This is the peer reviewed version of the following article: Montez J. V. (2013). Inefficient sales delays by a durable-good monopoly facing a finite number of buyers. *The RAND Journal of Economics*, 44(3), 425-437., which has been published in final form at 10.1111/1756-2171.12025. This article may be used for non-commercial purposes in accordance with Wiley Terms and Conditions for Self-Archiving.

# Inefficient Sales Delays by a Durable-Good Monopoly Facing a Finite Number of Buyers

João Montez<sup>\*</sup>

#### Abstract

This article offers a new explanation for unscheduled price cuts and slow adoption of durable-goods. We study a standard durable-goods monopoly model with a finite number of buyers and show that this game can have multiple perfect equilibria in addition to the Pacman outcome—including the Coase conjecture. Of particular interest is a class of equilibria where the seller first charges a high price, and only lowers that price once some—but not all—high-valuation buyers purchase. This price structure creates a war of attrition between those buyers, which delays market clearing and rationalizes unscheduled purchase and price cut dates.

# 1 Introduction

Many durable-goods are first introduced at a high price but buyers expect the seller to cut that price in the future. Price cuts tend however to be unscheduled and buyers cannot forecast their timing. "This is life in the technology lane" according to the late Steve Jobs, "there is always someone who bought a product before a particular cutoff date and

<sup>\*</sup>London Business School, jmontez@london.edu. I am grateful to Luis Cabral, David Myatt, Dezsö Szalay, Thomas von Ungern-Sternberg, Lucy White, the editor Benjamin E. Harmalin, and two anonymous referees for very careful comments that have greatly benefited the article. I also thank Patrick Bolton, Yeon-Koo Che, Eloy Perez, John Sutton and participants at several conferences and also seminars at Bonn, Columbia, LSE, NYU and Pompeu Fabra. The Web appendix for the article will remain available on the author's personal website.

misses the new price" he wrote in a press release following a \$200 price cut on Apple's i-Phone in September 2007. Besides consumer electronics, this pattern is shared by many durable-goods—for example music, movies and books.

Yet the existing durable-goods literature offers little to explain why the date of price cuts cannot be anticipated and why purchases at a high price spread out over time—after all, wouldn't a buyer, who discounts the value of future consumption, purchase once the price at which he has decided to buy is first offered? This article provides an explanation for these phenomena.

Naturally, once high-valuation buyers have purchased and left the market, the seller of a durable-good will want to lower the price to reach low-valuation buyers as well. Coase (1972) argued that high-valuation buyers, anticipating a price cut, may then become reluctant to accept a high price and, to avoid delaying his sales, the seller may finally need to offer a low opening price. This idea has been formalized by studying subgame perfect equilibria (SPE) of a benchmark infinite-horizon model of complete information where a durable-good monopoly seller, producing at a constant marginal cost, posts a price in each period and buyers with a unit demand simultaneously decide to either accept or reject each price.

When demand is made of a *continuum* of buyers this model has generally a unique equilibrium path. As Coase conjectured, when the time between each offer converges to zero the opening price converges to the lowest buyer valuation and the competitive quantity is sold in a twinkle of an eye (e.g. Gul, Sonnenschein, and Wilson, 1986). With a *finite number* of buyers the result changes dramatically. As each sale has a non-negligible effect on the profit, the seller may then credibly condition price reductions on single purchases. If prices can be revised sufficiently frequently, there exists a SPE in which the seller achieves virtually the perfect discrimination profit by selling sequentially at a price equal to the valuation of each buyer—as if eating down the demand curve, an outcome known as *Pacman* after the classic computer game (Bagnoli, Salant, and Swierzbinski, 1989). The Pacman outcome is in fact the unique SPE when the valuation of each buyer is large relative to the sum of the valuations of all buyers with a lower valuation (Fehr and Kuhn, 1995).

Despite the different predictions on market power, both results challenge the classic association of monopoly and inefficiency with a captivating idea: that a seller, uncommitted to future prices, will clear the market in a proverbial twinkle of an eye. Given the implications of this research to regulation and antitrust, there is a large literature that tries to identify sources of inefficiency. Authors have studied the role of reputation (e.g. Ausubel and Deneckere, 1989), the effect of time varying demand (e.g. Sobel, 1991 and Biehl, 2001) and challenged technology and informational assumptions—for example accounting for imperfect durability (e.g. Deneckere and Liang, 2008), decreasing returns and limited capacity (e.g. Bulow, 1982, Kahn, 1986 and McAfee and Wiseman, 2008), private information on cost (e.g. Ausubel and Deneckere, 1992) or the seller's ability to commit to prices (Inderst, 2005 and Kim, 2009).

This article offers two new results to this literature. First we find that when the difference between high and low valuations is not too large even in the most canonical model with a finite number of buyers there exist SPE satisfying the Coase conjecture—in addition to the Pacman outcome. In fact, in that case any profit level between the Coasian and the perfect discrimination level can be supported by efficient SPE.

Second, we find that this model can also support SPE that rationalize stochastic price cut dates and slow adoption of durable-goods. These equilibria are inefficient as the market clearing date remains bounded away from zero even as the interval between offers shrinks to zero.

The latter SPE can be described as follows. The seller initially sets a high price. Then, once some—but not all—high-valuation buyers have purchased, the seller finds it optimal to lower the price to sell to all remaining buyers and clear the market. For this reason a high-valuation buyer who purchases early creates a positive externality for the remaining buyers and, from the high-valuation buyers' perspective, the game resembles a war of attrition in which buyers delay purchases in the hope another buyer will purchase first.

In equilibrium the high-valuation buyers randomize between purchasing and not purchasing in each period or, equivalently, they use a distribution function to choose the exact date at which to purchase if the price remains high. The seller sets a high initial price despite the buyer war of attrition delaying the sales to the low-valuation segment—as the alternative of lowering the price results in the players coordinating in a Coasian outcome thereafter with an even lower profit. Eventually, as if succumbing to the temptation, someone ends up purchasing at the high price, and the price is therefore reduced to finally clear the market—but the exact date at which this happens is stochastic.

A Coasian equilibrium supporting additional SPE is also a feature of the inefficient SPE identified by Ausubel and Deneckere (1989) in a model with a continuum of buyers. There are however several important differences. One is that in a model with a continuum of buyers the Coasian outcome is unique unless the profit made in that outcome is arbitrarily close to zero, i.e. it is unique unless the lowest buyer valuation does not exceed the marginal cost—the no-gap case. On the other hand, in the present model with a finite number of buyers a Coasian outcome only exists if the profit made by selling to the low-valuation segment is non-negligeble, i.e. when the difference between the cost and the lowest buyer valuation is sufficiently high—otherwise the Pacman outcome is unique.

