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Strategies for written additions in adults

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Studies about strategies used by adults to solve multi-digit written additions are very scarce. However, as advocated here, the specificity and characteristics of written calculations are of undeniable interest. The originality of our approach lies in part in the presentation of two-digit addition problems on a graphics tablet, which allowed us to precisely follow and analyse individuals' solving process. Not only classic solution times and accuracy measures were recorded but also initiation times and starting positions of the calculations. Our results show that adults largely prefer the fixed columnar strategy taught at school rather than more flexible mental strategies. Moreover, the columnar strategy is executed faster and as accurately as other strategies, which suggests that individuals' choice is usually well adapted. This result contradicts past educational intuitions that the use of rigid algorithms might be detrimental to performance. We also demonstrate that a minority of adults can modulate their strategy choice as a function of the characteristics of the problems. Tie problems and additions without carry were indeed solved less frequently through the columnar strategy than non-tie problems and additions with a carry. We conclude that the working memory demand of the arithmetic operation influences strategy selection in written problem-solving.

Keywords: Arithmetic; Flexibility; Numerical cognition.

Strategies in mental arithmetic are extensively studied in the field of numerical cognition (Ashcraft & Guillaume, 2009 for a review). In contrast, researchers are less interested in investigating strategies used by individuals when they perform written calculations. This might be due to the common idea that written and mental calculations share common components and that, consequently, specific research on written calculations is necessarily redundant. Nevertheless, there are several reasons to posit that these two skills are distinct. First, they involve different capacities. Using a battery of tests to evaluate verbal and visual-spatial performance as well as written and mental arithmetic skills, Solan (1987) revealed that mental arithmetic correlates significantly with verbal but not with visual-spatial capacities. Conversely, written arithmetic correlates with visual-spatial but not with verbal performance. Second, Thevenot and Castel (2012) investigated the relationship between mental and written additions and examined whether training one skill transfers to the other. In one experiment, adults were trained in mental addition and were tested before and after training

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on written addition. The reverse was done in a second experiment wherein adults were trained on written addition and tested on mental addition. The results showed that training written addition led to an improvement in mental addition whereas the opposite was not observed. Given that there is a transfer from written to mental addition, the authors first concluded that the two skills necessarily share common components. Nevertheless, because mental addition does not transfer to written addition, the authors concluded further that some characteristics of written addition are not trained during mental addition. Thus, compared to the latter, specific components and mechanisms seem to be proper to written addition. Further support for a distinction between mental and written calculations is provided by neuropsychological studies. For example, Semenza, Miceli, and Girelli (1997) reported the case of a patient who was able to solve complex mental calculations but not complex written calculations. This dissociation was interpreted as difficulties for the patient to apply the required algorithmic written procedures. In fact, such dissociation has been observed both in developmental dyscalculia (Temple, 1991) and in brain-damaged patients (e.g., McCloskey, Aliminosa, & Sokol, 1991).

Therefore, it seems that written calculation is a cognitive skill that needs specific attention from researchers. Nevertheless, only a few experimental studies have described what children do when they can use paper and pencil to solve multi-digit operations. A pioneer study by Brown and Burton (1978) described the two main methods than can be used for written additions. The first method corresponds to the standard algorithm from right to left wherein the units are added first, the tens are carried if necessary, the tens are added and so on for additions involving numbers with more than two digits. The second method consists in processing the problem from left to right with any carries being below the answer in the next column to the left. If any carries occur in the problem, they must be added in a second addition. This second method has rarely been described in subsequent studies, which rather opposed the standard columnar algorithm to more mental strategies. As a matter of fact, more recent studies concur to the conclusion that children often use the columnar strategy in order to perform written calculation. More precisely, Lucangeli, Tressoldi, Bendotti, Bonami, and Siegel (2003) showed that the frequency of the columnar strategy use increases from 83% among 8-year-old children to 91% among 11-year-old children. Similarly, Ginsburg, Posner, and Russell (1981) reported variations from 79% among 8-year-olds to 96% among 10-year-old pupils. Of particular interest is the fact that the percentage of the columnar strategy use increases through practice and schooling. This could be considered as an improvement because standard columnar algorithms are more and more systematically implemented by children. However, the strict and systematic application of routines taught at school could also be considered as a drawback. In terms of performance, it might indeed be disadvantageous to engage in a columnar procedure to solve problems such as 50 + 50 or 11 + 12. As reminded by Verschaffel, Greer, and De Corte (2007), Thompson (1999) considered that algorithms encourage cognitive passivity and Kamii and Dominick (1998) even claim that teaching algorithms in elementary school could have harmful effect on cognitive flexibility. Indeed, Torbeyns and Verschaffel (2013) note that many reform-based documents encourage teachers to teach mental computation before and besides written algorithm. Nevertheless, Selter (2001, cited by Verschaffel et al., 2007), showed that 10–12-year-old German children solve multi-digit sums, even sums as 527 + 399 that could be solved easily using mental computation, using what could be considered as inefficient, cognitively costly, and inflexible written algorithms. Given the apparent lack of children's adaptivity in the appliance of standard algorithms, it might be envisioned that the quasi exclusive use of the columnar strategy in 10-year-old children is only a transitory developmental stage wherein children over-practice their new acquisition. In later years, the application of the written algorithm could be less systematic and more adaptively selected as a function of the characteristics of the problems. One argument for this scenario is the fact that the ability to represent the magnitude of numbers, a central aspect of basic number sense which sharpens through development (Halberda & Feigenson, 2008), is particularly linked to the flexible use of mental calculations (Linsen, Verschaffel, Reynvoet, & De Smedt, 2014, 2015).

In the present study, we examine the possibility that, in written calculation, the frequency use of flexible strategy increases with age by investigating strategy selection in a mature cognitive system. To this aim, we presented adults with two-digit additions on a graphics tablet, which enabled us to collect precise measures of individuals' solving process. As just described, the columnar written strategy is the one preferred by children to solve written calculations. Nevertheless, adults might use mental strategies as efficiently as written procedures in order to solve specific written calculations (as evoked earlier, e.g., 50 + 50 or 12 + 11). Lemaire and Callies (2009) identified four strategies to solve multi-digit additions mentally, namely direct retrieval from memory (e.g., 53 + 35 is associated to its answer 88), full decomposition (e.g., 53 + 35 = (50 + 30) + (3 + 5) = 80 + 8 = 88), partial decomposition (e.g., 53 + 35 = (53 + 30) + 5 = 83 + 5 = 88), and transformation (e.g., 53 + 35 = 50 + 38 = 88). The full and partial decompositions are also called the "decompose-tens-and-ones" and the "begin with one number" methods by Fuson et al. (1997), the 1010 and N10 procedures by Blöte, Van der Burg, and Klein (2001) or Beishuizen (1993) or the split and jump methods, respectively (see Green, Lemaire, & Dufau, 2007 for a review). Whether or not adults prefer one of these mental strategies over the columnar one to solve written calculations might strongly depend on the characteristics of the problems. For example, 30 + 30, in opposition to 48 + 56, might be more likely to be solved through retrieval than through a fixed columnar strategy. More generally, the difficulty of a problem is likely to impact the process solving (Klein et al., 2010). This difficulty is strongly influenced by the number of carries in the problem (Green et al., 2007; Imbo, Vandierendonck, & De Rammelaere, 2007) and the fact that the problem involves repeated operand or not (Campbell & Gunter, 2002; Groen & Parkman, 1972). In our experiment, we were therefore particularly interested in determining the role of these two factors on individuals' strategies.

