Health and (other) Asset Holdings*

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Abstract

The empirical literature on the asset allocation and medical expenditures of U.S. households consistently shows that risky portfolio shares are increasing in both wealth and health whereas health investment shares are decreasing in these same variables. Despite this evidence, most of the existing models treat financial and health-related choices separately. This paper bridges this gap by proposing a tractable framework for the joint determination of optimal consumption, portfolio and health investments. We solve for the optimal rules in closed form and show that the model can theoretically reproduce the empirical facts.

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1 Introduction

Conventional wisdom suggests that how wealthy and healthy we are determines how we allocate resources between financial and health-related investments. The empirical literature on asset allocation and health investments confirms that this is indeed the case and has outlined three main regularities. First, risky portfolio shares are increasing in both wealth (e.g. Wachter and Yogo, 2008; Carroll, 2002) and health (Guiso et al., 1996; Rosen and Wu, 2004, among others). Second, health investment shares are decreasing in both wealth (e.g. Meer et al., 2003; DiMatteo, 2003) and health (e.g. Smith, 1999; Yogo, 2009). Third, labor income is increasing in health (e.g. Smith, 1999; Rosen and Wu, 2004), although the sensitivity to health tends to decline after retirement (Smith, 1999). Such strong interactions indicate that the analysis of financial and health investments should be undertaken as that of a joint decision process. Yet, aside from rare exceptions (e.g. Edwards, 2008; Yogo, 2009), the two are almost always analyzed separately in theoretical frameworks. At the risk of over-simplifying, health investment models tend to abstract from financial investment decisions whereas health related considerations are usually absent from portfolio choice models.

The objective of this paper is to bridge this gap by proposing a tractable model of joint consumption, portfolio and health investment decisions. Our modeling strategy innovates by combining two well-accepted —but otherwise segmented —models from the Financial and Health Economics literatures within a unified setup. More precisely, we start from a standard Merton (1971) asset allocation problem with IID returns and intermediate consumption utility, and append to this model a costly health investment decision à la Grossman (1972) where better health improves labor income as well as reduces the agent's mortality risk through a decrease in his death intensity. We solve for the optimal rules in closed form and show that the model can theoretically generate portfolios that increase in health and wealth, along with health investment shares that decrease in these same variables.

This model provides new insights on the complex relations between health and wealth statuses on the one hand and financial and health decisions on the other. For instance, it remains unclear through which channel(s) a reduction in health translates into increased health investment shares and more conservative portfolio positions (Rosen and Wu, 2004). First the agent may associate a deteriorated health condition with a shorter life expectancy. This could increase the desirability of health investments (assuming that life is valuable). Moreover a shorter available time to recuperate from adverse financial shocks might reduce the attractiveness of risky assets. Second, cross effects of health on the marginal utility of wealth could result in increased risk aversion at poor health levels, inducing the agent to reduce exposure to both financial and mortality risks. Third, deteriorated health could translate into lower labor income and consequently lower human wealth, affecting available resources for both financial and health-related investments. Finally, low health levels could affect the relative returns to financial and health investments if the agent is subject to decreasing returns in adjusting health levels and/or mortality risks. Contrary to segmented frameworks of financial and health investments, our model can offer guidance as to which (if any) of these explanations is relevant. In particular, our closed-form solutions emphasize the reduction in human wealth to explain more conservative portfolios, as well as the shorter life expectancy and changes in relative returns to explain the increase in health expenditures when health deteriorates.

As is well-known (e.g. Shepard and Zeckhauser, 1984; Rosen, 1988, among others), the modeling of preferences is not innocuous in an endogenous mortality setting such as ours, since preferences are no longer invariant to monotone transformations. In the standard time-additive framework of Yaari (1965) and Hakansson (1969) utility is computed as a sum of discounted period utilities up to the random time of death. These models associate death with a utility level of zero and, thus, imply that the utility of any consumption schedule must be compared to zero in order to determine whether the agent is better off living or dying. In particular, power utility with a relative risk aversion coefficient larger than one, as is often the case in the literature, implies negative welfare levels and, thus, yields the counterintuitive result that death is preferable to life.[1] Our

[1] To avoid this outcome, proposed solutions include adding a sufficiently large positive constant to utility (see Rosen, 1988; Becker et al., 2005; Hall and Jones, 2007, among others) and restricting the relative risk aversion to be smaller than one (Shepard and Zeckhauser, 1984). Another possible solution is to equate death with full depreciation of the health stock and impose Inada conditions on the flow utility of health, see Yogo (2009).

approach to this problem innovates by resorting to recursive preferences which allow to measure utility and consumption in the same metric (Epstein and Zin, 1989; Duffie and Epstein, 1992). With such preferences death is associated with a consumption level of zero whereas life corresponds to strictly positive consumption and, since preferences are monotonic, it follows that life is always preferred to death, regardless of parameter values. A second important feature of our preference specification is non-homotheticity. Iso-elastic utility, coupled with IID returns and a constant investment set, counterfactually implies that portfolio shares are invariant to wealth levels (Merton, 1971). Introducing subsistence consumption is a convenient way to generate an endogenous liquidity constraint and thus risky portfolio shares that profactually increase in financial wealth (Ameriks and Zeldes, 2004; Carroll, 2002; Wachter and Yogo, 2008).

In our model, health investment is subject to diminishing returns to scale and enters the agent's decision problem through two channels. First, better health increases labor income through, for example, less frequent sick leaves and/or better access to promotions for more assiduous workers. Second, better health lowers —but does not completely eliminate —mortality risk by reducing the agent's death intensity. In order to gain some intuition on the respective impact of these two channels, we first abstract from the latter to focus on the former only. In this restricted case, the agent's mortality risk is independent from his health status and this allows us to solve for the optimal rules in closed form. These optimal rules reveal that the risky portfolio shares are proportional to the ratio of total disposable wealth (i.e. financial wealth plus the net present value of labor income net of investment and subsistence) to financial wealth. Since total disposable wealth increases in the health level, it follows that the risky portfolio shares increase in health as well. They profactually increase in financial wealth as well for certain levels of health and wealth. We also find that the health investment level is proportional to the health level. Health investment shares thus profactually fall in financial wealth, but counterfactually increase as the agent's health improves.

Allowing for a health-dependent death intensity makes the agent's mortality risk endogenous and, unfortunately, implies that the model can no longer be solved in closed form. To circumvent this difficulty, we resort to a perturbation analysis which uses the explicit solution of the health independent mortality case as a benchmark for a first order expansion with respect to the parameter which governs the health dependence of the agent's death intensity.[2] While it only provides an approximation of the optimal rules, this perturbation method nevertheless presents several advantages. In particular, it gives us explicit expressions for the optimal rules and, thus, allows us to interpret the comparative statics of the model without relying on numerically calibrated parameters. We find that, up to a first order approximation, portfolio shares remain independent of mortality and, therefore, conclude that most of the impact of health on portfolios obtains through the labor income channel. By contrast, we find that endogenous mortality risk does have a first order impact on the optimal health investment. In particular, diminishing returns to health in the control of mortality risk reduce the attractiveness of health investments at certain health and wealth levels. The comparative statics thus show that, contrary to its exogenous mortality version, the model with endogenous mortality now has the potential to reproduce the observed co-movements.

Perhaps the three most closely related papers are those of Edwards (2008), Yogo (2009) and Hall and Jones (2007). Edwards (2008) does allow for asset selection in the presence of health risks, but he completely abstracts from horizon uncertainty and health-dependent income. Moreover, his distributional assumptions on health are quite different from ours since, in his model, sickness is purely exogenous and requires constant expenditures once incurred. As a result, Edwards (2008) can neither address the joint determination of portfolios and health investment as we do, nor study the impact of endogenous life expectancy on the agent's decisions.

Yogo (2009) is closer to us in that he also considers portfolio implications of a model à la Grossman (1972) where health investments are subject to diminishing returns to scale. However, his focus on the welfare gains of actuarially fair annuities is quite different from ours. Moreover, he models health as generating direct utility flows instead of our health-dependent labor income approach.

Hall and Jones (2007) also consider an endogenous mortality model with costly health investment and positive service flows of better health. However, they do not

[2]Examples of papers which apply similar perturbation methods in asset pricing include Kogan and Uppal (2002), Kogan (2001), Chan and Kogan (2002) and Ferretti and Trojani (2005) among others.

consider portfolio allocations and their focus on the time series of aggregate health spending and longevity is very different. Our preference specification also differs as we advocate non expected utility to guarantee that life is valuable, instead of adding a positive constant to utility in order to raise the relative value of life.

The rest of this paper is organized as follows. Following a discussion of the empirical properties of health and financial investments in Section 2, we outline the theoretical model in Section 3. The solution to the model is presented in Section 4. Finally, a conclusion in Section 5 reviews the main findings and suggests potential research agenda.

2 Stylized facts

In order to motivate the construction of our model, we first look at the empirical facts that characterize the cross-section of risky asset and health investment shares for a subset of American households. In particular, we outline in this section how health, wealth and age affect risky asset and health investment shares as well as labor income. Although we rely on summary statistics to describe the relevant co-movements, our conclusions are reasonably robust to reduced-form econometric evaluations with socio-economic covariates.[3]

2.1 Data sources and details

In our analysis, we rely on the Health and Retirement Study (HRS) data set, a widely-used survey of American individuals aged 51 and over.[4] Although not fully representative of the entire age distribution of US agents, the HRS data allows for useful comparisons with other health investment studies that also analyze the same data set. It provides socio-economic variables, alternative measures of health status and health-

[3]Specifically, regressing portfolios or health investment on health and wealth levels, with gender, race, age, or education co-variates through Tobit estimators yields qualitatively similar results.

[4]We use the RAND Center for the Study of Aging version of the HRS data files. This data set includes the cohort associated with the Study of Assets and Health Dynamics among the Oldest Old (AHEAD), the Children of the Depression (CODA), the War Babies and the initial HRS. See RAND Corporation (2008) for details. The summary statistics that we report are robust to alternative sampling weights including uniform weights, household weights, or the person-level analysis weights provided in the RAND HRS data set.

related expenditures, in addition to holdings of financial and non-financial assets. We focus on a cross-sectional perspective using the fifth wave of HRS (respondents in 2000), with our sample including both married and single, male and female agents.[5]

Regarding the definition of risky portfolios, we follow Rosen and Wu (2004); Berkowitz and Qiu (2006), in using a four-item classification of financial assets into safe assets (checking and saving accounts, money market funds, CD's, government savings bonds and T-bills), bonds (corporate, municipal and foreign bonds and bond funds), risky assets (stock and equity mutual funds) and retirement accounts (IRAs and Keoghs). The first three items are the result of deliberate economic decisions and represent direct asset holdings, whereas retirement funds capture more indirect portfolio choices. Consistent with similar applications in the empirical asset pricing literature, we define net financial wealth as the sum of safe, risky and bond holdings minus debts, and omit pension as well as social security funds. Risky portfolio shares are simply the percentage of net financial wealth held in risky assets.

We follow a large strand of the Health Economics literature in selecting the self-reported general health status to measure the health of agents in the data set.[6] This polytomous variable is categorized as poor, fair, good, very good, or excellent. Finally, the RAND HRS data reports the agents' total health expenditures and we will use this variable as our measure of health investment. It includes medical expenditures (doctor visits, outpatient surgery, hospital and nursing home, home health care, prescription drugs and special facilities), as well as out-of-pocket medical expenses (uninsured cost over the two previous years). The health investment share is the percentage of net financial wealth spent on total health expenditures.

Table 1 presents summary statistics of the key variables, including demographic characteristics, health-related measures and financial wealth holdings. We distinguish

[5]The summary statistics that we report below are robust to the choice of the wave in the HRS data. As pointed out by Yogo (2009) the fact that male and female, unmarried and married individuals maximize different objective functions could result in a source of heterogeneity. However, all of our results are robust to accounting for such effects.