A second important distinction lies with the models' predicted behavior. In the equilibria identified by Ausubel and Deneckere (1989) the seller adheres to a deterministic price path which must never clear the market.<sup>1</sup> We find that with a finite number of buyers there exist multiple equilibria where the market clears in finite time with probability one—including efficient ones—and there are also inefficient SPE where the price path is stochastic, despite the seller using a pure strategy.

The idea that a war of attrition can lead to delay in a durable-good monopoly is also present in the work of Inderst (2005) and Kim (2009). They consider a setting where a single buyer, with private information on its valuation, does not know if the seller is committed to its price or not, i.e. if the seller can change the initial price. They find that a war of attrition can take place between the buyer and the seller, so that in each period a player may concede with a positive probability—the buyer by purchasing at a high price and the seller by reducing his price. Therefore uncertainty about the seller's commitment offers another explanation for real-time delay. Delay does however vanish or becomes negligible once the probability of commitment is small. Two key distinctions are that in the present setting a war of attrition results from the strategic interaction between buyers and delay arises even if it is known that the seller is not committed—and therefore buyers know that the price will eventually be reduced for with probability one.

The remainder of the article is organized as follows. In section 2 we set-up a simple

<sup>&</sup>lt;sup>1</sup>If there exists a finite bound on the number of periods in which sales can occour, by backward induction the Coase conjecture implies that the opening price must be close to the lowest valuation.

model with a finite number of buyers. In section 3 we show that Coasian SPE exist when the differences in valuations is not too large, which also support a folk-theorem on the seller's profit. In section 4 we identify equilibria where buyers engage in a war of attrition. We conclude in section 5. The proofs of the main results are in Appendix A.

## 2 The model and preliminary analysis

We study a canonical durable-good monopoly model with a finite number of buyers. A seller, indexed by m, can produce a durable good in any period t = 0, 1, 2, ... at a constant marginal cost which, without loss of generality, is set to zero—interpret prices and valuations as net of the cost.

The game is the standard one. The seller in each period t posts a price  $p_t \ge 0$ . Buyers then simultaneously accept the current price (and leave the game) or reject it and continue to the next period—action  $a_t^i = 0$  denotes a rejection in period t by buyer i and  $a_t^i = 1$  an acceptance and each buyer i may choose  $a_t^i = 1$  at most once.

There is a set  $N = \{1, ..., n\}$  of buyers and each buyer has a valuation v(i) for a single unit of the good. The valuation of each buyer can be either high or low, i.e.  $v(i) \in \{L, H\}$ with H > L > 0. Let  $n_t^v > 0$  denote the number of buyers with valuation v in the market at date t. To capture the intuition in a simple model we assume that  $n_0^H = 2$ . In this standard model there is complete information, so all valuations, prices, and purchases are observed.<sup>2</sup>

The payoff of buyer i is

$$u^{i} = \begin{cases} \delta^{t}(v(i) - p_{t}) \text{ if } a_{t}^{i} = 1 \text{ for some } t \\ 0 \text{ if } a_{t}^{i} = 0 \text{ for all } t, \end{cases}$$

where  $\delta = e^{-\rho \Delta} \in (0, 1)$  is the discount factor,  $\Delta > 0$  is the real time between two successive offers and  $\rho > 0$  the discount rate. The seller's payoff is the discounted revenue

$$u^m = \sum_{t=0}^{\infty} \delta^t \left( p_t \sum_{i \in N} a_t^i \right).$$

 $<sup>^{2}</sup>$ Below we discuss how the analysis may be extended to multiple high-valuation buyers and the effect of alternative assumptions.

A *t*-period history is a list of prices and purchases from period 0 to t - 1. To simplify notation, we often make reference to the relevant history in the text and denote equilibrium actions (not the strategies) by  $p_t^*$  and  $a_t^{i*}$ . A pure strategy is a function specifying a player's action plan at each period for each history prior to that period. The vector of strategies is **s** and player *i*'s expected payoff is

$$\mu^{i}(\mathbf{s}) \equiv E\left[u^{i} | \mathbf{s}\right]$$

We study symmetric subgame perfect equilibria (SPE) of this game G, where buyers with the same valuation use similar strategies. We focus on pure strategy pricing equilibria, i.e. equilibria in which the seller uses a pure strategy on and off the equilibrium path. In Markov PE, or simply stationary equilibria (SE), strategies depend only on the payoff relevant history—the set  $I(t) \subseteq N$  of buyers remaining in the market at t and which, when we consider symmetric equilibria, can be summarized by  $h(t) = (n_t^H, n_t^L)$ .

In line with the literature we study the case where the time between offers  $\triangle$  is close to zero, i.e.  $\delta$  close to 1. In particular we study real-time efficiency:

**Definition 1.** A SPE with strategies  $\mathbf{s}^*$  is real-time efficient if all gains from trade are realized as  $\delta \to 1$ , i.e.

$$\lim_{\delta \to 1} \left[ \sum_{i \in N} \mu^i(\mathbf{s}^*) + \mu^m(\mathbf{s}^*) \right] = \sum_{i \in N} v(i).$$

Bagnoli, Salant, and Swierzbinski (1989) showed that the *Pacman* strategies always form a SPE of the present game—which is also a SE. With those strategies at any date tthe seller posts a price equal to the highest valuation in the market at t and all buyers with that valuation purchase immediately, i.e. for all  $I(t) \neq \emptyset$ 

$$p_t = \max \left\{ v(i) : i \in I(t) \right\} \text{ and } a_t^i = \begin{cases} 1 \text{ if } p_t \le v(i) \\ 0 \text{ otherwise} \end{cases}$$

In the present setting the market would clear in 2 periods, and as  $\delta \to 1$  the Pacman profit becomes that of a perfectly discriminating monopolist, i.e.

$$\lim_{\delta \to 1} \mu^m(\mathbf{s}^*) = \sum_{i \in N} v(i) \text{ and } \mu^i(\mathbf{s}^*) = 0 \text{ for all } i \in N.$$

Therefore this outcome is real-time efficient. Fehr and Kuhn (1995) find that this is also the unique SPE when every buyer has a valuation which is "large relative to the sum of valuations of all buyers with a lower willingness to pay." What remains unknown is what may happen when this condition is not satisfied.

We will show that the Pacman outcome is the unique SPE of G if and only if the difference in valuations is higher than the total profit the seller makes from the low-valuation segment, i.e.  $n_0^L L < H - L$ . Only then can the seller credibly delay the sales to that whole low-valuation segment to collect the premium a single H-buyer is willing to pay to avoid a period of delay in consumption.

When that condition is not satisfied we find that it can be optimal for the seller to charge an opening price that is close or equal to L and make a *Coasian profit*.

