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Energy saving technical progress and optimal capital stock: the role of embodiment

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Abstract

We study optimal capital accumulation at the firm level when technical progress is energy saving. Energy and capital are complementary. First we solve a benchmark case with disembodied technical progress. Then, we turn to the model with embodiment. We characterize the optimal replacement of obsolete capital, and the optimal capital stock. The latter is shown to be lower under embodiment compared to the benchmark case. Moreover, we demonstrate that a rising energy price has two opposite effects on the optimal capital stock under embodiment: the traditional direct negative effects, but also an indirect positive effect via the optimal scrapping rule. Nevertheless, the optimal capital stock is shown to remain a decreasing function of the energy cost.

JEL classifications: E22; E32; O40; C63

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1. Introduction

It is widely admitted that rising oil prices have a negative impact on economic activity. Indeed, eight of the nine recessions experienced by the US economy after the World War II (until the early 1990s) were preceded by an increase in the oil

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price (see Brown and Yucel, 2001, for a survey, and Hamilton, 1983, for a seminal inspection into how the energy cost affected the US economy over this period). Obviously, this argument is not correct in the opposite direction, as the declining oil prices in the mid-1980s did not induce any expansion for example. However, the inverse relationship between the oil price and economic activity when the former is rising sounds as a robust empirical regularity. It suggested a number of theoretical contributions, especially after the first oil shock in 1973, which caused a dramatic slowdown in the economic activity of the major industrialized countries.

There are several explanations of the inverse relationship between oil prices and economic activity (see again Brown and Yucel, 2001, for a survey). Some invoke income transfers from oil importing countries to oil exporting countries; others are based on the monetary policy implemented after the oil shocks, etc. The most known (and accepted) explanation relies on a classic supply side effect: rising oil prices are indicative of the reduced availability of basic inputs to production. This concerns the energy input itself but also and specially, the capital input as advocated by Baily (1981). In particular, Baily argued that the productivity slowdown experienced by the US economy and the other industrialized countries after the first oil shock might well be due to a reduction in the utilization rate of capital, namely in the decrease of the effective stock of capital.

The keywords, said Baily, are embodied technological change, obsolescence of capital goods and the energy cost. Technical advances are typically embodied in the capital goods, implying that investment is the unique channel through which these innovations could be incorporated into the productive sectors. As a corollary, the old capital goods get less and less efficient over time, which might well induce the firms to scrap them (obsolescence). Therefore, the implications of embodied technical change are extremely different from those of the typical neutral and *disembodied* technological progress specifications adopted in the neoclassical theory. According to Baily, embodiment is behind the productivity slowdown. The rising energy cost following the first oil shock caused a massive capital obsolescence and a subsequent decline in capital services: '...Energy-inefficient vintages of capital will be utilized less intensively and scrapped earlier following a rise in energy prices'. Robert Gordon (1981), after recognizing that Baily's hypothesis is indeed highly attractive, pointed at the difficulty of its empirical validation in the macroeconomy (as measuring the utilization rate is rather hard for certain sectors, like the non-farm non-manufacturing sectors) and reported that in any case, it does not seem to be supported systematically by the evidence available from certain energy-consuming industries like the airline industry.

Our paper is devoted to the study of the supply side effect depicted above in the presence of **energy saving** technological progress. Indeed, there are two major departures with respect to Baily's setting. In the latter, obsolescence is simply modelled through a decreasing effective output (at a given constant rate) as capital ages, and there is no explicit scrapping decision (of the oldest capital goods). In our model, the scrapping decision is endogenous, and since we assume complementarity between capital and energy inputs, finite scrapping time is indeed optimal. Secondly, in Baily's set-up, embodied technological progress makes capital goods less productive over time while in our model, technological progress is energy saving. Obviously, embodied technological progress may work in both directions, but as far as the energy-saving characteristic is accounted for, the implications of a more costly energy on optimal capital accumulation are naturally more complex. We will carefully investigate how the presence of an energy saving embodied technical change affects the optimal capital accumulation decisions at the firm level.

There is a growing evidence that energy-saving technological progress has been significant in the last two decades. In a recent contribution, Newell et al. (1999) studied whether the increase in the energy cost in the recent years induces energy saving innovations in the USA. Their conclusion is neat: the induced innovation hypothesis is more than plausible. More recently, Kuper and Soest (2002) found in a panel of sectors of the Dutch economy that energy saving technical progress is particularly significant in periods preceded by high and rising energy prices, while the pace of this form of technical change is definitely much slower in periods of low energy prices. Overall, the energy saving nature of technical progress is increasingly becoming a key descriptive feature of many innovations that have been taking place in the manufacturing and transport sectors in the recent years. Naturally, a more rigorous specification of this induced innovation mechanism would require endogenizing technical progress in terms of the energy price. Since we focus on capital accumulation at the firm level, technological progress is exogenous in our model. Nonetheless, we do model its energy saving characteristic through the embodiment assumption.

