Long-Term Care Models and Dependence Probability Tables by Acuity Level: New Empirical Evidence from Switzerland[*]

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Abstract

Due to the demographic changes and population aging occurring in many countries, the financing of long-term care (LTC) poses a systemic threat. The scarcity of knowledge about the probability of an elderly person needing help with activities of daily living has hindered the development of insurance solutions that complement existing social systems. In this paper, we consider two models: a frailty level model that studies the evolution of a dependent person through mild, moderate and severe dependency states to death and a type of care model that distinguishes between care received at home and care received in an institution. We develop and interpret the expressions for the state- and time-dependent transition probabilities in a semi-Markov framework. Then, we empirically assess these probabilities using a novel longitudinal dataset covering all LTC needs in Switzerland over a 20-year period. As a key result, we are the first to derive dependence probability tables by acuity level, gender and age for the Swiss population. We find that the transition probabilities differ significantly by gender, age and time spent in the frailty level and type of care states.

Key words long-term care · semi-Markov model · actuarial dependence tables

1 Introduction

One of the most dramatic challenges facing many high-income countries is population aging. Therefore, long-term care (LTC) delivered to elderly persons in need of assistance in activities of daily living (ADL, e.g., dressing, bathing, eating) is predicted to increase in the foreseeable future (United Nations, 2015). In many countries, over a 30-year horizon from the present, spending on formal LTC is expected to reach approximately 2% of GDP (Colombo et al., 2011; Rockinger and Wagner, 2016; Fuino and Wagner, 2018b) while the value of informal care delivered by relatives remains important (Pickard et al., 2000; Karlsson et al., 2006; Brown

and Finkelstein, 2009; Zhou-Richter et al., 2010; Courbage et al., 2018). This stresses the relevance of proper financing and pricing of LTC. At present, countries employ various approaches to distribute these costs (see, e.g., Colombo, 2012, Costa-Font et al., 2015). Often, one part is taken over by state social systems, either through comprehensive universal schemes offering basic coverage to the entire population or means-tested schemes that subsidize individuals' expenses. Such systems are typically financed through levies from salaries or tax contributions. Another part of the LTC costs is borne by health or other private insurance plans. However, the availability of such insurance is often limited, even in the most developed LTC markets (e.g., the US, the UK, or France). Indeed, private insurers face difficulties in determining proper pricing, which often entails higher premiums and re-pricing (Carrns, 2015). Finally, in many countries, households cover more than one-third of the formal LTC costs. For example, in Switzerland, no attractive or affordable insurance offering exists, and Switzerland ranks among the countries with the highest out-of-pocket spending (Swiss Re, 2014). The problem in pricing LTC solutions essentially results from a lack of knowledge on individuals' health paths. Additionally, the effect of gender, age and other sociodemographic factors such as culture (Eugster et al., 2011; Gentili et al., 2017) is often not well understood. In fact, in the Swiss LTC system, benefits from the first pillar of the old-age social insurance law add on the reimbursement of the pure medical costs covered by health insurance. On the one hand, old-age social insurance is financed by contributions from the salaries and taxes. Thus, no proper premium calculation is done by the State. On the other hand, private LTC insurance is very little developed in Switzerland due the scarcity of dependence data and little incentives for privates. Currently, no LTC dependence tables are available for Switzerland since no claims experience has been privately recorded yet.

The aim of our work is to reduce this gap by deriving dependence tables that provide a basis for the pricing of LTC solutions. With respect to the main cost drivers, which are the frailty level, the type of care and the time spent in dependence, we statistically describe individual transitions through the dependency states. To do so, we use a comprehensive longitudinal dataset covering the total dependent population in Switzerland over a 20-year period. We detail the transition probabilities by gender and by age as a function of the duration in dependency.

The actuarial valuation of LTC products is commonly based on Markov processes given the available data (see, e.g., Haberman and Pitacco, 1999; Pritchard, 2006; Ameriks et al., 2011; Christiansen, 2012; Brown and Warshawsky, 2013; Govorun et al., 2015; Ai et al., 2016). Thereby, the calculation of transition intensities between frailty states at different ages is often the focus (e.g., Levantesi and Menzietti, 2012; Fleischmann, 2015; Fong et al., 2015). This approach, however, only considers the previously visited state to be relevant information to determine the future state. Since the seminal work of Hoem (1972), several studies in the area of disability have identified two important factors: if the transition probability depends on the previous state, it also depends on the time spent in that state. In their study on German LTC insurance, Czado and Rudolph (2002) extend a Markov-type model by introducing time-dependent transition intensities along frailty levels and types of care. Within a similar setup, Helms et al. (2005) estimate transition probabilities and calculate insurance premiums for LTC plans. To consider both factors, the *semi*-Markov model extends the Markovian approach and allows to choose the duration law (Janssen and Manca, 2001, 2007; Denuit and Robert, 2007). This semi-Markov framework has long been applied to understand individuals' patterns with respect to health status (see, e.g., D'Amico et al., 2009; Adékambi and Christiansen, 2017). In the biomedical field for example, Foucher et al. (2005, 2007, 2010) calibrate a semi-Markov model on interval-censored data. Fur-

ther, the works of De Uña-Álvarez and Meira-Machado (2015) and Guibert and Planchet (2017) discuss the direct estimation of transition probabilities using non-parametric techniques. More recently, works on LTC have adopted the semi-Markov approach. When modeling reverse mortgages for the UK and US markets, Ji et al. (2012) consider LTC facilities among health-related reasons for terminating the mortgage. Based on a French LTC insurance portfolio, Planchet and Tomas (2013) study the mortality of LTC claimants while Biessy (2015b) discusses the transition intensities through four levels of dependency. Biessy (2016), using an illness-death model, studies the impact of pathologies on the evolution of LTC.

In this paper, we develop two semi-Markov models to address LTC pricing in Switzerland. Our work is most similar to Biessy (2015b); however, we consider two separate models, derive transition probabilities and apply our study to a larger empirical dataset. We address the question of the type of care received, basing our analyses on data on the total elderly population in need of LTC. The first model is a five-state model in which we distinguish autonomy, three frailty levels along mild, moderate and severe dependency, and death. This model permits to explain the evolution through the considered acuity levels. In the second model, our aim is to investigate the transitions between care received at home and care received in institutions because the costs associated with the two types of care are significantly different. To do so, we introduce a four-state model including autonomy, care at home, care in an institution, and death. We separately define age-dependent transition probabilities for men and women (cf. Fong et al., 2015). Our empirical analysis reveals that a Weibull duration law accurately models the time spent in the previous state. We formulate the likelihood for the semi-Markov model and find the solution by estimating the maximum likelihood. We provide an application using novel longitudinal data on the total old-age dependent population in Switzerland from 1995 to 2015. Our dataset provides complete information on the paths of $284\,482$ dependent individuals, including dates of transitions, dependency states, gender and age. This represents an extension of the studies mentioned above that solely use private insurance datasets with a more limited number of observations.

Our main results are two-dimensional transition probabilities defined by age and the elapsed time in the previous state for both men and women. To the best of our knowledge, we are the first to provide such a detailed study and to derive actuarial tables for Switzerland. We present significant results on the evolution of dependencies and the types of care received. Our key findings are as follows: First, we observe significant differences in transition probabilities between men and women and between individuals below and above 80 years of age. Further, the probabilities of staying in a dependency state decrease with age and duration, whereas the probabilities of leaving a state increase with duration. While the average total time spent in dependence is approximately three years (Fuino and Wagner, 2018a), we find that elderly persons cared for at home enter institutional care after approximately one year. Second, for short durations, mildly dependent individuals have a higher mortality relative to moderately and severely dependent persons. This may be linked to the fact that mildly dependent elderly are more often cared for at home and that different pathologies are underlying their dependence (see, e.g., Monod-Zorzi et al., 2007; Biessy, 2017). Third, we find that women, given their lower mortality, spend more time than men in any of the dependency states. This supports the study of Fong et al. (2017) which finds that female experience more years in severe dependence than male.

This article is organized as follows: Section 2 introduces our two models and the mathematical aspects of the semi-Markov framework. In Section 3, we provide descriptive statistics on the transitions and the time spent in the various dependency states. In Section 4, after a brief presentation of the numerical implementation, we empirically calibrate the parameters of the model, report robustness tests and present the dependence probability tables. In the two considered models, we discuss our findings by comparing the results for males and females for selected ages. We conclude in Section 5.

2 Model framework

In this paper, we consider a model framework conceptualizing the frailty levels and types of care as recorded in Switzerland. Typically, three frailty levels (mild, moderate, severe) are considered and care given is differentiated between at-home and institutional care. After the introduction of the models (see Section 2.1), we develop on a semi-Markov-type model to calculate the hazard rates and transition probabilities. We separately discuss the probability of entering dependence in Section 2.3. The model framework developed in this section can be directly applied to statistical data to pose the actuarial bases for pricing insurance products.

2.1 Old-age dependency models

We consider two models: the first is linked to the transition between frailty levels and death, and the second focuses on the transitions between states with different types of care. Overall, six different states of dependency are considered in our framework (Czado and Rudolph, 2002; Commenges and Joly, 2004). They result from the combination of the three dependency levels (mild, moderate, severe) and the two types of care (at-home and institutional care).

Our first model focuses on frailty levels. The assessment of care needs is usually based on the number of limitations in ADL (e.g., dressing, bathing, eating; see also Section 3.1). This international measure is used to determine a dependent person's need for care and is considered a rather accurate proxy for the hours of care required. In this sense, serious cases require more care and entail higher costs. In many developed countries such as Switzerland, the state offers an allowance based on the patient's dependency level. This motivates our first model, which analyzes the individual paths of a dependent person through three frailty levels while recording the time spent in each state from autonomy to death.

Figure 1 provides a representation of the frailty level model. We consider five states. The first state is the autonomous state (0). Then, three dependency states are distinguished by their respective severity: mild (1), moderate (2) or severe (3). The last state considered is the death state (4). In our model, we focus on nine transitions described by the plain arrows, omitting recovery and death from autonomy (dashed arrows). First, we study the transitions between acuity levels (three transitions) and from dependency to death (three transitions), where the probabilities are calculated using a semi-Markov model (Section 2.2). Second, regarding the three transitions from autonomy to dependency, we use prevalence rates and Markov probabilities (Section 2.3). We disregard total and partial recovery. This is a reasonable assumption because in practice recovery probabilities are negligible and such hypothesis is frequently used in the academic research (e.g., Foucher et al., 2010; Levantesi and Menzietti, 2012; Biessy, 2015a; Fong et al., 2017). Our assumption is further supported by the available empirical data covering the

whole of Switzerland where less than 0.5‰ are recovery transitions (see the later discussion in Section 3.1). The mortality of the autonomous population is outside the scope of our model because we are interested in modeling the characteristics of the dependent population. Furthermore, mortality statistics are not available for the sole autonomous population making an unbiased study impossible.
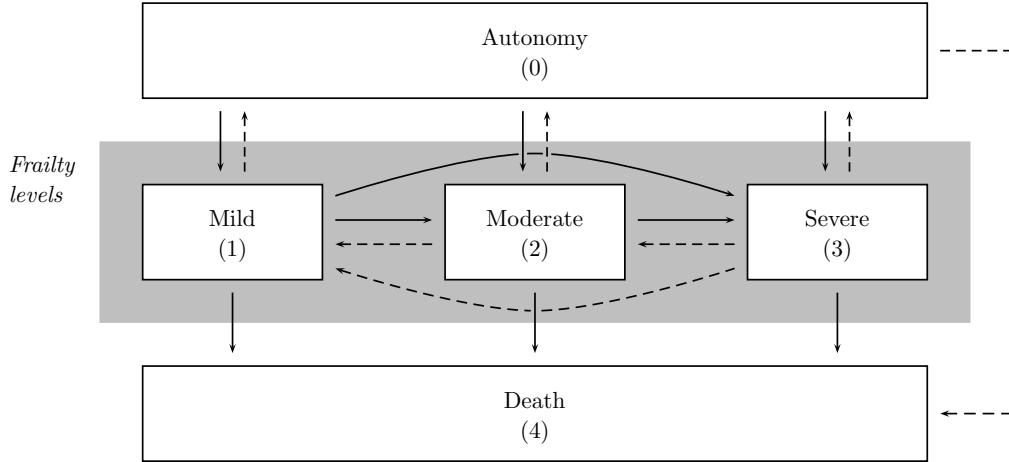

Figure 1: Illustration of the frailty level model.

While the frailty level model is best suited to studying the increasing dependency levels of the elderly, our second model focuses on the types of care. Depending on the country, different types of care facilities are available, and individuals may choose between receiving care at home and receiving care in an institution (Costa-Font and Courbage, 2012). Living in institutions produces higher costs than receiving care at home, especially due to the additional accommodation costs (e.g., laundry, feeding). This is highly relevant when pricing insurance solutions. Thus, adding on the information from the frailty level model, we distinguish the type of care facilities based on whether accommodation is included. The at-home care type represents individuals receiving care in their own place of residence without needing accommodation, while the institutional care type includes lodging and meals. Based on this cost driver, we elaborate a second model that considers four states: autonomy (0); two types of care, care at home (a) and care in an institution (b); and death (4). Similar to the first model, we do not consider recovery or returns to care at home from institutional care. Figure 2 illustrates this second model and the possible transitions.

2.2 The semi-Markov model framework

In the following, we introduce the general theoretical framework for a semi-Markov process first developed by Hoem (1972) (see also, e.g., Janssen and Manca, 2007, for theoretical foundations and applications). In contrast to a Markov process, which depends solely on the previously visited state, a semi-Markov process incorporates a time variable, that is, the transition probability is both affected by the previously visited state and the time spent in it. Our objective is to calculate the transition probabilities between the dependency and death states in the two models described above, separately with respect to gender and age, breaking each gender-age combination into a homogeneous model. Thus, we address the non-homogeneity by splitting the dataset by gender (male and female) and by age (integer ages) when conducting our estimations (see the implementation notes in Section 4.2).
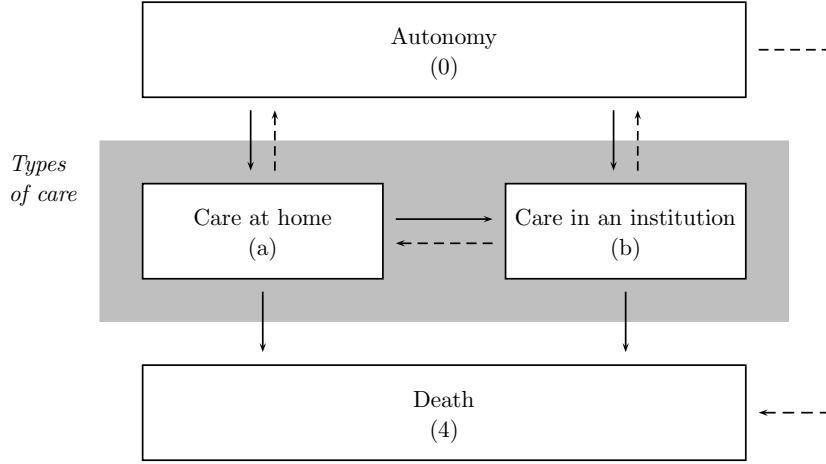
Figure 2: Illustration of the type of care model.