**Definition 2.** A SPE with strategies  $\mathbf{s}^*$  is Coasian if  $\lim_{\delta \to 1} p_0^* = L$  and

$$\lim_{\delta \to 1} \mu^m(\mathbf{s}^*) = nL \text{ and } \lim_{\delta \to 1} \mu^i(\mathbf{s}^*) = v(i) - L \text{ for all } i \in N.$$

The game has an infinite horizon but it is over when all buyers have purchased, i.e. once the market has cleared. We have:

## **Lemma 1.** In all SPE of G every buyer accepts with probability one the price $p_t = L$ .

For this reason the seller never sets the price below L because that would lower the revenue without generating additional sales. Therefore either all low-valuation buyers are in the market at t or the market has cleared. Although low-valuation buyers use a simple cut-off strategy—buying whenever the price does not exceed L—and will have zero surplus in equilibrium, they crucially affect the seller's cost of waiting to clear the market. As we shall see, this cost has important implications on the equilibrium behavior.

## **3** A Coase conjecture with a finite number of buyers

We can restrict our attention to those subgames where  $n_t^L = n_0^L$  as by Lemma 1 the price  $p_t = L$  would be immediately accepted by all buyers, and therefore in all relevant subgames either all low-valuation buyers are in the market or none are. Consider the subgame where a single high-valuation buyer remains in the market at t. The normalized difference in valuations is x = (H - L)/L. Then:

**Lemma 2.** The Pacman outcome is the unique SE of the subgame with one high-valuation buyer and  $n_0^L$  low-valuation buyers if and only if  $n_0^L < x$ . If  $n_0^L > x$ , then, for any  $\delta$  close to 1, besides the Pacman there exist two additional SE and both are Coasian, one with mixed strategy pricing and one with pure strategy pricing where  $p_0^* = L$ , while if  $n_0^L = x$ then the latter and the Pacman are the only SE.

We now work backwards to G with two high-valuation buyers. The novelty is not in the existence of the Pacman outcome—or that it is unique when the difference in valuations is large. The novelty concerns the case where the difference in valuations is not too large relative to the value of the low-valuation segment, i.e. when  $n_0^L L \ge H - L$ .

**Proposition 1.** If  $n_0^L < x$  the Pacman outcome is the unique SPE (and SE) of G, and as  $\delta \to 1$  the seller appropriates the perfect discrimination profit. If  $n_0^L \ge x$ , for  $\delta$  close to 1, there are exactly two symmetric SE with pure strategy pricing: the Pacman and a Coasian one where along the equilibrium path high-valuation buyers purchase immediately at

$$p_0^* = \begin{cases} L \text{ if } n_0^L \ge 2x \\ (1-\delta)H + \delta L \text{ if } n_0^L \in [x, 2x]. \end{cases}$$
(1)

This Coasian SE supports additional SPE, and as  $\delta \to 1$  any profit level between the Coasian and the perfect discrimination profit is supported in real-time efficient SPE of G.

The intuition for the reason why the price in the case where  $n_0^L \ge x$  is as given in (1) is the following. Suppose that players expect the price to be L when there is a single high-valuation buyer left in the market (an outcome from Lemma 2). Conditional on this expectation, suppose that some  $p^* > (1 - \delta)H + \delta L$  is part of a pure strategy pricing equilibrium when  $h(t) = (2, n_0^L)$ . Then the *H*-buyers must accept  $p^*$  with some positive probability in each period, which in a symmetric equilibrium must also be smaller than one.

When buyers use Markovian strategies, a one period price cut at t does not affect the buyers' behavior in subsequent periods but it has the following two effects: a negative effect of reducing the profit on the sales that are made today at a price below  $p^*$ , and a positive effect that reducing the price increases the high-valuation buyers' acceptance rate at t and this brings forward the sales to the remaining buyers. For  $\delta$  close to one, the loss associated with the former is always outweighed by a larger gain on the latter because without the price reduction the probability a buyer purchases at t is close to zero but that probability increases significantly for even a small price reduction. Since this is true for any  $p^* > (1 - \delta)H + \delta L$ , in equilibrium the seller would not choose such  $p^*$  in a SE with pure strategy pricing. For those prices in (1) there is however no gain from reducing the price as the high-valuation buyers already accept those prices with probability 1, hence those prices are sustained in a SE. This explanation is reminiscent of the intuition for the Coase conjecture provided by Gul, Sonnenschein, and Wilson (1986) in a setting with a continuum of buyers.

The folk-theorem on the seller profit is sustained by the threat of reverting to the unique Coasian SPE if the seller changes the price before both high-valuation buyers have purchased. This is reminiscent of Ausubel and Deneckere (1989) in the setting with a continuum of buyers. Like in their work, we support these equilibria with deterministic price paths and on the equilibrium path each buyer immediately accepts the first price that gives him his expected equilibrium payoff.

There are however important differences between the two settings. With a continuum of buyers the existence of non-Coasian equilibria relies on the existence of an equilibrium with profits arbitrarily close to zero, which only exist if the lowest buyer valuation does not exceed cost—known as the no-gap case. Otherwise, in the gap-case, the Coase conjecture is the unique equilibrium outcome in that setting.<sup>3</sup>

With a finite number of buyers only situations where the revenue made with each lowvaluation buyer exceeds the cost are relevant—the gap case. What we find is that here if that gap is small then the Pacman outcome is the unique SPE and multiple SPE exist precisely in the opposite situation where that gap between the cost and the low-valuation is large, as then the profit of selling to low-valuation segment is sufficiently high and therefore the seller finds it hard to credibly wait for every high-valuation buyer to purchase before reducing the price.

A second difference is that, in the setting with a continuum of buyers, non-Coasian

<sup>&</sup>lt;sup>3</sup>If there is a gap between the seller's cost and the lowest buyer valuation there will also exist a finite bound on the number of periods in which sales can occur and, by backward induction, the Coase conjecture implies that the opening price must be close to the lowest valuation.

SPE relies on sales necessarily occurring over infinite time, i.e. the market must never clear. Therefore any equilibrium outcome that is not Coasian must be inefficient. In contrast, in a setting with a finite number of buyers we find that different profit levels can be supported even when the market clears in a twinkle of an eye, i.e., in real-time efficient equilibria.

Fehr and Kuhn (1995) find in a reverse situation, i.e., with a continuum of buyers and a discrete price grid, that multiple competitive prices are sustained as SE. That multiplicity supports through trigger strategies any profit level between the Coasian and the perfect discrimination profit in both real-time efficient and inefficient SPE. Conceptually similar to Benoit and Krishna's (1985) folk theorem in finite games, their construction requires different competitive SE prices to clear the market on and off the equilibrium path. On the other hand with a continuos price grid, like in our and Ausubel and Deneckere's (1989) setting, a similar construction does not hold since there is a single SE competitive price—see Lemma 1.