[6]See Smith (1999); Deaton and Paxston (1998); Rosen and Wu (2004); Berkowitz and Qiu (2006); Love and Smith (2007); Yogo (2009) for examples. The validity of self-reported health has been discussed extensively. See discussions in Benítez-Silva and Ni (2008); Crossley and Kennedy (2002); Hurd and McGarry (1995). For robustness, we also considered other health-related measures, such as the self-reported health change or the number of diagnosed conditions with no qualitative effect on our results.

between younger (age less than 65) and older agents (age greater than or equal to 65) and associate the latter with retired individuals.[7] The average age is 58 for non-retired and 75 for retired, with a negligible difference between married and unmarried individuals. Females make up roughly 60% of the total sample of 19,571 observations. This percentage varies however with the marital status and is larger for singles.

[Insert Table 1 about here]

As can be seen from the second panel of Table 1 the self-reported health distribution is skewed towards good or better for younger agents, but displays a deterioration after age 65. The average health-related total expenditures increases from \$10,615 to \$15,862 for older agents. These expenditures are generally higher for singles. The cross-sectional distribution of medical, out-of-pocket, or total expenses is characterized by a long upper tail with much lower median values (French and Jones, 2004).

For those agents with positive financial wealth, we notice that almost all (85%) hold safe assets. These assets constitute by far the largest share of financial wealth and increase in the post-retirement period from 57 to 65%.[8] In comparison, only 9% hold corporate and government bonds corresponding to a meager 2% of financial wealth. Stock holdings are observed for roughly a third of individuals and represent 20% of wealth on average, with minor differences in the pre- and post-retirement periods. Finally, debt remains sizable for many agents and falls noticeably after retirement.

2.2 Relevant co-movements

Tables 2 (non-retired) and 3 (retired) categorize sample statistics in terms of increasing health levels and increasing financial wealth quintiles for the means of the net financial wealth, the probability of holding risky assets, the risky share and the median of the health investment shares.[9] Overall, both tables convey the same messages.

[7] Our results are robust to using the self-reported retirement status available in HRS instead of a definition based on age.

[8] The interpretation of this figure may be misleading since almost all individuals in our sample have Social Security wealth, which can be perceived as a safe asset (Rosen and Wu, 2004).

[9] We select the median for the heath investment share instead of the mean. Contrary to the risky probabilities and portfolio shares, the health investment share is not restricted to lie on the unit interval. As such, it is more sensitive to extreme measures.

[Insert Tables 2 and 3 about here]

First, consistent with similar findings by Michaud and van Soest (2008); Meer et al. (2003); Adams et al. (2003), we notice relatively minor variations in financial wealth as health changes. This can be interpreted as indicating that financial and human wealth are segmented measures. Put differently, contemporary health determines total wealth through its impact on human and not on financial wealth. Second, we notice that both the probability of holding risky assets and the risky asset shares increase sharply in financial wealth. Similar findings have been highlighted by Wachter and Yogo (2008); Guiso et al. (1996); Carroll (2002). Third, consistent with Guiso et al. (1996); Rosen and Wu (2004); Coile and Milligan (2006); Berkowitz and Qiu (2006); Goldman and Maestas (2005); Yogo (2009), there exists a positive correlation between health and risky asset holdings.[10] As the health level improves, both the probability of holding risky assets and the risky asset shares increase, especially for intermediate health level. Fourth, the health investment shares decrease when either financial wealth increases or health improves. Moreover, this finding is robust to the type of health investment under consideration, whether expenditures include insured spending (as in total expenditures) or uninsured spending (out-of-pocket expenditures). Similar findings are discussed in Meer et al. (2003); DiMatteo (2003) (decreasing in wealth) and in Smith (1999); Yogo (2009) (decreasing in health).

Table 4 next presents descriptive statistics in function of health and age when wealth is fixed at the third quintile.[11] We notice that both the probability of risky asset holdings and their portfolio shares fall at retirement, whereas the health investment shares tend to increase. However, conditional upon a given health level, the statistics show that both the risky asset holdings and the health investment shares are relatively independent of age in the post-retirement phase. The latter presents a flat pattern in the first 20 years following retirement but tends to increase over 85, probably reflecting the explosion in medical expenditures in the last period of life. Age-independent risky portfolios can

[10] Kochar (2004) finds similar health-portfolio links in developing countries. See however Love and Smith (2007); Fan and Zhao (2009) for a more reserved interpretation which accounts for unobserved heterogeneity.

[11] The results are relatively insensitive to the choice of the wealth quintile level.

also be found using the HRS data reported in Coile and Milligan (2006) to construct a measure of financial wealth that is similar to ours. Wachter and Yogo (2008); Gomes and Michælides (2005); Ameriks and Zeldes (2004); Rosen and Wu (2004); Poterba (2001) also document flat age profiles for risky portfolios. The initially flat profile for health investment shares is also consistent with weak age effects in international comparisons in health care expenditure shares once proximity to death is taken into account (Zweifel et al., 2004; Gerdtham and Jönsson, 2000; Felder et al., 2000).

[Insert Table 4 about here]

Finally, we are interested in measuring the impact of health on revenues. For this purpose, we regress total income on health in Table 5 in order to gauge its impact.[12] Overall, regardless of the type of estimator and/or the set of explanatory variables, the regressions convey the same message: income depends positively on the health level and this effect is larger in the pre- than in the post-retirement phase. French (2005); Smith (1999); Deaton and Paxston (1998); Bodie et al. (1992) also document the positive relation between health and labor income using American data whereas Kochar (2004) provides further evidence of strong health effects on income in developing economies. It can be explained by lower absenteeism for healthier workers, with potential implications for promotion decisions. The positive post-retirement health gradient highlights the fact that certain elders may still find it convenient to continue working after 65. The lower health gradient and higher constant term after that age reflects the increased importance of pension and other non-wage income in the post-retirement phase.

[Insert Table 5 about here]

To summarize, the analysis of the HRS data set reveals the following empirical regularities: (i) the contemporary financial wealth is independent of contemporary health level, (ii) the labor income depends positively on the health level, with a stronger effect in the pre- than in the post-retirement phase, (iii) both the risky portfolio and the health investment shares are relatively insensitive to age in the post-retirement phase.

[12] We measure total income as the sum of earnings (wages/salary), capital income, pensions or annuities, Social Security benefits as well as unemployment and other government transfers.

We have also shown that (iv) the risky portfolio shares of financial wealth increase in both health and in financial wealth. Conversely, (v) the health investment shares of financial wealth decrease in both health and in financial wealth. In the next section, our theoretical model will be constructed by assuming the first three regularities and will aim at reproducing the last two findings.

3 The model

This section describes an economic environment in which the agent has preferences over lifetime consumption plans in the presence of partially controllable mortality.

3.1 Survival and health dynamics

Following Ehrlich (2000); Ehrlich and Yin (2005) and Hall and Jones (2007) among others, we model the agent's mortality as a partially endogenous Poisson process whose intensity depends on the agent's health status. Specifically, we assume that the agent's death intensity is given by

$$\lim_{s\to 0}\frac{1}{s}P_t\big[t<\tau\leq t+s\big]=\lambda(H_t)=\lambda_0+\frac{\lambda_1}{H_t^{\xi}} \tag{1}$$

for some nonnegative constants λ_0, λ_1 and ξ, where τ is the random duration of the agent's lifetime or, equivalently, the agent's age of death, the nonnegative process H represents the agent's health status and $P_t(\cdot)$ is a conditional probability. The fact that the intensity function is decreasing in health ensures that the agent's survival probability:

$$P_0[\tau>t]=E_0\left[e^{-\int_0^t\lambda(H_s)\mathrm{d}s}\right] \tag{2}$$

is monotone increasing in the health status. As the agent's health status improves, his death intensity decreases but it never falls below the level λ_0. Intuitively, an agent can increase his survival probability by investing in his health and still die from an exogenous shock, such as an accident, or an illness that does not depend on controllable health (e.g. certain types of cancer). Alternatively, this incompressible part of the intensity can be interpreted as an endowed death probability that is determined by natural and/or biological factors.

The specification of the survival probability in equation (2) is closely related to that proposed by Ehrlich (2000) or by Ehrlich and Yin (2005). However, we differ along two dimensions. First, we make the incompressible part of the death intensity constant, rather than age-varying.[13] Second, we let the endogenous part of the death intensity be a function of the current health status rather than of the agent's health investment. This assumption implies that the agent cannot fully solve contemporaneous health-related problems by investing large amounts in times of need and, hence, reflects the path dependence of health-related decisions.

In the spirit of Grossman (1972), we model the agent's health status as a locally deterministic process whose growth/decay rate is a function of current health investment and health status. More precisely, we assume that the agent's health status evolves according to

$$\mathrm{d}H_t = \left(I_t^\alpha H_t^{1-\alpha} - \delta H_t\right)\mathrm{d}t, \qquad H_0 > 0, \tag{3}$$

for some constants $\alpha \in (0,1), \delta \geq 0$, where the nonnegative process I represents his health investment.[14]

The above dynamics specify that the agent's health status depreciates at a rate δ and that gross health investment $I^\alpha H^{1-\alpha}$ displays constant returns to scale. This last assumption implies that the growth rate of the agent's health status has decreasing returns in the investment to health ratio. Hence, a given health investment will have a larger impact on the growth rate of health when the agent is sick, than when he is healthy.[15] The constraint that health investment cannot be negative is standard

[13] This assumption can be relaxed by letting the incompressible part of the intensity be a function of the agent's age. Our results still hold in this more general setup but we maintain the assumption that λ_0 is constant for simplicity.

[14] The assumption of a constant δ is again made for simplicity but can be relaxed in favor of an age-dependent depreciation rate, with our main results still holding.

[15] Similar decreasing returns to health investments can be found in Ehrlich and Chuma (1990) and Ehrlich (2000); Ehrlich and Yin (2005). An equivalent interpretation of equation (3) is that the agent's is endowed with a health production function that is linear in gross health investment $I_g = I^\alpha H^{1-\alpha}$ but faces convex health adjustment costs given by $I = I_g^{1/\alpha} H^{1-1/\alpha}$.

in the Health Economics literature.[16] It reflects the irreversibility of health related expenditures and the fact that health is not a traded asset.

Our assumption of a (locally) deterministic process in (3) implies that we explicitly abstract from instantaneous health shocks. This hypothesis may appear restrictive, given that unanticipated changes in health have been shown to have strong impacts on both financial investments and health expenditures (Rosen and Wu, 2004; Smith, 1999) and should ideally be accounted for. Nonetheless, our decision to abstract from health shocks is motivated by three elements.

First, a deterministic process for $\mathrm{d}H_t$ allows us to derive closed-form solutions that are unattainable otherwise. Undiversifiable mortality risk paired with a health asset that cannot be bought and sold freely implies that our setup is characterized by incompleteness. Such incomplete-markets frameworks are notoriously difficult to solve analytically, even in settings that are much simplified compared to ours. Nevertheless, in Section 3.5 below, the Poisson assumption will be shown to imply an iso-morphism with a complete market and endogenous discounting problem whose solution can be obtained. Appending health risk to mortality risk would unfortunately break that iso-morphism. Although numerical solution would then remain possible, they lose the tractability offered by closed-form optimal rules.

Second, the locally deterministic health assumption does follow a long tradition in Health Economics of abstracting from minor health shocks when studying optimal health investment in the presence of partially endogenous mortality.[17] This hypothesis can be understood as ruling out inconsequential health shocks (e.g. suffering from a headache or a cold) that are non-life threatening and/or have limited income implications when focusing on mortality risk. Instead, uncontrollable mortal health shocks are subsumed in the λ_0 term, whereas controllable life-threatening shocks are captured by the $\lambda_1 H_t^{-\xi}$ term in (1).[18]

[16] See for example Grossman (1972); Ehrlich and Chuma (1990); Chang (1996); Picone et al. (1998); Ehrlich (2000); Edwards (2008); Hall and Jones (2007).

[17] See Grossman (1972); Ehrlich and Chuma (1990); Ehrlich (2000); Chang (2005) and Hall and Jones (2007) among others.

[18] Note that if the agent forgoes the possibility of health investment, then our specification of the intensity function conforms with Gompertz' law according to which human mortality rates increase exponentially with age. See Wetterstrand (1981) and Leung (1994) among others for details.