Theoretical framework

In the sequel, we follow the notations proposed by Janssen and Manca (2001) and Saint-Pierre (2005). While all the below quantities are calculated for each gender-age combination separately, we omit indices referring to gender and age to improve readability. With the state space $I = \{1, 2, \ldots, m\}$, let us consider the states J_n, $n \in \mathbb{N}$. Let T_n denote the time of the $(n+1)$th transition going from state J_n to state J_{n+1}. Then,

$$X_{n+1} = T_{n+1} - T_n > 0, \tag{1}$$

is the sojourn time in state J_{n+1}. We assume that the transition probability for someone staying from some time s to time $t + s$ reduces to the same probability as someone staying from time 0 to t; i.e., when arriving in a given state, time is reset to zero. Thus, the time variable t is interpreted as a duration. The semi-Markov kernel $Q_{ij}(t)$ completely defines the process as follows:

$$Q_{ij}(t) = \Pr(J_{n+1} = j, X_{n+1} \leq t \mid J_n = i). \tag{2}$$

The function $Q_{ij}(t)$ represents the probability for the process to travel from state i to state j at the $(n+1)$th transition before a duration t in state i. In each gender-age group, this expression is entirely defined by both the underlying embedded Markov chain and the duration law. The Markov chain describes the probability ϕ_{ij} to go from state i to state j disregarding the time spent in the states:

$$\phi_{ij} = \lim_{t \to +\infty} Q_{ij}(t) = \Pr(J_{n+1} = j \mid J_n = i). \tag{3}$$

The duration law $F_{ij}(t)$, characterizes the duration elapsed in state i before the transition to state j occurs, i.e.,

$$F_{ij}(t) = \Pr(X_{n+1} \leq t \mid J_n = i, J_{n+1} = j). \tag{4}$$

This distribution function describes the cumulative probability of the time spent in the previous state, knowing that the process traveled from state i to state j. Without loss of generality, the semi-Markov kernel $Q_{ij}(t)$ can be explicitly expressed as

$$Q_{ij}(t) = \phi_{ij} F_{ij}(t). \tag{5}$$

Furthermore, assuming that it exists, we define the density function $f_{ij}(t)$ of the duration law $F_{ij}(t)$ by

$$f_{ij}(t) = \frac{\partial F_{ij}(t)}{\partial t}. \tag{6}$$

By definition, the instantaneous transition probabilities $\lambda_{ij}(t)$ are obtained from (De Dominicis and Janssen, 1984):

$$\lambda_{ij}(t) = \frac{\phi_{ij}\, f_{ij}(t)}{\sum_{j=1}^{m} \phi_{ij}\,(1 - F_{ij}(t))} \qquad \text{if } \phi_{ij} \neq 0 \text{ and } F_{ij}(t) \neq 1, \tag{7}$$

and $\lambda_{ij}(t) = 0$ otherwise.

Transition probabilities

The above Equation (7) expresses the instantaneous probability that the process exits state i for state j in an infinitesimal time interval $]t, t+dt]$. Consequently, the transition probabilities $p_{ij}(t)$ from state i to state j after a duration t that solve the semi-Markov model are

$$p_{ij}(t) = \begin{cases} e^{-\int_0^t \sum_{k \neq i} \lambda_{ik}(\tau)\, d\tau} + \int_0^t p_{ii}(\tau) \sum_{k \neq i} \lambda_{ik}(\tau)\, p_{kj}(t - \tau)\, d\tau & \text{if } i = j, \\ \int_0^t p_{ii}(\tau) \sum_{k \neq i} \lambda_{ik}(\tau)\, p_{kj}(t - \tau)\, d\tau & \text{if } i \neq j. \end{cases} \tag{8}$$

This actuarial formulation of the transition probabilities can be obtained from the Chapman-Kolmogorov equation (see Pitacco, 1995, who provides a comprehensive approach for pricing disability benefits). In the case in which $i = j$ in Equation (8), the expression of the semi-Markov probability can be decomposed into two parts. The first term considers the case of staying in state i, while the second term defines the case of having made at least one transition. This term accounts for all the possible paths that return to i after having left it. When $i \neq j$, the expression in Equation (8) simplifies because the process has left the initial state at least once and only the second term remains. Using the hypothesis that recovery transitions are not possible in our model (Section 2.1), we have $p_{ij} = 0$ and $\lambda_{ij} = 0$ for $i > j$. Thus, Equation (8) becomes

$$p_{ij}(t) = \begin{cases} e^{-\int_0^t \sum_{k > i} \lambda_{ik}(\tau)\, d\tau} & \text{if } i = j, \\ \int_0^t p_{ii}(\tau) \sum_{k > i} \lambda_{ik}(\tau)\, p_{kj}(t - \tau)\, d\tau & \text{if } i < j. \end{cases} \tag{9}$$

Recall that in practice the above transition probabilities $p_{ij}(t)$ are calculated for each gender and each age separately.

We now apply the above results to the dependency models introduced in Section 2.1. From Equation (9), we can explicitly express the transition probabilities linked to the transitions from both models. For the frailty level model, we have the state space $I = \{1, 2, 3, 4\}$. We exclude the autonomous state (0) in I since we separately discuss the probability of losing autonomy in Section 2.3. The "staying" probabilities with $i = j$, where $i, j \in I$, are written as follows:

$$
\begin{aligned}
p_{11}(t) &= e^{-\int_0^t \lambda_{12}(\tau) + \lambda_{13}(\tau) + \lambda_{14}(\tau)\, d\tau}, \\
p_{22}(t) &= e^{-\int_0^t \lambda_{23}(\tau) + \lambda_{24}(\tau)\, d\tau}, \\
p_{33}(t) &= e^{-\int_0^t \lambda_{34}(\tau)\, d\tau}, \\
p_{44}(t) &= 1.
\end{aligned}
\tag{10}
$$

The probabilities $p_{11}(t)$, $p_{22}(t)$, $p_{33}(t)$ and $p_{44}(t)$ denote the probabilities of staying in the mild (1), moderate (2), severe (3) or death (4) states for a duration t. Note that state 4, representing death, is an absorbing state and leads to $p_{44} = 1$. The "leaving" probabilities when $i < j$ complement the definition of the semi-Markov chain:

$$p_{12}(t) = \int_0^t p_{11}(\tau)\,\lambda_{12}(\tau)\,p_{22}(t-\tau)\,\mathrm{d}\tau,$$

$$p_{13}(t) = \int_0^t p_{11}(\tau)\,[\lambda_{12}(\tau)\,p_{23}(t-\tau) + \lambda_{13}(\tau)\,p_{33}(t-\tau)]\,\mathrm{d}\tau,$$

$$p_{14}(t) = \int_0^t p_{11}(\tau)\,[\lambda_{12}(\tau)\,p_{24}(t-\tau) + \lambda_{13}(\tau)\,p_{34}(t-\tau) + \lambda_{14}(\tau)\,p_{44}(t-\tau)]\,\mathrm{d}\tau,$$

$$p_{23}(t) = \int_0^t p_{22}(\tau)\,\lambda_{23}(\tau)\,p_{33}(t-\tau)\mathrm{d}\tau,$$

$$p_{24}(t) = \int_0^t p_{22}(\tau)\,[\lambda_{23}(\tau)\,p_{34}(t-\tau) + \lambda_{24}(\tau)\,p_{44}(t-\tau)]\,\mathrm{d}\tau,$$

$$p_{34}(t) = \int_0^t p_{33}(\tau)\,\lambda_{34}(\tau)\,p_{44}(t-\tau)\,\mathrm{d}\tau.$$

$$(11)$$

These expressions deserve a brief interpretation. For example, the expression of the probability p_{12} considers the probability $p_{11}(\tau)$ of remaining for a time τ in state 1 and reaching state 2 through the direct transition from state 1 to state 2 after a duration t. The factor $p_{22}(t-\tau)$ expresses staying in state 2 during the remaining time $t - \tau$. The other transition probabilities follow the same reasoning.

Similarly, for the type of care model with $I = \{a, b, 4\}$, we have the probabilities of staying in care at home (a) and care in an institution (b),

$$p_{aa}(t) = e^{-\int_0^t \lambda_{ab}(\tau) + \lambda_{a4}(\tau)\,\mathrm{d}\tau},$$

$$p_{bb}(t) = e^{-\int_0^t \lambda_{b4}(\tau)\,\mathrm{d}\tau},$$

$$(12)$$

and, again, $p_{44}(t) = 1$. The "leaving" probabilities are as follows:

$$p_{ab}(t) = \int_0^t p_{aa}(\tau)\,\lambda_{ab}(\tau)\,p_{bb}(t-\tau)\,\mathrm{d}\tau,$$

$$p_{a4}(t) = \int_0^t p_{aa}(\tau)\,[\lambda_{ab}(\tau)\,p_{b4}(t-\tau) + \lambda_{a4}(\tau)\,p_{44}(t-\tau)]\,\mathrm{d}\tau,$$

$$p_{b4}(t) = \int_0^t p_{bb}(\tau)\,[\lambda_{b4}(\tau)\,p_{44}(t-\tau)]\,\mathrm{d}\tau.$$

$$(13)$$

The sets of Equations (10) and (11), respectively (12) and (13), describe the semi-Markov probabilities of transiting between the different dependency states and death for both models. The probabilities are evaluated on the basis of empirical data in Section 4.

2.3 Probability of losing autonomy

A common challenge in the estimation of transition probabilities for LTC concerns the "entry" probability, that is, the probability of losing autonomy and entering one of the acuity states.

This challenge arises because datasets focusing on LTC after only contain information about the dependent population while disregarding others. The lack of knowledge about the total population impedes the estimation of the probability of losing autonomy and leaves an important gap to fill. Moreover, as we only consider the old-age population, the semi-Markov model is not appropriate for these transitions (see e.g., Touraine, 2013; Helmer et al., 2016).

The estimation of the transition probabilities p_{0j} for a given gender and age reduces, on the one side, to the estimation of prevalence rates π and, on the other side, to the estimation of the Markov probabilities ϕ_{0j}. The prevalence rates π represent for a given age the ratio of the population entering one of the three acuity states over the total population. The estimated transition probabilities $p_{0j}(x)$ from autonomy (0) to any acuity state $j \in I \setminus \{4\}$ correspond to the product of the prevalence rate multiplied by the Markov probability,

$$p_{0j} = \pi \cdot \phi_{0j}, \tag{14}$$

for each gender and each age. As laid out in Section 2.1, we do not consider the transition from autonomy (0) to death (4).

3 Dataset and descriptive statistics

In this section, we introduce the data used for our analysis and describe the major characteristics that are relevant to the model and the interpretation of the results (see Section 3.1). In Section 3.2, we report descriptive statistics on the observed transitions between dependency states. Finally, we describe in Section 3.3 the distribution of the durations laying the basis for the choice of the duration law of the semi-Markov model.

3.1 Description of available data

The Swiss old-age care system provides LTC benefits for non-autonomous persons aged over 65 years. The first pillar of the Old-Age Social Insurance (OASI) law regulates those benefits. They are paid to all elderly persons permanently living in Switzerland, no matter their wealth or existence of private insurance, and suffering from limitations in ADL such as dressing, bathing, and eating, which require different levels of assistance and personal supervision. The conditions are fully described by the Swiss Federal Social Insurance Office (2015, FSIO). The amount of the allowance depends both on the acuity of the dependence and the canton of residence. The Swiss system distinguishes three levels of acuity. *Mild* acuity characterizes persons needing regular assistance in at least two ADL or permanent personal supervision. *Moderate* acuity defines dependents needing assistance in at least two ADL and permanent personal supervision, while *severe* acuity identifies insured persons in need of regular assistance with all the daily living activities and further entails permanent personal supervision.

The Swiss Central Compensation Office (CCO)[1] specializes in the benefits paid under the old-age social insurance scheme concerning both pension and disability benefits. We consider a dataset covering the period from 1995 to 2015 that reports information on persons aged 65+ years receiving old-age care benefits under the OASI law. Our data is nationally representative covering the whole dependent population registered for old-age care benefits in Switzerland. This

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[1] www.zas.admin.ch

dataset was purpose-built for our study by the CCO and contains, for each beneficiary, among others information on gender, year of birth, level of dependency and type of care received.[2] For the dependency states, the related start and end dates are reported at monthly precision. Each time a change in the individual status appears, e.g., a change in the acuity level, death or departure from Switzerland, a variable records the reason, and an updated entry appears in the data (except for death where the record ends).[3] A variable reports on the level of dependency specifying the acuity level (mild, moderate or severe) and the type of care (care at home or care in an institution). Note that elderly being cared for at home have received benefits only since 2001. Further, mildly dependent persons living at home are only recognized since 2011 by OASI. Thus, these persons do not appear in our dataset in the years before (see Table 1 below). Further, elderly persons living at home are sometimes unaware of the benefits they are entitled to or may forget to request them despite being eligible (**?**). This is less the case for elderly being cared in an institution since the institutions manage most administrative tasks. Overall, we need to be aware that there is a downward bias on transition probabilities from autonomy to dependence, affecting mostly care delivered at home and the mild frailty state. In fact, mildly dependent persons, although when receiving formal care, are less likely to register for the benefits they are entitled to. This affects to a lesser extend moderately and severely dependent persons that, by definition, receive permanent personal supervision and for whom the benefits are more critical and higher. Finally, note that the above factors also drive the number of available observations in our data.

Before reshaping the raw data from the CCO into a longitudinal dataset to be employed in the semi-Markov model (Section 4.2) and deriving transition probabilities (Section 4.4), some data cleaning is required. This introduces certain limitations of our data representing the total LTC needs in Switzerland. In the following, we discuss the issues linked to discontinuities in records, to censoring and to recovery. In fact, for a limited number of beneficiaries, the payment stream discontinues and we are unable to identify whether the introduced gaps are due to seasonal movements (e.g., living in an institution during winter and at home without being registered for care during summer) or to missing entries. By removing such entries, we not only eliminate the incomplete entry but also disregard the history of this individual to avoid potential outliers. We also remove beneficiaries who leave Switzerland because such persons can no longer be tracked.

Further, both left and right censoring affect our data.[4] When censoring is not informative, meaning that the censoring is not the consequence of a particular event (e.g., a change in the law), the inclusion of censored data reduces the precision of the estimation. In our case, the

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[2]The original data were compiled by the CCO in 2016 and contain information for the period from 1995 to 2015. However, at that time, the data for the year 2015 were still provisional because the figures are typically completed following updates from the cantonal instances. Therefore, we remove incomplete 2015 data omitting the year 2015 in the calculation of the prevalence rates (see below). However, with regard to the transitions that occurred in 2015, we consider all available fully recorded observations.

