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THREE ESSAYS ON MONETARY POLICY AND HOUSEHOLD HETEROGENEITY

Meichtry Pascal

UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT D'ÉCONOMIE

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THÈSE DE DOCTORAT

présentée à la

Faculté des Hautes Études Commerciales
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pour l'obtention du grade de
Docteur ès Sciences Économiques,
mention « Économie politique »

par

Pascal MEICHTRY

Directeur de thèse
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Jury

Prof. Rafael Lalive, président
Prof. Kenza Benhima, experte interne
Prof. Ricardo Reis, expert externe

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I M P R I M A T U R

Sans se prononcer sur les opinions de l'auteur, la Faculté des Hautes Etudes Commerciales de l'Université de Lausanne autorise l'impression de la thèse de Monsieur Pascal MEICHTRY, titulaire d'un bachelor en économie et d'un master en économie de l'Université de Saint-Gall (HSG), en vue de l'obtention du grade de docteur ès Sciences économiques, mention « économie politique ».

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All revisions that I or committee members
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Introduction

The role of monetary policy in shaping the macroeconomy has been a topic of extensive research in economics for decades. Conventional monetary policy tools such as interest rate adjustments have been the standard way for central banks to control inflation and stabilize economic growth. However, the recent global financial crisis has led to a widespread use of unconventional monetary policies. Those include forward guidance and quantitative easing which emerged to address the limitations of conventional policy tools.

In recent years, policymakers and researchers have become increasingly interested in the distributional implications of monetary policy. Heterogeneous-agent models have emerged as a popular way to study the effects of various central bank policy tools on different groups of individuals in the economy. Furthermore, differences in information updating across agents have been identified as a critical factor for the transmission and impact of monetary policy.

Against this backdrop, this doctoral thesis investigates the effects of both conventional and unconventional monetary policies on the macroeconomy and inequality. By shedding light on these crucial topics, the thesis aims to contribute to the ongoing discussion on the role of monetary policy in promoting macroeconomic stability and reducing economic inequality.

The first chapter explores how heterogeneity and information rigidities on the part of households impact the transmission of conventional monetary policy to aggregate demand. It investigates the relative importance of amplification and dampening effects resulting from the interaction of these two frictions and their implications for the effectiveness of monetary policy in quantitative macroeconomic models. I provide empirical evidence for considerable differences in the frequency of information updating among households. I also show in an analytical model that the amplified effects of monetary policy shocks might be substantially driven by the presence of information frictions.

In the second chapter, my co-author Giacomo Mangiante and I study the distributional effects of conventional monetary policy and forward guidance and provide an analytical model to rationalize their differences. While both policies impact aggregate macroeconomic variables in similar ways, they have opposite effects on consumption inequality.

Our results show that the diverse response of the government to the two types of policy shocks is a key factor in driving these differences. We highlight the importance of fiscal adjustments in understanding the cyclical variations in consumption inequality and mitigating the adverse distributional effects of central bank policy tools.

The third chapter, co-authored with Cristiano Cantore, studies how central bank asset market operations and their interaction with household heterogeneity affect macroeconomic outcomes. We compare large-scale asset purchases to asset sales and highlight an asymmetry between their respective impacts. The state dependency of asset market operations implies that, when the economy is close to a liquidity trap, increasing the policy rate before unwinding quantitative easing minimizes the economic costs of normalizing monetary policy. We also show that combining household heterogeneity with state dependency can amplify the asymmetry between the aggregate effects of asset purchases and sales.

Overall, this doctoral thesis provides valuable insights for policymakers and researchers alike. To understand the macroeconomic and distributional effects of monetary policy, it is important to disentangle and analyze the impacts of conventional and unconventional policies using both empirical and theoretical approaches. Incorporating household heterogeneity and information frictions in standard macroeconomic models can further enhance our understanding of these topics. Given that these elements significantly affect the transmission of monetary policy, it appears essential to consider them in the design of optimal monetary policy frameworks.

Chapter 1

Sticky Information, Heterogeneity, and Aggregate Demand

1.1 Introduction

Traditional New Keynesian models often assume that households have full up-to-date information at any point in time and can be represented by a single rational-expectations agent. Relaxing these two assumptions has considerable implications for consumption and demand both at the individual household and the aggregate level.

Heterogeneity in terms of households' income, wealth, or consumption and saving decisions crucially shapes the transmission mechanism of monetary policy. One result of this literature is the appearance of an amplified response of economic aggregates to monetary policy relative to the standard representative-agent New Keynesian (RANK) economy (see, among others, Auclert, 2019; Bilbiie, 2018, 2020; Bilbiie, Känzig, & Surico, 2022; Debortoli & Galí, 2017). A core component to achieve such *amplification* is heterogeneity across agents in terms of the marginal propensity to consume (MPC) out of a transitory income shock. On the other hand, how agents form their expectations is still a much-debated question in macroeconomics. The assumption of full-information rational expectations (FIRE), according to which economic agents are entirely aware of the structure of the economy and can perfectly observe and use all available information at hand to form expectations, has been the gold standard for a long time. However, there is pervasive evidence of large information rigidities for a broad spectrum of economic agents (Coibion & Gorodnichenko, 2012, 2015). These frictions cause rational individuals to have only imperfect information about economic conditions and to often underreact in response to macroeconomic shocks. This leads to delayed responses and generates *dampening* at the aggregate level.

Against this backdrop, this paper studies how household heterogeneity and information rigidities impact the transmission of conventional monetary policy to aggregate consumption or, equivalently here, aggregate demand. In this context, I explore the relative importance of amplification and dampening arising from the interaction of these two frictions, and their implications for the effectiveness of monetary policy in quantitative models. The target value is the aggregate-demand multiplier, which measures the quantitative effect of a change in the current real interest rate on aggregate consumption.

For this purpose, I build a tractable two-agent New Keynesian (TANK) model with heterogeneity in household income based on Bilbiie (2008). Households are divided according to their participation in asset markets: a fraction of agents are able to smooth consumption by saving in state-contingent bonds ("savers"), while the remaining households have no assets and consume their entire disposable income in each period ("hand-to-mouth households"). I extend this setup by introducing a rigidity in the form of sticky information. Following Mankiw and Reis (2006, 2007), a portion of households can only occasionally update their information about the state of the economy. Due to these elements, I term the framework a sticky-information two-agent New Keynesian (SI-TANK) model.

My modeling choice is motivated twofold. On the one hand, the first part of the paper provides empirical evidence for information frictions in household expectations. Estimates of the relation between households' inflation forecast errors and their forecast revisions indicate considerable differences in the degree of information rigidity across the income distribution in the U.S. In particular, there is strong evidence for little information acquisition at low income levels. This result forms the basis for the analytical model and justifies the assumption regarding the information structure. On the other hand, to obtain models that are consistent with both empirical microeconomic and macroeconomic moments, researchers recently relied on combining heterogeneous households with information frictions (Auclert, Rognlie, & Straub, 2020; Carroll, Crawley, Slacalek, Tokuoka, & White, 2020; Pfäuti & Seyrich, 2022). Even though such existing frameworks are successful in matching fundamental evidence in the data, it is not always straightforward to isolate impact channels and analyze interdependencies in them. A common challenge when deviating from a RANK economy and FIRE is the handling of mathematically and computationally complex models. For that reason, this paper goes one step back and explores what a small-scale two-agent framework implicates for interactions between the mentioned frictions and about the mechanisms at play. In this regard, the concept of sticky information is an appealing and simple way to introduce information rigidity. It only calls for a single alternative assumption in the spirit of the well-known Calvo staggered pricing, while retaining the rationality assumption regarding agents.

In the first main part of the paper after presenting the model, I discuss its main properties. My focus is thereby at first on the initial response of aggregate demand in the period where a monetary policy shock is announced. The SI-TANK framework combines two important propagation characteristics of such a shock as outlined above: amplification and dampening. Within the context of the two-agent framework at hand, amplification means that the effect of a change in the real interest rate on aggregate demand (i.e., the aggregate-demand multiplier) is higher than in a RANK model and increases in the share of hand-to-mouth households in the economy. If that effect is lower, there is dampening.

Therefore, one key component of SI-TANK that changes the size of real effects is the presence of constrained households that live hand-to-mouth, so that the MPC out of their own income is one. This increases the aggregate MPC in the economy relative to RANK. A *force of amplification* then emerges if the income of hand-to-mouth agents reacts more than one-to-one to changes in aggregate income. This condition results in countercyclical income inequality and a reinforced demand response: after an unexpected interest rate cut that implies an initial increase in aggregate demand, hand-to-mouth agents become disproportionately richer, leading to declining inequality between unconstrained and constrained agents together with a further demand boost. The feedback from individual income back to aggregate income is precisely what eventually amplifies the real effects of monetary policy. It is in line with the mechanisms in Bilbiie (2018, 2020) or Bilbiie et al. (2022).

At the same time however, I show that the effects of monetary policy are to a certain extent *dampened* within SI-TANK due to the introduction of information rigidities. In general, there are different ways how to depart from the full-information component of FIRE.[1] The goal of this paper is to keep the analytics tractable and likewise the model comparable to the standard RANK model and the recent literature in this field, which is why I adapt the concept of sticky information as in Mankiw and Reis (2002, 2006, 2007) or earlier in Gabaix and Laibson (2002). Among the two household types in the model, only the consumption-smoothing, unconstrained savers are subject to this friction. In each period, a constant fraction of savers update their information about the state of the economy. Based on this, they optimally choose a consumption plan that is just revised at some unknown point in the future. Between two planning dates, the household does not obtain new information and its consumption follows the pre-determined path. As a result, information about economic conditions diffuses slowly through the population. The lag in perception generates a sluggish aggregate-consumption response after a monetary policy shock compared to RANK, because savers' consumption only adjusts slowly to the arrival of news.

[1] For a compact overview, see Coibion, Gorodnichenko, and Kamdar (2018).

The SI-TANK model combines both of the described features. Household heterogeneity can amplify the initial aggregate-demand response, whereas sticky information attenuates it. By analyzing the IS curve (or aggregate Euler equation), I find that the net propagation effect is largely determined by the two main model parameters: the share of hand-to-mouth agents and the degree of information stickiness. Somewhat less obvious and different from what previous authors have found, dampening might arise even if income inequality is countercyclical. Hence, the overall effect of a monetary policy shock on aggregate demand may still be attenuated although hand-to-mouth agents' income reacts more than proportionally to aggregate income changes – precisely in the case in which the share of constrained agents (and therefore the amplifying component of SI-TANK) is not high enough compared to the degree of information stickiness. On the other hand, to achieve overall amplification of monetary policy effects, income inequality must be *substantially* countercyclical for a standard calibration of the model.

The magnitudes of the main model parameters are critical for the absolute effects of monetary policy. In a next step I show that this changes when studying household heterogeneity and sticky information both in isolation and jointly, and relating the respective aggregate-demand multipliers to each other. Considering an unexpected one-time change in the real interest rate, I demonstrate that the propagation of monetary policy shocks is shaped by an asymmetric interaction of amplification and dampening – irrespective of the selected parameter values. Sticky information attenuates the aggregate-consumption response more when added to a standard representative-agent model instead of a two-agent economy. What is even more striking is that household heterogeneity has a larger relative impact in combination with sticky information. In other words, it becomes proportionately more influential. Both asymmetries arise from the fact that in a two-agent model the intertemporally optimizing savers alone are affected by the information friction, while this is not the case in an economy where all households are identical. Amplification, in contrast, always involves and works through both types of households.

It is well-known from the heterogeneous-agent literature that hand-to-mouth agents constitute the main element of amplification. On the other hand, my finding about the relative strength of heterogeneity in rigid-information setups indicates that amplification might substantially be driven by the presence of information frictions rather than by high-MPC agents alone. The paper therefore contributes to a better understanding of the sources of propagation effects in quantitative models with more than one agent. Furthermore, it points at the importance to differentiate individual frictions and their role.

The second main part of this paper is dedicated to solving the SI-TANK model analytically. Incorporating sticky information in standard macroeconomic models gives rise to an infinite number of lagged expectations and thus an infinite state space. An analytical solution is therefore usually complex or not even possible. To overcome these difficulties, I provide a novel, albeit simple, way to solve a wide range of sticky-information models analytically when one is interested in isolating the aggregate-demand side. It allows me to derive reduced-form expressions for output and inflation that only depend on the monetary policy shock and model parameters. These expressions can then be used to verify the findings obtained from analyzing the effects of the policy shock on the familiar three-equations system of the SI-TANK model.

To complete the analytical part, I simulate impulse responses to an unanticipated monetary policy shock. The graphical representation not only confirms the preceding results about the response of aggregate consumption and output on impact of the shock, but it also facilitates a discussion about the periods subsequent to the shock. Among others, the presence of non-updated savers generates a hump-shaped response as documented in the macroeconomic literature.

Related literature. This work contributes to the growing literature on the effectiveness of monetary policy in heterogeneous-agent models.[2] That field exposes how different assumptions and elements of such models affect the propagation of monetary policy shocks and thereby shape amplification and dampening effects. My analysis draws in particular on the analytical TANK literature that makes simplifying assumptions to identify the driving forces at work in richer models.

The main framework is based on Bilbiie (2008). He builds an analytical TANK model with two types of agents differing in their degree of participation in asset markets as described above. The implied heterogeneity in MPCs changes the sensitivity of aggregate demand to monetary policy and gives room for amplification with respect to RANK. Bilbiie (2020) emphasizes that the net propagation effect hinges on the elasticity of hand-to-mouth households' income to aggregate income: when it is above one, amplification arises; otherwise, there is dampening. While the SI-TANK model in this paper implicates comparable effects, the sufficient conditions are different and dampening might arise even if constrained agents react disproportionately to changes in aggregate income.

Second, I build on the large literature that explores deviations from FIRE, in particular about the assumption of sticky information originating from Gabaix and Laibson (2002) and Mankiw and Reis (2002, 2006, 2007). In the context of representative-agent models, sticky information has been first and foremost applied on the part of firms to study price dynamics.[3] The seminal work of Mankiw and Reis (2002) proposes it as an alternative way to model price setting. Related to this and more recently, Bacchetta, van Wincoop,

[2] See, among others, Acharya and Dogra (2020); Alves, Kaplan, Moll, and Violante (2020); Auclert (2019); Auclert et al. (2020); Bilbiie (2018, 2020); Bilbiie et al. (2022); Debortoli and Galí (2017); Werning (2015).

[3] See, for example, Chung, Herbst, and Kiley (2014); Coibion (2006); Dupor, Kitamura, and Tsuruga (2010); Dupor and Tsuruga (2005); Mankiw and Reis (2002); Trabandt (2007).

and Young (2022) use a similar Calvo type friction in the context of modeling portfolio decisions and as a way to achieve gradual portfolio adjustment. Other prominent papers incorporate the assumption of sticky information in a fully-fledged DSGE framework to match U.S. business cycle facts or study monetary policy (Mankiw & Reis, 2006, 2007; Reis, 2009a, 2009b). They assume that households, firms, and workers are all subject to inattention when taking decisions. Estimates for the U.S. and the Euro area unveil a different degree of information stickiness to be present in various markets (goods, labor, financial), most notably for consumers. Due to the recent advances in the context of heterogeneous-agent models and the revived interest in aggregate demand, however, it appears appropriate to focus on the implications of sticky information on exclusively the household side within those models.

The literature closest related to this work combines concepts of limited information with household heterogeneity, mostly to match or explain microeconomic and macroeconomic evidence in the data. Similar to this paper, Broer, Kohlhas, Mitman, and Schlafmann (2021) unveil systematic heterogeneity in the macroeconomic expectations of U.S. households. They try to rationalize this fact in a quantitative heterogeneous-agent New Keynesian (HANK) framework with dynamic information choice. Unlike them, I model information exogenously to focus on its interaction with the pre-determined degree of household heterogeneity and because the way households acquire information is only of second-order importance here. Pfäuti and Seyrich (2022) discuss amplification and dampening channels within a New Keynesian model with household heterogeneity and bounded rationality and study what the interaction of these two elements implies for the IS curve. The model here can be seen as a simplified version of theirs, with an even simpler information friction and without idiosyncratic risk. Instead of explaining empirical facts like they do, I derive sufficient conditions that determine the net propagation effect of monetary policy, focus on the asymmetric interplay of amplification and dampening, and discuss implications for theoretical modeling.

Another strand of this literature combines heterogeneity in household income with sticky expectations about the macroeconomy, assuming that households can perfectly observe their personal circumstances or idiosyncratic shocks, while they perceive information about macroeconomic variables or aggregate shocks only infrequently. Applying this in the context of an estimated HANK model, Auclert et al. (2020) achieve realistic MPCs out of a transitory income shock and, at the same time, reproduce the empirical fact that the response of macroeconomic aggregates to monetary policy shocks tends to be hump-shaped. Prior to this, the assumption of sticky expectations was used in Carroll et al. (2020) who succeed in matching aggregate-consumption dynamics in both a micro-founded, small open economy model and a micro-founded HANK model. Unlike these papers, I assume that only part of the households are affected by the information rigidity. As a consequence, these households are eventually the driver of the sluggishness in aggregate consumption, whereas in the mentioned papers it is the imperfect attention to aggregate shocks of all households that counts.

Finally, by providing a simple approach to deal with sticky information, I also address the literature on solution methods for this friction. Mankiw and Reis (2007), Meyer-Gohde (2010), and Wang and Wen (2006) draw on infinite moving average representations and the method of undetermined coefficients to efficiently handle the infinite number of lagged-expectation terms. On the other hand, Trabandt (2007) as well as Verona and Wolters (2014) limit those terms to approximate the infinite with a finite state space. My work differs from these papers in its focus on analytical tractability. Although the solution method I propose premises a specific monetary policy rule, it is computationally straightforward and comprehensive enough to elaborate the implications of sticky information in various models – be it in combination with heterogeneous households or not.

Outline. The rest of the paper is organized as follows. Section 1.2 provides empirical evidence for information frictions across households. Section 1.3 presents the SI-TANK model and its reduced-form equilibrium conditions. Section 1.4 explains the (asymmetric) interplay of amplification and dampening of the aggregate-consumption response following a monetary policy shock. Section 1.5 then provides an analytical and graphical view on the model. Finally, Section 1.6 provides some practical implications and Section 1.7 concludes.

1.2 Evidence for information rigidities

In order to motivate the model structure below, I start by providing some survey-based evidence for information frictions. A popular data set choice in the literature are historical forecasts of U.S. consumer price inflation. I will use data from the Michigan Surveys of Consumers (MSC), which asks more than 500 U.S. households on a monthly basis about their consumption attitudes and expectations. Among other aspects, the University of Michigan interviews the participants about the average change in prices they expect over the next 12 months. It also collects information on each household's income which makes it convenient to study differences along the income distribution.

To demonstrate the presence of information rigidities in expectations data, I follow Coibion and Gorodnichenko (2015) and study the relation between the ex-post mean year-ahead inflation forecast errors across agents and the change in the ex-ante mean year-ahead forecast (which I call forecast revision for simplicity):[4]

$$\pi_{t+4,t} - F_t\, \pi_{t+4,t} = \alpha + \beta\,(F_t\, \pi_{t+4,t} - F_{t-1}\, \pi_{t+3,t-1}) + \varepsilon_t\ , \tag{1.1}$$

where $\pi_{t+4,t}$ denotes the inflation rate between $t+4$ and t, and $F_t\ \pi_{t+4,t}$ is the average forecast across agents at time t. Coibion and Gorodnichenko (2015) argue that the assumption of full information requires $\beta = 0$, but that information frictions are present as soon as $\beta > 0$.[5] The latter case can be visualized, for example, by a slow updating of information in the economy over time. In each period, some agents do not adjust their information set, which is why the average forecast only adjusts gradually and average forecast errors become predictable.

The mean forecast revisions are computed as the difference between the current mean forecast and the mean forecast lagged by one quarter. As the MSC provides one-year-ahead inflation expectations, I define the forecast error in equation (1.1) as the difference between the actual value of inflation and the average quarterly forecasts across survey respondents. As a first measure of inflation, I use year-on-year changes in the U.S. consumer price index (CPI), taken from the FRED database operated by the Federal Reserve Bank of St. Louis. However, given potential revisions of the realized inflation values, the CPI data might not be directly comparable to the historical consumer expectations. To take this into account, I use as a second measure quarterly real-time data from the first release of the actual personal consumption expenditures (PCE) price index one year ahead. These vintages are available from the Federal Reserve Bank of Philadelphia's real-time data set for macroeconomists.

The time horizons of the forecast data used in equation (1.1) do not fully overlap across periods. The error term ε_t is therefore not orthogonal to information at time t or earlier and the regression equation cannot be estimated by standard OLS. To overcome that issue, Coibion and Gorodnichenko (2015) propose an instrumental-variable (IV) approach, using the log change in the oil price as the instrument due to its high significance for the course of CPI inflation.

I estimate equation (1.1) using the average responses for inflation expectations across all households, but also for each of the four equally-sized groups along the income distribution for which the MSC data set provides mean responses. The results for the sample period from 1980-Q1 to 2019-Q4 are shown in Table 1.1.

[4] Compared to other surveys, the MSC only provides expectations data for one-year ahead inflation. Revisions in forecasts over identical forecasting horizons (for instance, $F_t\ \pi_{t+4,t} - F_{t-1}\ \pi_{t+4,t}$) can therefore not be computed.

[5] Absent any information frictions, the mean forecast should react to a shock just as much as future inflation. This would imply a zero response of forecast errors.

Table 1.1: IV estimates of information rigidity in consumers' inflation forecasts

	Inflation expectations along the income distribution				
Forecast error	Aggregate	Bottom 25%	Second 25%	Third 25%	Top 25%
CPI					
Forecast revision	0.953***	1.560***	0.691**	0.824***	0.804***
	(0.301)	(0.601)	(0.297)	(0.293)	(0.307)
Constant	−1.112***	−1.969***	−1.331***	−0.852***	−0.395***
	(0.155)	(0.186)	(0.159)	(0.155)	(0.149)
First stage F-statistic	36.32	10.49	40.30	29.85	27.93
Observations	160	160	160	160	160
PCE (real-time)					
Forecast revision	0.546**	1.032**	0.349	0.439*	0.426*
	(0.237)	(0.454)	(0.237)	(0.232)	(0.242)
Constant	−1.531***	−2.391***	−1.749***	−1.271***	−0.812***
	(0.132)	(0.153)	(0.133)	(0.134)	(0.133)
First stage F-statistic	36.32	10.49	40.30	29.85	27.93
Observations	160	160	160	160	160

Notes: Coefficient estimates of the instrumental variable regression equation (1.1) using MSC data, with Newey-West standard errors in parentheses. The dependent variable is the mean year-ahead forecast error for inflation and the forecast revision is defined as the change in the mean year-ahead forecast. The instrumental variable is the log change in the oil price. Sample period is 1980–2019. The F-statistic reports the first-stage fit and expresses the relevance of the instrument (Kleibergen-Paap rk Wald F-statistic).
* p<0.10, ** p<0.05, *** p<0.01

The results show that there is evidence for information frictions. The aggregate estimate of $\hat{\beta}$ when using the CPI as the inflation variable implies that an average household updates its information set roughly every six months.[6] Almost all estimates with CPI inflation point to a rejection of the null of full information at the one percent level. The point estimates and statistical significance are lower for the PCE data, but still indicate the presence of rigid information.

Analyzing the empirical findings across the income distribution provides strong evidence for a higher degree of information stickiness at the left tail as compared to other parts of the distribution. A representative household in the first quartile shows an average duration of seven to eight months since the last information update, while it is five to six

[6] Coibion and Gorodnichenko (2015) describe how the regression results can be mapped directly into the degree of information rigidity within a sticky-information model as presented in section 1.3. If agents update their information sets with probability δ in every period, we can write the degree of information rigidity as a function of the estimated coefficients in equation (1.1), $1 - \hat{\delta} = \hat{\beta}/(1 + \hat{\beta})$. From this, the average duration between two updates can be expressed by $1/\hat{\delta} = 1 + \hat{\beta}$.

months for higher quartiles of the income distribution.[7] In Appendix 1.A.1, I test if the estimated coefficients along the income distribution are statistically different from each other.

The lowest part of the distribution contains, among others, poor agents who are often borrowing-constrained or live hand-to-mouth. The results above suggest that those households tend to update their information much less frequently than richer households. I will use this fact for the theoretical model below and, for the sake of simplicity, take the empirical evidence to the extreme by assuming that agents living hand-to-mouth have fully rigid information. This seems intuitive as those agents tend to be much less informed and highly myopic. They undervalue information and therefore do not make an effort to acquire it.[8]

1.3 Model economy

I propose a model that unifies elements from two different strands of the literature. First, I introduce heterogeneity into a standard representative-agent New Keynesian model with sticky prices and flexible wages by dividing households according to their participation in asset markets. Drawing on the seminal work of Galí, López-Salido, and Vallés (2007) and Bilbiie (2008), I consider two different types of households: intertemporally optimizing savers and constrained agents living hand-to-mouth. Second, I build on Mankiw and Reis (2006, 2007) and assume that only part of the savers are fully informed about economic conditions every period. This assumption is motivated by the empirical evidence provided in the previous section. Savers alone value additional information to make their optimal decisions while agents at their borrowing constraint have no use for it. Following the respective terms used in the literature, I call this economy a sticky-information two-agent New Keynesian (SI-TANK) model.[9]

The model economy is based on a small-scale dynamic general equilibrium model without capital or a government where agents meet in three different markets: the goods market, where firms sell varieties of goods to households; the labor market, where households sell a representative type of labor to firms; and the financial market, where part of the households trade bonds among each other. To close the model, a monetary authority

[7]The instrument for the regression of the lowest quartile seems to be weaker. This could also explain the slightly higher standard errors.

[8]Broer et al. (2021) emphasize that information only becomes valuable when being away from the borrowing constraint. As long as households live hand-to-mouth, they have no benefit from additional information because they do not need to decide about their savings or consumption smoothing. In fact, the authors find empirically that households at low levels of wealth are less well informed.

[9]According to the groups of households present at each point in time (hand-to mouth agents, updated savers and non-updated savers), one might label this framework as a *three*-agent model. Instead, I think of the model as being composed of two types of households, one of which has two subtypes with respect to whether the information set is up-to-date.

controls the real interest rate. Appendix 1.B contains details on the derivation of the model.

1.3.1 Households

The economy is populated by a continuum of households indexed by $j \in [0,1]$. Out of this unit mass, an exogenous share ω has no access to financial markets and thus cannot smooth consumption over time. These households only consume their disposable income such that the marginal propensity to consume out of their own income is equal to one. Following the literature, I call this type of agent hand-to-mouth (H), or constrained, households. The remaining $1-\omega$ households hold all assets in the economy. They can save by trading state-contingent bonds among each other and equally own firms. I follow Bilbiie (2008) and call them savers (S).

In each period, a household decides how many varieties of goods to buy from firms and how many units of labor to provide in order to produce these varieties. Irrespective of its type $o \in \{H,S\}$, a households' period utility function is given by

$$U\left(C^o_{t,j}, L^o_t\right) = \ln C^o_{t,j} - \xi \frac{(L^o_t)^{1+\eta} + 1}{1+\eta} \,, \tag{1.2}$$

where $C^o_{t,j}$ is the consumption level of household j at time t, L^o_t are hours worked by a household, η is the inverse of the Frisch elasticity of labor supply and ξ captures the relative weight of the disutility in labor.

Each household decides on the optimal allocation of spending across the different varieties of goods in the economy.[10] For this, a household of type $o \in \{H,S\}$ has full information and solves the following problem:

$$\min_{\left\{C^o_{t,j}(i)\right\}_{i\in[0,1]}} \int_0^1 P_t(i) C^o_{t,j}(i) di \quad \text{s.t. } C^o_{t,j} = \left(\int_0^1 C^o_{t,j}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \,,$$

where $C^o_{t,j}$ is a household's consumption index for different varieties of goods indexed by $i \in [0,1]$, with an elasticity of substitution $\epsilon_p > 1$. $P_t(i)$ is the price of variety i. The solution to this problem is

$$C^o_{t,j}(i) = C^o_{t,j} \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \,, \tag{1.3}$$

where the aggregate price index is defined as $P_t^{1-\epsilon_p} = \int_0^1 P_t(i)^{1-\epsilon_p} di$.

[10] In the terminology of Mankiw and Reis (2006, 2007), this decision is made by the attentive shopper, whereas the decision about total expenditure is made by the inattentive planner.

Savers

Each unconstrained household wants to maximize its expected discounted utility drawing on (1.2) while facing the following period budget constraint:

$$P_t C_{t,j}^S + B_{t,j}^S = W_t L_t^S + (1 + i_{t-1}) B_{t-1,j}^S + \frac{1}{1-\omega} P_t D_t + P_t T_{t,j} \ ,$$

where P_t is the aggregate price level of goods, $B_{t,j}^S$ are nominal bond holdings, W_t is the common flexible nominal wage associated with the representative type of labor supply L_t^S, i_{t-1} is the nominal return at time t on a bond purchased in $t-1$, D_t are real dividend payoffs arising from firms' profits and equally distributed to the savers, and $T_{t,j}$ are real lump-sum transfers. Transfers arise from an insurance contract that all these types of households enter to ensure that they start with the same real wealth in every period.

As seen before, each saver is fully attentive when deciding about how to allocate total spending across differentiated goods. On the other hand, when it comes to the planning of total expenditure and savings, an unconstrained household faces costs of information that make him prone to being inattentive. It will make decisions only at irregular intervals. I follow Mankiw and Reis (2006, 2007) and assume that savers obtain new information about the current state of the economy with probability $\delta \in [0,1]$ every period, which is constant and independent across households.[11] Based on this information, updating households will choose a consumption plan into the far future. Agents that have not updated their information in a given period continue to make their decisions based on outdated information by following the pre-determined consumption path from when they last updated. Consequently, the mass of savers is divided into a share of δ agents with current information and $\delta(1-\delta)^i$ agents with information as old as i periods, where $i = 1, 2, \ldots$. The case of full information is nested for $\delta = 1$.

Savers only differ in the period in which they last updated their information set. I therefore redefine the index j accordingly: for this part, $C_{t,j}^S$ denotes expenditures at time t for a saver who last updated his information set j periods ago. The optimality conditions of the maximization problem (see Appendix 1.B.1) are then the following:

$$\left(C_{t,0}^S\right)^{-1} = \beta E_t \left[R_{t+1} \left(C_{t+1,0}^S\right)^{-1} \right] \ ,$$

$$\left(C_{t+j,j}^S\right)^{-1} = E_t \left[\left(C_{t+j,0}^S\right)^{-1} \right] \ ,$$

$$\xi C_{t,0}^S \left(L_t^S\right)^{\eta} = \frac{W_t}{P_t} \ ,$$

[11] In fact, to derive solutions for the policy experiments further below, I need to assume that $\delta \in (0,1]$.

where $R_{t+1} = (1+i_t)\frac{P_t}{P_{t+1}}$ denotes the gross real return on bonds between periods t and $t+1$. These conditions hold for all t and j. The first one is the Euler equation which specifies the optimal intertemporal consumption-savings choice between today and tomorrow of a consumer in an attentive household. The second expression is the Euler equation for an inattentive consumer. It states that the marginal utility of consumption of a saver at any point in time should be equal to the corresponding expectation of the attentive consumer's marginal utility. The last condition determines the labor-leisure choice.

Hand-to-mouth households

Constrained households do not hold assets, but only consume their current disposable income in every period. They maximize their utility $U\left(C_t^H, L_t^H\right)$ subject to

$$P_t C_t^H = W_t L_t^H \ .$$

As agents supply a representative type of labor and prices and wages are common to all agents, consumption will be the same across hand-to-mouth households, $C_{t,j}^H = C_t^H$.

The resulting optimality condition is

$$\xi C_t^H \left(L_t^H\right)^\eta = \frac{W_t}{P_t} \ .$$

Aggregation

Consumption of household type $o \in \{H, S\}$ is given by $C_t^o = \int_0^1 C_{t,j}^o dj$. Aggregate spending of all households is equal to $C_t = \omega C_t^H + (1-\omega)C_t^S$, whereas total labor supply is $L_t = \omega L_t^H + (1-\omega)L_t^S$. Finally, summing the individual demand for each variety in (1.3) over all agents of each household type and aggregating up leads to the total demand for variety i:

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \ , \tag{1.4}$$

where $C_t(i) = \omega C_t^H(i) + (1-\omega)C_t^S(i)$.

1.3.2 Firms

The firm side of the model is kept standard. There is a continuum of monopolistically competitive firms owned by savers, each of which produces one of the differentiated consumption goods i using labor as the only input. They take aggregate prices and wages as given, thereby facing the same market wage W_t.

Each firm minimizes its total variable cost of production $W_t N_t(i)$, given the production function $Y_t(i) = N_t(i)$. This brings along a real marginal cost $MC_t = \frac{W_t}{P_t}$. In addition, each firm maximizes nominal profits $P_t D_t(i) = P_t(i) Y_t(i) - W_t N_t(i)$. Summing over all firms leads to total nominal profits $P_t D_t = P_t Y_t (1 - MC_t)$, which are redistributed to savers by dividend payments. I assume a Calvo (1983) price setting, where each firm can reset the price of its good in every period with probability $1 - \lambda$, which is constant and independent across firms. The problem of a firm i at time t choosing the reset price that maximizes the current market value of the profits generated over the time that the price remains effective is

$$\max_{\tilde{P}_t(i)} E_t \sum_{k=0}^{\infty} \lambda^k \left[Q_{t,t+k} \left(\tilde{P}_t(i) Y_{t+k|t}(i) - W_{t+k} N_{t+k|t}(i) \right) \right] ,$$

$$\text{s.t. } Y_{t+k|t}(i) = N_{t+k|t}(i)^{1-\alpha} ,$$

$$Y_{t+k|t}(i) = \overline{C}_{t+k} \left(\frac{\tilde{P}_t(i)}{P_{t+k}} \right)^{-\epsilon_p} ,$$

where $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k,0}}{C_{t,0}} \right)^{-\gamma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal payoffs in period $t+k$, $\tilde{P}_t(i)$ is the price chosen by a firm that re-optimizes in period t, and $X_{t+k|t}(i)$ is the value of variable X at time $t+k$ for a firm that last reset its price in period t.

All producers face the same production function and the same probability of resetting prices for their goods, which is why all adjusting firms set the same adjustment price $\tilde{P}_t(i) = \tilde{P}_t$. Hence, $Y_{t+k|t}(i) = Y_{t+k|t}$, $N_{t+k|t}(i) = N_{t+k|t}$ and also $MC_{t+k|t}(i) = MC_{t+k|t}$. The resulting optimality condition for the reset price, written as a function of the real marginal cost, is

$$\tilde{P}_t = \frac{\epsilon_p}{\epsilon_p - 1} \frac{\sum_{k=0}^{\infty} \lambda^k E_t \left[Q_{t,t+k} Y_{t+k|t} P_{t+k} MC_{t+k|t} \right]}{\sum_{k=0}^{\infty} \lambda^k E_t \left[Q_{t,t+k} Y_{t+k|t} \right]} ,$$

where the aggregate price dynamic is governed by

$$P_t = \left[\lambda \left(P_{t-1} \right)^{1-\epsilon_p} + (1-\lambda) \left(\tilde{P}_t \right)^{1-\epsilon_p} \right]^{\frac{1}{1-\epsilon_p}} .$$

1.3.3 Monetary policy

The central bank strives to fix the real interest rate, where I assume that the Fisher equation holds. The nominal interest rate is determined by

$$i_t = \log \left[E_t \left(\frac{P_{t+1}}{P_t} \right) \right] + \varepsilon_t = \log \left[E_t \left(R_{t+1} \frac{P_{t+1}}{P_t} \right) \right] ,$$

where $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \nu_t$ is a policy shock with innovation $\nu_t \sim N(0, \sigma_\varepsilon^2)$ and persistence $\rho_\varepsilon \in [0,1]$. This policy rule implies that the real interest rate is exogenously determined by the monetary policy shock. It allows to isolate the mechanisms on the aggregate-demand side by prohibiting interactions with aggregate supply, in particular inflation, as will become evident later.

1.3.4 Market clearing

In the goods market for each variety $i \in [0,1]$, it holds that $C_t(i) = Y_t(i)$, where total output is defined as $Y_t^{\frac{\epsilon_p - 1}{\epsilon_p}} = \int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di$. Using the demand function in (1.4), it follows that $Y_t = C_t$. Furthermore, labor market clearing requires total labor supply to be equal to total labor demand, $L_t = \int_0^1 N_t(i) di$. This leads to $L_t = N_t$. Finally, financial assets are in zero net supply and so $\int_0^1 B_{t,j}^S dj = 0$.

1.3.5 Steady state

The model is approximated around a deterministic steady state. From the Euler equations of the saver, one gets $R = \beta^{-1}$ and $C_{\cdot,j}^S = C_{\cdot,0}^S = C^S$, which shows that different information sets do not play a role if variables are constant in steady state.

Assuming zero profits and zero lump-sum transfers in steady state, the budget constraint for both types of households evaluated at steady state collapses to $PC^o = WL^o$ and their labor supply condition becomes $\xi C^o (L^o)^\eta = W/P$, where $o \in \{H, S\}$. Combining these two expressions entails consumption and hours worked being equal across households in steady state, namely that $C^S = C^H = C$ and $L^S = L^H = L$. Moreover, by market clearing, $C = Y$ and $L = N$. Finally, due to zero profits in steady state, total nominal profits become $PY = WN$.

1.3.6 Equilibrium conditions and reduced-form representation

Table 1.2 contains the log-linearized equilibrium conditions. Small letters denote the log-linear deviation of the respective uppercase characters from a variable's non-stochastic steady state. Two exceptions are w_t and r_t, which are the log-linear deviations of the real wage $\frac{W_t}{P_t}$ and of the real return $E_t[R_{t+1}]$, respectively. In addition, profits and transfers are defined relative to total income, $d_t = \frac{D_t}{Y}$ and $t_t = \frac{T_t}{Y}$. Finally, inflation is defined as $\pi_t = p_t - p_{t-1}$.

In a next step, I present the key log-linearized equilibrium conditions for the demand and supply side of the SI-TANK model. See Appendix 1.B.2 for further details on the derivations. Important to emphasize, the framework here nests the representative-agent (for $\omega = 0$ and $\delta = 1$) and the basic two-agent (for $\delta = 1$) New Keynesian models.

Table 1.2: Equilibrium conditions for the SI-TANK model

Euler equation, attentive S	$c^S_{t,0} = E_t\left(c^S_{t+1,0} - r_t\right)$
Euler equation, inattentive S	$c^S_{t,j} = E_{t-j}\left(c^S_{t,0}\right)$
Consumption index, S	$c^S_t = \delta \sum_{j=0}^{\infty}(1-\delta)^j c^S_{t,j}$
Labor supply, S	$\eta l^S_t = w_t - c^S_{t,0}$
Budget constraint, S	$c^S_t = w_t + l^S_t + \frac{1}{1-\omega} d_t + t_t$
Labor supply, H	$\eta l^H_t = w_t - c^H_t$
Budget constraint, H	$c^H_t = w_t + l^H_t$
Production function	$y_t = l_t$
Real marginal cost	$mc_t = w_t$
Real profits	$d_t = -mc_t$
Phillips curve	$\pi_t = \beta E_t(\pi_{t+1}) + \frac{(1-\lambda)(1-\lambda\beta)}{\lambda} mc_t$
Monetary policy rule	$i_t = E_t(\pi_{t+1}) + \varepsilon_t$
Fisher equation	$i_t = r_t + E_t(\pi_{t+1})$
Aggregate consumption	$c_t = \omega c^H_t + (1-\omega) c^S_t$
Aggregate labor	$l_t = \omega l^H_t + (1-\omega) l^S_t$
Resource constraint	$y_t = c_t$

Aggregate demand. First, the log-linearized Euler equation for savers reads

$$c^S_t = -\delta \sum_{j=0}^{\infty} (1-\delta)^j E_{t-j}\left(R_t\right) \ , \tag{1.5}$$

where $R_t = E_t\left(\sum_{k=0}^{\infty} r_{t+k}\right)$ is the long-run real interest rate. The consumption of savers is determined by current and past expectations of R_t: lower (expected) real rates encourage consumers to save less and to spend more. The impact of unexpected shocks to these real interest rates is dampened due to the fact that only a share of consumers δ are fully informed.

Second, we can write consumption of hand-to-mouth households as a function of aggregate spending and past expectations of real interest rates:

$$c^H_t = y^H_t = \chi_{\textit{SI-TANK}}\, y_t + \Psi\delta \sum_{j=1}^{\infty} (1-\delta)^j E_{t-j}\left(R_t\right) \ , \tag{1.6}$$

where $\Psi = \left(\frac{1-\omega}{\omega+(1-\omega)\delta}\right)$ is a composite parameter decreasing in both ω and δ. Moreover, $\chi_{\textit{SI-TANK}}$ is one of the key parameters of this paper. It denotes the elasticity of constrained households' individual income to current aggregate income y_t, disregarding information sets last updated in the past, and is defined as

$$\chi_{\textit{SI-TANK}} = \frac{1+\delta\eta}{\omega+(1-\omega)\delta} \ .$$

A similar expression for individual spending can be derived for savers:

$$c_t^S = \frac{1 - \omega\chi_{\textit{SI-TANK}}}{1-\omega} y_t - (1-\delta\Psi)\delta \sum_{j=1}^{\infty} (1-\delta)^j E_{t-j}\left(R_t\right) \ . \tag{1.7}$$

The second term of (1.6) and (1.7) refers to agents with outdated information sets and becomes relevant when looking at past or anticipated shocks. It captures the spillover of expectations about R formed in the past to the consumption of both household types at time t. Savers expecting interest rates to be lower in the future stimulates spending in the past. By intertemporal substitution, this also increases the current consumption levels of savers and therefore affects hand-to-mouth households.[12]

Finally, the IS curve (or aggregate Euler equation) reads

$$c_t = y_t = -\mu \left\{ \omega R_t + (1-\omega)\delta \sum_{j=0}^{\infty} (1-\delta)^j E_{t-j}\left(R_t\right) \right\} \ , \tag{1.8}$$

where $\mu = \frac{1-\omega}{1-\omega(1+\eta)}$. The IS curve entails the usual inverse relationship between aggregate consumption (or output) today and expected real interest rates. Different from the standard New Keynesian literature, these expectations are split into two different parts: an undiscounted sum of future real interest rates, also present in RANK models, and a stream of current and past expectations about current and future real rates, emerging from incorporating sticky information. On top of this, the IS curve is shaped by μ, which increases in the share of hand-to-mouth households.

Aggregate supply. The Phillips curve of the SI-TANK model is given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa y_t - \Theta\Psi \left\{ \delta R_t - \delta \sum_{j=0}^{\infty} (1-\delta)^j E_{t-j}\left(R_t\right) \right\} \ , \tag{1.9}$$

where $\kappa = \Theta\chi_{\textit{SI-TANK}}$ and $\Theta = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}$. Current inflation depends not only on expected future inflation and output, but also on past expectations of the long-run real interest rate.[13] Just as for the individual consumption levels, the last term is redundant when ignoring past or anticipated shocks. Also note that (1.9) turns into the standard

[12]On closer examination of equation (1.6), it seems that the anticipation of a negative interest rate shock today by agents in the past (second part of the equation) attenuates the increase of c_t^H arising through larger aggregate spending (first part of the equation). However, considering that y_t itself is governed by past expectations, it can be shown that the net effect of this cut in the real rate is positive as long as $\eta > 0$. Consumption of hand-to-mouth households will eventually be higher relative to the case without anticipation of the shock; and likewise the spending of savers.

[13]Note that firms' marginal cost and thus inflation depend directly on monetary policy. A higher nominal interest rate affects the cost of working capital and leads to higher prices. This transmission mechanism known as the cost channel in the literature (see, among others, Barth & Ramey, 2002, and Ravenna & Walsh, 2006) entails a mitigated response of inflation after a policy shock.

New Keynesian Phillips curve and becomes independent of the share of hand-to-mouth households ω if agents are fully informed ($\delta = 1$).

1.4 Amplification and dampening in interaction

The combination of household heterogeneity and sticky information generates dynamics different from standard New Keynesian models. To get a deeper insight into the mechanisms at play within the SI-TANK model, I will now look separately at the impact of an exogenous monetary policy shock that changes the real interest rate. For the time being, I will exclusively focus on the response of aggregate consumption and on the period in which the change in the real rate actually occurs, that is, the *initial* impact of the shock. Further down in section 1.5, I will also elaborate on potential differences in peak impacts between the models and discuss the response of inflation.

1.4.1 Aggregate demand under the two frictions

A natural way to investigate the nature of consumption and output responses is through the aggregate Euler equation. Table 1.3 displays IS curves for different model alternatives, which will be discussed successively in the light of a change in the real interest rate.

The representative-agent New Keynesian (**RANK**) model with full information and the standard consumption-smoothing type of household serves as a benchmark. Output is completely negatively related to the long-run real interest rate R. Higher expected real rates encourage consumers to save more and to spend less, thus depressing aggregate consumption.

Dampening. Assuming imperfect attention to economic events and shocks results in a representative-agent economy with sticky information (**SI-RANK**). Similar to the model described in section 1.3, households are partly inattentive, meaning that only a fraction of them update their information about the state of the economy in any period. This leads to the case where output in equilibrium is no longer determined by current expectations of R alone, but also by past expectations. This implies a dampening effect as follows. A cut in interest rates encourages updating households to increase their consumption. However, only a share of households are fully informed in each period and learn about news. As a result, aggregate demand will react less to the occurrence of shocks to the real rate relative to RANK. It will only adjust slowly over time, always leaving behind some agents with outdated information sets. That is the result found by Mankiw and Reis (2006, 2007) in its purest form.

Note that a change in the real interest rate is attenuated as long as there is rigid information and hence lagged expectations, be the change unexpected or not. On the other

Table 1.3: IS curve for various model specifications

	Full information	Sticky information
RANK	$y_t = -R_t$	$y_t = -\delta \sum_{j=0}^{\infty}(1-\delta)^j E_{t-j}(R_t)$
TANK	$y_t = -\mu R_t$	$y_t = -\mu \left\{\omega R_t + (1-\omega)\delta \sum_{j=0}^{\infty}(1-\delta)^j E_{t-j}(R_t)\right\}$

Notes: The composite multiplier is defined as $\mu = \frac{1-\omega}{1-\omega\chi_{TANK}}$, where $\chi_{TANK} = 1+\eta$. Moreover, $R_t = E_t\left(\sum_{k=0}^{\infty} r_{t+k}\right)$.

hand, the more households update their information sets in the current period (higher δ), the more aggregate spending responds to changes in interest rates and the closer is the output response to the case without information frictions. In fact, the model nests RANK for $\delta = 1$.

Amplification. Adding hand-to-mouth households to the representative-agent model leads to the simplest version of a two-agent New Keynesian (**TANK**) economy. As Table 1.3 shows, this model's IS curve differs from the RANK case in μ. This composite parameter is affected by labor market characteristics (captured by the Frisch elasticity of labor supply) and the degree of heterogeneity (captured by the share of hand-to-mouth households). Looking at individual consumption levels, the TANK model is characterized by $c_t^H = \chi_{TANK} y_t$ and $c_t^S = \frac{1-\omega\chi_{TANK}}{1-\omega} y_t$, where $\chi_{TANK} = 1+\eta$ is the elasticity of constrained households' individual income to aggregate income. As a result, the multiplier reads $\mu = \frac{1-\omega}{1-\omega\chi_{TANK}}$.

Amplification requires the effect of a change in the real interest rate on aggregate demand to be higher than in RANK (i.e., $\mu > 1$) and to increase in the share of hand-to-mouth households ω. This is the case if and only if $\chi_{TANK} > 1$, namely when the individual income of constrained households responds more than proportionally to changes in aggregate income.[14] By contrast, the savers' income elasticity will be smaller than one in that case. This implies countercyclical income inequality as analyzed by Bilbiie (2018, 2020), meaning that inequality between unconstrained and constrained agents declines in a period of economic expansion.[15]

The intrinsic mechanism behind the amplification works through the specific distribution of profits I postulated, following Bilbiie (2008). Assume a cut in interest rates that induces an increase in aggregate demand. Even though agents then consume more

[14] Bilbiie (2008) shows that, depending on the proportion of hand-to-mouth households, the slope of the IS curve may turn positive and reverse the impact of the real interest rate on aggregate demand. In the present case, one needs $\omega < 1/(1+\eta)$ for μ to be positive for sure. This is achieved with empirically plausible values for η and respective estimates of hand-to-mouth shares in empirical studies. I therefore focus only on the common case where $\mu > 0$.

[15] For a complementary analysis within a more complex heterogeneous-agent framework, see the earnings heterogeneity channel in Auclert (2019). Moreover, Patterson (2022) provides estimates for the covariance between MPCs and individual earnings elasticities to GDP.

and work less at a given wage, sticky prices induce firms to increase labor demand. The result are higher wages. This increases the individual income of hand-to-mouth households, which they completely spend for consumption because they cannot intertemporally optimize. Thus, they respond to the initial shock with a higher demand – exactly where $\chi_{TANK} > 1$ is put into effect. This boosts aggregate spending, pushing up wages further, and so on. At the same time, the rise in wages translates into higher marginal costs for firms, shrinking their profits and therefore also each saver's dividend income. As their individual income goes down, savers are willing to bear the required increase in labor supply to meet the higher aggregate demand and work more.

It is apparent that the presence of constrained households that live hand-to-mouth is essential for the real effects of monetary policy to be different from RANK. The MPC out of their own income is one, which increases the aggregate MPC in the economy. In addition, the feedback mechanism from individual back to aggregate income described above is precisely what eventually leads to amplification.

Amplification and dampening. Incorporating household heterogeneity as well as sticky information in the standard RANK economy yields the **SI-TANK** model. The corresponding IS curve unifies amplification and dampening. On the one hand, both types of households react to a change in the real rate, leading to a reinforced impact on aggregate demand as described before. This mechanism is captured by the TANK multiplier μ. At the same time, the response of spending is attenuated because not all agents are aware of the change. However, different from the amplification element, δ just reaches part of the agents. This arises by construction of the model since only a fraction of savers $1-\omega$ is subject to the information friction. Unifying both frictions, the model naturally nests TANK (for $\omega = 0$) as well as SI-RANK (for $\delta = 1$).

