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The decisionalization of individualization

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ABSTRACT

Throughout forensic science and adjacent branches, academic researchers and practitioners continue to diverge in their perception and understanding of the notion of 'individualization', that is the claim to reduce a pool of potential donors of a forensic trace to a single source. In particular, recent shifts to refer to the practice of individualization as a *decision* have been revealed as being a mere change of label [1], leaving fundamental changes in thought and understanding still pending. What is more, professional associations and practitioners shy away from embracing the notion of decision in terms of the formal theory of decision in which individualization may be framed, mainly because of difficulties to deal with the measurement of desirability or undesirability of the consequences of decisions (e.g., using utility functions). Building on existing research in the area, this paper presents and discusses fundamental concepts of utilities and losses with particular reference to their application to forensic individualization. The paper emphasizes that a proper appreciation of decision tools not only reduces the number of individual assignments that the application of decision theory requires, but also shows how such assignments can be meaningfully related to constituting features of the real-world decision problem to which the theory is applied. It is argued that the *decisonalization of individualization* requires such fundamental insight to initiate changes in the fields' underlying understandings, not merely in their label.

"If you don't understand a problem from a Bayesian decision theory point of view, you don't understand the problem and trying to solve it is like shooting at a target in the dark." (Hermann Chernoff, from a personal communication to Martin McIntosh, quoted in [2, p. 6])
"Give me a place to stand, and I shall move the earth." (Sentence attributed to *Archimedes* [e.g., 3,4])[1]

1. Introduction

Academic researchers and practitioners in forensic science and other fields, such as medicine and the law, maintain divergent views about 'individualization', that is the reduction of a pool of potential donors of a forensic trace to a single source [5]. Viewpoints differ with respect to the definition, the scope and the practical feasibility of individualization [1,6,7]. As a hallmark in the last decade, the report of the US National Research Council in 2009 [8] considerably stirred up the discussion by drawing a rather critical picture of the current state of the field. It triggered diverse reactions from institutions, practitioners and scholars, inspired scientific research and received attention in courtrooms in the US and beyond [9], but the situation as of today remains ambivalent. While it is largely uncontroversial that forensic traces such as fingermarks and toolmarks can have – depending on their quality – a considerable potential to help discriminate between competing propositions regarding common source, and that there are practitioners who are able to demonstrate reliable practice in trials under controlled conditions, the field's main struggle remains conceptual. This touches on two fundamental issues: first, the question of what strength is to be assigned to a comparison conducted in a given case, and second how particular conclusions can be justified through an argument.

The former of these two issues, value of evidence, is not dealt with in this paper. In forensic science, value of evidence is defensibly approached in terms of likelihood ratios or, more generally, Bayes factors, that feature a unified underlying logic

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[1] The relevance of this quote in the context of decision theory and forensic individualization will be discussed in Section 4 in this paper.

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[10–13], although they may take different forms and degrees of technicality according to the domain of application (such as fingermarks [e.g., 14], DNA [e.g., 15], handwriting [e.g., 16], etc.). This paper concentrates on the latter of the above two issues – the justification of conclusions – by focusing on one recent movement in response to the NAS report, exemplified by the fingerprint profession. This movement gravitates around the notion of 'decision' as mentioned in the title of the document 'Guideline for the Articulation of the Decision-Making Process for the Individualization in Friction Ridge Examination'[2] issued by the Scientific Working Group on Friction Ridge Analysis, Study and Technology (SWGFAST).[3] In Section 3.1, this document acknowledges that "(...) it is now recognized that our conclusions are more appropriately expressed as a *decision*, rather than proof", and in Section 10.2.2, the following definition is given: "Individualization is the *decision* by an examiner that there are sufficient features in agreement to conclude that two areas of friction ridge impressions originated from the same source." [*italics* added by the authors] This prominent use of the term decision contributes to its more widespread adoption as standard terminology by many forensic practitioners across the so-called identification branches.

The field's shift to a new term, decision, remains dubious, however. In one of the most meticulous studies of the fingerprint profession's recent 'decision shift', Cole [1] reveals[4] that the term decision appears to be used merely as a new label without any fundamental change in conceptual understanding or actual practice. Most interestingly, in exchange with Cole, SWGFAST declared that it does *not* rely on decision theory as endorsed in papers such as [17,18], despite giving reference to such publications. While this can be seen as a deliberate choice that is open to any discussant of the topic, it is worth mentioning that such a choice is of no effect to the validity of decision theory itself, in particular its logic. Also, it is of no detriment to the interest that one may take in comparing current practices of the profession with the prescriptions that derive from (Bayesian) decision theory. The focus on such prescriptions on how to act under uncertainty represents an analytical approach to the notion of decision and is to be distinguished from the descriptive use of this notion for people's observable (decision) behaviour, intuitive or otherwise. In this article, we will concentrate on the analytical and normative approach to the notion of decision and argue that it can foster progress in fundamental understanding of core forensic science topics [e.g., 19] and, thus, should drive what we will propose to call the *decisionalization* (of individualization).

Besides the extreme position of those who do not endorse decision theory, there are others who are sensitive to the theory's logic but still refrain from applying the approach on grounds that they don't 'know' what numbers they ought to use in the various formulaic expressions, or what those numbers actually mean. In Bayesian decision theory, the numbers refer to probabilities and utilities[5] (or, alternatively, losses). While the meaning of probabilities in forensic science is well established, in particular the subjectivist belief type interpretation [18,20,21], the notion of utility is more recent and less well known [22,23].

Thus, in the current state-of-art, there is room for the study and discussion of the constituting elements of Bayesian decision theory – especially the utility component – from a forensic science point of view, which is the main aim of this paper. Section 2 recalls the principal elements of classical Bayesian decision theory, applied to the 'problem' of individualization, whereas Section 3 will focus on the choice of the utility scale and the subsequent derivation of the utility function. At this juncture, the paper will seek to justify the standpoint that the numbers to be assigned to utilities are *not* undefinable, and hence arbitrary, as claimed by critics, but can be given a clear interpretation. Most importantly, we will emphasize that this interpretation can embrace defining elements of the individualization task sketched at the outset, which represents a strong argument in favour of the relevance of Bayesian decision theory for inference and decision in forensic science. We will also point out that a close look at the decision theoretic formulation of individualization, under modest and reasonable assumptions, reduces the number of assessments that require the attention of the analyst. Section 4 will present a general discussion of the foregoing analyses and converge to conclusions highlighted in previous works, in particular the importance of understanding the normative character of the theory [24]. The discussion in Section 4 will also emphasize the natural role of traditional expressions of weight of evidence, in particular likelihood ratios, in the decision framework and the feasibility of illustrating the logic of Bayesian decision theory through fundamental insights from other fields, such as physics, that can be traced back to Archimedes in Ancient Greece. Readers well acquainted with decision theory may skip Section 2, but they should take notice briefly of the notation introduced there. Conclusions are presented in Section 5.