The embodiment of technological progress in capital goods is introduced via a vintage capital technology in line with the specifications adopted by Solow et al. (1966), and more recently by Malcomson (1975), Van Hilten (1991), Boucekkine et al. (1996, 1997, 1998). In addition to capital and labor, production involves energy expenditures. Vintage capital models with energy as an input have been intensively used in the late 1970s by some well-known US economists confronted with the productivity slowdown puzzle. Indeed, Baily (1981) himself (see also Shoven and Slepian, 1978) uses a vintage capital model with exogenous obsolescence rules. As argued above, we allow instead for an endogenous determination of the scrapping time of old capital goods, and this is likely to produce some very different economic mechanisms. Indeed, we show that an increase in the energy price level decreases the scrapping age, and the resulting lower scrapping time induces a rising, and not a falling, optimal effective capital stock for our optimal scrapping condition to hold even if the direct effect of the energy price level prevails: we show that in our model with complementary capital and energy and a rise in energy price generates a decrease in the effective capital stock.

In order to clearly show the implications of the embodiment of technological progress, we first analyze the optimal capital accumulation decision in the presence of a disembodied energy saving technical progress, i.e. when technical progress affects the whole capital stock independently of its age distribution. Explicitly comparative exercises between the embodiment case and the disembodiment benchmark will be conducted along the way. The paper is organized as follows. The next section analyzes the properties of the benchmark model with disembodied energy saving technical progress. The third section is devoted to study the counter-part model with energy saving embodied technical progress. The optimal scrapping rule is first derived; then, the determinants of the optimal effective capital stock are studied in detail with reference to Baily's work and a thorough comparison with the benchmark case. Section 4 concludes.

2. Optimal capital accumulation under disembodied technological progress

As a benchmark, we first consider that technical progress is purely disembodied. We consider a standard monopolistic competition economy (cf. Dixit and Stiglitz, 1977 or the production side of Boucekkine et al., 1996 for a vintage capital growth model) in which the firm has to solve the following problem:

$$\max\int_0^{\infty}\left[P(t)Q(t)-Pe(t)\ \ E(t)-w(t)L(t)-k(t)I(t)\right]\mathrm{e}^{-rt}\mathrm{d}t$$

subject to:

$$P(t)=bQ(t)^{-\theta}\quad \text{with}\ \ 0<\theta<1 \tag{1}$$

$$Q(t)=AK(t)^{\beta}L(t)^{1-\beta} \tag{2}$$

$$dK(t)=I(t)\quad \mathrm{d}t \tag{3}$$

$$Pe(t)=\overline{Pe}\ \ \mathrm{e}^{\mu t} \tag{4}$$

$$E(t)=K(t)\mathrm{e}^{-\gamma t} \tag{5}$$

$$w(t)=\bar{w}\ \ \mathrm{e}^{\varepsilon t} \tag{6}$$

$$I(t)\ \text{known for}\ t<0$$

$P(t)$ is the market price of the goods produced by the firm, $Q(t)$ is the production, the demand price elasticity is $(-1/\theta)$, $K(t)$ is capital, $L(t)$ is labor, $E(t)$ stands for the energy use and $I(t)$ is investment; $w(t)$ is the wage rate, $Pe(t)$ is energy price and $k(t)$ is the purchase cost of capital; r is the discount rate, μ is the energy price rate of growth, and $\gamma>0$ represents the rate of energy saving technical progress. We assume that there is no physical depreciation. Moreover, we assume that $\mu<r$ and $\gamma<r$. If $\mu>r$, the firm would have an incentive to infinitely get into debt to buy an infinite amount of energy. $\gamma<r$ is a standard assumption in the exogenous growth literature since it allows to have a bounded objective function.

The Cobb–Douglas production function exhibits constant returns to scale but there exists operating costs whose size depends on the energy requirement of the capital: to any capital use $K(t)$ corresponds a given energy requirement $K(t)\mathrm{e}^{-\gamma t}$.

Such a complementarity is assumed in order to be consistent with the results of several studies showing that capital and energy are complements (see for instance Hudson and Jorgenson, 1975; Berndt and Wood, 1974).