Finally, it should be kept in mind that a locally deterministic process in (3) does *not* imply a non-stochastic process for health along the optimal path. Indeed, as will become clear shortly, optimal health investments will remain subject to stochastic movements in wealth caused by financial shocks.[19] Although instantaneous movements in health are perfectly forecastable, their long-term evolution is not, such that health, income and the intensity of mortality risk all remain stochastic in the long run.

3.2 Income dynamics

To introduce life cycle considerations into the model, we identify time t with the agent's age and assume that, if alive, the agent is employed ($i = e$) up to some fixed age T and retired thereafter ($i = r$).

The agent's flow rate of income/pension per unit of time depends on his employment status and is defined by

$$Y_t = Y(t, H_t) = 1_{\{T>t\}} Y_t^{\mathrm{e}} + 1_{\{T \leq t\}} Y_t^{\mathrm{r}}. \tag{4}$$

In this equation, the nonnegative processes Y^i are related to the agent's employment status and are given by

$$Y_t^i = Y^i(H_t) = y^i + \beta^i H_t, \tag{5}$$

for some nonnegative constants y^{e}, y^{r}, β^{e} and β^{r}.[20]

Since β^i is nonnegative, the above specification implies that the agent's income increases with his health status. A natural interpretation is that employers offer higher wages to agents who are in better health and thus less subject to be absent from work. Equivalently, a healthier agent tends to miss less workdays and, hence, receives higher labor income. Since β^{r} can be different from zero, the income specification of (5) allows for health dependent post-retirement income. As explained in Section 2 this potential dependence is due to the fact that some agents continue working even after

[19] As equation (38) below makes it clear, optimal health investment remains wealth-dependent and therefore subject to financial shocks.

[20] The model can be generalized to allow for age varying, stochastic intercepts y^{e} and y^{r}. We restrict our analysis to constant intercepts for the sake of simplicity and in order to focus on the effect of health on the agent's portfolio, consumption and health investment decisions.

retirement. Consistent with the empirical evidence also presented in Section 2, we allow for differences in pre and post-retirement health elasticities of income.

3.3 Preferences

Starting with the seminal contributions of Yaari (1965) and Hakansson (1969), the standard way of specifying preferences in the presence of mortality risk has been to assume that the continuation utility, to an agent of age t, of a lifetime consumption plan c is given by

$$U_t = 1_{\{\tau > t\}} E_t \left[\int_t^\tau e^{-\rho(s-t)} u(c_s) \mathrm{d}s \right] \tag{6}$$

for some subjective rate of time preference $\rho \geq 0$ and some concave period utility function $u(\cdot)$ satisfying standard regularity conditions.[21]

As pointed out by Shepard and Zeckhauser (1984) and Rosen (1988), the level of the period utility function has important implications in such a specification since adding a nonnegative constant to $u(\cdot)$ raises the value that the agent places on longevity relative to consumption. Put differently, in the presence of an uncertain horizon, preferences are not invariant to affine and more generally monotone transformations as they are in the standard setting where the horizon is fixed and non random. This anomaly is due to the fact that equation (6) attributes a utility of zero to death and, hence, implies that the utility of any consumption schedule must be compared to zero in order to determine whether the agent is better off living or dying. In particular, if the period utility is taken to be of the HARA type

$$u(c) = \frac{(c-a)^{1-\gamma}}{1-\gamma}, \quad c \geq a, \tag{7}$$

for some nonnegative constants a and $\gamma \neq 1$ then the agent's preferences towards mortality depend on whether the risk aversion parameter γ is smaller or larger than unity. In the former case, the utility of any consumption is positive and it follows

[21] See for example Richard (1975); Shepard and Zeckhauser (1984); Rosen (1988); Ehrlich and Chuma (1990); Ehrlich (2000); Becker et al. (2005); Edwards (2008); Hall and Jones (2007) and Yogo (2009) among others.

that the agent always prefers life to death. On the contrary, if $\gamma > 1$, as is often found in empirical studies, then the utility of any consumption plan is negative and it follows that the agent counterfactually prefers death to life irrespective of his current consumption level. Preference for death over life is not only unrepresentative, but also implies that the agent may select negative levels for I_t in order to accelerate the timing of death. Technically, a negative I_t is not compatible with decreasing returns, such as encompassed by our Cobb-Douglas specification (3) for arbitrary levels of α. More importantly, although negative net investments $\mathrm{d}H_t$ are admissible and feasible, negative gross investments are not; we may adjust health levels downwards by letting it depreciate but certainly cannot sell our health continuously in markets for medical, ethical, and market incompleteness reasons.

To obtain sensible results, most of the authors who use the preference specification in equation (6) thus restrict their study to nonnegative period utility functions for which any life is always preferred to death. Following this approach, Rosen (1988); Becker et al. (2005) and Hall and Jones (2007) among others specify a period utility of the form

$$v(c) = b + u(c)$$

where the constant b is chosen in such a way as to guarantee that the function $v(\cdot)$ is nonnegative. Unfortunately, such a constant exists only if the utility function is bounded from below and it follows that this approach cannot be used to accommodate the case where the period utility function is given by equation (7) for some risk aversion parameter γ greater than unity.

Motivated by the above discussion and in particular by the fact that the specification in equation (6) cannot reconcile an empirically plausible level of risk aversion with a sensible behavior towards mortality, we will forgo the time additive specification. Instead we assume that the agent has recursive preferences of the type proposed by Kreps and Porteus (1979); Epstein and Zin (1989) and Weil (1989) in discrete time; and by Duffie and Epstein (1992); Schroder and Skiadas (1999) in continuous time. As we show below, when generalized to include a random horizon, such preferences allow to remedy the above deficiencies of time additive preferences while maintaining a tractable setup.

Let $U_t = U_t(c)$ denote for the continuation utility of a consumption plan c of an agent of age t. Introducing a random horizon into the Kreps–Porteus preference specification of Duffie and Epstein (1992), we assume that U satisfies the recursive integral equation

$$U_t = 1_{\{\tau > t\}} E_t \left[\int_t^\tau \left(f(c_s, U_s) - \frac{\gamma}{2U_s} |\sigma_s(U)|^2 \right) \mathrm{d}s \right] \tag{8}$$

where $\sigma(U)$ measures the instantaneous volatility of the continuation utility,[22] and $f(\cdot)$ is the intertemporal aggregator defined by

$$f(c, v) = \frac{v\rho}{1 - 1/\varepsilon} \left[\left(\frac{c - a}{v} \right)^{1 - 1/\varepsilon} - 1 \right]. \tag{9}$$

In the above equation the nonnegative constants ρ, a, ε and γ represent, respectively, the agent's subjective rate of time preferences, his subsistence consumption level, his elasticity of intertemporal substitution and his risk aversion over static gambles.[23]

As pointed out by Duffie and Epstein (1992), the fact that the aggregator function $f(\cdot)$ is homogenous of degree one with respect to $(c - a, v)$ implies that the continuation utility is homogenous of degree one with respect to $(c - a)$ and it follows that the agent's utility is measured in the same unit as excess consumption. In particular, the agent's utility is always nonnegative and, since death is associated with zero utility, it follows that the agent sees his own mortality as detrimental irrespective of whether his risk aversion coefficient, γ, is smaller or larger than unity.

The specification of the continuation utility in equation (8) captures non homothetic preferences for $a \neq 0$ and iso-elastic preferences otherwise. Non homotheticity is often advocated as a mean to generate risky portfolio shares that increase in wealth.[24]

[22] More precisely, $\sigma_t(U) = \mathrm{d}\langle U, Z\rangle_t / \mathrm{d}t$ measures the instantaneous covariance of the continuation utility process with the Brownian motion driving market returns.

[23] In the absence of mortality risk, our specification of the agent's preferences boils down to time additive power utility under the usual parametric restriction that $\varepsilon = 1/\gamma$.

[24] For example, Aït-Sahalia et al. (2004) and Wachter and Yogo (2008) assume that agents have non separable preferences in consumption and luxury goods. Campbell and Cochrane (1999) extend non separability to consumption and a habit stock, whereas Carroll (2002) adds a HARA bequest to a CRRA period utility function.

Our specification of the agent's continuation utility does not include bequest motives. The reason for this restriction is mainly technical: adding a bequest to our setup with endogenous mortality considerably complicates the analysis and prevents us from deriving closed form solutions even in the case where the death intensity is independent from the health status.[25]

3.4 Financial market and budget constraint

The financial market is frictionless and consists of two continuously traded securities. The first of these securities is a locally riskless bond whose time t price is given by

$$S_t^0 = e^{rt} \tag{10}$$

for some constant rate of interest $r > 0$. The second security is a risky stock and we assume that its price evolves according to

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}Z_t, \qquad S_0 > 0, \tag{11}$$

for some constant growth rate $\mu \geq r$ and constant volatility $\sigma > 0$ where the process Z is a one dimensional standard Brownian motion.[26]

Let w denote the agent's financial wealth at birth and π denote the fraction of his wealth that the agent invests in the stock. Under the usual self-financing requirement the agent's financial wealth evolves according to

$$\mathrm{d}W_t = (rW_t + Y_t - I_t - c_t)\mathrm{d}t + W_t \pi_t \sigma(\mathrm{d}Z_t + \theta \mathrm{d}t), \tag{12}$$

[25] Bequest motives are admittedly relevant in an endogenous mortality setting such as ours. For instance, Love et al. (2009) emphasize their role (in addition to uncertain lifetime and medical expenditures) in explaining observed life cycle for annualized comprehensive wealth in HRS data, notably the fact that it is *rising*, rather than declining as life horizon shortens. This being said, the panel data evidence is mixed concerning the role of bequest motives in explaining the behavior of retired agents. For example, Hurd (2002) finds no clear evidence of a bequest motive behind savings decisions, whereas Hurd (1987) finds no differences in the saving behavior of the elderly who have children compared to those who don't. Similarly, Hurd (1989) finds no significant difference in the net present value of expected bequests of elderly agents when there is a bequest motive compared to when there is none. Hurd and Smith (2002) document that the importance of post-retirement dis-saving implies low bequests.

[26] The assumption of a single risky security is made purely for expositional simplicity. Under the assumption of a constant investment opportunity set, the model can be easily generalized to include multiple risky securities.

subject to $W_0 = w$ where the constant $\theta = (\mu - r)/\sigma$ denotes the market price of risk induced by the prices of the traded assets.

3.5 The decision problem

In the first part of his life the agent is employed and has an horizon which is the minimum of τ and T. This implies that, prior to retirement, the indirect utility function and the optimal rules are functions of the agent's age, t, financial wealth, W and health status, H. In contrast, the indirect utility and optimal rules become age independent after retirement since our model does not distinguish age among retired agents. Taking this into account, we will from now on write all quantities as functions of (t, W, H) with the understanding that the age dependence terminates after retirement.

The agent's problem consists in choosing a portfolio, consumption and health investment strategy to maximize his lifetime utility. This implies that the indirect utility of the agent at age t is given by

$$V(t, W_t, H_t) = \sup_{(\pi,c,I)} U_t(c), \tag{13}$$

subject to equations (1), (3), (4) and (12), where $U(c)$ is the continuation utility associated with the consumption plan c through equation (8).

Our formulation of the agent's decision problem parallels the widely used approach of specifying a health dependent utility and omitting health dependent income.[27] To see this, abstract from the dependence of the agent's income on his employment status by letting $y^i = y$, $\beta^i = \beta$ in equation (5) and set $x = c - \beta H$. Straightforward manipulations reveal that

$$V(t, W_t, H_t) = \sup_{(\pi,x,I)} U_t(x + \beta H),$$

subject to equation (3) and the modified budget constraint

$$dW_t = (rW_t + y - I_t - x_t)\mathrm{d}t + W_t\pi_t\sigma(\mathrm{d}Z_t + \theta\mathrm{d}t).$$

[27] Examples include Grossman (1972), Ehrlich and Chuma (1990), Picone et al. (1998), Ehrlich (2000), Edwards (2008), Hall and Jones (2007) and Yogo (2009).

