DEBT-STABILIZING PROPERTIES OF GDP-LINKED SECURITIES: A MACRO-FINANCE PERSPECTIVE

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ABSTRACT. We study the debt-stabilizing properties of indexing debt to GDP using a consumption-based macro-finance model. To this end, we derive quasi-analytical pricing formulas for any type of bond/equity by exploiting the discretization of the state-space, making large-scale simulations tractable. We find that GDP-linked security prices would embed time-varying risk premiums of about 40 basis points. For a fixed budget surplus, issuing GDP-linked securities does not imply more beneficial debt-to-GDP ratios in the long-run, while the debt-stabilizing budget surplus is more predictable at the expense of being higher. Our findings call into question the view that such securities tame debt.

JEL: C32, E43, G12, G18, H63.

Keywords: GDP-linked securities, term structure, consumption-based model, Markov-switching, debt stabilization

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1. Introduction

With the Great Recession, general government debt-to-GDP ratios in advanced economies increased considerably, from 71% in 2007 to 103% in 2019. Such large debt-to-GDP ratios can be a source of vulnerabilities, which amplify macroeconomic and financial shocks and thus increase the likelihood of a decline in economic activity.[1] Discussions in policy circles suggest that indexing debt to GDP would reduce the severity of such an event by providing the government with an automatic stabilizer to its finance. By issuing state-contingent debt instruments, the government's burden of servicing its debt is higher in periods of strong economic growth, while payments are reduced in periods of economic downturn. In other words, if all of a country's public debt were indexed to GDP, one would expect the public debt-to-GDP ratio to be hedged against unforeseen changes in nominal GDP growth. However, other drivers, beyond economic growth, are key in determining the debt-to-GDP ratio, such as the quantity of debt issued across time, the bond prices associated with these issuances and other debt-related variables. For instance, we expect that the price of a state-contingent asset, such as a GDP-linked security, is procyclical and therefore a government might need to issue more GDP-linked bonds to meet a given funding requirement in an adverse state of the world. Do GDP-linked securities remain a relevant tool for the stabilization of debt once accounting for a wider set of determinants?

In this paper, we address this question by extending the representation of the debt-to-GDP process to allow for the issuance of any type of bond (conventional or GDP-linked) with any maturity.This generalization features several drivers of the debt-to-GDP process and notably requires information on the price dynamics of GDP-linked bonds, which is not directly observable given no large developed economy has ever issued them.To this end, we estimate an asset-pricing model that exploits observed macroeconomic and financial data that are informative about the pricing of GDP-linked bonds. Notably, we take advantage of the natural

[1]High debt levels can crowd out private capital investment and push the government to increase distortive taxes and to decrease its investment in order to facilitate repayment (Yared, 2019). Indeed, this explains why many countries have adopted fiscal rules to limit excessive debt buildups since the early 1990s. For instance, the United States has an expenditure rule that places caps on public spending, while the member states of the European Union are subject to Maastricht criteria which include staying within the limits on government deficit (3% of GDP) and debt (60% of GDP) (see Lledó et al., 2017).

relationship between equity and GDP-linked bonds, as highlighted by Athanasoulis, Shiller, and Wincoop (1999).This relationship is reflected by a strong correlation between stock returns and GDP surprises, as illustrated in Panel (a) of Figure 1, where GDP surprises correspond to the unexpected part of (hypothetical) one-year GDP-linked bonds issued a year ago. Panel (b) of the same figure shows that the payoffs of conventional bonds are far less procyclical – and hence weakly correlated with stock returns – relative to GDP-linked bonds.[2] Hence, to study the price dynamics of GDP-linked bonds, our model selection is driven by its ability to jointly capture the dynamics of stock and bond returns. Specifically, we rely on the consumption-based model with habits *à la* Campbell and Cochrane (1999) and Wachter (2006) and extend it to include (i) the term structure of hypothetical GDP-linked bond yields and (ii) several economic growth regimes allowing, in particular, for persistent changes in the dynamics of fundamentals. This augmented framework allows for time-varying investors' preferences (through habit persistence) and countercyclical risk premiums to successfully capture the joint dynamics of macroeconomic (GDP, consumption, inflation) and financial (stock returns, nominal and real yields of different maturities) data. Moreover, by introducing several regimes, we capture salient economic phenomena, such as changing economic conditions, different risks across economic states, and rare events, which all significantly impact asset prices. A crucial advantage of this type of model is that it is consistent with a positive average slope of the term structure of real interest rates. By contrast, most alternative equilibrium models predict downward sloping real interest rate curves (e.g. Piazzesi and Schneider, 2007; Bansal and Shaliastovich, 2013; Creal and Wu, 2020),[3] which is at odds with empirical evidence (Zhao, 2020). This is of particular importance in our context because we want to compare the funding costs associated with different debt instruments. Therefore, using a model that erroneously generates very low long-term real rates is likely to bias results in favor of inflation-linked bonds.

[2]The close relationship between equity and GDP-linked bonds is implicitly reckoned in asset-pricing studies where shares are represented by claims on future consumption (e.g. Abel, 1999; Campbell and Cochrane, 1999; Campbell, 2003; Wachter, 2006; Seo and Wachter, 2018): if one assumes a unitary elasticity between consumption and GDP, these models then imply that a share is the exact same asset as a perpetual coupon-paying GDP-linked bond.

[3]These three papers however manage to get nominal yield curves that are upward sloping (on average), by introducing a positive correlation between inflation and the stochastic discount factor.

We further use this model to simulate debt-to-GDP ratios resulting from different public debt strategies involving all possible types of government bonds (conventional and GDP-linked bonds) and investigate their cost and risk implications. However, such simulation-based exercises cannot be implemented using standard computational methods because a single resolution of the model requires computationally intensive numerical integration. To bypass this issue, our pricing formulas exploit the so-called risk-neutral dynamics of the variables of interest, which we approximate in the context of a discretized state-space. Model estimation and large-scale simulations become tractable thanks to the resulting pricing formulas. To our knowledge, this paper is the first to (i) explicitly derive an approximate (i.e., discretized) risk-neutral dynamics, and (ii) to use it to obtain quasi-analytical formulas to price assets and to compute model-implied moments. Importantly, this approach is not specific to the present consumption-based model. In any model where researchers employ grids to determine a stochastic discount factor and the associated approximate dynamics of state variables, similar tractable formulas —for pricing and moments computation— are readily available using our methodology.

We consider two extreme situations regarding the fiscal reaction function that differ in the way the primary budget surplus is determined.In the first exercise, the fiscal authority does not react to the debt-to-GDP ratio, and the primary budget surplus is set to a constant value. In the second one, the fiscal authority aims at perfectly stabilizing the debt-to-GDP ratio. This strategy consists in setting the primary budget surplus to its debt-stabilizing value. Papers on optimal public debt management are often based on similar budget smoothing arguments, i.e. stabilizing or targeting the level of debt, see for instance Faraglia, Marcet, and Scott (2008). However, this strand of the literature is known to struggle in providing practically-implementable strategies (Faraglia, Marcet, and Scott, 2010). In our setup, we abstract from any normative recommendations (as in Bohn, 1990; Angeletos, 2002; Buera and Nicolini, 2004), and rather assess the implications of using different debt instruments in stabilizing the debt-to-GDP ratio.

Three results stand out from our analysis. First, GDP-linked bond prices would embed sizable and time-varying risk premiums, estimated at about 40 basis points along the maturity spectrum, as investors require an additional compensation for holding them. Second, for a fixed budget surplus, issuing GDP-linked bonds instead of conventional ones does not necessarily imply more beneficial debt-to-GDP ratios in the medium to long run. Third, the debt-stabilizing budget surplus is more predictable under GDP-linked bond issuances at the expense of being higher on average. Hence, our findings call into question the view that GDP-linked bonds tame debt.

The remainder of the paper is organized as follows. Section 2 defines the three bonds we consider in our analysis (nominal, inflation-linked and GDP-linked) and their respective yields-to-maturity. This section also characterizes the debt dynamics under the issuance of these three bonds. Section 3 describes our model and estimation strategy. Section 4 presents empirical results of the effects of the issuance of GDP-linked bonds on debt-to-GDP ratios. Section 5 concludes. An Online Appendix encloses technical results and robustness exercises.

2. Issued bonds and debt dynamics

In this section, we define GDP-linked bonds (GDP-LBs) and study their influence on debt dynamics. To highlight the properties of GDP-LBs, we contrast them against two standard instruments, namely nominal and inflation-linked bonds. Subsection 2.1 details the payoffs stemming from each type of bonds and defines their respective yields-to-maturity. Subsection 2.2 exploits yields-to-maturities to derive the debt accumulation process. Subsection 2.3 shows how the standard law of motion of the debt-to-GDP ratio can be extended to the general case where the government issues any type of bonds. Importantly, the content of the present section is not model dependent and could be used in the context of any modeling framework.

2.1. Bonds and risk premium

Without loss of generality, we only consider zero-coupon bonds.[4] Let us denote by $i_{t,h}$, $r_{t,h}$ and $r^*_{t,h}$, the continuously-compounded yields-to-maturity respectively associated with zero-coupon nominal, inflation-linked and GDP-linked bonds with residual maturities of h periods.

[4] Coupon-bearing bonds can always be replicated using a combination of zero-coupon bonds.

The definition of any type of yield is mostly conventional: a yield unequivocally relates the bond price (as of date t) to expected payoffs (at date $t+h$).

In the case of nominal and inflation-linked bonds, future payoffs are trivial. For nominal bonds with a face value of 1, the expected future *nominal* payoff (settled at date $t+h$) corresponds to one unit of currency as of date $t+h$. Similarly, the expected future *real* payoff (also settled at date $t+h$) of an inflation-linked bond of face value of 1 is equal to one unit of currency as of date t. This corresponds, in nominal terms, to $\exp(h\pi_{t,t+h})$ units of currency as of date $t+h$, with $h\pi_{t,t+h}$ denoting the inflation rate between dates t and $t+h$, where $\pi_{t,t+h} = 1/h \log(P_{t+h}/P_t)$ and P_t is the price level.[5] Note that, for a nominal bond, the expected nominal maturity payoff as of date t is deterministic. However, this is not the case for its real payoff, given inflation between t and $t+h$ is uncertain as of date t. Vice versa for an inflation-linked bond, the real payoff is deterministic while the nominal one is uncertain as of date t. The fact that nominal and inflation-linked bonds respectively feature predetermined nominal and real payoffs is key for the definition of their (nominal and real) yields-to-maturity. The nominal yield-to-maturity $i_{t,h}$ is based on the logarithm of the ratio between the *nominal* face value of a nominal bond (which is 1) and its current price $P^n_{t,h}$, that is

$$i_{t,h} \equiv \frac{1}{h} \log\left(\frac{1}{P^n_{t,h}}\right), \tag{1}$$

and the real yield-to-maturity $r_{t,h}$ is based on the logarithm of the ratio between the *real* face value of an inflation-linked bond (which is 1) and its current price $P^r_{t,h}$, that is

$$r_{t,h} \equiv \frac{1}{h} \log\left(\frac{1}{P^r_{t,h}}\right). \tag{2}$$

Such standard definitions of yields-to-maturity are not replicable for bonds whose (real or nominal) payoffs are not known as of date t, as is the case for GDP-LBs, due to their state-contingent nature. All the more, pure GDP-LBs have not been issued as of yet, hence there is

[5]In practice, the nominal payoff of inflation-linked bonds is indexed to lagged realized inflation (typically with a lag of a few months). Refer to Section I of Gürkaynak, Sack, and Wright (2010) for an illustration on US Treasury Inflation-Protected Securities (TIPS). In our setup, we abstract from the so-called indexation lag. To the extent that the pricing influence of this feature mainly concerns short-dated bonds, our results would not be affected had it been taken into account.

no established convention regarding the definition of their associated yields-to-maturity. We thus propose a definition below.