The sequence of positions in which participants write down the solution on the graphics tablet will allow us to infer the strategy used. If individuals start writing the answer down from the units, they have necessarily applied a strict columnar algorithm. Oppositely, if they start from another position, they have used a more flexible mental strategy, which could be one of the mental procedures described earlier. As already mentioned, because problems involving repeated operands can be retrieved from long-term memory (e.g., Campbell & Gunter, 2002; Fayol & Thevenot, 2012), they should more often be solved through mental strategies than standard written algorithms. However, this prediction is based on research about single-digit operand problems and, to the best of our knowledge, strategy variations on multi-digit tie problems have never been studied. Our predictions about carry problems are more empirically grounded because Green et al. (2007) showed in the domain of mental arithmetic that starting to process a problem from the unit is more probable for problems involving at least one carry.

METHOD

Participants

Forty-eight first-year psychology students (42 females) from the University of Geneva were recruited for the experiment (Mean age = 22; ranging from 17 to 35). Their participation was one of the conditions required for course validation.

Material

Sixty additions involving two-digit numbers were presented vertically on the screen (see the Appendix). Thirty problems did not contain any carry (e.g., 83 + 34) whereas 30 others contain a carry (e.g., 57 + 29). Half of problems had a two-digit sum whereas the other half a three-digit sum. One-third of the additions were constructed with repeated addends (tie problems) whereas the other two-thirds did not (non-tie problems). In order to avoid ceiling or floor effects in performance, the sum of the additions was arbitrarily comprised between 40 and 160. For tie problems, no addends corresponded to a multiple of 10 or 11. For non-tie problems, the additions never contained two addends ending with a 0 (e.g., 20 + 30). In order to ensure that problems in each category mainly differed in the characteristics we manipulated, we verified that the sum of the addends was the same for tie (100.6) and non-tie problems (100), $t(58) = .07$, $p = .95$ and the same for carry (101.8) and no carry problems (98.6), $t(58) = .38$, $p = .70$.

Each participant was presented with the 60 problems in a different random order. For each addition, the starting position, the initiation and solution times as well as the accuracy were recorded. The starting position was defined according to the position that was written first (units or not units). The initiation time was the time elapsed from the presentation of a problem up to the moment when the participant put the stylus on the graphics tablet for the first time to work the problem out. The solution time was the total time between the presentation of a problem and the moment when the participant lifted the stylus from the tablet after having solved the problem. Both the initiation and solution times were measured in milliseconds. The correctness was judged and registered by the experimenter (first author). In order to familiarise the participants with the experiment, we started with 10 practice problems.

Apparatus and procedure

The design of the graphics-tablet experiment was carried out using the interactive pen display Wacom DTF-720, which was connected to a Dell Latitude E6410 laptop. The tablet was a 17-inch screen with a 1024 × 768 resolution, placed on a mount adjustable from 17° to 73°. The participant sat on a chair and the graphics tablet was placed on a table. We allowed the participant to adjust the tablet to a comfortable inclination. The experiment was controlled by Presentation (version 16.4).

Participants had to click with the stylus on the screen when they were ready and the first addition then appeared. The addends were presented in Times New Roman size 130, which corresponds to 2.2 cm vertically on the screen. On the left of the second addend, there was a " + " sign, indicating the operation they had to perform. Under the second addend there were three boxes of $4.9 \times 7.0\ cm^2$ or 150 × 200 pixels each. We asked the participants to write the answer to the addition in the given boxes. If the answer was a two-digit number, they were to use the middle and the right-most boxes. On top of the first addend, there was a box measuring $15.5 \times 2.8\ cm^2$ or 470 × 80 pixels, in which the participants could write the carry should they need to (see Figure 1). The whole stimulus covered an area of $15.5 \times 22.9\ cm^2$ or 470 × 650 pixels. Once the participants had finished writing the answer, the experimenter judged whether the response was correct by key press. A preparation screen for the next problem then appeared and the process was repeated until all the 60 problems had been presented.

The computer timer was activated as soon as the problem appeared on the screen and ran until it disappeared. As soon as the stylus touched the tablet for the first time, the time and the x and y coordinates of the stylus position were registered every 17 ms (i.e., screen refresh time). We operationalised initiation time as the time elapsed between the presentation of the problem and the first touch. Solution time was operationalised as the time elapsed between the presentation of the problem and the last time the stylus lost contact with the tablet screen. The last point of contact was identified when no more coordinates were recorded further. Once the problem was solved, the experimenter pressed one of the two labelled keys coding accuracy and the timer was reset. This procedure ensured that solution times was not contaminated by the time taken by the experimenter to judge the correctness of the response.

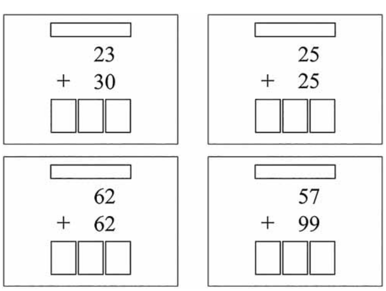

Figure 1. Examples of stimuli presented on the graphics tablet (with the outer frame representing the screen), for non-tie additions without a carry (top left), tie additions with a carry (top right), tie additions without a carry (bottom left) and non-tie additions with a carry (bottom right).

RESULTS

First, the results will be presented for the whole population. Second, we will focus on the participants who used both strategies (i.e., starting from the units or the tens) in order to determine whether the percentages of use of a specific strategy were modulated by the characteristics of the problems. Third, the results for participants who used only the unit strategy will be described, which will allow us to analyse the role of problem characteristics on performance when the dominant strategy is used. Fourth, performance on problems when the unit strategy is selected will be compared in mixed-strategy and unit-strategy users. This analysis will inform us about the potential benefit of strategy switch across problems. For the sake of concision and clarity, these last analyses will focus only on solution times. Finally, correlation and regression analyses on percentages of use of the unit strategy in mixed-strategy users will be conducted with carry, tie, sum of the problem, likelihood of a mental strategy, and parity of operands as the predictors. The rationale in considering these different predictors will be provided within the section devoted to these analyses.

