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Random assignments with uniform preferences: An impossibility result

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ABSTRACT

Agents have uniform preferences if a weakly decreasing utility function determines each agent's preference ranking over the same order of alternatives. We show that the impossibility in the random assignment problem between strategyproofness, ordinally efficiency, and fairness in the sense of equal division lower bound, prevails even if agents have uniform preferences. Furthermore, it continues to hold even if we weaken the strategyproofness to upper-contour strategyproofness, or the ordinal efficiency to robust ex-post Pareto efficiency.

1. Introduction

We study the assignment problem, which is concerned with allocating a set of objects, without monetary transfer, to agents who revealed their uniform ordinal preferences over objects and are entitled to only one. Whereas assignment context may lead to specific preferences structure, in some cases, restriction on preferences domain can be considered. For instance, a set of agents has single-peaked preferences if we could identify an underlying common order of the objects such that for each agent, objects further from her best one are preferred less. In contrast to these preferences, in a uniform preference profile, the order of objects is the same for all agents, while they differ in their strict preference relation.

The uniform preference domain is intuitively plausible, where the objects are arranged in the same order by all the agents, and each agent's preference ranking is determined by a weakly decreasing utility function over the objects. In the generic assignment problem, objects do not inherit an intrinsic objective value, and each agent might have a different perception of its subjective value and rank it differently. However, some cases have a clear public ranking over alternatives, e.g., unit-length jobs.

From a technical point of view, the study of impossibilities in the narrower domain is interesting because it indicates how severe the impossibility is and whether there is room to look for a possibility, at least in a specific domain. Moreover, the richer domain may yield false results in the narrow domain. There is a vast literature on impossibilities in the random assignment problem. No ordinally efficient and strategyproof mechanism satisfies equal treatment of equals [2]. Moreover, Nesterov [9] proved that a strategyproof mechanism does not exist that is ex-post efficient and envy-free, and ordinally efficiency is not compatible with strategyproofness and *equal division lower bound*. Sethuraman and Ye [11] showed that the first two impossibilities results hold for the uniform preference domain.

In this paper, we assess the last impossibility result on a uniform preference domain and show that it continues to hold even in this restricted domain. The paper is organized as follows. Section 2 reviews the related literature. Section 3 recalls the standard model and axioms of random assignments. In Section 4, we prove our impossibility result. Finally, Section 5 concludes.

2. Related literature

Our work is associated with the random assignment literature that considers agents to have almost similar preferences over objects. Bogomolnaia and Moulin [3] assumed that all agents have the same ordinal ranking over all objects, receiving no object may be preferable to some objects, and agents differ on which objects are worse than opting out. They characterized Probabilistic Serial (PS) and showed that the PS assignment improves upon the Random Priority assignment. Chang and Chun [5] investigated the impossibility of [2] for a more restricted domain. They showed that the three properties are still incompatible even if all agents have the same preferences except the ordinal ranking of one object.

Our work is also related to the random assignment literature on the weak preference domain that permits indifferences. The sem-

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inal paper [2] introduced ordinal efficiency and proved that the PS mechanism finds an envy-free ordinally efficient assignment for allocating a set of indivisible objects to agents with strict preferences. Bogomolnaia, Deb, and Ehlers [4] compared the outcomes from the Serially Dictatorial Rules with a market-based approach for the problem of efficiently allocating indivisible objects between agents whose preferences are private information with indifferences permitted.

For the full preference domain, Katta and Sethuraman [7] defined a similar algorithm and proved that even a weak strategyproof mechanism could not find a random assignment that is both ordinally efficient and envy-free. Yılmaz [12] constructed a recursive algorithm for the assignment problem with private endowments and a social endowment, which is a natural extension of one à la [7] for the weak preference domain and satisfies individual rationality, ordinal efficiency, and no justified-envy. Aziz, Luo, and Rizkallah [1] showed that for the full preference domain, there exists no extension of PS that is ex-post efficient and weak strategyproof.

