A structural analysis of the health expenditures and portfolio choices of retired agents[*]

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Abstract

Richer and healthier agents tend to hold riskier portfolios and spend proportionally less on health expenditures. Potential explanations include health and wealth effects on preferences, expected longevity or disposable total wealth. Using HRS data, we perform a structural estimation of a dynamic model of consumption, portfolio and health expenditure choices with recursive utility, as well as health-dependent income and mortality risk. Our estimates of the deep parameters highlight the importance of health capital, mortality risk control, convex health and mortality adjustment costs and binding liquidity constraints to rationalize the stylized facts. They also provide new perspectives on expected longevity and on the values of life and health.

Keywords: Asset allocation; Expected lifetime; Health production function; Mortality risk; Recursive utility; Value of health; Value of life.

JEL Classification. G11, I12.

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1   Introduction

The joint analysis of the financial and health-related decisions made by households reveals, among others, four important stylized facts. The share of wealth invested in risky assets is found to be increasing in both the wealth of the agents (e.g. Wachter and Yogo, 2008; Carroll, 2002), and in their health status (Guiso, Jappelli and Terlizzese, 1996; Rosen and Wu, 2004; Fan and Zhao, 2009, among others). Moreover, the share of wealth spent on medical expenditures is found to be decreasing in both wealth (e.g. Meer, Miller and Rosen, 2003; DiMatteo, 2003; Gilleskie and Mroz, 2004; Acemoglu, Finkelstein and Notowidigdo, 2009, for similar results based on the income shares), and in the health status (e.g. Smith, 1999; Yogo, 2009; Gilleskie and Mroz, 2004). In other words, richer and healthier agents tend to hold riskier financial portfolios and spend proportionally less on health expenditures.

To rationalize these stylized facts, most studies focus on the effects of wealth and health on disposable resources, on preferences and on exposure to health, mortality and financial risks (e.g. Smith, 1999; Meer et al., 2003; Rosen and Wu, 2004; Puri and Robinson, 2007; Guiso and Paiella, 2008; Love and Smith, 2010; Pang and Warshawsky, 2010). However, this line of research also highlights how difficult it can be to discriminate among these competing hypotheses without a solid anchoring in economic theory to guide the empirical evaluations. The main objective of this paper is to provide such a mapping between theory and econometrics by performing a structural empirical assessment of these potential explanations. More precisely, we attempt to better understand the channels through which health and wealth determine financial and health-related choices by estimating the deep parameters of a joint dynamic model of health expenditures and financial decisions.

The theoretical model that we rely upon to understand the comparative statics of portfolios and health expenditures is developed in Hugonnier, Pelgrin and St-Amour (2009). This framework regroups two standard building blocks of the Financial and of the Health Economics literature. More precisely, a dynamic portfolio problem à la Merton (1971) is combined with a human capital model of health expenditures à la Grossman (1972), with physical depreciation and convex adjustment costs. The model further embeds endogenous longevity as the agent can (partially) reduce his mortality risk by improving health, but faces diminishing returns in doing so. In addition to prolonging expected lifetime, health is further valuable in improving labor

income, e.g. by reducing workdays lost when sick. The agent selects optimal consumption, portfolio and health expenditures to maximize recursive preferences of the type advocated by Duffie and Epstein (1992) and by Schroder and Skiadas (1999), and with minimal subsistence consumption constraints appended.

As shown in Hugonnier et al. (2009), this joint model of financial and health-related decisions presents numerous advantages. First, the two channels of longevity and human capital values of health provide an alternative to utility-based approaches. The latter remain subject to debate with respect to the sign of the cross derivatives of health and consumption utility (e.g. Finkelstein, Luttmer and Notowidigdo, 2009), an effect that plays a key role on portfolios and health expenditures. Second, relying on a capital theory of health, with irreversible investments and positive effects of health on labor income and mortality, avoids treating health as an ordinary asset that can be bought and sold freely on markets. Third, the non-expected utility framework ensures an unconditional preference for life over death. In comparison, iso-elastic VNM preferences require rescaling the utility function at certain curvature level to avoid counterfactual preference for death. Fourth, minimal consumption entails binding liquidity constraints. More risky asset holdings can be chosen when additional resources relax these constraints. Fifth and most important, these features allow this model to potentially account for the four stylized facts outlined earlier.

We use the closed-form rules derived by Hugonnier et al. (2009) for the estimation and thus ensure a correspondence between the theoretical and empirical models. The structural econometric model of health expenditures and portfolio allocations is estimated for retired agents using data from the Health and Retirement Survey (HRS). The estimation identifies the parameters for the preferences, health dynamics, mortality risk and for the income process. Our main results can be summarized as follows. First, our preference parameters are consistent with a binding subsistence consumption constraint and realistic relative risk aversion, as well as an unconditional preference for life and low elasticity of intertemporal substitution. Second, the technological and longevity parameters confirm that health is subject to rapid depreciation when investment is insufficient, and show that both health and mortality risk can be adjusted, but that the two are increasingly costly to change. Third, health has positive effects on labor income, even after retirement, and is therefore a significant contributor to disposable resources.

Taken together, our results indicate that agents should and are able to adjust health and that these two channels of higher quantity (i.e. longevity) and quality (i.e. consumption) of life are crucial to understanding the effects of health on financial and health-related decisions.

These findings are fruitful to revisit the potential explanations for the stylized facts outlined earlier. Indeed, an improvement in health not only increases both expected lifetime and human capital, thus relaxing the minimal consumption constraint, but also lowers the returns to health investments. This encourages the agent to reduce the health investment shares. The concurrent change in the financial portfolio composition is mainly driven by the relaxation of the minimal consumption constraint which leads the agent to take more risky positions. A longer expected lifetime at better health plays no role in explaining more risky portfolios since the asset allocation is independent of the planning horizon when the financial investment opportunity set is constant. The model therefore reproduces the positive effects of health on portfolios and its negative effect on health expenditure shares. In comparison, an increase in financial wealth also slackens the liquidity constraint, but it affects neither the expected lifetime nor the returns to health investment. This allows the agent to substitute away from health capital (when health is sufficiently high that returns to health investment are low) while encouraging more risky financial asset positions. The model is thus able to reproduce the positive effects of financial wealth on risky portfolios and its negative effect on health investment shares.

We perform robustness checks along many dimensions. In part, the model is derived under the assumption of age-independent preferences, mortality risk and health dynamics parameters. We verify and confirm that this assumption is realistic by computing the predicted rules for pre-retired agents and contrasting them with observed portfolios and health expenditures for younger individuals in the HRS data base in an out-of-sample performance test. Second, our benchmark estimation relies on a cross-section of the HRS panel, but we verify and confirm that all our key results are robust to sampling different waves or age groups, incorporating socio-economic covariates, controlling for health insurance and allowing for unobserved heterogeneity in a panel estimation.

Our structural estimates also allow us to measure other variables of interest. First, we evaluate the closed-form expressions for expected lifetime at the point estimates to verify and confirm that the model is consistent with plausible longevity. Moreover, we make use of the

value function to compute the certainty equivalent value of life and the value of health explicitly. We find that an individual aged 75 with net financial wealth of $300,000 would be willing to pay between $35,000 (at poor health) and $276,000 (at excellent health) in exchange for a 1-year increase in expected lifetime. We also find that the value of one unit of health (i.e of moving from one health category to an improved one) for that same agent is between $67,000 (at poor health) and $60,000 (at excellent health) and is almost exclusively attributable to the human capital value (as opposed to mortality control value) of health. These results compare advantageously with those found in the literature and lend further support to the structural analysis.

The rest of the paper develops as follows. We outline the theoretical model in Section 2 and discuss the estimation strategy as well as the data in Section 3. In Section 4 we present and discuss the estimation results. Additional theoretical and empirical implications are presented in Section 5. Finally, a conclusion in Section 6 reviews the main findings and discusses potential research agendas.

2   A model of health expenditures and portfolio allocations

2.1   Theoretical model

Hugonnier et al. (2009) consider retired agents indexed $j = 1, 2, \ldots$ who select period-$t \geq 0$ consumption $c_{j,t} \geq a$, health expenditures $I_{j,t} \geq 0$, as well as risky portfolio $\pi_{j,t}$ so as to maximize:

$$U_{j,t} = 1_{\{\tau > t\}} E_t \left[\int_t^\tau \left(f(c_{j,s}, U_{j,s}) - \frac{\gamma}{2U_{j,s}} |\sigma_s(U_j)|^2 \right) \mathrm{d}s \right] \tag{1}$$

$$f(c,v) = \frac{v\rho}{1 - 1/\varepsilon} \left[\left(\frac{c - a}{v} \right)^{1 - 1/\varepsilon} - 1 \right], \tag{2}$$

subject to:

$$\mathrm{d}H_{j,t} = \left(I_{j,t}^\alpha H_{j,t}^{1-\alpha} - \delta H_{j,t} \right) \mathrm{d}t, \quad H_{j,0} > 0, \tag{3}$$

$$Y_{j,t}^{\mathrm{r}} = y^{\mathrm{r}} + \beta^{\mathrm{r}} H_{j,t}, \tag{4}$$

$$\mathrm{d}W_{j,t} = (rW_{j,t} + Y_{j,t}^{\mathrm{r}} - I_{j,t} - c_{j,t})\mathrm{d}t + W_{j,t}\pi_{j,t}\sigma(\mathrm{d}Z_t + \phi\mathrm{d}t), \tag{5}$$

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and

$$\lim_{s \to 0} \frac{1}{s} P_t \big[t < \tau \le t + s \big] = \lambda_0 + \frac{\lambda_1}{H_{j,t}^{\xi}}, \tag{6}$$

where the random time τ measures the time of death of the agent and $\sigma_t(U) = \mathrm{d}\langle U, Z \rangle_t / \mathrm{d}t$ denotes the instantaneous volatility of the continuation utility. Individuals are assumed to be heterogeneous with respect to their health $H_{j,t}$ and financial wealth $W_{j,t}$ levels. Conversely, the preference, risk distribution and technological parameters are assumed to be time-independent and identical across agents.