Whole population

Percentage of strategy use

Sixty-five per cent of the participants ($N = 31$) always started the problem-solving process from the units, 8% systematically from the tens ($N = 4$), whereas 27% mixed the strategy ($N = 13$). Among the latter, the unit-strategy was used on average 76% of the time, ranging from 5% to 98%.

Performance depending on the strategy

A one-way ANOVA with the strategy used (unit, tens, or mixed) as the random variable was carried out on accuracy, initiation, and solution times.

Accuracy. Accuracy did not vary as a function of the strategy and was very high whatever the condition (95%, 96%, and 96% for unit, mixed, and tens strategies, respectively), $F < 1$.

Initiation times. There was a significant effect of strategy on initiation times, $F(2,45) = 15.91$, $\eta_p^2 = .41$, $p < .001$. Participants who always started from the units were faster (1486 ms) than those who mixed strategies (2158 ms), $F(1, 45) = 11.04$, $\eta_p^2 = .20$, $p = .002$, who, in turn, were faster than participants who always started from the tens (3143 ms), $F(1, 45) = 7.90$, $\eta_p^2 = .15$, $p = .007$ (see the top and bottom panels of Figure 2 for the unit- and tens-strategy users).

Solution times. There was a marginal effect of strategy on solution times, $F(2, 45) = 2.50$, $\eta_p^2 = .10$, $p = .09$. Participants who started from the units solved the problems faster (4394 ms) than those who mixed the strategies (5077 ms) or who started always from the tens (5095 ms), $F(1, 45) = 4.05$, $\eta_p^2 = .08$, $p = .05$ (see the top and bottom panels of Figure 2 for the unit- and tens-strategy users).

Mixed-strategy users only

Percentage of strategy use

As already mentioned earlier, 13 participants out of 48 (27%) used both strategies. They used the unit strategy on average on 76% of the problems

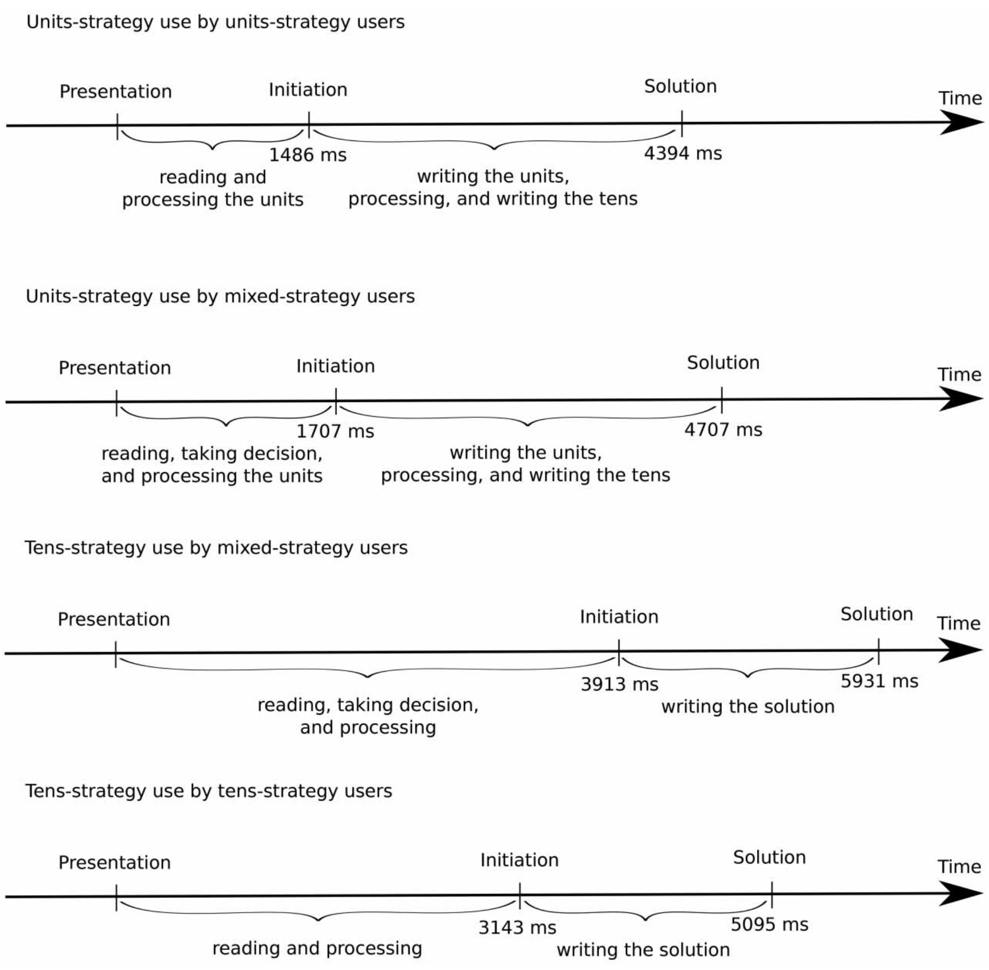

Figure 2. Time line for written addition solving, using the unit strategy performed by the unit-strategy group (top panel) and the mixed-strategy group (second panel) and using the tens strategy performed by the mixed-strategy group (third panel) and the tens-strategy group (bottom panel).

(ranging from 5% to 98%). The following analysis will elaborate those results as a function of the characteristics of the problems.

A 2 (carry: no-carry or carry) × 2 (problem type: tie or non-tie) ANOVA with the two variables as within measures was performed on the percentage of use of the unit strategy. Non-tie problems were more often solved using the unit strategy (80%, and therefore 20% using the tens strategy) than tie problems (69%), $F(1, 12) = 6.44$, $\eta_p^2 = .35$, $p = .03$. Likewise, problems with a carry were more often solved using the unit strategy (80%) than problems without a carry (69%), $F(1, 12) = 5.29$, $\eta_p^2 = .31$, $p = .04$.

Performance depending on the strategy

A 2 (strategy: units or tens) × 2 (carry: no-carry or carry) × 2 (problem type: tie or non-tie) ANOVA with the problems as the random variable was performed on accuracy, initiation, and solution times. The carry and tie factors corresponded to between measures whereas the strategy factor constituted a within measure because a problem could be solved either from the units or the tens. An analysis by participant was impossible due to the fact that some participants never used a specific strategy for a specific type of problems. For example, 6 participants out of the 13 never used the tens strategy for a non-tie problem with a carry.

Accuracy

In mixed-strategy users and in accordance with previous research (Green et al., 2007; Imbo, Vandierendonck, et al., 2007), additions without carry (97%) were performed marginally better than additions with a carry (91%), $F(1,56) = 3.14$, $\eta_p^2 = .05$, $p = .08$. However, there was no difference in accuracy depending on the strategy used (97% and 91% for problems started from the units and the tens, respectively, $F(1, 56) = 2.58$, $p = .11$). These two variables did not interact, $F < 1$. Regarding the type of the problem, there was no difference in performance between tie (95%) and non-tie problems (93%), $F < 1$, and again strategy did not impact this result, $F < 1$. Finally, the two characteristics of the problems did not interact and those factors did not interact either with the strategy, both $F_s < 1$.