3. Model

Let A be a finite set of objects which should be assigned to a finite set of agents, N, with $|A| = |N| = n$. Each agent $i \in N$ has a complete, transitive, and anti-symmetric *preference relation* $\succsim_i$ over A. We denote the domain of those preferences by $\mathcal{F}$ and a preference profile by $\succsim \equiv (\succsim_i)_{i\in N}$ on domain of $\mathcal{F}^n$. We consider the uniform preference domain in which $a \succsim_i b \succsim_i \ldots \succsim_i d$ for every agent $i \in N$. Agents differ in their preference ordering only in their strict preference relation $\succ_i$ and hence their indifference relation $\sim_i$. For ease of writing, all the objects within a non-singleton indifference class for an agent illustrated within braces, and a comma separates these indifference classes, and objects are always written in subscript order. Therefore, the preference ordering $a \succ_i b \sim_i c \succ_i d$ for agent i is written as $i : a, \{bc\}, d$.

We represent a **random assignment** by a *bistochastic matrix* $P = [p_{ia}]_{i\in N, a\in A}$, with agents on rows and objects on columns, where p_{ia} is the probability of assigning object a to agent i. We denote the domain of random assignments by R. A **random allocation** for some agent $i \in N$, P_i, is a probabilistic distribution over all objects in A where the sum of probabilities of assigning objects to the agent i equals to 1. Upon enumerating objects in A for agent i from best to worst according to $a_{i,1} \succ_i a_{i,2} \succ_i \ldots \succ_i a_{i,n}$, where $a_{i,k}$ is the k^{th} best object of agent i, we define $u^P_{ir} = \sum_{k=1}^{r} p_{ia_{i,k}}$. Given a preference ordering $\succ_i$ on A, the stochastic dominance relation is denoted by $\succ^{sd}_i$, where $P_i \succ^{sd}_i Q_i$ if and only if $u^P_{ir} \geq u^Q_{ir}$ for every $r = 1, \ldots, n$. Given a preference profile $\succ \in \mathcal{F}^n$, the random assignment P, is stochastically dominated by another random assignment $Q \neq P$, if $Q_i \succ^{sd}_i P_i$ for all $i \in N$. A random assignment P is **ordinally efficient** at a profile $\succ$ if and only if it is not stochastically dominated.

A **mechanism** $\mu(.)$ is a function from $\mathcal{F}^n$ into R, that associates each preference profile with some random assignment. If a random assignment P stochastically dominates the random assignment with equal division, i.e., $\forall i \in N$, $P_i \succ^{sd}_i 1/n$, then it satisfies **equal division lower bound**. A mechanism $\mu(.)$ is **strategyproof** whenever for each $i \in N$, $\mu_i(\succ_i, \succ_{-i}) \succ^{sd}_i \mu_i(\succ'_i, \succ_{-i})$ for all $\succ'_i \neq \succ_i$. For each pair $\succ_i, \succ'_i \in \mathcal{F}$, $\succ'_i$ is **adjacent** to $\succ_i$ if $\succ'_i$ is attained from $\succ_i$ by swapping two sequentially ranked objects without changing the rank of any other objects. A mechanism satisfies **swap monotonicity** if either $\mu_i(\succ'_i, \succ_{-i}) = \mu_i(\succ_i, \succ_{-i})$ or $\mu_{ib}(\succ'_i, \succ_{-i}) > \mu_{ib}(\succ_i, \succ_{-i})$.

Let $U(\succ_i, a) = \{b \in A | b \succ_i a\}$ be the (strict) **upper contour** set of a in $\succ_i$, and $L(\succ_i, a) = \{b \in A | a \succ_i b\}$ be the (strict) **lower contour** set of a in $\succ_i$. For $\succ_i \in \mathcal{F}$, each $i \in N$, each $\succ'_i \in \mathcal{F}$, and each $a, b \in A$, if $\succ'_i$ is adjacent to $\succ_i$, i.e., $a \succ_i b$, and $b \succ'_i a$, a mechanism satisfies **upper invariance**, if $\mu_{ic}(\succ'_i, \succ_{-i}) = \mu_{ic}(\succ_i, \succ_{-i})$ for each $c \in U(\succ_i, a)$, and meets **lower invariance** if $\mu_{ic}(\succ'_i, \succ_{-i}) = \mu_{ic}(\succ_i, \succ_{-i})$ for each $c \in L(\succ_i, b)$.

4. The impossibility result

For any given preference profile, Theorem 3 of [9] proved that, for more than four agents and objects, there does not exist a mechanism that is ordinally efficient, strategyproof, and satisfies equal division lower bound. Although this result is addressed for the general preference profile domain, the proof implicitly works for the single-peaked preferences by considering the order of objects in such a way that $c \succ a \succ b \succ d$. Now, we prove the same incompatibility for a uniform preference profile, using the decomposition of strategy-proofness into swap monotonicity, upper invariance, and lower invariance, à la [8].