However, similar to Fehr and Kuhn's (1995) argument, the existence of both the Pacman and the Coasian SE in the present setting supports additional SPE with delay. When  $n_0^L > x$ , for  $\delta$  close to 1, there is always some date t > 0 such that the monopolist posts some price larger than H before t and at that date the players coordinate on the Pacman outcome. This behavior is sustained by the threat to revert to the Coasian SE if the monopolist posts a price lower or equal to H before t.

## 4 Stochastic price cuts and inefficient delay

So far we have considered equilibria with deterministic equilibrium price paths. We now identify a class of equilibria where the seller uses a pure strategy to set prices but the equilibrium price path is nevertheless stochastic.

In these equilibria the seller posts a high price  $\hat{p}$  in every period until one of the high-valuation buyers makes a purchase, and only then lowers the price to L and clears the market. Conditional on this structure, the high-valuation buyers' payoff structure is similar to a war of attrition—the payoff is decreasing in the time of purchase but the one who purchases first gets a lower payoff. In equilibrium the high-valuation buyers randomize their acceptance and are indifferent between buying today or waiting to see if the other purchases first.

**Proposition 2.** If  $n_0^L \in [x, 2(x-1))$ , then, for any  $\delta$  close to 1 and  $\hat{p} \in (L, L(x-n_0^L/2)]$ , there exists a symmetric SPE of G with pure strategy pricing where

$$p_t^* = \begin{cases} \widehat{p} \text{ if } h(t) = (2, n_0^L) \\ L \text{ otherwise} \end{cases}$$

and when  $p_t^* = \hat{p}$  each high-valuation buyer uses a mixed strategy of acceptance, and its equilibrium probability of acceptance

$$q(\widehat{p}) = \frac{(1-\delta)(H-\widehat{p})}{\delta(\widehat{p}-L)}$$

converges to a Poisson process with parameter  $\rho(H - \hat{p})/(\hat{p} - L)$  as  $\delta \to 1$ . The expected profit and market surplus converge respectively to

$$\alpha(\widehat{p}) \cdot \left[\widehat{p} + (n_0^L + 1)L\right] \text{ and } \alpha(\widehat{p}) \cdot \sum_{i \in N} v(i) \text{ with } \alpha(\widehat{p}) = \frac{2(H - \widehat{p})}{2H - \widehat{p} - L}$$

Since  $\alpha(\hat{p}) < 1$ , these outcomes are real-time inefficient.

The high-valuation buyers' randomization delays purchases and makes the seller lose the financial interest on subsequent sales that are delayed. Similar to the argument of Proposition 1, this financial loss could tempt the seller to reduce the price to increase the acceptance rate. However, by offering a lower price the seller may lose his reputation of setting high prices as from that moment on the players coordinate in a Coasian equilibrium. This can be enough to incentivize the seller to keep a high price at the outset of the game.

Buyers using a mixed strategy on the equilibrium path does not however guarantee that the outcome is real-time inefficient—because the buyers' aggregate probability of acceptance could remain bounded away from zero as  $\delta$  converges to one. All purchases would then take place in the "twinkle of an eye" until the market clears. What creates real-time delay is the fact that the equilibrium acceptance rate of each high-valuation buyer converges to a Poisson process, which has an arbitrarily-slow (but positive) real-time rate of purchases. For example, if L = 1, H = 10 and  $n_0^L = 9$  the loss created by delay is close to 6% for  $\hat{p} = 2$ , 13% for  $\hat{p} = 3$  and 20% for  $\hat{p} = 4$ .

The reason the upper bound of  $\widehat{P}$  decreases with  $n_0^L$  is the following. The higher is  $\widehat{p}$  the lower is the high-valuation buyers' equilibrium rate of purchases, and therefore the expected time it takes to make the first sale increases with  $\widehat{p}$ . If  $\widehat{p}$  were too high then letting the high-valuation buyers select who makes the first purchase in a war of attrition would substantially delay the sales to the low-valuation segment. The seller would then prefer to clear the market immediately if the profit made on the low-valuation segment is sufficiently high—hence a  $\widehat{p}$  that is too high relative to  $n_0^L$  cannot be sustained in equilibrium.

Like the equilibria studied by Ausubel and Deneckere (1989) in the model with a continuum of buyers, our equilibrium construction consists of a main path and a punishment path. The main difference, in addition to those differences already mentioned in the previous section, lies in the predicted pricing and purchase behavior. In their work the equilibria price path and purchase dates are deterministic, so it cannot explain why the date of price cuts cannot be anticipated or why purchases made at a high price spread out over time. These are on the other hand the interesting features of the equilibria identified in this section.

### 4.1 Additional considerations

Unobservable purchases. The standard model assumes common knowledge of demand at each point in time. An arguably more realistic description is that buyers observe prices but do not know which buyers are still in the market—but the seller knows the number of buyers who purchase in each period. In that case buyers can still use prices to infer the current state of demand.

Call G' this game of incomplete information derived from G. Suppose the high-valuation buyers believe that  $n_t^H = 2$  if  $p_{t'} = \hat{p} > L$  for all  $t' \leq t$  and  $n_t^H = 1$  otherwise. We find that the outcomes of G characterized in Proposition 2 are also perfect-Bayesian equilibrium of G' (see Web appendix).

The reason is that in equilibrium the high-valuation buyers engage in an unobservable actions war of attrition, and thus over time one buyer will purchase and leave the market. Once this happens the seller is left with the option of lowering the price to L and clear the market or to continue charging  $\hat{p}$  and wait for the remaining high-valuation buyer to purchase before reducing the price to L. It turns out that for any  $\hat{p} \in \hat{P}$  the purchase rate with a single high-valuation buyer left in the market becomes sufficiently low that, following the first purchase, the seller prefers to immediately cut the price to L and clear the market. In turn, the anticipation of this price cut—although at an ex-ante unknown date—rationalizes the high-valuation buyers' randomization.

Valuation uncertainty. Suppose now that high-valuation buyers have some private information on their exact valuations in the sense that each high-valuation is an independent draw from a common differentiable distribution F(v) with  $F(H - \varepsilon) = 0$ , F(H) = 1. Assume that  $\varepsilon$  is "small". There are perfect-Bayesian equilibria similar to those identified in Proposition 2 in this perturbed version of the game G. This time, from the high-valuation buyer's perspective, the game resembles an incomplete information war of attrition.