The recursive preferences in equations (1) and (2) are of the type proposed by Duffie and Epstein (1992); Schroder and Skiadas (1999).[1] The nonnegative parameters ρ, a, ε and γ respectively capture the agent's subjective rate of time preference, his subsistence consumption level, his elasticity of intertemporal substitution, as well as his risk aversion over static gambles. As shown in Hugonnier et al. (2009), this specification avoids scaling problems in standard additive setups, thereby guaranteeing that life is always valuable regardless of parametric values.[2] Finally the nonnegativity constraint on health expenditures is standard in the Health Economics literature and reflects the irreversibility of health investments by ruling out the possibility of selling one's health in markets.

In the spirit of Ehrlich (2000); Ehrlich and Chuma (1990); Hall and Jones (2007), the endogenous mortality is assumed to follow a Poisson process whose death intensity is declining in health. The parameter $\lambda_0 \ge 0$ captures the health-independent (or endowed) death probability, whereas $\lambda_1 \ge 0$ encompasses the controllable components, and $\xi \ge 0$ measures the degree of costs convexities in adjusting the death intensity. The locally deterministic process for health

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[1]See also Kreps and Porteus (1979); Epstein and Zin (1989) and Weil (1989) for discrete-time analogs.

[2]For an endogenous mortality problem with standard time additive power utility we have

$$U_t = 1_{\{\tau > t\}} E_t \left[\int_t^{\tau} e^{-\rho(s-t)} \frac{c_s^{1-\gamma}}{1-\gamma} \mathrm{d}s \right],$$

and it follows that the agent strictly prefers life ($U_t > 0$) to death ($U_t = 0$) only when $\gamma < 1$. Since risk aversion is often estimated to be above 1, the model counter-intuitively assumes preference for death. To avoid this outcome, a constant $\bar{u} \gg 0$ is often added to the CRRA functional: $u(c) = \bar{u} + (1-\gamma)^{-1} c^{1-\gamma}$ to insure that the agent prefers life (e.g. Rosen, 1988; Becker, Philipson and Soares, 2005; Hall and Jones, 2007, among others). In comparison, recursive preferences (1) and (2) unconditionally imply $U_t > 0$ when the agent is alive, and consequently preference for life.

in (3) is similar to Grossman (1972); Ehrlich (2000), with the parameter $\alpha \in (0,1)$ capturing convexities in health adjustment costs, and $\delta \geq 0$ representing a depreciation rate.

The post-retirement income process $Y_{j,t}^{\mathrm{r}}$ in (4) has $y^{\mathrm{r}} \geq 0$ for the health-independent (e.g. pension) income, and $\beta^{\mathrm{r}} \geq 0$ the sensitivity of labor income to the agent's health. Specifically, a healthier agent has an increased ability to work and receives higher income even after retirement. As a result, health serves a dual purpose: improved health reduces mortality risk and at the same time increases labor income. Finally, the wealth process (5) involves a single risky asset with constant mean return $\mu \geq r$ and volatility $\sigma > 0$ on a univariate Brownian process $\mathrm{d}Z_t$ and one riskless asset with return $r \geq 0$, with $\phi \equiv (\mu - r)/\sigma$ denoting the market price of risk.

2.2 Main theoretical results

The endogeneity of the death intensity (6) unfortunately implies that the model has no closed-form solutions for the general case of $\lambda_1 \neq 0$. To circumvent this difficulty, Hugonnier et al. (2009) resort to an expansion analysis to approximate the optimal rules through an expansion centered on the parameter λ_1 governing the health dependence of the Poisson intensity (6).[3] More precisely, an explicit solution can be obtained for the restricted case of exogenous mortality ($\lambda_1 = 0$). This solution is then used as a benchmark for a $n^{th}-$order expansion around λ_1 under the assumption of a small value for that parameter.[4] Adapting the theoretical results in Hugonnier et al. (2009) to the post-retirement phase reveals the following:

Theorem 1 *Assume that the following conditions hold:*

$$A \equiv \varepsilon\rho + (1 - \varepsilon)\left(r - \lambda_0 + \frac{1}{2\gamma}\theta^2\right) > 0, \tag{7}$$

$$\beta^{\mathrm{r}} < (r + \delta)^{\frac{1}{\alpha}}, \tag{8}$$

$$A > \left(r - \lambda_0 + \frac{1}{\gamma}\theta^2\right)^+, \tag{9}$$

$$\Delta^{-1} \equiv A + \xi\left((\alpha B)^{\frac{\alpha}{1-\alpha}} - \delta\right) > r - \lambda_0 + \frac{\theta^2}{\gamma}. \tag{10}$$

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[3]See Kogan (2001), Kogan and Uppal (2002), Chan and Kogan (2002) and Ferretti and Trojani (2005) for applications of expansion analysis in different contexts.

[4]We verify and confirm in Section 4 that the estimated λ_1 is indeed numerically small, but nonetheless significant.

Then, up to a first-order approximation, the agent's indirect utility $V_{j,t}$, his net disposable total wealth $N_{j,t}$, his value of risky asset holdings $W_{j,t}^e \equiv \pi_{j,t} W_{j,t}$, and his health expenditures $I_{j,t}$ in the post-retirement phase are:

$$V_{j,t} = \rho \left(\frac{A}{\rho} \right)^{\frac{1}{1-\varepsilon}} N_{j,t} [1 - \lambda_1 H_{j,t}^{-\xi} \Delta], \tag{11}$$

$$N_{j,t} = W_{j,t} + BH_{j,t} + C, \tag{12}$$

$$W_{j,t}^e = \frac{\theta}{\gamma \sigma} N_{j,t}, \tag{13}$$

$$I_{j,t} = H_{j,t} (\alpha B)^{\frac{1}{1-\alpha}} + \lambda_1 H_{j,t}^{-\xi} \Delta (\alpha B)^{\frac{\alpha}{1-\alpha}} \eta N_{j,t}, \tag{14}$$

where B solves

$$\beta^{\mathrm{r}} - (r + \delta)B + \Phi B^{\frac{1}{1-\alpha}} = 0, \tag{15}$$

subject to

$$B > \frac{(r+\delta)^{1-\frac{1}{\alpha}}}{\alpha},$$

and where C, η and Φ are defined by:

$$C \equiv (y^{\mathrm{r}} - a)/r, \tag{16}$$

$$\eta \equiv \alpha \xi / (1 - \alpha),$$

$$\Phi \equiv (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}}.$$

The constants A, B, C, Δ denote respectively the agent's marginal propensity to consume out of disposable wealth, the order-0 shadow price (i.e. marginal-Q) of health,[5] the net present value (NPV) of the fixed portion of labor income above subsistence, and a first-order correction to the optimal rules. The theoretical restrictions (8), (9) and (10) guarantee that the agent's disposable wealth and value function are finite and can be interpreted as transversality conditions.

As shown in Hugonnier et al. (2009), this model can jointly reproduce the four main empirical facts. First, because health positively affects labor income, the capitalized value of labor revenues is also health-dependent and determines the human capital of the agent, thereby

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[5] As was mentioned earlier, health is valuable because of its dual effects on longevity and on labor income. The shadow price capturing the labor-income effect only obtains by abstracting from the former (i.e. imposing $\lambda_1 = 0$, or order-0 effect) and is given by B. See (24) and the discussion in Section 5 below for estimates of the total value of health.

contributing to disposable net worth $N_{j,t}$ in (12). Second, the minimal consumption creates an endogenous liquidity contraint whereby available resources must be kept sufficiently high to cover subsistence consumption. Improvements in health and/or wealth increase surplus net worth thereby allowing the agent to take on more risky asset positions in (13). This rationalizes the positive health and wealth gradients for risky portfolios found in the data. Third, an increase in health reduces detrimental mortality risk and, because of cost convexities, lowers the returns to health expenditures, thereby reducing the attractiveness of health investments. Higher wealth facilitates substitution in favor of other non-human assets in (14).[6] This rationalizes the negative health and wealth gradients in health expenditures shares data.

3   Empirical analysis

In this section, we present the data, as well as the empirical strategy that we use to conduct the estimation of the structural model (13) and (14) .

3.1   Data

For our benchmark case, we rely on a cross section composed of the fifth wave (respondents in 2000) of the Health and Retirement Survey data set, a survey of American individuals aged 51 and over.[7] The robustness analysis in Section 4.2 considers other waves, and also exploits the panel dimension in the HRS data.

We construct financial wealth as the sum of safe assets (checking and saving accounts, money market funds, CD's, government savings bonds and T-bills), bonds (corporate, municipal and foreign bonds and bond funds), risky assets (stock and equity mutual funds) and retirement accounts (IRAs and Keoghs). The risky portfolio share is then expressed as the percentage of financial wealth held in risky assets.

Health status is evaluated using the self-reported general health status. This variable has been shown to be a valid predictor of the objective health status (Benítez-Silva and Ni, 2008; Crossley and Kennedy, 2002; Hurd and McGarry, 1995). Health investments are obtained as the sum of medical expenditures (doctor visits, outpatient surgery, hospital and nursing home,

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[6]This last effect occurs unless health is so low that its high return justifies investing more in one's health when financial wealth increases.

[7]We use the HRS distribution made available by the RAND Center for the Study of Aging. See RAND Corporation (2008) for details.

home health care, prescription drugs and special facilities), and out-of-pocket (OOP) medical expenses (uninsured cost over the two previous years). Health investment shares are computed by dividing health investment by financial wealth. The estimates presented in Section 4 are obtained for a scaling of \$1M applied to all nominal variables (I_j, W_j, Y_j) and by expressing the polytomous self-reported health variable in integer values ranging from 1 (poor) to 5 (excellent).

[Insert Table 1 about here]

The summary statistics in Table 1 distinguish between non retired individuals (age less than 65) in columns (1)–(3) and retired agents (age 65 and over) in columns (4)–(6). They highlight a deterioration in health status as well as an increase in health expenditures in the post-retirement phase. We also notice that riskless assets clearly dominate the financial allocations. Direct holdings of stocks are found in about a third of our sample and correspond to roughly one fifth of the portfolios, with negligible variations between pre- and post-retirement phases.