While the initial response of output is amplified by the presence of hand-to-mouth agents relative to RANK, information rigidity tempers it relative to TANK. It appears natural to ask under which conditions the propagation of monetary policy shocks takes one or the other direction. To find a sufficient answer, the IS curve can be rewritten as

$$y_t = -\frac{1-\omega}{1-\omega\chi_{SI\text{-}TANK}}\delta R_t - \mu(1-\omega)\delta\sum_{j=1}^{\infty}(1-\delta)^j E_{t-j}\left(R_t\right) \ .$$

Focusing exclusively on real interest rate changes that are unanticipated, the second term equals zero because it refers to agents with outdated information sets.[16] The first term consists of two parts. The fraction is characterized by amplification for $\chi_{SI\text{-}TANK} > 1$, similar to TANK. Thus, disregarding any outdated expectations, the individual income of

[16]For *expected* changes in the real interest rate, the condition for amplification depends on when the change is announced. However, this case is left out of consideration here.

constrained households has to be more elastic to aggregate income than the one of savers. However, this condition is not sufficient to achieve overall amplification in the SI-TANK model: the presence of the stickiness parameter δ might eventually dampen the total effect of a change in real rates. Instead, the following (sufficient) threshold conditions hold:

$$\text{Amplification (on impact):} \quad \chi_{\textit{SI-TANK}} > \frac{1-\delta}{\omega} + \delta \geq 1 \; ;$$
$$\text{Dampening (on impact):} \quad \chi_{\textit{SI-TANK}} < \frac{1-\delta}{\omega} + \delta \; .$$

These expressions point out three things. First and to some extent obvious, what determines the net propagation effect in the SI-TANK model is the relative magnitude of ω as against δ, which in turn both shape $\chi_{\textit{SI-TANK}} = \frac{1+\delta\eta}{\omega+(1-\omega)\delta}$. Appendix 1.C approaches this interplay and the role of the labor supply elasticity graphically. For a given share of hand-to-mouth agents, the probability of getting dampening on impact increases if information becomes stickier. On the contrary, if the degree of information stickiness is fixed, amplification of aggregate demand is more likely with a higher proportion of constrained households. On top of that, this result does not only hold for the aggregate-demand response on impact, but also for the periods subsequent to the shock. I will elaborate more on this point in the graphical analysis of the model in section 1.5.3.

Second, amplification calls for countercyclical income inequality, as in Bilbiie (2018, 2020) or Patterson (2022). However, it may require $\chi_{\textit{SI-TANK}}$ to lie considerably above one, meaning that constrained agents' income reacts substantially to changes in current aggregate income.[17] Naturally, this is to outweigh the downward pressure caused by sticky information. The closer the model moves to the full information case, the lower $\chi_{\textit{SI-TANK}}$ will be.

Third, dampening might arise even if income inequality is countercyclical. This result is contrary to Bilbiie (2018, 2020), where attenuated aggregate demand presupposes procyclical inequality.[18] Unlike such a TANK model, a share of savers remain here uninformed about any news in each period. Since, as a result, only a fraction of them react to a monetary policy shock, the response of their spending behavior is relatively weak. This also depresses aggregate consumption relative to a simple TANK model with full information. In fact, the expressions above nest the latter case for $\delta = 1$.

[17] In comparison to the TANK model, the income elasticity of constrained agents is required to be higher to get amplification in SI-TANK. This can be ascertained by rewriting the threshold condition as $\chi_{\textit{TANK}} > 1 + (1-\delta)\frac{(1-\omega)^2}{\omega} \geq 1$.

[18] Note that in the present simple setup $\chi_{\textit{SI-TANK}} \geq 1$ always holds, independent of the parameter values.

1.4.2 Multiplier effects after a monetary policy shock

The interplay of amplification and dampening generates effects of various magnitudes. To narrow down the analysis to a common shock, I now consider the effect of an unexpected one-time cut in the current real interest rate on aggregate demand. This impact multiplier can be expressed by

$$\Phi_M = \frac{\partial y_t}{\partial\left(-r_t\right)}|_M \ ,$$

where M denotes the respective model specification. Table 1.4 shows the aggregate-demand multiplier Φ_M for the different cases.

Table 1.4: Impact of monetary policy on aggregate demand: formal expressions

	Full information	Sticky information
RANK	$\Phi_{RANK} = 1$	$\Phi_{SI\text{-}RANK} = \delta$
TANK	$\Phi_{TANK} = \mu$	$\Phi_{SI\text{-}TANK} = \mu\omega + \mu(1-\omega)\delta$

Notes: Multipliers Φ_M of the effects of an unexpected interest rate cut in the current period on aggregate demand in model specification M. It holds that $\mu = \frac{1-\omega}{1-\omega\chi_{TANK}}$, where $\chi_{TANK} = 1+\eta$.

As before, the starting point is the standard RANK model, which has an aggregate-demand multiplier of 1. Adding sticky information attenuates this multiplier. Not all households perceive the shock so that aggregate spending increases only partly. On the other hand, incorporating hand-to-mouth agents in RANK can induce an amplified response of output provided that income inequality is countercyclical (i.e., $\chi_{TANK} > 1$). Finally, in the SI-TANK model, amplification and dampening clash. We learned in the previous section that the magnitudes of ω and δ are critical to determine which of the two forces eventually prevails. However, by considering the ratios between various multipliers instead of absolute effects, the following proposition states some *universal* results regarding the propagation of monetary policy shocks.

Proposition 1 (Asymmetric effects of dampening and amplification)**.** ***(I)*** *Sticky information dampens the initial aggregate-consumption response associated with an unexpected one-time change in the real interest rate by a higher factor when added to RANK instead of TANK:*

$$\frac{\Phi_{RANK}}{\Phi_{SI\text{-}RANK}} \geq \frac{\Phi_{TANK}}{\Phi_{SI\text{-}TANK}} \ .$$

(II) *Household heterogeneity amplifies the initial aggregate-consumption response associated with an unexpected one-time change in the real interest rate by a higher factor when* *added to SI-RANK instead of RANK:*

$$\frac{\Phi_{SI\text{-}TANK}}{\Phi_{SI\text{-}RANK}} \geq \frac{\Phi_{TANK}}{\Phi_{RANK}} \, .$$

These asymmetries are independent of the parameter values.

Proof. Follows from the multipliers in Table 1.4 and $\delta \in [0, 1]$. ■

The impact of neither of the two rigidities is proportional across models. The dampening arising from incorporating sticky information is much less pronounced in a two-agent compared to a representative-agent framework. On the other hand, adding hand-to-mouth agents has a larger relative impact when information frictions are present at the same time.

Even though somewhat mechanical, it seems particularly striking that heterogeneity is proportionately more influential in the presence of information rigidities. Prominent TANK or HANK models show that hand-to-mouth agents are one of the most important elements to achieve amplification. The results above suggest that a lot of these amplifying effects might instead originate from information frictions, implying an overstatement of the importance of heterogeneity in this respect.

Both asymmetries are based on the different channels through which the propagation of the monetary policy shock works. The amplification mechanism in a two-agent economy involves both types of households, meaning that the adjustments in their optimal behavior jointly contribute to the boost in aggregate demand. This holds independent of the presence of information frictions.

It matters, in contrast, whether sticky information comes along with heterogeneous households or not. While all households are prone to being inattentive in a representative-agent framework, this turns out to be different in SI-TANK. In fact, (limited) inattention to information is intrinsically linked with intertemporal optimization. Not only are hand-to-mouth households unable to shift consumption across periods by saving, but they are also not subject to the information friction. These agents are extremely myopic in the sense that they do not care about the future or about how much and which information is revealed at each point in time. In fact, they have no use for additional information and therefore never acquire it. Savers instead benefit from it to make their optimal decisions, but given the cost to acquire it, they update their information set only infrequently.

The assertions above are best reflected by $\Phi_{SI\text{-}TANK}$ in Table 1.4. While the TANK multiplier μ that guides amplification reaches both types of households alike, only a share of savers $(1 - \omega)$ are affected by the information stickiness parameter. In SI-RANK, all households are impacted by δ instead. The respective two fractions for sub-propositions (I) and (II) in Proposition 1 might only be identical if one added some sort of information stickiness on the part of hand-to-mouth households – whatever its source. Otherwise, in the case at hand, the wedge between the ratios becomes wider with a larger share of constrained agents (higher ω) or more stickiness (lower δ). The degree of asymmetry thus becomes more pronounced. See Appendix 1.D for a graphical demonstration in this regard.

1.4.3 Complementarity between agent types and dynamic effects

Household heterogeneity and information rigidities have been studied so far without considering any potential complementarities between them. In particular, I have assumed that hand-to-mouth agents are fully inattentive to information because they are financially constrained and have no need for information. More in line with the empirical findings of Table 1.1, we could suppose instead that agents located at the lower end of the income distribution have a particularly high (but not infinite) degree of information rigidity. This would allow us to assess how the interaction of heterogeneity and information drives the dynamics of aggregate demand in the SI-TANK model.

To discuss the interaction between ω and δ, I consider two alternative setups. First, we could assume a third type of household with a very low probability of obtaining new information. It is close to being hand-to-mouth, but able to save a small portion of its income up to a certain limit. Such an intermediate agent is still highly sensitive to changes in its earnings due to the risk of becoming fully financially constrained. If the real wage increases after a cut in interest rates, however, only a small share of the intermediate agents will adjust their optimal behavior while the much larger fraction sticks to outdated consumption plans. Hence, if we introduce an intertemporal substitution component on the part of households with very infrequent information updates, we lack the strong amplifying effects known from fully constrained agents. Compared to the baseline model, the result is a relatively smaller response of aggregate demand which decreases further with lower δ.

In a second setup, we could start from the idea that constrained agents live hand-to-mouth precisely due to the presence of information frictions, meaning that they consume all their income in each period as long as there is no information update. Following again the results in Table 1.1 and assuming a low probability for such updates, the response of this type of household to an interest rate cut will be close to the reaction of the baseline hand-to-mouth agents. Only a small fraction will refrain from naively consuming all income gains. Once those agents learn about the shock and its persistence, they will start saving part of their higher individual income as their financial constraints have loosened.[19] Consequently, the amplification coming from hand-to-mouth agents will be slightly weaker

[19]We could assume instead that the degree of information rigidity is state-dependent. For instance, while households at the borrowing constraint do not value additional information, it tends to be very useful when wealth starts to increase because savings mistakes can become costly. As a result, their

due to the small fraction of agents that essentially switch from being fully constrained to a low-savings type. This translates into an aggregate demand response that is only marginally lower, at least as long as the degree of information rigidity of hand-to-mouth agents remains high.

1.5 Analytical insights for the effects of monetary policy

Taking advantage of the tractability of the SI-TANK model, this section strives to find an analytical solution for the impact of an unanticipated monetary policy shock, with the aim to isolate the aggregate-demand side. Solving sticky-information models can be tedious due to infinite lagged expectations. I follow hereafter a straightforward approach that leads to simple reduced-form equations for aggregate demand and inflation. Those kinds of solutions provide more insights into how monetary policy works in SI-TANK, but also confirm some of the results that have been found earlier from a different point of view. The section will be completed with a graphical analysis meant to discuss what happens in the periods following the initial occurrence of the policy shock.

1.5.1 The pitfall of expectations under sticky information

The difficulty in handling models with sticky information arises from the presence of an infinite number of lagged expectations, which leads to an infinite state space. A few papers try to deal with this problem, either by building on infinite moving average representations and the method of undetermined coefficients (Mankiw & Reis, 2007; Meyer-Gohde, 2010; Wang & Wen, 2006), or by implementing restrictions regarding the number of lagged-expectation terms (Trabandt, 2007; Verona & Wolters, 2014). Although these methods are valid from a computational point of view, they reveal the common issue that it is tedious or not possible at all to solve models with sticky information analytically.

In my simple model, I overcome the issue of lagged-expectation terms by the specific choice of the policy rule and by assuming an AR(1) shock process $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \nu_t$. The former implies that the central bank controls the real interest rate, which makes it possible to abstract from aggregate supply and hence to isolate the aggregate-demand side of the model – equivalently to what could be achieved through postulating fixed prices. The shock process then allows me to solve for the lagged expectations within the IS curve analytically, namely by feeding it directly into the IS curve and transforming the past expectations of future shocks into expressions that are only dependent on past shocks and their persistence.

attentiveness changes. See Broer et al. (2021) for a model that combines incomplete markets with dynamic, heterogeneous information choices.

Combining the monetary policy rule and the Fisher equation yields

$$r_t = \varepsilon_t \, .$$

The real rate in each period is completely determined by the policy shock, and the long-run real interest rate R_t is therefore exogenously determined. Inserting this into the IS curve (1.8) gives

$$y_t = -\mu \left\{ \omega E_t \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) + (1-\omega)\delta \sum_{j=0}^{\infty} (1-\delta)^j E_{t-j} \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) \right\} \, . \tag{1.10}$$

It is useful to expand the part with lagged expectations in different ways:

$$\begin{aligned}
&\delta \sum_{j=0}^{\infty} (1-\delta)^j E_{t-j} \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) \\
&= \delta \sum_{j=0}^{\infty} (1-\delta)^j \left\{ E_{t-j} \left(\varepsilon_t \right) + E_{t-j} \left(\varepsilon_{t+1} \right) + E_{t-j} \left(\varepsilon_{t+2} \right) + \dots \right\} \\
&= \delta E_t \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) + \delta(1-\delta) E_{t-1} \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) + \delta(1-\delta)^2 E_{t-2} \left(\sum_{k=0}^{\infty} \varepsilon_{t+k} \right) + \dots \, .
\end{aligned}$$

The second line emphasizes the role of the policy shocks. The curly brackets enclose a stream of expectations for the current shock, but also for all shocks up to the infinite future, captured by the index k. Compared to this, the third line rewrites the IS curve in a way to stress the presence of current and past information sets of the stream of shocks from time t on, weighted with the respective probabilities to update. This dimension is captured by the index j. As a result, consumption and output are determined by the current and past expectations of current and future policy shocks, resulting in an infinite stream of combinations of information sets and shocks.

1.5.2 Analytical solution for an unanticipated one-time innovation

To study the dynamics of the model at hand, I will look at an unanticipated one-time innovation that happens at time t, ν_t, and fades out thereafter. To isolate the effects of ν_t, I disregard any future anticipated and any past monetary policy shocks. For a more general solution including past shocks, see Appendix 1.E.

In order to obtain an analytical solution, I start by simplifying the expectation expressions for some $k \geq 0$. Forwarding the AR(1) process that was assumed for the shock gives the common result

$$\varepsilon_{t+k} = \rho_{\varepsilon}^k \varepsilon_t + \sum_{m=0}^{k-1} \rho_{\varepsilon}^m \nu_{t+k-m} \, ,$$

which holds for all t. Given that ν is assumed to have mean zero and I rule out future anticipated shocks, one gets $E_{t+i}\left(\varepsilon_{t+i+k}\right) = \rho_\varepsilon^k \varepsilon_{t+i}$ for any $i, j \geq 0$. Moreover, disregarding any past shocks means that $\varepsilon_t = \nu_t$ and that $E_{t+i-j}\left(\varepsilon_{t+i+k}\right) = \rho_\varepsilon^{j+k} \varepsilon_{t+i-j}$ is only non-zero for $0 \leq j \leq i$. The expectation of an agent with an information set older than time t about shocks at or after t will always be zero, as this agent's best guess would be based on (non-existent) shocks before t. With all this in mind, the latter expression can be simplified to

$$E_{t+i-j}\left(\varepsilon_{t+i+k}\right) = \rho_\varepsilon^{i+k} \nu_t \ ,$$

where $0 \leq j \leq i$. An agent's best guess of a future unanticipated shock is the last perceived shock, taking into account the persistence across time. From a date $t+i$ perspective, the left-hand side reflects the expectation about a shock k periods in the future of an agent who last updated his information set j periods ago. This expectation is equal to the product of two terms: the one-time policy shock that the agents observe in this experiment and the overall series of persistence coefficients between the date at which the shock happened (t) and the period of the respective future shock ($t+i+k$). Due to $\rho_\varepsilon \in [0,1]$, the weight on ν_t decreases if the time difference between t and the respective future shock increases. Note that apart from ensuring that $i - j \geq 0$, the index j is irrelevant for the final expression as all information sets before time t, where the innovation happens, can be neglected.

Aggregating over all k implies

$$E_{t+i-j}\left(\sum_{k=0}^{\infty} \varepsilon_{t+i+k}\right) = \frac{\rho_\varepsilon^i}{1-\rho_\varepsilon} \nu_t \ . \tag{1.11}$$

Aggregate demand. Inserting this last expression into the IS curve (1.10) yields, for all non-negative i,

$$y_{t+i} = -\mu \frac{\rho_\varepsilon^i}{1-\rho_\varepsilon} \left\{ \omega \nu_t + (1-\omega)\delta \sum_{j=0}^{i} (1-\delta)^j \nu_t \right\} \ ,$$

which can be further simplified to

$$c_{t+i} = y_{t+i} = -\mu \Big\{ 1 - (1-\omega)(1-\delta)^{i+1} \Big\} \frac{\rho_\varepsilon^i}{1-\rho_\varepsilon} \nu_t \ . \tag{1.12}$$

Instead of depending on partly unknown elements, aggregate consumption and output are now solely functions of the innovation at date t, whose quantitative impact depends on the two main parameters of the SI-TANK model. More persistent shocks in the past increase the effect of a policy shock. In addition, the larger the share of hand-to-mouth agents (higher ω) and the lower the degree of information stickiness (higher δ), the larger the impact of shocks from the past.

Mirroring earlier results, only the expectations of agents who updated their information set at or after time t determine spending in equation (1.12). Although the innovation that happened in t affects future consumption and output through the persistence of the shock, the strength of the effect is dampened by the mass of savers who last updated their information sets before the innovation happened. In other words, those savers might have updated $i+1$ periods ago, but clearly remained uninformed since then. Overall, the curly bracket therefore captures the mass of all households that, at time $t+i$, know about the occurred shock. And exactly this group of agents is hit by the TANK multiplier μ, amplifying the output response if income inequality is countercyclical. However, since the mass of informed households is smaller than in a model without information rigidities, the amplification effect is lower as well.

Aggregate supply. Using (1.11) and (1.12) in (1.9) leads to

$$\pi_{t+i} = \beta E_{t+i}(\pi_{t+i+1}) - \Theta\Big\{\chi_{\textit{SI-TANK}}\,\mu\Big[1-(1-\omega)(1-\delta)^{i+1}\Big] - (1-\delta)\Psi\Big[1-(1-\delta)^{i}\Big]\Big\}\frac{\rho_\varepsilon^i}{1-\rho_\varepsilon}\nu_t \; .$$

Similar to the IS curve, the response of inflation is in large part determined by the informed households, captured by the first square bracket. If the constrained agents' income elasticity to aggregate income $\chi_{\textit{SI-TANK}}$ and the TANK multiplier μ are considerable above one, that channel generates a strong amplifying effect. The remaining parts within the curly brackets include some counteractive small-sized effects due to the sluggishness coming from non-updating agents.

Solving the last expression forward and simplifying yields

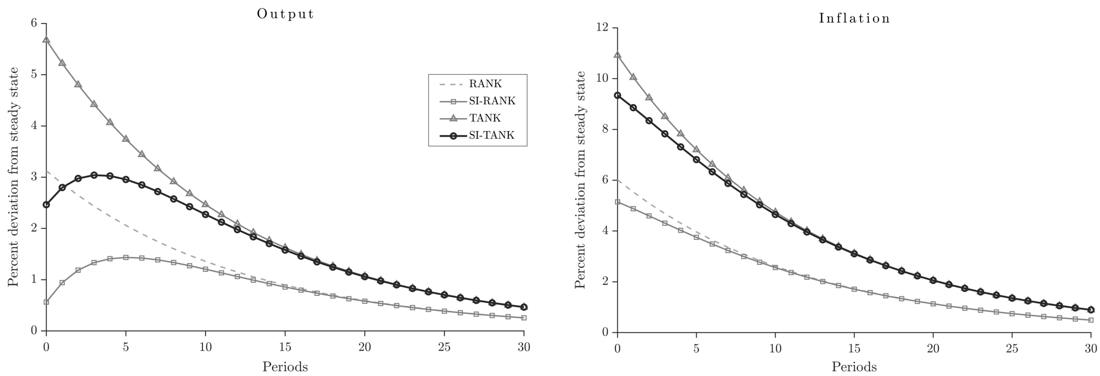
$$\begin{aligned}\pi_{t+i} = -\Theta\Big\{&\chi_{\textit{SI-TANK}}\,\mu\Big[\frac{1}{1-\beta\rho_\varepsilon} - \frac{1}{1-\beta\rho_\varepsilon(1-\delta)}(1-\omega)(1-\delta)^{i+1}\Big]\\ &-(1-\delta)\Psi\Big[\frac{1}{1-\beta\rho_\varepsilon} - \frac{1}{1-\beta\rho_\varepsilon(1-\delta)}(1-\delta)^{i}\Big]\Big\}\frac{\rho_\varepsilon^i}{1-\rho_\varepsilon}\nu_t \; .\end{aligned}$$

The effectiveness of monetary policy in this Phillips curve as before just depends on the model parameters. The response of inflation is reinforced with a higher ρ_ε (more persistent shock), higher ω (more constrained households), or higher δ (less information stickiness). Moreover, as in standard New Keynesian models, a higher share of firms resetting their goods' prices increases Θ and thus the inflation response to the innovation ν_t.

1.5.3 Graphical insights

With the simple reduced-form expressions at hand, I now move on to a graphical analysis and try to quantify the disproportionate effects of amplification and dampening. The object of study is an unanticipated monetary policy shock of 25 basis points that happens at time t and fades out thereafter. Apart from the benchmark representative-agent economy

Figure 1.1: Dynamic responses to monetary policy shock



Notes: Impulse response functions of output and inflation to an expansionary monetary policy shock of 25 basis points for different model specifications: representative-agent New Keynesian model without (RANK) or with sticky information (SI-RANK), and two-agent New Keynesian model without (TANK) or with sticky information (SI-TANK).

($\delta = 1, \omega = 0$) and the SI-TANK model, I am also interested in the individual role of household heterogeneity and sticky information. I isolate each of the two by studying the TANK ($\delta = 1$) and the SI-RANK ($\omega = 0$) models separately.

Figure 1.1 depicts the impulse responses of output (or, equally, aggregate demand) and inflation after a negative interest rate shock for the different model specifications. Appendix 1.F outlines the calibration of the model parameters. Starting with the left graph, several findings with regard to output arise.

First, sticky information dampens the effect of the monetary policy shock on impact and also for several periods after that. It is therefore able to replicate the inertial, hump-shaped impulse response behavior found in empirical studies. Output in the models without a lag in perception immediately jumps on impact and gradually declines thereafter due to the assumed persistence of the monetary policy shock. By contrast, having information frictions in the model leads to a more delayed response. Only households with an updated information set become aware of the policy shock, as observed in the analytical solution in the previous section. This implies that the maximum impact on output only occurs after a few periods. Once all agents have gotten to know the shock, the output response converges to the model alternative without sticky information.

However, sticky information has a disproportionate effect on output, depending on the underlying model economy. A look at the first row of Table 1.5 reveals that it attenuates the aggregate-consumption response on impact proportionally more in RANK than in TANK, namely by a factor of $\frac{\Phi_{RANK}}{\Phi_{SI\text{-}RANK}} = 5.6$ as opposed to $\frac{\Phi_{TANK}}{\Phi_{SI\text{-}TANK}} = 2.3$. This confirms the result from Proposition 1: hand-to-mouth agents do not acquire information and are thus not impacted by any information rigidity. As a result, the output response in SI-

Table 1.5: Impact of monetary policy on aggregate demand: dynamics

	Full information		Sticky information	
	RANK	TANK	RANK	TANK
Impact multiplier	1.00	1.82	0.18	0.79
Peak response	3.13	5.67	1.43	3.04
Cumulative response	33.10	60.10	22.98	47.42

Notes: Effects of an unexpected interest rate cut in the current period on aggregate demand. The table contains the multipliers on impact (Φ_M for model specification M), the responses cumulated over time ($\sum_{i=0}^{T} \beta^i y_i$, where y_i is the response of aggregate demand in period i and $T = 1000$), and the magnitude of the peak impact.

TANK peaks earlier than in SI-RANK where all households are affected. The aggregate degree of inattention to information in the economy is lower.

Second, adding heterogeneity between households permanently amplifies the response to the policy shock throughout all depicted periods in Figure 1.1. The cut in the interest rate triggers the mechanism described before: savers adjust their intertemporal consumption and labor supply decisions, which increases the demand of hand-to-mouth households and induces a multiplier effect. Heterogeneity has thereby a stronger relative impact when combined with sticky information. Table 1.5 implies that it amplifies the output response in RANK in the period of the shock $\frac{\Phi_{TANK}}{\Phi_{RANK}} = 1.8$ times, but in SI-RANK even $\frac{\Phi_{SI\text{-}TANK}}{\Phi_{SI\text{-}RANK}} = 4.4$ times. Although amplification works through both types of households in the two-agent models, it matters for the respective ratio that savers alone are prone to sticky information.

Third, given the current calibration, the output response *on impact* of the shock is lower in SI-TANK compared to RANK. Although $\chi_{SI\text{-}TANK} = 2.72$ and constrained agents' income therefore reacts strongly to changes in aggregate income, the amplifying component seems not to be strong enough and consumption at the aggregate level remains attenuated. However, the hump-shaped form makes it still possible to achieve amplification in a later period. The peak effects in RANK and SI-TANK are quantitatively similar in the present case, but a different set of parameters can make amplification more likely. In particular, more frequent updating would mean that a larger fraction of savers adjust their consumption plans in each period and that aggregate demand would therefore react more. This would in turn have implications for both the magnitude and the timing of the peak impact. I show in Appendix 1.G that a larger δ leads to a higher maximum response.[20]

[20] Likewise, more hand-to-mouth agents (a higher ω) would lead to a higher peak impact by reinforcing the amplification channel. These conclusions can be checked by means of the analytical solution for the IS curve (1.12).

Moreover, output then generally peaks earlier in both SI-RANK and SI-TANK, while its inertia in the latter case is comparable for a broad range of δ values.

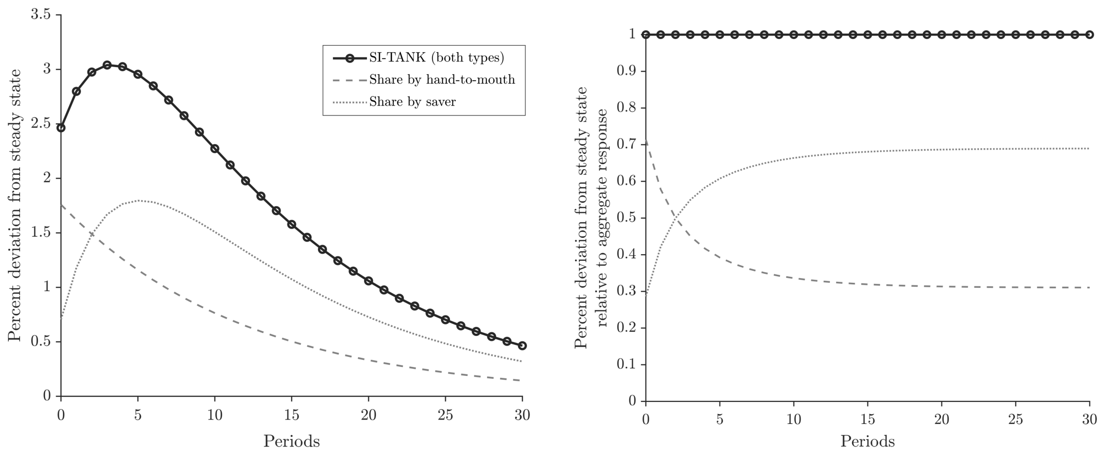
Fourth, even if output moves less on impact, the *cumulative* response can show a different picture. The second row of Table 1.5 illustrates that the present discounted value is much higher in SI-TANK (47.42) compared to RANK (33.10). Note further that Proposition 1 holds in cumulative terms as well. For instance, adding heterogeneity to RANK amplifies the output response by a factor of $\frac{60.10}{33.10} = 1.8$, but by $\frac{47.42}{22.98} = 2.06$ when combined in addition with sticky information. These two findings hold even for extreme calibration values.[21]

Turning to the price response, the right graph of Figure 1.1 reveals some features for the course of inflation after the expansionary monetary policy shock. Compared to aggregate demand, the implications of the considered frictions are rather modest. Especially the already substantial degree of information stickiness has only a limited impact given the strong counteractive amplification effects – as was indicated in the analytical solution for the Phillips curve. Moreover, due to missing information frictions on the part of firms, there is no delayed reaction of inflation. At the same time, asymmetric effects across models are absent: Sticky information dampens the initial inflation response by the same (minor) factor when added to RANK and TANK, respectively, and household heterogeneity amplifies that response similarly when incorporated in RANK or SI-RANK.

Shifting the focus back on the demand side, one might be interested in the individual contribution of each household type to the aggregate impulse response within SI-TANK. Figure 1.2 reveals that it varies over the periods following the monetary policy shock. The left graph displays the split of the aggregate into individual responses. Savers only gradually adjust consumption, while the peak impact for constrained households is in the period in which the shock happens. As one might expect, the sluggishness in aggregate consumption therefore originates alone from the behavior of the intertemporal optimizers who are subject to information frictions. In addition, the relative shares in the right graph indicate that hand-to-mouth agents significantly drive the output response in the periods right after the shock, because only part of the savers are already aware of the latter. The remaining share of savers still acts according to their outdated information sets. If time passes and more savers learn about the shock, the relative contribution of each household type converges to the calibrated value for the agent's share in the population.

[21] See Table 1.G1 in Appendix 1.G for a sensitivity analysis with respect to δ.

Figure 1.2: Unequal shares of household types in output response



Notes: Impulse responses of output to an expansionary monetary policy shock of 25 basis points for the SI-TANK model. Subfigures show absolute (left) and relative (right) shares of each type of household.

1.6 Implications for policymaking and theoretical modeling

The findings of the SI-TANK model reveal some implications for policymaking and the fine-tuning of aggregate demand in practice. Briefly speaking, empirical evidence for constrained households and the degree of information frictions in an economy should be considered together in the design of policy measures. Understanding their interaction is important due to the implications for the transmission and effectiveness of monetary policy and to avoid misleading policy recommendations.

The macroeconomic impact of information frictions and household heterogeneity should not be studied in isolation. As seen in section 1.4.1, the (absolute) impact of the propagation of a policy shock relative to a benchmark RANK model eventually hinges on the magnitude of the two main model parameters in SI-TANK. Various countries may therefore draw different conclusions for policymaking when using such a framework. Building on the figures estimated by Kaplan, Violante, and Weidner (2014), the SI-TANK model indicates that amplification is more likely if it is applied to countries with high shares of hand-to-mouth households such as the U.S., Canada, or the United Kingdom (above 30 percent). It is instead less likely for Euro area countries such as France, Italy, or Spain with smaller shares (around 20 percent), which, all else equal, require a larger monetary policy impulse to achieve a comparable response of aggregate demand. At the same time, a lower degree of information stickiness on the part of households may conduce to amplification: if a central bank tries to stimulate aggregate spending by cutting interest rates, more widespread updating of households' information sets can help.

This paper also opens room for the question of how to precisely model information frictions in macroeconomics. For instance, Auclert et al. (2020) and Carroll et al. (2020) assume that all households are subject to the same amount of stickiness. Each of them adjusts its expectations about macroeconomic variables only sluggishly. The SI-TANK model postulates instead that savers alone are affected by sticky information. This builds on the view that the economy is in part made up of a group of (hand-to-mouth) households that are always at their borrowing constraint and cannot shift consumption across periods by saving. As a consequence, these constrained agents are assumed to be overly short-sighted and ignore any kind of information regarding the state of the economy. Their degree of information rigidity should thus be much different in the data. Unfortunately, to the best of my knowledge, no study so far regards hand-to-mouth agents' degree of inattention separately. There is only evidence of large information rigidities for consumers as a whole (Coibion & Gorodnichenko, 2012, 2015; Mankiw & Reis, 2007; Reis, 2009a, 2009b). The findings I present in section 1.2 are a first rough attempt for a more detailed analysis, but further work in this direction would be needed to show empirically significant differences in the degree of information stickiness for precisely identified household groups.

It is only safe to say that the way of incorporating information frictions in macroeconomic models will influence the evaluation of monetary policy transmission in practice. My findings for the SI-TANK model point at the potential asymmetric interaction of amplification and dampening effects. The impact of monetary policy at the individual-household level will therefore depend on whether a specific group of households updates its information set at all and if so, how often.[22] As seen in the graphical analysis, intertemporal optimizers only get to know new information slowly, while constrained agents are those who drive the output response right after a monetary policy shock.

Finally, to influence the behavior of the public effectively, the way a monetary authority communicates is key. Likewise, it is important how economic agents react to the information provided to them and how they adjust their views about the (future) state of the economy accordingly. In fact, expectation formation is intrinsically linked to communication by nature and affects the formulation of optimal monetary policy. On the other hand, differences in MPCs, labor income or also the degree of information frictions induce households to form distinct expectations as a response to specific policy measures. Given the implications of these elements for aggregate demand, policymakers might actively investigate how to tackle them in the context of communication. Shaping the timing or simplicity of published content and especially the channels through which the public acquires information is clearly of high importance. As an example, reducing households'

[22]It remains to be verified if the revealed asymmetry also arises in setups where all households are affected by information frictions.

costs to gather information can make it easier for a central bank to boost aggregate spending during a recessionary period.

1.7 Conclusion

The literature on macroeconomic models increasingly tries to follow recent empirical evidence and relax the traditional assumption of a representative household with full information. This paper studies the transmission of monetary policy to aggregate demand when incorporating sticky information in a TANK model with heterogeneity in household income. The resulting SI-TANK framework features specific propagation characteristics of a monetary policy shock: the presence of constrained hand-to-mouth households with a high MPC amplifies the impact of a change in the real interest rate with respect to RANK, while the information friction attenuates it.

Focusing on the net response of aggregate consumption on impact of the shock, I find that the effects of monetary policy might be dampened even if income inequality is countercyclical, which is different from recent findings in the literature. As a consequence, amplification only arises if constrained agents' income reacts substantially more than one-to-one to changes in aggregate income. Even more essential, the interaction of sticky information and household heterogeneity generates asymmetric effects on demand. The former attenuates the aggregate-consumption response more in a model without heterogeneous households, while the latter is proportionately more influential in combination with sticky information.

As already shown by some recent work (Auclert et al., 2020; Carroll et al., 2020; Pfäuti & Seyrich, 2022), combining heterogeneous households and information frictions is a convenient way to match both microeconomic and macroeconomic evidence in the data. When using such models for policy analysis, my findings point at the importance to locate the exact source of amplification and how the asymmetry between the two frictions changes the effectiveness of monetary policy. Policymakers need to consider this in the design of policy measures and communication to the public. In addition, it remains crucial to further explore various approaches how to incorporate information frictions in macroeconomic models and whether they are supported in the data.

Appendix

1.A Empirical analysis

1.A.1 Test for the equality of regression coefficients

One can test for the statistical significance of differences between the coefficients of Table 1.1. To do so, I append the data on forecast errors and revisions related to the first quartile of the income distribution separately to the data for each of the other quartiles. I then define a dummy variable that equals 0 for data points from the first quartile and 1 otherwise. With this, we can estimate the following regression for each pair of quartiles:

$$\begin{aligned}\pi_{t+4,t} - F_t\,\pi_{t+4,t} = \alpha + \beta_1\,d_t + \beta_2\,(F_t\,\pi_{t+4,t} - F_{t-1}\,\pi_{t+3,t-1}) \\ + \beta_3\,d_t\,(F_t\,\pi_{t+4,t} - F_{t-1}\,\pi_{t+3,t-1}) + \varepsilon_t\,,\end{aligned} \tag{1.A.1}$$

where d_t is the dummy variable. Note that β_3 is the coefficient on an interaction term between the dummy and the forecast revision. With this specification, $\hat{\beta}_2$ and $\hat{\alpha}$ are going to equal the estimated coefficients for the bottom 25% in Table 1.1, whereas $\hat{\beta}_2 + \hat{\beta}_3$ and $\hat{\alpha} + \hat{\beta}_1$ are going to be consistent with the estimates for the respective other group.

Similar to Section 1.2, we follow Coibion and Gorodnichenko (2015) and estimate equation (1.A.1) using log changes in the oil price as an instrument to account for the correlation between the error term ε_t and variables at time t. The results are shown in Table 1.A1.

The coefficient of interest is β_3. Its estimates are negative for all specifications, but never significantly different from zero. For example, focusing on the Bottom|Top 25% specification with CPI inflation, the p-value of the mentioned coefficient equals 0.263. This value suggests that the estimates of the coefficients on the forecast revision of these two groups in Table 1.1 (1.560 and 0.804) are statistically different from each other with only a low probability. This finding is also reflected in the 95% confidence bands of the two estimates, which overlap in large part ($[0.382, 2.737]$ for the first quartile and $[0.203, 1.406]$ for the fourth quartile). The estimation results should therefore be treated with caution and only be seen as a rough guidance for the structure of the theoretical model.

Table 1.A1: IV estimates of information rigidity in consumers' inflation forecasts

	Inflation expectations for quartile pairs of income distribution		
Forecast error	Bottom|Second 25%	Bottom|Third 25%	Bottom|Top 25%
CPI			
Forecast revision	1.560***	1.560***	1.560***
	(0.601)	(0.601)	(0.601)
Interaction term	−0.869	−0.736	−0.756
	(0.670)	(0.668)	(0.674)
Dummy	0.638***	1.117***	1.575***
	(0.244)	(0.242)	(0.238)
Constant	−1.969***	−1.969***	−1.969***
	(0.186)	(0.186)	(0.186)
Observations	320	320	320
PCE (real-time)			
Forecast revision	1.032**	1.032**	1.032**
	(0.454)	(0.454)	(0.454)
Interaction term	−0.683	−0.593	−0.607
	(0.513)	(0.510)	(0.515)
Dummy	0.642***	1.120***	1.579***
	(0.203)	(0.203)	(0.202)
Constant	−2.391***	−2.391***	−2.391***
	(0.153)	(0.153)	(0.153)
Observations	320	320	320

Notes: Coefficient estimates of the instrumental variable regression equation (1.A.1) using MSC data, with Newey-West standard errors in parentheses. The dependent variable is the mean year-ahead forecast error for inflation and the forecast revision is defined as the change in the mean year-ahead forecast. The instrumental variable is the log change in the oil price. Sample period is 1980–2019.
* p<0.10, ** p<0.05, *** p<0.01

1.B Model derivations

1.B.1 Saver's optimization problem

Each saver has utility from consumption and leisure, $U\left(C^S_{t,j}, L^S_t\right) = \ln C^S_{t,j} - \xi \frac{\left(L^S_t\right)^{1+\eta}+1}{1+\eta}$, and is subject to the following budget constraint:

$$C^S_{t,j} + \frac{B^S_{t,j}}{P_t} = \frac{W_t}{P_t} L^S_t + (1+i_{t-1}) \frac{B^S_{t-1,j}}{P_t} + \frac{1}{1-\omega} D_t + T_{t,j} \; . \tag{1.B.1}$$

Since transfers make sure that all savers start into a period with the same level of real wealth, we can define the right-hand side as $X^S_{t,j} = X^S_t$. From this, rewriting the flow of budget constraints, the intertemporal optimization problem of an inattentive saver j choosing a plan for current and future consumption (with $i = 0, 1, 2, \ldots$) at time t is

$$\begin{aligned} V\left(X^S_t\right) = \max_{\{C^S_{t+i,i},\, L^S_{t+i}\}} & \left\{ \sum_{i=0}^{\infty} \beta^i (1-\delta)^i \left[\ln C^S_{t+i,i} - \xi \frac{\left(L^S_{t+i}\right)^{1+\eta}+1}{1+\eta} \right] \right. \\ & \left. + \beta\delta \sum_{i=0}^{\infty} \beta^i (1-\delta)^i E_t \left[V(X^S_{t+1+i}) \right] \right\} , \\ \text{s.t. } X^S_{t+1+i} = & \frac{W_{t+1+i,.}}{P_{t+1+i}} L^S_{t+1+i,.} + R_{t+1+i}\left(X^S_{t+i} - C^S_{t+i,.}\right) + D_{t+1+i} + T_{t+1+i,.} \; , \end{aligned}$$

where $V(.)$ denotes the agent's value function conditional on date t being a planning date, $R_{t+1} = (1+i_t)\frac{P_t}{P_{t+1}}$ is the gross real return on bonds between periods t and $t+1$, $\beta \in (0,1)$ is the discount factor, and $C^S_{t,j}$ denotes individual consumption at time t of a saver who last updated his information set j periods ago. The j-subscripts in the budget constraint were replaced by dots, indicating that all savers arrive in period t with the same real resources, regardless of when they last updated.

The value function consists of two parts. The first term captures the expected discounted utility that the saver gets if he does not update his information set in any period from time t on.[23] The second part contains the continuation value functions for the potential case in which the household updates again at some point in the future. This can happen with probability $\delta(1-\delta)^i$ in each period.

[23] Strictly speaking, the information friction exclusively affects the consumption decision. How many hours to work is decided without considering when information was last updated because all savers choose the same labor supply in each period. As a result, regardless of whether the problem of consumers and workers are separated as in Mankiw and Reis (2006, 2007), or combined into a single problem, the log-linearized equilibrium will be equal for the model specification here.

The optimality conditions (for $i = 0, 1, 2, \ldots$) are given by

$$\beta^i (1-\delta)^i \left(C^S_{t+i,i}\right)^{-1} = \beta\delta \sum_{k=i}^{\infty} \beta^k (1-\delta)^k E_t \left[V'(X^S_{t+1+k}) \overline{R}_{t+i,t+1+k}\right] ,$$

$$\beta^i (1-\delta)^i \xi \left(L^S_{t+i}\right)^{\eta} = \beta\delta \sum_{k=i}^{\infty} \beta^k (1-\delta)^k E_t \left[V'(X^S_{t+1+k}) \overline{R}_{t+i,t+1+k}\right] \frac{W_{t+i}}{P_{t+i}} ,$$

$$V'(X^S_t) = \beta\delta \sum_{k=0}^{\infty} \beta^k (1-\delta)^k E_t \left[V'(X^S_{t+1+k}) \overline{R}_{t,t+1+k}\right] ,$$

with $\overline{R}_{t+i,t+1+k} = \prod_{z=t+i}^{t+k} R_{z+1}$ being the compound return between two periods $t+i$ and $t+1+k$. Setting the first condition for $i = 0$ equal to the envelope condition yields $V'(X^S_t) = \left(C^S_{t,0}\right)^{-1}$. Inserting this result into the three optimality conditions gives the Euler equations for both attentive and inattentive households and the usual intratemporal condition:

$$1 = \beta E_t \left[R_{t+1} \left(\frac{C^S_{t+1,0}}{C^S_{t,0}} \right)^{-1} \right] ,$$

$$1 = E_t \left[\left(\frac{C^S_{t+j,0}}{C^S_{t+j,j}} \right)^{-1} \right] ,$$

$$\frac{W_t}{P_t} = \xi C^S_{t,0} \left(L^S_t\right)^{\eta} .$$

1.B.2 Derivation of the reduced-form model representation

The non-linear model is approximated around a non-stochastic steady state as described in section 1.3.6. The resulting log-linearized model conditions are the following:

$$c^S_{t,0} = E_t \left(c^S_{t+1,0} - r_t\right) \tag{1.B.2}$$

$$c^S_{t,j} = E_{t-j} \left(c^S_{t,0}\right) \tag{1.B.3}$$

$$c^S_t = \delta \sum_{j=0}^{\infty} (1-\delta)^j c^S_{t,j} \tag{1.B.4}$$

$$\eta l^S_t = w_t - c^S_{t,0} \tag{1.B.5}$$

$$\eta l^H_t = w_t - c^H_t \tag{1.B.6}$$

$$c^H_t = w_t + l^H_t \tag{1.B.7}$$

$$c^S_t = w_t + l^S_t + \frac{1}{1-\omega} d_t + t_t \tag{1.B.8}$$

$$y_t = l_t \tag{1.B.9}$$

$$mc_t = w_t \tag{1.B.10}$$

$$d_t = -mc_t \tag{1.B.11}$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \Theta mc_t, \;\; \Theta = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda} \tag{1.B.12}$$

$$i_t = E_t(\pi_{t+1}) + \varepsilon_t \tag{1.B.13}$$

$$i_t = r_t + E_t(\pi_{t+1}) \tag{1.B.14}$$

$$c_t = \omega c_t^H + (1-\omega)c_t^S \tag{1.B.15}$$

$$l_t = \omega l_t^H + (1-\omega)l_t^S \tag{1.B.16}$$

$$y_t = c_t \tag{1.B.17}$$

Using these conditions, I can derive the aggregate demand (IS curve) and aggregate supply (Phillips curve) equations in reduced form.

Aggregate demand. I start by iterating equation (1.B.2) forward. In the limit as time goes to infinity, all agents will be fully informed. Therefore, $\lim_{i\to\infty} E_t\left(r_{t+i}\right) = \lim_{i\to\infty} E_t\left(r_{t+i}^n\right) = 0$ and $\lim_{i\to\infty} E_t\left(c_{t+i,0}^S\right) = \lim_{i\to\infty} E_t\left(y_{t+i}^{S,n}\right) = 0$, where the superscript n is used to denote the natural equilibrium without any frictions such that all agents are attentive. This leaves us with $c_{t,0}^S = -R_t$, where $R_t = E_t\left(\sum_{k=0}^{\infty} r_{t+k}\right)$. Inserting this expression into (1.B.3), combining the result with (1.B.4) and using (1.B.17) leads to the Euler equation (1.5) of the main text that governs the bond holding decision of savers.

To derive an aggregate Euler equation, first note that the labor supply of hand-to-mouth households is fully inelastic because I assumed unity for the intertemporal elasticity of substitution, as shown in Bilbiie (2008). In other words, their hours worked are constant, such that $l_t^H = 0$. Combining (1.B.5), (1.B.7), (1.B.9) and (1.B.16) yields

$$c_t^H = \eta \frac{1}{1-\omega} y_t + c_{t,0}^S \, . \tag{1.B.18}$$

By (1.B.4), one gets $c_t^S = \delta c_{t,0}^S + \delta \sum_{j=1}^{\infty} (1-\delta)^j E_{t-j}\left(c_{t,0}^S\right)$. Combined with (1.B.15) and replacing $c_{t,0}^S$ in (1.B.18) leads to equation (1.6) of the main text, the consumption of constrained agents as a function of aggregate output and real interest rates.

Finally, using (1.6) together with (1.B.17) in (1.B.15) gives us an expression for c_t^S that can be combined with the Euler equation of savers to find the IS curve (1.8) of the SI-TANK model.

Aggregate supply. By (1.B.6) and (1.B.10), we get $mc_t = c_t^H$. Using (1.6) and replacing the real marginal cost in (1.B.12) results in the Phillips curve (1.9) of the SI-TANK model.

1.C Dependence of shock propagation on main parameters

The key driver for the propagation of monetary policy shocks in the SI-TANK model is the relative proportion between the share of hand-to-mouth agents (ω) and the degree of information stickiness $(1-\delta)$. Focusing on the *initial* consumption and output responses on impact of the shock, in order to get amplification of the effects of a change in real interest rates, the following threshold condition must hold:

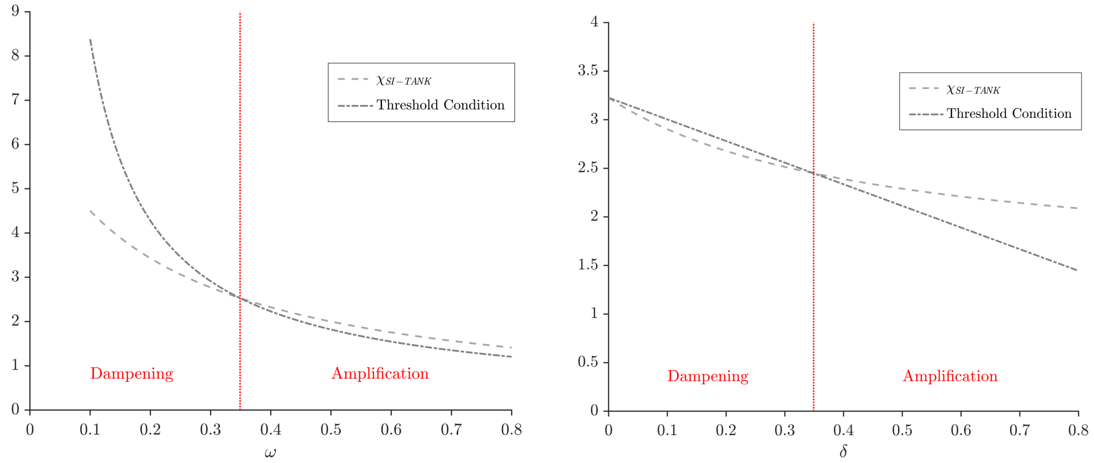
$$\chi_{\text{SI-TANK}} > \frac{1-\delta}{\omega} + \delta \geq 1 \ ,$$

where $\chi_{\text{SI-TANK}} = \frac{1+\delta\eta}{\omega+(1-\omega)\delta}$. Otherwise, there is dampening. See section 1.4.1 for more details.

Assuming conventional parameter values (see Table 1.F1), Figure 1.C1 shows how likely amplification arises relative to dampening. Given the baseline values of $\omega = 0.31$ and $\delta = 0.18$, respectively, the particular other parameter has to be relatively high to achieve amplification of monetary policy effects on impact. However, regarding the absolute values, one needs to consider the simplicity of the model at hand. An extended model including, for example, fiscal redistribution as in Bilbiie (2020) might narrow the dampening region. Moreover, as discussed in the graphical analysis in section 1.5.3, one also needs to consider the further course of the response. Even if the effect of the policy shock was attenuated on impact, the sticky-information assumption and the resulting hump-shaped behavior of output would still allow to get amplification in subsequent periods.

