage variation (represented by its standard deviation) or leverage persistence (captured by its first-order autocorrelation). As argued by Baranchuk and Xu

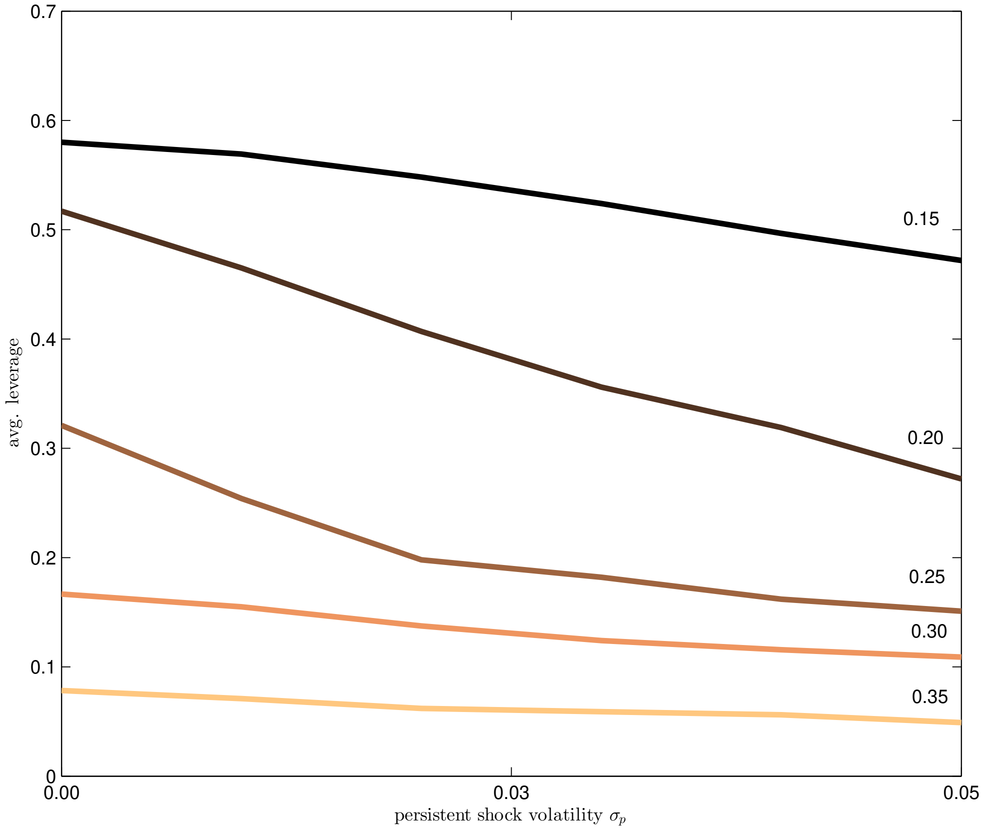


Figure 3.2 – **Fundamental volatility and average leverage.** The figure contains the model-implied average leverage as a function of total volatility σ (number above each line) and persistent risk volatility σ_P. The transitory shock volatility σ_T was computed such that the total volatility was constant on each line. In the graph, the transitory shock persistence ρ is set at 0.6.

(2007, 2011), these moments appear to vary at least as much as average debt ratios themselves. Standard leverage factors fail to explain their variation in the data and have even lower explanatory power than for average debt ratios. Figure 3.3 shows how these characteristics differ depending on the firm's exposure to persistent shock and the level of total volatility.

Leverage variation increases not only when total volatility rises, but also as persistent part of the firm's cash flows becomes more important. Furthermore, its sensitivity to σ_P is the greater, the higher the firm's total volatility, which highlights the fact that when persistent shock volatility constitutes a lower share of total volatility, their effect could still be visible in certain moments. The channel through which persistent shocks affect leverage volatility is related to variation in investment. As shown by the comparative statics in section 3.3, the firm's investment policy becomes more volatile when the share of σ_P in total volatility rises, as it tends to disinvest substantially more. In other words, the firm's investment policy is very

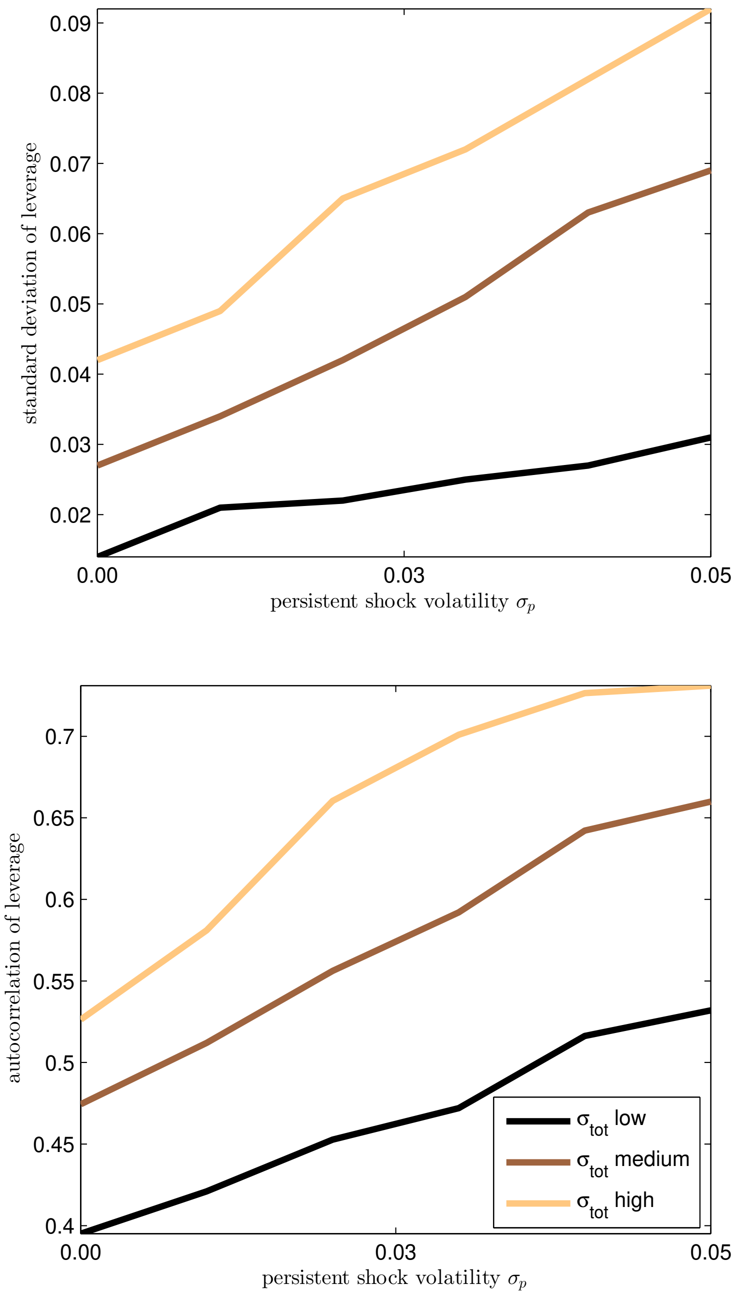


Figure 3.3 – **Fundamental volatility and leverage dynamics.** The figure contains the model-implied average standard deviation of leverage (left graph) or average first-order autocorrelation of leverage (right graph) as a function of total volatility σ (ranging from 0.15 to 0.25) and persistent shock volatility σ_P. The transitory shock volatility σ_T was computed such that the total volatility was constant along each line. The transitory shock persistence ρ is set at 0.6.

sensitive to the realizations of the persistent shock, which translates to highly variable debt policy.

Leverage persistence increases with both total volatility and persistent shock exposure, but is much more sensitive to the latter. The differences between various levels of total volatility remain relatively constant when varying the firm's shock exposure even when changes made to persistent shock volatility are small in comparison to the magnitude of varying total volatility. For very high values of total volatility, however, leverage persistence may also decrease with σ_P due to the fact that the firm is then increasingly more likely to hold low level of debt or even cash, which mutes the state-dependence of leverage.

3.3.3 The fundamental persistence channel

The analysis in previous subsection suggests that the composition of total volatility plays an important role in shaping the firm's debt policy. However, it focuses on only one particular dimension of the firm's fundamental risk related to fundamental volatility, which captures the magnitude of shocks. Another important characteristic of fundamental risk, and one that has not attracted much attention in the literature, concerns how long the effects of shocks are expected to affect cash flows. A firm is likely to behave differently if its cash flows are subject to shocks of large magnitude but which reverse quickly or shocks that may have lower magnitude but whose effects last for many periods. Thus, different persistence of cash flows is bound to result in different firm policies. For example, if hit by a positive shock, the firm may be incentivized to invest more if the effect on cash flows is more lasting to take advantage of the investment opportunity that persists. As such, shock persistence directly affects investment policy and thus the firm's financing choices, given that the firm has to raise internal or external funds to cover increased capital expenditure.

Even if these theoretical arguments appear sound, the data suggests that the between-industry variation in profit persistence is smaller than the variation in other firm characteristics. Figure 3.4 shows that the average estimated coefficients of profit persistence for different industries, computed assuming that the firm's log real profits follow an AR(1) process as usually done in practice, are strongly positively skewed and cluster around a high value or 0.8. For approximately 15% of industries they also assume values greater or equal than 1, which further highlights the need to consider a more flexible setting able to cover the potential non-stationarity of profits. Further examination of the data reveals that these estimated coefficients are not significantly related to leverage or other firm characteristics. This is evident when considering correlations between average firm size, leverage, investment, asset tangibility or market-to-book ratio, as well as other variables, and the estimated measures of profit persistence: all resulting values are negligibly small.[5] Moreover, even when extracting the value of ρ using structural estimates for different industries, as in DeAngelo, DeAngelo, and Whited (2011), its explanatory power for the cross-sectional variation in average leverage is still weak or modest at best. These findings are strikingly at odds with the evidence resulting from this model, in which the comparative statics of the transitory shock persistence ρ in Table 3.2 suggest that it has a strong and robust effect on model-implied moments.

[5] See Appendix C.II for detailed empirical evidence on the relationship between persistence and leverage as well as other firm characteristics.

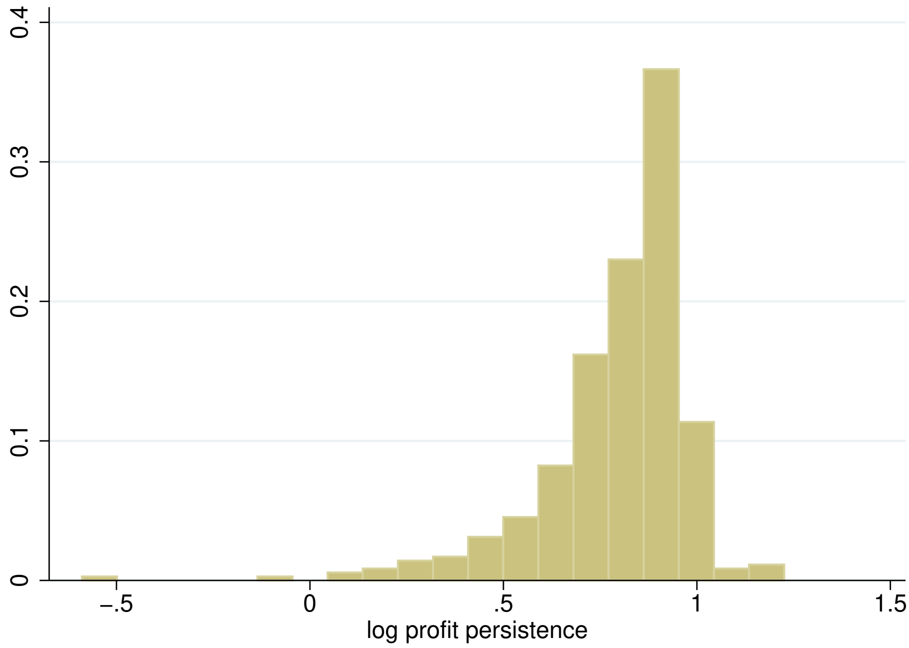


Figure 3.4 – **Histogram of the average estimated persistence parameter ρ of log real operating profits** $\log(\Pi)$ **of firms in 4-digit SIC industries.** The estimate of the persistence parameter ρ was computed using an AR(1) fit of log real profits for each firm and then averaged over all firms in an industry. Definitions of variables are provided in Appendix C.II. All variables are winsorized at 1% and 99%.

Based on these results and on the intuitive notion of the nature of uncertainty discussed earlier, we could suspect that changing persistence should play an important role in firms' investment and leverage decisions. However, the channel through which it takes effect must be different than the one implied by standard models with a single transitory shock in which all persistence comes from ρ. I argue that shock composition discussed in this paper offers a convincing alternative explanation for these phenomena and that it is also able to justify the disparity between model-based evidence and the data.

Decomposing fundamental persistence

There are two sources of cash flow persistence in the model. First, any realization of the persistent shock impacts all future values of cash flow, greatly increasing the persistence in firms policies. However, the share of persistent shock volatility in total volatility may be small, thus it is not clear whether their overall contribution to total persistence is always large. Second, transitory shocks can also affect the overall persistence, as they are path-dependent, but plausibly to a much lesser extent than persistent shocks given their transient nature. As such, both sources could be vital for determining the firm's debt policy.

Figure 3.5 illustrates the differential effect of the two persistence channels by plotting the model-implied average profit persistence as a function of transitory shock persistence ρ for different levels of persistent shock volatility σ_P. Both ρ and σ_P strongly affect the level of profit

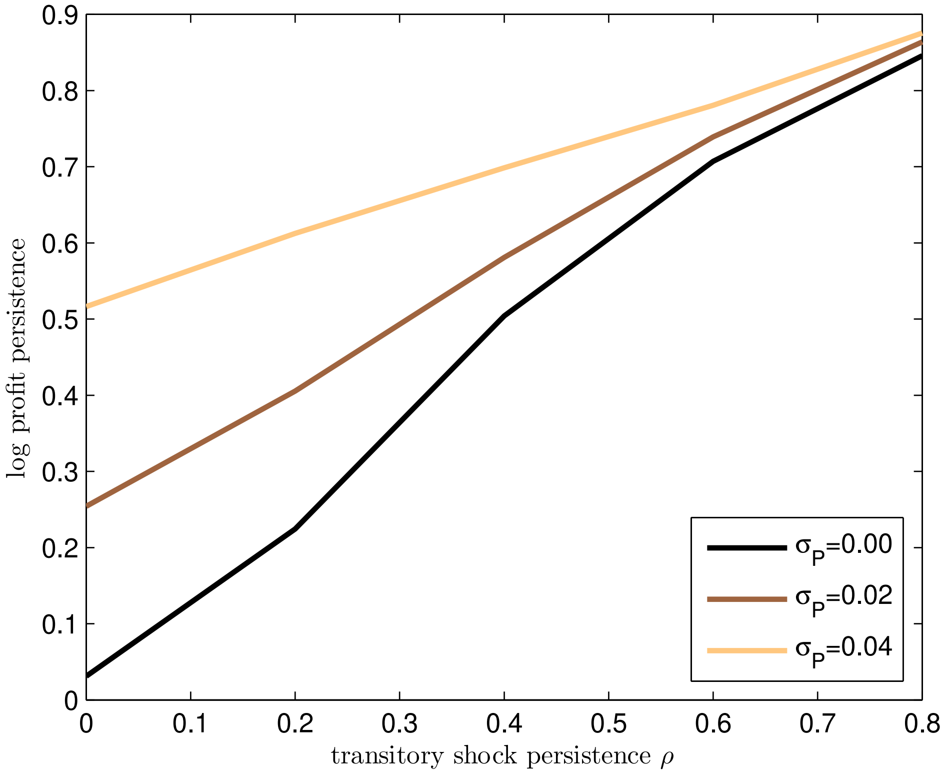


Figure 3.5 – **Model-implied average persistence of log profits** $\rho(\log(\Pi))$ **as a function of transitory shock persistence** ρ**.** Each line corresponds to a different level of persistent shock volatility σ_P. The total volatility is set to $\sigma = 0.15$.

persistence. However, it is also important to notice that observable profits can be substantially path-dependent even when $\rho = 0$ if the cash flow process contains a small persistent part. When ρ takes small to moderate values, the effect of risk composition on profit persistence is the stronger, the higher is the share of persistent shock volatility in total volatility. However, when the transitory shock persistence is very high, then the additional persistence stemming from persistent shock is fairly small.

The effect of transitory shock persistence ρ and persistent shock volatility σ_P on model-implied profit persistence may vary depending on the level of total volatility, which determines the relative importance of the two channels. To this end, I analyze the elasticities of model-implied profit persistence to changing these parameters for different values of total volatility σ. Figure 3.6 contains the results which suggest that the effect of changing σ_P on profit persistence is only important when ρ assumes low or modest values. Its significance also decreases as the total volatility is increased, as then transitory shocks become relatively more important. On the other hand, ρ appears to wield substantial influence on profit persistence for different levels of σ_P, but is less vital when the firm's persistent shock exposure is high. The fact that the elasticity of profit persistence to ρ decreases when the firm is not exposed to the

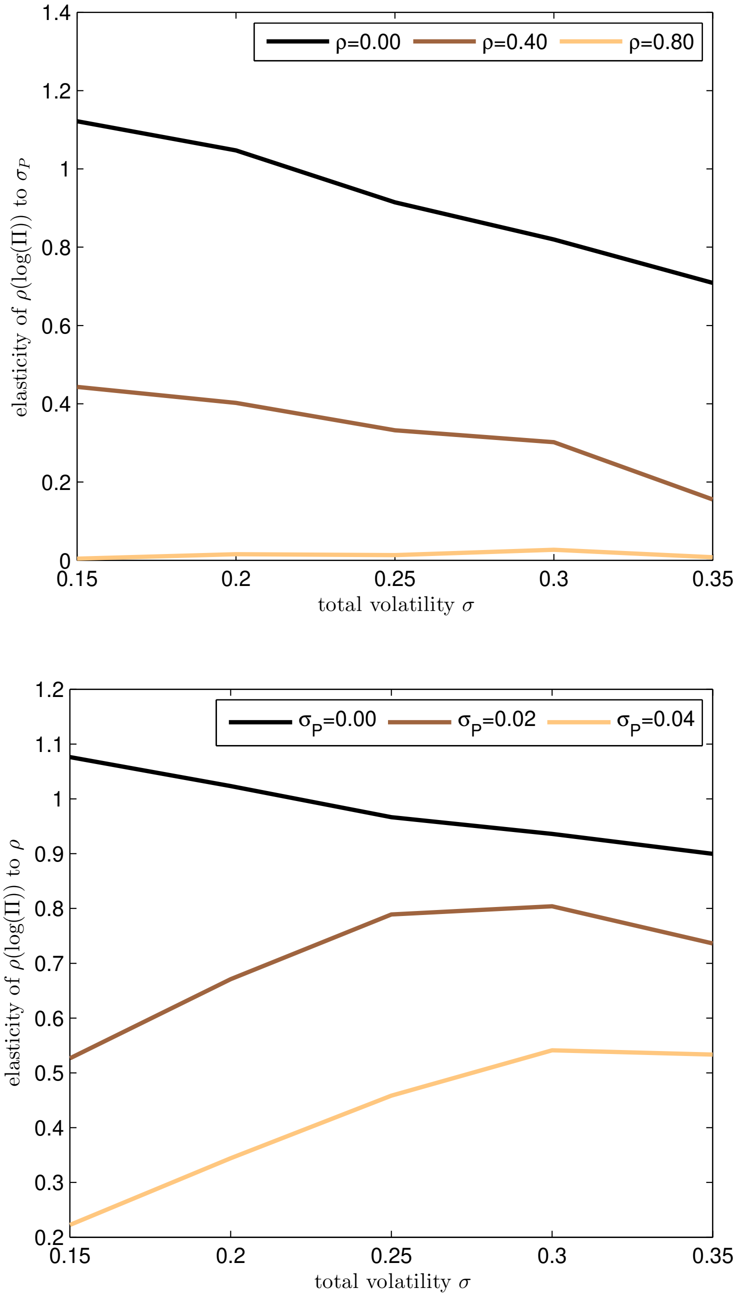


Figure 3.6 – **Elasticity at average moments** $(\partial m/\partial x) \times (\overline{m}/\overline{x})$ **of log profit persistence** $\rho(\log(\Pi))$ **to persistent shock volatility** σ_P**.** Each line corresponds to a different level of ρ (left graph) or to changing transitory shock persistence ρ for different levels of σ_P (right graph). The elasticities are computed as functions of total volatility σ.

persistent shock represents another important result not directly represented by the graphs, that is the negative relationship between total volatility and persistence. In general, higher values of σ result in lower average profit persistence, all else equal, implying that volatility composition plays a vital role in determining fundamental persistence as well.

Finally, the evidence presented in this subsection suggests that the model provides more flexibility in terms of being able to generate firms with the same level of profit persistence but different values of transitory shock persistence ρ. In particular, this implies that the *true*, unobservable ρ could vary widely between firms or industries despite observing very similar, potentially high, values of profit persistence. As such, it is not surprising that the empirical association between estimated profit persistence and firm characteristics is weak, because it is the variation in unobservable parameters describing the overall fundamental risk of the firms that wields more influence on their characteristics.