To do so, we first need to establish the exact payoff of such a bond. Following the literature, we consider payoffs that are linearly and unitarily indexed to nominal GDP growth between t and $t+h$. Consequently, the *nominal* maturity payoff of a GDP-linked bond issued at date t is of the form:

$$\alpha \exp(h[y_{t,t+h} + \pi_{t,t+h}]),$$

where $hy_{t,t+h}$ denotes the log real growth between dates t and $t+h$ (i.e., $\exp(hy_{t,t+h}) = Y_{t+h}/Y_t$ where Y_t is real GDP at date t). While the coefficient α can be set to any value, we have to make it precise in order to define a "unit face value"– a notion needed to define the yield-to-maturity. In what follows, a "unit face value" maturity-h GDP-linked bond issued at date t is defined as a bond characterized by $\alpha = 1/\mathbb{E}_t[\exp(hy_{t,t+h})]$, where $\mathbb{E}_t$ denotes the expectation operator conditional on information at date t. This bond provides the following *real* payoff to its bondholder at date $t+h$:

$$\frac{\exp(hy_{t,t+h})}{\mathbb{E}_t[\exp(hy_{t,t+h})]}. \tag{3}$$

It is important to note that, as of date t, the expected real maturity payoff of this bond is one. In other words, this bond has a "unit face value".

We now proceed to defining the yield-to-maturity of a GDP-linked bond. Inspired by Eq. (2), the date-t yield-to-maturity $r^*_{t,h}$ of a maturity-h GDP-linked bond is defined as the time-to-maturity-adjusted logarithm of the ratio between the expected *real* maturity payoff and the current bond price $P^*_{t,h}$:

$$r^*_{t,h} \equiv \frac{1}{h} \log\left(\frac{1}{P^*_{t,h}}\right), \tag{4}$$

where $P^*_{t,h} = \mathbb{E}_t\left(\mathcal{M}_{t,t+h}\frac{Y_{t+h}P_{t+h}}{\mathbb{E}_t(Y_{t+h})P_t}\right)$, and $\mathcal{M}_{t,t+h}$ is the nominal discount factor between t and $t+h$.

While the definition of the yield $r^*_{t,h}$ is conventional, its bond price is not once its payoff is defined as in Eq. (3). However, our definition admits two convenient features. First, it is easily seen that $r^*_{t,h}$ can be expressed as the date-t-expected log real return of a GDP-linked bond of maturity h issued at time t. Such a return can be expressed as:

$$\frac{1}{h}\log\left\{\mathbb{E}_t\left(\frac{\exp(hy_{t,t+h})}{\mathbb{E}_t[\exp(hy_{t,t+h})]}\right)\Big/P^*_{t,h}\right\},$$

which is also equal to the right-hand-side of Eq. (4).

Second, if investors were risk-neutral, then GDP-linked bond yields $r^*_{t,h}$ would coincide with the inflation-linked bond yields $r_{t,h}$, given that both rates are expressed in real terms. Specifically, Eq. (3) shows that the expected real maturity payoff of a unit face value GDP-linked bond is one, as is the case for a unit face value inflation-linked bond. Hence, risk-neutral agents would be indifferent between holding either bonds, implying $P^*_{t,h} = P^r_{t,h}$ and, further, $r^*_{t,h} = r_{t,h}$ (using Eqs. 2 and 4).

As explained above, this definition relies on the specification of the payoff of a unit face value GDP-linked bond, in the sense that $r^*_{t,h}$ is defined through the price $P^*_{t,h}$ of a bond whose payoff is given by Eq. (3). However, the latter payoff contains a term, $\mathbb{E}_t[\exp(hy_{t,t+h})]$, that is never perfectly observed in practice. One can circumvent this issue by setting a convention: the expectation at time t of GDP growth between t and $t+h$ could be proxied using survey-data such as the Survey of Professional Forecasters conducted by the Federal Reserve, or even using a smoothed average of past GDP growth rates.

In this context, $r^*_{t,h} - r_{t,h}$ is a direct measure of the GDP risk premium. Formally, denoting by $\mathcal{M}^r_{t,t+h} = \mathcal{M}_{t,t+h}P_{t+h}/P_t)$ the real stochastic discount factor, we have:

$$\begin{aligned}
P^*_{t,h} &= \mathbb{E}_t\left(\mathcal{M}^r_{t,t+h}\frac{Y_{t+h}}{\mathbb{E}_t\left(Y_{t+h}\right)}\right) = \mathbb{E}_t\left(\mathcal{M}^r_{t,t+h}\right)\mathbb{E}_t\left(\frac{Y_{t+h}}{\mathbb{E}_t\left(Y_{t+h}\right)}\right) + \mathbb{C}ov_t\left(\mathcal{M}^r_{t,t+h}, \frac{Y_{t+h}}{\mathbb{E}_t\left(Y_{t+h}\right)}\right)\\
&= \mathbb{E}_t\left(\mathcal{M}^r_{t,t+h}\right) + \mathbb{C}ov_t\left(\mathcal{M}^r_{t,t+h}, \frac{Y_{t+h}}{\mathbb{E}_t\left(Y_{t+h}\right)}\right)\\
&= \underbrace{P^r_{t,h}}_{\text{Inflation-linked bond}} + \underbrace{\mathbb{C}ov_t\left(\mathcal{M}^r_{t,t+h}, \frac{Y_{t+h}}{\mathbb{E}_t\left(Y_{t+h}\right)}\right)}_{\text{(level) GDP risk premium}}.
\end{aligned}$$

2.2. **The accounting of public debt**

Yields-to-maturity have important implications on public debt accounting. Specifically, they are involved in the recording of accrued interests, which play a crucial role in the determination of the outstanding debt. International statistical standards advocate for the use of the concept of "nominal valuation of debt securities", where the outstanding debt reflects the sum of funds originally advanced (the issue price), plus any subsequent advances, less any repayments, plus any accrued interest (see International Monetary Fund, Bank for International Settlements and European Central Bank, 2015, notably 5.51 on p. 39, 5.55 on p. 40).[6] According to the System of National Accounts (2008, paragraph 7.113), the accrued interest is the amount that issuers of debt securities become liable to pay over a given period of time without reducing the outstanding principal. This has important implications on the assessment of the indebtedness of a government because an inappropriate recording of accrued interest may result in a substantial undervaluation of government liabilities.

Using the above-mentioned accounting convention, we can provide the accrued debt generated by the issuances of the three types of bonds defined in Subsection 2.1. Table 1 reports the accrued nominal debt generated by each issuance between t (issuance date) and $t+h$ (maturity date). In the case of GDP-LBs, we make use of the following notation: $\exp(hy^e_{t,h}) = \mathbb{E}_t[\exp(hy_{t,t+h})]$.

2.3. **Debt accumulation process**

Assuming that the government only issues short-term nominal bonds, the dynamics of the nominal outstanding debt D_t, observed at the end of date t, is given by

$$D_t = D_{t-1}\exp(i_{t-1,1}) - BS_t, \tag{5}$$

where BS_t is the primary budget surplus at date t. Let us denote by small capitals a variable normalized by nominal GDP ($Y_t P_t$), where Y_t is the real GDP and P_t is the GDP deflator. Eq. (5) can therefore be written in terms of the debt-to-GDP ratio, as follows:

$$d_t = \exp(i_{t-1,1} - y_{t-1,t} - \pi_{t-1,t})d_{t-1} - bs_t, \tag{6}$$

where $d_t = D_t/(Y_t P_t)$ and $bs_t = BS_t/(Y_t P_t)$.

[6]This document is available at http://www.bis.org/publ/othp23.pdf.

This equation is often used in the GDP-LB literature, given it is always assumed that the government only issues short-term debt. However, this assumption being restrictive, we extend the representation of the dynamics of debt to allow for issuances of any maturity and bond type (nominal, inflation-linked, and GDP-linked bonds). This generalization necessitates the introduction of additional debt-related variables. Specifically, the outstanding debt depends on its own lagged values and also on the types and maturities of the debt instruments that have been issued in the past. At date t, the government issues the following amount of new debt:

$$I_t = \sum_{h=1}^{H} I_{t,h}^n + I_{t,h}^r + I_{t,h}^*, \tag{7}$$

where $I_{t,h}^n$, $I_{t,h}^r$ and $I_{t,h}^*$ are the amounts issued in the form of maturity-h nominal, real and GDP-linked bonds, respectively, and where H is the largest maturity of issued bonds. Using the information reported in the last column of Table 1 to account for the accruing of past issuances, we obtain the following expression for D_t:

$$D_t \quad = \quad I_t + \underbrace{\sum_{h=1}^{H}\sum_{k=1}^{h-1} I_{t-k,h}^n e^{k i_{t-k,h}} + I_{t-k,h}^r e^{k[r_{t-k,h}+\pi_{t-k,t}]} + I_{t-k,h}^* e^{k[r_{t-k,h}^* + (y_{t-k,t} - y_{t-k,h}^e) + \pi_{t-k,t}]}}_{\text{accrued (and non-repaid) debt}}. \tag{8}$$

This expression can be re-written as follows (see Online Appendix A for further details):[7]

$$\begin{aligned} & bs_t + d_t - d_{t-1} \\ = \quad & \sum_{h=1}^{H}\sum_{k=1}^{h} iss_{t-k,h}^n \left[e^{k(i_{t-k,h} - y_{t-k,t} - \pi_{t-k,t})} - e^{(k-1)(i_{t-k,h} - y_{t-k,t-1} - \pi_{t-k,t-1})}\right] + \\ & \sum_{h=1}^{H}\sum_{k=1}^{h} iss_{t-k,h}^r \left[e^{k(r_{t-k,h} - y_{t-k,t})} - e^{(k-1)(r_{t-k,h} - y_{t-k,t-1})}\right] + \\ & \sum_{h=1}^{H}\sum_{k=1}^{h} iss_{t-k,h}^* \left[e^{k(r_{t-k,h}^* - y_{t-k,h}^e)} - e^{(k-1)(r_{t-k,h}^* - y_{t-k,h}^e)}\right], \end{aligned} \tag{9}$$

where $iss_{t,k}^n = I_{t,k}^n/(Y_t P_t)$, $iss_{t,k}^r = I_{t,k}^r/(Y_t P_t)$, and $iss_{t,k}^* = I_{t,k}^*/(Y_t P_t)$.

It can be noted that the right-hand side of Eq. (9) corresponds to the value of the budget surplus bs_t that implies no change in the debt-to-GDP ratio between $t-1$ and t. We refer to this particular value as the "debt-stabilizing budget surplus" (DSBS). Note that the DSBS

[7] Naturally, this equation boils down to Eq. (6) when only short-term nominal bonds are issued.

value at time t does not depend on the choice of the type of bonds issued at the same date (i.e. $iss^*_{t,h}$, $iss^r_{t,h}$ and $iss^n_{t,h}$ do not appear on the right-hand side of Eq. 9).

3. Model

In this section, we introduce a model to analyze the debt-to-GDP ratio dynamics resulting from the issuance of different types of debt instruments. As shown above, the required framework needs to capture the joint fluctuations of bond prices, real GDP and inflation. To this end, we use an off-the-shelf consumption-based model with habits *à la* Campbell and Cochrane (1999) and Wachter (2006) and extend it to include the term structure of hypothetical GDP-linked bond yields.[8]

This setup features time-varying investors' preferences (through habit persistence) and is able to produce realistic risk premiums for both bonds and stocks. Our chosen framework is able to easily reproduce an upward-sloping term structure of real (TIPS) rates, as observed in the data. In contrast, most structural models mechanically generate a downward-sloping term structure of real rates.[9] The fact that our model delivers reliable average interest rates is a desirable characteristic since our study hinges on the average funding costs underlying the issuance of different types of bonds.

3.1. **Macroeconomic dynamics**

We consider a closed economy populated by an infinity of identical investors whose consumption level at time t is denoted by C_t. In order to capture persistent changes in the dynamics of fundamentals, we consider three growth regimes on consumption: it can be low when $z_{c,t} = [1,0,0]'$, intermediate when $z_{c,t} = [0,1,0]'$, and high when $z_{c,t} = [0,0,1]'$. Specifically, the logarithm of consumption, denoted by c_t, evolves according to:

$$c_t = c_{t-1} + g_c' z_{c,t-1} + \nu_t, \tag{10}$$

with $\nu_t \sim \ i.i.d. \mathcal{N}(0, \sigma_\nu^2)$, and where $g_c = [g_{c,\ell}, g_{c,i}, g_{c,h}]'$ is the expected consumption growth conditional on $z_{c,t-1}$, with $g_{c,\ell} < g_{c,i} < g_{c,h}$. The Markov chain $z_{c,t}$ is time-homogeneous. We

[8]Structural models impose underlying constraints across variables that allow us to discipline the dynamics of unobserved GDP-linked bond prices and deduce their implication on debt dynamics.

