

Abstract

Mechanical waves, which are commonly employed for the non-invasive characterization of fluid-saturated porous media, tend to induce pore-scale fluid pressure gradients. The corresponding fluid pressure relaxation process is commonly referred to as squirt flow and the associated viscous dissipation can significantly affect the waves' amplitudes and velocities. This, in turn, implies that corresponding measurements contain key information about flow-related properties of the probed medium. In many natural and applied scenarios, pore fluids are effectively non-Newtonian, for which squirt flow processes have, as of yet, not been analysed. In this work, we present a numerical approach to model the attenuation and modulus dispersion of compressional waves due to squirt flow in porous media saturated by Maxwell-type non-Newtonian fluids. In particular, we explore the effective response of a medium comprising an elastic background with interconnected cracks saturated with a Maxwell-type non-Newtonian fluid. Our results show that wave signatures strongly depend on the Deborah number, defined as the relationship between the classic Newtonian squirt flow characteristic frequency and the intrinsic relaxation frequency of the non-Newtonian Maxwell fluid. With larger Deborah numbers, attenuation increases and its maximum is shifted towards higher frequencies. Although the effective plane wave modulus of the probed medium generally increases with increasing Deborah numbers, it may, however, also decrease within a restricted region of the frequency spectrum.

I. INTRODUCTION

Mechanical waves are commonly employed for the non-invasive characterization of fluid-saturated biological [1], geological [2], and engineered [3] materials. In this context, the probed media are commonly conceptualized as a solid matrix comprising an interconnected void/pore space, which is occupied by a fluid phase [4]. In general, pore fluids are assumed to be Newtonian, implying that their viscosity η is a shear-stress and frequency-independent parameter. However, for a wide range of practical applications and natural scenarios, fluids present an effective non-Newtonian behavior [e.g., 5]. For example, fluids employed in hydraulic fracturing and/or drilling operations in porous geological formations are characterized by comprising large concentrations of polymers, surfactants, and/or colloids, which

* santiago.solazzi@unil.ch

34 result in non-Newtonian properties [e.g., 6, 7]. As of yet, there is a lack of comprehension
35 of the characteristic signatures of mechanical waves traveling in porous media saturated by
36 non-Newtonian fluids.

37 Biot’s theory of poroelasticity is arguably the most widely used formulation to study wave
38 propagation in porous media saturated by Newtonian fluids [8, 9]. Within the framework
39 of this theory, relative motion of the viscous pore fluid with respect to the pore walls can
40 occur in response of a passing wave which, in turn, experiences amplitude loss and phase
41 velocity dispersion due to viscous energy dissipation. Due to this inherent relation between
42 wave characteristics and fluid flow properties, there is significant interest in understanding
43 the physical mechanisms behind the attenuation and dispersion of mechanical waves, which
44 may provide information about the hydraulic properties of the explored media, such as the
45 permeability [e.g., 10].

46 There are two main fluid-related dissipation mechanisms that can take place in mono-
47 saturated porous media: (i) *global flow* [e.g., 8, 9] and (ii) *squirt flow* [e.g., 11]. Global flow
48 takes place when the solid frame is accelerated by a passing wave, thus inducing relative
49 fluid displacements with respect to the pore walls. This mechanism is driven by inertial
50 forces and, in the context of geophysical characterization of consolidated geological forma-
51 tions, tends to become relevant at frequencies that are much higher than those typically
52 employed in seismic exploration [e.g. 12]. On the other hand, squirt flow prevails in porous
53 media with locally contrasting compressibilities, such as, for example, interconnected cracks
54 embedded in an otherwise intact matrix, whose characteristic sizes are much smaller than
55 the prevailing wavelengths (i.e., microscopic-mesoscopic scale). Notably, squirt flow effects
56 prevail at much lower frequencies than those of global flow and, thus, may be an important
57 source of energy dissipation in the seismic and sonic frequency band [e.g., 13, 14]. The above
58 described dissipation mechanisms have been studied extensively for porous media saturated
59 with Newtonian fluids (see Müller *et al.* [10] for a comprehensive review). However, further
60 research is needed in order to understand how these important dissipation mechanisms are
61 affected by the presence of non-Newtonian pore fluids.