Hence, solving the agent's optimization problem using the non separable, health dependent intertemporal aggregator

$$g(c, H, v) = f(c + \beta H, v),$$

and abstracting from health dependent income is equivalent to solving our formulation of the agent's problem with health independent preferences and health dependent income.[28]

Since the uncertain duration of his lifetime cannot be hedged by trading in the market, the agent faces an incomplete market. However, under the assumption of Poisson mortality, the agent's problem can be conveniently transformed into an equivalent infinite horizon problem with endogenous discounting and complete financial markets. Indeed, using equation (2) and the law of iterated expectations we can write continuation utility as

$$U_t(c) = 1_{\{\tau > t\}} U_t^o$$

where the process $U^o = U^o(c)$ satisfies the infinite horizon, recursive integral equation given by

$$U_t^o = E_t\left[\int_t^\infty e^{-\int_t^s \lambda(H_u)\mathrm{d}u}\left(f(c_s, U_s^o) - \frac{\gamma}{2U_s^o}|\sigma_s(U^o)|^2\right)\mathrm{d}s\right]. \tag{14}$$

This formulation brings to light the two channels through which health enters the agent's problem. First, health can be interpreted as a durable good which generates service flows through the agent's income. Second, health determines the endogenous rate at which the agent discounts future consumption and continuation utilities. We show in the next sections that both the service flow and the discounting channels are necessary to generate the observed patterns of portfolio and health investment shares.

[28] Relaxing the employment status restrictions $y^i = y$, $\beta^i = \beta$ implies that the agent has an age and health dependent intertemporal aggregator whose health elasticity depends on the agent's age/employment status through the constants β^{e} and β^{r}.

4 Optimal rules

This section derives the solution to our model. As explained above, health enters the agent's problem through two channels: the mortality channel and the income channel. In order to gain some intuition on the respective impact of these two channels we start by analyzing the case where the agent's mortality is independent from his health status and, thus, cannot be controlled. We then turn to the analysis of the general case where the agent's health status influences both his mortality rate and his income.

4.1 Health independent mortality

When $\lambda_1 = 0$ the agent's mortality rate is constant and, as a result, his objective function is independent from his health. In conjunction with the fact that financial markets are complete, this implies that the problem can be solved in two steps as in the human capital model of Bodie et al. (1992). First, compute the optimal health investment by maximizing the present value of the agent's disposable income. Second, obtain the optimal portfolio and optimal consumption plan by solving the problem of an hypothetical agent who has no income and no subsistence consumption, but whose financial wealth is equal to the total disposable wealth of the original agent.

Since financial markets are complete, the present value of the agent's disposable income is given by

$$P(t, H_t) = \sup_{I \geq 0} E_t \left[\int_t^\infty \xi_{t,s} \big(Y(s, H_s) - a - I_s \big) ds \right] \tag{15}$$

subject to equation (3), where the nonnegative process

$$\xi_t = \exp\left(-rt - \theta Z_t - \frac{1}{2}\theta^2 t \right) \tag{16}$$

is the stochastic discount factor induced by the price of the traded securities and we have set $\xi_{t,s} \equiv \xi_s / \xi_t$. The following proposition derives an analytical solution for the present value of the agent's disposable income and for the optimal health investment.

Proposition 1 *Let $\lambda_1 = 0$ and assume that*

$$\beta^{\mathrm{r}} < (r+\delta)^{\frac{1}{\alpha}}. \tag{17}$$

Then the present value of the agent's disposable income and the optimal health investment strategy are given by

$$P(t,H) = B(t)H + C(t), \tag{18}$$

$$I_0(t,H) = H\big(\alpha P_H(t,H)\big)^{\frac{1}{1-\alpha}} = H\big(\alpha B(t)\big)^{\frac{1}{1-\alpha}}, \tag{19}$$

where a subscript denotes a partial derivative,

$$C(t) = \int_t^\infty e^{-r(s-t)}\big(Y(s,0) - a\big)ds, \tag{20}$$

and $B(\cdot)$ is a nonnegative function of the agent's age which is defined in the appendix.

The restriction imposed by equation (17) is a transversality condition which guarantees that the present value of the agent's disposable income is finite. Since the constants r, δ, β^{r} and α are all nonnegative by assumption, this restriction effectively limits the convexity of the agent's health adjustment technology and/or sets a minimal health depreciation rate to compensate for the sensitivity of the agent's income to his health status.

The present value of the agent's disposable income can be decomposed in two parts. First $C(\cdot)$ gives the present value of the health independent part of the agent's lifetime income net of subsistence consumption expenditures. Second, the product $B(\cdot)H$ gives the present value of the health dependent part of the agent's lifetime income net of optimal health investments, i.e. $B(\cdot)$ measures the shadow price (i.e. marginal q) of health. The fact that both this present value and the optimal investment are linear in the agent's health status follows from the linearity of the agent's income and the Cobb-Douglas specification of the health adjustment technology.[29]

[29] See Uzawa (1969), Hayashi (1982) and Abel and Eberly (1994) among others for similar results in the investment literature where this property is referred to as the equivalence between marginal and average q.

Having computed the present value of the agent's disposable income and the optimal health investment strategy, we now turn to the determination of the optimal portfolio and consumption strategy. Let

$$N_t = N(t, W_t, H_t) = W_t + P(t, H_t) \tag{21}$$

denote the agent's total wealth net of subsistence consumption expenditures. Using the result of Proposition 1 in conjunction with equation (12) we obtain that the agent's total disposable wealth evolves according to

$$\mathrm{d}N_t = (rN_t - x_t)\mathrm{d}t + N_t \nu_t \sigma(\mathrm{d}Z_t + \theta \mathrm{d}t), \tag{22}$$

where the modified portfolio and consumption strategy (ν, x) is related to the original one through $\nu = (W/N)\pi$ and $x = c - a$. This implies that the agent's indirect utility function can be written as

$$V_0(t, W, H) = F(t, N(t, W, H)) \tag{23}$$

where the function $F(\cdot)$ is defined by

$$F(t, N_t) = \sup_{(x,\nu)} U_t(x + a) \tag{24}$$

subject to the dynamic budget constraint in equation (22). The solution to this modified portfolio and consumption choice problem with recursive utility and IID returns is well-known and can be found in Svensson (1989) and Obstfeld (1994) among others. Using this solution to construct the agent's optimal strategy gives the following theorem.

Theorem 1 *Let* $\lambda_1 = 0$ *and assume that equation* (17) *and*

$$A = \varepsilon\rho + (1 - \varepsilon)\left(r - \lambda_0 + \frac{1}{2\gamma}\theta^2\right) > \left(r - \lambda_0 + \frac{1}{\gamma}\theta^2\right)^+ \tag{25}$$

hold. The indirect utility function of a living agent of age t is

$$V_0(t, W, H) = \rho(A/\rho)^{\frac{1}{1-\varepsilon}} N(t, W, H), \tag{26}$$

and generates the optimal portfolio, optimal consumption and optimal health investment strategy given by

$$\pi_0(t, W, H) = \frac{\theta}{\gamma\sigma W} N(t, W, H), \tag{27}$$

$$c_0(t, W, H) = a + A\, N(t, W, H), \tag{28}$$

and equation (19). Here, $N(\cdot)$ represents the agent's total disposable wealth as defined by equation (21) and $B(\cdot)$ is defined as in Proposition 1.

As explained in Smith (1996), the restriction imposed by equation (25) serves two purposes. On the one hand, it guarantees that the agent's marginal propensity to consume, A, is strictly positive and, hence, that the optimal consumption plan c_0 is feasible. On the other hand, it insures that the indirect utility indeed coincides with the continuation utility of the optimal consumption plan c_0 as defined by equation (8) and, thus, is a transversality condition. The exact form of the restriction is entirely standard (e.g. Svensson, 1989; Smith, 1996), except for the presence of the mortality rate λ_0 which reflects the impact of mortality on the agent's consumption decisions.

It is of interest to contrast the optimal rules in Theorem 1 with those obtained under separable VNM utility. To do so, assume that $\varepsilon = 1/\gamma$ so that, absent mortality risk, the agent has time additive preferences. In this case equation (25) gives

$$A^o = A\big|_{\varepsilon=1/\gamma} = \frac{\rho + \lambda_0}{\gamma} + \left(1 - \frac{1}{\gamma}\right)\left(r + \frac{1}{2\gamma}\theta^2\right) - \lambda_0$$

which is different from the marginal propensity to consume $A^{\mathrm{vnm}} = A^o + \lambda_0$ that would be obtained under the time additive HARA preference specification of equations (6) and (7). The reason for this difference is that we take into account the impact of mortality on the certainty equivalent of future consumption and not on its utility. This allows us to guarantee that mortality is detrimental to the agent.

Equation (25) reveals that introducing exogenous mortality is equivalent to decreasing the interest rate and thereby leads to two conflicting effects. First, a lower interest rate implies that more resources are needed to fund a given amount of future consumption and, thus, encourages the agent to consume less today in order to maintain the same level of future consumption. Second, a decrease in the interest rate implies that current consumption is less costly relative to future consumption and, thus, leads to consume more today through a substitution effect. When the elasticity of intertemporal substitution, ε, is smaller than unity, the first effect dominates and the agent reduces his marginal propensity to consume in response to exogenous mortality. Conversely, when the elasticity of intertemporal substitution is greater than unity, the substitution effect dominates and the agent increases his marginal propensity to consume in response to an increase in his mortality rate. Exact cancelation of the two effects occurs if $\varepsilon = 1$ in which case mortality has no impact on consumption.

Equations (26) and (27) show that, when the agent cannot control it, mortality risk decreases his indirect utility function but has no impact on the optimal portfolio. The intuition behind these results is straightforward. Indeed, an increase in the mortality rate implies a decrease in the utility of any given consumption schedule and, hence, a decrease in the indirect utility. On the other hand, the property that the optimal portfolio is independent from the agent's mortality rate follows from the well-known fact that, in the absence of hedging demands, the optimal portfolio is independent of the agent's planning horizon, (see Richard, 1975, for details).

The share of the agent's wealth allocated to risky assets has the usual mean variance efficient form in that it is increasing in the market Sharpe ratio, θ/σ and decreasing in the risk aversion parameter, γ. However and in stark contrast to the standard case of homothetic preferences without human capital ($a = \beta^i = y^i = 0$), the optimal portfolio share is not constant but a function of the agent's age, wealth and health status. As the agent's health improves, the present value of his disposable income increases and his portfolio share increases.[30] To understand this effect it suffices to observe that the presence of income and a non zero subsistence consumption level generates an endogenous liquidity constraint which forces the agent to keep his financial wealth, W, high enough for his total disposable wealth, N, to be nonnegative. As the agent's health status improves, this constraint softens due to the increase in the present value of future income and, consequently, more risky assets can be held today.

The impact of the agent's financial wealth on his portfolio share depends crucially on whether the present value of his disposable income is positive or negative. Since the agent's health is deterministic along the optimal path, the present value of his disposable income is also non stochastic. In the spirit of Bodie et al. (1992), this implies that receiving this disposable income is equivalent to holding a position in the bond. When the present value of his disposable income is positive (negative), the agent implicitly holds a long (short) position in the bond and thus can invest more (less) in the risky asset than he would have absent human capital. As the agent's financial wealth increases, the absolute value of this implicit position in the bond decreases in percentage of financial wealth and, as a result, his portfolio share gets closer to the constant share $\theta/(\sigma\gamma)$ that he would have chosen in the absence of human capital and subsistence consumption. This implies that the agent's portfolio share increases as a function of wealth as long as the present value of his disposable income is negative and decreases otherwise.

In order for the present value of the agent's disposable income to be negative, as required to generate the observed patterns of portfolio shares, it must be the case that (i) his subsistence consumption level is high enough relative to the health independent part of his labor income in the sense that $C(t) \leq 0$ and (ii) that his health status is not too high. In particular, if the agent's subsistence consumption is equal to zero, i.e. if the agent has homothetic preferences, then the present value of his disposable income is positive at all times since and, as a result, his portfolio share is a decreasing function of financial wealth.

[30]Throughout this discussion we maintain the assumption that the agent's financial wealth is nonnegative. If this is not the case then the comparative statics of the optimal portfolio share with respect to financial wealth and health status are simply reversed due the mechanical effect of dividing by a negative quantity.