[Insert Table 2 about here]

Table 2 reports summary statistics by health level and gross financial wealth quintiles. A first observation concerns the relative insensitivity of financial wealth to the health status (see also Michaud and van Soest, 2008; Meer, Miller and Rosen, 2003; Adams, Hurd, McFadden, Merrill and Ribeiro, 2003, for additional evidence). Second, we notice the very low participation rates in stock markets for the poorer and unhealthy agents which increases with health, and wealth. Direct risky portfolio shares are also increasing in both health and wealth. Similar positive effects of wealth on risky holdings have been identified in the literature (e.g. Wachter and Yogo, 2008; Guiso et al., 1996; Carroll, 2002) whereas positive effects of health have also been highlighted (e.g. Guiso et al., 1996; Rosen and Wu, 2004; Coile and Milligan, 2009; Berkowitz and Qiu, 2006; Goldman and Maestas, 2005; Fan and Zhao, 2009; Yogo, 2009). Health expenditures shares of financial wealth, whether total or out-of-pocket display the opposite patterns: They are sharply decreasing in wealth and in health levels. Similar findings with respect to wealth (e.g. Meer et al., 2003; DiMatteo, 2003; Gilleskie and Mroz, 2004; Acemoglu et al., 2009) and health (e.g. Smith, 1999; Gilleskie and Mroz, 2004; Yogo, 2009) have been discussed elsewhere.

3.2 Estimation strategy

Our objective is to estimate the deep parameters of the structural model (1)–(5) by using the optimal rules (13) and (14) subject to the transversality inequalities (8), (9) and (10). Since the dynamic model is fully amenable to a static as well as a dynamic perspective, we consider both the cross-sectional and panel perspectives.

The estimation of the structural parameters poses several challenges. First, a sizable percentage of the portfolio shares are reported at zero, whereas this is not the case for either the income or the health expenditures data.[8] This implies that some of our dependent variables are possibly left-censored at zero and that a censored-data (Tobit) estimator is warranted. Second, and related, the theoretical restrictions in Theorem 1 are highly nonlinear and these nonlinearities are compounded by the Tobit estimator, thereby making the estimation particularly challenging.

Fortunately, we may observe that, conditional upon a given value for the parameter ξ, the model in Theorem 1 can be rearranged and regrouped with the income process to reveal a linear econometric model:

$$W_{j,t}^e = \theta_{\pi,0} W_{j,t} + \theta_{\pi,1} H_{j,t} + \theta_{\pi,2} + \nu_{j,t}^\pi \tag{17}$$

$$I_{j,t} = \theta_{I,1} H_{j,t} + \theta_{I,2} H_{j,t}^{-\xi} W_{j,t} + \theta_{I,3} H_{j,t}^{1-\xi} + \theta_{I,4} H_{j,t}^{-\xi} + \nu_{j,t}^i \tag{18}$$

$$Y_{j,t}^{\mathrm{r}} = y^{\mathrm{r}} + \beta^{\mathrm{r}} H_{j,t} + \nu_{j,t}^y, \tag{19}$$

where $(\nu_{j,t}^\pi, \nu_{j,t}^i, \nu_{j,t}^y)$ are potentially correlated error terms, and where the semi-restricted reduced form (SRF) parameters $\theta_\pi \in \mathbb{R}^3$ and $\theta_I \in \mathbb{R}^4$ are related to the deep parameters as follows:

$$
\begin{aligned}
\theta_{\pi,0} &= \frac{\theta}{\gamma\sigma}, & \theta_{I,1} &= (\alpha B)^{\frac{1}{1-\alpha}}, \\
\theta_{\pi,1} &= \theta_{\pi,0} B, & \theta_{I,2} &= \frac{\alpha \lambda_1 \xi (\alpha B)^{\frac{\alpha}{1-\alpha}}}{\left(\xi\left((\alpha B)^{\frac{\alpha}{1-\alpha}} - \delta\right) + A\right)(1-\alpha)}, \\
\theta_{\pi,2} &= \theta_{\pi,0} C, & \theta_{I,3} &= \theta_{I,2} B, \\
& & \theta_{I,4} &= \theta_{I,2} C,
\end{aligned}
\tag{20}
$$

and A, B, C are defined as in Theorem 1.

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[8]Specifically, out of our full sample of 10,735 individuals aged 65 and over, 7,261 (67,6%) reported no direct portfolio holdings, compared to only 48 (0.4%) reporting no labor income, and 225 (2.1%) reporting no medical expenditures.

This formulation suggests adopting a two-stage approach. In the first stage, we compute the likelihood function for a joint mixture–ML model combining one censored density in (17) with two continuous densities in (18) and (19).[9] This tri-variate econometric model estimates the SRF parameters θ_π, θ_I, as well as the structural parameters y^r, β^r, imposing the functional form, but not the full set of parametric constraints. In the second stage, we estimate the remaining structural parameters by using a minimum distance estimator between the estimated SRF parameters and their theoretical counterparts given in (20). The standard errors are computed by implementing the Delta method. This two-step procedure presents important advantages over a single-step, fully structural estimation. First, it is considerably easier to implement.[10] Second, and as is discussed below, it provides a set of first checks of the theoretical model through the signs of the SRF parameters.

The first-stage estimation yields seven free parameters. In order to ensure identification and full compliance with theory, we append the relevant theoretical constraints to (20) in the second step such that the structural parameters are identified subject to the nonlinear transversality inequalities (8), (9) and (10).[11] Since the model is under-identified, we follow standard approaches in calibrating a subset of the parameters, estimating the remaining deep parameters, and verifying robustness to key calibrated values. First, a natural choice is to calibrate the parameters of the financial markets (r, σ, μ), as well as the subjective discount rate (ρ) for which data and ample literature both provide guidance. Second, fixing the convexity of the Poisson intensity (ξ) implies that the SRF model is conditionally linear and considerably facilitates the estimation. We consequently calibrate that parameter and verify robustness in Section 4.2. Table 3 summarizes the calibrated and estimated parameter subsets.

[Insert Table 3 about here]

Several hypotheses can be tested through the joint estimation of the structural parameters. First, the estimated preference parameters in (1) and (2) allow us to test the null hypotheses

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[9]An alternative interpretation is to consider the zeroes in the portfolios as deliberate choices, rather than reflecting a binding non-negativity (e.g. short-sales) constraint. To take that perspective into consideration, we also estimated the trivariate model with a continuous density for (17). The results in Section 4 remain qualitatively robust.

[10]We also verify and confirm that the results in Section 4 are robust to using a single-step ML estimation of the fully restricted structural model instead of the two-step procedure.

[11]It is worth noting that exact identification can be achieved if the structural parameters of interest satisfy the (strict) inequality constraints without imposing them. Otherwise, the inequality constraints will preclude for such an exact scheme, as in our setting.

of no aversion to a-temporal risk ($\gamma = 0$), of no subsistence consumption ($a = 0$), of inelastic inter-temporal substitution of deterministic consumption paths ($\varepsilon = 0$), as well as the null of time additive preferences in the absence of mortality risk ($\varepsilon = 1/\gamma$). Second, the mortality intensity parameters in (6) make it possible to test the null hypotheses that there is no exogenous mortality component ($\lambda_0 = 0$), and that the agent has no control over mortality risk ($\lambda_1 = 0$). Third, the health dynamics parameters in (3) can be used to test whether the agent has some control over the evolution of his health status ($\alpha \neq 0$) and whether he faces convex adjustment costs ($\alpha \neq 1$) in health investments, in addition to testing that health does not depreciate exogenously ($\delta = 0$) in the absence of health expenditures. Finally, we can test for the presence of health-independent income ($y^{\mathrm{r}} = 0$) and of health-dependent labor income in the post-retirement phase ($\beta^{\mathrm{r}} = 0$) using the income process parameters in (4). From (15) observe that the latter is equivalent to testing for zero shadow value of health ($B = 0$).

4 Results

We first discuss the estimation results for the SRF parameters (Table 4), followed by the structural parameter estimates (Table 5). In Section 4.1, we present the benchmark case reported in column (1) for both tables. In Section 4.2, we address in-sample and out-of-sample performance, followed by a discussion of the various robustness checks that are reported in columns (2)-(9).

4.1 Benchmark case

Table 4 presents the maximum likelihood estimates of the SRF parameters θ_π, θ_I for the risky asset levels (17) and health investments levels (18).[12] We also indicate the expected sign for each parameter which is obtained by combining the definition of the SRF parameters with the theoretical restrictions summarized in (20).

The estimated SRF parameters provide a first indication on the model's ability to reproduce the data. First, the parameter $\theta_{\pi,0}$ is significantly positive and, given a positive financial risk premia (see the calibrated values in Table 3), is consistent with strictly positive risk

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[12]The deep parameters of the income process (19), which are estimated jointly in the trivariate mixture model, are reported with the other structural parameters in Table 5. For brevity, we omit the full reporting of the variance-covariance parameters.

aversion. Second, the theoretical restrictions outlined in Hugonnier et al. (2009) indicate that the parameters $\theta_{\pi,1}, \theta_{I,1}, \theta_{I,2}, \theta_{I,3}$ should be positive, which we do observe, with the exception of $\theta_{I,2}$ which has the correct sign but is not statistically significant. Third, we should also expect $\theta_{\pi,2} < 0$ and $\theta_{I,4} < 0$ for portfolios to be profactually increasing in financial wealth.[13] Our estimates confirm that this is indeed the case.

[Insert Table 4 about here]

The SRF parameters thus provide an encouraging first evaluation of the model. In particular, all the seven reduced-form parameters have the correct signs and are (with one exception) all significant. Our next objective is to recover the structural parameters in the second step of our estimation strategy. This is achieved by minimizing the distance between the SRF parameters and their theoretical counterparts listed in (20), subject to the transversality inequalities (8), (9), and (10). Importantly, the preferences, mortality, health dynamics and income process parameters in Table 5 are all significant at the 1% level, and all have the required sign (i.e. positive).

[Insert Table 5 about here]

The estimate for γ in (1) is indicative of aversion to atemporal risk and is realistic compared with the usual standards (e.g. Mehra and Prescott, 1985). The estimate of the elasticity of intertemporal substitution ε in (2) is significantly lower than 1 and accords with similar findings in the literature (e.g. Engelhardt and Kumar, 2009; Lee, 2008; Biederman and Goenner, 2008; Saltari and Ticchi, 2007; Vissing-Jørgensen, 2002, provide recent examples). Moreover, the null hypothesis of time additive preferences in the absence of mortality risk ($\varepsilon - 1/\gamma = 0$) is strongly rejected. Finally, the subsistence consumption parameter a in (2) corresponds to a minimal consumption of $0.0248 \times 10^6 = \$24,800$ which is lower than the mean labor income of \$28,709 for agents over 65, but remains larger than the fixed part of the income process for retired agents ($y^r = \$9,096$). This is consistent with agents having to work and/or hold positive financial balances to finance subsistance consumption.