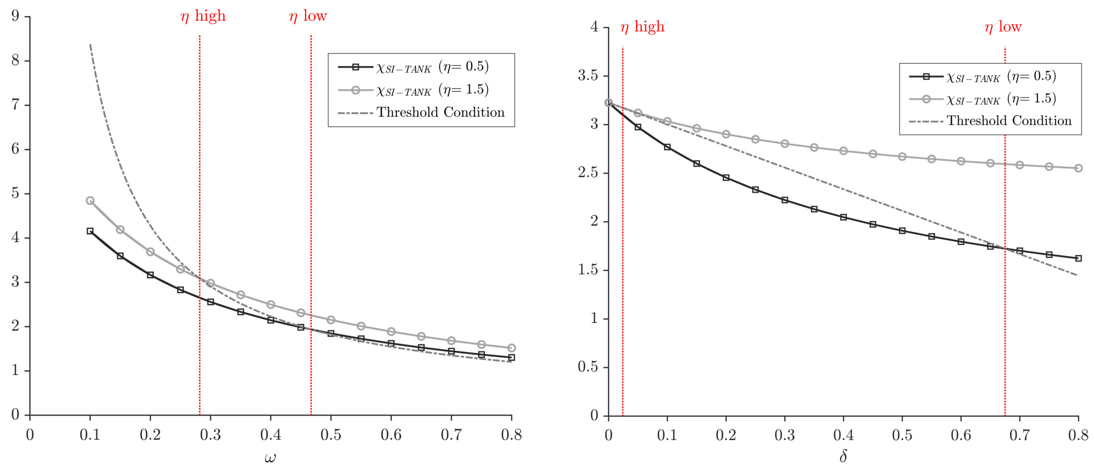
Another parameter that determines the propagation regions is η, the inverse of the Frisch elasticity of labor supply. As Figure 1.C2 shows, the higher its value, the lower is the threshold between dampening and amplification along the distributions of both ω and δ. Thus, a more inelastic labor supply elasticity (i.e., a lower $1/\eta$) compared to the benchmark case in Figure 1.C1 implies that amplification becomes more likely to arise in economies with even a low amount of hand-to-mouth households and a high degree of information stickiness, respectively.

Figure 1.C1: Impact of main model parameters on the propagation of monetary policy



Notes: Propagation regions along the distribution of the share of hand-to-mouth households ω (left graph; $\delta = 0.18$) or of the information stickiness parameter δ (right graph; $\omega = 0.31$). Labor supply elasticity is set to $1/\eta = 1$. The amplification region is characterized by the constrained agents' income elasticity to aggregate income $\chi_{SI\text{-}TANK}$ being larger than the threshold condition $\frac{1-\delta}{\omega} + \delta$. Left graph: The lowest part of the distribution is not depicted because the threshold condition is exploding for very small values of ω.

Figure 1.C2: Impact of labor supply elasticity on the propagation of monetary policy



Notes: Propagation regions along the distribution of the share of hand-to-mouth households ω (left graph; $\delta = 0.18$) or of the information stickiness parameter δ (right graph; $\omega = 0.31$), given alternative values for the inverse of the Frisch elasticity of labor supply. Left graph: The lowest part of the distribution is not depicted because the threshold condition is exploding for very small values of ω.

1.D Dependence of multiplier wedges on main parameters

The degree of asymmetry regarding the *initial* impact of monetary policy shocks on aggregate consumption and output in the SI-TANK model is significantly influenced by the share of hand-to-mouth agents (ω) and the degree of information stickiness ($1-\delta$). Assuming conventional parameter values (see Table 1.F1), Figure 1.D1 shows how the wedge between the ratios of different aggregate-demand multipliers varies with these two parameters.

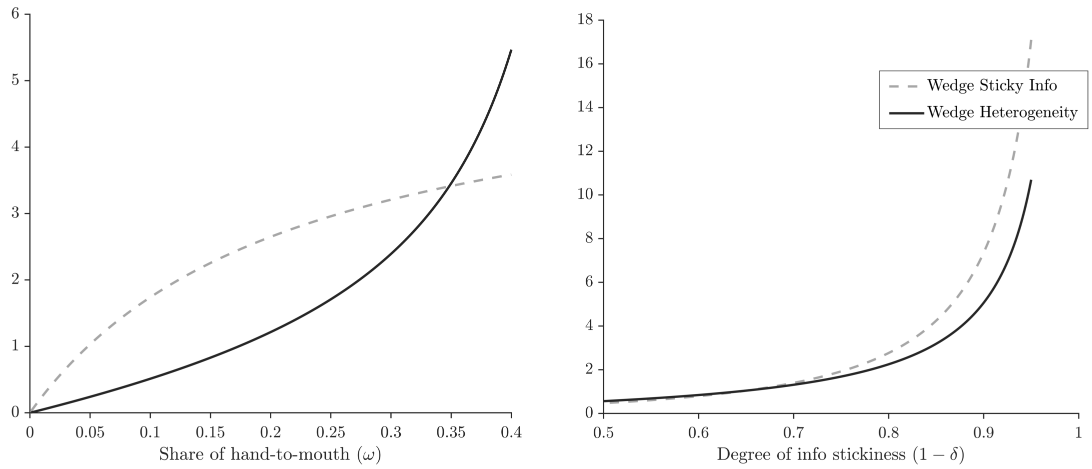
The ratios in question arise from Proposition 1. In particular, I define two wedges:

(i) Wedge from sticky information: $\frac{\Phi_{RANK}}{\Phi_{SI\text{-}RANK}} - \frac{\Phi_{TANK}}{\Phi_{SI\text{-}TANK}} \geq 0$

(ii) Wedge from heterogeneity: $\frac{\Phi_{SI\text{-}TANK}}{\Phi_{SI\text{-}RANK}} - \frac{\Phi_{TANK}}{\Phi_{RANK}} \geq 0$

where Φ_M is the aggregate-demand multiplier on impact of a monetary policy shock. A larger difference between any two fractions indicates a higher degree of asymmetry, which is equivalent to a more pronounced inequality between the ratios in sub-propositions (I) and (II) of Proposition 1. As Figure 1.D1 shows, the differences increase in both ω and $1-\delta$. A higher share of hand-to-mouth households results in stronger amplification, whereby the latter is relatively more influential in SI-TANK compared to TANK. Similarly, stickier information translates into a more dampened output response in SI-TANK, but even more attenuation in SI-RANK.

Figure 1.D1: Impact of the main model parameters on the multiplier ratios



Notes: Differences in ratios between aggregate-demand multipliers of various model specifications. Left graph: wedges along the distribution of ω with information stickiness set to $\delta = 0.18$. Right graph: wedges along the distribution of δ with a share of hand-to-mouth households of $\omega = 0.31$.

1.E General analytical solution of lagged expectations

For the general solution of the expectation expressions, I only rule out future anticipated shocks, but shocks in the past (i.e., before t) are considered. Using the forwarded AR(1) process for the shock and the assumption that the innovation ν has mean zero, one gets $E_t(\varepsilon_{t+k}) = \rho_\varepsilon^k \varepsilon_t$, $E_{t-1}(\varepsilon_{t+k}) = \rho_\varepsilon^{k+1}\varepsilon_{t-1}$, etc. More general, for any $j \geq 0$ and $t = t + i$ with $i \geq 0$,

$$E_{t+i-j}(\varepsilon_{t+i+k}) = \rho_\varepsilon^{j+k}\varepsilon_{t+i-j} \, .$$

An agent's expectation at any point in time of current and future policy shocks is equal to the product of the shock that the agent observed at the time he last updated and the overall series of persistence coefficients since then. The latter accumulate over $(t+i+k) - (t+i-j) = j+k$ periods.

Summing over all $k \geq 0$,

$$E_{t+i-j}\left(\sum_{k=0}^{\infty} \varepsilon_{t+i+k}\right) = \frac{\rho_\varepsilon^j}{1-\rho_\varepsilon}\varepsilon_{t+i-j} \, .$$

Combined with the IS curve yields, for all non-negative i,

$$c_{t+i} = y_{t+i} = -\frac{\mu}{1-\rho_\varepsilon}\left\{\omega\varepsilon_{t+i} + (1-\omega)\delta\sum_{j=0}^{\infty}(1-\delta)^j\rho_\varepsilon^j\varepsilon_{t+i-j}\right\} \, .$$

Aggregate consumption and output are not anymore functions of expectation expressions and future policy shocks shocks that are partly unknown, but only depend on current and past shocks that were observed by an agent being in period $t+i$. Spending after time t thereby reacts more with a larger persistence of the shock (higher ρ_ε), more hand-to-mouth households (higher ω), and stickier information (smaller δ).

1.F Parameterization for the graphical analysis

Table 1.F1 shows the calibration of the model parameters used for the analysis in section 1.5.3. I take the baseline value for the information stickiness parameter from the estimates for the United States in Mankiw and Reis (2007). The value implies that consumers update their information on average about every 16 months.[24] The baseline value for the share of hand-to-mouth households, taken from Kaplan et al. (2014), is an average value for the United States over the period 1989-2010. I assume a Calvo stickiness parameter of 0.75,

[24] Reis (2009a, 2009b) confirms the relatively high inattentiveness to information of consumers. He estimates $\delta = 0.08$ for the U.S. and $\delta = 0.21$ for the Euro area. A slightly more moderate value of $\delta = 0.25$ is found by Mankiw and Reis (2006), which indicates that consumers update their information on average once a year.

implying average price duration of four quarters. Labor supply elasticity is set to one. Finally, the persistence of the policy shock is assumed to be 0.92, just as in Mankiw and Reis (2006).

Table 1.F1: Calibration

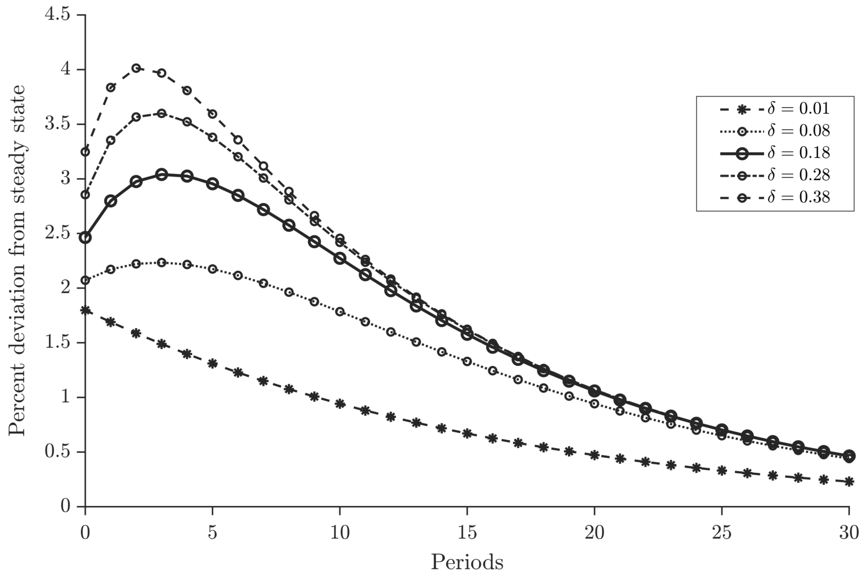
Parameter	Value	Description
δ	**0.18**, 1.0	Probability of updating information set
ω	0, **0.31**	Share of hand-to-mouth households
λ	0.75	Probability of not resetting price
β	0.99	Household discount factor
$1/\eta$	1	Frisch labor supply elasticity
ρ_ε	0.92	Persistence of monetary policy shock

Note: Baseline values in bold.

1.G Sensitivity analysis for information stickiness

Information frictions are an important driver of the dynamic effects of a monetary policy shock. Table 1.G1 lists different response measures for alternative values of the information stickiness parameter δ. We can draw several conclusions from it. First and intuitively, changes in δ leave the impact, cumulative, and peak responses under full information all unaffected. Second, more frequent information updating makes a larger fraction of savers aware of the policy shock and thus implies higher values for all listed responses of the SI-RANK and SI-TANK models. This result is also visible in Figure 1.G1. Third, an increase in δ also leads to earlier peaks of output in SI-RANK, while the period of the maximum impact is less variable in SI-TANK due to the smaller group of households for which the information frictions matter. The only exception is the case where savers are almost fully inattentive to information (i.e., $\delta = 0.01$) and overall updating in the economy is very slow, which is why the peak occurs already on impact of the shock. See also Figure 1.G1. Fourth, comparing the ratios between various responses implies that Proposition 1 not only holds for initial aggregate-consumption responses, but also at the cumulative level.

Figure 1.G1: Dynamic response of output to monetary policy shock



Notes: Impulse response function of output in thhe SI-TANK model to an expansionary monetary policy shock of 25 basis points for alternative values of the information stickiness parameter δ. A lower value of δ denotes a smaller probability of obtaining new information and thus more rigid information.

Table 1.G1: Impact of monetary policy on aggregate demand: sensitivity of dynamics

		Full information		Sticky information	
		RANK	TANK	RANK	TANK
$\delta = 0.01$	Response on impact	3.13	5.67	0.03	1.80
	Cumulative response	33.10	60.10	2.90	22.27
	Peak response	3.13	5.67	0.14	1.80
	Peak period	1	1	11	1
$\delta = 0.08$	Response on impact	3.13	5.67	0.25	2.07
	Cumulative response	33.10	60.10	15.43	37.97
	Peak response	3.13	5.67	0.85	2.23
	Peak period	1	1	8	4
$\delta = 0.18$	Response on impact	3.13	5.67	0.56	2.46
	Cumulative response	33.10	60.10	22.98	47.42
	Peak response	3.13	5.67	1.43	3.04
	Peak period	1	1	6	4
$\delta = 0.28$	Response on impact	3.13	5.67	0.88	2.86
	Cumulative response	33.10	60.10	26.56	51.91
	Peak response	3.13	5.67	1.81	3.60
	Peak period	1	1	5	4
$\delta = 0.38$	Response on impact	3.13	5.67	0.88	2.86
	Cumulative response	33.10	60.10	28.65	54.52
	Peak impact	3.13	5.67	2.07	4.01
	Peak period	1	1	4	3

Notes: Effects of an unexpected interest rate cut in the current period on aggregate demand. The table contains the responses both on impact and cumulated over time ($\sum_{i=0}^{T} \beta^i y_i$, where y_i is the response of aggregate demand in period i and $T = 1000$), and the magnitude and period of the peak impact.

Chapter 2

On the Distributional Effects of Conventional Monetary Policy and Forward Guidance[†]

2.1 Introduction

The relationship between monetary policy and inequality has been a core topic in macroeconomics in recent years. At the same time, the policy tools available to monetary authorities to achieve their mandates have increased in scope and complexity. Despite the fact that several central banks have increasingly relied on unconventional monetary policies like forward guidance, only little is known about their distributional impact.[1] Understanding the different channels through which monetary policy can impact households and firms beyond the standard aggregate macroeconomic effects has especially become of utmost importance in the post-Covid-19 period when inflation rates worldwide reached historically high levels. To tackle the current surge in prices, monetary authorities need to decide about the optimal set of policies to implement and this debate cannot abstract from considering the second-order effects that particular policy tools might involve.

In this paper, we study empirically and theoretically the distributional effects of forward guidance as compared to conventional monetary policy. We document that the two policies have a similar impact on aggregate macroeconomic variables, but opposite effects on the cross-sectional distribution of consumption: a contractionary conventional policy shock leads to an increase in consumption inequality whereas forward guidance decreases it. We then evaluate the potential driving forces of this result through the lens of a two-

[†]This chapter is co-authored with Giacomo Mangiante from the University of Lausanne.

[1]See Colciago, Samarina, and de Haan (2019) for a comprehensive summary of the existing evidence.

agent New Keynesian (TANK) model with household heterogeneity. A transfer system in which the fiscal authority reacts to changes in the government's debt burden and in the business cycle allows us to replicate the empirical evidence.

Our first contribution is to evaluate the diverse macroeconomic and distributional implications of conventional monetary policy and forward guidance empirically. We exploit U.S. household-level survey data from the Consumer Expenditure Survey (CEX) to compute a measure of consumption inequality defined as the cross-sectional standard deviation of real consumption across households. We include this measure together with macroeconomic and financial variables in a common vector autoregressive (VAR) model and use monetary policy factors extracted by Swanson (2021) from high-frequency asset price movements around monetary policy events to disentangle the impact of the two policies.

This approach enables us to document three stylized facts about the effects of conventional monetary policy and forward guidance. First, aggregate macroeconomic variables show *similar* and significant responses to both policies. Following a contractionary shock of either type, real output decreases persistently over time and inflation shows a gradual fall after a few quarters. Second, with respect to the business cycle, consumption inequality is *countercyclical* after a conventional monetary policy shock but *procyclical* after a forward guidance announcement. The reaction is immediate in both cases and particularly strong after an announcement of an interest rate change in the future. Third, we document that the opposite inequality responses emerge from the different sensitivity to each shock at the two tails of the consumption distribution. Households at the *bottom* of the distribution disproportionately reduce their spending in response to a contractionary conventional policy shock, leading to an increase in inequality. In contrast, following a forward guidance shock, households at the *top* of the consumption distribution are those that decrease their consumption the most, thus reducing inequality.

In the next step, we provide a potential explanation for the various cyclicality of inequality observed in the data, namely the fiscal response to the two shocks. Bonds issued by the government to finance its expenditures are one natural example of an asset that is directly affected by interest rate movements. A central bank policy rate hike increases interest payments on government debt or can decrease the price of newly issued bonds through higher yields. This impacts the government's budget and, all else equal, limits its spending capability, resulting in fiscal adjustments. In our VAR model, we use transfer income received by households as a proxy for the government's response to monetary shocks because it is directly linked to households' budget constraints and therefore affects consumption decisions.

We find that there are clear differences in the impulse responses to conventional monetary policy and forward guidance, respectively, both for aggregate transfers and for the average transfer income of households at the bottom of the consumption distribution. In particular, the data imply that households at the left tail of the distribution considerably drive the response of total transfers in the periods after the shock in the case of conventional monetary policy, but are almost unresponsive to forward guidance.

The second main contribution of the paper appears in the form of a theoretical framework with the aim of rationalizing the facts uncovered in the data. We build an analytical TANK model as in Bilbiie (2008, 2020) with heterogeneity in household income and with debt in positive net supply. The setup comprises two types of households. The first type can smooth consumption by saving in government bonds, while the other agent lives hand-to-mouth and consumes its entire disposable income in each period. Households of the latter type are financially constrained through their lack of access to asset markets, which makes their individual income oversensitive to changes in monetary policy. The fiscal authority will try to partly attenuate any income fluctuations by means of a particular fiscal policy mix comprising two elements: a redistribution of monopoly profits between households and a lump-sum transfer scheme that adjusts in response to changes in the government's budget and to cyclical variations.[2] The transfer in lump sum form will determine the inequality response endogenously, together with the profit redistribution that determines it in the absence of such transfers.

We use the model to provide a set of analytical results. We first derive a closed-form solution for consumption inequality as a function of transfers to hand-to-mouth households and (expected) real interest rates. This allows us to determine analytically the condition required for any *arbitrary* transfer function to replicate the cyclical behavior of inequality after a policy rate change today or in the future. Drawing on this result, we propose an example of a function that determines the transfer income received by financially constrained agents. It consists of a debt component linking it directly to the fiscal budget, but also of a cyclical component making the government react to fluctuations in output.

We calibrate the model to the U.S. economy and show that it can broadly replicate the empirical facts about the responses of macroeconomic variables, consumption inequality, and transfers. In particular, following a contemporaneous rise in the real interest rate, the government's debt burden increases immediately and triggers an instant fiscal adjustment which affects the consumption response of households. In comparison, the fiscal authority only partially adjusts transfers after a forward guidance shock as the actual rate hike and

[2] Heathcote, Perri, and Violante (2010) document for the U.S. that public transfers are particularly important to stabilize income variations and compress inequality for households at the bottom of the income distribution.

thus the higher interest payments on public debt lie in the future. The fiscal adjustment differing in timing and magnitude explains the increase in inequality under conventional monetary policy and its decrease under forward guidance.

As an extension of the baseline model, we evaluate whether our results continue to hold and whether the same mechanisms are present in a more complex framework with two assets of different liquidity and investment – a setup that comprises well-known channels of standard heterogeneous-agent New Keynesian (HANK) models, but is still tractable enough to examine the underlying transmission mechanisms. We thereby draw on the two-agent version of the benchmark HANK model developed by Kaplan, Moll, and Violante (2018). The findings are consistent with those of the simpler one-asset model, not only in terms of the sign and shape of the macroeconomic and consumption inequality responses, but largely also in magnitudes.

Central banks around the world have responded to the recent increase in inflation rates by raising their key interest rates considerably and with different mixtures of policy tools. Similarly, governments have announced new fiscal transfers to compensate households for the increase in energy costs. Against this backdrop, our paper sheds new light on the interaction between such monetary and fiscal policies. The timing and magnitude of the fiscal adjustment to a central bank's decisions are of utmost importance to reduce the negative second-order effects and to counteract an increase in economic inequality. At the same time, inequality matters for the transmission of monetary policy and taking it into account when deciding about the optimal monetary policy mix can turn out to be beneficial.

Related literature. This paper contributes to three strands of the literature. First, the results complement the large body of empirical evidence on the effects of monetary policy on consumption and income inequality.[3] Using the same survey data as us and various measures of dispersion, Coibion, Gorodnichenko, Kueng, and Silvia (2017) show that consumption and income inequality in the U.S. have a countercyclical response to contractionary monetary policy shocks. This result has been confirmed for the United Kingdom (Mumtaz & Theophilopoulou, 2017) and, with respect to income inequality, for the euro area (Guerello, 2018; Samarina & Nguyen, 2019) and for a panel of 32 advanced and emerging economies (Furceri, Loungani, & Zdzienicka, 2018). Other authors, however, find procyclical responses, namely for consumption inequality in the U.S. (Chang & Schorfheide, 2022) or income inequality in the U.S. and the United Kingdom (Cloyne, Fer-

[3] See Attanasio and Pistaferri (2016) for a discussion about the evolution of U.S. consumption inequality and a comparison with trends in income inequality. Moreover, Colciago et al. (2019) provide a recent summary of empirical evidence and theoretical literature regarding the relationship between (unconventional) monetary policy and income and wealth inequality.

reira, & Surico, 2020). In contrast, consumption inequality shows only a minor response to monetary policy shocks in Japan (Inui, Sudo, & Yamada, 2017).

Turning to the distributional consequences of unconventional policies, the empirical evidence is much scarcer and sometimes conflicting in its conclusions. Authors often focus on large-scale asset purchases within the context of quantitative easing programs. For instance, Guerello (2018) and Lenza and Slacalek (2021) provide evidence that quantitative easing reduced the income dispersion in several European countries, while Montecino and Epstein (2015) and Mumtaz and Theophilopoulou (2017) find the opposite for the U.S. and the United Kingdom, respectively. Saiki and Frost (2014) document that expansionary unconventional policy measures implemented in Japan increased income inequality, while Inui et al. (2017) find insignificant effects.

We extend this literature by analyzing the aggregate and distributional responses to forward guidance in comparison to conventional monetary policy for the case of the U.S. economy. To the best of our knowledge, this paper is the first to study empirically the separate impact of this unconventional policy tool on the consumption distribution of households. Our results suggest that a standard monetary contraction increases consumption inequality, but a contractionary forward guidance announcement decreases it.

Second, we borrow from the literature that uses high-frequency asset price movements around monetary policy events to identify monetary shocks (Altavilla, Brugnolini, Gürkaynak, Motto, & Ragusa, 2019; Andrade & Ferroni, 2021; Bundick & Smith, 2020; Ferreira, 2022; Gertler & Karadi, 2015; Gürkaynak et al., 2005; Jarociński & Karadi, 2020; Kuttner, 2001; Lakdawala, 2019).[4] The general idea is to extract the surprise component of policy actions on days with monetary policy announcements. To disentangle conventional monetary policy shocks from forward guidance shocks, we use the monetary policy surprises computed by Swanson (2021). These are decomposed into different factors which measure unexpected variations in asset prices at short, intermediate, and long maturities, respectively. We complement the existing studies on the macroeconomic effects of forward guidance (e.g., Bundick & Smith, 2020; Ferreira, 2022; Lakdawala, 2019) by investigating its distributional aspects.

Third, this paper also contributes to the growing literature on the transmission of monetary policy in heterogeneous-agent models. Part of this literature studies the propagation of conventional monetary policy and the interaction with different household characteristics (e.g., Auclert, 2019; Auclert et al., 2020; Kaplan et al., 2018; Luetticke, 2021). Other work focuses specifically on the transmission of forward guidance and addresses the magnitude of its aggregate effects (Acharya & Dogra, 2020; Bilbiie, 2018; Farhi & Werning, 2019;

[4]See Ramey (2016) for a comprehensive overview of alternative identification approaches for monetary policy and other shocks.

Ferrante & Paustian, 2019, 2020; Hagedorn, Luo, Manovskii, & Mitman, 2019; McKay, Nakamura, & Steinsson, 2016; Werning, 2015).

Our paper relates in particular to the studies that assign a key role to how fiscal policy, in terms of transfers or the redistribution of monopolistic firms' profits, responds to monetary policy changes. As shown for the two-agent models in Bilbiie (2008, 2018, 2020) or Bilbiie et al. (2022), the extent to which fiscal redistribution results in a procyclical or countercyclical response of inequality is critical for several features of these models, such as the transmission of monetary policy to aggregate demand or the power of forward guidance. The latter also crucially depends on the degree of countercyclical transfers as illustrated by Gerke, Giesen, and Scheer (2020). The importance of the government's response is also well-known in fully-fledged heterogeneous-agent models. Kaplan et al. (2018) show that the type of fiscal response to a monetary policy shock considerably shapes its macroeconomic effects. Kaplan, Moll, and Violante (2016) extend the analysis to forward guidance shocks and Evans (2022) emphasizes that various profit distribution schemes significantly affect the sensitivity of income and consumption to monetary shocks.

We contribute to the literature on heterogeneous-agent models by studying how the interaction between monetary and fiscal policy influences the inequality response after conventional monetary and forward guidance shocks. We do this in a standard two-agent model that allows us to derive analytical solutions and to illustrate the role of (fiscal) redistribution. Our empirical and theoretical analysis suggests that the government's response under the two monetary policies is key for their propagation and to understanding the cyclicality of consumption inequality.

Outline. The remainder of the paper is organized as follows. Section 2.2 describes the data we use for the empirical analysis and Section 2.3 the empirical specification we adopt to evaluate the effects of the monetary shocks on consumption inequality. Section 2.4 reports the main results of the empirical analysis. In Section 2.5, we present the analytical model and the resulting impulse responses. Section 2.6 discusses some policy implications of our findings. Finally, Section 2.7 concludes.

2.2 Data and identification

This section presents the aggregate and household-level data used for the empirical analysis. We also discuss how we disentangle the effects of monetary policy and identify the structural shocks of interest.

2.2.1 Macroeconomic and financial variables

Our empirical analysis focuses on the U.S. economy. The main macroeconomic and financial variables for the baseline model are the real Gross Domestic Product (GDP), the GDP price deflator, the Excess Bond Premium (EBP) from Gilchrist and Zakrajsek (2012), the Federal Funds Rate (FFR), and the 2-year constant-maturity Treasury yield. In the robustness checks, we will use a few alternative variables: industrial production to measure real activity, the Consumer Price Index (CPI) as a price variable, and the 1-year constant-maturity Treasury yield as short-term rate. All these data series are taken from the FRED database operated by the Federal Reserve Bank of St. Louis, except for the EBP data which are from the website of the Federal Reserve System.

2.2.2 Household-level data

We compute the measures of consumption inequality from the Consumer Expenditure Survey (CEX). The CEX, provided by the Bureau of Labor Statistics (BLS) since 1980, is the most comprehensive and granular data source on household consumption in the U.S. and is used for constructing U.S. CPI weights. The survey consists of two separate modules: the Interview Survey and the Diary Survey. The first provides information on up to 95% of a typical household's consumption expenditures whereas the second covers only expenditures on small items from stores. Therefore, in our analysis, we only use data from the Interview Survey.[5]

The CEX is a monthly rotating panel where households are interviewed once per quarter, for at most five consecutive quarters (although the first interview is not publicly available). In each round, the respondents report their expenditures for the three months prior to the interview. In line with the literature, we aggregate monthly into quarterly expenditures to alleviate a few weaknesses in measuring inequality at higher frequencies. First, households sometimes tend to report values for past expenditures that are smoothed over time, which decreases the reliability of monthly data. Second, aggregation reduces sampling errors arising from the relatively small cross section compared to administrative-level data. Third, unusual or large one-time purchases might lead to biased estimates at monthly level whereas they are partially smoothed out at quarterly level. Finally, a lower frequency considers seasonal patterns better.

To compute the measures of consumption inequality, we closely follow Coibion et al. (2017).[6] Household consumption is defined as the sum of non-durables, services, and some durable goods, for example, household appliances, entertainment goods like televisions,

[5] See Bee, Meyer, and Sullivan (2013) for an assessment of the quality of the consumer dataset and its limitations.

[6] We refer the reader to the appendix in Coibion et al. (2017) for a detailed description of the cleaning procedure performed on the data.

and furniture. Large durable expenditures such as house and car purchases are excluded since they are considered a form of investment rather than consumption. All nominal variables are deflated using the CPI for all urban consumers (CPI-U) from FRED and survey sample weights are consistently applied. Real consumption is winsorized at the bottom and top one percent to reduce the influence of outliers and the series are seasonally adjusted.

The baseline measure of inequality we compute is the cross-sectional standard deviation of real consumption across households. As a robustness check, we will use the Gini coefficient of the cross-sectional distribution of household-level real consumption. The advantage of the standard deviation relative to this alternative measure is that it is less sensitive to the behavior of extreme values at the tails of the distribution.

In this paper, we decided to focus on consumption inequality rather than income or wealth inequality for several reasons. First, the data quality is higher for expenditures. In fact, the CEX is specifically designed to collect information on household spending over time. Although the BLS provides some measures of income and wealth, they are mainly imputed from expenditure and demographic data. Moreover, the consumption distribution is a good proxy for income and wealth distributions as well. Second, consumption is connected to the households' well-being since it directly enters their utility functions. In fact, it is the primary reason to earn income and build up wealth in the first place and fluctuates generally less than either of these, allowing an assumingly more stable assessment of differences across households. Third, Coibion et al. (2017) show that contractionary monetary shocks have a negligible effect on income inequality, but that consumption responds strongly.

The CEX also reports data on total income from transfers at household-level. As a proxy for the government's response to monetary shocks, we compute the amount of transfer income received by the households at the bottom of the consumption distribution. Following Coibion et al. (2017), transfer income includes Supplemental Security income and Railroad Retirement before deductions, unemployment insurance, workers' compensation and veterans' benefits, public assistance, contributions from alimony and child support, and other monetary income (scholarships, fellowships, stipends, etc.).[7] The series is deflated, seasonally adjusted, and winsorized as for consumption inequality.

Finally, we use as an aggregate fiscal transfer measure the personal current transfer receipts, as reported by the Bureau of Economic Analysis, deflated by the CPI-U. As with the household-level data, it mainly consists of government social benefits received by people for whom no current services are provided (Social Security, unemployment insurance, Medicare, Medicaid, veterans' benefits, and other federal programs). This data

[7] Most of these variables are available only until 2012.

series is equal or similar to those used by comparable papers (e.g., Amberg, Jansson, Klein, & Picco, 2022; Coibion et al., 2017; Evans, 2022).

2.2.3 Monetary policy shocks

To identify the structural shocks of interest for our purposes, we draw on the concept of high-frequency identification. The goal is to monitor changes in market-based measures at dates with a policy event (so-called monetary policy surprises) to isolate the *unexpected* variation in monetary policy. One can then estimate unobserved factors that together explain the variations in the market-based measure around the policy events. Eventually, the idea is to use these exogenous monetary policy surprises or factors to instrument changes in interest rates.

We rely on different measures of U.S. monetary policy surprises and factors. In our baseline specification, we use the factors computed by Swanson (2021) who extends the high-frequency approach of Gürkaynak et al. (2005). The author collects the changes in specific asset prices in a 30-minute window around each Federal Open Market Committee (FOMC) announcement between 1991 and 2019 and computes the first three principal components of those responses, which together describe the vast majority of market movements. Among all possible rotations of these principal components, he considers that in which the first factor can be thought of as corresponding to changes in the Federal Funds Rate (or FFR), the second to changes in forward guidance, and the third to changes in large-scale asset purchases (LSAPs).[8] These factors represent the three elements of monetary policy that had the largest systematic impact on asset prices. Drawing on this, Swanson (2021) decomposes the changes in asset prices around FOMC announcements into a Federal Funds Rate (or FFR) factor, a Forward Guidance (or FG) factor, and a LSAP factor, each measuring surprises at short, intermediate, and long maturities, respectively.[9] In particular, the FG factor captures the revision in market expectations about the future path of policy rates that are orthogonal to the current policy surprise.

For our analysis, we use the first two factors (FFR and FG) as measures of the structural monetary shocks. The series are available at a daily frequency and we sum up the data points within each quarter to convert them to quarterly frequency.[10]

[8] Swanson (2021) imposes three restrictions to identify the respective factors. First, changes in forward guidance have no impact on the current FFR. Second, neither do changes in LSAPs. Finally, LSAPs had only a minor impact in the time preceding the zero lower bound period.

[9] The factor capturing surprise changes in the FFR is sometimes termed Target factor. Likewise, the factor capturing changes in forward guidance is called Path factor elsewhere. See, for instance, the seminal work by Gürkaynak et al. (2005).

[10] Adopting the alternative approach from Gertler and Karadi (2015), who cumulate the surprises on any FOMC meeting days during the last 93 days and then take the quarterly averages, barely changes the results.

As a robustness check, we use the original two factors computed by Gürkaynak et al. (2005), which we extend to 2019. On top of that, we also clean the factors of Gürkaynak et al. (2005) and Swanson (2021) from the superior-information component of the Federal Reserve by regressing the surprises on Greenbook forecasts and revisions, as proposed by Miranda-Agrippino and Ricco (2021b). These results are reported in Appendix 2.A.

2.3 Econometric approach

We adopt a standard VAR specification with p lags:

$$y_t = B_0 + B_1 y_{t-1} + \ldots + B_p y_{t-p} + u_t \ , \tag{2.1}$$

where y_t is the vector of variables of dimension $n \times 1$, u_t the vector of reduced-form innovations with covariance matrix $\mathrm{Var}\,(u_t) = \Sigma_u$, B_0 is the vector of constant terms, and $B_1, ..., B_p$ are $n \times n$ coefficient matrices.

The VAR model can be written in its structural form by multiplying each side of the reduced form by A_0:

$$A_0 y_t = C_0 + C_1 y_{t-1} + \ldots + C_p y_{t-p} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma) \ , \tag{2.2}$$

where $C_0 = A_0 B_0$ and $C_j = A_0 B_j$ for $j = 1, \ldots, p$. The reduced-form residuals are a function of the structural shocks $u_t = A_0^{-1} \varepsilon_t$. Therefore, it is possible to write the reduced-form variance-covariance matrix as $\mathbb{E}\,(u_t u_t') = \Sigma_u = A_0^{-1} A_0^{-1\prime}$.

The conventional monetary and forward guidance shocks are identified by executing a Cholesky factorization of the reduced-form variance-covariance matrix Σ_u. As in Coibion (2012), Cloyne and Hürtgen (2016) and Lennard (2018), the FFR and the forward guidance factor are integrated directly into the vector autoregressive model and ordered first.[11] By ordering the factor of interest first, we allow all the other variables in the system to contemporaneously respond to the shock.[12]

The remaining variables included in the baseline model specification are: (i) Real GDP; (ii) GDP price deflator; (iii) Excess Bond Premium; (iv) Federal Funds Rate; (v) 2-year

[11]The small sample size and the relatively low frequency of the aggregate data hamper the use of the factors as direct instruments. For instance, the first stage of a proxy VAR with the factors used as external instruments for changes in interest rates results in low F-statistics, in particular for the forward guidance factor, suggesting that the factors are weak instruments. This result is confirmed for alternative factors such as those discussed in Appendix 2.A.

[12]Our results are insensitive to a different ordering of the variables in the VAR. The same holds for including one factor at a time because the two factors are orthogonal to each other.

Treasury yield; and (vi) consumption inequality measure.[13] The Excess Bond Premium, the FFR, and the Treasury yield enter the model in percentage points (ppt.), while the other variables are in log levels, transformed by multiplying their log value by 100. The data are at quarterly frequency for the period 1991-Q3 to 2019-Q2. We include three lags ($p = 3$) for each independent variable as indicated by the corrected Akaike information criterion (AICc).[14] Standard errors are computed using a residual-based moving block bootstrap following Jentsch and Lunsford (2019) with block size set to 16.

2.4 Empirical results

This section reports the impulse responses resulting from the baseline SVAR model to both a conventional monetary policy and a forward guidance shock. We present the results for macroeconomic and financial variables, consumption inequality, and for differences along the consumption distribution. Our findings are robust to different sets of variables, including other factors and inequality measures, or alternative VAR settings. Appendix 2.A provides more details.

2.4.1 Aggregate responses

We start by analyzing how the macroeconomic and financial variables react to conventional monetary policy and forward guidance shocks. The impulse responses to a one-standard-deviation increase in the respective factor are reported in Figure 2.1. The blue dashed lines are the point estimates and the shaded areas are the 68 percent confidence bands based on 10,000 residual-based moving block bootstrap replications.

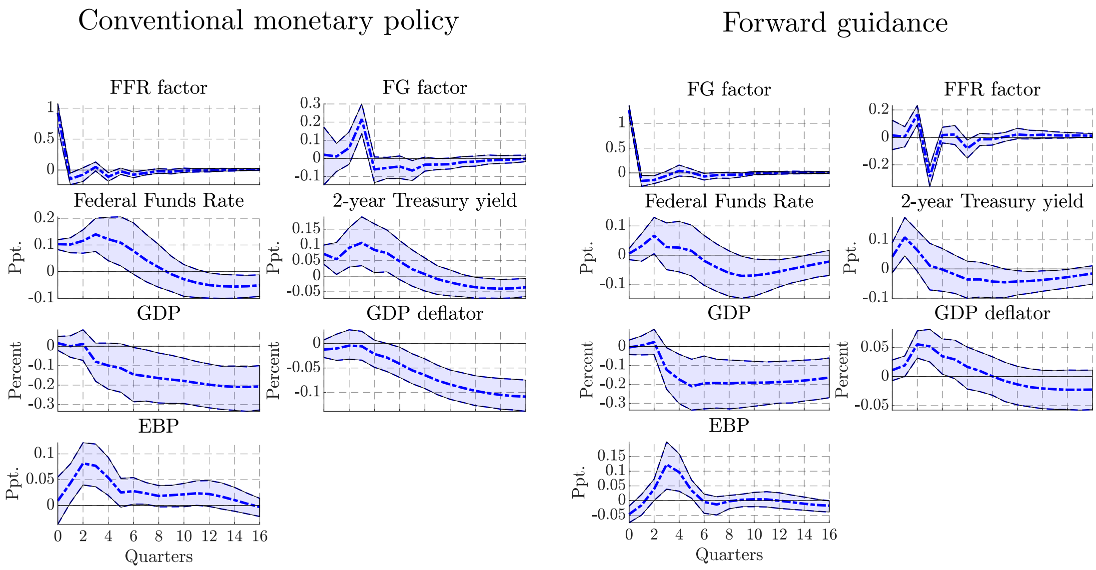
Following a contractionary monetary policy shock, the Federal Funds Rate increases as expected whereas the impact on the 2-year Treasury yield is more muted. GDP and inflation start to decrease persistently around a year after the shock while the EBP signalizes tighter financial conditions.

A positive forward guidance shock causes an increase in the Treasury yield, but the Federal Funds rate does not respond by much as could have been expected given the construction of the factors. The shock also leads to a sizable decrease in GDP and an increase in the EBP a few quarters after the shock. However, prices show a positive response for several quarters (the so-called price puzzle). The same result is found by

[13] Some authors have argued to employ the 1-year Treasury yield instead of the FFR in setups like ours (see, among others, Gertler & Karadi, 2015; Jarociński & Karadi, 2020). A longer-term rate might have the advantage to remain a valid measure of monetary policy even during times when nominal rates are close to or at the zero lower bound. However, our results barely change when using the 1-year Treasury rate instead of the FFR.

[14] When confronted with small samples like ours, the AICc performs better than the more common AIC. However, the impulse responses are much the same when using four lags ($p = 4$), which is a common choice in VARs for monetary analysis with quarterly data.

Figure 2.1: Macroeconomic responses to monetary policy shocks



Notes: This figure depicts the impulse responses of macroeconomic variables to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

Barakchian and Crowe (2013) and Lakdawala (2019). As we show in Appendix 2.A.4, once we control for central bank private information, the response of inflation turns negative without affecting the sign of the consumption inequality response.

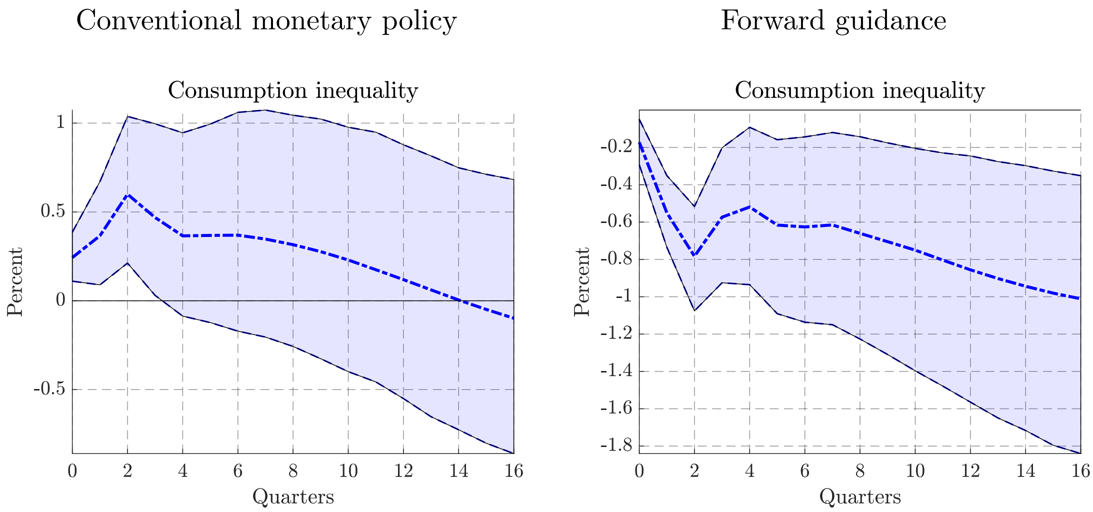
As discussed in Andrade and Ferroni (2021), the sign of the price response to a contractionary forward guidance shock depends on how the shock is perceived. If markets see the announcements as Delphic (news on future macroeconomic conditions), prices will increase, whereas if markets see them as Odyssean (news about the future stance of monetary policy), prices will decrease. Once we clean the shocks from the Delphic component we obtain the expected response that prices decrease after a contractionary forward guidance shock.

2.4.2 Consumption inequality responses

We now focus on the cumulative response of our measure of inequality, namely the log of the cross-sectional standard deviation of real consumption. The impulse responses to a conventional monetary policy and a forward guidance shock are reported in Figure 2.2.

The two shocks have opposite effects on inequality. A contractionary monetary shock results in an increase in consumption inequality, implying a countercyclical behavior with respect to the output response. This result is in line with those in Coibion et al. (2017),

Figure 2.2: Consumption inequality responses to monetary policy shocks



Notes: This figure depicts the cumulated impulse responses of consumption inequality to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

Furceri et al. (2018), Mumtaz and Theophilopoulou (2017), or Samarina and Nguyen (2019). In contrast, a forward guidance shock causes an immediate sizable decrease in consumption inequality and hence a procyclical response.[15] The latter is thereby much stronger in magnitude compared to the response after a conventional shock. In relative terms, both impulse responses are around the same size as the peak impact on output after the respective shock.[16]

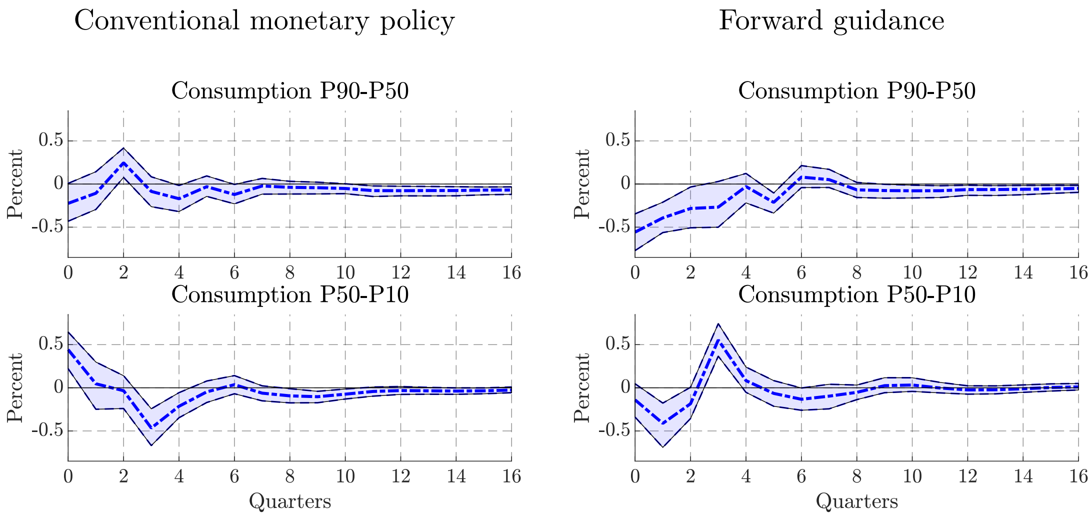
To shed further light on which households drive this result, we replace our inequality measure in the SVAR model with two variables: the difference between log consumption at the 90th and 50th percentiles of the household consumption distribution (the right tail minus the median) and the difference between log consumption at the 50th and 10th percentile (the median minus the left tail). The impulse responses are reported in Figure 2.3.

The top left panel shows that, in response to a contractionary conventional policy shock, the households at the top 10% of the distribution reduce their consumption slightly more than those at the median such that the difference is negative, but not significantly. As expected, the households at the bottom 10% of the distribution remarkably decrease

[15] There have been different types of forward guidance announcements used by central banks over time. See Appendix 2.A.8 for a sensitivity check of the procyclical response of consumption inequality after forward guidance.

[16] Appendix 2.A.9 presents a forecast error variance decomposition and a historical decomposition which quantifies the impact of each shock on inequality.

Figure 2.3: Consumption responses to monetary policy shocks by percentiles



Notes: This figure depicts impulse responses to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. The variable of interest in the panels of the top row is the difference in log real consumption between the 90th and the 50th percentiles of the household consumption distribution. In the panels of the bottom row, it is the difference in log real consumption between the 50th and the 10th percentiles. Impulse responses are from a SVAR computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

their consumption so that the distance to the median households further increases. This insight might be explained by the fact that a large share of these households are usually close to or even at their borrowing constraint and so their consumption is very sensitive to current interest rate changes. Overall, the considerable decrease in consumption of the left tail leads to a rise in inequality.

The right panel tells a different story. In response to a forward guidance shock, the consumption of households at the bottom 10% of the distribution reacts similarly to the consumption of the median households – at least in the first few periods after the shock. However, the consumption of the right tail substantially decreases and so the difference to the 50th percentile goes down as well. This implies that the cross-sectional standard deviation of real consumption significantly decreases after a forward guidance shock.

To sum up, the empirical analysis so far allows us to draw three main conclusions regarding the overall effects of conventional monetary policy and forward guidance. First, the macroeconomic variables show *similar* and significant responses to the two monetary policies. Second, consumption inequality is *countercyclical* under conventional monetary policy, but *procyclical* under forward guidance. Its response to the latter is thereby much stronger. Third, the opposite inequality responses emerge from the different sensitivity to each shock at the two tails of the consumption distribution: inequality increases after a contractionary conventional shock because the consumption of households at the *bottom* of the consumption distribution decreases relatively more than for the rest of the distribution. Under forward guidance, however, households at the *top* of the distribution decrease their consumption disproportionately and so inequality goes down.

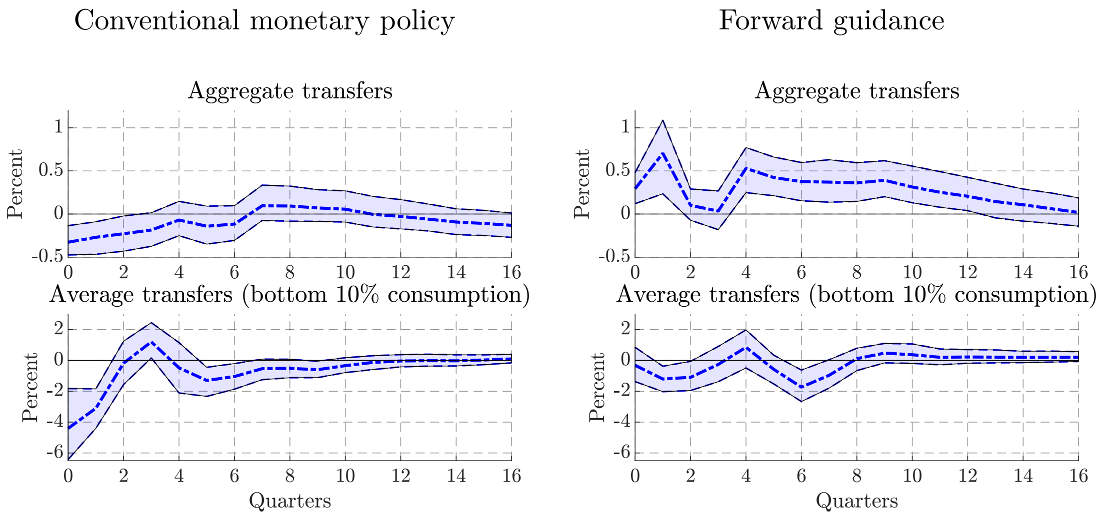
2.4.3 Fiscal transfers as explanatory factor

What could explain the finding that consumption inequality is countercyclical after conventional monetary policy, but procyclical after forward guidance shocks? The macroeconomic responses reported in Figure 2.1 showed that the two monetary shocks have comparable effects on the real economy and on financial conditions, in terms of both shape and magnitude. Instead, an element that might provide an explanation for the various cyclicality of inequality is the fiscal response to the two shocks. Government bonds are one natural example of an asset that is directly affected by interest rate movements, namely through implied changes in interest payments on public debt or changes in the price of newly issued bonds. This impacts the government's budget and its spending capability and calls for fiscal adjustments, taking into account updates in the (macroeconomic) outlook. A fiscal variable directly linked to households' budget constraints, and thus their consumption decisions, is transfers.