[9]See e.g. Piazzesi and Schneider (2007), in particular their Subsections 3.4 and 3.5, and Zhao (2020).

assume that, from one period to the other, we cannot go from the low to the high growth rate, and vice versa. As a result, $z_{c,t}$'s dynamics is characterized by four parameters: the probability to stay in each of the three regimes ($p_{\ell\ell}$, p_{ii}, and p_{hh}), and by $p_{i\ell}$, the probability to go from the intermediate to the low growth regime (we then have $p_{ih} = 1 - p_{ii} - p_{i\ell}$). Note that the formulas provided in this paper are general and are valid for any number of regimes or any matrix of transition probabilities.

Real GDP growth, defined as $\Delta y_t = \log(Y_t/Y_{t-1})$, and inflation, defined as $\pi_t = \log(P_t/P_{t-1})$, follow the processes:

$$\Delta y_t = g_c' z_{c,t-1} + \rho_y \nu_t + \varepsilon_t^y \tag{11}$$

$$\pi_t = (1-\psi)\overline{\pi} + \psi \pi_{t-1} + \rho_\pi \nu_t + \varepsilon_t^\pi, \tag{12}$$

where $\overline{\pi}$ is the average inflation, $\varepsilon_t^y \sim i.i.d.\mathcal{N}(0,\sigma_y^2)$, $\varepsilon_t^\pi \sim i.i.d.\mathcal{N}(0,\sigma_\pi^2)$, and ψ is a smoothing parameter. If $\rho_y \neq 0$ (respectively $\rho_\pi \neq 0$), consumption and GDP growth (resp. inflation) are correlated.

3.2. Agents' preferences

Following Campbell and Cochrane (1999) and Wachter (2006), each investor has the following utility over consumption:

$$\mathbb{E}_t\left(\sum_{h=0}^{+\infty} \delta^h \frac{(C_{t+h} - X_{t+h})^{1-\gamma} - 1}{1-\gamma}\right), \tag{13}$$

where X_t is the level of habit (a reference point), δ is the subjective discount factor, γ is the utility curvature.[10] Habit is defined through the dynamics of surplus consumption S_t, as follows:

$$S_t \equiv \frac{\widetilde{C}_t - X_t}{\widetilde{C}_t}, \tag{14}$$

where $\widetilde{C}_t$ denotes average consumption. Habits are external and therefore each investor's habit is determined by everyone else's consumption. This simplifies the analysis given that

[10] We also estimate a model *à la* Bansal and Yaron (2004) and Bansal and Shaliastovich (2013), in which investors feature Epstein and Zin (1989)'s preferences. This class of models mechanically generates a downward sloping real yield curve (see the third panel of Table J.3 in the Online Appendix). A promising avenue to overcome this limitation might be to combine external habits with Epstein and Zin (1989)'s preferences.

current consumption is not affected by future habit.[11] In equilibrium, identical investors choose the same level of consumption, and hence $\widetilde{C}_t = C_t$. Based on these assumptions, the stochastic discount factor between t and $t+1$ is given by:

$$M_{t,t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{15}$$

We now specify the joint dynamics of S_t and C_t. Since surplus consumption S_t is strictly positive, its dynamics can be defined through its logarithm s_t, given by:[12]

$$\begin{aligned} s_{t+1} &= (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - \mathbb{E}_t(c_{t+1})) \\ &= (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)\nu_{t+1} \end{aligned} \tag{16}$$

$$\lambda(s_t) = \begin{cases} \dfrac{1}{\exp(\bar{s})}\sqrt{1-2(s_t-\bar{s})} - 1 & \text{if } s_t \leq s_{max} \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

where ϕ controls the persistence of habit, $s_{max} = \bar{s} + \frac{1}{2}(1-\exp(\bar{s})^2)$, which ensures that $\lambda(s_t) \geq 0$, and with

$$\bar{s} = \log\left(\sqrt{\mathbb{V}ar(\nu)\frac{\gamma}{1-\phi-b/\gamma}} \right). \tag{18}$$

To calibrate the model, we make use of stock return moments. To this purpose, we complement our model with the specification of real dividends Div_t, which have the following law of motion:

$$\log Div_t = \log Div_{t-1} + g_d{}' z_{c,t-1} + \rho_d \nu_t + \varepsilon_t^d, \tag{19}$$

where the vector of expected dividend growth rates (conditional on $z_{c,t-1}$), $g_d = [g_{d,\ell}, g_{d,i}, g_{d,h}]'$, is such that the expected change in the dividend growth rate is proportional to the expected change in the consumption growth rate, in the spirit of Abel (1999), and $\varepsilon_t^d \sim i.i.d.\mathcal{N}(0, \sigma_d^2)$.[13]

[11] Note that the distinction between internal and external habits typically matters when analysing welfare properties but is immaterial for the macroeconomic effects of policies (Broer et al., 2023). Moreover, in the macro-finance literature, it has been shown that the use of either of the two leads to similar results (Kehoe et al., 2022), which is why papers that use the consumption-based asset pricing framework usually favour external habits (Campbell, Pflueger, and Viceira, 2020; Pflueger, 2023).

[12] Using the notation $x_t = \log(X_t)$, Eq. (16) implies that $\frac{dx_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda(s_t)}{\exp(-s_t)-1}$. As shown by Eq. (17), $\lambda(\bar{s}) = \dfrac{1}{\exp(\bar{s})} - 1$, therefore $\frac{dx_{t+1}}{dc_{t+1}} = 0$. This implies that habit is predetermined at the steady state.

[13] The resulting stock return formulas are detailed in the Online Appendix D.

Introducing regimes allows for persistent changes in the conditional mean and variance (i.e., uncertainty) of growth of consumption, real GDP and real dividends.[14]

3.3. **Model discretization and pricing formulas.**

The derivation of quasi-analytical pricing formulas hinges on the derivation of the approximate risk-neutral dynamics of the variables of interest, which we derive in the context of a discretized state space. Importantly, our pricing methodology is general and can be used to obtain explicit formulas in the context of any model whose dynamics has been discretized (i.e., as soon as a grid approach is resorted to). Key to our approach are two ingredients: (i) a selection vector z_t, that indicates the (discretized) state prevailing on date t, and (ii) the matrix of transition probabilities. Let us describe these two objects in the present framework.

We consider the following discretization of s_t: $s_t \approx \mu_s' z_{s,t}$, where $z_{s,t}$ is a N_s-dimensional selection vector, i.e. $z_{s,t} \in \{e_1, \dots, e_{N_s}\}$, where e_i is the i^{th} column vectors of the $N_s \times N_s$ identity matrix. Hence, vector μ_s contains the discretized values of s_t. (The largest entry of μ_s is lower than s_{max}.) Similarly, we introduce vector λ_s, whose i^{th} component is $\lambda_{s,i} \equiv \lambda(\mu_{s,i})$, function $\lambda(\bullet)$ being defined in (17). Therefore, we have $\lambda_t \approx \lambda_s' z_{s,t}$.

The dynamics of $z_{s,t}$ is defined by the matrix of transition probabilities $\Pi_s = \{p_{i,j}^{(s)}\}_{i,j\in[1,N]^2}$, with:[15]

$$\mathbb{P}(z_{s,t+1} = e_j | z_{s,t} = e_i) = p_{i,j}^{(s)}.$$

We then combine the surplus-regime selection vector ($z_{s,t}$) with the growth-regime one ($z_{c,t}$) to obtain an aggregated selection vector z_t:

$$z_t = z_{c,t} \otimes z_{s,t}, \tag{20}$$

whose dynamics is defined by the following matrix of transition probabilities:

$$\Pi = \Pi_c \otimes \Pi_s, \quad \text{where} \quad \Pi_c = \begin{bmatrix} p_{\ell\ell} & 1 - p_{\ell\ell} & 0 \\ p_{i\ell} & p_{ii} & 1 - p_{i\ell} - p_{ii} \\ 0 & 1 - p_{hh} & p_{hh} \end{bmatrix}. \tag{21}$$

[14] For instance, both $\mathbb{E}_t(\Delta y_{t+1})$ and $\mathbb{V}ar_t(\Delta y_{t+1})$ depend on $z_{c,t}$ (see Online Appendix E).

[15] The computation of the transition probabilities $p_{i,j}^{(s)}$ is based on the (non-discretized) dynamics of s_t (Eq. (16)). See Online Appendix C.

It is easily seen, in particular, that we have $\mathbb{E}_t(z_{t+1}) = \Pi' z_t$. Therefore, the knowledge of Π is equivalent to the knowledge of $\mathbb{E}_t(z_{t+1})$. Equivalently, the discretized risk-neutral dynamics $\mathbb{Q}$ is characterized by a matrix Q that satisfies $\mathbb{E}_t^{\mathbb{Q}}(z_{t+1}) = Q' z_t$. Since $\mathbb{E}_t^{\mathbb{Q}}(z_{t+1}) = \mathbb{E}_t\left[\{M_{t,t+1}/\mathbb{E}_t M_{t,t+1}\} z_{t+1}\right]$, determining the risk-neutral dynamics amounts to computing the latter conditional expectation. In the context of the present consumption-based model, tedious but simple algebra leads to the following expression of the risk-neutral matrix of transition probabilities Q (see Online Appendix C):[16]

$$Q = \frac{\exp\left(-\gamma\left\{1+\dfrac{1}{\lambda}\right\}\mu'\right)}{\left[\exp\left(-\gamma\left\{1+\dfrac{1}{\lambda}\right\}\mu'\right)\odot\Pi\right]\mathbf{1}\mathbf{1}'}\odot\Pi,$$

where vectors μ and λ are such that $s_t \approx \mu' z_t$ and $\lambda_t \approx \lambda' z_t$.[17]

To our knowledge, our paper is the first to exhibit the approximate risk-neutral dynamics in the context of a discretized state-space model. Knowing the approximate risk-neutral dynamics (i.e., knowing Q) dramatically simplifies asset pricing. Specifically, we provide quasi-analytical pricing formulas for inflation-linked, nominal and GDP-linked bonds of any maturity, as well as stock returns. Table 2 provides the resulting expressions for zero-coupon debt instruments. To simplify the exposition of formulas, we only provide their general form, while the details of some variables are only defined in Online Appendix D.

Moreover, the stock return is given by:

$$\frac{P_{t+1}^d + Div_{t+1}}{P_t^d} - 1 = \exp\left(\varepsilon_{t+1}^d - \frac{\sigma_d^2}{2}\right) z_t' M z_{t+1} - 1,$$

with

$$\mathbb{E}_t\left(\frac{P_{t+1}^d + Div_{t+1}}{P_t^d} - 1\right) \quad = \quad \mathbf{1}'(M\odot\Pi)' z_t - 1.$$

[16] It can be checked that the row coefficients of Q sum to one, and therefore that Q indeed specifies a probability measure.

[17] Formally, $\mu = [1,1]' \otimes \mu_s$ and $\lambda = [1,1]' \otimes \lambda_s$, where μ_s and λ_s are themselves such that $s_t \approx \mu_s' z_{s,t}$ and $\lambda_t \approx \lambda_s' z_{s,t}$.

3.4. **Data**

We use US data spanning the period from 1985Q1 to 2018Q1. Macroeconomic data comprise real GDP, consumption and inflation, and are extracted from the FRED database (Federal Reserve Bank of St. Louis). Consumption encompasses non-durables and services, and is deflated by the GDP deflator. Inflation is the log growth rate of the GDP deflator. Financial data include nominal and real yields at different maturities and stock returns. Nominal zero-coupon yields are taken from the updated database of Gürkaynak, Sack, and Wright (2007) and we focus on the 3-month, 10-year and 30-year maturities. Real zero-coupon yields are taken from Gürkaynak, Sack, and Wright (2010)'s updated database and we focus on the 2-year and 10-year maturities. The real short-term rate is defined as the 3-month nominal rate minus a corresponding inflation expectation taken from the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia. Series pertaining to stocks are from the updated database of Shiller (2015). Finally, we use survey-based expectations for real GDP and the GDP deflator, stemming from the SPF. Specifically, we convert probabilistic responses for GDP (PRGDP) and its deflator (PRPGDP) to mean forecasts at a 1- and 2-year horizon. In doing so, we account for the fact that the bins of probabilistic responses change over time.