Since $\lambda_1 = 0$ the agent's mortality rate is purely exogenous and, as a result, so is the effective discount rate. Since consumption and investment are then separated decisions, exogenous discounting does not affect investment and mortality is irrelevant for health investment rules. From equation (19), the optimal health investment is proportional to the agent's health status but independent of his financial wealth. As a result, the health investment share

$$I_0^s(t, W, H) = \frac{I_0(t, H)}{W} = \frac{H}{W}\left(\alpha B(t)\right)^{\frac{1}{1-\alpha}} \tag{29}$$

decreases in wealth, as required to explain the data, but counterfactually increases in health. Equations (3) and (19) together imply that the optimal growth rate of health is deterministic and independent from the agent's health status. It follows that changes in the agent's health status have no impact on the returns to health investments. On the other hand, the income effect of health is still present and is the only driving force behind the comparative statics of health investment. As the agent's health status improves, the present value of his disposable income increases and, since returns are unaffected, the agent increases his health investment.

The restriction of the model studied in this section abstracts from the mortality reducing effects of health investments and only accounts for the impact of health through the agent's income. This specification allows to reproduce the fact that empirical portfolio shares increase in both health and financial wealth, with the latter property obtaining when the agent's human capital does not cover his subsistence consumption needs. In accordance with the data, the model also produces health investment shares that are decreasing in financial wealth. However, the model counterfactually predicts that healthier agents should invest more in their health. In the next section we reintroduce health dependent mortality in order to verify whether or not this additional channel of health dependence is able to explain the observed patterns of health investment without altering the favorable performance of the model with respect to portfolio.

4.2 Health dependent mortality

When $\lambda_1 \neq 0$ the agent's mortality rate depends on his health status. In this case, one can no longer determine the optimal heath investment policy independently of the optimal portfolio and consumption strategy since, as shown by equation (14), the agent's objective function now depends on his health status through the mortality rate's impact on the endogenous discount rate.

Using standard calculations (e.g. Duffie and Epstein, 1992) we obtain that the Hamilton-Jacobi-Bellman (HJB) equation associated with the agent's optimization problem is given by

$$\lambda(H)V = \max_{(\pi,c,I)} \left\{ L^{\pi,c,I}V + f(c,V) - \frac{\gamma}{2V}(\pi\sigma W V_W)^2 \right\} \tag{30}$$

where

$$\begin{aligned} L^{\pi,c,I} &= \partial_t + (H^{1-\alpha}I^\alpha - \delta H)\partial_H \\ &\quad + ((r + \pi\sigma\theta)W + Y - c - I)\partial_W + \frac{1}{2}(\pi\sigma W)\partial_{WW} \end{aligned}$$

is the differential operator associated to the agent's wealth and health status under the strategy (π, c, I). In the above equations, an alphabetical subscript indicates a partial derivative and we have omitted the dependence of the functions on the agent's age, wealth and health status for ease of notation.

Maximizing the right hand side of the HJB equation (30) with respect to π, c and I we find that the optimal rules can be computed as

$$\pi^* = \frac{\theta V V_W}{\sigma W(\gamma V_W^2 - V V_{WW})}, \tag{31}$$

$$c^* = a + V\left(\frac{\rho}{V_W}\right)^{\varepsilon}, \tag{32}$$

$$I^* = H\left(\frac{\alpha V_H}{V_W}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

Substituting these expressions into the HJB equation and simplifying the resulting expression gives a nonlinear partial differential equation for the agent's indirect utility.[31] Unfortunately, no closed form solution to this equation is known except for the case of health independent mortality which was solved in the previous section. Nonetheless, as

[31] For details see equation (54) in the appendix.

we now explain, we can use the explicit solution to this special case as a benchmark in order to obtain an approximate solution for the general case by expanding the indirect utility in powers of λ_1.

Denote by n the order of the expansion and let us expand the indirect utility function of a living agent as

$$V \approx V_0 + \lambda_1 V_1 + \cdots + \frac{1}{m!}\lambda_1^m V_m + \cdots \frac{1}{n!}\lambda_1^n V_n.$$

Here $V_0 = V_0(t, W, H)$ is the agent's indirect utility function in the case of health independent mortality and

$$V_m = V_m(t, W, H) = \frac{\partial^m}{\partial \lambda_1^m} V(t, W, H)\Big|_{\lambda_1=0}$$

is the m^{th} order correction induced by the presence of health dependent mortality. Substituting this approximation of the indirect utility function into the HJB equation and expanding the resulting equation in powers of λ_1 gives a sequence of partial differential equations which can be solved recursively starting from the known function $V_0(\cdot)$. Once the correction terms have been computed up to the desired order of expansion, one can obtain an approximation of the optimal portfolio, consumption and health investment by substituting the expansion of the indirect utility function into equations (31), (32) and (33) and expanding the resulting expressions in powers of λ_1.

In order to implement the above method it is necessary to select the accuracy of the approximation by fixing the number of terms to include in the expansion. Since the intensity parameter λ_1 is expected to be small, [32] we can be reasonably confident that the expansion method already delivers good approximations of the indirect utility function and optimal rules at the first order. While higher order approximations can also be computed, we will restrict ourselves to this first order solution because it allows for an intuitive analysis of the optimal rules.

[32]This conjecture is confirmed by preliminary structural estimation results which show that the value of λ_1 needed to fit the HRS data is of the order of 10^{-3} (Hugonnier et al., 2009).

Theorem 2 *Assume that equations (17), (25) and*

$$A + \xi \left((\alpha B(T))^{\frac{\alpha}{1-\alpha}} - \delta \right) > r - \lambda_0 + \frac{\theta^2}{\gamma} \geq 0 \tag{34}$$

hold where A and the function $B(\cdot)$ are defined as in Theorem 1. Up to a first order approximation, the agent's indirect utility is

$$V(t, W, H) = V_0(t, W, H) - \frac{\lambda_1}{H^\xi} \Delta(t) V_0(t, W, H) \tag{35}$$

and generates the approximate optimal portfolio, consumption and health investment strategy given by

$$\pi_1(t, W, H) = \pi_0(t, W, H), \tag{36}$$

$$c_1(t, W, H) = c_0(t, W, H) - \frac{\lambda_1}{H^\xi} \Delta(t)(1 - \varepsilon) A N(t, W, H), \tag{37}$$

$$I_1(t, W, H) = I_0(t, H) + \frac{\lambda_1}{H^\xi} \Delta(t)(\alpha B(t))^{\frac{\alpha}{1-\alpha}} \eta N(t, W, H). \tag{38}$$

In the above equations, $\Delta(\cdot)$ is a nonnegative function of the agent's age which is defined in the appendix, $\eta = \alpha\xi/(1-\alpha)$ and the functions $V_0(\cdot)$, $\pi_0(\cdot)$, $c_0(\cdot)$, $I_0(\cdot)$ and $N(\cdot)$ are defined as in Theorem 1.

The restriction imposed by equation (34) is a transversality condition which guarantees that the first order correction to the agent's indirect utility is well defined. Under this restriction, equation (35) and the nonnegativity of the functions $V_0(\cdot)$ and $\Delta(\cdot)$ imply that the presence of health dependent mortality reduces the agent's indirect utility. The intuition behind this result is straightforward. Indeed, adding a positive health dependent component to the mortality rate triggers a mechanical decrease in the survival probability and hence also a decrease in the indirect utility since the agent has a preference for life.

Compared to the case of health independent mortality, the presence of health dependent mortality decreases the agent's life expectancy and leads to two conflicting effects whose interaction determines the change in the agent's optimal consumption. First, an increase in the mortality intensity reduces the implicit value of the agent's income and, hence, incites him to decrease his consumption through a wealth effect. Second, a reduction in the agent's life expectancy reduces the need to save for the future and, thus, leads the agent to increase his consumption through an intertemporal substitution effect. When the elasticity of intertemporal substitution is smaller than unity, the wealth effect dominates and the agent decreases his consumption relative to the health independent mortality case. Conversely, when the elasticity of intertemporal substitution is greater than unity, the substitution effect dominates and the agent increases his consumption. Exact cancelation of the two effects occurs when $\varepsilon \equiv 1$ in which case mortality, be it health dependent or not, has no impact on the optimal consumption plan.

Equation (36) shows that, although it affects the repartition of the agent's wealth between financial and human capital, the presence of health dependent mortality has no first order effect on his optimal portfolio share. As in the case of health independent mortality, this is due to the fact that portfolio rules are insensitive to discounting considerations in the absence of dynamic hedging motives. Indeed, the first order portfolio is obtained by performing a perturbation of the agent's problem around the optimal rules associated with the case of health independent mortality. Since the optimal health status associated with that case evolves in a deterministic way, it follows that the investment opportunity set which is relevant to the determination of the first order rules is also deterministic and this implies that there is no first order correction to the agent's portfolio.

Since the constant η and the functions $B(\cdot)$, $\Delta(\cdot)$ and $N(\cdot)$ are nonnegative, equation (38) shows that health dependent mortality induces the agent to increase his health investment. The intuition behind this result is again straightforward. The presence of health dependent mortality mechanically increases the agent's death intensity and, thus, decreases his life expectancy compared to the case of health independent mortality. Since the agent has a preference for life, he responds to this decrease in life expectancy by increasing his health investment in order to improve his health status and, thus, partially mitigate the impact of health dependent mortality. As can be seen from the fact that the right hand side of equation (38) depends on the agent's wealth and health status, this in turn implies that health dependent mortality introduces substitution effects between health investment, consumption and financial investments at the optimum.

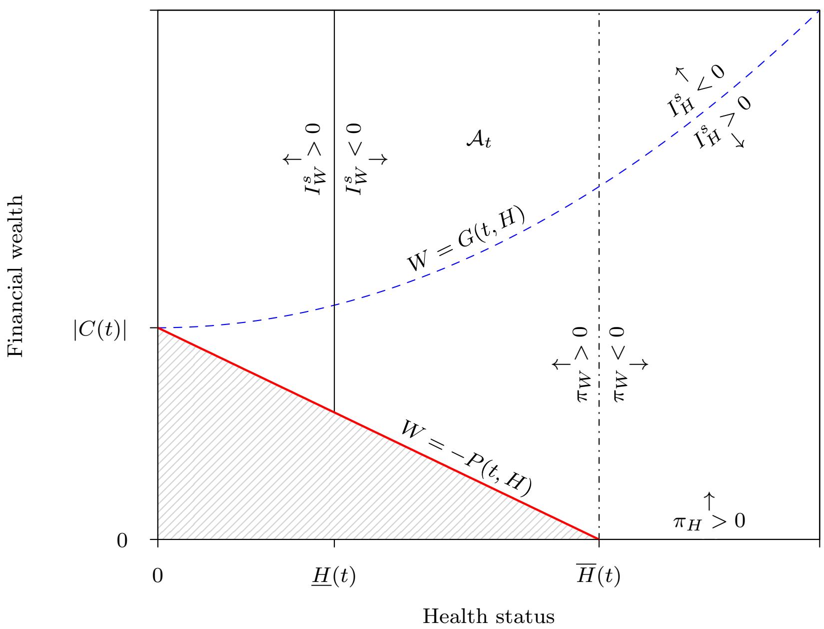
In order to better understand these substitution effects we now study the comparative statics of the health investment and portfolio shares with respect to the agent's financial wealth and health status. Assume that $C(\cdot)$ is negative, for otherwise the optimal portfolio is a monotone decreasing function of financial wealth and let

$$I^s(t, W, H) \equiv \frac{I_1(t, W, H)}{W} \tag{39}$$

denote the agent's health investment share. Using this definition together with equations (36) and (38) it is easily shown that the comparative statics of the agent's health investment and portfolio shares can be represented graphically as in Figure 1 for some functions $G(\cdot)$, $\overline{H}(\cdot)$ and $\underline{H}(\cdot) \leq \overline{H}(\cdot)$ which are defined in the appendix. Since health dependent mortality has no first order effect on the optimal portfolio share, the comparative statics of the first order portfolio share are exactly the same as those discussed in the previous section. We will therefore focus our analysis on the comparative statics of the health investment share.

