The effect of vertical measurement resolution on the correlation structure of a ground penetrating radar reflection image

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[1] Geostatistical analysis of a ground penetrating radar reflection image can be used to quantify the maximum correlation direction and the range of horizontal and sub-horizontal radar reflections. A review of previous work, and an analysis of a photograph of layered sediments, suggest that the vertical resolution of a radar image strongly affects its lateral correlation structure. Numerical modeling was used to generate synthetic radar sections and investigate the effect of the vertical resolution of the radar measurement on the link between the correlation structure of the radar reflections and the true correlation structure of subsurface water content. The horizontal range of the radar reflections decreased as the vertical resolution improved, closely matching that of the water content when the vertical resolution was approximately equal to the vertical range of the water content. *INDEX TERMS:* 0694 Electromagnetics: Instrumentation and techniques; 0994 Exploration Geophysics: Instruments and techniques; 1894 Hydrology: Instruments and techniques. **Citation:** Knight, R., P. Tercier, and J. Irving (2004), The effect of vertical measurement resolution on the correlation structure of a ground penetrating radar reflection image, *Geophys. Res. Lett.*, *31*, L21607, doi:10.1029/2004GL021112.

1. Introduction

[2] Studies of the near-surface (top ~100 m) of the earth, for environmental or engineering applications, often involve the measurement of subsurface properties which are then used in models designed to predict subsurface processes, such as groundwater flow and contaminant transport. As has been discussed, for example by *Beckie* [1996], there are a number of different spatial scales involved in measurement, all of which can affect the measured properties and, in turn, the predicted processes. Of interest in our research is the use of surface-based ground penetrating radar (GPR) data to characterize the spatial variation in subsurface hydrogeologic properties. We focus here on the role of measurement resolution in using GPR data for this purpose.

[3] Surface-based GPR data are acquired by sending a pulse of high frequency (1 MHz to 1 GHz) electromagnetic (EM) energy into the earth and recording energy reflected back to the earth's surface from interfaces across which there are changes in electrical properties, most notably the dielectric constant κ. After processing, the location of these interfaces in the subsurface and the magnitude of reflected energy are displayed as reflections in the radar image. The radar reflection image thus contains information about the way in which κ varies spatially throughout the sampled subsurface region. A review of numerous laboratory studies, as provided by *Knight* [2001], shows that the dominant factors controlling κ are the water content, the volume fraction of high surface area materials (such as clays), and the pore-scale geometry of the solid phase.

[4] Our hypothesis is that the correlation structure seen in the radar reflection image can be used to determine the correlation structure of the subsurface properties determining κ. Given this, surface-based GPR data could be used to obtain critical information that cannot currently be obtained with any other method. While wells can provide information about vertical changes in properties, there are never enough wells to adequately characterize the lateral spatial variability. Our past research has dealt with initial testing of the above hypothesis. The focus in this study is to determine the effect of the vertical resolution of a radar measurement on the link between the radar image and the spatial variation in subsurface properties.

2. Geostatistical Analysis of Radar Reflection Images

[5] A radar reflection image is a compilation of the radar traces recorded as a transmitter and receiver antenna are moved across the surface of the earth. Detailed discussions of the fundamental principles of GPR can be found in the publications by *Daniels et al.* [1988] and *Davis and Annan* [1989]. A single radar trace can be approximated by the convolution of the source EM pulse (or wavelet) with a series of reflection coefficients, defined at each subsurface interface as:

$$R = \frac{\sqrt{\kappa_1} - \sqrt{\kappa_2}}{\sqrt{\kappa_1} + \sqrt{\kappa_2}} \quad (1)$$

where subscripts 1 and 2 refer to the materials above and below the interface. Here, R represents the ratio of the amplitudes of reflected energy to incident energy for a normally incident EM wave at a planar interface, where the incident wave's electric field is polarized perpendicular to the plane of incidence (referred to as TE mode). In order to convert the reflection amplitude data, recorded as a function of time, to a radar reflection image displayed in terms of depth, the velocity at which an EM wave travels through the subsurface must be known. The EM wave velocity v is equal to $c/\sqrt{\kappa}$ where c is the speed of light (3.0×10^8 m/s). Note that both this expression for c and equation (1) assume EM wave propagation through a material with relatively low loss and with magnetic permeability equal to its value in free space.