Proposition 1. *For $n \geq 4$, there is no strategyproof mechanism satisfying ordinal efficiency and equal division lower bound in the uniform preferences domain.*

Remark. As we did not use swap monotonic in our proof, we indeed showed a stronger result: for more than four agents and objects, ordinal efficiency and equal division lower bound are incompatible with a weaker notion of strategyproofness, in the sense of Chun and Yun [6], called upper-contour strategyproofness, which is equivalent to the combination of upper invariance and lower invariance.

Proof. First, note that it is enough to prove the claim for the case with $n = 4$, where $N = \{1, 2, 3, 4\}$, and $A = \{a, b, c, d\}$. Suppose by contradiction that there exists a mechanism φ satisfying the required properties. For preference profile

$$Q1: \begin{array}{l} 1: a,\ b,\ c,\ d \\ 2: a,\ b,\ c,\ d \\ 3: a,\ \{b\ c\},\ d \\ 4: a,\ \{b\ c\},\ d \end{array},$$

by equal division lower bound (EDLB), we find that agents receive their first-best objects with the same probability, i.e., $\varphi_{1a}(Q1) = \varphi_{2a}(Q1) = \varphi_{3a}(Q1) = \varphi_{4a}(Q1) = 1/4$ since it is not possible to have $\varphi_{ia}(Q1) > 1/4$ for an agent i as it then implies $\varphi_{ja}(Q1) < 1/4$ for at least another agent j that contradicts EDLB. With the same line of reasoning, agents also receive their worst objects with the same probability, i.e., $\varphi_{1d}(Q1) = \varphi_{2d}(Q1) = \varphi_{3d}(Q1) = \varphi_{4d}(Q1) = 1/4$.

From ordinal efficiency, we get $\varphi_{3b}(Q1) = \varphi_{4b}(Q1) = 0$ (Otherwise, $\varphi_{ib}(Q1) > 0$ for $i = 3$ or $i = 4$ implies $\varphi_{3c}(Q1) + \varphi_{4c}(Q1) < 1$, which in turn indicates $\varphi_{jc}(Q1) > 0$ for $j = 1$ or $j = 2$. Now, agents i and j are better off exchanging b and c, which contradicts ordinal efficiency). Therefore, for remaining elements, we have $\varphi_{3c}(Q1) = \varphi_{4c}(Q1) = 1/2$ which means $\varphi_{1c}(Q1) = \varphi_{2c}(Q1) = 0$ and $\varphi_{1b}(Q1) = \varphi_{2b}(Q1) = 1/2$:

$$\varphi(Q1) = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \end{pmatrix}.$$

For preference profile

$$Q2: \begin{array}{l} 1: a,\ b,\ \{c\ d\} \\ 2: a,\ b,\ c,\ d \\ 3: a,\ \{b\ c\},\ d \\ 4: a,\ \{b\ c\},\ d \end{array},$$

we get $\varphi_{1a}(Q2) = \varphi_{2a}(Q2) = \varphi_{3a}(Q2) = \varphi_{4a}(Q2) = 1/4$ by EDLB, and $\varphi_{1b}(Q2) = \varphi_{1b}(Q1) = 1/2$ by upper invariance. Moreover, ordinal efficiency implies $\varphi_{1c}(Q2) = 0$ (Otherwise $\varphi_{1c}(Q2) > 0$ implies $\varphi_{id}(Q1) > 0$ for $i = 2$, $i = 3$, or $i = 4$, as $\varphi_{2d}(Q2) + \varphi_{3d}(Q2) + \varphi_{4d}(Q2) > 3/4$. Then, agents 1 and i are better off exchanging c and d, which contradicts ordinal efficiency).

Hence, $\varphi_{1d}(Q2) = 1/4$, and from EDLB we have $\varphi_{2d}(Q2) = \varphi_{3d}(Q2) = \varphi_{4d}(Q2) = 1/4$. Now, by applying the same argument as preference profile $Q1$, we can show that $\varphi_{3b}(Q2) = \varphi_{4b}(Q2) = 0$, and as a result $\varphi_{3c}(Q2) = \varphi_{4c}(Q2) = 1/2$. Therefore, $\varphi(Q2) = \varphi(Q1)$.