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[13]To understand this result, observe from (13) that:

$$\text{Sign}\left(\frac{\partial \pi_{j,t}}{\partial W_{j,t}}\right) = \text{Sign}\left(-\theta_{\pi,1} H_{j,t} - \theta_{\pi,2}\right),$$

which is positive when $\theta_{\pi,2} < 0$, given nonnegative health and that $\theta_{\pi,1}$ is expected to be positive. Because we expect and find $\theta_{\pi,0} > 0$ and that $\theta_{I,2} > 0$, (20) reveals that $C < 0$, and, consequently that $\theta_{I,4} < 0$.

Regarding the parameters of the death intensity (6), we find a significant λ_0 pointing towards an incompressible component to mortality risk. Furthermore, the estimate of the endogenous mortality parameter λ_1 is low, adding confidence in the expansion method of Hugonnier et al. (2009) constructed around a small value for that parameter. We nonetheless reject the null that λ_1 is zero, indicating that the agent can adjust mortality risk through health investments. Moreover, the estimates for the health dynamics (3) identify a low value for the Cobb-Douglas parameter α that is significantly different from both 0 and from 1 as well as a fairly high depreciation rate for the health stock δ. This is consistent with agents being able to offset rapid depreciation in the health status, but having to face large adjustment costs in doing so. Finally, the parameters of the income process (4) confirm the relevance of both the fixed and of the health-dependent components in post-retirement labor income.

Implications for health expenditures Our structural estimates are indicative of a dual motivation for investing in health. First, the strong rejection of the VNM hypothesis ($\varepsilon \neq 1/\gamma$) and the parameter for risk aversion ($\gamma > 1$) are jointly consistent with unconditional preference for life. In addition, we find that the marginal rate of substitution for consumption across periods is very low ($\varepsilon < 1$) indicating that our agents do not appear to substitute easily between quantity (i.e. length) and quality (i.e. consumption) of life. Taken together, these elements suggest that an agent reacts to increased mortality risk by investing more in his health, rather than by consuming more over his shorter time horizon. This can be observed from (7): the marginal propensity to consume A is lower following an increase in λ_0 when the elasticity of inter-temporal substitution is low, i.e. $\varepsilon < 1$. In particular, an increase in λ_0 can be interpreted as an decrease in the riskfree rate of interest. At low elasticity, the income effect outweighs the substitution effect and consumption decreases to finance more health investment. This response is consistent since investing more in one's health to increase life expectancy is not only desirable, but also feasible as the agent *can* reduce his death intensity ($\lambda_1 \neq 0$).

Second, better health is also valuable in that it allows for increased labor revenues ($\beta^{\mathrm{r}} \neq 0$), notwithstanding the fact that fixed part of labor income remains important in the post-retirement period ($y^{\mathrm{r}} \neq 0$). The latter likely reflects the importance of pension revenues while the former could indicate that many elders still find it profitable to continue working after age

65, but are forced to reduce hours (and therefore income) when health deteriorates.[14] Better health thus results in higher total disposable wealth (financial + human) and consequently allows for higher consumption. These two channels of higher quantity and quality of life are the main elements behind positive net investments in health capital. Preferences-based approaches play no role as health has no other intrinsic value in the model.

The technological constraints facing the agent indicate that health and mortality risk adjustments are feasible, but are both subject to strongly convex costs ($\xi \gg 0$ and $\alpha \ll 1$). Equivalently, these results imply that the return to health expenditures increases sharply when health is low and further deteriorates. This would prompt the agent to invest more in health in adverse health conditions, provided resources (whether financial or human wealth) are available. By a similar reasoning, increasing wealth reduces the health expenditure shares, unless health is so low (and therefore returns are high) that it becomes more profitable to invest in one's health than in other assets.

Finally, our findings point to the "long reach of childhood" effects (Smith, 1999, 2009) in mortality risk. Indeed, the bulk of expected longevity stems from the uncontrollable component, with health investments having much more modest effects ($\lambda_0 \gg \lambda_1 H^{-\xi}$). Taken together, these results are very likely related to our choice of sample of elderly agents for which investing in health is concurrently more urgent because of rapid depreciation, more costly because of strong convexities and less profitable because of the importance of endowed components in income and mortality compared to younger individuals.

Implications for risky portfolios Our results are also consistent with a binding liquidity constraint argument to understand how health and wealth affect risky portfolios. First, the low participation rate in the risky asset market is not explained by excessive risk aversion ($\gamma < 2$), but occurs because financial wealth must be sufficient to cover liquidity needs. Indeed, we find that the minimal consumption is quite important ($a \gg 0$). This implies that the present value of income net of health investments and net of subsistence consumption expenditures is negative ($BH + C < 0$) and that positive financial wealth balances have to be maintained. This has two consequences. First, if he could, a very poor and unhealthy agent would take short positions in both financial and health investments in order to cover subsistence. Whereas the former is

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[14]See French (2005) for similar findings.

feasible, irreversibility rules out the latter. Second, an increase in either health or wealth is tantamount to a relaxation of the agent's binding liquidity constraint. He thus increases risky asset holdings unless his health is sufficiently high that the liquidity contraint no longer binds.

Importantly, the positive effect of health on risky portfolios is neither related to mortality risk nor to risk aversion effects. Indeed, the predicted portfolio parameters $\theta_{\pi,0}, \theta_{\pi,1}, \theta_{\pi,2}$ in (17) are completely independent of the Poisson parameters $\lambda_0, \lambda_1, \xi$. Hence, changing mortality risk has no first-order impact on risky asset holdings.[15] The model neither relies on cross effects of health on risk aversion as the latter is a constant parameter (γ). Rather, the model resorts to effects of liquidity of health on income to explain why portfolios increase in health.

4.2 Performance and robustness

Performance In order to assess the in-sample performance of the model we compute the predicted portfolios and health investments and contrast them with observed levels. Specifically, for the model (13) and (14), we calculate the predicted portfolio and health investment levels for all retired agents in our sample.[16] We then compute the mean for each age $t = \{65, 66, \dots\}$ and compare those age-specific averages with their HRS counterparts. The results are plotted in the right-hand sides of Figures 1 (portfolio levels) and Figure 2 (health investment levels).

[Insert Figures 1 and 2 about here]

Overall, the estimated model appears to reproduce the data quite well.[17] In particular, the estimated risky portfolios in Figure 1 capture both the life-cycle profile and the age-to-age volatility in HRS sample means. Although less conclusive, the fit for the estimated health investment shares in Figure 2 remains acceptable in capturing the age-to-age variations. Admittedly, the model has more difficulty in reproducing the long-term age gradient, especially in the last periods of life where health expenditures tend to explode. This could suggest that appending age-dependent processes to the theoretical model (e.g. through age-specific health

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[15]It can be shown that the effects of mortality on risky portfolios are second- and higher-order effects in the approximation to the solution (Hugonnier et al., 2009). However, those effects are numerically negligible, given the low estimated value for λ_1. Similar findings are reported by Puri and Robinson (2007) who find that optimism (measured by self-reported life expectancy) has no impact on portfolio composition.

[16]Note that the predicted portfolio values incorporate a correction associated with censored data estimated by Tobit models. See Greene (1990); Cameron and Trivedi (2005) for discussions.

[17]A word of caution concerns the age-to-age variation which is clearly not indicative of any plausible life-cycle pattern but rather caused by differences in the sample means for each age group.

depreciation and/or mortality risk) might be fruitful. We leave such modifications on the research agenda.

One may legitimately inquire about the performance of the model with respect to younger agents. Indeed, the theoretical model of Hugonnier et al. (2009) is derived under the assumption of age-invariant deep parameters, except for the income process which varies with the employment status. This assumption can be gauged by computing and comparing the predicted optimal allocations with the data in the period preceding retirement. For that purpose, replace the agent's income (4) by:

$$Y_t = Y(t, H_t) = 1_{\{T > t\}} Y_t^{\mathrm{e}} + 1_{\{T \leq t\}} Y_t^{\mathrm{r}}. \tag{21}$$

where

$$Y_t^i = Y^i(H_t) = y^i + \beta^i H_t, \tag{22}$$

for some nonnegative constants y^{e}, y^{r}, β^{e} and β^{r}, and where $T = 65$ is the retirement age. We can then combine estimates for the pre-retirement income parameters $y^{\mathrm{e}}, \beta^{\mathrm{e}}$ in the general income process (21) and (22) with our post-retirement estimates $y^{\mathrm{r}}, \beta^{\mathrm{r}}$ as well as the other deep parameters to calculate the optimal pre-retirement portfolios and health expenditures using the age-dependent expressions for $B(t), C(t)$, and $\Delta(t)$ reported in Appendix A.[18] Since our parameter estimates are evaluated using post-retirement data exclusively, this approach can be interpreted as an out-of-sample evaluation of the model. The out-of-sample results plotted in the left-hand-side of Figures 1 and 2 confirm our earlier in-sample findings. The model continues to perform surprisingly well with respect to portfolios and reasonably well with respect to health expenditures. Overall, we may conclude that the constant deep parameter hypothesis of the model does not seem to be at odds with the out-of-sample data.

Robustness We next perform various robustness checks that are reported in columns (2)–(9) of Tables 4 and 5. Regarding the Poisson convexity parameter, we perform an extensive search procedure to verify the sensitivity in terms of fit and compliance with theoretical restrictions.

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[18]Hugonnier et al. (2009, Table 5) report pre-retirement constant term $y^{\mathrm{e}} = 0.0052$ (0.0051) and health sensitivity $\beta^{\mathrm{e}} = 0.0130$ (0.0014) (standard errors in parentheses) using the same data set.

To illustrate robustness, we provide results for $\xi \in \{4.2, 4.7\}$, our preferred range with respect to fit, in columns (2) and (3).