3.5. **Taking the model to the data**

The parameters of the model are set in two steps. First, some parameters are calibrated based on the literature. Following Campbell and Cochrane (1999) and Wachter (2006), we set the utility curvature parameter γ to 2. Moreover, the "leverage parameter" ρ_d, that is the semi-elasticity of the dividend growth rate to consumption growth (see Eq. 19), is set to 2. This value is in between the unit elasticity considered by Campbell and Cochrane (1999) or Wachter (2005) and the estimates obtained by Abel (1999), Collin-Dufresne, Johannes, and Lochstoer (2016) and Seo and Wachter (2018) – of 2.74, 2.5 and 2.6, respectively. The same value is used to link the entries of g_d to those of g_c. Specifically, we set $g_{d,\ell} = \overline{g}_c + \rho_d(g_{c,\ell} - \overline{g}_c)$, $g_{d,i} = \overline{g}_c + \rho_d(g_{c,i} - \overline{g}_c)$, and $g_{d,h} = \overline{g}_c + \rho_d(g_{c,h} - \overline{g}_c)$, where $\overline{g}_c$ is the unconditional mean of consumption.

Second, the remaining parameters are set to optimize a combination of moment and time-series matching. Specifically, we consider first- and second-order moments pertaining to inflation and real GDP growth, as well as nominal and real bond yields. Furthermore, we exploit the similarities between stocks (for which data exist) and GDP-linked bonds (for which data do not exist) by also targeting stock-related moments. Importantly, the feasibility of this strategy relies on various quasi-analytical pricing formulas (see Online Appendix D), which are based on an approximated state dynamics (see Online Appendix C). Specifically, consistent with the literature, our formulas rely on a discretization of the state space. However, unlike most studies resorting to numerical methods (e.g. Wachter, 2005), we propose explicit formulas for all considered moments by using the Markov-switching representation of the state dynamics (see Online Appendix C). In particular, our pricing formulas exploit the so-called risk-neutral dynamics of the variables of interest, which are derived in the context of a discretized state space.

Targeted moments are detailed in the first column of Table 3.[18] Note that we do not only consider unconditional means and variances: for stock returns and zero-coupon yields, we also consider the model's ability to replicate (average) conditional volatilities.[19] As mentioned above, our calibration strategy also controls for the quality of fit of some key variables, in the time dimension. Specifically, our approach involves the time series of the following variables: the 2-year nominal rate, the 10-year nominal rate, inflation, consumption growth, as well as 1- and 2-year-ahead inflation and real GDP forecasts based on surveys of professional forecasters.

Let us denote by θ the vector of parameters to be estimated (excluding γ and ρ_d). Let M and $\mathcal{M}(\theta)$ be the vectors of targeted moments and model-implied moments, respectively. Similarly, let $Y_{1:T}$ and $\mathcal{Y}_{1:T}(\theta)$ be the respective matrices of observed and model-implied variables. The parameter values $\hat{\theta}$ are obtained by minimizing a distance between $\{M, Y_{1:T}\}$ and $\{\mathcal{M}(\theta), \mathcal{Y}_{1:T}(\theta)\}$, that is:

$$\hat{\theta} \quad = \quad \underset{\theta}{\text{argmin}} \left\{ \underbrace{(\mathcal{M}(\theta) - M)'\Omega(\mathcal{M}(\theta) - M)}_{\text{moment-based loss}} - \underbrace{\omega\mathcal{L}(\mathcal{Y}_{1:T}(\theta) - Y_{1:T})}_{\text{time-series-based loss}} \right\},$$

[18] The formulas used to compute model-implied moments are detailed in Online Appendices E and G.

[19] Observed averages of conditional volatilities are sample averages of realized volatilities, which are based on daily data of stock returns and yields.

where Ω is a diagonal matrix whose non-zero entries are weights that are arbitrarily set in order to have a satisfying overall fit of moments, and where function $\mathcal{L}$ denotes the log-likelihood function associated with i.i.d. Gaussian measurement errors. The parameter ω specifies the relative importance given to the fit of the time series with respect to that of the moments. Computational details are given in the online appendix.

The estimated (and calibrated) parameter values are summarized in Table 4. Moreover, the last column of Table 3 reports the model-implied first- and second-order moments. The fit is predominantly satisfactory despite the breadth of moments we account for.[20] This translates into model-implied time series of nominal and real yields that are very close to the data, as reported in Figure 6. Note that the real yields are not directly included in the fitting approach; in that sense, they can be seen as "out-of-sample". Consequently, the ability of our model to replicate the joint dynamics of observed macroeconomic and financial variables gives us confidence in using it to assess the debt-stabilizing properties of GDP-linked bonds.[21]

4. Results

In this section, we use the estimated model of Subsection 3.5 and present the resulting term structure of GDP-linked bonds. Moreover, we simulate debt-to-GDP ratios resulting from different public debt strategies involving nominal, inflation-linked and GDP-linked bonds, and investigate their respective cost and risk implications.[22]

[20] Table H.1 of the Online Appendix compares the data and model-implied moments. Notably, our average fit is comparable to that of Wachter (2006), despite the fact that we fit a much wider set of data (e.g., inflation and GDP surveys). Moreover, Figure H.1 in the Online Appendix reports the good fit of inflation, consumption growth and inflation and GDP growth surveys across horizons. Importantly, accounting for different regimes enhances the fit of GDP growth surveys.

[21] We compute return predictive regressions on the slope of the yield curve (see Figure H.3 of the Online Appendix). Our findings are consistent with sign switches of model-implied coefficients observed in Table 3 of Campbell, Pflueger, and Viceira (2020) and Table 2 of Pflueger (2023), over a similar sample period. We also produce predictive regressions on the price-dividend ratio (see Figure H.4 of the Online Appendix), which are in line with Table 2 of Campbell, Pflueger, and Viceira (2020) and Table 2 of Pflueger (2023).

[22] A discussion of several questions that arise when analyzing the usefulness of GDP-linked bonds (international risk sharing, novelty and liquidity premium, moral-hazard and financial stability) is offered in Section I of the Online Appendix.

4.1. **The term structure of GDP-linked bond yields**

We first build the model-implied average term structures of interest rates of nominal, inflation-linked and GDP-linked bonds, for maturities up to 50 years, which are displayed on Figure 2. The gray dashed line depicts average nominal rates ($i_{t,h}$, Eq. 1) across maturities, while the gray and black solid lines represent average inflation-linked and GDP-linked bond rates ($r_{t,h}$ and $r^*_{t,h}$, Eqs. 2 and 4), respectively. The white and gray dots represent sample averages of nominal and real yields, respectively. All three average yield curves are upward sloping. This is in line with empirical evidence, as shown by the fit of the average nominal and real (inflation-linked) yield curves. As discussed in Subsection 2.1, $r_{t,h}$ and $r^*_{t,h}$ coincide when there is no uncertainty about real GDP. As a consequence, the wedge between the two solid lines represents the additional compensation (known as the risk premium) required by investors to hold GDP-linked bonds. This GDP risk premium amounts to 40 basis points at short- to medium-term maturities and 30 basis points at long-term maturities.[23]

Looking at the model-implied probability density functions (p.d.f.) of inflation-linked and GDP-LB yields offers additional insights. As illustrated in Figure 3, the model-implied distributions of both inflation- and GDP-linked yields at a 2-year (left plot) and 10-year (right plot) maturity are positively skewed. Moreover, in line with Figure 2, the distributions of GDP-LB yields are shifted to the right by the average GDP risk premium (about 40 basis points), in comparison with the distributions of real yields.

As shown in Wachter (2006), in our framework, the risk premium is time-varying and countercyclical. More precisely, it appears to be higher in bad states of the economy (i.e. when surplus consumption is low) than in good states (i.e. when surplus consumption is high). Indeed, Figure 5 indicates that, following a negative shock, surplus consumption shrinks towards zero as average consumption $\tilde{C}_t$ gets closer to habit X_t. Such a situation is associated

[23] Note that our estimates are at the bottom of the range of available positive values found in the literature, including Borensztein and Mauro (2004) (40 bps), Kamstra and Shiller (2009) (150 bps), Barr, Bush, and Pienkowski (2014) (35 bps), Blanchard, Mauro, and Acalin (2016) (100 bps), Consiglio and Zenios (2018) (50 to 100 bps) and Eguren Martin, Meldrum, and Yan (2021) (-600 to 200 bps). Moreover, our structural model is able to generate nominal term premiums that exhibit (i) time variation, (ii) countercyclicality, and (iii) a downward trend, as in standard term structure decompositions stemming from latent-factor models (e.g., Kim and Wright, 2005). Note that the correlation between the 2-year (10-year) nominal term premium of Kim and Wright (2005) and our counterparty is equal to 81% (75%). Refer to Figure H.2 in the Online Appendix.

with high real and GDP-linked-bond yields, represented by gray and black solid lines, respectively. Importantly, GDP-linked yields are higher than real yields at any level of surplus consumption and for all maturities. Therefore, given the inverse relationship between bond prices and yields, it would require a larger issuance of GDP-LBs, relative to inflation-linked bonds, to raise a particular amount. In other words, issuing GDP-LBs is on average more costly than issuing inflation-linked bonds. Specifically, following a negative shock, surplus consumption shrinks towards zero as average consumption $\widetilde{C}_t$ gets closer to habit X_t. Such a situation is associated with high real and GDP-linked-bond yields,Importantly, GDP-linked yields are higher than real yields at any level of surplus consumption and for all maturities. Therefore, given the inverse relationship between bond prices and yields, it would require a larger issuance of GDP-LBs, relative to inflation-linked bonds, to raise a particular amount. In other words, issuing GDP-LBs is on average more costly than issuing inflation-linked bonds.

These findings suggest that treasuries may be prompted to issue GDP-LBs during expansions. However, the treasury – that issues government bonds– complies with principles that preclude such opportunistic issuances and encourage issuing debt in a regular and predictable manner. Not complying would have negative repercussions on the liquidity of the sovereign marketable debt, ultimately affecting its price (Bank for International Settlements, 1999; Favero, Missale, and Piga, 2000; Mylonas et al., 2000; OECD, 2018). Accordingly, in our simulation exercises, we preclude active management strategies and only focus on passive ones.

4.2. **The debt-stabilizing properties of GDP-linked bonds**

In the previous subsection, we established that, on average, GDP-linked bonds command higher yields to maturity than their inflation-linked bond counterparts, and that this wedge reflects a positive and time-varying risk premium. Although indexing debt to GDP seems to be more costly on average, we now examine whether this additional cost has the benefit of superior debt-stabilizing properties.

To this end, we run Monte-Carlo simulations using the approach depicted in Figure 4. Specifically, we first draw ν_t, ε_t^y and ε_t^π shocks from their respective estimated distributions (see

Subsection 3.5).[24] This provides us with the simulated paths of: (i) macroeconomic variables (using Eqs. 10, 11, 12, 16 and 17) and (ii) bond yields (using Eqs. a.12, a.13 and a.14). In a second step, we implement a specific issuance strategy in order to obtain the debt trajectory. More formally, a debt strategy is a function that associates the issued amounts $\{iss^n_{t,h}, iss^r_{t,h}, iss^*_{t,h}\}_{h=1,\ldots,H}$ with the observed current state of the economy. As mentioned earlier, we focus on simple "constant" strategies whereby the government issues a single type of bonds (either nominal, inflation-linked or GDP-linked) with maturities comprised between 1 and H quarters, with the issuance proceeds being evenly spread across maturities. For instance, in one of the strategies we consider, the government issues GDP-linked bonds with maturities of 3 months, 6 months, $\ldots$, 10 years, that is: $iss^n_{t,h} = iss^r_{t,h} = 0$ for all maturities h, and $iss^*_{t,h} = I_t/(hY_tP_t)$ if $h \leq 40$ (quarters), and 0 otherwise.