2. The Bayesian decision theoretic answer to the 'problem' of individualization

2.1. The basic elements of the decision problem

In Bayesian decision theory, the basic components of a decision problem are formalized in terms of three elements. Consider these elements in the context of forensic individualization as defined at the beginning of this paper (Section 1). In particular, suppose that there is trace material collected on a crime scene, such as a fingermark, and reference material is available from an individual (the suspect), considered to be a potential source of the fingermark. After comparative examinations between the fingermark and the fingerprints taken from the suspect under controlled conditions, individualization – our decision problem – may be brought up as an issue.[6]

The first decision theoretic element are the feasible decisions d, which define the decision space. To keep the discussion on a moderately technical level, let there be only two decisions, d_1, short for 'individualize', and d_2, short for 'not individualize'. For a development with the decision 'not individualize' broken down to the decisions 'exclusion' and 'inconclusive' see, for example, [17,19]. Note that the simple negation of the first decision is rarely a concise approach because, generally, there *are* explicit alternatives available and their respective merit ought to be appreciated [26]. Stated otherwise, the alternative must specify what to do if not individualizing.

When a choice has to be made, it is usually not known which state of nature actually holds. A second element, thus, is the list of uncertain events, also called states of nature, denoted θ. Clearly, in an individualization scenario, the states of nature that are

[2] Version 1.0, available at http://www.swgfast.org/documents/articulation/130427_Articulation_1.0.pdf, page last accessed 15 July 2015.

[3] The discussion in this paper will mainly refer to the formative documents of SWGFAST in order to acknowledge the original source. Notice, however, that SWGFAST has undergone changes and became the Subcommittee on Friction Ridge, which is part of the Organization of Scientific Area Committees (OSAC).

[4] Cole's study [1] is based, in part, on SWGFAST replies on comments submitted during a public consultation process for one of its guideline drafts.

[5] A utility, in the context of the current discussion, is an expression of an individual's desirability for a given consequence, that is a result of a decision in the light of a particular state of nature. Section 2 will elaborate further on these terms.

[6] Note that another decision, not studied in this paper, relates to the question of whether or not to search for fingermarks on a receptor surface. See [25] for further details.

uncertain to the decision-maker[7] are those formulated more commonly in terms of the propositions 'the person of interest is the source of the crime mark (or, trace)' (θ_1) and 'an unknown person is the source' (θ_2). The couple {θ_1, θ_2} forms the set of possible states of nature denoted Θ. Taking a decision d_i in the light of a state of nature θ_j leads to a consequence C_{ij}. The set of all consequences is written C, for short. It represents the third element of the decision problem. Using this notation, C_{11} is the consequence of an individualization (d_1) when the suspect is truly the source of the fingermark (θ_1) and C_{12} is the consequence of the same decision when the suspect is *not* the source (θ_2). Thus, C_{11} and C_{12} represent correct and false individualizations, respectively. Analogously, C_{21} and C_{22} denote a missed individualization and a correct non-individualization,[8] respectively. Note also that only states of nature are uncertain, whereas consequences are not: the combination of a state of nature with a particular action leads to a consequence that is – in our case – sure.

Clearly, if the actual state of nature would be known with certainty, that is whether or not the suspect is the source of the crime mark, there would be no decision problem. One could directly choose the decision to individualize (d_1) if the suspect is the source of the crime mark, and choose not to individualize (d_2) otherwise. In both cases, one would reach correct conclusions. When it is not known which state of nature actually holds, it may not be obvious to decide at a glance. Notwithstanding, it is of obvious interest to make an optimal decision given the elements of the decision problem outlined above. What is needed, thus, are decision criteria (Section 2.2) that incorporate the assessment of both the desirability (undesirability) of possible consequences and also uncertainty about which state of nature actually holds, to compare the merit of available decisions and to avoid incoherent proceedings.

2.2. The Bayesian decision rule

The Bayesian decision theoretic approach to the problem of decision is based on two additional concepts, besides the elements introduced in the previous section. The first is a measure of uncertainty about the states of nature, given by probability. In the current discussion, the states of nature θ are discrete, hence a probability mass function $\Pr(\theta \mid I)$ is applicable, with I denoting the information available at the time when the decision is to be taken. The second concept is a measure of the desirability of consequences. This measure takes the form of a so-called *utility function*, denoted $U(\cdot)$. With the utility function, one assigns utility values to each consequence on a numerical scale. When there is uncertainty about the states of nature, one can – for each decision – multiply the desirability of each consequence with the probability of obtaining that consequence, as given by the probability of the state of nature of interest, and then take the sum of these products. The result is known as the *expected utility* (EU) of the decision. For example, the expected utility of the decision to individualize (d_1) is equal to the utility of a correct individualization $U(C_{11})$ multiplied by $\Pr(\theta_1 \mid I)$, the probability that the suspect truly is the source, plus the utility of a false identification $U(C_{12})$, multiplied by the probability that the suspect is not the source of the mark, $\Pr(\theta_2 \mid I)$:

$$EU(d_1) = U(C_{11})\Pr(\theta_1|I) + U(C_{12})\Pr(\theta_2|I). \quad (1)$$

The expected utility of the alternative action, not identifying the suspect, is obtained with the same procedure:

$$EU(d_2) = U(C_{21})\Pr(\theta_1|I) + U(C_{22})\Pr(\theta_2|I), \quad (2)$$

where $U(C_{21})$ and $U(C_{22})$ are, respectively, the utilities of a missed individualization and a correct non-individualization (Table 1). The latter two utilities are weighted, as for $EU(d_1)$, by the probabilities $\Pr(\theta_1 \mid I)$ and $\Pr(\theta_2 \mid I)$, the suspect being and not being the source of the mark.

Eqs. (1) and (2) quantify the overall value that one may expect to obtain as a consequence of taking one or the other of the available decisions. These expected utilities characterize the available decisions, allows one to compare them and formulate a decision rule: taking the decision with *the maximum expected utility*. Thus, if one does not know which state of nature actually holds, and hence it is not trivial to tell which decision to take in order to obtain the best consequence, the reasonably best way to proceed is to choose the decision that has the highest expected utility. This is the general criterion referred to as *maximum expected utility* (MEU) in which one selects that action for which one has the highest expected utility [e.g., 26].