Initiation times

In mixed-strategy users, additions without a carry were initiated faster (1942 ms) than additions with a carry (3678 ms), $F(1, 56) = 24.60$, $\eta_p^2 = .31$, $p < .001$. Moreover, problems started from the units were initiated faster (1707 ms) than problems started from the tens (3913 ms), $F(1, 56) = 41.68$, $\eta_p^2 = .43$, $p < .001$ (see the second and third panels of Figure 2). This was true whether there was a carry, $F(1, 56) = 49.56$, $\eta_p^2 = .47$, $p < .001$, or not, $F(1, 56) = 4.37$, $\eta_p^2 = .07$, $p = .04$. However, the effect of strategy was more pronounced for carry problems than for no carry problems, $F(1, 56) = 12.24$, $\eta_p^2 = .18$, $p < .001$.

As far as the type of the problem was concerned, tie additions were initiated faster (2203 ms) than non-tie additions (3417 ms), $F(1, 56) = 12.02$, $\eta_p^2 = .18$, $p = .001$, and this variable also modulated the effects of strategies, $F(1, 56) = 5.51$, $\eta_p^2 = .09$, $p = .02$. Problems started from the units were initiated faster than problems started from the tens both for non-tie, $F(1, 56) = 58.12$, $\eta_p^2 = .51$, $p < .001$, and non-tie problems, $F(1, 56) = 6.33$, $\eta_p^2 = .10$, $p = .01$. However, the effect of strategy was more pronounced for non-tie problems than for tie problems, $F(1, 56) = 5.51$, $\eta_p^2 = .09$, $p = .02$.

Finally, the two characteristics of the problems did not interact, $F(1, 56) = 2.10$, $p = .15$, and those factors did not interact either with the strategy, $F < 1$ (Figure 3).

Solution times

In mixed-strategy users, additions without a carry were solved faster (3998 ms) than additions with a carry (6640 ms), $F(1, 56) = 37.25$, $\eta_p^2 = .40$, $p < .001$. Moreover, problems started from the units were solved faster (4707 ms) than problems started from the tens (5931 ms), $F(1, 56) = 12.79$, $\eta_p^2 = .19$, $p < .001$ (see the second and third panels of Figure 2). However, this was true only for problems with a carry, $F(1, 56) = 17.05$, $\eta_p^2 = .23$, $p < .001$ ($F < 1$ when there was no carry and $F(1, 56) = 5.12$, $\eta_p^2 = .08$, $p = .03$ for the interaction).

Regarding a problem type, tie additions were solved faster (4444 ms) than non-tie additions (6194 ms), $F(1, 56) = 16.35$, $\eta_p^2 = .23$, $p < .001$, and this variable also modulated the effects of strategies, $F(1, 56) = 4.64$, $\eta_p^2 = .07$, $p = .04$. Again, the effect of strategy was only significant for non-tie problems, $F(1, 56) = 24.27$, $\eta_p^2 = .30$, $p < .001$ ($F < 1$ for tie problems).

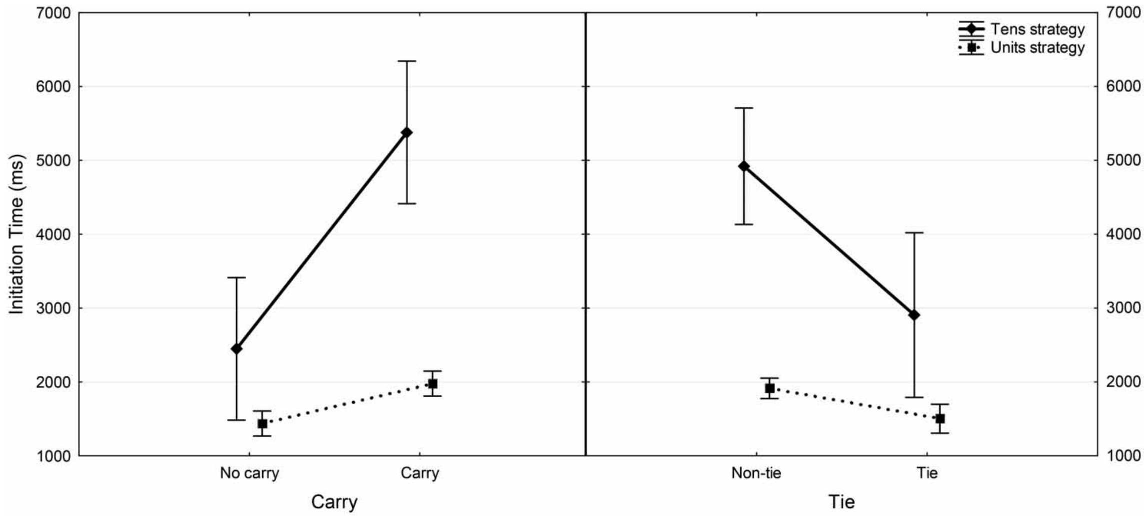

Figure 3. Initiation time for problems solved from the units (dotted line) and from the tens (solid line) as a function of carry (left panel) and tie (right panel) by mixed-strategy users.

Finally, we did not find an interaction between the two problem characteristics, $F(1, 56) = 2.81$, $p = .10$, or a two-way interaction with the strategy, $F < 1$ (Figure 4).

Unit-strategy users only

Performance depending on the problem characteristics

A 2 (problem type: tie or non-tie) × 2 (carry: carry or no carry) ANOVA with both variables as within measures was performed on accuracy, initiation, and solution times (Table 1).

Accuracy

In unit-strategy users, additions without a carry (97%) were performed better than additions with a carry (94%), $F(1, 30) = 5.52$, $\eta_p^2 = .16$, $p = .03$. Moreover tie problems (98%) were performed better than non-tie problems (93%), $F(1, 30) = 14.86$, $\eta_p^2 = .33$, $p < .001$. Finally, the interaction between the two characteristics of the problems was marginally significant, $F(1, 30) = 3.35$, $\eta_p^2 = .10$, $p = .08$, and