62 Previous efforts to explore the effects of non-Newtonian fluids on the wave signatures of
63 porous media were mainly focused on global flow. Del Rio *et al.* [15] studied the effects of
64 an oscillating non-Newtonian fluid on a capillary tube to explore the corresponding effects
65 on the *dynamic permeability*. For this purpose, the non-Newtonian viscosity behavior was

66 modeled using a linearly viscoelastic Maxwell-type model. Tsiklauri and Beresnev [6, 16]
67 connected this model to Biot’s poroelasticity theory [9] to study global flow dissipation
68 experienced by rotational and dilatational elastic waves, conceptualizing the pore space as
69 a bundle of capillary tubes. These authors demonstrated that the non-Newtonian behavior
70 of the fluid can significantly affect the wave signatures. More recently, the approach of
71 Tsiklauri and Beresnev [6] was used to study Rayleigh wave signatures [17] and guided
72 waves generated in a fluid-filled borehole [18] in porous media saturated with Maxwell-type
73 non-Newtonian fluids. As opposed to global flow effects, which were addressed in the works
74 mentioned above, the effects of squirt flow in porous media saturated by non-Newtonian
75 fluids do, however, remain largely unexplored.

76 Here, we study squirt flow effects on compressional wave attenuation and dispersion in
77 porous media saturated by a linearly viscoelastic Maxwell-type non-Newtonian fluid. We
78 provide a procedure to include the effects of such a non-Newtonian Maxwell fluid on the
79 squirt flow modeling approach proposed by Quintal *et al.* [19], which permits to analyze
80 the associated compressional wave signatures for fluid viscosities with different intrinsic
81 relaxation characteristics. To illustrate these effects, we consider a simple model of a porous
82 medium, whose representative elementary volume (REV) consists of two orthogonal and
83 intersecting cracks saturated with a non-Newtonian linear Maxwell fluid embedded in an
84 elastic impervious background.

85 II. THEORY

86 In the following, we describe a set of equations that permit us to compute squirt flow ef-
87 fects on the compressional wave signatures of porous media for a known pore space topology.
88 For this, we introduce the method of Quintal *et al.* [19], which considers that the embedding
89 frame is an elastic solid hosting cracks/pores saturated with a Newtonian fluid. Then, we
90 present a simple procedure to include non-Newtonian behavior in the corresponding formu-
91 lation.

92 **A. Governing equations for squirt flow with a Newtonian pore fluid**

93 Let us consider a porous medium whose matrix is an isotropic elastic linear solid hosting
 94 a pore space that is saturated by a viscous and compressible Newtonian fluid. Let us assume
 95 that this medium is deformed by a passing compressional wave characterized by presenting
 96 small strains ($\sim 10^{-6}$) and a wavelength that is large compared with the characteristic pore
 97 size. Furthermore, we assume that the flow within the pores is such that viscous forces dom-
 98 inate over inertial forces [20]. Note that this latter assumption is fulfilled provided that the
 99 prevailing frequencies are much smaller than the so-called *Biot's frequency*, which is associ-
 100 ated with the onset of global flow dissipation. In this context, the linearized and quasi-static
 101 coupled Lamé-Navier and Navier-Stokes (LNS) equations can be employed to derive the ef-
 102 fective frequency-dependent bulk and shear moduli of the system [19]. The corresponding
 103 set of equations, which is detailed below, consists of the conservation of momentum and a
 104 generalized constitutive equation.

105 The conservation of momentum is given by

$$106 \quad \nabla \cdot \boldsymbol{\sigma} = 0, \quad (1)$$

107 where $\boldsymbol{\sigma}$ denotes the total stress tensor. As the considered medium comprises both solid and
 108 fluid domains, it is possible to discriminate between a solid and a fluid contribution within
 109 the total stress tensor [19]

$$110 \quad \boldsymbol{\sigma} = \varphi \boldsymbol{\sigma}^s + (1 - \varphi) \boldsymbol{\sigma}^f, \quad (2)$$

111 where φ is a spatially variable parameter, which is equal to 1 and 0 in the solid and fluid
 112 domains, respectively. The total stress tensor can be divided into a bulk (volumetric) and a
 113 deviatoric (shear) part

$$114 \quad \boldsymbol{\sigma} = -p \mathbf{I} + \mathbf{s}, \quad (3)$$

115 with p denoting the pressure or hydrostatic stress, \mathbf{I} the identity, and \mathbf{s} the so-called *excess*
 116 *stress tensor*. Note that Eq. (3) is valid both for the fluid and solid domains.

117 On the other hand, the strain tensor for both the solid and the fluid corresponds to
 118 $\boldsymbol{\epsilon} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$, with \mathbf{u} denoting the displacement vector and T the transpose. $\boldsymbol{\epsilon}$ can
 119 also be divided into a bulk and a deviatoric part

$$120 \quad \boldsymbol{\epsilon} = \frac{\text{tr}[\boldsymbol{\epsilon}]}{3} \mathbf{I} + \boldsymbol{\varepsilon}, \quad (4)$$

121 where $\boldsymbol{\varepsilon}$ is the deviatoric strain.