The probabilities $\Pr(\theta_1 \mid I)$ and $\Pr(\theta_2 \mid I)$ denote the decision-maker's personal beliefs about the states of nature, given all the information available, at the time when the decision must be made. These probability assignments are the same for both Eqs. (1) and (2), and any changes[9] in their magnitude may affect the expected utilities of the decisions. Hence, they are one source that impacts on the decision that has the maximum expected utility. The second obvious influence stems from the utilities: they are *distinct* terms in Eqs. (1) and (2). To ensure an informed and meaningful use of the Bayesian decision rule, the question of how to understand these utility terms, and assign values to them represents a relevant topic of inquiry.

Table 1
A decision matrix using utilities with d_1 and d_2 denoting, respectively, the decisions to individualize and not to individualize a suspect. The states of nature θ_1 and θ_2 are, respectively, the suspect is the source of the crime mark and the suspect is not the source of the crime mark. The notation $U(C_{ij})$ with $i, j = \{1, 2\}$ represents the utility of the consequence C_{ij} when taking decision d_i and the state of nature θ_j holds.

	States of nature: Suspect is	. . . the source (θ_1)	. . . not the source (θ_2)
Decisions	Individualize (d_1)	$U(C_{11})$	$U(C_{12})$
	Do not individualize (d_2)	$U(C_{21})$	$U(C_{22})$

3. The choice of a scale for the valuation of consequences

3.1. The utility view

Table 1 summarizes the main components of the individualization scenario in decision theoretic terms. To operationalize Eqs. (1) and (2), that is finding the decision with the maximum expected utility, the decision analyst must somehow conceive of a way to express the desirability of the various consequences C_{ij}. In Table 1, the desirability (or, preference) is expressed, more formally, in terms of utilities $U(C_{ij})$. A related concept is 'loss', considered later in Section 3.3. It is worth mentioning that decision theory merely says that utilities are part of the formulation of the problem, and

[7] Throughout this paper, terms such as 'decision-maker' and 'decision analyst' are used interchangeably. In fact, the theory presented is entirely general and is applicable by any person facing a decision problem in their own way regarding, in particular, their beliefs about states of nature and preferences among consequences.

[8] The term 'non-individualization' may seem awkward, but it is a consequence of the fact that the decisions other than an individualization are not specified in further detail in this discussion.

[9] Note that, as evidence E accumulates, $\Pr(\theta_j \mid I)$ becomes $\Pr(\theta_j \mid E, I)$ through Bayes' theorem.

how utilities are connected with the other elements of the decision theoretic formulation, but it does not say how numerical assignments for utilities ought to be made. This seems to be a major disturbing factor in the application of decision theory in forensic science. Below, we present several considerations that help substantiate utility assignments in a way that is tailored to the situation faced by the forensic decision analyst.

Following an approach that goes from the general to the particular, one does not need to start by focusing on any particular values. Indeed, a common claim among forensic practitioners is that utilities cannot be assigned in principle, so it appears relevant to ask: 'Can we really tell *nothing* about the consequences C_{ij}?' This seems a restrictive view because people can reasonably be expected to have, at least, an ordering of the various consequences. That is, it should be possible to designate at least one of the consequences as the most favourable, and one that is the least favourable. Clearly, in a decision scenario regarding individualization, correct consequences C_{11} (individualizing if the suspect is the source of the trace) and C_{22} (not individualizing if the suspect is not the source of the trace) are the best consequences. Conversely, no one would wish a suspect to be wrongly associated with the trace, hence C_{12}, a false individualization, is the worst consequence. These considerations leave us with only one intermediate consequence, C_{21}, a missed individualization. Thus, a question to be dealt with is how to position this consequence with respect to the best and the worst consequences, respectively.

Turning now to the issue of numbers, start by considering how to assign numbers for the best and the worst consequence, respectively, that is fixing the maximum and the minimum of scale of preferences. To deal with this question, one can invoke a property of the measurement procedure for subjective utilities developed by Ramsey [27] and von Neumann and Morgenstern [28]: their utility functions are unique up to linear transformations. This means that if $U(\cdot)$ is a utility function, then $aU(\cdot) + b$ is another utility function which maintains the same ordering as $U(\cdot)$, changing only the origin of the utility measure [e.g., 29]. Practically, this means that the maximum and the minimum of the utility scale can be fixed at one and zero, respectively. More formally, $U(C_{11}) = U(C_{22}) = 1$ and $U(C_{12}) = 0$. Hence, one can see that all but one cell in the decision matrix (Table 1) are already assigned through the sole effort of specifying a qualitative ordering of preference among consequences and choosing the endpoints of the preference scale. The latter is greatly facilitated by the function's mathematical property of being invariant under linear transformations. The former – qualitative ordering – should be largely uncontroversial and intersubjectively agreeable.

To assign a value to $U(C_{21})$, the remaining intermediate consequence, the procedure for subjective utility measurement proceeds as follows (Fig. 1). Start by considering the intermediate consequence, a missed individualization (C_{21}), compared to the gamble in which the best consequence (e.g., C_{11}, a correct identification) is obtained with probability α and the worst consequence (C_{12}, a false individualization) is obtained with probability $1-\alpha$. To render this probability α explicit, one can consider, for example, a so-called 'probability wheel' [e.g., 30] as shown in Fig. 1, where the spun of a pointer can stop in either of two sectors with angles that correspond to the ratio $\alpha : (1-\alpha)$. A common alternative procedure to measure α consists of imagining the drawing a ball from a urn with balls that have one of two colours (e.g., red and white) [e.g., 31], and where the drawing of a coloured ball depends on the proportion of red balls, which is to be equated with α. Doing so, the last step of the procedure (Fig. 1) consists of finding one's probability α that makes one indifferent between the sure consequence C_{21} and the gamble, that is:

$$C_{21} \sim \alpha C_{11} + (1-\alpha) C_{12}. \quad (3)$$

When relation (3) is satisfied, it can be proved that the utility of C_{21} can be derived accordingly as [32]:

$$U(C_{21}) = \alpha U(C_{11}) + (1-\alpha) U(C_{12}). \quad (4)$$

However, as the best consequence C_{11} has utility one and the worst consequence C_{12} utility zero, one immediately obtains $U(C_{21}) = \alpha$. Thus, according to this scheme, the numerical measure of the desirability of the intermediate consequence C_{21} equals the probability α that could lead one to the best consequence (e.g., a correct individualization) and, conversely, with probability $1-\alpha$ to be worst consequence (i.e., false individualization).