We consider two extreme situations regarding the fiscal reaction function. To this end, we conduct two exercises that differ in the way the primary budget surplus is determined.[25] In the first exercise, the fiscal authority does not react to the debt-to-GDP ratio, and the primary budget surplus is set to a constant value. For a given issuance strategy and model-implied economic path, the dynamics of the debt-to-GDP ratio is determined by Eq. (9). In the second one, the fiscal authority aims at perfectly stabilizing the debt-to-GDP ratio. This strategy consists in setting the primary budget surplus to its debt-stabilizing value (previously defined as DSBS in Subsection 2.3). Notice that in this context, the stationarity of debt is guaranteed.

Although none of these two exercises should be seen as realistic, an issuance strategy that achieves either a constant debt-to-GDP ratio (first exercise) or a constant DSBS (second exercise) is consistent with the notion of fiscal insurance developed in the optimal public debt management literature (Faraglia, Marcet, and Scott, 2008, 2010). Indeed, all else being equal,

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[24] Instead of directly drawing from the distribution of the shocks ν_t, we simulate the trajectory of the discretized values of s_t, which indirectly depends on ν_t. Specifically, the discretized values of s_t are defined through the selection vector z_t, such that $s_t \approx s_{z,t} = \mu' z_t$ (see Online Appendix C). Moreover, the dynamics of the (discretized) state variable z_t is entirely defined by its matrix of transition probabilities that is made explicit in Online Appendix C (Eq. a.10). The transition probabilities are based on Eq. (16) and consequently on the distribution of ν_t. In our simulations, we therefore retrieve the simulated values of ν_t by inverting Eq. (16) – hence, by replacing the s_t's by $s_{z,t}$'s. Note that the z_ts are particularly useful because they determine the values of the simulated yields, as shown in Eqs. (a.12), (a.13) and (a.14).

[25] See Leeper, Plante, and Traum (2010) for an analysis of alternative fiscal rules. Notably, they show that: (i) their estimated policy rules respond strongly to government debt and (ii) strong automatic stabilizers impose a long-run cost, despite reducing short-run fluctuations.

such strategies reduce the need for aggressive fiscal adjustments to ensure the government's intertemporal solvency.[26]

4.2.1. *Simulations under a constant budget-surplus.* This exercise compares different issuance strategies by simulating model-implied paths for macroeconomic variables and bond yields over a twenty-year period. In the context of this exercise, budget surpluses are set to a constant fraction of GDP (equal to -1%) and the initial level of debt is set at 100% of GDP. The resulting debt-to-GDP paths are obtained from Eq. (9).[27]

We simulate 10.000 macro-finance trajectories covering a 20-year period and employ kernel approaches to approximate the distributions of the debt-to-GDP ratios, at different horizons, and for different issuance strategies, where the government only issues one type of zero-coupon bonds – either nominal, inflation-linked or GDP-linked – with maximum maturities of 1 and 10 years. Panel A of Figure 7 depicts the distributions of these simulated debt-to-GDP paths. We evaluate these distributions after 2 years (left-hand side charts) and 20 years (right-hand side charts) of the government following these strategies. Each row represents the issuance of bonds of a given maturity (1-year bonds with maturities ranging between 1 to 4 quarters on the first row; 10-year bonds with maturities ranging between 1 to 40 quarters on the second row) and each column is associated with a horizon over which the strategy is studied.

We observe that, given the existence of strictly positive risk premiums associated with GDP-LBs, debt-to-GDP ratios are, on average, higher when the government issues GDP-LBs relative to the issuance of the two other bonds. Moreover, the variance of the distribution of debt-to-GDP ratios at a horizon of 2 years is far lower when GDP-LBs are issued, hence suggesting that the debt-to-GDP ratio is easier to predict. This increased predictability is particularly clear when long-term bonds are issued (bottom left plot) because, in this context, a relatively

[26] Empirical studies suggest that primary budget balances positively react to debt-to-GDP ratios (see e.g. Bohn, 1998; Mendoza and Ostry, 2008; Ghosh, Ostry, and Qureshi, 2013). However, the data also suggest that the adjustment weakens for large debt-to-GDP ratios. The latter phenomenon, dubbed as "fiscal fatigue" implies the existence of fiscal limits (see Ghosh et al., 2013).

[27] It is assumed that the initial outstanding debt results from issuances that are evenly distributed over the period preceding the start of our simulation. Therefore, while the initial debt-to-GDP ratios are the same across simulations and issuance strategies, the initial debt repayment schedules are not. One could alternatively imagine that all simulations start form the same situation, if interest-rate derivatives –namely interest-rate swaps, inflation-linked swaps and (hypothetical) GDP-linked swaps– are employed to obtain the desired debt structure.

smaller fraction of the debt has to be rolled over during the two-year period. However, at a 20-year horizon, the distributions of debt-to-GDP ratios resulting from the issuance of GDP-LBs are almost as dispersed as when conventional bonds are issued. Moreover, the distributions of debt-to-GDP ratios are shifted to the right in the case of GDP-LBs issuances, due to higher average funding costs. Let us stress that this result can only be detected in a modeling environment that accounts for time-varying bond prices because their fluctuations are the only stochastic driver of the debt-to-GDP ratio in an environment of constant budget surpluses. To the best of our knowledge, none of the existing literature performing Monte-Carlo simulations to study the smoothing properties of GDP-LBs allows for time-varying bond prices (or yields-to-maturity), thereby mechanically overestimating the stabilization property of these bonds.

The debt-to-GDP ratio is less volatile when issuing GDP-LBs because the debt service (i.e. the increase in debt, absent of budget surpluses) resulting from these bonds is pro-cyclical.[28] In other words, having a smooth debt-to-GDP ratio comes at the cost of a volatile debt service. This is illustrated in Panel B of Figure 7, which displays the simulated distributions of the debt services. The distributions of future debt service are evidently flatter under GDP-linked bond issuances, thus indicating more dispersed interest payments. Moreover, due to positive risk premiums associated with GDP-LBs, interest payments under this issuance strategy also tend to be higher. Therefore, the servicing of debt is on average more expensive and more volatile when GDP-linked bonds are issued.

To get a better grasp of whether the issuance of GDP-LBs prevents high debt-to-GDP ratios, we look at high quantiles of the distribution of debt-to-GDP ratios. Table 5 reports the 50th, 90th and 95th quantiles of the debt-to-GDP ratio over a 2-year and 20-year horizon. The last lines of each panel indicate that, at a 2-year (resp. 20-year) horizon, the lowest (resp. highest) debt-to-GDP ratio is obtained under the issuance of GDP-LBs. Therefore, it seems that issuing GDP-LBs can tame debt-to-GDP ratios only at short horizons.

[28]Formally, the dynamics of the debt-service-to-GDP ratio is given by Eq. (9).

4.2.2. *Simulations with debt-stabilizing budget surpluses (DSBS).* This exercise studies the moments of the debt-stabilizing budget surplus pertaining to different issuance strategies, which is in line with empirical studies showing that primary budget balances positively react to debt-to-GDP ratios (Bohn, 1998; Mendoza and Ostry, 2008; Ghosh, Ostry, and Qureshi, 2013). Recall that the DSBS is the budget surplus the government needs to deliver in order to keep the debt-to-GDP ratio constant over time, which is obtained by setting $d_t - d_{t-1} = 0$ in Eq. (9). We simulate a very long sample (covering 10.000 years) of DSBS and approximate the population moments using its sample-based counterparts. Debt-stabilizing budget surpluses are stationary processes when expressed as a fraction of GDP.

Figure 8 displays several cost/risk performances associated with different issuance strategies. Each issuance strategy is represented by a circle whose coordinates are: cost measures on the x axis and risk measures on the y axis. The average DSBS is the only cost measure we consider, as it represents the average funding cost. We consider four different risk measures: the standard deviation of the DSBS, the smoothness of the DSBS (defined as the standard deviation of the annual change in the DSBS) and, the 90th and 95th quantiles of the DSBS, which provide information about the right tail of the DSBS distribution.

At first glance, several findings arise. First, the higher the maturity, the larger the cost measure (x-axis). This holds true for any issuance strategy (nominal, inflation-linked or GDP-linked), and reflects the existence of positive *term premiums*.[29] Second, GDP-LB strategies imply larger average funding costs, for any maturity, because there is an additional positive *GDP risk premium*.

Regarding risk measures, we observe that, in order to stabilize the debt-to-GDP ratio, the budget surplus has to be less volatile under GDP-LB issuances, in contrast to conventional (nominal or inflation indexed) bond issuances. This result is more striking when looking at the smoothness measure. Specifically, the annual change in DSBS is dramatically smaller when issuing GDP-LBs, as is reflected by the standard deviation of the change in the DSBS, which is four times lower under this strategy. In other words, to stabilize the debt-to-GDP ratio in the short-run (with a one-year horizon), the government would have to run budget surpluses that

[29] Positive term premiums graphically correspond to the positive slopes of the average yield curves shown in Figure 2.

are far easier to predict under a GDP-LB issuance. However, the right tail of the distribution of the DSBS paints a less favorable picture of GDP-linked bonds. Indeed, in a bad scenario (characterized by the need to run a *high* budget surplus to stabilize debt), achieving debt stabilization requires larger budget-surpluses under GDP-LB issuances. For instance, focusing on the 95th percentile, we find that budget surpluses as high as 3.3% of GDP have to be run in 5% of the simulated periods when up-to-10-year GDP-linked bonds are issued, as opposed to 3.1% when up-to-10-year conventional bonds are issued.

5. Concluding remarks

Discussions in policy circles suggest that indexing debt to GDP would provide the government with an automatic stabilizer to its finance. The present paper calls into question this view. Using a consumption-based macro-finance model, which includes the term structure of hypothetical GDP-linked bonds, we study their debt stabilizing properties. Three results stand out.

First, issuing GDP-linked bonds generates, on average, higher interest payments for the government. This is because such bonds provide lower payoffs in periods of economic downturn and hence investors are willing to hold them only if they provide an average excess return, called GDP risk premium. We find that the GDP risk premium approximately amounts to 40 basis points along the entire maturity spectrum. Hence, for a given path of primary surpluses, issuing GDP-linked bonds, rather than conventional ones, is on average costlier and inflates debt-to-GDP ratios.

Second, the prices of GDP-linked bonds are state contingent and therefore not perfectly predictable. This implies that a large fraction of the debt will have to be rolled over before a given (long-run) horizon, and since future issuance prices are not deterministic, the long-run debt-to-GDP ratio is partly unknown as of today. Our simulation exercises indicate that, at a horizon of 20 years, the debt-to-GDP ratio is not more predictable when GDP-linked bonds are issued, relative to conventional ones.

Third, the debt stabilizing budget surplus is more predictable under GDP-linked bond issuances at the expense of being higher on average. Moreover, in a bad state of the world, characterized by the need to run a *high* budget surplus to stabilize debt, achieving debt stabilization requires larger budget-surpluses under GDP-linked bond issuances. Our findings, thus, call into question the view that GDP-linked bonds tame debt.

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TABLE 1. Characteristics of zero-coupon debt instruments

Instrument	h-period yield (at issuance)	Accrued nominal value at date $t+k$
Nominal bond	$i_{t,h}$	$\exp(ki_{t,h})$
Inflation-linked bond	$r_{t,h}$	$\exp(k[r_{t,h}+\pi_{t,t+k}])$
GDP-linked bond	$r^*_{t,h}$	$\exp(k[r^*_{t,h}+(y_{t,t+k}-y^e_{t,h})+\pi_{t,t+k}])$

Note: The last column reports the outstanding debt, recorded at date $t+k$, resulting from the issuance of a zero-coupon bond (nominal, inflation-linked, or GDP-linked) at date t. This issuance provides unitary proceeds to the government at the issuance date t. The accrued amounts are expressed in nominal terms. We use the notation: $y^e_{t,h} = \frac{1}{h}\log \mathbb{E}_t[\exp(y_{t,t+h})]$; where $y^e_{t,h}$ denotes the expected annualized continuously-compounded GDP growth rate between t and $t+h$ (expected as of date t). We also denote by $\pi_{t,t+h}$ the realized annualized continuously-compounded inflation rate between t and $t+h$. At date $t+h$, the accrued value is equal to the amount repaid by the issuer to the bondholder.