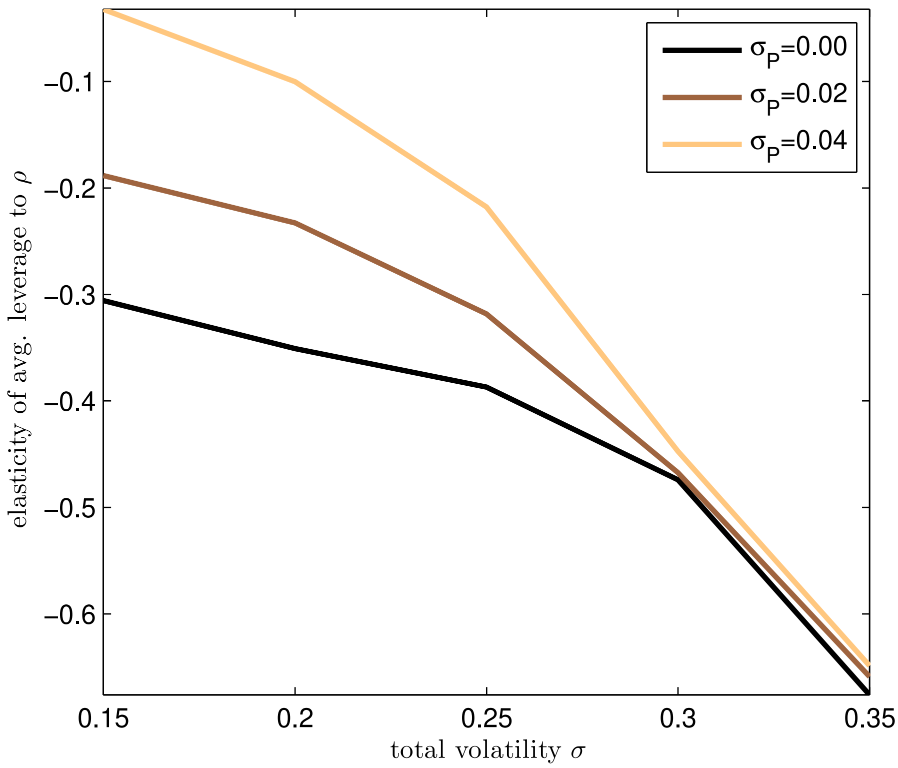


Figure 3.7 – **Elasticity at average moments of leverage to transitory shock persistence** $(\partial lev/\partial\rho)\times(\overline{lev}/\overline{\rho})$**.** Each line corresponds to a different level of total volatility σ and persistent shock volatility σ_P.

Fundamental persistence and leverage

Having examined the extent to which the two channels generate model-implied profit persistence and documented that different parameters affect profit persistence to different extent, it is also important to ask how the two channels of persistence affect the firm's leverage policy. Figure 3.7 contains the elasticity of the model-implied average leverage to changing transitory shock persistence ρ for different level of total volatility σ and persistent shock volatility σ_P. While the elasticities are always negative, highlighting that persistence and leverage are negatively related, the graph reaffirms the claim that each source of persistence may have a distinctive quantitative effect on the firm's debt policy, depending on its overall fundamental risk. In particular, it shows that ρ can have a different effect on leverage depending on σ_P.

First, changing ρ appears to affect leverage for any given risk composition. Second, firms not exposed to persistent shocks are always more sensitive to changing ρ than firms whose cash flow also contains a small persistent component. Finally, when σ_P constitutes a large share of total volatility (for example when $\sigma = 0.15$ and $\sigma_P = 0.04$), changing ρ may have negligible effect on the firm's average leverage. In other cases, however, the effect is expected to be sizeable.

3.3.4 Fundamental risk and model-implied moments

The last important issue related to examining the main mechanisms of the model concerns analyzing how risk composition influences model-implied quantities. Table 3.2 contains the values of selected moments resulting from simulating the model using different values of the three parameters describing the nature of risk faced by the firm: the persistent shock volatility σ_P, the transitory shock persistence ρ and the transitory shock volatility σ_T.[6]

The results in Table 3.2 imply that the firm invests more, on average, as its persistent shock exposure increases. This result, consistent with Gourio (2008), is important given that investment constitutes the main reason for debt issuance in the model, driving the dynamics of leverage. Persistent shocks also have a large influence on the dynamics of investment, as shown by its variance and correlations between investment and other moments. Importantly, persistent shocks substantially increase the long-run persistence in investment, which is consistent with the evidence from Gourio (2008) and DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013) that the effect of persistent shocks can be seen in higher order autocorrelations. Finally, higher persistent shock exposure increases the incidence of disinvestment, in line with the intuition that the firm's policies are more sensitive to persistent shock realizations. Intuitively, if the firm experiences a negative persistent shock, then it is more likely to conduct an asset sale, because its cash flows will be forever affected by this shock realization.

The moments related to the firm's debt policy, that have largely been discussed in Subsections 3.3.2 and 3.3.3, reveal the negative relationship between persistent shock exposure and average leverage. Similarly as in case of investment, higher persistent shock volatility increases leverage variation and leverage persistence and these outcomes are closely related to the behavior of investment-related moments.

Not unexpectedly, shock composition also holds significance for the dynamics of profits and profitability as well as for the correlations between profit and growth variables. However, it is important to note that persistent shocks affect profitability and log profits differently. While their importance for the former is limited, it is fairly substantial for the latter. This

[6] Appendix C.IV contains the comparative statics of parameters related to real or financing frictions such as the capital adjustment cost ψ, the external equity issuance cost η or the collateral constraint parameter ω.

happens due to the fact that profits Π are detrended by capital K, both of which are affected by persistent shocks. Both numerator and denumerator also interact with transitory shock persistence in a subtle way, making it difficult to distinguish between the effects of these two channels. Similar rationale explains the decreasing correlation between investment and profitability as well as between (log) growth and profitability.

Finally, it is important to note that persistent shock exposure greatly increases the degree of dispersion in the simulated moments, which is documented by the median absolute deviations of average investment and leverage. This result shows that controlling for risk composition can help explain the dispersion in model-implied moments.

3.3.5 Identifying fundamental risk

Seeing that the effects of different parameters describing the firm's fundamental risk may wield similar influence on its policies, it is important to ask whether we can infer the firm's risk composition by observing model-implied moments Ideally, we would want to identify each parameter by a single moment, in which case changing the parameter would cause only that moment to vary. However, many directional effects of parameters are similar (cf. Table 3.2). The main concerns that have to be addressed are related to examining the relationship between persistent shock volatility σ_P and capital adjustment cost ψ as well as distinguishing between the effect of persistent shock Z_P and the transitory shock Z_T. Therefore, we have to consider the overall relationship between risk characteristics and resulting corporate policies.[7]

First, the firm's capital and debt policies should be respond smoothly to a persistent shock realization, as compared to a transitory shock. As such, we could expect that changing persistent shock exposure could have similar implications as those of increasing the convex capital adjustment cost ψ. However, Table 3.2 suggests that the effect of changing persistent shock volatility σ_P is significantly different than the effect of changing ψ, which increase average leverage and decrease investment variation, thus resulting in lower leverage volatility. These effects are qualitatively different than those of varying the firm's exposure to persistent shock.

Second, changing volatility and persistence has a similar effect on model-implied moments, no matter whether the source of change comes from the transitory shock or the persistent shock. Considering these characteristics jointly is crucial given the two-faceted nature of persistent shocks, which affect both at the same time. Therefore it is important to ask if we can infer the relative importance of persistent shock volatility σ_P in total volatility σ. Table 3.2 contains several moments that could provide insight about the parameters describing the firm's fundamental risk. For example, distinguishing between profit persistence and profitability persistence is informative: the former is greatly affected by persistent shocks,

[7] In this subsection I only discuss the identification of parameters describing the firm's risk exposure. Identification of the remaining parameters is fairly standard, e.g. as in DeAngelo, DeAngelo, and Whited (2011) or Nikolov and Whited (2014). For example, the external equity financing cost η can be identified off its effect on investment and leverage while the collateral constraint parameter ω from the dynamics of leverage, which it significantly affects.[8]

		Total volatility σ	0.15	0.15	0.15	0.15	0.15	0.15	0.25	0.25	0.25	0.25	0.25	0.25
		Transitory shock persistence ρ	0.20	0.20	0.20	0.60	0.60	0.60	0.20	0.20	0.20	0.60	0.60	0.60
		Persistent shock volatility σ_P	0.00	0.02	0.04	0.00	0.02	0.04	0.00	0.02	0.04	0.00	0.02	0.04
1.	Investment	Average investment (i/k)	0.151	0.152	0.153	0.154	0.155	0.155	0.151	0.152	0.153	0.156	0.158	0.159
2.		Standard deviation of investment (i/k)	0.031	0.041	0.069	0.093	0.095	0.099	0.041	0.046	0.065	0.106	0.122	0.131
3.		Autocorrelation of investment $\phi_1(i/k)$	0.007	0.212	0.315	0.192	0.256	0.285	-0.109	0.132	0.258	0.225	0.214	0.248
4.		Autocorrelation of investment $\phi_3(i/k)$	-0.103	-0.028	-0.007	-0.106	-0.106	-0.066	-0.083	-0.029	0.002	-0.095	-0.106	-0.088
5.		Frequency of disinvestment $\#(i/k<0)$	0.000	0.000	0.016	0.042	0.044	0.062	0.000	0.001	0.040	0.053	0.086	0.106
6.		Corr. inv. and profitability corr$(i/k,\pi/k)$	0.899	0.693	0.500	0.877	0.883	0.775	0.700	0.603	0.432	0.855	0.846	0.836
7.		Autocorrelation capital stock $\phi_1(K)$	0.667	0.871	0.896	0.775	0.817	0.863	0.564	0.847	0.893	0.794	0.799	0.832
8.		Dispersion of average investment $MAD(\overline{i/k})$	0.002	0.010	0.020	0.010	0.014	0.022	0.003	0.010	0.020	0.012	0.016	0.023
9.	Leverage	Average leverage (p/k)	0.596	0.590	0.375	0.578	0.548	0.497	0.567	0.512	0.381	0.320	0.201	0.172
10.		Standard deviation of leverage (p/k)	0.003	0.003	0.030	0.015	0.029	0.030	0.021	0.034	0.041	0.043	0.064	0.074
11.		Persistence of leverage $\rho(p/k)$	0.108	0.267	0.539	0.396	0.455	0.518	0.226	0.491	0.632	0.528	0.693	0.729
12.		Volatility of leverage $\sigma(p/k)$	0.003	0.003	0.024	0.014	0.020	0.024	0.016	0.028	0.023	0.035	0.043	0.047
13.		Dispersion of average leverage $MAD(\overline{p/k})$	0.001	0.001	0.011	0.005	0.013	0.020	0.008	0.012	0.025	0.012	0.025	0.034
14.	Profits	Persistence of profitability $\rho(\pi/k)$	0.082	0.089	0.110	0.337	0.335	0.352	0.079	0.088	0.093	0.347	0.337	0.339
15.		Volatility of profitability $\sigma(\pi/k)$	0.066	0.066	0.067	0.069	0.067	0.067	0.123	0.121	0.121	0.144	0.141	0.139
16.		Persistence of log profits $\rho(\log(\Pi))$	0.225	0.406	0.613	0.707	0.739	0.781	0.191	0.259	0.411	0.620	0.661	0.702
17.		Volatility of log profits $\sigma(\log(\Pi))$	0.149	0.160	0.175	0.156	0.157	0.161	0.257	0.268	0.278	0.263	0.266	0.267
18.		Corr. prof. and growth corr$(\pi/k,k'/k)$	0.903	0.696	0.499	0.880	0.886	0.780	0.704	0.434	0.607	0.858	0.849	0.839
19.		Corr. prof. and log gth. corr$(\pi/k,\log(k'/k))$	0.903	0.695	0.498	0.872	0.882	0.776	0.699	0.431	0.598	0.843	0.831	0.819

Table 3.2 – **Summary statistics of model-implied moments for different values of σ, ρ and σ_P.** Remaining parameters are taken as specified in Table 3.1. An AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ was fit for each simulated firm to compute persistence ρ_x or volatility σ_x of variable x. ϕ_k is the k^{th} order autocorrelation. The measure of dispersion MAD is median absolute deviation. The numerical solution and simulation of the model is described in Appendix C.II.

while the latter being insensitive, as detrending of profits Π by capital K removes a part of their effect.

There are also other channels, however, which may help tell the two shocks apart. Most importantly, volatility composition as well as total volatility level, while holding transitory shock persistence ρ constant, greatly affect investment, leverage and profit autocorrelation and the direction of change is different for the two shocks. Therefore, these moments are informative about the magnitude of volatility parameters. The distinction between ρ and σ_P can be seen in Table 3.2 by considering their effect on higher-order autocorrelations (which increase with persistent shock exposure but decrease when ρ rises), on correlations between investment and profitability or on correlations between profitability and log growth (which decrease with σ_P but rise when transitory shock volatility is increased). Furthermore, in spite of the same qualitative effect of both of these parameters on model-implied moments, their sensitivity, that is the quantitative effect, may be different.

3.3.6 Implications for capital structure heterogeneity

A vast amount of corporate finance research focusing on understanding the variation in corporate leverage ratios recognizes that capital structure heterogeneity remains largely unexplained by existing factors based on firm characteristics such as size, profitability, asset tangibility or the degree of financial constraints.[9] Lemmon, Roberts, and Zender (2008) and Graham, Leary, and Roberts (2015) examine the underlying reasons as to why these factors fare poorly and point out that firm fixed effects provide substantial incremental explanatory power. It is therefore important to understand what these fixed effects contain. Examining deep structural parameters of the model, for example these governing the firm's cash flow process, could therefore help rationalize the importance of the time-invariant effects seen in the data. The evidence presented in this section suggests that studying the nature of risk affecting firms is likely to provide further insight regarding the variation in corporate policies.

There are two main reasons why studying risk exposure could provide further explanatory power above and beyond standard leverage factors. First, the evidence in section 4.1 suggest that while two firms may have similar observable risk, their leverage could be markedly different, because the composition of their fundamental volatility is distinctive. As such, separating total volatility into transitory and persistent components would enhance the ability of the standard cross-sectional regression in explaining the data.

Second, composition of fundamental persistence is also informative of the firm's leverage policy. The model has the appealing feature of allowing to change the transitory shock persistence ρ without changing profit persistence $\rho(\log(\Pi))$ much. To illustrate how this helps in explaining variation in firms' policies, let us consider the following example. Suppose that we fit an AR(1) model to firms' log real profits, as frequently assumed in empirical studies. Suppose further that we observe $\widehat{\rho} \approx 0.6$ for two firms which also adopt different leverage ratios of 0.59 and 0.36. The ability of $\widehat{\rho}$ to explain capital structure variation would be low in

[9] The factors used in empirical studies are based on various theories of capital structure, summarized e.g. in Harris and Raviv (1991). Titman (1984) and Rajan and Zingales (1995) examine these factors extensively while Strebulaev and Yang (2013) consider whether they can explain the zero-leverage puzzle.

this case. However, the model suggests that it could very well be the case that the composition of firms' risk is such that the first one has $\rho = 0.4$ and $\sigma_P = 0$ while the other $\rho = 0$ and $\sigma_P = 0.04$. In this case, both persistent shock volatility σ_P as well as transitory shock persistence ρ would be able to provide additional insight concerning leverage heterogeneity.

Overall, I argue that the three parameters governing the processes (ρ, σ_T and σ_P) may not only provide explanatory power over σ_{total} or $\widehat{\rho}$ in explaining the capital structure variation in the data, but that their incremental contribution to explaining the heterogeneity will also vary. The marginal effect of each parameter characterizing the firm's fundamental risk will in general depend on the overall risk composition, as seen in the analysis of the elasticities of model-implied moments to these parameters. As an example, when the firm is much more exposed to the transitory shock, then changing its persistence has a much greater effect on firms policies than when the firm is more exposed to the persistent shock.

3.4 Conclusion

In this paper I argue that the nature of fundamental risk, captured by the composition of firms' cash flow process, has important implications for capital structure characteristics. Crucially, the two channels through which fundamental risk affects capital structure are fundamental volatility and fundamental persistence. The model is able to explain several empirical patterns, in particular the dispersion in the risk-leverage relationship and the low observed variation in profit persistence. It also highlights that risk composition may provide additional explanatory power for capital structure heterogeneity, above and beyond standard leverage factors.

The paper is silent, however, on the actual extent to which fundamental risk affects firms and it has to be taken to the data. Measuring fundamental risk could prove challenging, given that it is inherently unobservable. Empirical studies often resort to stock returns to measure a firm's risk, but any stock price-based measure is unlikely to provide much insight about profitability shock persistence, as stock returns are approximately iid. Statistical filtering of cash flow into different components, while well-suited to decompose aggregate processes, may be inappropriate for firm-level analysis given the low number of firm-level observations available and the ambiguity about its true dynamics. It may be unable to identify the correct magnitude of fundamental risk characteristics, because it does not take into account policies, which are informative about risk composition. To alleviate some of these concerns, one can employ structural estimation to extract a measure of risk by quantifying how firms perceive their own risk exposure. In this estimation method, the theoretical structure of the model is used to interpret the observed data by ascertaining that it resembles the model-generated behaviour. In short, the observed corporate policy choices are used to infer the magnitudes of risk exposures of an average firm.

Appendices

A Product Market Strategy and Corporate Policies

Appendix A consists of four parts. Section A.I provides more details about the product data and sample selection. Section A.II contains the definitions of variables, additional empirical results, and robustness checks. Section A.III provides additional details regarding the solution of the model. Section A.IV describes the estimation procedure and contains in-depth estimation results.

A.I Product data

A.I.1 Data description

I use the AC Nielsen Homescan (NH) data to obtain information about firms' product market strategies. An extensive description of the database can also be found in e.g. Broda and Weinstein (2010) or Hottman, Redding, and Weinstein (2016). The data has three dimensions: household, product and time. Each household in the sample reports the prices and quantities of items purchased during each shopping trip and any potential discounts or deals associated with the purchases. Overall, the data contains a representative sample of approximately 40,000-60,000 households stratified into 61 geographic areas in the US. The sample is designed so that it can be projected to the total US population (projection factors are available). In total, the data spans over the period of 15 years (2004–2018).[1]

Product classification in NH

Each product in the data belongs to specific categories, varying in their granularity. There are categories such as Departments (10), Product Groups (≈125), Modules (≈1,075) and UPC codes (≈4.3 million out of which ≈2.3 million are present in the consumer panel files). Most products also have a specific brand. An example of product classification can be found in Table A.1.

The most granular level of product categories is the UPC code. Each UPC code is 12-digit long and the first 6 to 11 digits are a unique identifier of the firm to which the product belongs ('GCP code'). However, firms can have many GCP codes. To obtain all possible combinations, all GCP codes are collected using the GLN code, issued by GS1 (which also manages the issuance of the UPC codes). The GLN code is used to identify physical locations or legal

[1] It should be noted that Nielsen Homescan database by construction focuses on nondurable consumer goods, so most apparel, electronics and home furnishing purchases may not be recorded.

Compustat identifier	GS1 identifier	product identifier	product department	product group	product module	product brand
(6 digits)	(13 digits)	(12 digits)	DRY GROCERY	CANDY	CANDY – CHOCOLATE	HERSHEY'S KISSES

Table A.1 – **Example of a product in the data.** Details of 'NESTLE USA 8.47 OZ (240g) Nescafe Frothe Latte Coffee Drink'

entities of the firms. The key is 12 digits long and comprises a GS1 Company Prefix, Location Reference, and Check Digit. Both the GCP code as well as the GLN codes are obtained from the GEPIR database provided by POD, which additionally contains the full name and the address of each firm. Overall, the POD database is able to match 3.4 millions UPCs (≈ 78% of all available products and ≈ 96% of product data available in the consumer panel files) which belong to 51,592 firms (37,492 firms in the panel data).

Table A.2 contains the summary statistics of firm-level number of products per category. It indicates the large degree of heterogeneity in the data: while an average firm owns roughly 8.82 products, a typical (median) firm owns only 2. Similar conclusions can be drawn from looking at other classifications of products.

	mean	sd	q1	med	q3	min	max
# UPCs	63.8	493.43	2	5	17	1	36352
# brand-modules	8.82	37.28	1	2	5	1	1738
# brands	4.67	17.69	1	1	3	1	1126
# product modules	4.83	22.62	1	2	3	1	795
# product groups	2.44	5.12	1	1	2	1	111
# departments	1.37	0.89	1	1	1	1	11

Table A.2 – **Summary statistics of different product classifications.** Each classification contains data from 37,492 uniquely identified firms in Nielsen Homescan.

Merging with Compustat

The matched firm-product data can be merged with accounting data from Compustat by using text matching of firm names. However, many public firms own multiple subsidiaries and the firm-product data could thus contain their name rather the one of the ultimate parent. For example, in the data P&G directly 'owns' most if not all of the products while Newell Brands only owns its products through some of the 121 subsidiaries. As such, I obtain the names of subsidiaries of each firm in Compustat from Capital IQ.

The text matching procedure is conducted using fuzzy merging based on several 'similarity' functions and the matches were manually verified. The matching scores are based on GED-SCORE, SPEDIS and % of the same 3-character combinations of one company name in the other. All punctuation, special characters and common words are removed before conducting the comparison. After the merge, I manually add firms with at least 200 UPC codes to the data (out of all unmatched firms with more than 800 UPC codes – 477 firms – 15% turned out to be public or subsidiaries of public firms). In total, I was able to merge 1376 GLN-level firms (or subsidiaries) from the firm-product data, which correspond to 720 US-headquartered Compustat firms.

To verify that the matching procedure is reasonable, I analyze the 'sales share', i.e. the ratio of the projected sales (to the whole US; computed using the projection factors in the data) of each matched Compustat firm-quarter to its actual sales in that quarter, which are available in the data. I only focus on 2-digit SIC industries with at least two matched firms. Table A.3 contains the summary statistics on sales share for the whole sample as well as for 2-digit SIC industries.

	mean	sd	p25	median	p75	N
Agricultural Production - Crops	0.530	0.221	0.316	0.529	0.723	110
Food & Kindred Products	0.580	0.469	0.221	0.487	0.797	2573
Tobacco Products	0.181	0.093	0.099	0.141	0.264	118
Chemical & Allied Products	0.503	0.564	0.103	0.263	0.861	616
Rubber & Miscellaneous Plastics Products	0.608	0.632	0.076	0.419	1.106	123
Electronic & Other Electric Equipment	0.369	0.339	0.098	0.264	0.469	206
Total	0.542	0.481	0.173	0.404	0.784	3746

Table A.3 – **Sales' shares of matched firms.** All variables are winsorized at 2.5% and 97.5% percentile.