If  $\varepsilon$  is "large", in the sense that  $H - \varepsilon = L$ , then all buyers could have a low-valuation and we expect the outcome to be Coasian (see e.g. Fudenberg et al., 1985). Less extreme and perhaps more realistic is the case where some, but not all, buyers may have a lowvaluation. In that case we intuitively expect that a war of attrition among high-valuation buyers can persist, because the price must then also drop to L once a critical number of sales have taken place—but we have not formally explored that possibility.

*Behavioral motivations.* We have found Coasian SPE in a model with a finite number of buyers using the standard assumptions. Coasian equilibria do become more relevant if we take behavioral considerations into account—similar to those of the recent bargaining literature (e.g. Abreu and Gul, 2000).

Suppose for example that there is a small probability the high-valuation buyers may be stubborn, in the sense that any high-valuation buyer may refuse to accept prices higher than L when he is the only such buyer left in the market. We know from Fudenberg et al. (1985) that the unique SPE of the subgame where  $h(t) = (1, n_0^L)$  is then Coasian. It follows from the proofs that in that case a Coasian SE of G always exists, independently of the difference in valuations—i.e. for any level of x. These behavioral considerations also make the behavior described in Proposition 2 more prevalent, as the constraint  $n_0^L \ge x$  is then no longer required and for that reason there always exist a set prices that sustain equilibria where high-valuation buyers initially engage in a war of attrition. Multiple high-valuation buyers. It is no surprise to those familiar with the theoretical literature on the war of attrition that extending the analysis to multiple high-valuation buyers can raise a few technical issues. One way we can do this is to consider a situation where the seller drops the price to L once k high-valuation buyers remain in the market, and he charges a decreasing sequence of high prices before that critical mass is reached. The latter ensures that we do not disturb the essential feature of the complete information war of attrition, i.e. that those who purchase later are better off than those who purchase early (see Kapur, 1995). It can be shown that in this case there are equilibria where some real-time elapses between each purchase, as a war of attrition among multiple high-valuation buyers also takes place.

For  $\delta$  close to 1, if the gap between the high prices becomes small then all but k + 1 high-valuation buyers purchase almost immediately and "one too many" high-valuation buyers are soon left trying to get the low price.<sup>4</sup> This seems to capture, albeit in a crude way, frenzies at the launch of new durables that are followed by a market slowdown before a large price cut finally takes place.

Markovian strategies and delay. The equilibrium actions described in Proposition 2 are stationary in the sense that they are time invariant. However the strategies supporting those actions are not Markovian—they depend not only on current demand but also directly on past prices. An extension of theoretical interest is that a similar behavior can be sustained with Markovian strategies.

To show this we have introduced in G an additional buyer with an intermediate valuation. We find that there are instances in which a small price cut induces the intermediatevaluation buyer to purchase immediately, transitioning the game to a state with only the high-valuation and low-valuation buyers. The seller may then find it optimal to avoid the transition to that state if that subgame is Coasian.

Instead the seller initially keeps a high price that the intermediate-valuation buyer refuses but that high-valuation buyers accept with a positive probability. The result is that

<sup>&</sup>lt;sup>4</sup>As the rate at which each high-valuation buyers purchases when more than k+1 are still in the market converges to a Poisson process with a "large" parameter, but the rate of purchases decreases substantially once only k+1 are left in the market. This feature is similar to that found in a generalized incomplete information war of attrition, as studied by Bulow and Klemperer (1999).

a war of attrition among high-valuation buyers is supported by Markovian strategies (see Web appendix).

In a sense the presence of buyers with intermediate valuation can replace trigger strategies by creating a discontinuity on the seller's payoff, which makes it optimal for the seller to keep a high price at the outset of the game and let a buyer war of attrition determine who purchases first.

## 5 Conclusion

The extreme prediction of the Pacman outcome, and the fact that it crucially relies on the seller being able to wait for *every* high-valuation buyer to purchase before reducing his price, has always raised some concern over its practical relevance. By looking beyond the Pacman outcome, our work suggests that the model with a finite number of buyers can still enhance our understanding of durable-goods markets with alternative and empirically sensible predictions.

Durable-goods are often first priced high and only once some buyers have purchased the seller will drop the price. A seller may however be unable to wait out for every highvaluation buyer to purchase before decreasing the price, as the premium he can collect on just a few high-valuations buyers may not be sufficient to cover the cost of delaying the sales to all remaining low-valuation buyers.

In that case each high-valuation buyer may hope to get a bargain if he waits and enough of the remaining buyers purchase early. Such expectations over the price process could create a war of attrition between high-valuation buyers, thus slowing down the sales process.

In this article we have explored this mechanism, developing its basic intuition in the simplest durable-good monopoly model with a finite number of buyers. We showed that the considerations raised above can lead the seller to offer a low price at the outset of the game to avoid delaying his sales. Therefore the Coase conjecture may prevail also when the collection of buyers is finite.

There are also equilibria where the seller offers a high initial price and buyers randomize the dates of their purchases as in a war of attrition, trying to benefit from the eventual price cut. The latter can help to explain unanticipated price cuts and slow adoption in durable-goods markets.

#### Appendix A

**Proof of Lemma 1.** Denote by  $\overline{p}$  the minimum among the highest price every buyer  $i \in N$ would accept with probability 1 in all subgames where  $I(t) \neq \emptyset$ . By profit maximization we must have that  $p_t \geq \overline{p}$ . So for any  $s^*$  we have  $0 \leq \mu^i(s^*) \leq v(i) - \overline{p}$  for all  $i \in N$ . Buyer i will then accept with probability 1 any price  $p_t$  such that

$$v(i) - p_t \ge \delta(v(i) - \overline{p}) \Leftrightarrow p_t \le (1 - \delta)v(i) + \delta\overline{p}.$$

Since every buyer *i* refuses prices larger than v(i) and  $v(i) \ge L$ , by the definition of  $\overline{p}$  we have  $\overline{p} = L$ .

**Proof of Lemma 2.** Here we consider the case where  $h(t) = (1, n_0^L)$ . We first show that the Pacman outcome is the unique SPE—and therefore SE—if  $n_0^L < x$  (Step 1). We then show by construction that if  $n_0^L \ge x$  a Coasian equilibrium with  $p_h^* = L$  also exists (Step 2). Finally we show that if  $n_0^L > x$  an additional mixed strategy equilibrium exists but it is also Coasian (Step 3).