Furthermore, we conduct the following additional estimations. First, we assess the impact of socio-economic covariates that are omitted from the theoretical analysis in column (4). Indeed, whereas financial covariates are explicitly (wealth) and implicitly (income) incorporated into the model, other variables that are known to predict portfolios and health expenditures are not. We thus re-estimate the SRF model (17)–(18) by appending age, gender, race, education and marital status. Second, since the HRS study mainly involves older individuals with an important attrition rate, potential cohort effects are to be anticipated. To control for these, we re-estimate the model for the fourth (1998) and sixth (2002) waves in columns (5) and (6).

Third, in order to account for the effects of unobserved heterogeneity, we re-estimate the model in the panel dimension with results reported in column (7). Specifically, we construct a balanced panel of 5,736 individuals using the fourth, fifth and sixth waves of HRS (17,208 observations).[19] Unobserved heterogeneity is modeled using random, rather than fixed effects. This choice is motivated by several concerns. Indeed, fixed effects are both more complicated to estimate than random effects in mixture models, and more difficult to justify in the absence of a constant term in the health investment equation (18). In comparison, the second moments of the error terms for the optimal rules are not restricted by the theoretical model, thereby allowing complete flexibility in modeling the scedastic structure. Finally, not resorting to fixed effects is consistent with the assumed representativeness of the HRS sample with respect to the entire US population of retirees.

Fourth, we control for the impact of health insurance by re-estimating the parameters using out-of-pocket health expenditures only in column (8). Finally, several studies document a flat age profile in health expenditures after retirement, but a very rapid increase during the last periods of life (e.g. Zweifel, Felder and Werblow, 2004; Gerdtham and Jönsson, 2000; Felder, Meier and Schmitt, 2000). This increase is apparent in Figure 2 where the theoretical model fares less well at later ages. To take these differences into account, we re-estimate the model using agents aged 65-79 only in column (9).

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[19]Using the full HRS set of waves proved impractical for our purposes. As is well known, differences in construction methods exists between the initial (waves 1-3) and posterior (waves 4-8) waves (e.g. Love and Smith, 2010). Also, the attrition rate when using the full set of waves proved to be too important to allow for the construction of a representative balanced panel of retired agents. For these reasons we restrict our analysis to waves 4, 5 and 6.

Overall, all our qualitative results (i.e. compliance with theoretical and sign restrictions, significance) are remarkably robust to the choice of specification, whereas the quantitative impact can be considered as rather limited. In particular, the preference, mortality dynamics as well as income process parameters are the least sensitive to alternative specifications. On the other hand, the health dynamics estimates do vary moderately when socio-economic covariates are included, for the 1998 wave (although not for the 2002 wave) and when the very old are omitted. Since the OOP health expenditures are smaller than the total expenditures in column (8), this has no impact on the portfolio parameters, and only scale effects on the health expenditure parameters. Nonetheless, the deep parameters remain globally unaffected. In light of these results we conclude that the estimates are qualitatively robust to potential sample and mis-specification errors.

5   Extensions: Longevity, values of life and of health

The following theoretical and empirical results show how the model can be extended to study the expected lifetime and the willingness to pay for additional longevity and better health. In addition to providing for new insights on the effects of health and wealth on these variables, they can act as useful ex-post checks on the estimated parameters.

5.1   Additional theoretical results

The explicit expression for the agent's value function (11) makes it possible to compute the implied value of health and of life as the maximum amount of wealth that the agent would be willing to give-up in order to improve either his health or his life expectancy. Resorting to the compensating variation approach, we define the value of τ^h units of additional health as the solution Δ^h to the indifference equation

$$V(W_{j,t} - \Delta^h, H_{j,t} + \tau^h; (\lambda_0, \lambda_1)) = V(W_{j,t}, H_{j,t}; (\lambda_0, \lambda_1)) \tag{23}$$

where $V(W, H; \lambda_0, \lambda_1)$ denotes the value function of an agent with wealth W, health H and mortality parameters (λ_0, λ_1). The following proposition relies on an expansion technique similar to that of Theorem 1 to derive an explicit approximation for the value of health.

Proposition 1 *Assume that equations* (8), (9) *and* (10) *hold true. Up to a first order approximation the value of one unit of additional health is given by*

$$\Delta_{j,t}^h = B\tau^h + \lambda_1 N_{j,t} \Delta \left[H_{j,t}^{-\xi} - (H_{j,t} + \tau^h)^{-\xi} \right] \tag{24}$$

with B, $N_{j,t}$ *and* Δ *as in Theorem* 1.

By a similar reasoning, if

$$\ell(W, H; (\lambda_0, \lambda_1)) = E_t[\tau] \tag{25}$$

denotes the life expectancy of an agent with wealth W, health H and mortality parameters (λ_0, λ_1), then we can obtain the value of τ^ℓ additional units of life expectancy as the solution Δ^ℓ to the indifference equation

$$V(W_{j,t} - \Delta^\ell, H_{j,t}; (\lambda_0^*, \lambda_1)) = V(W_{j,t}, H_{j,t}; (\lambda_0, \lambda_1)) \tag{26}$$

where the modified intensity $\lambda_0^* < \lambda_0$ solves

$$\ell(W, H; (\lambda_0^*, \lambda_1)) = \tau^\ell + \ell(W, H; (\lambda_0, \lambda_1)). \tag{27}$$

We thus reduce the endowed death intensity so as to gain τ^ℓ units of longevity, and the value of life is the willingness to pay for that reduction in mortality risk. We again resort to an expansion technique to derive an explicit approximation for both the life expectancy and the value of life implied by the theoretical model.

Proposition 2 *Assume that equations* (8), (9), (10) *and*

$$\Psi^{-1} \equiv \lambda_0 + \xi \left((\alpha B)^{\frac{\alpha}{1-\alpha}} - \delta \right) > 0 \tag{28}$$

hold true. Then up to a first order approximation, the life expectancy and the value of τ^ℓ *additional units of life expectancy are given by*

$$\ell(W_{j,t}, H_{j,t}; \lambda_0, \lambda_1) = \frac{1}{\lambda_0} \left(1 - \frac{\lambda_1}{H_{j,t}^\xi} \Psi \right), \tag{29}$$

and

$$\Delta_{j,t}^{\ell} = (1 - \chi_0)N_{j,t} + \lambda_1\chi_0 N_{j,t}H_{j,t}^{-\xi}\left(\Delta - \Delta^* - K^*\right) \tag{30}$$

where the constants $\chi_0 \in (0,1)$, Δ^* *and* K^* *are defined by*

$$A^* = \varepsilon\rho + (1 - \varepsilon)\left(r - \frac{\lambda_0}{1 + \tau^{\ell}\lambda_0} + \frac{\theta^2}{2\gamma}\right),$$

$$\chi_0 = (A/A^*)^{\frac{1}{1-\varepsilon}},$$

$$\Delta^* = A^* - A + \Delta,$$

$$K^* = \tau^{\ell}\frac{\Psi}{A^*}\left(\frac{\lambda_0}{1 + \tau^{\ell}\lambda_0}\right)^2\left(\frac{1 + \lambda_0(\tau^{\ell} + \Psi)}{1 + \tau^{\ell}\lambda_0(1 - \lambda_0\Psi)}\right)$$

and Δ, $N_{j,t}$ *are as in Theorem 1.*

To understand these results it is useful to distinguish between the two attributes of health: labor income enhancement and mortality control. If we abstract from the latter by imposing $\lambda_1 = 0$, the expected longevity in (29) is then the inverse of the endowed death intensity, and the value function $V_{j,t}$ in (11) is then proportional to net disposable wealth $N_{j,t} = W_{j,t} + BH_{j,t} + C$. An increase in health raises net disposable wealth by $B\tau_h$ which must also be deducted from financial wealth in (24) so as to leave the agent indifferent. On the other hand, a reduction in the endowed mortality risk λ_0 necessary to increase expected lifetime in (27) has no effect on net disposable wealth, but changes the marginal propensity to consume A in (7). Since the marginal propensity to consume determines the factor of proportionality in the value function, the compensating variation for added longevity in (30) will also be proportional to net disposable wealth $N_{j,t}$.

Allowing for $\lambda_1 \neq 0$ reinstates the mortality control value of health. The expected longevity in (29) is then lower because of the mechanical increase in the death intensity by $\lambda_1 H_{j,t}^{-\xi}$. Because life is always valuable, this increase is detrimental to the agent and the indirect utility $V_{j,t}$ in (11) is lower for any given disposable net wealth. The agent is thus willing to pay more for better health in order to offset that effect, with the increment in valuation reflecting the effectiveness of health in reducing the death intensity. The effect on the value of life, however, is less apparent under endogenous mortality because of two conflicting influences. On the one hand the lower endowed intensity is a welcomed counter-measure to the mechanical increase

in the endogenous intensity. This increases the willingness to pay for longevity. On the other, a reduction in λ_0 is not as valuable when the agent can affect longevity through his health decisions, compared to when he cannot. This reduces the willingness to pay. In particular, at the estimated low elasticity of inter-temporal substitution $\varepsilon < 1$, the marginal propensity to consume increases following a reduction in the death intensity (i.e. $A^* > A$, implying $\Delta < \Delta^*$). However, because of the mechanical increase in the death intensity, the required decline in λ_0 is more important under endogenous mortality, i.e. $(\lambda_0^*)'(\lambda_1) < 0$ which implies that $K^* < 0$ in (45). The net effect on the value of life is captured by the term $(\Delta - \Delta^* - K^*)$ in (30) whose sign remains uncertain.

5.2 Implied variables

We first calculate the value of health corresponding to a 1 unit increase in H, i.e. $\tau^h = 1$, using (24). The corresponding values are plotted in Figure 3 for various health and wealth levels.[20] These plots reveal that health values rapidly converge to their labor income value $B = \$60,205$, an estimate which is comparable to those found in the literature.[21] An exception is observed for the richer, but unhealthy individuals, who value health more. This can be understood from (24) where the mortality control value increases at low health levels but rapidly becomes negligible when health improves given both the predominance of the endowed death probability and the very high degree of cost convexities faced by elders in adjusting mortality.