[3]Following the federal ordinance on health care benefits (Federal Department of Home Affairs, 2018), the medical assessment defining the level of dependence is done through the completion of a two-page form in collaboration with a recognized instance. For persons being cared for at home, the state-recognized home care provider is responsible for assessing changes in the health status. When persons are being cared for in an institution, the latter takes this responsibility.

[4]Left censoring characterizes data for which the starting date of dependency is unknown or lies before the beginning of the observation period in 1995. Right censoring defines data for which the end date of dependency is unknown, i.e., the individual remains alive and in a state of dependency at the end of the observation period in 2015.

dataset is not affected by informative censoring, and given the large sample size, we remove censored data. This means that, for example, we do not include dependent persons that were disabled before retirement and we restrict our observations to ages starting at 66 years. Finally, we observe only a very small number of recovery transitions which also justifies their exclusion from the models introduced in Section 2.1. In fact, the 100 recovery transitions correspond to less than 0.5‰ of all observed transitions in our data and do not allow for model estimation when split by gender and by age. Overall, the original dataset is reduced by approximately ten percent, essentially following the removal of censored data. The final dataset used in our analysis covers *all* available completely defined transitions.

In Table 1, we report the number of observations on the transitions between the different states for the years from 1995 to 2015. Over the period, the CCO dataset records 284 482 dependent individuals. For our analysis, we can rely on 269 082 and 219 540 uncensored transitions in the frailty level and the type of care models respectively. These numbers can be compared with the data available in other LTC studies, e.g., Guibert and Planchet (2017) where 12 800 dependent persons are observed and Biessy (2016) where a total of 20 988 dependent individuals (of whom about 16 000 are uncensored) are recorded in France. Furthermore, recall that our transitions reflect the total population of Switzerland, while most other studies focus on private insurance data (e.g., D'Amico et al., 2009). In the frailty level model, we consider three transitions from autonomy to dependency and six transitions within dependency and death (see Section 2.1 and Figure 1). In the type of care model, we have two transitions from autonomy to dependency and three transitions within dependency and death (Figure 2). For readability, we use the notations 0 to 4, or 0, a, b and 4, respectively, when referring to the dependency states (see Figures 1 and 2 for the definitions).

When comparing the years before and after 2011, Table 1 reports significant differences in the number of observed transitions from the mild state of dependency ($1 \rightarrow 2$, $1 \rightarrow 3$ and $1 \rightarrow 4$) as well as from the care received at home ($a \rightarrow b$ and $a \rightarrow 4$). This relies on the late recognition of dependent persons with mild dependency living at home as laid out above. Most available observations concern the transitions moderate to severe (total 49 352), moderate to death (63 588) and severe to death (145 050), respectively. After the year 2011, we also account for a significant number of transitions leaving mild dependency. In the type of care model, we observe the largest number of transitions for the individuals dying while being cared for in an institution. In terms of numbers, the transitions from the severe dependency state to death as well as the transitions from the care received in an institution to death (208 795) are the transitions with the highest number of observations.

From the CCO data, we compile annualized information about the dependent population by gender and by age. To derive the prevalence rates required for the evaluation of the probability of losing autonomy, we add information about the total old-age population in Switzerland available from the Federal Statistical Office (FSO).[5] In fact, the prevalence rates are calculated by dividing the number of dependent elderly by the population for each gender and each age.[6]

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[5] www.bfs.admin.ch

[6] The figures for the yearly cross-sectional view constructed from the detailed CCO data slightly differ from the less detailed aggregate figures published by the FSO. Differences in the numbers may arise from the exact registration dates of the acuity levels, how up-to-date the sources are, the processes for aggregation used, and the cleaning of incomplete entries. More details on the corrections applied in our approach can be found in **?**.

	Frailty level							Type of care			
Year	$1 \to 2$	$1 \to 3$	$1 \to 4$	$2 \to 3$	$2 \to 4$	$3 \to 4$	Total	$a \to b$	$a \to 4$	$b \to 4$	Total
1995	6	4	21	1 211	1 532	6 632	9 406	0	0	8 185	8 185
1996	9	16	13	1 626	1 629	6 338	9 631	0	0	7 980	7 980
1997	7	10	17	1 685	1 735	6 370	9 824	0	0	8 122	8 122
1998	5	7	12	1 759	1 780	6 187	9 750	0	0	7 979	7 979
1999	13	6	14	1 712	2 255	6 673	10 673	0	0	8 942	8 942
2000	2	9	9	1 949	2 365	6 503	10 837	0	0	8 877	8 877
2001	6	3	13	2 166	2 401	6 878	11 467	0	0	9 292	9 292
2002	4	5	7	2 346	2 674	7 277	12 313	0	0	9 958	9 958
2003	9	2	7	2 406	2 927	7 546	12 897	0	0	10 480	10 480
2004	5	4	10	2 474	2 859	7 020	12 372	3	5	9 884	9 892
2005	6	1	9	2 561	3 104	7 461	13 142	0	5	10 569	10 574
2006	3	2	9	2 790	3 194	7 223	13 221	3	5	10 421	10 429
2007	3	4	9	2 677	3 291	7 249	13 233	3	4	10 545	10 552
2008	3	3	2	2 617	3 307	7 078	13 010	1	8	10 379	10 388
2009	5	4	12	2 807	3 412	7 285	13 525	9	8	10 701	10 718
2010	5	0	10	2 771	3 586	7 083	13 455	7	8	10 671	10 686
2011	801	212	209	2 921	3 780	6 815	14 738	984	197	10 607	11 788
2012	987	318	527	2 781	4 253	7 095	15 591	1 301	523	11 352	13 176
2013	1 292	441	645	2 889	4 427	7 123	16 817	1 735	643	11 552	13 930
2014	1 475	503	729	2 832	4 424	6 504	16 467	1 967	723	10 934	13 624
2015	1 430	373	805	2 372	4 653	6 710	16 343	1 800	803	11 365	13 968
Total	6 076	1 927	3 089	49 352	63 588	145 050	269 082	7 813	2 932	208 795	219 540

Table 1: Number of observed transitions by calendar year in the frailty level and the type of care models.

Figure 3 illustrates the 1995 to 2014 average prevalence rates $\bar{\pi}$ (cf. Footnote 2) as a function of age for both gender. Independently from the acuity level, prevalence rates grow with age for males and females prevalence (see also Guibert et al., 2014; ?). Until the age of 80 years, we do not observe significant differences in the prevalence rates between gender. At the age of 85 years, the female prevalence rate is about 9% and exceeds the male's one by 3%. This difference increases with the age since the steepest increase, showing exponential behavior, is observed for the females. At age 90, on average, 15% of the female population and 10% of the male population is in dependence.

Figure 3: Prevalence rates $\bar{\pi}$ by gender and by age (averaged 1995–2014).

3.2 Descriptive statistics

In the following, we present descriptive statistics to provide first insights in the data over the whole period 1995–2015. In Table 2, we characterize the transitions in the frailty level and the type of care models. We provide statistics on the transitions of elderly persons entering and leaving each of the states and on the time spent therein. Thereby, for each transition $i \rightarrow j$, we report the total number of observations, the share of female, the average age at transition by gender as well as the time spent in state i. For the transitions from autonomy ($0 \rightarrow j$), the reported age refers to the average age when becoming dependent. For the other transitions ($i \rightarrow j$, $i \neq 0$), the age represents the average age at transition, i.e. the age when entering state j.

Among the 284 482 individuals entering dependency, we observe 21 933 from autonomy to the mild dependency state ($0 \rightarrow 1$), 134 263 to the moderate dependency state ($0 \rightarrow 2$) and 128 286 to the severe dependency state ($0 \rightarrow 3$). When considering the type of care model, we record 21 561 transitions from autonomy to care at home ($0 \rightarrow a$) and 262 921 from autonomy to care in an institution ($0 \rightarrow b$). In the analysis of these transitions, we also note an entry age in dependency of about 80 years. The differences in the total number of transitions

Model	Transition $i \rightarrow j$	N	Share of female (%)	Age at transition (yrs.)				Duration in state i (mth.)	
				Male		Female			
Frailty level	$0 \rightarrow 1$	21 933	51.3	78.1	(10.9)	79.5	(10.6)	–	–
	$0 \rightarrow 2$	134 263	53.5	79.9	(10.1)	81.4	(10.6)	–	–
	$0 \rightarrow 3$	128 286	55.7	79.9	(10.3)	81.4	(10.8)	–	–
	Total	284 482							
	$1 \rightarrow 2$	6 076	59.5	82.5	(7.3)	84.7	(7.1)	15.5	(19.1)
	$1 \rightarrow 3$	1 927	62.4	83.4	(7.7)	86.5	(6.9)	21.9	(29.0)
	$1 \rightarrow 4$	3 089	57.2	83.9	(7.8)	86.8	(7.0)	30.4	(35.0)
	$2 \rightarrow 3$	49 352	69.4	83.4	(7.1)	86.7	(6.9)	27.1	(26.7)
	$2 \rightarrow 4$	63 588	65.0	85.2	(7.1)	88.5	(6.5)	32.4	(27.1)
	$3 \rightarrow 4$	145 050	69.5	85.1	(7.0)	89.2	(6.6)	35.1	(31.8)
	Total	269 082							
Type of care	$0 \rightarrow a$	21 561	48.5	80.9	(11.3)	84.0	(10.5)	–	–
	$0 \rightarrow b$	262 921	54.9	79.2	(10.2)	80.6	(10.6)	–	–
	Total	284 482							
	$a \rightarrow b$	7 813	60.6	82.9	(7.3)	85.3	(7.1)	14.9	(13.0)
	$a \rightarrow 4$	2 932	58.3	84.1	(8.0)	87.1	(6.9)	24.1	(14.7)
	$b \rightarrow 4$	208 795	68.1	85.1	(7.0)	89.0	(6.6)	39.5	(33.4)
	Total	219 540							

Table 2: Descriptive statistics on the transitions in the frailty level and the type of care models in the period 1995–2015. For each transition $i \rightarrow j$ the number N of observations, the percentage share of females, the average age in years by gender and the time spent in state i are provided. Numbers in brackets report the standard deviation for the ages and duration respectively.

between entrance in dependency (284 482) and the total flows within dependency and death in the frailty level model (269 082) and the type of care model (219 540) arise from right-censoring and multiple transitions. Indeed, a same individual may be observed through more than one transition (e.g., an individual may experience several frailty levels without changing the type of care).

The transitions from moderate to severe dependency $(2 \rightarrow 3)$, moderate dependency to death $(2 \rightarrow 4)$ and severe dependency to death $(3 \rightarrow 4)$ concern more than 95% of the total number of observations. On average, the transitions from severe to death $(3 \rightarrow 4)$ occur at an age of 85.1 years for male and 89.2 years old for female. These observed life expectancies (conditional upon entering severe dependency) are particularly high when compared to the Swiss conditional life expectancy at age 65 which is about 84.4 years for male and 87.4 years for female in 2014 (figures from FSO). When considering the type of care model, we see that the vast majority of the transitions concerns the flow from institutional care to death $(b \rightarrow 4)$. The average age is comparable to the one observed in the frailty level model.

When considering the time spent in the previous state before the transition, we observe that on average, elderly dependent individuals remain 15.5 months in the mild state before entering the moderate state of dependency $(1 \rightarrow 2)$, 21.9 months in the mild state before entering the severe state of dependency $(1 \rightarrow 3)$ and 30.4 months before death $(1 \rightarrow 4)$. For the ones leaving the moderate state of dependency, they remain there for 27.1 months before becoming severely dependent $(2 \rightarrow 3)$ and 32.4 months before death $(2 \rightarrow 4)$. Finally, severely dependent individuals remain 35.1 months before death $(3 \rightarrow 4)$. The results differ more between the states in the type of care model: we observe that persons being cared at home remain 14.9 months before going for institutional care $(a \rightarrow b)$ and 24.1 months before death $(a \rightarrow 4)$. For the ones being cared for in an institution, they remain on average 39.5 months before death $(b \rightarrow 4)$. We also note that the standard deviation for the duration is very high when compared to the average evidencing the variability in durations. In fact, the standard deviation is not fully appropriate for commenting on the distribution of the duration (see Section 3.3).

3.3 Empirical evidence for the choice of the duration law

In the following, we provide empirical evidence that may support the choice of the duration law in the semi-Markov model. For this purpose, we present the empirical density of the elapsed number of months spent in the respective state *before* transiting to a next state for each transition in both dependency models. Figures 4 and 5 report these empirical duration densities in the frailty level and the type of care model, respectively, considering the aggregate data for both genders and all ages. For example, the duration density reported in the first graph of Figure 4a illustrates the probability density of the sojourn times in the mild frailty state (1) before transiting to the moderate frailty state (2) for all individuals independent of their gender and age. The total number of observations underlying each graph can be taken from Tables 1 or 2. Note that in the application used to calculate the dependence tables (see Section 4), we will estimate the Weibull parameters of the distribution separately for each data subset by gender and by age.