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[6] We have adopted a geostatistical framework for quantifying the correlation structure of radar reflection images. We use the software GSLIB [*Deutsch and Journel*, 1998] to obtain an experimental semivariogram, which is described by the following equation [*Journel and Huijbregts*, 1978]:

$$\gamma(\mathbf{h}) = \frac{1}{2N}\sum_{i=1}^{N}[z(\mathbf{x_i}+\mathbf{h}) - z(\mathbf{x_i})]^2, \qquad (2)$$

where $\mathbf{h}$ is the lag, or separation vector, between two data points $z(\mathbf{x}_i + \mathbf{h})$ and $z(\mathbf{x}_i)$, and N is the number of data pairs in each summation. For a data set with a defined correlation structure, γ increases at small lag values and then asymptotically approaches the sill, where γ equals the variance of the data. The data values we use in analyzing radar reflection images are the reflection amplitudes. When working with field data, these amplitudes are equalized in strength throughout the radar image using automatic gain control (AGC). As suggested by *Journel and Huijbregts* [1978], we limit the lag vector to one half the domain size of the data set.

[7] Modeling can be used to provide an analytic description of the experimental semivariogram. We typically fit our semivariogram data using an exponential model, which is the one often assumed by researchers in stochastic hydrology [*Woodbury and Sudicky*, 1991]. The exponential model is given by the following equation:

$$\begin{aligned} &\gamma(h) = C\left(1 - e^{-\frac{3h}{a}}\right), \qquad \mathit{if}\ h > 0 \\ &\gamma(0) = 0 \end{aligned} \qquad (3)$$

where C is the sill and a, referred to as the range of the data set, is the maximum distance at which the data are spatially correlated. The correlation length of the data set is equal to $a/3$ and is the average distance over which the data are correlated.

[8] In our previous work we have used geostatistical analysis to determine the maximum correlation direction and the range of the reflections in a radar image. We have dealt exclusively with radar data acquired over sediments ranging in size from fine-grained silts and clays to gravels, and with systems in which the maximum correlation direction ranged from horizontal to a maximum angle of $\sim$20 degrees below the horizontal. When the lag vector plunges at angles greater than $\sim$45 degrees, the length scale associated with the radar wavelet dominates geostatistical analysis; we thus limit ourselves to the analysis of structure oriented closer to horizontal.

[9] Our hypothesis, stated earlier, is that the correlation structure of the radar reflections can be used to determine the correlation structure of subsurface properties controlling κ at a site. Two previous field studies, designed to test this hypothesis, yielded very encouraging results. In the study by *Rea and Knight* [1998] we found good agreement between the maximum correlation direction and range of the reflections in a 100 MHz radar image and the correlation structure of grain size, as quantified by analysis of a binary photograph of the radar-imaged cliff-face. In the second field study conducted by Knight and others [*Knight et al.*, 2003] we found good agreement between the horizontal range of the 100 MHz radar reflections and that determined for water content data derived from neutron probe measurements at the field site. These studies suggest that the correlation structure determined for a radar reflection image in the horizontal, or sub-horizontal, direction is closely related to the correlation structure of subsurface properties. In our latest study, reported here, we addressed a critical issue that affects the general applicability of the results from our earlier work. The issue is the effect of the vertical resolution of the radar measurement on the observed correlation structure of a radar image.

3. Observed Dependence on Measurement Resolution

[10] In a study of surface soil moisture by *Western and Blöschl* [1999], it was shown that as the area sampled by a measurement increased, the determined correlation length of the sampled system increased. In a similar way, we propose, the resolution of a radar measurement should affect our ability to determine the true correlation structure of subsurface properties from the observed or apparent correlation structure in a radar image.

[11] The vertical resolution of a radar measurement is commonly taken to be one quarter of the dominant wavelength λ of the transmitted EM pulse [*Davis and Annan*, 1989]; with λ related to the EM wave velocity and the dominant frequency f by:

$$\lambda = \frac{v}{f}. \qquad (4)$$

Quantifying the horizontal resolution of a GPR reflection image is much more challenging and involves accounting for the radiation pattern of the antennas and the subsurface variation in EM properties. The value that most commonly is used as an estimate of horizontal resolution is the width of the first Fresnel zone [e.g., *Yilmaz*, 1987]. In this case, there is not only a dependence on frequency but a strong dependence on depth, with the width of the first Fresnel zone increasing with increasing depth.

[12] In the study by *Knight et al.* [1999] we determined the horizontal and sub-horizontal ranges of the reflections in radar data collected at three field sites using different frequency antennas. At each of the sites, we found that as the antenna frequency decreased, the range of the reflections increased. Given the relationship between antenna frequency and the vertical resolution of the radar measurement, these results led us to propose that a loss of vertical resolution caused the increase in the horizontal or sub-horizontal range of the reflections.

