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THREE ESSAYS IN MICROECONOMICS: RISK-TAKING IN TOURNAMENTS, ENTRY IN VERTICALLY DIFFERENTIATED MARKETS, AND BANNING COMMISSIONS

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FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT D'ÉCONOMIE

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THÈSE DE DOCTORAT

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pour l'obtention du grade de
Docteure ès Sciences Économiques,
mention « Économie politique »

par

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Prof. Luís Pedro Santos-Pinto

Jury

Prof. Marianne Schmid Mast, présidente
Prof. Rustamdjan Hakimov, expert interne
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I M P R I M A T U R

Sans se prononcer sur les opinions de l'autrice, la Faculté des Hautes Etudes Commerciales de l'Université de Lausanne autorise l'impression de la thèse de Madame Noëmi Sara JACOBER, titulaire d'un bachelor en Économie, business et droit de l'Université de Berne, et d'un master en Économie de l'Université de Lausanne, en vue de l'obtention du grade de docteure ès Sciences économiques, mention « économie politique ».

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Contents

Introduction

The field of economics has long been concerned with understanding how individuals and firms make decisions, and how these decisions affect market outcomes. Behavioral economics and industrial organization are two subfields that have grown in recent years to shed new light on these questions. This thesis is an exploration of the theoretical foundations of these fields, with a focus on how they can be used to analyze real-world market situations. The chapters in this thesis cover a range of topics, including the role of behavioral biases in decision-making, the impact of market structure on firm behavior, and the implications of these findings for public policy. Through a careful examination of these topics, this thesis aims to provide new insights into the workings of markets and the behavior of the actors within them.

All three chapters of this thesis employ theoretical approaches to model certain market situations and contribute to topics in the field of behavioral economics and industrial organization. In the first chapter of this thesis, Luís Santos-Pinto and I study the role of overconfidence in tournaments where players choose effort provision as well as risk exposure. We show that, against the common idea, overconfidence can raise effort provision and leads to lower risk-taking. In the second chapter, João Montez and I explore the speed of copycat entry in vertically differentiated markets in which consumers differ in their willingness to pay. We identify welfare-reducing incentives to enter the market which lead to an extended monopoly situation despite costless entry. In the third chapter, I analyze the welfare implications of switching from a commission-based to a fee-based remuneration model in markets where consumers rely on expert advice. This study identifies a mechanism through which the number of consumers buying in equilibrium decreases and provokes the question of whether a paradigm shift away from compensation through commissions towards transactional fees is worth the drawbacks.

Chapter 1

Risk Taking and Effort Provision in Tournaments with Overconfident Players

Abstract This paper investigates the role of overconfidence in tournaments where players choose effort as well as risk. We obtain three main results in tournaments where two homogeneous players compete against each other. First, players may adopt less risky strategies when they are overconfident than when they are rational. Second, players may exert more effort when they are overconfident than when they are rational. Third, the players' overconfidence can make the tournament organizer better off. We obtain four main results in tournaments where an overconfident player competes against a rational player. First, the overconfident player exerts less effort than the rational player. Second, the overconfident player is less likely to win the tournament when risk is normally distributed. Third, the overconfident player may choose a less risky strategy than the rational player but the reverse cannot happen. Fourth, the presence of an overconfident player may lead a rational player to take less risk. Our study goes against the idea that overconfident individuals take more risks than rational ones and uncovers a new mechanism whereby overconfidence raises effort provision.

This chapter was written in collaboration with Luís Santos-Pinto.

1.1 Introduction

The evidence that humans are overconfident is widespread and well-established. For example, most drivers believe they are better than the median driver (Svenson, 1981). Overconfidence affects the behavior of decision-makers such as CEOs (Malmendier and Tate, 2005, 2008, 2015), fund managers (Menkhoff et al., 2006), poker and chess players (Park and Santos-Pinto, 2010), CFOs (Ben-David et al., 2013), and marathon runners (Krawczyk and Wilamowski, 2017). These decision-makers are often engaged in rank-order tournaments, that is, competitions where rewards are based on relative performance. Who gets promoted to CEO, who wins a prize in an athletic competition (Szymanski, 2003), or who wins a sales bonus (Murphy et al., 2004), is typically decided on the basis of relative performance.[1]

Studies analyzing the role of overconfidence in tournaments have so far focused on how it influences either risk choices (Goel and Thakor, 2008) or effort provision (Santos-Pinto, 2010). However, in many tournaments, players decide not only how much risk they take but also how much effort they exert. For example, a CEO can choose whether her firm has an innovative or a conservative research and development strategy in addition to how hard she works. A fund manager can choose the risk exposure of her portfolio and how much time and resources to spend on collecting and analyzing stock information. Similarly, a poker player chooses risk taking and the effort she puts into computing conditional probabilities.

This paper investigates the implications of overconfidence for tournaments where players choose risk as well as effort. We ask the following questions: Does overconfidence lead players to adopt more or less risky strategies? Does overconfidence raise or lower effort provision? Can the tournament organizer benefit from the players' overconfidence? Is an overconfident player more likely to win a tournament than a rational player? Can the presence of an overconfident player lead a rational to lower risk taking?

To answer these questions, we consider two-player tournaments where each player's

[1] Moore and Healy (2008) distinguish between three types of overconfidence: (i) overestimation of one's absolute performance, (ii) overestimation of one's relative performance (overplacement), and (iii) excessive confidence in the precision of one's private information, estimates, and forecasts (overprecision or miscalibration). In our study, we use the term overconfidence in the sense of overestimation of absolute and relative performance in a tournament.

production function or output is additively separable in talent, effort, and a random shock. At the first stage (risk stage), both players simultaneously choose the risk of their production functions which can be either low or high. At the second stage (effort stage), each player observes the chosen risks and then decides on how much effort to exert. The player who attains the highest output or performance is the winner of the tournament and receives the winner's prize whereas the other player receives the loser's prize. The players are homogeneous in talent, cost of effort, and utility function. An overconfident player overestimates his absolute talent and, as a consequence, his relative performance, and winning probability.

We study the implications of overconfidence using two different setups. In the first set-up, the players are equally overconfident and the tournament is symmetric. In the second set-up, the tournament is asymmetric since one player is overconfident and the other one is rational. To be able to derive equilibria when players hold mistaken beliefs we follow Squintani (2006). Since this is a two-stage game of complete information we look for subgame perfect equilibria (SPE). For some of our results, we assume the random shocks follow a normal distribution and effort costs are exponential.

We obtain three main results for tournaments where two homogeneous players compete against each other. First, players may adopt less risky strategies when they are overconfident than when they are rational. The intuition behind this rather counter-intuitive result is as follows. In the unique SPE of a tournament where two rational players compete against each other, both players choose the high risk strategy in the first stage and low effort in the second stage (Hvide, 2002). Now, consider a tournament where two overconfident players compete against each other. In any pure-strategy SPE, both players choose the same risk and the same effort. However, due to the (mis)perceived talent advantage, each player thinks, mistakenly, that he is the favorite and the opponent is the underdog. When the overconfidence bias is large, a player's perceived talent advantage over his opponent is also large. In such a situation, the outcome of the tournament is (mis)perceived to be more dependent on the large talent gap but less so on effort. Hence, it is beneficial for both players not to imperil their (mis)perceived favorite position. They do so by selecting the low risk strategy in the first stage.[2] This finding goes against the idea that overconfi-

[2]When the bias is small, overconfident players choose the high risk strategy in the first stage.

dence always raises risk taking. On the contrary, it shows that highly overconfident individuals may innovate less than rational ones (interpreting an investment in new technology as a high risk strategy).

Second, players may exert more effort when they are overconfident than when they are rational. As we have seen, overconfident players with a large bias choose the low risk strategy in the first stage. Since the equilibrium risk taking in the first stage is low, luck plays a minor role in determining the winner. Consequently, overconfident players with a large bias choose a high equilibrium effort in the second stage. This finding provides a novel mechanism whereby overconfidence raises effort provision. Note that we rule out complementarities between self-confidence and effort (Bénabou and Tirole, 2002, 2003) by assuming that talent and effort are additively separable. In fact, this assumption implies that self-confidence and effort are substitutes, that is, an increase in self-confidence lowers the equilibrium effort in the second stage for any risk strategy profile chosen in the first stage.[3]

Third, the players' overconfidence can make the tournament organizer (firm or principal) better off. As we have seen in the two previous paragraphs, overconfidence may lower risk taking as well as raise the effort provision of both players. This makes a risk-neutral tournament organizer better off: keeping total compensation (the sum of tournament prizes) fixed, the tournament organizer obtains higher output when the players are overconfident than when they are rational. Of course, if the tournament organizer is risk averse, there is the additional benefit from the fact that lower risk taking by players lowers the tournament organizer's risk exposure. An implication of this finding is that it might not be in the best interests of the tournament organizer to debias overconfident players. The paper clarifies when this is the case. When the

In this case, a player's perceived talent advantage over the opponent is small. In such a situation, the outcome of the tournament is (mis)perceived to be less dependent on the small talent gap but more so on effort. Hence, it is beneficial for both players to limit the effort exerted in the second stage. They do so by selecting the high risk strategy in the first stage.

[3]Overconfident players with a small bias choose the high risk strategy in the first stage. In this case, overconfident players exert less effort than rational ones in the second stage since, taking risk strategies as given, an increase in overconfidence always lowers effort. In contrast, overconfident players with a very large bias choose a low risk strategy in the first stage. However, the very large bias implies that the negative effect of overconfidence on effort provision dominates the positive effect of lower risk taking. Hence, overconfident players with a very large bias exert less effort than rational ones.

players' bias is large, both players provide more effort when they are overconfident than when they are rational, and a risk neutral tournament organizer is better off. In contrast, when the players' bias is either small or very large, both players exert less effort when they are overconfident than when they are rational, and a risk-neutral tournament organizer is worse off.

We obtain four main results for tournaments where an overconfident player competes against a rational player. First, in any SPE the overconfident player exerts less effort than the rational player. The intuition behind this result is straightforward. When players are equally talented but one is overconfident and the other is rational, the tournament is asymmetric and so is any SPE. Since the overconfident player thinks, mistakenly, that he has a talent advantage over his opponent, he prefers to lower his effort to save on effort costs. The rational player anticipates this and decides to lower her effort too but to a lesser extent.

Second, the overconfident player is less likely to win the tournament when risk is normally distributed. As we have seen, in any SPE the overconfident player exerts less effort than the rational player in the second stage. Hence, in any SPE where both workers choose the same risk strategy in the first stage, risk taking cancels out and the rational player has a higher objective probability of winning the tournament due to her higher effort. Matters are not so straightforward in any SPE where players choose different risk strategies. However, when the random shocks are normally distributed, only the sum of risks matters to determine how risk taking influences the players' objective probabilities of winning the tournament. Since both players face the same sum of risks, regardless of their chosen risk strategies, it follows that the overconfident player, by exerting lower effort, has a smallest objective winning probability than the rational player. This result shows that in tournaments where players can choose risk as well as effort, overconfident players may be less likely to be promoted than rational ones. This finding is relevant to the literature on how overconfidence can affect the selection of managers for CEO positions and stands in contrast to Goel and Thakor (2008) who show that overconfident managers (defined as those who underestimate the risk of their projects) are more likely to be promoted to CEO than rational ones.

Third, the overconfident player may choose a less risky strategy than the rational player but the reverse cannot happen. The intuition behind this result is as follows.

As the overconfident player becomes increasingly overconfident, both players exert lower efforts, and the effort gap increases. When the overconfident player's bias is large, he thinks, mistakenly, that he has a large talent advantage over the rational player, even considering that he ends up exerting less effort than the rational player. Thinking, mistakenly, that he is the favorite, the overconfident player chooses the low risk strategy. The rational player, aware of her opponent's overconfidence, and knowing that the effort gap in her favor is small, still prefers the high risk strategy. Therefore, when the overconfident player's bias is large, the overconfident player chooses the low risk strategy and the rational player the high risk strategy.[4]

Fourth, the presence of an overconfident player may lead a rational player to take less risk. As we have seen in the previous paragraph, when the overconfident player's bias is large, the rational player chooses the high risk strategy whereas the overconfident player switches from the high to the low risk strategy. When the overconfident player's bias is very large, there is a large effort gap in favor of the rational player. Nevertheless, the overconfident player chooses a low risk strategy since he thinks, mistakenly, that his very large talent advantage more than compensates the large effort gap in favor of the rational player. The rational player, aware that her opponent's overconfidence, and knowing that the effort gap in her favor is large, thinks correctly she has a clear advantage and switches from the high to the low risk strategy. Thus, when the overconfident player's bias is very large, both players choose the low risk strategy. This finding shows that overconfident individuals can lead rational individuals to innovate less than they would if everyone were rational. This stands in contrast to the idea that overconfident individuals take more risks, which can spur innovation (See Malmendier and Tate (2008); Nosić and Weber (2010); Hirshleifer et al. (2012); Goldberg et al. (2020)).

Finally, we discuss the impact of overconfidence on the tournament organizer's welfare when he knows about the players' overconfidence bias and selects the tournament

[4] When the overconfident player's bias is small, he thinks, mistakenly, that he has a small talent advantage whereas the rational player thinks, correctly, that she has an effort advantage (due to the small effort gap in her favor). The small (mis)perceived talent advantage of the overconfident player and the small effort advantage of the rational player make the players exert similar efforts. Due to the small effort gap, players have an incentive to lower effort costs by choosing the high risk strategy. Thus, when the overconfident player's bias is small, both players choose the high risk strategy.

prizes optimally. We show that if the players are risk neutral, the tournament organizer is able to counteract any unfavorable effect of overconfidence on effort by raising the prize spread while keeping total compensation fixed. Hence, if the players are risk neutral, the tournament organizer always benefits from overconfidence. This is no longer the case when the players are risk averse since raising the prize spread exposes players to increasing risk which they dislike. These welfare results extend those obtained in Santos-Pinto (2010) for tournaments where players only choose effort.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 sets up the model. Section 4 studies tournaments where two homogeneous players compete against each other. Section 5 studies tournaments where an overconfident player competes against a rational player. Section 6 concludes the paper. All proofs are in the Appendix.

1.2 Related Literature

Our paper contributes to two strands of literature. First, it contributes to the growing literature on the impact of overconfidence on labor and financial markets.

Malmendier and Tate (2005, 2008, 2015) study how the behavior of overconfident CEOs differs from that of rational CEOs. They show that overconfidence leads to a general overestimation of the ability to raise a company's stock prices and consequently leads CEOs to hold stock options for too long. Their empirical analysis supports the theoretical predictions. Goel and Thakor (2008) study tournaments where overconfident and rational managers compete against each other to be promoted to CEO. The managers compete by choosing the level of risk of their projects and overconfident managers underestimate the risk of their projects. They find that overconfident managers have a higher likelihood of being promoted to CEO than rational ones.

Using a principal-agent model, Santos-Pinto (2008) shows that when effort is observable, worker overconfidence is always favorable to the firm. He also provides conditions under which worker overconfidence makes the firm better off when effort is unobservable. Santos-Pinto (2010) finds that overconfidence can raise effort provision in tournaments with homogeneous workers. He also shows how the firm should set tournament prizes to exploit workers' overconfidence. Daniel and Hirshleifer (2015) discuss the role of overconfidence (defined as miscalibration, where an investor overestimates the precision of her signal) as an explanation for aggressive trading resulting in high risk and low net returns. They also evaluate empirically to what extent overconfidence is a key factor in financial decision-making. Hoffman and Burks (2020) show that truck drivers tend to systematically and persistently overpredict their productivity. While overconfidence moderately decreases the workers' welfare, it also substantially increases companies' profits as overconfidence makes drivers less likely to quit.

Our study contributes to this literature by showing that overconfidence can lower risk taking in tournaments where managers compete for promotion to CEO. We also document a new mechanism whereby worker overconfidence makes the firm better off: By lowering risk taking, overconfidence can raise workers' effort provision. This finding is in line with previous studies which identify positive effects of worker overconfidence for firms (Fang and Moscarini, 2005; Gervais and Goldstein, 2007; Santos-Pinto, 2008, 2010; De la Rosa, 2011). In contrast to Goel and Thakor (2008), our study shows that overconfident managers are less likely to be promoted to CEO than rational ones. Finally, we show overconfident managers are less likely to be promoted to CEO than rational ones when they compete in a tournament where they choose risk as well as effort.

Second, our paper contributes to the literature on tournaments started by Lazear and Rosen (1981). The most closely related papers within this literature are Hvide (2002), Kräkel and Sliwka (2004), and Kräkel (2008). Hvide (2002) shows that a tournament might break down as an incentive scheme when players can choose risk as well as effort. He finds that, in a tournament where two homogeneous players compete against each other and where there are no limits to possible risk taking, the players choose an infinite amount of risk and zero effort in equilibrium. The intuition for this result is that since equilibrium efforts are always identical the winning probability of each player is always one half irrespective of the risk level. As a consequence, players have a common incentive to increase the level of risk (noise) in the tournament to lessen the importance of differences in effort to the winning probability. And, when effort becomes less important to determine the winning probability, players have an incentive to exert less effort. He also shows that this result holds when there are limits to risk taking (in this case players choose maximum risk and minimum effort) and when cost of effort asymmetries between the players are not too large.

Kräkel and Sliwka (2004) study tournaments where players are heterogeneous in talent (one player is more talented than the other) but homogeneous in cost of effort and utility function. They find that, even though there is a talent difference, equilibria in the effort stage are symmetric. They also show that the amount of risk taking is determined by two separate effects. First, risk taking influences a player's effort and the cost of effort (effort effect). Second, risk taking affects a player's probability of winning (likelihood effect). Kräkel and Sliwka (2004) find that whether the players prefer high or low risk depends on the magnitude of the favorite's lead. Moreover, if the lead is small (large) both players choose high (low) risk. The impact of risk on effort levels crucially depends on the magnitude of talent difference, i.e., the equilibrium efforts increase in risk only if the players' talents are sufficiently different. In particular, they show that depending on the relative size and the interplay of the effort and likelihood effect, different SPE are possible.[5]

Kräkel (2008) studies tournaments where players are heterogeneous in their utility functions (one player is more risk averse than the other) but homogeneous in terms of talent and cost of effort. He finds that when the players' utility functions differ, only asymmetric pure-strategy equilibria can exist at the effort stage. In these asymmetric equilibria, the less risk averse player exerts the highest effort and hence is the favorite. In addition, he shows that in these tournaments a reverse likelihood effect can occur, that is, a higher risk leads to an increased winning probability of the favorite which is impossible in the tournaments studied by Hvide (2002) and Kräkel and Sliwka (2004). As a consequence, there can exist SPE where the favorite chooses the high risk strategy and the underdog the low risk strategy.

Tournaments with heterogeneous players are also related to contests in which players differ.[6] In this context, Singh and Wittman (2001) look at a contest where players differ in the marginal productivity of effort. They find that output increases in ability and that effort provision decreases in effort costs. Santos-Pinto and Sekeris

[5] Bronars (1986) shows, in a sequential tournament setting, that the player who has an advantage prefers a low risk strategy to secure his favorable position whereas the opponent lagging behind has incentives to choose a high risk strategy as this opens the possibility to catch up.

[6] Tullock contests can be equivalently represented as tournaments with the Gumbel distribution of noise (Drugov et al., 2022).

(2022) investigate the role of overconfidence in contests, where players are assumed to have different technologies and preferences. They show that for any advantage, a player may have regarding his contest technology or cost function, a large enough overconfidence bias can always make that player's winning odds smaller than 1/2.

Our study contributes to this literature by being the first to analyze how overconfidence influences risk taking and effort provision in tournaments. We find that overconfident players may adopt less risk and exert higher effort than rational ones. We also show that a tournament no longer breaks down as an incentive scheme when players are overconfident and choose risk as well as effort. This stands in contrast to Hvide (2002). In our study, just like in Kräkel and Sliwka (2004), risk taking influences a player's effort (effort effect) and a player's perceived winning probability (likelihood effect). We find that the two effects are completely separate in a tournament where two equally overconfident players compete against each other. In contrast, the two effects are strictly interrelated in a tournament where an overconfident player competes against a rational player. Hence, an interrelatedness between the effort and the likelihood effects might not only exist due to heterogeneity in players' utility functions, as in Kräkel (2008), but also due to heterogeneity in players' beliefs about talent.

1.3 Set-up

We consider a two-stage tournament played between two players. The winner of the tournament receives the winning prize y_w while the loser receives the losing prize y_l, with $y_w > y_l \geq 0$. The winner of the tournament is the player who attains the highest output or performance. Hence, winning or losing the tournament only depends on relative performance. The players have identical utility functions U which are separable in income y_i and effort a_i.

$$U(y_i, a_i) = u(y_i) - c(a_i).$$

Note that the von Neumann-Morgenstern utility function $u(y_i)$ and the cost of effort function $c(a_i)$ are identical for both players. We assume $u'(y_i) > 0$, $c'(a_i) > 0$, and $c''(a_i) > 0$. A player's outside option is normalized to zero. We assume that player i's performance or output is linearly additive in talent $t \geq 0$, effort a_i, and an individual noise term ϵ_i. That is, if player i exerts effort a_i her output is given by

$$Q_i = t + a_i + \epsilon_i,$$

We assume ϵ_1 and ϵ_2 are zero mean random variables with variances σ_1^2 and σ_2^2, respectively, are stochastically independent, and symmetric around their mean. The difference $\epsilon_2 - \epsilon_1$ has the cumulative distribution function $G(\epsilon_2 - \epsilon_1)$ and density $g(\epsilon_2 - \epsilon_1)$. We assume $G(.)$ is continuous and twice differentiable. Note that $g(x)$ is symmetric around zero since ϵ_1 and ϵ_2 are symmetric. Further, $g(x)$ satisfies $g'(x) > 0$ for $x < 0$ and $g'(x) < 0$ for $x > 0$. This specification is chosen for its analytical simplicity and is often used in the tournament literature (see Lazear and Rosen (1981), Green and Stokey (1983), and Akerlof and Holden (2012)).

Players are assumed to be homogeneous except for potential differences in their overconfidence levels. Thus, players can differ from one another in terms of the perception of their own talent. Player 1 is considered to be overconfident and, thus, overestimates his talent. Player 2 can be either overconfident or rational. Following Squintani (2006) we assume that: (1) a player who faces an overconfident opponent is aware that the latter's perception of his own talent (and probability of winning) is mistaken, (2) each player thinks that his own perception of his talent (and probability of winning) is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players' agree to disagree about their talents (and winning probabilities).[7]

Player 1's perceived production function, hence, can be described by

$$\tilde{Q}_1 = \lambda_1 + t + a_1 + \epsilon_1,$$

where $\lambda_1 > 0$ is the parameter that captures player 1's overconfidence. Player 2 perceives her stochastic production function to be equal to

$$\tilde{Q}_2 = \lambda_2 + t + a_2 + \epsilon_2,$$

where $\lambda_2 \geq 0$ is the parameter that captures player 2's overconfidence. Under this specification, an overconfident player overestimates his total productivity of effort but holds a correct assessment of his marginal productivity of effort.

[7] These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin and Ross, 2002; Pronin and Kugler, 2007).

The timing of the tournament is as follows. At the first stage (risk stage) players simultaneously choose their risk exposure $\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}$, where $i = 1, 2$ and $0 < \sigma_L^2 < \sigma_H^2 < \infty$. By choosing the high risk strategy, a player induces a mean preserving spread of her output Q_i through an increase of the variance of ϵ_i. At the second stage (effort stage), each player observes the chosen risks and decides about her effort a_i. Thereby, we reflect the timing of many tournaments, where players decide about a risk strategy beforehand. For instance, managers have the option to implement a new (and riskier) production technology or to stick to the old, more standard technology before they actually start producing. Also in many sports, the coach of a team or a player formulates a game plan before he or she decides how much effort to put into the game later on. The game plan can be seen as a risk choice when the coach or player decides on an offensive (riskier) or defensive (less risky) strategy. The solution concept we employ is SPE.

1.4 Two Equally Overconfident Players

This section studies a tournament between two homogeneous and overconfident players. Hence, we assume $\lambda_1 = \lambda_2 = \lambda > 0$. We start by solving the second stage (effort stage) and then solve the first stage (risk stage). In the second stage, player 1 chooses the optimal effort level that maximizes his perceived expected utility

$$\begin{aligned} E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] &= u(y_l) + P_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)\Delta u - c(a_1) \\ &= u(y_l) + \Pr(\tilde{Q}_1 \geq Q_2)\Delta u - c(a_1) \\ &= u(y_l) + \Pr(\lambda + a_1 + \epsilon_1 \geq a_2 + \epsilon_2)\Delta u - c(a_1) \\ &= u(y_l) + \Pr(\epsilon_2 - \epsilon_1 \leq \lambda + a_1 - a_2)\Delta u - c(a_1) \\ &= u(y_l) + G(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1). \end{aligned}$$

where $\Delta u \equiv u(y_w) - u(y_l) > 0$ denotes the utility prize spread. Similarly, in the second stage, player 2 chooses the optimal effort level that maximizes her perceived expected utility

$$\begin{aligned} E[U_2(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] &= u(y_l) + P_2(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(\tilde{Q}_2 \geq Q_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(\lambda + a_2 + \epsilon_2 \geq a_1 + \epsilon_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(\epsilon_2 - \epsilon_1 \geq a_1 - a_2 - \lambda)\Delta u - c(a_2) \end{aligned}$$

$$= u(y_l) + \left[1 - G(a_1 - a_2 - \lambda; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2).$$

The pure-strategy Nash equilibrium (a_1^*, a_2^*) of the second stage satisfies the two first-order conditions simultaneously, and is given by

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*)$$

and

$$g(a_1^* - a_2^* - \lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*).$$

Thus, the second-order conditions of the effort stage are given by

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1^2} = g'(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c''(a_1) < 0$$

and

$$\frac{\partial^2 E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2^2} = -g'(a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c''(a_2) < 0,$$

respectively. A sufficient condition for a unique pure-strategy Nash equilibrium to exist at the effort stage is that

$$g'(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u < c''(a_1), \quad \forall a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2$$

and

$$-g'(a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u < c''(a_2), \quad \forall a_1, a_2, \sigma_1^2, \sigma_2^2.$$

As it is known in the tournament literature, a pure-strategy Nash equilibrium only exists if there is sufficient noise and the cost function $c(a)$ is sufficiently convex (Lazear and Rosen, 1981). Therefore, the existence of a pure-strategy Nash equilibrium is assured when

$$\Delta u \sup_x g'(x) < \inf_{a>0} c''(a). \tag{1.1}$$

Condition (1.1) ensures that the second-order conditions are satisfied. Note that the lower is $\sup_x g'(x)$ the flatter is $g(x)$ and hence the higher is the noise in the tournament. Note also that $0 < c_0 = \inf_{a>0} c''(a)$ defines a class of cost functions with a second derivative bounded away from zero.

Lemma 1 shows that there exists a unique symmetric pure-strategy equilibrium of the effort stage.

Lemma 1. *In a tournament between two homogeneous and equally overconfident players, the effort stage has a unique symmetric pure-strategy equilibrium.*

Lemma 1 tells us that in the any SPE of a tournament between two homogeneous and equally overconfident players, both players exert the same effort for any risk strategy profile (σ_1^2, σ_2^2) chosen in the risk stage. Since $a_1^* = a_2^* = a^*$ the first-order condition of the representative player's optimization problem in the effort stage becomes

$$g(\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*). \tag{1.2}$$

Equation (1.2) shows that in equilibrium players should increase their effort level up to the point where the perceived marginal benefit of doing so - the perceived marginal probability of winning the tournament times the utility differential between winning and losing - equals its incremental cost - the marginal disutility of effort. Differentiation of (1.2) gives us

$$\frac{\partial a^*}{\partial \lambda} = -\frac{g'(\lambda; \sigma_1^2, \sigma_2^2)\Delta u}{g'(\lambda; \sigma_1^2, \sigma_2^2)\Delta u - c''(a^*)}. \tag{1.3}$$

The denominator in (1.3) is the second-order condition and is negative. Since the utility prize spread Δu is always positive the relation between overconfidence and effort is given by the sign of $g'(\lambda; \sigma_1^2, \sigma_2^2)$, that is, how overconfidence influences the perceived marginal probability of winning the tournament for any given risk strategy profile (σ_1^2, σ_2^2). Since $\lambda > 0$ it follows that $g'(\lambda; \sigma_1^2, \sigma_2^2) < 0$ and therefore $\partial a^*/\partial \lambda < 0$. Hence, the equilibrium effort in the second stage is always decreasing in the overconfidence bias λ for any given utility prize spread and risk strategy profile. In other words, in the second stage, self-confidence and effort are substitutes. This result is driven by the assumption that output is additively separable in talent and effort.

In the first stage, players choose their risk strategies simultaneously. Hence, players 1 and 2 solve the following maximization problems

$$\max_{\sigma_1^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + G(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1^*),$$

$$\max_{\sigma_2^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + \left[1 - G(a_1^* - a_2^* - \lambda; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2^*),$$

respectively. Since in any SPE $a_1^* = a_2^* = a^*$ for any given (σ_1^2, σ_2^2), and the symmetry of $G(x)$ implies $G(\lambda) = 1 - G(-\lambda)$, the two problems are identical and the representative player's maximization problem is

$$\max_{\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + G(\lambda; \sigma_1^2, \sigma_2^2)\Delta u - c(a^*(\sigma_1^2, \sigma_2^2)). \tag{1.4}$$

Problem (1.4) shows that a player's risk choice has two effects on his perceived expected utility. On the one hand, it changes the player's perceived winning probability (likelihood effect). On the other hand, it changes the player's effort in the second stage and therefore the cost of effort (effort effect).[8]

Since the tournament is symmetric, we have two possible candidates for pure-strategy risk strategy profiles in a SPE: $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$ and $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$. It follows from (1.4) that both players choose the high risk strategy as long as

$$c(a^*(\sigma_L^2, \sigma_H^2)) - c(a^*(\sigma_H^2, \sigma_H^2)) \geq \left[G(\lambda; \sigma_L^2, \sigma_H^2) - G(\lambda; \sigma_H^2, \sigma_H^2)\right] \Delta u. \tag{1.5}$$

Inequality (1.5) tells us that both players choose the high risk strategy when a unilateral deviation to a low risk strategy raises the cost of effort more than it increases the perceived probability of winning times the utility prize spread. In other words, both players choose the high risk strategy when the unfavorable effort cost effect is greater than the favorable likelihood effect of switching to a low risk strategy. Hence, when inequality (1.5) holds, there exists a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$. It also follows from (1.4) that both players choose the low risk strategy as long as

$$\left[G(\lambda; \sigma_L^2, \sigma_L^2) - G(\lambda; \sigma_H^2, \sigma_L^2)\right] \Delta u \geq c(a^*(\sigma_L^2, \sigma_L^2)) - c(a^*(\sigma_H^2, \sigma_L^2)) \tag{1.6}$$

Inequality (1.6) tells us that both players choose the low risk strategy when a unilateral deviation to a high risk strategy lowers the cost of effort less than it lowers the perceived probability of winning times the utility prize spread. In other words, both players choose the low risk strategy when the favorable effort cost effect is smaller than the unfavorable likelihood effect of switching to a high risk strategy. Hence, when inequality (1.6) holds, there exists a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$.

As we shall show next, there exist threshold values for the bias λ such that inequalities (1.5) and (1.6) bind. Moreover, we will show that when the players' bias is small, both choose the high risk strategy but, when the players' bias is large, both choose

[8]This terminology for the two effects was introduced by Kräkel and Sliwka (2004),

the low risk strategy. To do this, we specialize the model by assuming that ϵ_1 and ϵ_2 follow a normal distribution with zero mean and variance σ_1^2 and σ_2^2, respectively. This implies $x = \epsilon_1 - \epsilon_2$ is also normally distributed with zero mean and variance $\sigma_1^2 + \sigma_2^2$. We use $\Phi(x)$ to denote the cumulative distribution function of x and $\phi(x)$ its density. In addition, we assume an exponential cost of effort, that is, $c(a_i) = e^{a_i}$.[9] These two assumptions allow us to derive a closed-form solution for the equilibrium effort and to obtain unique threshold values for λ under which (1.5) and (1.6) hold as equalities. This leads us to Proposition 1.

Proposition 1. *In a tournament between two homogeneous and equally overconfident players, where ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, the equilibrium effort is given by*

$$a_1^*(\sigma_1^2, \sigma_2^2) = a_2^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}. \tag{1.7}$$

The equilibrium effort is strictly increasing in the utility prize spread Δu, and decreasing in the overconfidence bias λ. Furthermore, the equilibrium effort is decreasing (increasing) in the sum of risks $\sigma_1^2 + \sigma_2^2$ when $\lambda^2 < \sigma_1^2 + \sigma_2^2$ ($\lambda^2 > \sigma_1^2 + \sigma_2^2$).

Proposition 1 shows that the equilibrium effort is increasing in the utility prize spread and decreasing in the overconfidence bias. Intuitively, the higher a player's overconfident bias becomes the more he trusts his (perceived) advantage to get himself a lead in the tournament. Consequently, he becomes more slack and decreases his effort level up to a point, where the overconfidence bias is so dominant that he exerts zero effort. Further, the equilibrium effort levels are only positive if the overconfident bias λ is sufficiently small or the utility prize spread Δu is sufficiently large. Henceforth, we assume that $\Delta u > \sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}e^{\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}$.[10] Whether this threshold is increasing or decreasing in the sum of risks $\sigma_1^2 + \sigma_2^2$ depends on the relative size of

[9]Note, that this specific cost function exhibits fixed costs as $c(0) > 0$. Fixed costs can be motivated by the fact that tournament players often face costs before participating in the actual tournament. Athletes may have to pay for a gaming license or travel to a specific sports contest. Also, the preparation in form of training prior to a tournament can be seen as fixed costs (Kräkel and Sliwka, 2004).

[10]This assumption permits for an endogenous choice of the prize spread, which indicates that the tournament organizer can choose the prize spread such that players exert positive efforts.

the overconfident bias λ. Proposition 1 also provides an interesting result regarding the relation between equilibrium effort and risk taking captured by the sum of risks $\sigma_1^2+\sigma_2^2$. When the square of the bias is less than the sum of risks, an increase in risk taking by either player lowers the equilibrium efforts. However, when the square of the bias is greater than the sum of risks, an increase in risk taking by either player raises the equilibrium efforts.

Let us now consider stage 1, the risk stage. As we have seen, the level of risk affects a player's perceived winning probability (likelihood effect) as well as his effort in the second stage (effort effect). Depending on the size and direction of these two effects we obtain different SPE outcomes. This finding is summarized below.

Proposition 2. *Consider a tournament between two homogeneous and equally overconfident players, where ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential. Let $\bar{\lambda}_1$ denote the unique solution to (1.5) and $\bar{\lambda}_2$ the unique solution to (1.6).*

(i) If $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, then there is a unique SPE where both players choose the high risk strategy.

(ii) If $\lambda \in (\bar{\lambda}_1, \bar{\lambda}_2)$, then there is a unique SPE where players mix between the high and the low risk strategies.

(iii) If $\lambda \in (\bar{\lambda}_2, \bar{\lambda}_1)$, then there are three SPE; In one SPE both players choose the high risk strategy. In another SPE both players choose the low risk strategy. In another SPE players mix between the high and the low risk strategies.

(iv) If $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$, then there is a unique SPE where both players choose the low risk strategy.

In all of the above SPE, the players' equilibrium effort is given by (1.7).