Table A.3 suggests that not all industries are equally well-represented in the data. For example, Agriculture and Food industries are relatively well matched. Other industries, such as Electronic and Chemicals, are characterized by large degree of within-industry dispersion in sales share. This is partially related to the fact that AC Nielsen data focuses on particular product categories, which are only partially present in some industries (e.g. one could think about Procter and Gamble, whose product portfolio is relatively well-captured in the data, but which is primarily in the Chemicals sector also containing other firms with low sales share). However, the matched sample contains primarily firms from the 'food' industry, which comprises two SIC2 codes: 01 and 20, and these firms rely heavily on the retail channel in generating sales.

A.I.2 Sample selection

To refine the data for further empirical analysis, I first apply the standard Compustat data filters: I remove firms with missing data on any variables used in structural estimation, market-to-book larger than 15 or negative book equity. Based on the matched sample, I remove all financial conglomerates (SIC≥ 6000). I require that firm's projected sales share (that is, the ratio of Homescan-based sales to accounting measures of sales reported in Compustat) be at least 5% and no more than 150% of its total sales on average. Moreover, I only keep firms in industries in which the average projected sales share is at least 10%. I also remove all firms which have on average less than 20 UPC codes. Even though I control for the sales share in the empirical analysis, this filter is important as it disposes of firms that do no rely on retail channel (so the main mechanism is unlikely to matter) or that were plausibly mismatched. As an example, this filter removes firms such as American Crystal Sugar Co or Archer-Daniels-Midland Co. for whom the retail channel is clearly of secondary if not tertiary importance. I winsorize the remaining data at 2.5% and 97.5% level. The final sample spans 2004Q1 to 2017Q4 and contains 2,366 firm-quarter observations.

A.II Data and stylized facts

A.II.1 Definitions of variables

This section presents the definitions of variables used throughout the paper.

1. **Product portfolio age** – weighted share of old products in the portfolio:

$$\text{age}_{it} = \frac{\text{weighted \# of products with age exceeding 50\% of lifespan}(it)}{\text{total \# of products}(it)}, \tag{A.II.1}$$

   where the weights correspond to product-specific revenues.

2. **Product portfolio size** – effective number of products at level x, $x \in [\text{upc}, \text{bm}]$:

$$eff\_no\_prod_{it} = 1/rc\_x_{it}, \tag{A.II.2}$$

   where rx_x is the revenue concentration at level x:

$$rc\_x_{it} = \frac{H_{it} - 1/N_{it}}{1 - 1/N_{it}}, \text{ with } H_{it} = \sum_{x=1}^{N_{it}} \left(\frac{r\_sale_{xt}}{\sum_{x=1}^{N_{it}} r\_sale_{xt}} \right)^2, \tag{A.II.3}$$

   where r_sale_{xt} are the estimated aggregate retail sales for each $x \in \{\text{upc}, \text{bm}\}$ product at time t. To see where the measure for the effective number of products stems from, suppose that a firm supplies N products. Then its revenue concentration is $rc = \sum_{i=1}^{N} s_i^2$, where s_i is the share of product i in the firm's sales. Assuming all products provide equal revenue, their revenue share is $s_i = s = 1/N$, thus the HHI is now $rc = \sum_{i=1}^{N} 1/N^2 = N/N^2 = 1/N$, which implies that we can back out the 'effective' number of products as $eff\_no\_prod = 1/rc$.

3. **Product portfolio adjustments** – net product entry:

$$ne_{it} = (\text{\# product introductions}(it) - \text{\# product withdrawals}(it))/\text{total \# products}(it),$$

   where an 'introduction' indicates a new product that has never been offered by firm i before time t and a 'withdrawal' indicates that a product was no longer supplied by firm i after time t. I also consider net product creation:

$$nc_{it} = (\text{revenue of entering products}(it) - \text{revenue of exiting products}(it))/\text{total revenue}(it),$$

where an 'entry' indicates an introductions of a new product that has never been offered by firm i before time t and an 'exit' indicates a product that was no longer supplied by firm i after time t.

4. **Market-to-book**: book value of debt plus market value of equity over total assets.
5. **Investment**: capital expenditure minus asset sales over gross plant, property and equipment.
6. **Net book leverage**: book debt minus cash and short-term investments over book debt plus book equity.
7. **Cash**: cash and short-term investments over total assets.
8. **Firm size**: natural logarithm of real total assets.
9. **Profitability**: operating income over total assets.
10. **Cost of sales**: general and administrative expense over total assets.
11. **Implied competition**:

$$ihhi_{it} = \sum_{m=1}^{M} s_{mit} HHI_{mt}, \tag{A.II.4}$$

where $s_{m,i,t}$ is the share of firm's i sales in market m at time t, and $HHI_{m,t}$ is the Herfindahl of market m at time t, computed using *all* firms available in the sample, both public and private.

12. **Firm age**:

$$\text{f.age}_{it} = -\frac{1}{1 + \text{listing age}_{it}}, \text{ where listing age}_{it} = t - t_{i0}, \tag{A.II.5}$$

with t_{i0} being the first appearance of firm i in CRSP, as in Pástor and Veronesi (2003) or Loderer, Stulz, and Waelchli (2017).

13. **Cash flow volatility**: the rolling standard deviations of profitability, computed over the past 8 quarters.
14. **Log sales**: natural logarithm of real sales.
15. **Log product sales**: natural logarithm of the real estimated product sales.
16. **Product durability**: average calendar age of products at exit.

A.II.2 Robustness: defining product portfolio age

Table A.1 documents that the stylized fact about investment and product portfolio age is qualitatively robust to adopting a different definition of a product or a different definition of product portfolio age.

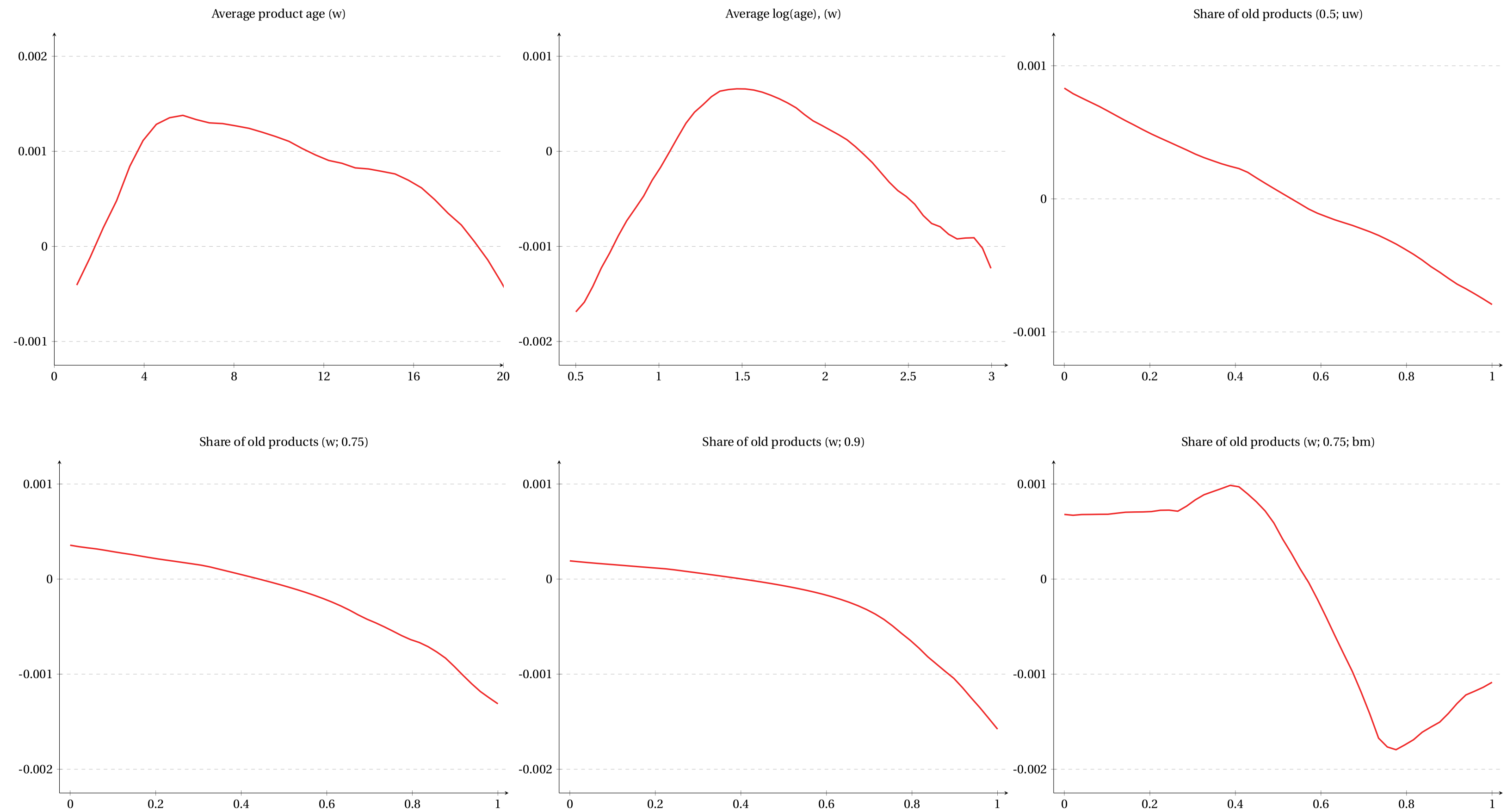


Figure A.1 – **Robustness check: defining product portfolio age.** Each graph is obtained from local polynomial regression of the residuals from an investment regression on a given product portfolio age variable, using an Epanechnikov kernel function with a rule-of-thumb bandwidth estimator and local-mean smoothing. The controls used to compute the investment residuals include size, cash flow, and market-to-book. w indicates that product-level revenue weights were used (uw - not used). $0.5, 0.75, 0.9$ are the tresholds used to define an old product, i.e. one that exceeds 50%, 75% and 90% of its lifespan, respectively. bm indicates that brand-module level was used rather than UPC-level.

A.II.3 Further evidence about product portfolio characteristics

In this part of the appendix I investigate basic empirical relationships between three product portfolio characteristics: age, size, and adjustments, and corporate policies.

The first column of Figure A.2 documents that product portfolio age is largely negatively related to firm value, except for the firms with oldest product portfolios which are also predominantly riskier. Capital investment tends to decline with product portfolio age, which suggests that investment and product introductions act to a large extent as complements. Finally, leverage is a hump-shaped function of product portfolio age (and cash a u-shaped one), meaning that firms with youngest and oldest product portfolios adopt lower leverage ratios. It should be noted, however, that these relationships are 'contaminated' by other firm characteristics. For example, the fact that leverage initially increases with product portfolio age could be attributed to firm entry and their initial growth, rather than within-firm changes in product portfolio composition. For this reason, in the following subsection I investigate the relationship between product portfolio age and corporate policies in more detail given that this characteristic will be the key ingredient in the model to follow.

I consider the number of products supplied by firms as the measure of their product portfolio size. Firms differ greatly in the number of products they supply: as shown in Table 1.1, the average number of products is 441. A comparison with Table 1.2 suggests that not all products are equally important for firms, as the average 'effective' number of products (58) is much lower than the raw one (441). This result indicates that the majority of firms' revenues can be attributed to a small number of products, supporting the notion that product revenues are fairly concentrated. Thus, rather than only using the raw number of products supplied by firms, I focus on the *effective* number of products, equal to the inverse of their product revenue concentration measured using the normalized HHI of each firm's product revenue:

$$\text{portfolio size}_{it} = 1/\tilde{H}_{it}, \text{ with } H_{it} = \sum_{p=1}^{P_{it}} \left(\frac{rev_{pit}}{\sum_{p=1}^{P_{it}} rev_{pit}} \right)^2, \tag{A.II.6}$$

where P_{it} is the number of products supplied by firm i in quarter t and rev_{pit} is the revenue of product p. The effective number of products can be interpreted as the number of products supplied by the firm assuming all of its products generate the same revenue. As such, it better reflects the number of products that contribute to the firm's total sales as opposed to the raw number which may contain many small products contributing little. However, the effective number of products also varies substantially across- and within firms, which is documented by its distribution in the top panel of Figure A.2. Notably, the shape of the distribution of product portfolio size resembles that of firm size, which is intuitive given that firm- and product portfolio size are positively, but not perfectly, correlated ($\rho \approx 0.45$).

I measure the product portfolio adjustments by computing the extent of net product entry, that is the difference between firm-level product entry and exit, similar to to Argente, Lee, and Moreira (2019). Each quarter, I count the share of new products introduced by each firm, that is ones that have never been supplied before, and the share of products that are withdrawn,

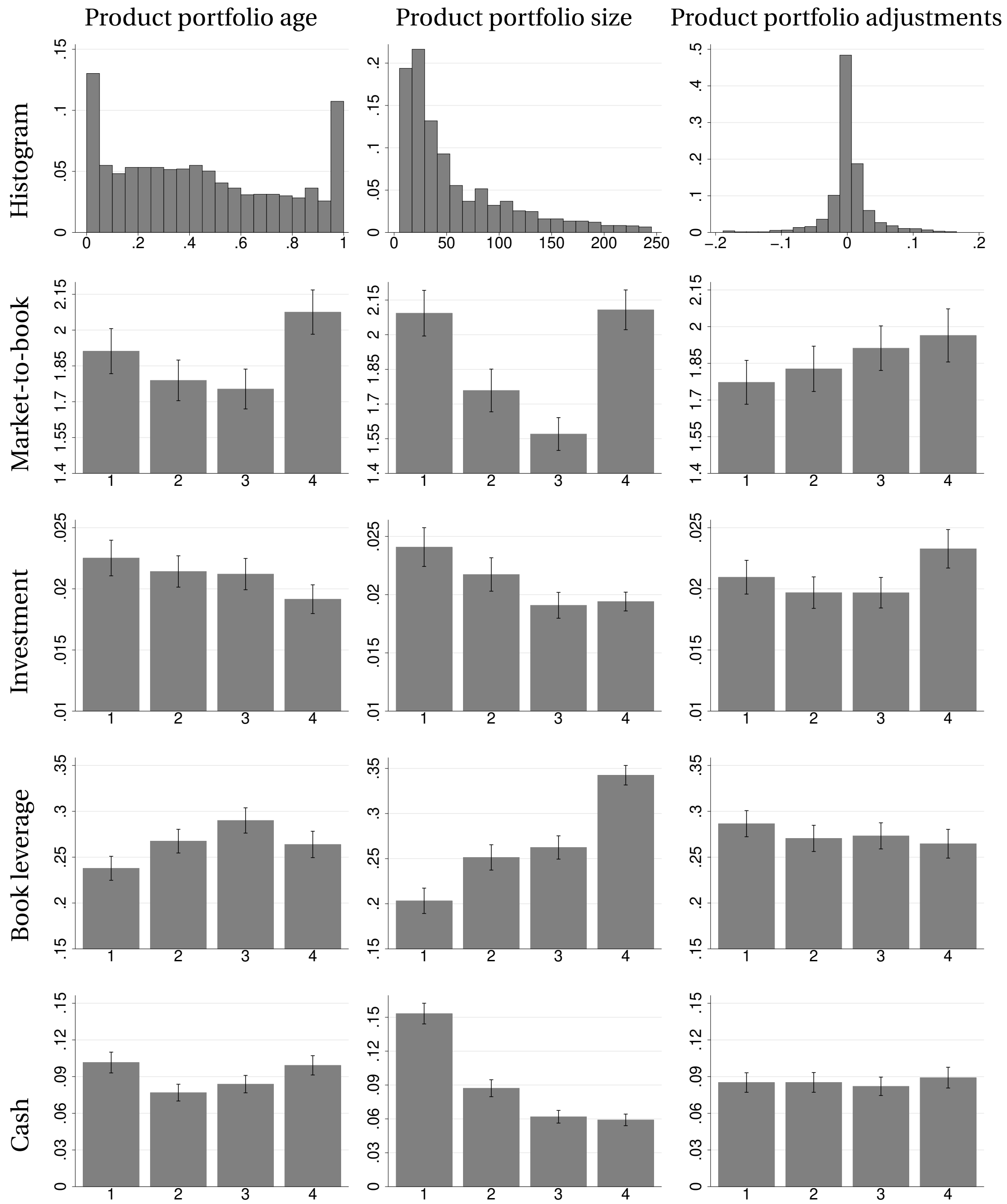


Figure A.2 – **Product portfolio structure and the relationship between firms' corporate policies and product characteristics.** The first row contains the histograms of product portfolio age, size and adjustments. Rows two to 5 contain the relationship between each product portfolio- and firm characteristic. In each of these graphs, every product portfolio characteristic is divided in four equally-sized bins and the corresponding average firm characteristic in every bin is computed. Each bar contains the 95% confidence interval.

i.e. that are never supplied again in the future (relative to the total number of products):[2]

$$\text{net entry}_{it} = \frac{\text{weighted \# product introductions}(it) - \text{\# product withdrawals}(it)}{\text{total \# products}(it)}. \quad \text{(A.II.7)}$$

The histogram of product portfolio adjustments in the top panel of Figure A.2 shows that 50% of time firms' product portfolios do not change, which indicates that product portfolio adjustments take place relatively infrequently. This result is 'the other side' of the evidence of Argente, Lee, and Moreira (2019), who document that product reallocation is very large in the aggregate: while the average net product entry equals 0.9% per quarter, not all firms adjust their product portfolios all the time. This indicates that a large degree of between- and within-firm variation in product portfolios is necessary to reconcile the two findings. Moreover, in Table 1.2 I report that the average net entry amounts to 0.26% each quarter, thus more than 3 times lower than the aggregate one, implying that a vast majority of product creation and destruction takes place in private firms. In practice, these numbers correspond to an average sample firm introducing 3.5 products each quarter, which increase its retail sales by roughly 1.2%, suggesting that within-firm product-level dynamics have important implications for cash flow dynamics.

The second column of Figure A.2 indicates that firms with smaller product portfolios invest more and adopt lower leverage ratios (or hold more cash) than firms with larger product portfolios. The u-shaped relationship between product portfolio size and market-to-book suggests that firms with many products also have higher valuations. This result is at odds with the standard notion that market-to-book declines with firm size and is consistent with the notion of product portfolio size increasing firms' market power, e.g. through differentiation (e.g. Feenstra and Ma, 2007). The third column of Figure A.2 shows that firm value increases in the extent of net product entry. This reaffirms the notion that managing product portfolios is important for firms. The graphs also show that firms invest more when withdrawing or introducing new products. This suggests that capital investment could serve as a substitute or a complement for product introductions.

The first column of Figure A.2 documents that product portfolio age is largely negatively related to firm value, except for the firms with oldest product portfolios which are also predominantly riskier. Capital investment tends to decline with product portfolio age, which suggests that investment and product introductions act to a large extent as complements. Finally, leverage is a hump-shaped function of product portfolio age (and cash a u-shaped one), meaning that firms with youngest and oldest product portfolios adopt lower leverage ratios. It should be noted, however, that these relationships are 'contaminated' by other firm characteristics. For example, the fact that leverage initially increases with product portfolio age could be attributed to firm entry and their initial growth, rather than within-firm changes in product portfolio composition. For this reason, in the following subsection I investigate the relationship between product portfolio age and corporate policies in more detail given that this characteristic will be the key ingredient in the model to follow.

[2]Given the definition of the proxy, I exclude first- and last year of the data to make sure that product entry and exit are correctly captured.

A.III Model solution

A.III.1 Product transition matrix

To get the product transition matrix T_{Φ}, I have to consider all possible states of the products in the future $\Phi' = (P'_n, P'_o)$ conditional on $\Phi = (P_n, P_o)$. We know that $P_e = (P'_n + P'_o) - (P_n + P_o)$ products exit. There are 3^2 cases in total to consider. Two examples of how these are computed are as follows, note that for the purpose of computing the transition matrix I also allow old products to transition to being new (which in the main specification is not allowed) hence I need to know both $q_{o\to o}$ and $q_{o\to e}$:

- $P'_n = P_n$ and $P'_o = P_o$:

$$\Pr(\Phi'|\Phi) = \sum_{k=0}^{\min(P_o,P_n)} Bin(\max(P_n - k, 0), P_n, p_{n\to n}) \times Trin(k, P_e, P_o, q_{o\to o}, q_{o\to e}). \quad \text{(A.III.1)}$$

- $P'_n = P_n$ and $P'_o < P_o$:

$$\Pr(\Phi'|\Phi) = \sum_{k=0}^{\min(P'_n, P_o - P_e)} Bin(\max(P_n - k, 0), P_n, p_{n\to n}) \times Trin(k, P_e, P_o, q_{o\to o}, q_{o\to e}). \quad \text{(A.III.2)}$$

Given that solving the model on the grid means that that the firm can have at most $\bar{P_n}$ new products and $\bar{P_o}$ old products, the transition matrix will be ill-defined in certain states, as the probabilities will not sum to one. To alleviate this issue, I normalize each such state by distributing the residual probability across all states with non-zero probability, with weights proportional to ex ante transition probabilities to these states. The results are robust to considering alternative normalization schemes, e.g. attributing the residual probability to current state.

