On the non-optimality of proportional reinsurance according to the dividend criterion

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In a recent paper in this journal, Beveridge et al. (2008) considered the classical compound Poisson risk model. They studied the effect of (static) proportional and excess-of-loss reinsurance on the expected difference between the discounted dividends until ruin and the discounted penalty at ruin. According to this criterion, and based on their numerical investigations, they conjecture that no reinsurance is better than proportional reinsurance. The goal of this note is to give a theoretical background to this conjecture and to show that it is true in a more general setting and under fairly general conditions.

Without reinsurance, the surplus of the company at time t is

$$U(t;u) = u + c\,t - S(t),$$

where u is the initial surplus, $S(t)$ the aggregate claims up to time t, and $c > S(1)$ is the constant rate at which the premiums are received. The maximal value function $W(u)$ is defined as follows. For given u, $W(u)$ is the maximal expected difference between the discounted dividends until ruin and the discounted penalty at ruin; the maximum is taken with respect to all dividend strategies.

Proportional reinsurance is available; the retained fraction of the claims is denoted by a, $0 < a < 1$. First we assume that the relative loading contained in the reinsurance premium is the same as in c. Then, with proportional reinsurance corresponding to the parameter a, the surplus of the company at time t is

$$U_a(t;u) = u + a \cdot c\,t - a \cdot S(t) = a \cdot U\left(t; \frac{u}{a}\right).$$

With proportional reinsurance, the maximal expected difference between the discounted dividends and the discounted penalty is denoted as $W_a(u)$. By a change of scale, we see that

$$W_a(u) = a \cdot W\left(\frac{u}{a}\right).$$

As a consequence, $W_a(u)$ can be obtained by the geometric construction that is shown in Figure 1. Suppose now that the function $W(u)$ satisfies the following Condition C: for any $x > 0$, the ray between the origin and the point $(x, W(x))$ is below the graph of the function. Then $W_a(u) < W(u)$ for all a and u, and we conclude that for any u no reinsurance ($a = 1$) is better than any proportional reinsurance. We note that Condition C is for instance satisfied in the particular case, where the graph of $W(u)$ is concave (see the recent paper of Loeffen and Renaud (2010) for general results on the shape of $W(u)$). Finally, if the relative loading of the reinsurance premium exceeds the one contained in c, the conclusion is a fortiori the same.

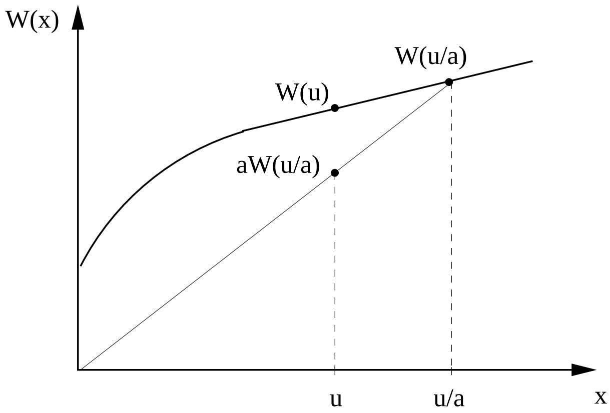

Figure 1: Comparison of $W_a(u)$ and $W(u)$

References

[1] C.J. Beveridge, D.C.M. Dickson and X. Wu. Optimal Dividends under Reinsurance. Bulletin of the Swiss Association of Actuaries, Heft 1& 2, 149–166, 2008.

[2] R. Loeffen and J.-F. Renaud. De Finetti's optimal dividends problem with an affine penalty function at ruin. Insurance: Mathematics & Economics, Vol. 46 (1), to appear, 2010.

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