Technical progress is assumed to make machines becoming less energy consuming over time. In the disembodied case, the capital goods become more and more energy saving over time, whatever their age. This is a rather unrealistic assumption, which will be relaxed in the next section. We assume that labor may be adjusted immediately and without any cost and this standard problem reduces to the following conditions for optimal inputs use:

$$L^*(t) = \left[\frac{A^{1-\theta} b(1-\beta)(1-\theta)}{\bar{w}}\right]^{\frac{1}{1-(1-\beta)(1-\theta)}} K(t)^{\frac{\beta(1-\theta)}{1-(1-\beta)(1-\theta)}} \tag{7}$$

$$K^{D*}(t) = \left[\frac{\alpha B}{\left(rk(t) - \dot{k}(t)\right) + \overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}}\right]^{\frac{1}{1-\alpha}} \tag{8}$$

with

$B = (A^{(1-\theta)} b)^{\frac{1}{1-(1-\beta)(1-\theta)}} \left[1-(1-\beta)(1-\theta)\right] \times \left[(1-\beta)(1-\theta)\right]^{\frac{(1-\beta)(1-\theta)}{1-(1-\beta)(1-\theta)}} \bar{w}^{\frac{(1-\beta)(1-\theta)}{(1-\beta)(1-\theta)-1}}$, and $\alpha = \left[\beta(1-\theta)\right] / \left[1-(1-\beta)\times(1-\theta)\right]$.
Note that $0<\alpha<1$.

The corresponding optimal investment may be written:

$$I^*(t) = -\frac{(\alpha B)^{\frac{1}{1-\alpha}}}{1-\alpha} (\mu-\gamma)\overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}\left[\overline{uc}\ \ \overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}\right]^{\frac{\alpha-2}{1-\alpha}}$$

assuming that the user cost of capital is constant and positive. Such an assumption will be needed for technical reasons when we will consider embodied technical progress in Section 3. Because we aim at comparing in a rigorous way the outcomes of the latter case with those of the disembodied model of the current section, we introduce this simplification here. Observe that as the user cost, $\overline{uc}$, is given by $\left(rk(t) - \dot{k}(t)\right) = \overline{uc} > 0$, we are indeed assuming that the price of capital $k(t)$ is the sum of an exponential e^{rt} and a constant. By construction, the user cost of capital variable has no trend. However, it does vary over time (see for example, Fig. 1 in Hasset and Hubbard, 1996, on the US case). Since our main results are derived for the long run (or permanent) regime, our assumption on the constancy of the user cost is innocuous. Without loss of generality, we also assume that the real cost of labor is constant: $w(t) = \bar{w}$. Under these simplifying assumptions, we are able to bring out the following analytical results.

- If $\gamma = \mu$, then $I^*(t) = 0$ and $K^{D*}(t)$ is constant. The behavior of the optimal capital stock with respect to the model parameters is such that:

$$\frac{\partial K^{D*}}{\partial \overline{Pe}} < 0 \quad \frac{\partial K^{D*}}{\partial \overline{\overline{uc}}} < 0 \quad \frac{\partial K^{D*}}{\partial \overline{w}} < 0$$

An increase in the energy price level decreases the optimal capital stock. This is due to the fact that the firm should invest until the marginal productivity of the capital equals the sum of the user cost of capital and the operating cost Pe. Note that such a result is not inconsistent with the interpretation proposed by Baily (1981) of the productivity slowdown observed in 1970s, namely that the lower growth rate of total factor productivity may well be attributed to a drop in the (optimal) capital stock as energy gets more expensive. However, no obsolescence scheme is so far involved in the story, and the results come from a direct operation cost effect. The remaining comparative statics exercises are completely standard. The user cost of capital and the real cost of labor negatively affect the optimal capital stock since a higher uc would require a higher marginal productivity of capital and a higher w would reduce the marginal productivity of capital.

- If $\gamma < \mu$, then $I^*(t) < 0$ and $K^{D*}(t)$ is decreasing with time. Moreover:

$$\lim_{t \to +\infty} K^{D*}(t) = 0 \quad \text{and} \quad \lim_{t \to +\infty} I^*(t) = 0$$

- If $\gamma > \mu$, then $I^*(t) > 0$ and $K^{D*}(t)$ is increasing with time. Moreover,

$$\lim_{t \to +\infty} K^{D*}(t) = \left[\frac{\alpha B}{\overline{\overline{uc}}}\right]^{\frac{1}{1-\alpha}} \text{ which is constant and } \lim_{t \to +\infty} I^*(t) = 0$$

Note the asymmetry of the results successively obtained for the cases $\gamma < \mu$ and $\gamma > \mu$. The optimal capital stock is produced once its marginal productivity is equal to the sum of the user cost and the operating cost. If $\gamma < \mu$ the operating cost increases indefinitely which leads to an optimal capital stock tending to zero. In the opposite case, namely if $\gamma > \mu$, this operating cost vanishes over time but the user cost is constant, and so the total cost of holding capital does not vanish. As a consequence, the optimal capital stock tends to a strictly positive constant. Note that the obtained limit capital stock is independent of the energy price's level and growth rate, which is precisely due to the fact that the operating cost vanishes asymptotically. We shall abstract from this trivial case hereafter, since it is definitely not useful regarding our main objective, namely the analysis of the relationship between optimal capital accumulation and the energy price in the presence of energy saving technological progress. We also omit the symmetric case $\gamma < \mu$ since it yields a zero capital stock asymptotically. We shall focus on the *balanced* case $\gamma = \mu$

where a well-behaved optimal capital stock exists in the disembodied case, and in the embodied case as it is demonstrated in Section 3.[1]