Step 1: It is known that Pacman is always a SPE and it is supported by Markovian strategies. Let  $p^1 \ge L$  be the lowest equilibrium price when  $h(t) = (1, n_0^L)$ . The *H*-buyer then accepts with probability 1 any price lower than  $(1 - \delta)H + \delta p^1$ . Let  $w = (1 - \delta)H + \delta L$  be the price an *H*-buyer is willing to accept to avoid delay of one period if he expects the price in the next period to be *L*. The profit of selling at *L* and clear the market immediately is lower than selling to the *H*-buyer first at *w* before lowering the price to the remaining buyers if

$$L(n_0^L + 1) < w + \delta L n_0^L \Leftrightarrow n_0^L < x.$$

So  $p^1 > L$  if  $n_0^L < x$ . Also, for any  $p^1 \in (L, H)$  there always exists a  $p > p^1$  such that

$$H - p > \delta(H - p^1)$$

which the *H*-buyer will also accept with probability 1. By the definition of  $p^1$ , this implies that  $p^1 \notin (L, H)$ . Therefore  $p^1 = H$  if  $n_0^L < x$  and the unique SPE is the Pacman outcome.

Step 2: Suppose now that  $n_0^L \ge x$ . The following Markovian strategies form a SE: in the relevant states the seller charges L and each buyer i accepts any price  $p_t$  such that

$$p_t \le (1 - \delta)v(i) + \delta L$$

and refuses higher prices. By the one deviation principle buyers do not wish to deviate as the lowest price they can hope to get is L (see Lemma 1). The seller also does not wish to use his best deviation, to charge a price of w, as when  $n_0^L \ge x$  the equilibrium profit is higher than the discriminating profit, i.e.  $L(n_0^L + 1) \ge w + \delta L n_0^L$ .

Step 3: In a SE the seller cannot set prices that are accepted with probability zero. If the *H*-buyer was to accept a price  $p_t > L$  with a probability less than one he would need to be indifferent between accepting or rejecting that price. But then there always exists a slightly lower price that is accepted with probability one and which gives the seller a higher profit. So only prices that are accepted with probability one can be part of a SE.

Suppose that some price  $p \in (L, H)$  is part of the seller's equilibrium price strategy but that L does not belong to the support of the seller's strategy. By the argument of step 1 we have that  $p \in (L, H)$  can not be part of a SE when  $h(t) = (1, n_0^L)$ .

Suppose now that some price  $p \in (L, H)$  is part of the seller's equilibrium price strategy but L is now part of the seller's strategy support. Then it must be that the seller is indifferent between charging p and L, given the H-buyers response to accept either with probability 1. The buyer also needs to be indifferent between accepting and rejecting that p, given that the seller charges the price L with probability q. So in a mixed strategy equilibrium we must have that

$$(n_0^L + 1)L = p + \delta n_0^L L$$
 and  $H - p = \delta(q(H - L) + (1 - q)(H - p))$ 

or

$$p = (1 - \delta)n_0^L L + L$$
 and  $q = \frac{H - L - (1 - \delta)n_0^L L}{\delta n_0^L L}$ .

Note that  $q \in (0,1)$  if  $n_0^L > x$  and q = 1 if  $n_0^L = x$  (thus degenerate). So for  $n_0^L \ge x$  the strategies where the seller sets the price L with probability q and p with 1 - q, and the H-buyer accepts with probability 1 any price lower or equal to p (and rejects higher prices) form a third and last SE of the game. Note that this third SE is Coasian and real-time efficient.

**Proof of Proposition 1.** Here we consider the case where  $h(t) = (2, n_0^L)$ . We first show that the Pacman outcome is the unique SPE of G, and therefore SE, if  $n_0^L < x$  (Step 1).

Then we study additional SE of G with pure strategy pricing when  $n_0^L \ge x$  (Step 2 and Step 3). Finally we prove a folk-theorem on the seller's profits for  $n_0^L \ge x$  (Step 4).

Step 1: When  $n_0^L < x$  the unique equilibrium price when  $h(t) = (1, n_0^L)$  is H. H-buyers should then accept with probability one the price  $\underline{p'} \ge L$ , the lowest equilibrium price when  $h(t) = (2, n_0^L)$ , and also accept with probability 1 any price lower than  $(1 - \delta)H + \delta \underline{p'} > \underline{p'}$ . It follows that, for  $\delta$  sufficiently close to 1, the H-buyers should accept with probability one any price below H, and the profit of pricing at L and clear the market immediately is always lower than selling to the H-buyers first at H and to the L-buyers only in the next period. Therefore the Pacman outcome is the unique SPE of G conditional on the price being H when  $h(t) = (1, n_0^L)$ , and it is therefore the unique SPE of G if  $n_0^L < x$ .

Step 2: From Lemma 2, when  $n_0^L \ge x$  there is, in addition to the Pacman, a unique SE with pure strategy pricing when  $h(t) = (1, n_0^L)$  and its equilibrium price is L. We first show that the following strategies form a SE equilibrium of  $G(2, n_0^L)$  conditional on  $p_h^* = L$ when  $h(t) = (1, n_0^L)$ : Let  $p_h^*$  in state  $h(t) = (2, n_0^L)$  be given by (1) and for each H-buyer the probability that  $a_h^{i*} = 1$  be given by

$$q^{*}(p_{t}) \begin{cases} = 1 \text{ if } p_{t} \leq w = (1 - \delta)H + \delta L \\\\ = \frac{(H - p_{t}) - \delta(H - p_{h}^{*})}{\delta(p_{h}^{*} - L)} \in (0, 1) \text{ if } p_{t} \in (w, (1 - \delta)H + \delta p_{h}^{*}] \\\\ = 0 \text{ if } p_{t} > (1 - \delta)H + \delta p_{h}^{*} \end{cases}$$

If v(i) = L then  $a_h^{i*} = 1$  if  $p_t \leq L$  and 0 otherwise.

By the one period deviation principle we have that a buyer cannot improve his expected utility by changing his strategy for one period given the strategies of the other players. It remains to check that the seller has no incentive to charge a price higher than w, which leads to a probability of acceptance lower than 1 (it is simple to check that the optimal prices in the set [L, w] are either L or w).

If  $n_0^L > 2x$  then in (1) we have that  $p_h^* = L$  and so this deviation results in no sales and it is therefore unprofitable to charge  $p_t > w$ —and it is also better to charge L rather than w. If  $x \le n_0^L \le 2x$  then  $p_h^* = w$  and we need to look at the profit of charging  $p_t$  for one period which is

$$q^{*}(p_{t})^{2}(2p_{t}+\delta n_{0}^{L}L)+2(1-q^{*}(p_{t}))q^{*}(p_{t})(p_{t}+\delta(1+n_{0}^{L}L))+(1-q^{*}(p_{t}))^{2}\delta(2w+\delta n_{0}^{L}L).$$

Differentiating twice with respect to  $p_t$  we find that it is convex for  $p \ge w$  and  $\delta$  close to 1. Since the profit evaluated at  $p_t = (1 - \delta)H + \delta w$  is lower than at  $p_t = w$ , a deviation to any price  $p_t > w$  is unprofitable.