122 The matrix and the fluid present an elastic response under bulk deformation and the
 123 corresponding constitutive equations are given by

$$124 \quad -p = K_\beta \text{tr}[\boldsymbol{\epsilon}], \quad \text{with } \beta = s, f, \quad (5)$$

125 where K_s and K_f denote the bulk moduli of the solid and fluid, respectively. However, the
 126 stress-strain relationship for shear deformation differs in the solid and fluid domains. For
 127 the solid, it is given by

$$128 \quad \mathbf{s}^s = 2\mu_s \boldsymbol{\varepsilon}, \quad (6)$$

129 where μ_s is the shear modulus of the solid matrix. Conversely, the stress-strain relationship
 130 for the fluid, in the space-frequency domain, is given by

$$131 \quad \mathbf{s}^f = 2\eta_0 i \omega \boldsymbol{\varepsilon}, \quad (7)$$

132 with η_0 the shear viscosity of the Newtonian fluid, i the imaginary unity, and ω the angular
 133 frequency. The generalized constitutive equation is thus given by

$$134 \quad \boldsymbol{\sigma} = \varphi (2\mu_s \boldsymbol{\varepsilon} + K_s \text{tr}[\boldsymbol{\epsilon}] \mathbf{I}) + (\varphi - 1) (2\eta_0 i \omega \boldsymbol{\varepsilon} + K_f \text{tr}[\boldsymbol{\epsilon}] \mathbf{I}). \quad (8)$$

135 Eqs. (1) and (8) can be used to describe the mechanical response of a porous medium
 136 comprising solid and fluid domains. At the boundaries between these domains, complex-
 137 valued solid and fluid displacements $\mathbf{u}(\mathbf{x}, \omega)$ are considered to be continuous and are thus
 138 naturally coupled. For further details regarding the finite-element procedure used to solve
 139 the corresponding equations we refer to the work of Quintal *et al.* [19]. It is important to
 140 remark here that, due to the capacity of the viscous Newtonian fluid to flow within the pores
 141 in response to a macroscopic deformation, the effective elastic moduli of the medium are
 142 complex-valued and frequency-dependent which, in turn, results in attenuation and modulus
 143 dispersion of compressional waves, as further explained in subsection IID.

144 B. Non-Newtonian Maxwell fluid with shear relaxation

145 Experimental evidence shows that several non-Newtonian fluids, such as some surfactant
 146 solutions, exhibit the rheological behavior of a linear Maxwell fluid [e.g., 21, 22]. In the

147 space-frequency domain, the relationship between the excess stress tensor \mathbf{s} and the shear
 148 rate $i\omega\boldsymbol{\varepsilon}$ for a linear Maxwell fluid responds to [e.g., 5]

$$149 \quad \mathbf{s}^f + \tau_m i\omega \mathbf{s}^f = 2\eta_0 i\omega \boldsymbol{\varepsilon}, \quad (9)$$

$$150 \quad \mathbf{s}^f = 2\eta_m i\omega \boldsymbol{\varepsilon}, \quad (10)$$

151 where τ_m is the relaxation time of the corresponding fluid and $\eta_m = \eta_0/(1 + \tau_m i\omega)$ is the
 152 frequency-dependent and complex-valued viscosity. We define $\omega_m = 2\pi/\tau_m$ as the char-
 153 acteristic angular frequency of the intrinsic relaxation of the fluid and, thus, η_m responds
 154 to

$$155 \quad \eta_m(\omega) = \frac{1}{(1 + 2\pi i \frac{\omega}{\omega_m})} \eta_0. \quad (11)$$