3. Optimal capital stock under embodied technological progress

We now consider that the technical progress is embodied in the new capital goods acquired by the firm. The firm's problem becomes:

$$\max \int_0^\infty \left[P(t)Q(t) - Pe(t)\ E(t) - w(t)L(t) - k(t)I(t)\right] \mathrm{e}^{-rt}\ \mathrm{d}t \tag{9}$$

subject to constraints taking embodiment into account:

$$P(t) = bQ(t)^{-\theta} \quad \text{with } \theta < 1 \tag{10}$$

$$Q(t) = AK(t)^{\beta} L(t)^{1-\beta} \tag{11}$$

$$K(t) = \int_{t-T(t)}^{t} I(z)\mathrm{d}z \tag{12}$$

$$Pe(t) = \overline{Pe}\ \mathrm{e}^{\mu t} \quad \text{with } \mu < r \tag{13}$$

$$E(t) = \int_{t-T(t)}^{t} I(z)\mathrm{e}^{-\gamma z}\ \mathrm{d}z \quad \text{with } \gamma < r \tag{14}$$

$$w(t) = \overline{w} \tag{15}$$

The unique additional variable with respect to the benchmark model is $T(t)$ which denotes the age of the oldest machine still in use at t or scrapping age. Also, the capital variable is now effective capital, since only active machines are taken into account in the definition of the capital stock. Note that only the new machines incorporate the latest technological advances, i.e. are more energy-saving than the machines acquired in the past. Such an assumption is consistent with Terborgh (1949) and Smith (1961) set-ups in which it is hypothesized that the operation cost of a machine is a decreasing function of its vintage.[2] However, the rate of technical progress γ enters linearly into their operation costs functions while it is exponential in our model.

It is not hard to see that the optimal labor used as a function of the amount of capital remains the same as in the previous section. The vintage structure does

[1] The assumption $\gamma = \mu$ can be justified in the context of intertemporal equilibrium model of optimal extraction of a non-renewable resources (see Epaulard and Pommeret, 2002, for a full analytical characterization of the dynamics in a stochastic set-up). In such a case, it is generally shown that the resource price evolves according to the same process as technological progress. We warmly thank an anonymous referee for suggesting this argument.

[2] On the contrary, Brems (1967) assumes a constant operation cost.

matter in capital accumulation decisions, investment and scrapping. By using the same definitions for B and α as in the previous section, and by noting $J(t)=T(t+J(t))$ the lifetime of a machine of vintage t, the problem may be transformed, following Malcomson (1975), into a more tractable one (Appendix A) and it then leads to the following first order conditions:

$$\int_t^{J(t)+t}\left\{B\alpha\left[K(\tau)\right]^{\alpha-1}-\overline{Pe}\ \mathrm{e}^{\mu\tau-\gamma t}\right\}\mathrm{e}^{-r(\tau-t)}\ \mathrm{d}\tau=k(t) \tag{16}$$

$$\alpha B\left[K(t)\right]^{\alpha-1}=\overline{Pe}\ \mathrm{e}^{-\gamma[t-T(t)]+\mu t} \tag{17}$$

Eq. (16) gives the optimal investment rule according to which the firm should invest at time t until the discounted marginal productivity during the whole lifetime of the capital acquired in t exactly compensates for both its discounted operation cost and its marginal purchase cost in t. Eq. (17) is the scrapping condition: It states that a machine should be scrapped as soon as its marginal productivity (which is the same for any machine whatever its age) no longer covers its operating cost (which rises with its age). Since the condition Eq. (16) must hold for any t, so must its derivative with respect to t:

$$-\left[\alpha BK(t)^{\alpha-1}-\overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}\right] + \int_t^{t+J(t)}[r\alpha BK(\tau)^{\alpha-1}-(r-\gamma)\overline{Pe}\ \mathrm{e}^{\mu\tau-\gamma t}]\mathrm{e}^{-r(\tau-t)}\mathrm{d}\tau=\dot{k}(t)$$

Using Eq. (17) and then Eq. (16), we obtain:

$$\Leftrightarrow -\left[\alpha BK(t)^{\alpha-1}-\overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}\right]+\frac{\gamma\overline{Pe}\ \mathrm{e}^{(\mu-\gamma)t}}{\mu-r}\left[\mathrm{e}^{(\mu-r)J(t)}-1\right] = \left(\dot{k}(t)-rk(t)\right)\Rightarrow\left[\mathrm{e}^{\gamma T(t)}-1\right]-\frac{\gamma}{\mu-r}\left[\mathrm{e}^{(\mu-r)J(t)}-1\right]=\left(rk(t)-\dot{k}(t)\right)\frac{\mathrm{e}^{(\gamma-\mu)t}}{\overline{Pe}} \tag{18}$$

Using the first order condition Eq. (17), one may deduce a characterization of the optimal capital stock as a function of the optimal scrapping age:

$$K^{E*}(t)=\left[\frac{\alpha B}{\overline{Pe}}\mathrm{e}^{\gamma[t-T^*(t)]}\mathrm{e}^{-\mu t}\right]^{\frac{1}{1-\alpha}} \tag{19}$$

The optimal scrapping age may be determined by going further into the model resolution, as explained just below. From now, and as announced in Section 2, we shall concentrate on the balanced case $\gamma=\mu$.[3]

[3] A treatment of the non-balanced cases is provided in the discussion paper version of this paper, DP 2001–2023 at IRES, Université catholique de Louvain. These trivial cases, as mentioned in the previous section, are of a very limited economic interest.