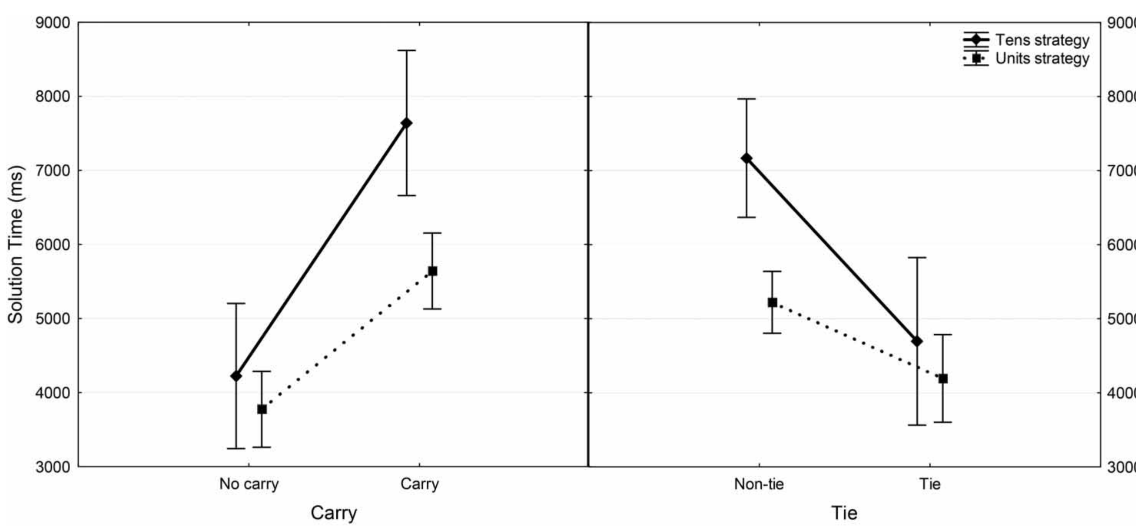

Figure 4. Solution time for problems solved from the units (dotted line) and from the tens (solid line) as a function of carry (left panel) and tie (right panel) by mixed-strategy users.

TABLE 1
The effect of characteristics on accuracy, initiation, and solution times in unit-strategy users

	Tie problems		*Non-tie problems*	
	Carry	*No carry*	*Carry*	*No carry*
Accuracy (%)	97	98	91	95
Initiation time (ms)	1378	1033	2063	1188
Solution time (ms)	4443	3068	5623	3758

showed that the carry effect was observed only for non-tie problems, $F(1, 30) = 9.32$, $\eta_p^2 = .24$, $p = .004$, but not for tie-problems, $F < 1$.

Initiation times

In unit-strategy users, initiation times were longer for carry problems (1721 ms) than for problems without carry (1111 ms), $F(1, 30) = 113.84$, $\eta_p^2 = .79$, $p < .001$. Moreover they were longer for non-tie problems (1623 ms) than for tie problems (1206 ms), $F(1, 30) = 54.85$, $\eta_p^2 = .65$, $p < .001$. Finally, the interaction between the two characteristics of the problems was significant, $F(1, 30) = 41.10$, $\eta_p^2 = .58$, $p < .001$, and showed that the carry effect was more pronounced for non-tie problems, $F(1, 30) = 101.49$, $\eta_p^2 = .77$, $p < .001$, than for tie problems, $F(1, 30) = 49.26$, $\eta_p^2 = .62$, $p < .001$.

Solution times

In unit-strategy users, solution times were longer for carry problems (5033 ms) than for problems without a carry (3413 ms), $F(1, 30) = 214.87$, $\eta_p^2 = .88$, $p < .001$. Moreover they were longer for non-tie problems (4690 ms) than for tie problems (3756 ms), $F(1, 30) = 88.47$, $\eta_p^2 = .75$, $p < .001$. Finally, the interaction between the two characteristics of the problems was significant, $F(1, 30) = 10.59$, $\eta_p^2 = .26$, $p = .003$, and showed that the carry effect was more pronounced for non-tie problems, $F(1, 30) = 215.53$, $\eta_p^2 = .88$, $p < .001$, than for tie problems, $F(1, 30) = 96.36$, $\eta_p^2 = .76$, $p < .001$.

Mixed-strategy users vs. unit-strategy users

To investigate the use of the dominant unit strategy by different strategy users, a 2 (strategy group: unit- vs. mixed-strategy users) × 2 (carry: carry vs. no carry) × 2 (problem type: tie vs. non-tie) repeated-measures ANOVA with first variable as a between measure and the two last variables as within measures was conducted on solution times. Because two mixed-strategy users did not use the unit strategy in all problem-type categories, only the data of 11 participants in this group were included in this analysis. The results revealed that solution times did not differ significantly between mixed- (4736 ms) and unit-strategy users (4223 ms), $F(1, 40) = 2.12$, $p = .15$, and this was true whatever the characteristics of the problems (both $Fs < 1$).

Correlation and regression analyses

Naturally, we considered tie (coded 1 and non-tie 0) and carry (coded 1 and non-carry 0) as predictors of percentages of use of the unit-strategy in the 13 participants who used both strategies. Problem sum was also entered as a predictor because the problem size is well known to impact the addition-solving process (e.g., Campbell & Xue, 2001; LeFevre, Sadesky, & Bisanz, 1996; Thevenot, Fanget, & Fayol, 2007). Parity of operands is also known to influence strategy choice (e.g., Lemaire & Reder, 1999; Masse & Lemaire, 2001) and was entered in the analyses as a variable with 1 coding for two odd operands, 2 coding for two even operands, and 3 coding for problems with one odd and one even operand. Finally, Torbeyns and Verschaffel (2013) examined the effect of the likelihood of the use of a mental strategy for specific problems. We adapted their classification and considered problems with one of the operands with 0, 1, and 9 as the unit as easily solvable mentally. They were coded 1 against 0 for all other problems. These problems are indeed easy to solve through a round-up or round-down strategy (e.g., Imbo, Duverne, & Lemaire, 2007) and might limit the use of a strict written algorithm.

Percentages of use of the unit-strategy were highly correlated with the presence of a carry ($r = .40$, $p = .001$), the type (tie vs. non-tie) of the problem ($r = .38$, $p = .003$), and the sum of the problem ($r = .28$, $p = .03$). The other factors were not related to percentages of use of the unit strategy. In order to determine the amount of variance explained by each of those factors, we ran a stepwise regression analysis, introducing a carry in the first step, the problem type (tie vs. non-tie) in the second step, and the sum of the problem in the third step. The presence of a carry explained 16% of the variance in the percentages of use of the unit-strategy,

$F(1, 58) = 11.21$, $p = .001$. The introduction of the problem type in the model added a significant 15% to the explained variance, $F(1, 57) = 12.08$, $p = .001$. The last step consisted in entering the sum of the problem to the model. Again, the addition of this factor increased significantly the percentage of explained variance by 7%, $F(1, 56) = 6.40$, $p = .01$. Another regression analysis was conducted with the introduction of the remaining variables as next steps, but no other factor led to a significant increase of R^2.

DISCUSSION

This research was conducted in order to examine the strategies used by adults when they solve written additions. We were particularly interested in the factors that may influence the choice of a strategy over another. The originality of the study lies in part in the use of a graphics tablet, which allowed precise tracking and description of individuals' solving process. If participants started to solve the problem from the units, we could infer that they used the columnar strategy taught at school. In contrast, if participants started from the tens or the hundreds, they obviously did not use this rigid strategy but a more flexible one closer to mental strategies.