Step 3. We now check that there are no other SE with pure strategy pricing conditional on the seller setting L when  $h(t) = (1, n_0^L)$ . Suppose that in state  $h(t) = (2, n_0^L)$  there exists such equilibrium price  $\hat{p} \in (w, H]$ . The probability of a typical H-buyer i accepting an offer  $p_t$  at t conditional on reaching that period with  $h(t) = (2, n_0^L)$  is denoted by  $q^i(p_t)$ . In a mixed strategy equilibrium each buyer must be indifferent between purchasing today or waiting to see if the other buyer purchases first and purchase tomorrow. H-buyer i's best response function is then

$$q^{i}(p_{t}) = 1 \text{ if } H - p_{t} > 
 q^{i}(p_{t}) \in [0, 1] \text{ if } H - p_{t} = 
 q^{i}(p_{t}) = 0 \text{ if } H - p_{t} < 
 }
 
$$\delta \left[ q^{j}(p_{t})(H - L) + (1 - q^{j}(p_{t}))(H - \hat{p}) \right].$$$$

For a given  $\hat{p}$  there is a unique symmetric best response, which is

$$q^{i}(p_{t}) = q^{j}(p_{t}) = q(p_{t}) \begin{cases} = 1 \text{ if } p_{t} < w \\ = \frac{(H-p_{t})-\delta(H-\widehat{p})}{\delta(\widehat{p}-L)} \in (0,1) \text{ if } p_{t} \in [w,(1-\delta)H + \delta\widehat{p}] \\ = 0 \text{ if } p_{t} > (1-\delta)H + \delta\widehat{p} \end{cases}$$
(2)

If the seller charges  $p_t = \hat{p} \in (w, (1 - \delta)H + \delta \hat{p})$  in state  $h(t) = (2, n_0^L)$  the *H*-buyers acceptance probability in each period is

$$q(\hat{p}) = \frac{(1-\delta)(H-\hat{p})}{\delta(\hat{p}-L)},\tag{3}$$

which is decreasing in  $\delta$  and close to zero for  $\delta$  close to one. The seller's expected payoff when he offers a price  $p_t \in (w, (1 - \delta)H + \delta \hat{p})$  is

$$\mu^{m}(\mathbf{s}) \equiv q(p_{t})^{2}(2p_{t} + \delta n_{0}^{L}L) + 2q(p_{t})(1 - q(p_{t}))(p_{t} + \delta(n_{0}^{L} + 1)L) + (1 - q(p_{t}))^{2}\delta\mu^{m}(\widehat{p}),$$

With (2), we find that for all  $\hat{p} \in (w, (1-\delta)H + \delta \hat{p})$ 

$$\lim_{\delta \to 1} \left. \frac{d\mu^m(\mathbf{s})}{dp_t} \right|_{p_t = \widehat{p}} = -\frac{2}{\widehat{p} - L} \left( 1 - \frac{2(H - \widehat{p})}{2H - \widehat{p} - L} \right) \left[ \widehat{p} + (n_0^L + 1)L \right] < 0 \text{ as } \frac{2(H - \widehat{p})}{2H - \widehat{p} - L} < 1.$$

Therefore it is always optimal to undercut any price  $\hat{p} \in (w, H]$ , hence no such price can be part of a SE with pure strategy pricing of G conditional on the price being L when  $h(t) = (1, n_0^L)$ . The reason is that when buyers' strategies are stationary the seller can increase the acceptance rate at t by offering a slightly lower price at t without affecting future play. He then i) loses the profit on buyers who accept  $p_t$  but ii) gains the additional interest on the sales to all remaining buyers that are expected to be made earlier. As  $\delta \to 1$  the effect i), given by the negative of the partial derivative of  $\mu^m(\mathbf{s})$  with respect to  $p_t$ , converges to zero but the effect ii), given by  $-\frac{d\mu^m(\mathbf{s})}{dq(p_t)}\frac{dq(p_t)}{dp_t}$ , remains strictly positive.

Step 4. We now show by construction that the existence of the Coasian SE derived in Step 2 can sustain additional SPE with pure strategy pricing. Take the following strategies: The seller charges some  $\hat{p} \in ((1 - \delta)n_0^L L + L, H)$  if t = 0 or if  $p_{t'} = \hat{p}$  for all t' < t and  $n_t^H \neq 0$ , and otherwise use the strategy of the SE outlined in step 2. The *H*-buyers accept  $p_t = \hat{p}$  if  $p_{t'} = \hat{p}$  for all  $t' \leq t$ . Otherwise they play the strategies of the SE derived in Step 2. *L*-buyers accept any price  $p_t \leq L$  and refuse it otherwise.

We use the one period deviation principle to show that these strategies form a SPE of G for  $\delta$  close to 1. The H-buyers would not want to refuse  $\hat{p}$  when it is offered as they expect the price to be  $\hat{p}$  in the future as well if either of them does not accept. On the other hand the seller by deviating can make at most  $(n_0^L + 1)L$  when h(t) = $(1, n_0^L)$  and max  $\{2w + \delta n_0^L, (n_0^L + 2)L\}$  when  $h(t) = (2, n_0^L)$  (the profit it would make in the continuation of the game, which can be checked to be larger than deviations to any alternative price when the buyers expect the continuation game to be the Coasian SE). Both deviations are unprofitable if  $\hat{p} > (1 - \delta)n_0^L L + L$  (from the comparison with the profits made if the seller plays its equilibrium strategy). Therefore for  $\delta$  sufficiently close to 1 any price  $\hat{p} \in ((1 - \delta)n_0^L L + L, H)$  can be sustained in a SPE as the opening price of a SPE of G, which is immediately accepted by both H-buyers and the price then drops to L to clear the market. In these SPE the seller makes  $2\hat{p} + \delta n_0^L L$ . Since the Pacman and the Coasian outcomes are also SPE, as  $\delta \to 1$  any profit between  $(n_0^L + 2)L$  and  $2H + n_0^L L$  can be sustained in a SPE of G.

**Proof of Proposition 2.** Step 1: When  $n_0^L \ge x$  and  $\delta$  is close to 1, the Coasian outcome described in Proposition 1 is a SE of the subgame  $h(t) = (2, n_0^L)$  which gives the seller a profit which is certain of max  $\{(n_0^L + 2)L, 2w + \delta n_0^L L\}$ . This outcome can be used as a punishment if the seller charges a price  $p_t \neq \hat{p}$  when  $h(t) = (2, n_0^L)$ . Therefore the seller adheres to the price path described in this proposition if the assured profit above is less than the resulting equilibrium profit, which is derived below.