3.1. Optimal scrapping

If $\gamma=\mu$, $J^*(t)$ and $T^*(t)$ are then determined by the system:

$$T^*(t)=f(J^*(t))=\frac{1}{\gamma}\ln\left[1+\left(rk(t)-\dot{k}(t)\right)\frac{1}{\overline{Pe}}+\frac{\gamma}{\gamma-r}\left[e^{(\gamma-r)J^*(t)}-1\right]\right], \tag{20}$$

$$J^*(t)=T^*(t+J^*(t)), \tag{21}$$

where Eq. (20) may be derived from Eq. (18). As in the previous section, we assume that the user cost of capital is constant and positive: $(rk(t)-k(t))=\overline{uc}$. In fact Van Hilten (1991) has shown that such a condition has to be satisfied to allow the use of a fixed-point argument, which is crucial in the analytical characterization of optimal capital accumulation in the embodiment case.[4] It can be easily shown that since function $f(J^*(t))$ is strictly increasing and concave, with $f(0)>0$ and $f(J^*(t))$ admitting a finite limit when its argument goes to infinity, it must admit a unique strictly positive fixed-point. The fixed-point argument of Van Hilten (1991) follows: The forward-looking system Eqs. (20) and (21) has a unique strictly positive solution, which is precisely the fixed-point of function $f(\cdot)$. Therefore, the Terborgh–Smith result $T^*(t)=J^*(t)=T$ is also reproduced in our case with T given by Proposition 1 below:

$$e^{-rT}=\frac{\gamma-r}{\gamma}\left[1-e^{-\gamma T}\left(\frac{r}{r-\gamma}+\frac{\overline{uc}}{\overline{Pe}}\right)\right] \tag{22}$$

Proposition 1. The optimal scrapping age $T^*(t)$ as well as the optimal lifetime of the machine $J^*(t)$ are such that: $T^*(t)=J^*(t)=T \quad \forall t\geq 0$, with T being the fixed-point of the function $f(\cdot)$.

Some results concerning the behavior of the optimal scrapping age with respect to the model parameters may be derived from Eq. (22):

$$\frac{\partial T}{\partial \overline{Pe}}=-\frac{\overline{uc}}{\overline{Pe}^2\left[\gamma e^{\gamma T}(1-e^{-rT})\right]}<0,$$
$$\frac{\partial T}{\partial \overline{uc}}=\frac{1}{\overline{Pe}\left[\gamma e^{\gamma T}(1-e^{-rT})\right]}>0,$$

As suggested by Baily (1981), the higher the energy price level, the sooner a machine has to be scrapped. Moreover, the higher the user cost of this machine, the longer a machine has to be kept in order to be profitable. Nevertheless, both the rate of technical progress γ and the interest rate affect the optimal scrapping age in

[4] Terborgh (1949) and Smith (1961) assume that the price of capital is constant.

an ambiguous way. This is a standard characteristic of the vintage capital models (cf. Boucekkine et al., 1998). For example, an increase in r will raise the unit cost of capital $k(t)$, and decrease the discounting factor of the profits stemming from the use of a particular machine. A quick look at the optimal investment rule Eq. (16) is sufficient to understand that the resulting overall effect on the optimal lifetime is completely ambiguous.[5] Therefore, to summarize:

Proposition 2. In the balanced case $\gamma = \mu$, the optimal scrapping age is such that $T = T(uc, r, Pe, \gamma)$. It decreases with the energy price level and increases with respect to the user cost of capital. The effect on optimal scrapping of a change in the interest rate or in the rate of technical progress is ambiguous.

3.2. The nature of technical progress and optimal capital accumulation

The optimal capital stock (given by Eq. (19)) becomes:

$$K^{E*}(t) = \left[\frac{\alpha B}{Pe} e^{-\gamma T}\right]^{\frac{1}{1-\alpha}} = K^{E*} \tag{23}$$

Recall the results obtained in the disembodied technical progress model in this same balanced case $\mu = \gamma$. The optimal capital stock is constant whatever is the nature of technological progress. However, its size does depend on the latter characteristic. Indeed, when technical progress is embodied, the firms are likely to acquire more new machines (since they are increasingly efficient over time) but also to scrap old machines (possibly in a massive way). These two effects work in opposite directions and there is no a priori reason to believe that one effect will systematically dominate the other. Nevertheless, it can be shown (Appendix A) that the second effect, applying through the endogenous scrapping rule, always prevails, so that the optimal capital stock is lower when technical progress is embodied. This leads to Proposition 3:

Proposition 3. (i) The optimal stock of capital, as the optimal scrapping time, remains constant over time: $K^{E*}(t) = K^{E*} \ \ \forall t \geq 0$. (ii) The optimal capital stock is lower in the embodied case: $K^{E*} < K^{D*}$.