3.2. Discussion of the utility approach

The utility view presented in the previous section involves the consideration of one's probability for a false individualization $(1-\alpha)$. This is a personal probability defined as part of the utility elicitation procedure. It is important to emphasize that it is conceptually different from – and ought not to be confounded with – one's probability for the proposition that the suspect is the source of the trace material, denoted $Pr(\theta_1 \mid I)$ in Section 2.2.

It might be considered objectionable to measure the desirability of a missed individualization (i.e., intermediate consequence C_{21}) in terms of the probability for obtaining the best consequence (correct individualization) and, conversely, the expression and acceptance of a probability for a false individualization. In particular, it might be argued that it is inacceptable to maintain such a probability in principle and that, as a consequence, one should require $\alpha = 1$. That is, one ought to be indifferent between the sure consequence C_{21} and the imaginary gamble only if the latter does *not* lead to a false individualization, a situation in which $(1-\alpha) = 0$. However, consider what happens if one assigns $U(C_{21}) = \alpha = 1$. The expected utility of decision d_2, not individualizing the suspect, would be one (Eq. (2)), whatever the probability $Pr(\theta_1 \mid I)$. Hence, it would always be greater than the expected utility of decision d_1, given by $EU(d_1) = Pr(\theta_1 \mid I)$ (Eq. (1)). But if it is

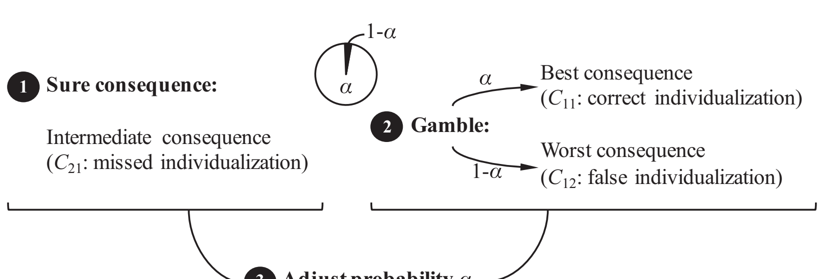

Fig. 1. Illustration of the procedure for subjective utility elicitation in terms of three steps: ❶ the sure consequence for which a utility is to be elicited, ❷ a gamble in which the best and the worst consequences are obtained with probabilities α and $1-\alpha$ respectively, ❸ adjustment of the probability α to the point at which one is indifferent between the sure consequence and the gamble.

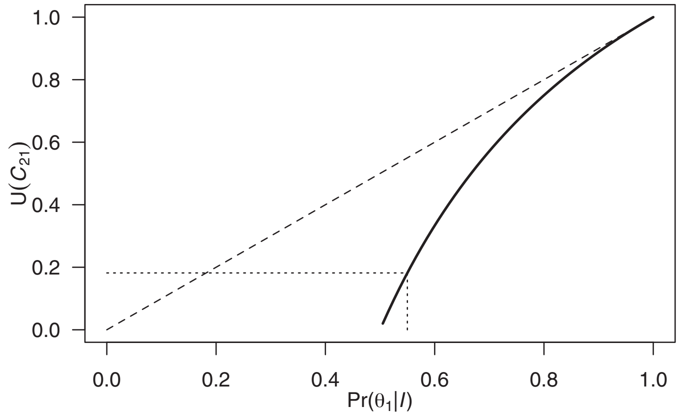

Fig. 2. Representation of the maximum value that the utility of a missed individualization, U(C_{21}), may take (bold solid line), as a function of the probability that the potential source is the true source (Pr(θ_1 | I)), in order for the decision to individualize (d_1) to be the preferred decision. This maximum value is given by 1 minus the odds in favour of θ_2 (Eq. (8)) and is strictly smaller than Pr(θ_1 | I) (dashed line). The dotted line illustrates an example for a case in which Pr(θ_1 | I) = 0.55 as discussed in the main body of the text.

always the case that EU(d_2) ≥ EU(d_1), then this means that one will *always* decide d_2, that is not individualize. Practical decision-makers *do* however make individualizations. Hence, their utility U(C_{21}) for a missed identification exists and is clearly smaller than one, and so is their probability α which makes the imaginary gamble acceptable to them.[10]

Presumably, the utility U(C_{21}) = α takes a value that is closer to the upper limit of the 0–1 utility scale, than to the lower limit. This follows from the view that the probability (1 − α) for the worst consequence (a false individualization) in the imaginary gamble should be clearly low. How low is a matter of judgment in the realm of the individual decision-maker. This conclusion might seem unsatisfactory for the practical decision-maker who requires an explicit assignment in order to determine the decision with maximum expected utility (Eqs. (1) and (2)). One way to approach this conceptual difficulty is to relate U(C_{21}) = α to the decision-maker's beliefs about the target propositions θ_1 and θ_2. For example, the decision maker might ask:

> Given my current state of belief about the truth or otherwise of θ_1, what ought to be my utility for a missed individualization, U(C_{21}), in order for an individualization (decision d_1) to be warranted in decision theoretic terms (i.e., the maximum expected utility principle)?

More formally, one needs to inquire about the values of U(C_{ij}) so that EU(d_1) > EU(d_2), that is

$$\mathrm{U}(C_{11})\Pr(\theta_1|I)+\mathrm{U}(C_{12})\Pr(\theta_2|I)>\mathrm{U}(C_{21})\Pr(\theta_1|I)+\mathrm{U}(C_{22})\Pr(\theta_2|I). \quad (5)$$

By rearranging terms one obtains

$$\frac{\Pr(\theta_2|I)}{\Pr(\theta_1|I)}<\frac{\mathrm{U}(C_{11})-\mathrm{U}(C_{21})}{\mathrm{U}(C_{22})-\mathrm{U}(C_{12})}. \quad (6)$$

Note that whenever a 0–1 scale is chosen for the utility function, the utilities for the best and the worst consequence (C_{11} and C_{12}) are assigned as, respectively, one and zero (see also Section 3.1) and Eq. (6) reduces to

$$\frac{\Pr(\theta_2|I)}{\Pr(\theta_1|I)}<1-\mathrm{U}(C_{21})=1-\alpha. \quad (7)$$

[10] This argument has also been presented in the context of legal decisions of guilt, regarding the utility assessment for a false acquittal [33]. Here, the above result EU(d_2) > EU(d_1) implies that acquittals (decision d_2) are to be preferred to convictions (decision d_1), which would not reflect *actual* judicial practice.