To approximate the government's reaction to monetary shocks, we therefore add fiscal transfer measures separately for the aggregate and the household level to the vector of variables of the SVAR model in equation (2.2). The top row of Figure 2.4 shows the impulse responses of total transfer income, measured by the personal current transfer receipts. It reacts procyclically to conventional monetary policy, in line with the results in Amberg et al. (2022), Coibion et al. (2017), or Evans (2022). However, forward guidance has the opposite effect, leading to an increase in income from transfers. In relative terms, the response lies significantly above the curve for conventional monetary policy over almost the entire horizon.

A similar result can be observed at the household level. The bottom row of Figure 2.4 reports the impulse responses of the average transfer income received by households belonging to the bottom 10% of the consumption distribution. Transfers to these agents significantly decrease following a conventional monetary shock. The drop is large in magnitude and around twice as much as the average response of, for example, the bottom 50% of the distribution, which implies that the left tail considerably drives total transfers. On the other hand, transfer income fluctuates around zero after a forward guidance shock, notably in the first few quarters after the shock, indicating a modest response of households with low consumption levels.

Figure 2.4: Transfer income responses to monetary policy shocks



Notes: This figure depicts impulse responses to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. The variable of interest in the panels of the top row is the log of real total transfers. In the panels of the bottom row, it is the log of real average transfer income for households in the bottom 10th percentile of the consumption distribution. Impulse responses are from a SVAR computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2012Q4. Shaded areas represent the 68% confidence interval.

The results suggest that the fiscal response to the two policies plays a key role in the opposite cyclicality of consumption inequality. There are clear differences in the impulse responses of aggregate transfers and the transfer income of households at the left tail of the consumption distribution, respectively. In the analytical model, we will capture this fact by a more generic element – government transfers in lump sum form – which allows us to replicate the empirical findings regarding aggregate variables and the cyclicality of inequality highlighted above.

2.5 Theoretical framework

In this section, we illustrate a specific channel within a standard heterogeneous-agent model that can replicate the main conclusions from the empirical analysis, in particular the diverse cyclical responses of consumption inequality to different monetary shocks. The main element is a particular fiscal policy mix comprising two elements: a redistribution of profits between households and a lump-sum transfer scheme that adjusts in response to changes in the government's budget and to cyclical variations.

We start with a simple model to derive analytical closed-form solutions for the response of consumption inequality and to explain the fiscal channel in a transparent way. The model combines a two-agent household side as in Bilbiie (2008, 2020) with a fiscal policy similar to Kaplan et al. (2018). We then clarify the impact of forward guidance on the yield curve and discuss how its effectiveness depends on the maturity structure of government debt. Finally, we evaluate whether our results still hold in a more complex setup such as a fully-fledged two-agent version of the benchmark HANK model from Kaplan et al. (2018). This framework comprises well-known channels of the HANK literature but is still tractable enough to examine the underlying transmission mechanisms.

2.5.1 Simple analytical two-agent model

The model economy includes four types of agents: households, firms, a government, and a monetary authority. Households are divided into constrained agents living hand-to-mouth and unconstrained savers. Firms are modeled in a standard New Keynesian fashion, with a nominal rigidity that implies sticky prices. The fiscal authority makes lump-sum transfers financed by short-term debt and conducts redistributive policies by taxing firms' profits. Finally, the central bank controls the real interest rate and sets an exogenous time path for it. Appendix 2.B provides further details regarding the model derivation and its equilibrium conditions.

Households. The unit mass of households is divided into two types: a share λ are hand-to-mouth households (H), while the remaining share $1-\lambda$ are savers (S). All households share the same period utility function over consumption C and labor L. For $j = \{H, S\}$,

$$U\left(C_t^j, L_t\right) = \frac{\left(C_t^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \varphi^j \frac{L_t^{1+\nu}}{1+\nu} \, ,$$

with discount factor $\beta \in (0,1)$ and where σ is the elasticity of intertemporal substitution, $\frac{1}{\nu}$ denotes the Frisch elasticity of labor supply, and $\varphi^j > 0$ indicates how strong each agent values leisure relative to consumption. We assume that both household types supply the same amount of hours worked.[17]

Savers. Unconstrained households hold all assets in the economy. They can save in risk-free real bonds issued by the government and get uniform labor income, transfers, and dividends from profits made by the monopolistic firms they own. Each saver solves the following problem:

$$\max_{C_t^S, L_t, B_{t+1}^S} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U\left(C_t^S, L_t\right) \qquad \text{subject to}$$

[17] One way to achieve equal hours worked across household types is to assume a centralized labor market. For example, Bilbiie et al. (2022) impose that a union consolidates labor inputs by households and sets the wage on their behalf.

$$C_t^S + B_{t+1}^S = (1 + r_{t-1})B_t^S + W_t L_t + \Gamma_t^S + T_t^S \ ,$$

where B_{t+1}^S are a saver's end-of-period-t holdings of liquid one-period government bonds issued in t, W_t is the real wage, Γ_t^S are dividends from monopolistic firms' profits net of taxes (specified below), T_t^S are real lump-sum government transfers, and r_t is the real interest rate on bonds, where $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$ with net inflation rate $\pi_t = \frac{P_t}{P_{t-1}} - 1$.

The optimality conditions for this problem result in the following Euler equation for bonds and labor supply conditions:

$$1 = \beta\, \mathbb{E}_t \left[\left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\frac{1}{\sigma}} (1 + r_t) \right] \ ,$$

$$W_t = \varphi^S \left(L_t \right)^{\nu} \left(C_t^S \right)^{\frac{1}{\sigma}} \ .$$

Hand-to-mouth households. Constrained households have no access to asset markets and simply consume their labor income and transfers from the government. Their budget constraint reads

$$C_t^H = W_t L_t + \Gamma_t^H + T_t^H \ .$$

Redistributed dividend income Γ_t^H and lump-sum transfers T_t^H will together play a key role in the mechanism as explained below. They substantially govern the direction of the inequality response to a monetary policy or a forward guidance shock.

The labor supply choice of hand-to-mouth agents is characterized by

$$W_t = \varphi^H \left(L_t \right)^{\nu} \left(C_t^H \right)^{\frac{1}{\sigma}} \ .$$

Firms. The supply side of the economy is standard and features monopolistically competitive producers that provide intermediate goods to perfectly competitive final goods firms.

Final goods producers. A representative firm in the final goods sector aggregates differentiated intermediate inputs j to a final good according to the CES production function $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}$ with elasticity of substitution across goods ϵ. Profit maximization yields the demand for each input, $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t$, where $P_t(j)$ is the price of intermediate good j and $P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} \, dj$ the aggregate price index.

Intermediate goods producers. There is a continuum of monopolistically competitive firms, each of which produces a variety j of the intermediate good using labor N as input. Their production function reads $Y_t(j) = N_t(j)$ and cost minimization implies real marginal cost $MC_t = W_t$. Each producer faces quadratic price adjustment costs as in Rotemberg

(1982) of the form $\Theta_t = \frac{\theta}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$. Real profits of firm j are then given by

$$D_t(j) = \left(1+\tau^S\right)\frac{P_t(j)}{P_t}\, Y_t(j) - W_t\, N_t(j) - \Theta_t - T_t^F \ ,$$

where $P_t(j)$ is the price set by firm j and P_t denotes the aggregate price level. Following Bilbiie (2020), we assume that the government pays a subsidy on sales, financed by a lump-sum tax on firms such that $T_t^F = \tau^S\, Y_t(j)$. With this, total profits over all firms are

$$D_t = \left(1 - MC_t - \frac{\theta}{2}\pi_t^2\right) Y_t \ .$$

An intermediate goods producer sets its price $P_t(j)$ to maximize the discounted stream of expected profits subject to the demand for its good. Appendix 2.B.1 shows the solution to this pricing problem which yields the following New Keynesian Phillips curve:

$$\pi_t(1+\pi_t) = \mathbb{E}_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\,\theta\pi_{t+1}(1+\pi_{t+1})\,\frac{Y_{t+1}}{Y_t}\right] + \frac{1}{\theta}\left[\epsilon MC_t - (1+\tau^S)(\epsilon-1)\right] \ .$$

Government. The fiscal authority issues one-period real bonds, only held by savers, to finance the repayment of existing debt and transfers to households. Its budget constraint is given by

$$B_{t+1} = (1+r_{t-1})B_t + T_t \ ,$$

where B_{t+1} are new bonds issued at time t, such that $B > 0$ denotes debt, with real interest rate r_t, and T_t are lump-sum transfers. We assume that bonds are in positive net supply in equilibrium.

The key instrument of fiscal policy is a tax and transfer system comprising two elements. First, the government levies taxes on the profits of monopolistic firms owned by savers and redistributes the revenues as a transfer to hand-to-mouth agents. This policy is balanced in every period such that the following conditions hold:

$$\Gamma_t^H = \frac{\tau^D}{\lambda} D_t$$
$$\Gamma_t^S = \frac{1-\tau^D}{1-\lambda} D_t \ ,$$

where τ^D is the proportional tax on profits that governs the magnitude of the redistribution. If $\tau^D > \lambda$, hand-to-mouth agents receive a disproportionate share of the profits and are therefore more exposed to changes in them.

Second, there is a lump-sum transfer scheme in place where total transfers are given by

$$T_t = \lambda T_t^H + (1-\lambda)T_t^S \ .$$

The exact functional form of individual transfers will be specified in Section 2.5.3. For now, we should think of them as functions that depend, for instance, on interest rates, the level of debt, or the business cycle.

For this simple model, we assume that the government adjusts lump-sum transfers to maintain a constant level of debt over time, so that we can illustrate the fiscal adjustment clearly. In other words, $B_t = B$ for all t, such that

$$-(r_{t-1} - r)B = \lambda \left(T_t^H - T^H\right) + (1-\lambda)\left(T_t^S - T^S\right) \ ,$$

where variables without time indices denote steady-state values. If the economy starts from a steady state, an expansionary monetary policy shock that moves the real rate below its long-run value r will imply lower interest payments on government debt and allow for higher transfers to households.

Monetary authority. Following McKay et al. (2016) and Kaplan et al. (2016), we assume that the central bank controls the real interest rate. It implements monetary policy by setting and committing to a path for the interest rate, $\{r_k\}_{k\geq 0}$, that is perfectly credible and foreseen by agents.

Once the central bank changes the real interest rate at some arbitrary point in time $\mathcal{T} > 0$, monetary policy will be governed by an exogenous rule. Prior to $\mathcal{T}$, the real rate remains fixed at its steady-state value r. Formally, for $\mathcal{T} \geq 0$:

$$r_t = \begin{cases} r, & t < \mathcal{T} \\ r + \rho^{t-\mathcal{T}} \varepsilon_{\mathcal{T}}, & t \geq \mathcal{T} \end{cases}$$

with policy shock $\varepsilon_{\mathcal{T}} = r_{\mathcal{T}} - r$ and persistence ρ.[18] Therefore, we have $\mathcal{T} = 0$ under conventional monetary policy shock and $\mathcal{T} > 0$ under forward guidance shock, respectively. Moreover, the Fisher equation holds:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \ .$$

Aggregation and market clearing. Aggregate consumption and labor market clearing are given by $C_t = \lambda C_t^H + (1-\lambda) C_t^S$ and $N_t = L_t$, respectively. Goods clearing requires $Y_t = C_t + \frac{\theta}{2}\pi_t^2\, Y_t$ and the bond market clears if $B_{t+1} = (1-\lambda) B_{t+1}^S$.

[18] An alternative setup would be to assume that the nominal interest rate is set according to a standard Taylor rule. Then there exists a sequence of anticipated shocks to the policy rule that implies the same path for the real rate that we set exogenously above. See Appendix 2.B.8 for further details.

2.5.2 Cyclical inequality through redistribution between households

We now study the key equilibrium conditions of our TANK model, log-linearized around a steady state without inequality ($C^H = C^S = C$), zero steady-state dividends ($\Gamma^S = \Gamma^H = 0$), and zero transfers to hand-to-mouth agents ($T^H = 0$). In general, small letters denote the log deviation of a variable from its non-stochastic steady state. See Appendices 2.B.2 and 2.B.3 for more details on the steady state and a summary of the log-linearized equilibrium conditions, respectively. In what follows, we build on previous work by Bilbiie, Monacelli, and Perotti (2020) and extend it for our purposes.

First, it is possible to write the individual consumption of households as a function of aggregate income and transfers to constrained households:

$$c_t^H = \chi c_t + t_t^H \tag{2.3}$$

$$c_t^S = \frac{1-\lambda\chi}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} t_t^H , \tag{2.4}$$

where

$$\chi \equiv 1 + (\sigma + \nu)\left(1 - \frac{\tau^D}{\lambda}\right) ,$$

which captures the elasticity of hand-to-mouth agents' income to total income.[19] This parameter, discussed in detail by Bilbiie (2020), expresses the *profit redistribution* from savers to hand-to-mouth households (as long as $\tau^D > 0$). If $\chi > 1$, the individual income of constrained households responds more than proportionally to changes in aggregate income. This is the case if and only if $\tau^D < \lambda$, meaning that constrained agents receive a proportion of profits that is lower than their share in the population.

The appearance of t_t^H entails that transfers to households immediately react to changes in the fiscal debt burden (through the government's budget constraint) and have a direct impact on individual spending levels. Even more important, (2.3) and (2.4) imply that those transfers are another source of redistribution: if $t_t^H > 0$, savers pay for the additional income of hand-to-mouth agents.

Second, aggregate demand is characterized by the (forwarded) aggregate consumption Euler equation:

$$c_t = \frac{\lambda}{1-\lambda\chi} t_t^H - \sigma\frac{1-\lambda}{1-\lambda\chi}\sum_{k=0}^{\infty}\mathbb{E}_t r_{t+k} . \tag{2.5}$$

[19] The elasticity expression slightly differs from that in Bilbiie (2020) who defined $\chi = 1 + \nu\left(1 - \frac{\tau^D}{\lambda}\right)$. This difference is due to our assumption of uniformly allocated hours worked, while he assumes that each household type provides a separate labor supply.

This expression nests the standard Euler equation of a representative-agent model if $t_t^H = 0$ (zero response of transfers or no transfers) and $\chi = 1$ (hand-to-mouth agents' income moves one-to-one with total income).

Third, consumption inequality can be written as follows:

$$\Phi_t \equiv c_t^S - c_t^H = -\frac{1}{1-\lambda\chi}\, t_t^H - \sigma \frac{1-\chi}{1-\lambda\chi} \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} \,. \tag{2.6}$$

The first part of the equation arises from the fact that transfers to households and hence their consumption decision instantly react to changes in the government's debt burden. The second part captures the common channel of intertemporal substitution, brought about by the Euler equation of savers. Overall, changes in either the contemporaneous or future real rates will have a direct effect on inequality.

Suppose now that the monetary authority announces at time 0 that it will change the real interest rate either today or at some future time $\mathcal{T}$. The instant response of inequality to this policy, for $\mathcal{T} \geq 0$, is

$$\frac{\partial\, \Phi_0}{\partial\, r_{\mathcal{T}}} = -\frac{1}{1-\lambda\chi}\frac{\partial\, t_0^H}{\partial\, r_{\mathcal{T}}} + \sigma \frac{\chi - 1}{1-\lambda\chi}\,\frac{1}{1-\rho} \,. \tag{2.7}$$

As becomes obvious from this expression, after a real interest rate change today or in the future, the transfer function t^H will determine the response of inequality endogenously, together with χ that determines it in the absence of such transfers.[20]

Relating these two elements to each other, we can derive a formal expression that defines the cyclical behavior of inequality.

Proposition 2 (Cyclicality of inequality for arbitrary transfer)**.** *In a simple TANK model with an arbitrary transfer t^H between the two agents that modulates inequality, there is countercyclical consumption inequality in response to a one-time change in the real interest rate at time $\mathcal{T}$ if*

$$\frac{\partial\, t_0^H}{\partial\, r_{\mathcal{T}}} < \sigma(\chi - 1)\,\frac{1}{1-\rho} \,. \tag{2.8}$$

In contrast, consumption inequality is procyclical with an opposite sign.

Proof. Assuming that $\lambda\chi < 1$, the proposition follows from (2.7). ■

In the case studied in this paper, the arbitrary transfer mentioned in the proposition and the associated redistribution occur through the government, in the form of lump-sum transfers. However, it needs to be stressed that this mechanism is only one out of a broader

[20] Throughout the paper, we assume that $\lambda < 1/\chi$ like, among others, Bilbiie (2020) did. If that condition does not hold, Bilbiie (2008) demonstrates how the slope of the IS curve might flip such that an expansionary monetary policy negatively affects aggregate consumption through the intertemporal substitution channel.

class of redistribution schemes that might work in this context. In fact, any mechanism in which the size and the timing of the government's intervention differ between the two types of monetary policy can generate similar income effects and achieve the desired cyclicality of inequality.

2.5.3 Inequality and the impact of monetary shocks

To determine analytically the responses of inequality to an interest rate change, we specify now a transfer function. As a result of Proposition 2, in order to achieve countercyclical consumption inequality on impact of a real interest rate change today (i.e., $\mathcal{T} = 0$), it has to hold that $\frac{\partial t_0^H}{\partial r_\mathcal{T}} < \sigma(\chi - 1)\frac{1}{1-\rho}$. Conversely, for inequality to respond procyclically after a forward guidance shock (i.e., $\mathcal{T} > 0$), we require $\frac{\partial t_0^H}{\partial r_\mathcal{T}} > \sigma(\chi - 1)\frac{1}{1-\rho}$.

We assume in our baseline specification that the transfer function for hand-to-mouth agents comprises both a debt element and a cyclical component:

$$t_t^H = -\phi_1 r_t B_Y - \phi_2 y_t \ , \tag{2.9}$$

where $\phi_1 > 0$ and $\phi_2 > 0$. The motivation for this function is twofold. First, the transfer scheme in our model is closely interlinked with fiscal debt. A look at the government's budget constraint unveils the channel: a rise in the real interest rate increases the public debt burden $r_t B_Y$ and triggers an instant fiscal adjustment in the form of fewer lump-sum transfers.[21] Hence, $\phi_1 > 0$. If the rate change is announced to happen later instead, the fiscal authority does not immediately adjust its transfers because the higher interest payments on government debt are in the future. This story mirrors the considerations in Kaplan et al. (2016).

Second, following a shock to the real rate, the government will adjust transfer payments to stabilize the income of hand-to-mouth agents over time. It does so to offset the fluctuations in output y_t so that transfers act here as an automatic stabilizer and $\phi_2 > 0$.[22] This setup is similar to the countercyclical transfer scheme proposed by Gerke et al. (2020).

Combined with the aggregate consumption Euler equation (2.5), the transfer rule (2.9) can be rewritten as

$$t_t^H = -\phi_1 \frac{1 - \lambda\chi}{\Upsilon} r_t B_Y + \phi_2 \frac{\sigma(1-\lambda)}{\Upsilon} \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} \ , \tag{2.10}$$

[21] In Appendix 2.B.10, we relax the constant-debt assumption and study alternative forms of transfer functions.

[22] See McKay and Reis (2016) for an example of a theoretical model that studies the implications of automatic fiscal stabilizers for the business cycle.

where

$$\Upsilon \equiv 1 - \lambda\chi + \phi_2\lambda \,.$$

Plugged into the equation for consumption inequality (2.6), we get

$$\Phi_t = \frac{\phi_1}{\Upsilon} r_t B_Y - \sigma \left[\frac{1-\chi}{1-\lambda\chi} + \phi_2 \frac{1-\lambda}{(1-\lambda\chi)\Upsilon} \right] \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} \,. \tag{2.11}$$

We are now interested in how much inequality changes if the central bank announces a one-time change in the real interest rate that is going to happen either today at $\mathcal{T} = 0$ (conventional monetary policy shock) or $\mathcal{T} > 0$ periods from now (forward guidance shock). As described in the model outline, the central bank implements such monetary policy by setting a perfectly credible path for the real interest rate: it keeps the real rate at its steady-state value prior to $\mathcal{T}$ (i.e., $r_t = 0$ in log-linear terms) and follows an exogenously given rule with some persistence ρ after that (i.e., $r_t = \rho^{t-\mathcal{T}} \varepsilon_{\mathcal{T}}$).

Evaluating the last equation above at time 0, the response of inequality *on impact* of a conventional monetary policy and a forward guidance shock is

$$\frac{\partial \Phi_0}{\partial r_{\mathcal{T}}} = \begin{cases} \frac{\phi_1}{\Upsilon} B_Y + \sigma \left[\frac{\chi-1}{1-\lambda\chi} - \phi_2 \frac{1-\lambda}{(1-\lambda\chi)\Upsilon} \right] \frac{1}{1-\rho}, & \mathcal{T} = 0 \\ \sigma \left[\frac{\chi-1}{1-\lambda\chi} - \phi_2 \frac{1-\lambda}{(1-\lambda\chi)\Upsilon} \right] \frac{1}{1-\rho}, & \mathcal{T} > 0 \end{cases} \tag{2.12}$$

We can notice a few points. First, if bonds are in zero net supply ($B_Y = 0$) or transfers to financially constrained agents are not directly linked to debt ($\phi_1 = 0$), inequality will respond by exactly the same amount regardless of when the policy shock happens. This stresses not only the importance of the debt burden and any fiscal adjustment for the response of households, but also for the role of income sensitivity.

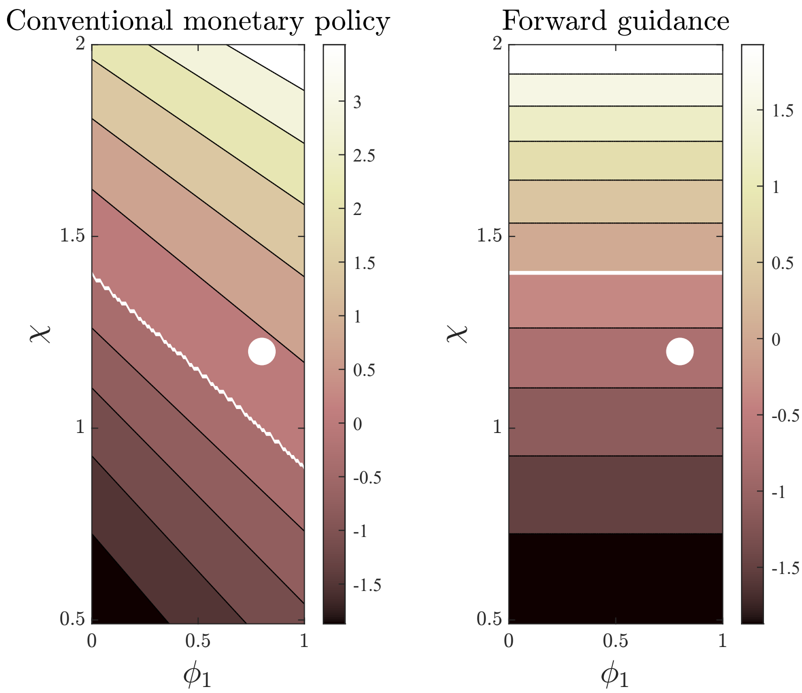
Second, given conventional values for σ, λ, and ρ, the sign and magnitude of the inequality response is determined by the three key parameters χ, ϕ_1, and ϕ_2. Drawing on Proposition 2, we can determine in which cases the proposed transfer function (2.9) will be able to replicate the cyclical behavior of inequality found in the data. The following proposition summarizes the necessary condition, which is derived in Appendix 2.B.5.

Proposition 3 (Opposed cyclicality of inequality for particular transfer)**.** *Given a transfer function of the form $t_t^H = -\phi_1 r_t B_Y - \phi_2 y_t$, the consumption inequality response on impact of a shock is countercyclical for conventional monetary policy and, at the same time, procyclical for forward guidance, if the following condition holds:*

$$-\phi_1 \frac{(1-\rho)}{\sigma} B_Y + \phi_2 < \chi - 1 < \phi_2 \,. \tag{2.13}$$

Proof. See Appendix 2.B.5. ∎

Figure 2.5: Sensitivity of the inequality response to redistribution and transfers



Notes: These heat maps show the response of inequality on impact of a conventional monetary policy and a forward guidance shock, respectively, for different combinations of χ (the elasticity of the constrained household's income to aggregate income) and ϕ_1 (the coefficient on debt burden in the constrained agent's transfer function). The bars next to each plot label the colors, where values above (below) zero refer to a positive (negative) inequality response. The white lines indicate the threshold with zero inequality response. The white dots mark the parameter values implied by the baseline calibration (see Table 2.B2).

Figure 2.5 depicts graphically how the three parameters influence the cyclicality of income. The heat map reports the contemporaneous responses of consumption inequality for different combinations of χ and ϕ_1 to a conventional monetary policy shock (left panel) and a forward guidance shock (right panel). Positive and negative responses are separated by the white line. ϕ_2 is kept fixed at 0.4 and the white dots mark the parameter values that we use as a baseline to compute the dynamic responses in the analytical TANK model ($\phi_1 = 0.8$, $\chi = 1.2$).

As recognizable from equation (2.3), the higher the value of χ the stronger the elasticity of hand-to-mouth agents' income to total income. In line with Bilbiie (2020), this implies that consumption inequality reacts more positively under both contractionary conventional monetary policy and forward guidance.

Similarly, the responsiveness of consumption inequality increases in ϕ_1 under conventional monetary policy. This is due to the fact that the amount of transfers the constrained agents receive is proportional to the debt burden. Under forward guidance, the interest rate hike happens only in the future so that there is no instant increase in the debt burden. Therefore, the value of ϕ_1 plays no role in this case.

Looking at the sign of the responses, we can notice that the higher the value of ϕ_1 under conventional monetary policy the lower χ can be to still achieve a positive response of inequality. Comparable empirical evidence from Auclert (2019) and Patterson (2022) suggests a value of $\chi > 1$, which implies $\tau^D < \lambda$ in our model.[23] In that case, constrained agents get a proportion of profits that is numerically below their share in the population. However, their individual income reacts disproportionately more to changes in aggregate income, which ensures that consumption inequality responds countercyclically. Conversely, assuming $\chi < 1$ would require an extremely high ϕ_1, far above one for an otherwise standard calibration. Constrained agents would get a relatively high share of profits compared to savers. To ensure that the two individual incomes do not diverge too much, transfers would therefore need to be more sensitive to changes in debt. Finally, note that if hand-to-mouth agents are too sensitive to changes in aggregate income (i.e., χ is very large), then inequality is countercyclical under both types of monetary policy regardless of the value of ϕ_1.[24]

In the next step, we study the *dynamic* response of inequality after a one-time unexpected monetary shock with some exogenous persistence. Assume that the central bank either raises the real rate today by 25 basis points (i.e., $\varepsilon_0 = 0.0025$) or promises an increase of the same size in two years from now (i.e., $\varepsilon_8 = 0.0025$). Figure 2.6 shows the main impulse responses to these shocks under a standard set of parameter values. More details on the calibration and the remaining impulse responses can be found in Appendices 2.B.6 and 2.B.7, respectively.

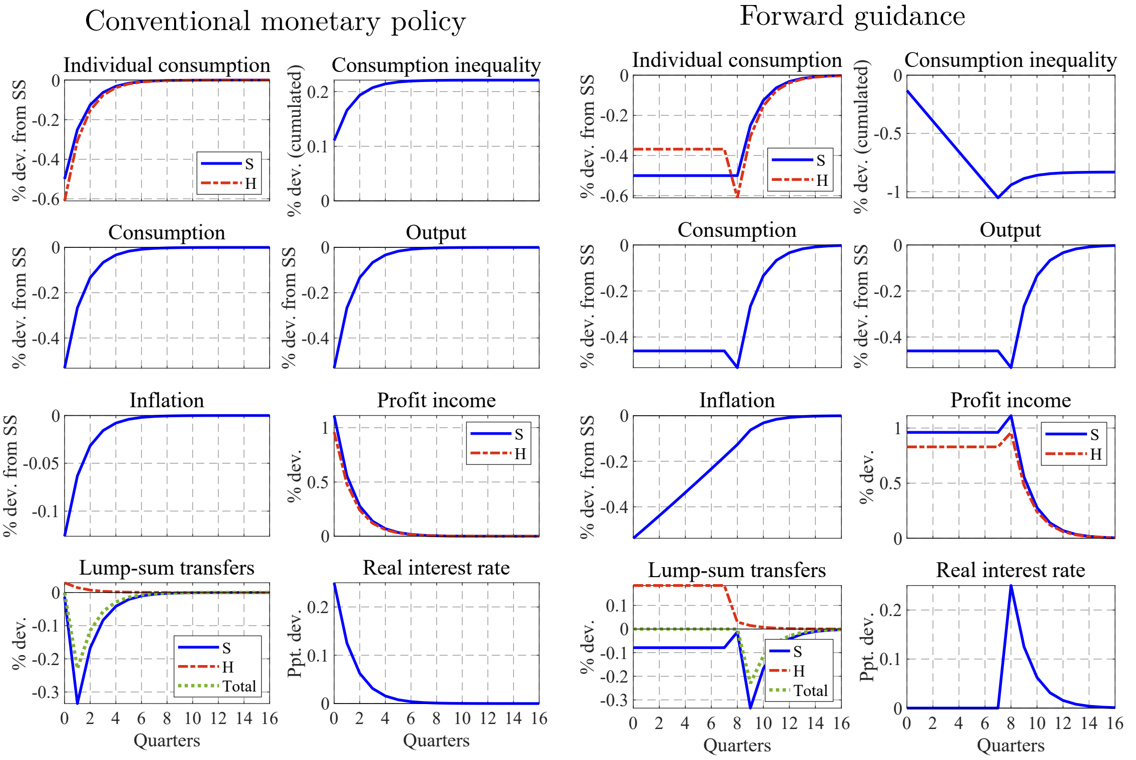
Both types of monetary policy lead to a comparable decrease in aggregate consumption and output on impact of each shock. In contrast, inflation shows a stronger decline after forward guidance. This comes from the permanently lower marginal costs in the periods up to the real rate change, which affects prices through the forward-looking nature of the Phillips curve.

The amount of profits redistributed is such that the individual consumption responses are similar. On top of that, due to the automatic stabilizer component of the transfer rule (2.9), the government partially offsets the decrease in consumption experienced by the hand-to-mouth agents by increasing the amount of transfers to them and letting the savers pay more for the recession. However, only a contemporaneous increase in the real

[23] Auclert (2019) demonstrates that low-income households tend to have higher marginal propensities to consume (MPCs). Patterson (2022) documents a positive covariance between the individual MPCs of workers and the sensitivities of their income to movements in output.

[24] Appendix 2.B.7 contains an alternative heat map in which the weight on the cyclical component in the transfer rule (2.9) is set higher. That setup implies then a relatively higher value for χ to replicate the empirical results on the cyclicality of income.

Figure 2.6: Impulse responses to monetary policy shocks: Analytical TANK model



Notes: This figure depicts selected impulse responses for the analytical TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). Responses of profit income and transfers are in deviations from their steady-state levels, relative to steady-state output. Individual responses for savers (S) and hand-to-mouth agents (H) are shown in per-capita terms.

interest rate (left panel of Figure 2.6) leads to an immediate higher debt burden. Under forward guidance (right panel of Figure 2.6), the interest rate change happens in the future and so does the adjustment in transfers owed to the component related to changes in the debt burden. What remains is only the cyclical part of lump-sum transfers which leads to a stronger reaction of the latter.[25]

How the government responds to the two monetary shocks is crucial in determining whether consumption inequality is procyclical or countercyclical. In line with the evidence of Section 2.4, both the aggregate as well as the household-level response of lump-sum transfers differ following a conventional monetary and forward guidance shock. It is important to stress that the purpose of the transfer rule we consider is not to perfectly match the sign of the empirical responses of transfers, but rather to qualitatively capture the different magnitude of the responses. The empirical evidence is only a partial proxy of the

[25] Note that the small response of transfers to constrained agents after conventional monetary policy arises from the relatively higher weight on the debt burden in the transfer function ($\phi_1 = 0.8$) compared to the weight on output ($\phi_2 = 0.4$). This leads overall to a downward pressure on these transfers.

government's overall reaction we consider in the model. In addition, the values for ϕ_1 and ϕ_2 in the transfer function (2.9) might not be constant over the business cycle and vary with changes in the economic conditions. Under the baseline calibration, the presence of the cyclical component avoids that hand-to-mouth agents pay (too much) in the form of negative transfers during the recession that follows the contractionary shock. This supports the view of fiscal transfers being a helpful tool to stabilize variations in income and to compress inequality for financially constrained agents, in line with the evidence for the U.S. in Heathcote et al. (2010).

Once the announced real rate change actually occurs, hand-to-mouth agents will cut back their consumption slightly more because of the suspended transfers from the government. The difference in magnitude and timing of the fiscal response is such that the consumption of hand-to-mouth agents decreases relatively more than that of savers under conventional monetary policy, but proportionally less under forward guidance. Eventually, this leads to a consumption inequality increase in the former and a decrease in the latter case.

To sum up, the consumption of hand-to-mouth agents is always more sensitive to any type of monetary shock because of their lacking access to asset markets. However, all else equal, the profit redistribution scheme and the presence of countercyclical profits make the consumption responses of the two household types close enough such that the fiscal response determines the sign of the inequality response. With these elements, the model can replicate the cyclicality of inequality and their origin as observed in the data.

2.5.4 Forward guidance and the maturity structure of debt

We assumed so far that government debt is entirely short-term such that an announcement of a future policy rate change leaves today's interest expenses unaffected. However, public debt is typically more long-term and forward guidance has an immediate impact on its market value by influencing yields. An announced future increase in the real interest rate leads to higher long-term bond yields and can create capital losses for the government in the short-term. Different from the baseline model, forward guidance has a direct impact on the government budget through the responsiveness of the yield curve. Today's economic impact of forward guidance therefore depends substantially on the maturity structure of government debt.[26]

Appendix 2.B.9 outlines the details of an alternative framework which comprises non-constant long-term debt, modeled as in Woodford (2001), with price Q_t and coupon payments that decay geometrically at rate $\kappa \in [0, 1]$. This parameter controls the maturity

[26] Among others, Filardo and Hofmann (2014) show empirically that forward guidance on policy rates had an impact on the expected path of future interest rates in different countries.

of debt where $\kappa = 0$ corresponds to a short-term bond as in the baseline model. In equilibrium, savers are indifferent between saving in a short-term, one-period bond and a long-term bond today. Therefore, the one-period real return of the long-term asset is equal to the return of the short-term asset (a no-arbitrage condition). Formally, $\mathbb{E}_t r^L_{t,t+1} = r_t$, where the return of long-term bonds is linked to their price by $r^L_{t-1,t} = \kappa\beta q_t - q_{t-1} - \pi_t$.

Since the government now issues longer-term assets, it cares about any changes in the long-term yield caused by forward guidance. To see why, we can derive an expression for the bond price as a function of future coupon payments (see Appendix 2.B.9):

$$q_t = -\sum_{i=0}^{\infty} (\kappa\beta)^i \, \mathbb{E}_t \left(r_{t+i} + \pi_{t+1+i} \right) \ .$$

All else equal, an announcement of an increase in the future real interest rate by the central bank would lead to an immediate decrease in today's bond price. However, while the real rate will be higher for only one period in the future, inflation is lower already from today onwards. This situation affects the bond price positively and dominates the downward pressure by the real rate. Overall, it implies a higher $r^L_{t-1,t}$ and therefore a larger value of the government's outstanding debt. Forward guidance will thereby be more effective with a larger bond maturity (i.e., a higher κ). This mechanism was termed by Ferrante and Paustian (2020) as the debt revaluation channel, but in the context of a fully-fledged heterogeneous-agent model where households were allowed to borrow in long-term bonds.[27]

Given the before-mentioned, to maintain a balanced budget after a contractionary forward guidance shock, the government can either increase its borrowing activity in long-term bonds or cut lump-sum transfers to households. If debt follows an exogenous rule and transfers to savers are adjusted, that would have a direct impact on households' consumption behavior and thus inequality. If the government adjusts the level of debt instead, transfers and consumption inequality could respond as in the baseline model if wanted even if capital losses are higher. In order to model this latter case, we would need to define a transfer function for savers. Appendix 2.B.10 presents two alternative setups assuming non-constant debt. They are designed for the case of the baseline framework, but can be easily adapted to the model with long-term debt at hand.

In summary, it can be said that the maturity structure of debt is important to assess the effectiveness of forward guidance today. However, the implications on households' budget constraints and so on consumption inequality depend heavily on which variable

[27] Note that Ferrante and Paustian (2020) argue that, when bonds are real instead of nominal, the effects of inflation are absent. In our case, the long-term bond price would therefore be lower, decreasing the government's debt burden. Moreover, forward guidance would become less effective as the bond maturity increases.

adjusts to balance the fiscal budget with non-constant debt and also on how the individual fiscal transfer functions are specified.

2.5.5 Fully-fledged two-asset TANK model with investment

The baseline analytical TANK model has shown that a combination of profit and lump-sum transfer redistribution can replicate the cyclicality of consumption inequality found in the data. To evaluate whether this finding still holds in a more complex setup, we implement our mechanism in a next step in a widely used framework of the heterogeneous-agent literature: the model by Kaplan et al. (2018). We focus on the two-agent version of their benchmark HANK model to make it more comparable to our analytical model. Such a framework comprises the well-known channels of standard HANK models, but is still tractable enough to examine the underlying transmission mechanisms. In fact, the model presented in Section 2.5.1 can be seen as a simplified two-agent version of the fully-fledged HANK model in Kaplan et al. (2018). Appendix 2.C provides a full description of the model and further explanations about the differences between our setup here and Kaplan et al. (2018). Furthermore, the appendix outlines the calibration values and comprises additional impulse responses not shown hereafter.

Model outline

The two major features that are added to the analytical TANK model are a multiple-asset structure and investment. Unconstrained households can save in two types of assets with different degrees of liquidity. There is a liquid asset with a low return, similar to the one-period government bond in the simple model.[28] In addition, there is a high-return illiquid asset. Deposits into or withdrawals from an agent's illiquid account are subject to a transaction cost. However, each saver can invest their illiquid savings either in capital or in equity shares. Capital is used by monopolistically competitive producers, together with the labor provided by individual households, to manufacture intermediate goods.[29] Shares figure as a claim to a fraction of intermediate firms' profits. That part is reinvested directly into the illiquid account, while the remaining fraction of profits is paid lump-sum to the savers' liquid account.

Finally, the two main instruments of fiscal policy are modeled as before. Savers pay taxes on monopolistic firms' total profits and the revenue is redistributed as a transfer to hand-to-mouth agents. Second, the government runs a transfer scheme in which transfers

[28] Besides short-term government bonds, liquid assets are understood as also comprising deposits in financial institutions and corporate bonds. On the other hand, the illiquid asset class captures elements like housing, consumer durables, and equity.

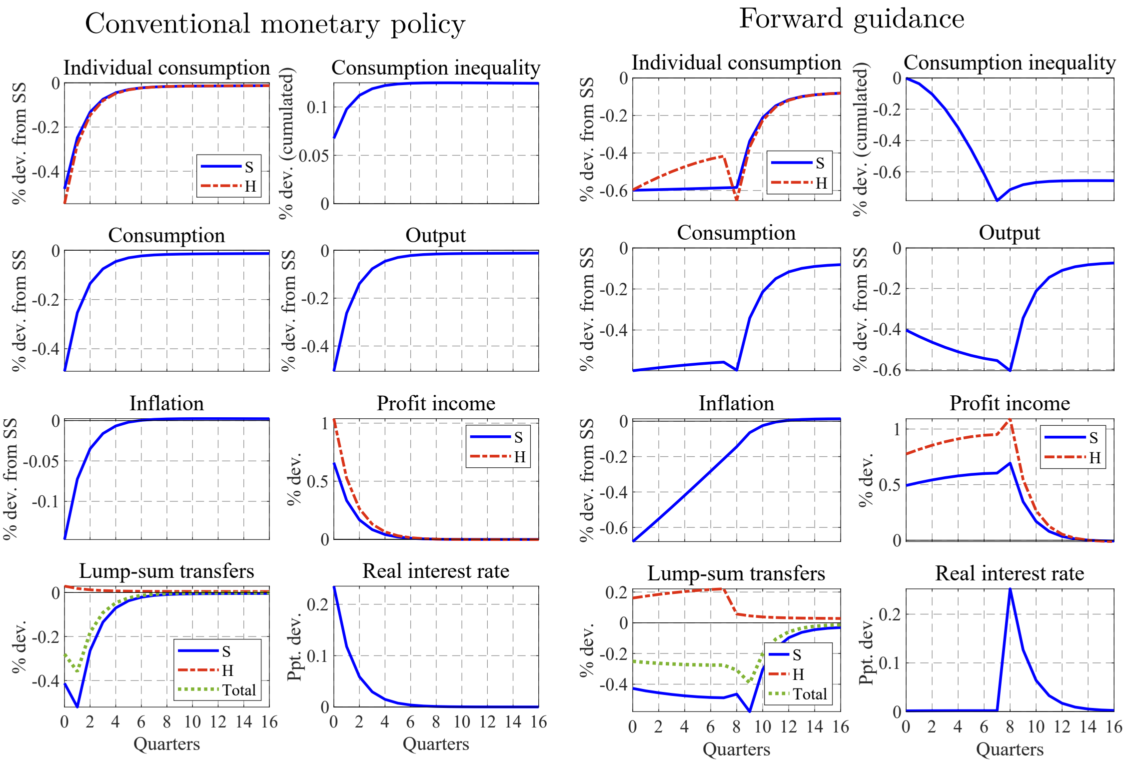
[29] The distinct labor earnings of each household type are now taxed by the government at a proportional rate.

to constrained agents depend on the amount of interest payments on public debt and also contains an automatic stabilizer element.

Impulse responses for the extended model

Equivalent to the simple TANK model, suppose now a 25 basis points increase in the real interest rate, either today or eight quarters from now. Figure 2.7 shows the main impulse responses to these two shocks. Both the positive monetary policy and the forward guidance shock lead to a decrease in consumption, output, and inflation on impact, where the latter sees again a stronger drop after forward guidance due to persistently lower marginal costs. The drop in consumption for the hand-to-mouth agents is partially offset by profit redistribution and the fiscal adjustment through transfers.

Figure 2.7: Impulse responses to monetary policy shocks: Fully-fledged TANK model



Notes: This figure depicts selected impulse responses for the extended TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). Responses of profit income and transfers are in deviations from their steady-state levels, relative to steady-state output. Individual responses for savers (S) and hand-to-mouth agents (H) are shown in per-capita terms.

As in the simple model, the government's response varies between the two policy tools. After a contemporaneous change in the real rate, both components of the transfer function – that is, the parts related to the automatic stabilizer and the debt burden –

react to the shock. However, only the first component is affected by a positive forward guidance shock, leaving us with countercyclical lump-sum transfers that are higher for hand-to-mouth agents.

The difference in timing and magnitude of the fiscal response leads to the heterogeneous responses of inequality under conventional monetary policy and forward guidance. The consumption of hand-to-mouth agents decreases relatively more under the former and proportionally less under the latter. Therefore, consumption inequality is countercyclical in the first case and procyclical in the second case, in line with the empirical evidence we provide.

Overall, the findings from the fully-fledged two-asset TANK model are consistent with those of the analytical TANK model, not only in terms of the sign and shape of the macroeconomic and consumption inequality responses, but largely also in magnitudes. It seems that the additional model elements (illiquid asset and investment) has only a negligible impact in this respect. However, this might clearly change with a different calibration of the main model parameters.

2.6 Policy implications

In this section, we discuss some policy implications that can be drawn from our empirical and theoretical findings. First of all, our results emphasize the role that the fiscal-monetary policy mix plays in shaping the second-order implications of policy rate changes, such as an increase in consumption inequality. Even though central banks and governments act independently from each other, their activities are deeply intertwined and a certain level of coordination therefore appears to be beneficial to limit negative side effects.

Second, our empirical evidence suggests that the fiscal adjustments of governments after monetary shocks might not always be fully appropriate. Cutting transfers in response to a contractionary policy rate change, for instance, contributes to an increase in consumption inequality. For fiscal authorities to be able to more flexibly and optimally adapt to monetary policy tools or regimes, transfer schemes are best to be kept flexible. Rather than strongly focusing on the debt burden, putting more emphasis on ongoing macroeconomic conditions could especially help during an economic downturn, where targeted fiscal redistribution to households at the bottom of the consumption, income, or wealth distributions can help to maintain an adequate expenditure level. In the theoretical framework with the transfer rule we propose, this corresponds to an increase in the weight on the business cycle (ϕ_2) relative to the weight on the debt burden (ϕ_1). However, all this strongly depends on how well the fiscal authority understands the macroeconomic and distributional consequences of various policy tools. This is key to setting up appropriate fiscal support through lump-sum schemes, unemployment benefits, tax cuts, or more.

Third, against this backdrop, it appears beneficial that the central bank communicates the expected aggregate effects of its policies through forecasts and reports in a transparent way such that they can be internalized, among others, in the government's decision-making process. Even though price stability is their main goal, monetary authorities could systematically report how inequality affects the efficiency of their policies, and how their policies themselves affect the distribution of income or wealth in the economy.

These policy recommendations are particularly crucial for the high-inflation environment we are currently facing. To reduce the increase in price growth, central banks have started to tighten their monetary policy by increasing their key interest rates. This can lead to a severe contraction in the aggregate economy. Our results suggest that the government's response determines to some extent how inequality will react. The fiscal authority can oppose an increase in inequality by implementing sizable transfer schemes in favor of the most financially constrained households instead of, for instance, adjusting tax rates regressively. In addition to this, central banks can use contractionary forward guidance announcements to dampen the negative distributional effects of the fast monetary policy normalization, thereby shaping the expectations of economic agents.

2.7 Conclusion

The relationship between monetary policy and inequality has been studied intensively in the recent past. At the same time, central banks have extensively used unconventional monetary policy tools like forward guidance when nominal interest rates have been trapped at the lower bound. However, there is still limited and often conflicting empirical evidence regarding the distributional effects those various monetary policies might have.

This paper investigates the macroeconomic and distributional impact of forward guidance as compared to conventional monetary policy. We compute a measure of consumption inequality from U.S. household-level expenditure data and include it in a SVAR model. The two monetary policies are identified using the latent factors extracted by Swanson (2021) from high-frequency monetary policy surprises in asset prices. We find that the aggregate effects of both policies on real and financial variables are similar in magnitude and shape. However, consumption inequality is countercyclical under conventional monetary policy and procyclical under forward guidance.

We rationalize these empirical findings through a standard New Keynesian model with heterogeneous households. Drawing on empirical evidence, the key element is the fiscal response in the form of lump-sum transfers that depend on the public debt burden and the business cycle. The timing of the interest rate change matters for the government's interest rate payments on its debt and thus results in fiscal adjustments differing in timing and magnitude for the two monetary policies. This ultimately results in opposite responses of consumption inequality to conventional monetary policy and forward guidance.

Our findings suggest that, from an aggregate point of view, an interest rate policy or announcements about the future stance of monetary policy have similar effects. However, both policies can involve negative second-order effects and the way governments react to different central bank tools is key to counteract those effects.

Appendix

2.A Empirical analysis: Robustness checks

In order to strengthen the validity of our findings in Section 2.4, we present here some sensitivity analysis in the form of alternative empirical model specifications. In Section 2.A.1, we use the Gini coefficient of real consumption as an alternative measure of consumption inequality. Second, in Sections 2.A.2 to 2.A.4, we adopt a series of alternative measures of conventional monetary policy and forward guidance shocks: the factors from Swanson (2021) cleaned from central bank information by using the procedures proposed by Miranda-Agrippino and Ricco (2021b), the raw and cleaned factors from Gürkaynak et al. (2005) and the cleaned path factor from Lakdawala (2019). Third, we use the same empirical model as Bundick and Smith (2020) to study the effects of forward guidance shocks in Section 2.A.5. Fourth, in Section 2.A.6, we compute the responses to a conventional monetary policy shock and a forward guidance shock using Bayesian local projections. Fifth, Section 2.A.7 presents sensitivity results for different parameter-variable combinations of our SVAR model. Finally, in Sections 2.A.8 and 2.A.9, we assess the historical importance of our identified shocks by comparing episodes of different forward guidance types and by performing both a variance and a historical decomposition.

2.A.1 Alternative inequality measures

We start by showing that the choice of the measure of consumption inequality plays no role in our results. In the main analysis, we measure inequality with the cross-sectional standard deviation of real consumption across households. Alternatively, we can compute the Gini coefficient of the cross-sectional distribution of household-level real consumption.

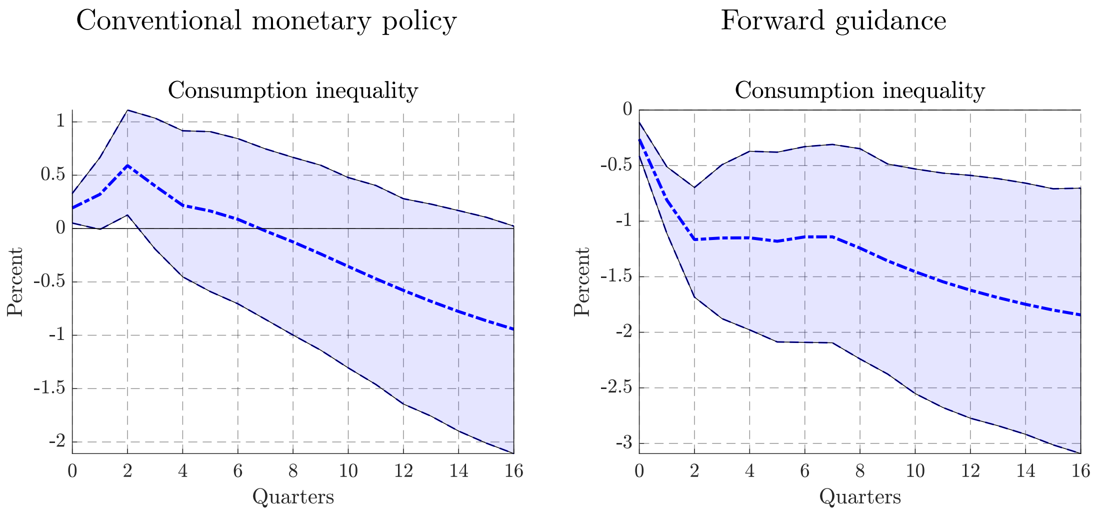
Figure 2.A1 shows that the sign of each consumption inequality response is unaffected: contractionary monetary shocks increase inequality whereas forward guidance shocks decrease it.[30]

2.A.2 Swanson (2021): Cleaned FFR factor

Central banks and market participants have different information about the state of the economy. Due to this asymmetry, market participants try to infer the potentially superior information that the policymakers might have through its policy actions (e.g., a change in policy rate). Therefore, as shown by Miranda-Agrippino and Ricco (2021b), raw monetary policy surprises tend to include both the true policy shock as well as an information

[30] The impulse responses of the macroeconomic variables are basically unaffected by the choice of the inequality measure. So for ease of exposition, we only show the inequality responses.