The majority of the transitions occur within the first 60 months, leading to a right-skewed empirical distribution. This is typically the shape one observes from the Weibull distribution. Chosing the Weibull distribution offers the important advantage of requiring the calibration of

(a) Transitions among frailty levels. (b) Transitions from frailty to death.
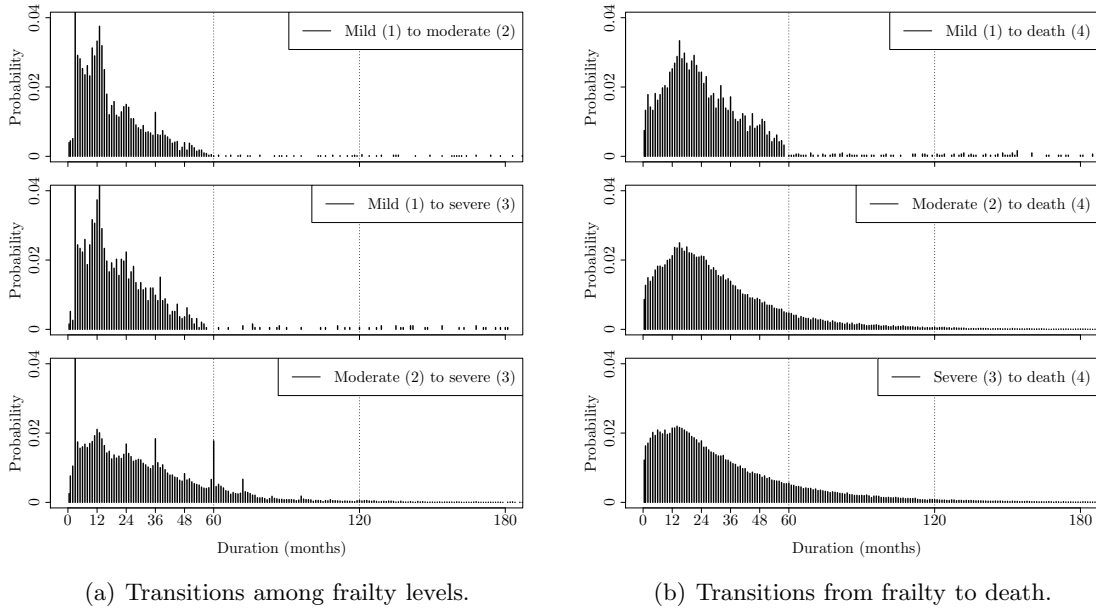
Figure 4: Empirical duration density (months before transition) in the frailty level model.
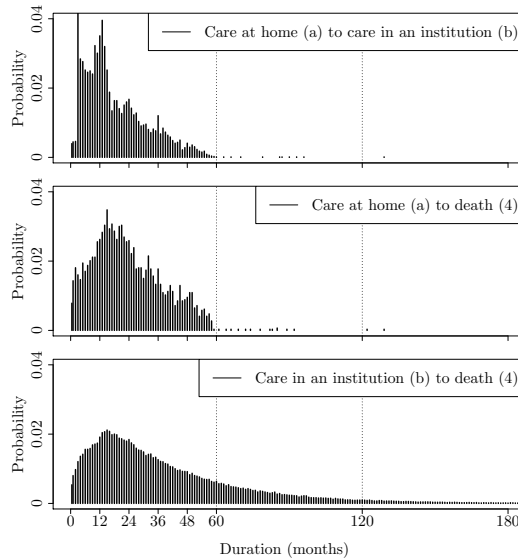
Figure 5: Empirical duration density (months before transition) in the type of care model.

only two parameters. Alternative distribution laws are possible (see, e.g., Foucher et al., 2005), but for our analysis, we will employ this simple framework (see Section 4.2 and the robustness tests in Section 4.3). The observed empirical duration densities are relatively smooth, underlining the continuous health assessment process, and merely show several spikes at specific durations. For example, in Figure 4a, for the transition from moderate to severe dependency, such spikes can be observed at the durations of 3, 12, 24, 36, etc. months. These may be related to the registration and reassessment process where specific periodic health appraisals (e.g., quarterly, semi-annually, yearly) are performed on top of the continuous assessments.

4   Application of the model and presentation of results

In this section, we present details on the numerical implementation of the model and discuss the results produced by applying it to the empirical data. First, we calculate the transition probabilities from autonomy to any dependency state for both LTC models (Section 4.1). Then, we report parameter estimates of the semi-Markov model and the first numerical results for selected ages in Section 4.2. For one of the transitions, we provide detailed parameter results for all ages and show the differences between males and females. In Section 4.3, we present the results from robustness tests regarding the choice of the distribution law in the semi-Markov model and the stability of the model parameters. In Section 4.4, we provide the transition probabilities in both models which are the main results of our paper.

4.1   Estimation of the probability of losing autonomy

Following the methodology described in Section 2.3 and the available prevalence data presented in Section 3.1, we determine the probability of losing autonomy and entering one of the dependency states with the help of Equation (14). For the prevalence rates π by gender and by age we use the 1995–2014 average prevalence rates $\bar{\pi}$ reported in Figure 3. The Markov probabilities ϕ_{0j} are obtained as the ratio between the number of new entrants in each dependency state j over the total number of new entrants by gender and by age using the CCO data. Since mildly dependent persons being cared for at home are only recognized under the OASI law after 2011 (cf. Section 3.1), those elderly do not appear in the statistics for the years before 2011. Thus, and only for calculating the ratios of new entrants in the different states j we focus on the observations from the period 2011–2015.[7] Note that the sum of ϕ_{0j} over all dependency states j equals one ($\sum_j \phi_{0j} = 1$, $j \in I \setminus \{4\}$). We report numerical values for selected ages in Table 3 and present a graphical illustration for all ages in Figures 6 and 7. The variables p_{01}, p_{02} and p_{03} denote the transition probabilities from autonomy to the three frailty levels in the frailty level model, while p_{0a} and p_{0b} are the probabilities of entering a type of care in our second model. The sum of the probabilities yields the same number in both models and is reported in the row labeled $\sum_j p_{0j}$. This sum corresponds to the age-gender specific prevalence

Model	Transition $0 \to j$	Age	Male			Female		
			70	80	90	70	80	90
Frailty level	$0 \to 1$	p_{01}	0.0051	0.0100	0.0265	0.0060	0.0147	0.0363
	$\to 2$	p_{02}	0.0053	0.0145	0.0425	0.0045	0.0145	0.0681
	$\to 3$	p_{03}	0.0038	0.0099	0.0284	0.0029	0.0104	0.0495
Type of care	$0 \to a$	p_{0a}	0.0054	0.0107	0.0288	0.0061	0.0157	0.0395
	$\to b$	p_{0b}	0.0088	0.0236	0.0687	0.0073	0.0239	0.1144
	$\bar{\pi} = \sum_j p_{0j}$		0.0142	0.0344	0.0974	0.0134	0.0396	0.1539

Table 3: Probability of losing autonomy by gender at the ages of 70, 80 and 90 years.

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[7]In the period 2011–2015, the number of new entrants in the states 1, 2 and 3 are respectively $N = 21\,385$, $31\,278$ and $22\,631$ (to be compared with the figures reported in Table 2). Along the types of care we count $N = 21\,357$ and $48\,178$ data points for lte states a and b, respectively. This restriction does not affect the later calibration of the semi-Markov model based on the full data from 1995–2015 where a lower number N of certain transitions only influences the precision of the estimates (also refer to the robustness tests presented in Section 4.3, Figure 10).

rates $\bar{\pi}$.[8] After age 80, we observe that the increase in the total transition probabilities from autonomy becomes much more important. For example, for women, $\sum_j p_{0j}$ increases by 2.62% and 11.43% for the ages from 70 to 80 and from 80 to 90 years, respectively. This outcome is not surprising because major degenerative illnesses implying dependence appear primarily at higher ages (see, e.g., Kaeser, 2012; Guibert and Planchet, 2017).

By comparing the probabilities for both genders, we note that their numbers are similar at the ages of 70 and 80 years. In total, approximately 1.42% of men and 1.34% of women become dependent at age of 70 and between 3.44%, and 3.96% do so at age 80. A difference can be observed at age 90: the probability for males is approximately 10%, while that for females reaches 15%. This higher probability can be explained both by the substantial number of women surviving to higher ages and the higher chance to present cognitive disease in comparison to men. In fact, male mortality is much higher at older ages. As mentioned earlier, we disregard the mortality of autonomous individuals and only focus on the dependent population and their transitions. We observe that the transition probabilities from autonomy to any state of dependency are positively correlated with the age. For example, an 80-years-old man has a probability p_{03} of about 1% of entering the severe dependency state (3), while at 90 years this probability is 2.84%. In the following, we analyze the impact of age in greater detail.

The graphs in Figure 6 report the values by age for males (Fig. 6a) and females (Fig. 6b) in the frailty level model. For both genders, the probability of losing autonomy increases with age, and the analysis of the results reveals that two transitions prevail especially at higher ages: the probabilities p_{02} and p_{03} of entering in a moderate (2) or severe (3) state of dependency are significantly higher than that of entering mild dependency (1). At lower ages, the latter is close to the one entering in a moderate (2) state of dependency and higher than entering in a severe (3) state of dependency. The corresponding probabilities depict exponential shapes as age increases. For both genders, the values until age 80 are similar (see the discussion above). While the transition probability p_{01} from autonomy to mild dependency is close to the other two transition probabilities at ages below 80 years, it remains rather flat at higher ages for both genders.

(a) Male   (b) Female
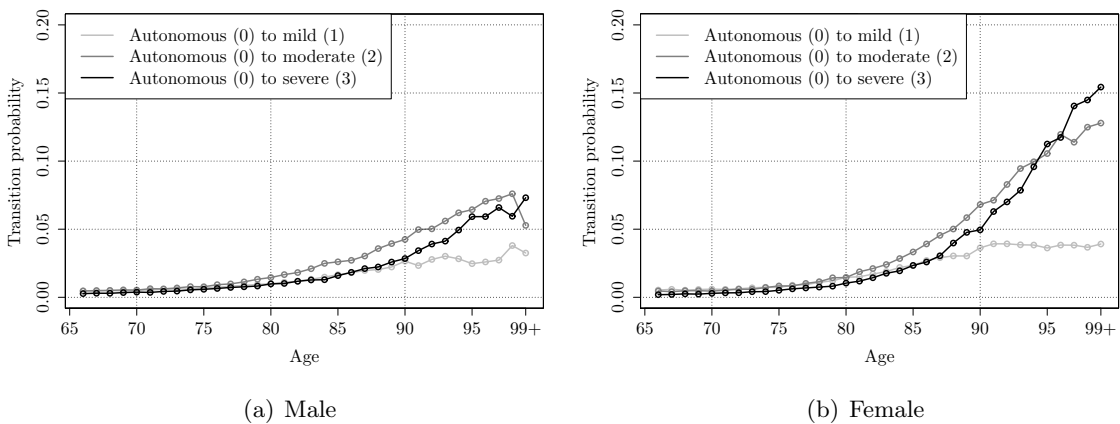
Figure 6: Transition probabilities from autonomy in the frailty level model by gender and age.

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[8]Note that the probability $1 - \sum_j p_{0j} = 1 - \bar{\pi}$, with $j \in I \setminus \{4\}$, i.e., $j \in \{1, 2, 3\}$ or $j \in \{a, b\}$, does not yield the probability p_{00} of staying autonomous since the mortality p_{04} of autonomous individuals is also included.

Figure 7 presents the results for the type of care model. Again, the two graphs (a) and (b) illustrate the transition probabilities for males and females by age. Our results show that the probability of receiving care in an institution exceeds that of receiving care at home. They also underline the dependence in age and women's higher probability of receiving care than men. From ages 70 to 95, the probability p_{0b} of receiving care in an institution grows from 1% to 12% for men and from 1% to 21% for women. The transition to care at home (a) remains at much lower levels across ages. In fact, persons cared for at home often receive care by relatives (which is not registered in our data), and, as mentioned above, the lower number may also stem from an unawareness of the availability of this allowance (see Section 3.1).

(a) Male   (b) Female
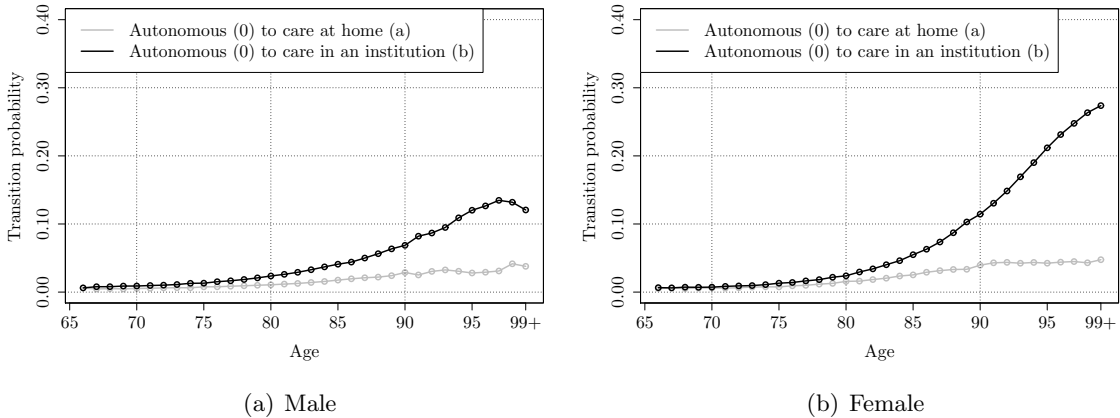
Figure 7: Transition probabilities from autonomy in the type of care model by gender and age.

4.2 Parameter estimation of the semi-Markov model

The specification of the semi-Markov model requires the choice of the duration law and the estimation of different parameters.

Duration law

In the semi-Markov model, the duration law $F_{ij}(t)$ introduced in Equation (4) plays an important role. This function is a stochastic representation of the time spent in the previous state that defines the probability distribution for the sojourn times. In other words, the duration law attributes a probability to each realization of the positive random variable X_n from Equation (1). In our application, based on the statistical description in Section 3.3 (see Figures 4 and 5) we use a Weibull duration law: it is skewed to the right and only requires the calibration of two parameters, thereby reducing the estimation errors. The Weibull duration law is expressed as follows:

$$F_{ij}(t) = 1 - e^{-(t/\theta_{ij})^{\sigma_{ij}}}, \tag{15}$$

where $\sigma_{ij} > 0$ represents the shape parameter and $\theta_{ij} > 0$ the scale parameter for a transition from i to j occurring after a duration $t \geq 0$ in state i. The corresponding density function $f_{ij}(t)$, see Equation (6), yields the following:

$$f_{ij}(t) = \frac{\sigma_{ij}}{\theta_{ij}} \left(\frac{t}{\theta_{ij}} \right)^{\sigma_{ij}-1} e^{-(t/\theta_{ij})^{\sigma_{ij}}}. \tag{16}$$

In Section 4.3, we provide robustness tests supporting the choice of the Weibull distribution (see Figure 9).

Maximum likelihood estimation

We base the model calibration on a maximum likelihood estimation (MLE). Thereby, the Markov transitions ϕ_{ij} and the two parameters of the Weibull duration law, the shape σ_{ij} and scale θ_{ij} parameters are estimated. In the following, we offer some technical remarks to explain the estimation of the parameters. MLE is a method that calibrates parameters such that the likelihood of the observations is maximized. In our model, for each gender and age, we calibrate the Markov probabilities ϕ_{ij} and the parameters σ_{ij} and θ_{ij} of the Weibull distribution for each transition (ij) from state i to state j. For this purpose, we divide the sample into subsets by gender (male and female) and by age (integer ages) and perform MLE, where each set of data contains the transitions realized and the times spent in the previous state. Thereby, for a transition (ij), the age over the years refers to the individual's age when entering state i.[9]

Based on Equation (5), $C_{(ij)}^h$ defines the marginal contribution to the likelihood of each individual h of a certain gender and age for the transition (ij). It is calculated as follows:

$$C_{(ij)}^h = \phi_{ij}^h f_{ij}^h(t). \tag{17}$$

The contribution to the likelihood represents information contained in the data that is relevant for the parameter calibration. The likelihood contribution is calculated for each individual by gender and by age. The likelihood function $\mathcal{L}$ aggregates the individual contributions $C_{(ij)}^h$ for all h and over all transitions (ij):

$$\mathcal{L} = \prod_h \prod_{(ij)} C_{(ij)}^h. \tag{18}$$

For computational reasons, we use the log-likelihood function ℓ given by the logarithm of the likelihood function $\mathcal{L}$:

$$\ell = \log \mathcal{L} = \sum_h \sum_{(ij)} \log C_{(ij)}^h. \tag{19}$$

For each contribution of the individuals' gender and age, the above problem yields a homogeneous semi-Markov model. This feature allows us to apply the R package "semi-Markov"(Król and Saint-Pierre, 2015) to estimate the model parameters (see Tables 4 and 5).