Our results clearly show that solving the additions from the units was the preferred strategy of adult participants. Therefore, similar to children (Ginsburg et al., 1981; Lucangeli et al., 2003; Torbeyns & Verschaffel, 2013), adults mainly use the strategy taught at school rather than more flexible mental strategies such as partial or full decomposition or even direct retrieval of the answer from memory. This seems to be an adaptive choice because solution times were shorter when problems were solved by the columnar strategy than when they were started from the tens. In the latter case, initiation times were logically longer than for the columnar strategies. Indeed, when processing starts from the tens, participants have to solve the problems mentally first before they start writing the solution. More interesting is the fact that this lag in initiation times was not caught up later on during the solving process (see Figure 2). The shorter initiation and solution times when additions are solved from the units was, however, not detrimental to the accuracy, as both strategies led to a comparable number of correct answers. Therefore, and contrary to what we suggested in the introduction of this article, the use of the columnar strategy is not a transient developmental stage and, moreover, it seems to be the best strategy. This result in adults is original and suggests that standard written algorithms are less inefficient and less cognitively costly than it was suggested in the past (Selter, 2001, cited by Verschaffel et al., 2007). This is coherent with Hickendorff, Heiser, Van Putten, and Verhelst (2009) observations in 11–12-year-old Dutch children who received reform-orientated mathematics instruction insisting on the use of mental strategies. Mental computation was indeed frequently implemented by children but this change in strategy choices was associated with a decrease in performance compared to the previous situation wherein children classically apply standard written algorithms. Our results are also perfectly in line with the observations made by Torbeyns and Verschaffel (2013) in 10-year-old Flemish children who largely prefer the written algorithm strategies over mental calculations, whatever the characteristics of multi-digit additions and subtractions. Again, this choice appeared to be highly adaptive because children were more performant when they were forced to choose written rather than mental strategies. Altogether, our results in adults and previous results in children support the conclusion that flexible and innovative strategies do not always result in better performance than more rigid and less cognitively elaborated solution methods.

Following Torbeyns and Verschaffel's conclusions (2013) in children, our results suggest that, even for adults, the working memory demand put on the cognitive system during multi-digit calculation is high enough to benefit from a strategy that releases memory resources. Indeed, mental calculation requires the temporary storage of transient results before the final answer can be reached (DeStefano & LeFevre, 2004). On the contrary, working memory is necessarily less taxed when intermediary results can be put down directly in writing. Furthermore, no matter how many digits there are in the addends, the calculation is reduced into one single-digit addition at a time. The advantage of the columnar strategy is attested in our experiment by the fact that participants who always started to process the problems from the units were faster than participants who always started from the tens or even from participants who mixed the two strategies. This last result demonstrates that strategy switching between problems is not advantageous. However, strategy switch-costs (e.g., Ardiale, Hodzik, & Lemaire, 2012; Lemaire & Lecacheur, 2010) are not likely to explain that the mixed strategy is not the optimum one because when solution times are examined only for the unit strategy, there is no difference between mixed-strategy and unit-strategy users.

Switch-costs would have necessarily resulted in shorter solution times when the unit strategy is systematically repeated than when it is intermixed with mental strategies. Contrary to past intuitions, the conclusion here is again that, generally, mental strategies are time-consuming and cognitively demanding in comparison to written algorithms. However, as we will develop later, this reasoning holds only if selection effects do not bias our results or, in other words, if the characteristics of participants (fast or slow for example) are not determinant of strategy choices.

Still, the analyses we conducted on mixed-strategy users showed that the percentages of use of the unit strategy can be modulated depending on the characteristics of the problems. The unit strategy was indeed more often used for carry and non-tie problems. However, the effects of the characteristics of the problems never interacted. To the best of our knowledge, our research is the first revealing an impact of multi-digit tie problems on individuals' strategies. Tie problems were therefore more often used through the implementation of mental strategies and, maybe, through the use of direct retrieval of the answer from long-term memory. If our participants knew that 32 + 32 is 64, there was no need for them to engage in the sequential columnar routine. Again, this variation in strategy choice seemed very adaptive as tie problems were initiated and solved faster than non-tie problems and were also more accurately solved. Furthermore, regression analyses allowed us to study the influence of factors that were not manipulated in the analysis of variance. These analyses confirmed the role of carry and tie problems in determining the choice of the strategy. Moreover, the sum of the problem turned out to be also a good predictor of the strategy used. Interestingly and in perfect accordance with Torbeyns and Verschaffel's (2013) observations, problems that were easy to round up or round down did not elicit a particular strategy. Finally, parity of the operands did not operate either as a predictor of percentages of use of the unit strategy.

The fact that the unit strategy was more often used for tie and carry problems strengthens our conclusion that the unit strategy is particularly efficient when the working memory demand of the problem is high. In the same line and even more interestingly, the use of the unit strategy seems to reduce the intrinsic difficulty of the problem. This is suggested by the fact that for easy problems (i.e., tie and no carry), solution times did not differ as a function of the strategy used by mixed-strategy users. In contrast, for difficult problems (i.e., non-tie or with a carry), solution times were longer than for easy problems only when the problems were solved by mental calculations. When standard columnar algorithms were used, there was no more difference in percentages of correct answer as a function of the difficulty of the problem. Of course, this is not to say that the unit strategy always annihilates problem difficulty because individuals who always use the unit strategy still suffer from the cognitive demand of the calculations. Indeed, carry and tie effects interacted within this population and carry problems led to lower percentages of correct answers, were initiated and solved slower than non-carry problems, especially when the problems were non-ties.

To conclude, we showed here that adults mainly rely on the columnar strategy to solve two-digit additions presented vertically. However, the choice of a strategy to solve the problems was influenced by the presence of carry and the nature of the operands involved in the problems, namely repeated or different operands. This shows that individuals do not always blindly follow a fixed and standard written routine but can also adapt their strategy to the characteristics of the problem. Still, we failed to demonstrate here that this adaptivity to the problem characteristics was associated with better performance in the task. On the contrary, our set of results concurs to the conclusion that systematically using rigid algorithms in order to solve written calculations is the best strategy. These results were obtained using a promising technique of investigation ensuring an ecological observation of individuals' behaviours. Indeed and contrary to verbal reports, known to be potentially problematic (e.g., Ericsson & Simon, 1980; Kirk & Ashcraft, 2001; Lucidi & Thevenot, 2014; Thevenot, Castel, Fanget, & Fayol, 2010), our experimental setting cannot influence participants' strategies by drawing their attention on the object of the study. Moreover, a description of our participants' production revealed that out of 44 adults who used the unit strategy, only 19 made use of the carry box. Interestingly, the carry was systematically written down after the unit. The fact that a written record of the carry processing was not available for 25 of our participants demonstrates that post-production observation is not a reliable method of investigation in written arithmetic. Finally, an online observation of participants' behaviours in a classic paper-and-pencil task would have been more informative but precise solution and initiation time collection would not have been possible.