To understand the intuition behind Proposition 2, let us start by looking at the SPE of a tournament with two rational players. In this case, there is a unique SPE where both players choose the high risk strategy and exert efforts of

$$a_1^*(\sigma_1^2, \sigma_2^2) = a_2^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right).$$

Since the tournament is symmetric both players exert the same effort and are therefore equally likely to win, i.e., $P_i = 1/2$, $\forall i$. This holds independently of the risk strategy configurations in the first stage because players take the sum of risks as given in the second stage. Since the equilibrium effort in the second stage depends negatively on the sum of risks, choosing a high risk strategy in the first stage is a dominant strategy. In other words, a unilateral deviation to a low risk strategy in the first stage does not alter a player's probability of winning but raises the cost of effort. This result was first shown by Hvide (2002).

Let us now consider a tournament with two overconfident players. When the players' bias is small, i.e., $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, a player's perceived advantage over the opponent is also small. In this case, the outcome of the tournament is less dependent on the perceived talent gap and more so on effort. Hence, it is beneficial for the players to limit the effort exerted. They can do so by selecting the high risk strategy. In contrast, when the players' bias is large, i.e., $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$, a player's perceived advantage over the opponent is also large. Now, the outcome of the tournament is more dependent on the perceived talent gap but less on effort. Hence, it is beneficial for players to lower the role played by risk and not to imperil their perceived large advantage. They can do so by selecting the low risk strategy.[11]

Our next result compares the equilibrium efforts in a tournament with overconfident players to those in a tournament with rational players.

Proposition 3. *Consider a tournament between two homogeneous and equally overconfident players, where ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential. Let $\bar{\lambda}_1$ denote the unique solution to (1.5) and $\bar{\lambda}_2$ the unique solution to (1.6).*

(i) If $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, then effort provision is lower than if both players were rational.

(ii) If $\lambda \in (\max\{\bar{\lambda}_1, \bar{\lambda}_2\}, 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)})$, then effort provision is higher than if

[11] Part (ii) of Proposition 2 shows that when $\lambda \in (\bar{\lambda}_1, \bar{\lambda}_2)$, there exists a unique SPE where players mix between the low and the high risk strategy. In addition, part (iii) of Proposition 2 shows that when $\lambda \in (\bar{\lambda}_2, \bar{\lambda}_1)$ there are two pure-strategy SPE and a SPE where players mix between the low and high risk strategies. We focus throughout on pure-strategy SPE and hence skip the full characterization of mixed-strategy SPE.

both players were rational.

(iii) If $\lambda > 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)}$*, then effort provision is lower than if both players were rational.*

Proposition 3 shows that the size of the players' overconfidence bias leads to different equilibrium risk and effort strategy profiles. If the bias is small, both players choose a high risk strategy and low effort. If the bias is large, both players choose a low risk strategy and high effort. Finally, if the bias is very large, both players choose a low risk strategy and low effort.

The intuition behind this result is as follows. In a tournament between two rational players, both players choose the high risk strategy. Since the equilibrium risk taking is high in the first stage, luck plays a major role in determining the winner. Consequently, rational players choose low equilibrium efforts in the second stage. Overconfident players with a small bias, i.e., $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, also choose the high risk strategy. Since a small bias lowers effort provision without changing risk taking, overconfident players exert less effort than rational ones. In contrast, overconfident players with a large bias, i.e., $\lambda \in (\max\{\bar{\lambda}_1, \bar{\lambda}_2\}, 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)})$, choose the low risk strategy. Since the equilibrium risk taking is low, luck plays a minor role in determining the winner. Consequently, overconfident players with a large bias choose high equilibrium efforts. Finally, overconfident players with a very large bias, i.e., $\lambda > 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)}$, also choose a low risk strategy. However, the very large bias implies that the direct negative effect of overconfidence on effort provision dominates the indirect positive effect from lower risk taking. Hence, overconfident players with a very large bias exert less effort than rational ones.

This result is illustrated below. Figure 1.1 shows a jump in effort provision when overconfident players switch from the high risk strategy to the low risk strategy. Further, as depicted, there exist values for λ for which overconfident players exert more effort than rational ones. In other words, for a large overconfidence bias, the equilibrium effort a^* of overconfident players lies above that of rational players, where the latter is represented by the dashed line. However, as the bias increases further, effort provision of overconfident players falls below that of rational players. This is because, for a given risk strategy, effort provision a^* decreases in the overconfidence bias λ.

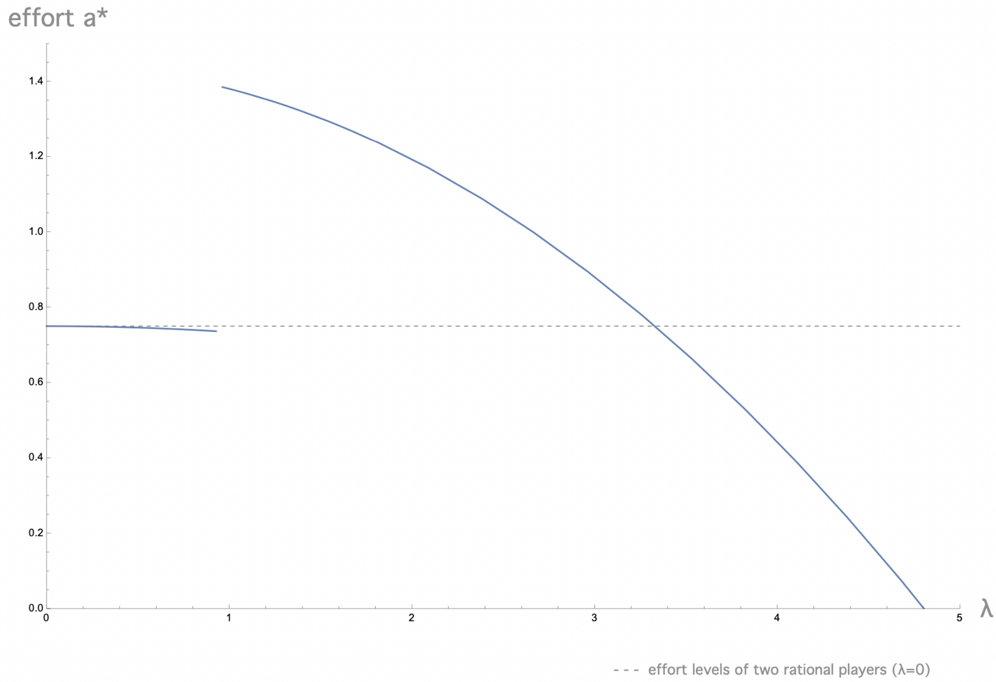

Figure 1.1: Relationship between equilibrium effort and overconfidence level, for $\sigma_L^2, \sigma_H^2 = (4, 16)$ and $\Delta u = 30$. In this case, the thresholds $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are equal to 0.963 and 0.927 , respectively.

These results imply that under specific circumstances a risk-neutral tournament organizer benefits from players' overconfidence. Assuming that the tournament organizer cares about total effort, he favors a large overconfidence bias as largely overconfident players exert higher effort than rational ones.[12]

Finally, we find that overconfidence has the following welfare implications for the players. Either a small or a very large bias makes both players better off since the lower effort leads to lower effort costs. In contrast, a large bias makes both players worse off since the higher effort leads to higher effort costs.

[12]Note that if instead, the tournament organizer wishes to maximize the winner's output there is a trade-off. On the one hand, a large overconfidence bias increases the effort provision of the winner. On the other hand, a large overconfidence bias lowers risk taking which thus decreases the output of the winner.

1.5 Overconfident vs Rational Player

This section studies tournaments where an overconfident player 1 competes against a rational player 2. To do so, we assume $\lambda_1 = \lambda > 0$ and $\lambda_2 = 0$. We start by solving the second stage (effort stage) and continue to solve the first stage (risk stage). In the second stage, player 1 chooses the optimal effort level that maximizes his perceived expected utility

$$E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] = u(y_l) + G(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1).$$

Similarly, player 2 chooses the optimal effort level that maximizes her expected utility

$$\begin{aligned} E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)] &= u(y_l) + P_2(a_1, a_2, \sigma_1^2, \sigma_2^2)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(Q_2 \geq Q_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(a_2 + \epsilon_2 \geq a_1 + \epsilon_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(\epsilon_2 - \epsilon_1 \geq a_1 - a_2)\Delta u - c(a_2) \\ &= u(y_l) + \left[1 - G(a_1 - a_2; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2). \end{aligned}$$

The pure-strategy Nash equilibrium (a_1^*, a_2^*) of the second stage satisfies the two first-order conditions simultaneously and is given by

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*),$$

and

$$g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*).$$

The second-order conditions of the effort stage are satisfied when the cost function is sufficiently convex (see Appendix). Our next result characterizes the pure-strategy equilibrium efforts (a_1^*, a_2^*).

Proposition 4. *In a tournament where player 1 is overconfident and player 2 is rational, the overconfident player 1 exerts less effort than the rational player 2, i.e., $a_1^* < a_2^*$. Moreover, the efforts of both players are decreasing in player 1's overconfidence bias λ, with $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$, such that the effort gap increases in λ, i.e., $\partial(a_2^* - a_1^*)/\partial\lambda > 0$.*

The intuition for this result is that while trusting the (perceived) advantage in his ability to get himself a lead in the tournament, the overconfident player becomes slack relative to the rational player. This effect holds for any risk strategy profiles $\left(\sigma_1^2, \sigma_2^2\right)$.

Proposition 5. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the overconfident player 1 has a lower objective probability of winning the tournament than the rational player 2.*

To understand the intuition behind Proposition 5, let P_i denote the objective winning probability of player i, with $i = 1, 2$. Note that, in any pure-strategy SPE, the overconfident player 1 wins the tournament with probability $P_1 = G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$ and the rational player 2 with probability $P_2 = G(a_2^* - a_1^*; \sigma_1^2, \sigma_2^2)$.

When both players are rational, the tournament is symmetric and players exert the same efforts given by $a_1^* = a_2^* = a^*$ where a^* solves $g(0; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*)$. Symmetry of $g(x)$ implies $P_1 = P_2 = G(0; \sigma_1^2, \sigma_2^2) = 1/2$. Hence, when both players are rational, each player is equally likely to win the tournament (i.e., the winner is purely random). This is true for any cumulative distribution G that satisfies the assumptions we made.

When player 1 is overconfident and player 2 is rational, the tournament is asymmetric and the overconfident player 1 exerts less effort than the rational player 2, i.e., $a_1^* < a_2^*$. Hence, in any pure-strategy SPE where both players choose the same risk strategy, the overconfident player 1 is less likely to win the tournament due to his lower effort. However, in any pure-strategy SPE where the players choose different risk strategies that might no longer be the case due to different likelihood effects. Still, Proposition 5 shows that when G is the normal cumulative distribution only the sum of risks $\sigma^2 = \sigma_1^2 + \sigma_2^2$ matters to determine the likelihood effect. Since both players face the same sum of risks, the likelihood effect is identical and the overconfident player 1, who exerts less effort, has a lower objective probability of winning the tournament than the rational player 2.

In the first stage, players 1 and 2 solve the following maximization problems, respectively,

$$\max_{\sigma_1^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + G(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1^*) \tag{1.8}$$

and

$$\max_{\sigma_2^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + \left[1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2^*). \tag{1.9}$$

Problems (1.8) and (1.9) show that a player's risk choice influences his perceived expected utility through the effort and likelihood effects identified previously. However, these two effects are now interrelated: risk taking influences both the shape of the perceived cumulative distribution function G and the position of $a_1^* - a_2^*$ at which the perceived winning probability is computed, namely the gap between the equilibrium efforts. This interrelatedness is due to the heterogeneity in players' beliefs about talent and has important consequences for the SPE as we shall illustrate next.

It follows from (1.8) and (1.9) that both players choose the high risk strategy as long as

$$\begin{aligned} &G(\lambda + a_1^*(\sigma_H^2, \sigma_H^2) - a_2^*(\sigma_H^2, \sigma_H^2); \sigma_H^2, \sigma_H^2)\Delta u - c(a_1^*(\sigma_H^2, \sigma_H^2)) \\ &\geq G(\lambda + a_1^*(\sigma_L^2, \sigma_H^2) - a_2^*(\sigma_L^2, \sigma_H^2); \sigma_L^2, \sigma_H^2)\Delta u - c(a_1^*(\sigma_L^2, \sigma_H^2)), \end{aligned} \tag{1.10}$$

and

$$\begin{aligned} &\left[1 - G(a_1^*(\sigma_H^2, \sigma_H^2) - a_2^*(\sigma_H^2, \sigma_H^2); \sigma_H^2, \sigma_H^2)\right] \Delta u - c(a_2^*(\sigma_H^2, \sigma_H^2)) \\ &\geq \left[1 - G(a_1^*(\sigma_H^2, \sigma_L^2) - a_2^*(\sigma_H^2, \sigma_L^2); \sigma_H^2, \sigma_L^2)\right] \Delta u - c(a_2^*(\sigma_H^2, \sigma_L^2)). \end{aligned} \tag{1.11}$$

Both players choose the low risk strategy as long as

$$\begin{aligned} &G(\lambda + a_1^*(\sigma_L^2, \sigma_L^2) - a_2^*(\sigma_L^2, \sigma_L^2); \sigma_L^2, \sigma_L^2)\Delta u - c(a_1^*(\sigma_L^2, \sigma_L^2)) \\ &\geq G(\lambda + a_1^*(\sigma_H^2, \sigma_L^2) - a_2^*(\sigma_H^2, \sigma_L^2); \sigma_H^2, \sigma_L^2)\Delta u - c(a_1^*(\sigma_H^2, \sigma_L^2)), \end{aligned} \tag{1.12}$$

and

$$\begin{aligned} &\left[1 - G(a_1^*(\sigma_L^2, \sigma_L^2) - a_2^*(\sigma_L^2, \sigma_L^2); \sigma_L^2, \sigma_L^2)\right] \Delta u - c(a_2^*(\sigma_L^2, \sigma_L^2)) \\ &\geq \left[1 - G(a_1^*(\sigma_L^2, \sigma_H^2) - a_2^*(\sigma_L^2, \sigma_H^2); \sigma_L^2, \sigma_H^2)\right] \Delta u - c(a_2^*(\sigma_L^2, \sigma_H^2)). \end{aligned} \tag{1.13}$$

Player 1 chooses the low risk strategy and player 2 the high risk strategy when inequalities (1.10) and (1.13) hold in opposite directions. Finally, player 2 chooses the low risk strategy, and player 1 the high risk strategy when inequalities (1.11) and (1.12) hold in opposite directions.

To be able to characterize the SPE of this asymmetric tournament, we specialize the model as in the previous section. Our next result characterizes the equilibrium efforts in the specialized model.

Proposition 6. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, the equilibrium efforts are:*

$$a_1^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_1^2 + \sigma_2^2))^2}{8(\sigma_1^2 + \sigma_2^2)(\lambda + (\sigma_1^2 + \sigma_2^2))^2} \tag{1.14}$$

$$a_2^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^4}{8(\sigma_1^2 + \sigma_2^2)(\lambda + (\sigma_1^2 + \sigma_2^2))^2} \tag{1.15}$$

The equilibrium efforts are strictly increasing in Δu and decreasing in player 1's overconfidence bias λ.

Indeed, in equilibrium, the overconfident player always exerts less effort, i.e., $a_2^* > a_1^*$. Note, in the case where $\lambda = 0$, we reach the symmetric case of Kräkel and Sliwka (2004), in which both players choose identical efforts in equilibrium. However, as the asymmetry grows stronger, i.e. λ increases, both efforts decrease. As the efforts decrease with different speeds, the tournament outcome becomes more asymmetric with higher overconfidence levels, and the effort gap increases. While the overconfidence of player 1 is not affecting the best response of player 2, an increase in λ shifts the best response function of player 1 and thereby the asymmetric equilibrium away from the symmetric equilibrium.

Before moving on to the risk stage, we take a closer look at the effect of risk taking on players' equilibrium efforts.

Lemma 2. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the cost of effort is exponential, and $\lambda^2 \leq \sigma^2 = \sigma_1^2 + \sigma_2^2$, the equilibrium efforts are decreasing in risk taking, that is,*

$$\frac{\partial a_1^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_1^2} = \frac{\partial a_1^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_2^2} < 0 \text{ and } \frac{\partial a_2^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_1^2} = \frac{\partial a_2^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_2^2} < 0.$$

Thus, for a given risk taking r_j of the other player, we have $a_i^*(\sigma_H^2, \sigma_{r_j}^2) \leq a_i^*(\sigma_L^2, \sigma_{r_j}^2)$.

Lemma 2 describes the effect of risk taking on the equilibrium effort levels.[13] If the level of overconfidence of player 1 is sufficiently small, i.e. $\lambda^2 \leq \sigma^2$, the high risk strategy will reduce the equilibrium effort of both players. Lemma 3 describes the effect of risk taking on the equilibrium effort gap.

Lemma 3. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, the equilibrium effort gap is decreasing in the sum of risks, i.e., $\partial(a_2^* - a_1^*)/\partial\sigma^2 < 0$.*

Lemma 3 shows that an increase in the sum of risks lowers the equilibrium effort gap. Finally, Lemma 4 describes how a change in the sum of risks affects player 1's perceived, and player 2's objective winning probability.

Lemma 4. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, player 1's perceived probability of winning decreases in the sum of risks $\sigma_{r_1}^2 + \sigma_{r_2}^2$, independently of his overconfidence bias. Further, player 2's objective probability of winning also decreases in the sum of risks.*

Recall, that the objective probability of winning depends on the relative size of the effort levels of the two players, i.e. the effort gap. As mentioned in Lemma 3, the effort gap decreases in the number of risks. Therefore, ceteris paribus, switching from a high to a low risk strategy increases the effort gap and thus the objective probability of winning the rational player but decreases the objective winning probability of the overconfident player accordingly.

Ultimately, the SPE outcome depends on the relative importance of effort and likelihood effects and their interrelatedness. Proposition 7 characterizes the SPE of the specialized model.

Proposition 7. *Consider a tournament where player 1 is overconfident and player*

[13] Note, that the derivatives with respect to σ_i^2 always coincide with the derivatives with respect to σ^2. For this reason, what matters ultimately is the total variance.

2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the cost of effort is exponential, and $\lambda < \sigma^2 = \sigma_1^2 + \sigma_2^2$. Let $\bar{\lambda}_{hh1}$ denote the unique solution to (1.10), $\bar{\lambda}_{hh2}$ the unique solution to (1.11), $\bar{\lambda}_{ll1}$ the unique solution to (1.12), and $\bar{\lambda}_{ll2}$ the unique solution to (1.13).

(i) If $\lambda < \min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}$, then there is a unique SPE where both players choose the high risk strategy.

(ii) If $\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$, then there is a unique SPE where the overconfident player chooses the low risk strategy and the rational player the high risk strategy.

(iii) If $\lambda < \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\}$, then there is a unique SPE where both players choose the low risk strategy.

In all of the above SPE the equilibrium efforts of players 1 and 2 are given by (1.14) and (1.15), respectively.

In Proposition 7 we show that there is a unique asymmetric SPE for any overconfidence bias λ. Depending on the size of λ, this SPE consists of different strategy profiles. To give some intuition, consider a small bias, i.e., $\lambda < \min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}$. In this case, both players choose the high risk strategy. When player 1 is just slightly overconfident, both players see themselves as being almost equally talented and therefore choose very similar efforts and have very similar winning probabilities. In such a situation, the outcome of the tournament is less dependent on the perceived talent gap but more so on effort. Hence, it is beneficial for the players to limit the effort exerted. They do so by selecting the high risk strategy.

Now, consider a large bias, i.e., $\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$. In this case, the overconfident player chooses the low risk strategy whereas the rational player chooses a high risk strategy. Now player 1 thinks, mistakenly, that he has a large talent advantage over player 2. This mistaken perception holds even after player 1 takes into account that player 2 will exert more effort than him. Since player 1 thinks, mistakenly, that he has a large advantage, he goes for the low risk strategy. Player 2, being aware that 1 is overconfident, knows that she is going to exert only a slightly higher effort than player 1. Since, from player 2's perspective, talent is the same and efforts are very close, she chooses the high risk strategy.

Finally, consider a very large bias, i.e., $\lambda > \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\}$. In this case, both players opt for the low risk strategy. For player 1 the above reasoning applies again. Player 2 knows she exerts much more effort than player 1. This large effort advantage makes her want to play it safe and therefore she opts for the low risk strategy. Also, player 2, by choosing the low risk strategy, increases the effort gap even further which pushes up player 2's probability of winning.

With Proposition 7, we show that for any overconfidence bias $\lambda < \sigma^2 = \sigma_1^2 + \sigma_2^2$, there does not exist a SPE where the overconfident player chooses a high risk strategy and the rational player a low risk strategy. Hence, we show that the idea of overconfident individuals choosing riskier strategies than rational ones (see Malmendier and Tate (2008); Nosić and Weber (2010); Hirshleifer et al. (2012); Goldberg et al. (2020)) does not hold in our model. Instead, either both players choose the same risk strategy or, when the overconfidence bias is large, the overconfident player chooses the low risk strategy while the rational player chooses the high risk strategy.

1.6 Conclusion

Our study shows that overconfidence can lead to a drastic change in the nature of the equilibrium of a tournament where players choose risk as well as effort.

We obtain three main results for tournaments with homogeneous players. First, players may adopt less risky strategies when they are overconfident than when they are rational. Second, players may exert higher effort when they are overconfident than when they are rational. Third, the players' overconfidence can make the tournament organizer better off.

We obtain four main results for tournaments where an overconfident player competes against a rational player. First, the overconfident player exerts less effort than the rational player. Second, the overconfident player is less likely to win the tournament when risk is normally distributed. Third, the overconfident player can choose a less risky strategy than the rational player but the reverse cannot happen. Fourth, the presence of an overconfident player can lead a rational player to take less risk.

Our findings go against the idea that overconfident individuals take more risks than rational ones. We also uncover a new mechanism whereby overconfidence can raise effort provision.

1.7 Appendix

First and Second-Order Conditions of the Effort Stage

General Model

In a tournament where both players are equally overconfident, the first-order condition of the effort stage for player i is

$$\frac{\partial E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i} = g(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u - c'(a_i) = 0.$$

The second-order condition of the effort stage for player i is

$$\frac{\partial^2 E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i^2} = g'(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u - c''(a_i) < 0.$$

Hence, a sufficient condition for a pure-strategy Nash equilibrium to exist at the effort stage is that

$$g'(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u < c''(a_i),\ \forall a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2.$$

In a tournament where player 1 is overconfident and player 2 is rational, the first-order conditions of the effort stage for players 1 and 2 are

$$\frac{\partial E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1} = g(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2} = g(a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c'(a_2) = 0,$$

respectively. The second-order conditions of the effort stage for players 1 and 2 are

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1^2} = g'(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c''(a_1) < 0,$$

and

$$\frac{\partial^2 E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2^2} = -g'(a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c''(a_2) < 0,$$

respectively. Hence, a sufficient condition for a pure-strategy Nash equilibrium to exist at the effort stage is that

$$g'(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u < c''(a_1), \forall a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2$$

and

$$-g'(a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u < c''(a_2), \forall a_1, a_2, \sigma_1^2, \sigma_2^2.$$

Specialized Model

In a tournament where both players are equally overconfident, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the first-order condition of the effort stage for player i is

$$\frac{\partial E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i} = \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{(\lambda+a_i-a_j)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c'(a_i) = 0.$$

The second-order condition of the effort stage for player i is

$$\frac{\partial^2 E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i^2} = -\frac{\lambda + a_i - a_j}{\sqrt{2\pi(\sigma_i^2+\sigma_j^2)}(\sigma_i^2+\sigma_j^2)} e^{-\frac{(\lambda+a_i-a_j)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c''(a_i) < 0.$$

The first term attains a maximum value of $e^{-1/2}\Delta u/(\sqrt{2\pi}(\sigma_1^2+\sigma_2^2))$ at $\lambda + a_i - a_j = -\sqrt{\sigma_1^2+\sigma_2^2}$. Hence, the existence of a pure-strategy Nash equilibrium will be guaranteed if

$$\frac{e^{-1/2}}{\sqrt{2\pi}(\sigma_1^2+\sigma_2^2)} \Delta u < \min_{a_i} c''.$$

In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the first-order conditions of the effort stage for players 1 and 2 are

$$\frac{\partial E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1} = \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{(\lambda+a_1-a_2)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2} = \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{(a_1-a_2)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c'(a_2) = 0,$$

respectively. The second-order conditions of the effort stage for players 1 and 2 are

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1^2} = -\frac{\lambda + a_1 - a_2}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}(\sigma_1^2+\sigma_2^2)} e^{-\frac{(\lambda+a_1-a_2)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c''(a_1) < 0,$$

and

$$\frac{\partial^2 E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2^2} = \frac{a_1 - a_2}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}(\sigma_1^2+\sigma_2^2)} e^{-\frac{(a_1-a_2)^2}{2(\sigma_1^2+\sigma_2^2)}} \Delta u - c''(a_2) < 0,$$

respectively. The first term in the first-order condition of player 1 attains a maximum value of $e^{-1/2}/(\sqrt{2\pi}(\sigma_1^2+\sigma_2^2))$ at $\lambda + a_1 - a_2 = -\sqrt{\sigma_1^2+\sigma_2^2}$. The first term in the first-order condition of player 2 attains a maximum value of $e^{-1/2}\Delta u/(\sqrt{2\pi}(\sigma_1^2+\sigma_2^2))$ at $a_2 - a_1 = -\sqrt{\sigma_1^2+\sigma_2^2}$. Hence, the existence of a pure-strategy Nash equilibrium will be guaranteed if

$$\frac{e^{-1/2}}{\sqrt{2\pi}(\sigma_1^2+\sigma_2^2)}\Delta u < \min_{a_i} c''(a_i).$$

Proof of Lemma 1

Assume, by contradiction, $a_1^* > a_2^*$. This implies $\lambda + a_1^* - a_2^* > a_1^* - a_2^* - \lambda > -\lambda - a_1^* + a_2^*$. Since $g(x) = g(-x)$ we have $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = g(-\lambda - a_1^* + a_2^*; \sigma_1^2, \sigma_2^2)$. Since $g'(x) < 0$ for $x > 0$ and $g'(x) > 0$ for $x < 0$ it follows from $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = g(-\lambda - a_1^* + a_2^*; \sigma_1^2, \sigma_2^2)$ that $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) > c'(a_2^*)$. Now, assume, by contradiction $a_1^* < a_2^*$. This implies $\lambda + a_2^* - a_1^* > \lambda + a_1^* - a_2^* > -\lambda + a_1^* - a_2^*$. Since $g(x) = g(-x)$ we have $g(\lambda + a_2^* - a_1^*; \sigma_1^2, \sigma_2^2) = g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$.Since $g'(x) < 0$ for $x > 0$ and $g'(x) > 0$ for $x < 0$ it follows from $g(\lambda + a_2^* - a_1^*; \sigma_1^2, \sigma_2^2) = g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$ that $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) > g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) > c'(a_2^*)$ which contradicts $c'(a_1^*) < c'(a_2^*)$. Hence, the unique pure-strategy equilibrium of the effort stage is given by $a_1^* = a_2^*$. It is easy to see that this symmetric equilibrium satisfies the first-order conditions. Setting $a_1^* = a_2^* = a^*$ in the first-order conditions we obtain

$$g(\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*)$$

and

$$g(-\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*).$$

These two first-order conditions are equivalent since $g(x) = g(-x)$.

Proof of Proposition 1

The perceived expected utilities that players 1 and 2 maximize are given by

$$E[U_1(a_1, a_2, \sigma_1^2, \sigma_2^2)] = u(y_l) + \Phi\left(\frac{\lambda + a_1 - a_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\Delta u - e^{a_1},$$

$$E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)] = u(y_l) + \left[1 - \Phi\left(\frac{a_1 - a_2 - \lambda}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\Delta u - e^{a_2}.$$

The first-order conditions are

$$\frac{\partial E[U_1(a_1,a_2,\lambda,\sigma_1^2,\sigma_2^2)]}{\partial a_1} = \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{(\lambda+a_1-a_2)^2}{2(\sigma_1^2+\sigma_2^2)}}\Delta u - e^{a_1} = 0$$

$$\frac{\partial E[U_2(a_1,a_2,\lambda,\sigma_1^2,\sigma_2^2)]}{\partial a_2} = \frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{(a_1-a_2-\lambda)^2}{2(\sigma_1^2+\sigma_2^2)}}\Delta u - e^{a_2} = 0.$$

Since efforts must be identical, the equilibrium effort a^* must satisfy

$$\frac{1}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}\Delta u = e^{a^*}.$$

Taking logs we have

$$a^* = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}.$$

This expression is positive if the utility prize spread Δu is sufficiently big, i.e.

$$\begin{aligned}
\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)} &> 0 \\
\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) &> \frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)} \\
\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} &> e^{\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}} \\
\Delta u &> e^{\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}.
\end{aligned}$$

The equilibrium effort, further, a^* is increasing with the utility prize spread Δu and decreasing with the overconfidence bias λ. To determine how a change in risk affects the equilibrium effort a^* let $\sigma^2 = \sigma_1^2 + \sigma_2^2$. We have

$$\frac{\partial a^*}{\partial \sigma} = \frac{\partial}{\partial \sigma}\left[\ln\left(\frac{\Delta u}{\sqrt{2\pi}}\right) - \ln\sigma - \frac{\lambda^2}{2\sigma^2}\right] = -\frac{1}{\sigma} + \frac{\lambda^2}{\sigma^3} = \frac{1}{\sigma}\left[-1 + \left(\frac{\lambda}{\sigma}\right)^2\right].$$

Hence, the equilibrium effort a^* decreases with risk as long as $\lambda < \sigma = \sqrt{\sigma_1^2+\sigma_2^2}$.

Proof of Proposition 2

For the specialized model we have

$$E[U_1(a_1^*,a_2^*,\sigma_1^2,\sigma_2^2)] = u(y_l) + G(\lambda + a_1^* - a_2^*;\sigma_1^2,\sigma_2^2)\Delta u - c(a_1^*)$$

$$
\begin{aligned}
&= u(y_l) + \Phi\left(\frac{\lambda + a_1^* - a_2^*}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - e^{a_1^*} \\
&= u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - e^{\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}} \\
&= u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}
\end{aligned}
$$

and

$$
\begin{aligned}
E[U_2(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2)] &= u(y_l) + \left[1 - G(a_1^* - a_2^* - \lambda; \sigma_1^2, \sigma_2^2)\right]\Delta u - c(a_2^*) \\
&= u(y_l) + \left[1 - \Phi\left(\frac{a_1^* - a_2^* - \lambda}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - e^{a_2^*} \\
&= u(y_l) + \left[1 - \Phi\left(\frac{-\lambda}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - e^{\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}} \\
&= u(y_l) + \left[1 - \Phi\left(\frac{-\lambda}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}
\end{aligned}
$$

Thus, the maximization problems of the risk stage for both players, respectively, are

$$
\max_{\sigma_1^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{(\sigma_1^2 + \sigma_2^2)}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}
$$

$$
\max_{\sigma_2^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + \left[1 - \Phi\left(\frac{-\lambda}{\sqrt{(\sigma_1^2 + \sigma_2^2)}}\right)\right]\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2+\sigma_2^2)}}
$$

Proof of (i): The high risk equilibrium $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$ takes place when the expected utility of the high risk strategy is higher than a unilateral deviation to the low risk strategy, that is, if

$$
\begin{aligned}
&u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} \\
&\geq u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},
\end{aligned}
$$

or

$$
\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} \geq \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right). \quad (1.16)
$$

Setting $\lambda = 0$ in the LHS of (1.16) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (1.16) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Hence, the inequality is satisfied when $\lambda = 0$, i.e., when both players are rational they both choose the high risk strategy. Note that RHS of (1.16) is always non-negative. Note also that the LHS of (1.16) is equal to zero when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} = \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}} = e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)} + \frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$\frac{1}{2}\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right) = -\frac{\lambda^2}{2(\sigma_H^2 + \sigma_H^2)} + \frac{\lambda^2}{2(\sigma_L^2 + \sigma_H^2)},$$

or

$$\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right) = \lambda^2\left(\frac{1}{\sigma_L^2 + \sigma_H^2} - \frac{1}{\sigma_H^2 + \sigma_H^2}\right),$$

or

$$\lambda = \sqrt{\frac{(\sigma_L^2 + \sigma_H^2)(\sigma_H^2 + \sigma_H^2)}{\sigma_H^2 - \sigma_L^2}\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right)}.$$

Taking the derivative of the LHS of (1.16) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}LHS(\lambda) &= \frac{\partial}{\partial\lambda}\left(\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right) \\
&= -\frac{\frac{\lambda}{\sigma_L^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} + \frac{\frac{\lambda}{\sigma_H^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}} \\
&= -\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)^3}} + \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)^3}}
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\frac{\partial}{\partial \lambda} LHS(\lambda)\bigg|_{\lambda=0} = 0$$

The derivative is negative when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{(\sigma_H^2+\sigma_H^2)^3}} < \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{(\sigma_L^2+\sigma_H^2)^3}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} < \sqrt{\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)^3},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} - \frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)} < \frac{3}{2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{(\sigma_H^2+\sigma_H^2)-(\sigma_L^2+\sigma_H^2)}{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{\sigma_H^2-\sigma_L^2}{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\lambda < \sqrt{3\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}.$$

Taking the derivative of the RHS of (1.16) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial \lambda} RHS(\lambda) &= \frac{\partial}{\partial \lambda}\left[\Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\right] \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \frac{1}{\sqrt{\sigma_H^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_H^2+\sigma_H^2}}\right) \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} \\
&\qquad - \frac{1}{\sqrt{\sigma_H^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} \\
&= \frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} - \frac{1}{(\sigma_H^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}.
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\frac{\partial}{\partial \lambda} RHS(\lambda)\bigg|_{\lambda=0} = \frac{1}{(\sigma_L^2 + \sigma_H^2)\sqrt{2\pi}} - \frac{1}{(\sigma_H^2 + \sigma_H^2)\sqrt{2\pi}} > 0.$$

The derivative is positive when

$$\frac{1}{(\sigma_L^2 + \sigma_H^2)} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} > \frac{1}{(\sigma_H^2 + \sigma_H^2)} e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} - \frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} < \frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2},$$

or

$$\frac{\lambda^2}{2\left(\sigma_L^2 + \sigma_H^2\right)} - \frac{\lambda^2}{2\left(\sigma_H^2 + \sigma_H^2\right)} < \ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right),$$

or

$$\frac{\lambda^2}{\left(\sigma_L^2 + \sigma_H^2\right)} - \frac{\lambda^2}{\left(\sigma_H^2 + \sigma_H^2\right)} < 2\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right),$$

or

$$\left[\frac{1}{\left(\sigma_L^2 + \sigma_H^2\right)^2} - \frac{1}{\left(\sigma_H^2 + \sigma_H^2\right)^2}\right]\lambda^2 < 2\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right),$$

or

$$\frac{\sigma_H^2 - \sigma_L^2}{\left(\sigma_H^2 + \sigma_H^2\right)\left(\sigma_H^2 + \sigma_L^2\right)}\lambda^2 < 2\ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right),$$

or

$$\lambda < \sqrt{2\frac{(\sigma_L^2 + \sigma_H^2)(\sigma_H^2 + \sigma_H^2)}{\sigma_H^2 - \sigma_L^2} \ln\left(\frac{\sigma_H^2 + \sigma_H^2}{\sigma_L^2 + \sigma_H^2}\right)}.$$

We have shown the LHS of (1.16) is strictly positive at $\lambda = 0$, decreases in λ, and is equal to zero at $\lambda = \sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2} \ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}$. Furthermore, we have shown the RHS of (1.16) is non-negative, is equal to zero at $\lambda = 0$, first increases and then decreases in λ, and attains its maximum at $\lambda = \sqrt{2\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2} \ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}$. Hence, it follows that there is a unique positive value for λ that satisfies (1.16) as an equality. Let $\bar{\lambda}_1$ denote the unique solution to:

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}} = \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right).$$

Figure 1.2 shows the LHS and RHS of (1.16) for $(\sigma_L^2, \sigma_H^2) = (4, 16)$. As the plot

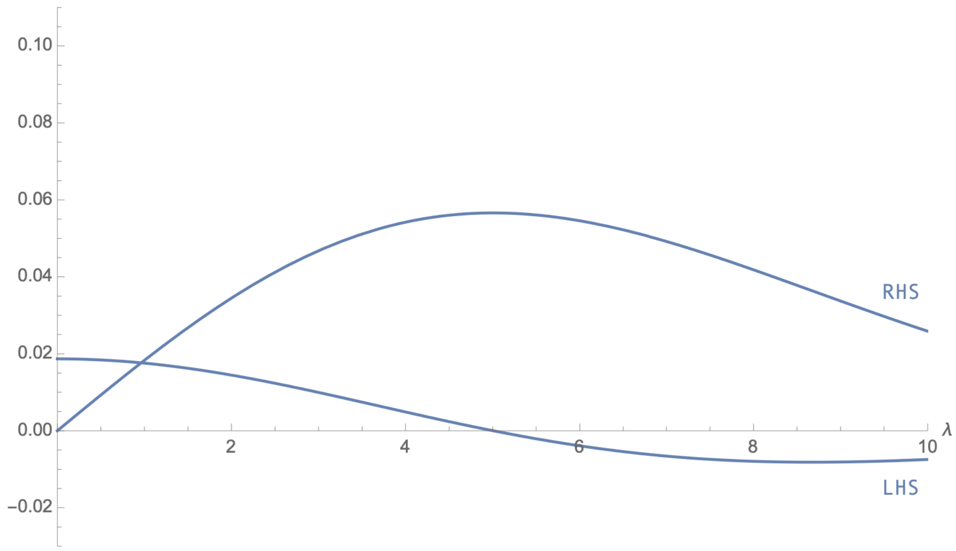

Figure 1.2: LHS and RHS of inequality (1.16)

shows, the LHS and RHS of (1.16) cross only once. We, thus, have a unique threshold for the overconfidence bias λ for which (1.16) holds as an equality. The point of intersection between the LHS and RHS represents the threshold $\bar{\lambda}_1$.
Hence, we have shown that if $\lambda < \bar{\lambda}_1$, then there exists a pure-strategy SPE where both players choose the high risk strategy in the first stage and where the equilibrium effort in the second stage is given by (1.7) with $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$.