Consider first the effect of a change in the agent's financial wealth. As financial wealth increases, the liquidity constraint induced by the presence of minimal consumption expenditures softens and, consequently, the agent has additional capital that can be allocated to health investment, consumption and/or financial investments. To decide on the allocation of this surplus, the agent compares the return on health investments to those of the other available opportunities. The latter are independent of the agent's health status but the former is not. In particular, the specification of the health production function and of the mortality rate imply that the return on health investments decreases as the agent's health status improves. The convexity of both the health adjustment costs and the intensity function implies that, for low health status (below $\underline{H}(\cdot)$) this return is very high and encourages the agent to increase his health investment share in response to an increase in financial wealth. In contrast, for high health status (above $\underline{H}(\cdot)$) the return on health investments is much lower and this prompts the agent to decrease his health investment share at the benefit of

Figure 1: Comparative statics of the first order rules



Notes: This figure illustrates the comparative statics of the first order portfolio and health investment shares predicted by Theorem 2 under the assumption that $C(\cdot) \leq 0$. The solid line plots the minimal level of financial wealth needed to cover subsistence consumption needs; the vertical dotted lines plot the loci of points where $I^s_W \equiv 0$ and $\pi_W \equiv 0$; and the dashed curve indicates the financial wealth and health status levels where $I^s_H \equiv 0$.

portfolio, consumption and/or savings. Exact cancelation of the two effect occurs along the vertical locus $H = \underline{H}(\cdot)$ where the optimal health investment share is insensitive to changes in financial wealth.

An improvement in the agent's health status leads to two opposite effects. First, a better health status implies a lower return to health investments through a lower mortality rate and a lower effectiveness of health investments. Both reductions encourage the agent to decrease his health investment. Second, an improvement in health status relaxes the agent's liquidity constraint through an increase in the present value of disposable income and, hence, encourages the agent to increase his health investment share. At low financial wealth (below $G(\cdot)$) the marginal relaxation of the agent's liquidity constraint dominates the decrease in the return to health investments and, consequently, the agent increases his health investment share in response to an improvement in his health status.

Conversely, at high financial wealth (above $G(\cdot)$) the marginal relaxation of the agent's constraint is dominated by the decrease in the return to health investments and the agent decreases his health investment share. Exact cancelation of the two effects occurs along the locus $W = G(\cdot)$ where the optimal health investment share is insensitive to changes in the agent's health status.

Interestingly, the region of the state space below the locus $W = G(\cdot)$ and to the left of the locus $H = \underline{H}(\cdot)$ is characterized by health investment shares that increase with respect to both financial wealth and health status. In this region, an adverse financial shock is followed by a reduction in the health investment share since the agent's resources are barely sufficient to cover his minimal consumption needs. This results in lower future health and hence in lower future income to which the agent reacts by reducing his health investment share even further. Put differently, this region of the state space can be seen as a poverty trap in which adverse shocks get amplified through reductions in the optimal health investment share.

The above discussion shows that, contrary to the partial success of its health independent mortality counterpart, the model with health dependent mortality can potentially reproduce the empirical patterns of portfolio and health investment shares. In particular, there exists a non trivial region of the state space given by:

$$\mathcal{A}_t = \left\{ (W, H) \in \mathbb{R} \times \mathbb{R}_+ : W \geq G(t, H) \text{ and } \underline{H}(t) \leq H \leq \overline{H}(t) \right\}$$

where predicted portfolio shares profactually increase with financial wealth and health status while predicted health investment shares profactually decrease with financial wealth and health status.

As a final check, it is also of interest to gauge the model's performance at reproducing observed links between health, wealth and mortality. For that purpose, we can obtain the closed-form life expectancy of a retired agent that is predicted by the model as follows:

Proposition 2 *Assume that equations (17), (25), (34) and*

$$\Phi^{-1} = \lambda_0 + \xi \left((\alpha B(T))^{\frac{\alpha}{1-\alpha}} - \delta \right) > 0 \tag{40}$$

hold true. Up to a first order approximation, the life expectancy of a retired agent is independent of wealth and given by

$$\ell(H) = \frac{1}{\lambda_0} \left(1 - \frac{\lambda_1}{H^{\xi}} \Phi \right). \tag{41}$$

Proposition 2 reveals that, up to a first order, the remaining time horizon of a retired agent is given as an increasing function of the agent's health status and is independent of financial wealth. Note also that longevity increases in the shadow price of health evaluated at retirement, $B(T)$. These two results are consistent with the findings of De Nardi et al. (2009) who find positive effects of both health and permanent income on longevity. Indeed, recall from equation (18) that the net present value of the agent's disposable income is $P(t, H) = B(t)H + C(t)$, an increasing function of the shadow price and health status. The independence of life expectancy from financial wealth W_t is also confirmed empirically for AHEAD data by Hurd et al. (2001), once current health status is accounted for.[33]

5 Conclusion

This paper shows that the observed co-movements between financial and health statuses and investments can jointly be explained by a parsimonious model that integrates two well-accepted, but otherwise segmented, Health and Financial Economics frameworks. Our theoretical results confirm that endogenous mortality, positive health elasticities of labor income, convex health adjustment costs and generalized recursive, non-homothetic preferences are all key ingredients in understanding individual financial and health choices. Admittedly a more complete investigation of the model's properties will have to rely on a thorough econometric evaluation, yet preliminary results based on cross-

[33]See in particular Table 20 in Hurd et al. (2001). Note that Attanasio and Emmerson (2003) however find a joint effect of financial wealth on mortality using British Retirement Survey BRS data.

sectional perspectives appear very encouraging and confirm the model's good performance (Hugonnier et al., 2009).

Despite its parsimony, we find it reassuring that the proposed model fares so well. We believe it speaks in favor of it being used as a workhorse for additional developments either along the lines we have suggested or along others. One avenue that we find particularly promising is the cyclical implications of the model. If adverse economic conditions in recessions lead to adjustments in health investments and if the resulting changes in the health status affect the willingness to hold risky financial assets, then this might point towards interesting predictions for the business cycle properties of asset markets. We leave these issues for a future research agenda.

We have voluntarily selected a high level of abstraction at a cost of realism. Many interesting and possibly relevant issues have thus been set aside. For example, our choice of deterministic health, although standard in Health Economics, may fruitfully be questioned. We could conjecture that increased background health risk leads to even more prudent portfolios or to the maintenance of a health buffer stock. Conversely, introducing health, work or life insurance as well as bequests might affect the results in the opposite way. Finally, by focusing on the cross-sectional dimension for our descriptive statistics, we have not fully exploited all the information contained with the data set or the model. Estimating the latter in the panel dimension could provide for interesting subsequent empirical work.

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A Proofs

Proof of Proposition 1. Using equation (16) and the fact that, in the absence of wealth effects, the health status is deterministic we deduce that the present value of the agent's disposable income can be written as

$$P(t, H_t) = C(t) + \sup_{I \geq 0} \int_t^\infty e^{-r(s-t)}(\beta(s)H_s - I_s)\mathrm{d}s, \tag{42}$$

where $\beta(\cdot)$ is the deterministic function defined by

$$\beta(t) = Y_H(t, H) = \beta^{\mathrm{e}} + 1_{\{T \leq t\}}(\beta^{\mathrm{r}} - \beta^{\mathrm{e}}).$$

Let $\mathcal{P}(t, H) = P(t, H) - C(t)$ denote the value function of the optimization problem on the right hand side of equation (42). The dynamics of the health status and the linearity of the objective function imply that $\mathcal{P}(\cdot)$ is increasing and homogenous of degree one with respect to health. Using these properties it can be shown that $\mathcal{P}(\cdot)$ and the optimal investment policy are given by

$$\begin{aligned}
&\mathcal{P}(t, H) = B(t)H, \\
&I_0(t, H) = H(\alpha \mathcal{P}_H(t, H))^{\frac{1}{1-\alpha}} = H(\alpha B(t))^{\frac{1}{1-\alpha}},
\end{aligned}$$

for some nonnegative function $B(\cdot)$ of the agent's age which solves the Hamilton–Jacobi–Bellman equation

$$\begin{aligned}
(r + \delta)B(t) &= B'(t) + \beta(t) + \max\nolimits_{x \geq 0} (x^\alpha B(t) - x) \\
&= B'(t) + \beta(t) + \Phi B(t)^{\frac{1}{1-\alpha}},
\end{aligned} \tag{43}$$

subject to the transversality condition

$$\lim_{t \to \infty} e^{-rt} B(t) H_t^0 = 0, \tag{44}$$

where H^0 denotes the path of the agent's health status under the optimal investment strategy and we have set $\Phi = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}$.

Since the function $\beta(\cdot)$ becomes age independent after the retirement date, the solution to the above ODE is given by

$$B(t) = 1_{\{T>t\}} B_{\mathrm{e}}(t) + 1_{\{T\leq t\}} B_{\mathrm{r}}$$

for some pair $(B_{\mathrm{r}}, B_{\mathrm{e}}(\cdot))$ such that $B_{\mathrm{r}} \geq 0$ solves the steady state equation

$$g(B_{\mathrm{r}}) = \beta^{\mathrm{r}} - (r+\delta) B_{\mathrm{r}} + \Phi B_{\mathrm{r}}^{\frac{1}{1-\alpha}} = 0, \tag{45}$$

and the function $B_{\mathrm{e}}(\cdot)$ solves equation (43) on the time interval $[0, T]$ subject to the value matching condition $B_{\mathrm{e}}(T) = B_{\mathrm{r}}$. Furthermore, for any such solution the associated health status process satisfies

$$\mathrm{d}H_t^0 = H_t^0 \left((\alpha B_{\mathrm{r}})^{\frac{\alpha}{1-\alpha}} - \delta \right) \mathrm{d}t, \tag{46}$$

for all $t \geq T$ and it follows that the transversality condition (44) is equivalent to the requirement that $g'(B_{\mathrm{r}}) < 0$.

Summarizing the above results, the problem reduces to finding a constant $B_{\mathrm{r}} \geq 0$ and a function $B_{\mathrm{e}}(\cdot) \geq 0$ such that

$$g(B_{\mathrm{r}}) = 0 > g'(B_{\mathrm{r}}), \tag{47}$$

$$B_{\mathrm{e}}'(t) = (r+\delta) B_{\mathrm{e}}(t) - \beta^{\mathrm{e}} - \Phi B_{\mathrm{e}}(t)^{\frac{1}{1-\alpha}}, \tag{48}$$

$$B_{\mathrm{e}}(T) = B_{\mathrm{r}}. \tag{49}$$

Unfortunately, these equations do not admit closed form solutions in general so we will have to prove that, under condition (17), there exists a unique solution which can then be computed numerically.

Straightforward analysis shows that the function $g(\cdot)$ satisfies $g(0) = \beta^{\mathrm{r}} > 0$ as well as $g'(0) = -(r+\delta) < 0$ and attains a unique minimum over the positive real line. The value of this minimum is explicitly given by

$$\min_{x\geq 0} g(x) = g\left((1/\alpha)(r+\delta)^{\frac{1}{\alpha}-1}\right) = \beta^{\mathrm{r}} - (r+\delta)^{\frac{1}{\alpha}}.$$

Under condition (17), the minimal value is negative and it follows that there exists a unique nonnegative B_{r} which satisfies the requirements of equation (47). Given B_{r}, the existence of a unique nonnegative solution to the boundary value problem in equations (48) and (49) follows from standard results on first order ODEs. □

Proof of Theorem 1. When the mortality rate is independent of the health status, the agent's problem is equivalent to that of equation (23) given the initial capital $N(t, W, H)$. In particular, the optimal investment is the one of Proposition 1 and the optimal consumption, optimal portfolio and value function can be computed as

$$\begin{aligned} c_0 &= a + x_0, \\ \pi_0 &= \nu_0 N(t, W, H)/W, \\ V_0(t, W, H) &= F(t, N(t, W, H)), \end{aligned}$$

where $F(\cdot)$ and (x_0, ν_0) denote, respectively, the value function and the optimal strategy for the problem of equation (23).

