The Origin and Significance of Zero

An Interdisciplinary Perspective

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Much Ado about Nothing or, How Much Philosophy Is Required to Invent the Number Zero?

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Abstract

This article argues that the search for a philosophy that gave rise to the number zero is misguided. No philosophy is required to invent this number. The article further shows that there are good reasons to accept that Buddhism did not play a role in this invention. It further points out that the notion of number as developed in Indian philosophy had no place for zero.

Keywords

zero – empty – philosophy – numerical place-value – Buddhism – numbers in Indian philosophy

1 Introduction

The question what philosophy made the invention of zero possible has often been raised. A question less often raised is whether any philosophy at all is required for this invention. This article will raise this question and find that it is far from obvious that philosophy has to play a role here. It will subsequently look for whatever evidence there might be for a connection between Buddhist philosophy and the invention of zero and repeat the, by now, well-known conclusion that there is none. It will also briefly survey what ideas about numbers were produced in Indian philosophy and point out that these ideas could not possibly give rise to the number zero.

2 Zero and Philosophy

One of the uses of a written numerical place-value system – perhaps the most important one – is that it facilitates arithmetical operations that would be complicated without it. The contrast between Roman and Hindu-Arabic numerals illustrates this sufficiently:

> The addition 123 + 234 (= 357) is easily carried out with Arabic numerals by adding the digits in each of the three columns, but becomes complex with Roman numerals: CXXIII + CCXXXIV = CCCLVII. Subtraction and multiplication become even more complex.

Let us assume, for argument's sake, that certain cultures have adopted a written numerical place value system at least in part for this reason: to facilitate arithmetical operations. If so, they cannot have done without a way of indicating that certain places are 'empty'. Consider the following:

> $123 = 1 \times 100 + 2 \times 10 + 3 \times 1$. The value of each digit ('1', '2', or '3' in this case) depends on its position.

In the analysis of 103, there must be a way to indicate that the second 'column', which represents the value 10, is empty. This can be done by leaving that place open: 1/ /3. It will be less confusing to put some kind of visible marker at that place, for example 1/*/3, or simply 1*3. Having such a marker does not imply that one has accepted the number 0, far from it. However, in specifying how addition, subtraction, and multiplication can be carried out, rules about how to deal with that marker will be necessary. Consider 103 + 234 = 337. Our hypothetical mathematician does not have the number 0. To him this equation looks like this: 1*3 + 234 = 337. There is yet no doubt about the outcome of this addition. But how can the digits of the second column be added if there is there no digit, as in 1*3? How can the known result (viz. 337) be obtained?

Clearly the digits in the '1 category' must be added, and the sum of this addition will occupy the '1 category' of the result: -3 + -4 = -7. The same rule applies for the '100 category': 1- + 2- = 3-. But how to deal with the '10 category', which is empty in one case? The answer is straightforward. It cannot but be: -*- + -3- = -3-.

An analysis of other cases, including subtraction and multiplication, will reveal that the empty placeholder * follows rules that may be expressed as follows:

X + * = X
X − * = X
X × * = *

Note that these rules can be obtained without the help of a zero *concept*. Perhaps one should say: *Zero needs no concept in order to be useful in arithmetic*. Anyone who knows the rules can calculate with zero and profit from its usefulness.

3 Origin

When did the written numerical place value system arise in India?[1] In a short article that came out many years ago (Bronkhorst, 1994), attention was drawn to a passage in Vasubandhu's *Abhidharmakośa Bhāṣya* that is ascribed to a certain Bhadanta Vasumitra. This same passage occurs in the Chinese translations of the *Mahāvibhāṣā* and the *Vibhāṣā*. These texts may have been composed during the reign of Kaniṣka, the *Vibhāṣā* presumably somewhat earlier. All this supports the claim that Vasumitra is to be dated at that same period. Well, the passage attributed to him illustrates his position with the help of a *vartikā* that in the unit position has the value of a unit, in the hundreds position that of a hundred, and in the thousands position that of a thousand. This was presented as evidence for the existence of a written numerical place value system during the early centuries of the Common Era. This conclusion seems no longer valid. Dominik Wujastyk (2018, p. 41) rightly makes the following observation: 'Vasubandhu's description may refer not to writing but to placing a strip or tube on a marked board, perhaps analogous to an abacus. The word *vartikā* that he used means a wick, stalk, paintbrush, or twist of cloth. It is not clear what Vasumitra was describing.' In short, this passage provides no proof that a written numerical place value system existed at that time.