Proof of (iii): The low risk equilibrium $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$ takes place when the expected utility of the low risk strategy is higher than a unilateral deviation to the high risk strategy, that is, if

$$u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}$$
$$\geq u(y_l) + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} \leq \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right). \quad (1.17)$$

Setting $\lambda = 0$ in the LHS of (1.17) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (1.17) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Hence, the inequality is violated when $\lambda = 0$, in other words, when both players are rational the low risk equilibrium does not exist. Note that the RHS of (1.17) is always non-negative. Note also that the LHS of (1.17) is equal to zero when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} = \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_L^2 + \sigma_H^2}{\sigma_L^2 + \sigma_L^2}} = e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} + \frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}},$$

or

$$\ln\left(\frac{\sigma_L^2 + \sigma_H^2}{\sigma_L^2 + \sigma_L^2}\right) = \left(\frac{1}{\sigma_L^2 + \sigma_L^2} - \frac{1}{\sigma_L^2 + \sigma_H^2}\right)\lambda^2,$$

or

$$\lambda = \sqrt{\frac{(\sigma_L^2 + \sigma_H^2)(\sigma_L^2 + \sigma_L^2)}{\sigma_H^2 - \sigma_L^2} \ln\left(\frac{\sigma_L^2 + \sigma_H^2}{\sigma_L^2 + \sigma_L^2}\right)}.$$

Taking the derivative of the LHS of (1.17) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda} LHS(\lambda) &= \frac{\partial}{\partial\lambda}\left(\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\right) \\
&= -\frac{\frac{\lambda}{\sigma_L^2+\sigma_L^2} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} + \frac{\frac{\lambda}{\sigma_L^2+\sigma_H^2} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} \\
&= -\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)^3}} + \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)^3}}
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\left.\frac{\partial}{\partial\lambda} LHS(\lambda)\right|_{\lambda=0} = 0$$

The derivative is negative when

$$\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)^3}} > \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)^3}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} < \sqrt{\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)^3},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)} - \frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} < \frac{3}{2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{(\sigma_L^2+\sigma_H^2)-(\sigma_L^2+\sigma_L^2)}{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{\sigma_H^2-\sigma_L^2}{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\lambda < \sqrt{3\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}.$$

Taking the derivative of the RHS of (1.17) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}RHS(\lambda) =& \frac{\partial}{\partial\lambda}\left[\Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)\right] \\
=& \frac{1}{\sqrt{\sigma_L^2+\sigma_L^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right) - \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) \\
=& \frac{1}{\sqrt{\sigma_L^2+\sigma_L^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}} \\
& \qquad - \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} \\
=& \frac{1}{(\sigma_L^2+\sigma_L^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}} - \frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}.
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\frac{\partial}{\partial\lambda}RHS(\lambda)\bigg|_{\lambda=0} = \frac{1}{(\sigma_L^2+\sigma_L^2)\sqrt{2\pi}} - \frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}} > 0.$$

The derivative is positive when

$$\frac{1}{(\sigma_L^2+\sigma_L^2)}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}} > \frac{1}{(\sigma_L^2+\sigma_H^2)}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} < \frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)} - \frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} < \ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{\lambda^2}{(\sigma_L^2+\sigma_L^2)} - \frac{\lambda^2}{(\sigma_L^2+\sigma_H^2)} < 2\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\lambda < \sqrt{2\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}.$$

We have shown the LHS of (1.17) is strictly positive at $\lambda = 0$, decreases in λ, and is equal to zero at $\lambda = \sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_L^2+\sigma_L^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}$. Furthermore, we have shown the RHS of (1.17) is non-negative, is equal to zero at $\lambda = 0$, first increases and then decreases in λ, and attains its maximum at $\lambda = \sqrt{2\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}$. Hence, it follows that there is a unique positive value for λ that satisfies (1.17) as an equality. Let $\bar{\lambda}_2$ denote the unique solution to:

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} = \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right).$$

Figure 1.3 shows the LHS and RHS of (1.17) for $(\sigma_L^2, \sigma_H^2) = (4, 16)$. As the plot shows, the LHS and RHS of (1.17) cross only once. We, thus, have a unique threshold for the overconfidence bias λ for which (1.17) holds as an equality. The point of intersection between the LHS and RHS represents the threshold $\bar{\lambda}_2$. Hence, we have shown that if $\lambda > \bar{\lambda}_2$, then there exists a pure-strategy SPE where both players choose the low risk strategy in the first stage and where the equilibrium effort in the second stage is given by (1.7) with $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$.

To complete the proof we need to determine what is the SPE outcome for the remaining values of λ. Since we are unable to show whether $\bar{\lambda}_1$ is always less than $\bar{\lambda}_2$ or vice-versa, we distinguish between two cases: (a) $\bar{\lambda}_1 < \lambda < \bar{\lambda}_2$ and (b) $\bar{\lambda}_2 < \lambda < \bar{\lambda}_1$.

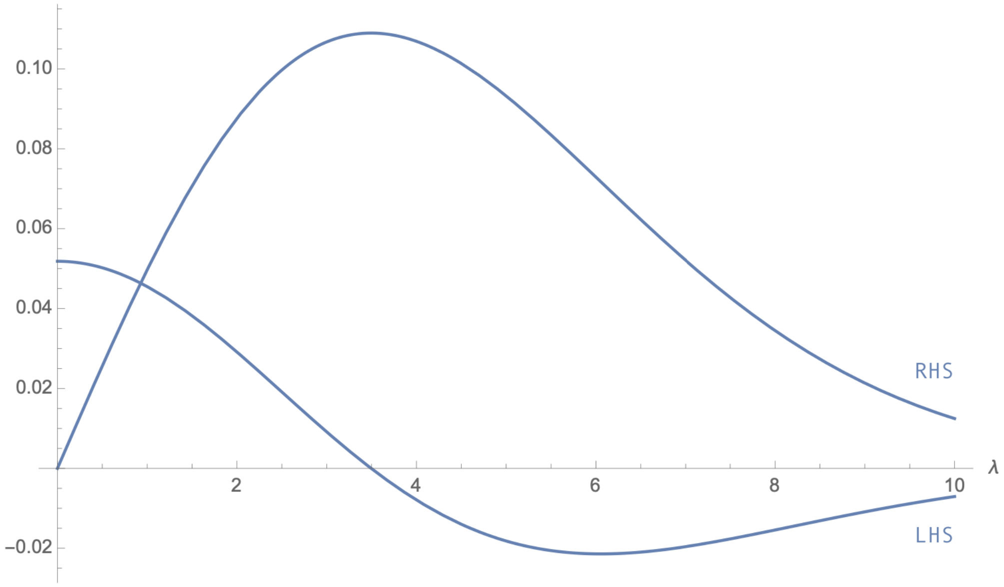

Figure 1.3: LHS and RHS of inequality (1.17)

In case (a) there are no symmetric pure-strategy SPE for $\bar{\lambda}_1 < \lambda < \bar{\lambda}_2$. However, existence is guaranteed by standard arguments. Hence, there exists a SPE where players mix between the low and the high risk strategies in the first stage and where the equilibrium effort in the second stage is given by (1.7). In case (b) there are three SPE for $\bar{\lambda}_2 < \lambda < \bar{\lambda}_1$. There exists one pure-strategy SPE where both players choose the high risk strategy in the first stage and where the equilibrium effort in the second stage is given by (1.7) with $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$. There exists another pure-strategy SPE where both players choose the low risk strategy in the first stage and where the equilibrium effort in the second stage is given by (1.7) with $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$. Finally, there is a SPE where players mix between the low and the high risk strategies in the first stage and where the equilibrium effort in the second stage is given by (1.7).

Proof of Proposition 3

The equilibrium effort in a tournament with rational players is

$$a^*(\sigma_H^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right). \tag{1.18}$$

The equilibrium effort in a tournament with overconfident players in a SPE where both players choose the high risk strategy is

$$a^*(\sigma_H^2, \sigma_H^2; \lambda) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right) - \frac{\lambda^2}{2(\sigma_H^2 + \sigma_H^2)} \tag{1.19}$$

The equilibrium effort in a tournament with overconfident players in a SPE where both players choose the low risk strategy is

$$a^*(\sigma_L^2, \sigma_L^2; \lambda) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}}\right) - \frac{\lambda^2}{2(\sigma_L^2 + \sigma_L^2)} \tag{1.20}$$

It follows directly from (1.18) and (1.19) that if $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, then $a^*(\sigma_H^2, \sigma_H^2) > a^*(\sigma_H^2, \sigma_H^2; \lambda)$. This proves part (i). It follows from (1.18) and (1.20) that for $a^*(\sigma_L^2, \sigma_L^2; \lambda) > a^*(\sigma_H^2, \sigma_H^2)$ we must have

$$\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}}\right) - \frac{\lambda^2}{2(\sigma_L^2 + \sigma_L^2)} > \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right),$$

or

$$\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}}\right) - \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right) > \frac{\lambda^2}{2(\sigma_L^2 + \sigma_L^2)},$$

or

$$\ln\frac{1}{\sqrt{2\sigma_L^2}} - \ln\frac{1}{\sqrt{2\sigma_H^2}} > \frac{\lambda^2}{2(\sigma_L^2 + \sigma_L^2)},$$

or

$$\ln\frac{1}{\sigma_L} - \ln\frac{1}{\sigma_H} > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$-\ln\sigma_L + \ln\sigma_H > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$\ln\sigma_H - \ln\sigma_L > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$\ln\frac{\sigma_H}{\sigma_L} > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$4\sigma_L^2\ln\frac{\sigma_H}{\sigma_L} > \lambda^2,$$

or

$$\lambda < 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}.$$

Hence, if $\max\{\bar{\lambda}_1, \bar{\lambda}_2\} < \lambda < 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}$, then $a^*(\sigma_L^2, \sigma_L^2; \lambda) > a^*(\sigma_H^2, \sigma_H^2)$. This proves part (ii). Finally, this last result implies that if $\lambda > 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}$, then $a^*(\sigma_L^2, \sigma_L^2; \lambda) < a^*(\sigma_H^2, \sigma_H^2)$. This proves part (iii).

Proof of Proposition 4

The first-order conditions of players 1 and 2 are

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*),$$

and

$$g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*),$$

respectively.

Assume, by contradiction, $a_1^* = a_2^*$. This, $\lambda > 0$, and $g'(x; \sigma_1^2, \sigma_2^2) < 0$ for $x > 0$ imply $g(\lambda; \sigma_1^2, \sigma_2^2) < g(0; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) = c'(a_2^*)$. Next, asssume, by contradiction, $a_1^* > a_2^*$. Since, $\lambda > 0$ this implies $\lambda + a_1^* - a_2^* > a_1^* - a_2^* > 0$. However, this and $g'(x; \sigma_1^2, \sigma_2^2) < 0$ for $x > 0$, in turn, imply $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) > c'(a_2^*)$. Hence, it must be that $a_1^* < a_2^*$. Finally, note that $a_1^* < a_2^*$ and the first-order conditions imply $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality, $\lambda + a_1^* - a_2^* > a_1^* - a_2^*$ and $g'(x; \sigma_1^2, \sigma_2^2) > 0$ for $x < 0$ imply $\lambda + a_1^* - a_2^* > 0$. Hence, in equilibrium we have $\lambda + a_1^* > a_2^* > a_1^*$.

The impact of overconfidence on the pure-strategy Nash equilibrium efforts is obtained from total differentiation of the first-order conditions of players 1 and 2:

$$g'(\lambda + a_1^* - a_2^*)(\partial\lambda + \partial a_1^* - \partial a_2^*)\Delta u = c''(a_1^*)\partial a_1^*$$

and

$$g'(a_1^* - a_2^*)(\partial a_1^* - \partial a_2^*)\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial\lambda$ we obtain

$$g'(\lambda + a_1^* - a_2^*)\left(1 + \frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}, \tag{1.21}$$

and

$$g'(a_1^* - a_2^*) \left(\frac{\partial a_1^*}{\partial \lambda} - \frac{\partial a_2^*}{\partial \lambda} \right) \Delta u = c''(a_2^*) \frac{\partial a_2^*}{\partial \lambda}. \tag{1.22}$$

Solving (1.22) for $\partial a_2^* / \partial \lambda$ we have

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)} \frac{\partial a_1^*}{\partial \lambda}. \tag{1.23}$$

Substituting (1.23) into (1.21) we obtain

$$g'(\lambda + a_1^* - a_2^*) \left[1 + \frac{\partial a_1^*}{\partial \lambda} - \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)} \frac{\partial a_1^*}{\partial \lambda} \right] \Delta u = c''(a_1^*) \frac{\partial a_1^*}{\partial \lambda}$$

Solving this equation for $\partial a_1^* / \partial \lambda$ we obtain

$$\frac{\partial a_1^*}{\partial \lambda} = \frac{1}{D^*} \left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*) \right] g'(\lambda + a_1^* - a_2^*)\Delta u, \tag{1.24}$$

where

$$\begin{aligned} D^* = & \left[g'(\lambda + a_1^* - a_2^*)\Delta u - c''(a_1^*) \right] \left[-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*) \right] \\ & + g'(\lambda + a_1^* - a_2^*) g'(a_1^* - a_2^*)(\Delta u)^2. \end{aligned}$$

Substituting (1.24) into (1.23) we obtain

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{1}{D^*} g'(a_1^* - a_2^*) g'(\lambda + a_1^* - a_2^*)(\Delta u)^2. \tag{1.25}$$

Note that the two terms inside square brackets in D^* are the second-order conditions of workers 1 and 2, respectively, and their signs are negative. Hence, the sign of the product of the terms inside square brackets is positive. Now, $\lambda + a_1^* > a_2^* > a_1^*$ implies $g'(\lambda + a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$. Hence, the last term in D^* is negative. However, simplifying D^* we obtain

$$D^* = -g'(\lambda + a_1^* - a_2^*) c''(a_2^*)\Delta u + g'(a_1^* - a_2^*) c''(a_1^*)\Delta u + c''(a_1^*) c''(a_2^*). \tag{1.26}$$

When $g'(\lambda + a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, the first and second terms in (1.26) are positive. The third term in (1.26) also is positive since $c'' > 0$. Hence, if $\lambda + a_1^* > a_2^* > a_1^*$, then $D^* > 0$. Thus, we have shown that $D^* > 0$. It follows from (1.24), (1.25), $\lambda + a_1^* > a_2^* > a_1^*$, and $D^* > 0$, that $\partial a_1^* / \partial \lambda < \partial a_2^* / \partial \lambda < 0$.

Proof of Proposition 5

When the random terms are normally distributed, players 1 and 2 objective probabilities of winning the tournament are

$$P_1(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right),$$

and

$$P_2(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = 1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = 1 - \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right),$$

respectively. We know from Proposition 4 that in the pure-strategy Nash equilibrium of the effort stage the rational player 2 exerts higher effort than the overconfident player 1, i.e., $a_2^* > a_1^*$. This implies

$$P_1(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{(\sigma_1^2 + \sigma_2^2)}}\right) < \Phi(0) = \frac{1}{2}.$$

Proof of Proposition 6

Player 1 chooses the optimal effort level that maximizes

$$E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] = u(y_l) + \Phi\left(\frac{\lambda + a_1 - a_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - e^{a_1},$$

and player 2 chooses the optimal effort level that maximizes

$$E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)] = u(y_l) + \left[1 - \Phi\left(\frac{a_1 - a_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - e^{a_2}.$$

The first-order conditions for players 1 and 2, respectively, are

$$\frac{\partial E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1} = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{(\lambda + a_1 - a_2)^2}{2(\sigma_1^2 + \sigma_2^2)}} \Delta u - e^{a_1} = 0 \tag{1.27}$$

$$\frac{\partial E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2} = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{(a_1 - a_2)^2}{2(\sigma_1^2 + \sigma_2^2)}} \Delta u - e^{a_2} = 0. \tag{1.28}$$

Taking logs and rearranging the first-order conditions yields the following expressions.

$$a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 - 2\ln(r)\sigma^2 = 0 \tag{1.29}$$

$$a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2 - 2\ln(r)\sigma^2 = 0, \tag{1.30}$$

where $r = \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2$. Rearranging further and we obtain

$$\begin{aligned} a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 &= 2\ln(r)\sigma^2 \\ a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2 &= 2\ln(r)\sigma^2 \end{aligned}$$

Thus, it must be that

$$a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 = a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2$$

Simplifying this expression yields

$$\begin{aligned} \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 &= \left(2\sigma^2 - 2a_1\right) a_2 \\ 2a_1\sigma^2 + 2a_1\lambda - 2a_1a_2 + \lambda^2 - 2\lambda a_2 &= 2a_2\sigma^2 - 2a_1a_2 \\ 2a_1\sigma^2 + 2a_1\lambda + \lambda^2 - 2\lambda a_2 &= 2a_2\sigma^2 \\ a_1(2\sigma^2 + 2\lambda) + \lambda^2 &= a_2(2\sigma^2 + 2\lambda) \end{aligned}$$

Thus,

$$a_2 = a_1 + \frac{\lambda^2}{2\sigma^2 + 2\lambda}, \tag{1.31}$$

where we define $z = \frac{\lambda^2}{2\sigma^2+2\lambda}$. Inserting into (1.30) we obtain

$$\begin{aligned} (a_1 + z)^2 + \left(2\sigma^2 - 2a_1\right)(a_1 + z) + a_1^2 - 2\ln(r)\sigma^2 &= 0 \\ a_1^2 + 2a_1z + z^2 + 2a_1\sigma^2 + 2z\sigma^2 - 2a_1^2 - 2a_1z + a_1^2 - 2\ln(r)\sigma^2 &= 0 \\ z^2 + 2a_1\sigma^2 + 2z\sigma^2 - 2\ln(r)\sigma^2 &= 0 \end{aligned}$$

Solving for a_1 yields

$$\begin{aligned} a_1 &= \ln r - \frac{(2\sigma^2 + z)}{2\sigma^2} z \\ &= \ln(r) - \frac{(2\sigma^2 + \frac{\lambda^2}{2\sigma^2+2\lambda})}{2\sigma^2}\left(\frac{\lambda^2}{2\sigma^2 + 2\lambda}\right) \\ &= \ln(r) - \frac{2\sigma^2(2\sigma^2 + 2\lambda) + \lambda^2}{2\sigma^2(2\sigma^2 + 2\lambda)}\left(\frac{\lambda^2}{2\sigma^2 + 2\lambda}\right) \\ &= \ln(r) - \frac{2\sigma^2(2\sigma^2 + 2\lambda) + \lambda^2}{2\sigma^2(2\sigma^2 + 2\lambda)^2}\lambda^2 \\ &= \ln(r) - \frac{4\sigma^4 + 4\sigma^2\lambda + \lambda^2}{8\sigma^2(\sigma^2 + \lambda)^2}\lambda^2 \\ &= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2(2\sigma^2 + \lambda)^2}{8\sigma^2(\sigma^2 + \lambda)^2}. \end{aligned}$$

Which coincides with equation (1.14). The equilibrium effort level for player 2 we obtain be inserting the previous result into (1.31).

$$\begin{aligned}
a_2 &= \ln(r) - \frac{\lambda^2(2\sigma^2+\lambda)^2}{8\sigma^2(\sigma^2+\lambda)^2} + \frac{\lambda^2}{2\sigma^2+2\lambda} \\
&= \ln(r) - \frac{\lambda^2(2\sigma^2+\lambda)^2}{8\sigma^2(\sigma^2+\lambda)^2} + \frac{4\sigma^2\lambda^2(\sigma^2+\lambda)}{8\sigma^2(\sigma^2+\lambda)^2} \\
&= \ln(r) - \frac{\lambda^2(4\sigma^4+4\sigma^2\lambda+\lambda^2) - 4\sigma^4\lambda^2 - 4\sigma^2\lambda^3}{8\sigma^2(\sigma^2+\lambda)^2} \\
&= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^4}{8\sigma^2(\sigma^2+\lambda)^2}.
\end{aligned}$$

Which coincides with equation (1.15). When analyzing the effect of the overconfidence level λ on the equilibrium values, we find a negative relationship, independent of the size of λ:

$$\begin{aligned}
\frac{\partial a_1^*(\sigma_1^2,\sigma_2^2)}{\partial\lambda} &= -\frac{\left(2\lambda(2\sigma^2+\lambda)^2 + 2\lambda^2(2\sigma^2+\lambda)\right)8\sigma^2(\sigma^2+\lambda)^2 - \lambda^2(2\sigma^2+\lambda)^2 16\sigma^2(\sigma^2+\lambda)}{(8\sigma^2(\sigma^2+\lambda)^2)^2} \\
&= -\frac{\left(\lambda(2\sigma^2+\lambda)^2 + \lambda^2(2\sigma^2+\lambda)\right)(\sigma^2+\lambda) - \lambda^2(2\sigma^2+\lambda)^2}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\frac{\left(\lambda(2\sigma^2+\lambda)\right)\left(\left(2\sigma^2+2\lambda\right)(\sigma^2+\lambda) - \lambda(2\sigma^2+\lambda)\right)}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\frac{\left(2\lambda\sigma^2+\lambda^2\right)\left(2\sigma^4+4\lambda\sigma^2+2\lambda^2-2\lambda\sigma^2-\lambda^2\right)}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\underbrace{\frac{\left(2\lambda\sigma^2+\lambda^2\right)\left(2\sigma^4+2\lambda\sigma^2+\lambda^2\right)}{4\sigma^2(\sigma^2+\lambda)^3}}_{>0} < 0 \\
\frac{\partial a_2^*(\sigma_1^2,\sigma_2^2)}{\partial\lambda} &= -\frac{32\lambda^3\sigma^2(\sigma^2+\lambda)^2 - 16\lambda^4\sigma^2(\sigma^2+\lambda)}{(8\sigma^2(\sigma^2+\lambda)^2)^2} \\
&= -\frac{2\lambda^3(\sigma^2+\lambda)-\lambda^4}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\underbrace{\frac{2\lambda^3\sigma^2+\lambda^4}{4\sigma^2(\sigma^2+\lambda)^3}}_{>0} < 0
\end{aligned}$$

Proof of Lemma 2

Consider the derivatives of the equilibrium efforts of player 1 and 2 with respect to their respective risk choice.

$$\frac{\partial a_1^*(\sigma_1^2,\sigma_2^2)}{\partial\sigma_1^2} = \frac{\partial}{\partial\sigma_1^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}\right)$$

$$
\begin{aligned}
&= \frac{\partial}{\partial \sigma^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right) - \frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\frac{\Delta u}{\sqrt{2\pi}}\left(-\frac{1}{2}\right)}{\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\left(\sigma^2\right)^{\frac{3}{2}}} - \frac{32\lambda^2(\lambda+2\sigma^2)\sigma^2(\lambda+\sigma^2)^2}{\left(8\sigma^2(\lambda+\sigma^2)^2\right)^2} \\
&\qquad - \frac{8\lambda^2(\lambda+2\sigma^2)^2\left((\lambda+\sigma^2)^2+2\sigma^2(\lambda+\sigma^2)\right)}{\left(8\sigma^2(\lambda+\sigma^2)^2\right)^2} \\
&= \frac{1}{2\sigma^2}\left[-1+\frac{\lambda^2}{\sigma^2}\frac{4\sigma^8+8\sigma^6\lambda+7\sigma^4\lambda^2+4\sigma^2\lambda^3+\lambda^4}{4\sigma^8+16\sigma^6\lambda+24\sigma^4\lambda^2+16\sigma^2\lambda^3+4\lambda^4}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial a_2^*(\sigma_1^2,\sigma_2^2)}{\partial \sigma_2^2} &= \frac{\partial}{\partial \sigma_2^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}\right) \\
&= \frac{\partial}{\partial \sigma^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right) - \frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\frac{\Delta u}{\sqrt{2\pi}}\left(-\frac{1}{2}\right)}{\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\left(\sigma^2\right)^{\frac{3}{2}}} - \frac{-8\lambda^4\left((\lambda+\sigma^2)^2+2\sigma^2(\lambda+\sigma^2)\right)}{\left(8\sigma^2(\lambda+\sigma^2)^2\right)^2} \\
&= \frac{1}{2\sigma^2}\left[-1+\frac{\lambda^2}{\sigma^2}\frac{3\sigma^4\lambda^2+4\sigma^2\lambda^3+\lambda^4}{4\sigma^8+16\sigma^6\lambda+24\sigma^4\lambda^2+16\sigma^2\lambda^3+4\lambda^4}\right]
\end{aligned}
$$

A sufficient condition for the derivatives $\partial a_1^2(.)/\partial \sigma_1^2$ and $\partial a_2^*(.)/\partial \sigma_2^2$ to be negative is that

$$-1+\frac{\lambda^2}{\sigma^2} \leq 0$$

and thus

$$\lambda^2 \leq \sigma^2 = \sigma_1^2+\sigma_2^2. \tag{1.32}$$

Hence, if we assume (1.32), it follows that

$$a_1^*(\sigma_H^2,\sigma_H^2) < a_1^*(\sigma_L^2,\sigma_H^2) = a_1^*(\sigma_H^2,\sigma_L^2) < a_1^*(\sigma_L^2,\sigma_L^2),$$

and

$$a_2^*(\sigma_H^2,\sigma_H^2) < a_2^*(\sigma_L^2,\sigma_H^2) = a_2^*(\sigma_H^2,\sigma_L^2) < a_2^*(\sigma_L^2,\sigma_L^2) < 0. \quad \square$$

Proof of Lemma 3

$$
\begin{aligned}
\frac{\partial a_2^* - a_1^*}{\partial \sigma^2} &= \frac{\partial}{\partial \sigma^2}\left(\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2} - \frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\partial}{\partial \sigma^2}\left(\frac{4\lambda^2\sigma^2(\lambda+\sigma^2)}{8\sigma^2(\lambda+\sigma^2)^2}\right)
\end{aligned}
$$

$$= \frac{\partial}{\partial \sigma^2}\left(\frac{\lambda^2}{2(\lambda+\sigma^2)}\right)$$
$$= -\frac{\lambda^2}{2(\lambda+\sigma^2)^2} \quad \square$$

Proof of Lemma 4

We determine how player 1's perceived probability of winning is influenced by the total variance. If only considering the probability of winning the tournament, the overconfident player chooses the low risk instead of the high risk if

$$G(\lambda + a_1^* - a_2; \sigma_L^2, \sigma_{r_2}^2) > G(\lambda + a_1^* - a_2; \sigma_H^2, \sigma_{r_2}^2)$$

$$\frac{1}{2}\left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_L^2+\sigma_{r_2}^2))\sqrt{2(\sigma_L^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau\right)$$
$$> \frac{1}{2}\left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_H^2+\sigma_{r_2}^2))\sqrt{2(\sigma_H^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau\right)$$

$$\int_0^{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_L^2+\sigma_{r_2}^2))\sqrt{2(\sigma_L^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau > \int_0^{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_H^2+\sigma_{r_2}^2))\sqrt{2(\sigma_H^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau$$

The left hand side is greater if its integral's upper limit is larger than the one of the right hand side. Thus, if we have that

$$\frac{\lambda(\lambda + 2(\sigma_L^2 + \sigma_{r_2}^2))}{2(\lambda + (\sigma_L^2 + \sigma_{r_2}^2))\sqrt{2(\sigma_L^2 + \sigma_{r_2}^2)}} > \frac{\lambda(\lambda + 2(\sigma_H^2 + \sigma_{r_2}^2))}{2(\lambda + (\sigma_H^2 + \sigma_{r_2}^2))\sqrt{2(\sigma_H^2 + \sigma_{r_2}^2)}}.$$

As it is the total variance that matter, the probability of winning is the same in either of the two asymmetric risk choice profiles, i.e. $G(\lambda + a_1^* - a_2; \sigma_L^2, \sigma_H^2) = G(\lambda + a_1^* - a_2; \sigma_H^2, \sigma_L^2)$. Further, as $\frac{\partial G(\lambda + a_1^* - a_2^*, \sigma_1^2; \sigma_2^2)}{\partial \sigma_1^2} < 0$, player 1's perceived probability of winning decreases with higher values of the total variance. The same applies for the probability of winning of player 2. If only taking into account the probability of winning the tournament, player 2 chooses the low risk instead of the high risk if

$$G(a_1^* - a_2; \sigma_{r_1}^2, \sigma_L^2) > G(a_1^* - a_2; \sigma_{r_1}^2, \sigma_H^2)$$

$$\frac{1}{2}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}}} e^{-\tau^2} d\tau\right)$$
$$> \frac{1}{2}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}} e^{-\tau^2} d\tau\right)$$

$$\int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}}} e^{-\tau^2} d\tau < \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}} e^{-\tau^2} d\tau$$

The right hand side is greater if its integral's upper limit is larger than the one of the left hand side. Thus, we have that

$$\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}} < \frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}$$
$$\frac{1}{(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}} > \frac{1}{(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}$$
$$(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)} > (\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)} \quad \square$$

Proof of Proposition 7

Given the Nash equilibrium efforts of the effort stage (1.14) and (1.15), we can write the perceived expected utilities of both players at the risk stage as

$$\begin{aligned}
E[U_1(a_1^*, a_2^*, \lambda, \sigma_1^2, \sigma_2^2)] &= u(y_l) + G(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1^*) \\
&= u(y_l) + \Phi\left(\frac{\lambda + a_1^*(\sigma_1^2, \sigma_2^2) - a_2^*(\sigma_1^2, \sigma_2^2)}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\Delta u - e^{a_1^*(\sigma_1^2, \sigma_2^2)} \\
&= u(y_l) + \Phi\left(\frac{\lambda + \frac{\lambda^4 - \lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\Delta u \\
&\qquad - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u \\
&= u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_1^2+\sigma_2^2))}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u,
\end{aligned}$$

and

$$\begin{aligned}
E[U_2(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2)] &= u(y_l) + [1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)]\Delta u - c(a_2^*) \\
&= u(y_l) + \left[1 - \Phi\left(\frac{a_1^*(\sigma_1^2, \sigma_2^2) - a_2^*(\sigma_1^2, \sigma_2^2)}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_1^2, \sigma_2^2)} \\
&= u(y_l) + \left[1 - \Phi\left(\frac{\frac{\lambda^4 - \lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\Delta u
\end{aligned}$$

$$
\begin{aligned}
&\quad - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u \\
&= u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\Delta u \\
&\quad - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u.
\end{aligned}
$$

Thus, the maximization problems of the risk stage for players 1 and 2, respectively, are

$$
\max_{\sigma_1^2\in\{\sigma_L^2,\sigma_H^2\}} \quad u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_1^2+\sigma_2^2))}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u
$$

$$
\max_{\sigma_2^2\in\{\sigma_L^2,\sigma_H^2\}} \quad u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u
$$

(i) Let us consider a SPE where

$$
(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2),
$$

and

$$
\begin{aligned}
a_1^*(\sigma_H^2, \sigma_H^2) &= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) - \frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2} \\
a_2^*(\sigma_H^2, \sigma_H^2) &= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) - \frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}.
\end{aligned}
$$

In a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, that is,

$$
E[U_1(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \lambda, \sigma_H^2, \sigma_H^2)] \geq E[U_1(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \lambda, \sigma_L^2, \sigma_H^2)],
$$

or

$$
u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_H^2))}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\Delta u
$$

$$\geq u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_H^2))}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}\Delta u,$$

or

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}$$
$$\geq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_H^2))}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_H^2))}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right). \quad (1.33)$$

Setting $\lambda = 0$ in the LHS of (1.33) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (1.33) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Note that the RHS of (1.33) is non-negative. Note also that the LHS of (1.33) is equal to zero when

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} = \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}} = e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}+\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right) = \frac{(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2} - \frac{(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}. \quad (1.34)$$

The LHS and the RHS of (1.34) are both strictly positive. Moreover, the LHS and the RHS of (1.34) are both monotonically decreasing in λ. Since the LHS starts at a higher value and decreases at a faster rate with λ than the RHS, there is a unique value for λ that satisfies (1.34). Denote this value by $\bar{\lambda}_{0hh1}$. Since the LHS of (1.33) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0hh1})$ such that (1.33) holds as an equality. Denote this value by $\bar{\lambda}_{hh1}$. Hence, inequality (1.33) is satisfied when $\lambda < \bar{\lambda}_{hh1}$ and is violated when $\lambda > \bar{\lambda}_{hh1}$.

