The ultimate problem of the tribes (Part 2)

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In the previous issue of the TILT, you can find Gus Simmons' challenge :

Four people on an island

On an island there are three tribes. The members of one tribe tell only the truth. The members of another tribe always lie. The members of the third tribe, like most of us, tell the truth sometimes and lie sometimes. The social custom on the island is that in any gathering of three or more individuals each tribe must be represented.

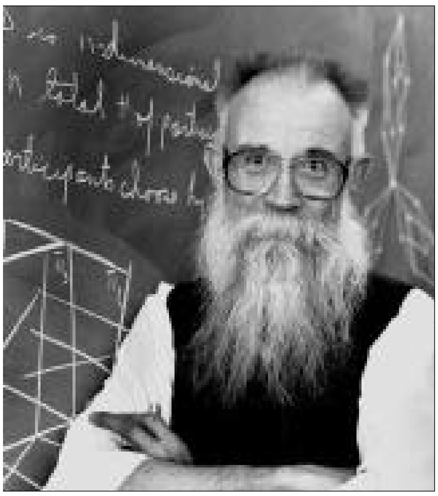
An explorer lost on the island, but familiar with the strange customs of the natives, comes on three individuals in a clearing in the jungle ; call them A, B and C for convenience. While he is trying to figure out what questions to ask, and to whom, to get directions to go back to his camp that he can trust, the natives speak to him.

A says, "Ask C, he always tells the truth".

B says, "Oh no, you can't believe anything C says".

C says, "That advice ought to confuse you".

The explorer now knows who to ask for directions. Do you ?



Gustavus J. Simmons

Gustavus J. Simmons is usually considered as the "*Father of Authentication Theory*". He is "*Senior Fellow for National Security Studies*" at the Sandia National Laboratories, Albuquerque (NM), USA. Earlier he was "Manager of the Applied Mathematics Department" and **"Supervisor" of one of two divisions at Sandia devoted to the command and control of nuclear weapons.**

He received his Ph.D. degree in mathematics for his research in combinatorics and graph theory. Then he focused his research on applied topics of information theory and cryptography. His scientific contributions have been applied in several domains where information integrity arises at a national security level : identification of individuals at sensitive facilities, command and control of nuclear weapons, verification of compliance with various arm control treaties.

Dr. Simmons received two major distinctions in 1986: the "*U.S Government's E. O. Lawrence Award*" and the "*Department of Energy Weapons Recognition of Excellence Award*". In 1991 he was awarded an "*honorary Doctorate of Technology*" by the University of Lund (Sweden). In 1996 Cambridge University in England named him "*Rothschild Professor of Mathematics at the Newton Institute of Mathematics*" and Trinity College made him a "*visiting Fellow Commoner*".

Dr. Simmons has published more than 120 papers and books. He is the author of the section on cryptology in the 16th edition of the Encyclopaedia Britannica. He is also an editor for some of the most famous publications in cryptology, in particular, for the Journal of Cryptology and Codes, Designs and Cryptography.

Solution:

As the group of natives contains exactly three individuals, we know that there must be one representative of each tribe.
A says, "Ask C, he always tells the truth".
We deduce that **A** is not the one who always tells the truth. Otherwise we would have two representatives of the same tribe in this gathering of three individuals.
Therefore, the one who always tells the truth is either **B** or **C**. Let's consider now the second affirmation :

B says, "Oh no, you can't believe anything C says".

- either **B** is the one who always tells the truth
  (Then **C** would be the liar and **A** should be the one who tells the truth sometimes and lies sometimes ; in this case **A** would have lied.)
- or **C** is the one who always tells the truth
  (Then **A** would have told the truth and would be the one who tells the truth sometimes and lies sometimes. Consequently **B** should be the liar.)

At this point both solutions are still compatible with what has been said.

Therefore, when **C** says, "That advice ought to confuse you", he tells the truth. This eliminates the first solution and it remains a unique solution for the challenge :

- **A** tells the truth sometimes and lies sometimes,
- **B** always lies,
- **C** always tells the truth.

The explorer should ask **C**.

Is this challenge optimal?

After having solved Gus Simmons' challenge I built a modified version with four tribes on the island (instead of three), and I succeeded in keeping only three sentences. This new version gives therefore a counterexample to the optimality of the original challenge.

How is it possible?
It is true that three sentences can uniquely distinguish at most 2*2*2=8 different subsets of solutions. However, these subsets do not need to have the same number of elements. The condition simply requires that the subset of all compatible solutions contains actually only one element.

After reading the problem, Gus Simmons wrote to me : "Your puzzle […] deserves the title [of] the ultimate island problem."

> On an island there are four tribes. The members of one tribe tell only the truth. The members of another tribe always lie. The members of the third tribe, like most of us, tell the truth sometimes and lie sometimes. The member of the fourth tribe always lie, but never answer a question…
> The social custom on the island is that in any gathering of four or more individuals each tribe must be represented.
> An explorer lost on the island, but familiar with the strange customs of the natives, comes on four individuals in a clearing in the jungle ; call them A, B, C and D for convenience. While he is trying to figure out what questions to ask, and to whom, to get directions to go back to his camp that he can trust, the natives speak to him.
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> A says, "Ask D, he always tells the truth".
> B says, "Don't ask me anything, I never answer a question".
> C says, "That's true! B never answers a question".
>
> The explorer now knows who to ask for directions. Do you?

Question: Can you deduce to which tribes A, B, C and D belong? Show that the solution is unique and have fun!

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