This result reveals two points. First, if the odds against θ_1 are greater than 1, that is Pr(θ_2 | I) > Pr(θ_1 | I), then the inequality can not be satisfied since the utility $U(C_{21})$ should be negative, that is beyond the 0–1 utility scale (see also Fig. 2). Hence, decision d_1 cannot be the preferred decision in this framework for probabilities Pr(θ_1 | I) below 0.5. This is in line with the general idea that one would not be prepared to decide in favour of a proposition that does not have the preponderance of probability. The second point is that, for d_1 to be the preferred decision, the utility U(C_{21}) must not exceed one minus the odds against θ_1. That is, rewriting Eq. (7):

$$1-\frac{\Pr(\theta_2|I)}{\Pr(\theta_1|I)}>\mathrm{U}(C_{21}). \quad (8)$$

In principle, from the above, the scheme allows d_1 to be the preferred decision for probabilities even slightly above 0.5, but this would result in a rather low limiting value for U(C_{21}). This may conflict with one's assessment of U(C_{21}) = α through Eq. (4), which involves the probability 1 − α for a false identification. For example, if one's probability Pr(θ_1 | I) is only 0.55, then U(C_{21}) must be smaller than 1 − (0.45/0.55) = 0.18 in order for d_1 to be the preferred decision (see dotted line in Fig. 2). However, from Eq. (4), α = 0.18 implies a probability of 1 − 0.18 = 0.82 for a false identification, which is considerable. Thus, in order to decide d_1, the probability of θ_1 ought not only be higher than 0.5, as noted in the previous paragraph, but clearly preponderant (i.e., values close to one). Generally, as Pr(θ_1 | I) tends to one, the upper limit of U(C_{21}) approaches one, too. This should resolve possible conflicts with the preferences expressed through Eq. (4). These tendencies seem entirely reasonable.

An essential conclusion from the above is that any decision to individualize (d_1) made in a state of belief Pr(θ_1 | I) > 0.5 can be reconstructed in terms of an assignment of a utility value α, smaller than one, to the intermediate consequence 'missed individualization' which, in turn, is related to a probability for a false identification (1 − α) greater than zero.

3.3. The loss approach

The choice of the utility scale considered throughout Sections 3.1 and 3.2 is subtle and conceptually intricate. To find, for example, the decision with maximum expected utility, one needs to consider one's probability for a false identification (1 − α), but this value does *not* correspond to the probability that the decision to individualize (d_1) in the case at hand is erroneous. Clearly, the latter is given by Pr(θ_2 | I), the probability of the trace not coming from potential source. Note, in particular, that in order

Table 2
Reformulation of Table 1 in terms of losses $L(C_{ij})$, with $i, j=\{1, 2\}$, assigned to consequences C_{ij} resulting from decisions d_i under possible states of nature θ_j.

	States of nature: Suspect is	... the source (θ_1)	... not the source (θ_2)
Decisions	Individualize (d_1)	$L(C_{11})$	$L(C_{12})$
	Do not individualize (d_2)	$L(C_{21})$	$L(C_{22})$

to individualize (decision d_1) in a given state of uncertainty about the true state of affairs (θ), one's probability for a false identification ($1-\alpha$), as defined in the elicitation procedure (Fig. 1), must actually be *greater* than $\Pr(\theta_2 \mid I)$. These distinctions may not be easy to accommodate.

An alternative way to specify the decision matrix is to valuate consequences C_{ij} in terms of losses, denoted $L(C_{ij})$ in Table 2. In this view, the best consequences – correct individualization (C_{11}) and correct exclusion (C_{22}) – are assigned the value zero: no loss is associated to these because they do not represent undesirable consequences. In turn, let the consequences C_{12} (false individualization) and C_{21} (missed individualization) of an erroneous decision have values different from zero. For the time being, let particular numerical assignments aside and focus only on the general properties of the development. In fact, the very advantage of the loss function considered in the application here is that explicit numerical assignments are not needed in order to further the understanding of forensic individualization from a decision analytic point of view.

Start by rewriting Eqs. (1) and (2) in terms of losses instead of utilities. This leads to the expected loss EL of decisions d_1 and d_2, respectively:

$$EL(d_1) = L(C_{11})\Pr(\theta_1|I) + L(C_{12})\Pr(\theta_2|I), \quad (9)$$

$$EL(d_2) = L(C_{21})\Pr(\theta_1|I) + L(C_{22})\Pr(\theta_2|I). \quad (10)$$

The decision criterion now is to choose the option that *minimizes* expected loss. For example, individualization (d_1) is the preferred decision if its expected loss is lower than that of not individualizing (d_2), that is $EL(d_1) < EL(d_2)$. To determine the conditions under which this is the case, it is necessary to take a closer look at the assignments $L(C_{12})$ and $L(C_{21})$ for adverse consequences. One can concentrate on these because the other two losses $L(C_{11})$ and $L(C_{22})$ are zero and cancel out. Writing $EL(d_1) < EL(d_2)$ in full length and eliminating the terms involving the zero losses, $L(C_{11})$ and $L(C_{22})$, leads to the following:

$$\begin{aligned} &L(C_{12})\Pr(\theta_2|I) < L(C_{21})\Pr(\theta_1|I), \\ &\frac{\Pr(\theta_1|I)}{\Pr(\theta_2|I)} > \frac{L(C_{12})}{L(C_{21})}. \end{aligned} \quad (11)$$

Eq. (11) states that the decision to individualize (d_1) is to be preferred if and only if the odds in favour of θ_1 (the proposition according to which the trace comes from the potential source) is greater than the ratio of the losses $L(C_{12})$ and $L(C_{21})$ for, respectively, a false individualization and a missed individualization.

3.4. Discussion of the loss approach

It is worth mentioning that Eq. (11) is a standard result in Bayesian decision theory regarding the choice between any two rival theories or models [e.g., 29].[11] For example, it is readily seen that if decisions for the wrong state of nature (i.e., consequences C_{12} and C_{21}) are considered equally undesirable, that is assigned the same loss value, then the Bayesian decision rule is to decide d_1 if and only if θ_1 is considered more probable than θ_2. This is sometimes illustrated with reference to the civil process where the result would be to decide for a party if the probability of their 'case' is larger than 0.5, and hence the probability of the adversary party's case is smaller than 0.5, and deciding wrongly for either side is considered equally undesirable [e.g., 35].

The comparison implied by Eq. (11) is essentially qualitative and reduces to a single factor, call it x for simplicity, that states how much greater one loss value is compared to the other. Assuming that a false identification (C_{12}) is worse that a missed individualization (C_{21}), one has $L(C_{12}) > L(C_{21})$ and one can define

$$L(C_{12}) = xL(C_{21}), \text{ for } x > 0. \quad (12)$$

Eq. (12) shows that the central factor is x and that for a given x, using the 0–1 scale, one can set $L(C_{12})$ – the loss associated with the worst consequence – and then divide by x to get immediately $L(C_{21})$, or alternatively set $L(C_{21})$ first and then get $L(C_{12})$ through multiplication by x. The practical conclusion thus is that decision-makers only need to specify *how much worse* they consider a wrong identification compared to a missed individualization.