Calibration of the frailty level model

In Table 4, we present the parameter estimates for the transitions (ij), denoted $i \to j$, in the frailty level model. For each transition, we report the estimates of the Markov probabilities ϕ_{ij} and of the Weibull shape (σ_{ij}) and scale (θ_{ij}) parameters. We support the precision of the estimates by reporting the standard deviation. We also calculate the expected staying time $\mathbb{E}(X)$ in the state before the transition[10] and report the number of underlying observations N. The results are presented for both genders at the ages of 70, 80 and 90. We note that the number N

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[9]In other words, for a given transition, we refer to this (constant) entrance age through the duration t in state i and for the transition (ij). Since t takes values beyond one year, the actual individual's age changes. However, in our approach, we do not take this change into account, and our results always refer to the age at entry to the state i. This assumption, although it introduces a deviation, allows for a smooth solution with the duration t.

[10]With the notation X, we omit the index n in X_n (see Equation 1) since the order of the transitions is not the focus of our study.
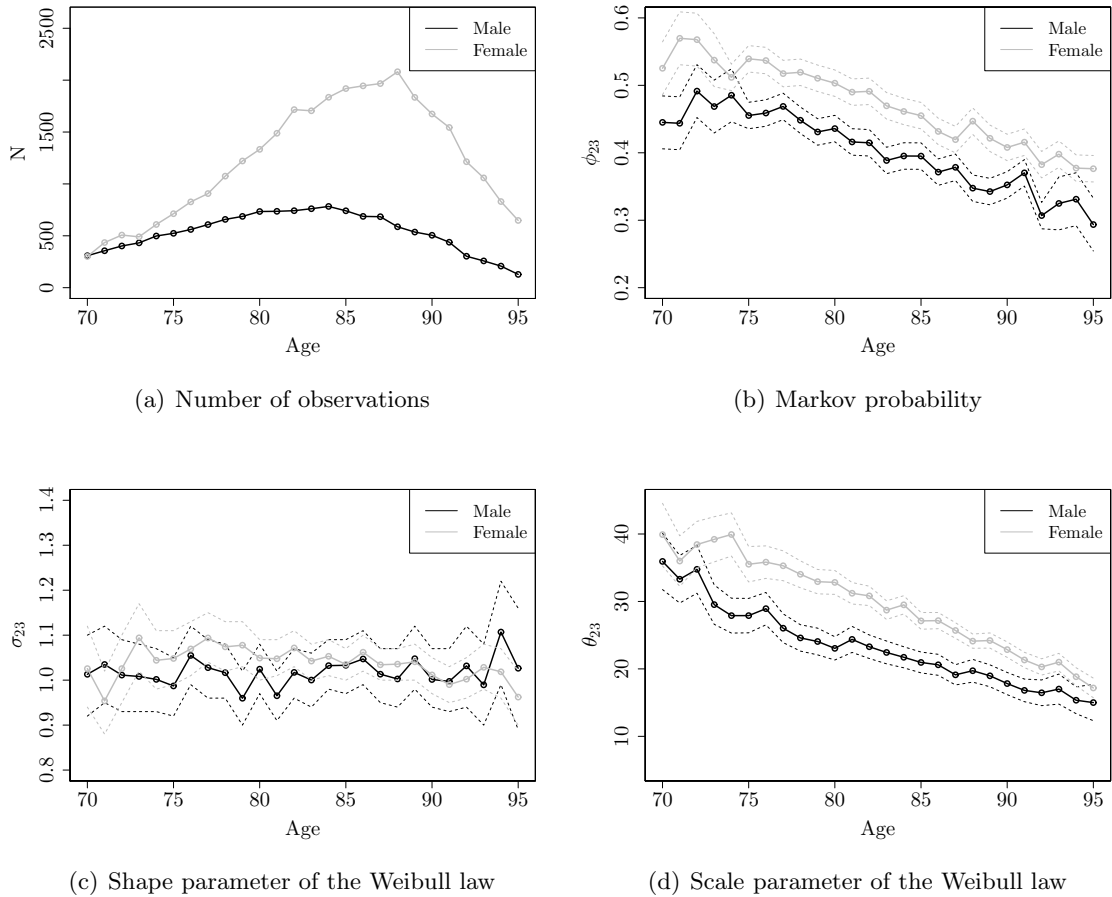
		Male						Female				
	Age	70		80		90		70		80		90
	ϕ_{ij}	0.476	(0.054)	0.564	(0.036)	0.495	(0.035)	0.596	(0.054)	0.619	(0.031)	0.491	(0.026)
	σ_{ij}	0.856	(0.092)	1.051	(0.078)	1.170	(0.089)	0.852	(0.087)	1.143	(0.070)	1.145	(0.065)
$1 \to 2$	θ_{ij}	14.514	(2.819)	13.183	(1.302)	14.188	(1.285)	19.275	(3.401)	14.312	(1.062)	14.581	(1.012)
	$\mathbb{E}(X)$	15.720		12.926		13.437		20.943		13.644		13.895	
	N	41		105		100		50		156		178	
	ϕ_{ij}	0.198	(0.043)	0.124	(0.024)	0.168	(0.026)	0.190	(0.043)	0.171	(0.024)	0.198	(0.021)
	σ_{ij}	0.923	(0.170)	1.124	(0.180)	1.117	(0.153)	0.771	(0.146)	1.363	(0.170)	1.379	(0.128)
$1 \to 3$	θ_{ij}	34.097	(9.496)	14.336	(2.819)	15.285	(2.485)	38.561	(13.260)	19.924	(2.342)	18.199	(1.639)
	$\mathbb{E}(X)$	35.409		13.738		14.678		44.903		18.242		16.627	
	N	17		23		34		16		43		72	
	ϕ_{ij}	0.326	(0.051)	0.312	(0.034)	0.337	(0.033)	0.214	(0.045)	0.210	(0.026)	0.311	(0.024)
	σ_{ij}	0.959	(0.138)	1.168	(0.114)	1.716	(0.170)	0.690	(0.128)	1.735	(0.184)	1.677	(0.126)
$1 \to 4$	θ_{ij}	56.444	(11.757)	24.061	(2.846)	27.419	(2.033)	68.371	(24.656)	29.351	(2.443)	28.364	(1.675)
	$\mathbb{E}(X)$	57.500		22.818		24.449		87.703		26.154		25.332	
	N	28		58		68		18		53		113	
	ϕ_{ij}	0.445	(0.019)	0.436	(0.012)	0.352	(0.013)	0.525	(0.021)	0.503	(0.009)	0.408	(0.008)
	σ_{ij}	1.013	(0.046)	1.024	(0.030)	1.001	(0.034)	1.026	(0.046)	1.049	(0.023)	1.012	(0.019)
$2 \to 3$	θ_{ij}	35.936	(2.130)	23.045	(0.880)	17.826	(0.839)	39.912	(2.367)	32.823	(0.903)	22.857	(0.584)
	$\mathbb{E}(X)$	35.744		22.799		17.815		39.500		32.204		22.747	
	N	308		731		505		301		1 332		1 672	
	ϕ_{ij}	0.555	(0.000)	0.564	(0.000)	0.648	(0.000)	0.475	(0.000)	0.497	(0.000)	0.592	(0.000)
	σ_{ij}	1.242	(0.049)	1.283	(0.032)	1.349	(0.034)	1.109	(0.052)	1.363	(0.029)	1.402	(0.022)
$2 \to 4$	θ_{ij}	47.847	(2.072)	33.682	(0.899)	25.189	(0.645)	52.734	(3.041)	42.782	(0.911)	30.870	(0.471)
	$\mathbb{E}(X)$	44.630		31.192		23.102		50.748		39.163		28.130	
	N	384		946		928		272		1 315		2 427	
	ϕ_{ij}	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–
	σ_{ij}	1.129	(0.033)	1.175	(0.020)	1.193	(0.023)	1.179	(0.037)	1.243	(0.017)	1.251	(0.013)
$3 \to 4$	θ_{ij}	51.887	(1.822)	34.071	(0.679)	23.038	(0.497)	69.332	(2.445)	47.729	(0.716)	32.045	(0.353)
	$\mathbb{E}(X)$	49.648		32.226		21.704		65.523		44.509		29.838	
	N	706		2 022		1 674		636		3 183		5 815	

Table 4: Parameter estimates and standard deviation (in brackets) for the Markov probabilities, the Weibull duration law with expected duration (in months) and the number of underlying observations for the transitions in the frailty level model. The results are represented by gender at the ages of 70, 80 and 90 years.

of observations strongly depends on the transition. This number is important since it drives the reliability of the estimates. For the transitions mild (1) to moderate (2), mild (1) to severe (3), and mild (1) to death (4), fewer than 100 data points are available for both genders. The situation is different for the other transitions, where we count between 500 and 6 000 data points at the ages of 80 and 90.

Considering the estimates, we first discuss the Markov probabilities ϕ_{ij}. These probabilities introduced in Equation (3) correspond to the total transition probabilities disregarding the time spent in the states. For example, at age 70, 47.6% of the men in the mild state will enter the moderate state, whereas 19.8% and 32.6% will join the severe and death states, respectively. At the same age, 59.6% of the mildly dependent women enter the moderate state, 19.0% enter the severe state, and 21.4% die. For the transitions leaving the mild state (1), we observe that the share of individuals entering a more severe frailty state (2 or 3) is higher than those dying (4). This holds for both genders and even at higher ages. Regarding the transition from moderate (2) to severe (3), the Markov probabilities ϕ_{ij} decrease with the age of the person. This decrease is of course complemented by the increasing probability of the transition from moderate (2) to death (4).

After the Markov probabilities, we focus on the estimates of the Weibull duration law. In most of the reported cases, the shape parameter σ_{ij} yields similar values close to 1. The situation is different for the scale parameter θ_{ij} because we observe a high sensitivity with respect to the transition and gender considered. A specific trend appears when comparing the changes with the ages. In all of the transitions from the moderate and the severe states ($2 \to 3$, $2 \to 4$ and $3 \to 4$), an increase in the entrance age comes with a decrease in θ_{ij}. Since the shape parameter σ_{ij} is close to 1, the scale parameter approximates the expected duration $\mathbb{E}(X)$. In this case, smaller values of θ_{ij} correspond to a reduction in the expected duration. For example, for a 70-years-old man and the transition from moderate (2) to severe (3), the scale parameter θ_{ij} is 35.936 and decreases to 23.045 and 17.826 at the ages of 80 and 90, respectively. In comparison, the corresponding expected durations $\mathbb{E}(X)$ are approximately 36, 23 and 18 months (see Section 4.4 for a discussion of these results). Finally, the small standard deviations of the parameters for the transitions with more than 300 underlying observations confirm the quality of our estimation.

(a) Number of observations

(b) Markov probability

(c) Shape parameter of the Weibull law

(d) Scale parameter of the Weibull law

Note: Dashed lines indicate the confidence interval at the 95% level.

Figure 8: Estimates of the number of observations, the Markov probabilities, the shape and scale parameters of the Weibull law by gender and age for the transition from moderate (2) to severe (3).

In Figure 8, we choose to illustrate the above estimates for the transition from moderate (2) to severe (3) in the frailty level model through the ages of 70 to 95 for both genders. We present (a) the number of observations N, (b) the Markov probabilities ϕ_{ij}, (c) and (d) the Weibull duration law shape and scale parameters σ_{ij} and θ_{ij}. In graphs (b) to (d), the 95%-confidence

interval is given. For any transition, the number of observations of women always exceeds that of men. We also observe a significant difference in the Markov probabilities when comparing the genders, and the same holds for the shape parameter of the Weibull law. This finding supports the decision to separately consider males and females throughout our study. The scale parameter takes values close to one for both genders at all ages. We note that the values for men and women cannot be distinguished. Furthermore, at ages above 90, the estimates become more erratic, as a result of the lower number of observations. This limited number of data points drove our decision to graphically present results only between ages 70 and 95, although less precise estimates are available at higher and lower ages.

Calibration of the type of care model

The estimates for the type of care model are shown in Table 5. The majority of our data cover the transition from care in an institution (b) to death (4). At age 90, we observe $2\,494$ men and $7\,951$ women for this transition. The difference between the two figures underlines the higher proportion of women living in institutions at advanced ages. For the other two transitions (a $\rightarrow$ b, a $\rightarrow$ 4), the number of data points is below 300. The Markov probabilities ϕ_{ij} are decreasing with age for the elderly moving from care at home (a) to care in an institution (b). This is the case for both genders. We note that 77.3% of the 70-year-old men receiving care at home transition to institutional care (the remaining 22.7% die). In comparison, for an 80- and 90-year-old, these percentages are 69.2 and 66.7, respectively. This is in line with the increasing mortality.

			Male						Female					
		Age	70		80		90		70		80		90	
Transitions	a → b	ϕ_{ij}	0.773	(0.052)	0.692	(0.034)	0.667	(0.033)	0.829	(0.045)	0.795	(0.026)	0.686	(0.024)
		σ_{ij}	1.270	(0.136)	1.063	(0.072)	1.166	(0.078)	1.149	(0.117)	1.184	(0.065)	1.188	(0.058)
		θ_{ij}	12.236	(1.429)	13.410	(1.183)	14.613	(1.147)	14.769	(1.788)	15.605	(0.991)	15.522	(0.873)
		$\mathbb{E}(X)$	11.357		13.094		13.854		14.061		14.731		14.641	
		N	51		128		134		58		198		251	
	a → 4	ϕ_{ij}	0.227	(0.003)	0.308	(0.001)	0.333	(0.001)	0.171	(0.002)	0.205	(0.001)	0.314	(0.001)
		σ_{ij}	1.870	(0.418)	1.400	(0.152)	1.709	(0.171)	1.014	(0.226)	1.892	(0.214)	1.696	(0.127)
		θ_{ij}	20.815	(2.976)	22.427	(2.221)	27.547	(2.065)	20.451	(6.126)	28.394	(2.203)	28.588	(1.654)
		$\mathbb{E}(X)$	18.480		20.441		24.570		20.334		25.200		25.512	
		N	15		57		67		12		51		115	
	b → 4	ϕ_{ij}	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–
		σ_{ij}	1.240	(0.028)	1.306	(0.018)	1.332	(0.020)	1.234	(0.031)	1.368	(0.015)	1.373	(0.012)
		θ_{ij}	58.724	(1.446)	38.611	(0.567)	27.471	(0.435)	73.638	(1.975)	53.559	(0.599)	36.359	(0.313)
		$\mathbb{E}(X)$	54.798		35.628		25.254		68.793		49.003		33.243	
		N	1 191		3 027		2 494		1 012		4 742		7 951	

Table 5: Parameter estimates and standard deviation (in brackets) for the Markov probabilities, the Weibull duration law with expected duration (in months) and the number of underlying observations for the transitions in the type of care model. Results are represented by gender at the ages of 70, 80 and 90 years.