[Insert Figure 3 about here]

The implied life expectancy (29) can be compared with expected lifetime estimated by Lubitz, Cai, Kramarow and Lentzner (2003) for an individual aged 70, again at various self-reported health levels. The results reported in Table 6 show that our model is able to fit the expected lifetime quite well even though we somewhat underestimate the health gradient. This last caveat may be explained by the relatively high value of the convexity parameter ξ which tends to dampen the effect of health on the mortality intensity. Nonetheless, the life expectancy (29) remains an increasing function of health , and, given health, is independent of wealth. Both

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[20]Very low health and wealth levels correspond to negative disposable to total wealth $N_{j,t}$ and are not reported.
[21]For example, Murphy and Topel (2006, Fig. 9, p. 901) report estimates for health benefits unrelated to mortality control (referred to as "quality of life" or "type-H" benefits) below \$100,000 at age 85.

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facts are consistent with empirical findings by De Nardi, French and Jones (2009) and by Hurd, McFadden and Merrill (2001).

[Insert Table 6 about here]

Using (30), we next look at the annuitized values of life corresponding to a 1-year increase in expected lifetime, i.e. $\tau^\ell = 1$, and plot the results in Figure 4. We find that an agent aged 75 with \$300,000 financial wealth would be willing to pay between \$35,000 (poor health) and \$276,000 (excellent health) for an additional year of longevity. Interestingly, these can be contrasted with age-adjusted value of statistical life year (VSLY) estimates, confirming that our results are again close to those found in the literature.[22]

[Insert Figure 4 about here]

The estimates are consistent with wealthier and healthier agents willing to pay more for an increase in expected lifetime. Indeed, the low estimated λ_1 and high calibrated ξ imply that the value of life (30) is dominated by the exogenous mortality term $(1 - \chi_0)N_{j,t}$ and that the endogenous mortality component is negligible. Since net disposable wealth increases in both health and wealth, it follows that richer and healthier agents value longevity more.

6   Conclusion

Financial wealth and health status are strong predictors of risky portfolios and health expenditure shares of wealth with the former increasing and the latter falling in both variables. Potential explanations include longevity, utility-based and human-capital arguments. This paper proposes a structural empirical analysis to distinguish among these competing hypotheses. Using survey data on retired agents' health expenditures and financial allocations, we estimate the preference, technological and risk distribution parameters of the closed-form solutions to a joint dynamic model of health accumulation, consumption and asset allocations.

We find that health and mortality adjustments remain both desirable and feasible, but subject to steeply decreasing returns and rapid depreciation. Taken together, these results imply

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[22]In particular, Aldy and Viscusi (2008, Fig. 2, p. 579) report VSLY estimates between \$100,000 and \$350,000 at age 62. Murphy and Topel (2006, p. 886) find VSLY estimates of \$373,00 at age 50 and falling by 50% at age 80.

that the effects of health and wealth on health investment shares can be entirely accounted for by longevity and human-capital arguments, whereas the corresponding effects on portfolios are explained by the latter exclusively without resorting to utility-based rationales, such as health-dependent risk aversion. Some important extensions can also be obtained from the estimated parameters. Indeed, our corresponding estimates of expected longevity, as well as the values of life and health compare advantageously with other estimates in the literature. They also provide new insights on how these variables are affected by health and wealth levels.

Future research could fruitfully relax some of the restrictions that are necessary to solve the model. For instance, the locally deterministic health process could be replaced by a stochastic one. We also pointed out that incorporating age-dependent processes for health depreciation and/or health-independent mortality risk could prove useful additions to capture the steep age gradient found in health expenditures. Moreover, inelastic labor supply could be replaced by preference for leisure. This last modification would, in our mind, allow for interesting analysis of the observed co-movements between macro cycles, health expenditures and labor supply decisions that have been identified in the recent literature.

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A General closed-form solutions

The general case with pre- $(T > t)$ and post-retirement $(T \leq t)$ periods is developed in Hugonnier et al. (2009) and is reproduced here for completeness. Specifically, consider the pre- and post-retirement income process given by (21) and (22). The general solution is obtained by replacing age-independent B, C, Δ by their age-dependent values $B(t), C(t), \Delta(t)$ in Theorem 1 (the marginal propensity A in (7) remains unaffected). The age-dependent shadow price of health $B(t)$ is given as:

$$B(t) = 1_{\{T>t\}} B_{\mathrm{e}}(t) + 1_{\{T \leq t\}} B_{\mathrm{r}} \tag{31}$$

where $B_{\mathrm{r}}, B_{\mathrm{e}}(\cdot) \geq 0$ solve

$$g(B_{\mathrm{r}}) = \beta^{\mathrm{r}} - (r+\delta)B_{\mathrm{r}} + \Phi B_{\mathrm{r}}^{\frac{1}{1-\alpha}} = 0, \tag{32}$$

$$B_{\mathrm{e}}'(t) = (r+\delta)B_{\mathrm{e}}(t) - \beta^{\mathrm{e}} - \Phi B_{\mathrm{e}}(t)^{\frac{1}{1-\alpha}}, \tag{33}$$

$$B_{\mathrm{e}}(T) = B_{\mathrm{r}}, \tag{34}$$

subject to $g'(B_{\mathrm{r}}) < 0$ and where we define $\Phi \equiv (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$. The age-dependent NPV of the health-independent part of income net of subsistence $C(t)$ is given as:

$$C(t) = \int_t^{T \wedge t} e^{-r(s-t)}(y^{\mathrm{e}} - a)ds + \int_{T \wedge t}^{\infty} e^{-r(s-t)}(y^{\mathrm{r}} - a)ds, \tag{35}$$

Finally, the nonnegative and age-dependent function $\Delta(t)$ is given by:

$$\Delta(t) = 1_{\{T>t\}} \Delta_{\mathrm{e}}(t) + 1_{\{T \leq t\}} \Delta_{\mathrm{r}}, \tag{36}$$

where $\Delta_{\mathrm{r}}, \Delta_{\mathrm{e}}(\cdot)$ solve:

$$0 = \Delta_{\mathrm{r}} \left(\xi(\alpha B_{\mathrm{r}})^{\frac{\alpha}{1-\alpha}} - \xi\delta + A \right) - 1, \tag{37}$$

$$\Delta_{\mathrm{e}}(t) = e^{-\int_t^T \zeta(s)ds} \Delta_{\mathrm{r}} + \int_t^T e^{-\int_t^s \zeta(u)du}ds, \tag{38}$$

$$\Delta_{\mathrm{e}}(T) = \Delta_{\mathrm{r}} \tag{39}$$

and where

$$\zeta(t) = \xi(\alpha B(t))^{\frac{\alpha}{1-\alpha}} - \xi\delta + A.$$

It is straightforward to verify that the corresponding post-retirement values B, C, Δ are given by (15), (16) and (10).

B   Propositions 1 and 2

Value of health   Consider an agent with wealth W, health H and mortality parameters (λ_0, λ_1) and denote by $\Delta^h = \Delta^h(\lambda_1)$ the value of to this agent of τ^h units of additional health. Expanding both sides of equation (23) to the first order in λ_1 we obtain

$$\begin{aligned}
o(\lambda_1^2) = & V(W - \Delta^h(0), H + \tau^h; (\lambda_0, 0)) - V(W, H; (\lambda_0, 0)) \\
& + \lambda_1 [V_W(W - \Delta^h(0), H + \tau^h; (\lambda_0, 0))(-\Delta^h)'(0) \\
& + V_{\lambda_1}(W - \Delta^h(0), H + \tau^h; (\lambda_0, 0)) - V_{\lambda_1}(W, H; (\lambda_0, 0))].
\end{aligned}$$

On the other hand, using the results of Theorem 1 we get that the value function satisfies

$$V(W, H; (\lambda_0, \lambda_1)) = \rho(A/\rho)^{\frac{1}{1-\varepsilon}}(W + BH + C)\left(1 - \lambda_1 \Delta H^{-\xi}\right) + o(\lambda_1^2). \tag{40}$$

Inserting this into the above expansion and simplifying shows that

$$\begin{aligned}
o(\lambda_1^2) = & B\tau^h - \Delta^h(0) - \lambda_1(\Delta^h)'(0) \\
& + \lambda_1 \Delta \left[(W + BH + C)H^{-\xi} - \left(W - \Delta^h(0) + B(H + \tau^h) + C\right)(H + \tau^h)^{-\xi}\right].
\end{aligned}$$

Setting both terms on the right hand side to zero then gives

$$\Delta^h(0) = \tau^h B, \tag{41}$$

$$(\Delta^h)'(0) = \Delta \left[H^{-\xi} - (H + \tau^h)^{-\xi}\right] \tag{42}$$

and it follows that up to a first order approximation the value of health is given by

$$\Delta^h(\lambda_1) = \Delta^h(0) + \lambda_1(\Delta^h)'(0) = \tau^h B + \lambda_1 \Delta \left[H^{-\xi} - (H + \tau^h)^{-\xi}\right] \tag{43}$$

as claimed in the statement.

Expected longevity   The approximation of the life expectancy in equation (29) is derived in Hugonnier et al. (2009, Proposition 2). We omit the details.

Value of life   Consider an agent with wealth W, health H and mortality parameters (λ_0, λ_1). Denote by $\Delta^\ell = \Delta^\ell(\lambda_1)$ the value of to this agent of τ^ℓ units of additional life expectancy and by $\lambda_0^*(\lambda_1)$ the solution to equation (27). Expanding equations (27) and (26) to the first order in λ_1 we obtain

$$
\begin{aligned}
o(\lambda_1^2) = {} & \ell(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0)) - \ell(W, H; (\lambda_0, 0)) - \tau^\ell \\
& + \lambda_1 [\ell_W(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0))(-\Delta^\ell)'(0) + \ell_{\lambda_0}(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0))(\lambda_0^*)'(0) \\
& + \ell_{\lambda_1}(W - \Delta^\ell(0), H + \tau^\ell; (\lambda_0^*(0), 0)) - \ell_{\lambda_1}(W, H; (\lambda_0, 0))], \\
o(\lambda_1^2) = {} & V(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0)) - V(W, H; (\lambda_0, 0)) \\
& + \lambda_1 [V_W(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0))(-\Delta^\ell)'(0) + V_{\lambda_0}(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0))(\lambda_0^*)'(0) \\
& + V_{\lambda_1}(W - \Delta^\ell(0), H; (\lambda_0^*(0), 0)) - V_{\lambda_1}(W, H; (\lambda_0, 0))].
\end{aligned}
$$

Inserting the first order expansion of the life expectancy into the first equation and setting both sides to zero shows that

$$
\begin{aligned}
\lambda_0^*(0) &= \frac{\lambda_0}{1 + \tau^\ell \lambda_0}, \\
(\lambda_0^*)'(0) &= (\lambda_0^*(0))^2 H^{-\xi} \left(\frac{\Psi}{\lambda_0} - \frac{\Psi^*}{\lambda_0^*(0)} \right)
\end{aligned}
$$

where we have set

$$
(\Psi^*)^{-1} = \Psi^{-1} + \lambda_0^*(0) - \lambda_0. \tag{44}
$$

Inserting this as well as the explicit expression for the value function into the second equation and setting both sides of the resulting equation to zero then gives

$$
\begin{aligned}
\Delta^\ell(0) &= (1 - \chi_0)(W + BH + C), \\
(\Delta^\ell)'(0) &= \chi_0(W + BH + C)H^{-\xi} \left[\Delta - \Delta^* - \frac{(\lambda_0^*)'(0)}{H^{-\xi} A^*} \right]
\end{aligned}
$$

where the constants Δ, Δ^*, A, A^* and χ_0 are defined as in the statement. The desired result now follows by observing that

$$K^* = \frac{(\lambda_0^*)'(0)}{H^{-\xi} A^*} \tag{45}$$

does not depend on the agent's health.