In a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$, that is,

$$E[U_2(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \sigma_H^2, \sigma_H^2)] \geq E[U_2(a_1^*(\sigma_H^2, \sigma_L^2), a_2^*(\sigma_H^2, \sigma_L^2), \sigma_H^2, \sigma_L^2)],$$

or

$$u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\Delta u$$
$$\geq u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2 + \sigma_L^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}}\Delta u,$$

or

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}} - \frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}$$
$$\geq \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right) - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2 + \sigma_L^2}}\right). \tag{1.35}$$

Setting $\lambda = 0$ in the LHS of (1.35) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (1.35) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Note that the RHS of (1.35) is non-negative. Note also that the LHS of (1.35) is equal to zero when

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}} = \frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2 + \sigma_H^2}{\sigma_H^2 + \sigma_L^2}} = e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2} + \frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_H^2+\sigma_L^2}\right)=\frac{\lambda^2}{(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}-\frac{\lambda^2}{(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}. \tag{1.36}$$

The LHS and the RHS of (1.36) are both strictly positive. Moreover, the LHS of (1.36) is monotonically decreasing in λ whereas the RHS of (1.36) is monotonically increasing in λ. Since the LHS starts at a higher value than the RHS, there is a unique value for λ that satisfies (1.36). Denote this value by $\bar{\lambda}_{0hh2}$. Since the LHS of (1.35) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0hh2})$ such that (1.35) holds as an equality. Denote this value by $\bar{\lambda}_{hh2}$. Hence, inequality (1.35) is satisfied when $\lambda < \bar{\lambda}_{hh2}$ and is violated when $\lambda > \bar{\lambda}_{hh2}$.

(iii) Let us consider a SPE where

$$(\sigma_1^2,\sigma_2^2)=(\sigma_L^2,\sigma_L^2),$$

and

$$a_1^*(\sigma_L^2,\sigma_L^2)=\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right)-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}$$
$$a_2^*(\sigma_L^2,\sigma_L^2)=\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right)-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}.$$

In SPE where $(\sigma_1^2,\sigma_2^2)=(\sigma_L^2,\sigma_L^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2,\sigma_2^2)=(\sigma_H^2,\sigma_L^2)$, that is,

$$E[U_1(a_1^*(\sigma_L^2,\sigma_L^2),a_2^*(\sigma_L^2,\sigma_L^2),\lambda,\sigma_L^2,\sigma_L^2)]\geq E[U_1(a_1^*(\sigma_H^2,\sigma_L^2),a_2^*(\sigma_H^2,\sigma_L^2),\lambda,\sigma_H^2,\sigma_L^2)],$$

or

$$u(y_l)+\Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)\Delta u-\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\Delta u$$
$$\geq u(y_l)+\Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right)\Delta u-\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}\Delta u,$$

or

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}-\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}$$

$$\leq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)-\Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right). \quad (1.37)$$

Setting $\lambda = 0$ in the LHS of (1.37) we obtain

$$LHS(\lambda=0)=\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}-\frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}>0.$$

Setting $\lambda = 0$ in the RHS of (1.37) we obtain

$$RHS(\lambda=0)=\Phi(0)-\Phi(0)=0.5-0.5=0.$$

Note that the RHS of (1.33) is non-negative. Note also that the LHS of (1.37) is equal to zero when

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}=\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2+\sigma_L^2}{\sigma_L^2+\sigma_L^2}}=e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}+\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2+\sigma_L^2}{\sigma_L^2+\sigma_L^2}\right)=\frac{(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}-\frac{(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}. \quad (1.38)$$

The LHS and the RHS of (1.38) are both strictly positive. Moreover, the LHS and the RHS of (1.38) are both monotonically decreasing in λ. Since the LHS starts at a higher value and decreases at a faster rate with λ than the RHS, there is a unique value for λ that satisfies (1.38). Denote this value by $\bar{\lambda}_{0ll1}$. Since the LHS of (1.37) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0ll1})$ such that (1.37) holds as an equality. Denote this value by $\bar{\lambda}_{ll1}$. Hence, inequality (1.37) is satisfied when $\lambda > \bar{\lambda}_{ll1}$ and is violated when $\lambda < \bar{\lambda}_{ll1}$.

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, that is,

$$E[U_2(a_1^*(\sigma_L^2,\sigma_L^2),a_2^*(\sigma_L^2,\sigma_L^2),\sigma_L^2,\sigma_L^2)]\geq E[U_2(a_1^*(\sigma_L^2,\sigma_H^2),a_2^*(\sigma_L^2,\sigma_H^2),\sigma_L^2,\sigma_H^2)],$$

or

$$u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\Delta u$$
$$\geq u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}\Delta u,$$

or

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}$$
$$\leq \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right). \quad (1.39)$$

Setting $\lambda = 0$ in the LHS of (1.39) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (1.39) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Note that the RHS of (1.39) is non-negative. Note also that the LHS of (1.39) is equal to zero when

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} = \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}} = e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}+\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right) = \frac{\lambda^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2} - \frac{\lambda^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}. \quad (1.40)$$

The LHS and the RHS of (1.40) are both strictly positive. Moreover, the LHS of (1.40) is monotonically decreasing in λ whereas the RHS of (1.40) is monotonically increasing in λ. Since the LHS starts at a higher value than the RHS, there is a unique value for λ that satisfies (1.40). Denote this value by $\bar{\lambda}_{0ll2}$. Since the LHS of (1.39) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0ll2})$ such that (1.39) holds as an equality. Denote this value by $\bar{\lambda}_{ll2}$. Hence, inequality (1.39) is satisfied when $\lambda > \bar{\lambda}_{ll2}$ and is violated when $\lambda < \bar{\lambda}_{ll2}$.

(ii) Let us consider a SPE where

$$(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2),$$

and

$$a_1^*(\sigma_L^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_L^2 + \sigma_H^2))^2}{8(\sigma_L^2 + \sigma_H^2)(\lambda + (\sigma_L^2 + \sigma_H^2))^2}$$
$$a_2^*(\sigma_L^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\right) - \frac{\lambda^4}{8(\sigma_L^2 + \sigma_H^2)(\lambda + (\sigma_L^2 + \sigma_H^2))^2}.$$

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, that is,

$$E[U_1(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \lambda, \sigma_L^2, \sigma_L^2)] \geq E[U_1(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \lambda, \sigma_H^2, \sigma_L^2)]. \tag{1.41}$$

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$, that is,

$$E[U_2(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \sigma_L^2, \sigma_H^2)] \geq E[U_2(a_1^*(\sigma_L^2, \sigma_L^2), a_2^*(\sigma_L^2, \sigma_L^2), \sigma_L^2, \sigma_L^2)]. \tag{1.42}$$

Note that inequality (1.41) states the contrary of inequality (1.37) and that inequality (1.42) states the contrary of inequality (1.39). Hence, when

$$\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$$

this SPE holds.

Finally, we show that the strategy profile

$$(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2),$$

and

$$a_1^*(\sigma_H^2, \sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_H^2 + \sigma_L^2))^2}{8(\sigma_H^2 + \sigma_L^2)(\lambda + (\sigma_H^2 + \sigma_L^2))^2}$$

$$a_2^*(\sigma_H^2, \sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}}\right) - \frac{\lambda^4}{8(\sigma_H^2 + \sigma_L^2)(\lambda + (\sigma_H^2 + \sigma_L^2))^2}.$$

cannot be a SPE. If $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$ were a SPE, then

$$E[U_1(a_1^*(\sigma_H^2, \sigma_L^2), a_2^*(\sigma_H^2, \sigma_L^2), \lambda, \sigma_H^2, \sigma_L^2)] \geq E[U_1(a_1^*(\sigma_L^2, \sigma_L^2), a_2^*(\sigma_L^2, \sigma_L^2), \lambda, \sigma_L^2, \sigma_L^2)],$$

and

$$E[U_2(a_1^*(\sigma_H^2, \sigma_L^2), a_2^*(\sigma_H^2, \sigma_L^2), \sigma_H^2, \sigma_L^2)] \geq E[U_2(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \sigma_H^2, \sigma_H^2)].$$

These inequalities are given by

$$\Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+\sigma_H^2+\sigma_L^2)}}{\sqrt{\sigma_H^2 + \sigma_L^2}}\right)\Delta u - e^{a_1^*(\sigma_H^2,\sigma_L^2)} \geq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+\sigma_L^2+\sigma_L^2)}}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right)\Delta u - e^{a_1^*(\sigma_L^2,\sigma_L^2)}, \quad (1.43)$$

and

$$\left[1 - \Phi\left(\frac{-\frac{\lambda^2}{2(\lambda+\sigma_H^2+\sigma_L^2)}}{\sqrt{\sigma_H^2 + \sigma_L^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_H^2,\sigma_L^2)} \geq \left[1 - \Phi\left(\frac{-\frac{\lambda^2}{2(\lambda+\sigma_H^2+\sigma_H^2)}}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_H^2,\sigma_H^2)}, \quad (1.44)$$

respectively. For (1.43) to be satisfied, we need that $\Phi(.)\Delta u - e^{a_1^*(.)}$ is increasing in the sum of risks σ^2, i.e.,

$$\frac{\partial}{\partial\sigma^2}\left[\Phi\left(\frac{\frac{\lambda(\lambda+2\sigma^2)}{2(\lambda+\sigma^2)}}{\sqrt{\sigma^2}}\right)\Delta u - e^{\log\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right) - \frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}}\right] > 0$$

$$-e^{-\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}}\left[\frac{\lambda(\lambda^2 + \lambda\sigma^2 + 2\sigma^4)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}} - \frac{1}{2\sigma^2\sqrt{2\pi\sigma^2}} + \frac{\lambda^2(\lambda+2\sigma^4)(\lambda^2+\lambda\sigma^2+2\sigma^2)}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}}\right]\Delta u > 0$$

$$-\frac{\lambda(\lambda^2 + \lambda\sigma^2 + 2\sigma^4)}{2(\lambda+\sigma^2)^2} + 1 - \frac{\lambda^2(\lambda+2\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4)}{4\sigma^2(\lambda+\sigma^2)^3} > 0$$

$$4\sigma^2(\lambda+\sigma^2)^3 > \lambda\sigma^2(\lambda+\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4) + \lambda^2(\lambda+2\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4)$$

$$4\sigma^2(\lambda+\sigma^2)^3 > \lambda(\lambda^2+\lambda\sigma^2+2\sigma^4)\left(\sigma^2(\lambda+\sigma^2) + \lambda(\lambda+2\sigma^2)\right) \quad (1.45)$$

For (1.44) to hold, we need that $[1-\Phi(.)]\Delta u - e^{a_2^*(.)}$ is decreasing in the sum of risks σ^2, i.e.,

$$\frac{\partial}{\partial\sigma^2}\left[\left[1-\Phi\left(\frac{\frac{-\lambda^2}{2\lambda+\sigma^2}}{\sqrt{\sigma^2}}\right)\right]\Delta u - e^{\log\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right)-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\right] < 0$$

$$-e^{-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\frac{\lambda^2(\lambda+3\sigma^2)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}}\Delta u - e^{-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}}\Delta u < 0$$

$$-\frac{\lambda^2(\lambda+3\sigma^2)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}} - \frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}} < 0$$

$$-\lambda^2(\lambda+3\sigma^2) - \frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{2\sigma^2(\lambda+\sigma^2)} < 0$$

$$4\sigma^2(\lambda+\sigma^2)^3 < 2\sigma^2\lambda^2(\lambda+\sigma^2)(\lambda+3\sigma^2) + \lambda^4(\lambda+3\sigma^2)$$

$$4\sigma^2(\lambda+\sigma^2)^3 < \lambda^2(\lambda+3\sigma^2)\left(2\sigma^2(\lambda+\sigma^2)+\lambda^2\right) \tag{1.46}$$

For (1.45) and (1.46) to be satisfied and we would need that,

$$\begin{aligned}
\lambda\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)\left(\sigma^2(\lambda+\sigma^2)+\lambda(\lambda+2\sigma^2)\right) &< \lambda^2(\lambda+3\sigma^2)\left(2\sigma^2(\lambda+\sigma^2)+\lambda^2\right)\\
\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)\left(\lambda^2+3\lambda\sigma^2+\sigma^4\right) &< \lambda\left(\lambda^2+2\lambda\sigma^2+2\sigma^4\right)(\lambda+3\sigma^2)\\
(\lambda^2+\lambda\sigma^2+2\sigma^4)\sigma^4 &< \lambda^2\sigma^2(\lambda+3\sigma^2)\\
(\lambda^2+\lambda\sigma^2+2\sigma^4)\sigma^2 &< \lambda^2(\lambda+3\sigma^2)\\
\lambda^2\sigma^2+\lambda\sigma^4+2\sigma^6 &< \lambda^3+3\lambda^2\sigma^2\\
\lambda\sigma^4+2\sigma^6 &< \lambda^3+2\lambda^2\sigma^2\\
\frac{\sigma^2}{\lambda}+\frac{2\sigma^4}{\lambda^2} &< \frac{\lambda}{\sigma^2}+2\\
0 &< \frac{\lambda}{\sigma^2}-\frac{\sigma^2}{\lambda}+2\left(1-\frac{\sigma^4}{\lambda^2}\right)\\
0 &< \frac{\lambda^2-\sigma^4}{\lambda\sigma^2}+2\frac{\lambda^2-\sigma^4}{\lambda^2}\\
0 &< (\lambda^2-\sigma^4)\frac{1}{\lambda}\left(\frac{1}{\sigma^2}+\frac{2}{\lambda}\right)
\end{aligned}$$

Since we consider values of λ such that $\lambda < \sigma^2$ this is a contradiction. Therefore, there are no values of $\lambda < \sigma^2$ for which (1.43) and (1.44) hold simultaneously and $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$ can be a part of a SPE.

Chapter 2

The Speed of Entry in Vertically Differentiated Markets

Abstract This paper explores a model of vertically differentiated markets, where products differ in their quality levels and consumers differ in their willingness to pay. In the market, there is an incumbent and two potential entrants. Observing that higher quality is always more profitable, we find that when entrants are entering simultaneously, there is a unique pure strategy equilibrium where all active firms choose the highest quality and price at marginal cost. Thus, firms make zero profits and two entrants are enough to guarantee competition. When assuming endogenous sequential entry, we observe a second-mover advantage. Entrants, thus, have no incentives to enter a market first despite costless entry and entering becomes a game with "war-of-attrition"-features. Applying a mixed strategy at the entry stage yields a delayed market entry. This idleness to enter the market extends the monopoly position of the incumbent and can deteriorate consumer surplus. Whether one, two, or no entrant is socially desirable depends on the consumers' level of willingness to pay. Finally, we investigate practices with a potentially accelerating effect on the speed of entry, like exclusivity periods for the first entrant or the taxation of late entrants.

This chapter was written in collaboration with João Montez.

2.1 Introduction

In today's world, being a copycat company is a way of doing business. Increasingly, one can observe the crowded fields of technology, where different companies' gadgets like smartwatches, smartphones, and tablets have become close copies of their rivals' products. Also in other fields such as the fashion industry, copying is as old as the industry itself. Fast fashion companies have built multi-billion-dollar businesses by reproducing the latest catwalk creations for a fraction of the original price. Today, copying is more prevalent than ever before.

The omnipresence of copycats thus raises the question of how the potential entry of copycat companies affects the market and what challenges it entails. Vertical product differentiation (VPD) models intend to explain these challenges and investigate the quality choice behavior of the economic agents active in a specific market. Mussa and Rosen (1978), Gabszewicz and Thisse (1979), Shaked and Sutton (1982, 1983), and Tirole (1988: 296-298) provide convenient tools which have been applied extensively in different contexts.

This paper investigates the timing of copycat entry into a market with an established incumbent when entry is free. We rely on a setting of an oligopolistic market with vertical product differentiation with an incumbent firm and two potential copycat entrants, for which the most closely related study is Peitz (2002). The latter study considers a model of vertical product differentiation with sequential quality choice and simultaneous price setting among two incumbents and one potential entrant. Although our analysis draws heavily from his work, there are several important differences. First, unlike Peitz (2002), we consider that there is only one incumbent but two potential entrants. Second, potential entrants do not face any entry costs, i.e. entry is free. Finally, we do not assume an exogenously determined sequence of entry, but let copycat entrants decide endogenously when to enter the market. Thus, we provide a model of vertical product differentiation with sequential or simultaneous quality choice depending on the timing of entry, and simultaneous price setting among an incumbent and two potential copycat entrants. The game is played repeatedly over an infinite number of discrete time periods.

When analyzing the one-shot entry game, we find that when copycats enter simultaneously, there is a unique pure strategy equilibrium where they choose the highest quality possible and make zero profits. When we allow for sequential entry, the second entrant chooses to produce a quality that represents the average of the incumbent's and the first entrant's quality. Taking into account the behavior of the second entrant, the first entrant locates at the bottom of the quality space. We find that there is a considerable profit advantage for the copycat entering second. This stems from the fact that the second copycat entrant sells higher quality to a higher price and that he serves a bigger fraction of consumers. Like in many two-stage quality-price game models with vertical product differentiation and zero cost, we thus observe that higher-quality firms reap larger profits (Wang, 2003).

This second-mover advantage destroys the urge to enter the market first and entering becomes a game with *war of attrition* characteristics. Thus, when giving firms the possibility to decide for themselves when to enter the market, we find that copycats have no incentive to enter the market immediately but to delay their entry with the hope that their rival will enter before them. Anticipating a consequent higher profit, the desire to stay out of the market induces a mixed strategy equilibrium, with the incumbent earning monopoly profits until a random time period. Allowing for endogenous sequential entry, consequently, distorts the market with an extended monopoly situation of the incumbent. This result is in line with observations in the pharmaceutical industry, where incumbent firms enjoy the absence of generic competitors for a certain period (Grabowski and Kyle, 2007). Against intuition, introducing competition among copycat firms delays first entry. Having examined this effect of competition on entry times, we turn to address welfare. Surprisingly, we find that in some cases, an additional entrant decreases welfare. Whether it is better to have one or two copycat entrants, depends on the parameters of the game.

The remainder of this paper is organized as follows. Section 2.2 discusses the previous literature. Section 2.3 sets up the general model. In Section 2.4 we state our preliminary results on the characterization of the equilibrium. Section 2.5 identifies the delay of entry. In Section 2.6 we study the welfare implications of an additional copycat entrant compared to the benchmark model of only one entrant. Section 2.7 discusses the effects of an exclusivity period as a remedy to speed up entry. Section 2.8 concludes and provides some directions for future work. All proofs that are not treated in the main text can be found in the Appendix.

2.2 Related Literature

Our paper contributes to two different strands of literature. First, it contributes to the vast literature of markets with differentiated products. A standard result in the analysis of markets with horizontally differentiated products is that with free entry firms make zero profits. This is due to intense Bertrand price competition. From a theoretical point of view, this result is one of the reasons why we observe vertical product differentiation: firms use vertically differentiated products to soften intense price competition and make positive profits. In addition, diversity in consumer tastes, disparities in consumers' income or timing of firms' entry into the industry explain the extent of product differentiation (Donnenfeld and Weber, 1992). As a result, vertical product differentiation became a basic principle in industrial organisation. One defining characteristic that basic models with vertical product differentiation (VPD) exhibit, is that for any two of the goods in question offered at an identical price, consumers would agree in choosing the higher quality good (Shaked and Sutton, 1983).

In the standard product-differentiation models, of which (Mussa and Rosen, 1978) and (Gabszewicz and Thisse, 1979) probably are the most notable contributors, consumers have heterogeneous preferences. Preferences are specified by a linear indirect utility function and consumers are assumed to be uniformly distributed within a certain range. It is further implicitly assumed that each consumer purchases at most one unit of good per period. In such frameworks, it is, therefore, possible to express explicitly how demands are affected by quality differences. Due to the explicit form of demand and the possibility to obtain an explicit solution of the game, Mussa and Rosen - type utility functions have been predominately used to model vertically differentiated markets.

Previous quality-choice models mostly focus on a pure duopoly case. The most basic quality-differentiation models are based on the following two-stage game between two firms: quality competition followed by simultaneous price choice, where each firm is offering one quality only. Consumers then choose which firm to purchase from. Entry, however, comprises a number of different variants: firms may enter one after another, they may enter simultaneously, or initially, some firms enter simultaneously and are seen as incumbents by later entrants (Donnenfeld and Weber, 1992). Different variants of sequential entry have also been studied. These sequential entry models were analyzed and extended to comprise the effects of potential entry in markets with dominant incumbents who face the threat of potential entry (Donnenfeld and Weber, 1992). Even when the timing in the quality choice stage differs between models, assuming a covered market results in maximal product differentiation. In other words, when assuming that every consumer purchases one unit of either good, each firm settles at one end of the quality spectrum to maximally differentiate their products and avoid intensive price competition (see (Tirole, 1988) and (Shaked and Sutton, 1982)). This holds even in the absence of entry costs.

Besides the order of entry, also the number of entrants or incumbents, respectively, and hence the total number of firms varies rather strongly between different models. (Tirole, 1988), for example, proposes a modified version of (Shaked and Sutton, 1982), where, in a covered market, two firms produce distinct goods and face zero production cost. This standard differentiation duopoly model with simultaneous quality choice predicts an equilibrium with maximum quality differentiation over the available range of qualities. (Choi and Shin, 1992) look at the same problem in an uncovered market setting, i.e. a market where firms do not sell to some fraction of consumers. They find that the low-quality firm produces some medium quality that is a fraction of the high-quality firm. Even though both models display the absence of costs, price competition gives them a strong incentive to differentiate their products. Thus, we observe some distortion at market equilibrium as it moves away from the ideal of perfect competition, which intervenes with maximizing social welfare.

(Donnenfeld and Weber, 1992) introduce a model with two established firms and one late entrant. The firms, therefore, differ in the order in which they enter the market. Incumbents choose qualities simultaneously. Assuming Mussa and Rosen - type utility functions, (Peitz, 2002) looks at a similar market environment with two incumbent firms but a different timing of the game: incumbents choose quality sequentially. In both articles, they find that the incumbents choose maximal product differentiation, whereas the entrant always chooses the average quality of the two incumbents. Although in both models competition is introduced by allowing entry, perfect competition is still absent.

Our study contributes to this literature by assuming endogenous entry into a market with a single incumbent and two potential copycat entrants. By doing so, we allow for sequential, simultaneous entry, as well as the possibility of no entry, and can thus better capture entry decisions in real-life markets.

Second, our paper contributes to the literature which observes mechanisms of war of attrition. (Henry and Ponce, 2011) use a model of competition among imitators to show that the incumbent innovator can strategically create lead time over her competitors even in the absence of patent rights. They analyze a theoretical model in which the inventor can sell specific knowledge on an invention to potential imitators. In equilibrium, the inventor chooses to sell her technology in a way that allows acquiring firms to resell the knowledge to other firms. As a result, once the first imitator has acquired the knowledge and entered the market, he will compete with the innovator in the market for knowledge. This drives prices for the knowledge to zero. This is nevertheless optimal for the inventor because potential imitators do not have incentives to immediately enter the market, but wait in the hope that another firm enters first and drives down the price of the required knowledge. This produces a situation in which the inventor is protected from imitators and enjoys a temporary monopoly position without the existence of a patent.

In the presence of patents, (Marxen and Montez, 2020) study an entry game of firms where potential generic entrants can sign an early entry agreement with the incumbent, which leads to entry just before patent expiry. An early entry agreement ensures that only a single generic firm enters the market and enables the incumbent to extract the entrant's profit. Yet, signing an early entry agreement is always welfare improving. In the absence of such an agreement, entry has a "grab the dollar structure" and is not profitable. Then, as long as no entry has occurred, each generic chooses to enter the market in each period with some probability. As a result, the incumbent remains a monopolist post-patent expiry, which hurts the consumers.

Our study contributes to this literature by showing that the outcome of the natural forces of vertically differentiated markets not only prevent the market from an efficient outcome (perfect competition) but can aggravate the situation with a delayed entry, even in the absence of patents. Unlike (Henry and Ponce, 2011) and (Marxen and Montez, 2020), we abstract from any contracts and agreements between the incumbent and potential entrants. By doing so, we are able to focus on the sophisticated entry behavior of potential entrants into a market with vertical product differentiation. The delay we observe in our paper is consequently entirely strategic. To mitigate the negative implications of this delay and prevent the distortions of late entry, we look at different policy variables with a potentially positive effect on the speed of entry.

2.3 The Model

We model a game in a vertically differentiated market with an incumbent, firm 1, and two potential copycat entrants, firms 2 and 3. The set of players, hence, is $I = \{1, 2, 3\}$. Firm 1 is the branded producer, while firms 2 and 3 are copycat producers who potentially enter at a later stage. Despite the sequence of entering the market, the three firms are identical in all respects. In particular, each of them is constrained to offer only one quality of a non-durable good, and each of the firms faces the same constant marginal costs of developing the technology that enables the provision of quality q. The firms compete for consumers by offering packages of quality and price (q_i, p_i), where $q_i \in \Omega = [\underline{q}, \overline{q}] = [0, 10]$ and $p_i \in \mathbb{R}_0^+$. For the choice of the bounds of the quality space, it is only essential that the lowest possible quality is a non-negative number. We favor a 0-10 quality scale, not only because we believe that it matches how customers tend to evaluate different qualities but also for the sake of legibility of the results. We assume that the quality of the incumbent is exogenous and that she produces the highest quality possible, i.e. $q_1 = \overline{q}$.[1] We also assume that marginal costs are constant and equal to zero. This assumption is without loss of generality to the extent that the market is covered, i.e. that all consumers end up buying a unit of produce. When we observe at least one entrant, we thus restrict our attention to covered market configurations. Consequently, we guarantee that any kind of competition makes the good available for all consumers in the market.

The demand side of the underlying market consists of a continuum of consumers of mass 1, where consumers' quality valuations θ are uniformly distributed over $[0, 1]$. We use the Mussa and Rosen (1978) specification of preferences.[2] Thus, the utility

[1]Imposing the highest quality for the incumbent firm is shown to be reasonable. The branded producer typically was a patent holder and in the market for a longer time. For this reason, she has a quality advantage and produces the highest quality in the market. It is further in line with the fact, that higher quality is always more profitable, independent on whether other firms enter the market or not (Peitz, 2002).

[2]Other applications in vertically differentiated market models that use this specification can for example be found in Tirole (1988).

function at each period t of a consumer of type θ takes the following form.

$$u = \begin{cases} V + \theta q_i - p_i & \text{, if she buys one unit of good of quality } q_i \text{ at price } p_i \\ 0 & \text{, otherwise} \end{cases}$$

The utility specification implies that all consumers prefer high quality, but a consumer with a higher θ is willing to pay more for it. Hence, θ can also be interpreted as the taste parameter. The reservation value V stands for the willingness to pay for a good of basic quality $\underline{q} = 0$. Assuming that one entrant is enough to ensure a covered market, in equilibrium each consumer is buying exactly one unit of product. This assumption reads $V > \frac{10}{3}$, i.e. when the incumbent faces a competitor, we need a sufficiently high reservation value for the market to be covered. We comment on this in further detail below.

We consider a model with infinite periods and complete information. Firms make decisions at each period $t \geq 0$. Each period has three stages. In the first stage, entrants 2 and 3 decide simultaneously whether to enter the market at t (if they have not done so before), or to wait and potentially enter at a later period. A strategy for player i specifies an entry time t_i, or alternatively a distribution $G_i(t)$. Therefore, an entrant i enters the market with a probability $g_i(t)$, for $i = 2, 3$. Firm i is said to be active as of the entry date, i.e. being active in t is denoted by $a_i^{t'} = 1$ for some $t' \leq t$, and inactive in t if $a_i^{t'} = 0$ for all $t' \leq t$. Note that the incumbent, or firm 1, is active from the beginning, i.e. $a_1^t = 1 \quad \forall t$.

In the second stage, firms that have just become active make their definite quality choice, producing each a unit of quality q_i^t. Firms are assumed to be committed to their quality, meaning firms choose their quality once and for all, i.e. $q_i^t = q_i^{t'}$ for all $t' \leq t$ if $a_i^{t'} = 1$.[3] With abuse of notation, we denote the quality of a non-active firm i by $q_i^t = \{\emptyset\}$. In the third stage, active firms compete in the market by simultaneously choosing prices p_i^t. With abuse of notation, we denote the price of a non-active firm i by $p_i^t = \{\emptyset\}$.

The actions in each period $t \geq 0$, therefore, are a pair (q_1^t, p_1^t) with $q_1^t = 10$ and

[3]The technology requires that the firms adhere to a certain level of quality, such that it is either not possible or very expensive to increase or decrease the quality by damaging the product (as described in Deneckere and Preston McAfee (1996)). This commitment to quality is reasonable in markets where the need for a different quality requires a significant change in the product's design, making it impractical to lower or increase the quality (Peitz, 2002).

$p_1^t \in \mathbb{R}_0^+$ for the incumbent, and a triplet (a_i^t, q_i^t, p_i^t) with $a_i^t \in \{0,1\}$, $q_i^t \in [0,10] \cup \{\emptyset\}$ and $p_i^t \in \mathbb{R}_0^+ \cup \{\emptyset\}$ for each entrant $i \in \{2,3\}$. For both, incumbent and entrants, strategies only depend on the set of active firms. Firms make decisions at discrete time periods, but the market operates in continuous time, thus payoffs are generated in real time. Firms discount the future exponentially at a per-period rate $\delta = e^{-r\Delta}$ with $\delta \in (0,1)$, where $\Delta \geq 0$ denotes the time length of a period. r denotes the discount rate, which we normalize to 1. There is no outside option for potential entrants. Thus, copycats make zero profits until they decide to enter the market. The present value of profit is given by $\Pi_i = \int_{t=0}^{\infty} e^{-\Delta t} p_i^t X_i^t a_i^t$, where X_i^t for $i = 1,2,3$ is the demand firm i is facing in period t. The solution concept we employ is a Markov perfect equilibrium. Thus, the firm's strategies depend on the current state only and are not influenced by the strategic decisions of previous periods (Fudenberg & Tirole, 1991).

2.4 Preliminary Results

For intuitive purposes, we first look at all potential outcomes in the one-shot game. As shown in Figure 2.1, in the entry stage, there are three possible situations: no copycat enters (monopoly), only one copycat enters (duopoly), and both copycats enter simultaneously. As we will show below, duopoly is not an absorbing state, meaning that it cannot be sustained in the long run. For this reason, we also consider the case where copycats enter sequentially over time (displayed in grey). We look at quality and price decisions in these one-shot games. In all relevant subgames, we start by solving the last stage of period t, in which the entrant(s) and the incumbent compete in prices.

Monopoly: No Copycat Entry

Consider the situation where no copycat $j = 2,3$ enters the market such that $a_j^t = 0$. The incumbent then remains a monopolist for another period. The incumbent's monopoly quality-price-pair (q_1^{m*}, p_1^{m*}) maximizes the profit $\Pi_1^m = p_1 X_1^m = p_1(1 - \frac{p_1 - V}{10})$, which yields a monopoly price of $p_1^{m*} = \frac{10+V}{2}$. Note that, if V is sufficiently high, a monopoly is enough to cover the entire market. This condition reads $V \geq 10$. In this case, the monopolist is better off charging a price equal to the valuation of the consumers, i.e. $p_1^{m*} = V$.

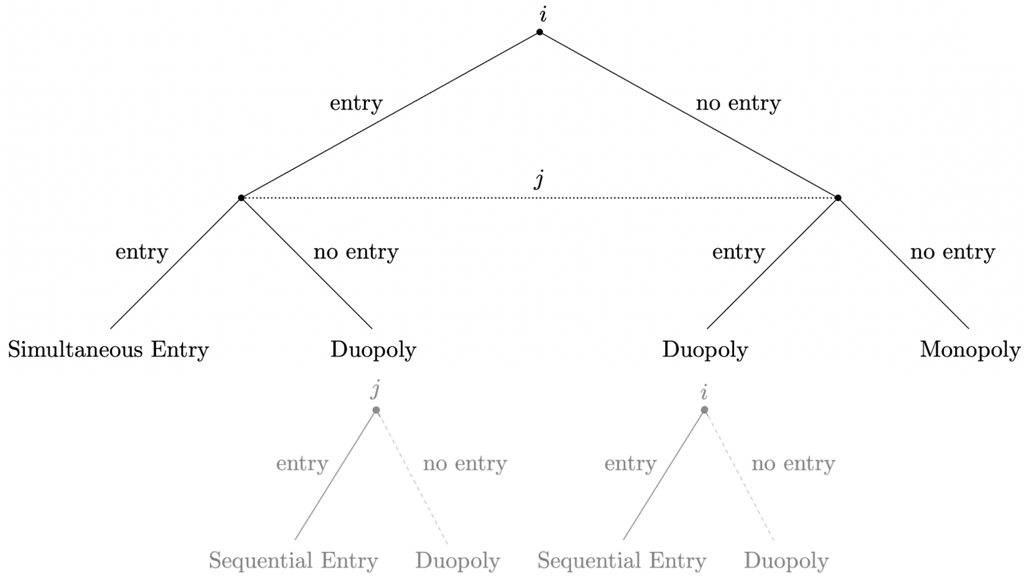

Figure 2.1: Game Tree One-Shot Game

The equilibrium profit of the monopolist is

$$\Pi_1^{m*} = \begin{cases} \frac{(10+V)^2}{40} & \text{, for } V < 10 \\ V & \text{, for } V \geq 10 \end{cases}$$

The consumer surplus, thus, is

$$CS^m = \begin{cases} \frac{(10+V)^2}{80} & \text{, for } V < 10 \\ 5 & \text{, for } V \geq 10 \end{cases}$$

and social welfare is given by

$$W^m = \begin{cases} \frac{3}{80}(10+V)^2 & \text{, for } V < 10 \\ V+5 & \text{, for } V \geq 10. \end{cases}$$

Note that, when V is sufficiently high, i.e. $V \geq 10$, from a social welfare point of view an efficient outcome is reached even with a monopoly. For simplicity, we thus focus on the case where a monopoly is not welfare maximizing and not enough to cover the market, i.e. $V < 10$.

Duopoly: Single Copycat Entry

Consider next a duopoly case where besides the incumbent only one entrant, say firm 2, is active. This case has been studied by Peitz (2002). He shows that the profit of incumbent 1 and incumbent 2 (here a copycat entrant) are given by $\Pi_1^d = p_1 X_1^d = p_1(1 - \frac{p_1 - p_2}{10 - q_2})$ and $\Pi_2^d = p_2 X_2^d = p_2(\frac{p_1 - p_2}{10 - q_2})$, respectively. He finds a unique pure strategy equilibrium in the simultaneous duopoly pricing game between the the two firms, which is equal to $(p_1^{d*}, p_2^{d*}) = \left(\frac{2}{3}(10 - q_2^d), \frac{1}{3}(10 - q_2^d)\right)$. Maximising the reduced form profits of firm 2 leads to maximal product differentiation, that is, firm 2 locates at the bottom of the quality space $q_2^{d*} = \underline{q} = 0$. The equilibrium prices are given by $(p_1^{d*}, p_2^{d*}) = \left(\frac{20}{3}, \frac{10}{3}\right)$. To ensure a covered market, the consumer who is indifferent between buying a good of quality 0 at a price of $\frac{10}{3}$ needs a quality valuation of at most $\overline{\theta} = 0$, i.e. $V + \overline{\theta} q_2^d - p_2^{d*} = 0$. With a sufficiently high reservations value $V > \frac{3}{10}$ we ensure that every consumer is ending up buying the good in equilibrium. For the remainder we focus on the case where the duopoly market is fully covered in price equilibrium, that is, all consumers buy in the differentiated market, i.e. $\frac{10}{3} < V \leq 10$. Recall that, we observe an uncovered market in the monopoly situation. Nevertheless, one entrant is sufficient to ensure that every consumer is served in equilibrium.