Our results have also some implications in terms of vintage capital theory. Indeed, they depart to some extent from those established for general equilibrium growth models (see Boucekkine et al., 1997; Caballero and Hammour, 1996, for example). First of all, in our set-up, as the production function is strictly concave with respect to (effective) capital, it is possible to define an optimal value for the stock of

[5] Obviously, we can always set some sufficient conditions on the parameters to get any desired comparative statics (see for example, Boucekkine et al., 1998). We do not do this here since we are only interested in the energy price variable.

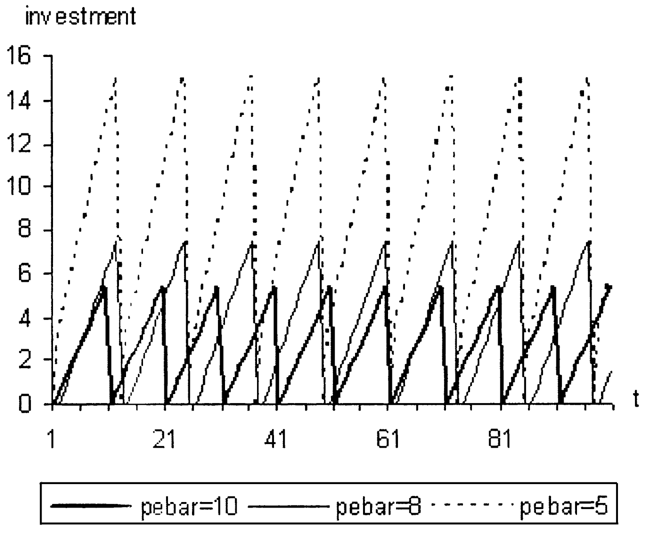

Fig. 1. Investment dynamics.

capital in contrast to the general equilibrium models mentioned above which typically use linear technologies. Second, the constant optimal scrapping rule combined with the constant optimal capital stock can obtain results in a periodic investment rule as illustrated in Fig. 1.[6] One can notice in this graph that the optimal scrapping age which provides the periodicity of investment is a decreasing function of the energy price, and that the optimal capital stock which gives the amplitude of the investment process is decreasing with the energy price as well (see Section 3.3). Periodicity of investment already appears in Boucekkine et al. (1997) but for completely different reasons. Indeed, as both optimal scrapping and capital stock are constant, investment is T-periodic by simple differentiation of the definition of the capital stock, $K^{E*} = \int_{t-T}^{t} I(z) \, \mathrm{d}z$, at the optimum. The same property is generated in the above mentioned paper thanks to the equilibrium condition in the labor market (with fixed labor supply), and to the constancy of the optimal scrapping rule. Naturally, since non-monotonic investment patterns are primarily observed at the firm level, our mechanism seems more relevant at least at this level.

It should be noted that the periodicity property derived above is by far different from the optimal investment rule in the balanced case $\mu = \gamma$ when technical progress is disembodied. In such a case, investment is done once for all. The ability of vintage models to generate such a markedly different behavior with respect to the basic neoclassical Jorgenson-like investment model is now very well known (see the papers mentioned just above). Our model allows in getting the same properties

[6] To construct this illustration, it has been assumed for each value of Pe that the firm has the optimal capital stock at time $t = 0$ and that the adjustment to this optimal value (before $t = 0$) is linear.

through different channels. In terms of investment patterns, we, therefore, get the expected differences. We now discuss the relationship between the energy price level and optimal capital accumulation under embodiment with reference to Baily's set-up and a comparison to the benchmark disembodied case.

3.3. Energy price level and optimal stock of capital

The behavior of the optimal capital stock with respect to the parameters of the model is as follows:

$$\begin{aligned}
&\frac{\partial K^{E*}}{\partial T} < 0\\
&\frac{\partial K^{E*}}{\partial \overline{Pe}} = \frac{\partial K^{E*}(T,\overline{Pe})}{\partial \overline{Pe}} + \frac{\partial K^{E*}(T,\overline{Pe})}{\partial T}\frac{\partial T}{\partial \overline{Pe}} < 0\\
&\frac{\partial K^{E*}}{\partial \overline{uc}} = \frac{\partial T}{\partial \overline{uc}} < 0\\
&\frac{\partial K^{E*}}{\partial \overline{w}} = \frac{\partial K^{E*}}{\partial B}\frac{\partial B}{\partial \overline{w}} < 0
\end{aligned} \tag{24}$$