156 The Newtonian regime (Eq. 7) prevails when the fluid has enough time to relax during a
 157 wave cycle, that is,

$$158 \quad \lim_{\omega/\omega_m \rightarrow 0} \mathbf{s}^f = \lim_{\omega/\omega_m \rightarrow 0} \frac{2\eta_0 i\omega}{(1 + 2\pi i\omega/\omega_m)} \boldsymbol{\varepsilon}, \quad (12)$$

$$159 \quad = 2\eta_0 i\omega \boldsymbol{\varepsilon}, \quad (13)$$

$$160 \quad (14)$$

161 On the other hand, the elastic regime (Eq. 6) prevails when the angular frequency ω of the
 162 traveling wave is such that $\omega_m \ll \omega$

$$163 \quad \lim_{\omega/\omega_m \rightarrow \infty} \mathbf{s}^f = \lim_{\omega/\omega_m \rightarrow \infty} \frac{2\eta_0 i\omega}{(1 + 2\pi i\omega/\omega_m)} \boldsymbol{\varepsilon}, \quad (15)$$

$$164 \quad = \lim_{\omega/\omega_m \rightarrow \infty} \frac{2\eta_0 \omega_m}{2\pi (\omega_m/i2\pi\omega + 1)} \boldsymbol{\varepsilon}, \quad (16)$$

$$165 \quad = \frac{2\eta_0 \omega_m}{2\pi} \boldsymbol{\varepsilon}, \quad (17)$$

$$166 \quad = \frac{2\eta_0}{\tau_m} \boldsymbol{\varepsilon}, \quad (18)$$

$$167 \quad = 2\mu_f \boldsymbol{\varepsilon}, \quad (19)$$

168 with $\mu_f = \frac{\eta_0}{\tau_m}$ denoting the *shear modulus* of the fluid. Consequently, a direct replacement
 169 of η_0 for η_m in Eq. (8) permits to obtain the constitutive equation for a saturating fluid
 170 presenting Maxwell-type non-Newtonian shear behavior.

171 C. Deborah number

172 Squirt flow occurs in response to a fluid pressure diffusion process whose characteristic
 173 time, when the medium is saturated with a Newtonian fluid, can be defined as $\tau_c = 2\pi/\omega_c$,

174 with ω_c being the corresponding Newtonian squirt flow characteristic frequency. When
 175 considering a Maxwell-type non-Newtonian pore fluid, the interrelationship between squirt
 176 flow and the intrinsic shear relaxation of the fluid is a key aspect determining the effective
 177 response of the medium. Following previous works [6, 15, 16], we define the so-called *Deborah*
 178 *number* χ of the system, which is determined here as the ratio between the relaxation time of
 179 squirt flow with Newtonian fluids τ_c and the intrinsic relaxation time of the non-Newtonian
 180 Maxwell fluid τ_m , that is,

$$181 \quad \chi = \frac{\tau_c}{\tau_m} = \frac{\omega_m}{\omega_c}. \quad (20)$$

182 The Deborah number χ determines the pore fluid flow regime. Beyond a certain critical
 183 value χ^* , the fluid's intrinsic relaxation occurs faster than fluid pressure diffusion and, thus,
 184 the fluid behaves as Newtonian during squirt flow. Conversely, for $\chi < \chi^*$ the fluid exhibits
 185 non-Newtonian viscoelastic behavior during the fluid pressure diffusion process. In this
 186 context, it is important to note that Eq. (11) can be expressed as a function of the Deborah
 187 number

$$188 \quad \eta_m = \frac{\chi\omega_c}{(\chi\omega_c + 2\pi i\omega)}\eta_0. \quad (21)$$

189 **D. Dispersion and attenuation of compressional waves**

190 To estimate the compressional wave attenuation and plane wave modulus dispersion,
 191 we solve Eqs. (1) and (8) using suitable boundary conditions in a rectangular REV of the
 192 porous medium of interest. The boundary conditions can be conceptualized as an oscillatory
 193 relaxation test, which emulates the effects of a vertically traveling compressional wavefield
 194 (Figs. 1a and 1b). Recall that we are under the assumption that the prevailing wavelengths
 195 λ are much larger than the REV side-length L ($\lambda \gg L$). The corresponding test consists in
 196 applying a harmonic downward-oriented displacement homogeneously at the upper boundary
 197 of the REV. The displacements in the vertical and horizontal directions at the bottom and
 198 along the lateral boundaries of the model, respectively, are set to zero (Fig. 1c) [e.g 23, 24].

199 The upscaled elastic properties of the porous medium saturated with a mobile fluid phase
 200 are complex-valued and frequency dependent and, thus, the medium can be regarded as
 201 an effective homogeneous viscoelastic solid. Consequently, we can calculate the effective
 202 attenuation and dispersion using volume averages of the frequency-dependent stress and
 203 strain fields [e.g., 23, 25]. In this context, the complex-valued and frequency-dependent

204 plane wave modulus H , associated with a compressional wave propagating in the vertical
 205 direction \mathbf{x}_3 , can be approximated by

$$206 \quad H(\omega) = \frac{\langle \sigma_{33}(\omega) \rangle}{\langle \epsilon_{33}(\omega) \rangle}, \quad (22)$$

207 where $\langle \cdot \rangle$ denotes the volume average of the corresponding parameters. The attenuation
 208 experienced by the wave in such a medium, expressed as the inverse of the quality factor, is
 209 given by [e.g., 26]

$$210 \quad \frac{1}{Q_p(\omega)} = \frac{\Im\{H(\omega)\}}{\Re\{H(\omega)\}}, \quad (23)$$

211 where \Im and \Re denote the real and imaginary parts, respectively. This approach to compute
 212 attenuation and modulus dispersion of compressional waves due to squirt flow has previously
 213 been validated and verified [e.g., 19, 24, 27].