The preference profile

$$Q2' : \begin{array}{l} 1 : a,\ b,\ c,\ d \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ \{b\ \ c\},\ d \\ 4 : a,\ \{b\ \ c\},\ d \end{array},$$

can be derived from $Q2$ by swapping agents 1 and 2. Therefore, we have $\varphi(Q2') = \varphi(Q2) = \varphi(Q1)$.

For the preference profile

$$Q3 : \begin{array}{l} 1 : a,\ b,\ \{c\ \ d\} \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ \{b\ \ c\},\ d \\ 4 : a,\ \{b\ \ c\},\ d \end{array},$$

by EDLB, we have $\varphi_{1a}(Q3) = \varphi_{2a}(Q3) = \varphi_{3a}(Q3) = \varphi_{4a}(Q3) = 1/4$, and by upper invariance we get $\varphi_{2b}(Q3) = \varphi_{2b}(Q2) = 1/2$ and $\varphi_{1b}(Q3) = \varphi_{1b}(Q2') = 1/2$, and thus $\varphi_{3b}(Q3) = \varphi_{4b}(Q3) = 0$. Moreover, ordinal efficiency implies $\varphi_{1c}(Q3) = \varphi_{2c}(Q3) = 0$ (Otherwise if $\varphi_{ic}(Q3) > 0$ for $i \in \{1, 2\}$, we have $\varphi_{jd}(Q3) > 0$ for $j = 3$ or $j = 4$ as $\varphi_{3d}(Q3) + \varphi_{4d}(Q3) > 1/2$. Then, agents i and j are better off exchanging c and d, which contradicts ordinal efficiency). Therefore, $\varphi(Q3) = \varphi(Q1)$.

For the preference profile

$$Q4 : \begin{array}{l} 1 : a,\ b,\ \{c\ \ d\} \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ b,\ c,\ d \\ 4 : a,\ b,\ c,\ d \end{array},$$

we have $\varphi_{1a}(Q4) = \varphi_{2a}(Q4) = \varphi_{3a}(Q4) = \varphi_{4a}(Q4) = 1/4$ and $\varphi_{1b}(Q4) = \varphi_{2b}(Q4) = \varphi_{3b}(Q4) = \varphi_{4b}(Q4) = 1/4$ by EDLB. With same argument as preference profile $Q3$, ordinal efficiency implies $\varphi_{1c}(Q3) = \varphi_{2c}(Q3) = 0$. Since $\varphi(Q4)$ is a bistochastic matrix, we can fill in the random assignment for remaining agents and objects to get

$$\varphi(Q4) = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}.$$

For the preference profile

$$Q5 : \begin{array}{l} 1 : a,\ b,\ \{c\ \ d\} \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ b,\ c,\ d \\ 4 : a,\ \{b\ \ c\},\ d \end{array},$$

by EDLB, we have $\varphi_{1a}(Q5) = \varphi_{2a}(Q5) = \varphi_{3a}(Q5) = \varphi_{4a}(Q5) = 1/4$, and by lower invariance, we get $\varphi_{4d}(Q5) = \varphi_{4d}(Q4) = 1/4$ and $\varphi_{3d}(Q5) = \varphi_{3d}(Q3) = 1/4$. Moreover, ordinal efficiency implies that $\varphi_{4b}(Q5) = 0$ (Otherwise, we have $\varphi_{ic}(Q5) > 0$ for $i = 1$, $i = 2$, or $i = 3$ as $\varphi_{1c}(Q5) + \varphi_{2c}(Q5) + \varphi_{3c}(Q5) > 1/2$, and agents 4 and i can beneficially exchange their share from b and c that violates ordinal efficiency). Hence, we have $\varphi_{4c}(Q5) = 1/2$. Moreover, with same lines of argument as the preference profile $Q3$, we get $\varphi_{1c}(Q5) = \varphi_{2c}(Q5) = 0$ by ordinal efficiency. Therefore, $\varphi(Q5) = \varphi(Q1)$.