Two main results are worth pointing out here:

i. We might think of a simple direct effect between the scrapping age and the optimal stock of capital: The longer machines are kept, the larger the optimal capital stock is. This is indeed consistent with the spirit of Baily's arguments. However, Eq. (24) shows that there exists in fact a **negative** relationship between K^{E*} and a given scrapping age T! The underlying mechanism is the following. The higher the age of the operated machines, the bigger the operation cost associated with those machines, and thus the higher the marginal productivity required for all machines whatever their age, by the optimality condition Eq. (17). Since the production function has decreasing returns with respect to capital, a higher marginal productivity can only be achieved by lowering the stock of capital.
ii. Concerning $\frac{\partial K^{E*}}{\partial \overline{Pe}}$, the sign results from two opposite effects. In addition to the traditional direct negative effect (for a given optimal scrapping age, a higher initial energy price leads to a lower optimal capital stock), we have an effect with respect to the disembodiment case. Indeed, when the energy price is rising, the optimal scrapping age decreases (Proposition 2), thus leading to a higher optimal capital stock (Property (i) just above). So the overall effect on capital accumulation of a raising energy cost is ambiguous. The indirect effect is inherent to the endogenous nature of the scrapping rule.

Obviously, it does not arise when technical progress is disembodied, and it does not appear in Baily's vintage model neither since the obsolescence rule is exogenous in the latter. This indirect effect reduces the sensitivity of capital accumulation to change in the energy cost in comparison to the disembodiment case or to Baily's model. Could it be the case that this effect more than compensates the usual negative price effect? The answer is no. Developing a bit more of the algebra, one gets:

$$\frac{\partial K^{E*}}{\partial \overline{Pe}} = \frac{\alpha B \mathrm{e}^{-\gamma T}}{\overline{Pe}^2(1-\alpha)}\left(\frac{\alpha B \mathrm{e}^{-\gamma T}}{\overline{Pe}}\right)^{\frac{\alpha}{1-\alpha}}\left[-1-\gamma\overline{Pe}\frac{\partial T}{\partial \overline{Pe}}\right].$$

Denote by $C = \frac{\alpha B \mathrm{e}^{-\gamma T}}{\overline{Pe}^2(1-\alpha)} \times \left(\frac{\alpha B \mathrm{e}^{-\gamma T}}{\overline{Pe}}\right)^{\frac{\alpha}{1-\alpha}}, \quad C > 0$. The indirect (positive) effect comes from the term $-\gamma\overline{Pe}\frac{\partial T}{\partial \overline{Pe}}$. This term will dominate if and only if it is bigger than one. However, developing more of the algebra, it allows us to find that:

$$-\gamma\overline{Pe}\frac{\partial T}{\partial \overline{Pe}} = \frac{(r-\gamma)\frac{\overline{uc}}{\overline{Pe}}}{r(1-\mathrm{e}^{(\gamma-r)T}) + (r-\gamma)\frac{\overline{uc}}{\overline{Pe}}}.$$

Hence, the indirect effect is always dominated. Therefore, though the usual price effect is mitigated, it is not offset, which is the most reasonable outcome anyway.

Finally, note that the user cost of capital only affects the optimal capital stock through the optimal scrapping age (cf. Proposition 2 for the effect of these parameters on T). Also, observe that the wage rate negatively affects the optimal capital stock, which in turn reduces the optimal labor use (see Eq. (7)). Proposition 4 below sums up the main comparative statics properties of the optimal capital stock:

Proposition 4. The optimal capital stock is a decreasing function of the scrapping time. An increase in the energy price level has a direct negative effect on the optimal capital stock, and an indirect positive effect via the scrapping time. Though the negative effect prevails, the sensitivity of the optimal capital stock with respect to the energy price is likely to be lower in the embodiment case. Finally, the optimal capital stock decreases with the wage and the user cost of capital.

4. Conclusion

In this paper, we have proposed a vintage capital model at the firm level in which energy and capital are complements, the returns to (effective) capital are decreasing, and technological progress is energy saving. We study two versions of the model, with disembodied and with embodied technical progress. Several lessons can be brought out from our analysis.

Beside the increase in analytical difficulty, the vintage structure with endogenous scrapping induces some additional worthwhile mechanisms. Compared to the disembodiment case, the firms have one more control (scrapping). As the available capital goods get more and more efficient, they can decide to invest massively in the new vintages and to scrap a significant fraction of the old vintages at the same time. The overall effect on the optimal capital stock is a priori ambiguous. We show that the scrapping effect prevails, so that the optimal capital stock is lower in the embodiment case. Second, we show that the traditional supply side discussion around the inverse relationship between the energy cost and economic activity can be notably enriched if one manages to study properly the vintage effect. In particular, we identify an indirect positive effect of higher energy prices on optimal capital accumulation, which operates via endogenous scrapping. Though the latter effect cannot compensate the direct negative price effect as expected, it may explain why Baily's argument does not work so neatly in certain microeconomic cases (as outlined by Gordon, 1981).