Following Svensson (1989), Duffie and Epstein (1992) and Duffie and Lions (1992) among others we have that the Hamilton-Jacobi-Bellman equation associated with the latter problem is given by

$$\lambda_0 F = \max_{(x,\nu)} \left\{ L_N^{x,\nu} F + f(x+a, F) - \frac{\gamma}{2F}(\nu\sigma N F_N)^2 \right\}$$

subject to the transversality condition

$$\lim_{t\to\infty} e^{-\lambda_0 t} E_0[F(t, N_t^0)] = 0, \tag{50}$$

where N^0 denotes the path of the agent's total disposable wealth under the optimal consumption and portfolio strategy and

$$L_N^{x,\nu} = \partial_t + ((r + \nu\sigma\theta)N - x)\partial_N + \frac{1}{2}(\nu\sigma N)^2\partial_{NN}$$

is the differential operator associated to (t, N) under the strategy (x, ν). Since the agent's faces an infinite horizon problem with a constant investment opportunity set and does not receive any income through time, it must be the case that $F(\cdot)$ is time independent. On the other hand, the specification of the agent's preferences and the dynamics of the process N in equation (22) imply that $F(\cdot)$ is increasing and homogenous of degree one with respect to the agent's total disposable wealth.

Using these properties in conjunction with HJB equation, we obtain that the value function and the optimal strategy are given by $F(N) = \phi N$ and

$$x_0 = \rho^{\varepsilon}\phi^{1-\varepsilon}N, \tag{51}$$

$$\nu_0 = \theta/(\gamma\sigma), \tag{52}$$

for some nonnegative constant ϕ which solves

$$\lambda_0\phi = \max_{(x,\nu)}\left\{\phi(r + \nu\sigma\theta - x) + f(x + a, \phi) - \frac{\gamma}{2}\phi(\nu\sigma)^2\right\}$$

This equation admits a well-defined solution if and only if the constant A of equation (25) is strictly positive. In this case, $\phi = \rho(A/\rho)^{\frac{1}{1-\epsilon}}$ and substituting this into the definition of the optimal consumption plan we conclude that $x_0 = AN$ as required.

In order to complete the proof we need to show that under condition (25) the above solution satisfies (50). Using equation (22) and the definition of the candidate optimal strategy we obtain that the agent's total disposable wealth evolves according to

$$\mathrm{d}N_t^0 = N_t^0(r - A)\mathrm{d}t + N_t^0(\theta/\gamma)(\mathrm{d}Z_t + \theta\mathrm{d}t).$$

Combining this expression with well-known results on the expectation of the geometric Brownian motion gives

$$e^{-\lambda_0 t}E_0[F(t,N_t^0)] = e^{-\lambda_0 t}E_0[vN_t^0] = e^{-(\lambda_0 - r - \theta^2/\gamma + A)t}vN_0,$$

and it follows that condition (25) is necessary and sufficient for both the feasibility of x_0 and the validity of the transversality condition. □

Proof of Theorem 2. The Hamilton-Jacobi-Bellman equation associated with the agent's problem is given by (30) subject to

$$\lim_{t\to\infty} E_0\left[e^{-\int_0^t \lambda(H_s^*)\mathrm{d}s}V(t,W_t^*,H_t^*)\right] = 0, \tag{53}$$

where W^* and H^* denote the path of the agent's wealth and health status under the optimal strategy. Maximizing the right hand side of the HJB equation with respect to π, c and I gives the candidate optimal strategy of equations (32)–(33) and substituting these back into equation (30) shows that the HJB equation can be written as

$$\lambda(H)V = L^*V + f(c,V) - \frac{\gamma\theta^2 V V_W^4}{2(\gamma V_W^2 - VV_{WW})^2} \tag{54}$$

where

$$\begin{aligned} L^* = \partial_t &+ ((I^*/H)^\alpha - \delta)H\partial_H \\ &+ ((r + \pi^*\sigma\theta)W + Y - c^* - I^*)\partial_W + \frac{1}{2}(\pi^*\sigma W)^2\partial_{WW} \end{aligned}$$

is the differential operator associated to the process (t,H,W) under the optimal strategy. Consider the approximate value function given by

$$V(t,W,H) = V_0(t,W,H) + \lambda_1 V_1(t,W,H) + o(\lambda_1^2) \tag{55}$$

where the function

$$V_1(t,W,H) = \frac{\partial V}{\partial \lambda_1}(t,W,H)\bigg|_{\lambda_1=0}$$

is the first order correction induced by the presence of health-dependent mortality. Substituting this approximation of $V(\cdot)$ into the HJB equation and expanding the resulting expression to the first order in λ_1 we obtain that the functions $V_0(\cdot)$ and $V_1(\cdot)$ solve the system of partial differential equations given by

$$\lambda_0 V_0 = L^0 V_0 + f(c_0, V_0) - \frac{\gamma\theta^2 V_0 V_{0W}^4}{2(\gamma V_{0W}^2 - V_0 V_{0WW})^2}, \tag{56}$$

$$\lambda_0 V_1 = L^0 V_1 + f_V(c_0, V_0) V_1 - \frac{V_0}{H^\xi} + \frac{\gamma\theta^2 (V_1 V_{0W}^4 - 2V_0^2 V_{0W}^3 V_{1W})}{2(\gamma V_{0W}^2 - V_0 V_{0WW})^2}, \tag{57}$$

where

$$L^0 = \partial_t + (H^{1-\alpha} I_0^\alpha - \delta H)\partial_H + ((r + \pi_0 \sigma\theta)W + Y - c_0 - I_0)\partial_W + \frac{1}{2}(\pi_0 \sigma W)^2 \partial_{WW}$$

is the differential operator associated to the optimal strategy (π_0, c_0, I_0) of the health-independent mortality case. Similarly, substituting equation (55) into equations (32)–(33) and expanding the resulting expressions shows that a first order approximation of the optimal strategy is

$$\pi_1 = \frac{\theta N(t, W, H)}{\gamma\sigma W} + o(\lambda_1^2), \tag{58}$$

$$c_1 = c_0 + \lambda_1 (V_{0W}/\rho)^{-\varepsilon} \frac{V_1 V_{0W} - \varepsilon V_0 V_{1W}}{V_{0W}} + o(\lambda_1^2), \tag{59}$$

$$I_1 = I_0 + \lambda_1 \alpha I_0^\alpha \frac{V_{0H} V_{1W} - V_{1H} V_{0W}}{(1-\alpha) V_{0W}^2} + o(\lambda_1^2), \tag{60}$$

where the function $N(\cdot)$ is defined as in (21). Using the result of Theorem 1, the solution to (56) is given by

$$V_0(t, W, H) = \rho (A/\rho)^{\frac{1}{1-\varepsilon}} N(t, W, H).$$

Substituting this function into equation (57) and inspecting the resulting linear partial differential equation, we deduce that

$$V_1(t, W, H) = -\frac{\Delta(t)}{H^\xi} V_0(t, W, H) \tag{61}$$

for some function $\Delta(\cdot)$ of the agent's age which solves the first order ordinary differential equation

$$\Delta'(t) = \Delta(t)\left(\xi(\alpha B(t))^{\frac{\alpha}{1-\alpha}} - \xi\delta + A\right) - 1. \tag{62}$$

Since the function $B(\cdot)$ becomes age independent after the retirement date, the solution to the above ODE is given by

$$\Delta(t) = 1_{\{T>t\}}\Delta_{\mathrm{e}}(t) + 1_{\{T\le t\}}\Delta_{\mathrm{r}},$$

where Δ_{r} is the unique solution to the steady state equation

$$0 = \Delta_{\mathrm{r}}\left(\xi(\alpha B_{\mathrm{r}})^{\frac{\alpha}{1-\alpha}} - \xi\delta + A\right) - 1, \tag{63}$$

and the function $\Delta_{\mathrm{e}}(\cdot)$ solves the ordinary differential equation (62) on the time interval $[0, T]$ subject to the value matching condition $\Delta_{\mathrm{e}}(T) = \Delta_{\mathrm{r}}$. The unique solution to this equation is given by

$$\Delta_{\mathrm{e}}(t) = e^{-\int_t^T \zeta(s)\mathrm{d}s}\Delta_{\mathrm{r}} + \int_t^T e^{-\int_t^s \zeta(u)\mathrm{d}u}\mathrm{d}s,$$

where

$$\zeta(t) = \xi(\alpha B(t))^{\frac{\alpha}{1-\alpha}} - \xi\delta + A.$$

Now, substituting this function as well as $V_0(\cdot)$ and $V_1(\cdot)$ into equations (58), (59) and (60) gives the result reported in the statement and the proof will be complete once we show that the restrictions imposed by equations (25) and (34) are sufficient to guarantee that the approximate value function satisfies a suitable transversality condition.

To unveil the nature of this approximate condition, consider the original transversality condition in equation (53) and expand the quantity inside the expected value to the first order in λ_1. This gives

$$\begin{aligned} e^{-\int_0^t \lambda(H_s^*)\mathrm{d}s}V(t, W_t^*, H_t^*) &= e^{-\lambda_0 t}V_0(t, W_t^0, H_t^0) \\ &\quad + \lambda_1 e^{-\lambda_0 t}\left(V_1(t, W_t^0, H_t^0) - \mathcal{H}_t^0 V_0(t, W_t^0, H_t^0)\right) + o(\lambda_1^2), \end{aligned} \tag{64}$$

where W^0 and H^0 denote the path of the agent's wealth and health status under the optimal strategy (π_0, c_0, I_0) of the health independent mortality case and we have set

$$\mathcal{H}_t^0 = \int_0^t (H_s^0)^{-\xi} \mathrm{d}s.$$

Since the expectation of the first term on the right of equation (64) converges to zero due the transversality condition of the health-independent mortality case, this implies that the transversality conditions associated to the first order approximation can be formulated as

$$\lim_{t\to\infty} E_0 \left[e^{-\lambda_0 t} \left(V_1(t, W_t^0, H_t^0) - \mathcal{H}_t^0 V_0(t, W_t^0, H_t^0) \right) \right] = 0. \tag{65}$$

Let $t \geq T$ and denote by Γ_t the quantity in the above expectation. Using the definition of $\mathcal{H}_0$ in conjunction with the dynamics of H^0 in equation (46) we obtain

$$\mathcal{H}_t^0 = \mathcal{H}_T^0 + (1/\zeta)\Big((H_T^0)^{-\xi} - (H_t^0)^{-\xi} \Big).$$

where we have set $\zeta = \zeta(T)$. Combining this expression with equation (61) and using the definition of the function $N(\cdot)$ we obtain

$$\begin{aligned} E_0 \left[\Gamma_t \right] &= e^{-\lambda_0 t} \phi\, E_0 \Big[\left((1/\zeta) - \Delta_{\mathrm{r}} \right) (H_t^0)^{-\xi} N_t^0 - \left(\mathcal{H}_T^0 + (1/\zeta)(H_T^0)^{-\xi} \right) N_t^0 \Big] \\ &= e^{-(\lambda_0 - r - \theta^2/\gamma + A)t} \phi \left(e^{-\zeta(t-T)} C_1 - C_2 \right) N_0 \end{aligned}$$

where ϕ and N^0 are defined as in the proof of Proposition 1, the second equality follows from the law of iterated expectations, the dynamics of the agent's health status in equation (46) and the fact that the process N^0 is a geometric Brownian motion; and we have set

$$\begin{aligned} C_1 &= \left((1/\zeta) - \Delta_{\mathrm{r}} \right) (H_T^0)^{-\xi}, \\ C_2 &= \mathcal{H}_T^0 + (1/\zeta)(H_T^0)^{-\xi}. \end{aligned}$$

Finally, taking the limit on both sides of the above expression and using the fact that the constants C_1 and C_2 are finite shows that conditions (17) and (17) are necessary and sufficient for the validity of equation (65). $\square$