Divide  $q(\hat{p})$  from (3) by  $\triangle$  and, as  $\delta = e^{-\rho \triangle}$ , we have

$$\lim_{\Delta \to 0} \frac{q(\widehat{p})}{\Delta} = \lim_{\Delta \to 0} \frac{(H - \widehat{p})}{(\widehat{p} - L)} (\frac{1 - e^{-\rho\Delta}}{\Delta (e^{-\rho\Delta})}) = \rho(H - \widehat{p})/(\widehat{p} - L).$$

That is, in the limit each *H*-buyer's mixed strategy over purchasing dates is characterized by an exponential distribution with parameter  $\rho(H-\hat{p})/(\hat{p}-L)$ . In addition when the seller always charges the price  $\hat{p}$  when  $h(t) = (2, n_0^L)$ , the expected equilibrium profit, denoted by  $\mu^m(\mathbf{s}^*)$ , is the solution to

$$\mu^{m}(\mathbf{s}^{*}) = q(\hat{p})^{2} \left[ 2\hat{p} + \delta n_{0}^{L}L \right] + 2q(\hat{p})(1 - q(\hat{p})) \left[ \hat{p} + \delta(n_{0}^{L} + 1)L \right] + (1 - q(\hat{p}))^{2} \delta \mu^{m}(s^{*})$$

or

$$\mu^{m}(\mathbf{s}^{*}) = \frac{1}{q(\hat{p})(2-q(\hat{p}))} \left[ q(\hat{p})^{2} \left[ 2\hat{p} + \delta n_{0}^{L}L \right] + q(\hat{p})(1-q(\hat{p})) \left[ \hat{p} + \delta(n_{0}^{L}+1)L \right] \right]$$

In the limit we have

$$\lim_{\delta \to 1} \mu^m(\mathbf{s}^*) = \alpha(\widehat{p}) \cdot \left[\widehat{p} + (n_0^L + 1)L\right] \text{ where } \alpha(\widehat{p}) = \frac{2(H - \widehat{p})}{2H - \widehat{p} - L} \in (0, 1) \text{ for all } \widehat{p} \in (L, H).$$

So the seller's expected profit is a share  $\alpha(\hat{p})$  of the profit he would get in the case one *H*-buyer accepts  $\hat{p}$  immediately.

Step 2: For  $\delta$  arbitrarily close to 1, the deviating profit  $(n_0^L + 2)L$  is smaller than the expected equilibrium profit  $\alpha(\hat{p}) \cdot \left[\hat{p} + (n_0^L + 1)L\right]$  if  $\hat{p} \in \left(L, H - \frac{n_0^L + 2}{2}L\right]$ . Also  $H - \frac{n_0^L + 2}{2}L > L$  if  $n_0^L < 2(x-1)$ , and  $n_0^L \ge x$  ensures that the Coasian outcome in the punishment path is credible. Therefore when  $x \le n_0^L < 2(x-1)$  there exists a set of prices  $\hat{P} = \left(L, L(x - n_0^L/2)\right)$  that sustain the equilibria of Proposition 2. To see that those outcomes are real-time

inefficient take the profit derived above and, as in equilibrium the H buyers are indifferent between accepting and rejecting the offers, we also have that  $\mu^i(s^*) = H - \hat{p}$  if v(i) = Hand  $\mu^i(\mathbf{s}^*) = 0$  if v(i) = L. Simplifying we find

$$\lim_{\delta \to 1} \left[ \sum_{i \in N} \mu^i(\mathbf{s}^*) + \mu^m(\mathbf{s}^*) \right] = \alpha(\widehat{p})(2H + n_0^L L)$$

which is less than  $\sum_{i\in N} v(i)$  as  $\alpha(\widehat{p}) < 1$  for all  $\widehat{p} \in (L, H)$ .

#### References

Abreu, D. and Gul, F. "Bargaining and Reputation." Econometrica, Vol. 68 (2000), pp. 85-117.

Ausubel, L.M. and Deneckere, R.J. "Reputation in Bargaining and Durable Goods Monopoly." Econometrica, Vol. 57 (1989), pp. 511–531.

Ausubel, L.M. and Deneckere, R.J. "Durable Goods Monopoly with Incomplete Information." Review of Economic Studies, Vol. 59 (1992), pp. 795–812.

Bagnoli, M., Salant, S.W., and Swierzbinski, J.E. "Durable-Goods Monopoly with Discrete Demand." Journal of Political Economy, Vol. 97 (1989), pp. 1459–1478.

Biehl, A. "Durable-Goods Monopoly with Stochastic Values." Rand Journal of Economics, Vol 32 (2001), pp. 565–577.

Bulow, J. "Durable Goods Monopolists." Journal of Political Economy, Vol 90 (1982), pp. 314-332.

Bulow, J., Klemperer, P. "The Generalized War of Attrition." American Economic Review,Vol. 89 (1999), pp. 175-189

Coase, R. "Durability and Monopoly." Journal of Law and Economics, Vol. 15 (1972), pp. 143-149.

Deneckere, R. and Liang, M. "Imperfect Durability and the Coase Conjecture." Rand Journal of Economics, Vol 39 (2008) pp. 1-19.

von der Fehr, N.M. and Kuhn, K.U. "Coase versus Pacman: Who Eats Whom in the Durable-Goods Monopoly?" Journal of Political Economy, Vol. 103 (1995), pp. 785-812.

Fudenberg D., Levine D., Tirole J. "Infinite Horizon Models of Bargaining with One-Sided Incomplete Information." In: Roth A., editor. Game Theoretic Models of Bargaining. Cambridge: Cambridge University Press; (1985). Gul, F., Sonnenschein, H., and Wilson, R. "Foundations of Dynamic Monopoly and the Coase Conjecture." Journal of Economic Theory, Vol. 39 (1986), pp. 155–190.

Haigh, J. and Cannings, C. "The n-Person War of Attrition." Acta Applicandae Mathmaticae, Vol 14 (1989), pp. 59-74.

Inderst, R. "Bargaining with a Possibly Committed Seller." Review of Economic Dynamics, Vol 8 (2005), pp. 927-944.

Kapur, S. "Markov Perfect Equilibria in an n-player War of Attrition." Economics Letters, Vol 47 (1995), pp. 149-154.

Kim, K. "The Coase Conjecture with Incomplete Information on the Monopolist's Commitment." Theoretical Economics, Vol 4 (2009), pp. 17-44.

McAfee, P and Wiseman, T. "Capacity Choice Counters the Coase Conjecture." Review of Economic Studies, Vol. 75 (2008), pp. 317-332.

Sobel, J. "Durable Goods Monopoly with Entry of New Consumers." Econometrica, Vol. 59 (1991) pp. 1455-85.