TABLE 2. Pricing formulas for zero-coupon debt instruments

Instrument	Bond price	h-period yield
Nominal bond	$P^n_{t,h}=\exp(-b_h\pi_t)F^{n\prime}_h z_t$	$i_{t,h}=\frac{b_h}{h}\pi_t-\frac{1}{h}\log(F^n_h)'z_t$
Inflation-linked bond	$P^r_{t,h}=\left(\{Q\times \mathrm{Diag}[\exp(-\mu_r)]\}^{h-1}\exp(-\mu_r)\right)'z_t$	$r_{t,h}=-\frac{1}{h}\log P^r_{t,h}$
GDP-linked bond	$P^*_{t,h}=(H^h\mathbf{1})'z_t$	$r^*_{t,h}=-\frac{1}{h}\log(H^h\mathbf{1})'z_t$

Note: More details on pricing formulas are provided in Online Appendix D.

TABLE 3. Fitted moments

Moment description		Data	Model
Mean of GDP growth rate Δy_t	$\times 10^2$	0.65	0.80
Mean of consumption growth rate Δc_t	$\times 10^2$	0.47	0.80
Mean of inflation π_t	$\times 10^2$	0.54	0.65
Std. dev. of GDP growth rate Δy_t	$\times 10^2$	0.57	0.59
Std. dev. of consumption growth rate Δc_t	$\times 10^2$	0.48	0.56
Std. dev. of inflation π_t	$\times 10^2$	0.24	0.29
Mean of short-term nom. rate	$\times 10^2$	2.32	2.78
Mean of 10-year nom. rate	$\times 10^2$	5.43	4.12
Mean of 30-year nom. rate	$\times 10^2$	5.77	6.01
Std. dev. of short-term nom. rate	$\times 10^2$	2.17	2.26
Std. dev. of 10-year nom. rate	$\times 10^2$	2.33	2.08
Std. dev. of 30-year nom. rate	$\times 10^2$	1.95	1.72
Auto-correl. of short-term nom. rate		0.99	0.91
Auto-correl. of 10-year nom. rate		0.99	0.98
Auto-correl. of 30-year nom. rate		0.98	0.98
Mean of slope of the nom. yd curve (3m-30yrs)	$\times 10^2$	2.45	3.23
Std. dev. of slope of the nom. yd curve (3m-30yrs)	$\times 10^2$	1.43	0.96
Mean of short-term real rate	$\times 10^2$	0.14	0.20
Mean of 2-year real rate	$\times 10^2$	0.08	0.50
Mean of 10-year real rate	$\times 10^2$	1.67	1.75
Std. dev. of short-term real rate	$\times 10^2$	1.95	2.26
Std. dev. of 2-year real rate	$\times 10^2$	1.36	2.15
Std. dev. of 10-year real rate	$\times 10^2$	1.29	2.08
Auto-correl. of short-term real rate		0.81	0.92
Auto-correl. of 2-year real rate		0.86	0.97
Auto-correl. of 10-year real rate		0.98	0.98
Mean of slope of the real yd curve (3m-10yrs)	$\times 10^2$	2.01	1.34
Std. dev. of slope of the real yd curve (3m-10yrs)	$\times 10^2$	1.25	0.73
Mean of condi. var. of the short-term nom. rate	$\times 10^5$	1.39	6.19
Mean of condi. var. of the 30-year nom. rate	$\times 10^5$	2.22	1.40
Mean of condi. var. of the 10-year real rate	$\times 10^5$	1.33	2.94
Average expected excess return (annualized)	$\times 10^2$	7.78	7.03
Average cond. volat. of stock return (annualized)	$\times 10^2$	15.69	20.92
Average P/D		40.93	43.91
Std. dev. of P/D		17.32	15.30

Note: This table reports sample moments and their model-implied counterparts. “condi.” stands for “conditional”, “var.” stands for “variance”, “nom.” stands for “nominal”, “Std. dev.” stands for “Standard deviation”, “correl.” stands for “correlation”.

TABLE 4. Calibrated and estimated parameter values

Rate of preference for present	δ		0.999
Risk aversion parameter	γ		2.0
Consumption growth (eq. 10 and Subsection 3.5)	$g_{c,l}$	$\times 10^2$	-0.334
	$g_{c,i}$	$\times 10^2$	0.580
	$g_{c,h}$	$\times 10^2$	0.801
	p_{ll}	$\times 10^2$	11.790
	p_{hh}	$\times 10^2$	98.537
	p_{ii}	$\times 10^2$	97.162
	p_{il}	$\times 10^2$	1.337
GDP growth shocks (eq. 11)	ρ_y		1.000
	σ_y	$\times 10^3$	1.969
Inflation dynamics (eq. 12)	$\overline{\pi}$	$\times 10^2$	0.647
	ψ		0.997
	ρ_π		0.035
	σ_π	$\times 10^4$	0.590
Dynamics of consumption ratio (eqs. 16, 17 and 18)	ϕ		0.977
	b	$\times 10^2$	1.800
Growth rate of dividends (eq. 19)	ρ_d		2.000
	σ_d	$\times 10^4$	0.396

TABLE 5. Influence of issuance strategies on debt-to-GDP quantiles

Panel A – Maturity of issued bonds: 1 year

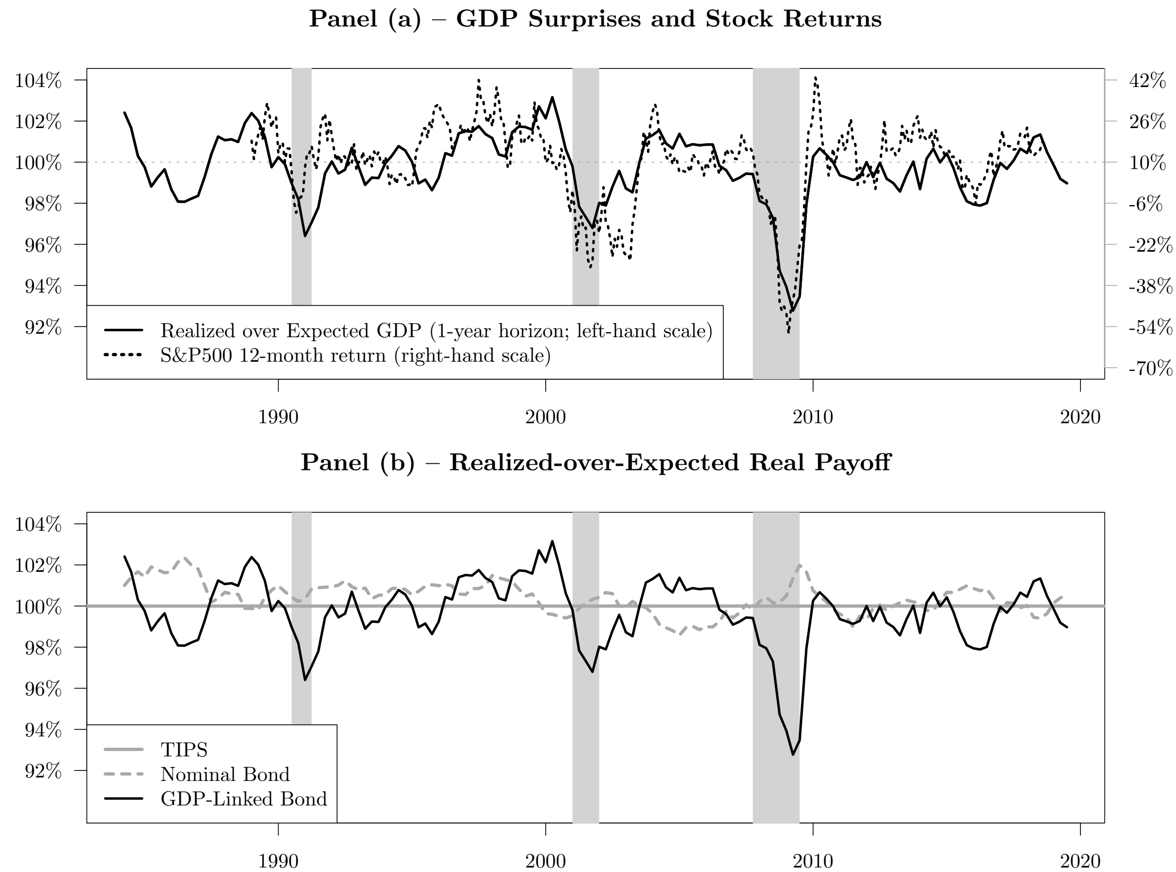
Quantile	Horizon: 2 years			Horizon: 20 years		
	Nominal	Infl.-linked	GDP-linked	Nominal	Infl.-linked	GDP-linked
50%	0.98	0.98	0.98	0.75	0.75	0.76
90%	1.01	1.01	0.99	1.19	1.19	1.18
95%	1.02	1.02	1.00	1.41	1.41	1.39

Panel B – Maturity of issued bonds: 10 years

Quantile	Horizon: 2 years			Horizon: 20 years		
	Nominal	Infl.-linked	GDP-linked	Nominal	Infl.-linked	GDP-linked
50%	0.98	0.99	0.99	0.86	0.88	0.90
90%	1.01	1.01	1.00	1.31	1.32	1.32
95%	1.02	1.01	1.00	1.52	1.53	1.51

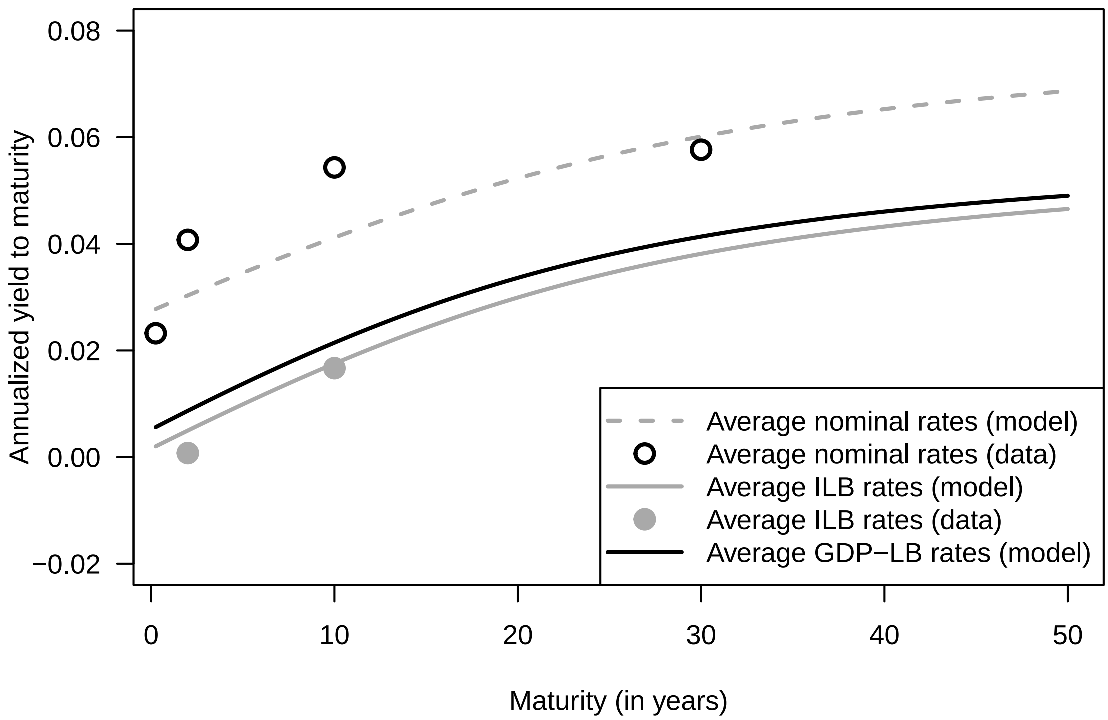
Note: This table reports the quantiles of the distributions of debt-to-GDP ratios resulting from different issuance strategies. The issuance strategies are basic strategies whereby the government, on each period, issues the same type of bonds to meet its funding needs.