Figure 2.A1: Consumption inequality responses to monetary policy shocks: Gini



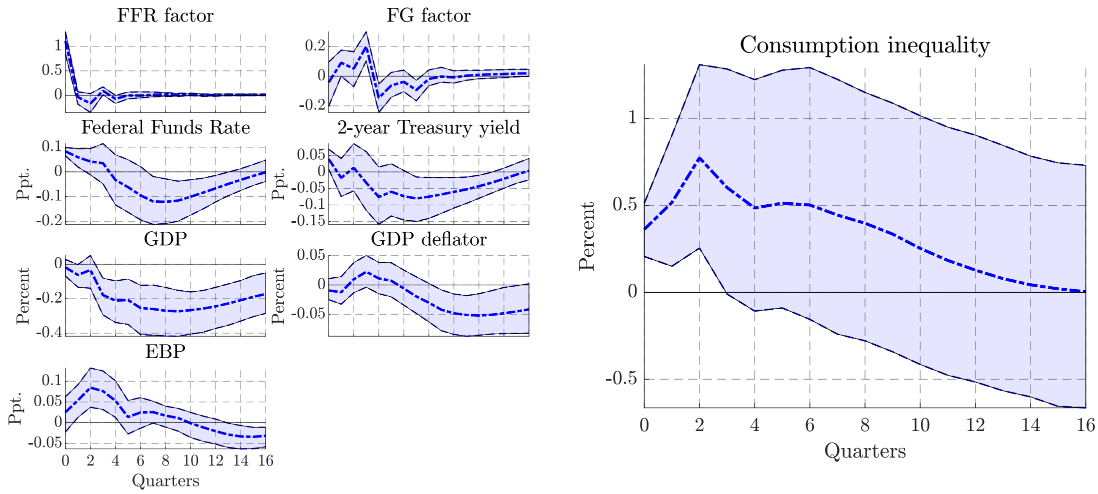
Notes: This figure depicts the cumulated impulse responses of consumption inequality to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. Consumption inequality is measured by the Gini coefficient of the cross-sectional distribution of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence intervals.

component about fundamentals of the economy. This signaling effect of monetary policy can give rise to empirical puzzles.

To correct for the presence of this information friction in our target factor, we adopt the approach proposed by Miranda-Agrippino and Ricco (2021b) and Degasperi and Ricco (2021). In particular, we isolate the pure monetary shocks which are orthogonal to both the central bank's economic projections and to past market surprises by regressing the target factor from Swanson (2021) on the Greenbook forecasts and forecast revisions for real output growth, inflation (measured as the GDP deflator), and the unemployment rate. The residuals of the regression are the exogenous and unpredictable component of the monetary surprises since we control for the central bank's private information and hence for the central bank information channel. Since the Greenbook forecasts are published after a five-year lag, the most recent data series stops in 2016Q4.

Figure 2.A2 reports the responses of the aggregate variables and consumption inequality to the cleaned target factor. Using the cleaned measure in the SVAR model does not change the fact that the response of inequality is countercyclical under conventional monetary policy. Apart from that, results are much in line with the baseline results, except for the 2-year Treasury yield which turns negative almost immediately after the shock.

Figure 2.A2: Impulse responses to the cleaned target factor from Swanson (2021)



Notes: This figure depicts the impulse responses of macroeconomic variables (left panel) and the cumulated impulse response of consumption inequality (right panel) to a one-standard-deviation increase in the cleaned target factor from Swanson (2021). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1992Q3-2016Q4. Shaded areas represent the 68% confidence interval.

2.A.3 Gürkaynak et al. (2005): Raw and cleaned factors

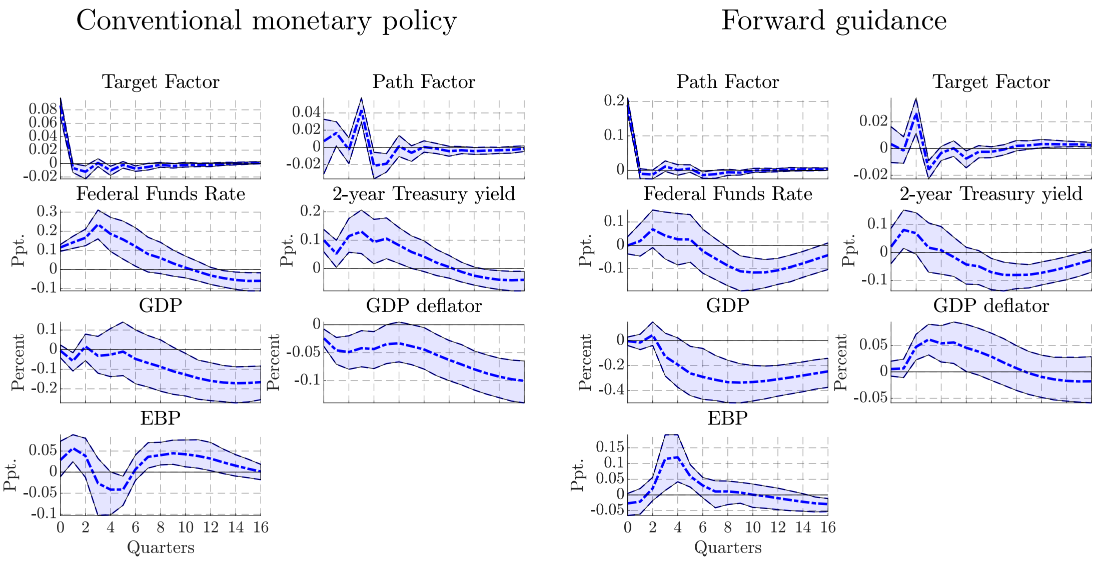
As an alternative measure of conventional monetary policy and forward guidance, we use the two factors (target and path) computed by Gürkaynak et al. (2005), which we extend to 2019.

Figure 2.A3 reports the impulse responses to the two policy shocks. Similarly to the baseline specification with the Swanson (2021) factors, following a contractionary conventional policy shock GDP and inflation decrease whereas EBP increases although the responses are less statistically significant. After a forward guidance shock, GDP decreases but inflation shows a price puzzle similar to the baseline model.

The corresponding consumption inequality responses are shown in Figure 2.A4. The results are consistent with the main results presented in Section 2.4. An increase in the target factor rises inequality whereas an increase in the path factor decreases it.

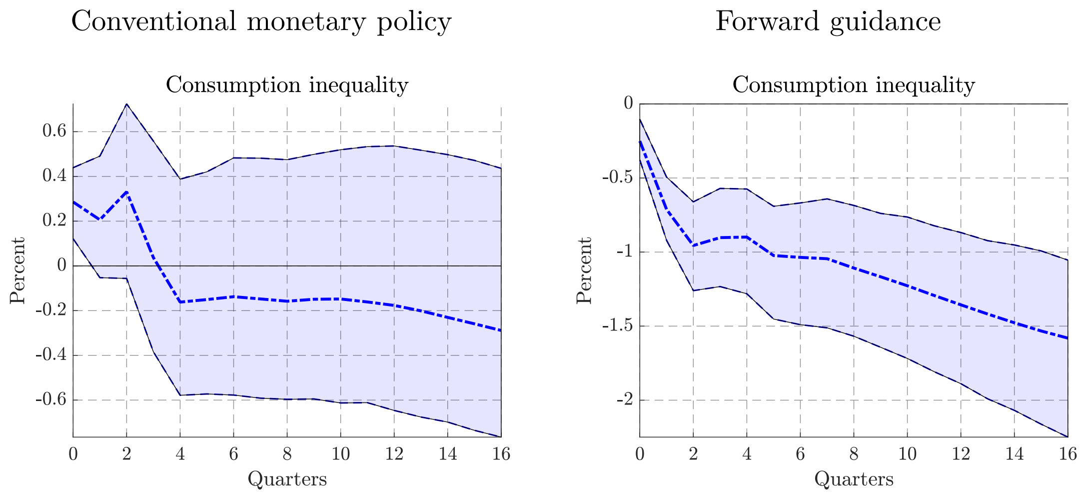
To remove the information component, we adopt the cleaning approach proposed by Miranda-Agrippino and Ricco (2021b) on the target factor computed by Gürkaynak et al. (2005) as well. The responses are reported in Figure 2.A5. Our main findings hold also under this alternative specification.

Figure 2.A3: Macroeconomic responses to the Gürkaynak et al. (2005) factors



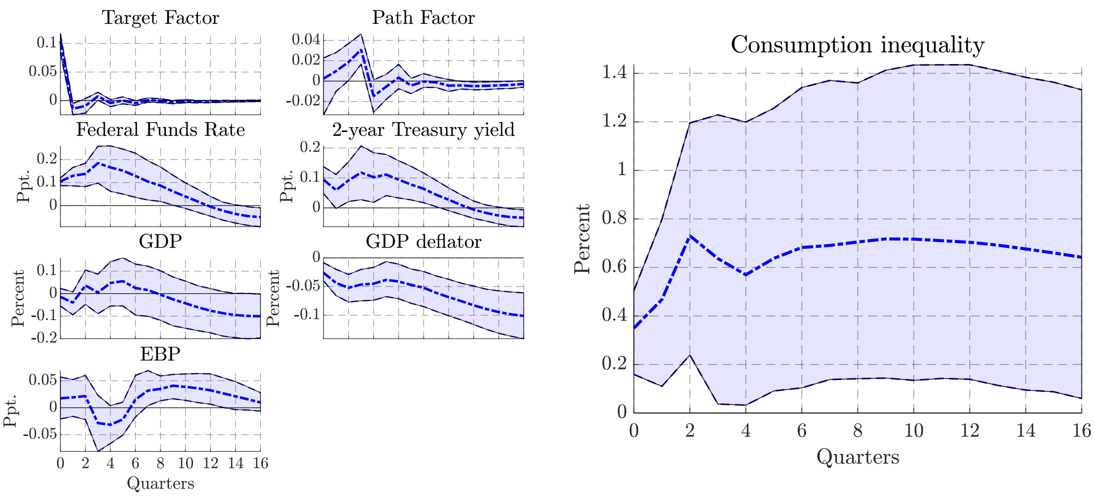
Notes: This figure depicts the impulse responses of macroeconomic variables to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Gürkaynak et al. (2005), respectively. Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q1-2016Q4. Shaded areas represent the 68% confidence interval.

Figure 2.A4: Consumption inequality responses to the Gürkaynak et al. (2005) factors



Notes: This figure depicts the cumulated impulse responses of consumption inequality to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Gürkaynak et al. (2005), respectively. Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q1-2016Q4. Shaded areas represent the 68% confidence interval.

Figure 2.A5: Impulse responses to the cleaned target factor from Gürkaynak et al. (2005)



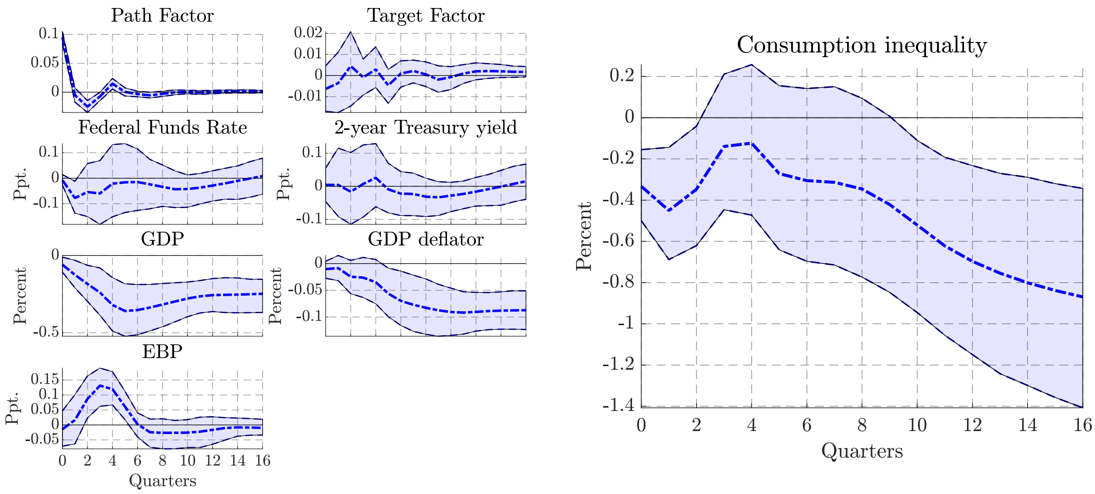
Notes: This figure depicts the impulse responses of macroeconomic variables (left panel) and the cumulated impulse response of consumption inequality (right panel) to a one-standard-deviation increase in the cleaned target factor from Gürkaynak et al. (2005). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q1-2016Q4. Shaded areas represent the 68% confidence interval.

2.A.4 Lakdawala (2019): Cleaned path factor

Lakdawala (2019) proposes a different approach to remove from the factors any component that is capturing the release of private information by the Federal Reserve. The author uses the residuals from a regression where the factors are the dependent variable and controls for the Federal Reserve as well as the market information sets are included. In particular, the Greenbook dataset is used to capture the Federal Reserve's forecasts and the consensus forecasts from the Blue Chip survey is used as an indicator of the market's expectations. The main idea is that the difference between the Greenbook forecasts and the Blue Chip forecasts can be considered as a measure of Federal Reserve private information. The cleaned measures are available from 1991Q1 to 2011Q4.

The responses from the SVAR model with the cleaned path factor from Lakdawala (2019) as exogenous variables are reported in Figure 2.A6. Once the information component is removed from the factor, both GDP and inflation decrease after a contractionary forward guidance shock. On top of that, the shock results in procyclical consumption inequality, again confirming our baseline results.

Figure 2.A6: Impulse responses to the cleaned path factor from Lakdawala (2019)



Notes: This figure depicts the impulse responses of macroeconomic variables (left panel) and the cumulated impulse response of consumption inequality (right panel) to a one-standard-deviation increase in the cleaned path factor from Lakdawala (2019). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q1-2011Q4. Shaded areas represent the 68% confidence interval.

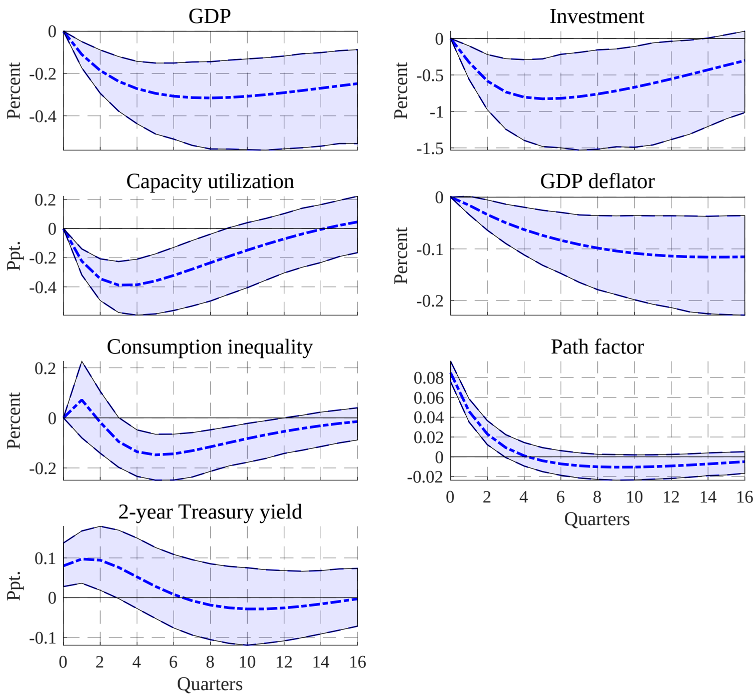
2.A.5 SVAR model specification from Bundick and Smith (2020)

We compare our findings from the SVAR model with a similar specification used in the literature. Bundick and Smith (2020) evaluate the effect of a forward guidance shock on the economy in a structural VAR with a recursive identification scheme. The variables included in the VAR are the real GDP, a proxy for real equipment investment, capacity utilization, the GDP deflator, the cumulative sum of the path factor, and the 2-year Treasury yield. The authors assume that macroeconomic conditions adjust slowly to changes in expected policy rates but financial markets may respond immediately. They order therefore the forward guidance shock measure after real activity and the price level but before the 2-year Treasury yield. Finally, Bundick and Smith (2020) use the pre-zero lower bound period as a pre-sample to form the priors for the VAR parameters during the zero lower bound period (although uninformative priors lead to similar results).

We compute the impulse responses to path factor shock from the same VAR specification, with the same controls and the same measure of forward guidance. The only differences are that the VAR is computed at quarterly frequency and that we add our baseline measure of consumption inequality.

The results are reported in Figure 2.A7. The responses of the macroeconomic variables are similar to those obtained by Bundick and Smith (2020). An increase in the path factor

Figure 2.A7: Impulse responses to a forward guidance shock: Bundick and Smith (2020) approach



Notes: This figure depicts the impulse responses to a one-standard-deviation increase in the path factor from Bundick and Smith (2020). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are computed at quarterly frequency using aggregate-level and inequality data for the period 1994Q1-2015Q4. Shaded areas represent the 90% confidence interval.

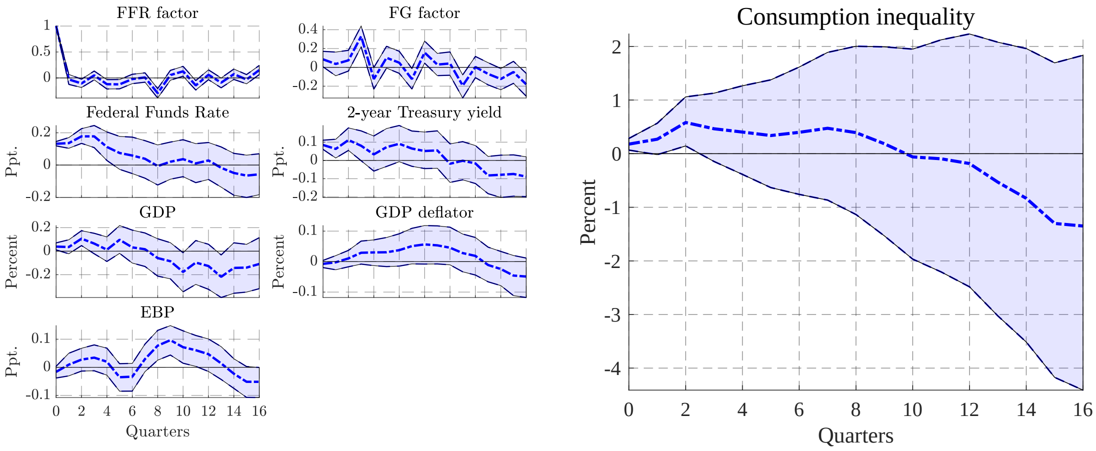
leads to a decrease in output, investment, capital utilization, and the price level. In line with the results from our baseline analysis, consumption inequality significantly decreases in response to forward guidance.

2.A.6 Bayesian local projections

The impulse response functions estimated using a VAR model can suffer from model misspecification, especially if the sample size is small. This might arise, for instance, if some important interactions are neglected, the number of lags is inappropriate or non-linearities are not taken into account. As further robustness check for our results, we compute the responses to conventional monetary policy and forward guidance using the local projection approach by Jordà (2005) which is regarded as more robust to misspecification and imposes fewer assumptions on the empirical model structure.

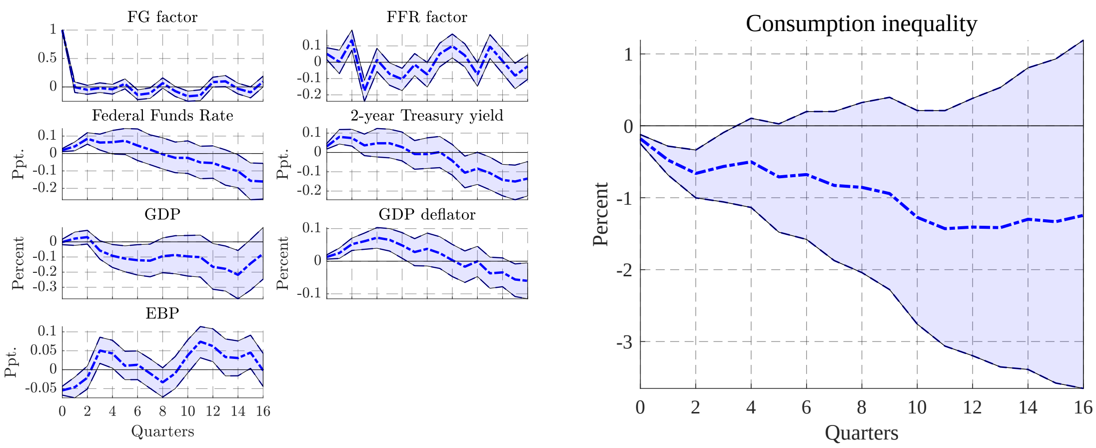
In our specific setup, standard local projections might deliver imprecise estimates given the small sample size. This potential problem is overcome using Bayesian local projections

Figure 2.A8: Impulse responses to the target factor from Swanson (2021): Bayesian local projections



Notes: This figure depicts the impulse responses of macroeconomic variables (left panel) and the cumulated impulse response of consumption inequality (right panel) to a one-standard-deviation increase in the target factor from Swanson (2021). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from Bayesian local projections computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

Figure 2.A9: Impulse responses to the path factor from Swanson (2021): Bayesian local projections



Notes: This figure depicts the impulse responses of macroeconomic variables (left panel) and the cumulated impulse response of consumption inequality (right panel) to a one-standard-deviation increase in the path factor from Swanson (2021). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from Bayesian local projections computed at quarterly frequency using aggregate-level and inequality data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

as proposed by Miranda-Agrippino and Ricco (2021a). Their approach allows us to obtain more precise estimates by specifying a prior for the local projection coefficients at each horizon.

The results to a contractionary conventional monetary policy shock and a forward guidance shock are reported in Figure 2.A8 and Figure 2.A9, respectively. Overall, the responses of the macroeconomic and the financial variables are qualitatively similar to those obtained using the baseline VAR model. Regarding consumption inequality, the alternative specification confirms the different cyclicality in responses under the two monetary shocks.

2.A.7 Alternative empirical specifications

In this exercise, we evaluate if alternative model specifications in terms of the variables used in the VAR or the selected lag length do significantly affect our main result. We compute the consumption inequality responses to conventional monetary policy and forward guidance shocks for all the possible combinations of the Swanson (2021) and the Gürkaynak et al. (2005) factors with either GDP or industrial production as real activity variable, either GDP deflator or CPI as price variable, either the Federal Funds Rate or the 1-year Treasury yield as short-term interest rate variable, either including EBP in the VAR or not, and lag lengths from 2 to 4 lags. The nearly 100 impulse responses are reported in Figure 2.A10.

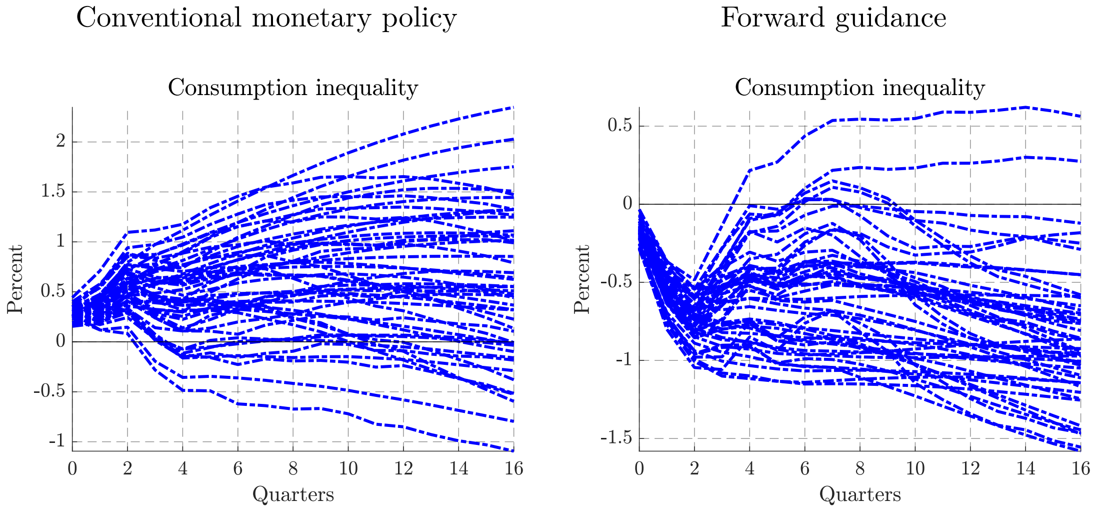
The combination of variables and lags chosen clearly influence the shape and magnitude of the inequality responses to the two monetary policies. However, the majority of simulations point to countercyclical (procyclical) inequality after monetary policy (forward guidance). Even more relevant appears that conventional monetary policy always leads to a contemporaneous increase in inequality whereas forward guidance always decreases it. This finding implies that irrespective of the chosen specification, the main finding in terms of the cyclicality of inequality still holds.

2.A.8 Type-dependency of forward guidance

The nature of forward guidance used by central banks has changed over time. In this section, we therefore assess if the procyclical response of consumption inequality to forward guidance announcements depends on their specific form.

The main types identified in the literature are open-ended guidance, calendar-based guidance, and state-contingent guidance (see, e.g., Ehrmann, Gaballo, Hoffmann, & Strasser, 2019; Moessner & Rungcharoenkitkul, 2019). Open-ended forward guidance is characterized by qualitative statements about the future policy path, time-dependent guidance entails more explicit statements with reference to calendar time, whereas the state-contingent

Figure 2.A10: Consumption inequality responses for various parameter-variable combinations



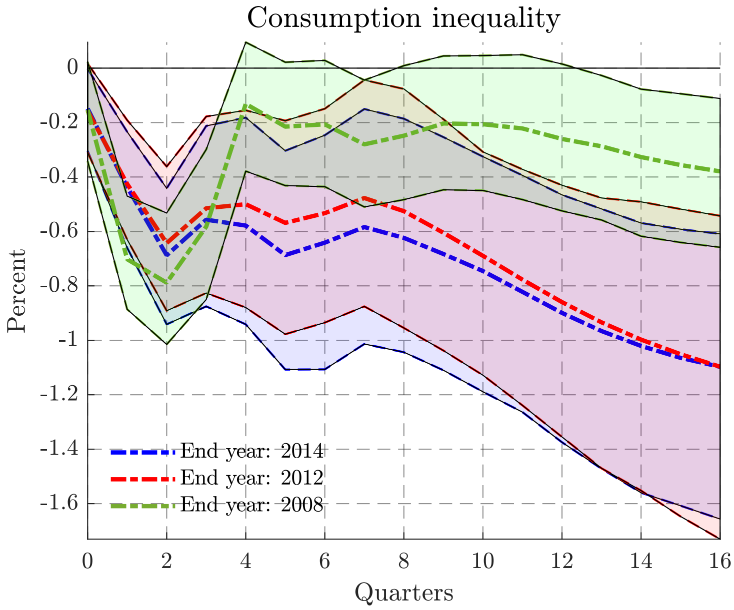
Notes: This figure depicts the cumulated impulse responses of consumption inequality to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. The impulse responses arise from various SVAR models computed for all the possible combinations of the Swanson (2021) and the Gürkaynak et al. (2005) factors with either GDP or industrial production, either GDP deflator or CPI, either the federal funds rate or the 1-year Treasury yield , either including EBP in the VAR or not, and lag lengths from 2 to 4 lags.

type links the policy path to economic developments or outcomes. This categorization is typically applied to the period since policy rates approached the effective lower bound for the first time.

The Federal Reserve in the U.S. has relied on all three types over different subperiods. Its forward guidance can be roughly categorized as open-ended from end-2008 to mid-2011, after that as time-dependent until end-2012, and then state-contingent until 2014. To compare these different forward guidance periods, we compute the responses of consumption inequality to our baseline forward guidance shock ending the sample in 2008, 2012, and 2014, respectively.

The results are reported in Figure 2.A11. The procyclical response of inequality is overall unaffected by the considered subperiod. However, focusing on the sample up to 2008, it seems that the impact in the first few quarters after the shock is marginally stronger but then fades in the longer term. After 2008, there are no significant differences visible and the magnitudes are almost equivalent to the full-sample responses in Figure 2.2.

Figure 2.A11: Consumption inequality responses for different types of forward guidance



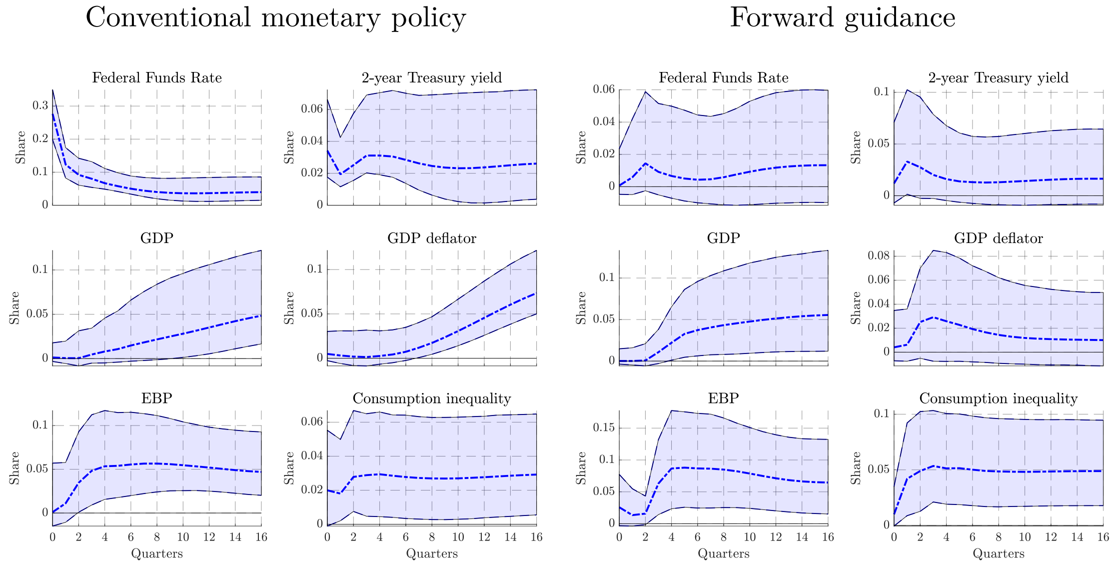
Notes: This figure depicts the cumulated impulse responses of consumption inequality to a one-standard-deviation increase in the path factor from Swanson (2021). It considers different time periods related to the type of forward guidance (see text). Consumption inequality is measured by the cross-sectional standard deviation of household-level real consumption. Impulse responses are from a SVAR model computed at quarterly frequency using aggregate-level and inequality data starting from 1991Q3 onwards. Shaded areas represent the 68% confidence interval.

2.A.9 Quantitative importance of shocks

We have shown how monetary policy shocks significantly influence macroeconomic and financial variables as well as consumption inequality. To evaluate the quantitative importance of these effects, we first compute the forecast error variance decomposition of the baseline VAR model. It captures the share of the variance of our main variables accounted for by each shock at different horizons. The results are reported in Figure 2.A12.

As expected, conventional monetary policy shocks explain a sizable portion of the variance in the Federal Funds Rate, while forward guidance shocks explain a higher share of movements in the 2-year Treasury yield. However, the contribution of both shocks to output and inflation in the short run is relatively small. This is due to the fact that the size of the estimated shocks is quite small since they are computed using the somewhat selective exogenous component of high-frequency changes in asset prices. In terms of consumption inequality, the two shocks account for as much of its variance as they explain movements in medium-term output, suggesting that they both have some non-negligible

Figure 2.A12: Forecast error variance decomposition



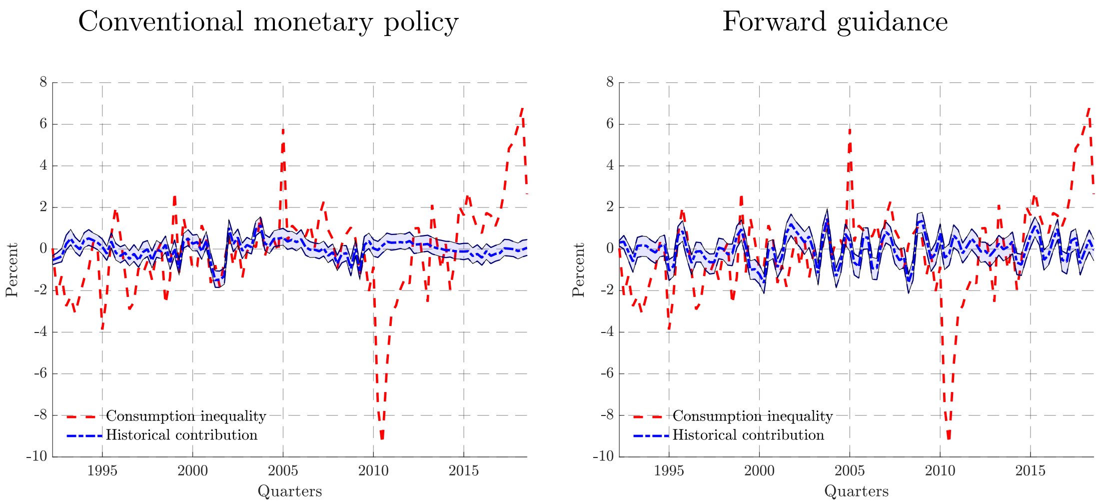
Notes: This figure depicts the forecast error variance decomposition to a one-standard-deviation increase in the target factor (left panel) and the path factor (right panel) from Swanson (2021), respectively. They are computed from a SVAR model at quarterly frequency using aggregate-level data for the period 1991Q3-2019Q2. Shaded areas represent the 68% confidence interval.

distributional effects. The forward guidance shock explains thereby a relatively higher portion of the inequality dynamics.

An alternative way to evaluate the significance of our empirical results is to investigate how important the two monetary shocks are in explaining variations in consumption inequality over time. To do so, Figure 2.A13 reports the historical contribution of conventional monetary policy and forward guidance shocks together with the actual consumption inequality.

Both shocks contribute in particular to the dynamics in inequality after 2000. The magnitudes are sometimes small but overall not negligible, stressing the historical importance of central bank policies for the economy. As expected, the contribution of conventional shocks to the course of inequality is extremely minor during the zero lower bound period after 2008, while forward guidance kept playing a role.

Figure 2.A13: Historical decomposition of consumption inequality



Notes: This figure depicts the historical contribution of conventional monetary policy and forward guidance shocks to consumption inequality together with the actual consumption inequality (in percentage deviations from the mean) for the period 1992–2017. Shaded areas represent the 68% confidence interval.

2.B Analytical TANK model: Derivations and figures

This appendix provides details on the derivations of the simple two-agent model presented in Section 2.5.1 and derives its key analytical expressions. Furthermore, it contains a summary of selected parameter values and additional impulse responses.

2.B.1 Problem of the intermediate goods producers

The price-setting problem of each intermediate goods producer looks as follows:

$$\max_{\{P_{t+k}(j)\}_{k=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left\{ \left[(1+\tau^S)\frac{P_{t+k}(j)}{P_{t+k}} - MC_{t+k} \right] Y_{t+k}(j) - \Theta_{t+k}(j) - T^F_{t+k} \right\}$$

$$\text{subject to} \quad Y_{t+k}(j) = \left(\frac{P_{t+k}(j)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$$

$$\Theta_{t+k}(j) = \frac{\theta}{2} \left(\frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1 \right)^2 Y_{t+k} \ ,$$

where $\Lambda_{t,t+k} = (\beta^S)^k \left(\frac{U^S_{c,t+k}}{U^S_{c,t}} \right)$ is the stochastic discount factor for payoffs in period $t+k$. The optimality condition of this problem is

$$\mathbb{E}_t \Bigg\{ \Lambda_{t,t} \Bigg[\left(1+\tau^S\right)(1-\epsilon)\, P_t(j)^{-\epsilon}\, P_t^{\epsilon-1}\, Y_t + MC_t\, \epsilon\, P_t(j)^{-\epsilon-1}\, P_t^{\epsilon}\, Y_t$$
$$- \theta \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} \Bigg] + \Lambda_{t,t+1}\, \theta \left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{P_t(j)^2}\, Y_{t+1} \Bigg\} = 0 \ .$$

Note that in steady state, if adjustment costs are zero ($\theta = 0$), the last expression reduces to $MC = (1+\tau^S)\frac{\epsilon-1}{\epsilon}$, so that the optimal subsidy τ^S that induces marginal cost pricing in steady state ($MC = 1$) turns out to be $(\epsilon-1)^{-1}$.

Since all firms are identical and face the same demand, they will all make the same decisions and set the same price such that $P_t(j) = P_t$ and $Y_t(j) = Y_t = N_t$. Rewriting the last expression then leads to the Phillips curve:

$$(1+\tau^S)(1-\epsilon) + \epsilon MC_t - \theta(1+\pi_t)\pi_t + \mathbb{E}_t \left[\frac{\Lambda_{t+k}}{\Lambda_t}\, \theta(1+\pi_{t+1})\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \ .$$

2.B.2 Steady state

We consider a steady state with net inflation rate $\pi = 0$, where we normalize output to one by setting $N = 1$ and thus $Y = C = 1$. The Euler equation yields the steady-state real interest rate $r = \beta^{-1} - 1$, which in turn equals the discount rate. We assume that the subsidy on firms' sales is set to its optimal value ($\tau^S = (\epsilon-1)^{-1}$), which induces marginal cost pricing ($MC = W = 1$) and leads to zero profits ($D = 0$) and thus zero dividend income for households ($\Gamma^S = \Gamma^H = 0$) in steady state. Given a calibrated value for the debt-to-GDP ratio $B_Y \equiv B/Y$, we have $B_Y^S = B_Y/(1-\lambda)$ and, through the government budget constraint, $T_y = -rB_y$. Furthermore, we assume that hand-to-mouth agents only consume their labor income in steady state, so that $T^H = 0$ and that steady-state consumption is the same across household types ($C^H = C^S = C$). This also pins down transfers to savers through $T_Y^S = T_Y/(1-\lambda)$. Finally, the weights on hours worked in the utility function are given by $\varphi^j = W\,(L)^{-\nu}(C^j)^{-1}$ for $j = \{H, S\}$.

2.B.3 Log-linearized model

The simple TANK model is approximated around the non-stochastic steady state just described before. Table 2.B1 contains the log-linearized equilibrium conditions, where we have already imposed our assumption that debt is constant over time. Small letters denote the log deviation of a variable from its deterministic steady state. Exceptions are profits, transfers, and debt, each of whose deviation from steady state is considered relative to total income $\left(x_t^j = \frac{X_t^j - X^j}{Y} \text{ for } j = \{H, S\}\right)$, and interest and inflation rates which are expressed in absolute deviations from steady state. Finally, we denote steady-state debt as a fraction of aggregate steady-state income by $B_Y \equiv B/Y$.

Table 2.B1: Model overview of the analytical TANK model

Euler equation, S	$c_t^S = \mathbb{E}_t c_{t+1}^S - \sigma r_t$
Budget constraint, S	$c_t^S = \frac{1}{1-\lambda} r_{t-1} B_Y + w_t + l_t + \frac{1-\tau^D}{1-\lambda} d_t + t_t^S$
Budget constraint, H	$c_t^H = w_t + l_t + \frac{\tau^D}{\lambda} d_t + t_t^H$
Labor supply	$\nu l_t = w_t - \sigma c_t$
Real marginal cost	$mc_t = w_t$
Phillips curve	$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{\epsilon}{\theta} mc_t$
Production function	$y_t = n_t$
Real profits	$d_t = -mc_t$
Government constraint	$-r_{t-1} B_Y = \lambda t_t^H + (1-\lambda) t_t^S$
Aggregate consumption	$c_t = \lambda c_t^H + (1-\lambda) c_t^S$
Labor market clearing	$n_t = l_t$
Resource constraint	$y_t = c_t$
Fisher equation	$r_t = i_t - \mathbb{E}_t \pi_{t+1}$
Monetary policy	$r_t = \rho^{t-\mathcal{T}} \varepsilon_{\mathcal{T}}, \quad t \geq \mathcal{T}$

Notes: This table summarizes the log-linearized equilibrium conditions for the analytical TANK model. The government's lump-sum transfers to individual households, t_t^h and t_t^S, are specified in the main text (see Section 2.5.3).

2.B.4 Reduced-form model equations for consumption and inequality

This section derives reduced-form expressions for the log-linearized analytical model, namely for individual and aggregate consumption and for inequality. The derivations in the first part resemble the ones in Bilbiie et al. (2020). We develop them further in the main part of the paper and determine the condition required for any arbitrary transfer function to replicate the cyclical behavior of inequality found in the empirical analysis.

Drawing on Table 2.B1, the expression for labor supply can be rewritten as $w_t = (\sigma+\nu)c_t$. We can use this together with the condition for profits in the budget constraint of hand-to-mouth agents to get

$$c_t^H = \chi c_t + t_t^H \ ,$$

where $\chi = 1 + (\sigma+\nu)\left(1 - \frac{\tau^D}{\lambda}\right)$. Replacing c_t^H in the equation for aggregate consumption by the last expression leads to

$$c_t^S = \frac{1-\lambda\chi}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} t_t^H \ .$$

By using the above equations, consumption inequality can be written as

$$\Phi_t \equiv c_t^S - c_t^H = \frac{1-\chi}{1-\lambda} c_t - \frac{1}{1-\lambda} t_t^H \ .$$

If we iterate forward the Euler equation and assume $\lim_{i\to\infty} \mathbb{E}_t c_{t+i}^S = 0$, we get $c_t^S = -\sigma \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k}$. Replacing the saver's consumption with the previous expression and solving for aggregate consumption results in the aggregate Euler equation:

$$c_t = -\sigma \frac{1-\lambda}{1-\lambda\chi} \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} + \frac{\lambda}{1-\lambda\chi} t_t^H \ . \tag{2.B.1}$$

Finally, the stream of real interest rates can be rewritten as $\sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} = \sum_{k=0}^{\infty} \mathbb{E}_t \rho^{t+k-\mathcal{T}} \varepsilon_{\mathcal{T}} = 1/(1-\rho)\, \varepsilon_{\mathcal{T}}$, for $t \geq \mathcal{T}$. Combining the previous equations then leads to the expression for consumption inequality (2.6).

2.B.5 Proof of Proposition 3

Combining the proposed transfer function for constrained households, $t_t^H = -\phi_1 r_t B_Y - \phi_2 y_t$, with the aggregate Euler equation (2.B.1) yields

$$t_t^H = -\phi_1 \frac{1-\lambda\chi}{1-\lambda\chi+\phi_2\lambda} r_t B_Y + \phi_2 \frac{\sigma(1-\lambda)}{1-\lambda\chi+\phi_2\lambda} \sum_{k=0}^{\infty} \mathbb{E}_t r_{t+k} \ .$$

Let $\mathcal{T} \geq 0$ denote the period of the real interest rate change. According to Proposition 2, to achieve countercyclical consumption inequality on impact of a conventional monetary policy shock ($\mathcal{T} = 0$) and, at the same time, for inequality to respond procyclically to forward guidance ($\mathcal{T} > 0$), the transfer function above must fulfill the following conditions simultaneously:

$$\frac{\partial t_0^H}{\partial r_{\mathcal{T}}} \begin{cases} < \sigma(\chi - 1)\,\frac{1}{1-\rho}, & \text{if } \mathcal{T} = 0 \\ > \sigma(\chi - 1)\,\frac{1}{1-\rho}, & \text{if } \mathcal{T} > 0 \end{cases}$$

For the first condition to hold, we require

$$-\phi_1 \frac{(1-\lambda\chi)}{1-\lambda\chi+\phi_2\lambda} B_Y + \phi_2 \frac{\sigma(1-\lambda)}{1-\lambda\chi+\phi_2\lambda}\frac{1}{1-\rho} < \sigma(\chi-1)\,\frac{1}{1-\rho}\ .$$

We assume again that $\lambda < 1/\chi$ and further that $\phi_2 > 0$ as argued in Section 2.5.3, which together imply $1 - \lambda\chi + \phi_2\lambda > 0$. Simplifying the last equation then leads to

$$-\phi_1(1-\rho)B_Y + \phi_2\sigma < \sigma(\chi-1)\ . \tag{2.B.2}$$

On the other hand, for the second condition above to be fulfilled, it has to hold that

$$\phi_2 \frac{\sigma(1-\lambda)}{1-\lambda\chi+\phi_2\lambda}\frac{1}{1-\rho} > \sigma(\chi-1)\,\frac{1}{1-\rho}\ .$$

which simplifies to

$$\phi_2 > \chi - 1\ . \tag{2.B.3}$$

Combining (2.B.2) with (2.B.3) concludes the proof.

2.B.6 Calibration for the analytical TANK model

Table 2.B2 summarizes the parameterization for the simple TANK model. Most parameter values are either based on convention or taken from Kaplan et al. (2018), except for the demand elasticity ϵ which is chosen to match a price markup of 20%. The transfer rule coefficients and the tax rate on profits are jointly determined within the range of possibilities that fulfill Proposition 3. In particular, $\tau^D = 0.27$ is in line with the model-implied computations in Bilbiie (2020). Moreover, $\tau^D < \lambda$ implies that $\chi > 1$, which is in line with the comparable empirical results from Auclert (2019) and Patterson (2022).

2.B.7 Additional figures for the baseline analytical TANK model

Figure 2.B1 shows an alternative specification which adds to the remarks in Section 2.5.3. The baseline parameterization for the tax rate on profits and the transfer rule coefficients has been replaced by $\tau^D = 0.2$, $\phi_1 = 0.4$, and $\phi_2 = 0.8$, such that there is a higher weight on the cyclical component in the transfer function and a lower weight on the debt burden.

Table 2.B2: Parameter values for the simple TANK model

Parameter	Description	Value	Source / Target
λ	Share of hand-to-mouth	0.3	Kaplan et al. (2018)
β	Discount factor	1.0125^{-1}	Kaplan et al. (2018). Annual steady-state interest rate of 5%
σ	Intertemporal elasticity of substitution	1	Conventional
$1/\nu$	Frisch elasticity of labor supply	1	Conventional
ϵ	Elasticity of substitution between goods	6	Price markup of 20%
θ	Rotemberg price adjustment cost	100	Kaplan et al. (2018)
τ^D	Tax rate on profits	0.27	Own choice based on empirical evidence
ϕ_1	Transfer rule coefficient on debt	0.8	Own choice based on empirical evidence
ϕ_2	Transfer rule coefficient on output	0.4	Own choice based on empirical evidence
$\|B\|/(4Y)$	Steady-state debt to annualized GDP	0.23	Kaplan et al. (2018)
ρ	Persistence of policy shock	0.5	Kaplan et al. (2018)
$\varepsilon_{\mathcal{T}}$	Shock impact	0.0025	Annualized change of 1%

Compared to Figure 2.5 this setup implies a lower τ^D and therefore a higher elasticity of constrained agents' income to total income. Namely, $\chi = 1.67$.

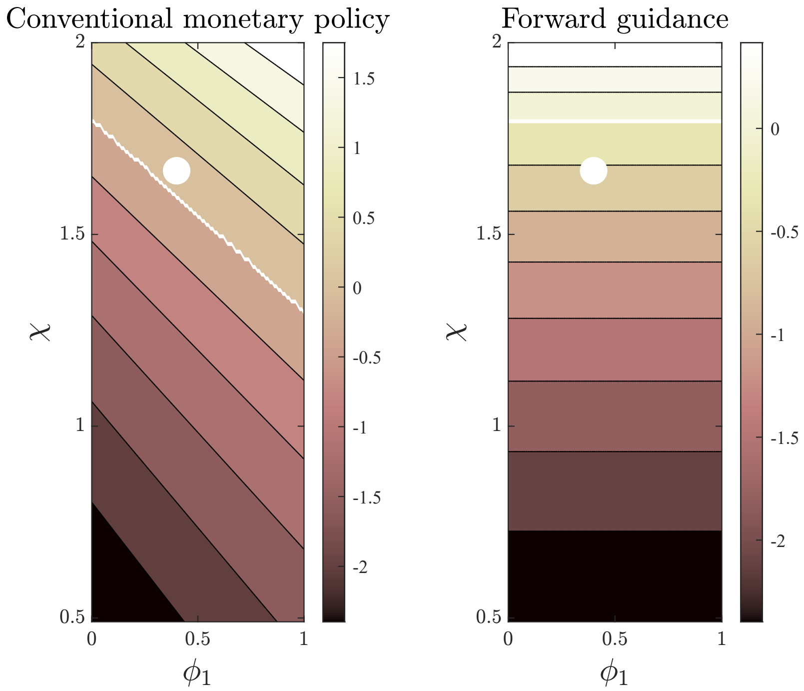
Figure 2.B2 complements the set of impulse responses for the simple TANK model, with the main graphs located in Figure 2.6. Note that the response of debt is not shown because it is assumed to be constant and remains at its steady-state level over the full horizon.