Example. Suppose that a decision-maker considers a wrong individualization (C_{12}) fifty[12] times worse than a missed individualization (C_{21}). So x in Eq. (12) is 50. To individualize with such a preference structure, Eq. (11) requires the decision-maker to have odds of at least 50 in favour of θ_1 (the suspect being the source of the crime mark), which corresponds to a probability $\Pr(\theta_1 \mid I)$ of approximately 0.98.

Example. Suppose that the decision-maker's odds for the proposition θ_1 (the suspect being the source of the crime mark) over θ_2 (an unknown person is the source of the crime mark) is 1000, corresponding to a probability $\Pr(\theta_1 \mid I)$ of 0.999. Given this state of beliefs about θ_1 and θ_2, the decision criterion of Eq. (11) entitles the decision-maker to individualize (decision d_1) if and only if the loss $L(C_{12})$ for a wrong individualization is *less* than one thousand times greater than the loss $L(C_{21})$ of a missed individualization.

Note that the factor x is sometimes thought of in terms of Blackstone's "it is better that ten guilty persons escape, than that one innocent suffer" [36, at p. 352]. However, as Kaye [35] notes, this sentence expresses actual error rates rather than a ratio of losses (i.e., relative losses) for a given case, represented by the righthand side of Eq. (11).

While the interpretation of the loss values through Eq. (12) is intuitively clear, it is still of interest to consider the relationship between loss and utility values so as to ensure overall coherence of the decision-maker's preference structure. A standard way to obtain a loss function is to consider, for each state of nature θ (i.e., the columns in Table 1), the difference between the utility of the best consequence under the given state of nature and the utility of a consequence of interest. The loss assigned to a given consequence thus expresses the penalty for not choosing the best decision under the state of nature at hand. More formally, the loss for any consequence C_{ij} can be written as $L(C_{ij}) = \max\{U(C_{\{\cdot\}j})\} - U(C_{ij})$. Table 3 shows how to derive loss values from the utilities assigned in Section 3.1. For a given state of nature θ_j, one starts by identifying the maximum utility (e.g., 1 under θ_1). Next one subtracts, for each consequence, the utility $U(C_{ij})$ of the consequence. This will transform utilities of 1 for the best consequences to losses of 0, expressing the view that no loss is incurred by taking the best action. Note that the resulting loss function is closely related to the utility function in the sense that it is confined to

[11] See [34] for an application of this result for Bayesian classification in forensic science.

[12] Readers are invited to consider their own numbers.

Table 3
A decision matrix with utilities and losses for the consequences of decisions d_1 and d_2 under states of nature θ_1 (the suspect is the source of the crime mark) and θ_2 (an unknown person is the source). Utilities are assigned according to the discussion presented in Section 3.1 with α denoting the probability to obtain the best consequence (see also Fig. 1).

States of nature:	The suspect is	the source θ_1	not the source θ_2	the source θ_1	not the source θ_2
		Utilities		Losses	
Decisions	Individualize (d_1)	1	0	0	1
	Do not individualize (d_2)	α	1	$1-\alpha$	0

values in the interval between 0 and 1. It is also worth noting that the utility of the intermediate consequence C_{21} (missed individualization), that is the probability α (Fig. 1), becomes the loss $(1-\alpha)$, which has been interpreted as the probability of a false individualization in the utility elicitation procedure (Section 3.1).

With the above loss structure in mind, one can again ask a question similar to the one considered in Section 3.2:

> Given my current state of belief about the truth or otherwise of θ_1, what are the logical constraints on my loss for a missed individualization, $L(C_{21})$, in order for an individualization (decision d_1) to be warranted in decision theoretic terms (i.e., the minimum expected loss principle)?

With the losses assigned in Table 3, one can rewrite Eq. (11) as:

$$\frac{\Pr(\theta_1|I)}{\Pr(\theta_2|I)} > 1/(1-\alpha)$$

from which it follows that

$$\frac{\Pr(\theta_2|I)}{\Pr(\theta_1|I)} < (1-\alpha). \tag{13}$$

In words, thus, the decision to individualize (d_1) ought to be selected if one's probability for a false individualization $(1-\alpha)$ is *greater* than one's odds against θ_1. Fig. 3 provides a visual summary of this condition and also illustrates that $(1-\alpha)$ ought to be strictly greater than $\Pr(\theta_2 \mid I)$. The figure also illustrates a situation in which one's probability for θ_1 is rather moderate, that is 0.55, which would require a probability for a false identification $(1-\alpha)$ (in the gamble defined as in Fig. 1) of at least 0.45/0.55 = 0.82 (dotted line) in order for the decision to individualize (d_1) to be the preferred decision according to the principle of minimizing expected loss. Note also that such a high value for $(1-\alpha)$ implies a low probability α, and thus a low utility for a missed individualization. Clearly, as noted in Section 3.2, it would seem more appropriate to maintain a low probability $(1-\alpha)$ for a false individualization, which would make an individualization (d_1) preferable only when $\Pr(\theta_1 \mid I)$ tends towards 1. Insofar, thus, the decision theoretic model translates an intuitively acceptable perspective.

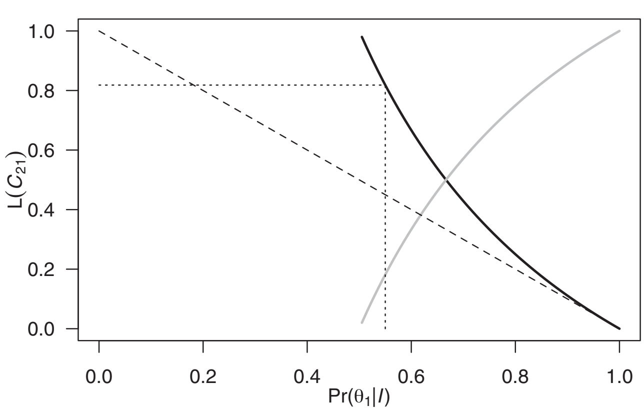

Fig. 3. Representation of the minimum value that the loss of a missed individualization, $L(C_{21})$, must have (bold black line), as a function of the probability that the potential source is the true source ($\Pr(\theta_1 \mid I)$), in order for the decision to individualize (d_1) to be the preferred decision. This minimum value is given by the odds against θ_1 (Eq. (13)) and is strictly greater than $\Pr(\theta_2 \mid I)$ (dashed line). For the purpose of comparison, the grey solid line reproduces the maximum value that the utility of a missed individualization ($U(C_{21})$) may take according to Fig. 2. The dotted line illustrates an example for a case in which $\Pr(\theta_1 \mid I) = 0.55$ as discussed in the main body of the text.