FIGURE 1. Realized vs expected real payoffs of one-year bonds with unit face value



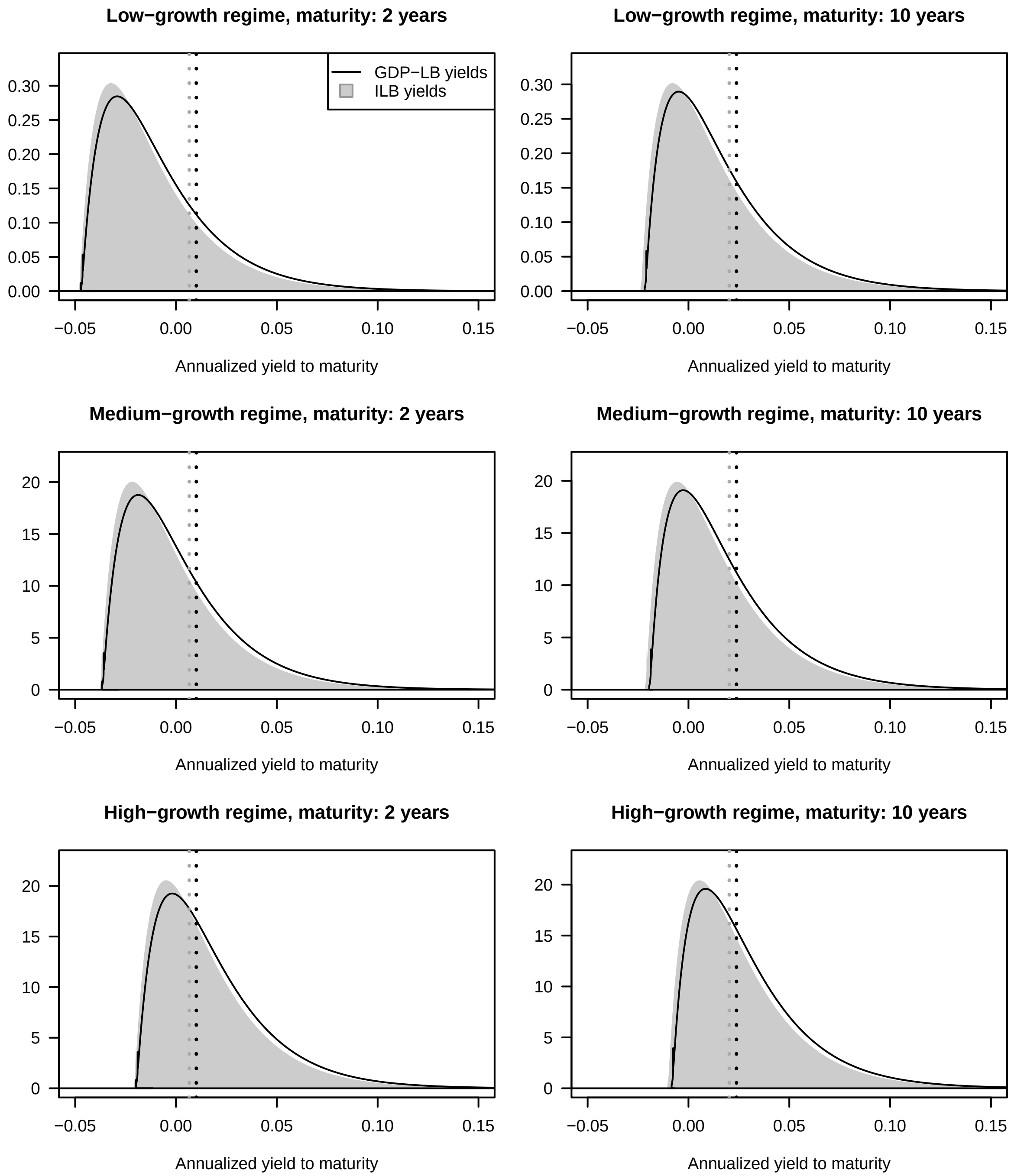
Note: Panel (a) shows that GDP surprises are highly correlated with the one-year returns of the S&P500 stock price index. The Realized-over-Expected GDP ratio is defined by $(1 + y_{t-4,t} + \pi_{t-4,t})/(1 + \mathbb{E}_{t-4}[y_{t-4,t} + \pi_{t-4,t}])$, where $\pi_{t-4,t}$ and $y_{t-4,t}$ respectively denote inflation and real GDP growth rates between quarters $t-4$ and t. The (model-free) computation of this ratio involves expectations stemming from the Survey of Professional Forecasters conducted by the Philadelphia Fed. Panel (b) displays (model-free) realized-versus-expected ratios of payoffs associated with three types of one-year bonds: a nominal bond, an inflation-linked bond (TIPS) and a (hypothetical) GDP-linked bond. Based on the definition of a TIPS, the realized real payoff of such a bond is equal to its expectation, which implies a constant ratio of 100% for this type of bond. For the nominal bond, this ratio is computed as $(1 + \mathbb{E}_{t-4}[\pi_{t-4,t}])/(1 + \pi_{t-4,t})$. For a GDP-linked bond, this ratio is equal to the Realized-over-Expected GDP ratio shown in Panel (a). Shaded areas indicate NBER recession periods.

FIGURE 2. Term structures of interest rates



Note: This figure displays model-implied yield curves. Nominal, real, and GDP-linked bond yields-to-maturity are respectively defined by Eqs. (1), (2) and (4); the corresponding pricing formulas are Eqs. (a.13), (a.12) and (a.14) of Online Appendix D. Dots represent sample averages of nominal and real yields. Yields are annualized.

FIGURE 3. Model-implied distributions of real and GDP-linked bond yields



Note: This figure displays the model-implied distributions of real (ILB) and GDP-linked (zero-coupon) bond yields. Vertical bars indicate means. Yields are annualized. The rows correspond to the different regimes of consumption (low, medium, high).

FIGURE 4. Schematic representation of our approach

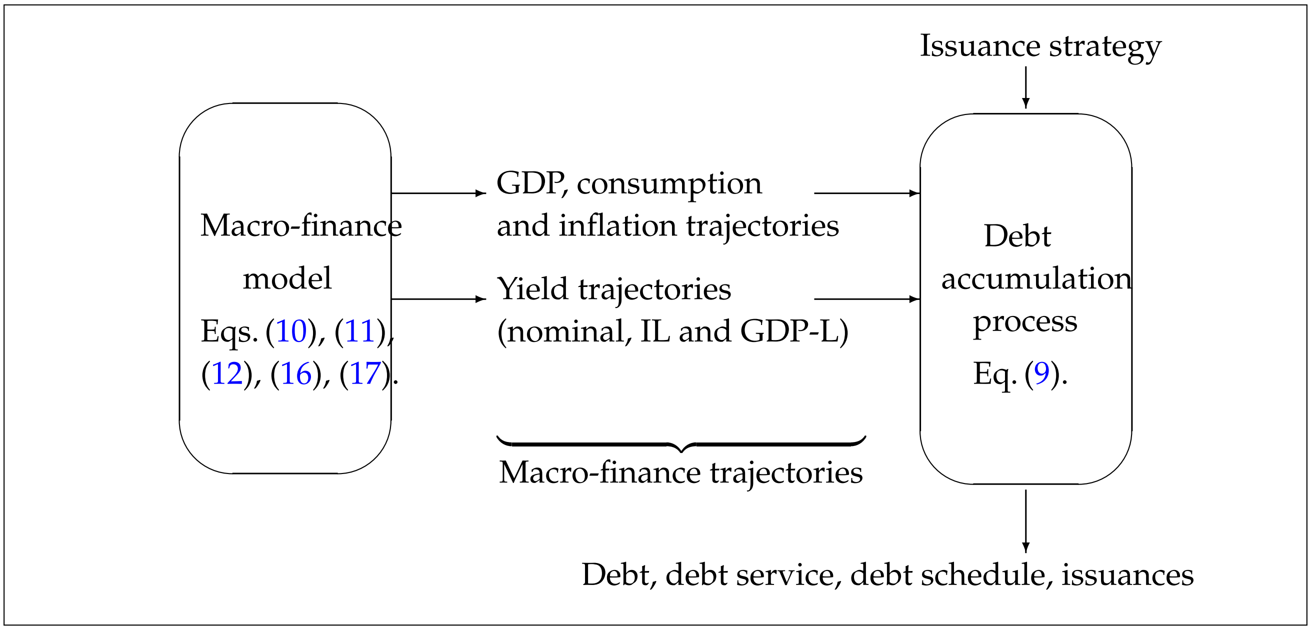
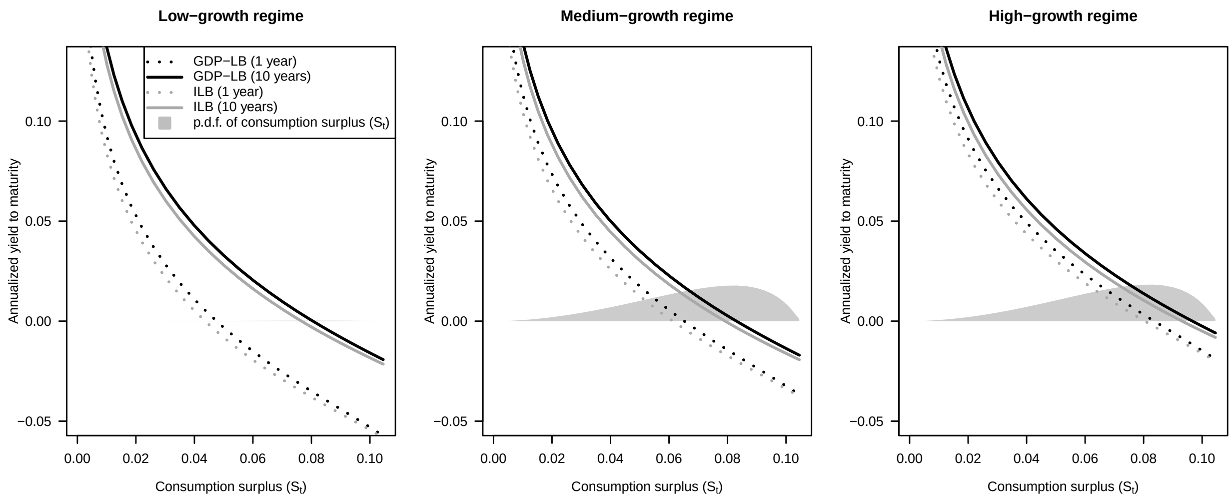
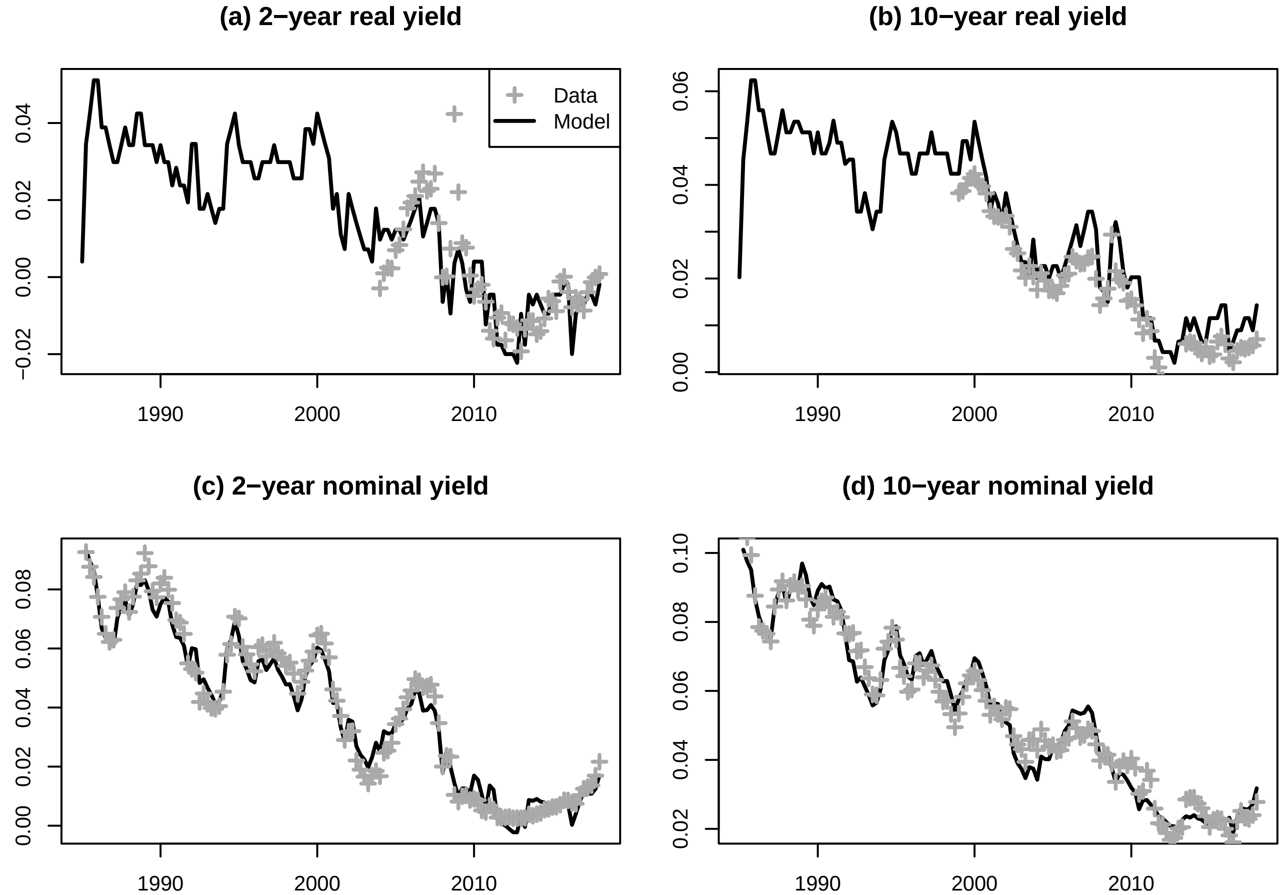