214 III. RESULTS

215 A. Squirt flow effects in a cracked medium

216 Following Quintal *et al.* [19], we consider the scenario of a 2D medium whose REV is
 217 a square of side-length L comprising two interconnected orthogonal cracks embedded in
 218 an elastic homogeneous background (Fig. 1). The cracks constitute the pore space and are
 219 characterized by a length l_f and aperture h_f , such that the aspect ratio is given by $\alpha = h_f/l_f$
 220 [19]. In the following, we take $\alpha = 3.6 \times 10^{-3}$. The side-length of the REV is such that the
 221 porosity of the system is $\phi = 2h_f l_f / L^2 = 0.35\%$. As long as the geometrical configuration,
 222 α , and ϕ are maintained, and the underlying assumptions are valid, the physical process is
 223 completely scalable.

225 When a compressional wave propagates vertically through the cracked medium (Fig.
 226 1a), it compresses the horizontal cracks, thus increasing the fluid pressure within them,
 227 while leaving the fluid pressure in vertical cracks essentially unperturbed. The thus induced
 228 fluid pressure gradients arising between the horizontal and the vertical cracks relax through
 229 viscous fluid flow. For sufficiently small frequencies, the medium is in a relaxed state, that
 230 is, the pressure gradients have time to relax in a half-wave-cycle. In this regime, viscous
 231 dissipation is virtually null and the medium presents its lowest stiffness. Conversely, for
 232 sufficiently high frequencies, fluid pressure does not have enough time to equilibrate in a

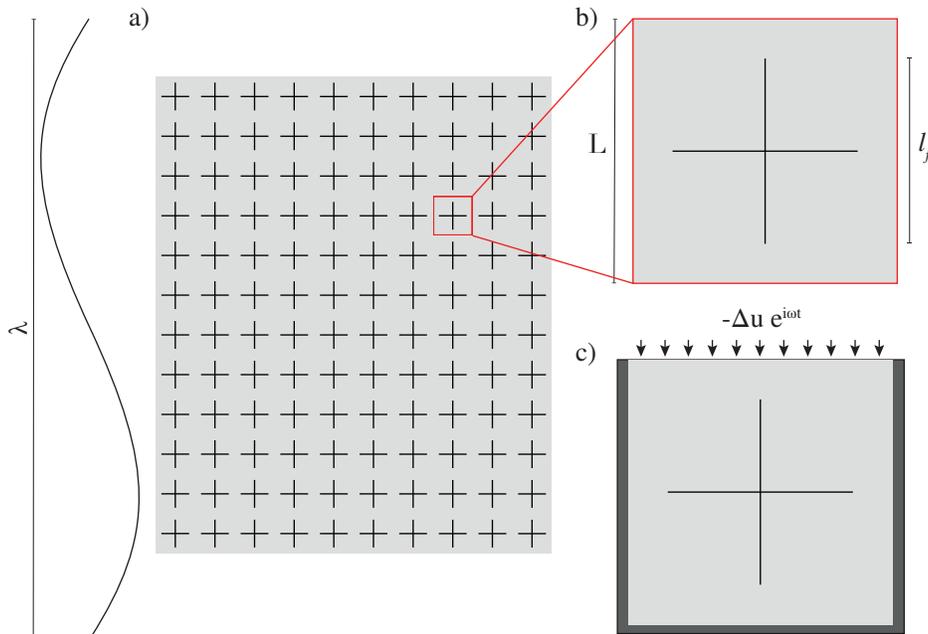


FIG. 1. Schematic illustration of: (a) the probed porous medium; (b) an REV of such medium, which contains a set of interconnected orthogonal cracks; and (c) the oscillatory relaxation test employed to obtain the frequency-dependent overall stress and strain of the medium. Note that the side-length L of the REV is considered to be much smaller than the prevailing wavelengths λ .