A.III.2 Further details on computing the investment Euler equation

To compute the investment Euler equation, I first take the first-oder condition of Equation (1.10) with respect to K', which yields

$$(1 + \Lambda(E(\cdot)))(-1 - \Psi_{K'}(K, K')) + \beta\mathbb{E}\left[V_{K'}(K', D', \Phi', Z')\right] = 0 \quad \text{(A.III.3)}$$

as well as the envelope condition that gives

$$V_K(K, D, \Phi, Z) = (1+\Lambda(E(\cdot)))(1-\tau)[(1-\phi(1-\xi))\theta K^{\theta-1} Z - \eta\Delta_P] + \tau\delta + (1-\delta) - \Psi_K(K, K'). \quad \text{(A.III.4)}$$

Combining them both yields the investment Euler equation

$$\begin{aligned} 1 \quad = \quad \beta\mathbb{E}\Bigg[\mathscr{F}\Lambda \frac{1}{1+\frac{\psi}{2}i}\Bigg(& (1-\tau)\theta K'^{\theta-1} Z' + 1 - (1-\tau)\delta + \psi i' \left(\frac{1}{2} i' + 1 - \delta\right) \\ & - (1-\tau)\left(\phi'(1-\xi)\theta K'^{\theta-1} Z' + \eta\Delta'_P\right)\Bigg)\Bigg], \end{aligned} \quad \text{(A.III.5)}$$

where

$$\mathscr{F}_{\Lambda} = \frac{(1 + \Lambda'(E(\cdot)))}{(1 + \Lambda(E(\cdot)))} \tag{A.III.6}$$

is the 'external financing discount factor,' see e.g. Eisfeldt and Muir (2016). Thus, the marginal benefit (MB_i) to investment in physical capital and the marginal cost (MC_i) are

$$MB_i = (1-\tau)\theta K'^{\theta-1} Z' + 1 - (1-\tau)\delta + \psi i' \left(\frac{1}{2} i' + 1 - \delta\right), \tag{A.III.7}$$

$$MC_i = 1 + \psi i, \tag{A.III.8}$$

$$MB_i^{\Phi}(\cdot) = -(1-\tau)\left(\phi'(1-\xi)\theta K'^{\theta-1} Z' + \eta \Delta'_P\right), \tag{A.III.9}$$

where $i = I/K$. Thus, we derived Equation (1.12)

$$1 = \beta \mathbb{E}\left[\frac{(1 + \Lambda'(E(\cdot)))}{(1 + \Lambda(E(\cdot)))}\left(\frac{MB_i}{MC_i} + \frac{MB_i^{\Phi}(K', Z', \Delta'_P, \Phi')}{MC_i}\right)\right]. \tag{A.III.10}$$

Verifying that $\partial MB_i^{\Phi}(\cdot)/\partial \phi' < 0$ follows from a direct computation

$$\frac{\partial MB_i^{\Phi}(\cdot)}{\partial \phi'} = -(1-\tau)(1-\xi)\theta K'^{\theta-1} Z' < 0. \tag{A.III.11}$$

A.IV Structural estimation

A.IV.1 Estimation procedure

I follow Lee and Ingram (1991) when estimating the model using structural method of moments. As in Hennessy and Whited (2007), I extract as much of observed heterogeneity from data as possible to make the model- and data-implied moments comparable, that is I use within-transformed variables to compute all moments except for means, which are computed using the raw data. Let the pooled time series of all firms be $x_i = x_1, \ldots, x_N$, where $N = n \times T$ is the total number of firm-year observations. Using the transformed data, I compute a set of moments $h(x_i)$.

I create the simulated moments by first solving solving the model given a vector of parameters $\beta = (\theta, \sigma, \rho, \psi, \eta, \omega, \xi)$ and then generating simulated data y from the model. I simulate $S = 10$ datasets of $N = 2,000$ firm-quarters, following Michaelides and Ng (2000), who find that a simulation estimator behaves well in finite samples if the simulated sample is approximately ten times as large as the actual data sample. The resulting moments in a given simulated sample are given by the vector $h(y_s, \beta)$.

The simulated methods of moments estimator $\widehat{\beta}$ is then the solution to

$$\widehat{\beta} = \arg\min_{\beta} \left[g(x) - g(y, \beta)\right]' W \left[g(x) - g(y, \beta)\right], \tag{A.IV.1}$$

where $g(x) = \frac{1}{N}\sum_{i=1}^{N} h(x_i)$ and $g(y, \beta) = \frac{1}{S}\sum_{s=1}^{S} h(y_s, \beta)$ are the sample means of the actual and model-implied data, and W a positive definite weight matrix. I use the optimal clustered weight matrix constructed as in Bazdresch, Kahn, and Whited (2017). I use simulated annealing to find the optimum to the minimization problem.

Under mild regularity conditions, the SMM estimator is asymptotically normal

$$\sqrt{N}(\widehat{\beta} - \beta) \xrightarrow{d} \mathcal{N}(0, V), \tag{A.IV.2}$$

where V is the covariance matrix adjusted for sampling variation induced by estimating a number of parameters outside of the model, see Newey and McFadden (1994).

A.IV.2 Estimation diagnostics

I compute the diagnostic measure of Andrews, Gentzkow, and Shapiro (2017) to investigate whether the model parameters are locally identified by the underlying moments. The key benefit of the measure is that a reported high sensitivity means not only that the moment is sensitive to the underlying parameter, but also that the parameter is precisely estimated. The results are presented in Table A.4, in which each column corresponds to a structural parameter and each row to a moment. The sensitivities in the table are trimmed at 0.5 in absolute value to ease the presentation of key relationships, similar to Michaels, Page, and Whited (2018).

The results confirm the intuition behind the identification of the structural parameters. For example, the standard deviation and persistence of the profit shock are sensitive to variance and serial correlation of profitability, the depreciation rate δ is closely linked to the mean of investment and the collateral constraint parameter ω is strongly positively related to the mean of net leverage. More importantly, the product-related moments are sensitive to product characteristics ξ and η. It should be noted, however, that the elasticities are only local and, moreover, highly sensitive to the numerical properties of the gradient. Because of that it might appear that some moments are not informative about the underlying parameter while in reality they do provide substantial identifying information. One example of that are the product introduction cost η and the old-product specific revenue discount ξ: while the Andrews, Gentzkow, and Shapiro (2017) sensitivities are smaller than 0.5 in absolute value, over a wider range of the parameter values they are substantial. Moreover, the sign and magnitudes of these elasticities for product-related moments are different, in line with the intuition outlined in Section III.

A.IV.3 Additional results: sample splits

In this subsection I present additional details concerning the cross-sectional estimates. In particular, for each sample split I provide the data- and model-implied moments in addition to the structural estimates.

Moments	θ	σ	ρ	δ	ψ	ω	η	ξ
Mean operating profits	-0.960	-0.542					0.544	
Variance of operating profits		0.650						
Serial correlation of operating profits			0.824				-0.655	
Mean investment	0.805			0.917			-0.686	0.624
Variance of investment	-0.537			-0.512	-0.501			
Serial correlation of investment			-0.591				0.715	
Mean net leverage						0.914		
Variance of net leverage								
Serial correlation of net leverage							0.516	-0.530
Mean old product share			-0.544	0.512				
Variance of old product share			0.521		-0.588			
Serial correlation of old product share								

Table A.4 – Local sensitivity of parameters to moments. The table presents the sensitivities of structural parameters to moments using the normalized diagnostic tool of Andrews, Gentzkow, and Shapiro (2017). Blank entries indicate sensitivities lower than 0.5 in absolute value. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount.

Panel A: Moments

	Small product portfolio		Large product portfolio	
	Simulated	Actual	Simulated	Actual
Mean operating profits	0.0441	0.0391	0.0365	0.0415
Variance of operating profits	0.0012	0.0014	0.0005	0.0005
Serial correlation of operating profits	0.1001	0.1005	0.1969	0.4092
Mean investment	0.0265	0.0239	0.0195	0.0190
Variance of investment	0.0009	0.0011	0.0003	0.0003
Serial correlation of investment	0.1904	0.1137	0.1828	0.1573
Mean net leverage	0.1145	0.1022	0.2027	0.2396
Variance of net leverage	0.0080	0.0111	0.0073	0.0062
Serial correlation of net leverage	0.6137	0.7649	0.6327	0.6961
Mean old product share	0.4427	0.4611	0.4310	0.4278
Variance of old product share	0.0825	0.1074	0.0772	0.0736
Serial correlation of old product share	0.4073	0.3897	0.3908	0.6292

Panel B: Parameters

Small product portfolio								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.7062	0.3294	0.1070	0.1041	0.5509	0.2385	0.0096	0.4334
Std. error	(0.0383)	(0.0452)	(0.0196)	(0.0086)	(0.1087)	(0.0428)	(0.0017)	(0.0547)
Large product portfolio								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.6636	0.2610	0.2799	0.0773	0.5967	0.3043	0.0053	0.6141
Std. error	(0.0520)	(0.0237)	(0.0620)	(0.0028)	(0.1996)	(0.0291)	(0.0007)	(0.0213)

Table A.5 – **Structural estimates and model-implied moments: firms with small and large product portfolios.** This table reports the estimation results for subsamples of firms with small and large product portfolios, classified using the median breakpoint of the effective number of products. The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the estimated parameters and their standard errors. Standard errors are clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix D.1 provides the details about the estimation procedure.

Panel A: Moments

	More competitive		Less competitive	
	Simulated	Actual	Simulated	Actual
Mean operating profits	0.0335	0.0338	0.0458	0.0469
Variance of operating profits	0.0008	0.0010	0.0004	0.0007
Serial correlation of operating profits	0.2626	0.2141	0.2113	0.2519
Mean investment	0.0208	0.0215	0.0218	0.0210
Variance of investment	0.0007	0.0008	0.0004	0.0004
Serial correlation of investment	0.1882	0.1501	0.1934	0.1634
Mean net leverage	0.2238	0.1820	0.1825	0.1598
Variance of net leverage	0.0084	0.0092	0.0059	0.0077
Serial correlation of net leverage	0.5800	0.8213	0.6076	0.7242
Mean old product share	0.4411	0.4322	0.4312	0.4567
Variance of old product share	0.0789	0.0835	0.0763	0.0848
Serial correlation of old product share	0.3882	0.3198	0.3871	0.3331

Panel B: Parameters

Firms exposed to more competitive product markets								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.7457	0.3558	0.3802	0.0810	0.6455	0.3429	0.0064	0.4484
Std. error	(0.0452)	(0.0404)	(0.0997)	(0.0056)	(0.1711)	(0.0397)	(0.0010)	(0.0535)
Firms exposed to less competitive product markets								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.5873	0.1878	0.2813	0.0865	0.5315	0.2720	0.0063	0.6310
Std. error	(0.0347)	(0.0237)	(0.0614)	(0.0056)	(0.0953)	(0.0269)	(0.0012)	(0.0575)

Table A.6 – **Structural estimates and model-implied moments: firms exposed to more and less competitive product markets.** This table reports the estimation results for subsamples of firms exposed to more and less competitive product markets, computed using the exposure of each firm's sales to the HHI of each market, defined by product groups (see Appendix A). The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the estimated parameters and their standard errors. Standard errors are clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix D.1 provides the details about the estimation procedure.

Panel A: Moments

	Less durable		More durable	
	Simulated	Actual	Simulated	Actual
Mean operating profits	0.0422	0.0391	0.0397	0.0412
Variance of operating profits	0.0005	0.0010	0.0006	0.0007
Serial correlation of operating profits	0.2010	0.2224	0.2770	0.0571
Mean investment	0.0230	0.0213	0.0235	0.0212
Variance of investment	0.0006	0.0006	0.0006	0.0006
Serial correlation of investment	0.1480	0.2053	0.2550	0.1980
Mean net leverage	0.2842	0.1574	0.1951	0.1855
Variance of net leverage	0.0047	0.0081	0.0060	0.0086
Serial correlation of net leverage	0.4682	0.8528	0.6066	0.7401
Mean old product share	0.4482	0.4688	0.4188	0.4201
Variance of old product share	0.0948	0.0957	0.0721	0.0742
Serial correlation of old product share	0.3011	0.3026	0.4495	0.8090

Panel B: Parameters

	Firms supplying less durable products							
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.6473	0.2238	0.2905	0.0904	0.4003	0.3757	0.0067	0.6004
Std. error	(0.0429)	(0.0423)	(0.0846)	(0.0031)	(0.1271)	(0.0368)	(0.0011)	(0.0705)
	Firms supplying more durable products							
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.7113	0.2299	0.3270	0.0926	0.6709	0.3104	0.0091	0.4208
Std. error	(0.0434)	(0.0373)	(0.1130)	(0.0069)	(0.0990)	(0.0367)	(0.0010)	(0.0277)

Table A.7 – **Structural estimates and model-implied moments: firms supplying more and less durable products.** This table reports the estimation results for subsamples of firms supplying more and less durable products, computed using the products' average calendar age at exit. The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the estimated parameters and their standard errors. Standard errors are clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix D.1 provides the details about the estimation procedure.

Panel A: Moments

	Lower cost of sales		Higher cost of sales	
	Simulated	Actual	Simulated	Actual
Mean operating profits	0.0307	0.0353	0.0422	0.0447
Variance of operating profits	0.0002	0.0005	0.0012	0.0014
Serial correlation of operating profits	0.3117	0.4049	0.0796	0.1113
Mean investment	0.0178	0.0191	0.0209	0.0233
Variance of investment	0.0003	0.0004	0.0006	0.0009
Serial correlation of investment	0.3123	0.2148	0.2277	0.1962
Mean net leverage	0.2953	0.2179	0.1916	0.1248
Variance of net leverage	0.0044	0.0079	0.0050	0.0091
Serial correlation of net leverage	0.6444	0.7740	0.6749	0.7513
Mean old product share	0.4318	0.4673	0.4381	0.4216
Variance of old product share	0.0780	0.0906	0.0886	0.0867
Serial correlation of old product share	0.4069	0.5927	0.3763	0.3720

Panel B: Parameters

Firms with lower cost of sales								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.7203	0.1283	0.1897	0.0708	0.8094	0.4248	0.0080	0.4114
Std. error	(0.0743)	(0.0556)	(0.0356)	(0.0056)	(0.2971)	(0.0355)	(0.0012)	(0.0588)
Firms with higher cost of sales								
Parameter	θ	σ	ρ	δ	ψ	ω	η	ξ
Estimate	0.6050	0.3756	0.1294	0.0822	0.3190	0.3330	0.0087	0.5586
Std. error	(0.0390)	(0.0334)	(0.0542)	(0.0086)	(0.2499)	(0.0334)	(0.0026)	(0.1030)

Table A.8 – **Structural estimates and model-implied moments: firms with higher and lower cost of sales.** This table reports the estimation results for subsamples of firms with higher and lower selling-related expenses, computed using Compustat item `xsga` scaled by total assets. The estimation is done using simulated method of moments, which chooses model parameters by minimizing the distance between the moments from a simulated panel of firms and their data counterparts. Panel A reports the simulated and actual moments, while Panel B the estimated parameters and their standard errors. Standard errors are clustered at firm-level. θ is the production function curvature; σ is the standard deviation of the profitability shock; ρ is the persistence of the profitability process; δ is the capital depreciation rate; ψ is the investment adjustment cost; ω is the parameter governing the collateral constraint; η is the product introduction cost; ξ is the old-product specific revenue discount. Appendix D.1 provides the details about the estimation procedure.

B Debt, Innovation, and Growth

Appendix B consists of six parts. We solve the static debt case (Theorem 1 and Proposition 1) in Section B.I. Section B.II embeds this static debt model into an industry equilibrium (Theorem 2 and Proposition 2). Section B.III derives the steady state firm size distribution. Section B.IV closes the model in general equilibirum. Section B.V solves the model with refinancing (Theorem 3 and Theorem 4). Section B.VI provides additional numerical results from the model with refinancing.

B.I Debt Financing

First, we establish the individual firm results (Theorem 1) and intermediate results that show that the equity value is continuous and decreasing in f and c (Lemma 1). Finally, we prove the comparative statics results (Proposition 1).

In the static debt model an incumbent's coupon is constant. Therefore, we write the equity value as

$$E(p) = E(p,c) \tag{B.I.1}$$

and use this notation when it does not lead to confusion. Furthermore, the equity value indirectly depends on the parameters f and c. When necessary, we make this dependence explicit by writing $E(p|f,c)$.

Theorem 1 (Equity Value)**.** *A unique solution to the equity value* (2.7) *exists. Equity value is non-decreasing in p and therefore the optimal default strategy is a barrier default strategy* $\tau_D = \inf\{t > 0 | P_t \le p_D\}$. *If the optimal level of R&D is interior* $((\lambda,\theta) \in (0,\bar{\lambda}) \times (0,1))$, *it solves*

$$\mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c)\right] - E(p,c) = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial \lambda}, \tag{B.I.2}$$

$$\lambda\frac{\partial \mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c)\right]}{\partial \theta} = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial \theta}. \tag{B.I.3}$$

Proof. The proof has several steps. First, we establish existence of the equity value. Then we show that it is increasing in the number of product lines p. Finally, we derive the first-order conditions for the internal optimal level of R&D.

1. Equation (2.8) shows that the equity value for $p \in \{1, ..., \bar{p}\}$ can be rewritten as

$$E(p) = \sup_{\theta,\lambda,\tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} (1-\pi)(p-c-q(p,\lambda,\theta)) dt \right] \right. \tag{B.I.4}$$

$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \left(\lambda \mathbb{E}^\theta \left[E(\min\{p+x, \bar{p}\}) \right] + pfE(p-1) \right) dt \right] \right\}. \tag{B.I.5}$$

with $E(0) = 0$. Define $\mathcal{M}(E)$ as the mapping

$$\mathcal{M}(E) = \sup_{\theta,\lambda,\tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} (1-\pi)(p-c-q(p,\lambda,\theta)) dt \right] \right. \tag{B.I.6}$$

$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \left(\lambda \mathbb{E}^\theta \left[E(\min\{p+x, \bar{p}\}) \right] + pfE(p-1) \right) dt \right] \right\}. \tag{B.I.7}$$

Any fixed point of this mapping is bounded from above by $\bar{p}/r$ and from below by zero. Furthermore, the mapping is monotone in E and finally,

$$\mathcal{M}(E+L) = \sup_{\theta,\lambda,\tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} (1-\pi)(p-c-q(p,\lambda,\theta)) dt \right] \right. \tag{B.I.8}$$

$$+ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \lambda \mathbb{E}^\theta \left(\left[E(\min\{p+x, \bar{p}\}) \right] + L \right) dt \right] \tag{B.I.9}$$

$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} pf \left(E(p-1) + L \right) dt \right] \right\}, \tag{B.I.10}$$

$$\mathcal{M}(E+L) \leq \mathcal{M}(E) + \frac{\bar{\lambda} + \bar{p}f}{r + \bar{\lambda} + \bar{p}f} L, \tag{B.I.11}$$

because $\lambda \leq \bar{\lambda}$ by assumption. Therefore, the mapping $\mathcal{M}(E)$ satisfies Blackwell's sufficient conditions for a contraction (see Theorem 3.3 on page 54 in Stokey, Lucas, and Prescott, 1989) and it is a contraction mapping, which implies that a fixed point exists and is unique. The equity value is the fixed point of this mapping.

2. The next step is to show that equity value is non-decreasing in p. We do this by showing that having one extra product line improves a firm's cash flows even if shareholders run the firm as if it does not have this extra product line. Assume today the firm has $p+1$ product lines and that it separates one product line and runs the firm as if it had only p product lines. The firm receives cash flows from this extra product line until the product line becomes obsolete, the firm's non-separated number of product lines reaches $\bar{p}$ or zero, or the firm defaults. The firm receives the extra (gross) profits from this separated product line but it also incurs higher R&D costs (since they depend on P_t). The equity value of this $p+1$ firm with a separated product line is given by

$$E(p) + \mathbb{E}_p \left[\int_0^{\tau_D(p) \wedge \tau_0(p) \wedge \tau_{\bar{p}}(p)} e^{-(r+f)t} (1-\pi) \left(1 - q(P_t+1, \lambda_t, \theta_t) + q(P_t, \lambda_t, \theta_t) \right) dt \right],$$

(B.I.12)

where $\tau_D(p)$ is the optimal default time of a firm that starts with p product lines, $\tau_0(p)$ is the first time the firm has zero product lines if it starts with p product lines, and $\tau_{\bar{p}}(p)$ is the first time a firm with p product lines has $\bar{p}$ product lines. The first term is the cash flows from the p product line firm, and the second term is the cash flow from the separated product line minus the changes in R&D costs. The conditions on the R&D cost function ensure that the second term is non-negative. Furthermore, the optimal R&D and default strategy followed by a $p+1$ product line firm (weakly) dominates the one chosen by a firm that separates one product line and uses the strategy from a p product line firm. Therefore,

$$E(p) \leq E(p) \tag{B.I.13}$$

$$+\mathbb{E}_p\left[\int_0^{\tau_D(p)\wedge\tau_0(p)\wedge\tau_{\bar{p}}(p)} e^{-(r+f)t}(1-\pi)\left(1-q(P_t+1,\lambda_t,\theta_t)+q(P_t,\lambda_t,\theta_t)\right)dt\right] \tag{B.I.14}$$

$$\leq E(p+1), \tag{B.I.15}$$

which shows that the equity value $E(p)$ is non-decreasing in p. This also implies that a barrier default strategy is the optimal default strategy.

3. Finally, the (internal) optimal levels of R&D should satisfy the first-order conditions that follow from equation (2.8).

□

Lemma 1. *The equity value $E(p|f,c)$ is continuous and non-increasing in f and c. If $E(p|f,c) > 0$ then the equity value is decreasing in f and c.*

Proof. We first show that equity value decreases with the rate of creative destruction f.

