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A NOTE ON ZERO AND THE NUMERICAL PLACE-VALUE SYSTEM IN ANCIENT INDIA

The second volume of the Kalātattvakośa, which has recently (1992) come out, refers to zero and the numerical place-value system in its section called "śūnya: mathematical aspect" written by S.R. Sarma. Sarma is of the opinion that "there is enough indirect evidence to say that the decimal place-value system with symbols for 1 to 9 and zero developed in India much before the beginning of the Christian era" (p. 403). This evidence is, according to Sarma, constituted by a passage in the Chandaḥ-Sūtra of Piṅgala, and by another one in the Jaina canonical text Anuyogadvāra.

Chandaḥ-Sūtra 8.28-31, Sarma points out, uses the symbols 2 and 0 in a meaningful way. He concludes: "The fact ... that Piṅgala uses them goes to show the existence of a well-recognised symbol for zero at his time, which is variously placed between 400 to 200 B.C." The problem here lies with the date he ascribes to the Chandaḥ-Sūtra. Various authors have pointed out that this date is not all that certain. This is what Renou (1947-49: 307) had to say about it: "Les Sūtra de Piṅgala sur la métrique, les Chandaḥsūtra, ... sont récents puisqu'ils traitent de mètres purement classiques."[1] Note further that the passage referred to by Sarma does not even occur in the portion of the text dealing with Vedic metres, which some claim to be older. It will be clear that Sarma's chronological conclusions are not justified by the use of zero in the Chandaḥ-Sūtra.

Regarding the Anuyogadvāra, Jarl Charpentier (who refers back to Albrecht Weber) pointed out long ago (1914: 29) that it contains a list (section 49) which refers to the Vaiśeṣika, Sāṃkhya and Lokāyata systems of philosophy. Of these the Vaiśeṣika system is, according to Charpentier, the most modern, and indeed there is no evidence that this system existed before the beginning of our era. The Sāṃkhya philosophy is referred to by the name Ṣaṣṭitantra, which appears to have been the name of a work composed by Vārṣagaṇya, who may have lived in the beginning of the 4th century of our era (Frauwallner, 1958: 131 (270)). Charpentier considers it "possible that Ṣaṣṭitantra is here only a name for the Sāṅkhya system of philosophy, which is one of the very oldest amongst the Hindu [1040] philosophical schools, being mentioned already by Kauṭilya" (p. 28), but this position is not tenable, for various reasons. It is

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[1] Cf. Van Nooten, 1993: 33: "... nor is it possible to prove that Piⁿgala's work existed before the third century A.D."

not at all certain that Sāṃkhya as a system is all that old; no evidence to that extent seems to exist. Kauṭilya's Arthaśāstra is known to be a composite work, which reached completion long after the period of the Mauryas.[2] And even if the Sāṃkhya system were to be old, it is never referred to in early texts as Ṣaṣṭitantra. This is not without reason, for the term Ṣaṣṭitantra refers to the sixty principles which, if Frauwallner (1956: 320) is to be believed, did not enter the system until a late date.

A late date for the Anuyogadvāra — or at least for this passage of it — is confirmed by the reference to a Pātañjalī(ya) in it. Charpentier (p. 29) observes: "What Pātañjalī(ya) means is doubtful; it may refer to the Mahābhāṣya — vyākaraṇa, however, is specially mentioned — or rather to the Yoga-sūtra's, which are certainly a late work." What Charpentier did not yet know, is that the terms Patañjali and Pātañjala are used in the earlier literature with reference to Yoga Sūtra and Bhāṣya together, never with reference to the Yoga Sūtra alone (Bronkhorst, 1985: 203 ff.). Pātañjalī(ya) looks therefore even more recent than Charpentier realized.

It appears, then, that the evidence for a mathematical zero in India before the beginning of our era is very weak indeed. I should mention here in passing the recent publication by M.D. Pandit, Zero in Pāṇini (1990). Pandit is of the opinion that there is a resemblance between the mathematical zero and certain techniques used by Pāṇini. Pāṇini lived most probably in the fourth century before our era (Hinüber, 1990: 34). Does this imply that the mathematical zero existed before Pāṇini, and therefore well before our era? Pandit is careful to avoid this conclusion (p. 116): "It is also possible that the ancient Indian mathematicians might also have borrowed certain techniques from Pāṇini." We reach therefore the same conclusion as above: there is no solid evidence to believe that zero and the numerical place-value system existed in India before the beginning of our era.[3]

For the more recent period Sarma refers to a number of texts which testify to the use of zero and a decimal place-value system: Āryabhaṭīya, Puliśasiddhānta, Pañcasiddhāntikā, Bṛhatkṣetrasamāsa, Siddhasena Gaṇi's commentary on the Tattvārtha Sūtra, the Yoga Bhāṣya and Śaṅkara's commentary on the Brahma Sūtra. Sarma does not mention a passage in [1041] Vasubandhu's Abhidharmakośa Bhāṣya (5.26; p. 296, l. 21-23), referred to by Ruegg (1978: 172-73). This passage is part of an opinion ascribed to a certain Bhadanta Vasumitra, and illustrates his position with the help of a marker or counter (*vartikā*) which in the unit position has the value of a unit, in the hundred's position that of a hundred, and in the thousand's position that of a thousand. Ruegg

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[2] See Asiatische Studien / Études Asiatiques 45 (2), 1991, p. 214 f.
[3] Van Nooten (1993: 42-44) draws attention to the inscriptions of Nåneghåṭ (ca. 1st century B.C.E.), but admits that they do not contain a pure place-value system.

considers it probable that Vasumitra is "to be identified with one of the leading figures at the time of Kaniṣka's Great Council" and concludes that he "could be the contemporary of Nāgārjuna (if not older)" (p. 175).

   This conclusion is confirmed by the presence of the same — or a closely similar — passage in several earlier texts. It occurs in both the Chinese translations of the Mahāvibhāṣā (T. 1545, vol. 27, p. 396b l. 3-4; T. 1546, vol. 28, p. 295c l. 18-20) and in the Vibhāṣā (T. 1547, vol. 28, p. 466b l. 18-19).[4] The Mahāvibhāṣā may have been composed during the reign of Kaniṣka, and the Vibhāṣā may be slightly older (Nakamura, 1980: 107). It is therefore safe to conclude that the decimal place-value system was known in India during the early centuries of our era.[5]

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[4] It also occurs in Saĺghabhadra's Nyåyånusåra (T. 1562, vol. 29, p. 631a l. 26), the Yoga Bhå∑ya (3.13), and several later works (see Ruegg, 1978: 173-174).

[5] Joseph (1991: 241) remarks: "If ... the original version of the Bakhshali Manuscript dates from the third century AD, it would be the earliest evidence of a well-established number system with a place-value scale and zero which is also recognizably an ancestor of our present-day number system." However, L'Inde Classique characterizes this manuscript as "relativement tardif", although "d'abord considéré à tort comme très ancien" (Filliozat, 1953: 175).

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