However, in order to gather more robust conclusions about the adaptivity of adults' strategy choices for written calculations, future research using our experimental setting with the graphics tablet could apply the choice/no-choice methodology developed by Siegler and Lemaire (1997). Indeed, a possible bias in studies wherein only a choice condition is examined is called the selection effect according to which, for example, a strategy used mainly on easy items will seem more efficient than a strategy that is almost exclusively used on difficult items. In order to avoid this selection effect, participants can be forced to use a specific strategy for all items and their performance in the choice and no-choices condition is compared. The choice/no-choice methodology seems quite appropriate for our research question because we have identified two specific strategies (i.e., starting from the units or the tens), which constitutes the exhaustive list of strategies that can be applied in the choice condition. As noted by Luwel, Onghena, Torbeyns, Schillemans, and Verschaffel (2009), the choice/no-choice methodology can be more problematic when the task under study or the way the task is apprehended can elicit numerous and various strategies. Additionally, future research could address the role played by the orientation of the problem on strategy choice selection. Within our design, the vertical presentation of the addends may have increased the frequency of the strict columnar strategy (Lemaire & Callies, 2009). It might therefore be interesting to contrast vertical and horizontal presentations. Another line of research that would deserve further investigation is the exploration of the mental strategy that was used when participants did not start the solving process from the units. It is indeed a limitation of the present study that strategies can only be classified as mental or written. As described in our Introduction, mental strategies in multi-digit calculations are numerous and learning further how they are implemented depending on the characteristics of the problems would provide a more comprehensive picture of strategy use in written arithmetic. To this aim, combining the use of the graphics tablet with differed verbal report collection and on-line observations could be useful. Moreover, carry and tie problems were studied here but other problem characteristics could also influence the solving process and should be investigated in the future (i.e., problems with 0, tie problems on operands with a repeated number (22 + 22), problems with a sum equal to 10 on the units …). Finally, a factor that could also influence strategy choices is the succession of strategies. It has indeed been shown repeatedly that using a strategy on a specific problem can impact the choice of the strategy on the subsequent one (Lemaire & Hinault, 2014; Lemaire & Lecacheur, 2010; Luwel, Schillemans, Onghena, & Verschaffel, 2009; Uittenhove, Poletti, Dufau, & Lemaire, 2013). All those avenues for research suggest that written calculation could benefit from more attention from researchers and our first attempt for a precise description of adults' behaviour in this field might constitute a starting point. Developmental studies could also be worthwhile and the replication of our study with the graphics tablet in children might be valuable on theoretical and educational point of views.

REFERENCES

Ardiale, E., Hodzik, S., & Lemaire, P. (2012). Aging and strategy switch costs: A study in arithmetic problem solving. *L'AnnéePsychologique*, *112*, 345–360. doi:10.4074/S0003503312003028

Ashcraft, M. H., & Guillaume, M. M. (2009). Mathematical cognition and the problem size effect. In B. H. Ross (Ed.), *The psychology of learning and motivation* (pp. 121–151). San Diego: Elsevier Academic Press.

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education*, *24*, 294–323. doi:10.2307/749464

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effect. *Journal of Educational Psychology*, *93*, 627–638. doi:10.1037/0022-0663.93.3.627

Brown, J. S., & Burton, R. R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*, *2*(2), 155–192. doi:10.1207/s15516709cog0202_4

Campbell, J. I. D., & Gunter, R. (2002). Calculation, culture, and the repeated operand effect. *Cognition*, *86*, 71–96. doi:10.1016/S0010-0277(02)00138-5

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, *130*, 299–315. doi:10.1037/0096-3445.130.2.299

DeStefano, D., & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, *16*, 353–386. doi:10.1080/09541440244000328

Ericsson, K. A., & Simon, H. A. (1980). Verbal reports as data. *Psychological Review*, *87*, 215–251. doi:10.1037/0033-295X.87.3.215

Fayol, M., & Thevenot, C. (2012). The use of procedural knowledge in simple addition and subtraction problems. *Cognition*, *123*, 392–403. doi:10.1016/j.cognition.2012.02.008

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., … Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and