1. Fix $f_2 < f_1$. Let P_t^1 be the number of product lines of a firm facing a rate of creative destruction f_1. We know that

$$E(p|f_1) = \mathbb{E}_p\left[\int_0^{\tau_D^1\wedge\tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - q(P_t^1,\lambda_t^1,\theta_t^1)\right)dt\right], \tag{B.I.16}$$

where $\{\lambda_t^1,\theta_t^1\},\tau_D^1$ are shareholders optimal strategy given f_1. The dynamics of P_t^1 are

$$dP_t^1 = dI_t^1 - dO_t^1 = \max\left(Y_t^1, \bar{p} - P_{t^-}^1\right)dN_t^1 - dO_t^1 \tag{B.I.17}$$

with

$$\mathbb{E}\left[dP_t^1\right] = \lambda_t^1\mathbb{E}^{\theta_t^1}\left[\max\left(Y_t^1,\bar{p}-P_{t^-}^1\right)\right]dt - f_1P_{t^-}^1dt. \tag{B.I.18}$$

2. Define $\tilde{P}_t^2$ as,

$$d\tilde{P}_t^2 = d\tilde{I}_t^1 - X_t dO_t^1 - dH_t, \tag{B.I.19}$$

where

$$\tilde{I}_t^1 = \max\left(Y_t^1, \bar{p} - \tilde{P}_{t^-}^2\right) dN_t^1, \tag{B.I.20}$$

$$X_t \sim Bin\left(1, \frac{f_2}{f_1}\right), \tag{B.I.21}$$

$$H_t \sim Poisson\left(f_2\left(\tilde{P}_t^2 - P_t^1\right)\right). \tag{B.I.22}$$

The construction of X_t and H_t implies that,

$$\mathbb{E}_t\left[X_t d\tilde{O}_t^1 - dH_t\right] = \frac{f_2}{f_1} f_1 P_{t^-}^1 dt - f_2\left(\tilde{P}_t^2 - P_{t^-}^1\right) dt = f_2 \tilde{P}_{t^-}^2 dt. \tag{B.I.23}$$

These dynamics imply that $\tilde{P}_t^2$ evolves according to the R&D strategy $\{\lambda_t^1, \theta_t^1\}$ given a failure intensity of f_2. The construction $\tilde{P}_t^2$ ensures that

$$P_t^1 \le \tilde{P}_t^2. \tag{B.I.24}$$

If $\tilde{P}_{t^-}^2 = P_{t^-}^1$ then innovation dynamics are the same $dI_t^1 = d\tilde{I}_t^2$. Furthermore, product line failure is higher for $P_{t^-}^1$ since $f_2/f_1 < 1$ and if a product line fails for $\tilde{P}_{t^-}^2$ then it fails for $P_{t^-}^1$. Therefore, if $\tilde{P}_{t^-}^2 = P_{t^-}^1$ then $\tilde{P}_t^2 \ge P_t^1$. If $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then product line failure can never imply $\tilde{P}_t^2 < P_t^1$ since product lines drop by only one. Furthermore, by construction innovation happens at the same time and the number of product lines created for both is either Y_t or $\bar{p}$ is reached. This implies that if at time t product lines are created and $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then $\tilde{P}_t^2 = \min\left(\tilde{P}_{t^-}^2 + Y_t, \bar{p}\right) \ge \min\left(P_{t^-}^1 + Y_t, \bar{p}\right) = P_t^1$. Therefore, if $\tilde{P}_{t^-}^2 > P_{t^-}^1$ then $\tilde{P}_t^2 \ge P_t^1$.

3. Given the assumptions on the cost function the equity value satisfies

$$E(p|f_1) = \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - q(P_t^1, \lambda_t^1, \theta_t^1)\right) dt\right] \tag{B.I.25}$$

$$\le \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(\tilde{P}_t^2 - c - q(\tilde{P}_t^2, \lambda_t^1, \theta_t^1)\right) dt\right] \tag{B.I.26}$$

$$\le E(p|f_2). \tag{B.I.27}$$

If the equity value is positive then $\tau_D^1 \wedge \tau_0^1 > 0$, and the second inequality becomes a strict inequality. This shows that $E(p|f)$ is non-increasing in f and strictly decreasing in f when $E(p|f) > 0$.

4. The next step is showing that the equity value is continuous in f. The mapping $\mathcal{M}(E|f)$ is continuous in f. Therefore, for every $\epsilon > 0$ there exists a $\delta > 0$ such that for $f' \in$

$(f-\delta, f+\delta)$,

$$\|\mathcal{M}(E(p|f)|f') - E(p|f)\| = \|\mathcal{M}(E(p|f)|f') - \mathcal{M}(E(p|f)|f)\| < \epsilon. \quad \text{(B.I.28)}$$

Fix one such ϵ. Define $\mathcal{M}^m(E|f)$ as applying the mapping $\mathcal{M}(\cdot|f)$ m times to E. Applying the mapping $\mathcal{M}$ again leads to,

$$\|\mathcal{M}^2(E(p|f)|f') - \mathcal{M}(E(p|f)|f')\| < U\|\mathcal{M}(E(p|f)|f') - E(p|f)\| < U\epsilon. \quad \text{(B.I.29)}$$

where

$$U = \frac{\bar{\lambda} + \bar{p}f'}{r + \bar{\lambda} + \bar{p}f'}. \quad \text{(B.I.30)}$$

This process can be repeated and leads to

$$\|\mathcal{M}^{m+1}(E(p|f)|f') - \mathcal{M}^m(E(p|f)|f')\| < U^m\epsilon. \quad \text{(B.I.31)}$$

Therefore, the distance between $E(p|f)$ and $E(p|f')$ is bounded by

$$\begin{aligned}
\|E(p|f) - E(p|f')\| &= \|E(p|f) - \mathcal{M}^\infty(E(p|f)|f')\| && \text{(B.I.32)}\\
&\le \sum_{i=0}^{\infty} \|\mathcal{M}^{i+1}(E(p|f)|f') - \mathcal{M}^i(E(p|f)|f')\| && \text{(B.I.33)}\\
&< \epsilon \sum_{i=0}^{\infty} U^i && \text{(B.I.34)}\\
&= \epsilon \frac{1}{1-U} && \text{(B.I.35)}\\
&= \epsilon \frac{r + \bar{\lambda} + \bar{p}\left(f + \left(f' - f\right)\right)}{r} && \text{(B.I.36)}\\
&= \epsilon \frac{r + \bar{\lambda} + \bar{p}\left(f + \delta\right)}{r}. && \text{(B.I.37)}
\end{aligned}$$

Take an $\tilde{\epsilon} > 0$ and set

$$\epsilon = \tilde{\epsilon}\frac{r}{r + \bar{\lambda} + \bar{p}(f+1)}. \quad \text{(B.I.38)}$$

Then define $\tilde{\delta} = \min\{\delta, 1\}$. We get that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$

$$\frac{r + \bar{\lambda} + \bar{p}\left(f + \left(f' - f\right)\right)}{r} \le \frac{r + \bar{\lambda} + \bar{p}\left(f + 1\right)}{r} = \tilde{\epsilon}. \quad \text{(B.I.39)}$$

This implies that for every $\tilde{\epsilon} > 0$ there exists a $\tilde{\delta} > 0$ such that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$,

$$\|E(p|f) - E(p|f')\| < \tilde{\epsilon} \quad \text{(B.I.40)}$$

Therefore, $E(p|f)$ is continuous in f. The same argument shows that $E(p|c)$ is continuous in c.

5. The final step is showing that the equity value is non-increasing in c and decreasing if $E(p|c) > 0$. The mapping $\mathcal{M}(E|c)$ is non-increasing in c and non-decreasing in E. Therefore, for a $c < c'$ we have that

$$
\begin{aligned}
E(p|c) &= \mathcal{M}(E(p|c)|c) && \text{(B.I.41)}\\
&\geq \mathcal{M}(E(p|c)|c') && \text{(B.I.42)}\\
&\geq \mathcal{M}^2(E(p|c)|c') && \text{(B.I.43)}\\
&\geq \mathcal{M}^{n>2}(E(p|c)|c') && \text{(B.I.44)}\\
&\geq \mathcal{M}^{\infty}(E(p|c)|c') && \text{(B.I.45)}\\
&= E(p|c'), && \text{(B.I.46)}
\end{aligned}
$$

which proves the result. The first inequality becomes a strict inequality when $E(p|c) > 0$, which shows the decreasing result.

□

Proposition 1 (Comparative Statics: Equity Value)**.** *If $E(p,c) > 0$, equity value is decreasing in the tax rate π, the coupon c, the rate of creative destruction f, and the cost $q(p,\lambda,\theta)$ of performing R&D.*

Proof. The result for c and f follows from Lemma 1. Take any other parameter (or the function $q(p,\lambda,\theta)$) and call it Ξ. If $E(p|\Xi) > 0$ then the mapping $\mathcal{M}(E|\Xi)$ is decreasing in Ξ and increasing E. Therefore, we have

$$
\begin{aligned}
E(p|\Xi) &= \mathcal{M}(E(p|\Xi)|\Xi) && \text{(B.I.47)}\\
&> \mathcal{M}(E(p|\Xi)|\Xi') && \text{(B.I.48)}\\
&\geq \mathcal{M}^2(E(p|\Xi)|\Xi') && \text{(B.I.49)}\\
&\geq \mathcal{M}^{n>2}(E(p|\Xi)|\Xi') && \text{(B.I.50)}\\
&\geq \mathcal{M}^{\infty}(E(p|\Xi)|\Xi') && \text{(B.I.51)}\\
&= E(p|\Xi'), && \text{(B.I.52)}
\end{aligned}
$$

which proves the result. □

B.II Industry Equilibrium

We first establish the existence of an industry equilibrium (Theorem 2). We then derive conditions under which there is a unique rate of creative destruction (Proposition 2).

To establish the existence of an equilibrium, we make the following assumption:

Assumption 1. *For the firm value, the order of the limit with respect to f and the supremum over c can be interchanged:*

$$\lim_{f'\to f} \sup_c \left\{E(p,c|f') + (1-\xi)D(p,c|f')\right\} = \sup_c \lim_{f'\to f} \left\{E(p,c|f') + (1-\xi)D(p,c|f')\right\}. \quad \text{(B.II.1)}$$

Theorem 2 (Equilibrium Existence)**.** *If Assumption 1 holds then there exists an industry equilibrium* Ψ^*.

Proof. The proof has several steps:

1. The first step is showing that the equity value converges to zero when $f \to \infty$. Assume this is not the case then for some p we have that $E(p|f) > 0$ when $f \to \infty$. From equation (2.8) it follows that for any $p > 0$ with $E(p|f) > 0$

$$0 = \frac{-rE(p|f) + (1-\pi)(p-c)}{f} \tag{B.II.2}$$

$$+ \frac{\max_{(\lambda,\theta)} \left\{\lambda \left(\mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\})\right] - E(p|f)\right) - (1-\pi)q(p,\lambda,\theta)\right\}}{f} \tag{B.II.3}$$

$$+ p\left\{E(p-1|f) - E(p|f)\right\}. \tag{B.II.4}$$

Given that $E(p|f) \leq \bar{p}/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$0 = p\left\{E(p-1|f=\infty) - E(p|f=\infty)\right\} \tag{B.II.5}$$

and therefore that

$$E(p|f=\infty) = E(p-1|f=\infty) \tag{B.II.6}$$

for any p for which $E(p|f=\infty) > 0$. Given that $E(0|f=\infty) = 0$ this implies that

$$E(p|f=\infty) = 0, \tag{B.II.7}$$

which is a contradiction. Therefore, the equity value does converge to zero.

2. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. Therefore, firm value $V(f,\theta)$ and also entrant value $E^e(f)$ goes to zero as $f \to \infty$.

3. Define firm value as

$$F(p_0|f,c) = E(p_0|f,c) + (1-\xi)D(p_0|f,c). \tag{B.II.8}$$

4. By Lemma 1, equity value is continuous in f and therefore

$$\lim_{f'\to f} \|E(p|f,c) - E(p|f',c)\| = 0. \tag{B.II.9}$$

As a result, the dynamics of P_t will also be the same under f and $f' \to f$. If in addition the default threshold is the same then

$$\lim_{f'\to f} \|D(p|f,c) - D(p|f',c)\| = 0 \tag{B.II.10}$$

since the default times will converge. Since the equity value is continuous in f, if the default threshold is not the same then at f shareholders must be exactly indifferent between default and no default. Take an arbitrary small ϵ, because the equity value is decreasing in c, for either $c-\epsilon$ or $c+\epsilon$ the default threshold under $f' \to f$ will be the same as the default threshold under f (and c). Furthermore, because the equity value is continuous in f and c the dynamics of P_t will be continuous in both as well. This implies that

$$\lim_{\epsilon\to 0}\lim_{f'\to f} \|D(p|f,c) - D(p|f',c\pm\epsilon)\| = 0 \tag{B.II.11}$$

since the default time will converge. This implies that

$$\lim_{\epsilon\to 0}\lim_{f'\to f} |F(p_0|f,c) - F(p_0|f',c\pm\epsilon)| = 0, \tag{B.II.12}$$

5. The previous step shows that for a given f, c, and $f' \to f$ there exists an $c' = \lim_{\epsilon\to 0} c\pm\epsilon$ such that the firm value is continuous in f. This implies that

   $$\sup_c F(p_0|f',c) = \sup_c \lim_{f'\to f} F(p_0|f',c) = \lim_{f'\to f}\sup_c F(p_0|f',c). \tag{B.II.13}$$

   The last step follows from Assumption 1. This shows that $\sup_c F(p_0|f,c)$ is continuous in f.

6. The above also implies that

   $$V(f,\theta) = \mathbb{E}^{\theta}\left[\sup_c \{F(p_0|f,c)\}\right] \tag{B.II.14}$$

   with $p_0 = \min(y,\bar{p})$ and $y \sim Bin(n,\theta)$ is continuous in f and θ.

7. If there exists an $E^e(f) \geq H(1-\pi)$ then the intermediate value theorem ensures existence of an f such that $E^e(f) = H(1-\pi)$, which is an industry equilibrium.

8. If for all f $E^e(f) < H(1-\pi)$ then entry is never optimal. Given the fact that P_t is non-decreasing for $f = 0$, it follows that for $p > 0$ and $c = 0$ the equity value is positive $E(p|c=0, f=0) > 0$. Therefore, a steady state equilibrium exists in which all firms have $\bar{p}$ product lines and no one innovates.

□

Proposition 2 (Uniqueness of the Rate of Creative Destruction)**.** *If the debt value is strictly decreasing in f then all equilibria have the same rate of creative destruction f^*.*

Proof. The proof has several steps:

1. First, we show the entrant value is strictly decreasing in f. Since the equity value (for any positive value) and debt value are strictly decreasing in f, the optimal firm value

$V(f,\theta)$ must be strictly decreasing in f as well. Take an $f_1 < f_2$ then

$$V(f_2,\theta) = \mathbb{E}^{\theta}\left[E(\min\{x,\bar{p}\}|f_2,c_2) + D(\min\{x,\bar{p}\}|f_2,c_2)\right] \quad \text{(B.II.15)}$$

$$< \mathbb{E}^{\theta}\left[E(\min\{x,\bar{p}\}|f_1,c_2) + D(\min\{x,\bar{p}\}|f_1,c_2)\right] \quad \text{(B.II.16)}$$

$$\leq V(f_1,\theta), \quad \text{(B.II.17)}$$

where c_2 is the firm value maximizing coupon given θ and f_2. Because the entrants value is

$$E^e(f) = \sup_{\{\lambda,\theta\}}\left(\frac{\lambda V(f,\theta) - (1-\pi)q_E(\lambda,\theta)}{r+\lambda}\right), \quad \text{(B.II.18)}$$

it is also strictly decreasing in f.

2. There are now two cases. If $E^e(0) \leq H(1-\pi)$ then $E^e(f) < H(1-\pi)$ for all $f > 0$ and the only equilibrium rate of creative destruction is $f^* = 0$. If $E^e(0) > H(1-\pi)$ then there exists a unique f^* such that

$$E^e(f^*) = H(1-\pi), \quad \text{(B.II.19)}$$

which is a condition that needs to be satisfied in equilibrium if $f^* > 0$. This proves that any equilibrium must have a rate of creative destruction f^*.

□

Proposition 3 (Debt versus No Debt)**.** *Let $f^*_{No\ Debt}$ be the equilibrium rate of creative destruction in case firms are restricted to have no debt. Then there exists an industry equilibrium with a rate of creative destruction*

$$f^* \geq f^*_{No\ Debt}. \quad \text{(B.II.20)}$$

Proof. The proof has several steps:

1. By assumption the option to issue debt increases shareholder value. This implies that,

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}). \quad \text{(B.II.21)}$$

2. If $f^*_{No\ Debt} = 0$ then from Theorem 2 it directly follows that there exists an

$$f^* \geq f^*_{No\ Debt}. \quad \text{(B.II.22)}$$

3. If $f^*_{No\ Debt} > 0$ then

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}) = H(1-\pi). \quad \text{(B.II.23)}$$

The proof of Theorem 2 shows that the entrant value is continuous in f and that

$\lim_{f\to\infty} E^e(f) = 0$. Therefore, there exists an $f^* \geq f^*_{No\,Debt}$ such that

$$E^e(f^*) = H(1-\pi). \tag{B.II.24}$$

This f^* is an industry equilibrium.

□

B.III Steady State Distribution

In this appendix, we derive the steady state firm size distribution. Let $S(p|p_0)$ be the steady state distribution of firms that started initially with p_0 product lines and coupon $c^*(p_0)$. If firms with p product lines decided to default, then $S(p|p_0) = 0$. Assuming the firm does not default, the steady state distribution for p product lines $S(p|p_0)$ solves

$$0 = -\underbrace{\lambda(p|p_0) * (1-\psi(p,0,n,\theta(p|p_0))) * S(p|p_0)}_{\textit{Exit: Product line creation}} \tag{B.III.1}$$

$$- \underbrace{f * p * S(p|p_0)}_{\textit{Exit: Product line becomes obsolete}} \tag{B.III.2}$$

$$+ \underbrace{\sum_{i=1}^{\min(n,p)} \lambda(p-i|p_0) * \psi(p-i,i,n,\theta(p-i|p_0)) * S(p-i|p_0)}_{\textit{Entry: Product line creation}} \tag{B.III.3}$$

$$+ \underbrace{f * (p+1) * S(p+1|p_0)}_{\textit{Entry: Product line becomes obsolete}} \tag{B.III.4}$$

$$+ \underbrace{s * \mathbb{1}_{\{p=p_0\}}}_{\textit{Entry: Entrants}}, \tag{B.III.5}$$

where $\psi(p,X,n,\theta)$ is the pdf of $\min(X, \bar{p}-p)$ with $X \sim Bin(n,\theta)$.

Firms can exit for two reasons. First, they can create new product lines (first term). Second, one of their product lines can become obsolete (second term). Firms can enter for three reasons. First, a firm with less than p product lines can create new product lines and become a p-product line firm (third term). Second, a product line of a firm with $p+1$ product lines can become obsolete (fourth term). Third, there is endogenous entry (fifth term).

The term s determines the flow of entrants that become incumbents with p_0 product lines. In steady state, the constant s ensures that the outflow of firms is equal to the inflow of firms. The mass of firms that flow out is given by

$$f * (\min(p_D(p_0),0)+1) * S(\min(p_D(p_0),0)+1|p_0), \tag{B.III.6}$$

where $p_D(p_0)$ is the optimal default threshold. Given that the optimal default strategy is of a barrier type, firms can exit by either flowing into the default state or into the state with zero product lines, which one of the two happens first. Setting

$$s = f * (\min(p_D(p_0),0)+1) * S(\min(p_D(p_0),0)+1|p_0) \tag{B.III.7}$$

ensures that the inflow of entering firms is equal to the outflow of defaulting firms.

Given that $p_0 \sim \min\{x, \bar{p}\}$ with $x \sim Bin(n, \theta_e)$, the steady state firm size distribution is

$$S(p) = \sum_{p_0=1}^{n} \frac{\psi(0, p_0, n, \theta_e) * S(p|p_0)}{1 - \psi(0, p_0, n, \theta_e)}. \tag{B.III.8}$$

B.IV General Equilibrium Setup

In this appendix, we embed our model into a general equilibrium setup. This endogenizes the growth rate of the economy, the labor supply, and the interest rate. The general equilibrium setup is similar to Klette and Kortum (2004) and leads to a stationary equilibrium with a balanced growth path.

Production

There is a unit mass of differentiated goods in the economy, which are indexed by $i \in [0,1]$. A measure L^P of labor is used for production, a measure $L^{R\&D}$ of labor performs R&D, and a meausre L^E of labor is used to generate entrants. Labor supply L^S is perfectly elastic, and it receives a wage w per unit supplied in each of these activities.

Incumbent firms use labor and installed product lines to produce goods. An improvement in the production technology increases the amount of the consumption good that one unit of labor produces.

For each type of product there is a leading producer, as in the industry equilibrium model. The production technology of good i's leading producer is q_t^i and determines the number of products that one unit of labor produces.

A firm that innovates on product i improves the production technology and becomes the leading producer. Each innovation is a quality improvement applying to a good drawn at random. The innovation increases the production technology proportionally. That is, when an innovation arrives at time t, the production technology increases from $q_{t^-}^i$ to $q_t^i = (1+\delta) q_{t^-}^i$ with $\delta > 0$.