Given the equilibrium prices and qualities, the respective equilibrium profits are

$$\Pi_1^{d*} = \frac{40}{9} \qquad \text{and} \qquad \Pi_2^{d*} = \frac{10}{9}.$$

These profits do not depend on V, the willingness to pay for a basic quality 0. As we assume that all consumers are buying in equilibrium, V has no effect on the market shares and therefore neither on the equilibrium prices nor profits. Note that, if $V < \frac{10}{3}$, the market in equilibrium would be uncovered, that is, not all consumers would buy in equilibrium. In this case, in order to maximize her profit, the entrant would have an incentive to increase her market share by locating away from the lower quality bound and towards the quality of the incumbent.

The consumer surplus in the covered duopoly situation is given by

$$CS^d = V - \frac{10}{9}$$

and social welfare is equal to

$$W^d = V + \frac{40}{9}.$$

Note that, due to the assumption that copycats face no entry costs, a duopoly can only be a temporary market situation. We will show next that it is never optimal to stay out of the market forever. In the long-term, a duopoly is thus an unachievable equilibrium outcome.

Oligopoly: Sequential Entry of Two Copycats

Consider now the case where only one copycat enters in t and the remaining copycat has the possibility to enter at a later stage, i.e. copycats enter sequentially over time. Peitz (2002) has discussed this case with two incumbents and a potential entrant, where the incumbents' quality choice is modeled as sequential. For the potential entrant, taking into account the incumbents' quality choices, entering and choosing a quality is modeled as a simultaneous decision. As Peitz (2002) abstracts from any discounting, the model can be considered a one-period game.

Here, we assume that firm i has entered the market in t. Firm $j \neq i$ which has not entered yet has two options in $t+1$: entering in $t+1$ or staying out for another period. Staying out leaves firm j with zero profits. However, when entering in $t+1$, firm j has the possibility to differentiate and can set a positive price. Entering in $t+1$ and making a strictly positive profit, therefore, is a dominant strategy. Thus, if entrant i's entry is observed in t, entrant j enters in the subsequent period $t+1$, turning the market into a competitive oligopoly with three firms. Based on this observation, we assume that a copycat, say firm 2, enters in t and firm 3 enters in $t+1$. Solving the problem backward, we find that the simultaneous pricing game in $t+1$ has a unique equilibrium in pure strategies which is (p_1^c, p_2^c, p_3^c). Maximising the reduced form profits of firm 3 yields a unique reaction function $q_3^c(q_2^c) = \frac{10+q_2^c}{2}$.

Taking into account the quality decision of firm 3 and the pricing strategies in both periods, firm 2 maximizes her intertemporal discounted present value.

$$\max_{q_2} \int_0^{\Delta} p_2^d \left(\frac{p_1^d - p_2^d}{10 - q_2^c} \right) e^{-\tau} d\tau + \int_{\Delta}^{\infty} p_2^c \left(\frac{p_3^c - p_2^c}{q_3^c - q_2^c} \right) e^{-\tau} d\tau \tag{2.1}$$

The next result characterizes the equilibrium quality choice when firms enter sequentially.

Lemma 1. *If entrant 2 enters in t and entrant 3 stays out for one more period, it is a dominant strategy for firm 2 to locate at the bottom of the quality space $q_2^c = \underline{q} = 0$.*

In $t+1$, firm 3 then chooses an intermediate quality of $q_3^c = \frac{10+q_2^c}{2} = 5$.

This result is in line with Peitz (2002). Anticipating the behavior of firm 3, firm 2 tries to relax price competition as much as possible. Moving closer to the maximum quality would lead to intense price competition with the incumbent. Moving to the bottom quality, however, slackens price competition. Thus, to reach maximum product differentiation, firm 2 chooses a location at $\underline{q} = 0$. Firm 3 then always chooses the average quality of the firms already active in the market. Note that, because offering higher quality always leads to higher profits, choosing a quality above entrant 2 is a dominant strategy. Choosing a quality exactly in the middle of the already active firms and thereby another maximal differentiation formalizes the effect of strategic behavior in an extreme way and prevents the market from an efficient outcome.

Like in Peitz (2002), the equilibrium prices are given by $(p_1^{c*}, p_2^{c*}, p_3^{c*}) = \left(\frac{35}{12}, \frac{5}{12}, \frac{5}{6}\right)$ and the respective equilibrium profits in $t+1$ are

$$\Pi_1^{c*} = \frac{245}{144} \quad , \quad \Pi_2^{c*} = \frac{5}{144} \quad \text{and} \quad \Pi_3^{c*} = \frac{40}{144} = \frac{5}{18}.$$

Since $\Pi_2^{c*} < \Pi_3^{c*} < \Pi_1^{c*}$ and $\Pi_2^{d*} < \Pi_1^{d*}$, higher quality is indeed more profitable, and firm 3 earns higher profits despite being the last mover. We, therefore, observe a 'second-mover advantage 'by the late entrant. Note that, the equilibrium prices and profits are independent of the willingness to pay for basic quality V. The reason for this is again the fact that V is sufficiently high. Given that $V > \frac{3}{10}$ the willingness to pay is higher than any equilibrium price, all consumers end up buying a unit of good and the market is covered.

The consumer surplus in the competitive oligopoly is given by

$$CS^c = V + \frac{365}{144}$$

and social welfare is

$$W^c = V + \frac{655}{144}.$$

Oligopoly: Simultaneous Entry of Two Copycats

Finally, suppose both copycats enter simultaneously, i.e. $a_2^t = a_3^t = 1$, where $a_2^{t'} = a_3^{t'} = 0 \quad \forall t' \leq t$. Copycat entrants, hence, will choose their quality simultaneously. The results can be summarized as follows.

Lemma 2. *If entrants 2 and 3 enter simultaneously in t, there is a unique pure strategy subgame perfect equilibrium in which entrants choose the highest quality* $q_i^{sim} = \overline{q} = 10$*, and all active firms price at marginal cost* $p_i^{sim} = 0$*, for* $i = 1, 2, 3$.

Proof. Consider, for example, $q_2^* < q_3^* < 10$. Given the fact that higher quality firms reap larger profits, we have $\Pi_2 < \Pi_3$. Firm 2, then, has the lowest profit in the market. On the other hand, if firm 2 chooses $q_2 = q_3^* + \epsilon$ (where ϵ is positive and infinitesimal small), she obtains a higher profit. Therefore, firm 3 cannot be acting in her own interest if she chooses q_3^*. Now, suppose that $q_2^* = q_3^* < 10$. Firms share a demand schedule, which is equal to the market demand at the common quality, without other copycats in the market. Copycats make, however, zero profits due to intense Bertrand competition in the pricing stage. If firm 2 increases her quality slightly to $q_3^* + \epsilon$ her profit becomes again positive. This will drive qualities up. Because no copycat can choose a quality above $\overline{q} = q_1 = 10$, we are left with firms 2 and 3 choosing a quality exactly equal to $q_2^* = q_3^* = 10$. Competing a la Bertrand will lead to zero profits, as prices are driven down to marginal costs equal. Finally, assume that $q_2^* = q_3^* = 10$. A deviation to a lower quality $q_i = q_i^* - \epsilon$ for $i = 2, 3$ is not profitable, then consumers would not pay a positive price for a lower quality as the highest quality is available at a price zero too.

When copycats enter simultaneously the respective equilibrium profits are $\Pi_i^{sim} = 0$, for $i = 1, 2, 3$. Therefore, under simultaneous entry, two entrants are enough to guarantee perfect competition. Introducing a third firm in a vertically differentiated market with zero entry costs where firms choose quality simultaneously, thus, can eliminate the distortion induced by vertical product differentiation. Simultaneous entry destroys the maximal differentiation outcome and rebuts the theory of strategic behavior.

Since firms make zero profits, consumer surplus and welfare coincide and are equal to

$$CS^{sim} = W^{sim} = V + 5$$

Although simultaneous entry is a reasonable assumption in some cases[4], the assumption of sequential entry seems more reasonable. Indeed, entry is usually a process taking place over time with at least a bit of a sequential element. Letting copycats endogenously decide when to enter the market compromises both possibilities and, thus, reflects best real-life cases.

Welfare Comparison

In the following, we compare the levels of social welfare in the four possible outcomes of the one-shot game. When the willingness to pay for a basic quality V is low, the monopoly situation with a welfare of $W^m = \frac{3}{80}(10+V)^2$ is the least desirable outcome. Upon an entry of a copycat firm onto the market, the former monopolist has to give up some shares to the entrant. Though fewer consumers are served the high-quality good, contrary to the monopoly situation, in duopoly, the total market size is expanding, and all consumers are served. The latter, positive welfare effect, which a covered market entails, outweighs the former welfare loss from a decreasing quality mix, resulting in a welfare of $W^d = V + \frac{40}{9}$. Upon sequential entry of a second copycat onto the market, the quality improves and the more intensive price competition forces prices to drop. This results in slightly higher welfare, thus welfare in oligopoly after sequential entry exceeds welfare in competition, i.e. $W^c = V + \frac{655}{144} < W^d = V + \frac{40}{9}$. Yet, most welfare gains are exhausted after a single entry. The reason why the quality improvement of a third firm does not entail much more welfare gain comes from the covered market assumption. Assuming the market is covered in the duopoly case, a third firm on the market does not expand the size of the market. Adding a higher quality to the already existing quality mix does not excessively compensate for the loss in profits due to more intense price competition and, thus, does not induce significant welfare enhancement Finally, an efficient market outcome is observed upon simultaneous entry of copycats, as it guarantees perfect competition. Consequently, welfare in oligopoly with simultaneous entry is always higher than welfare in the case of oligopoly with sequential entry or duopoly, i.e. $W^{sim} = V + 5 < V + \frac{40}{9} = W^d$.

Being a quadratic function of the willingness to pay for basic quality, welfare with only an incumbent on the market expands and the monopoly situation becomes more

[4]In 1968, for example, Lockheed and McDonnell Douglas entered the market for wide-body aircraft approximately at the same time (Cabral, 2004).

socially beneficial as V increases. Depending on the value of V, we observe a change in the rank order of social welfare in the different one-shot game outcomes. Proposition 1 summarizes these results and identifies the values for which the rank order of social welfare changes.

Proposition 1. *In the one-shot game, the rank order of social welfare changes depending on the value of the willingness to pay for a basic quality V:*
(i) for $V < \frac{1}{9}(30 + 20\sqrt{6}) \approx 8.7766$ *the rank order is* $W^m < W^d < W^c < W^{sim}$
(ii) for $\frac{1}{9}(30+20\sqrt{6}) < V < \frac{1}{9}(30+5\sqrt{105}) \approx 9.0261$ *the rank order is* $W^d < W^m < W^c < W^{sim}$
(iii) for $\frac{1}{9}(30 + 5\sqrt{105}) < V < 10$ *the rank order of social welfare is* $W^d < W^c < W^m < W^{sim}$
(iv) for $V = 10$ *the rank order is* $W^d < W^c < W^m = W^{sim}$.

Contrary to consumer surplus, social welfare is not monotonically increasing in the number of entries. As V becomes higher, the welfare loss stemming from a lower quality mix outweighs the welfare gain from an increasing market size. When consumers are willing to pay more for a good, an additional good at the bottom of the quality range does not improve social welfare. For sufficiently high values of V it, thus, becomes beneficial to no longer have any copycats entering (sequentially) the market. As V becomes sufficiently high, the willingness to pay for a good of basic quality increases and makes low-quality suppliers redundant. Entry and, with it, more competition, therefore, is welfare harming as V increases.

2.5 War of attrition

Our preliminary analysis shows that entering with higher quality leads to higher profits and, thus, is in line with (Peitz, 2002) and (Wang, 2003). As the second entrant enters with an intermediate quality and, so, makes higher profits than the first entrant, she benefits from a second-mover advantage. Entrants, therefore, have no incentives to enter the market first. This finding turns the entry game into a game with 'war-of-attrition'-features in which copycat entrants delay entry in the hope that the rival entrant will enter before them.

Given the entrant's profits obtained in all possible outcomes, we can simplify the game and focus on the endogenous entry stage. As mentioned above, to solve the game, we focus on Markov Perfect Equilibra (MPE), specifically symmetric ones (SMPE). Note that, a strategic asymmetry would be captured by a predetermined sequential order of moves in the quality decision, like in Peitz (2002). However, unlike Peitz (2002), we do not designate *a priori* the entry time and so the type of quality for each firm. In our model, whether and when copycat entrants choose to enter a market is determined endogenously. As mentioned before, by allowing for endogenous entry we try to capture way real-life situations in a more precise manner.

Each copycat entrant $i \in \{2,3\}$ chooses $a_i^t = 1$ with a probability g as long as no entry has taken place. We assume that all firms observe the history of the game up to the beginning of time period t. The game continues in this manner as long as no copycat enters the market. Since we know all future profits obtained by firms, the game is modeled such that payoffs are determined as soon as one player enters the market. Thus, even with an infinite horizon, the game ends as soon as one copycat decides to enter.

In equilibrium, entrants randomize their entry decision and are indifferent between entering today and staying out for another period. Thus, it must hold that the expected gain from entering today $t = 0$ (LHS) equals the expected gain from staying out (RHS), i.e.

$$\begin{aligned}
&(1-g)\Big(\int_0^{\Delta}\Pi_2^d e^{-\tau}d\tau + \int_{\Delta}^{\infty}\Pi_2^c e^{-\tau}d\tau\Big) + g\int_0^{\infty}\Pi^{sim}e^{-\tau}d\tau \\
&= ge^{-\Delta}\int_{\Delta}^{\infty}\Pi_3^c e^{-\tau}d\tau + (1-g)e^{-\Delta}\left((1-g)\left(\int_0^{\Delta}\Pi_2^d e^{-\tau}d\tau + \int_{\Delta}^{\infty}\Pi_2^c e^{-\tau}d\tau\right)\right. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\left. + g\int_0^{\infty}\Pi^{sim}e^{-\tau}d\tau\right)
\end{aligned}$$

, where $\Pi_2^d = \frac{10}{9}$, $\Pi_2^c = \frac{5}{144}$, $\Pi_3^c = \frac{5}{18}$ and $\Pi^{sim} = 0$. When solving for g we obtain the mixed strategy of the entry game.

Proposition 2. *There exists a mixed strategy Markov perfect equilibrium, in which each copycat entrant $i \in \{2,3\}$ chooses to enter the market in t, i.e. $a_i^t = 1$, with a probability equal to*

$$g = \frac{-32 + 95e^{-\Delta} - 70e^{-2\Delta} + \sqrt{1024 - 1984e^{-\Delta} + 1473e^{-2\Delta} - 1520e^{-3\Delta} + 1056e^{-4\Delta}}}{e^{-\Delta}(64 - 62e^{-\Delta})}. \tag{2.2}$$

This equilibrium entry probability converges to a Poisson process as $\Delta \to 0$. The first entrants, say firm 2, chooses the lowest quality $q_2^* = \underline{q} = 0$. Firm 3, in the subsequent period, chooses an intermediate quality $q_3^* = \frac{\overline{q}+q_2}{2} = 5$.

Figure 2.2 shows the mixed strategy of the entry stage as a function of the period length Δ. Note that, it is independent of the valuation for a good of basic quality V. The probability to enter the market g is positively depending on Δ and as $\Delta \to 0$ the probability distribution g converges to a Poisson process. When the time length of a period becomes very small, the entry of a copycat becomes arbitrarily slow. The randomization delays entry and makes consumers pay the monopoly price for a longer time period. The fact that entrants are using a mixed strategy on the equilibrium path, however, does not guarantee that the outcome is inefficient. Then, delay vanishes or becomes negligible once the period length becomes infinitely long. As $\Delta \to \infty$, the probability of entering the market converges to one and entry takes place in the "twinkle of an eye". What creates a real-time delay is the fact that the equilibrium entry rate converges to a Poisson process as $\Delta \to 0$.

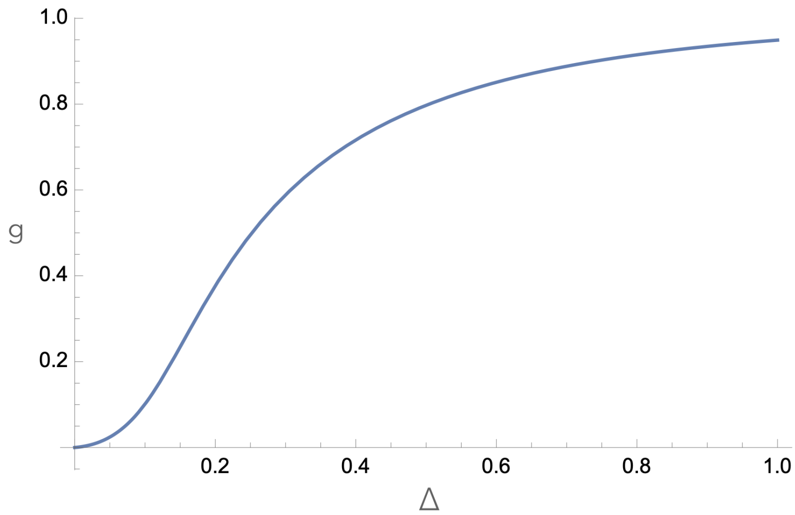

Figure 2.2: Mixed strategy in the entry stage

This real-time delay raises the question of how long the incumbent can expect to keep her monopoly position. We know if no entry occurred in the periods before, with a probability of $2g(1-g)+g^2 = 1-(1-g)^2$ the game ends up in a state with at least one entry. The incumbent can, therefore, expect to keep her monopoly position beyond t for $(1-(1-g)^2)^{-1}$ periods, before eventually one or both copycat firms enter the market. The exact date t at which at least one copycat entrant chooses to enter the market, then, is given by the respective cumulative distribution function.

Once an entry happens, all active firms choose their qualities forever after and set their prices according to the subgame perfect equilibra. In case only one copycat enters, the remaining copycat enters in the subsequent period, chooses an intermediate quality and all active firms compete in prices (cf. Section 2.4). The expected payoff of the incumbent at the outset of that period t then is

$$\mathbb{E}[\Pi_1] = g^2\Big(\int_0^\infty \Pi^{sim} e^{-\tau} d\tau\Big) + 2g(1-g)\Big(\int_0^\Delta \Pi_1^d e^{-\tau} d\tau + \int_\Delta^\infty \Pi_1^c e^{-\tau} d\tau\Big) \\ + (1-g)^2\Big(\int_0^\Delta \Pi_1^m e^{-\tau} d\tau + \mathbb{E}[\Pi_1]e^{-\Delta}\Big).$$

Solving for $\mathbb{E}[\Pi_1]$ yields

$$\mathbb{E}[\Pi_1] = \frac{g^2\Pi^{sim} + 2g(1-g)\Big((1-e^{-\Delta})\Pi_1^d + e^{-\Delta}\Pi_1^c\Big) + (1-g)^2(1-e^{-\Delta})\Pi_1^m}{1-(1-g)^2 e^{-\Delta}}.$$

Next, the expected payoff of copycat firm $i = 2, 3$ applying a mixed strategy is given by

$$\mathbb{E}[\Pi_i] = g^2\Big(\int_0^\infty \Pi^{sim} e^{-\tau} d\tau\Big) + g(1-g)\Big(\int_0^\Delta \Pi_2^d e^{-\tau} d\tau + \int_\Delta^\infty \Pi_2^c e^{-\tau} d\tau\Big) \\ + (1-g)g\Big(\int_\Delta^\infty \Pi_3^c e^{-\tau} d\tau\Big) + (1-g)^2\Big(\mathbb{E}[\Pi_i]e^{-\Delta}\Big).$$

When solving for $\mathbb{E}[\Pi_i]$ we obtain

$$\mathbb{E}[\Pi_i] = \frac{g^2\Pi^{sim} + g(1-g)\Big((1-e^{-\Delta})\Pi_2^d + e^{-\Delta}\Pi_2^c\Big) + (1-g)g\Big(e^{-\Delta}\Pi_3^c\Big)}{1-(1-g)^2 e^{-\Delta}},$$

with $\Pi_1^m = \frac{(10+V)^2}{40}$, $\Pi_1^d = \frac{40}{9}$, $\Pi_2^d = \frac{10}{9}$, $\Pi_1^c = \frac{245}{144}$, $\Pi_2^c = \frac{5}{144}$, $\Pi_3^c = \frac{5}{18}$ and $\Pi^{sim} = 0$. Note that, since copycat firms play a mixed strategy, the expected payoff is equal to the expected payoff when entering the market or the expected payoff when staying out. The firms' expected payoffs become a weighted average of their payoffs in the different possible states of no entry, sequential or simultaneous entry, with weights that reflect the probability of single and double entry.

Corollary 1. *Conditional on the game's history, firms play according to Proposition 2 and to Lemmas 1 and 2. The incumbent remains a monopolist for an expected real-time length of* $\Delta(1-(1-g)^2)^{-1}$ *before eventually one or both copycat firms enter. The expected payoff of the incumbent is* $\mathbb{E}[\Pi_1]$*, while the entrants' expected profit from entering is* $\mathbb{E}[\Pi_i]$*.*

It is intuitive to consider the limiting cases where the time between periods is close to zero, i.e. $\Delta \to 0$. Periods become extremely short, such that we can study the continuous-time version of the game and real-time efficiency. As the probability distribution converges to a Poisson process, the probability of entering becomes infinitesimally small. The probability that at least one firm enters converges to zero, i.e.

$$\lim_{\Delta \to 0^+} 1-(1-g(\Delta))^2 = 0.$$

The speed of copycat entry slows down, and the average time of the monopoly period converges to

$$\lim_{\Delta \to 0^+} \frac{\Delta}{1-(1-g(\Delta))^2} = \frac{7}{2}.$$

The average time until an entry occurs, thus, converges to a constant number. Nevertheless, the time until the first entry is still randomly spaced. We might observe an immediate entry, but we could also go numerous periods without entry due to the randomness of the process. As periods become extremely short, the expected payoffs of the incumbent and the active firms become

$$\lim_{\Delta \to 0^+} \mathbb{E}[\Pi_1] = \frac{1}{9}\left(2 \cdot \Pi_1^c + 7 \cdot \Pi_1^m\right) = \frac{245}{648} + \frac{7}{360}\left(10+V\right)^2$$

and

$$\lim_{\Delta \to 0^+} \mathbb{E}[\Pi_{i=2,3}] = \frac{1}{9}\left(\Pi_2^c + \Pi_3^c\right) = \frac{5}{144},$$

respectively. While the expected payoff of the incumbent depends positively on the willingness to pay for basic quality V, the expected payoff of an active copycat is constant. When periods become extremely short, the copycats' expected payoffs converge to a weighted average of the payoffs in the absorbing state, i.e. the oligopoly situation.

2.6 Welfare Analysis

In this section, we consider the normative aspects of the underlying problem. First, we investigate how the market equilibrium level of consumer surplus and social welfare with two entrants compares to the situation when there is only one entrant.

2.6.1 Consumer Surplus & Social Welfare

Studying consumer surplus and social welfare typically involves considering all potential states of a game: no entry, sequential entry and simultaneous entry. We have seen that with probabilities $2g(1-g) = g^2 = 1-(1-g)^2$ the game transitions to absorbing states with two entrants. With probability $(1-g)^2$ there is still no entry at t. The incumbent can expect to keep her monopoly position beyond t for $(1-(1-g)^2)^{-1}$ periods, before copycats eventually enter, either simultaneously or sequentially. Once entry occurs, all active firms choose their qualities and prices as shown in Section 2.4. The expected consumer surplus in t then is given by

$$CS = g^2(V+5) + 2g(1-g)\Big((1-e^{-\Delta})(V-\frac{10}{9}) + e^{-\Delta}(V+\frac{365}{144})\Big) + (1-g)^2\Big((1-e^{-\Delta})\frac{(10+V)^2}{80} + e^{-\Delta}CS\Big).$$

Recall that $\delta = e^{-\Delta}$. Solving for CS yields

$$CS = \frac{g^2(V+5) + 2g(1-g)\Big((1-e^{-\Delta})(V-\frac{10}{9}) + e^{-\Delta}(V+\frac{365}{144})\Big)}{1-(1-g)^2e^{-\Delta}} + \frac{(1-g)^2(1-e^{-\Delta})\frac{(10+V)^2}{80}}{1-(1-g)^2e^{-\Delta}},$$

where g is given by expression (2). The consumer surplus increases monotonically in the willingness to pay for a good of basic quality V. For the latter, the same intuition as for the present value of payoffs applies. Further, the consumer surplus depends negatively on $\delta = e^{-\Delta}$ and therefore positively on Δ. As discussed in Section 2.5, the copycat entrants' probability of entering the market depends positively on Δ. Therefore, when the period lengths Δ become shorter, i.e. when δ becomes higher, the probability of observing an entry decreases. Due to the delayed introduction of competition, consumers are less likely to benefit from more variety and lower prices, and, thus, suffer from a drop in consumer surplus. As the expected length of the incumbent's monopoly situation lasts longer, an increase in variety and competition is only observed later in time.

Then, the expected social welfare at the time of period 0 is given by

$$W = g^2(V+5) + 2g(1-g)\Big((1-e^{-\Delta})(V+\frac{40}{9}) + e^{-\Delta}(V+\frac{655}{144})\Big) \\ + (1-g)^2\Big((1-e^{-\Delta})\frac{3}{80}(10+V)^2 + e^{-\Delta}W\Big).$$

Solving for W yields

$$W = \frac{g^2(V+5) + 2g(1-g)\Big((1-e^{-\Delta})(V+\frac{40}{9}) + e^{-\Delta}(V+\frac{655}{144})\Big)}{1-(1-g)^2e^{-\Delta}} \\ + \frac{(1-g)^2(1-e^{-\Delta})\frac{3}{80}(10+V)^2}{1-(1-g)^2e^{-\Delta}},$$

where g is again given by expression (2). Again, social welfare increases in the consumers' willingness to pay for basic quality V. Further, we have seen that with a decreasing Δ, the probability of entering the market decreases too (see Figure 2). A shorter period length, thus, delays entry and extends the monopoly duration during which consumers face higher prices. Especially, when V is low, this effect heavily predominates, and welfare depends negatively on δ and thus positively on Δ.

2.6.2 Comparison

In standard models, we normally expect that multiple entry is more desirable as competition decreases prices and thereby increases consumer surplus. However, we have seen that by introducing a second copycat entrant, consumers must expect a delay in entry. This delay of competition extends the monopoly power of the incumbent and makes consumers pay the monopoly price for longer. It is, thus, natural to ask whether there are situations in which a single copycat entry is more desirable. In the following, we compare the benchmark model (BM) with only one copycat entrant with the situation in which there are two potential copycat entrants.

If there is only one entrant, it is a dominant strategy for the copycat to enter the market in the first period, independent of the period length Δ. The same reasoning as in the proof of Lemma 1 applies. Further, from Section 4 (cf. duopoly outcome), we know that in equilibrium we observe maximal product differentiation. Thus, when there is only one copycat entrant the consumer surplus is given by

$$CS^{BM} = \int_0^\infty e^{-\tau}\Big[\int_0^{\frac{1}{3}} V + \theta\cdot 0 - \frac{10}{3}d\theta + \int_{\frac{1}{3}}^1 V + 10\cdot\theta - \frac{20}{3}d\theta\Big]d\tau = V - \frac{10}{9}$$

and social welfare is

$$W^{BM} = CS^{BM} + \int_0^\infty e^{-\tau}\left[\frac{40}{9} + \frac{10}{9}\right]d\tau = V + \frac{40}{9}.$$

Both, consumer surplus and social welfare have a constant positive rate of change in V.

To analyze under which conditions it is better to have only one entrant, we look at the ratio of social welfare and consumer surplus of one copycat entrant to the welfare of two copycat entrants, i.e. $\frac{CS^{BM}}{CS}$ and $\frac{W^{BM}}{W}$, respectively. Figures 2.3 and 2.4 illustrate these ratios of consumer surplus and social welfare, respectively. For sufficiently large values of Δ and thereby sufficiently small values of δ, the prospect of more variety and stronger competition in the market is more promising for consumers. For longer period lengths, the probability of a copycat entry is higher, and hence the expected waiting period until a first entry is smaller. Therefore, when δ (Δ) isn't too high (low), having a second copycat entrant is the most favorable situation for consumers.

However, as the time length between two successive periods becomes extremely small, i.e. Δ converges to 0 and δ converges to 1, the benchmark model with only one entrant becomes more beneficial to consumers. With Δ converging to 0 and therefore δ converging to 1, we study real-time efficiency and copycat entrants fight more intensively for winning the war of attrition. The copycat firms' probability of entering becomes so small that the disadvantage of a delayed entry outweighs the advantage of having an additional firm fighting to enter the market. As V increases, the threshold of this trade-off decreases, that is the entry of only one copycat is reinforcing the consumer surplus for smaller values of δ.

When looking at social welfare, the effect for rather low values of δ are similar: having a second copycat entrant is better in terms of social welfare. Interestingly, the benefits from an additional entrant decrease with shorter time lengths between periods, but only for lower values of V. As shown in Figure 2.4, for sufficiently large values of V, having two copycat entrants is always more beneficial. The intuition behind this is that when V becomes sufficiently high, social welfare is maximized when observing a monopoly. Thus, having two potential copycats is a favorable situation because it reinforces the war of attrition and delays entry.

The interpretation of this result stems from the fact that when having two copycat

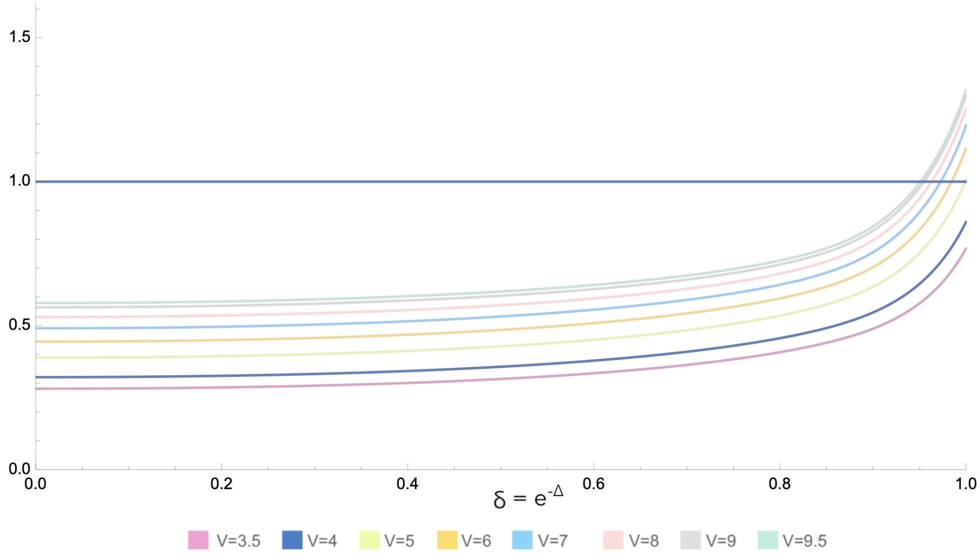

Figure 2.3: Ratio of Consumer Surplus with One Entrant to Two Entrants

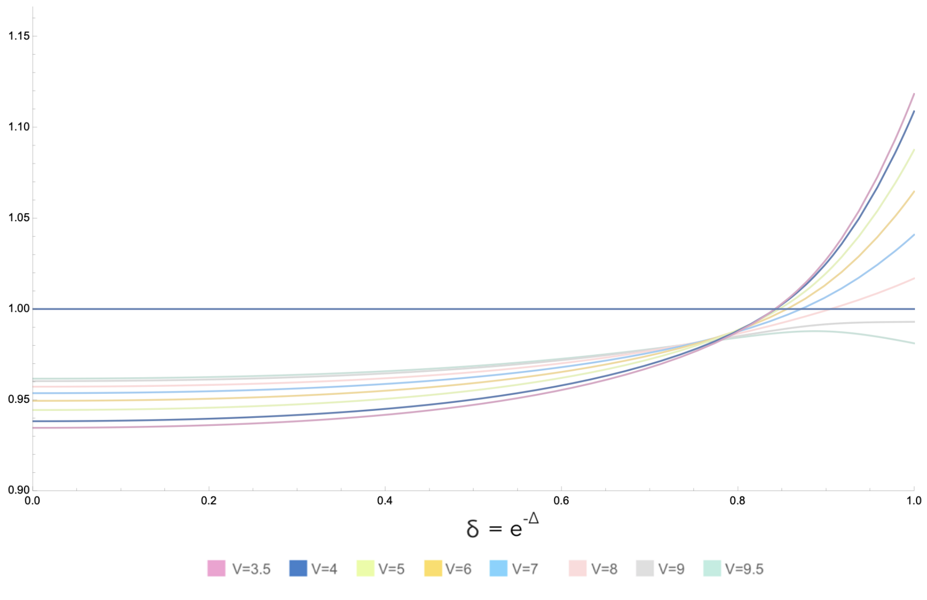

Figure 2.4: Ratio of Welfare with One Entrant to Two Entrants

entrants, first entry may be delayed. Normally, the resulting monopoly period is an undesired outcome. However, when V is sufficiently high, we have shown that it becomes beneficial to no longer have any copycats entering the market. Thus, when assuming high values of V, the effect of potentially having a longer monopoly situation and thereby higher welfare prevails.

To further study real-time efficiency, we determine the thresholds of V for which it is beneficial to only have one copycat entrant.

Proposition 3. *As the length between periods becomes extremely short, i.e.* $\Delta \to 0$, *the ratio of consumer surplus with one entrant to two entrants converges to*

$$\frac{720(9V-10)}{9950+2700V+63V^2}.$$

The threshold value of V for which one copycat entrant becomes more beneficial for consumers, namely for which the ratio is greater than 1, is $V \geq \frac{5}{3}(18-\sqrt{226}) \approx 4.9445$.

As V becomes sufficiently high, the price depressing effect due to higher competition introduced by the second entry is not strong enough to compensate for the delayed entry. This delay emerges from the fact that copycat entrants use a mixed strategy when taking their entry decision. We have shown in Section 2.5 that when the period length becomes extremely short, the expected average time of the monopoly situation converges to $\frac{7}{2}$. During this time period, consumers expect to pay the monopoly price and to face a surplus of CS^m, but miss out on CS^d. Thus, in the period before the first entry is observed, consumers' expected loss in surplus from introducing a second copycat entrant and facing delayed entry is given by

$$\int_0^{\frac{7}{2}} e^{-t}\Big(CS^d - CS^m\Big)dt = \frac{(3V-10)(170-3V)}{720}\left(1-e^{-\frac{7}{2}}\right).$$

Once copycat entrants enter the market, the effect of more competition induces prices to drop and causes consumer surplus to rise from CS^d to CS^c in every period after the second entry. Thus, the entry of a second copycat, and thereby lower prices, results in an additional surplus of

$$\int_{\frac{7}{2}}^{\infty} e^{-t}\Big(CS^c - CS^d\Big)dt = \frac{175}{48}e^{-\frac{7}{2}}$$

in the time after the first entry. Note that we can ignore the period, in which the first copycat enters the market. In both scenarios, we observe the same duopoly situation and consequently, consumers are equally well off. Therefore, when comparing the two cases, the duopoly period cancels out.

Finally, comparing the two effects in the limit case where Δ converges to 0, we find that

$$\frac{(3V-10)(170-3V)}{720}\left(1-e^{-\frac{7}{2}}\right) > \frac{175}{48}e^{-\frac{7}{2}}$$

holds for $V \geq \frac{5}{3}(18-\sqrt{226})$ (see Proposition 3). However, as illustrated in Figure 2.3, for sufficiently small values of δ consumer surplus is monotonically increasing in the value of V but also in the number of entering firms. Then, by assuming that $V > \frac{10}{3}$ we make sure consumer surplus increases with additional copycat entrants.