[13] In contrast to this proposed dependence on vertical measurement resolution, the recent analysis of radar reflection images by Dafflon and co-workers [*Dafflon et al.*, 2004] did not reveal a dependence of the horizontal range on the horizontal measurement resolution. In their study, the horizontal correlation length was found to vary with depth, due to changes in the sampled geologic facies, but there was not the consistent increase with depth that would occur if horizontal resolution were a dominant factor. In addition, migration of the radar data, a processing step that serves to reduce the size of the Fresnel zone and thus improve horizontal resolution, had no effect on the determined

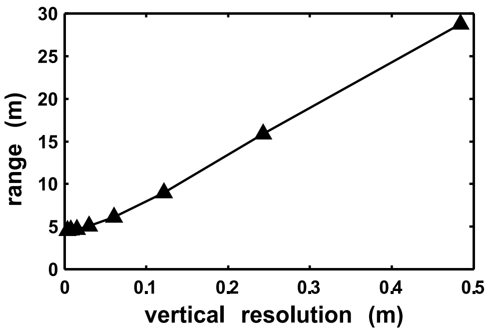

Figure 1. The observed change in the horizontal range of the cliff face photograph as the vertical resolution was varied.

correlation structure. What we observe in radar data, therefore, is a dependence of the horizontal correlation structure on the vertical measurement resolution.

[14] We can gain useful insight into the importance of the vertical resolution of a radar measurement by observing the way in which vertical resolution affects the horizontal range of a photograph of a cliff face of layered sediments. The cliff face, which we analyzed, has an anisotropic correlation structure similar to that which we image in many of our GPR studies of sedimentary depositional sequences. In the starting image, each pixel was square with side dimensions of 3.80×10^{-3} m. We defined the vertical resolution as the vertical dimension of a pixel. To investigate the effect of vertical resolution on the horizontal range of the image, we produced 7 more images by averaging over 2, 4, 8, 16, 32, 64, and 128 pixels in the vertical direction. The GSLIB software was used to conduct a geostatistical analysis of all the images, with the gray scale as the data values and the lag vector oriented in the horizontal direction. Each experimental semivariogram was fit with an exponential model to obtain the horizontal range.

[15] The effect that degrading the vertical resolution of the image had on the determined value of the horizontal range is clearly seen in Figure 1. Even though we are considering the spatial correlation of the data in the horizontal direction, the range was highly sensitive to changes in vertical resolution, increasing from 4.6 m to 28.8 m as vertical resolution changed from 3.8×10^{-3} m to 4.8×10^{-1} m. A loss of vertical resolving ability had the effect of connecting thin, laterally continuous features to form a much more correlated structure in the horizontal direction.

[16] This example provides an explanation for the dependence that we observed [*Knight et al.*, 1999] of the range of radar reflections on frequency, and the lack of observed dependence [*Dafflon et al.*, 2004] on horizontal resolution. We believe that in radar images of sedimentary sequences, with anisotropic structures similar to the cliff face, the vertical resolution, which is governed by frequency, will have a dominant effect on the determined horizontal, or sub-horizontal, range. The horizontal resolution, in contrast, will not play as significant a role; it will typically be less than the horizontal (or sub-horizontal) range so unlikely to have a large impact on the lateral correlation structure. For this reason, we focused specifically in this study on the dependence of the range of radar reflections on vertical resolution. We used a numerical example to develop an understanding of the effect of vertical resolution on the relationship between the true correlation structure of subsurface properties and the correlation structure captured in the radar image.

4. Numerical Example: Radar Imaging on a Known Correlation Structure

[17] In many field studies it is assumed that water content θ_w is the subsurface property determining the magnitude of κ due to the large contrast between κ of water (80) and that of air (κ = 1) and most solid minerals ($\kappa \sim 5$). For our numerical example, we considered the simple case where the Topp equation [*Topp et al.*, 1980] describes the relationship between κ and θ_w:

$$\kappa = 3.03 + 9.30(\theta_w) + 146.00(\theta_w)^2 - 76.70(\theta_w)^3. \qquad (5)$$

We generated a subsurface model of θ_w that was 100 m across, 14 m deep, and had a mean water content of 0.20 with a standard deviation of 0.03. We assumed an exponential correlation structure with a horizontal range of 30 m and a vertical range of 0.24 m. This model was intended to be representative of an anisotropic near-surface region where the horizontal range is much greater than the vertical range. We obtained the corresponding model of dielectric constant using the Topp relationship. This produced a dielectric model with the same horizontal range as the model of water content.