The preference profile

$$Q5' : \begin{array}{l} 1 : a,\ b,\ \{c\ \ d\} \\ 2 : a,\ b,\ c,\ d \\ 3 : a,\ b,\ \{c\ \ d\} \\ 4 : a,\ \{b\ \ c\},\ d \end{array}$$

can be derived from $Q5$ by swapping agents 2 and 3. Thus, we get

$$\varphi(Q5') = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \end{pmatrix}.$$

The preference profile

$$Q5'' : \begin{array}{l} 1 : a,\ b,\ c,\ d \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ b,\ \{c\ \ d\} \\ 4 : a,\ \{b\ \ c\},\ d \end{array},$$

can be derived from $Q5$ by swapping agents 1 and 3. Hence, we have

$$\varphi(Q5'') = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \end{pmatrix}.$$

Finally, for the preference profile

$$Q6 : \begin{array}{l} 1 : a,\ b,\ \{c\ \ d\} \\ 2 : a,\ b,\ \{c\ \ d\} \\ 3 : a,\ b,\ \{c\ \ d\} \\ 4 : a,\ \{b\ \ c\},\ d \end{array},$$

EDLB implies $\varphi_{1a}(Q6) = \varphi_{2a}(Q6) = \varphi_{3a}(Q6) = \varphi_{4a}(Q6) = 1/4$. Moreover, by upper invariance, we find that $\varphi_{3b}(Q6) = \varphi_{3b}(Q5) = 0$, $\varphi_{2b}(Q6) = \varphi_{2b}(Q5') = 0$, and $\varphi_{1b}(Q6) = \varphi_{1b}(Q5'') = 0$. From ordinal efficiency, we have $\varphi_{4b}(Q6) = 0$ (Otherwise, $\varphi_{4b}(Q6) > 0$ implies $\varphi_{4c}(Q6) < 1$, which in turn indicates $\varphi_{ic}(Q6) > 0$ for $i = 1$, $i = 2$, or $i = 3$. Now, agents 4 and i are weakly better off exchanging b and c, which contradicts ordinal efficiency),

$$\varphi(Q6) = \begin{pmatrix} \frac{1}{4} & 0 & - & - \\ \frac{1}{4} & 0 & - & - \\ \frac{1}{4} & 0 & - & - \\ \frac{1}{4} & 0 & - & - \end{pmatrix},$$

however, it contradicts with the fact that $\varphi(Q6)$ is a bistochastic matrix.

For the case $n > 4$, it suffices to reconsider the profiles such that for the first four agents, the profiles Qt be the same at each $t \in \{1, 2, \dots, 6\}$ which is extended by strict preference relation over all remaining objects $o_i \in A \backslash \{a, b, c, d\}$ for $i \in \{5, 6, \dots, n\}$. For agents $i \geq 5$, the preference orders over objects correspond to their index:

$$\succ_i : \{a\ b\ c\ d\ \dots\ k_i\}, o_{i+1}, \dots, o_n.$$

Therefore, due to ordinal efficiency we have $\varphi_{io_i}(Qt) = 1$. Then, similar arguments lead to the same contradiction. □

Robust ex-post Pareto efficiency is an intermediate notion of efficiency weaker than ordinal efficiency but stronger than ex-post

Pareto efficiency [10]. An assignment is robust ex-post Pareto efficient whenever *for all* of its lottery decomposition, *each* deterministic assignment in its support is Pareto efficient. An assignment is robust ex-post Pareto efficient whenever each deterministic assignment in support of any of its ex-post decomposition is Pareto efficient. In the proof, whenever we used ordinal efficiency, we could have used Lemma 3 of [10], to prove that the impossibility prevails even if we weaken the ordinal efficiency.

Lemma 3 of [10]. *Let* $N = \{i, j, h, k\}$ *be a set of agents and* $\succ$ *be a profile of preferences over objects* $A = \{a, b, c, d\}$*. Suppose for two arbitrary objects, without loss of generality say a and b, and two arbitrary agents, without loss of generality say i and j, we have* $a \succ_i b$ *and* $b \succ_j a$*. For every robust ex-post Pareto efficient random assignment P if* $p_{ib} > 0$ *and* $p_{ja} > 0$*, then either for object c,* $p_{hc} = p_{kc} = 0$*, or for object d,* $p_{hd} = p_{kd} = 0$.

Indeed, Ramezanian and Feizi [10] showed that, for more than four agents and objects, there is no strategy proof mechanism satisfying robust ex-post Pareto efficiency and equal division lower bound. Our result could be read as its special case for a uniform preference profile.

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Data availability

No data was used for the research described in the article.

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