Figure 2.4 shows that social welfare, on the contrary, is not monotonically increasing in the willingness to pay for basic quality and neither in the number of copycat entries, for any value of δ. Our findings are summarised below.

Proposition 4. *As the length between periods becomes extremely short, i.e. $\Delta \to 0$, the ratio of social welfare with one entrant to two entrants converges to*

$$\frac{720(9V+40)}{25550+5220V+189V^2}.$$

The socially beneficial number of firms in the market then depends on the willingness to pay for a good of basic quality V:

(i) The range of values of V for which one copycat entrant is most socially beneficial is

$$\frac{10}{3} < V < \frac{5}{63}(42+\sqrt{4494}) \approx 8.6538.$$

(ii) Two copycat entrant becomes most socially beneficial for intermediate values of V, i.e.

$$8.6538 \approx \frac{5}{63}(42+\sqrt{4494}) < V < \frac{1}{9}(30+5\sqrt{111}) \approx 9.1865.$$

(iii) A monopoly situation is most socially desirable when V takes sufficiently high values, i.e.

$$V > \frac{1}{9}(30+5\sqrt{111}) \approx 9.1865.$$

When analyzing the model in real-time, we find that the optimal number of copycat entrants depends on the valuation for a good of basic quality V, regardless of whether one considers welfare or consumer surplus. The value of V influences whether the price-depressing effect of an additional entrant or the effect of delayed entry due to copycats' mixed strategies prevails. Surprisingly, and against intuition, in some cases, an additional entrant decreases consumer surplus and social welfare even with zero entry costs. For some very high values of V social welfare is even maximized when the incumbent remains a monopolist.

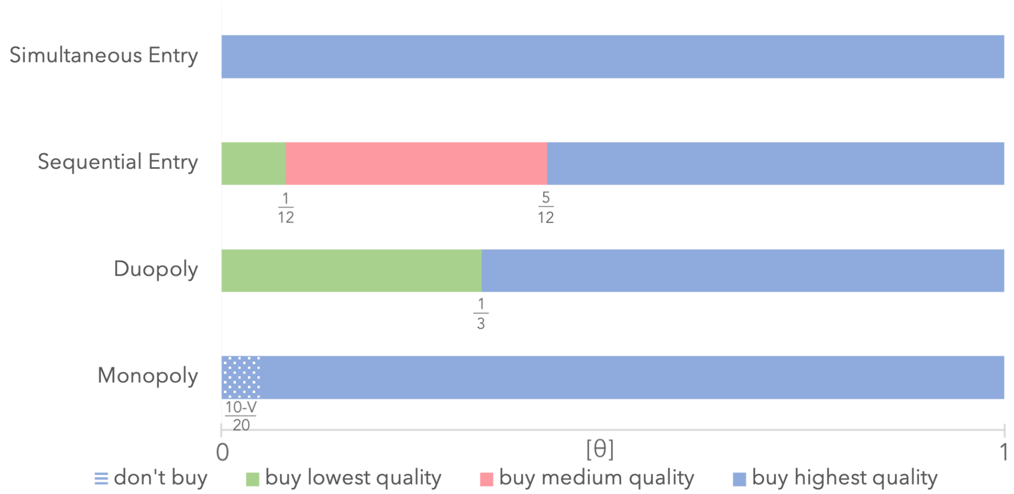

Figure 2.5: Quality Mix

Figure 2.5 shows the intuition behind this and illustrates the quality mix in the different market outcomes. It depicts what fraction of consumers consumes the lowest, the medium, and the highest quality good, and which consumers don't buy in equilibrium. When the willingness to pay V is close to 10, i.e. $\frac{10-V}{20}$ close to zero, almost all consumers benefit from consuming the highest quality good. In this case, a single firm in the market is all it takes to maximize welfare. A launch of a new good on the lower quality spectrum, which we observe in duopoly, would involve a deadweight loss caused by a lower quality mix. Even though an additional firm in the market introduces competition and thereby pushes prices down, entry does not offset the welfare loss. As we assume a covered market, consumers always buy in equilibrium. Hence, when a lower quality good is introduced, consumers with $\theta \in [0, \frac{1}{3}]$ will experience a welfare loss by ending up consuming a lower quality good. However, when V decreases and is closer to $\frac{10}{3}$, a higher fraction of consumers do not buy in equilibrium. In this case, the introduction of a good on the lower quality spectrum is enough to considerably increase consumer surplus and offset the loss in firms' aggregated profits. First, consumers benefit from lower prices due to introduced competition. Second, since, in the duopoly case, firms maximally differentiate in quality, the copycat is facing enough demand from consumers with a lower willingness to pay. These two effects are enough to make up for the loss in the incumbent's profit, and optimally solve the trade-off between lower prices and lower profits when facing more competition.

Lastly, the optimal number of copycat entrants is to be equal to 2 for intermediate values of V. On one side, as the monopoly price becomes relatively cheaper with an increasing V, an intermediate value of V makes sure that consumers do not suffer too much during the expected monopoly period. On the other side, it sees to it that all firms face enough demand, and make enough profits even though price competition becomes more intense.

2.7 Exclusivity Period for First Entrant

An efficient production structure in the underlying setting has to deal with two main problems: First, it must introduce competition to avoid too high market prices for consumers, and second, solve the problem of delayed market entry. In the following, we discuss a potential remedy involving an exclusivity period for the first entrant. In markets where incumbents hold patents, monopoly rents can be secured for a limited amount of time. In the pharmaceutical industry, moreover, it has been asserted that incumbent firms enjoy the absence of generic competitors for an increasing period of time after patent expiration (Grabowski and Kyle, 2007). For this reason, the law of 180-days exclusivity for generic drug applicants was enacted by the the US congress in the 1984 Hatch-Waxman amendments to the Federal Food, Drug, and Cosmetic Act (FDCA) (Center for Drug Evaluation and Research (CDER), 1998). Under current law, the 180-day exclusivity rewards generic companies that take on the risk and costs of challenging patents protecting brand-name drugs of an incumbent. Specifically, the basis of this exclusivity is that the first generic entrant to challenge an incumbent's patent is rewarded with a six months window of exclusivity against subsequent patent challengers (Lietzan and Korn, 2007). The incentive structure has proven to be successful: Generic companies intensely compete with one another to quickly develop more affordable versions of brand-name drugs, and more consumers have access to treatments that previously were unaffordable.

In our analysis, we have shown that even in an off-patent environment, the incumbent enjoys extended monopoly power. Inspired by the above-mentioned policy intervention, we thus analyze exclusivity periods for first entrants. The basis of this policy intervention is to counteract the monopoly situation, fight against the war-of-attrition-features and thereby speed up entry by increasing incentives for entry.

An easy way to model an exclusivity period is to use its length as a policy choice variable. We model the length of the exclusivity period as the additional time after first entry during which no further copycat can enter the market. We denote the length of the exclusivity period by $\phi > 0$. In line with the *multiple first applicant* approach, a shared exclusivity is provided to entrants entering in the same period.[5] According to Center for Drug Evaluation and Research (CDER) (2003), this approach avoids the random aspect of a lottery and prevents conflicts over the question of who was first.

In the underlying model, the exclusivity period only has an effect if copycats enter sequentially, i.e. if $a_i^t = 1$ and $a_j^{t'} = 0$ for all $t' \leq t$ where $i \neq j$. In the event of both copycats entering simultaneously and sharing the exclusivity, firms still price at marginal costs and the socially optimal outcome is achieved. Given the modified game, the indifference condition of an entrant is the following.

$$\begin{aligned}
&(1-g)\Big(\int_0^{\Delta+\phi} \Pi_2^d e^{-\tau} d\tau + \int_{\Delta+\phi}^{\infty} \Pi_2^c e^{-\tau} d\tau\Big) \\
&\quad = g e^{-(\Delta+\phi)} \int_{\Delta+\phi}^{\infty} \Pi_3^c e^{-\tau} d\tau + (1-g)^2 e^{-\Delta}\Big(\int_0^{\Delta+\phi} \Pi_2^d e^{-\tau} d\tau + \int_{\Delta+\phi}^{\infty} \Pi_2^c e^{-\tau} d\tau\Big)
\end{aligned}$$

For simplicity reasons, we neglected the listing of $\Pi^{sim} = 0$. When solving for g we obtain

$$\begin{aligned}
g^{excl} =& \frac{1}{64e^{-\Delta} - 62e^{-(2\Delta+\phi)}} \Big(-32 + 64e^{-\Delta} - 62e^{-2\Delta-\phi} + 31e^{-\Delta-\phi} \\
& -8e^{-2\Delta-2\phi} + \Big[1024 + 64e^{-4\Delta-4\phi} + 992e^{-4\Delta-3\phi} - 496e^{-3\Delta-3\phi} \\
& -1024e^{-3\Delta-2\phi} + 1473e^{-2\Delta-2\phi} - 1984e^{-\Delta-\phi}\Big]^{\frac{1}{2}}
\end{aligned}$$

[5] The *multiple first applicant* approach provides all applicants submitting patent challenges on the same day an opportunity to share an exclusivity (Center for Drug Evaluation and Research (CDER), 2003).

As ϕ converges to infinity, the probability of entering the market converges to one and consumers play a pure strategy in which they enter immediately. Consequently, the probability that at least one firm enters converges to one.

$$\lim_{\phi \to \infty} 1 - (1 - g^{excl})^2 = 1$$

To be consistent with our welfare analysis, we further investigate the real-time efficiency of an exclusivity period, i.e. $\phi > 0$. As depicted in Figures 2.6 and 2.7, we find that consumer surplus as well as social welfare first decrease in ϕ, but then increase until they remain constant as ϕ becomes sufficiently large.

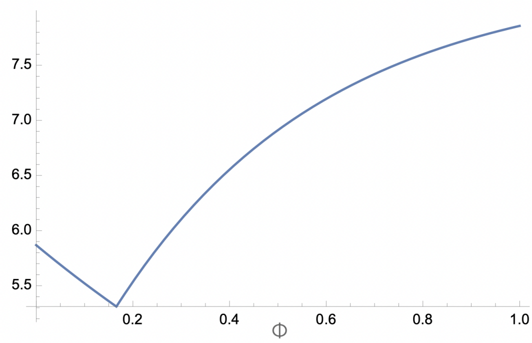

Figure 2.6: Consumer Surplus in the Limit where $\Delta \to 0$ and $V = \frac{10}{3}$

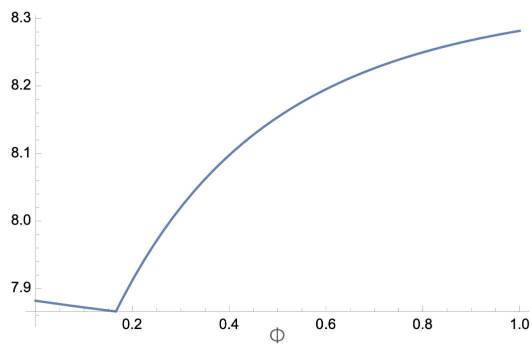

Figure 2.7: Welfare in the Limit where $\Delta \to 0$ and $V = \frac{10}{3}$

To get an intuitive understanding, consider the case where an exclusivity period is introduced. When an exclusivity period ϕ is implemented, it incentivizes potential copycat entrants to increase their probability of entry which accelerates the speed of entry. Consequently, the expected monopoly period decreases and consumers are expected to benefit from more competition and lower prices earlier in time. The larger ϕ grows, the sooner first entry is expected to happen, which is considerably beneficial to consumers. However, a higher ϕ not only introduces competition faster, it also induces an extended duopoly period. This extended duopoly period leaves consumers with higher prices than in a competitive setting with three firms and prevents the second copycat entry to enter the market.

When ϕ takes sufficiently low values, the latter effect prevails and drives down consumer surplus and welfare. Once, the exclusivity period becomes sufficiently large, the former effect becomes stronger and starts to dominate. Is ϕ sufficiently large, consumer welfare and social welfare are increasing in the length of the exclusivity period. The increase in surplus is diminishing and becomes constant once ϕ becomes very large. Note that, both, consumer surplus and social welfare, are monotonically increasing in the willingness to pay for basic quality V.

2.8 Conclusion

This paper studies the strategic behavior of copycat firms wanting to enter a market. We show that they do not have any incentives to enter the market first. While entering first gives the copycat the possibility to secure positive profits from the first period on, it also reduces the profits in the long-run as the second entrant makes higher profits. In equilibrium, this trade-off results in mixed strategies in the entry stage. We find, surprisingly, that even when entry is free, copycats delay their entry and the incumbent benefits from an extended monopoly situation. Monopoly prices induce consumers with lower valuations to not buy until prices are driven down by introduced competition. In the limit, where time between periods converges to zero, it turns out that for sufficiently low values of consumers' willingness to pay it is socially desirable to restrict competition and only have one entrant. For sufficiently high values of consumers' willingness to pay, social welfare is maximized in the monopoly situation. Thus, against intuition, multiple entry and thereby more competition is not always socially favorable. Furthermore, we discuss the adoption of potential remedies to overcome the issues of delayed market entry and too light competition. We find that the introduction of a positive exclusivity period has a positive effect on welfare when the exclusivity period is sufficiently long. Is the exclusivity period too short, we observe a dampened effect on welfare. The negative effect of the extended duopoly period, namely paying higher prices for consumers and missing out on profit for the second copycat entrant, prevails the positive effects of an exclusivity period.

Limitations and further research
Several limitations in this paper open opportunities for further research. First, we use the standard vertical differentiation framework of (Mussa and Rosen, 1978), in which the consumption value of each consumer is a multiplicative factor of its taste for quality by the good's quality. We, therefore, disregard the possibility of consumers assigning intrinsic value to the good. We also assume that all consumers have the same willingness to pay. Allowing for consumers to have uniformly distributed willingness to pay would account for an additional dimension with potential interesting findings.

Further, in our approach, we assume a uniform distribution of consumer's taste. Unfortunately, the vast vertical differentiation literature does not offer a clear characterization of oligopoly outcomes for other taste distributions. However, we conjecture that if relatively more consumers have taste for intermediate quality, the incentives to enter last increase and so the probability to stay out for one more period. The negative welfare effect of additional competition would, thus, be intensified. Still, this remains an open issue.

Lastly, in our vertical differentiation model, symmetry is an important component. We assume the same constant marginal costs of developing technologies of firms and focus on symmetric equilibria. Allowing for different marginal costs depending linearly on quality and asymmetric equilibria offers potential for future research.

2.9 Appendix

Proof of Lemma 1

Assume that in period t, besides the incumbent, only firm 2 is active, that is firm 3 has not yet entered the market. From Lemma 1 we know that firm 3 will enter in the subsequent period $t+1$. Thus, when maximizing her intertemporal profit, firm 2 is taking into account the entry behavior of firm 3, the reaction function of firm 3 as well as the pricing strategies in the duopoly situation (p_1^d, p_2^d) and in the case of competition (p_1^c, p_2^c, p_3^c), with

$$\begin{aligned}
p_1^d(q_2) &= \frac{2(10-q_2)}{3} \\
p_2^d(q_2) &= \frac{10-q_2}{3} \\
p_1^c(q_2,q_3) &= \frac{(10-q_3)(30-4q_2+q_3)}{6(10-q_2)} \\
p_2^c(q_2,q_3) &= \frac{(10-q_3)(q_3-q_2)}{6(10-q_2)} \\
p_3^c(q_2,q_3) &= \frac{(10-q_3)(q_3-q_2)}{3(10-q_2)}
\end{aligned}$$

Using these, the intertemporal maximization problem becomes

$$\max_{q_2^c} \int_0^\Delta p_2^d \left(\frac{p_1^d - p_2^d}{10-q_2^c}\right) e^{-\tau} d\tau + \int_\Delta^\infty p_2^c \left(\frac{p_3^c - p_2^c}{q_3^c - q_2^c}\right) e^{-\tau} d\tau$$

$$\max_{q_2^c} \int_0^\Delta \frac{10-q_2^c}{3} \left(\frac{\frac{2(10-q_2^c)}{3} - \frac{10-q_2^c}{3}}{10-q_2^c}\right) e^{-\tau} d\tau$$

$$+ \int_\Delta^\infty \frac{(10-q_3^c)(q_3^c-q_2^c)}{6(10-q_2^c)} \left(\frac{\frac{(10-q_3^c)(q_3^c-q_2^c)}{3(10-q_2^c)} - \frac{(10-q_3^c)(q_3^c-q_2^c)}{6(10-q_2^c)}}{q_3^c - q_2^c}\right) e^{-\tau} d\tau$$

$$\max_{q_2^c} \int_0^\Delta \frac{10-q_2^c}{3} \left(\frac{\frac{10-q_2^c}{3}}{10-q_2^c}\right) e^{-\tau} d\tau + \int_\Delta^\infty \frac{(10-q_3^c)(q_3^c-q_2^c)}{6(10-q_2^c)} \left(\frac{\frac{(10-q_3^c)(q_3^c-q_2^c)}{6(10-q_2^c)}}{q_3^c-q_2^c}\right) e^{-\tau} d\tau$$

$$\max_{q_2^c} \int_0^\Delta \frac{10-q_2^c}{9} e^{-\tau} d\tau + \int_\Delta^\infty \frac{(10-q_3^c)^2 (q_3^c-q_2^c)}{36\,(10-q_2^c)^2} e^{-\tau} d\tau$$

Using firm 3's reaction function regarding the quality choice $q_3^c(q_2^c) = \frac{10+q_2^c}{2}$, we get

$$\max_{q_2^c} \int_0^\Delta \frac{10-q_2^c}{9} e^{-\tau} d\tau + \int_\Delta^\infty \frac{\left(10 - \frac{10+q_2^c}{2}\right)^2 \left(\frac{10+q_2^c}{2} - q_2^c\right)}{36\,(10-q_2^c)^2} e^{-\tau} d\tau$$

$$\max_{q_2^c} \int_0^{\Delta} \frac{10 - q_2^c}{9} e^{-\tau} d\tau + \int_{\Delta}^{\infty} \frac{\left(\frac{10-q_2^c}{2}\right)^3}{36\left(10 - q_2^c\right)^2} e^{-\tau} d\tau$$
$$\max_{q_2^c} \int_0^{\Delta} \frac{10 - q_2^c}{9} e^{-\tau} d\tau + \int_{\Delta}^{\infty} \frac{10 - q_2^c}{288} e^{-\tau} d\tau$$

Since firm 2's intertemporal profit is depending negatively on her quality, she will go for the lowest quality possible and locates at the bottom of the quality spectrum.

Proof of Proposition 1

The obtained social welfare values in the different scenarios are given by

$$\begin{aligned} W^m &= \frac{3}{80}\left(10 + V\right)^2 \\ W^d &= V + \frac{40}{9} \\ W^c &= V + \frac{655}{144} \\ W^{sim} &= V + 5. \end{aligned}$$

Independent of the value V, we have that $W^d < W^c < W^{sim}$. How W^m ranks in the comparison, on the other hand, depends on V.

(i) First, we observe $W^m < W^d$ if

$$\begin{aligned} W^m = \frac{3}{80}\left(10 + V\right)^2 &< V + \frac{40}{9} = W^d \\ \frac{3}{80}\left(100 + 20V + V^2\right) &< V + \frac{40}{9} \\ \frac{3}{80}(100 + V^2) + \frac{3}{4}V &< V + \frac{40}{9} \\ \frac{30}{8} + \frac{3}{80}V^2 - \frac{1}{4}V - \frac{40}{9} &< 0 \\ \frac{3}{80}V^2 - \frac{1}{4}V - \frac{25}{36} &< 0 \end{aligned}$$

This is a quadratic inequality, and we can solve it by finding the roots of the corresponding equation. Setting the quadratic equal to zero, we get

$$\frac{3}{80}V^2 - \frac{1}{4}V - \frac{25}{36} = 0.$$

Hence, the roots are $\frac{1}{9}\left(30 - 20\sqrt{6}\right) \approx -2.1100$ and $\frac{1}{9}\left(30 + 20\sqrt{6}\right) \approx 8.7766$. Thus, for $V < \frac{1}{9}\left(30 + 20\sqrt{6}\right) \approx 8.7766$ the rank order of social welfare is $W^m < W^d < W^c < W^{sim}$.

(ii) Second, we observe $W^m < W^c$ if

$$\begin{aligned}
W^m = \frac{3}{80}(10+V)^2 &< V + \frac{655}{144} = W^c \\
\frac{3}{80}\left(100+20V+V^2\right) &< V + \frac{655}{144} \\
\frac{3}{80}(100+V^2) + \frac{3}{4}V &< V + \frac{655}{144} \\
\frac{30}{8} + \frac{3}{80}V^2 - \frac{1}{4}V - \frac{655}{144} &< 0 \\
\frac{3}{80}V^2 - \frac{1}{4}V - \frac{115}{144} &< 0
\end{aligned}$$

This is a quadratic inequality, and we can solve it by finding the roots of the corresponding equation. Setting the quadratic equal to zero, we get

$$\frac{3}{80}V^2 - \frac{1}{4}V - \frac{115}{144} = 0.$$

Hence, the roots are $\frac{1}{9}\left(30+5\sqrt{105}\right) \approx 9.0261$ and $\frac{1}{9}\left(30-5\sqrt{105}\right) \approx -2.3594$. Thus, combined with the results from (i), for $8.7766 \approx \frac{1}{9}\left(30+20\sqrt{6}\right) < V < \frac{1}{9}\left(30+5\sqrt{105}\right) \approx 9.0261$the rank order of social welfare is $W^d < W^m < W^c < W^{sim}$.

(iii) Third, we observe $W^m < W^{sim}$ if

$$\begin{aligned}
W^m = \frac{3}{80}(10+V)^2 &< V + 5 = W^{sim} \\
\frac{3}{80}\left(100+20V+V^2\right) &< V + 5 \\
\frac{3}{80}(100+V^2) + \frac{3}{4}V &< V + 5 \\
\frac{30}{8} + \frac{3}{80}V^2 - \frac{1}{4}V - 5 &< 0 \\
\frac{3}{80}V^2 - \frac{1}{4}V - \frac{5}{4} &< 0
\end{aligned}$$

This is a quadratic inequality, and we can solve it by finding the roots of the corresponding equation. Setting the quadratic equal to zero, we get

$$\frac{3}{80}V^2 - \frac{1}{4}V - \frac{5}{4} = 0.$$

Hence, the roots are $V = 10$ and $V = -\frac{10}{3} \approx -3.3333$. Thus, combined with the results from (ii), for $9.0261 \approx \frac{1}{9}\left(30+5\sqrt{105}\right) < V < 10$ the rank order of social welfare is $W^d < W^c < W^m < W^{sim}$.

(iv) Finally, given (iii) for $V = 10$, the rank order of social welfare is $W^d < W^c < W^m = W^{sim}$.

Proof of Proposition 2

$$\begin{aligned}(1-g)\Big(\int_0^{\Delta}\Pi_2^d e^{-\tau}d\tau + \int_{\Delta}^{\infty}\Pi_2^c e^{-\tau}d\tau\Big) + g\int_0^{\infty}\Pi^{sim}e^{-\tau}d\tau \\ = ge^{-\Delta}\int_{\Delta}^{\infty}\Pi_3^c e^{-\tau}d\tau + (1-g)e^{-\Delta}\left((1-g)\left(\int_0^{\Delta}\Pi_2^d e^{-\tau}d\tau\right.\right. \\ \left.\left. + \int_{\Delta}^{\infty}\Pi_2^c e^{-r\tau}d\tau\right) + g\int_0^{\infty}\Pi^{sim}e^{-\tau}d\tau\right)\end{aligned}$$

For simplicity reasons, define $a = \int_0^{\Delta}\Pi_2^d e^{-\tau}d\tau + \int_{\Delta}^{\infty}\Pi_2^c e^{-\tau}d\tau$ and $b = \int_{\Delta}^{\infty}\Pi_3^c e^{-\tau}d\tau$. Thus, we have

$$(1-g)a = ge^{-\Delta}b + (1-g)^2 e^{-\Delta}a$$

or with $\delta = e^{-\Delta}$ that

$$(1-g)a = g\delta b + (1-g)^2\delta a.$$

Solving for g yields

$$g = -\frac{a - 2\delta a + \delta b - \sqrt{a^2 + 2\delta ab - 4\delta^2 ab + \delta^2 b^2}}{2\delta a}$$

Using $\Pi_2^d = \frac{10}{9}$, $\Pi_2^c = \frac{5}{144}$, $\Pi_3^c = \frac{5}{18}$ and $\Pi^{sim} = 0$ we have

$$\begin{aligned}a &= \frac{10}{9}\left(1 - e^{-\Delta}\right) + \frac{5}{144}e^{-\Delta} = \frac{10}{9}(1-\delta) + \frac{5}{144}\delta \\ b &= \frac{5}{18}e^{-\Delta} = \frac{5}{18}\delta\end{aligned}$$

and thus

$$\begin{aligned}g = -\frac{1}{2\delta\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right)}\Bigg(\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right) - 2\delta\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right) + \delta\frac{5}{18}\delta \\ - \left[\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right)^2 + 2\delta\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right)\frac{5}{18}\delta\right. \\ \left. -4\delta^2\left(\frac{10}{9}(1-\delta) + \frac{5}{144}\delta\right)\frac{5}{18}\delta + \delta^2\left(\frac{5}{18}\delta\right)^2\right]^{\frac{1}{2}}\end{aligned}$$

$$g = -\frac{1}{2\delta\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right)}\left(\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right) - 2\delta\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right) + \delta^2\frac{5}{18}\right.$$
$$-\left[\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right)^2 + 2\delta\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right)\frac{5}{18}\delta\right.$$
$$\left.\left. -4\delta^2\left(\frac{10}{9}(1-\delta)+\frac{5}{144}\delta\right)\frac{5}{18}\delta + \delta^2\left(\frac{5}{18}\delta\right)^2\right]^{\frac{1}{2}}\right)$$

$$g = -\frac{1}{\delta\left(\frac{20}{9}-\frac{155}{72}\delta\right)}\left(\left(\frac{10}{9}-\frac{155}{144}\delta\right) - 2\delta\left(\frac{10}{9}-\frac{155}{144}\delta\right) + \delta^2\frac{5}{18}\right.$$
$$\left. -\sqrt{\left(\frac{10}{9}-\frac{155}{144}\delta\right)^2 + 2\delta\left(\frac{10}{9}-\frac{155}{144}\delta\right)\frac{5}{18}\delta - 4\delta^2\left(\frac{10}{9}-\frac{155}{144}\delta\right)\frac{5}{18}\delta + \delta^2\left(\frac{5}{18}\delta\right)^2}\right)$$

$$g = -\frac{1}{\delta\left(\frac{20}{9}-\frac{155}{72}\delta\right)}\left(\frac{10}{9}-\frac{155}{144}\delta-\frac{20}{9}\delta+\frac{155}{72}\delta^2+\frac{5}{18}\delta^2\right.$$
$$\left. -\sqrt{\frac{24025}{20736}\delta^2-\frac{775}{324}\delta+\frac{100}{81}+\frac{50}{81}\delta^2-\frac{775}{1296}\delta^3+\frac{775}{648}\delta^4-\frac{100}{81}\delta^3+\frac{25}{324}\delta^4}\right)$$

$$g = -\frac{1}{\delta\left(\frac{20}{9}-\frac{155}{72}\delta\right)}\left(\frac{10}{9}-\frac{475}{144}\delta+\frac{175}{72}\delta^2-\sqrt{\frac{100}{81}-\frac{775}{324}\delta+\frac{12275}{6912}\delta^2-\frac{2375}{1296}\delta^3+\frac{275}{216}\delta^4}\right)$$

$$g = -\frac{1}{\delta\left(\frac{4}{9}-\frac{31}{72}\delta\right)}\left(\frac{2}{9}-\frac{95}{144}\delta+\frac{35}{72}\delta^2-\sqrt{\frac{4}{81}-\frac{31}{324}\delta+\frac{491}{6912}\delta^2-\frac{95}{1296}\delta^3+\frac{11}{216}\delta^4}\right)$$

$$g = -\frac{1}{\delta\left(4-\frac{31}{8}\delta\right)}\left(2-\frac{95}{16}\delta+\frac{35}{8}\delta^2-\sqrt{4-\frac{31}{4}\delta+\frac{1473}{256}\delta^2-\frac{95}{16}\delta^3+\frac{33}{8}\delta^4}\right)$$

Finally, we get

$$g = \frac{-32+95\delta-70\delta^2+\sqrt{1024-1984\delta+1473\delta^2-1520\delta^3+1056\delta^4}}{\delta(64-62\delta)}$$

Proof that entry process converges to a Poisson process

The decision to enter the market is per unit time and happens completely at random. At $t = 0$ we don't observe an entry from potential entrants yet, so $N(0) = 0$. We then divide the period $(0, t]$ to tiny subintervals of length Δ. Thus, there are $n \approx \frac{t}{\Delta}$ periods. In each period, copycat firms decide with a probability g to enter the market, and with a probability $1-g$ to stay out another period. Thus, $N(t) \sim Binomial(n, g)$, where

$$g = \frac{-32 + 95e^{-\Delta} - 70e^{-2\Delta} + \sqrt{1024 - 1984e^{-\Delta} + 1473e^{-2\Delta} - 1520e^{-3\Delta} + 1056e^{-4\Delta}}}{e^{-\Delta}(64 - 62e^{-\Delta})}.$$

Note that the probability of entering takes the form $g = \lambda\Delta$, so

$$\begin{aligned} ng &= n\lambda\Delta \\ &= \frac{t}{\Delta}\lambda\Delta \\ &= \lambda t. \end{aligned}$$

Thus, by the Poisson limit theorem we have that $N(t)$ converges to a $Poisson(\lambda t)$ as $n \to \infty$, that is as $\Delta \to 0$.

Proof of Corollary 1

We know if no entry occurred in the periods before, with a probability of $2g(1-g) + g^2 = 1-(1-g)^2$ the game ends up in a state with at least one entry. The incumbent can, therefore, expect to keep her monopoly position beyond t for $(1-(1-g)^2)^{-1}$ periods, before eventually one or both copycat firms enter the market. To get the real-time monopoly length, i.e. the real-time length of no copycat entering, the expected monopoly period needs to be multiplied by the real-time length, i.e. $\Delta(1-(1-g)^2)^{-1}$.

Proof of Proposition 3

$$\lim_{\Delta \to 0} \frac{CS}{CS^{BM}} = \frac{720(9V-10)}{9950 + 2700V + 63V^2}$$

Thus, $CS > CS^{BM}$ if the above expression is greater than 1, i.e.

$$\frac{720(9V-10)}{9950 + 2700V + 63V^2} > 1$$

This is the case for $V \geq \frac{5}{3}(18 - \sqrt{226}) \approx 4.9445$

Proof of Proposition 4

The social welfare for the monopoly case, duopoly case and competition case with 2 entrants in the limiting case where $\Delta \to 0$ are given by

$$\begin{aligned}
\lim_{\Delta\to 0} \quad & \int_0^\infty W^m e^{-t} dt = \frac{3}{80}(10+V)^2 \\
\lim_{\Delta\to 0} \quad & W^{BM} = (V + \frac{40}{9}) \\
\lim_{\Delta\to 0} \quad & W = \frac{25550 + 5220V + 189V^2}{6480},
\end{aligned}$$

respectively. Thus, one copycat entrant is most socially beneficial if

$$\frac{25550 + 5220V + 189V^2}{6480} < (V + \frac{40}{9})$$

which holds if

$$\lim_{\Delta\to 0} \frac{W}{W^{BM}} = \frac{720(9V+40)}{25550 + 5220V + 189V^2} > 1$$

This is the case for values of V which satisfy

$$\frac{10}{3} < V < \frac{5}{63}(42 + \sqrt{4494}) \approx 8.6538.$$

A monopoly situation is most socially desirable when

$$\frac{25550 + 5220V + 189V^2}{6480} < \frac{3}{80}(10+V)^2$$

which is holds for values of V which satisfy

$$V > \frac{1}{9}(30 + 5\sqrt{111}) \approx 9.1865.$$

Therefore, finally, two copycat entrants are most socially beneficial when

$$V + \frac{40}{9} < \frac{25550 + 5220V + 189V^2}{6480}$$

and

$$\frac{25550 + 5220V + 189V^2}{6480r} < \frac{3}{80}\frac{(10+V)^2}{r}$$

This is the case for values of V which satisfy

$$8.6538 \approx \frac{5}{63}(42 + \sqrt{4494}) < V < \frac{1}{9}(30 + 5\sqrt{111}) \approx 9.1865.$$

Chapter 3

The Price of Banning Commissions

Abstract This paper analyzes the welfare impact of switching from a commission-based to a fee-based remuneration model in markets where consumers rely on expert advice when choosing between two different products. Starting from the baseline scenario with commissions, I ban commissions paid by producers. Instead, I introduce a fixed transaction fee which is paid by consumers when buying a product. I show that the number of consumers buying the good in equilibrium decreases. Depending on the social preferences of the intermediary advisor, prices might increase or decrease under a new regime. Further, I show that banning commissions is not welfare-enhancing, even when the intermediary has the possibility to impose a transaction fee.

3.1 Introduction

Today, consumers face an overwhelming task when choosing an investment portfolio or medical treatment. Due to a high degree of complexity and the variety of products available consumers find it difficult to assess which product best suits them (Lusardi and Mitchell, 2011; Atkinson and Messy, 2012). To overcome their poor literacy, consumers commonly seek professional assistance and turn to intermediary advisors, which due to their expertise and strategic advantages support consumers in their decision-making (Judge, 2015). In the financial industry, intermediary advisors take a notable role when consumers make important financial decisions, such as saving for their retirement or taking out a life insurance policy (BEUC, 2019). Naturally, intermediaries are also observed in the medical industry, where physicians advise on treatments or drugs, and in markets for services with credence goods such as legal or repair services (Darby and Karni, 1973). Independent of the sector, the primary role of an intermediary is to reduce prevailing information asymmetries and support consumers in making the right choice.

In this service landscape, consumers usually do not pay directly for advice. Instead, a commission culture prevails where advisors are compensated through commissions. Channeled by producing firms to intermediaries these payments are conditional on the sale of their respective products. Concerning the medical industry, Rodwin (1995) writes: *"The medical care system has become a competitive, revenue-seeking industry in which many physicians have an economic interest that goes beyond their personal services. [...] The physician [...] has become the target of all kinds of financial arrangements designed to influence his recommendations [...]."* These financially motivated interests unquestionably affect a doctor's decision and ultimately might have problematic consequences on a patient's welfare. In the financial industry, the influence of commissions on the recommendation of advisors is evident as commissions account for a rather large proportion of the revenue for intermediary advisors (Fox, 2017; Finansinspektion, 2016). The Australian Securities and Investment Commission further stated that *"[t]he commission-based salary structures created an incentive for representatives to emphasize [...] a culture in which the best interests and appropriate advice duties were more likely to be overlooked"*.[1] It thus stands to reason that in the presence of such commission structures, the incentives of the intermediary and consumers may not necessarily be aligned (BEUC, 2019). As a reaction, regulators in different countries recommended an end to payments from financial product providers to financial advisors.[2] Ultimately, these policies should help to ensure that intermediaries give objective advice to their customers (Fox, 2017).

[1] Federal Court of Australia (2017): Australian Securities and Investments Commission v NSG Services Pty Ltd, in the matter of NSG Services Pty Ltd [2017] FCA 345.

[2] In 2013, the Netherlands introduced a ban on commissions on complex financial products (de Jong, 2018; Kramer, 2018). The United Kingdom implemented a ban on commissions for retail investment advice at the end of 2012 (The Financial Services Authority 2011). Australia introduced a prospective ban on conflicted remuneration structures, including commissions and volume-based payments in June 2012 (Batten and Pearson, 2014; ASIC, 2012).