FIGURE 5. Yields and consumption surplus



Note: This figure shows the relationship between real and GDP-LB yields on the one hand and consumption surplus on the other hand. The grey area is the p.d.f. of consumption surplus. Yields are annualized.

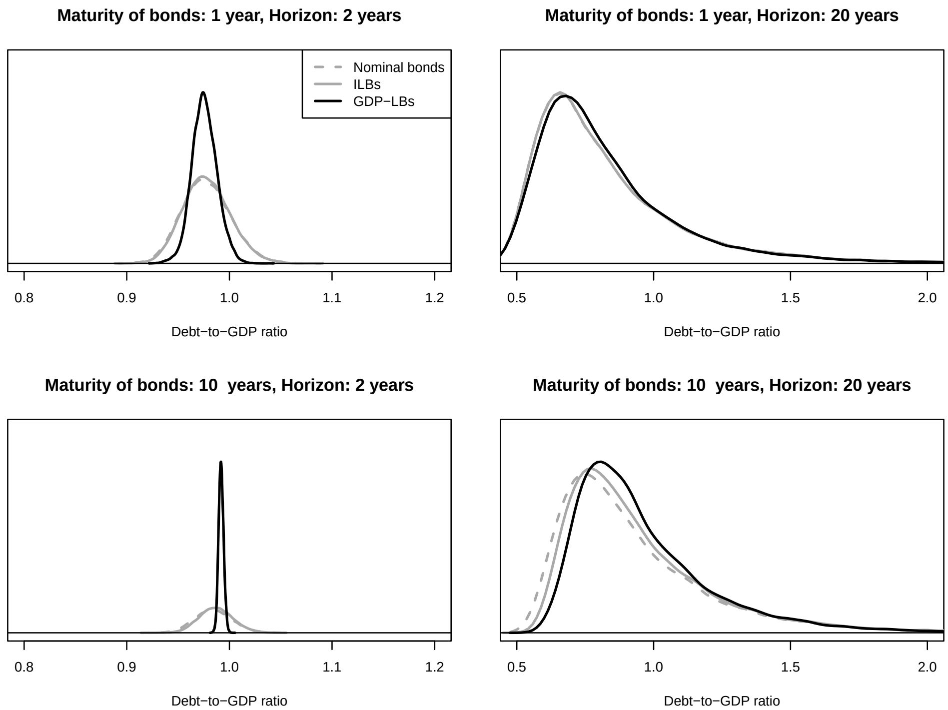
FIGURE 6. Yields: data versus model



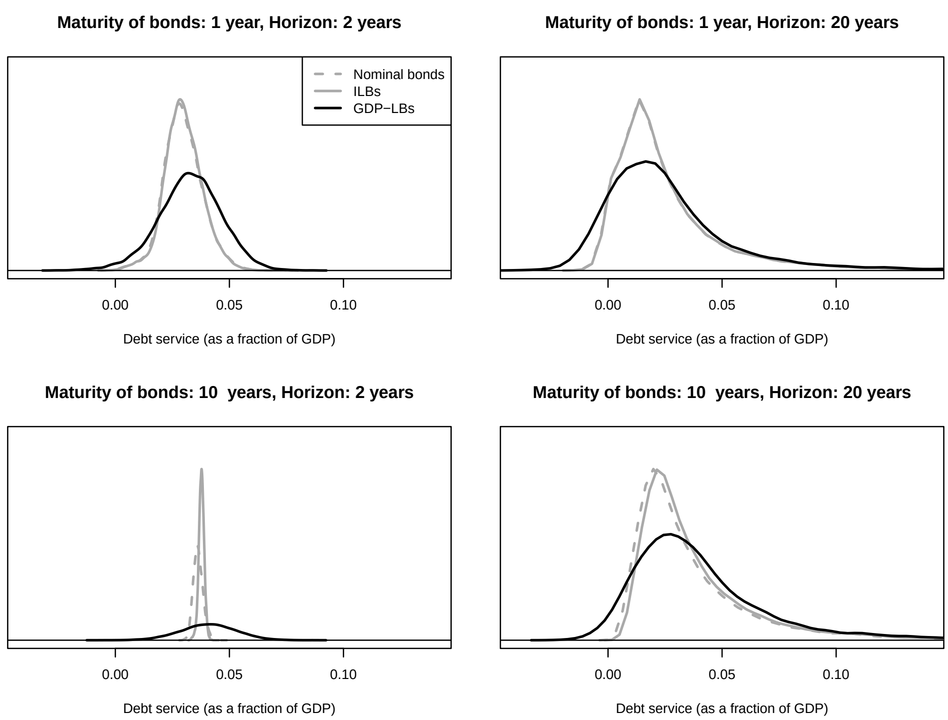
Note: This figure presents the model fit. Lines (respectively crosses) correspond to model-implied variables (respectively observed variables). Note that the real rates are not directly included in the fitting approach; in that sense, they can be seen as "out-of-sample."

FIGURE 7. Distribution of future debt-to-GDP ratios and future debt service

Panel A. Debt-to-GDP ratios

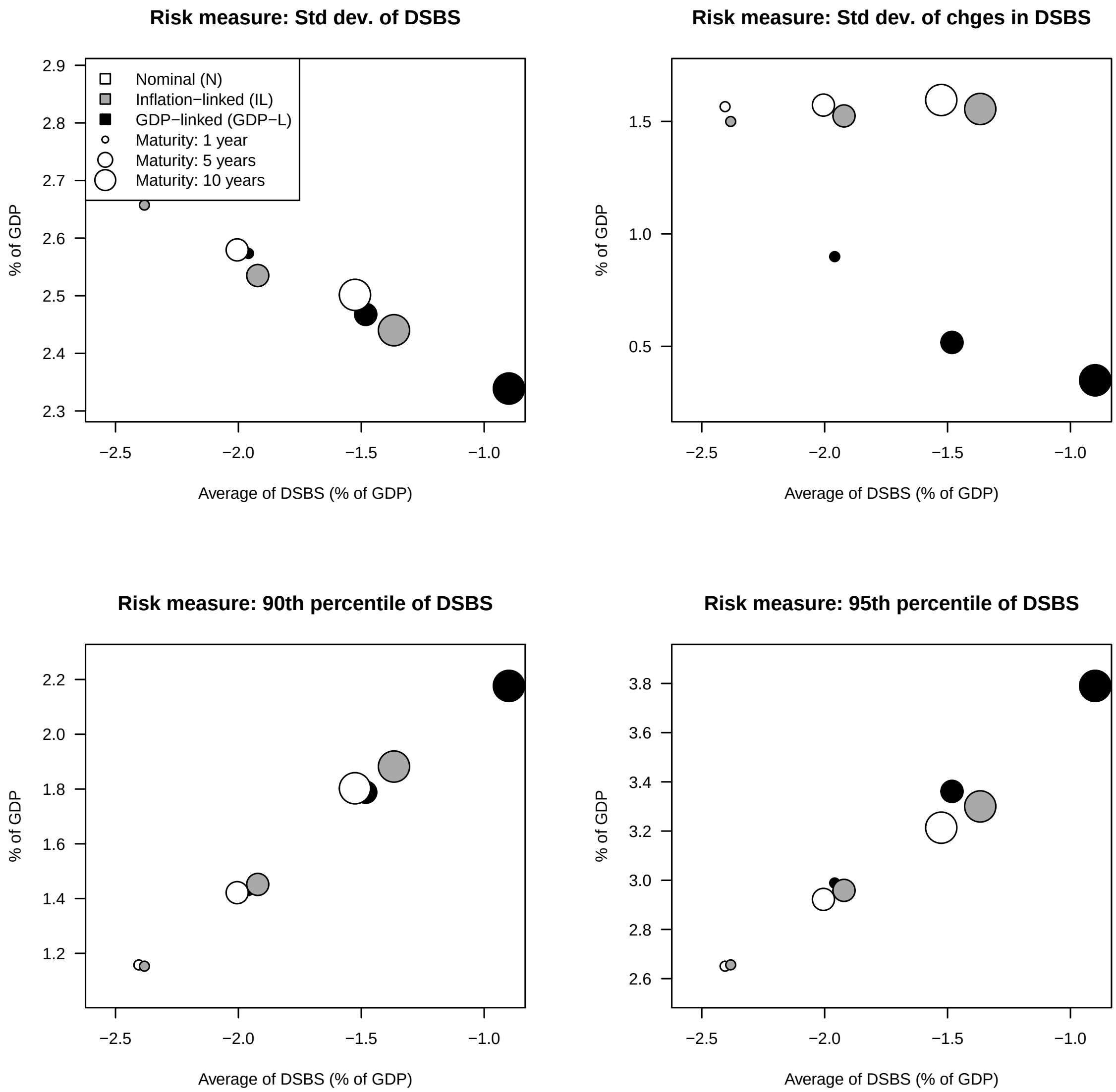


Panel B. Debt service



Note: Panel (a) (resp. Panel (b)) of this figure displays the distributions of debt-to-GDP ratios (resp. the future debt service) after periods of 2 years (left column of plots) and 20 years (right column of plots) during which the government implements basic issuance strategies: specifically, the government issues bonds of the same type (nominal, inflation-linked or GDP-linked) and of the same maximum maturity (1 or 10 years). We simulate 10.000 macro-finance trajectories over a period of 80 quarters. The primary budget surplus is equal to -1% of nominal GDP.

FIGURE 8. Effect of issuance strategies on DSBS risk measures



Note: This figure depicts risk measures associated with the debt-stabilizing budget surplus (DSBS) of different issuance strategies. The DSBS is the budget surplus that the government needs to deliver in order to keep the debt-to-GDP ratio constant over time. Each dot corresponds to a given issuance strategy characterized by a type of bond issued (nominal, inflation-linked or GDP-linked) and a maturity. The four risk measures are: the standard deviation of the DSBS (top-left plot), the standard deviation of the annual change in the DSBS (top-right plot), the 90th percentile (bottom-left plot) and the 95th percentile (bottom-right plot) of the DSBS distribution. We simulate a macro-finance trajectory of 40.000 periods.