However, a variant of Vasubandhu's passage occurs in the *Yogaśāstra* on sūtra 3.13. It uses *rekhā* ('line, scratch') instead of *vartikā*, which could suggest some kind of writing. 'This passage has been cited as the earliest unambiguous description of written place-value notation using digits, and Patañjali's version

1 Lam (1986; 1987; 1988) argues that a written numerical place value system with zero existed in China, too. Martzloff (1995) shows that these conclusions have to be looked at with suspicion.

is datable to the period 375–425' (Wujastyk, 2018: p. 41). This would then be evidence for a written place value system that existed around 400 CE.[2]

A century after Patañjali, the author on Yoga, the situation becomes clearer. A number of texts from the middle of the first millennium onward present clear evidence for a written place value system (See, for example, Gāṅguli, 1932; 1933). One of these is Varāhamihira's *Pañcasiddhāntikā*, which 'incidentally states two fundamental arithmetical operations by the zero … viz. addition and subtraction, in more than one place' (Datta, 1926, p. 451). Other texts, too, show acquaintance with both the written numerical place value system and with the rules how to manipulate zero.

4 Zero and Philosophy Again

Is zero anything more than this? Consider the following observation:

> If zero merely signified a magnitude or a direction separator, the Egyptian zero, **nfr**, dating back at least four thousand years, amply served these purposes. If zero was merely a placeholder symbol, then such a zero was present in the Babylonian positional number system before the first recorded occurrence of the Indian zero. If zero was represented by just an empty space within a well-defined positional number system, such a zero was present in Chinese mathematics a few centuries before the beginning of the Common Era. The dissemination westwards of the Indian zero as an integral part of the Indian numerals is one of the most remarkable episodes in the history of mathematics … the Indian zero was a multi-faceted mathematical object: a symbol, a number, a magnitude, a direction separator, and a placeholder, all in one operating within a fully established positional numeration system. (Joseph, 2008, pp. 37–38)

This is not the place to evaluate these claims. However, it is far from obvious where – in the development of these aspects of zero – philosophy comes in. To my eye, these features considerably facilitated arithmetic operations and were

2 Certain Jaina texts were apparently acquainted with a written numerical place value system. It is unfortunately extremely difficult (if not impossible) to date these texts, so that it is impossible to draw chronological conclusions. The *Lokavibhāga* (Joseph, 2016, p. 105) might be an exception. The fact that it only survives in a Sanskrit translation that is younger than the original text makes it, once again, difficult to draw chronological conclusions.

a plausible consequence of the introduction of a written numerical place value system, and an intelligent extension of it. *But where is the philosophy*?

Philosophy did, of course, play a role in the history of zero, if not in its invention. As is well known, zero did not find a warm welcome in Europe when it was introduced there in the thirteenth century. Aristotelian philosophy had no room for empty space, and the arrival of the number zero was felt as a threat by many a churchman, delaying its full establishment by centuries. Lots of philosophy here, but philosophy that stood in the way of zero, not philosophy that promoted its invention or use.

5 Buddhism and Zero

Returning to India, no one has yet shown that there is any connection between the Buddhist philosophy of emptiness and the invention of zero. Scholars go on and on about it, no doubt because that philosophy frequently uses the term *śūnya*. That same term is used in mathematical literature to designate zero. By itself, this proves nothing. The word *śūnya* means 'empty'. 'Empty' is a common word and can be applied to numerous altogether different situations, both in India and in the West. No arguments to support the connection between Buddhist emptiness and zero are known to me, apart from pure (one would almost say 'empty') speculation.

The lack of evidence for a link between Buddhism and zero is not surprising. Elsewhere attention has been drawn to the virtual non-existence of Buddhist treatises on astrology, astronomy, and mathematics. This non-existence, it was there proposed, is due to the fact that Buddhism in classical India had taken the position that there were occupations that were best left to Brahmins, and these included astrology, astronomy, and mathematics.[3]

Consider at this point some of the other words that were used in mathematical texts to refer to zero; these include *kha*, *ambara*, *antarikṣa*, *gagana*, *abhra*, *viyat*, *nabhas*, *ākāśa*. These words have one philosophically loaded meaning in common: 'sky, ether'. One might think that empty space is meant, but this would not be correct. Ether, in Indian philosophical thought, is not empty. It is an omnipresent element and therefore an existing element. This is even so in Buddhist scholasticism, where ether is an unconditioned element (*asaṃskṛta dharma*) and therefore an existing 'thing'.