233 half-wave-cycle and, thus, the medium presents its highest stiffness and viscous flow and
 234 dissipation are negligible. Interestingly, for intermediate frequencies, significant fluid flow
 235 occurs and, thus, traveling waves can be largely affected by squirt flow.

236 We study the compressional wave attenuation by analyzing the inverse quality factor
 237 Q_p^{-1} for different Deborah numbers χ (Fig. 2). Note that, in Fig. 2, the values of the
 238 quality factor are normalized with respect to the maximum attenuation associated with the
 239 Newtonian scenario, that is,

$$240 \quad \bar{Q}_p(\omega, \chi) = \frac{Q_p(\omega, \chi)}{Q_p(\omega_c, \infty)}. \quad (24)$$

241 This characteristic, combined with a normalized frequency ω/ω_c , renders the results inde-
 242 pendent of K_s , K_f , and η_0 . We observe that when χ is sufficiently high ($\chi \geq 10^4$), the
 243 attenuation curve follows that of a squirt flow process in the presence of a Newtonian fluid.
 244 However, with decreasing χ -values, the non-Newtonian behavior of the fluid becomes more
 245 pronounced. As a result, attenuation decreases and the frequency associated with the peak
 246 attenuation ω_{\max} is shifted towards lower values (Fig. 2). Interestingly, when the non-

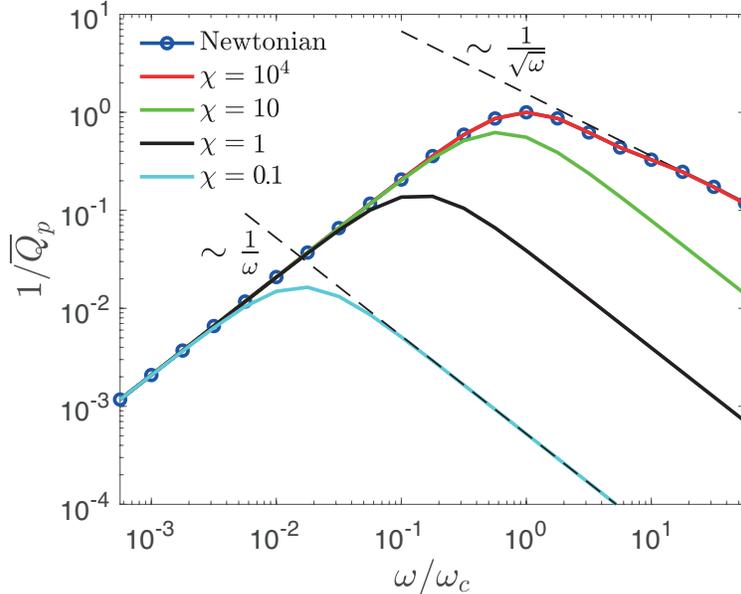


FIG. 2. Normalized inverse quality factor \overline{Q}_p^{-1} as a function of the normalized frequency ω/ω_c for the model shown in Fig. 1. We illustrate the results considering a Newtonian pore fluid (blue line with circles) and Maxwell-type non-Newtonian fluids for different Deborah numbers (colored solid lines). Dashed black lines denote the high-frequency asymptotic behavior of the Newtonian and non-Newtonian fluids.

247 Newtonian behavior of the fluid becomes dominant, the high-frequency asymptotic behavior
 248 of the attenuation changes from $\sim 1/\sqrt{\omega}$, which is the typical asymptote of squirt flow for
 249 Newtonian fluids [13], to $\sim 1/\omega$. This is an interesting characteristic that may permit, in
 250 well constrained scenarios, to discern whether the saturating fluid presents a Newtonian or
 251 non-Newtonian characteristics from wave arrival observations.

252 As a consequence of the squirt flow process, the real part of the plane wave modulus
 253 H increases, evidencing a stiffening effect with increasing frequencies. This characteristic
 254 is illustrated in Fig. 3, which shows the real part of the plane wave modulus normalized
 255 respect to its low-frequency Newtonian counterpart

$$256 \quad \Re\{\overline{H}(\omega, \chi)\} = \frac{\Re\{H(\omega, \chi)\}}{\Re\{H(0, \infty)\}}. \quad (25)$$

257 We note that dispersion is more pronounced for high values of the Deborah number χ ,
 258 emulating the Newtonian behavior (Fig. 3). With decreasing values of χ , the dispersion
 259 decreases, as does the inflection of the dispersion curve moves towards lower frequencies.
 260 This is expected, as the inflection in the dispersion curve is associated with the frequency