Last but not least, this paper can be considered as a contribution to the vintage capital models literature. Indeed, it deals with optimal capital accumulation in a vintage capital partial equilibrium framework with a concave technology while the recent literature has focused on general equilibrium set-ups with linear preferences and technologies. As a consequence, we are able to define an optimal (effective) capital stock, and then to establish the periodicity of the investment paths at the interior solution of the firm's problem, a feature that typically comes from the labor market specifications in the general equilibrium related models. Since non-monotonic investment patterns are primarily observed at the firm level, our mechanism seems more relevant at least at this level.

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Appendix A: First order conditions associated with program (9)

The program (9) may be rewritten:

$$\max_{T(t),I(t)} \int_{0}^{t-\mathrm{d}t} \left\{ B\left[\int_{\tau-T(\tau)}^{\tau} I(z)\,\mathrm{d}z\right]^{\alpha} - Pe(\tau)\int_{\tau-T(\tau)}^{\tau} I(z)\mathrm{e}^{-\gamma z}\,\mathrm{d}z - k(\tau)I(\tau)\right\}\mathrm{e}^{-r\tau}\,\mathrm{d}\tau$$

$$+\int_{t}^{t+J(t)} \left\{ B\left[\int_{\tau-T(\tau)}^{\tau} I(z)\,\mathrm{d}z\right]^{\alpha} - Pe(\tau)\int_{\tau-T(\tau)}^{\tau} I(z)\mathrm{e}^{-\gamma z}\,\mathrm{d}z - k(\tau)I(\tau)\right\}\mathrm{e}^{-r(\tau-t)}\,\mathrm{d}\tau$$

$$+\int_{t+J(t)+\mathrm{d}t}^{\infty}\left\{B\left[\int_{\tau-T(\tau)}^{\tau} I(z)\,\mathrm{d}z\right]^{\alpha}\right.$$
$$\left.-Pe(\tau)\int_{\tau-T(\tau)}^{\tau} I(z)\mathrm{e}^{-\gamma z}\,\mathrm{d}z-k(\tau)I(\tau)\right\}\mathrm{e}^{-r(\tau-J(t)-t-\mathrm{d}t)}\,\mathrm{d}0\tau$$

$$=\max_{T(t),I(t)}\int_{t}^{t+J(t)}\left\{B\left[\int_{\tau-T(\tau)}^{\tau} I(z)\,\mathrm{d}z\right]^{\alpha}\right.$$
$$\left.-Pe(\tau)\int_{\tau-T(\tau)}^{\tau} I(z)\mathrm{e}^{-\gamma z}\,\mathrm{d}z-k(\tau)I(\tau)\right\}\mathrm{e}^{-r(\tau-t)}\mathrm{d}\tau$$

since the first and third integrals do not involve any control variable. The first order conditions are then:

$$k(t)=\int_{t}^{t+J(t)}\left\{B\alpha\left[\int_{\tau-T(\tau)}^{\tau} I(z)\,\mathrm{d}z\right]^{\alpha-1}-Pe(\tau)\mathrm{e}^{-\gamma t}\right\}\mathrm{e}^{-r(\tau-t)}\,\mathrm{d}\tau$$

and

$$\alpha B\left[\int_{t-T(t)}^{t} I(z)\,\mathrm{d}z\right]^{\alpha-1}=Pe(t)\mathrm{e}^{-\gamma[t-T(t)]}$$

from which Eqs. (16) and (17) in the text may easily be deduced.

Appendix B: Proof of $K^{E*}<K^{D*}$

First we, using both the expression Eqs. (23) and (8) which, respectively, give K^* and K^{D*}, it can be shown that

$$K^{E*}\geq K^{D*}\Leftrightarrow \mathrm{e}^{-\gamma T}\geq\frac{\overline{Pe}}{\overline{Pe}+\overline{uc}} \tag{25}$$

Second, the implicit expression for the optimal scrapping age provides some restriction for T:

Since it has been assumed that $r>\gamma$, we also have $\mathrm{e}^{-rT}<\mathrm{e}^{-\gamma T}$. Using Eq. (22), we then have the following inequality:

$$\frac{\gamma-r}{\gamma}\left[1-\mathrm{e}^{-\gamma T}\left(\frac{\gamma-r}{\gamma}+\frac{\overline{uc}}{\overline{Pe}}\right)\right]<\mathrm{e}^{-\gamma T}\quad\Leftrightarrow \mathrm{e}^{-\gamma T}<\frac{\overline{Pe}}{\overline{Pe}+\overline{uc}}$$

which contradicts Eq. (25). We, therefore, deduce that we always have $K^{E*}<K^{D*}$.

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