Regarding the duration law, we find a similar situation to the frailty level model. The shape parameter σ_{ij} is approximately one, inducing that the value of the scale parameter θ_{ij} is close to the expected duration $E(X)$. Our estimates reveal that persons receiving care at home change to care in an institution after approximately one year. The values of the expected duration vary between 11.357 and 14.731 months for the reported ages. For the elderly being cared for at home, the duration before death increases with the age. A 70-year-old male remains on average

18.480 months at home before dying, 20.441 months if he is 80 years old and 24.570 months at age 90. An explanation for this increase may be linked to the pathology of the person. For example, individuals affected by cancer exhibit lower expected lifetimes than those affected by mental diseases (see, e.g., Kaeser, 2012). The latter are usually diagnosed at higher ages (typically above 80 years), justifying the trend that we observe.

4.3 Robustness tests

In this section, we investigate on the robustness of our results. We first analyze the appropriateness of using a Weibull distribution for modeling the duration law introduced in Equation (15). Then, we test the stability of the semi-Markov estimates presented in Tables 4 and 5 by reestimating the model on two subsamples covering the observed transitions in the two periods 1995–2010 and 2011–2015.

From the graphical representation laid out in Figures 4 and 5, we have derived the choice of the Weibull distibution for modeling the time spent in dependence. In the following, we statistically support the choice of this duration law using the quantile-quantile plots reported in Figure 9. For illustration purpose, we choose to describe the results for the transition from the moderate (2) to the severe state of dependency (3) in the frailty level model for men at the ages of 70, 80 and 90 years. The graphs (a–c) in Figure 9 illustrate the goodness-of-fit of the duration law at these three ages. For the presented cases, we observe that the number of data points is large and that the fit is rather appropriate for durations below 60 months since the sample quantiles follow relatively well the theoretical quantiles. Note that 82.1% of the transitions at the age of 70 years occur at times below 60 months, they are 92.5% at the age of 80 years and 97.2% at the age of 90 years. We decide to present all further results in Section 4.4 for durations up to five years.

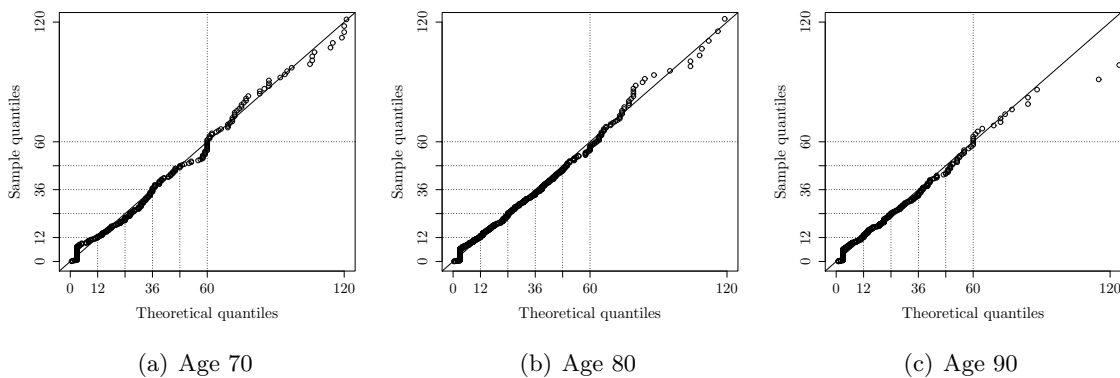
| (a) Age 70 | (b) Age 80 | (c) Age 90 |

Figure 9: Q-Q plots of the duration (in months) before the transition from moderate (2) to severe (3) in the frailty level model for males at the ages of 70, 80 and 90 years.

We also assess the stability of the semi-Markov parameter estimates by reevaluating the model on a first sample covering the years from 1995 to 2010 and a second sample covering the years from 2011 to 2015. This decomposition reflects the periods before and after the recognition in 2011 of the mildly dependent persons cared at home (see the discussion in Section 3.1). Considering again the transition $2 \rightarrow 3$ and male persons, we present the estimates of the Markov probabilities ϕ_{23}, the shape and the scale parameters σ_{23} and θ_{23} of the Weibull distribution

law by age in Figure 10. We display the estimates obtained on the overall period (1995–2015) with its 95% confidence interval (cf. Figure 8) together with the estimates obtained for the periods 1995–2010 and 2011–2015. Our results report relatively stable estimates among the periods. We observe identical shapes for the three presented parameters as a function of age. Looking at the Weibull shape and scale parameters, we observe that the results remain in the confidence interval of the period from 1995 to 2015. A similar comment can be made for the Markov probability. However, the estimates over the period 2011–2015 are lower when compared to the 1995–2010 period.

(a) Markov probability   (b) Weibull shape parameter   (c) Weibull scale parameter
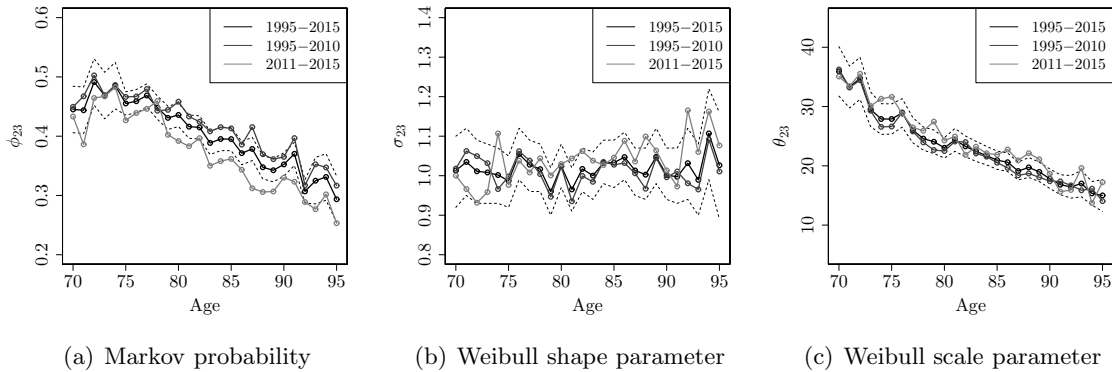
Note: Dashed lines indicate the confidence interval at the 95% level.

Figure 10: Robustness test on the estimates of the Markov probabilities, the shape and scale parameters of the Weibull law for the transition from moderate (2) to severe (3) in the frailty level model for males.

4.4 Transition probabilities

Using the parameter estimates derived in Section 4.2, the calculation of the transition probabilities requires the evaluation of the $p_{ij}(t)$ expressions given in Equations (10) and (11) and Equations (12) and (13) for the two dependency models. We evaluate the time integrals contained in these expressions using numerical integration. To do so, we apply a trapezoidal rule with $1\,000$ steps per month. We first compute the staying probabilities $p_{ii}(t)$ (Equations 10 and 12). Next, in the frailty level model, the leaving probabilities are calculated in the following order: $p_{34}(t)$, $p_{23}(t)$, $p_{24}(t)$, $p_{12}(t)$, $p_{13}(t)$ and $p_{14}(t)$. In the type of care model, these probabilities are evaluated in the following order: $p_{b4}(t)$, $p_{a4}(t)$ and $p_{ab}(t)$. For illustration, we provide numerical results for selected durations t up to 60 months spent in the states.

Transition probabilities in the frailty level model

Table 6 presents an excerpt from the actuarial dependence table for the states of the frailty level model. The dependence table is an important consideration for the pricing of LTC insurance products since it represents the technical basis for premium calculations. In our case, the table corresponds to a dependence table for the 1995–2015 period that, in contrast to a cohort table, assigns the same transition probability to persons of the same age regardless of the year of birth. This approach ensures that the values for each transition are supported by sufficient data. For both genders and at the ages of 70, 80 and 90, we report the transition probabilities for the durations $t \in \{3, 6, 12, 18, 24, 36, 48, 60\}$ in months. The numerical values

Months	Male								Female							
	3	6	12	18	24	36	48	60	3	6	12	18	24	36	48	60
Age 70																
p_{11}	0.8523	0.7496	0.5983	0.4899	0.4079	0.2931	0.2179	0.1658	0.8415	0.7391	0.5907	0.4850	0.4054	0.2950	0.2238	0.1754
p_{12}	0.0260	0.0442	0.0719	0.0927	0.1093	0.1336	0.1502	0.1617	0.0301	0.0518	0.0862	0.1137	0.1366	0.1730	0.2009	0.2227
p_{13}	0.0265	0.0443	0.0700	0.0882	0.1021	0.1229	0.1384	0.1510	0.0391	0.0644	0.1005	0.1255	0.1440	0.1692	0.1857	0.1975
p_{14}	0.0952	0.1619	0.2598	0.3292	0.3807	0.4504	0.4935	0.5215	0.0893	0.1447	0.2226	0.2758	0.3140	0.3628	0.3896	0.4044
p_{22}	0.9479	0.8925	0.7840	0.6833	0.5920	0.4384	0.3198	0.2304	0.9450	0.8891	0.7836	0.6880	0.6028	0.4602	0.3497	0.2647
p_{23}	0.0110	0.0220	0.0437	0.0647	0.0850	0.1231	0.1576	0.1882	0.0157	0.0317	0.0629	0.0930	0.1215	0.1741	0.2202	0.2596
p_{24}	0.0411	0.0855	0.1723	0.2520	0.3230	0.4385	0.5226	0.5814	0.0393	0.0792	0.1535	0.2190	0.2757	0.3657	0.4301	0.4757
p_{33}	0.9608	0.9163	0.8259	0.7390	0.6580	0.5159	0.4002	0.3078	0.9756	0.9457	0.8812	0.8155	0.7510	0.6302	0.5230	0.4303
p_{34}	0.0392	0.0837	0.1741	0.2610	0.3420	0.4841	0.5998	0.6922	0.0244	0.0543	0.1188	0.1845	0.2490	0.3698	0.4770	0.5697
Age 80																
p_{11}	0.8469	0.7054	0.4828	0.3280	0.2223	0.1021	0.0473	0.0220	0.8880	0.7652	0.5468	0.3773	0.2527	0.1039	0.0378	0.0122
p_{12}	0.0114	0.0227	0.0428	0.0594	0.0728	0.0917	0.1027	0.1088	0.0180	0.0380	0.0752	0.1058	0.1297	0.1603	0.1755	0.1823
p_{13}	0.0166	0.0313	0.0531	0.0674	0.0766	0.0863	0.0902	0.0918	0.0232	0.0476	0.0887	0.1189	0.1399	0.1640	0.1745	0.1786
p_{14}	0.1251	0.2406	0.4213	0.5452	0.6283	0.7199	0.7598	0.7774	0.0708	0.1492	0.2893	0.3980	0.4777	0.5718	0.6122	0.6269
p_{22}	0.9244	0.8445	0.6934	0.5611	0.4490	0.2797	0.1690	0.0996	0.9477	0.8891	0.7716	0.6609	0.5602	0.3926	0.2676	0.1783
p_{23}	0.0077	0.0155	0.0311	0.0462	0.0606	0.0873	0.1104	0.1294	0.0108	0.0224	0.0460	0.0696	0.0928	0.1368	0.1765	0.2104
p_{24}	0.0679	0.1400	0.2755	0.3927	0.4904	0.6330	0.7206	0.7710	0.0415	0.0885	0.1824	0.2695	0.3470	0.4706	0.5559	0.6113
p_{33}	0.9441	0.8782	0.7458	0.6235	0.5156	0.3441	0.2240	0.1430	0.9684	0.9269	0.8355	0.7427	0.6535	0.4945	0.3653	0.2647
p_{34}	0.0559	0.1218	0.2542	0.3765	0.4844	0.6559	0.7760	0.8570	0.0316	0.0731	0.1645	0.2573	0.3465	0.5055	0.6347	0.7353
Age 90																
p_{11}	0.8931	0.7746	0.5602	0.3899	0.2619	0.1060	0.0370	0.0112	0.9030	0.7903	0.5791	0.4066	0.2757	0.1148	0.0419	0.0135
p_{12}	0.0031	0.0071	0.0157	0.0242	0.0321	0.0446	0.0525	0.0568	0.0061	0.0133	0.0279	0.0415	0.0532	0.0708	0.0814	0.0868
p_{13}	0.0051	0.0103	0.0192	0.0257	0.0305	0.0368	0.0407	0.0430	0.0099	0.0208	0.0404	0.0561	0.0681	0.0832	0.0905	0.0934
p_{14}	0.0987	0.2080	0.4049	0.5602	0.6755	0.8126	0.8698	0.8890	0.0810	0.1756	0.3526	0.4958	0.6030	0.7312	0.7862	0.8063
p_{22}	0.9098	0.8123	0.6281	0.4714	0.3454	0.1750	0.0833	0.0378	0.9288	0.8504	0.6961	0.5563	0.4359	0.2550	0.1415	0.0754
p_{23}	0.0026	0.0053	0.0107	0.0162	0.0218	0.0330	0.0437	0.0530	0.0059	0.0119	0.0240	0.0361	0.0482	0.0713	0.0924	0.1101
p_{24}	0.0876	0.1824	0.3612	0.5124	0.6328	0.7920	0.8730	0.9092	0.0653	0.1377	0.2799	0.4076	0.5159	0.6737	0.7661	0.8145
p_{33}	0.9159	0.8180	0.6317	0.4747	0.3499	0.1821	0.0907	0.0436	0.9497	0.8844	0.7464	0.6152	0.4984	0.3145	0.1905	0.1117
p_{34}	0.0841	0.1820	0.3683	0.5253	0.6501	0.8179	0.9093	0.9564	0.0503	0.1156	0.2536	0.3848	0.5016	0.6855	0.8095	0.8883

Table 6: Dependence table by gender for selected ages (70, 80, 90 years) and durations (3 to 60 months) in the frailty level model.

correspond to the transition probabilities $p_{ij}(t)$ for the transition $i \to j$ and the duration t. For each state i, we consider the staying probability $p_{ii}(t)$ and the leaving probabilities $p_{ij}(t)$. The staying probabilities are highlighted, and we have $p_{ii}(t) = 1 - \sum_j p_{ij}(t)$, where the index j takes values in $\{1, 2, 3, 4\}$.