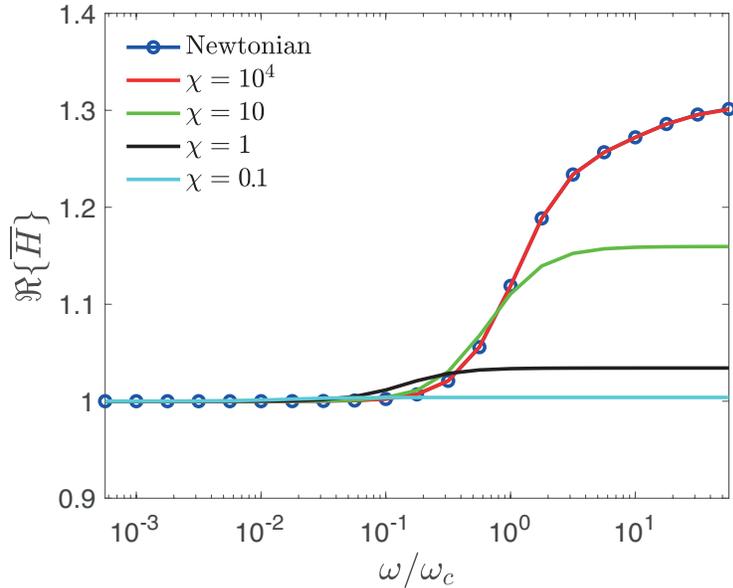


FIG. 3. Real part of the normalized effective bulk modulus $\Re\{\overline{H}\}$ as a function of the normalized frequency ω/ω_c for the model illustrated in Fig. 1. We illustrate the results considering a Newtonian pore fluid (blue line with circles) and non-Newtonian fluids with different Deborah numbers (colored solid lines).

261 corresponding to the attenuation peak ω_{\max} , which also moves towards lower frequencies
 262 for decreasing values of χ (Fig. 2). It is interesting to observe that, even though $\Re\{\overline{H}\}$
 263 generally increases with χ , this is not true across the entire frequency band. In a narrow
 264 frequency range near the inflection of the dispersion curve, smaller χ -values are associated
 265 with slightly higher $\Re\{H\}$ values, which, in turn, could result in higher compressional wave
 266 velocities.

268 B. Deborah number and peak frequency

269 The frequency associated with the maximum attenuation ω_{\max} changes with the Deborah
 270 number χ (Fig. 2). In the classic squirt flow mechanism for porous/cracked media saturated
 271 with Newtonian fluids, the peak frequency, which, in this case, is given by ω_c , fulfills [e.g.,
 272 28, 29]

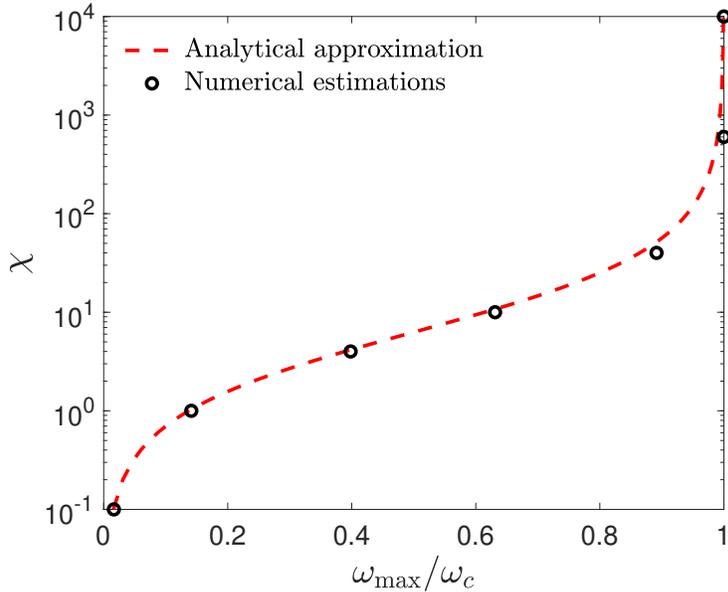
$$273 \omega_c \propto \frac{K_s}{\eta_0} a^3. \quad (26)$$

274 This implies that the peak frequency depends on the geometrical characteristics of the fluid-
 275 filled pore space. In the particular case studied here, ω_{\max} depends on the aspect ratio
 276 of the cracks α . Further knowledge regarding the relationship between ω_{\max} and χ may
 277 permit to discern variations in the compressional wave signatures related to the presence of
 278 non-Newtonian fluids from those associated with changes the crack aspect ratio.