[18] Vertical radar reflection coefficients were calculated (with equation (1)) to produce a reflection coefficient model. Using geostatistical analysis, we found the range of the reflection coefficient model to be 4.5 m, much shorter than the range of 30 m for the water content and dielectric models. This reduction in the range is because the calculation of reflection coefficients acts like a vertical differencing filter, disrupting the continuity of the dielectric model in the horizontal direction. We note that there can be some systems with a spatial structure such that the horizontal correlation structure of the dielectric model is preserved when transformed to a reflection coefficient model. One simple example would be a system that could be represented as a mixture of two materials, where discrete regions of the one are embedded in a background of the other. In general, however, we expect the horizontal range of the reflection coefficient model to be less than that of the dielectric model.

[19] The reflection coefficient model was convolved with four radar wavelets, having central frequencies of 50, 100, 200 and 450 MHz, to produce four synthetic radar sections. The convolution model, while a very simple form of modeling, captures the essential features of the radar method relevant to our study. The vertical resolution of each synthetic radar section was calculated, with v set equal to the average EM wave velocity for the water content model, and ranged from 0.05 m for the 450 MHz data to 0.46 m for the 50 MHz data. We used geostatistical analysis to obtain the horizontal range for each of the synthetic radar reflection images.

[20] Figure 2 shows the observed dependence of the horizontal range of the radar images on the vertical resolution, which is governed by the change in frequency of the radar data. Also marked on Figure 2 are the horizontal

ranges of the starting water content model and the reflection coefficient model. If the radar wavelet were an impulse function in the time domain, the correlation structure of the radar image would be the same as that of the reflection coefficient model; and the horizontal ranges would be equal. But the radar wavelet is much broader than an impulse function. As a result, the convolution of the four radar wavelets with the reflection coefficient model results in spatial averaging that produces radar images with horizontal ranges that are greater than the range of the reflection coefficient image.

[21] The results shown in Figure 2 demonstrate what we believe to be a critical issue in further developing the use of radar images for estimating the range of subsurface properties in many near-surface environments: the frequency of the radar measurement, due to its control on the vertical resolution. The radar data with the best resolution are the 450 MHz data. The horizontal range for this radar reflection image is close to that of the reflection coefficient model. As the frequency of the radar wavelet decreases, the vertical measurement resolution degrades, and the spatial averaging causes the range of the radar reflections to increase until it approximately equals and then exceeds that of the water content. The best agreement between the range for the radar data and the range for subsurface water content is obtained when the frequency of measurement is equal to 100 MHz.

[22] For this numerical example, we found a remarkably simple explanation for the close match between the horizontal range of the 100 MHz radar reflection image and that of the water content model. The 100 MHz image has a vertical measurement resolution (0.23 m) that is approximately equal to the vertical range (0.24) of the water content model. In other words, when the vertical resolution of the radar measurement was close to the vertical range of the imaged system, the lateral correlation structure of the radar reflections was close to that of the imaged system. While this same result was found in repeated testing with more than 20 other numerical examples, it is premature to conclude that this is a general result that could be used as a guideline for estimating subsurface correlation structure from radar data.

5. Conclusions

[23] What do our results suggest about the use of radar reflection images for characterizing the correlation structure of subsurface properties controlling κ in a region? We conclude that radar reflection images capture information about the spatial structure of the subsurface properties. The relationship between the horizontal correlation structure of a radar image and that of the imaged, anisotropic subsurface system, however, is highly sensitive to the vertical resolution of the radar measurement. We suggest that in order to recover information about the true structure of subsurface properties from radar images, we need to use detailed information from wells about the true vertical correlation structure. While further field tests are planned to fully explore this concept, we believe that accounting for the vertical resolution of a radar measurement will allow us to use radar images to describe the lateral correlation structure of subsurface properties, information that is needed to develop accurate models of subsurface processes.

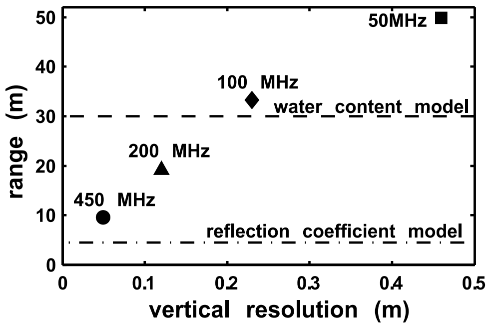

Figure 2. The horizontal range of each synthetic radar reflection image, plotted as function of the vertical resolution of the image. Also shown are the horizontal ranges of the starting water content model and the derived model of reflection coefficients.

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