A firm that is the leading producer for product i is a monopolist for that good and can choose to supply or not supply that good. If the firm supplies the good then it uses one unit of labor to generate q_t^i units of the product. If the firm does not supply the good, its output and profits are zero.[1]

Let y_t^i be the amount of good i produced at time t. As in Klette and Kortum (2004) or Aghion, Bloom, Blundell, Griffith, and Howitt (2005), the aggregate consumption good is produced using a logarithmic aggregator

$$\ln(Y_t) = \int_0^1 \ln\left(y_t^i\right) di, \tag{B.IV.1}$$

[1] We can obtain equivalent results when each production line has as production function $q_t^i(l - \mathbb{1}_{\{l \geq 1\}} k(l-1))$ where l is the amount of labor used, $k(0) = 0$, $k'(\cdot) > 0$, and the firm produces the maximum amount of the good among production quantities that maximize its profits.

with Y_t the aggregate production of the consumption good.[2]

Innovation

Firms can invest in R&D. Investment in R&D leads to product innovations, which improve the amount of a product that one unit of labor produces. R&D investment costs come in the form of labor costs. Innovation costs are a function of the wage rate multiplied by the number of hours spend on R&D:

$$q(p,\lambda,\theta) = w * \tilde{q}(p,\lambda,\theta). \tag{B.IV.3}$$

Therefore, a firm with p products that has an R&D policy (λ,θ) requires $\tilde{q}(p,\lambda,\theta)$ units of labor.[3] We define the innovation cost function for an entrant in a similar way:

$$q_E(\lambda,\theta) = w * \tilde{q}_E(\lambda,\theta). \tag{B.IV.5}$$

Default and Entry

Debt distorts investment in R&D and can lead to default. If a firm with profitable product lines defaults, creditors continue producing these goods until the products become obsolete after which they exit. Furthermore, creditors do not perform R&D and run the firm as an all-equity financed firm. Their expected payoff in default is therefore

$$\frac{(1-\pi)P_{\tau_D}(1-w)}{r+f}. \tag{B.IV.6}$$

This setup implies that the debt value is the same as in the industry equilibrium model with $\alpha = 0$; see equation (2.14). In this model, default costs are therefore uniquely related to the distortions in investment policy triggered by default (and debt overhang).

Because firms exit, in a stationary equilibrium there must be entry. As in the industry equilibrium model, entrants have no product lines but perform R&D in the hope of developing innovations, so they can become the leading producer for at least one product. In the industry equilibrium model, entrants pay a fixed entry cost $H(1-\pi)$ to become an entrant. In our general equilibrium model, these fixed costs are replaced by labor costs (as in e.g. Klette and Kortum, 2004 or Lentz and Mortensen, 2008). An entrepreneur can hire one unit of labor, which costs him after tax $w(1-\pi)$, and that generates an idea with Poisson intensity h. Once

[2] This is a limiting case of the Dixit and Stiglitz (1977) aggregator when the elasticity of substitution ϵ goes to 1

$$\lim_{\epsilon\to 1}\ln\left(\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right) = \lim_{\epsilon\to 1}\frac{\ln\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)}{\frac{\epsilon-1}{\epsilon}} = \lim_{\epsilon\to 1}\frac{\int_0^1 \ln\left(y_t^i\right)\left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di}{\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di} = \int_0^1 \ln\left(y_t^i\right) dt. \tag{B.IV.2}$$

[3] The condition on the R&D cost that ensures that the equity value is non-decreasing in p, see (2.5), in the general equilibrium framework boils down to

$$q(p+1,\lambda,\theta) - q(p,\lambda,\theta) \leq 1 - w. \tag{B.IV.4}$$

the entrepeneur has generated this idea he can become an entrant. Since in equilibrium the cost and benefits should equate for an entrepreneur, the free entry condition becomes

$$E^e(f)h = w(1-\pi). \tag{B.IV.7}$$

Representative Household

There is a representative household with logarithmic preferences:

$$U_0 = \int_0^\infty e^{-rt}\left(\ln(\mathscr{C}_t) - wL_t^S\right)dt \tag{B.IV.8}$$

where $\mathscr{C}_t$ is aggregate consumption and r is the discount rate. The representative household's labor supply L_t^S is perfectly elastic at a wage rate w.

Equilibrium Properties

Since our model is a closed economy and all costs come in the form of labor costs, consumption equals production for each good i, and therefore aggregate consumption and production are also equal

$$\mathscr{C}_t = Y_t. \tag{B.IV.9}$$

The logarithm of aggregate consumption $\ln(\mathscr{C}_t)$ is the numeraire in this economy. The representative household owns all (financial) assets in the economy and receives all labor income.

Using the logarithm of consumption $\ln(\mathscr{C}_t)$ as the numeraire, the representative household's optimal consumption across goods implies that the price of good i should be

$$\frac{1}{y_t^i} = p_t^i, \tag{B.IV.10}$$

where the marginal benefit of good i is equal to its marginal cost. The average cost of production are

$$\frac{w}{q_t^i}. \tag{B.IV.11}$$

Therefore, the profits on product i are given by

$$\pi_t^i = q_t^i\left(\frac{1}{q_t^i} - \frac{w}{q_t^i}\right) = 1 - w. \tag{B.IV.12}$$

This result implies that the equity value is as in the industry equilibrium framework (see equation (2.7)), except that the profit flow from a product line is $1 - w$ instead of 1 and the R&D cost depend on the wage rate w (see equation (B.IV.3)).

In equilibrium, the growth rate g, the interest rate $\tilde{r}$, and the labor supply L^S are determined by market clearing. Since we use the logarithm of consumption as the numeraire, the agent is effectively risk-neutral in the numeraire and therefore,[4]

$$\tilde{r} = r. \tag{B.IV.20}$$

Consumption grows at a rate of

$$d\ln(\mathscr{C}_t) = d\int_0^1 \ln(y_t^i)di = \ln(1+\delta)f\,dt = g\,dt \tag{B.IV.21}$$

where f is the rate of creative destruction in the economy, which results from innovations by incumbents and entrants.

Finally, there is a labor supply L^S which is used for production L^P, for research $L^{R\&D}$, and to generate entrants L^E:

$$L^P = 1, \tag{B.IV.22}$$

$$L^{R\&D} = \int_{\mathscr{F}_t^I} \tilde{q}(P_t^j, \lambda_t^j, \theta_t^j)dj + \tilde{q}_E(\lambda_E, \theta_E)\int_{\mathscr{F}_t^E} dj, \tag{B.IV.23}$$

where subscript j indicates firm j, $\mathscr{F}_t^I$ is the set of active incumbents, and $\mathscr{F}_t^E$ is the set of active entrants. The labor supply is set such that the labor market clears at a wage w:

$$L^S = L^P + L^{R\&D} + L^E. \tag{B.IV.24}$$

[4]The risk-free interest rate $\tilde{r}$ should be set such that a household is indifferent between consuming today or tomorrow. Given that there is no aggregate uncertainty, the Hamiltonian for the consumption smoothing problem, with $\hat{\mathscr{C}}_t = \ln(\mathscr{C}_t)$ logarithm of aggregate consumption, $\hat{Y}_t = \ln(Y_t)$ logarithm of aggregate production, S_t savings, and κ_t the co-state, is

$$H(\hat{\mathscr{C}}, \hat{Y}, S, \tilde{r}, \kappa, t) = e^{-rt}u(\hat{\mathscr{C}}) + \kappa[\tilde{r}S + \hat{Y} - \hat{\mathscr{C}}] \tag{B.IV.13}$$

where

$$u(\hat{\mathscr{C}}) = \hat{\mathscr{C}} = \ln(\mathscr{C}). \tag{B.IV.14}$$

The optimal solution satisfies the following conditions

$$H_{\hat{\mathscr{C}}}(\hat{\mathscr{C}}_t, \hat{Y}_t, S_t, \tilde{r}_t, \kappa_t, t) = e^{-rt}u'(\hat{\mathscr{C}}_t) - \kappa_t = 0, \tag{B.IV.15}$$

$$H_S(\hat{\mathscr{C}}_t, \hat{Y}_t, S_t, \tilde{r}_t, \kappa_t, t) = \kappa_t\tilde{r}_t = -\frac{d\kappa_t}{dt}, \tag{B.IV.16}$$

see Chapter 7 in Acemoglu (2009). Taking the total derivative yields

$$0 = -re^{-rt}u'(\hat{\mathscr{C}}_t)dt + e^{-rt}u''(\hat{\mathscr{C}}_t)d\hat{\mathscr{C}}_t - d\kappa_t \tag{B.IV.17}$$

$$= -r\kappa_t dt + 0 + \tilde{r}_t\kappa_t dt \tag{B.IV.18}$$

$$\tilde{r}_t = r, \tag{B.IV.19}$$

which is the Euler equation that the interest rate $\tilde{r}_t$ solves.

The utility of the representative household is

$$U_0 = \int_0^\infty e^{-rt}(\ln(\mathscr{C}_0) + gt - wL^S)dt \tag{B.IV.25}$$

$$= \frac{\ln(\mathscr{C}_0) - wL^S}{r} + \left[\frac{-1}{r}e^{-rt}gt\right]_0^\infty + \int_0^\infty \frac{1}{r}e^{-rt}g\,dt \tag{B.IV.26}$$

$$= \frac{r\ln(\mathscr{C}_0) + g - rwL^S}{r^2}. \tag{B.IV.27}$$

The higher the growth rate in the economy the higher the representative household's utility.

The formal equilibrium definition is

Definition 2 (General Equilibrium)**.** *The parameters and policies*

$$\Psi^* = \{g^*, L^{S*}, r^*, f^*, c^*(p_0), \lambda^*(p|p_0), \theta^*(p|p_0), p_D^*(p_0), \lambda_e^*, \theta_e^*\} \tag{B.IV.28}$$

are a general equilibrium if:

1. ***Incumbents:*** *Given the rate of creative destruction f^*, the interest rate r^*, and coupon $c^*(p_0)$, incumbents production decision, level of R&D $(\lambda^*(p|p_0), \theta^*(p|p_0))$, and default decision $p_D^*(p_0)$ maximize their equity value.*

2. ***Entrants:*** *Given the rate of creative destruction f^* and the interest rate r^*, entrants level of R&D $(\lambda_e^*, \theta_e^*)$ and capital structure upon becoming an incumbent $c^*(p_0)$ maximize their equity value.*

3. ***Entry:*** *The free entry condition holds:*

$$E^e(f^*) \leq \frac{w(1-\pi)}{h}, \tag{B.IV.29}$$

and the inequality binds when there is creative destruction $f^* > 0$.

4. ***Labor:*** *The labor supply L^{S*} ensures that the labor market clears:*

$$L^{S*} = L^P + L^{R\&D} + L^E \tag{B.IV.30}$$

for a wage rate w.

5. ***Growth and interest rate:*** *The growth and interest rate follow from the Euler equation and the rate of creative destruction:*

$$d\ln(\mathscr{C}_t) = g^*dt = \ln(1+\delta)f^*dt, \tag{B.IV.31}$$

$$r^* = r. \tag{B.IV.32}$$

B.V Debt Refinancing

This appendix extends the model by allowing firms to dynamically optimize their capital structure. Notably, firms that perform well may releverage to exploit the tax benefits of debt.

For simplicity, we assume that firms can only reduce their indebtedness in default.[5] We consider that firms can call their debt at price $\rho\left(p^I\right)c$ with $\rho(p^I) > 0$, where p^I is the number of product lines the firm had when it previously issued debt. The ability to buyback the debt for $\rho(p^I)$ implies that we have to keep track of the number of product lines the firm had the last time it issued debt p^I. We restrict the firm to refinance at most K times and assume that $c \leq \bar{c}$. In this section, we present the solution for the stationary case when $K \to \infty$. Our results also hold for any finite K.

Define firm value as the equity value plus the debt value minus the issuance cost:

$$F\left(p,c,p^I\right) = E\left(p,c,p^I\right) + (1-\xi)\,D\left(p,c,p^I\right). \tag{B.V.1}$$

The exact definition of the equity and debt value in case the firm can refinance its debt is given below. The payoff to shareholders of restructuring the firm's debt is given by the value of the firm after refinancing minus the cost of buying back the debt:

$$\sup_{c'>c} F(p,c',p) - \rho\left(p^I\right)c. \tag{B.V.2}$$

This implies that the equity value, with the possibility to dynamically optimize the firm's capital structure, is given by

$$\begin{aligned} E\left(p,c,p^I\right) = \sup_{\{\lambda_t,\theta_t\}_{t\geq 0},\tau_D,\tau_R} \Bigg\{ & \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}(1-\pi)\left(P_t - c - q(P_t,\lambda_t,\theta_t)\right)dt\right] \\ & + \mathbb{E}_p\left[\mathbb{1}_{\{\tau_R<\tau_D\wedge\tau_0\}}e^{-r\tau_R}\left(\sup_{c'>c}F(P_{\tau_R},c',P_{\tau_R}) - \rho\left(p^I\right)c\right)\right]\Bigg\}, \end{aligned} \tag{B.V.3}$$

where τ_R is the restructuring time chosen by shareholders. Shareholders receive the revenues generated by the portfolio of products minus the coupon payments, the R&D cost, and corporate taxes until either the firm defaults or changes its capital structure. In default, equity value drops to zero. When refinancing, shareholders repurchase existing debt at price $\rho\left(p^I\right)c$ and obtain the (after issuance cost) optimal firm value with a larger coupon $F(P_{\tau_R},c',P_{\tau_R})$.

Debt value also takes into account the possibility that the firm refinances and is given by:

$$D\left(p,c,p^I\right) = \mathbb{E}_p\left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}c\,dt + \mathbb{1}_{\{\tau_D\wedge\tau_0\leq\tau_R\}}e^{-r(\tau_D\wedge\tau_0)}(1-\alpha)\frac{(1-\pi)P_{\tau_D\wedge\tau_0}}{r+f}\right] \tag{B.V.4}$$

$$+\,\mathbb{E}_p\left[\mathbb{1}_{\{\tau_R<\tau_D\wedge\tau_0\}}e^{-r\tau_R}\rho\left(p^I\right)c\right]. \tag{B.V.5}$$

This equation shows that creditors receive coupon payments until either the firm defaults or refinances its debt. When the firm defaults ($\tau_D \wedge \tau_0 \leq \tau_R$), creditors get the present value of the firm cash flows net of the proportional default costs α. When the firm refinances its debt ($\tau_R < \tau_D \wedge \tau_0$), creditors get $\rho\left(p^I\right)c$.

[5] While in principle management can both increase and decrease debt levels, Gilson (1997) finds that transaction costs discourage debt reductions outside of renegotiation. Hugonnier, Malamud, and Morellec (2015) show in a Leland-type model that reducing debt is never optimal for shareholders if debt holders are dispersed and have rational expectations. That is, there is no deleveraging along the optimal path.

In the numerical analysis, we set $\rho\left(p^I\right)$ such that debt is called at a fraction κ of its risk-free value. The buyback price $\rho(p^I)$ therefore solves

$$\rho(p^I)c = \frac{\kappa c}{r}. \tag{B.V.6}$$

The entrant value is the same as in equation (2.16) with $V(f,\theta_e)$ defined as

$$V(f,\theta_e) = \mathbb{E}^{\theta_e}\left[\sup_{c\geq 0}\left\{E\left(p_0,c,p_0\right)+(1-\xi)\,D\left(p_0,c,p_0\right)\right\}\right], \tag{B.V.7}$$

An industry equilibrium is defined as before, except that firms' optimal policies additionally depend on the number of product lines the firm had the last time it issued debt p^I.

In the next part of this appendix, we establish existence of the equity value, which is the equivalent of Theorem 1 in the model with static debt, and existence of an equilibrium.

B.V.1 Proof of Equilibrium Existence

First, we establish existence of the equity and debt values (Theorem 3). Next, we establish the existence of an industry equilibrium under Assumption 2 (Theorem 4).

In this appendix we denote by

$$E_K(p,c,p^I) \tag{B.V.8}$$

the equity value for a firm that can still restructure its debt K times. The debt value $D_K(p,c,p^I)$ and firm value $F_K(p,c,p^I)$ are similarly defined. Furthermore, define

$$E(p,c,p^I) = \lim_{K\to\infty} E_K(p,c,p^I), \tag{B.V.9}$$

$$D(p,c,p^I) = \lim_{K\to\infty} D_K(p,c,p^I), \tag{B.V.10}$$

$$F(p,c,p^I) = \lim_{K\to\infty} F_K(p,c,p^I). \tag{B.V.11}$$

Theorem 3. *The equity and debt values exist. If the optimal level of R&D is internal* $((\lambda,\theta)\in(0,\bar{\lambda})\times(0,1))$ *then it solves*

$$\mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c,p^I)\right] - E(p,c,p^I) = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial\lambda}, \tag{B.V.12}$$

$$\lambda\frac{\partial\mathbb{E}^{\theta}\left[E(\min\{p+x,\bar{p}\},c,p^I)\right]}{\partial\theta} = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial\theta}. \tag{B.V.13}$$

Proof. We establish existence of the equity and debt value recursively.

1. From Theorem 1 it follows that the equity value for a firm that does not have the option to refinance exists. Therefore, also the debt value exists. Let this equity and debt values define the firm value:

$$F_0(p,c,p^I) = E_0(p,c,p^I) + (1-\xi)D_0(p,c,p^I). \tag{B.V.14}$$

The state variable p^I plays no role if the firm cannot restructure.

2. Assume that $F_{K-1}(p,c,p^I)$ exists. First, observe the equity value $E_K(p,c,p^I)$ does not depend on $D_K(p,c,p^I)$ since the price at which the existing debt is bought back is $\rho(p^I)c$. The equity value for a firm that has K restructuring options is

$$E_K(p,c,p^I) = \sup_{\{\lambda_t,\theta_t\}_{t\geq 0},\tau_D,\tau_R} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D\wedge\tau_0\wedge\tau_R} e^{-rt}(1-\pi)\left(P_t - c - q\left(P_t,\lambda_t,\theta_t\right)\right) dt \right] \right. \tag{B.V.15}$$

$$\left. + \mathbb{E}_p \left[\mathbb{1}_{\{\tau_R<\tau_D\wedge\tau_0\}} e^{-r\tau_R} \left(\sup_{c'>c} F_{K-1}(P_{\tau_R},c',P_{\tau_R}) - \rho(p^I)c \right) \right] \right\}. \tag{B.V.16}$$

Given $F_{K-1}(p,c,p^I)$, this implies that the equity value $E_i(p,c,p^I)$ is a fixed point of the mapping

$$\mathcal{M}_K(E) = \sup_{\lambda,\theta,\tau_D,\tau_R} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D\wedge\tau_R} e^{-(r+\lambda+pf)t}(1-\pi)\left(p - c - q\left(p,\lambda,\theta\right)\right) dt \right] \right. \tag{B.V.17}$$

$$\mathbb{E}_p \left[\int_0^{\tau_D\wedge\tau_R} e^{-(r+\lambda+pf)t} \lambda \mathbb{E}^\theta \left[E(\min(p+x,\bar{p}),c,p^I) \right] dt \right] \tag{B.V.18}$$

$$\mathbb{E}_p \left[\int_0^{\tau_D\wedge\tau_R} e^{-(r+\lambda+pf)t} f p E(p-1,c,p^I) dt \right] \tag{B.V.19}$$

$$\left. + \mathbb{E}_p \left[\mathbb{1}_{\{\tau_R<\tau_D\}} e^{-(r+\lambda+pf)\tau_R} \left(F_{K-1}(p,c',p) - \rho(p^I)c \right) \right] \right\} \tag{B.V.20}$$

with $E_K(0,c,p^I) = 0$. The equity value is bounded from above by

$$\frac{(1-\pi)\bar{p} + \pi\bar{c}}{r} \tag{B.V.21}$$

and from below by zero, it is increasing in E, and

$$\mathcal{M}_K(E+L) \leq \mathcal{M}_K(E) + \frac{\bar{\lambda} + f\bar{p}}{r + \bar{\lambda} + f\bar{p}} L, \tag{B.V.22}$$

which holds even if the firm restructures its debt. Therefore, the mapping $\mathcal{M}_K(E)$ satisfies Blackwell's sufficient conditions for a contraction, see Theorem 3.3 on page 54 in Stokey, Lucas, and Prescott (1989), and it is a contraction mapping, which implies that a fixed point exists and is unique. Let $E_K(p,c,p^I)$ be the fixed point of this mapping.

3. The debt value $D_K(p,c,p^I)$ follows from the optimal policies of the firm and therefore firm value $F_K(p,c,p^I)$ also exists. These steps recursively establish existence of the value functions.

4. Optimality of an internal R&D policy implies that they solve the first-order conditions, which shows the last result.

□

We need the following assumption for the equilibrium existence proof, which generalizes Assumption 1 from the static debt case:

Assumption 2. *For the firm value, the order of the limit with respect to f and the supremum over c can be interchanged:*

$$\lim_{f'\to f}\sup_{c}\left\{E_K(p,c,p|f')+(1-\xi)D_K(p,c,p|f')\right\}=\sup_{c}\lim_{f'\to f}\left\{E_K(p,c,p|f')+(1-\xi)D_K(p,c,p|f')\right\}. \tag{B.V.23}$$

Lemma 2. *The entrant value $E_e(f)$ is continuous in f.*

Proof. Continuity is shown recursively.