279 The relationship between ω_{\max} and χ is displayed in Fig. 4 (black circles). This relation-
 280 ship is retrieved by computing the attenuation curves as functions of frequency for several
 281 χ values, including those illustrated in Fig. 2, and selecting the maxima of the correspond-
 282 ing curves. This is done following the approach described in Section II. We find that the
 283 relationship between ω_{\max} and χ can be approximated by the following empirical formula

$$284 \quad \frac{\omega_{\max}}{\omega_c} \simeq \left(1 + \frac{2\pi}{\chi}\right)^{-1}, \quad (27)$$

285 which is illustrated in Fig. 4 using red dashed lines.



286 FIG. 4. Relationship between the normalized maximum frequency ω_{\max}/ω_c and the Deborah num-
 287 ber χ . We compare the values obtained from the numerical estimations (black circles) and the
 288 analytical approximation given by Eq. (27) (red dashed line).
 289

290

291 IV. DISCUSSION

292 We have explored squirt flow on attenuation and plane wave modulus dispersion of me-
293 chanical waves in presence of Maxwell-type non-Newtonian pore fluids using an approach
294 based on continuum mechanics. To this end, we consider small strains, wavelengths which
295 are much larger than the pore scale, and Poiseuille-type flow. These assumptions have been
296 proven to be valid and pertinent to model squirt flow effects in a great variety of scenarios
297 in earth sciences, where cracks/pores range from μm - to mm-scale. However, some of the
298 assumptions may be inadequate for very small pores (i.e., molecule scale). For instance,
299 in nanometer-scale pores, the molecules of certain fluids can present a non-zero tangential
300 velocity at the solid-fluid interface, thus rendering the no-slip condition invalid [e.g., 30].

301 Several aspects of the mechanical response of rocks saturated by linear Maxwell-type
302 fluids were analyzed by Tsiklauri and Beresnev [6, 16]. These authors explored the effects
303 of non-Newtonian viscoelastic fluids on wave attenuation and dispersion due to global flow,
304 that is, when inertial forces prevail over viscous forces. The results of these studies are
305 therefore complemented by our work, which focuses on corresponding squirt flow effects.
306 It is important to remark here that our analysis is based on viscoelastic Maxwell fluids
307 with stress relaxation, which are a pertinent representation of some polymeric liquids [e.g.,
308 5]. However, an extension of the corresponding results to non-Newtonian fluids in general
309 (i.e., colloidal suspensions and/or polymeric fluids) is not straightforward. In this sense,
310 alternative non-Newtonian fluid behaviors, such as, shear thinning, shear thickening, and
311 viscoplastic fluids, would require different modelling approaches and upscaling techniques.

312 V. CONCLUSIONS

313 We have presented an approach that permits to include non-Newtonian behavior of
314 Maxwell-type fluids into the viscous dissipation mechanism associated with squirt flow.
315 Our results show that the resulting attenuation and modulus dispersion strongly depend
316 on the Deborah number, which is a measure of the relaxation time of the Maxwell-type
317 fluids with regard to that of the the Newtonian squirt flow process. For increasing Debo-
318 rah numbers, that is, for non-Newtonian fluids with relatively fast Maxwell-type relaxation,
319 the compressional wave attenuation tends to increase and its peak frequency moves towards

320 higher frequencies. Interestingly, the non-Newtonian behavior of pore fluids affects the high-
321 frequency asymptote of the attenuation curves, which becomes inversely proportional to the
322 angular frequency. This characteristic is important as, under well-controlled conditions,
323 it might help to discern whether the medium is effectively saturated by a non-Newtonian
324 fluid based on wave arrival observations. We also note that the plane wave modulus of the
325 medium increases with the Deborah number. However, the plane wave modulus may also
326 decrease within a restricted frequency range around the inflection of the dispersion curve for
327 increasing Deborah numbers. Within this frequency range, the velocity of the compressional
328 waves could therefore increase in response to the displacement of a Newtonian pore fluid
329 by a non-Newtonian phase. Finally, we show that a non-linear relationship exists between
330 the Deborah number and the frequency of the maximum attenuation. The results of this
331 study fundamentally improve our understanding of the squirt flow attenuation mechanism in
332 porous media saturated by non-Newtonian fluids and, thus, provide the basis for advancing
333 corresponding detection and interpretation techniques for a wide range of applications.

334 VI. ACKNOWLEDGMENTS

335 This study was completed within the Swiss Competence Center for Energy Research-
336 Supply of Electricity with support of Innosuisse.

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