Motivated by these regulations, I build on a simple model of intermediation activity with commissions paid by product providers. Starting from a pre-policy situation where commissions are not regulated, I use a theoretical framework to analyze the impact of a policy that forbids these payments. In doing so, I can analyze the welfare effects of switching from a commission-based to a fee-based remuneration model. The questions I am addressing are the following: (i) How does a ban on commissions influence the upstream firms' pricing behavior? (ii) How is the number of consumers buying in equilibrium affected when banning commissions and introducing transaction fees? (iii) Is a welfare improvement observed? (iv) What role do non-financial motivators play?

To answer these questions, I consider a model with two firms selling horizontally differentiated goods, a single intermediary, and a mass one of heterogeneous consumers. The two firms set the prices of their respective goods and sell them through an intermediary to consumers. This intermediary has private information about the suitability of products available to consumers and holds other-regarding preferences which make him care about a product's suitability. Consumers seek advice from the intermediary to get a recommendation on which product to buy. In the commission-based model, the intermediary receives commissions for each recommended purchase. When introducing a fee-based policy, these payments are banned, but the intermediary sets a transaction fee targeting consumers who purchase a recommended good.

The most closely related studies are Inderst and Ottaviani (2012a) and Schuler (2020). Inderst and Ottaviani (2012a) investigate a market with a single intermediary and horizontally differentiated product providers.They develop several foundations for the intermediary's concern for suitability, one in which the intermediary is directly concerned about a consumer's well-being. They specify passive beliefs implying that consumers do not react to prices by changing their expectations about firms' unobserved commissions. The authors compare two situations: the baseline scenario in which commissions are allowed, but not observed by consumers, and the policy scenario in which commissions are disclosed. They find that disclosure leads to a decrease in commissions paid to the intermediary, but that the impact of disclosure on welfare is ambiguous and depends on the concern for product suitability. Schuler (2020) analyzes a market with a single intermediary who advises consumers about available choices from product providers. He assumes that the intermediary cares for a suitable recommendation through supervision, which is independent of the consumer's utility. He further assumes that consumers hold naive beliefs that commissions for both firms are zero. Consumers, thus, expect that the quality of the intermediary's recommendation is not affected in any way. In contrast to Inderst and Ottaviani (2012a), he assumes a downward-sloping demand and thereby introduces heterogeneity of consumers. Schuler (2020), then, allows for a policy that caps commission. Choosing a symmetric setting, he shows that a simple ban on commissions affects total welfare only through prices and that a general ban is not welfare-enhancing.

As in Inderst and Ottaviani (2012a) and Schuler (2020), I capture the belief that the intermediary is acting in consumers' best interest by assuming that consumers are naive. In addition, I introduce a moral constraint affecting the advisor's utility directly when recommending a product. The preference for product suitability may result from a host of factors, referring to concerns over good reputation (Judge, 2015), recommendation actions to establish or maintain a relationship with a customer (Bolton and Chen, 2018), or genuine concerns about the well-being of a consumer. Receiving commissions, the intermediary, therefore, has the characteristics of a homo moralis who is torn between selfishness and morality (Alger and Weibull, 2013).

Unlike Inderst and Ottaviani (2012a), who capture the concern for product suitability by assigning some weight to the respective realization of the consumer's utility from consuming the suitable good, I introduce a general function that allows a more universal analysis of preferences. To examine different degrees, ranging from an entirely caring comportment to completely selfish behavior (homo oeconomicus), I capture the intermediary's concern for suitability in a flexible way. By imposing that the intermediary obtains some utility from recommending truthfully even when the consumer doesn't buy the product, I capture the authenticity of concern, that is, no matter what the consumer decides, the intermediary experiences some satisfaction by knowing he acted in the consumer's best interest. The scope of an intermediary's concern for product suitability clearly has an effect on the firms' incentives to increase commissions and takes an interesting aspect when comparing the benchmark scenario and the post-regulation scenario where commissions are prohibited.

In the commission-based model, which is a sub-case of Schuler (2020), the intermediary receives commissions from firms for each recommended purchase. Therefore, the intermediary faces a trade-off between the commissions he receives for a recommended purchase and a bad conscience from a potentially unsuitable sale. Depending on the intermediary's concern for his customers, this trade-off goes in one direction or the other. Then, I introduce a policy that bans commissions. In this scenario, a so-called “fee-only" advisor is paid a set rate for the products he sells rather than getting paid by commissions. By putting a price on his recommendation service, the advisor asks consumers to pay a transaction fee when buying a product.

Banning commissions and instead introducing transaction fees in a model with two product providers, an intermediary who holds social preferences, and heterogeneous consumers is novel to this literature. Comparing the fee-based model to a post-policy situation with commissions represents the primary departure from previous literature. This comparison allows me to investigate the impact of switching from a commission-based to a fee-based payment structure with the objective of deriving an extensive picture of its consequences. Clearly, when banning commissions, the intermediary loses his main source of income, though is no longer steered. Commission bans thus not only target the transparency of commissions but dampen the concerns that commissions lead to biased advice. Thus, under the removal of commission structures and the prevention of any steering behavior, consumers should be provided with greater quality of advice. Judged by this effect alone, banning commissions should account for an increase in consumer surplus. This effect, however, must be weighed up against the additional charges from the transaction fee and a change in the pricing behavior of firms. To abstract from biased advice and focus solely on the effect of prices, I choose a symmetric setting where firms face identical marginal costs. By assuming symmetry, firms choose identical commissions in equilibrium, which leaves the intermediary evenly influenced. Therefore, commissions do not impact the decision behavior of the intermediary but affect consumer surplus and total welfare through the price level, which allows me to disentangle the distortive effect of commissions.

I find that prices are higher in the fee-based model when consumers' utility from product suitability is low. In contrast, prices are higher in the commission-based model when consumers' utility from a suitable product is high. The pricing behavior of firms thus depends on the utility a consumer derives when buying the suitable product. The intuition behind this is the following. In the commission-based model, a consumer's utility from consuming a suitable product has a two-folded effect on prices. The first effect is straightforward: prices increase with consumers' willingness to pay. The second effect happens through commissions. As a consumer's utility from product suitability increases, the intermediary's concern for suitability becomes more pronounced. Firms, thus, have to pay higher commissions to be able to steer the intermediary's recommendation behavior. As a reaction, firms increase prices even more to compensate for higher commissions. In the fee-based model, however, firms know that the willingness to pay not only pushes up prices but also the transaction fee. Thus, to maximize the probability of the good being sold the price increase is rather reluctant. Consequently, when the consumers' utility from consuming the suitable good is sufficiently large the prices are higher in the commission-based model.

When looking at the equilibrium demand, the results are less ambiguous. My main result suggests that introducing a ban on commissions clearly decreases the share of consumers buying in equilibrium. This finding implies that the willingness to get advice is considerably lower under the new policy. Under the new regime, consumers not only have to pay the price of the product but also face a cost in form of a transaction fee. The total cost a consumer is facing when commissions are banned is weakly higher than in the commission-based model. Therefore, banning commissions results in a decrease in consumer surplus.

Next, I find that the effect of the policy intervention on the payoffs of firms and intermediary is ambiguous and depends on the social preferences of the intermediary. Only if the intermediary acts completely selfishly and merely takes into account financial incentives, the effects are clear. In the extreme case where the intermediary does not care about the well-being of his customers, firms are better off in the fee-based policy model. Being completely selfish, the intermediary, on the contrary, prefers the commission-based model. When the intermediary is not concerned about the suitability of his recommendation, firms' incentives to raise commissions are enhanced. In line with the findings in Inderst and Ottaviani (2012a) this is, first, due to the intermediary's amplified responsiveness to commissions and, second, because commissions are strategic complements. An increase in one firm's commission encourages an increase in the other firm's commission, to a point where both firms make zero profits. Firms, therefore, prefer the fee-based policy model, whereas for the same reason, the intermediary prefers the commission-based model.

Finally, when looking at the implications on social welfare, I find that society is worse off under a regime that bans commissions. This finding is independent of the intermediary's concern about suitability and implies that the role of social preferences does not influence the effect of such policies on social welfare. Similar to (Schuler, 2020), I can show that a ban on commissions is not welfare increasing, even when introducing transaction fees.

This study contributes to the growing economic literature that studies the existence and consequences of intermediary advisors and relates mainly to two strands of research. First, it closely relates to studies that investigate intermediaries who direct consumers to products of firms from which they receive compensations in form of commissions and (hidden) kickbacks. In this context, Inderst and Ottaviani (2012a) provide a theoretical framework that combines compensations for an intermediary with the quality of advice. They focus on commissions that are paid to the advisor and their responsiveness to these supply-side incentives. They find that in equilibrium firms have incentives to influence the recommendations made by the intermediary by increasing commission payments. This behavior is called "steering" and leads to distorted advice, as product compatibility is often neglected. Also, (Armstrong and Zhou, 2011) find that intermediaries are tempted to recommend products that primarily put themselves in a favorable position. De Corniere and Taylor (2019) look at a similar market structure, where the intermediary is integrated with one of the sellers. The intermediary adviser, therefore, has an incentive to bias his advice in favor of his own product offering. Schuler (2020) focuses on the advice behavior of an intermediary when introducing downward-sloping demand. He also turns to regulations and analyzes the welfare implications of capping on commissions.

This brings me to the second strand of literature to which this article is related, namely the literature on policies involving bans on commissions. Schuler (2020) shows that a general ban on commissions is not welfare-maximizing. He, however, neglects the fact that intermediaries are no longer remunerated by product providers and need instead another source of income, namely a separate fee for the cost of advice to the consumer. Considering these fee-based remunerations, different economists have been involved in empirically investigating the impact of a commission ban on financial advice-seeking taking (Kramer, 2018; de Jong, 2018). Inderst and Ottaviani (2012c) look at the concept of fees in a theoretical manner. However, analyzing the switch from a commission-based to a fee-based remuneration model as in the underlying model is yet new to the literature.

This study casts new light on the consequences of banning commissions and introducing transaction fees on the number of advice-seeking consumers who buy in equilibrium. In line with the general apprehensions, the implementation of a fee-based policy model decreases the share of consumers proceeding with a purchase after getting a recommendation. Furthermore, the introduction of an intermediary with social preferences offers valuable insights into how prices change, depending on the concern for suitability.

This paper proceeds by formulating the theoretical model in Section 2. Section 3 then characterizes the baseline scenario with commissions, while section 4 analyzes the regime where commissions are banned and transaction fees are introduced. In Section 5, the two regimes are compared and welfare implications are studied. Section 6 provides some final conclusions and directions for future work. All proofs can be found in the Appendix.

3.2 The Model

The model consists of a vertical contracting market in which two firms, $i \in \{A, B\}$ are competing with each other in prices p_i and commissions f_i. Prices are set directly for consumers, meaning that firms are deciding on the price a consumer pays for a specific good. Closely following Schuler (2020), in the commission-based model an intermediary advisor receives commissions for every recommended product sold. In the fee-based policy model, these payments are banned. Instead, the intermediary asks a price for going through with a purchase, namely, he sets a transaction fee that consumers pay when deciding to buy a good.

A consumer's valuation from buying product i depends on the binary state variable denoted by $\theta \in \{A, B\}$. The consumer derives utility $z + v_h$ if the bought product matches the state and $z + v_l$ if the product doesn't match the state. The parameter z describes an idiosyncratic change in a consumer's valuation for good i. Thus, z refers to the part of a consumer's valuation, the intermediary cannot observe. For a given consumer, z is randomly drawn from a uniform distribution over the interval $[\underline{z}, \overline{z}] = [0, 1]$, i.e. $z \sim U[0, 1]$. This distribution is known to the intermediary and firms, and generates a downward-sloping demand. As the extent and utility from product suitability are driven only by the difference between v_h and v_l, I simplify the problem by assuming that v_l takes the value zero and that v_h is strictly positive, i.e. $0 = v_l < v_h$. Note that, consumers do not know anything about which of the two products is the better fit. This ignorance of the product's suitability motivates the need for advice. When deciding to buy the recommended good, the consumer pays the price p_i for $i \in \{A, B\}$. In the commission-based model, the consumer's expected utility from consuming good i, hence, is

$$U^C = E[v_i] + z - p_i,$$

where ex- ante $E[v_i] = \frac{v_l + v_h}{2}$. In addition, when banning commissions and introducing a fee-based policy model, the consumer faces a (fixed) transaction fee t when buying the good. Her expected utility in the fee-based policy model, therefore, is given by

$$U^F = E[v_i] + z - p_i - t.$$

When the consumer chooses the outside option and doesn't buy either product, her utility is normalized to zero.

Like Schuler (2020), I specify naive consumer beliefs. Contrary to wary consumers, who understand that product providers have incentives to pay commissions to steer an adviser's recommendation behavior, naive customers are unaware of commissions and believe that advisors are unbiased. They do not understand how the price of a specific good is influenced by firms' incentives to boost sales by paying commissions to the intermediary (Inderst and Ottaviani, 2012c). In other words, consumers are fully unaware that the intermediary receives commissions from the producing firms. This assumption finds justification in the fact that consumers generally have high trust in advising intermediaries.[3]

[3]The US Department of Treasury (2009:68) for example stated that "Consumers [...] retain faith that the intermediary is working for them and placing their interests above his or her own, even if the conflict of interest is disclosed". Chater et al. (2010) show that the majority of consumers either trust the advice mostly or completely and do not view the advisor as biased. Pearson (2017), in addition, states that consumers are not aware of how remunerations influence their purchases and do not understand the inherent conflict of interest commissions create. Consequently, naivety implies that consumers expect the intermediary not to be influenced and not to steer them to a particular product.

The intermediary, who closes the gap between firms and consumers, recommends on the basis of some private information about which of the two products presents a better fit for a particular consumer. His private information is represented by a posterior belief that product A is more suitable, i.e. $q = Pr(\theta = A)$, which ex-ante is distributed according to a uniform distribution over $q \sim U[0, 1]$ and depends on the binary state variable θ. As q is uniformly distributed, posterior beliefs are symmetric around the (common) prior belief $q = \frac{1}{2}$, meaning that ex-ante and without advice, both products are equally likely to be suitable. The posterior belief of the intermediary results from updating the prior probability $q = \frac{1}{2}$ with the private information he has regarding the suitability of a product when learning about a consumer's specific circumstances and preferences. Note that due to naivety, even after observing prices, consumers expect the intermediary's cut-off probability to be equal to the prior of $q = \frac{1}{2}$.

Furthermore, I assume that the intermediary exhibits other-regarding behavior. Similar to Inderst and Ottaviani (2012a), he is concerned about product suitability and directly influenced by a consumer's obtained utility when consuming the suitable good. The intermediary's concern for suitability can be attributed to various causes and might be due to conscience or altruistic motives. I capture these motives by positing that the intermediary derives utility $w(v_h)$ when recommending the suitable good. I assume that $\frac{\partial w}{\partial v_h} \geq 0$, imposing that the more the consumer cares about the suitability of the good, the more the intermediary benefits when recommending the suitable good. By assuming social preferences, the intermediary advisor genuinely cares about the well-being of the consumer which directly influences his utility through $w(v_h)$, even when the consumer doesn't buy the product. Consequently, the other-regarding preferences influence the probability of the intermediary recommending a transaction that is well-suited to a consumer's needs. In the commission-based model, as the intermediary is receiving commission f_i when product i is sold, he is steered by the proposed remunerations. The intermediary therefore also cares about the suggested commissions when recommending a product.

To determine the payoff of the intermediary, assume that the intermediary recommends good i. Given that good i is the suitable good and the consumer buys good i, he earns a payoff of $f_i + w(v_h)$. If the consumer, however, decides not to buy, he earns a payoff of $w(v_h)$. This is due to the intermediary's social preferences which endow him with positive utility when recommending the suitable good. When, however, good j was the suitable good, the intermediary earns a payoff of f_i when the consumer buys the good, and 0 if the consumer doesn't buy the good. Thus, his expected payoff when recommending good i is

$$(1 - z_i)f_i + Pr(\theta = i)w(v_h),$$

where $(1 - z_i)$ is the consumer's probability of buying good i and $Pr(\theta = i)$ is the probability of product i being the suitable one. When deciding which good to recommend, i.e. which message $m \in \{A, B\}$ to send to consumers, the intermediary faces a binary choice and eventually chooses the option that gives a higher expected payoff, i.e.

$$\max_{m \in \{A,B\}} \Big\{(1 - z_A)f_A + qw(v_h), (1 - z_B)f_B + (1 - q)w(v_h)\Big\}.$$

I comment on the intermediary's binary choice in more detail below.

Eventually, when banning commissions and introducing a fee-based policy model, the intermediary asks a consumer to pay a transaction fee t, when she decides to buy either product. I restrict this fee to take a non-negative value, $f \geq 0$. Under the newly introduced policy, after observing firms' prices, the intermediary first chooses a transaction fee t and subsequently makes a product recommendation to the consumer. Again, when determining the payoff of the intermediary several cases need to be taken into account. Assume that the intermediary recommends good i. If good i is the suitable good and the consumer buys good i, he earns a payoff of $t + w(v_h)$. If the consumer, however, decides not to buy, he earns a payoff of $w(v_h)$. If, however, good j is the suitable good, the intermediary earns a payoff of t when the consumer buys the good, and 0 if the consumer doesn't buy the good. The intermediary's expected payoff from recommending good i is given by

$$(1 - z_i)t + Pr(\theta = i)w(v_h).$$

Thus, in a first step, when choosing the transaction fee, the intermediary faces the following optimization problem.

$$\max_{t} \quad [(1 - z_A) + (1 - z_B)] \cdot t + (1 - q)w(v_h) + qw(v_h)$$

In a second step, when preparing a recommendation, the intermediary faces a binary choice problem. When deciding which product to recommend, he considers the expected social utility from the concern for product suitability and chooses the option that gives a higher expected payoff, i.e.

$$\max_{m\in\{A,B\}}\Big\{qw(v_h),(1-q)w(v_h)\Big\}.$$

I assume that firms are equally cost-efficient, i.e. $c_A = c_B = c$. Such a symmetric setting allows me to abstract from biased advice and the distortive effects of commissions. Competing in prices p_i and commissions f_i firm's i profit in the commission-based model is given by

$$\Pi_i^C = [1-z_i]Pr(\theta=i)(p_i-f_i-c),$$

where $[1-z_i]Pr(\theta=i)$ represents the demand. In the fee-based policy model, firms only compete in prices, which simplifies the profit of firm i to

$$\Pi_i^F = [1-z_i]Pr(\theta=i)(p_i-c).$$

Following Inderst and Ottaviani (2012a) and Schuler (2020), to make advice essential for selling a product of either firm, I assume that

$$E[v_A]+\overline{z} = E[v_B]+\overline{z} = \frac{v_l+v_h}{2}+\overline{z} < c. \tag{3.1}$$

The marginal costs of firms are assumed to be sufficiently high, which guarantees that firms cannot get around the intermediary advisor and sell directly to consumers. The assumption used to rule out the possibility that firms can profitably deviate by sufficiently undercutting their rival's price and, thereby, persuading the consumer to buy their product even against the intermediary's recommendation. It finds justification in the medical industry, where patients cannot buy drugs directly from the producer but have to go through a specialist, a general practitioner, or, in the case of non-prescription drugs, a pharmacist. Also in financial markets, securities are traded by brokers or banks on the secondary market. As a result, investing consumers typically must reach out to brokers for the execution of their financial transactions.

To further guarantee that either firm's product can be sold with *good* advice, it must hold that

$$E[v_A|q\geq\frac{1}{2}]+\overline{z} = E[v_B|q<\frac{1}{2}]+\overline{z} > c. \tag{3.2}$$

This assumption ensures that when the intermediary advisor recommends the most suitable product, the expected conditional valuation exceeds the costs for some consumers.

The underlying game consists of 4 stages. In $\tau = 1$, both firms, A and B, simultaneously choose their prices p_A and p_B, and commissions, f_A and f_B, respectively. The prices are aimed directly at consumers and are observed by the intermediary as well as the consumer. In the commission-based model, commissions are paid to the advisor conditional on the sale of a product and are not observed by the consumer. In the fee-based policy model, these payments are prohibited, and firms only choose prices. At period $\tau = 2$, the intermediary's role is to provide advice to consumers. On the basis of his private information, he sends message $m \in \{A, B\}$ to consumers. In the fee-based policy model, the intermediary also chooses the value of the transaction fee t, which is aimed at consumers. Subsequently, when observing his private signal, he makes a recommendation to the consumer by sending a message $m \in \{A, B\}$. Finally, in stage $\tau = 3$, the consumer decides between buying the recommended product and going for the outside option of not buying anything. In the fee-based model, the consumer also needs to pay the transaction fee t, conditional on the purchase of a product. All payoffs are realized in the final stage, $\tau = 4$. I abstract from any discounting and risk considerations by assuming that all parties are risk-neutral.

The solution concept used is perfect Bayesian equilibrium. I focus only on symmetric pure strategy equilibria in which advice is informative, i.e. when deciding to buy a product, the consumer follows the recommendation of the intermediary. Note that, with assumptions (3.1) and (3.2), a purchase only takes place in informative equilibria, where both products are recommended with positive probability (see Schuler (2020)). Throughout the paper, I compare two situations: the pre-ban baseline scenario in which commissions are allowed ($f_i \geq 0$) but transaction fees are non-existent ($t = 0$), and the policy scenario with banned commissions ($f_i = 0$) but an introduced transaction fee ($t \geq 0$).

3.3 Commission-Based Model

This section discusses the commission-based model, which has been studied by Schuler (2020). In this case, where commissions are allowed, the intermediary is remunerated by a firm whose product he successfully recommends. The transaction fee, in contrast, is assumed to be zero, i.e. $t = 0$.

Starting with the consumer's purchase decision, the consumer's expected valuation from consuming product i when receiving message $m = i$ can be written as $E[v_i|m = i]$. A consumer, thus, follows the recommendation of the intermediary m and buys the recommended product at price p_i when her expected utility is weakly positive, i.e.

$$z + E[v_i|m = i] - p_i \geq 0,$$

or, to put it differently, when z is sufficiently high, i.e.

$$z \geq z_i^* = p_i - E[v_i|m = i]. \tag{3.3}$$

Otherwise, when $z < z_i^*$, she chooses the outside option and does not buy. From the binary nature of suitability, the intermediary only considers two messages, namely $m = A$ and $m = B$. Taking into account the consumer's behavior and applying the respective thresholds z_A^* and z_B^*, the intermediary faces a binary choice problem. If the intermediary recommends product A, he expects to realize a payoff of $[1-z_A^*]f_A + qw(v_h)$, where $[1 - z_A^*]$ is the probability that, given message $m = A$, a consumer buys product A. If, however, product B is recommended, the intermediary expects to get $[1 - z_B^*]f_B + (1 - q)w(v_h)$.

Thus, when both products are recommended with positive probability, the advisor recommends product A rather than product B when

$$[1 - z_A^*]f_A + qw(v_h) \geq [1 - z_B^*]f_B + (1 - q)w(v_h).$$

The intermediary, therefore, prefers to recommend product A for all posterior beliefs, when

$$[1 - z_A^*]f_A - [1 - z_B^*]f_B \geq w(v_h) \tag{3.4}$$

and prefers to recommend product B when

$$[1 - z_B^*]f_B - [1 - z_A^*]f_A \geq w(v_h) \tag{3.5}$$

Schuler (2020) shows, when (3.4) and (3.5) do not hold, there exists a threshold $0 < q^* < 1$,

$$q^* = \frac{1}{2} - \frac{(1 - z_A^*)f_A - (1 - z_B^*)f_B}{2w(v_h)} \tag{3.6}$$

such that the intermediary recommends product A when $q \geq q^*$ and recommends product B when $q < q^*$. Therefore, the threshold is defined as $q^* = 0$ in case of (3.4) and $q^* = 1$ in case of (3.5). Here, v_h captures the responsiveness of advice to commissions, meaning that when v_h becomes very high, firms need to increase commissions to still be able to steer the intermediary. Note that in the present analysis, it is not the size of p_i itself that influences the intermediary, but the expected value of the payment, taking into account the probability with which the consumer follows the advice, i.e. $1 - z_i^*$.

Given that consumers hold naive beliefs they do not internalize the effect of higher commissions or prices and believe that the cut-off probability is equal to $q^* = \frac{1}{2}$. This follows Inderst and Ottaviani (2012c). The consumer's expected valuations (net of z) simplify to

$$E[v_A|m = A] = E[v_A|q \geq \frac{1}{2}] = \int_{\frac{1}{2}}^{1} \frac{v_h q}{\frac{1}{2}} dq = \frac{3}{4} v_h, \tag{3.7}$$

and

$$E[v_B|m = B] = E[v_B|q < \frac{1}{2}] = \int_{0}^{\frac{1}{2}} \frac{v_h(1 - q)}{\frac{1}{2}} dq = \frac{3}{4} v_h. \tag{3.8}$$

The demands for firms A and B then take the following form

$$D_A = Pr[q \geq q^*] \cdot Pr[z \geq z_A^*] = [1 - q^*][1 - z_A^*]$$

and

$$D_B = Pr[q < q^*] \cdot Pr[z \geq z_B^*] = q^*[1 - z_B^*],$$

where $z_i^* = p_i - E[v_i|m = i]$. It is apparent that product prices affect a firm's demand through two channels. Prices have a direct effect on market shares through demand via $1 - z_i^*$, but impact also the intermediary's recommendation behavior q^*. Then, even if the intermediary doesn't care directly about the level of prices, he takes into account the risk of ending up with no sale at all when recommending a more expensive product.

The firms' respective maximization problems become

$$\max_{p_A, f_A} \Pi_A = [1 - q^*][1 - z_A^*](p_A - f_A - c) \tag{3.9}$$

and

$$\max_{p_B, f_B} \Pi_B = q^*[1 - z_B^*](p_B - f_B - c). \tag{3.10}$$

The optimal choices for firms A and B, respectively, are given by the following first-order conditions

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{\partial D_A}{\partial p_A}(p_A - f_A - c) + D_A = 0,$$
$$\frac{\partial \Pi_A}{\partial f_A} = \frac{\partial D_A}{\partial f_A}(p_A - f_A - c) - D_A = 0,$$

and

$$\frac{\partial \Pi_B}{\partial p_B} = \frac{\partial D_B}{\partial p_B}(p_B - f_B - c) + D_B = 0,$$
$$\frac{\partial \Pi_B}{\partial f_B} = \frac{\partial D_B}{\partial f_B}(p_B - f_B - c) - D_B = 0.$$

In this context, Schuler (2020) shows that the optimality condition for prices is familiar from oligopoly pricing and takes the form as follows

$$p_i^* = \frac{D_i}{\frac{\partial D_i}{\partial p_i}} + f_i + c. \tag{3.11}$$

Given the competitor's choice, the optimal price consists of the commissions f_i and the firm's markup resulting from the trade-off between higher demand and a higher margin. Since the profit function is composed of the product of demand and margin, it must hold that

$$\frac{\partial D_i}{\partial p_i} = -\frac{\partial D_i}{\partial f_i}. \tag{3.12}$$

Schuler (2020) shows that the optimal price-commission pair is given by

$$p_i^* = (1 - z_i^*) + c \tag{3.13}$$

and

$$f_i^* = (1 - z_i^*) - \frac{w(v_h)}{(1 - z_i^*)}. \tag{3.14}$$

Given (3.3), (3.7) and (3.8), the unique equilibrium price and commission simplify to

$$p_i^C = \frac{1 + \frac{3}{4}v_h + c}{2}, \tag{3.15}$$

and

$$f_i^C = \frac{1 + \frac{3}{4}v_h - c}{2} - \frac{2w(v_h)}{1 + \frac{3}{4}v_h - c}. \tag{3.16}$$

When looking at (3.15), clearly, prices are strictly increasing in v_h. The more a consumer values a suitable good, the more firms increase the price of their respective good to skim the consumers' increased willingness to pay. Although the optimal margin $p_i^* - f_i^* - c$ depends on the optimal commission, the optimal price does not. Therefore, at the optimal choice, it is as if firms set their prices like monopolists, ignoring their commissions (Schuler, 2020). As commissions are not passed down to consumers, the optimal price does neither depend on the scope of $w(v_h)$. Since consumers hold naive beliefs, they are unaware of commissions and thereby do not know the actual threshold of the intermediary nor how the intermediary's concern for suitability influences his recommendation behavior.

Then, having a look at the optimal commissions, I can make several observations. The first term in (3.16) indicates that commissions are positively influenced by the price and thus depend positively on v_h. The more a consumer cares about a suitable fit, the higher the price and the profit margin become. The scale on which a firm can choose its commissions becomes larger and gives the firm more room to steer the intermediary toward a particular good. The second term in (3.16) implies that the optimal commissions are also negatively influenced by v_h through $w(v_h)$. As v_h increases, not only the consumer but also the intermediary cares more about the suitability of a specific product. When the intermediary's social preferences are more pronounced, firms decrease their commissions. This is because firms are aware of these preferences and recognize the limited steering effect of commissions as v_h increases. The optimal commission solves the firms' trade-off between persuading the intermediary to steer consumers toward their direction and higher marginal costs in the form of higher payments to the intermediary.

From (3.16), it is apparent that if the intermediary's other-regarding preferences $w(v_h)$ becomes sufficiently high, firms set commissions equal to zero. If, however, the intermediary's concern for suitability is sufficiently weak, then it is optimal for firms to set positive commissions. Thus, if the intermediary's concern for product suitability $w(v_h)$ is smaller than some upper bound $\overline{w}(v_h)$ for a specific value of v_h, the commissions are strictly positive in equilibrium, i.e. if

$$w(v_h) < \overline{w}(v_h) = \frac{\left(1 + \frac{3}{4}v_h - c\right)^2}{4},$$

then $f_i^* > 0$. For the remainder of the paper, I assume that this inequality is satisfied. If the other-regarding preferences of the intermediary take on a sufficiently low magnitude, commissions are positive. However, when the intermediary's concern for the good's suitability becomes sufficiently strong, firms anticipate that the advisor cannot be influenced by higher commissions. Commissions as a steering device, thus, become useless and firms set $f_i^C = 0$.

In the other extreme case, where the intermediary is entirely selfish, I observe the following.

Lemma 1 *If the intermediary is purely selfish, i.e. $w(v_h) = 0$, the equilibrium commissions coincide with the difference of the price minus the marginal cost*

$$f_i^C|_{w(v_h)=0} = \frac{1 + \frac{3}{4}v_h - c}{2} = \frac{1 + \frac{3}{4}v_h + c}{2} - c = p_i^C - c,$$

and firms make zero profits in equilibrium.

When the intermediary doesn't care about the well-being of the consumer, he expects to realize a payoff of $[1 - z_i^*]f_i^*$ when recommending product i. Since the size of commissions is not passed down, retail prices for consumers stay constant, and the only way to influence the intermediary's behavior is through commissions. Firm i, thus, has an incentive to increase its commissions as it can maximally steer the intermediary towards recommending its product. By continuously overbidding its rival firm with higher commissions to secure the sale of the product, firm i eventually ends up with zero profits. The reason for this is that higher commissions reduce firms' margins similar to an increase in costs.

Since firms are equally cost-efficient, i.e. $c_A = c_B = c$, in equilibrium, firms set the same prices and commissions, and consequently the symmetric outcome $q^* = \frac{1}{2}$ arises. When firm i optimally sets price p_i^C and commission f_i^C according to (3.15) and (3.16), the equilibrium profit is given by $\Pi_i^C = \frac{w(v_h)}{2}$. Striking is the fact that due to the assumption of other-regarding preferences, in equilibrium, a firm's profit is increasing in $w(v_h)$, although a firm's power to steer the intermediary drops as $w(v_h)$ increases. Under individual rationality, firms behave self-interested and wish for a selfish intermediary to be able to maximally steer the recommendation. Under collective rationality, however, firms benefit from an intermediary with a pronounced concern for suitability. This is because firms fail to internalize the actual cost of higher commissions and the negative influence on their final profit. This observation can be attributed to the assumption of naive consumers and the fact that there is no pass-on of commissions to consumers. Facing a rather selfish intermediary, firms have strong incentives to increase their commissions to influence the intermediary's recommendation behavior. However, firms alone bear the additional marginal costs represented by these payments as their not passed down. Therefore, in equilibrium, firms are better off selling through an intermediary with strong social preferences.

Finally, I can determine the equilibrium threshold z_i^* above which a consumer buys the recommended good.

$$z_i^C = \frac{1 - \frac{3}{4}v_h + c}{2} \tag{3.17}$$

Due to assumption (3.2), it holds that $z_i^C < 1$. It follows that the share of consumers buying in equilibrium is given by

$$1 - z_i^C = \begin{cases} \frac{1+\frac{3}{4}v_h - c}{2} & \text{if } 1 + c > \frac{3}{4}v_h \\ 1 & \text{if } 1 + c \leq \frac{3}{4}v_h. \end{cases}$$

Therefore, if v_h becomes sufficiently large all consumers buy in equilibrium. Note that, the share of consumers who buy in equilibrium, i.e. $1 - z_i^C$, does not directly depend on commissions. Once again, due to naivety, consumers are not aware of any payments from firms to the intermediary. However, higher commissions are ultimately passed on to consumers in form of higher prices, similar to an increase in marginal costs.

3.4 Fee-Based Policy Model

In this section, I analyze the scenario where a ban on commissions ($f_i = 0$) is introduced but the intermediary has the opportunity to ask for a fixed transaction fee t when selling the good.

Following the same structure of analysis as in section 3, I start with the consumer's purchasing decision. Given a price p_i and transaction fee t, a consumer follows the recommendation of the intermediary m when

$$z \geq z_i^*(p_i, t) = p_i + t - E[v_i|m = i]. \tag{3.18}$$

Otherwise, i.e. when $z < z_i^*$, she chooses not to buy. The consumer's decision, therefore, is based on the total cost p_i+t, i.e. the sum of good i's price and the transaction fee. Given the respective threshold, z_A^* and z_B^*, the advisor considers two messages which correspond to the products A and B, and faces a binary choice problem. Given the realization of this posterior belief q that product A is more suitable, it is optimal for the intermediary to recommend product A whenever he receives a higher expected payoff when the consumer buys product A instead of product B. Recall, that the problem is solved backward. When deciding what good to recommend, the intermediary takes into account that the previously set transaction fee maximizes his expected utility. Thus, having already included a product's probability to be bought, the intermediary only considers his expected social utility of a subsequent match $w(v_h)$ resulting from his other-regarding preferences. Therefore, when the intermediary recommends product A, he expects to realize an additional payoff of $qw(v_h)$. If, however, he recommends product B, he expects to get $[(1-q)w(v_h)]$. When both products are recommended with positive probability, the advisor recommends product A rather than product B when

$$qw(v_h) \geq (1-q)w(v_h). \tag{3.19}$$

Therefore, the threshold above which the intermediary recommends product A rather than product B is given by $q^* = \frac{1}{2}$. This result is summarized as follows.

Lemma 2 *In a fee-based model where the intermediary takes into account an optimal transaction fee, he recommends product A rather than product B if $q \geq q^* = \frac{1}{2}$, independent of the extent of his social preferences $w(v_h)$.*

It becomes apparent that due to his other-regarding preferences, the intermediary is driven to recommend the suitable good to the consumer. Since commissions are banned, the intermediary is not steered towards any particular product and suggests the best fit. Note that, if the intermediary behaves purely selfishly with $w(v_h) = 0$, he will recommend randomly with a threshold $q^{**} \in [0, 1]$. As the random threshold q^{**} is assumed to be continuously random it is said to be uniformly distributed and ultimately converges to $q^* = \frac{1}{2}$. Given that consumers hold naive beliefs, their expected cut-off probability coincides with the actual cut-off probability equal to $q^* = \frac{1}{2}$. Again, the consumer's expected valuation follows (3.7) and (3.8).