1. From the proof of Theorem 2 it follows that $\sup_{c'>c} F_0(p,c,p^I|f)$ is continuous in f and c.

2. Assume that $\sup_{c'>c} F_{K-1}(p,c,p^I|f)$ is continuous in f and c. The mapping $\mathscr{M}_K(E|f)$ is continuous in f. Therefore, for every $\epsilon>0$ there exists a $\delta>0$ such that for $f'\in(f-\delta,f+\delta)$ we have

$$\begin{aligned}&\|\mathscr{M}_K(E_K(p,c,p^I|f)|f')-E_K(p,c,p^I|f)\| && \text{(B.V.24)}\\ =&\|\mathscr{M}_K(E_K(p,c,p^I|f)|f')-\mathscr{M}_K(E_K(p,c,p^I|f)|f)\| && \text{(B.V.25)}\\ <&\epsilon. && \text{(B.V.26)}\end{aligned}$$

Fix one such ϵ. Applying the mapping $\mathscr{M}_K$ again leads to,

$$\begin{aligned}&\|\mathscr{M}_K^2(E_K(p,c,p^I|f)|f')-\mathscr{M}_K(E_K(p,c,p^I|f)|f')\| && \text{(B.V.27)}\\ \le&U\|\mathscr{M}_K(E_K(p,c,p^I|f)|f')-E_K(p,c|f)\| && \text{(B.V.28)}\\ <&U\epsilon. && \text{(B.V.29)}\end{aligned}$$

where,

$$U=\frac{\bar{\lambda}+\bar{p}f'}{r+\bar{\lambda}+\bar{p}f'}. \tag{B.V.30}$$

This process can be repeated and leads to

$$\|\mathscr{M}_K^{m+1}(E_K(p,c,p^I|f)|f')-\mathscr{M}_K^{m}(E_K(p,c,p^I|f)|f')\|<U^m\epsilon. \tag{B.V.31}$$

Therefore, the distance between $E_K(p,c,p^I|f)$ and $E_K(p,c,p^I|f')$ is bounded by

$$
\begin{aligned}
&\|E_K(p,c,p^I|f) - E_K(p,c,p^I|f')\| && \text{(B.V.32)}\\
&= \|E_K(p,c,p^I|f) - \mathscr{M}_K^{\infty}(E_K(p,c,p^I|f)|f')\| && \text{(B.V.33)}\\
&\leq \sum_{i=0}^{\infty} \|\mathscr{M}_K^{i+1}(E_K(p,c,p^I|f)|f') - \mathscr{M}_K^{i}(E_K(p,c,p^I|f)|f')\| && \text{(B.V.34)}\\
&< \epsilon \sum_{i=0}^{\infty} U^i && \text{(B.V.35)}\\
&= \epsilon \frac{1}{1-U} && \text{(B.V.36)}\\
&\leq \epsilon \frac{r+\bar{\lambda}+\bar{p}\left(f+\left(f'-f\right)\right)}{r}. && \text{(B.V.37)}
\end{aligned}
$$

Take an $\tilde{\epsilon} > 0$ and set

$$
\epsilon = \tilde{\epsilon} \frac{r}{r+\bar{\lambda}+\bar{p}(f+1)} \qquad \text{(B.V.38)}
$$

then define $\tilde{\delta} = \min\{\delta, 1\}$. We get that for $f' \in (f-\tilde{\delta}, f+\tilde{\delta})$

$$
\frac{r+\bar{\lambda}+\bar{p}\left(f+\left(f'-f\right)\right)}{r} \leq \frac{r+\bar{\lambda}+\bar{p}\left(f+1\right)}{r} = \tilde{\epsilon}. \qquad \text{(B.V.39)}
$$

This implies that for every $\tilde{\epsilon} > 0$ there exists a $\tilde{\delta} > 0$ such that for $f' \in (f-\tilde{\delta}, f+\tilde{\delta})$,

$$
\|E_K(p,c,p^I|f) - E_K(p,c,p^I|f')\| < \tilde{\epsilon}. \qquad \text{(B.V.40)}
$$

Therefore, $E_K(p,c,p^I|f)$ is continuous in f. The same argument shows that $E_K(p,c,p^I)$ is continuous in c.

3. Since the equity value $E_K(p,c,p|f)$ is continuous in f, similar steps as in the proof of Theorem 2 show that for $F_K(p_0,c,p_0|f)$ there exists an ϵ such that

$$
\lim_{\epsilon\to 0} \lim_{f'\to f} |F_K(p_0,c,p_0|f) - F_K(p_0,c\pm\epsilon,p_0|f')| = 0. \qquad \text{(B.V.41)}
$$

4. The previous step shows that for a given f, c, and $f' \to f$ there exists a coupon $c' = \lim_{\epsilon\to 0} c \pm \epsilon$ such that the firm value is continuous in f. This implies that

$$
\sup_c F_K(p_0,c,p_0|f') = \sup_c \lim_{f'\to f} F_K(p_0,c,p_0|f') = \lim_{f'\to f} \sup_c F_K(p_0,c,p_0|f'). \qquad \text{(B.V.42)}
$$

The last step follows from Assumption 2. This shows that $\sup_c F(p_0|f,c)$ is continuous in f.

5. Applying the previous steps recursively ensures that

$$\sup_{c'>c} F_K(p, c', p|f) \tag{B.V.43}$$

is continuous in f. This result ensures that

$$V(f, \theta_E) = \mathbb{E}^{\theta_e}\left[\sup_{c\geq 0}\left\{F\left(p_0, c, p_0\right)\right\}\right] \tag{B.V.44}$$

is continuous in f and therefore that the entrant value $E^e(f)$ is continuous in f.

□

Theorem 4 (Equilibrium Existence with Debt Refinancing)**.** *If Assumption 2 holds, then there exists an industry equilibrium ψ^* in the model with debt refinancing.*

Proof. The proof has several steps

1. It follows from Theorem 2 that $F_0(p, c, p^I)$ converges to zero as $f \to \infty$. Assume $F_{K-1}(p, c, p^I|f)$ converges to zero as $f \to \infty$. If $E_K(p, c, p^I|f)$ does not converge to zero as $f \to \infty$ then for some p we have that $E_K(p, c, p^I|f) > 0$ when $f \to \infty$. This directly implies that the firm does not restructure for this p. Furthermore, from equation (2.8) it follows that for any $p > 0$ with $E_K(p, c, p^I|f) > 0$

$$0 = \frac{-rE_K(p, c, p^I|f) + (1-\pi)(p-c)}{f} \tag{B.V.45}$$

$$+\frac{\max_{(\lambda,\theta)}\left\{\lambda\left(\mathbb{E}^{\theta}\left[E_K(\min\{p+x, \bar{p}\}, c, p^I)\right] - E_K(p, c, p^I|f)\right) - (1-\pi)q(p, \lambda, \theta)\right\}}{f} \tag{B.V.46}$$

$$+p\left\{E_K(p-1, c, p^I|f) - E_K(p, c, p^I|f)\right\}. \tag{B.V.47}$$

Given that $E_K(p, c, p^I|f) \leq \left((1-\pi)\bar{p} + \pi\bar{c}\right)/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$0 = p\left\{E_K(p-1, c, p^I|f=\infty) - E_K(p, c, p^I|f=\infty)\right\} \tag{B.V.48}$$

and therefore that

$$E_K(p, c, p^I|f=\infty) = E_K(p-1, c, p^I|f=\infty) \tag{B.V.49}$$

for any p for which $E_K(p, c, p^I|f=\infty) > 0$. Given that $E_K(0, c, p^I|f=\infty) = 0$ this implies that

$$E_K(p, c, p^I|f=\infty) = 0 \tag{B.V.50}$$

which is a contradiction. Therefore, the equity value goes to zero as $f \to \infty$. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. This result implies that $F_K(p,c,p^i)$ goes to zero as $f \to \infty$. Recursively applying this argument ensures that the entrant value $E^e(f)$ goes to zero as $f \to \infty$.

2. If $\exists f$ such that $E^e(f) > H(1-\pi)$ then Lemma 2, the previous step, and the intermediate value theorem imply there exists an f^* such that

$$E^e(f^*) = H(1-\pi), \tag{B.V.51}$$

which is an industry equilibrium.

3. If $\nexists f$ such that $E^e(f) > H(1-\pi)$ then $f^* = 0$ is an industry equilibrium.

□

B.VI Dynamic Debt Model: Additional Results

We calibrate the dynamic debt model using an additional parameter κ, that captures the repurchase price of debt $\rho(p^I) = \frac{\kappa}{r}$. We set $\kappa = 87\%$.

Comparative statics. All values are in %.					
	Leverage Mean	**Leverage Variance**	**Value p.p. Mean**	**Tax benefit**	**Turnover rate**
Max # new products per innovation					
$n = 2$	29.20	2.67	0.31	4.38	0.75
$n = 3$	28.21	2.51	0.40	4.23	0.94
$n = 4$	27.93	1.62	0.47	4.19	1.13
After-tax entry cost					
$H = 4$	28.82	1.72	0.39	4.32	2.47
$H = 5$	28.21	2.51	0.40	4.23	0.94
$H = 6$	23.17	1.49	0.39	3.48	0.48
Innovation cost scale					
$\beta = 23$	28.11	1.66	0.50	4.22	1.03
$\beta = 26$	28.21	2.51	0.40	4.23	0.94
$\beta = 29$	27.64	1.84	0.45	4.15	1.09
Innovation cost curvature					
$\gamma = 0.333$	27.47	1.94	0.43	4.12	1.16
$\gamma = 0.345$	28.21	2.51	0.40	4.23	0.94
$\gamma = 0.357$	26.69	2.10	0.38	4.00	0.78
Tax rate					
$\pi = 0.10$	23.12	1.58	0.41	2.31	0.76
$\pi = 0.15$	28.21	2.51	0.40	4.23	0.94
$\pi = 0.20$	30.45	2.37	0.40	6.09	1.06
Debt issuance cost					
$\xi = 0\%$	28.62	2.17	0.41	4.29	0.99
$\xi = 1.09\%$	28.21	2.51	0.40	4.23	0.94
$\xi = 4.36\%$	25.24	2.55	0.41	3.79	0.77

Table B.1 – **Comparative statics of selected moments (dynamic debt model).**

C Fundamental Risk and Capital Structure

Appendix C consists of four parts. Section C.I provides more detail concerning the numerical solution of the model. Section C.II describes data processing and contains additional empirical results. Section C.III discusses whether the persistent and transitory shock model is equivalent to one with two transitory shocks with different persistence. Section C.IV provides comparative statics of additional parameters from the model.

C.I Model solution

C.I.1 Transforming the problem

Given the non-stationarity of permanent shocks, the state space is unbounded. However, as in Gourio (2008, 2012), one can define 'detrended' variables to make the state space bounded and reduce the dimensionality of the problem. Given the homogeneity of the value function, we can make (and verify) the following guess

$$V(K,P,Z_T,Z_P) = Z_P^{\frac{1}{1-\theta}} v(k,p,Z_T), \tag{C.I.1}$$

which implies the following laws of motion for:

1. Profit function:

$$\begin{aligned} &\Pi = (1-\tau)ZK^{\theta} = (1-\tau)Z_P Z_T K^{\theta} \\ \Longleftrightarrow\ &\pi = \Pi / Z_P^{\frac{1}{1-\theta}} = (1-\tau)Z_T\left(K/Z_P^{\frac{1}{1-\theta}}\right)^{\theta} = (1-\tau)Z_T k^{\theta}, \end{aligned} \tag{C.I.2}$$

   where $k = K/Z_P^{\frac{1}{1-\theta}}$.

2. Capital stock:

$$k' = \frac{K'}{Z_P'^{\frac{1}{1-\theta}}} = \frac{K'}{Z_P^{\frac{1}{1-\theta}}}\frac{Z_P^{\frac{1}{1-\theta}}}{Z_P'^{\frac{1}{1-\theta}}} = (k(1-\delta)+i)\exp\left(-(1-\theta)^{-1}\sigma_P\varepsilon_P'\right), \tag{C.I.3}$$

   where $i = I/Z_P^{\frac{1}{1-\theta}}$.

3. Net debt stock. Let $\Delta P \equiv P' - P$:

$$p' = \frac{P'}{Z_P'^{\frac{1}{1-\theta}}} = \frac{P'}{Z_P^{\frac{1}{1-\theta}}} \frac{Z_P^{\frac{1}{1-\theta}}}{Z_P'^{\frac{1}{1-\theta}}} = \left(p + \Delta p\right) \exp\left(-(1-\theta)^{-1}\sigma_P \varepsilon_P'\right), \tag{C.I.4}$$

where $\Delta p = \Delta P / Z_P^{\frac{1}{1-\theta}}$ and $p = P / Z_P^{\frac{1}{1-\theta}}$.

The resulting problem is:

$$\begin{aligned} v(k,p,Z_T) = \max_{k',p'} \{ & e(k,k',p,p',Z_T) + \phi(e(k,k',p,p',Z_T)) \\ & + \frac{1}{1+r} \mathbb{E}_{Z_T',\varepsilon_P'} \left[\exp\left((1-\theta)^{-1}\sigma_P \varepsilon_P'\right) v(k',p',Z_T') \right] \Big\}. \end{aligned} \tag{C.I.5}$$

To remove the problem of the dependence of k' and p' on ε_P', I transform the problem into an equivalent one by maximizing over i and Δp rather than over k' and p'. This transformation of the problem is without loss of generality given the laws of motion for capital and net debt. The ultimate formulation of the problem is thus:

$$\begin{aligned} v(k,p,Z_T) = \max_{i,\Delta p} \Big\{ & e(k,i,p,\Delta p,Z_T) + \phi(e(k,i,p,\Delta p,Z_T)) \\ & + \frac{1}{1+r} \mathbb{E}_{Z_T',\varepsilon_P'} \Big[e^{(1-\theta)^{-1}\sigma_P \varepsilon_P'} \times \\ & v\Big((k(1-\delta)+i) e^{-(1-\theta)^{-1}\sigma_P\varepsilon_P'}, (p+\Delta p) e^{-(1-\theta)^{-1}\sigma_P\varepsilon_P'}, Z_T' \Big) \Big] \Big\}, \\ \text{s.t. } & e(k,i,p,\Delta p,Z_T) = (1-\tau) Z_T k^\theta + \tau\delta k - i - \frac{\psi}{2k} i^2 + \Delta p - r(1-\tau)p, \\ & \Delta p \le \omega\,[k(1-\delta)+i] - p, \\ & \log(Z_T') = \rho \log(Z_T) + \varepsilon_T', \\ & \varepsilon_T' \sim \ i.i.d.\ \mathcal{N}(0,\sigma_T^2),\ \varepsilon_P' \sim \ i.i.d.\ \mathcal{N}(0,\sigma_P^2),\ \varepsilon_T' \perp \varepsilon_P'. \end{aligned} \tag{C.I.6}$$

The equivalent representation of the problem admits a standard numerical solution (described in the following subsection), as the state-space is bounded and the remaining shocks are stationary.

C.I.2 Numerical solution

The firm's problem is solved by value function iteration on a discrete state-space of k, p, i, Δp, Z_T. As in Gomes (2001), the equivalent specification of the problem implies that k lies in a compact set, with upper bound defined by $(1-\tau)\pi(\bar{k}, \bar{Z}_T) - \delta\bar{k} = 0$ and where $\bar{Z}_T$ is the highest level of the transitory shock. Therefore capital is discretized into the following grid (containing 81 points):

$$[\bar{k}(1-\delta)^{40}, \ldots, \bar{k}(1-\delta), \bar{k}(1-\delta)^{1/2}, \bar{k}]. \tag{C.I.7}$$

Net debt is discretized into an equally-spaced grid of 61 points over the interval $[-\bar{k}/2, \bar{k}]$. A less coarse grid was used for the control variables: investment was discretized over $[-\bar{k}/10, \bar{k}/10]$ and debt changes over $[-\bar{k}/15, \bar{k}/15]$, using 31 points for each. All grids were chosen so that the optimal choice of investment or debt change never hits the lower/ upper thresholds. The transitory shock was discretized into a Markov chain with 9 grid points using the method of Tauchen (1986). The process for the persistent shock was approximated with a truncated standard normal distribution using 5 grid points. The numerical procedure is implemented as follows:

1. Initial value for the value function in set.
2. Linear interpolation of the value function is used to compute the continuation value

$$\mathbb{E}_{Z_T',\varepsilon_P'}\left[e^{(1-\theta)^{-1}\sigma_P\varepsilon_P'}v\left((k(1-\delta)+i)e^{-(1-\theta)^{-1}\sigma_P\varepsilon_P'},(p+\Delta p)e^{-(1-\theta)^{-1}\sigma_P\varepsilon_P'},Z_T'\right)\right]. \quad \text{(C.I.8)}$$

3. For every k, p and Z_T the values of i and Δp are chosen such that the value function in (4) is maximized.
4. A new starting value is chosen and the procedure is repeated until convergence. Policy function iteration is also used as a part of the algorithm to speed it up.

As in Gourio (2012) the Blackwell's sufficient conditions for the contraction mapping may not be satisfied when the volatility of the persistent shock is too large, however in practive the convergence is achieved for most reasonable values.

The solution produces a value function $v(k, p, Z_P)$ and policy function $\{i, \Delta p\} = h(k, p, Z_T)$, which is used to compute k' and p' according to the law of motions derived in section 3 (and while making sure that the values stay within the specified grids). When simulating the model, the state space for Z_P is further extended to 120 points and interpolation is used to find the corresponding values of the value function and policy functions. I generate a simulated panel with $N = 10000$ firms over $T = 200$ periods and keep the 20 last observations for each firm to make sure that the realized values do not depend on the initial condition of k set at the steady state capital level, $p = 0$, $Z_P = 1$ and Z_T simulated from its stationary distribution.

C.II Data

C.II.1 Data processing and variable definitions

I use the annual Compustat data file for the sample period of 1965–2014. I remove all observations of firms that are not based in the US. As usual in the literature on leverage, I exclude firms in the financial and utility industries. I also exclude all variables with less than $10M of total book assets, negative book equity or market-to-book ratio above 15. I further remove all observations with missing data on the key variables: total book assets `at`, debt in current liabilities `dlc`, long-term debt `dltt`, total liabilities `lt` or operating income before depreciation `oibdp`. This leaves a dataset of roughly 192141 observations of 16490 firms.

I use the following definitions of variables, with `defl` being the CPI deflator:

1. quasi-market leverage (QML): (lt+pstkl-txditc)/(lt+pstkl-txditc+csho*prcc_f),
2. book leverage: (dltt+dlc)/at,
3. book net leverage: (dltt+dlc-che)/(at-che),
4. real profits: log(oibdp/defl),
5. profitability: oibdp/at,
6. cash flow growth: (oibdp$_t$-oibdp$_{t-1}$)/(0.5(sale$_t$+sale$_{t-1}$)),
7. investment: capx/at
8. collateral: (invt+ppent)/at,
9. asset tangibility: ppent/at,
10. size: log(sale/defl),
11. market-to-book: (csho*prcc_f+lt+pstkl-txditc)/at,
12. dividend dummy: 1 if dvt/defl>0.1.

I require that any firm has at least 10 observations when computing persistence (ρ) and volatility (σ) of variables of interest by means of fitting the AR(1) models of the form

$$x_{i,t+1} = a_i + \rho_i x_{i,t} + \sigma_i \varepsilon_{x;i,t+1}. \tag{C.II.1}$$

I use the following risk proxies (computed for each firm):

1. standard deviation of profitability, $\sigma_{\Pi/K}$,
2. volatility of profitability, $\sigma_{\Pi/K}$,
3. volatility of log real profits, $\sigma_{\log(\Pi)}$,
4. log volatility of cash flow growth (log of 10-year rolling st. dev. of cash flow growth), $\sigma_{\log(\Delta\Pi)}$.

These proxies are winsorized at 1% and 99% and (in most of the analysis) aggregated to industry level by averaging across all firms.

C.II.2 Further empirical evidence on the risk-leverage relationship

In this section I provide more evidence on the risk-leverage trade-off by considering different risk proxies. Table C.1 contains the correlation coefficients between four different risk proxies and four 'measures' of leverage, while Figure C.1 contains the corresponding scatter plots for four selected pairs. The evidence suggests what while the correlations are consistently negative, their magnitudes tend to vary. One implication of the data is that the correlations are always lower for the 'residual' leverage, which captures the fact that risk is likely to be jointly determined with other firm characteristics, therefore if one removes a part of leverage heterogeneity due to these observable factors, which results in lower correlation (in absolute terms). Another important issue concerns the fact that different proxies may capture different

Average...	$\overline{lev_i}$	$\overline{\widehat{lev}_i}$	$\overline{nlev_i}$	$\overline{\widehat{nlev}_i}$
St. dev. of profitability, std(Π/K)	−0.505	−0.278	−0.590	−0.282
Volatility of profitability, $\sigma_{\Pi/K}$	−0.406	−0.220	−0.489	−0.269
Volatility of log real profits, $\sigma_{\log(\Pi)}$	−0.106	−0.067	−0.079	−0.052
Log cash flow growth volatility, $\sigma_{\log(\Delta\Pi)}$	−0.193	−0.049	−0.411	−0.206

Table C.1 – **Correlations between averages of risk proxies and average annual book (net) leverage**. Each proxy was computed as the average in a 4-digit SIC industry. The residual (net) book leverage was computed as the difference between observed (net) leverage and a fitted value from a fixed-effect model of book leverage regressed on standard leverage factors (size, profitability, asset tangibility, market-to-book). The risk proxies include the standard deviation of profitability, the volatility of profitability, the volatility of log real profits, and the log volatility of cash flow growth (computed as the log of 10-year rolling st. dev. of cash flow growth). All variables are winsorized at 1% and 99%.

notions of risk. As an example, cash flow growth volatility and log real profit volatility behave differently from profitability volatility, which scales profits using total assets. Table C.2 reaffirms this claim by considering the pairwise correlations of risk measures. This gives hope that these measures could in fact be more informative about different parts of riskiness, which motivates this study. In particular, the volatility of log real profits appears to be less related to the other three measures, which on the one hand is a natural consequence of the way in which it was computed (using level variables rather than ratios), but on the other hand suggests that the stark difference between levels and ratios may be suggestive of the presence of some phenomenon that drives both the profits as well as the total assets in the same way. This paper argues that this phenomenon reveals itself by the means of persistent shocks.

Average...	std(Π/K)	$\sigma_{\Pi/K}$	$\sigma_{\log(\Pi)}$	$\sigma_{\log(\Delta\Pi)}$
St. dev. of profitability, std(Π/K)	1.000			
Volatility of profitability, $\sigma_{\Pi/K}$	0.774	1.000		
Volatility of log real profits, $\sigma_{\log(\Pi)}$	0.408	0.474	1.000	
Log cash flow growth volatility, $\sigma_{\log(\Delta\Pi)}$	0.635	0.631	0.329	1.000

Table C.2 – **Correlations of four risk proxies.** Each proxy was computed as the average in a 4-digit SIC industry. The risk proxies include the standard deviation of profitability, the volatility of profitability, the volatility of log real profits, and the log volatility of cash flow growth (computed as the log of 10-year rolling st. dev. of cash flow growth). All variables are winsorized at 1% and 99%.