4. Discussion and conclusions

4.1. Archimedes' Law of Lever and Bayesian decision theoretic individualization

Throughout Section 3, it has been shown that the *decision to individualize* relies, in essence, on a comparison between, on the one hand, the odds in favour of the proposition that the potential source is the true source (proposition θ_1), rather than a unknown person (proposition θ_2), and, on the other hand, the relative losses of wrong determinations, that is the ratio of the loss of a wrong individualization ($L(C_{12})$) and a missed individualization ($L(C_{21})$). Whenever the former ratio exceeds the latter, the Bayesian decision criterion is to select decision d_1 (individualization).

In order to help apprehend this formal result, it is of interest to convey the logic of the Bayesian decision point in an illustrative way. This can be achieved in terms of Archimedes' Law of Lever, illustrated in Fig. 4(i) and (ii). In brief, this law states that "*(...) magnitudes (...) will be in equilibrium at distances reciprocally proportional to the magnitudes*" [4, p. 305]. That is, for example, two equal magnitudes A and B are at equilibrium if the lengths R and S of the lever that pivots on a fulcrum are the same (situation shown in Fig. 4(i)). If the magnitude B were greater than A, then the length R would need to be increased in order to maintain an equilibrium: as shown in Fig. 4(ii), if the magnitude B is twice as large as A, the establishment of an equilibrium requires the length R to be two times that of S. More generally, A × R = B × S, from which follows that

$$\frac{A}{B} = \frac{S}{R}, \tag{14}$$

that is the ratio of the two magnitudes A and B being equal to the reciprocal of their distances R and S.

It is obvious to see that Eq. (14) has the same structure as the Bayesian decision criterion, Eq. (11), so that the magnitudes can be interpreted as the losses of adverse consequences and the lengths R and S as the probabilities of the propositions about which a decision needs to be made. This is illustrated in Fig. 4(iii): clearly, if the loss of a wrong individualization ($L(C_{12})$) times the probability of the suspect not being the source of the crime mark, $\Pr(\theta_2 \mid I)$, is smaller than the loss of a missed individualization times the probability of the suspect being the source of the mark, $\Pr(\theta_1 \mid I)$, the balance will drop to the left, indicating that the expected loss of not individualizing (decision d_2) is greater than the expected loss of individualizing (decision d_1). The decision to be taken thus is d_1, because it has the smaller expected loss. Likewise, it is possible to interpret the magnitudes as the probabilities of the main propositions θ_1 and θ_2, and their distances to the fulcrum as the

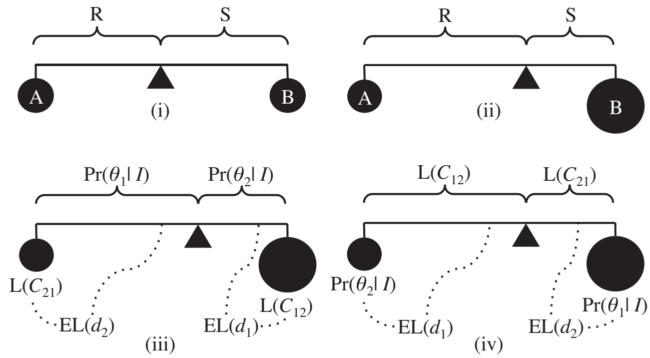

Fig. 4. (i and ii) Illustration of Archimedes' Law of Lever for two magnitudes A and B at distances R and S from a pivot. (iii and iv) Interpretation of the Law of Lever in terms of the probabilities for propositions θ_1 and θ_2 and the losses L of wrong decisions in an individualization scenario, as defined by the Bayesian decision criterion in Eq. (11).

losses of wrong decisions, as shown in Fig. 4(iv). It is clear from this perspective that when the two losses $L(C_{12})$ and $L(C_{21})$ are equal, then the equilibrium requires the two magnitudes – that is the probabilities $Pr(\theta_1 \mid I)$ and $Pr(\theta_2 \mid I)$ – to be equal as well, for as they are not, the lever will pivot to one side or the other, depending on which probability is larger. This precisely illustrates the reference to the civil process mentioned at the beginning of Section 3.4, where the decision is made on what is also known as the 'balance of probabilities'.

The Law of Lever provides a telling generalization of the more commonly known illustration of the scales (of justice). In particular, the Law of Lever illustrates that even though the magnitude $Pr(\theta_1 \mid I)$ (the probability of the suspect being the source) may be clearly greater than the probability of the alternative proposition θ_2, a sufficiently large loss associated to C_{12} (a wrong individualization), may make the lever pivot to the left, meaning that the expected loss $EL(d_1) = L(C_{12})\, Pr(\theta_2 \mid I)$ being greater than the expected loss of d_2 (not to individualize), making the latter decision preferable to d_1 (individualization) from a Bayesian decision point of view. Stated otherwise, even though one may have a case of a preponderance of probabilities in favour of the proposition according to which the suspect is the source of the crime stain, that is $Pr(\theta_1 \mid I) > Pr(\theta_2 \mid I)$, individualizing may *not* be the optimal decision when the loss of a false individualization $L(C_{12})$ is large enough compared to the loss of a missed individualization $L(C_{21})$, and capable to tip the pivot to the left in Fig. 4(iv), making $EL(d_1)$ greater than $EL(d_2)$.[13]

4.2. Likelihood ratios in the decision framework

It is worth to mention and hence to illustrate that the Bayesian decision theoretic framework for individualization is not incompatible with the likelihood ratio approach to evaluating results of comparative forensic examinations. This can be pointed out by reconsidering Eq. (11) and writing the posterior odds in favour of θ_1, that is the proposition according to which the suspect is the source of the crime mark, as the product of the prior odds and the likelihood ratio for the forensic results E:

$$\frac{Pr(\theta_1|I,E)}{Pr(\theta_2|I,E)} = \underbrace{\frac{Pr(\theta_1|I)}{Pr(\theta_2|I)}}_{\text{prior odds}} \times \underbrace{\frac{Pr(E|\theta_1,I)}{Pr(E|\theta_2,I)}}_{\text{likelihood ratio}} > \underbrace{\frac{L(C_{12})}{L(C_{21})}}_{\text{loss ratio}} \quad (15)$$

Recall that Eq. (15) defines the requirement that makes the decision to individualize (d_1) to be preferable to d_2 (not to individualize), that is when the product on the left is greater than the relative losses on the right. The Bayesian decision criterion can thus be reformulated with an emphasis on the likelihood ratio:

> The decision to individualize d_1 is to be preferred if the product of the likelihood ratio and the prior odds is larger than the ratio of the loss of an erroneous individualization to the loss of a missed individualization (i.e., losses associated with adverse consequences).