The intermediary, then, chooses t according to the following maximization problem.

$$\max_t \quad [(1 - z_A^*) + (1 - z_B^*)] \cdot t + w(v_h),$$

where z_A^* and z_B^* are linear functions of t and p_A and p_B, respectively, given by (3.18). When solving for the optimal transaction fee, I get the following result.

Lemma 3 *Given the prices of firms A and B, the optimal reaction function of the intermediary is given by*

$$t(p_A, p_B) = \frac{1 + \frac{3}{4}v_h - \frac{1}{2}(p_A + p_B)}{2}. \tag{3.20}$$

Before I investigate the equilibrium behavior of firms, I first analyze the intermediary's optimal strategy for given prices chosen by the firms. The extent of the transaction fee depends positively on the consumer's valuation for a suitable good v_h. The higher the consumer benefits from a suitable good, the more a consumer is willing to pay. Consequently, the intermediary has an incentive to increase the transaction fee and skim some of that willingness to pay. Further, the optimal transaction fee clearly depends negatively on the firms' prices. Then, if firm i increases the price of its respective good, the residual willingness to pay of a consumer shrinks, which leaves the intermediary with less freedom when choosing the scope of the transaction fee.

Next, I consider the behavior of the producing firms. Recall that a consumer purchases only when her private valuation exceeds a certain threshold. The intermediary, anticipating her behavior, sets a transaction fee according to (3.20) and applies a cut-off rule q^* when advising according to Lemma 4. Firm A and B's maximization problems are

$$\max_{p_A} \quad \Pi_A = (1 - q^*)(1 - z_A^*)(p_A - c)$$

and

$$\max_{p_B} \quad \Pi_B = q^*(1 - z_B^*)(p_B - c),$$

respectively.

Note that the price influences the firms' profits through several channels. First, it positively influences the margin of firm i directly via p_i. Second, the choice of price has an effect on the probability with which the consumer is ultimately buying the good. Clearly, $1 - z_i^*$ is directly affected by a p_i, but also through the transaction fee t, which reacts to changes in prices.

Once I restrict the attention to symmetric pure-strategy equilibria, the unique equilibrium price is given by

$$p_i^F = \frac{2}{5}\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right). \tag{3.21}$$

The optimal price is strictly increasing in v_h. Given p_i^*, the unique equilibrium value for the optimal transaction fee becomes

$$t^F = \frac{3}{10}\left(1 + \frac{3}{4}v_h - c\right). \tag{3.22}$$

Note that, due to (3.2), in equilibrium, the transaction fee is always non-negative.

Proposition 1 *When commissions are banned and instead a fee-based remunerative structure is introduced, in the unique equilibrium, firm i optimally sets price p_i^F according to (3.21) and the intermediary chooses a transaction fee t^F given by (3.22). The equilibrium profit of firm i becomes $\Pi_i^F = \frac{3}{50}\left(1 + \frac{3}{4}v_h - c\right)^2$.*

The equilibrium prices as well as the equilibrium transaction fee depend positively on the consumer's valuation for a suitable good v_h. As the additional valuation from consuming the suitable good increases, the intermediary and firms have more room to ask for higher prices and a higher transaction fee, respectively. Whereas the prices positively depend on the marginal cost c, the transaction fee is negatively influenced by the cost firms face. When the marginal cost c increases, firms increase their prices to mitigate the loss in profit. It is then in the intermediary's best interest to accommodate the price increase by lowering t^*. By asking for a lower transaction fee he ensures, that the consumer is still buying the good in equilibrium and that he realizes a positive profit.

Given the optimal transaction fee and the equilibrium prices, I can determine the equilibrium threshold z_i^* above which a consumer buys the recommended good, i.e.

$$\begin{aligned}
z_i^* &= p_i^* + t - \frac{3}{4}v_h; \\
&= \frac{2}{5}\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right) + \frac{3}{10}\left(1 + \frac{3}{4}v_h - c\right) - \frac{3}{4}v_h; \\
&= (\frac{2}{5} + \frac{3}{10}) + (\frac{2}{5} + \frac{3}{10} - 1)\frac{3}{4}v_h + (\frac{2}{5}\frac{3}{2} - \frac{3}{10})c; \\
&= \frac{7}{10} - \frac{3}{10}\frac{3}{4}v_h + \frac{3}{10}c;
\end{aligned}$$

$$z_i^F = \frac{7 - \frac{9}{4}v_h + 3c}{10}. \tag{3.23}$$

Due to assumption (3.2), it holds that $z_i^F < 1$. It follows that the share of consumers buying in equilibrium is given by

$$1 - z_i^F = \begin{cases} \frac{3+\frac{9}{4}v_h-3c}{10} & \text{if } \frac{7}{3} + c > \frac{3}{4}v_h \\ 1 & \text{if } \frac{7}{3} + c \leq \frac{3}{4}v_h. \end{cases}$$

Therefore, if v_h becomes sufficiently large all consumers buy in equilibrium.

3.5 Comparison and Welfare Analysis

In this section, I analyze the effect of a ban on commissions on prices, the share of consumers buying in equilibrium and social welfare.

Prices Banning commissions has an effect on the pricing behavior of firms. The equilibrium prices in the commission-based model and fee-based policy model are given by (3.15) and (3.21), respectively. When comparing the two prices, I obtain

$$\begin{aligned}
p_i^C = \frac{1}{2}\left(1 + \frac{3}{4}v_h + c\right) &> \frac{2}{5}\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right) = p_i^F; \\
5\left(1 + \frac{3}{4}v_h + c\right) &> 4\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right);
\end{aligned}$$

$$v_h > \frac{4}{3}(c-1).$$

Therefore, when v_h is sufficiently large, the prices in the commission-based model are higher, instead, when v_h is small, the prices in the fee-based model are higher. The intuition behind this is the following. When v_h is high, prices in the commission-based model increase due to an increase in the consumers' willingness to pay. In addition, as v_h grows stronger, the intermediary cares more about the suitability of the consumer and firms need to pay higher commissions to be able to steer the intermediary towards their respective product. This channel pushed up prices even further. In the fee-based model, naturally, a higher v_h also increases the consumers' willingness to pay. Since a higher willingness to pay pushed up prices as well as the transaction fee, firms are reluctant to increase the prices too much, as it would substantially decrease the probability of their good being sold.

This finding is shown in Figure (3.1). The prices are plotted conditional on a consumer's utility when the product matches the realized state v_h for different values of firms' marginal cost c. I can make two different observations. The first observation is that firms' marginal costs obviously tend to increase prices. More so, the equilibrium prices under the fee-based policy react stronger to an increase in marginal cost than the prices under the commission-based model, i.e.

$$\frac{\partial p_i^F}{\partial c} = \frac{3}{5} > \frac{1}{2} = \frac{\partial p_i^C}{\partial c}.$$

The reason for this is that in the commission-based model, an increase in costs not only affects the equilibrium prices but also the extent of equilibrium commissions paid to the intermediary. As the effect of an increase in cost is split, the effect of higher marginal costs on prices is consequently less pronounced in the commission-based model.

When the marginal cost of firms is sufficiently low, the equilibrium prices in the commission-based model are always higher than the prices in the fee-based model. For higher marginal costs, however, whether the equilibrium price in the commission-based model or the fee-based model is higher depends on the consumers' utility from product suitability v_h. This brings me to my second observation: It is apparent that both prices are increasing in v_h. However, the equilibrium price in the commission-

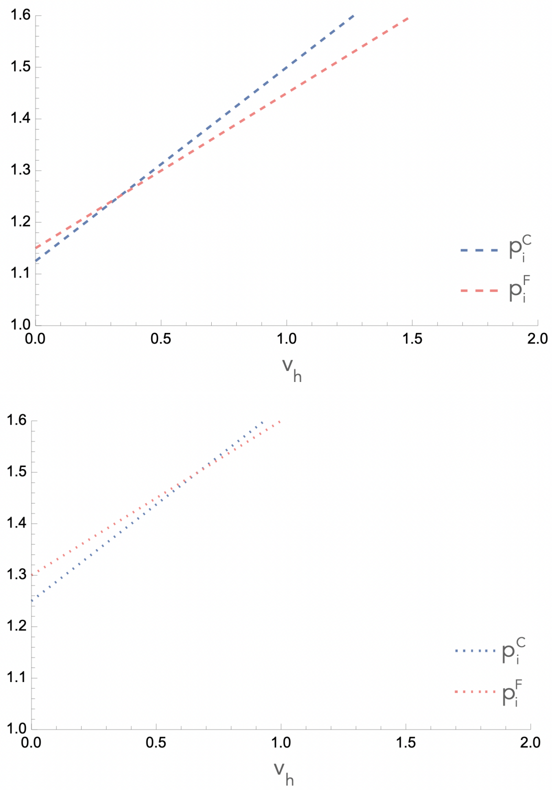

Figure 3.1: **Equilibrium prices under commission-based and fee-based structure.** This figure depicts the equilibrium prices in the commission-based model and the fee-based policy model, i.e. p_i^C and p_i^F, for different values of the marginal cost of firms. I assume the following parameters to generate this figure: c=1.25 (dashed), c=1.5 (dotted).

based model reacts stronger to an increase in v_h. i.e.

$$\frac{\partial p_i^C}{\partial v_h} = \frac{3}{8} > \frac{3}{10} = \frac{\partial p_i^F}{\partial v_h}.$$

Consequently, depending on the magnitude of the utility consumers receive when consuming the suitable good, they might face higher prices after the introduction of a ban on commissions.

Share of consumers buying In the fee-based policy model, when deciding to buy the product, consumers not only face product prices but also a transaction fee. To get a more robust comparison, I thus look at the effect of the introduction of a commission ban on the share of consumers buying in equilibrium. From (3.3) and (3.18), I know that a consumer follows the intermediary's recommendation and buys in equilibrium if her expected valuation for product i when receiving advice $m = i$ is higher than the price and, in case of the fee-based policy model, the sum of the price and the transaction fee she pays. Given the thresholds in (3.3) and (3.18) above which a consumer chooses to purchase the recommended good, I determined the shares of consumers who buy in equilibrium. Assuming positive shares, I compare

$$\begin{aligned} 1 - z_i^C = \frac{1 + \frac{3}{4}v_h - c}{2} &> \frac{3 + \frac{9}{4}v_h - 3c}{10} = 1 - z_i^F; \\ 5\left(1 + \frac{3}{4}v_h - c\right) &> 3 + \frac{9}{4}v_h - 3c; \\ 1 + \frac{3}{4}v_h &> c. \end{aligned}$$

which holds due to assumption (3.2).

Figure (3.2) shows these shares, depending on the consumer's utility when the product matches the realized state v_h, for different values of firms' cost c. An increase in costs clearly lowers the share of consumers buying in equilibrium. This follows from the effect of increasing costs on prices. As the marginal costs of firms get more pronounced, the prices increase accordingly. With a downward-sloping demand, higher prices are associated with fewer consumers buying the good in equilibrium (and hence higher cut-off values z_i^C and z_i^F).

In addition, under both regimes, the number of consumers buying in equilibrium increases in v_h. In equilibrium, however, the share of consumers buying in equilibrium is always higher in the commission-based model, where commissions are allowed.

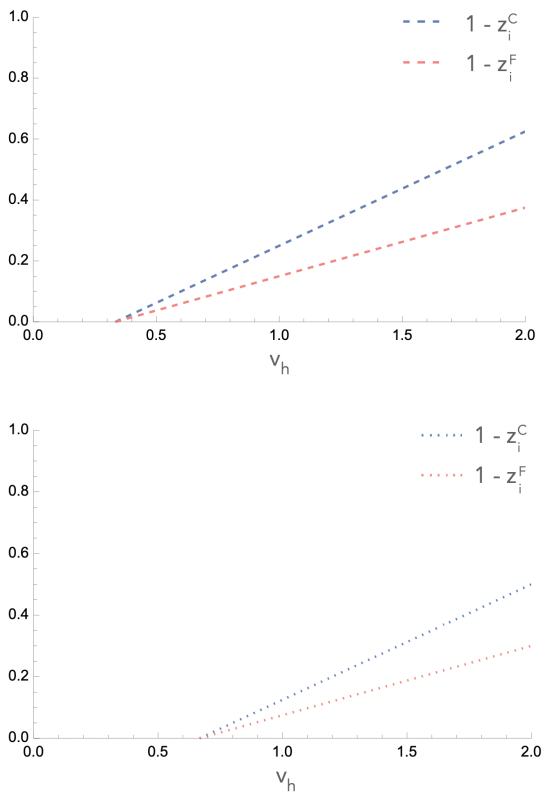

Figure 3.2: **Share of consumers who buy in equilibrium.** This figure depicts the shares of consumers who buy in equilibrium in the commission-based model and the fee-based policy model, i.e. $1-z_i^C$ and $1-z_i^F$, for different values of the marginal cost of firms. I assume the following parameters to generate this figure: c=1.25 (dashed), c=1.5 (dotted).

Consumer surplus Given this result, it is natural to ask how consumer surplus is affected. Given the threshold above which consumers buy in equilibrium, I can calculate the consumer surplus for the two scenarios. I obtain

$$
\begin{aligned}
CS^C &= q^* \int_{z_A^C}^{\overline{z}} z + E[v_A|q^* \geq \frac{1}{2}] - p_A^C dz + (1-q^*) \int_{z_B^C}^{\overline{z}} z + E[v_B|q^* < \frac{1}{2}] - p_B^C dz; \\
&= \int_{\frac{1-\frac{3}{4}v_h+c}{2}}^{1} z + \frac{3}{4}v_h - \frac{1+\frac{3}{4}v_h+c}{2} dz; \\
&= \left[\frac{1}{2}z^2 - z\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)\right]_{\frac{1-\frac{3}{4}v_h+c}{2}}^{1}; \\
&= \frac{1}{2} - \left(\frac{1-\frac{3}{4}v_h+c}{2}\right) - \frac{1}{2}\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)^2 + \left(\frac{1-\frac{3}{4}v_h+c}{2}\right)^2; \\
&= \frac{1}{2} - \left(\frac{1-\frac{3}{4}v_h+c}{2}\right) + \frac{1}{2}\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)^2,
\end{aligned}
$$

and

$$
\begin{aligned}
CS^F = q^* &\int_{z_A^F}^{\overline{z}} z + E[v_A|q^* \geq \frac{1}{2}] - p_A^F - t^F dz; \\
&\qquad + (1-q^*) \int_{z_B^F}^{\overline{z}} z + E[v_B|q^* < \frac{1}{2}] - p_B^F - t^F dz; \\
&= \int_{\frac{7-\frac{9}{4}v_h+3c}{10}}^{1} z + \frac{3}{4}v_h - \frac{2}{5}\left(1+\frac{3}{4}v_h+\frac{3}{2}c\right) - \frac{3}{10}\left(1+\frac{3}{4}v_h - c\right) dz; \\
&= \left[\frac{1}{2}z^2 - z\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)\right]_{\frac{7-\frac{9}{4}v_h+3c}{10}}^{1}; \\
&= \frac{1}{2} - \left(\frac{7-\frac{9}{4}v_h+3c}{10}\right) - \frac{1}{2}\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2 + \left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2; \\
&= \frac{1}{2} - \left(\frac{7-\frac{9}{4}v_h+3c}{10}\right) + \frac{1}{2}\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2,
\end{aligned}
$$

respectively. The results of this comparison can be summarized in the following proposition.

Proposition 2 *When abstracting from the distortive effect of commissions, moving from a commission-based to a fee-based remuneration model leads to a drop in the number of consumers following the intermediary's advice and buying in equilibrium. The introduction of a transaction fee increases the consumer's total cost and leads to a decrease in consumer surplus.*

The intuition of this result is straightforward, then consumers face higher total costs in the fee-based model than in the commission-based model, i.e.

$$\begin{aligned} p^C &< p^F + t \\ \frac{1}{2}\left(1+\frac{3}{4}v_h+c\right) &< \frac{2}{5}\left(1+\frac{3}{4}v_h+\frac{3}{2}c\right)+\frac{3}{10}\left(1+\frac{3}{4}v_h-c\right) \\ 5\left(1+\frac{3}{4}v_h+c\right) &< 4\left(1+\frac{3}{4}v_h+\frac{3}{2}c\right)+3\left(1+\frac{3}{4}v_h-c\right) \\ c &< 1+\frac{3}{4}v_h, \end{aligned}$$

which holds due to assumption (3.2). Consequently, fewer consumers buy in equilibrium which creates a dead-weight loss and decreases consumer welfare when switching from a commission-based to a fee-based remuneration model.

This is in line with the apprehension that after a ban on commissions and an introduction of transaction fees, some consumers are not willing to pay for financial advice. In a report by the National Institute for Family Finance Information of the Netherlands, van Gaalen et al. (2017) indicate that especially low-income consumers are more likely not to consult a financial advisor. The reason for this is the direct price of financial advice. Then, in the fee-based policy structure, consumers clearly see the cost of financial advice which may previously have appeared to be non-existing since the charges were part of the commission payments made to the intermediary. Consequently, introducing a transaction fee spells out the cost of financial advice and puts more consumers off seeking advice. Due to the higher cost, a consumer is facing when deciding to buy the recommended good, the consumer surplus decreases when banning commissions.

This result is sensitive to the assumption of symmetric firms. With naive customers, generally, there is a clear benefit of a policy intervention that requires firms to make consumers pay directly for advice (Inderst and Ottaviani, 2012b). A ban on commissions increases consumer surplus by limiting firms' abilities to steer the intermediary's recommendation behavior and therefore by restricting the extent to which consumers' naive beliefs can be exploited. In the underlying model, firms are symmetric which on the one hand allows for omitting the distortive effects of commissions. On the other hand, however, symmetry leaves the intermediary evenly influenced, which eliminates the part of the consumer surplus arising from the policy intervention.

Firms' profits The equilibrium profits of firm i in the baseline setting and in the fee-based model are given by $\Pi_i^C = \frac{w(v_h)}{2}$ and $\Pi_i^F = \frac{3}{50}\left(1 + \frac{3}{4}v_h - c\right)^2$, respectively. Whether firms make higher profits in the commission-based model or in the fee-based policy model, clearly, depends on the specification of the other-regarding preferences of the intermediary. The intermediary's preferences, therefore, have the capacity to shape the market distinctively and have ambiguous welfare effects. However, when I assume the extreme case where the intermediary is fully selfish, i.e. $w(v_h) = 0$, firm i's profit in the fee-based policy model is always higher than in the commission-based model. As seen in Lemma 1, in the commission-based model when the intermediary is purely selfish firms make zero profits, whereas in the fee-based model profits are always positive due to assumption (3.2).

Intermediary's payoff Recall that the symmetry in competition between firms creates balanced incentives for the intermediary when recommending a product. Therefore, in equilibrium, the optimal cut-off rule for the intermediary is $q^* = \frac{1}{2}$, irrespective of the extent of his concern for suitability and, hence, irrespective of the level of commissions that prevails in equilibrium. Consequently, the intermediary's advice is always informative, i.e. he always recommends the suitable good and obtains utility from good conscience $w(v_h)$. When analyzing the change in the intermediary's expected payoff, I can therefore neglect the utility arising from his social preferences and focus on the expected payments obtained when selling a good. In the commission-based model, the payoff results from the commission f_i^C, whereas in the fee-based policy model, he is remunerated by the transaction fee t^F. Both payments need to be weighed with the probability of selling the good in the respective scenario, i.e. I compare $(1 - z_i^C)f^C$ and $(1 - z_i^F)t^F$. The intermediary is better off under the commission-based model if it holds that

$$(1 - z_i^C)f^C > (1 - z_i^F)t^F;$$

$$\left(1 - \frac{1 - \frac{3}{4}v_h + c}{2}\right)\left(\frac{1 + \frac{3}{4}v_h - c}{2} - \frac{2w(v_h)}{1 + \frac{3}{4}v_h - c}\right)$$
$$> \left(1 - \frac{7 - \frac{9}{4}v_h + 3c}{10}\right)\left(\frac{3 + \frac{9}{4}v_h - 3c}{10}\right).$$

Even if I can neglect the direct effect of the intermediary's utility resulting from his other-regarding preferences, his concern for suitability affects the commissions in the commission-based model. Consequently, whether the intermediary is better off in the commission-based model or in the fee-based policy model depends on the specification of his social preferences. When, however, assuming that the intermediary is entirely selfish with $w(v_h) = 0$ he is better off in the scenario where commissions are allowed:

$$\left(1 - \frac{1 - \frac{3}{4}v_h + c}{2}\right)\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right) > \left(1 - \frac{7 - \frac{9}{4}v_h + 3c}{10}\right)\left(\frac{3 + \frac{9}{4}v_h - 3c}{10}\right);$$
$$\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right)\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right) > \frac{3}{5}\left(\frac{3 + \frac{9}{4}v_h - 3c}{10}\right)\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right);$$
$$\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right) > \frac{9}{25}\left(\frac{1 + \frac{3}{4}v_h - c}{2}\right).$$

The economic intuition behind this finding is the following. When assuming the intermediary to be entirely selfish, commissions can influence his recommendation behavior maximally. The pronounced incentives of firms to increase commissions and steer the recommendation favors the intermediary to such an extent that he is always better off in the commission-based model.

Social welfare Finally, I can analyze the effect of the introduced policy on social welfare. Social welfare refers to the sum of utility across all actors in the model. Since, in equilibrium, advice is informative, the intermediary experiences the same utility sprout by his social preferences in both scenarios. Thus, this concern does not affect the difference in social welfare and can be neglected. The expressions for social welfare in the commission-based model and in the fee-based policy model are given by

$$\begin{aligned} W^C &= CS^C + 2 \cdot \Pi_i^C + \left(1 - z_i^C\right) f^C; \\ &= \frac{1}{2} - \left(\frac{1 - \frac{3}{4}v_h + c}{2}\right) + \frac{1}{2}\left(\frac{1 - \frac{3}{4}v_h + c}{2}\right)^2 + w(v_h) \end{aligned}$$

$$+\left(1-\frac{1-\frac{3}{4}v_h+c}{2}\right)\left(\frac{1+\frac{3}{4}v_h-c}{2}-\frac{2w(v_h)}{1+\frac{3}{4}v_h-c}\right);$$
$$=\frac{3}{8}\left(1+\frac{3}{4}v_h-c\right),$$

and

$$\begin{aligned}
W^F &= CS^F + 2\cdot\Pi_i^F + \left(1-z_i^F\right)t^F; \\
&= \frac{1}{2}-\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)+\frac{1}{2}\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2+\frac{3}{25}\left(1+\frac{3}{4}v_h-c\right)^2 \\
&\quad +\left(1-\frac{7-\frac{9}{4}v_h+3c}{10}\right)\left(\frac{3+\frac{9}{4}v_h-3c}{10}\right); \\
&= \frac{51}{200}\left(1+\frac{3}{4}v_h-c\right),
\end{aligned}$$

respectively. Consequently, social welfare in the commission-based model prevails. Since commissions and transaction fees are simply transfers between firms and the intermediary, and the intermediary and consumers, respectively, these effects cancel out. The only effect that lasts is the deadweight loss associated with fewer consumers buying the good in equilibrium. Thus, it is self-evident that the policy implementation is accompanied by a reduction in social welfare.

3.6 Conclusion

Over the past decade, there has been a paradigm shift away from commission-based financial guidance to fee-based remuneration structures. This shift has been largely predicated on the perception that a fee-based structure is morally superior to commission-based guidance. The purpose of this paper is to analyze how a ban on commissions influences the advisory landscape in financial markets and to assess the validity of this perception by examining the welfare impacts of such a shift. In a first step, I study the commission-based model, which is a sub-case of Schuler (2020), where firms supplying financial products have the possibility to steer an intermediary advisor through rewarding payments. In equilibrium, firms adapt their commissions depending on the extent of the intermediary's social preferences. The stronger the intermediary's concern for product suitability becomes the lesser a firm tries to steer the intermediary toward its product. This is to an extent that, when the intermediary's social preferences become sufficiently strong, firms find it optimal to set commissions equal to zero. On the other extreme, when the intermediary is purely selfish, commissions become so high, that firms make zero profits in equilibrium.

In a second step, I introduce a ban on commissions. With the introduction of this policy, the intermediary advisor has the possibility to charge a transaction fee to his consumers. Not being steered by any remunerations, in equilibrium, the intermediary is always recommending the product with the best fit.

When comparing the two scenarios, I find that depending on the consumers' utility from product suitability, prices might decrease or increase after the ban. What is unambiguous, is that with the ban on commissions the number of consumers buying in equilibrium decreases. Looking at social welfare, the results are clear: switching from a commission-based to a fee-based remuneration model comes along with a decrease in social welfare. This insight proves key for the analysis as it conflicts with the perception that a fee-based structure puts consumers in a more favorable position and reflects the insights of studies that empirically investigated the effects of commission bans in different countries. It provokes the question of whether a paradigm shift away from compensation through commissions towards transactional fees is worth the drawbacks.

These results require strong assumptions regarding the structure of the market. Relaxing these assumptions could provide interesting issues that go beyond the scope of the underlying analysis.

An extension one might want to consider are different marginal costs for firms. With asymmetric firms, the optimal recommendation cut-off changes in favor of the firm with lower marginal costs. The intermediary, therefore, is biased and steers more consumers towards the more cost-efficient firm. The extent to which the intermediary is steered clearly depends on these social preferences. In equilibrium, the share of consumers buying in equilibrium, thus, depends more or less on the difference in marginal cost. In the fee-based model, the recommendation is still unbiased. The price of the more cost-efficient firm is lower relative to its rival firm. When choosing the transaction fee, the intermediary takes into account the average of the two prices. The share of consumers buying in equilibrium, thus, is different for each product. Whether these shares are lower or higher than the ones in the commission-based model will depend on the extent of the intermediary's social preferences.

Next, because the market is limited to only one intermediary, commissions are paid with one goal in mind: steering the intermediary's recommendation behavior. Introducing multiple intermediaries might distort the impact of rewarding payments as several competitive effects could prevail. However, it has been argued that in markets with complex products, competition is restraint (Hunter, 2005). The complexity of the underlying products makes it difficult for naive consumers to understand their needs and to distinguish the service quality of intermediaries. Making a recommendation in the commission-based model given the same product selection at identical prices, thus, results in consumers randomly choosing an intermediary. Consequently, the introduction of competition on the intermediary level would make sense, especially in the context of sophisticated consumers who understand how intermediaries are influenced through commissions. Considering different types of consumers, whereas one part of consumer shows naive traits but another part consists of sophisticated consumers, thus, provides another open opportunity for further research.

Finally, one of the assumptions of the fee-based model is that the intermediary charges a flat transaction fee. Allowing for percentage fees or firm-specific transaction fees could generalize the model. The former usually is levied on expenses where the intermediary collects a small percentage of the total transaction, whereas the latter could be charged for services that are unusual or require additional processing. Allowing the intermediary to charge a firm-specific fee, he would need to take into account the price-quality ratio of each product when recommending the good. Thus, the intermediary not only takes into account the suitability of a good but would need to consider the probability of a good being sold. When the intermediary is entirely selfish, I expect the firms to intensely compete in prices. A lower product price gives the intermediary more margin to set a higher transaction fee and make higher profits. By decreasing their prices firms forgo some profit to indirectly influence the intermediary towards their product. When the intermediary, however, greatly cares about the suitability of the good he will always recommend the suitable good, independent of the prices. Firms, thus, don't have any influence on the intermediary's recommendation behavior. Similar to double marginalization, I expect firms to choose prices similar to a monopoly situation and the intermediary to skim consumers' surplus while taking into account the consumers' probability of buying each good. I leave a rigorous analysis of this issue for future research.

3.7 Appendix

Proof of Lemma 1

If the intermediary becomes purely selfish, i.e. $w(v_h) = 0$, the optimal commission becomes

$$f_i^*|_{w(v_h)=0} = p_i^* - c.$$

The profit of firms A and B, respectively, are given by

$$\Pi_A = [1-q][1-z_A^*](p_A - f_A - c),$$
$$\Pi_B = q[1-z_B^*](p_B - f_B - c).$$

Inserting the optimal commission given that $w(v_h) = 0$, I get

$$\Pi_A = [1-q][1-z_A^*](p_A - (p_A^* - c) - c) = 0,$$
$$\Pi_B = q[1-z_B^*](p_B - (p_B^* - c) - c) = 0.$$

Proof of Lemma 2

The intermediary recommends product A rather than product B when

$$qw(v_h) \geq (1-q)w(v_h)$$
$$q \geq (1-q)$$
$$2q \geq 1$$
$$q \geq \frac{1}{2} = q^*.$$

If the intermediary is purely selfish, i.e. $w(v_h) = 0$, he has the choice between the following two options

$$\{q \cdot 0, (1-q) \cdot 0\} = \{0, 0\}$$

Thus, recommending good A or good B yields the same expected social utility, meaning he doesn't care about his recommendation. Consequently, he will recommend the goods randomly with $q \in [0, 1]$.

Proof of Lemma 3

The maximization problem of the intermediary is given by

$$\max_t \quad [(1-z_A^*) + (1-z_B^*)] \cdot t + w(v_h).$$

The corresponding first-order condition is

$$\begin{aligned}
\frac{\partial[(1-z_A^*)+(1-z_B^*)]\cdot t+w(v_h)}{\partial t} &= 0 \\
\frac{\partial[(1-p_A-t+\frac{3}{4}v_h)+(1-p_B-t+\frac{3}{4}v_h)]\cdot t+w(v_h)}{\partial t} &= 0 \\
\frac{1}{2}\left(1-p_A-2t+\frac{3}{4}v_h+1-p_B-2t+\frac{3}{4}v_h\right) &= 0 \\
2+\frac{6}{4}v_h-p_A-p_B-4t &= 0.
\end{aligned}$$

Solving for t yields

$$t(p_A,p_B)=\frac{1+\frac{3}{4}v_h-\frac{1}{2}(p_A+p_B)}{2}$$

Proof of Proposition 1

The maximization problem of firm A is given by

$$\begin{aligned}
\max_{p_A}\quad \Pi_A &= (1-q^*)(1-z_A^*)(p_A-c) \\
&= \frac{1}{2}\left(1-p_A-t(p_A,p_B)+\frac{3}{4}v_h\right)(p_A-c) \\
&= \frac{1}{2}\left(1-p_A-\frac{1+\frac{3}{4}v_h-\frac{1}{2}(p_A+p_B)}{2}+\frac{3}{4}v_h\right)(p_A-c) \\
&= \frac{1}{2}\left(\frac{1}{2}-p_A+\frac{3}{8}v_h+\frac{1}{4}(p_A+p_B)+\right)(p_A-c).
\end{aligned}$$

The corresponding first-order condition is given by

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A} = \frac{1}{2}\cdot\left(-1+\frac{1}{4}\right)(p_A-c)+\frac{1}{2}\left(\frac{1}{2}-p_A+\frac{3}{8}v_h+\frac{1}{4}(p_A+p_B)+\right) &= 0 \\
\left(-\frac{3}{4}\right)(p_A-c)+\left(\frac{1}{2}-p_A+\frac{3}{8}v_h+\frac{1}{4}(p_A+p_B)\right) &= 0 \\
-\frac{3}{4}p_A+\frac{3}{4}c+\frac{1}{2}-p_A+\frac{3}{8}v_h+\frac{1}{4}(p_A+p_B) &= 0
\end{aligned}$$

Due to the symmetry of firms, in equilibrium, we have that $p_A=p_B$, and thus

$$\begin{aligned}
-\frac{3}{4}p_A+\frac{3}{4}c+\frac{1}{2}-p_A+\frac{3}{8}v_h+\frac{1}{2}p_A &= 0 \\
\frac{3}{4}c+\frac{1}{2}+\frac{3}{8}v_h &= \frac{5}{4}p_A \\
\frac{3}{2}c+1+\frac{3}{4}v_h &= \frac{5}{2}p_A
\end{aligned}$$

$$\rightarrow \qquad p_A^* = p_B^* = \frac{2}{5}\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right).$$

Taking into account the equilibrium prices of firms, the intermediary sets the following transaction fee in equilibrium.

$$\begin{aligned}
t^* &= \frac{1 + \frac{3}{4}v_h - \frac{1}{2}(p_A + p_B)}{2} \\
&= \frac{1 + \frac{3}{4}v_h - \frac{2}{5}\left(1 + \frac{3}{4}v_h + \frac{3}{2}c\right)}{2} \\
&= \frac{1 + \frac{3}{4}v_h - \frac{2}{5} - \frac{6}{20}v_h - \frac{6}{10}c}{2} \\
&= \frac{\frac{3}{5} + \frac{9}{20}v_h - \frac{6}{10}c}{2} \\
&= \frac{3}{10} + \frac{9}{40}v_h - \frac{3}{10}c \\
&= \frac{3}{10}\left(1 + \frac{3}{4}v_h - c\right).
\end{aligned}$$

Proof of Proposition 2

If the share of consumers in the fee-based model is smaller than in the commission-based model, it must hold that

$$\begin{aligned}
1 - z_i^C &> 1 - z_i^F \\
1 - \frac{1}{2}\left(1 - \frac{3}{4}v_h + c\right) &> 1 - \frac{1}{10}\left(7 - \frac{9}{4}v_h + 3c\right) \\
7 - \frac{9}{4}v_h + 3c &> 5\left(1 - \frac{3}{4}v_h + c\right) \\
7 - \frac{9}{4}v_h + 3c &> 5 - \frac{15}{4}v_h + 5c \\
1 + \frac{3}{4}v_h - c &> 0.
\end{aligned}$$

This is true due to assumption (3.2).

If the consumer surplus in the commission-based model is greater than in the fee-based model, it must hold that

$$CS^C > CS^F$$

$$\frac{1}{2} - \left(\frac{1 - \frac{3}{4}v_h + c}{2}\right) + \frac{1}{2}\left(\frac{1 - \frac{3}{4}v_h + c}{2}\right)^2 > \frac{1}{2} - \left(\frac{7 - \frac{9}{4}v_h + 3c}{10}\right) + \frac{1}{2}\left(\frac{7 - \frac{9}{4}v_h + 3c}{10}\right)^2$$

$$-\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)+\frac{1}{2}\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)^2 > -\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)+\frac{1}{2}\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2$$
$$-\frac{1-\frac{3}{4}v_h+c}{2}+\frac{7-\frac{9}{4}v_h+3c}{10} > \frac{1}{2}\left(\left(\frac{7-\frac{9}{4}v_h+3c}{10}\right)^2-\left(\frac{1-\frac{3}{4}v_h+c}{2}\right)^2\right).$$

This has the same structure as

$$-a+b > \frac{1}{2}(b^2-a^2)$$
$$b-a > \frac{1}{2}(b+a)(b-a).$$

Due to assumption (3.1) and (3.2), it is true that $b > a$ and sufficient to show that

$$2 > (b+a).$$

Thus, I have

$$2 > \frac{7-\frac{9}{4}v_h+3c}{10}+\frac{1-\frac{3}{4}v_h+c}{2}$$
$$20 > 7-3\frac{3}{4}v_h+3c+5-5\frac{3}{4}v_h+5c$$
$$8 > -8\frac{3}{4}v_h+8c$$
$$1+\frac{3}{4}v_h > c,$$

which, again, holds due to (3.2).

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