2.B.8 Taylor rule and implied policy shocks

Instead of setting an exogenous path for the real interest rate as was assumed in the baseline model, we could alternatively assume that the central bank adjusts the nominal interest rate by following a Taylor rule of the form $i_t = \psi\pi_t + \tilde{\varepsilon}_{\mathcal{T}}$, with $\psi > 1$ and $\mathcal{T} \geq 0$. In that case, a particular sequence of implied anticipated shocks $\tilde{\varepsilon}_{\mathcal{T}}$ exists that generates the same path for the real rate as in the baseline model and also leaves all the other impulse responses unchanged.

Figure 2.B3 depicts the responses of nominal and real interest rates (for $\psi = 1.5$) together with the implied path for $\tilde{\varepsilon}_{\mathcal{T}}$. We can make a few observations. First, the course of the real rate is equivalent to the baseline responses as was intended. Second, the nominal interest rate initially reacts negatively to the contractionary forward guidance shock. This

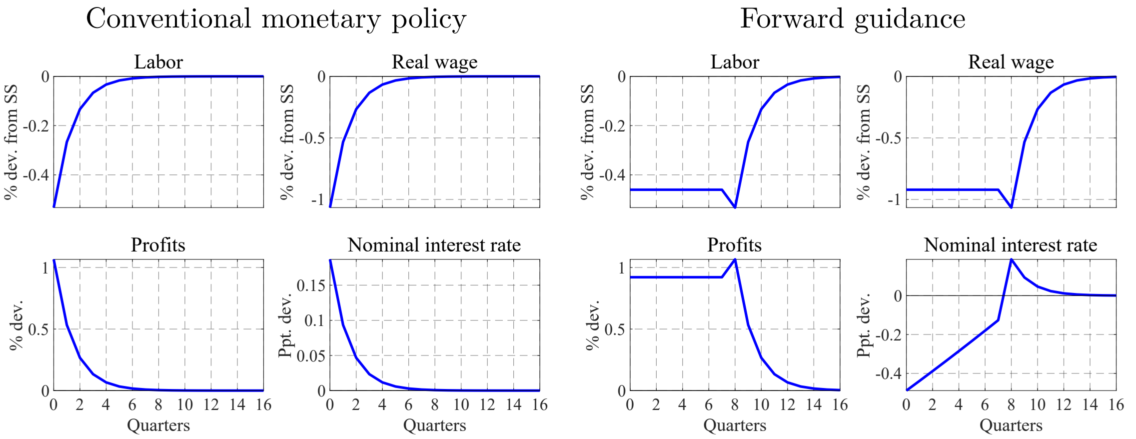
Figure 2.B1: Sensitivity of the inequality response: Alternative calibration



Notes: These heat maps show the response of inequality on impact of a conventional monetary policy and a forward guidance shock, respectively, for different combinations of χ (the elasticity of the constrained household's income to aggregate income) and ϕ_1 (the coefficient on debt burden in the constrained agent's transfer function). The bars next to each plot label the colors, where values above (below) zero refer to a positive (negative) inequality response. The white lines indicate the threshold with zero inequality response. The white dots mark the parameter values implied by an alternative calibration with $\tau^D = 0.2$, $\phi_1 = 0.4$, and $\phi_2 = 0.8$.

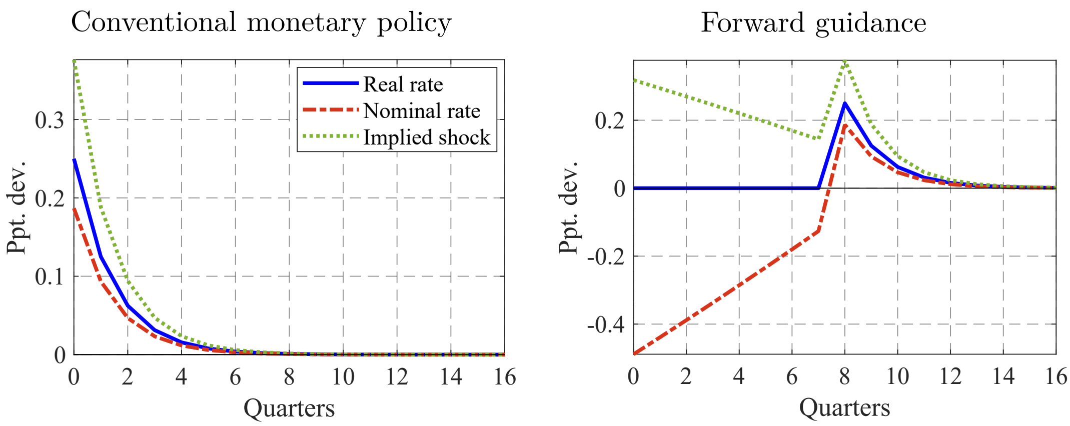
result is different from the empirical findings in Section 2.4. It arises from the large drop in inflation due to permanently lower marginal costs while the real rate remains unchanged for the first seven quarters (see Figure 2.6). Finally, the implied shocks are positive over the entire horizon for both types of policies, reflecting their contractionary nature and making them therefore comparable to the shocks to the FFR and the FG factor in the empirical part.

Figure 2.B2: Additional impulse responses: Analytical TANK model



Notes: This figure depicts the remaining impulse responses for the analytical TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). It complements the results in Figure 2.6. The response of profits is in deviations from their steady-state level, relative to steady-state output.

Figure 2.B3: Impulse responses of interest rates and implied policy shocks



Notes: This figure depicts selected impulse responses for the analytical TANK model alternative to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). The alternative model assumes that the nominal interest rate follows a Taylor rule and replicates the baseline exogenous path for the real rate through a sequence of implied anticipated shocks to the policy rule.

2.B.9 Model with long-term bonds

The core structure and equations of this alternative model are as in the baseline framework presented in Section 2.5.1. The main modification is the introduction of long-term bonds that replace short-term bonds. In what follows, we borrow in parts from the derivations in Harrison (2017) and Bonciani and Oh (2021).

We follow Woodford (2001) and model long-term bonds as perpetuities with coupon payments that decay geometrically at rate $\kappa \in [0,1]$. A nominal bond $\widetilde{B}^L_{t+1}$ issued at date t pays the stream of coupons $1, \kappa, \kappa^2, \ldots$ in the following periods. Its price at time t is Q_t and the real market value of long-term bonds can be defined as $B^L_{t+1} = Q_t \frac{\widetilde{B}^L_{t+1}}{P_t}$. Note that this setup also nests short-term bonds, namely for $\kappa = 0$.

The modification above affects the budget constraint of a saver which now looks as follows:

$$P_t C^S_t + Q_t \widetilde{B}^{S,L}_{t+1} = (1+\kappa Q_t)\widetilde{B}^{S,L}_t + P_t W_t L_t + P_t \Gamma^S_t + P_t T^S_t \ ,$$

where $\widetilde{B}^S_{t+1}$ are the end-of-period-t holdings of nominal long-term bonds by saver S. The last equation can be rewritten in real terms:

$$C^S_t + B^{S,L}_{t+1} = \frac{1+\kappa Q_t}{Q_{t-1}} \frac{1}{1+\pi_t} B^{S,L}_t + W_t L_t + \Gamma^S_t + T^S_t \ ,$$

We can then define the gross nominal one-period return on a long-term bond purchased at time $t-1$ as

$$R^{L,n}_{t-1,t} = \frac{1+\kappa Q_t}{Q_{t-1}} \ ,$$

or its real counterpart is given by

$$R^L_{t-1,t} = \frac{R^{L,n}_{t-1,t}}{1+\pi_t} \ .$$

The Euler equation for bonds therefore becomes

$$1 = \beta\, \mathbb{E}_t \left[\left(\frac{C^S_{t+1}}{C^S_t} \right)^{-\frac{1}{\sigma}} R^L_{t,t+1} \right] \ .$$

The setup above implies that the gross yield to maturity at time t on a long-term bond is given by

$$RL^n_t = \frac{1}{Q_t} + \kappa \ ,$$

and so the price of a long-term bond can be expressed by $Q_t = \frac{1}{RL^n_t - \kappa}$. Moreover, we can show that the one-period return is directly related to the yield to maturity by the following expression:

$$R^{L,n}_{t-1,t} = RL^n_t \frac{Q_t}{Q_{t-1}} \ .$$

Finally, in the absence of frictions and between two consecutive periods, there is a no-arbitrage condition between short-term, one-period debt and long-term debt:

$$\mathbb{E}_t R^L_{t,t+1} = R_t \ ,$$

where $R_t = 1 + r_t$ is the gross short-term real rate as used in the baseline model.

In log-linear terms, we have the following equations:

$$c^S_t = \mathbb{E}_t c^S_{t+1} - \sigma r^L_{t,t+1} \tag{2.B.4}$$

$$r^{L,n}_{t-1,t} = \kappa\beta q_t - q_{t-1} \tag{2.B.5}$$

$$= rl^n_t + q_t - q_{t-1} \tag{2.B.6}$$

$$rl^n_t = -(1-\kappa\beta) q_t \tag{2.B.7}$$

$$r^L_{t-1,t} = r^{L,n}_{t-1,t} - \pi_t \tag{2.B.8}$$

$$r_t = \mathbb{E}_t r^L_{t,t+1} \tag{2.B.9}$$

where interest rates are defined in log deviations from their non-stochastic steady state and where we used that $R^L = R^{L,n} = RL^n = \beta^{-1}$ holds in steady state. Note that due to the no-arbitrage condition (2.B.9), the Euler equation (2.B.4) is equivalent to the one from the baseline model (see Table 2.B1). All else equal, any changes in individual consumption levels will therefore originate from variations in transfers from the government.

With the equations above at hand, we can derive an expression for the price of the long-term bond as a function of expected nominal one-period returns. From (2.B.5), we have $\mathbb{E}_t r^{L,n}_{t,t+1} = -q_t + \kappa\beta \, \mathbb{E}_t q_{t+1}$. Solving for q_t and forwarding leads to

$$\begin{aligned} q_t &= -\sum_{i=0}^{\infty} (\kappa\beta)^i \, \mathbb{E}_t r^{L,n}_{t+i,t+1+i} \\ &= -\sum_{i=0}^{\infty} (\kappa\beta)^i \, \mathbb{E}_t \left(r^L_{t+i,t+1+i} + \pi_{t+1+i} \right) \ . \end{aligned}$$

Note that (2.B.9) implies $\mathbb{E}_t r^L_{t+i,t+1+i} = \mathbb{E}_t r_{t+i}$, and so a immediate impact of forward guidance on the bond price. Using the last equation in (2.B.7) relates the yield to maturity to expected future rates:

$$rl^n_t = (1-\kappa\beta) \sum_{i=0}^{\infty} (\kappa\beta)^i \, \mathbb{E}_t r^{L,n}_{t+i,t+1+i}$$

$$= (1-\kappa\beta)\sum_{i=0}^{\infty}(\kappa\beta)^i\,\mathbb{E}_t\left(r^L_{t+i,t+1+i}+\pi_{t+1+i}\right) \; .$$

The other main element that is affected by the introduction of long-term bonds is the budget constraint of the government, which is now given by

$$Q_t\widetilde{B}^L_{t+1} = (1+\kappa Q_t)\widetilde{B}^L_t + P_tT_t \; ,$$

or in real terms by

$$B^L_{t+1} = \frac{1+\kappa Q_t}{Q_{t-1}}\frac{1}{1+\pi_t}B^L_t + T_t \; .$$

Approximated around the non-stochastic steady state, we get

$$b_{t+1} = \beta^{-1}b_t + \beta^{-1}r^L_{t-1,t}B^L_Y + t_t \; ,$$

with debt-to-GDP ratio $B^L_Y \equiv B^L/Y$. We can assume for simplicity that $B^L_Y = B_Y$ to make the analytical results more easily comparable to the baseline model.

2.B.10 Transfer functions with non-constant debt

For illustration purposes, we have assumed in the baseline TANK model that the fiscal authority maintains a constant level of debt over time. Relaxing that assumption brings back the simple government budget constraint $B_{t+1} = (1+r_{t-1})B_t + T_t$, or in log-linear form $b_{t+1} = \beta^{-1}b_t + r_{t-1}B_Y + t_t$, where $t_t = t^H_t + (1-\lambda)t^S_t$ and $B_Y \equiv B/Y$. In order for the assumption of non-constant debt to have an economic impact beyond the fiscal budget, we need to modify the transfer function for hand-to-mouth agents. Moreover, assuming that the government now adjusts debt to balance its budget, we also have to define a rule that governs transfers to savers.

We follow the baseline specification in equation (2.9) and assume transfer functions with a debt element and a cyclical component. For the first specification, staying close to (2.9) again, the debt element consists of the interest expenses but in their non-constant form now:

$$t^H_t = -\phi_1\left(\beta^{-1}b_{t+1} + r_tB_Y\right) - \phi_2 y_t \qquad (2.B.10)$$

$$t^S_t = -\phi_1\left(\beta^{-1}b_{t+1} + r_tB_Y\right) + \phi_2 y_t \; . \qquad (2.B.11)$$

Note that we have simply assumed the same functional form for both agents, but with an opposed sign in front of ϕ_2 due to the idea of that second part being an automatic stabilizer intended to smooth fluctuations in constrained agents' income. We could alternatively assume $\phi_2 = 0$ and still achieve the findings below.

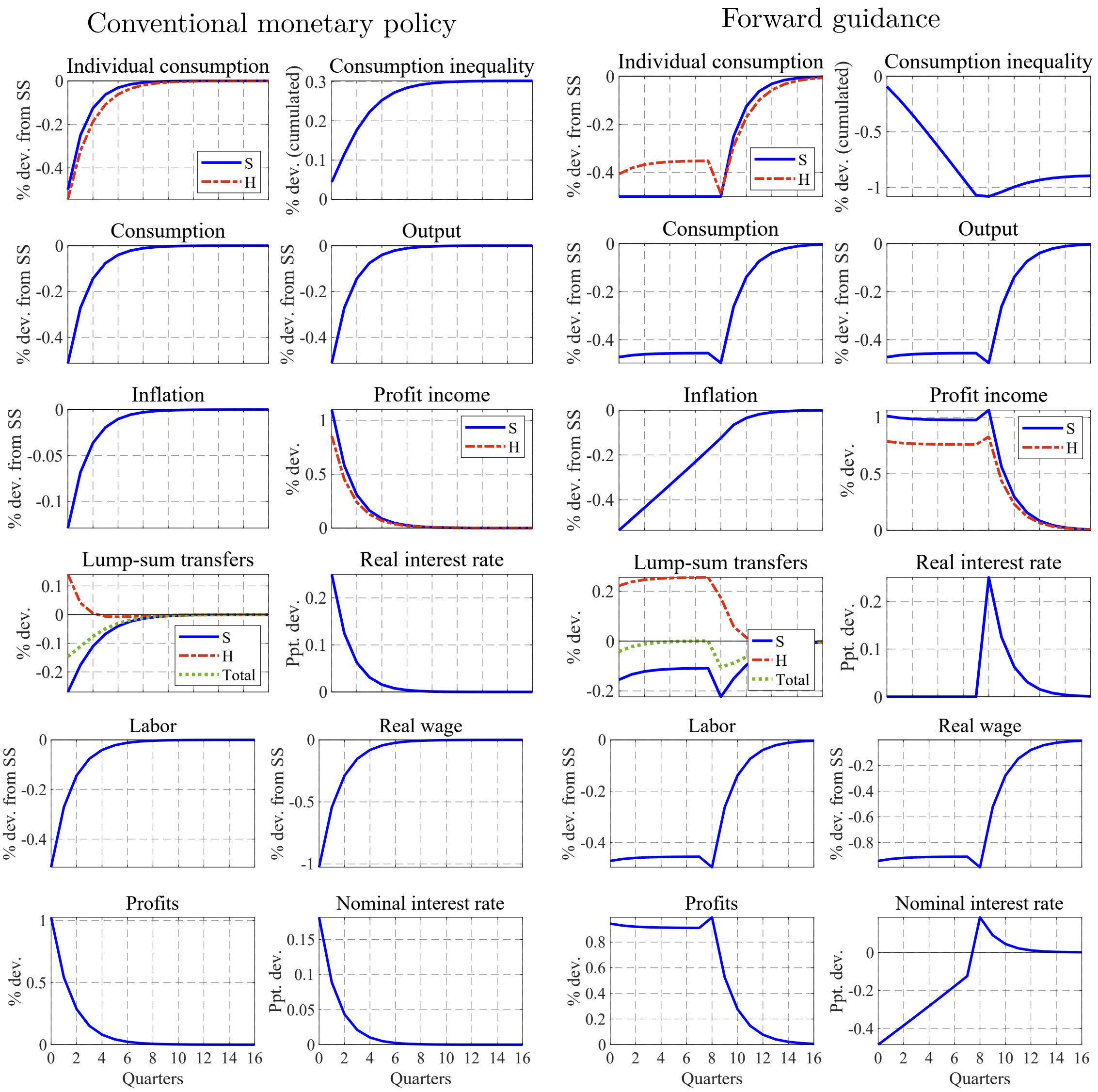
As an alternative specification, we consider a functional form where the first component is directly linked to the level of debt instead of the interest payments on debt:

$$t_t^H = -\phi_1 b_{t+1} - \phi_2 y_t \tag{2.B.12}$$

$$t_t^S = -\phi_1 b_{t+1} + \phi_2 y_t \; . \tag{2.B.13}$$

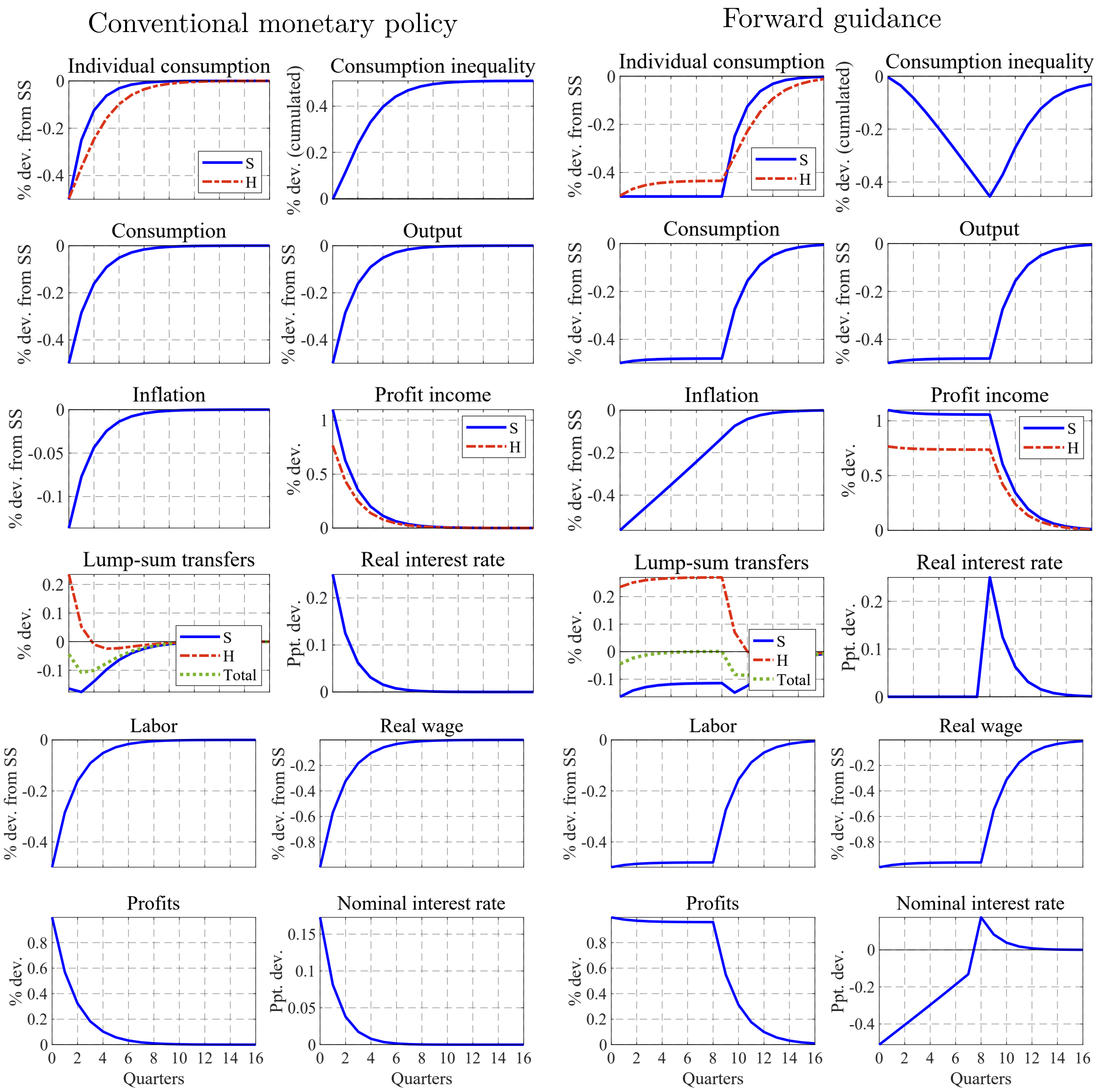
Figures 2.B4 and 2.B5 show the impulse responses from the two simulations, where we used the baseline calibration from Table 2.B2. The only exception is the tax rate on profits which is set slightly lower to $\tau^D = 0.25$ (first case) or $\tau^D = 0.23$ (second case), respectively, to be able to replicate the opposite cyclicality of inequality. The results are qualitatively similar to the ones from the baseline model. One main difference can be seen in the transfer responses which are more immediate for savers and larger for both agents. Moreover, inequality after a conventional monetary policy shock responds by more in the medium-term, in particular for the second specification (Figure 2.B5). At the same time, it responds by less to forward guidance. Overall, these findings show that the main results from the baseline model can even be achieved under non-constant debt.

Figure 2.B4: Impulse responses to monetary policy shocks: Non-constant debt specification 1



Notes: This figure depicts alternative impulse responses for the analytical TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). Different from the baseline model, debt is non-constant and individual transfers evolve according to equations (2.B.10) and (2.B.11). Responses of profit income and transfers are in deviations from their steady-state levels, relative to steady-state output. Individual responses for savers (S) and hand-to-mouth agents (H) are shown in per-capita terms.

Figure 2.B5: Impulse responses to monetary policy shocks: Non-constant debt specification 2



Notes: This figure depicts alternative impulse responses for the analytical TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). Different from the baseline model, debt is non-constant and individual transfers evolve according to equations (2.B.12) and (2.B.13). Responses of profit income and transfers are in deviations from their steady-state levels, relative to steady-state output. Individual responses for savers (S) and hand-to-mouth agents (H) are shown in per-capita terms.

2.C Fully-fledged TANK model: Derivations and figures

This appendix provides details on the derivations of the two-asset TANK model presented in Section 2.5.5. It also contains a summary of the parameterization and additional impulse responses.

2.C.1 Model

This section outlines the model structure of the extended TANK model. It builds for the most part on the two-agent version of the heterogeneous-agent model by Kaplan et al. (2018). The main differences or novelties with respect to their model are: i) a tax and transfer system applied by the government that redistributes income between households (through either profit taxation or in a lump-sum fashion); and ii) a different monetary policy setup where the central bank commits to a path for the real interest rate rather than sets the nominal rate according to a Taylor rule. All deviations are explained in detail along the model description.

Households. There is a continuum of households with an exogenous share $1-\lambda$ of savers (S) who hold and price all assets in the economy. The remaining share λ of households have no access to financial markets and live hand-to-mouth (H) by consuming their total income in each period.[31]

Each household has preferences over utility from consumption C and disutility from supplying labor L:

$$U\left(C_t, L_t\right) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \varphi \frac{L_t^{1+\nu}}{1+\nu} \ ,$$

where σ denotes the elasticity of intertemporal substitution, $\frac{1}{\nu}$ the Frisch elasticity of labor supply, and $\varphi > 0$ represents the relative weight of leisure in the utility function.

Savers. Unconstrained agents can save and borrow in a liquid real government bond B at the real interest rate r^B. They can also hold illiquid assets A at rate r^A, but need to pay a transaction cost χ for depositing into or withdrawing from that account.[32] The presence of this cost implies that, in equilibrium, the illiquid asset return will be higher than the liquid asset return. Besides this, savers consume, earn labor and dividend income, and

[31] This type of household is labeled as spenders by Kaplan et al. (2018).

[32] In the HANK model of Kaplan et al. (2018), the two assets are used by households to self-insure against idiosyncratic labor income risk. In this paper, we dispense with cyclical risk and precautionary savings.

pay taxes. They each solve the following problem:

$$\max_{C_t^S, L_t^S, D_t, B_{t+1}^S, A_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U\left(C_t^S, L_t^S\right) \quad \text{subject to}$$
$$C_t^S + B_{t+1}^S + D_t + \chi_t = (1 + r_{t-1}^B) B_t^S + (1-\tau) W_t L_t^S + \Gamma_t^S + T_t^S$$
$$A_{t+1} = (1 + r_t^A) A_t + D_t \ ,$$

where the notation for assets captures end-of-period values such that B_{t+1}^S and A_{t+1} denote savings in liquid and illiquid assets, respectively, at the end of period t. Moreover, D_t denotes deposits into ($D > 0$) or withdrawals from ($D < 0$) the illiquid account, W_t is the real wage, where labor income is taxed at rate τ, Γ_t^S are dividends from monopolistic firms' profits net of taxes (specified below), and T_t^S are real lump-sum transfers from the government.[33] The functional form of the transaction cost depends on the deposit decision:

$$\chi_t = \chi_1 \left|D_t\right|^{\chi_2} \ ,$$

where $\chi_1 > 0$ and $\chi_2 > 1$ make sure that deposit rates are finite. The optimality conditions for this problem are:

$$\begin{aligned}
(C_t^S)^{-\frac{1}{\sigma}} &= \Lambda_t \\
\varphi (L_t^S)^{\nu} &= \Lambda_t (1-\tau) W_t \\
\Psi_t &= 1 + \text{sgn}(D_t) \left\{ \chi_1 \chi_2 \left|D_t\right|^{\chi_2 - 1} \right\} \\
\Lambda_t &= \mathbb{E}_t \left[\Lambda_{t+1} (1 + r_t^B) \right] \\
\Lambda_t \Psi_t &= \mathbb{E}_t \left[\Lambda_{t+1} \Psi_{t+1} (1 + r_{t+1}^A) \right] \ ,
\end{aligned}$$

where Λ_t and $\Lambda_t \Psi_t$ define the Lagrangian multipliers on the budget constraint and the illiquid asset accumulation equation, respectively, and sgn(.) is a function that extracts the sign of D_t. By combining the expressions above, we can derive Euler equations for liquid and illiquid assets, respectively, and the standard intratemporal condition:

$$\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\frac{1}{\sigma}} (1 + r_t^B) \right] \\
1 &= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\frac{1}{\sigma}} \frac{1 + \text{sgn}(D_{t+1}) \left\{ \chi_1 \chi_2 \left|D_{t+1}\right|^{\chi_2 - 1} \right\}}{1 + \text{sgn}(D_t) \left\{ \chi_1 \chi_2 \left|D_t\right|^{\chi_2 - 1} \right\}} (1 + r_{t+1}^A) \right] \\
W_t &= \frac{\varphi}{1-\tau} \left(L_t^S\right)^{\nu} \left(C_t^S\right)^{\frac{1}{\sigma}} \ .
\end{aligned}$$

[33] Different from the simple TANK model presented in Section 2.5.1, firms' profits are denoted here by Π_t and D_t captures deposits instead.

Hand-to-mouth. Constrained households own no assets and just consume in every period their total after-tax labor income $W_t L_t^H$ together with transfers from the government. The latter consists of two parts: a redistributed part arising from taxed profits Γ_t^H and a lump-sum transfer T_t^H. Each hand-to-mouth household, therefore, solves the problem

$$\max_{C_t^H, L_t^H} U\left(C_t^H, L_t^H\right) \quad \text{subject to}$$
$$C_t^H = (1-\tau) W_t L_t^H + \Gamma_t^H + T_t^H \ .$$

The optimality condition is

$$W_t = \frac{\varphi}{1-\tau} \left(L_t^H\right)^{\nu} \left(C_t^H\right)^{\frac{1}{\sigma}} \ .$$

Firms. The supply side of the economy features monopolistically competitive producers that provide intermediate goods to perfectly competitive final goods firms.

Final goods producers. A representative firm in the final goods sector aggregates differentiated intermediate inputs j to a final good according to the CES production function $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \, di\right)^{\frac{\epsilon}{\epsilon-1}}$ with elasticity of substitution across goods ϵ. Profit maximization yields the demand for each input, $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$, where $P_t(j)$ is the price of intermediate good j and $P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} \, dj$ the aggregate price index.

Intermediate goods producers. There is a continuum of monopolistically competitive firms, each of which produces a variety j of the intermediate good using capital K and labor N as inputs:

$$Y_t(j) = K_t(j)^{\alpha} N_t(j)^{1-\alpha} \ ,$$

where α is the capital share and $1-\alpha$ is the labor share. Each firm rents capital and hires labor in competitive factor markets at rate r_t^K and wage W_t, respectively. Cost minimization results in the following conditions for the optimal factor shares:

$$r_t^K = \alpha \frac{Y_t(j)}{K_t(j)} MC_t$$
$$W_t = (1-\alpha) \frac{Y_t(j)}{N_t(j)} MC_t \ ,$$

where the real marginal cost is given by

$$MC_t = \left(\frac{r_t^K}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \ .$$

An intermediate goods producer sets its price $P_t(j)$ to maximize profits subject to consumers' demand and a quadratic price adjustment cost as in Rotemberg (1982):

$$\Theta_t = \frac{\theta}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \ .$$

Considering the above, the price-setting problem looks as follows:

$$\max_{\{P_{t+k}(j)\}_{k=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}\Psi_{t,t+k}\left\{\left[\frac{P_{t+k}(j)}{P_{t+k}} - MC_{t+k}\right] Y_{t+k}(j) - \Theta_{t+k}\right\} \qquad \text{subject to}$$

$$Y_{t+k}(j) = \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \ ,$$

where P_t denotes the aggregate price level and $\Lambda_{t,t+k}\Psi_{t,t+k} = \frac{\Lambda_{t+k}\Psi_{t+k}}{\Lambda_t\Psi_t}$ is the stochastic discount factor for payoffs in period $t+k$. Since dividends will be categorized as illiquid asset streams below, we discount the flow of future profits by the respective interest rate r^a, captured here by the Lagrangian multipliers from the saver's optimization problem.

Since all firms are identical and face the same demand, they will all set the same price P_t and we can drop the j subscripts. It also implies that we can write the aggregate production function as $Y_t = K_t^{\alpha} N_t^{1-\alpha}$. All this eventually leads to the following Phillips curve, with inflation defined by $\pi_t = \frac{P_t}{P_{t-1}} - 1$:

$$\pi_t(1+\pi_t) = \mathbb{E}_t\left[\frac{\Lambda_{t+k}\Psi_{t+k}}{\Lambda_t\Psi_t}\,\pi_{t+1}(1+\pi_{t+1})\,\frac{Y_{t+1}}{Y_t}\right] + \frac{1}{\theta}\left[\epsilon MC_t - (\epsilon - 1)\right] \ .$$

Finally, aggregating over firms yields total profits

$$\Pi_t = \left(1 - MC_t - \frac{\theta}{2}\pi_t^2\right) Y_t \ .$$

Profit distribution and illiquid assets. The portfolio of illiquid assets available to savers is composed of capital K_t^S and equity shares S_t^S. The latter figures as a claim to a fraction ω of intermediate firms' profits that are reinvested directly into the illiquid account. A saver's end-of-period-t stock of illiquid assets can therefore be written as

$$A_{t+1} = K_{t+1}^S + q_t S_{t+1}^S \ ,$$

where end-of-period-t shares S_{t+1}^S are priced in period t by q_t. To keep the focus on the illiquid account as a whole, it is assumed that savers can allocate between the two illiquid asset types for free. Therefore, the return on equity must be equal to the return on capital (no-arbitrage condition):

$$\frac{\omega\Pi_t + (q_t - q_{t-1})}{q_{t-1}} = r_t^K - \delta \equiv r_t^A \ ,$$

where δ is the depreciation rate of capital. This expression considers changes in the share price, which will restore equality between the returns from shares and capital after a shock to the economy. The share price itself evolves according to

$$q_t = \frac{1}{1 + r_{t+1}^A} \left(\omega\Pi_{t+1} + q_{t+1}\right) \ ,$$

which justifies the choice of the interest rate r^a for the discounting of future profits of intermediate firms.

Drawing on the expression above, the law of motion for illiquid assets, $A_{t+1} = (1 + r_t^A)A_t + D_t$, can be rewritten as

$$A_{t+1} = (1 + r_t^K - \delta)K_t^S + (\omega\Pi_t + q_t)S_t^S + D_t \ .$$

Aggregated over all savers and imposing market clearing for capital and shares (see below), the last expression becomes

$$(1-\lambda)A_{t+1} = (1 + r_t^K - \delta)K_t + (\omega\Pi_t + q_t) + (1-\lambda)D_t \ .$$

The remaining share of profits $1-\omega$ not reinvested in the illiquid account is transferred lump-sum in liquid form to savers. However, the government taxes the shareholders on the total amount of profits at rate τ^D. Hence, each saver receives an after-tax dividend income of

$$\Gamma_t^S = \frac{(1-\omega) - \tau^D}{1-\lambda}\Pi_t \ .$$

In the two-agent model version of Kaplan et al. (2018), even though only savers have an illiquid account, the fraction $(1-\omega)\Pi_t$ is assumed to be equally distributed lump-sum to both household types and then to be taxed at the same rate as labor income (τ). Here we assume instead that, in the first place, savers receive all the profits net of the share that is reinvested into the illiquid account. At the same time, however, they can be taxed on total profits (if $\tau^D > 0$) and hand-to-mouth agents would receive the revenues from this through the government (see below).

Government. The fiscal authority issues liquid real bonds B and collects taxes on households' labor income to finance public expenditures G_t, lump-sum transfers T_t, and interest payments on pre-existing debt. Its budget constraint is given by

$$B_{t+1} = (1 + r^B_{t-1})B_t - \tau W_t N_t + T_t + G_t \ ,$$

where B_{t+1} is end-of-period-t outstanding debt. We assume that the government adjusts transfers to balance its budget, while debt and expenditures remain fixed at their steady-state levels.

Besides labor income and equivalent to the analytical TANK model in Section 2.5.1, the government levies taxes on monopolistic firms' profits, paid by savers who own those firms, and redistributes the revenues to financially constrained households. This policy is balanced in every period such that

$$\Gamma^H_t = \frac{\tau^D}{\lambda}\Pi_t \ .$$

Furthermore, the government runs a second lump-sum scheme with total transfers given by

$$T_t = \lambda T^H_t + (1 - \lambda)T^S_t \ .$$

Unlike Kaplan et al. (2018) who model individual transfers as a fixed share of total transfers, we draw on the alternative specification from the analytical part and assume that transfers to constrained agents are dependent on the course of debt and the business cycle:

$$T^H_t = -\phi_1 r^B_t B - \phi_2 Y_t \ .$$

Monetary authority. Following McKay et al. (2016) and Kaplan et al. (2016), we assume that the central bank controls the real interest rate. More precisely, it implements monetary policy by setting and committing to a path for the interest rate, $\{r^B_k\}_{k\geq 0}$, that is perfectly credible and foreseen by agents. Prior to $\mathcal{T}$, the real rate remains fixed at its steady-state level r^B. After the change, monetary policy will be given by an exogenous rule. Formally, for $\mathcal{T} \geq 0$:

$$r^B_t = \begin{cases} r^B, & t < \mathcal{T} \\ r^B + \rho^{t-\mathcal{T}}\varepsilon_{\mathcal{T}}, & t \geq \mathcal{T} \end{cases}$$

where $\varepsilon_{\mathcal{T}} = r^B_{\mathcal{T}} - r^B$ denotes the policy shock and ρ its persistence. Moreover, the Fisher equation holds:

$$1 + r^B_t = \frac{1 + i_t}{1 + \pi_{t+1}} \ .$$

Aggregation and market clearing. Aggregate consumption and aggregate labor are given by

$$C_t = \lambda C_t^H + (1-\lambda) C_t^S$$
$$N_t = \lambda L_t^H + (1-\lambda) L_t^S \ .$$

Liquid asset market clearing requires

$$B_{t+1} = (1-\lambda) B_{t+1}^S \ .$$

Aggregating capital and equity shares yields

$$K_{t+1} = (1-\lambda) K_{t+1}^S$$
$$1 = (1-\lambda) S_{t+1}^S \ ,$$

where we normalized the total number of shares to 1. The illiquid asset market then clears when

$$(1-\lambda) A_{t+1} = K_{t+1} + q_t \ .$$

Finally, the goods market clearing condition reads

$$Y_t = C_t + I_t + G_t + (1-\lambda)\chi_t + \Theta_t \ ,$$

where investment evolves according to $I_t = K_{t+1} - (1-\delta)K_t$. By combining the law of motion and market clearing for illiquid assets, this can be rewritten as

$$I_t = r_t^K K_t + \omega \Pi_t + (1-\lambda) D_t \ .$$

2.C.2 Calibration for the extended TANK model

Table 2.C1 summarizes the parameterization for the extended TANK model. Besides the paper-specific parameters, all values are taken from Kaplan et al. (2018) except for the demand elasticity ϵ which is chosen to match a price markup of 20%. It is worth mentioning that the transfer rule coefficients as well as the tax rate on profits are set to the same values as in the analytical model.

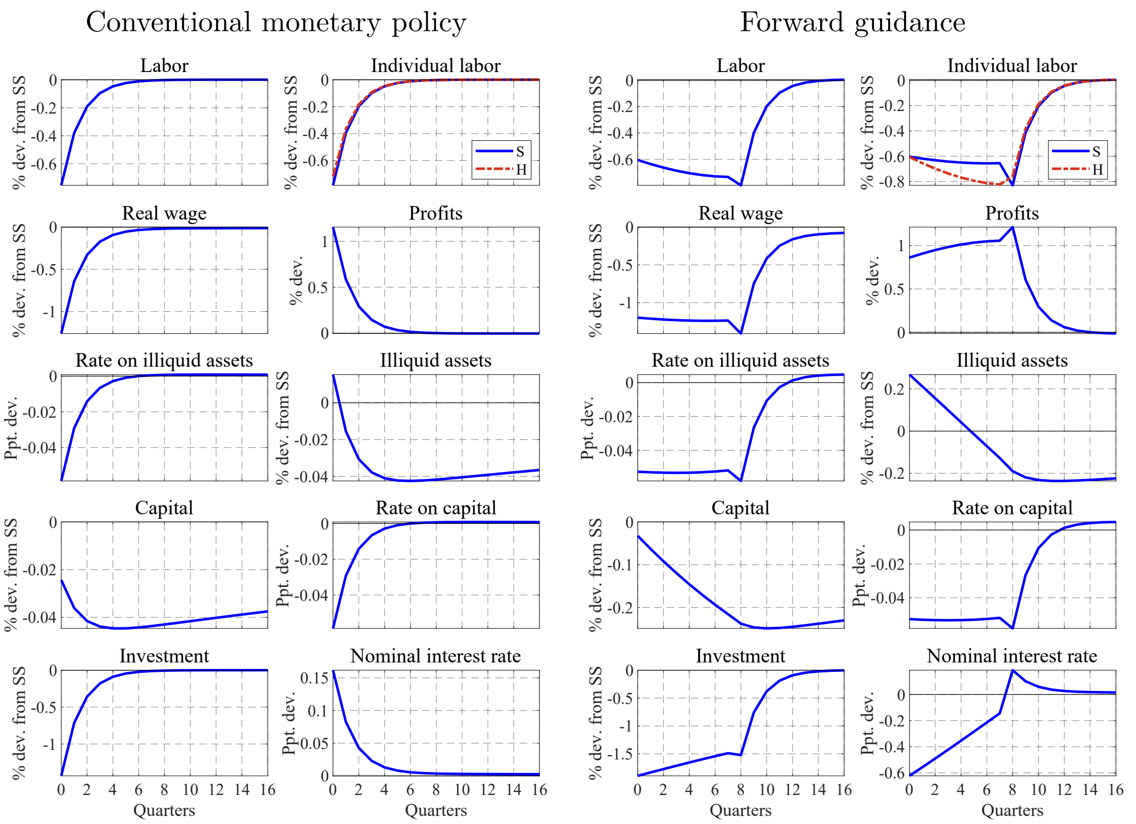
Table 2.C1: Parameter values for the fully-fledged TANK model

Parameter	Description	Value
λ	Share of hand-to-mouth	0.3
β	Discount factor	1.0125^{-1}
σ	Intertemporal elasticity of substitution	1
$1/\nu$	Frisch elasticity of labor supply	1
$\chi_1 \mid \chi_2$	Deposit cost parameters	0.956 | 1.402
ϵ	Elasticity of substitution between goods	6
α	Capital share	0.33
δ	Depreciation rate	0.017
θ	Rotemberg price adjustment cost	100
ω	Share of profits reinvested into illiquid account	0.33
τ	Labor tax rate	0.25
τ^D	Tax rate on profits	0.27
ϕ_1	Transfer rule coefficient on debt	0.8
ϕ_2	Transfer rule coefficient on output	0.4
T	Steady-state lump-sum transfer (% of GDP)	0.06
$\lvert B^G\rvert/(4Y)$	Steady-state debt to annualized GDP	0.23
r^b	Steady-state real liquid return (p.a.)	0.05
ρ	Persistence of policy shock	0.5
$\varepsilon_{\mathcal{T}}$	Shock impact	0.0025

2.C.3 Additional figures for the extended TANK model

Figure 2.C1 complements the set of impulse responses for the fully-fledged TANK model, with the main graphs located in Figure 2.7. Note that the responses for debt and government spending are not shown because both remain at their steady-state level over the full horizon.

Figure 2.C1: Additional impulse responses: Fully-fledged TANK model



Notes: This figure depicts the remaining impulse responses for the extended TANK model to a 25-basis-points increase in the contemporaneous real interest rate (left panel) or in the real rate eight quarters in the future (right panel). It complements the results in Figure 2.7. The response of profits is in deviations from their steady-state level, relative to steady-state output. Individual responses for savers (S) and hand-to-mouth agents (H) are shown in per-capita terms.

Chapter 3

Unwinding Quantitative Easing: State Dependency and Household Heterogeneity[†]

3.1 Introduction

In recent years, large-scale asset purchases have considerably increased the size of central banks' balance sheets. At the same time, as interest rates can fall back to low levels, future crises might once more call for similar unconventional policy tools to stabilize the economy. Central banks are therefore inclined to reduce the quantity of long-term bonds in their books to have sufficient leeway for monetary stimulus when the next negative shock happens.

Various studies have investigated the macroeconomic impact of quantitative easing (QE), generally finding evidence for meaningful effects on output and inflation (see, e.g., Baumeister & Benati, 2013; Haldane, Roberts-Sklar, Wieladek, & Young, 2016; Joyce, Miles, Scott, & Vayanos, 2012; Kapetanios, Mumtaz, Stevens, & Theodoridis, 2012; Weale & Wieladek, 2016). In contrast, evidence on unwinding asset purchases is scarce, mainly because there have to date only been a few attempts to actively do it.

Nevertheless, it seems reasonable to assume that balance sheet reductions do not necessarily have macroeconomic effects that are equal but opposite to expansions. For example, the Federal Reserve's unwind experience in 2017-2019 revealed strong asymmetries in the form of larger liquidity effects compared to periods of balance sheet expansion (Smith & Valcarcel, 2021). Furthermore, the effectiveness of unwinding might be closely linked to the state of the economy and financial markets — similar to QE itself having worked

[†]This chapter is co-authored with Cristiano Cantore from the Sapienza University of Rome.

particularly well when frictions in financial markets were high (Bailey, Bridges, Harrison, Jones, & Mankodi, 2020; Haldane et al., 2016). Finally, unwinding past asset purchases is most likely executed at a slower pace and more gradually and its impact would probably be different from entering QE because of the interaction with policy rates (Vlieghe, 2018, 2021).

Understanding the implications of reducing the central bank's balance sheet is key to dampening the negative side effects on the economy and deciding when and how fast to take that step. Given the lack of empirical evidence on the subject, this issue has to be studied theoretically.

In this paper, we therefore present a two-agent New Keynesian model with borrowers and savers (TANK-BS) that we use to study: i) the asymmetric macroeconomic effects of QE and quantitative tightening (QT) driven by state dependency in the form of a zero lower bound (ZLB) on the nominal short-term interest rate; and ii) the interactions between QE/QT, the ZLB, and household heterogeneity. We thereby define QT as an *active* reduction of a central bank's balance sheet in the form of a sale of assets back to the secondary market, aimed to decrease the amount of liquidity within the economy. Our focus will be on long-term bonds from the government only.

Similar to QE, tightening works through different transmission mechanisms. This paper focuses on the portfolio balance channel.[1] Asset purchases or sales by a central bank change the relative supply of assets the private sector holds, implying movements in relative asset prices and yields. Various studies show that QE programs have indeed raised financial asset prices and reduced longer-term interest rates, often substantially (Christensen & Rudebusch, 2012; Joyce et al., 2011; Krishnamurthy & Vissing-Jorgensen, 2011).

In our model, the two types of agents can borrow and save in short-term and long-term government bonds. The key assumption for the portfolio balance channel of QE/QT to work is the imperfect substitutability between assets, according to which investors value bonds along the yield curve differently (Andrés, López-Salido, & Nelson, 2004). Following Harrison (2017), we capture this idea using portfolio adjustment costs that investors have to pay whenever their preferred relative portfolio composition changes. Since asset market operations alter the relative supply and prices between short-term and long-term bonds, they incentivize asset holders to rebalance their portfolios. This, in

[1] There is a debate regarding the relative importance of the different transmission channels of asset market operations. Several papers have demonstrated the significance of the portfolio balance channel for the effectiveness of QE (see, e.g., D'Amico & King, 2013; Joyce, Lasaosa, Stevens, & Tong, 2011). We deem it as equally important for large-scale asset sales as those will also change the relative supply of assets in the economy and the portfolio composition of households, hence implying potentially considerable real effects.

turn, directly affects their average returns, because any adjustment is costly, and implies changes in their demand.

A large-scale asset sale in the model has an effect on bond returns which translates into an increase in the long-term interest rate and a decrease in the short-term real rate. These effects propagate to the real economy through changes in the portfolio allocation of all households and general-equilibrium effects on real wages, driving down individual consumption. The *direct* effects of QT through the bond market contribute thereby more persistently to the drop in consumption for both agents compared to the *indirect* effect through net labor income changes, among others due to a favorable tax cut. A major difference across the two household types is (countercyclical) profit income. It has a strong positive impact on savers' income such that their consumption drops by much less in relative terms compared to the case of borrowers.

Assuming the presence of state dependency in the form of a (non-)binding ZLB, we are then interested in how doing QE and unwinding it affects aggregate variables such as consumption and real output. The role of the lower bound and whether the nominal interest rate is available as an additional policy tool of the central bank will thereby be the main driver of the asymmetry we focus on.[2] As previous research has found, asset purchases are most effective if the ZLB is binding (Gertler & Karadi, 2013), but there also seems to be a role for asset market operations if policy rates are unconstrained (Sims & Wu, 2021). By analyzing the impact of state dependency on unwinding QE, we thus also address the question of when central banks should actually unwind.

In line with standard intuition, we find that a binding ZLB magnifies the macroeconomic effects of asset market operations by central banks. The response of the short-term real interest rate when the economy is in (or close to) a liquidity trap flips sign and is larger in magnitude. After a QT shock, the short-term real rate decreases when away from the ZLB, while it increases when at the lower bound, generating a further decrease in aggregate demand. As a result, when dealing with the risk of hitting the ZLB, our model implies that a central bank can minimize the economic costs of monetary policy normalization by prioritizing a policy rate hike before starting to sell assets. The likelihood of ending in a liquidity trap is thereby higher when the policy rate is close to the lower bound and QT starts too early or if the tightening is done too fast relative to the normalization of the short-term rate.

The second aim of the paper is to study the interaction between state dependency of QE/QT and household heterogeneity. The empirical literature provides evidence of heterogeneous effects of QE on households across the income distribution (Montecino & Epstein, 2015; Mumtaz & Theophilopoulou, 2017; Saiki & Frost, 2014). On the other

[2] Away from the ZLB, the TANK-BS model is symmetric.

hand, quantitative models have recently found strong distributional effects of QE (Cui & Sterk, 2021). Moreover, there is a large literature showing how heterogeneity can amplify the real effects of conditional monetary policy (see, among others, Auclert, 2019; Bilbiie, 2018, 2020; Bilbiie et al., 2022; Debortoli & Galí, 2017). Against this backdrop, we want to study how the presence of heterogeneous households affects the asymmetry between QE and QT.

We find that household heterogeneity alone does not amplify the aggregate effects of asset market operations when the economy is off the ZLB. This result is in line with that in the complementary work of Sims, Wu, and Zhang (2022b). Differently from us, they use a heterogeneous-agent New Keynesian (HANK) model with uninsurable income risk and QE introduced from the firm's side, as in Gertler and Karadi (2013). The lack of amplification for QE in their framework arises because most agents of the economy react in the same way as in the representative-agent New Keynesian (RANK) counterpart. Only very few households at the bottom of the wealth distribution behave differently and increase their consumption in response to a QE shock. Given that those agents represent a very small share of the population in the economy, it only has a marginal effect on aggregate consumption.

Our story here is different. We show that the lack of amplification via heterogeneity is due to a composition effect of changes in the balance sheet of the two household types and those changes almost entirely cancel out when moving from RANK to TANK-BS. Without borrowers in the model, all the impact of a QT shock on aggregate demand comes from a combination of direct effects (drop in bond demand and interest income) and indirect general-equilibrium effects (drop in real wage due to lower aggregate demand) on the income of the representative agent buying bonds from the central bank. When moving to TANK-BS, borrowers replace part of the savers in the population. While the latter behave like the representative agent in RANK, their share and thus their relative contribution to total spending are lower. The attenuated drop in aggregate demand through savers is compensated by a decrease in the labor income of borrowers who have a larger marginal propensity to consume (MPC). The net effect of the lower cut in spending coming from savers and the additional decrease through borrowers is almost neutral. In the background, profit income is as before an essential element because the higher the proportion of savers the less each agent benefits from the increased (countercyclical) earnings of firms.