3 Bronkhorst, 2011, section 3.1, esp. p. 109 (but cf. Bronkhorst, 2018, p. 319).

6 Numbers in Indian Philosophy

No one seems to have paid attention to what Indian philosophers actually thought about numbers and related matters. The texts that inform us about zero and the numerical place value system are, most of them, Brahmanical texts. No Buddhist texts betray acquaintance with them. It will therefore be appropriate to look at what Brahmanical thinkers thought about the nature of numbers.

Vaiśeṣika is the school to look at. Its vision of the world influenced other schools of thought. Vaiśeṣika had sophisticated ideas on the nature of numbers from *one* onward. These numbers were thought of as qualities that inhere in substances. 'One tree', for example, refers to a tree that has the quality 'one'; 'two trees' refers to two trees that share the quality 'two'; etc. The story as to how the numbers from two onward come about is complicated and involves the observer. It is not necessary to deal with it in detail.[4] More important for us at present is that these numbers (1, etc.) cannot exist independently of the substances in which they inhere. What is more, this understanding of numbers has no place for zero.

Incidentally, Vaiśeṣika had no room for infinity either. According to its classical text (called *Padārthadharmasaṃgraha* or *Praśastapādabhāṣya*), there is a highest number, called *parārdha*.[5]

4 By way of example, here is a description of how the number two ('duality') and its cognition come about according to the fourteenth century *Compendium of All Philosophies* (*Sarvadarśanasaṃgraha*), ed. Abhyankar p. 221: (1) First there is connection between sense and object. (2) It gives rise to knowledge of the universal one-ness. (3) After this a combining cognition comes about. (4) From this duality arises. (5) This gives rise to knowledge of the universal duality-ness. (6) It gives rise to knowledge of the quality duality. (7) After this the idea 'these are two substances' comes about. (8) This gives rise to a mental trace.

5 Bronkhorst & Ramseier, 1994: xxx: *ekādivyavahārahetuḥ saṃkhyā/ sā punar ekadravyā cānekadravyā ca/…/ anekadrvavyā tu dvitvādikā parārdhāntā/.* Ganeri (2001, p. 435 n. 6) makes the following perceptive observation about *parārdha*: 'Praśastapāda states that, after one, the numerical qualities run from two to a large but finite number *parārdha*. The number *parārdha* is mentioned in many texts as the highest decimal place name. The precise value of *parārdha* varies: in the *Taittirīya Saṃhitā* and the *Maitrāyaṇī Saṃhitā* it is given as 10^{12}, while the *Kāṭhaka Saṃhitā* records both 10^{12} and 10^{13}. Among the mathematicians, it is always 10^{17}. There are names for higher decimal powers in Buddhist and Jaina texts. Only the Nyāya-Vaiśeṣika takes *parārdha* to be the highest number, and not merely the highest named place value. There is no room for the idea of a maximal finite number if one thinks of the number series as generated by recursive application of the successor function, but among the Nyāya-Vaiśeṣika authors, only Bhāsarvajña attempts so to construct the number series. Within a conception of number as qualities of substances, indeed, it seems that there has to be a largest number, if the number of things in the cosmos is finite.' '*Parārdha* is not a number *per se*, but the number of substances' (Lysenko, 1994, p. 786).

Some years ago, there was occasion to observe that early mathematical texts in India betray no acquaintance with philosophical literature, even though their acquaintance with grammatical literature is strong and evident. The earliest mathematical work of some length, Bhāskara's commentary on Āryabhaṭa's *Āryabhaṭīya*, 'often cites grammatical and generally linguistic texts … astronomical texts, some religious and literary treatises, but not a single philosophical work' (Bronkhorst, 2001, p. 64). The Vaiśeṣika ideas about numbers existed well before the texts that provide evidence for numerical place value and zero. It is yet possible to exclude that our mathematicians drew inspiration from these ideas; if they had, they would not have been able to develop their new methods. Influence in the opposite direction did not take place either. Vaiśeṣika remained unperturbed by the new arithmetic and held on to its understanding of the nature of numbers.[6]

7 Conclusion

It seems highly unlikely that there was any link between the invention of zero and Indian philosophy in any of its forms. The importance of zero in the history of mathematics (and much more) cannot be underestimated, but it is important to remember that this discovery did not need philosophy. Worse, when philosophy got involved – i.e., when zero and the written place value system reached Western Europe – it stood in the way of this marvelous invention and unnecessarily delayed its adoption by several centuries.

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6 Cp. Ganeri (2001, p. 430): 'The theory of number developed by the philosophers is not based on any great awareness of the work of mathematicians, nor is its purpose to study the philosophical foundations of mathematics or scientific practice.' Lysenko (1994, p. 784): '[W]e cannot find in our [Vaiśeṣika] texts any account of mathematical operations like addition, subtraction, multiplication and division.'

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