In addition to the numerical values reported in Table 6, we illustrate the transition probabilities for male and female in Figures 11 and 12, respectively. In both figures, we present the transition probabilities affecting the mild (1), moderate (2) and severe (3) states. The corresponding graphs for each dependency level i are displayed in rows. In a given row, we present the graphs related to ages 70, 80 and 90. The graphs related to a state i list the probabilities $p_{ij}(t)$ for the given i and all possible $j \geq i$ after a duration $t \in [0, 60]$ in months.

For a mildly dependent person, the probability $p_{11}(t)$ of remaining in the mild state decreases both with the duration t and the person's age. We observe an important reduction in the probability p_{11} during the first 24 months. A *70-year-old man* has a 59.83% probability of staying in the mild dependency state after one year. This probability becomes 40.79% after two years and continues to decrease over time. When not remaining in the mild dependency state, he can either become moderately dependent with probability p_{12}, severely dependent with probability p_{13} or die with probability p_{14}. After 12 months, the probabilities are $p_{12} = 7.19\%$, $p_{13} = 7.00\%$, and $p_{14} = 25.98\%$. These three probabilities are 10.93%, 10.21% and 38.07% after 24 months and 13.36%, 12.29% and 45.04% after 36 months. As expected, we observe that these three transition probabilities increase with the time spent in the mild state. Age is also a relevant factor. In fact, the probabilities p_{11}, p_{12} and p_{13} decrease with a person's age because the mortality p_{14} meaningfully increases. For example, after a 36-months duration in the mild state of
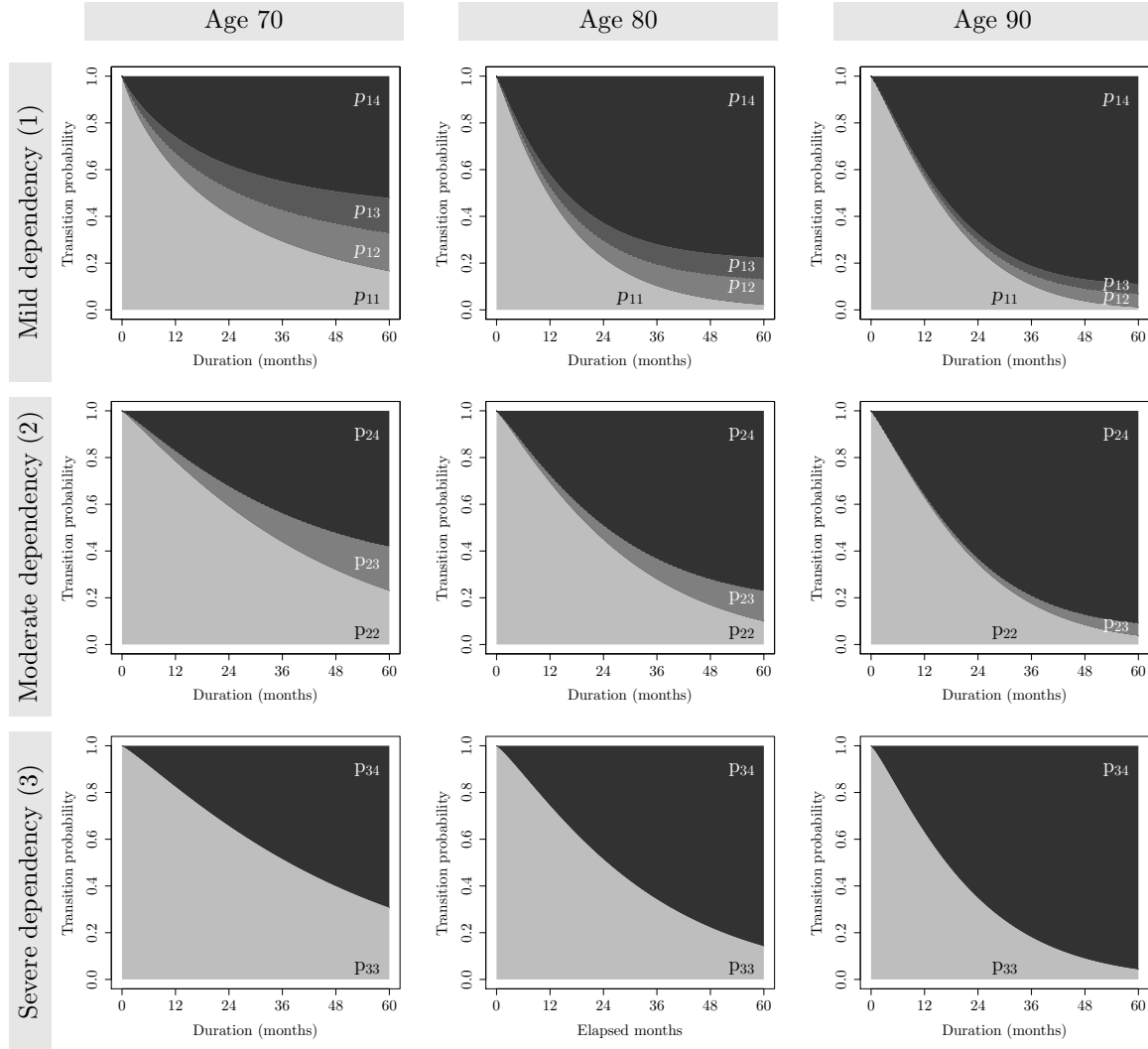
Figure 11: Transition probabilities for males at the ages of 70, 80 and 90 in the frailty level model.

dependency, a 70-year-old man has a 45.04% probability of dying. This probability increases to 71.99% for an 80-year-old man and to 81.26% for a 90-year-old man.

Analyzing the results for moderately and severely dependent males aged 70 years, we observe the same trends as described above. On the one hand, the probabilities of remaining in the moderate state p_{22} or of remaining in the severe state p_{33} are both decreasing with age and duration. After a 12-month duration, the values of p_{22} are 78.40%, 69.34% and 62.81% for a 70-, 80- and 90-year-old man, respectively. For p_{33}, they are 82.59%, 74.58% and 63.17%, respectively. On the other hand, the leaving probability p_{23} and the death probabilities p_{24} and p_{34} increase with the duration. For a 70-year-old moderately dependent male, the probability p_{23} of entering the severe state is 4.37% after 12 months, 8.50% after 24 months and 12.31% after 36 months. After the same durations, the death probabilities p_{24} and p_{34} are 17.23%, 32.30%, and 43.85% and 17.41%, 34.20%, and 48.41%, respectively. Finally, for short durations, we observe that mildly dependent individuals have a higher mortality p_{14} relative to the moderately and severely dependent persons (p_{24} and p_{34}). At a first glance, this may appear counterintuitive since limitations in ADL are typically linked to poorer health. However, two effects may

be concealed behind this observation. First, mildly dependent persons are more often cared for at home with no permanent assistance and no professional care infrastructure. Second, pathologies such as cancer may entail a very high mortality but express only few limitations in ADL. Other pathologies including cognitive diseases entail important ADL limitations without having a specific impact on the mortality (see also, e.g., Biessy, 2016).

For elderly females, the three main trends discussed above for males hold. First, the staying probabilities $p_{ii}(t)$ are decreasing with the time t spent in state i. Second, the leaving probabilities $p_{ij}(t)$, $i \neq j$, are increasing with the duration t. Third, given the increasing mortality $p_{i4}(t)$ with the age, the sum $\sum_{j \neq 4} p_{ij}(t)$ of all the other probabilities, i.e., the probabilities of staying and leaving for another frailty state decrease with age. This can be observed in Figure 12. Further, from the right-hand side of Table 6, we find, for example, that a mildly dependent *70-year-old woman* has a $p_{11} = 59.07\%$ probability of remaining mildly dependent after 12 months, a probability that decreases to 40.54% after 24 months and to 29.50% after 36 months. The leaving probability $p_{12} = 8.62\%$ for entering the moderate dependency state after 12 months becomes 13.66% after 24 months. Finally, the probability of dying increases
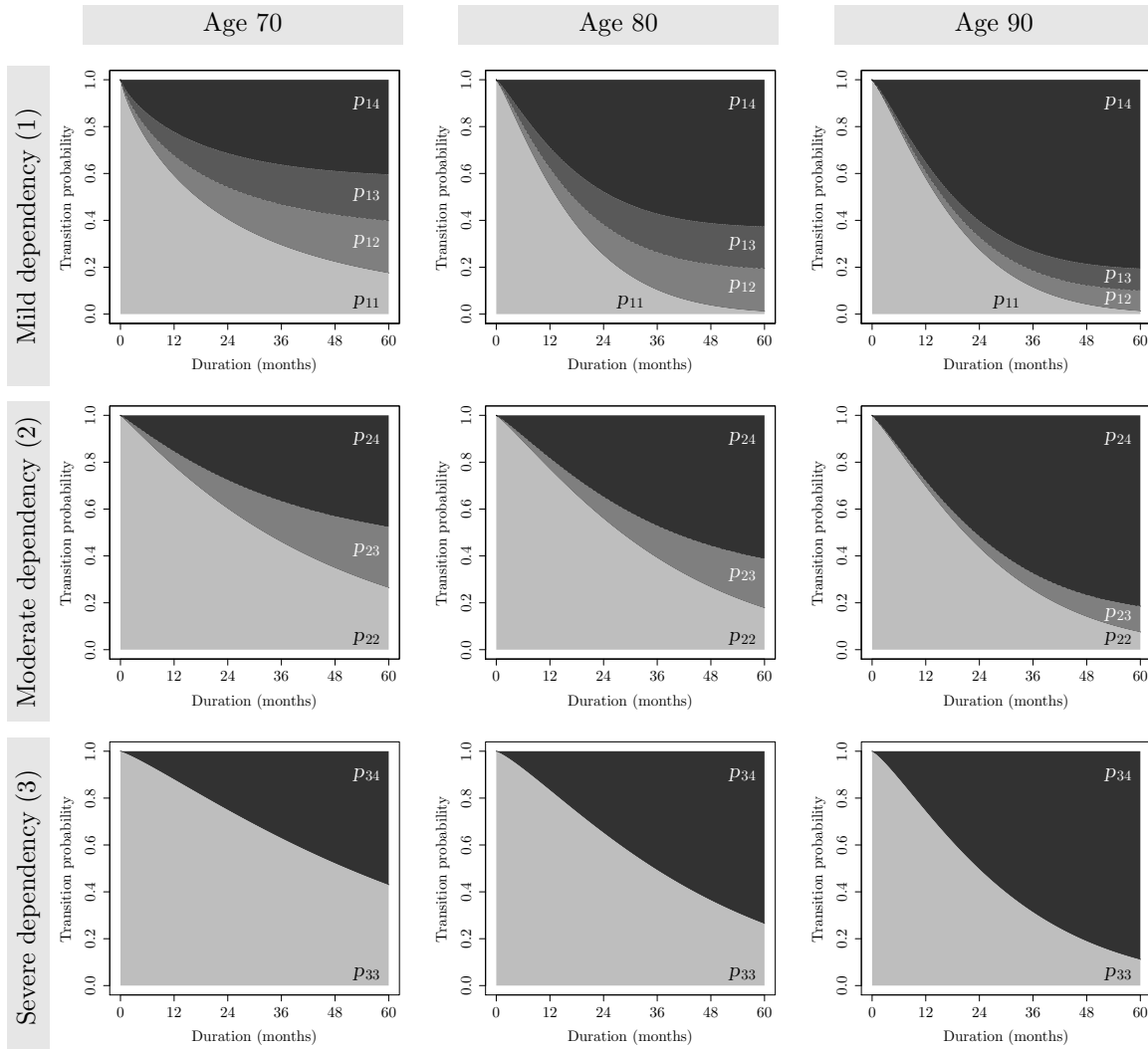

Figure 12: Transition probabilities for females at the ages of 70, 80 and 90 in the frailty level model.

from $p_{14} = 22.26\%$ after 12 months to 31.40% after 24 months for a 70-year-old woman. For $t = 12$ months, p_{14} increases to 28.93% at age 80 and to 35.26% at age 90. In these cases, the complementary probability, i.e., the sum of p_{11}, p_{12} and p_{13}, decreases.

By comparing the male transition probabilities with the female ones, we observe gender differences. At the three reported ages and for any duration in any dependency state, women show higher values than men for all the probabilities of leaving for another frailty state (p_{12}, p_{13}, p_{23}), while their mortality (p_{14}, p_{24}, p_{34}) is lower. Further, at ages 80 and 90, women show higher values for the staying probabilities (p_{11}, p_{22}, p_{33}) than men. This may be explained by the significantly lower female mortality at higher ages.

Figure 13 details the above probabilities in the example of the moderate (2) to severe (3) transition through ages from 70 to 95 for both genders. The graphs show the transition probabilities for the durations of 3, 12, 24, 36, 48 and 60 months. Recall that the ages presented in the graphs correspond to the entrance ages in state 2. Thus, along a given curve on the graphs, the actual age is obtained by summing the entrance age (reported on the x-axis) and the time spent in the state. For short durations, e.g., for individuals having spent up to three months in the moderate state (2), the transition probabilities to the severe state (3) seem quasi-independent of age since they stay close to zero for both genders. For longer durations, e.g., greater than 12 months, the transition probability is much higher at lower ages. In fact, both genders show a decreasing transition probability p_{23} with increasing entrance age. This effect becomes more important for longer durations. For example, the transition probability p_{23} for a 70-year-old man having spent 60 months in the moderate state (2) is approximately 19%, while for a 90-year-old man it is 5%. In fact, for the latter man, the mortality p_{24} is significantly higher (cf. Figure 11). Our results also allow us to identify the combined effect of an individual's (entrance) age and the duration on the transition probability. For example, a man who entered state 2 at age 70 attains an effective age of 75 years after a 60-month duration and bears a 19% transition probability p_{23}. This compares to a nearly zero transition probability for a 75-year-old man entering state 2. This example illustrates the important additional effect of the duration beyond the sole consideration of (entrance) age.
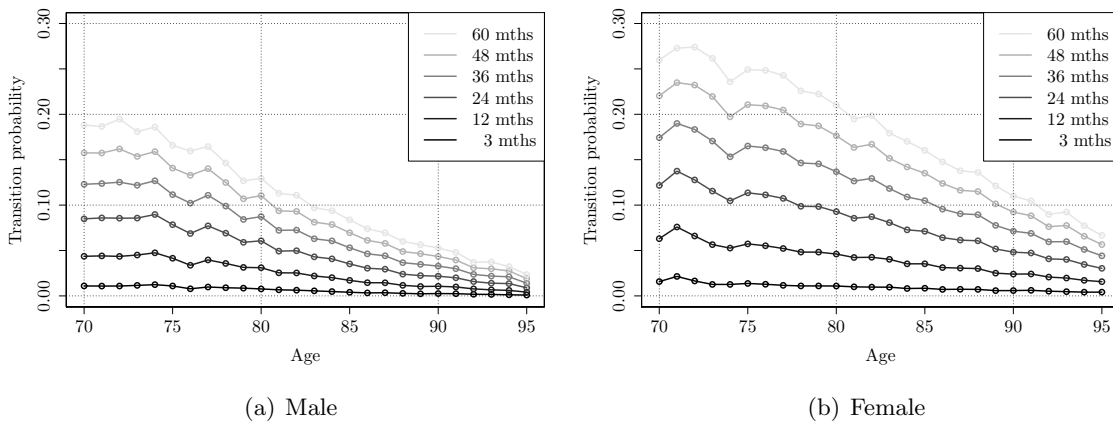

(a) Male   (b) Female

Figure 13: Illustration of the transition probability $p_{23}(t)$ for selected durations t and at the ages from 70 to 95 years for both genders.