Finally, we show that household heterogeneity, when combined with state dependency, amplifies the aggregate effects of asset market operations. When asset sales are performed at the ZLB, the direct and indirect effects on borrowers discussed above together generate a stronger decline in labor income of high-MPC borrowers than the decline in the spending contributed by savers.

Related literature. Our paper is related to several strands of the literature on asset market operations which we summarize hereafter.[3] On the empirical side, the literature has identified various channels through which QE affects the macroeconomy. See Bernanke (2020) and Bhattarai and Neely (2022) for comprehensive reviews. As discussed in the motivation, we focus here on the portfolio balance channel which is one of the key transmission mechanisms through which QE worked in the past.[4]

From a theoretical perspective, QE has mainly been studied in RANK setups (see, among others, Chen, Cúrdia, & Ferrero, 2012; Falagiarda, 2014; Gertler & Karadi, 2013; Harrison, 2012, 2017; Harrison, Seneca, & Waldron, 2021; Sims & Wu, 2021; Sims, Wu, & Zhang, 2022a). On the other hand, the bulk of the literature on household heterogeneity and monetary policy (e.g., Auclert, 2019; Bilbiie, 2008, 2020; Kaplan et al., 2018) has mostly focused on conventional monetary policy. The only two papers we are aware of that merge these two pieces of literature are Cui and Sterk (2021) and Sims et al. (2022b). As discussed in the motivation, while we find a similar result as in the latter, our setup is different because we focus mainly on the effect of asset market operations coming through the balance sheet of households. In Cui and Sterk (2021) instead, the impact of QE on the macroeconomy emerges from the household side as well. They use a model with liquid and illiquid wealth, in the HANK tradition, and focus on the different MPCs out of the two types of wealth. Hence, in their model, household heterogeneity plays a direct role in the transmission mechanism of QE, which they show to be significant on output and inflation. Here we use a much simpler setup, allowing only for two types of agents as in Eggertsson and Krugman (2012) or Bilbiie, Monacelli, and Perotti (2013) and abstracting from liquid and illiquid wealth, while focusing on the impact of QE on households' bonds positions at different maturities. Furthermore, differently from Cui and Sterk (2021) and Sims et al. (2022b), we are not just interested in the interaction of heterogeneity and QE but also on the effects of the ZLB, which both papers abstract from.[5]

The works cited so far are primarily focused on QE. Empirically, this is obviously due to the lack of enough episodes of large-scale asset sales or, more generally, central bank balance sheet reductions. On the theoretical side, a few exceptions are Benigno and Benigno (2022), Cui and Sterk (2021), Karadi and Nakov (2021), Sims et al. (2022a), Wei (2022), and Wen (2014). To the best of our knowledge, Wen (2014) is the first theoretical attempt about QE exit strategies and its impact on firms. We focus instead

[3] A thorough review of the literature is beyond the scope of this paper.

[4] See Christensen and Rudebusch (2012), D'Amico and King (2013), Froemel, Joyce, and Kaminska (2022), and Joyce et al. (2011) for empirical evidence on the portfolio balance channel. Related to this, see Andrés et al. (2004) and Vayanos and Vila (2009, 2021) for the theoretical foundation of imperfect substitutability between assets along the yield curve and preferred-habitat theory, respectively.

[5] Cui and Sterk (2021) assume in their model simulations for QE that the interest rate is pegged at zero. However, they do not compare simulations with and without the peg.

on households and the impact of unwinding QE on their portfolios. Cui and Sterk (2021) analyze the impact of the speed of QE exit, captured by the persistence of the policy in the model. They show that the quicker the exit, the lower the real impact of the policy, which is driven by agents anticipating the dampening effects of exiting QE. By keeping the nominal interest rate pegged, however, they do not look at the interaction between conventional and unconventional monetary policy as we do in this paper. Karadi and Nakov (2021) and Sims et al. (2022a) look at the optimal conditions to exit QE. The former present a model in which banks' balance sheet constraints bind only occasionally, so that asset purchases are not always effective. Unlike them, we are not conducting a normative analysis and we focus on the implications of asset market operations via the portfolio rebalancing of households' assets. Wei (2022) uses the preferred-habitat model of Vayanos and Vila (2021) to quantify how many interest rate hikes QT is equivalent to. Our focus is instead on the macroeconomic implications and we study the interaction of asset market operations with conventional monetary policy rather than treating the two as substitutes. A similar idea is advocated by Benigno and Benigno (2022) who study optimal monetary policy normalization when exiting a liquidity trap. Besides the policy rate, they view reserves as an additional tool of monetary authorities to influence macroeconomic aggregates, while we disregard liquidity in order to keep the central bank balance sheet simple and to stress the transmission through portfolio rebalancing. Somewhat contrary to our finding, their analysis implies that efforts to reduce the size of the central bank balance sheet ideally start before the policy rate is raised.

The last strand of the literature this paper addresses is related to state-dependent QE/QT and possible asymmetries between the two. Policymakers have discussed at length the possible causes and effects of state dependency, focusing mostly on different states of financial markets (Bailey et al., 2020; Haldane et al., 2016; Vlieghe, 2021). To maintain tractability and because our focus is on household portfolio compositions, we abstract in this paper from financial markets and focus on state dependency driven by the ZLB. With respect to asymmetries, we directly address the idea of policymakers that QT is likely to impact the economy by less than asset purchases. Potential explanations for this view include a milder reaction of bond markets as visible during the Federal Reserve's 2017-2019 unwind (Neely, 2019), the vanishing of the signaling effects of asset market operations once policy rates are well above zero (Bullard, 2019), or differences in the nature and scope of QE/QT episodes and the prevailing economic and financial conditions (Smith & Valcarcel, 2021; Vlieghe, 2018, 2021)

Outline. The rest of the paper is organized as follows. Section 3.2 presents the TANK-BS model economy and describes the calibration and the solution method. Section 3.3 discusses the simulation results and section 3.4 concludes.

3.2 Asset market operations in a borrower-saver model

This section presents the main elements of the model used for our analysis. Further details on the derivation, a thorough description of the steady state, and an overview of all model equations can be found in Appendix 3.A.

The model economy consists of four sectors: households, firms, a government and a central bank. The household sector is populated by two different types, savers and borrowers, who differ in their degree of patience, modeled as in Bilbiie et al. (2013) and Eggertsson and Krugman (2012). Firms are modeled as in standard New Keynesian models, with nominal frictions that generate sticky prices. The government finances public spending by issuing bonds and levying lump-sum taxes. It also implements redistributive policies by taxing firms' profits. Finally, the monetary authority follows a Taylor rule to set the nominal interest rate and participates in the market for long-term bonds. The design of asset market operations follows Harrison (2017).

3.2.1 Households

There is a continuum of households with a share λ being borrowers (B) who are constrained in terms of how much they can borrow. The remaining $1-\lambda$ are savers (S) with unconstrained access to asset markets. Borrowers are assumed to be less patient than savers, such that $\beta^S > \beta^B$. As will become clear later, this difference in the discount factors will induce lending from S to B in equilibrium.

The period utility function of household type $j = \{B, S\}$ is given by

$$U\left(c_t^j, N_t^j\right) = \theta_t \left(\frac{(c_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \zeta^j \, \frac{(N_t^j)^{1+\varphi}}{1+\varphi} \right) ,$$

where c_t is real consumption, N_t are hours worked, θ_t is a preference shock that follows an AR(1) process, σ is the elasticity of intertemporal substitution, $\frac{1}{\varphi}$ is the Frisch elasticity of labor supply, and ζ indicates how leisure is valued relative to consumption.

Both household types have access to bonds issued by the government. Following Harrison (2017), we differentiate between real short-term (b^j) and long-term ($b^{j,L}$) bonds. The former are one-period assets: a bond purchased in period $t-1$ pays a real return $r_{t-1} = \frac{R_{t-1}}{\Pi_t}$ at time t, where R is the gross nominal interest rate and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. On the other hand, we assume that longer-term government debt is captured by perpetuities with coupon payments that decay exponentially over time as in Woodford (2001). Denoting by $\widetilde{B}_t^{j,L}$ the nominal long-term bond holdings of a saver and by V_t the nominal price of each of these bonds, we can write the value of long-term bond holdings as $B_t^{j,L} = V_t \widetilde{B}_t^{j,L}$. By defining χ as the long-term bond coupon decay rate, Harrison (2017) then shows that the (ex-post) nominal return on long-term bonds is $R_t^L = \frac{1+\chi V_t}{V_{t-1}}$. This formulation allows us to express long-term bonds in the budget constraint in terms of a single stock variable and a single (one-period) bond return instead of having to keep track of issued bonds and their prices over time. In real terms, a long-term bond $b_{t-1}^{j,L}$ therefore pays $r_t^L = \frac{R_t^L}{\Pi_t}$ in interest one period later.

Households face portfolio adjustment costs whenever they change the allocation of their assets between short-term and long-term bonds. In the style of Chen et al. (2012) and Harrison (2017), this adjustment cost is specified as

$$\Psi_t^j = \frac{\nu}{2}\left(\delta^j \frac{b_t^j}{b_t^{j,L}} - 1\right)^2,$$

where $\delta^j = \frac{b^{j,L}}{b^j}$ is the steady-state ratio of long-term bonds to short-term bonds and $\nu > 0$ captures how costly deviations from a household's preferred steady-state portfolio mix are.[6]

Introducing adjustment costs implies a direct role for asset market operations to stimulate the economy, namely through the portfolio balance channel. If the central bank purchases bonds of a specific maturity, it thereby lowers the relative supply of those assets and so increases their price. Investors will rebalance their portfolios, which is costly due to the presence of Ψ and affects their average portfolio returns, thus implying a real impact through changes in individual and aggregate demand.[7] The adjustment cost captures in a parsimonious way the preferred-habitat theory which assumes that investors have preferences for specific maturities (Vayanos & Vila, 2009, 2021). In other words, these agents view different assets along the yield curve as imperfect substitutes (Andrés et al., 2004).

Savers

Unconstrained agents can save and borrow in both short-term and long-term bonds and receive dividends from their share holdings in monopolistically competitive firms. Apart from these asset returns, savers also earn labor income and pay taxes. They each maximize their lifetime utility from consumption and leisure subject to their budget constraint in

[6]The proposed adjustment cost function only captures the impact of changes in the relative supply of an asset and thus deviations from a household's desired portfolio composition (so-called stock effects). Harrison (2017) or Harrison et al. (2021) consider in addition the impact of fundamental changes in that portfolio mix (flow effects).

[7]Asset market operations prove to be ineffective in baseline New Keynesian models. Changes in the portfolio allocation of households have no impact on real economic variables as shown, among others, by Eggertsson and Woodford (2003).

real terms, taking prices and wages as given:

$$\max_{c_t^S, N_t^S, b_t^S, b_t^{S,L}} \mathbb{E}_t \sum_{t=0}^{\infty} \left(\beta^S\right)^t \theta_t \left(\frac{(c_t^S)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \zeta^S \frac{(N_t^S)^{1+\varphi}}{1+\varphi} \right) \qquad \text{subject to}$$

$$c_t^S + b_t^S + b_t^{S,L} = r_{t-1}\, b_{t-1}^S + r_t^L\, b_{t-1}^{S,L} + w_t\, N_t^S + \frac{1-\tau^D}{1-\lambda} d_t - t_t - \Psi_t^S - \frac{tr}{1-\lambda} \,,$$

where b_t^S and $b_t^{S,L}$ are real short-term and long-term government bonds held by a saver, respectively, with corresponding interest rates r and r^L as described above. Furthermore, w_t is the real wage, d_t are real dividends from firms' profits equally distributed to savers, t_t are real lump-sum taxes levied by the government, Ψ_t^S are portfolio adjustment costs described above, and tr are steady-state transfers from savers to hand-to-mouth agents that ensure consumption equality between the two household types in steady state.[8] Profits of intermediate firms that are owned by savers are taxed at a rate of τ^D. The government redistributes the tax revenues as a direct transfer to constrained households.

Solving the decision problem (see Appendix 3.A.1) results in the following consumption-leisure choice condition and Euler equations for short-term and long-term bonds:

$$\begin{aligned}
w_t &= \zeta^S\, (N_t^S)^{\varphi}\, (c_t^S)^{\frac{1}{\sigma}} \,, \\
1 &= \beta^S\, R_t\, \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^S}{c_t^S} \right)^{-\frac{1}{\sigma}} \frac{1}{\Pi_{t+1}} \right] - \frac{\nu\, \delta^S}{b_t^{S,L}} \left(\delta^S \frac{b_t^S}{b_t^{S,L}} - 1 \right) \,, \\
1 &= \beta^S\, \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^S}{c_t^S} \right)^{-\frac{1}{\sigma}} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \frac{\nu\, \delta^S\, b_t^S}{\left(b_t^{S,L}\right)^2} \left(\delta^S \frac{b_t^S}{b_t^{S,L}} - 1 \right) \,.
\end{aligned}$$

Borrowers

Constrained households have access to both types of government bonds as well and consume their disposable income together with transfers (net of taxes) from the government. Different from savers, they face a borrowing constraint such that the total amount borrowed in each period cannot exceed a given limit.[9] Each borrower therefore solves the following problem:

$$\max_{c_t^B, N_t^B, b_t^B, b_t^{B,L}} \mathbb{E}_t \sum_{t=0}^{\infty} \left(\beta^B\right)^t \theta_t \left(\frac{(c_t^B)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \zeta^B \frac{(N_t^B)^{1+\varphi}}{1+\varphi} \right) \qquad \text{subject to}$$

[8] We use a symmetric steady state with $c^B = c^S = c$ as a benchmark, modeled similar to Bilbiie et al. (2022).

[9] In equilibrium, constrained agents will borrow in both short-term and long-term bonds. Although they are termed government bonds, borrowers actually borrow from savers so that b_t^B and $b_t^{B,L}$ can alternatively be interpreted as bonds issued by B to S. Hence, the implicit assumption here is that public and private bonds are perfect substitutes.

$$
\begin{aligned}
c_t^B + b_t^B + b_t^{B,L} &\leq r_{t-1}\, b_{t-1}^B + r_t^L\, b_{t-1}^{B,L} + w_t\, N_t^B + \frac{\tau^D}{\lambda} d_t - t_t - \Psi_t^B + \frac{tr}{\lambda} \,, \\
-b_t^B - b_t^{B,L} &\leq \overline{D} \,,
\end{aligned}
$$

where $\overline{D} \geq 0$ is the exogenous borrowing limit. We assume that this constraint binds for all periods and borrowers thus have a high MPC.

Apart from the borrowing constraint, the optimality conditions are very similar to those of the savers, yielding:

$$
\begin{aligned}
w_t &= \zeta^B\, (N_t^B)^{\varphi}\, (c_t^B)^{\frac{1}{\sigma}} \,, \\
1 &= \beta^B\, R_t\, \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^B}{c_t^B} \right)^{-\frac{1}{\sigma}} \frac{1}{\Pi_{t+1}} \right] - \frac{\nu\, \delta^B}{b_t^{B,L}} \left(\delta^B\, \frac{b_t^B}{b_t^{B,L}} - 1 \right) + \psi_t^B \,, \\
1 &= \beta^B\, \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^B}{c_t^B} \right)^{-\frac{1}{\sigma}} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \frac{\nu\, \delta^B\, b_t^B}{\left(b_t^{B,L}\right)^2} \left(\delta^B\, \frac{b_t^B}{b_t^{B,L}} - 1 \right) + \psi_t^B \,,
\end{aligned}
$$

where $\psi_t^B \geq 0$ is the Lagrangian multiplier on the borrowing constraint, with complementary slackness condition $\psi_t^B \left(b_t^B + b_t^{B,L} + \overline{D} \right) = 0$. If the constraint is binding, $\psi_t^B > 0$ so that the marginal utility of consuming today is larger than the expected marginal utility of saving in any of the two bonds.

3.2.2 Firms

The firm sector is standard and features two different types of agents: monopolistically competitive intermediate goods producers and perfectly competitive final goods firms.

Final goods producers. The final goods sector aggregates differentiated intermediate goods according to a CES production function:

$$
y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di \right)^{\frac{\epsilon}{\epsilon-1}} \,,
$$

where ϵ is the elasticity of substitution. Final goods producers maximize their profits, resulting in a demand for each intermediate input of

$$
y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} y_t \,,
$$

where $P_t(i)$ is the price of intermediate good i and $P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon}\, di$ the aggregate price index.

Intermediate goods producers. Varieties of intermediate goods i are produced by a continuum of monopolistically competitive firms with production function $y_t(i) = z_t\, N_t(i)$, where technology z_t follows an AR(1) process. Cost minimization implies real marginal costs $mc_t = \frac{w_t}{z_t}$.

Intermediate goods firms set prices subject to a quadratic adjustment cost à la Rotemberg (1982) with the degree of nominal price rigidity governed by ϕ_p:

$$\Psi_t^p = \frac{\phi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 y_t \ .$$

Following Bilbiie (2020), we also assume that the government imposes an optimal subsidy on sales, τ^S, to induce marginal cost pricing in steady state. This subsidy is financed by a lump-sum tax on firms such that $t_t^F = \tau^S y_t$. Thus, real profits of each intermediate goods producer i are given by

$$d_t(i) = \left(1 + \tau^S\right)\frac{P_t(i)}{P_t}\, y_t(i) - w_t\, N_t(i) - \Psi_t^p - t_t^F \ .$$

Appendix 3.A.2 shows the solution to the price-setting problem which leads to the standard Phillips curve:

$$\left(1+\tau^S\right)(1-\epsilon)+\epsilon\, mc_t - \phi_p\left(\Pi_t - 1\right)\Pi_t + \beta^S\, \mathbb{E}_t\left[\frac{\theta_{t+1}}{\theta_t}\left(\frac{c_{t+1}^S}{c_t^S}\right)^{-\frac{1}{\sigma}}\phi_p\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{y_{t+1}}{y_t}\right] = 0 \ .$$

Abstracting from price adjustment costs, the optimal subsidy that induces marginal cost pricing turns out to be $\tau^S = (\epsilon - 1)^{-1}$. Finally, using the expression for the lump-sum tax and aggregating over firms yields total real profits:

$$d_t = \left[1 - mc_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2\right] y_t \ .$$

3.2.3 Government and Monetary Policy

Monetary and fiscal policy are combined in one entity. The government budget constraint is given by

$$b_t + b_t^L = r_{t-1}\, b_{t-1} + r_t^L\, b_{t-1}^L + \Omega_t + g_t - t_t \ ,$$

where b_t and b_t^L are total real short-term and long-term bonds issued by the government, respectively, Ω_t are net purchases of long-term bonds by the central bank, and g_t is real government spending which follows an AR(1) process. Note that subsidy expenses and tax revenues from firms' profits are balanced in every period and thus do not appear in the budget constraint above.

We assume that lump-sum taxes are set by the following rule:

$$\frac{t_t}{t} = \left(\frac{t_{t-1}}{t}\right)^{\rho^{\tau,t}} \left(\frac{b_t + b_t^L}{b + b^L}\right)^{\rho^{\tau,b}} \left(\frac{g_t}{g}\right)^{\rho^{\tau,g}} .$$

Moreover, total supply of long-term bonds follows an AR(1) process:

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{BL} \log\left(\frac{b_{t-1}^L}{b^L}\right) + \epsilon_t^{b^L} .$$

Turning to the central bank, net asset purchases of long-term bonds are defined as

$$\Omega_t = b_t^{CB,L} - r_t^L \, b_{t-1}^{CB,L} ,$$

where $b_t^{CB,L}$ denotes the value of long-term bonds purchased by the central bank. The inclusion of central bank asset purchases in the consolidated budget constraint implies that asset market operations are financed by the central government, which itself will pay for it with either tax revenues from households or through the issuance of new short-term debt.

The central bank has two policy tools. First, it conducts QE/QT by deciding on which fraction q_t of the total market value of long-term bonds to buy/sell:

$$b_t^{CB,L} = q_t \, b_t^L ,$$

where we model q_t as a AR(1) process:

$$\log\left(\frac{q_t}{q}\right) = \rho_q \log\left(\frac{q_{t-1}}{q}\right) + \epsilon_t^q .$$

Apart from asset market operations, the monetary authority can implement conventional monetary policy by setting the nominal short-term interest rate, R, according to a standard Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right)\right] + \epsilon_t^m ,$$

where ϵ_t^m is an i.i.d. policy shock.

3.2.4 Aggregation and market clearing

Aggregate consumption and aggregate hours are given by

$$c_t = \lambda c_t^B + (1-\lambda) c_t^S ,$$

$$N_t = \lambda N_t^B + (1-\lambda) N_t^S \ .$$

Market clearing for short-term and long-term bonds, respectively, requires

$$b_t = b_t^H \ ,$$
$$b_t^L = b_t^{H,L} + b_t^{CB,L} \ ,$$

with households' total demand for short-term bonds $b_t^H = \lambda b_t^B + (1-\lambda) b_t^S$ and for long-term bonds $b_t^{H,L} = \lambda b_t^{B,L} + (1-\lambda)\, b_t^{S,L}$. By using the equation for asset market operations, we can write $b_t^{H,L} = (1-q_t)\, b_t^L$. This condition shows the direct impact of asset purchases and sales on long-term bond holdings and hence the portfolio mix of households.[10]

Finally, the aggregate resource constraint is given by

$$y_t = c_t + g_t + \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2 y_t \ .$$

3.2.5 Steady state

We approximate our model around a deterministic steady state with zero net inflation and output normalized to one. Our assumption $\beta^S > \beta^B$ implies that the borrowing constraint will always bind in steady state:

$$\psi^B = \left(c^B\right)^{-\frac{1}{\sigma}} \left[1 - \frac{\beta^B}{\beta^S}\right] > 0 \ .$$

As a result, patient (impatient) agents will be net lenders (borrowers) in steady state.

The Euler equations of the saver yield for the nominal rates that $R = R^L = \left(\beta^S\right)^{-1}$ and we have $r = R$ and $r^L = R^L$. The presence of the optimal subsidy to firms results in zero profits ($d = 0$). Furthermore, we assume that labor supply is equalized across households ($N^B = N^S = N$), which implies that they will consume the same amount in steady state ($c^B = c^S = c$).

Regarding the steady-state ratio of bond holdings, δ^j, we impose the simplifying assumption that they are equal across household types such that individual demand variables can be replaced by their household-level counterparts:

$$\delta^S = \delta^B = \delta = \frac{b^{H,L}}{b^H} \ .$$

[10] In Appendix 3.A.1, we derive a no-arbitrage condition between short-term and long-term bonds. It shows that changes in households' portfolio composition caused by central bank asset market operations directly affect the long-term bond return, namely due to the presence of the portfolio adjustment cost.

We further define $\tilde{\delta} = \frac{b^L}{b}$ as the steady-state ratio between total long-term and short-term bonds. Finally, note that portfolio and price adjustment costs will be zero at steady state.

3.2.6 Calibration and simulation setup

Our calibration is summarized in Table 3.1. We target the case of the U.S. economy.

The parameters from the household sector are mostly taken from Bilbiie et al. (2013) who build a borrower-saver model similar to ours. In particular, we target a steady-state real interest rate of 4% annually. The baseline value for the savers' discount factor is therefore set to 0.99. Regarding the production side, it is worth mentioning that we set taxes on profits to zero in order to rule out any impact from redistribution on the income of borrowers.

For the bond-related parameters, we choose $\chi = 0.975$ to match, in the non-stochastic steady state, an average duration of ten-year US Treasury bonds of slightly more than seven years, following Harrison (2017) and Harrison et al. (2021) who in turn draw on D'Amico and King (2013). The same value is also used by Sims et al. (2022b). We therefore consider the long-term asset as a ten-year bond, but χ might also be increased to study longer maturities or durations. The adjustment cost parameter ν is chosen such that the model matches the empirical evidence by Weale and Wieladek (2016) on the impact of a QE shock on real output, as discussed hereafter. Finally, the value of central bank's long-term bond holdings in steady state implies that households hold a share of 0.75, namely three-quarters of the stock of long-term debt, which is equivalent to the calibration in Gertler and Karadi (2013) and Karadi and Nakov (2021).

These parameter values suggest that households hold a large fraction of the available long-term debt in steady state and also holds by construction all the short-term debt in the economy. In quantitative terms, the household sector holds more short-term than long-term debt in steady state ($b^H > b^{H,L}$), in particular due to the much larger value of short-term bonds ($\tilde{\delta} = 0.3$). Looking at the individual agent types and drawing on the steady-state equations of Section 3.2.5, we can show that constrained agents borrow in both bond types in steady state, while savers hold positive amounts of both. In fact, B borrows from S, whereby the amount of borrowing depends crucially on the choice of the borrowing limit: a higher $\overline{D}$ enables B to borrow more in both types of bonds as it is less constrained.

Output is normalized to one in steady state, while the target for net inflation is 0%, in line with Cui and Sterk (2021). Moreover, the persistence of the preference shock is set to 0.8, a high value as is common in the literature (see, e.g., Bianchi, Melosi, & Rottner, 2021). It allows to achieve a lasting ZLB spell of several quarters in our simulations.

Finally, the chosen QE smoothing reflects the high persistence of asset market operations and is similar to the value of 0.8 in Sims and Wu (2021) or Sims et al. (2022a).

In each simulation we run below, the shock size is such that the central bank buys or sells long-term bonds worth 1% of annualized nominal GDP. We then match the output response to empirical evidence from the United States. The simulation results used for the matching are the impulse responses of the net effect of a QE shock that happens when the economy is in a liquidity trap, a situation brought about by a negative preference shock. See section 3.3.2 for more details. All the other simulations build on the parameterization from this exercise.

Weale and Wieladek (2016) show that the peak impact on U.S. real GDP of an asset purchase in the size of 1% of annualized nominal GDP has been around 0.58%.[11] We take this number as our target for the average output response during the first four quarters subsequent to a QE shock at the ZLB, following the approach used in Cui and Sterk (2021). More specifically, we set the adjustment cost parameter ν accordingly to approximate this target.

To solve our model with the occasionally binding lower bound constraint, we use the dynareOBC toolbox developed by Tom Holden.[12] Given that we approximate the model at first order, our simulation results will be perfect foresight transition paths in response to a QE or QT shock.

[11]This number reflects the average of median peak effects of four different identification schemes in Weale and Wieladek (2016) that all leave the reaction of real GDP unrestricted.

[12]See Holden (2016, 2022) for theory and computational details.

Table 3.1: Parameter values

Parameter	Description	Value	Source / Target
λ	Proportion of borrowers	0.35	Bilbiie et al. (2013)
σ	Intertemporal elasticity of substitution	1	Conventional
$1/\varphi$	Frisch elasticity of labor supply	1	Conventional
β^S	Discount factor, saver	0.99	Annual steady-state interest rate of 4%; Bilbiie et al. (2013)
β^B	Discount factor, borrower	0.95	Bilbiie et al. (2013)
$\overline{D}$	Borrowing limit	0.5	Bilbiie et al. (2013)
ϵ	Elasticity of substitution between goods	6	Price markup of 20%
τ^D	Tax on profits	0	No redistribution
ϕ_p	Rotemberg price adjustment cost	42.68	3.5-quarters price duration
ϕ_π	Taylor rule coefficient on inflation	1.5	Conventional
χ	Long-term bond coupon decay rate	0.975	Average bond duration of 7-8 years
ν	Portfolio share adjustment cost	0.05	Empirical evidence on output response by Weale and Wieladek (2016)
$\tilde{\delta} = b^L/b$	Steady-state ratio of long-term to short-term bonds	0.3	Harrison (2017), Harrison et al. (2021)
$q = b^{CB,L}/b^L$	Steady-state CB long-term bond holdings	0.25	Households' long-term bond holdings
g/y	Steady-state government-spending-to-GDP ratio	0.2	Galí et al. (2007)
$(b + b^L)/y$	Steady-state total-debt-to-GDP ratio	0.8	U.S. average since 2009
Π	Steady-state gross inflation rate	1	Inflation target
Y	Steady-state output	1	Normalized
τ^S	Production subsidy	$(\epsilon - 1)^{-1}$	Marginal cost pricing
ρ_θ	Persistence of preference shock	0.8	Own choice
$\rho^{\tau,t}$	Tax smoothing in fiscal rule	0.7	Own choice
$\rho^{\tau,b}$	Tax response to total debt	0.33	Galí et al. (2007)
$\rho^{\tau,g}$	Tax response to government spending	0.1	Galí et al. (2007)
ρ_r	Interest rate smoothing	0.8	Sims and Wu (2019)
ρ_q	QE smoothing	0.9	Cui and Sterk (2021)

3.3 Results

In this section, we discuss the model simulations. We proceed in three steps. First, we study the impact of asset market operations when the economy is either close to or well above the ZLB and analyze the shock transmission to the real economy. Second, we examine the asymmetric macroeconomic effects of QE and QT due to state dependency. Finally, we compare our TANK-BS model to its representative-agent counterpart to isolate the implications of household heterogeneity.

3.3.1 Asset market operations and unwinding QE close to the ZLB

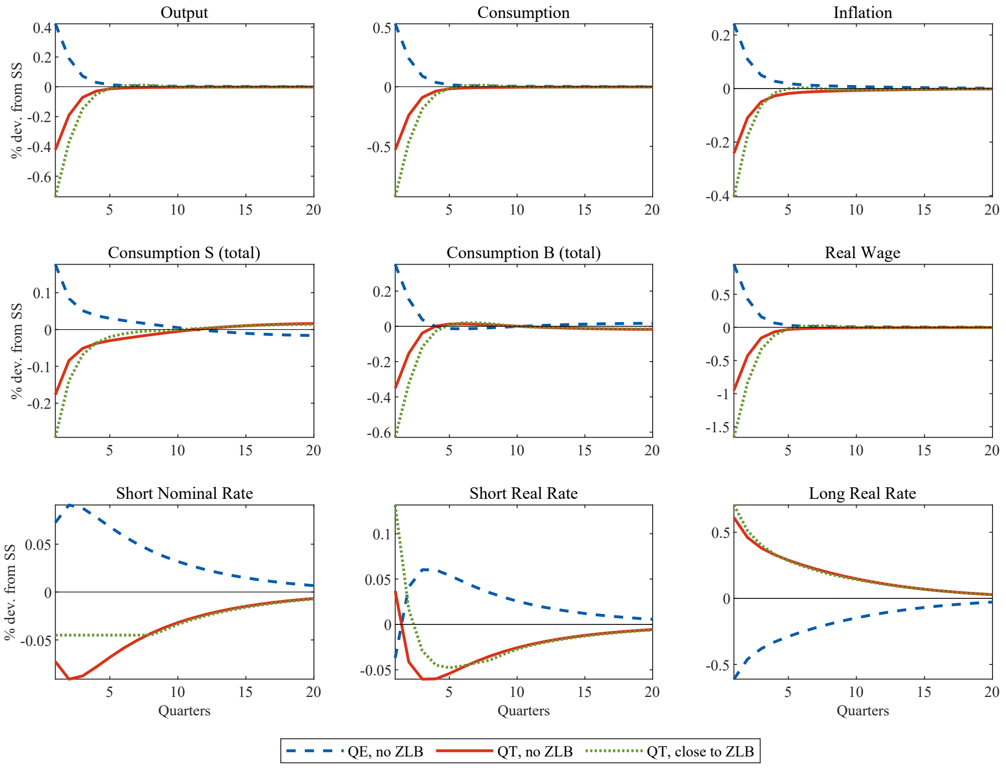
We start by illustrating what the TANK-BS model implies about the potential impact on macroeconomic aggregates of doing QE/QT and unwinding QE, conditional on an existing state dependency in the form of a lower bound on the nominal short-term interest rate. Figure 3.1 shows selected impulse responses to a QE and QT shock occurring when the economy is sufficiently far away from the ZLB and a QT shock which hits an economy that is already close to the ZLB. See Appendix 3.B.1 for the entire set of impulse responses.

To explain how the model works, we begin by analyzing a standard QT shock, captured by the solid red line in the figure. When the central bank sells long-term bonds, the amount of assets available to other agents in the economy increases. The return of those bonds goes up and their price decreases. Together with the lower short-term interest rate, both household types therefore demand more long-term and less short-term bonds. Constrained agents borrow now more in the short-term asset because it has become cheaper, while savers purchase the long-term asset sold by the central bank. Overall, the lower demand for long-term bonds from the central bank is exactly offset by the higher demand from households so that the supply of long-term bonds remains fixed.

The magnitude of the effects of QT will depend on the maturity structure of household portfolios. For instance, more short-term debt exposes borrowers to higher rollover risk and makes them more sensitive to changes in short-term interest rates. On the other hand, borrowing at the long-term rate includes valuation effects, as remarked by Auclert (2019). If a larger portion of assets in borrowers' portfolios consists of long-term bonds, central bank asset market operations will influence them more. Bond price changes induced by QE or QT directly affect the debt burden of the constrained agents, their wealth and so their consumption behavior. Elsewhere, this is what Ferrante and Paustian (2019) termed the debt revaluation channel in the context of forward guidance.

To understand the transmission of the shock to the real economy, it is useful to study the responses of the components of each agent's budget constraint to an asset market operation. Figure 3.2 shows that the individual consumption of both household types

Figure 3.1: Impulse responses to a QE/QT shock and a QT shock near the ZLB



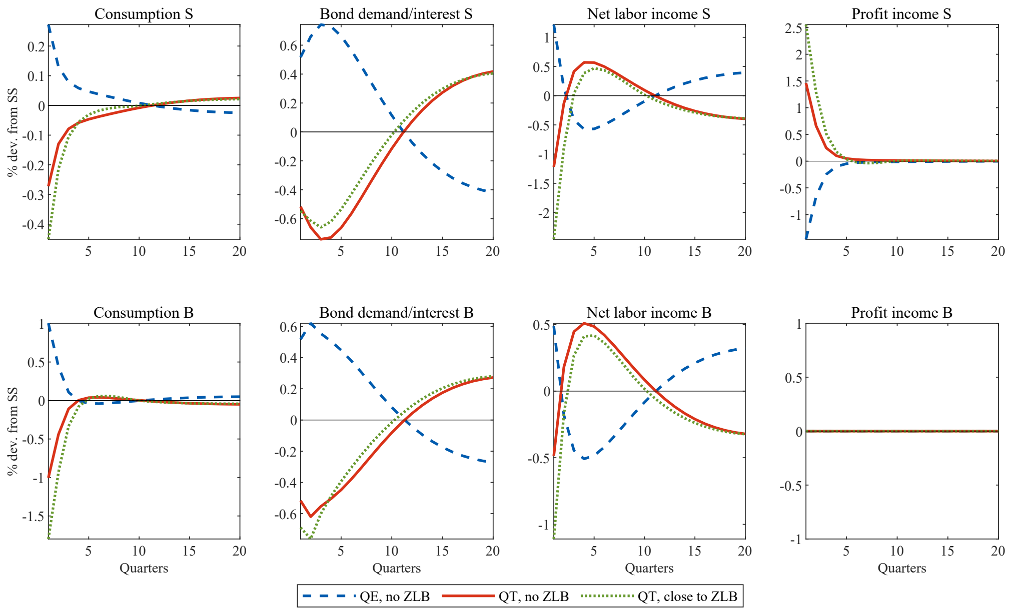
Notes: This figure depicts the impulse responses of selected variables to a QE (dashed blue line) and a QT (solid red line) shock occurring far enough above the ZLB, and a QT shock happening close to the ZLB (dotted green line, simulated with $\beta^S = 0.99955$). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

decreases in response to a QT shock far enough off the ZLB, but that the underlying driving forces differ. We distinguish between the direct effects of the asset sales (changes in bond demand and returns) and the indirect general-equilibrium effects (changes in the real wage and profits).[13]

The first panel reveals that the change in savers' labor income through general equilibrium has a negative effect on consumption, but only on impact of the shock. After that, the cut in lump-sum taxes and, in particular, the strong increase in countercyclical profits push savers' income up and leads to a quick recovery. Instead, the medium-term negative consumption response is mainly driven by developments in their portfolio allocation. By buying long-term bonds from the central bank, savers give up some of their income because changes in the bond portfolio are costly. This drop in income is larger than their gains from selling short-term bonds together with the increase in interest income coming

[13]The partition in Figure 3.2 can be captured by the budget constraints of the two household types: $c_t^j = \left[-b_t^j - b_t^{j,L} + r_{t-1}\, b_{t-1}^j + r_t^L\, b_{t-1}^{j,L} - \Psi_t^j\right] + \left[w_t\, N_t^j - t_t\right] + \left[d_t^j\right] + tr^j$, for $j = \{B, S\}$ and with $d_t^B = \frac{\tau^D}{\lambda} d_t$ and $d_t^S = \frac{1-\tau^D}{1-\lambda} d_t$. The square brackets represent the bond demand/interest, the net labor income, and the profit income component, respectively.

Figure 3.2: Households' budget components to a QE/QT shock and a QT shock near the ZLB



Notes: This figure shows grouped components of the budget constraints of savers (top) and borrowers (bottom) in response to a QE (dashed blue line) and a QT (solid red line) shock occurring far enough above the ZLB, and a QT shock happening close to the ZLB (dotted green line, simulated with $\beta^S = 0.99955$). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. Each panel consists of four columns, containing the responses of individual consumption, bond-related variables (bond demand, interest payments/income, net of adjustment cost), labor income net of taxes, and income from profits. All responses are shown in per-capita terms.

from more long-term bonds in their portfolio and the higher real rate on these assets.[14] This effect depresses the consumption of savers and thus aggregate demand.

The bottom panel of Figure 3.2 shows some commonalities for borrowers. Their bond demand and interest payments react similarly to those of savers. The other negative income effect comes as before through net labor income. While borrowers do not change labor supply by a lot because they cannot afford to work much less, the lower spending from savers hurts them through the drop in the real wage.[15] This effect on labor income is again short-lived due to the cut in taxes that causes a fast rebound.

Overall, the direct effects of QT and the ensuing changes in returns are considerable for all households and the indirect effect through the labor market is counterbalanced by a cut in taxes. The major difference that leads to a weaker drop in the individual consumption

[14]Strictly speaking, the rise in savers' long-term bond holdings is larger than the decrease in short-term bonds. Similarly, their interest income from long-term bonds increases by more than the income from short-term bonds falls. Combined, the former effect is larger and leads to a negative net effect out of the bond-related variables in the saver's budget constraint, as depicted in Figure 3.2.

[15]The weak reaction of borrowers' labor supply is also visible in the full set of impulse responses in Appendix 3.B.1.

of savers, however, is the response of profits. Those constitute a strong boost for savers such that their individual consumption drops by much less in relative terms.

A key point to mention here is that QT is modeled as the exact opposite of QE. Given the linearity of the model, both policies have therefore the same impact in absolute terms – as long as the economy is far enough away from the ZLB such that the QT shock cannot push it into a liquidity trap. This is also visible from Figure 3.1. QE decreases the long-term rate and increases the short-term rate. These effects then propagate to the real economy via households demanding more short-term and less long-term bonds, which translates into a higher aggregate demand and leads to a rise in all main aggregate variables.

Starting from a state of the world with symmetric effects of QE and QT makes it possible to isolate the asymmetry emerging from the presence of a ZLB. By allowing for a binding lower bound on the nominal short-term interest rate, we introduce state dependency that can generate asymmetric effects of asset market operations, similar to the literature about fiscal policy and the government-spending multiplier (see, e.g., Christiano, Eichenbaum, & Rebelo, 2011). This idea is also motivated by previous research that confirmed a stronger effectiveness of asset purchases if the ZLB was binding (see Gertler & Karadi, 2013).

Assuming that the economy is currently in a situation where the log interest rate is close to (but not at) zero, even a mild QT shock can push it into a liquidity trap.[16] We illustrate this case by a simulation using our baseline calibration except that we set $\beta^S = 0.99955$. The implied lower steady-state real rate (annual: 0.18%) ensures that the ZLB will bind right on impact of the QT shock and for a total of eight quarters, given the same shock size as before.

This case is captured by the dotted green impulse responses in Figure 3.1. If the policy rate were unconstrained, it would drop on impact of the shock and show a hump-shaped course, mitigating the contractionary implications of the asset sales. However, with a binding ZLB, it can no longer decrease by that much, while long-term rates are still at a higher level. As a consequence, the short-term real rate increases and both household types decrease their consumption more than in the unconstrained case, leading to larger drops in all aggregate variables and a deeper recession.

We can deduce from Figure 3.2 that the stronger decrease in savers' consumption right after the shock is substantially triggered by a magnified fall in labor income, which is again

[16] We do not discuss here the case of QE done near the ZLB. Due to its expansionary effects, such an asset market operation would move the economy in any case away from the lower bound. Asset purchases can therefore even be an effective policy tool if the policy rate is unconstrained.

partly absorbed by positive profits. Borrowers are particularly hurt through the higher borrowing costs and the larger drop in the real wage.[17]

The above unveils a distinct asymmetry in the macroeconomic effects of QE and QT, precisely arising from the different states of the world and the (non-)availability of the nominal short-term interest rate to help to stabilize the economy. It also addresses the question of when central banks should actually unwind. It is obvious to see that the central bank needs to be sure that any tightening will not bring the policy rate back to zero. Otherwise, it risks strong adverse effects on the aggregate economy. As a result, when dealing with the risk of hitting the ZLB, our model implies that minimizing the economic costs of normalizing monetary policy requires the monetary authority to first raise the policy rate before starting with active asset sales. Such an approach is less harmful to the overall economy.

The likelihood of staying away from the ZLB depends on the optimal co-ordination between interest rate increases and QT with respect to the order, timing, and pace of actions. Selling assets before normalizing the policy rate increases the probability of ending in a liquidity trap and staying there for an extended period of time. A similar outcome arises if QT starts when the short-term rate has not been raised enough or if the tightening is done too fast relative to the increases in the policy rate.

3.3.2 State-dependent asset market operations and their asymmetric impact

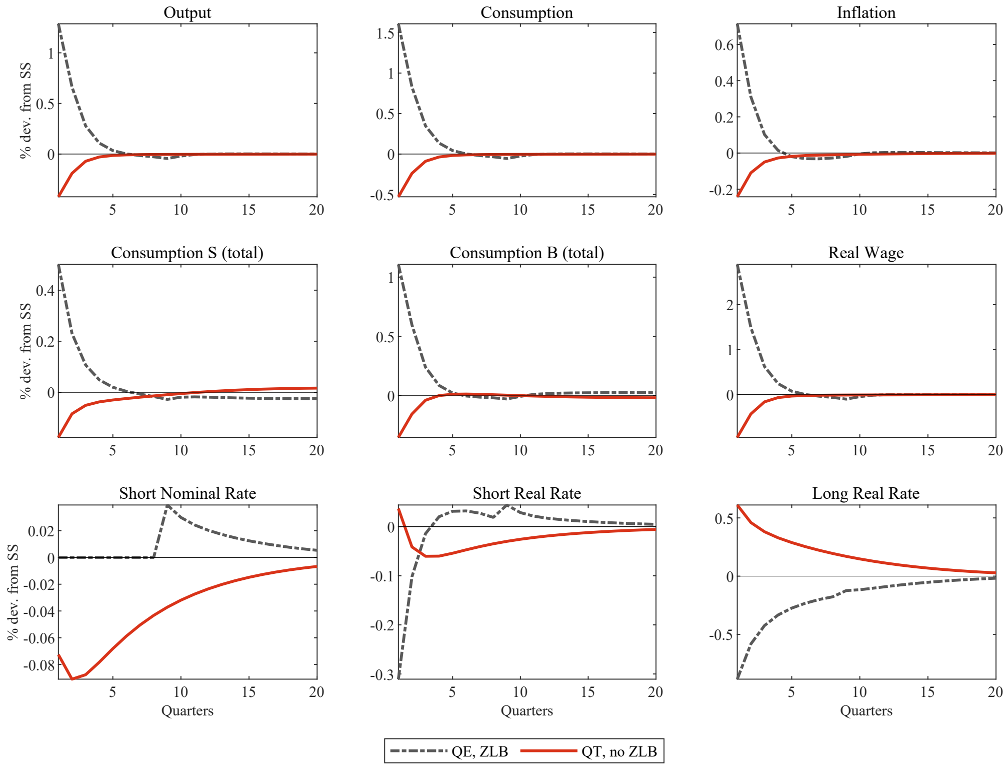
We now run a counterfactual exercise to compare QE and QT programs of similar size across different states of the economy. Based on the idea of state-dependent asset market operations, we compare two types of shocks: a QE shock that happens when the economy is in a liquidity trap, and a QT shock off the ZLB. Intuitively, central banks have heavily used large-scale asset purchase programs to fight the detrimental consequences of historically low interest rates in the past, often during times where the economy has been constrained at the ZLB. In contrast, we showed in the previous section that unwinding QE before the policy rate has reached a certain level is not advisable from our model's point of view.[18]

Figure 3.3 shows selected results of these simulations. Additional impulse responses are in Appendix 3.B.2. We model the net effect of the QE shock by first simulating an asset purchase together with a negative preference shock and then deduct the impact of a mere preference shock. The size of the latter shock is chosen such that the economy is brought to the ZLB on impact and remains constrained for eight quarters. Generating

[17] Overall, bond demand and supply variables respond similarly to QT, whether the economy is close to or away from the lower bound. See Appendix 3.B.1 for the respective impulse responses.

[18] There is a huge debate on whether a CB should raise the policy rate first or should start with some tapering or active asset sales. See Forbes (2021) for a recent consideration.

Figure 3.3: Impulse responses to a QE shock at the ZLB and a QT shock off the ZLB



Notes: This figure depicts the impulse responses of selected variables to a QE shock when the ZLB on the policy rate is binding (dash-dotted gray line, showing the impact of QE net of a negative preference shock), and a QT shock occurring far enough above the ZLB (solid red line). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. For QE, the size of the preference shock is chosen such that the ZLB binds for eight quarters. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

a liquidity trap by a preference shock is a simple and effective way for our purpose to isolate the effects of state dependency (see, e.g., Christiano et al., 2011). Otherwise, the QT shock is equivalent to the shock in the previous section where we discussed its effects on macroeconomic aggregates and the associated transmission mechanism.

The figure reveals clear differences in the macroeconomic implications of the two shocks. As before, QE has a positive effect on aggregate demand while QT affects the economy negatively. When QE is done at the ZLB, however, its positive effect is magnified compared to the findings from the previous section without the lower bound. The resulting uneven responses of aggregate variables emerge from the prevalent state dependency, best visible from the asymmetric behavior of interest rates. The long-term rate response shows only minor (absolute) differences across the two shocks. On the other hand, while the short-term real interest rate increases after a QE shock when the economy is away from the ZLB, it flips sign when at the lower bound and falls considerably due to the inability of the policy rate to react.[19]

Our findings highlight the significance of the occasionally binding lower bound for the asymmetric implications between QE and QT. If conventional monetary policy is constrained and the economy is stuck in a liquidity trap, QE helps to stimulate aggregate demand and will have a larger effect than in normal times. With the nominal interest rate being at the ZLB, the rise in output and prices following a QE shock decreases the real rate considerably and thus fosters spending by households and boosts real wages.[20] This, in turn, results in an even higher output and constitutes an expansionary spiral.

3.3.3 Household heterogeneity and state dependency in interaction

As a final exercise, we study how household heterogeneity affects the asymmetry between QE and QT. For this purpose, we compare the impulse responses resulting from our borrower-saver model (named TANK-BS) with those from a standard representative-agent framework (named RANK) without heterogeneity on the household side. See Appendices 3.B.3 and 3.B.4 for the entire set of impulse responses. The shocks we focus on are the same as in the previous section, namely an asset purchase at the ZLB and an asset sale away from it.

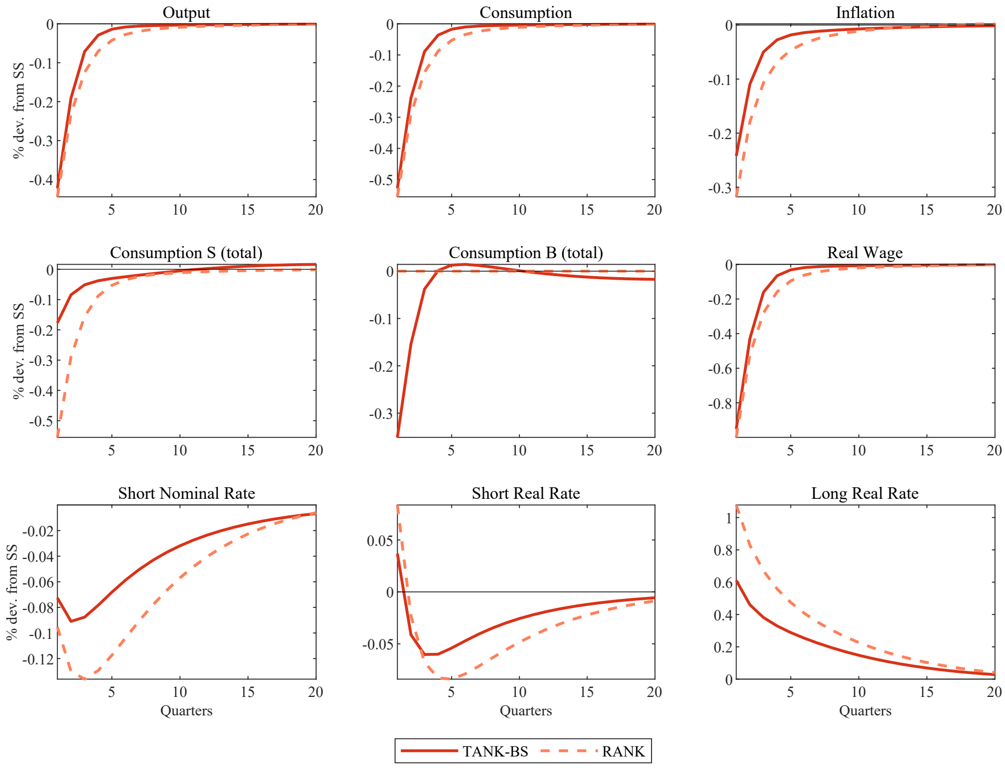
The motivation for such an exercise comes from the implications of heterogeneity in households' income, wealth, or consumption and saving decisions found in the literature. Studies focusing on conventional monetary policy find substantial amplification (e.g., Auclert, 2019; Bilbiie, 2018, 2020; Bilbiie et al., 2022; Debortoli & Galí, 2017), driven by heterogeneity in MPCs out of a transitory income shock. Sims et al. (2022b) instead focus on QE and find no amplification coming from household heterogeneity. In our setup, borrowers have a higher MPC than savers. Any policy measure that relaxes their borrowing constraint frees up some individual income which is spent immediately and boosts aggregate demand and consumption. It appears therefore natural to study if amplification also arises after asset market operations.