subtraction. *Journal for Research in Mathematics Education*, *28*, 130–162. doi:10.2307/749759
Ginsburg, H. P., Posner, J. K., & Russell, R. L. (1981). The development of knowledge concerning written arithmetic: A cross-cultural study. *International Journal of Psychology*, *16*, 13–34. doi:10.1080/00207598108247400
Green, H. J., Lemaire, P., & Dufau, S. (2007). Eye movement correlates of younger and older adults' strategies for complex addition. *ActaPsychologica*, *125*, 257–278. doi:10.1016/j.actpsy.2006.08.001
Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, *79*, 329–343. doi:10.1037/h0032950
Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the number sense: The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*, 1457–1465. doi:10.1037/a0012682
Hickendorff, M., Heiser, W. J., Van Putten, C. M., & Verhelst, N. D. (2009). Solution strategies and achievement in Dutch complex arithmetic: Latent variable modeling of change. *Psychometrika*, *74*, 331–350. doi:10.1007/s11336-008-9074-z
Imbo, I., Duverne, S., & Lemaire, P. (2007). Working memory, strategy execution, and strategy selection in mental arithmetic. *The Quarterly Journal of Experimental Psychology*, *60*, 1246–1264. doi:10.1080/17470210600943419
Imbo, I., Vandierendonck, A., & De Rammelaere, S. (2007). The role of working memory in the carry operation of mental arithmetic: Number and value of the carry. *The Quarterly Journal of Experimental Psychology*, *60*, 708–731. doi:10.1080/17470210600762447
Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1–4. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 NCTM yearbook* (pp. 130–140). Reston, VA: National Council of Teachers of Mathematics.
Kirk, E. P., & Ashcraft, M. K. (2001). Telling stories: The perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *27*, 157–175. doi:10.1037/0278-7393.27.1.157
Klein, E., Moeller, K., Dressel, K., Domahs, F., Wood, G., Willmes, K., & Nuerk, H.-C. (2010). To carry or not to carry—Is this the question? Disentangling the carry effect in multi-digit addition. *ActaPsychologica*, *135*, 67–76. doi:10.1016/j.actpsy.2010.06.002
LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *22*, 216–230. doi:10.1037/0278-7393.22.1.216
Lemaire, P., & Callies, S. (2009). Children's strategies in complex arithmetic. *Journal of Experimental Child Psychology*, *103*, 49–65. doi:10.1016/j.jecp.2008.09.007
Lemaire, P., & Hinault, T. (2014). Age-related differences in sequential modulations of poorer-strategy effects. A study in arithmetic problem solving. *Experimental Psychology*, *61*, 253–262.
Lemaire, P., & Lecacheur, M. (2010). Strategy switch costs in arithmetic problem solving. *Memory & Cognition*, *38*, 322–332. doi:10.3758/MC.38.3.322
Lemaire, P., & Reder, L. (1999). What affects strategy selection in arithmetic? The example of parity and five effects on product verification. *Memory & Cognition*, *27*, 364–382. doi:10.3758/BF03211420
Linsen, S., Verschaffel, L., Reynvoet, B., & De Smedt, B. (2014). The association between children's numerical magnitude processing and mental multi-digit subtraction. *ActaPsychologica*, *145*, 75–83. doi:10.1016/j.actpsy.2013.10.008
Linsen, S., Verschaffel, L., Reynvoet, B., & De Smedt, B. (2015). The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children. *Learning and Instruction*, *35*, 42–50. doi:10.1016/j.learninstruc.2014.09.003
Lucangeli, D., Tressoldi, P. E., Bendotti, M., Bonanomi, M., & Siegel, L. S. (2003). Effective strategies for mental and written arithmetic calculation from the third to the fifth grade. *Educational Psychology*, *23*, 507–520. doi:10.1080/0144341032000123769
Lucidi, A., & Thevenot, C. (2014). Do not count on me to imagine how I act: Behavior contradicts questionnaire responses in the assessment of finger counting habits. *Behavior Research Methods*, *46*, 1079–1087. doi:10.3758/s13428-014-0447-1
Luwel, K., Onghena, P., Torbeyns, J., Schillemans, V., & Verschaffel, L. (2009). Strenghts and weaknesses in the choice/no-choice method in research on strategy use. *European Psychologist*, *14*, 351–362. doi:10.1027/1016-9040.14.4.351
Luwel, K., Schillemans, V., Onghena, P., & Verschaffel, L. (2009). Does switching between strategies within the same task involve a cost? *British Journal of Psychology*, *100*, 753–771. doi:10.1348/000712609X402801
Masse, C., & Lemaire, P. (2001). On strategic combination: A case study of parity and five-rule effects in arithmetical problem solving. *Psychological Research*, *65*, 28–33. doi:10.1007/s004260000030
McCloskey, M., Aliminosa, D., & Sokol, S. M. (1991). Facts, rules and procedures in normal calculation: Evidence from multiple single-patient studies of impaired arithmetic fact retrieval. *Brain and Cognition*, *17*, 154–203. doi:10.1016/0278-2626(91)90074-I
Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's success, methods, and strategies. *Educational Studies in Mathematics*, *47*, 145–173.
Semenza, C., Miceli, L., & Girelli, L. (1997). A deficit for arithmetical procedures: Lack of knowledge or lack of monitoring? *Cortex*, *33*, 483–498. doi:10.1016/S0010-9452(08)70231-4
Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General*, *126*, 71–92. doi:10.1037/0096-3445.126.1.71
Solan, H. A. (1987). The effect of visual-spatial and verbal skills on written and mental arithmetic. *Journal of American Optometric Association*, *58*(2), 88–94.
Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology*, *8*, 155–176. doi:10.1080/02643299108253370
Thevenot, C., & Castel, C. (2012). Relationship and transfer between mental and written arithmetic. *Journal of*

Cognitive Psychology, *24*, 286–294. doi:10.1080/20445911.2011.617302

Thevenot, C., Castel, C., Fanget, M., & Fayol, M. (2010). Mental subtraction in high- and lower skilled arithmetic problem solvers: Verbal report versus operand-recognition paradigms. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *36*, 1242–1255. doi:10.1037/a0020447

Thevenot, C., Fanget, M., & Fayol, M. (2007). Retrieval or non-retrieval strategies in mental arithmetic? An operand-recognition paradigm. *Memory & Cognition*, *35*, 1344–1352. doi:10.3758/BF03193606

Thompson, I. (1999). Mental calculation strategies for additions and subtractions. Part 1. *Mathematics in School*, *28*, 2–4.

Torbeyns, J., & Verschaffel, L. (2013). Efficient and flexible strategy use on multi-digit sums: A choice/no-choice study. *Research in Mathematics Education*, *15*, 129–140. doi:10.1080/14794802.2013.797745

Uittenhove, K., Poletti, C., Dufau, S., & Lemaire, P. (2013). The time course of strategy sequential difficulty effects. An ERP study in arithmetic. *Experimental Brain Research*, *227*, 1–8. doi:10.1007/s00221-012-3397-9

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp. 557–628). Charlotte, NC: Information Age Publishing.

APPENDIX

The 60 additions and the corresponding problem types (tie and carry).

Addend 1	*Addend 2*	*Tie*	*Carry*	*Addend 1*	*Addend 2*	*Tie*	*Carry*
43	43	Tie	No carry	18	59	Non-tie	Carry
34	34	Tie	No carry	29	67	Non-tie	Carry
31	31	Tie	No carry	39	55	Non-tie	Carry
41	41	Tie	No carry	24	37	Non-tie	Carry
32	32	Tie	No carry	16	76	Non-tie	Carry
39	39	Tie	Carry	18	24	Non-tie	Carry
38	38	Tie	Carry	18	34	Non-tie	Carry
36	36	Tie	Carry	28	38	Non-tie	Carry
45	45	Tie	Carry	18	62	Non-tie	Carry
25	25	Tie	Carry	19	36	Non-tie	Carry
61	61	Tie	No carry	44	90	Non-tie	No carry
62	62	Tie	No carry	29	80	Non-tie	No carry
53	53	Tie	No carry	70	85	Non-tie	No carry
72	72	Tie	No carry	90	34	Non-tie	No carry
64	64	Tie	No carry	12	92	Non-tie	No carry
58	58	Tie	carry	64	73	Non-tie	No carry
69	69	Tie	carry	61	63	Non-tie	No carry
65	65	Tie	carry	33	96	Non-tie	No carry
79	79	Tie	carry	54	61	Non-tie	No carry
59	59	Tie	carry	33	75	Non-tie	No carry
12	30	Non-tie	No carry	58	97	Non-tie	Carry
23	30	Non-tie	No carry	73	77	Non-tie	Carry
60	24	Non-tie	No carry	68	78	Non-tie	Carry
10	57	Non-tie	No carry	12	98	Non-tie	Carry
16	73	Non-tie	No carry	24	89	Non-tie	Carry
34	51	Non-tie	No carry	28	84	Non-tie	Carry
23	43	Non-tie	No carry	55	65	Non-tie	Carry
16	83	Non-tie	No carry	54	77	Non-tie	Carry
11	64	Non-tie	No carry	57	99	Non-tie	Carry
32	41	Non-tie	No carry	54	66	Non-tie	Carry