The derivation of the dependence tables for the frailty level model has identified three important variables, the gender, the (entrance) age and the duration. We discover that women, compared to men, stay longer in the dependence states given their lower mortality. This is a classical result consistent with Mathers (1996), Mathers et al. (2001) and Fong et al. (2017), who find that elderly females live more years in dependence. As discussed above, the combined effect of the age and the duration impacts the transition probabilities. Finally, we argue that the type of care received and the specific pathologies inducing dependency may be key factors for explaining of the transition probabilities. In the following section, we focus on the influence of the type of care received by studying the transition probabilities in the type of care model.

Transition probabilities in the type of care model

Table 7 summarizes the transition probabilities in the states of the type of care model for males and females at ages 70, 80 and 90. The tables are constructed analogously to those for the frailty level model (cf. Table 6) and report the probabilities for durations between 3 and 60 months. Figures 14 and 15 graphically illustrate these results.

Months	Male								Female							
	3	6	12	18	24	36	48	60	3	6	12	18	24	36	48	60
Age 70																
p_{aa}	0.8747	0.7217	0.4504	0.2571	0.1352	0.0290	0.0045	0.0006	0.8544	0.7093	0.4726	0.3073	0.1973	0.0804	0.0332	0.0143
p_{ab}	0.0442	0.0985	0.1950	0.2630	0.3052	0.3422	0.3518	0.3538	0.0579	0.1196	0.2274	0.3088	0.3667	0.4328	0.4610	0.4720
p_{a4}	0.0811	0.1798	0.3546	0.4799	0.5596	0.6288	0.6437	0.6456	0.0877	0.1711	0.3000	0.3839	0.4360	0.4868	0.5058	0.5137
p_{bb}	0.9753	0.9426	0.8697	0.7939	0.7191	0.5798	0.4590	0.3581	0.9809	0.9557	0.8989	0.8388	0.7782	0.6613	0.5545	0.4599
p_{b4}	0.0247	0.0574	0.1303	0.2061	0.2809	0.4202	0.5410	0.6419	0.0191	0.0443	0.1011	0.1612	0.2218	0.3387	0.4455	0.5401
Age 80																
p_{aa}	0.8547	0.7153	0.4876	0.3240	0.2107	0.0840	0.0312	0.0109	0.8919	0.7703	0.5505	0.3776	0.2494	0.0967	0.0319	0.0091
p_{ab}	0.0228	0.0453	0.0849	0.1166	0.1412	0.1736	0.1908	0.1990	0.0341	0.0740	0.1499	0.2128	0.2611	0.3214	0.3495	0.3610
p_{a4}	0.1225	0.2394	0.4275	0.5594	0.6481	0.7424	0.7780	0.7901	0.0740	0.1557	0.2996	0.4096	0.4895	0.5819	0.6186	0.6299
p_{bb}	0.9651	0.9158	0.8046	0.6913	0.5842	0.4015	0.2648	0.1689	0.9808	0.9511	0.8787	0.7984	0.7163	0.5594	0.4228	0.3110
p_{b4}	0.0349	0.0842	0.1954	0.3087	0.4158	0.5985	0.7352	0.8311	0.0192	0.0489	0.1213	0.2016	0.2837	0.4406	0.5772	0.6890
Age 90																
p_{aa}	0.8951	0.7774	0.5629	0.3919	0.2634	0.1069	0.0373	0.0113	0.9024	0.7890	0.5780	0.4073	0.2775	0.1170	0.0433	0.0141
p_{ab}	0.0064	0.0143	0.0312	0.0478	0.0627	0.0858	0.1000	0.1071	0.0134	0.0299	0.0640	0.0952	0.1217	0.1590	0.1791	0.1883
p_{a4}	0.0985	0.2083	0.4059	0.5603	0.6739	0.8073	0.8627	0.8816	0.0842	0.1811	0.3580	0.4975	0.6008	0.7240	0.7776	0.7976
p_{bb}	0.9489	0.8764	0.7175	0.5658	0.4337	0.2385	0.1222	0.0590	0.9680	0.9192	0.8040	0.6833	0.5682	0.3729	0.2312	0.1368
p_{b4}	0.0511	0.1236	0.2825	0.4342	0.5663	0.7615	0.8778	0.9410	0.0320	0.0808	0.1960	0.3167	0.4318	0.6271	0.7688	0.8632

Table 7: Dependence table by gender for selected ages (70, 80, 90 years) and durations (3 to 60 months) in the type of care model.

Different conjectures can be drawn from the results. For a dependent elderly person receiving care at home, the probability $p_{aa}(t)$ of remaining in this type of care is decreasing with the duration t and increasing with age. We observe that a 70-year-old man has a $p_{aa} = 45.03\%$ probability of still being cared for at home after 12 months. This value decreases to 13.51% after 24 months and 2.91% after 36 months. At age 80, these probabilities are 48.76%, 21.07% and 8.41%. The probability p_{ab} of entering a care institution after having been cared for home increases from 19.50% (after 12 months) to 30.52% (after 24 months) at age 70. Both death probabilities p_{a4} and p_{b4} increase with the duration and age. After 36 months, we observe a 62.88% probability of dying for a 70-year-old man receiving care at home (a). This probability becomes 74.24% and 80.73% at the ages of 80 and 90 years, respectively. The corresponding mortality p_{b4} is lower for elderly persons living in an institution (b): a 70-year-old man has a 42.02% probability of dying after 36 months; at ages 80 and 90, the mortality is 59.85% and 76.15%, respectively. Regarding the male to female comparison, we can draw the same conclusion as in the frailty level model, i.e., the trends observed for men also hold for women. By further contrasting genders, we find lower death probabilities p_{a4} and p_{b4} for women.
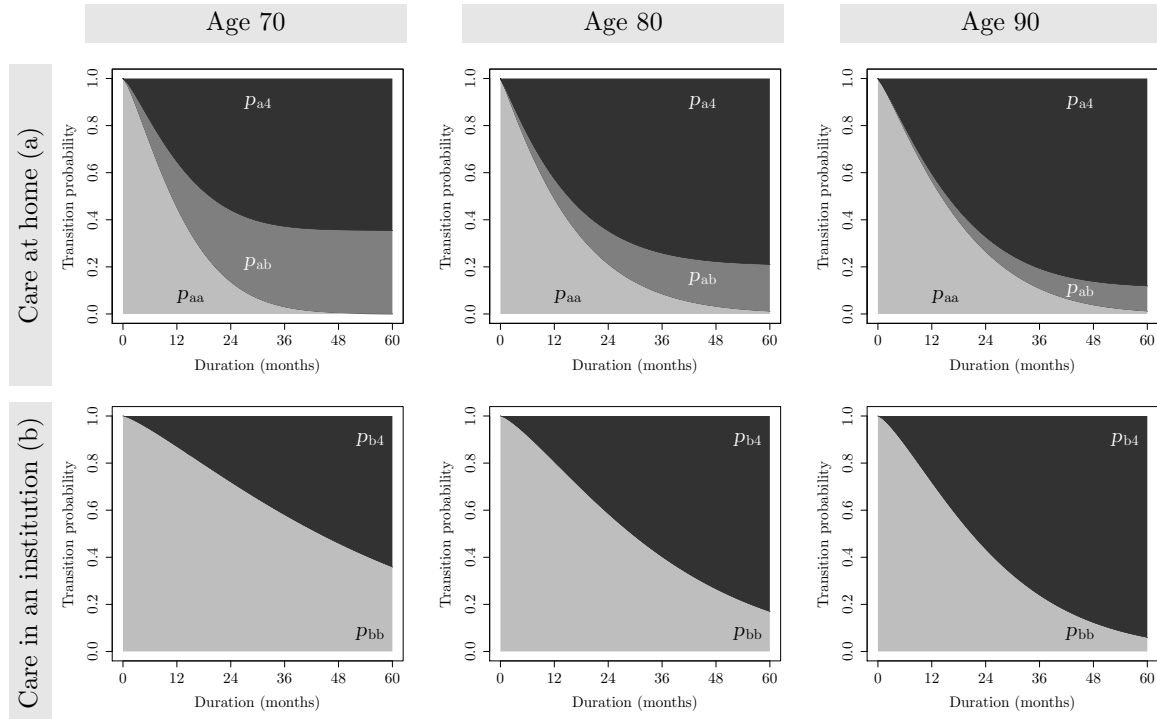
Figure 14: Transition probabilities for males at the ages of 70, 80, 90 in the type of care model.
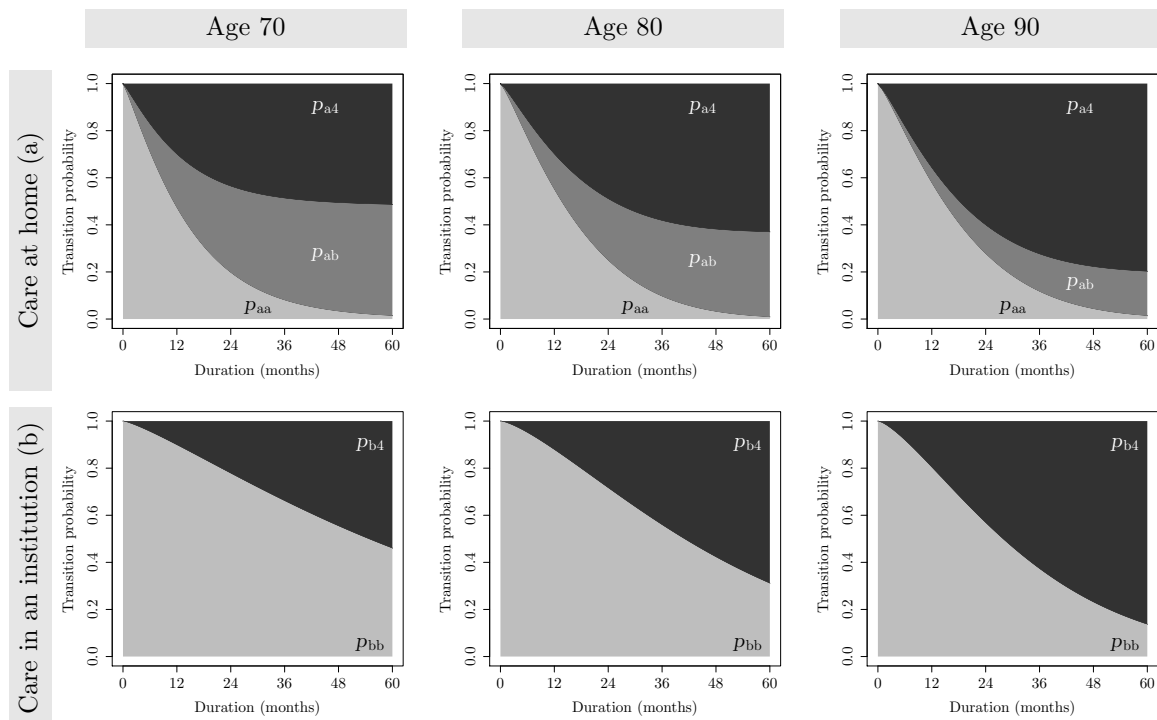

Figure 15: Transition probabilities for females at the ages of 70, 80, 90 in the type of care model.

The results from the type of care model are most relevant for the development of insurance products. In fact, the costs differ significantly between care at home and care in an institution. Similar to our findings in the frailty level model, we identify that the gender, the age and

the duration are three relevant variables for calculating transition probabilities. In particular, an important share of elderly persons cared for at home enters an institution after one year. At ages 70, 80 and 90, we conclude that elderly persons living in an institution have lower death probabilities than do those living at home (see e.g., Joly et al., 2009). This supports the hypothesis regarding the importance of the type of care made above (see the discussion of the results of the frailty level model). Institutions offer 24-hour supervision and more specialized infrastructure. Finally, an open point remains concerning the effect of the underlying pathologies of the dependent persons on the transition probabilities.

5   Conclusion

Due to limited data availability, most of the literature on LTC cannot account for the duration effect on the transition probabilities between different states of dependency. In this article, we develop dependence probability tables based on two models focusing on the frailty levels and the types of care received. In both models, we examine the paths followed by elderly persons from autonomy to death. In the frailty level model, we distinguish the three states of dependency, mild, moderate and severe, while in the type of care model, we concentrate on the types of care received, i.e., at home and in an institution. Our approach relies on the semi-Markov framework, and we derive analytical expressions for the transition probabilities. The proposed solution allows for a straightforward interpretation since it only depends on the estimation of the hazard rates. We reinforce the existing literature on LTC and insurance pricing (compare with the work of Biessy, 2015b) by applying this framework to two models and a unique longitudinal dataset that contains observations on the total population's LTC needs recorded over a 20-year period in Switzerland.

From the descriptive statistics on the paths followed and on the time spent by dependent persons in the considered states, we find that the average duration spent in severe LTC dependence or in institutional care is approximately three years. Then, we provide actuarial dependence tables by acuity level for both genders and selected ages. Our results show that transition probabilities depend on the individual's gender, age and duration in the previous state. In both models, we find that women spend more time than men in all of the dependency states (Fong et al., 2017). From the analyses in the type of care model, we learn that a major part of the dependent persons cared for at home switch to institutional care after one year. We conclude that receiving institutional care, compared to home-based care, is associated with lower death probabilities due to the specialized services offered. This argument, together with different underlying pathologies may explain why, for short durations, mildly dependent individuals have a higher mortality than the moderately and severely dependent persons.

Finally, we identify several directions for further research. The inclusion of data on the dependents' pathologies could help to improve the interpretation of the transition probabilities and the durations in the different acuity states (Biessy, 2016). Thereby, the development of prospective dependence tables given the future development of living conditions (e.g., Planchet and Tomas, 2015) as well as a more detailed understanding of the probability to lose autonomy are open problems. Moreover, socioeconomic factors such as former occupation or profession, the level of education, previous income and wealth may prove to be significant drivers (e.g., Szanton et al., 2010; Van den Bosch et al., 2013). Our work lays the basis for further development of LTC pricing and valuation that may lead to an assessment and further development

of the social systems and insurance solutions offered. The methodology and our findings are directly relevant for academics and insurance practice, including beyond Switzerland.

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