C Figures and tables

Figure 1: Actual vs predicted portfolio levels
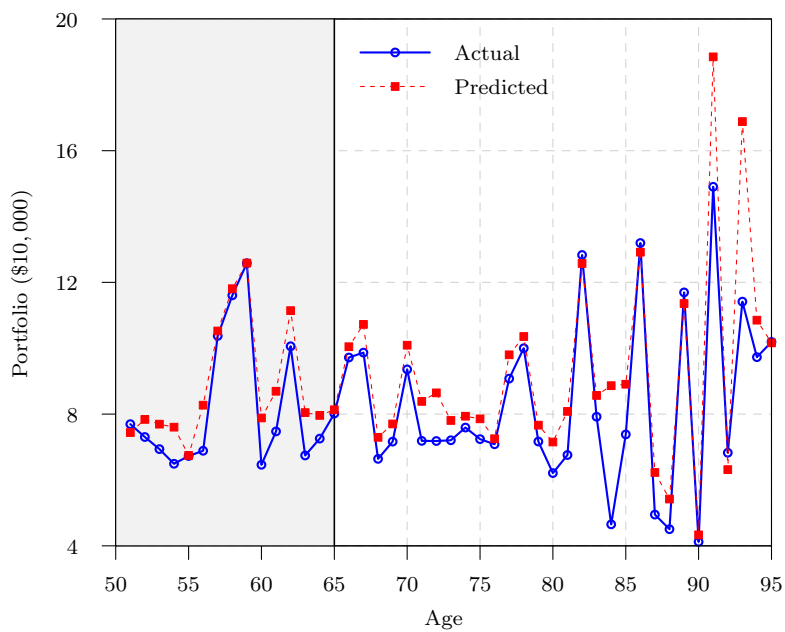
Notes: In sample, age 65+: Actual and predicted risky portfolio levels for optimal rule (13). Calibrated value of $\xi = 3.8$. Out-of-sample, age 51-64: Actual and predicted portfolio levels for structural model in Appendix A, evaluated at benchmark structural parameter estimates (column (1) of Table 5).

Figure 2: Actual vs predicted health investment levels
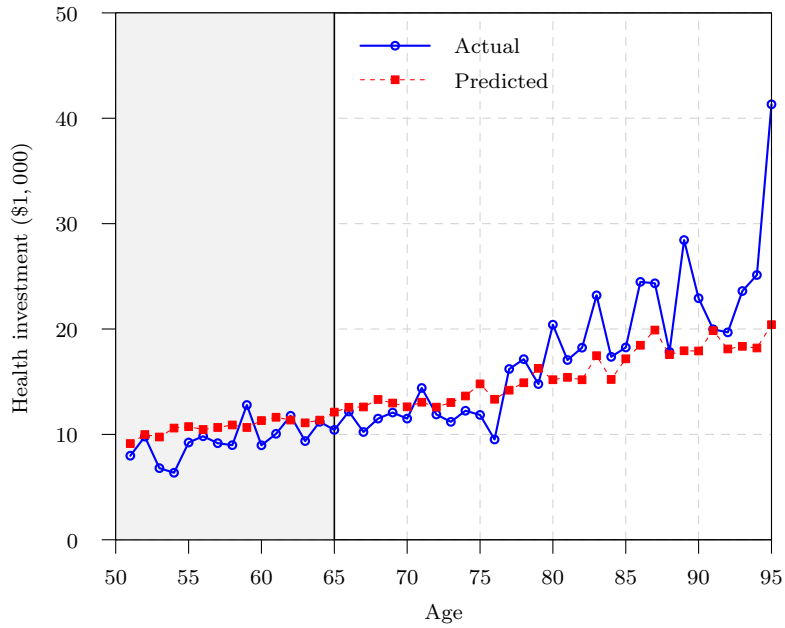
Notes: In sample, age 65+: Actual and predicted health investment levels, for optimal rule (14). Calibrated value of $\xi = 3.8$. Out-of-sample, age 51-64: Actual and predicted portfolio levels for structural model in Appendix A, evaluated at benchmark structural parameter estimates (column (1) of Table 5).

Figure 3: Value of health
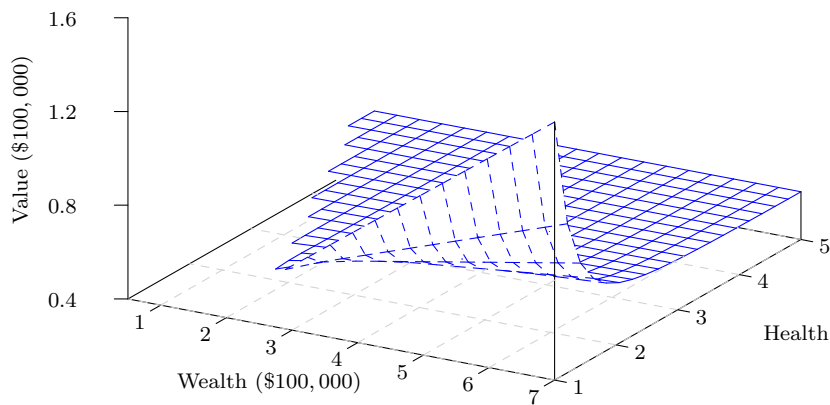
Notes: Value of 1 unit of additional health computed using (24), evaluated at benchmark structural parameter estimates (column (1) of Table 5). Low values of W, H correspond to negative disposable total wealth and are note reported.

Figure 4: Value of life
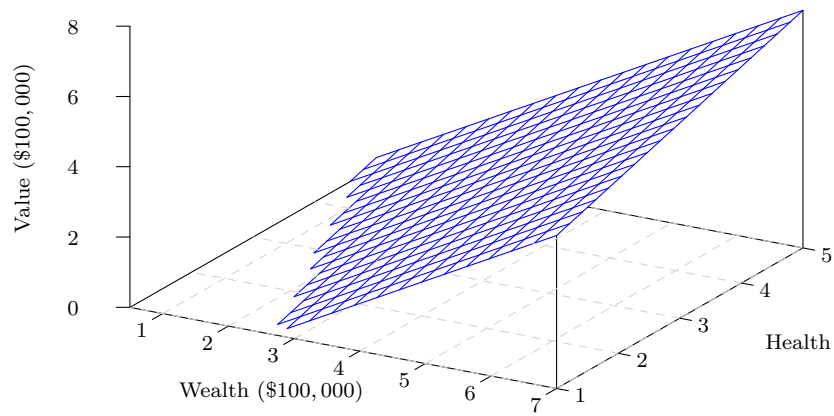
Notes: Value of 1-year extension in expected longevity computed using (30), evaluated at benchmark structural parameter estimates (column (1) of Table 5). Low values of W, H correspond to negative disposable total wealth and are note reported.

Table 1: Descriptive statistics

	Non retired (age < 65)			Retired (age ≥ 65)		
	(1)	(2)	(3)	(4)	(5)	(6)
	All sample	Single	Couple	All sample	Single	Couple
Socio-demographic						
Age	57.6	58.7	57.2	75.0	77.8	73.0
Male	40%	29%	44%	42%	22%	57%
Health status						
Poor ($H = 1$)	7%	11%	6%	11%	14%	9%
Fair ($H = 2$)	15%	20%	14%	22%	24%	20%
Good ($H = 3$)	29%	29%	29%	31%	31%	32%
Very good ($H = 4$)	32%	27%	34%	26%	23%	28%
Excellent ($H = 5$)	16%	14%	17%	10%	8%	11%
Health expenditures						
Medical	$8,755	$10,186	$8,337	$12,848	$15,517	$10,902
(median)	$1,350	$1,439	$1,331	$2,303	$2,680	$2,047
Out-of-pocket	$1,860	$1,911	$1,845	$3,014	$3,588	$2,593
(median)	$800	$700	$840	$1,040	$960	$1,120
Total	$10,615	$12,097	$10,182	$15,862	$19,105	$13,495
(median)	$2,779	$2,981	$2,729	$5,000	$5,000	$4,788
Asset holdings						
Hold safe asset	86%	75%	89%	85%	78%	90%
Hold bond	7%	4%	7%	9%	6%	11%
Hold risk asset	33%	20%	37%	32%	23%	39%
Have debt	38%	38%	38%	18%	16%	20%
Portfolio composition						
Financial wealth	$98,727	$62,330	$109,369	$122,573	$74,754	$157,524
Safe assets	57%	63%	55%	65%	70%	62%
Bonds	2%	1%	2%	2%	2%	3%
Risky assets	22%	14%	24%	20%	16%	23%
Debt	19%	22%	18%	12%	12%	13%
Observations	8,836	1,999	6,837	10,735	4,532	6,202

Notes: Data source is HRS (RAND version), 5th wave, respondents in 2000. The reported financial variables are conditional on non-zero holdings.