Proof of Proposition 2. Using standard results on Poisson point processes we have that the life expectancy of a retired agent is given by

$$\ell(W_t, H_t) = E_t[\tau] = E_t\left[\int_t^\infty e^{-\int_t^s \lambda(H_u^*)\mathrm{d}u}\mathrm{d}s\right]$$

where H^* denotes the path of the agent's health status under the optimal strategy. Applying the Feynman-Kac formula we have that the function E solves the partial differential equation

$$\lambda(H)\ell(W, H) = L^*\ell(W, H) + 1 \tag{66}$$

where L^* is the differential operator associated to (W, H) under the optimal strategy. In accordance with the method used in the rest of the paper, we consider an approximation of the form

$$\ell(W, H) = \ell_0(W, H) + \lambda_1\ell_1(W, H) + o(\lambda_1^2)$$

where $\ell_0(\cdot) = 1/\lambda_0$ is the life expectancy associated with the health independent mortality case and

$$\ell_1(W, H) = \frac{\partial \ell(W, H)}{\partial \lambda_1}\bigg|_{\lambda_1 = 0}$$

is the first order correction induced by the presence of health dependent mortality. Substituting this approximation into equation (66) and expanding the result in powers of λ_1 shows that the function $E_1(\cdot)$ is a solution to the partial differential equation

$$\lambda_0\ell_1 = L^0\ell_1 - \frac{1}{\lambda_0 H^\xi}$$

where L^0 is the differential operator associated to (W, H) under the optimal strategy of the health independent mortality case. Guessing a wealth independent solution of the form $\ell_1(W,H) = \varpi H^{-\xi}$ leads to

$$-\varpi\lambda_0 = \varpi\xi\left((\alpha B(t))^{\frac{\alpha}{1-\alpha}} - \delta\right) + \frac{1}{\lambda_0}$$

and solving that equation for ϖ gives the desired result. The restriction imposed by equation (40) is a transversality condition which guarantees that the first order correction to the agent's life expectancy is negative and increasing in the agent's health status. Its derivation being similar to that of equation (34) we omit the details. □

B Construction of Figure 1

Assume that the agent's financial wealth, W, is nonnegative for otherwise the comparative statics of the health investment share with respect to health are simply reversed. Differentiating equation (38) with respect to the agent's health status shows that the optimal health investment share decreases with the agent's health status if

$$W \geq G(t,H) = -P(t,H) + \frac{B(t)}{\xi}H + \frac{\alpha B(t)}{\lambda_1\eta\xi\Delta(t)}H^{1+\xi}$$

and increases otherwise. Similarly, differentiating equation (39) with respect to financial wealth shows that the optimal health investment share decreases with the agent's financial wealth if and only if $H \geq \underline{H}(t)$ where $\underline{H}(\cdot)$ is defined implicitly by

$$\alpha B(t)\underline{H}(t)^{1+\xi} + \lambda_1\eta\Delta(t)P(t,\underline{H}(t)) = 0. \tag{67}$$

In particular, the portfolio share increases with the health status as long as financial wealth is positive; and increasing in financial wealth if and only if the agent's future income is insufficient to cover his minimal consumption needs in the sense that $P(t,H) \leq 0$ or, equivalently,

$$H \leq \overline{H}(t) = -C(t)/B(t). \tag{68}$$

Figure 1 summarizes these results by providing a graphical representation of the comparative statics in the $(W,H)-$space.

Table 1: Descriptive statistics

	Non retired			Retired		
	All sample	Single	Couple	All sample	Single	Couple
Socio-demographic						
Age	57.6	58.7	57.2	75.0	77.8	73.0
Male	40%	29%	44%	42%	22%	57%
Female	60%	71%	56%	58%	78%	43%
Health status						
Poor ($H = 1$)	7%	11%	6%	11%	14%	9%
Fair ($H = 2$)	15%	20%	14%	22%	24%	20%
Good ($H = 3$)	29%	29%	29%	31%	31%	32%
Very good ($H = 4$)	32%	27%	34%	26%	23%	28%
Excellent ($H = 5$)	16%	14%	17%	10%	8%	11%
Health expenditures						
Medical	$8,755	$10,186	$8,337	$12,848	$15,517	$10,902
(median)	$1,350	$1,439	$1,331	$2,303	$2,680	$2,047
Out-of-pocket	$1,860	$1,911	$1,845	$3,014	$3,588	$2,593
(median)	$800	$700	$840	$1,040	$960	$1,120
Total	$10,615	$12,097	$10,182	$15,862	$19,105	$13,495
(median)	$2,779	$2,981	$2,729	$5,000	$5,000	$4,788
Asset holdings						
Hold safe asset	86%	75%	89%	85%	78%	90%
Hold bond	7%	4%	7%	9%	6%	11%
Hold risk asset	33%	20%	37%	32%	23%	39%
Hold debt	38%	38%	38%	18%	16%	20%
Portfolio composition						
Financial wealth	$98,727	$62,330	$109,369	$122,573	$74,754	$157,524
Safe assets	57%	63%	55%	65%	70%	62%
Bonds	2%	1%	2%	2%	2%	3%
Risky assets	22%	14%	24%	20%	16%	23%
Debt	19%	22%	18%	12%	12%	13%
Observations	8,836	1,999	6,837	10,735	4,532	6,202

Notes: The data source is HRS (RAND version), 5^{th} wave. Non-retired (retired) agents are individuals aged less than 65 (aged 65 and over). The reported financial variables are conditional on non-zero holdings.

Table 2: Summary statistics by net financial wealth and health for non-retired agents

	Net financial wealth quintile				
Health	1	2	3	4	5
Poor ($H = 1$)					
Wealth	−\$9,140	\$443	\$13,352	\$51,820	\$660,988
P(risky > 0)	4%	2%	25%	52%	77%
Risky assets	−4%	3%	18%	27%	54%
Health inv. share (total)	-387%	1553%	110%	34%	8%
(out-of-pocket)	-43%	186%	23%	4%	1%
Fair ($H = 2$)					
Wealth	−\$9,724	\$533	\$11,444	\$61,875	\$446,266
P(risky > 0)	5%	1%	20%	54%	82%
Risky assets	−12%	2%	13%	33%	54%
Health inv. share (total)	-101%	742%	42%	10%	2%
(out-of-pocket)	-29%	125%	13%	3%	1%
Good ($H = 3$)					
Wealth	−\$11,073	\$817	\$12,825	\$62,136	\$582,583
P(risky > 0)	6%	5%	25%	54%	81%
Risky assets	−5%	12%	15%	32%	54%
Health inv. share (total)	-62%	222%	28%	6%	1%
(out-of-pocket)	-15%	64%	9%	2%	0%
Very good ($H = 4$)					
Wealth	−\$16,170	\$1,070	\$13,063	\$63,597	\$461,875
P(risky > 0)	12%	6%	27%	63%	87%
Risky assets	−10%	12%	20%	39%	54%
Health inv. share (total)	-35%	143%	19%	4%	1%
(out-of-pocket)	-10%	47%	6%	1%	0%
Excellent ($H = 5$)					
Wealth	−\$19,909	\$1,007	\$12,743	\$64,163	\$501,961
P(risky > 0)	10%	6%	27%	64%	86%
Risky assets	−9%	9%	20%	39%	57%
Health inv. share (total)	-20%	98%	11%	3%	0%
(out-of-pocket)	-3%	28%	4%	1%	0%

Notes: Agents younger than 65 only. The reported values are respectively the mean of the net financial wealth (W_t), the mean of the probability of holding risky assets ($P(\pi_t > 0)$), the mean of the risky portfolio share (π_t) and the median of the health investment share out of net financial wealth (I_t/W_t).

Table 3: Summary statistics by net financial wealth and health for retired agents

	Net financial wealth quintile				
Health	1	2	3	4	5
Poor ($H = 1$)					
Wealth	−$10,088	$490	$13,310	$63,140	$426,698
P(risky > 0)	1%	2%	14%	42%	75%
Risky assets	0%	6%	9%	24%	45%
Health inv. share (total)	-568%	2366%	147%	28%	7%
(out-of-pocket)	-45%	142%	22%	6%	1%
Fair ($H = 2$)					
Wealth	−$6,114	$596	$12,683	$59,366	$514,602
P(risky > 0)	2%	1%	14%	42%	74%
Risky assets	−2%	1%	7%	24%	42%
Health inv. share (total)	-245%	710%	46%	12%	2%
(out-of-pocket)	-45%	96%	13%	3%	1%
Good ($H = 3$)					
Wealth	−$10,911	$718	$13,094	$64,108	$436,456
P(risky > 0)	5%	2%	19%	45%	77%
Risky assets	−5%	3%	12%	24%	45%
Health inv. share (total)	-79%	476%	31%	7%	1%
(out-of-pocket)	-18%	77%	10%	2%	0%
Very good ($H = 4$)					
Wealth	−$7,108	$960	$13,578	$64,905	$467,585
P(risky > 0)	7%	4%	24%	52%	82%
Risky assets	−61%	7%	12%	27%	50%
Health inv. share (total)	-86%	188%	21%	5%	1%
(out-of-pocket)	-14%	53%	7%	2%	0%
Excellent ($H = 5$)					
Wealth	−$6,469	$799	$13,195	$64,498	$554,980
P(risky > 0)	3%	2%	24%	44%	83%
Risky assets	−1%	10%	11%	25%	48%
Health inv. share (total)	-53%	177%	14%	3%	1%
(out-of-pocket)	-9%	41%	4%	1%	0%

Notes: Agents of age 65 and over only. The reported values are respectively the mean of the net financial wealth (W_t), the mean of the probability of holding risky assets ($P(\pi_t > 0)$), the mean of the risky portfolio share (π_t) and the median of the health investment share out of net financial wealth (I_t/W_t).

Table 4: Summary statistics by age and health

	Age			
Health	Less than 65	65 to 74	75 to 84	Over 85
Poor ($H = 1$)				
Wealth	\$13,352	\$12,599	\$14,640	\$12,033
P(risky > 0)	25%	18%	12%	8%
Risky assets	18%	11%	8%	5%
Health inv. share (total)	110%	151%	123%	156%
(out-of-pocket)	23%	21%	24%	18%
Fair ($H = 2$)				
Wealth	\$11,444	\$12,453	\$13,161	\$12,071
P(risky > 0)	20%	14%	15%	11%
Risky assets	13%	8%	6%	8%
Health inv. share (total)	42%	51%	45%	46%
(out-of-pocket)	13%	14%	13%	13%
Good ($H = 3$)				
Wealth	\$12,825	\$13,156	\$12,918	\$13,405
P(risky > 0)	25%	21%	18%	8%
Risky assets	15%	13%	14%	6%
Health inv. share (total)	28%	28%	37%	36%
(out-of-pocket)	9%	10%	13%	11%
Very good ($H = 4$)				
Wealth	\$13,063	\$13,704	\$13,537	\$12,613
P(risky > 0)	27%	23%	26%	25%
Risky assets	20%	13%	11%	14%
Health inv. share (total)	19%	20%	24%	25%
(out-of-pocket)	6%	7%	8%	6%
Excellent ($H = 5$)				
Wealth	\$12,743	\$13,691	\$12,403	\$11,411
P(risky > 0)	27%	26%	17%	29%
Risky assets	20%	12%	9%	15%
Health inv. share (total)	11%	11%	16%	42%
(out-of-pocket)	4%	3%	5%	3%

Notes: The reported values are respectively the mean of the net financial wealth, of the probability of holding risky assets ($P(\pi_t > 0)$), of the risky share (π_t) and the median of the health investment share of net financial wealth (I_t/W_t) for third quintile wealth level.

Table 5: Income and health regression

	All	Non-retired	Retired
	A. Individual income		
Constant	0.0047**	0.0052	0.0091***
	(0.0021)	(0.0051)	(0.0012)
Health	0.0104***	0.0130***	0.0065***
	(0.0006)	(0.0014)	(0.0004)
Obs.	19,571	8,836	10,735
	B. Household income		
Constant	0.0077**	0.0116***	0.0130***
	(0.0022)	(0.0053)	(0.0013)
Health	0.0141***	0.0174***	0.0082***
	(0.0007)	(0.0014)	(0.0004)
Obs.	19,571	8,836	10,735

Notes: Income is divided by 10^6 and health is measured between 1 (Poor) and 5 (Excellent). The results are robust to sampling weights, additional socio-demographic regressors, measurement errors, cohort effects and White's correction of the variance covariance matrix. Tobit estimations lead to the same conclusions. Standard errors are reported in parentheses, as well as statistical significance at the 1% level (***), 5% level (**) and at the 10% level (*).