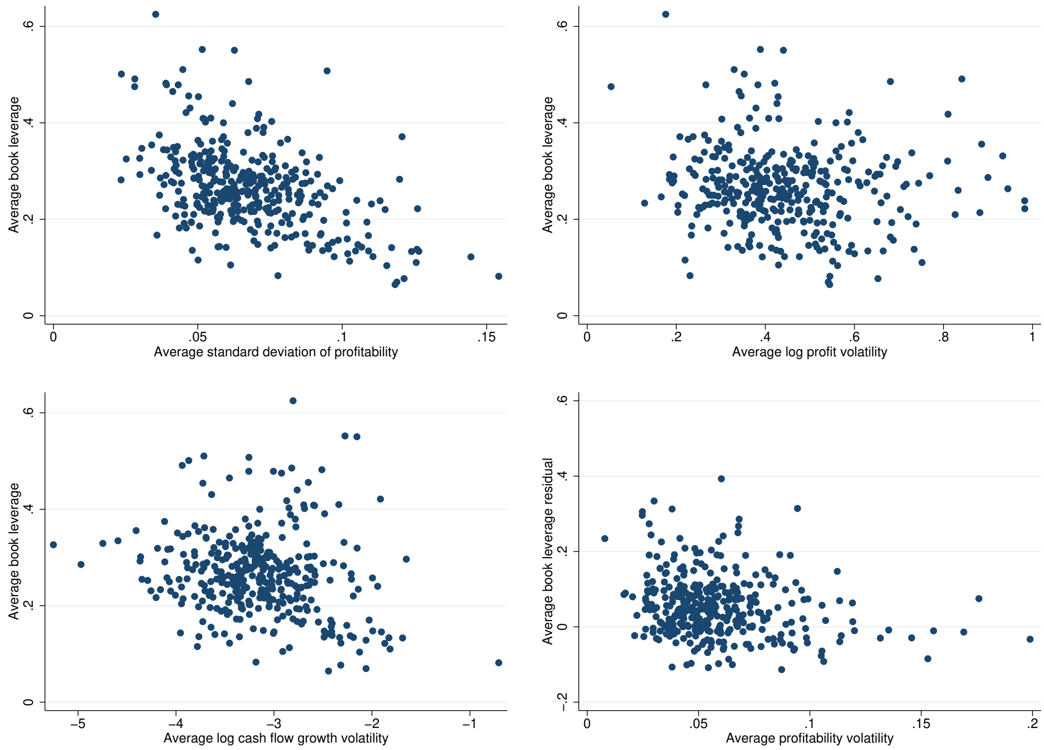


Figure C.1 – **Scatter plots of annual book leverage versus risk proxy.** Each proxy was computed as the average in a 4-digit SIC industry. The residual (net) book leverage was computed as the difference between observed (net) leverage and a fitted value from a fixed-effect model of book leverage regressed on standard leverage factors (size, profitability, asset tangibility, market-to-book). The risk proxies include the standard deviation of profitability, the volatility of profitability, the volatility of log real profits, and the log volatility of cash flow growth (computed as the log of 10-year rolling st. dev. of cash flow growth). All variables are winsorized at 1% and 99%.

C.II.3 Profit persistence as a leverage factor

I estimate profit persistence using an AR(1) model. To alleviate the concern that estimation bias may be at play, I consider different ways of aggregating the data. For example, I use profitability or log real profits computed for individual firms or industries using both firm-level and aggregate data. The results, presented in Table C.3, are similar in all cases and suggest that the association between persistence and firm characteristics is weak, in particular that of leverage. A similar conclusion can be drawn from running cross-sectional regressions of average book leverage on leverage factors and profit persistence. The results, presented in Table C.4, indicate that the regression coefficients of $\widehat{\rho}$ are nearly always statistically insignificant. They also provide very small incremental explanatory power compared to other variables (not reported).

Average...	Firms		Industries			
	$\rho(\Pi/K)$	$\rho(\log(\Pi))$	$\overline{\rho}(\Pi/K)$	$\overline{\rho}(\log(\Pi))$	$\rho_{agg}(\Pi/K)$	$\rho_{agg}(\log(\Pi))$
Book leverage	-0.018	-0.002	0.009	-0.108	-0.032	-0.139
Investment	-0.007	-0.033	0.047	-0.009	0.082	0.058
Market-to-book	0.016	0.037	-0.002	0.040	0.032	0.076
Size	0.013	0.029	0.070	0.085	0.011	-0.156
Asset tangibility	-0.006	-0.025	0.020	-0.045	0.031	-0.039
Collateral	-0.002	-0.002	-0.037	-0.058	-0.038	-0.127
Volatility of log real profits	-0.022	-0.028	-0.095	-0.191	-0.159	-0.128
Vol. of agg. log real profits	—	—	-0.059	-0.167	-0.312	-0.155

Table C.3 – **Correlations between firm characteristics and estimated profit persistence.** ρ is estimated as the persistence parameter from an AR(1) fit of log real profits $\log(\Pi)$ or profitability Π/K for each firm and then averaged over all firms in an industry. Industry-specific persistence parameters ρ_{agg} are estimated using the aggregate industry-level data. Industries are defined using the 4-digit SIC code. All variables are winsorized at 1% and 99%.

	Firms		Industries			
	$\overline{\rho}(\Pi/K)$	$\overline{\rho}(\log(\Pi))$	$\overline{\rho}(\Pi/K)$	$\overline{\rho}(\log(\Pi))$	$\rho_{agg}(\Pi/K)$	$\rho_{agg}(\log(\Pi))$
$\widehat{\rho}$	-0.001	0.001	-0.004	-0.007	-0.009	-0.011
t-stat	-1.89	1.30	-0.74	-0.69	-0.75	-0.86
Incr. $\overline{R}^2$ of $\widehat{\rho}$	0.001	0.000	0.002	0.001	0.001	0.000
$\overline{R}^2$	0.262	0.262	0.313	0.332	0.332	0.333
Industry FE	Yes, 4D-SIC	Yes, 4D-SIC	Yes, 2D-SIC	Yes, 2D-SIC	Yes, 2D-SIC	Yes, 2D-SIC
N	6387	6387	353	353	353	353

Table C.4 – **Coefficients from cross-sectional regressions of average book leverage on average leverage factors (size, profitability, asset tangibility, market-to-book, volatility of log real profits) and estimated profit persistence $\widehat{\rho}$.** Standard errors are robust and clustered at 4-digit or 2-digit industry level. ρ is estimated as the persistence parameter from an AR(1) fit of log real profits $\log(\Pi)$ or profitability Π/K for each firm and then averaging over all firms in an industry. Industry-specific persistence parameters ρ_{agg} are estimated using the aggregate industry-level data. The estimated profit persistence parameters are normalized by their full-sample standard deviation. Industries are defined using the 4-digit SIC code. All variables are winsorized at 1% and 99%.

C.III Is the model equivalent to one with two transitory shocks of different persistence?

One important question to ask is whether the main predictions of the model also prevail when we consider a related model in which the firm is exposed to two transitory shocks with different persistence parameter ρ. While certain features of both models are bound to be similar, given that each includes varying the 'overall persistence' of cash flow, there is a number of reasons as to why the persistent and transitory shock model is superior to one with two transitory shocks. I focus on three dimensions when comparing the two classes of models: the dynamics of the shocks and their implications for firm's policies, the effect of changing risk exposure on model-implied moments and the identification of risk characteristics.

C.III.1 Shock dynamics

The analysis in Section 3 shows that persistent and transitory shocks result in markedly different capital and debt policies. Their effects differ both quantitatively and qualitatively. Furthermore, if we considered the impulse response functions for two transitory shocks with different persistence, their shapes would be similar, but shifted. However, unlike persistent shocks, transitory shocks are unable to permanently affect firm's policies. From this point of view, the two classes of models are completely distinctive.

C.III.2 Effect on model-implied moments

For the purpose of this subsection, I solve an altered version of the model in which I set the profit function to $\Pi(K,Z) = (1-\tau)Z_1 Z_2^{1-\theta} K^{\theta}$ as in Belo, Lin, and Yang (2019), which allows to use a different solution method and consequently a wider range for σ_P's (there is still a one-to-one link between the extended model and the one considered in this paper). The Z_2 shock will be represented by either the persistent shock Z_P or a transitory shock Z_{T+} where the '+' represents a higher value of the persistence parameter ρ. The parametrizations of the models are summarized in Table C.5.

	Model 1 (T+P)		Model 2 (T_{-}+T_{+})	
	ρ_i	σ_i	ρ_i	σ_i
Z_1	0.2	0.24–0.15	0.2	0.24–0.15
Z_2	1.0	0.05–0.20	0.7	0.05–0.20

Table C.5 – **Parametrization of the extended models.** The assumed total volatility is constant and set to $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.25$. All remaining parameter values are taken as in Table 3.1.

The illustration in this subsection is qualitative in nature and demonstrates the nature of differences between the two classes of models. I solve the models for different exposures to the two types of shocks and compare the elasticity of model-implied moments to changing risk composition. The results of this exercise are presented in Table C.6.

The resulting elasticities suggest that while there are several moments that are affected in the same way in both models (the top panel of Table C.6), others behave differently. Moreover, the sensitivity of parameters to changing risk exposure is almost always higher for the model with a persistent shock, except for the variation investment and leverage and moments concerning profitability. This finding suggests that even despite their similarity, the way in which each set of shocks affect firm's policies is distinctive.

Furthermore, the model including a persistent shock appears to positively affect correlations of various moments, unlike the model with two transitory shocks. The intuition

	Moment	Elasticity	
		Model 1 (T+P)	Model 2 (T_{-}+T_{+})
Similar effect	Average investment (i/k)	0.073	0.065
	Standard deviation of investment (i/k)	0.383	0.618
	Average leverage (p/k)	-0.128	-0.105
	Standard deviation of leverage (p/k)	0.331	0.648
	MAD of average investment (i/k)	1.004	0.571
	MAD of average leverage (p/k)	1.205	0.362
	Persistence of profitability $\rho(\pi/k)$	0.103	0.379
	Volatility of profitability $\sigma(\pi/k)$	0.047	0.857
	Persistence of log profits $\rho(\log(\Pi))$	0.500	0.149
	Volatility of log profits $\sigma(\log(\Pi))$	0.536	0.081
	Corr. inv. and profitability $\text{corr}(i/k, \pi/k)$	-0.633	-0.043
Diff. effect	Autocorrelation of investment $\phi_1(i/k)$	1.323	-0.481
	Autocorrelation of investment $\phi_3(i/k)$	3.585	-0.907
	Autocorrelation of capital stock $\phi_1(K)$	0.243	-0.164
	Autocorrelation of investment $\phi_3(K)$	0.549	-0.361

Table C.6 – **Elasticity at average moments of model-implied moments to risk exposure** $(\partial m/\partial\beta) \times (\bar{m}/\bar{\beta})$**.** The assumed total volatility is constant and set to $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.25$. Elasticities are computed by changing risk composition according to Table C.5. *MAD* denotes the median absolute deviation, ϕ_k is the k^{th} order autocorrelation. volatility and persistence are the ε_i and rho_i parameters fron an AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ estimated using corresponding moments.

underlying this result is straightforward. While increasing exposure to the persistent shock or to the transitory shock with high ρ raises the overall persistence in the model, at least in terms of profits or profitability, the *type* of persistence is different. There are two forces at play affecting this outcome. On the one hand, higher persistent shock exposure increases the overall persistence of model-implied moments, especially when total volatility is not too high. On the other hand, higher persistence results in the firm being more sensitive to underlying shock realizations. A high shock is likely to be followed by another high realization, therefore the firm is likely to change its investment policy and fund its capital expenditure by issuing debt. Thus, the firm will alter its investment and leverage policies more frequently and with higher magnitude, which lowers the extent to which these policies are path-dependent. Here, the second effect dominates. However, in the other case when the firm is more exposed to a low-persistence transitory shock, then its policy response is in general muted, given that this shock is more 'iid-like', unless a really large and positive realization occurs.

The effect in which persistent shocks affect the persistence of firm's policies is also markedly different. Given that the impact of the shock is spread out over multiple periods, as shown by the analysis of IRFs in section 3.2, the autocorrelation of these policies is bound to increase. This is not to say that the logic of the previous paragraph does not apply here. To the contrary, the firm is also much more sensitive to persistent shock realizations when its exposure to this shock increases, which results in more variable policies. However, this effect does not overtake the impact of 'spreading out' the effect of a persistent shock.

Several remaining remarks concerning the elasticity of other moments:

- **Dispersion in firms' policies** It is important to notice that the sensitivity of dispersion (as measured by the median absolute deviation MAD) in firm characteristics to changing risk exposure is two or three times as large for the model with a persistent shock. This observation gives further backing to the claim of the paper that the differences in dispersion of within-industry firm characteristics for different industries can be explained to some extent by different exposure to persistent shock, rather than to changing ρ.

- **Persistence of log profits/ profitability** Transitory shocks affect profitability to a higher extent than log profits, while the opposite is true for persistent shocks. This distinction, already highlighted in Table 3.2 in the original model, results from de-trending profits Π by capital K. However, the quantitative differences remain large.

- **Correlation between investment and profitability** The value of the moment in a model with two transitory shocks is insensitive to changing firm's exposure to more persistent shocks, while the opposite is true in a model with a persistent shock. As already argued by Gourio (2008), we should be able to identify the extent to which the firm is exposed to persistent shocks by looking at how it responds to being hit by a profitability shock. Here, the negative elasticity of the correlation between investment and profitability to changing risk composition results from the fact that profitability is relatively insensitive to persistent shock exposure, given the de-trending.

- **Volatility of log profits/ profitability** Interestingly enough, the way in which each model affect these two moments is different. However, this result is intuitively related to how log profits and profitability are defined and has been discussed earlier when considering the comparative statics of the model in section 3.3. On the one hand, when we divide profits by capital, this implies that we de-trend profits, which removes most if not all of the variation due to persistent shocks (note that both profits and capital move with this variable). Therefore, this moment is more affected by transitory shocks. On the other hand, the volatility of log profits is much more sensitive to changing risk exposure in the model which includes a persistent shock. Again, this result in intuitive, given that persistent shock affect the growth rate of profits, unlike transitory shocks, which means that even small change in cash flow composition may result in big changes in the volatility of log profits.

C.III.3 Identification?

The last difference between the two classes of models is more subtle and relates to identifying the parameters governing the shock processes. In some sense, as implied by the results in

Table C.6 and the analysis in previous subsections, changing risk exposure in a model with two transitory shocks of different persistence is comparable to changing ρ in a standard dynamic capital structure model such as DeAngelo, DeAngelo, and Whited (2011). Therefore, it may be impossible to infer the exact composition of firm's cash flows in such a two-transitory-shock model. On the other hand, a model with persistent shock has a distinctive impact on several key moments, giving hope that the identification is possible.

Finally, apart from the purely technical concerns, there are also a few economically motivated reasons. Despite the fact that the exact nature of persistent and transitory shocks may be unknown, that is we do not know what this shock decomposition *exactly* represents, it is easier still to imagine a firm being exposed to these two sources of risk rather than two transitory shocks with differing persistence. It is not easy to imagine how to attribute the 'less persistent' and 'more persistent' features. It is also relatively easier to think of risk exposure in terms of a very persistent (permanent) shock and a transitory shock with small persistence, especially given the vast macroeconomics literature using persistent shocks to describe the evolution of technology shocks in the economy.

As a final remark it is important to mention that rather than investigating the differences between the models having $\rho = 1$ and $\rho \approx 0.99$, in this paper I am more interested in examining whether a model in which the firm is exposed to a small persistent shock and a transitory shock with lower persistence than usually assumed in the literature is able to provide additional insight regarding variation in observable corporate policies.

C.IV Comparative statics of other parameters

To conclude the analysis of the sensitivity of model-implied moments to model parameters, I analyze the effect of changing capital adjustment cost (ψ), external equity issuance cost (η) and the parameter governing tightness of the collateral constraint (ω). Table C.7 presents the resulting moments computed for a 'low' and 'high' value of each of the three specified parameters for different values of persistent shock volatility σ_P.

C.IV.1 Capital adjustment cost ψ

As convex capital adjustment costs increase, firms become less sensitive to shock arrival and, as a result, investment becomes less variable. Therefore, firms also increase their leverage, given that investment opportunities become more predictable and so they can manage their debt capacity less conservatively. One could be concerned that the effect of changing persistent shock volatility is equivalent to that of changing convex capital adjustment costs given the increased smoothness in capital that both induce. However, it turns out that it is possible to disentangle these two effects. For example, the average standard deviation of investment appears to distinguish the two sufficiently well: while it increases as firm's persistent shock exposure grows, it decreases as the magnitude of convex adjustment costs rises. The intuition for this result is related to the fact that while both parameters affect the 'smoothness' of investment, σ_P also impacts the overall time-series variation of firm policies (i.e. high convex costs affect mostly the overall smoothness of firm policies while firms act on shock realizations). Other moments which are affected differently by these parameters concern e.g. the average leverage or the dispersion in average investment or leverage: the dispersion generally increases with persistent shock exposure and decreases as adjustment costs become larger.

C.IV.2 Equity issuance cost η

Higher equity issuance costs result in lower leverage level, as in e.g. Hennessy and Whited (2005, 2007). They also result in more persistent and volatile leverage but less persistent investment. When equity issuance becomes more costly, firms' policies also become less dispersed. The effect of varying this parameter appears to be stronger when firm's persistent shock exposure is lower.

C.IV.3 Tightness of the collateral constraint ω

As expected, this parameter largely affects average leverage and to some extent also other moments related to firm's debt policy such as leverage variation or dispersion in average leverage, but its effect on other model-implied moments is relatively limited.

		Total volatility σ	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
		Persistent shock volatility σ_P	0.02	0.02	0.04	0.04	0.02	0.02	0.04	0.04	0.02	0.02	0.04	0.04
			Capital adj. cost ψ				Equity iss. cost η				Collateral constraint ω			
			low	high	low	high	low	high	low	high	low	high	low	high
1.	Investment	Average investment (i/k)	0.158	0.161	0.159	0.163	0.165	0.157	0.167	0.158	0.158	0.161	0.159	0.161
2.		Standard deviation of investment (i/k)	0.126	0.069	0.132	0.076	0.180	0.115	0.183	0.124	0.124	0.145	0.134	0.144
3.		Autocorrelation of investment $\phi_1(i/k)$	0.194	0.211	0.218	0.324	0.230	0.163	0.245	0.179	0.192	0.211	0.195	0.217
4.		Autocorrelation of investment $\phi_3(i/k)$	-0.107	-0.043	-0.090	-0.011	-0.109	-0.103	-0.105	-0.087	-0.105	-0.109	-0.085	-0.098
5.		Frequency of disinvestment $\#(i/k<0)$	0.078	0.035	0.105	0.052	0.156	0.068	0.170	0.105	0.098	0.087	0.121	0.103
6.		Corr. inv. and profitability $\text{corr}(i/k,\pi/k)$	0.834	0.696	0.812	0.666	0.879	0.784	0.864	0.787	0.825	0.866	0.808	0.840
7.		Autocorrelation capital stock $\phi_1(K)$	0.791	0.833	0.827	0.893	0.781	0.788	0.805	0.818	0.793	0.789	0.820	0.818
8.		Dispersion of average investment $MAD(\overline{i/k})$	0.016	0.014	0.023	0.021	0.021	0.015	0.027	0.021	0.016	0.018	0.023	0.024
9.	Leverage	Average leverage (p/k)	0.172	0.227	0.154	0.213	0.549	0.174	0.5511	0.142	0.154	0.296	0.118	0.227
10.		Standard deviation of leverage (p/k)	0.062	0.061	0.077	0.079	0.024	0.071	0.025	0.080	0.070	0.082	0.073	0.089
11.		Persistence of leverage $\rho(p/k)$	0.513	0.616	0.609	0.655	0.228	0.683	0.152	0.673	0.643	0.688	0.622	0.711
12.		Volatility of leverage $\sigma(p/k)$	0.045	0.041	0.059	0.054	0.023	0.049	0.024	0.055	0.051	0.054	0.055	0.058
13.		Dispersion of average leverage $MAD(\overline{p/k})$	0.022	0.018	0.036	0.024	0.006	0.036	0.006	0.037	0.024	0.031	0.024	0.041
14.	Profits	Persistence of profitability $\rho(\pi/k)$	0.333	0.432	0.336	0.377	0.300	0.342	0.304	0.341	0.332	0.324	0.332	0.329
15.		Volatility of profitability $\sigma(\pi/k)$	0.141	0.160	0.138	0.144	0.135	0.141	0.134	0.137	0.138	0.141	0.137	0.139
16.		Persistence of log profits $\rho(\log(\Pi))$	0.665	0.757	0.700	0.626	0.732	0.646	0.754	0.686	0.666	0.686	0.704	0.711
17.		Volatility of log profits $\sigma(\log(\Pi))$	0.267	0.288	0.270	0.267	0.274	0.267	0.276	0.270	0.268	0.268	0.271	0.270
18.		Corr. prof. and growth $\text{corr}(\pi/k, k'/k)$	0.838	0.707	0.816	0.667	0.882	0.790	0.868	0.791	0.830	0.869	0.813	0.844
19.		Corr. prof. and log gth. $\text{corr}(\pi/k, log(k'/k))$	0.823	0.723	0.799	0.661	0.866	0.780	0.849	0.776	0.818	0.849	0.797	0.826

Table C.7 – **Summary statistics of model-implied moments for different values of σ_P and parameters describing real or financing frictions.** I focus on the convex capital adjustment cost ψ, the linear equity financing costs η and the collateral constraint ω. Persistence of transitory shock ρ is set at 0.6. Remaining parameters are taken as specified in Table 3.1. An AR(1) model $x_{i,t+1} = a_i + \rho_i x_{i,t} + \varepsilon_{i,t+1}$ was fit for each simulated firm to compute persistence ρ_x or volatility σ_x of variable x. ϕ_k is the k^{th} order autocorrelation. The measure of dispersion MAD is median absolute deviation. The numerical solution and simulation of the model is described in detail in Appendix A.

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