Table 2: Summary statistics by gross financial wealth and health for retired agents

| | Gross financial wealth quintile | | | | |
Health	1	2	3	4	5
Poor ($H = 1$)					
Wealth	$24	$1,923	$18,296	$78,636	$463,592
$P(\text{risky} > 0)$	1%	4%	18%	45%	79%
Risky assets	2%	2%	10%	26%	46%
Health inv. share (total)	17659%	1221%	117%	21%	7%
(out-of-pocket)	938%	86%	15%	5%	1%
Fair ($H = 2$)					
Wealth	$29	$2,168	$18,929	$75,273	$560,434
$P(\text{risky} > 0)$	0%	3%	19%	47%	76%
Risky assets	0%	2%	10%	26%	43%
Health inv. share (total)	5885%	418%	32%	11%	2%
(out-of-pocket)	630%	63%	9%	3%	1%
Good ($H = 3$)					
Wealth	$27	$2,446	$18,467	$78,126	$477,701
$P(\text{risky} > 0)$	0%	7%	23%	49%	79%
Risky assets	0%	4%	12%	27%	46%
Health inv. share (total)	4090%	185%	22%	6%	1%
(out-of-pocket)	713%	42%	7%	2%	0%
Very good ($H = 4$)					
Wealth	$27	$2,603	$18,717	$80,850	$513,559
$P(\text{risky} > 0)$	1%	6%	30%	57%	83%
Risky assets	5%	3%	14%	31%	51%
Health inv. share (total)	2391%	111%	15%	4%	1%
(out-of-pocket)	601%	33%	6%	1%	0%
Excellent ($H = 5$)					
Wealth	$20	$2,534	$18,715	$79,227	$592,712
$P(\text{risky} > 0)$	1%	4%	29%	49%	84%
Risky assets	6%	2%	16%	25%	49%
Health inv. share (total)	3417%	86%	9%	2%	1%
(out-of-pocket)	185%	24%	3%	1%	0%

Notes: Data source is HRS (RAND version), 5th wave, respondents in 2000. Agents of age 65 and over only. The reported values are respectively the mean of the gross financial wealth, the mean of the probability of holding risky assets, the mean of the risky portfolio share and the median of the health investment share out of net financial wealth.

Table 3: Summary of calibrated and estimated parameters

	Notation	Calibrated	Estimated
Preferences (Eqs.(1),(2))			
Discount rate	ρ	0.025	
Risk aversion	γ		✓
EIS	ε		✓
Subsistence cons.	a		✓
Death intensity (Eq.(6))			
Convexity	ξ	$\in [3.8, 4.7]$	
Exogenous	λ_0		✓
Health sensitivity	λ_1		✓
Health dynamics (Eq.(3))			
Convexity	α		✓
Depreciation	δ		✓
Income dynamics (Eq.(4))			
Constant	y^{r}		✓
Health sensitivity	β^{r}		✓
Financial markets (Eq.(5))			
Interest rate	r	0.048	
Std. error risky return	σ	0.200	
Mean risky return	μ	0.108	

Notes: The calibrated and estimated parameters are for the econometric model of eqs. (17), (18) and (19).

Table 4: SRF parameter estimates

Parameter (exp. sign)	(1) $\xi = 3.8$	(2) $\xi = 4.2$	(3) $\xi = 4.7$	(4) $\xi = 4.1$	(5) $\xi = 3.8$	(6) $\xi = 3.9$	(7) $\xi = 4.5$	(8) $\xi = 4.2$	(9) $\xi = 4.2$
$\theta_{\pi,0}$ (+)	0.8514***	0.8514***	0.8514***	0.8313***	0.9401***	0.8945***	0.8007***	0.8480***	0.8146***
	(0.0060)	(0.0060)	(0.0060)	(0.0061)	(0.0045)	(0.0048)	(0.0049)	(0.0061)	(0.0066)
$\theta_{\pi,1}$ (+)	0.0222***	0.0222***	0.0222***	0.0110***	0.0251***	0.0250***	0.0173***	0.0210***	0.0204***
	(0.0022)	(0.0022)	(0.0022)	(0.0024)	(0.0031)	(0.0023)	(0.0021)	(0.0023)	(0.0023)
$\theta_{\pi,2}$ (−)	−0.2751***	−0.2751***	−0.2751***	−0.4950***	−0.3474***	−0.2968***	−0.2773***	−0.2668***	−0.2382***
	(0.0081)	(0.0081)	(0.0081)	(0.0160)	(0.0112)	(0.0081)	(0.0075)	(0.0083)	(0.0083)
$\theta_{I,1}$ (+)	0.0012***	0.0014***	0.0017***	−0.0001	0.0011***	0.0010***	0.0016***	0.0004***	0.0013***
	(0.0002)	(0.0002)	(0.0002)	(0.0004)	(0.0002)	(0.0001)	(0.0001)	(0.0000)	(0.0002)
$\theta_{I,2}$ (+)	0.0003	0.0002	0.0002	0.0002	−0.0027	−0.0002	0.0006	0.0009***	0.0024
	(0.0011)	(0.0010)	(0.0009)	(0.0010)	(0.0005)	(0.0003)	(0.0007)	(0.0002)	(0.0024)
$\theta_{I,3}$ (+)	0.0642***	0.0779***	0.0995***	0.0629***	0.0466***	0.0506***	0.0693***	0.0118***	0.0686***
	(0.0039)	(0.0046)	(0.0057)	(0.0052)	(0.0039)	(0.0030)	(0.0037)	(0.0009)	(0.0045)
$\theta_{I,4}$ (−)	−0.0545***	−0.0692***	−0.0918***	−0.0548***	−0.0387***	−0.0434***	−0.0633***	−0.0105***	−0.0607***
	(0.0039)	(0.0046)	(0.0057)	(0.0051)	(0.0039)	(0.0030)	(0.0037)	(0.0009)	(0.0045)
Wave(s)	2000	2000	2000	2000	1998	2002	1998–2002	2000	2000
Health expend.	total	total	total	total	total	total	total	OOP	total
Socio. econ. cova.	no	no	no	yes	no	no	no	no	no
Age group	65+	65+	65+	65+	65+	65+	65+	65	65–79
Panel + rand. ef.	no	no	no	no	no	no	yes	no	no

Notes: The parameters correspond to the semi-restricted trivariate estimation of mixture continuous (for Y_j, I_j) and Tobit (for $\pi_j W_j$) processes by maximum likelihood. Standard errors are reported in parentheses, as well as statistical significance at the 1% level (***), 5% level (**) and at the 10% level (*). The unreported scedastic parameters can be obtained from the authors upon request.

Table 5: Structural parameter estimates

Parameter	(1) $\xi = 3.8$	(2) $\xi = 4.2$	(3) $\xi = 4.7$	(4) $\xi = 4.1$	(5) $\xi = 3.8$	(6) $\xi = 3.9$	(7) $\xi = 4.5$	(8) $\xi = 4.2$	(9) $\xi = 4.2$
γ	1.7663***	1.7689***	1.7665***	1.8169***	1.5681***	1.6798***	1.8764***	1.7727***	1.8476***
	(0.0125)	(0.0125)	(0.0125)	(0.0132)	(0.0102)	(0.0113)	(0.0141)	(0.0133)	(0.0137)
ε	0.2807***	0.1748***	0.2968***	0.2314***	0.2309***	0.2292***	0.2234***	0.3985***	0.2350***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
a	0.0247***	0.0248***	0.0248***	0.0380***	0.0254***	0.0259***	0.0328***	0.0248***	0.0234***
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0004)	(0.0004)	(0.0005)	(0.0005)	(0.0005)
λ_0	0.0832***	0.0787***	0.0840***	0.0803***	0.0837***	0.0822***	0.0603***	0.0612***	0.0801***
	(0.0000)	(0.0002)	(0.0001)	(0.0001)	(0.0032)	(0.0001)	(0.0006)	(0.0000)	(0.0190)
λ_1	0.0037***	0.0059***	0.0057***	0.0078***	0.0029***	0.0038***	0.0135***	0.0019***	0.0053***
	(0.0009)	(0.0023)	(0.0010)	(0.0011)	(0.0011)	(0.0005)	(0.0004)	(0.0007)	(0.0011)
α	0.2255***	0.2147***	0.2275***	0.4277***	0.1076***	0.2360***	0.3315***	0.2057***	0.1114***
	(0.0580)	(0.0620)	(0.0565)	(0.0118)	(0.0368)	(0.0570)	(0.0540)	(0.0710)	(0.0344)
δ	0.2817***	0.2994***	0.2789***	0.0778***	0.5324***	0.2542***	0.1113***	0.3157***	0.5375***
	(0.0128)	(0.0191)	(0.0149)	(0.0174)	(0.1169)	(0.0149)	(0.0439)	(0.0816)	(0.1090)
y^{r}	0.0091***	0.0091***	0.0091***	0.0091***	0.0076***	0.0099***	0.0159***	0.0097***	0.0092***
	(0.0013)	(0.0013)	(0.0013)	(0.0013)	(0.0012)	(0.0011)	(0.0010)	(0.0014)	(0.0017)
β^{r}	0.0065***	0.0065***	0.0065***	0.0065***	0.0054***	0.0056***	0.0032***	0.0066***	0.0070***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0003)	(0.0003)	(0.0004)	(0.0005)
Wave(s)	2000	2000	2000	2000	1998	2002	1998–2002	2000	2000
Health expend.	total	total	total	total	total	total	total	OOP	total
Socio. econ. cova.	no	no	no	yes	no	no	no	no	no
Age group	65+	65+	65+	65+	65+	65+	65+	65	65–79
Panel + rand. ef.	no	no	no	no	no	no	yes	no	no

Notes: The parameters correspond to the second step estimates of structural parameters. Standard errors are reported in parentheses, as well as statistical significance at the 1% level (***), 5% level (**) and at the 10% level (*).

Table 6: Implied longevity

| | Implied expected lifetime | | | | |
	$H = 1$	$H = 2$	$H = 3$	$H = 4$	$H = 5$
Data	9.17	11.26	12.64	13.38	13.79
$\ell(H)$	11.57	11.98	12.01	12.01	12.01

Notes: The data row reports expected longevity at age 70 (source, Lubitz et al., 2003, Figure 2, p. 1052), the model values $\ell(H)$ are the implied longevity given by (29). All implied values and thresholds obtained for benchmark structural parameter estimates (column (1) of Table 5).