It is sometimes argued that the understanding of products can be eased when working with logarithms [e.g., 37], because this makes the terms additive. Applying the logarithm to Eq. (15), one obtains

$$\log\left[\frac{Pr(\theta_1|I)}{Pr(\theta_2|I)}\right] + \log\left[\frac{Pr(E|\theta_1,I)}{Pr(E|\theta_2,I)}\right] > \log\left[\frac{L(C_{12})}{L(C_{21})}\right], \quad (16)$$

and by re-arranging the terms one can isolate the likelihood ratio as follows:

$$\log\left[\frac{Pr(E|\theta_1,I)}{Pr(E|\theta_2,I)}\right] > \log\left[\frac{L(C_{12})}{L(C_{21})}\right] + \log\left[\frac{Pr(\theta_2|I)}{Pr(\theta_1|I)}\right]. \quad (17)$$

The logarithm of the likelihood ratio is commonly interpreted in terms of the weight of evidence, a term widely attributed to Good [37]. In the context of the Bayesian decision criterion for individualization (decision d_1), Eq. (17), it leads to the following requirement:

> Individualization (d_1) *is the preferred decision if and only if the weight of evidence is greater than the sum of the logarithm of the prior odds against the proposition of common source* (θ_1) *and the logarithm of the ratio of the loss of an erroneous individualization to the loss of a missed individualization (i.e., losses associated with adverse consequences).*

Table 4 illustrates examples of combinations of prior odds and threshold values that likelihood ratio must exceed to make – for

Table 4
Examples of minimum likelihood ratio (LR) values necessary to make the decision to individualize (d_1) preferable to not individualizing (d_2) for different combinations of prior odds (PO, odds in favour of the proposition that the suspect is the source of the crime stain) and relative losses (RL) as defined by Eq. (17). The values in columns four to six are the logarithms (base 10) of the values presented in the first three columns.

$PO = Pr(\theta_1 \mid I)/Pr(\theta_2 \mid I)$	LR	RL	log(PO)	log(LR)	log(RL)
1/10 = 0.1	100	10	−1	2	1
1/10 = 0.1	1000	100	−1	3	2
1/1000 = 0.001	10^5	100	−3	5	2
1/1000 = 0.001	10^6	1000	−3	6	3

[13] Archimedes' sentence "Give me a place to stand, and I shall move the earth." [e.g., 3,4] can thus be translated in the present context as, for example, 'any odds in favour of the proposition of common source can be levered if the loss of a missed individualization is sufficiently large'.

given loss ratios – an individualization preferable to not individualizing according to the Bayesian decision theoretic account (Eq. (16)).

5. Conclusions

The three main elements of Bayesian decision theory, that is propositions and their associated probabilities, decisions and preferences for consequences provide a rigorous framework through which the problem of individualization in forensic contexts can be approached in a disciplined manner. In particular, as Stoney [38] has noted, these three elements allow us to see that the traditional practice of forensic individualization took on a task that went beyond what it could justifiably do with science alone:

> For over 100 years the courts and the public have expected, and fingerprint examiners have provided, expert testimony that fuses these three elements: offering testimony not as evidence, but as proof, assuming priors and including decision-making preferences. This created an overwhelming and unrealistic burden, asking fingerprint examiners, in the name of science, for something that science cannot provide. As a necessary consequence, fingerprint examiners became unscientific. [38, p. 400]

Preference judgments for consequences, thus, hold a central position in the decision problem to which individualization amounts. Yet the very nature of these expressions of preferences, how they ought to be assigned and how they ought to be connected to the defining features of the individualization problem, remains a topic of lively discussion (see also reference in Section 1). Difficulties in how to answer these questions appear to be a major hindrance of a more widespread appreciation of the decision theoretic perspective.

Throughout this paper, the assignment of values to the decision matrix has been approached from two different perspectives, utilities and losses. It was emphasized that general mathematical properties of utility and loss functions not only ease the definition of the scope of the scale of value judgments, but also effectively reduce the number of assignments that require the analyst's active attention. In the particular context of individualization, the assignments that require primary care can actually be reduced to only a single element (e.g., when assuming a 2×2 decision matrix and a 0–1 utility (loss) function), that is the preference value for a missed individualization. It can be thought about in isolation or in a comparative way with respect to the preference value assigned to a false individualization. On a more general account, the decision to individualize can also be conceptualized as a comparison between relative beliefs in the main propositions (i.e., the suspect or some other person being the source of the crime stain) and relative losses for adverse consequences (i.e., Eq. (11)). The latter elements are all ingredients of what decision-makers already conceive of informally, hence the decision theoretic framework provides a way to make those elements explicit and formally precise. What is more, the established concept for weighing evidence in forensic applications, that is the likelihood ratio, has a clearly defined role in the decision theoretic framework for individualization: as shown through Eqs. (15)–(17), it compares against the relative losses of adverse consequences and the prior odds.

The above insights do not provide, nor do they intend to provide direct prescriptions for the roles that participants in practical proceedings ought to take. Also, there is no suggestion that the ultimate responsibility of decision-makers should be delegated to a formal theory. The theoretical framework merely intends to equip practical decision-makers with a powerful analytical and logical instrument to *help* them deal with the various factors thought to have a bearing on the decision problem they face. If, however, one accepts the precept that "(...) it is the utility function of the court that is appropriate (...)" [39, p. 141], then the Bayesian decision theoretic framework presented here provides a logically rigorous account of how utility may be framed, and by which participant in the legal proceeding.

The decisionalization of individualization may be nothing novel in the sense that, ever since, forensic practitioners may have *decided* on the conclusions they have rendered, which amounts to a *description* of what practitioners do, and what recent changes in terminology by professional associations reflect [1]. But, as argued by Stoney in the quote given above [38], this practice conflicts with a scientific approach and is beyond the scope of current guidelines [e.g., 13]. Contemporary means of Bayesian decision theory allow us to make these distinctions between probative value and decisional practice formally precise and articulate them in logically justifiable terms, providing thus a *normative* perspective. This should be of interest to professional associations and practitioners, as tying their considerations to normative precepts offers potential to gain credibility for their current practice, to scrutinize the role of experts and to rethink the scope of forensic expert reporting. This corresponds to a current need and helps counter critiques according to which changes in the disciplines of forensic individualization are mere changes of the label, rather than of the underlying practice.

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