Figure 3.4 shows the results for a QT shock when the economy is far enough off the ZLB such that the nominal short-term rate remains unconstrained. Adding household heterogeneity to a RANK model seems to have only a minor impact on the aggregate effects of QT (and due to the model linearity also of QE), which is in line with the finding in Sims et al. (2022b).

[19]This result resembles Gertler and Karadi (2013) who showed that central bank asset purchases lead to a larger drop in long-term rates the longer short-term rates are constrained.

[20]The boost originating from the drop in the short-term real rate is so large that it generates an increase in real wages that induces borrowers to work less when QE is done at the ZLB. On the contrary, asset purchases away from the ZLB would induce them to increase their labor supply. See Appendix 3.B.2 with the full set of impulse responses.

Figure 3.4: Impulse responses to a QT shock off the ZLB: RANK vs. TANK-BS



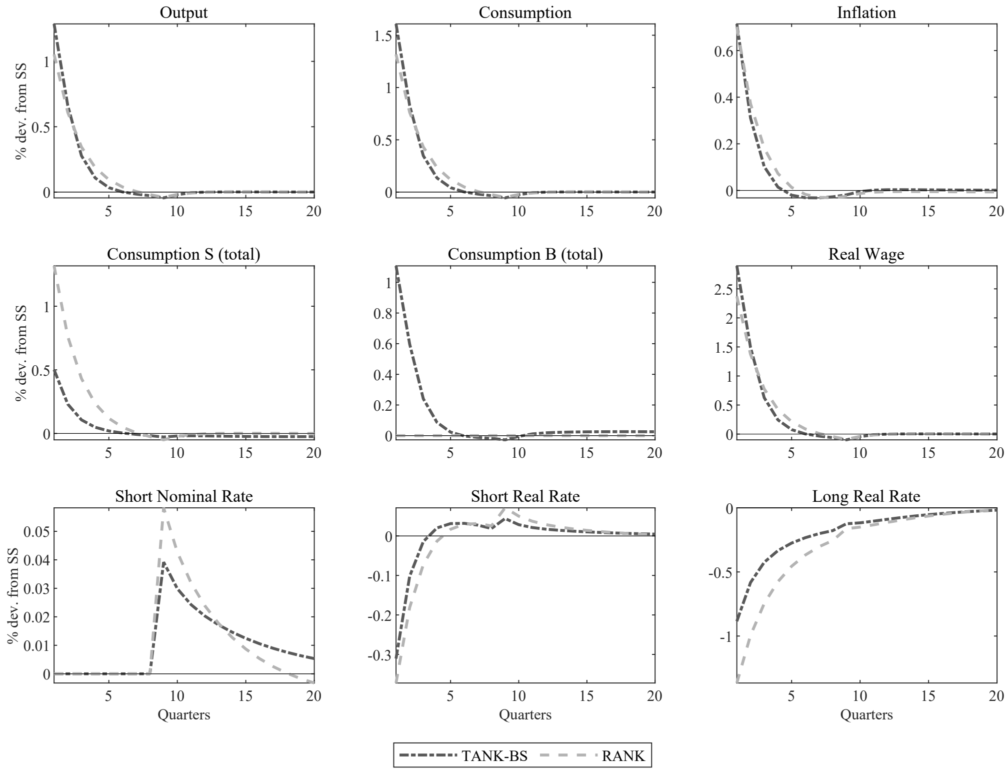
Notes: This figure depicts the impulse responses of selected variables to a QT shock occurring far enough above the ZLB, for the borrower-saver model (TANK-BS, solid red line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light red line). The shock for each simulation is an asset sale of size 1% of annualized GDP. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

The reason for this lack of amplification via heterogeneity lies in a composition effect of changes in households' balance sheets that roughly cancel out when moving from RANK to TANK-BS. Without borrowers in the model, the propagation of the shock works entirely through the income of the saver. The representative agent purchases the bonds sold by the central bank, which drives down their income and thus aggregate demand. Compared to TANK-BS, we observe a higher effect on the demands of short-term and long-term bonds of the total responses across savers (as they are the only household type) and a larger effect on the long-term real rate.[21] Together with the lower increase in gains out of firms' profits, this magnifies the income drop of savers from buying long-term bonds from the central bank, therefore decreasing their consumption more than in the TANK-BS case.

When moving to TANK-BS, savers behave like the representative agent in RANK. They affect, however, aggregate demand by relatively less given their lower share in the population and hence the higher profit income per agent. The reduced contribution to

[21] Bond demands of savers in the RANK model are only more sensitive in *total* terms. Once we look at *per-capita* bond demands, the effect of a shock will be lower in RANK compared to TANK-BS due to the higher share of savers.

Figure 3.5: Impulse responses to a QE shock at the ZLB: RANK vs. TANK-BS



Notes: This figure depicts the impulse responses of selected variables to a QE shock net of a negative preference shock when the ZLB on the policy rate is binding, for the borrower-saver model (TANK-BS, dash-dotted gray line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light gray line). The shock for each simulation is an asset purchase of size 1% of annualized GDP. The size of the preference shock is chosen such that the ZLB binds for eight quarters. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

the fall in spending is compensated by a decrease in the labor income of borrowers who have a larger MPC.[22] The net effect of the lower drop in aggregate consumption coming from savers and the additional decrease through borrowers is almost neutral. Even though this finding is consistent with the complementary work of Sims et al. (2022b), the story is different. The lack of amplification in their model arises because only very few households at the bottom of the wealth distribution respond other than the representative agent to a QE shock and their impact on aggregate consumption is therefore marginal.

Unlike a state of the world without a binding ZLB, household heterogeneity starts to matter more when combined with state dependency. Figure 3.5 shows that this case leads to amplified aggregate effects of asset purchases in TANK-BS.

The reasoning combines what has been described so far. First, the presence of the ZLB generates an asymmetric behavior of the short-term real rate, pushing the consumption of both household types and hence aggregate demand upwards. Second, there is an extra

[22] See Figure 3.B5 in Appendix 3.B.3 for more details on each agent's budget constraint components.

Table 3.2: Multipliers on impact and cumulated (in %)

	Output		**Inflation**		**Consumption**	
	QE	QT	QE	QT	QE	QT
RANK (impact)	1.05	-0.44	0.70	-0.32	1.32	-0.56
TANK-BS (impact)	1.29	-0.42	0.71	-0.24	1.61	-0.53
RANK (cumulative)	2.18	-0.86	1.32	-0.67	2.72	-1.08
TANK-BS (cumulative)	2.32	-0.71	1.14	-0.43	2.90	-0.89

Notes: This table summarizes the aggregate effects of a QE shock when the ZLB on the policy rate is binding and a QT shock occurring far enough above the ZLB, for the borrower-saver model (TANK-BS) and its representative-agent counterpart (RANK). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. The table contains the multipliers both on impact of the shock and cumulated over the first four periods after the shock.

boost from the presence of constrained households with a high MPC, such that an increase in their labor income through higher wages has a strong multiplier impact on aggregate demand. Together, these two elements lead to a larger increase in aggregate variables in TANK-BS.

Compared to the case of an asset market operation done off the ZLB discussed before, the direct and indirect effects of QE on borrowers together more than offset the reaction of savers in TANK-BS and the changes in their balance sheets no longer cancel out.[23] When an asset purchase is done when the lower bound binds, the impact of the increased labor income of borrowers with their high MPC exceeds the reduced contribution by savers in terms of spending, with a strong reaction of profits per agent being crucial again.

In order to quantify the asymmetry arising from state dependency in this model, Table 3.2 lists the responses of the main aggregate variables to the two shocks we have analyzed in this section, both on impact and cumulated over four periods, and for both the RANK and the TANK-BS model.

The impact multipliers reveal two results. First, as in the previous section, the impact of QE on macroeconomic aggregates is larger than the absolute impact of QT. This holds for both models and constitutes a *within-model asymmetry*. Doing QE at the ZLB instead of unwinding it off the ZLB has a macroeconomic effect on impact that is more than two times stronger in RANK and about three times stronger in TANK-BS.

Second, as discussed before, household heterogeneity amplifies the aggregate effects of asset market operations only when it appears in combination with state dependency. This *across-model asymmetry* is therefore very weak in the case of our simulated QT

[23]See Figure 3.B7 in Appendix 3.B.4 for more details on each agent's budget constraint components.

shock, but sizable for QE simulated at the ZLB.[24] Moving from RANK to TANK-BS, the macroeconomic impact multiplier of QE is around 20% higher for output and consumption, but about the same for inflation. This result might arise because heterogeneity affects the slope of the aggregate demand curve but not that of the Phillips curve. As a direct consequence, introducing household heterogeneity amplifies the within-model asymmetry.

3.4 Conclusion

In this paper, we present a two-agent New Keynesian model with borrowers and savers that is used to study the state dependency of asset market operations and their interactions with household heterogeneity. Central bank asset purchases and sales operate via portfolio rebalancing between short-term and long-term government bonds held by the two types of households in the economy. These assets are imperfect substitutes due to the portfolio adjustment costs in place. State dependency arises through the presence of an occasionally binding ZLB on the nominal short-term interest rate. Therefore, the asymmetry between QE and QT in this context is driven by whether the nominal rate is available as a policy tool or is constrained by the lower bound.

We find that a binding ZLB magnifies the macroeconomic effects of asset market operations by central banks. This is due to the behavior of the short-term real rate when the economy is at (or close to) the lower bound. Consequently, when dealing with the risk of hitting the ZLB, our simulations imply that a central bank can mitigate the adverse effects of monetary policy normalization by prioritizing a policy rate hike over asset sales and thus by avoiding to tighten too early or too fast.

Moreover, we find that the role of household heterogeneity in amplifying the effects of asset market operations also depends on the state of the economy. Away from the ZLB, household heterogeneity does not imply amplification. On the contrary, when asset market operations occur in a liquidity trap, we find substantial amplification for aggregate output and consumption.

Despite the lack of evidence, our model intends to contribute to a better understanding of the potential effects of balance sheet reductions. Given the widespread belief that the effects of QE and QT are not exactly of equal but opposite size, further work on the implications of monetary policy normalization are indispensable. In particular, it would be essential to analyze transmission channels other than portfolio rebalancing, to extend the heterogeneity dimension to a continuum of households, or to additionally consider frictions on the firms' side.

[24] Whether the aggregate effects of QT are slightly stronger or weaker depends on the calibrated parameter values. However, for a realistic calibration, QT has always around the same aggregate impact on output and total consumption in both models.

Appendix

3.A Borrower-saver model derivations

This part provides more details on the derivations of the model presented in section 3.2.

3.A.1 Household problem

Each household of type $j = \{B, S\}$ faces the following optimization problem:

$$\max_{c_t^j, N_t^j, b_t^j, b_t^{j,L}} \mathbb{E}_t \sum_{t=0}^{\infty} \left(\beta^j\right)^t \theta_t \left(\frac{(c_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \zeta^j \frac{(N_t^j)^{1+\varphi}}{1+\varphi} \right) \qquad \text{subject to}$$

$$c_t^j + b_t^j + b_t^{j,L} \leq r_{t-1}\, b_{t-1}^j + r_t^L\, b_{t-1}^{j,L} + w_t\, N_t^j + d_t^j - t_t - \frac{\nu}{2} \left(\delta^j \frac{b_t^j}{b_t^{j,L}} - 1 \right)^2 + tr^j \ ,$$
$$0 \leq \mathbb{I}^j \left(b_t^B + b_t^{B,L} + \overline{D} \right) \ ,$$

where $d_t^B = \frac{\tau^D}{\lambda} d_t$, $d_t^S = \frac{1-\tau^D}{1-\lambda} d_t$, $tr^B = \frac{tr}{\lambda}$, and $tr^S = -\frac{tr}{1-\lambda}$. Moreover, $\mathbb{I}^j$ is an indicator function with values $\mathbb{I}^S = 0$ and $\mathbb{I}^B = 1$.

The resulting optimality conditions for each agent are:

$$U_{c,t}^j = \theta_t (c_t^j)^{-\frac{1}{\sigma}} \ ,$$
$$U_{N,t}^j = -\theta_t\, \zeta^j\, (N_t^j)^{\varphi} \ ,$$
$$w_t = -\frac{U_{N,t}^j}{U_{c,t}^j} \ ,$$
$$U_{c,t}^j + U_{c,t}^j \frac{\nu\, \delta^j}{b_t^{j,L}} \left(\delta^j \frac{b_t^j}{b_t^{j,L}} - 1 \right) = \beta^j\, R_t\, \mathbb{E}_t \left[U_{c,t+1}^j \frac{1}{\Pi_{t+1}} \right] + \mathbb{I}^j\, \psi_t^B \ ,$$
$$U_{c,t}^j - U_{c,t}^j \frac{\nu\, \delta^j\, b_t^j}{\left(b_t^{j,L}\right)^2} \left(\delta^j \frac{b_t^j}{b_t^{j,L}} - 1 \right) = \beta^j\, \mathbb{E}_t \left[U_{c,t+1}^j \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \mathbb{I}^j\, \psi_t^B \ ,$$
$$0 = \mathbb{I}^j\, \psi_t^B \left(b_t^B + b_t^{B,L} + \overline{D} \right) \ ,$$

where $\psi_t^B \geq 0$ is the Lagrangian multiplier on the borrowing constraint. It holds that $\psi_t^B > 0$ whenever the constraint is binding.

From the expressions above, we can derive the following Euler equations for short-term and long-term bonds, where we already imposed $\delta^S = \delta^B = \delta$ as specified in the description of the steady state (see section 3.2.5):

$$1 = \beta^j R_t \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c^j_{t+1}}{c^j_t} \right)^{-\frac{1}{\sigma}} \frac{1}{\Pi_{t+1}} \right] - \frac{\nu\,\delta}{b^{j,L}_t} \left(\delta \frac{b^j_t}{b^{j,L}_t} - 1 \right) + \mathbb{I}^j \psi^B_t \,,$$

$$1 = \beta^j \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c^j_{t+1}}{c^j_t} \right)^{-\frac{1}{\sigma}} \frac{R^L_{t+1}}{\Pi_{t+1}} \right] + \frac{\nu\,\delta\, b^j_t}{\left(b^{j,L}_t\right)^2} \left(\delta \frac{b^j_t}{b^{j,L}_t} - 1 \right) + \mathbb{I}^j \psi^B_t \,.$$

Combining the two equations leads to an expression for the nominal return on long-term bonds as a function of the nominal rate on short-term bonds and the bond holdings of households:

$$\mathbb{E}_t R^L_{t+1} = \frac{1 - \frac{\delta\, b^j_t}{(b^{j,L}_t)^2}\, \widetilde{\Psi}^j_t - \mathbb{I}^j \psi^B_t}{1 + \frac{\delta}{b^{j,L}_t}\, \widetilde{\Psi}^j_t - \mathbb{I}^j \psi^B_t} R_t \,,$$

where $\widetilde{\Psi}^j_t = \nu \left(\delta \frac{b^j_t}{b^{j,L}_t} - 1 \right)$. This equation is a no-arbitrage condition between the two types of bonds and captures the key impact channel of asset market operations on bond returns. When the central bank buys or sells long-term bonds, it changes the quantity of assets available to the rest of the economy. Holding bond supply fixed, this implies that households' portfolio mix is not at its desired level and induces costly portfolio rebalancing. The impact of the adjustment cost and of changes in bond demands is directly visible from the equation above. It can be shown that the fraction is larger than one whenever $\delta < \frac{b^{j,L}_t}{b^j_t}$ and smaller than one otherwise.

3.A.2 Intermediate goods producer problem

The price-setting problem of an intermediate goods firm is

$$\max_{\{P_{t+k}(i)\}_{k=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[\left(1+\tau^S\right) \frac{P_{t+k}(i)}{P_{t+k}} y_{t+k}(i) - mc_{t+k}\, y_{t+k}(i) - \frac{\phi_p}{2} \left(\frac{P_{t+k}(i)}{P_{t-1+k}(i)} - 1 \right)^2 y_{t+k} - t^F_{t+k} \right]$$

$$\text{s.t.} \quad y_{t+k}(i) = \left(\frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon} y_{t+k} \,,$$

where $\Lambda_{t,t+k} = (\beta^S)^k \left(\frac{U^S_{c,t+k}}{U^S_{c,t}} \right)$ is the stochastic discount factor for payoffs in period $t+k$. The optimality condition of this optimization problem is

$$\mathbb{E}_t \left\{ \Lambda_{t,t} \left[\left(1+\tau^S\right)(1-\epsilon)\, P_t(i)^{-\epsilon} P_t^{\epsilon-1} y_t + mc_t\, \epsilon\, P_t(i)^{-\epsilon-1} P_t^{\epsilon} y_t - \phi_p \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{y_t}{P_{t-1}(i)} \right] + \Lambda_{t,t+1}\, \phi_p \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t(i)^2} y_{t+1} \right\} = 0 \,.$$

Since all firms are identical and face the same demand from final goods producers, they will all set the same price. This yields the following optimal price-setting condition:

$$\phi_p\left(\Pi_t-1\right)\Pi_t-\mathbb{E}_t\left[\Lambda_{t,t+1}\,\phi_p\left(\Pi_{t+1}-1\right)\Pi_{t+1}\,\frac{y_{t+1}}{y_t}\right]=\left(1+\tau^S\right)(1-\epsilon)+\epsilon\,mc_t\,.$$

3.A.3 Steady state

For the approximation of the model around a deterministic steady state, we assume a long-run inflation rate of unity ($\Pi = 1$), normalize output to one (by setting $z = N = 1$) and set $\theta = 1$.

The Euler equations of the saver gives $R = R^L = \left(\beta^S\right)^{-1}$. Using this in the Euler equations of the borrower implies that the borrowing constraint binds in steady state ($\psi^B > 0$) because we assumed $\beta^S > \beta^B$. We further impose for labor supply that $N^B = N^S = N$. Together with the steady-state transfer on the part of households, this results in $c^B = c^S = c$. Finally, the optimal subsidy induces $mc = 1$ and thus zero profits ($d = 0$).

For the real returns, we get $r = R$ and $r^L = R^L$, which pins down the nominal bond price $V = 1/\left(R^L - \chi\right)$. The weights on hours are found through the labor supply equations, $\zeta^j = w\,(N^j)^{-\varphi}(c^j)^{\sigma}$ with $j = \{B, S\}$ and where $w = y$ from the expression for labor demand. Due to equalized levels of labor supply and consumption across household types, $\zeta^S = \zeta^B$. Finally, as portfolio adjustment costs are zero in steady state ($\Psi^j = 0$), the aggregate resource constraint determines consumption through $c = \left(1 - \frac{g}{y}\right) y$.

With respect to the bond-related variables, we impose $\delta^S = \delta^B = \delta = \frac{b^{H,L}}{b^H}$. This expression can be rewritten by using bond market clearing as $b^L = \frac{\delta\, b}{1-q}$, where we define $\tilde{\delta} = \frac{b^L}{b}$. Moreover, we write the annual steady-state total government debt-to-GDP ratio (in quarterly terms) as $b^{tot}_y = \frac{b+b^L}{4y}$, where the denominator captures annualized output. In order to find an expression for short-term government debt, we rewrite the last equation as $b = 4\, b^{tot}_y \left(\frac{1-q}{1-q+\delta}\right) y$, or $b = 4\, b^{tot}_y \left(\frac{1}{1+\tilde{\delta}}\right) y$. Market clearing then gives $b^H = b$.

Regarding the central bank, bond holdings are $b^{CB,L} = q\, b^L$. This pins down net asset purchases $\Omega = \left(1 - r^L\right) b^{CB,L}$ and households' total demand for long-term bonds $b^{H,L} = b^L - b^{CB,L}$. A borrower's bond holdings are then determined through the (binding) borrowing constraint, with $b^B = -\frac{\overline{D}}{(1+\delta)}$ and $b^{B,L} = -\overline{D} - b^B$. A saver's holdings are pinned down by market clearing, with $b^S = \frac{b^H - \lambda b^B}{1-\lambda}$ and $b^{S,L} = \frac{b^{H,L} - \lambda b^{B,L}}{1-\lambda}$. Finally, lump-sum taxes are given by $t = g + \Omega - b(1 - r) - b^L(1 - r^L)$ and the steady-state transfer by $tr = \lambda[c^B + (1 - r)b^B + (1 - r^L)b^{B,L} - wN^B - \frac{\tau^D}{\lambda} d + t]$.

3.A.4 Model summary

Table 3.A1 lists all equations of the TANK-BS model.

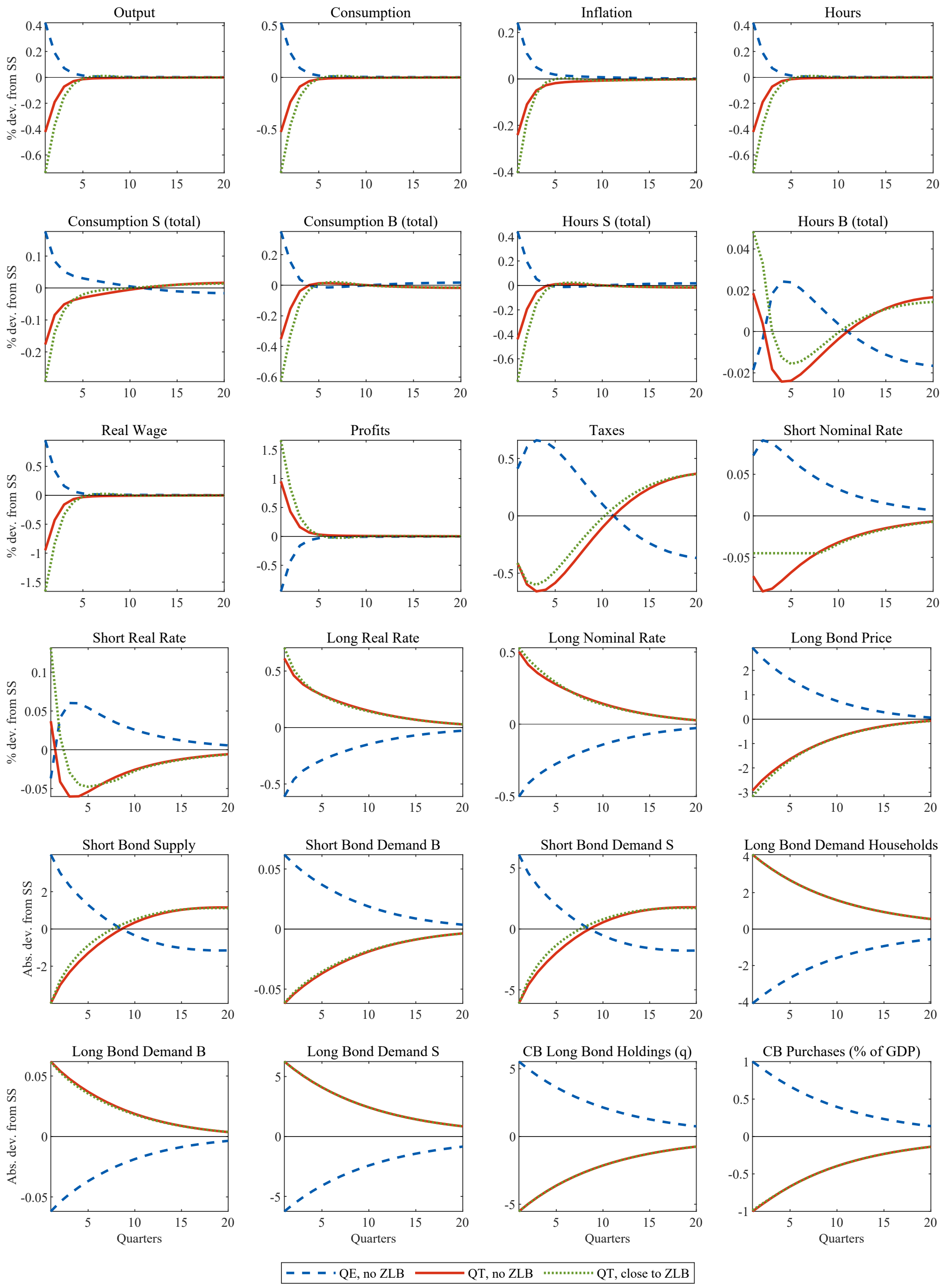
Table 3.A1: Model overview of the TANK-BS model with asset market operations

Labor supply	$w_t = \zeta^j \left(N_t^j\right)^{\varphi} \left(c_t^j\right)^{1/\sigma}, \quad j = \{B, S\}$
Euler short-term bonds, S	$1 = \beta^S \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^S}{c_t^S} \right)^{-1/\sigma} \frac{R_t}{\Pi_{t+1}} \right] - \frac{\nu\, \delta^S}{b_t^{S,L}} \left(\delta^S \frac{b_t^S}{b_t^{S,L}} - 1 \right)$
Euler long-term bonds, S	$1 = \beta^S \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^S}{c_t^S} \right)^{-1/\sigma} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \frac{\nu\, \delta^S\, b_t^S}{\left(b_t^{S,L}\right)^2} \left(\delta^S \frac{b_t^S}{b_t^{S,L}} - 1 \right)$
Budget constraint, S	$c_t^S + b_t^S + b_t^{S,L} = r_{t-1}\, b_{t-1}^S + r_t^L\, b_{t-1}^{S,L} + w_t\, N_t^S + \frac{1-\tau^D}{1-\lambda} d_t - t_t - \Psi_t^S - \frac{tr}{1-\lambda}$
Euler short-term bonds, B	$1 = \beta^B \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^B}{c_t^B} \right)^{-1/\sigma} \frac{R_t}{\Pi_{t+1}} \right] - \frac{\nu\, \delta^B}{b_t^{B,L}} \left(\delta^B \frac{b_t^B}{b_t^{B,L}} - 1 \right) + \psi_t^B$
Euler long-term bonds, B	$1 = \beta^B \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^B}{c_t^B} \right)^{-1/\sigma} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \frac{\nu\, \delta^B\, b_t^B}{\left(b_t^{B,L}\right)^2} \left(\delta^B \frac{b_t^B}{b_t^{B,L}} - 1 \right) + \psi_t^B$
Budget constraint, B	$c_t^B + b_t^B + b_t^{B,L} = r_{t-1}\, b_{t-1}^B + r_t^L\, b_{t-1}^{B,L} + w_t\, N_t^B + \frac{\tau^D}{\lambda} d_t - t_t - \Psi_t^B + \frac{tr}{\lambda}$
Borrowing constraint	$-b_t^B - b_t^{B,L} \leq \overline{D}$
Portfolio adjustment cost	$\Psi_t^j = \frac{\nu}{2} \left(\delta^j \frac{b_t^j}{b_t^{j,L}} - 1 \right)^2, \quad j = \{B, S\}$
Labor demand	$w_t = mc_t \frac{y_t}{N_t}$
Production function	$y_t = z_t\, N_t$
Profits, aggregate	$d_t = \left[1 - mc_t - \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2 \right] y_t$
Phillips curve	$\phi_p \left(\Pi_t - 1\right) \Pi_t = \epsilon\, mc_t + \left(1 + \tau^S\right)(1 - \epsilon)$ $+ \beta^S \mathbb{E}_t \left[\frac{\theta_{t+1}}{\theta_t} \left(\frac{c_{t+1}^S}{c_t^S} \right)^{-\frac{1}{\sigma}} \phi_p \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \frac{y_{t+1}}{y_t} \right]$
Government budget constraint	$b_t + b_t^L = r_{t-1}\, b_{t-1} + r_t^L\, b_{t-1}^L + \Omega_t + g_t - t_t$
Real short-term interest rate	$r_t = \frac{R_t}{\mathbb{E}_t \Pi_{t+1}}$
Nominal long-term bond return	$R_t^L = \frac{1 + \chi\, V_t}{V_{t-1}}$
Real long-term bond return	$r_t^L = \frac{R_t^L}{\Pi_t}$
Net bond purchases, CB	$\Omega_t = b_t^{CB,L} - r_t^L\, b_{t-1}^{CB,L}$
Value bond purchases, CB	$b_t^{CB,L} = q_t\, b_t^L$
Taylor rule	$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_r) \left[\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right)\right] + \epsilon_t^m$
QE shock rule	$\log\left(\frac{q_t}{q}\right) = \rho_q \log\left(\frac{q_{t-1}}{q}\right) + \epsilon_t^q$
Fiscal rule	$\frac{t_t}{t} = \left(\frac{t_{t-1}}{t}\right)^{\rho^{\tau,t}} \left(\frac{b_t + b_t^L}{b + b^L}\right)^{\rho^{\tau,b}} \left(\frac{g_t}{g}\right)^{\rho^{\tau,g}}$
Aggregate consumption	$c_t = \lambda c_t^B + (1 - \lambda) c_t^S$
Aggregate labor	$N_t = \lambda N_t^B + (1 - \lambda) N_t^S$
Short-term bonds market clearing	$b_t = \lambda b_t^B + (1 - \lambda) b_t^S$
Long-term bonds market clearing	$b_t^L = \left(\lambda b_t^{B,L} + (1 - \lambda)\, b_t^{S,L}\right) + b_t^{CB,L}$
Resource constraint	$y_t = c_t + g_t + \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2 y_t$
Other shock rules	$\log\left(\frac{x_t}{x}\right) = \rho_x \log\left(\frac{x_{t-1}}{x}\right) + \epsilon_t^x, \quad x = \{g, b^L, z, \theta\}$

3.B Full sets of impulse responses

3.B.1 QE/QT and QT near the ZLB

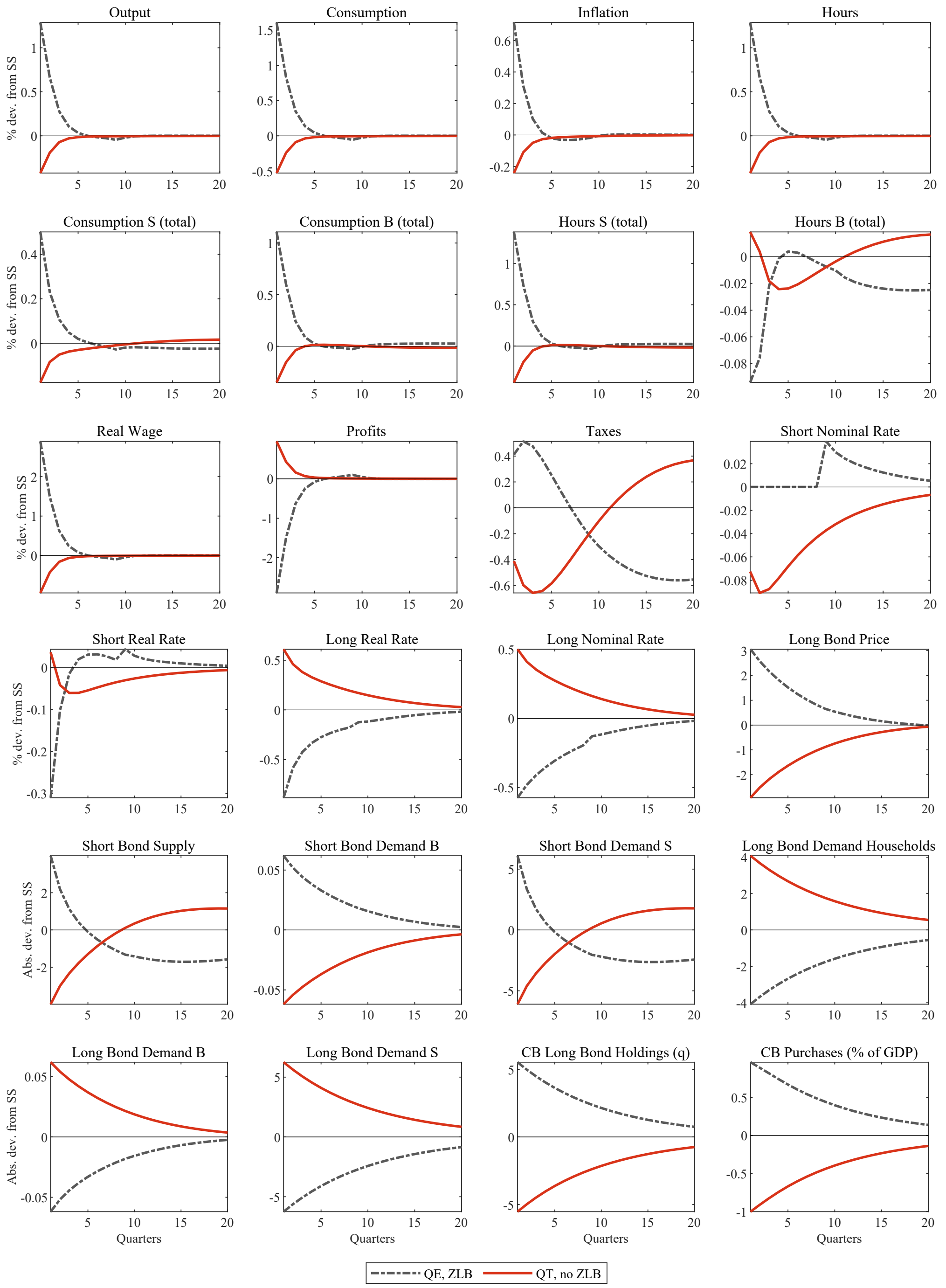
Figure 3.B1: Impulse responses to a QE/QT shock and a QT shock near the ZLB



Notes: This figure depicts the impulse responses to a QE (dashed blue line) and a QT (solid red line) shock occurring far enough above the ZLB, and a QT shock happening close to the ZLB (dotted green line, simulated with $\beta^S = 0.99955$). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

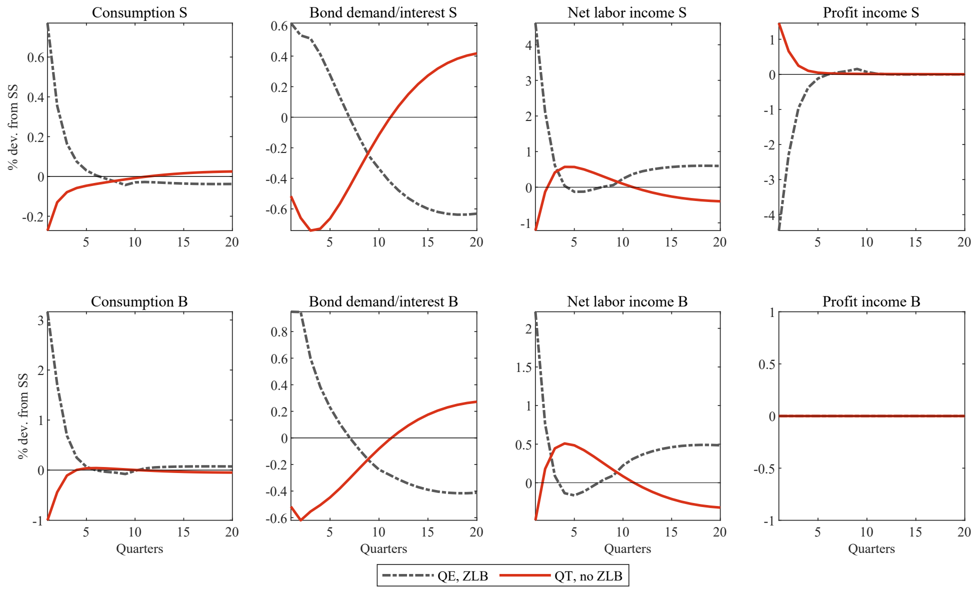
3.B.2 QE at the ZLB and QT off the ZLB

Figure 3.B2: Impulse responses to a QE shock at the ZLB and a QT shock off the ZLB



Notes: This figure depicts the impulse responses to a QE shock when the ZLB on the policy rate is binding (dash-dotted gray line, showing the impact of QE net of a preference shock), and a QT shock occurring far enough above the ZLB (solid red line). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. For QE, the size of the preference shock is chosen such that the ZLB binds for eight quarters. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

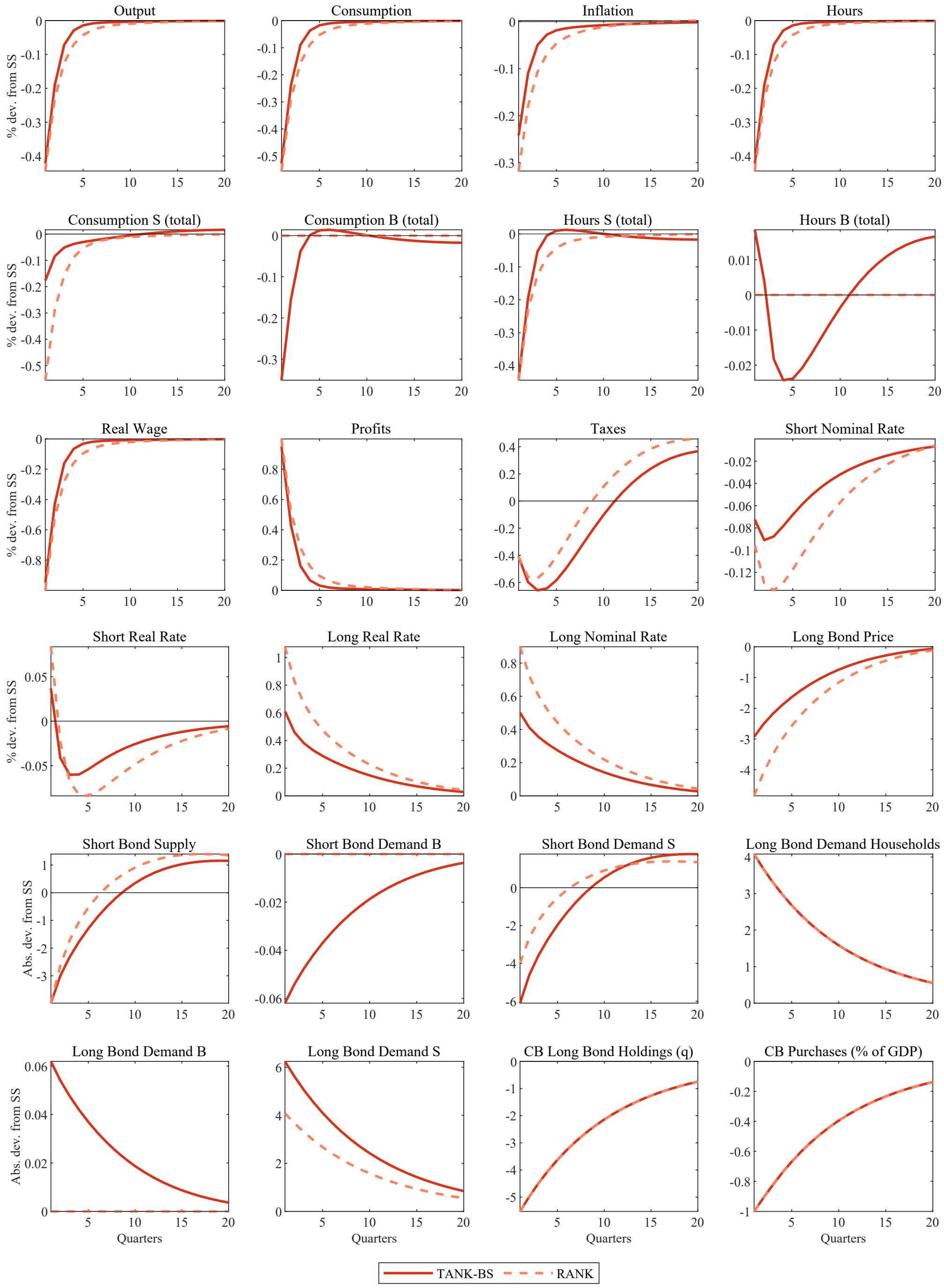
Figure 3.B3: Households' budget components to a QE shock at the ZLB and a QT shock off the ZLB



Notes: This figure shows grouped components of the budget constraints of savers (top) and borrowers (bottom) in response to a QE shock when the ZLB on the policy rate is binding (dash-dotted gray line, showing the impact of QE net of a negative preference shock), and a QT shock occurring far enough above the ZLB (solid red line). The shock for each simulation is an asset purchase/sale of size 1% of annualized GDP. For QE, the size of the preference shock is chosen such that the ZLB binds for eight quarters. Each panel consists of four columns, containing the responses of individual consumption, bond-related variables (bond demand, interest payments/income, net of adjustment cost), labor income net of taxes, and income from profits. All responses are shown in per-capita terms.

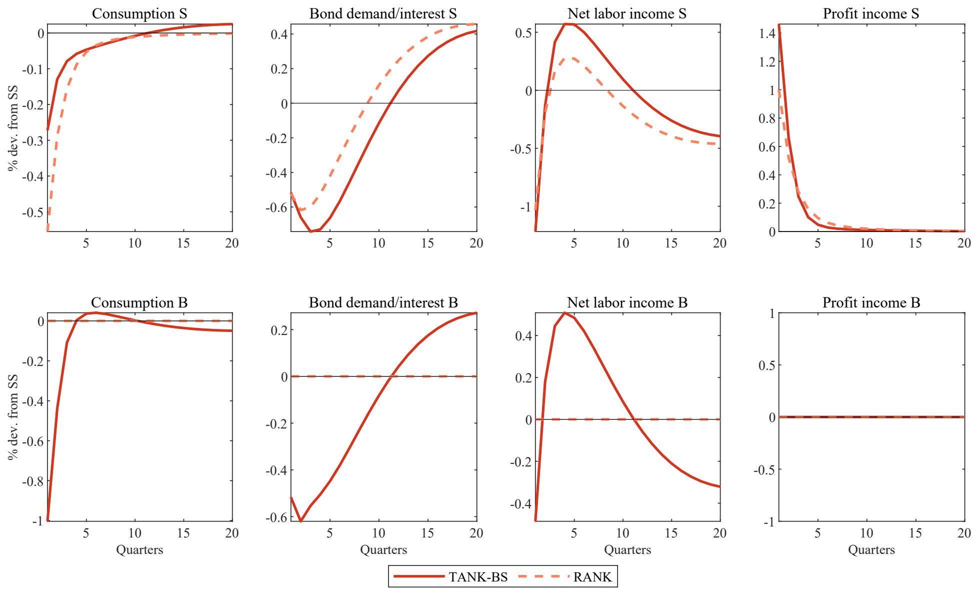
3.B.3 QT off the ZLB: RANK vs. TANK-BS

Figure 3.B4: Impulse responses to a QT shock off the ZLB: RANK vs. TANK-BS



Notes: This figure depicts the impulse responses to a QT shock occurring far enough above the ZLB, for the borrower-saver model (TANK-BS, solid red line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light red line). The shock for each simulation is an asset sale of size 1% of annualized GDP. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

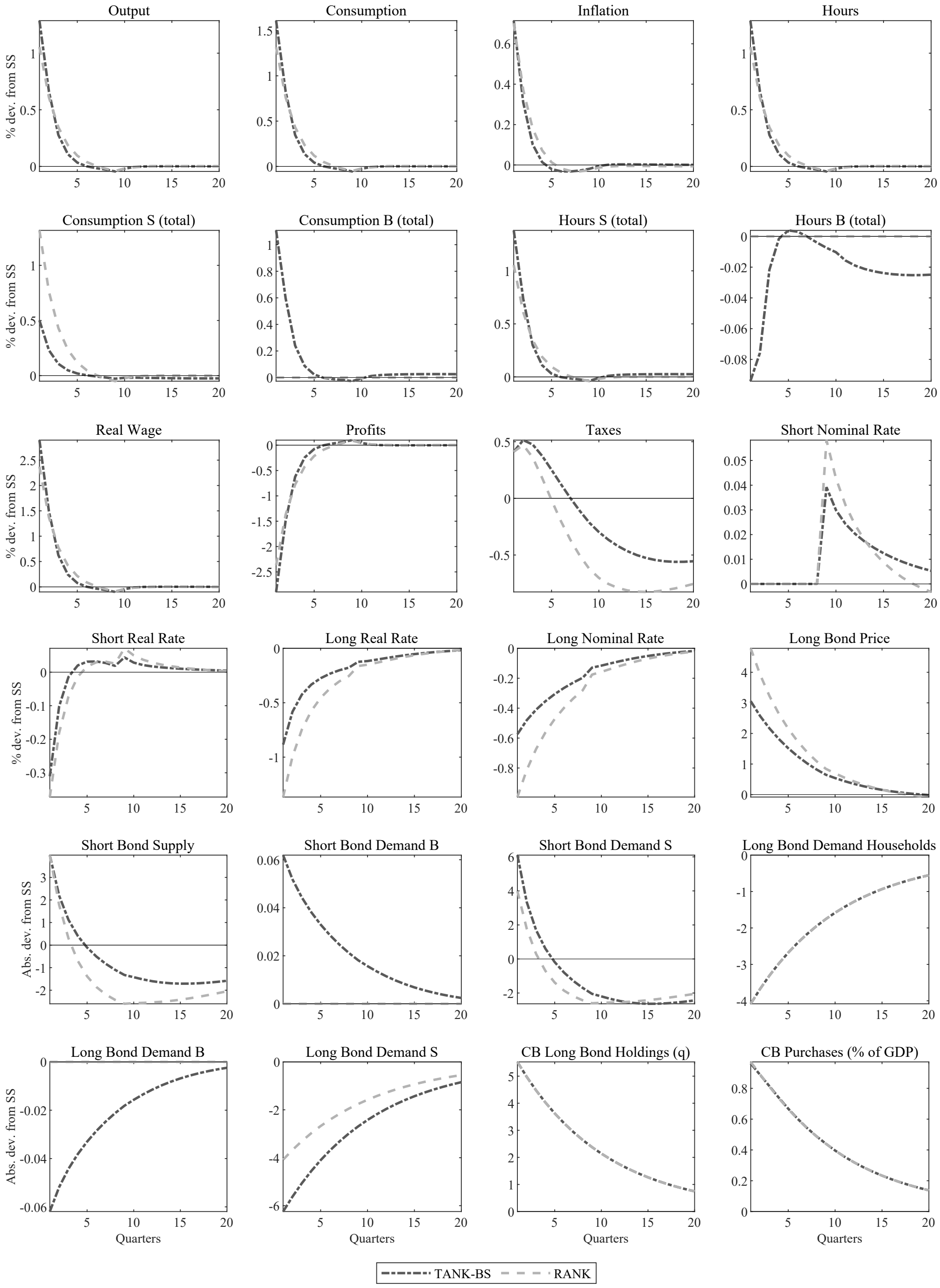
Figure 3.B5: Households' budget components to a QT shock off the ZLB: RANK vs. TANK-BS



Notes: This figure shows grouped components of the budget constraints of savers (top) and borrowers (bottom) in response to a QT shock occurring far enough above the ZLB, for the borrower-saver model (TANK-BS, solid red line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light red line). The shock for each simulation is an asset sale of size 1% of annualized GDP. Each panel consists of four columns, containing the responses of individual consumption, bond-related variables (bond demand, interest payments/income, net of adjustment cost), labor income net of taxes, and income from profits. All responses are shown in per-capita terms.

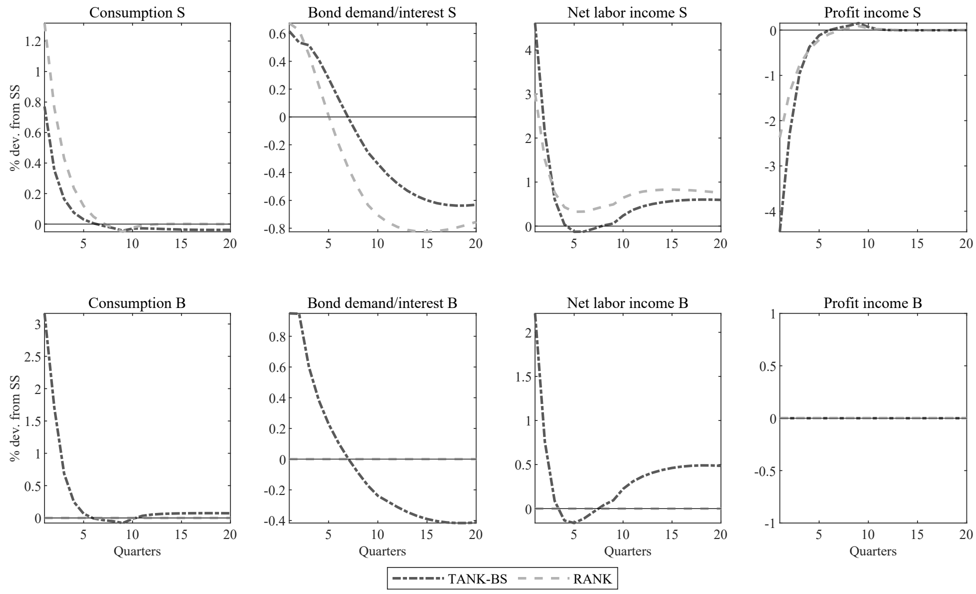
3.B.4 QE at the ZLB: RANK vs. TANK-BS

Figure 3.B6: Impulse responses to a QE shock at the ZLB: RANK vs. TANK-BS



Notes: This figure depicts the impulse responses to a QE shock net of a negative preference shock when the ZLB on the policy rate is binding, for the borrower-saver model (TANK-BS, dash-dotted gray line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light gray line). The shock for each simulation is an asset purchase of size 1% of annualized GDP. The size of the preference shock is chosen such that the ZLB binds for eight quarters. Responses for individual consumption levels and hours are weighted by population shares of savers (S) and borrowers (B), respectively, and thus represent total responses.

Figure 3.B7: Households' budget components to a QE shock at the ZLB: RANK vs. TANK-BS



Notes: This figure shows grouped components of the budget constraints of savers (top) and borrowers (bottom) in response to a QE shock net of a negative preference shock when the ZLB on the policy rate is binding, for the borrower-saver model (TANK-BS, dash-dotted gray line) and its representative-agent counterpart for $\lambda = 0$ (RANK, dashed light gray line). The shock for each simulation is an asset purchase of size 1% of annualized GDP. The size of the preference shock is chosen such that the ZLB binds for eight quarters. Each panel consists of four columns, containing the responses of individual consumption, bond-related variables (bond demand, interest payments/income, net of adjustment cost), labor income net of taxes, and income from profits. All responses are shown in per-capita terms.

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