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Research Article

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A probabilistic graphical model for assessing equivocal evidence

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Abstract

The Bayes' theorem can be generalized to account for uncertainty on reported evidence. This has an impact on the value of the evidence, making the computation of the Bayes factor more demanding, as discussed by Taroni, Garbolino, and Bozza (2020). Probabilistic graphical models can however represent a suitable tool to assist the scientist in their evaluative task. A Bayesian network is proposed to deal with equivocal evidence and its use is illustrated through examples.

Keywords: Bayes' Theorem; Jeffrey's Conditionalisation; Bayesian Networks; Probability Kinematics; Bayes Factor; Uncertain Evidence Evaluation.

> Credibility questions are among the most important in any conceivable context involving evidence-based reasoning. (Schum 2009) at p. 211

1. Introduction

In a recent paper (Taroni, Garbolino, and Bozza 2020), it was endorsed and detailed the point that the 'entire edifice of human knowledge rests on probability judgments, not on certainties' as mentioned by Galavotti (1996, at p. 253). This point of view dated back to de Finetti's pragmatism and was extended by Jeffrey in his seminal 1965 first edition book (Jeffrey 1983). It was noticed that the logic for reasoning is given by Bayes' theorem where the only constraint one has to face is to guarantee that the available evidence—upon which one should determine the probability of different hypotheses of interest—is judged as certain. In other words, this amounts to say that the acquisition of such evidence must be unequivocal. In such a situation, one is faced with what is also known as 'hard evidence' which is generally defined as the knowledge that some state of a variable (e.g. a given scientific feature of interest) definitely occurred, so that information arrives in the form of a proposition stating that event, say E, occurred. Unfortunately, the judiciary is often faced with situations where a scientist's degree of belief in the truth of proposition E, about a feature of a scientific finding, falls short of certainty. One is therefore faced with equivocal evidence so often called 'soft evidence' that is generally interpreted as evidence of uncertainty because there is uncertainty about the specific state of a variable and so there is a probability assignment associated with it. This aspect is strictly related to the doctrine called: no facts are known for certain, and experience does not 'speaks with the voice of an angel and gives you new total certainties' (van Fraassen 1989, p. 320).

Imagine, for the sake of illustration that one is interested in hypothesis H (and its negation $\bar{H}$) and that the event E is deemed as relevant for H. However, there is no certainty about the occurrence of the event E, and it is reported a probability equal to 0.7 that the event effectively has occurred (as in Dodson (1961)). What is the effect of such uncertain evidence upon the main hypotheses of interest, say H and $\bar{H}$? This has been described by Jeffrey (Jeffrey 1983)

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throughout a generalization of Bayes' theorem known as *Jeffrey Conditionalization* describing the updating mechanism called *Probability kinematics*.[1]

Taroni, Garbolino, and Bozza (2020) extended this line of reasoning by developing a generalized Bayes factor formula for equivocal (uncertain) evidence. The development takes also advantage of David Schum's works on cascaded inference where the author modelled situations involving the fact that the 'observation of evidence about an event is not diagnostically equivalent to the observation of the event itself' (Edwards, Schum, and Winkler 2008). This idea was adopted by Thompson, Taroni, and Aitken (2003) dealing with the problem of false association in cases involving DNA evidence. More generally, it can be said that Schum's work relates to the assessment of the credibility of evidence and its sources. The inferential steps describing the line of reasoning capture the different levels of credibility. Schum reported that:

> Our sensory capacities are limited in various ways, so devices have been designed to extend the range of things we can observe. […] No sensing device, human, mechanical, or electric, is infallible. We have natural questions concerning the inherent accuracy of sensing devices, but we also recognise that these devices can be influenced and tampered with in various ways. (Schum 1994, pp. 99–100)

Schum underlines the difference existing between his own approach and that of Jeffrey. He wrote (Schum 1994):

> In the Jeffrey situation, [a witness] W provides an assessment of his own credibility as far as his observation was concerned. He is uncertain about whether his observation was E or $\bar{E}$, and he expresses this uncertainty by means of $\Pr_1(E)$ and $\Pr_1(\bar{E})$. In [Schum's development], *we* make an assessment of the credibility of W's unequivocal testimony by means of [$\Pr(R|E)$ and $\Pr(\bar{R}|\bar{E})$] (at p. 353, *notation adapted*),

where R denotes the reported testimony about E. The topic of unreliable evidence (e.g. an eyewitness may not be 100% reliable) was addressed by Dawid and Mortera (1996) in forensic scenarios involving the selection of a suspect through a database search. The authors were interested in measuring the effect of unreliable evidence on the posterior probability of the hypothesis the person of interest is the source of the recovered stain.

Following Jeffrey's ideas, Taroni, Garbolino, and Bozza (2020) derived an extended formula for the Bayes factor computation that also takes equivocal evidence into account (see Equation (20) in Taroni, Garbolino, and Bozza (2020)). The novelty of the proposed equation consists in the possibility to model the uncertainty about the truth of a given feature of interest, in addition to the quantifications of the sensitivity and the specificity of a given analytical method, the capacity to report correctly the presence of a given feature when that feature does exist, and the capacity to detect potential false positive association (to not report a given feature when that feature does not exist) as presented in Schum's works.

In order to facilitate both the description of the probabilistic reasoning and the inferential computation that would require tedious calculations, a probabilistic graphical model (i.e. a Bayesian network) is proposed in this article. Bayesian networks are nowadays largely discussed and applied, either in judicial or forensic literature, as inferential and decisional tools. Examples can be found, e.g., in Taroni et al. (2014) and Taylor, Samie, and Champod (2019).

The article is structured as follows. Section 2 briefly describes the generalized Bayes' theorem equation (*Jeffrey's conditionalization*) and its impact in terms of evaluation of evidence. An extensive presentation of these arguments can be found in Sections 4 and 8 of Taroni, Garbolino, and Bozza (2020). In Section 3, a Bayesian network (BN) capable of handling this generalization and avoiding the tedious application of mathematical formulae is presented. Note that some algebraic expressions reported in Section 8 of Taroni, Garbolino, and Bozza (2020) have been reworked and adapted in the current article to facilitate the interpretation of the associated BN. Three examples of application are presented in Section 4 (*Case scenarios 1* and *2* were taken

[1] Probability kinematics or Jeffrey's conditionalization is a technique to update beliefs based upon uncertain results.

from Taroni, Garbolino, and Bozza (2020) for comparison purposes). Section 5, finally, concludes the article.

2. The generalization of Bayes' theorem and the Bayes factor for equivocal evidence

It is well known that Bayes' theorem represents the way to reply to the fundamental inferential question of how a new piece of evidence should be incorporated into one's knowledge about propositions of interest. As discussed in Taroni, Garbolino, and Bozza (2020), the process of evidence acquisition is modelled as a two-step process in time. At time t_0, it is planned to look for evidence E because E is believed to be relevant for proposition H. At time t_0, the change in degree of belief in H, if it were to be discovered that E were true, may be calculated by use of Bayes' theorem. For the sake of illustration, denote the degrees of belief about an event at time t_0 by $\Pr_0(\cdot)$. The Bayes' theorem is written as

$$\Pr_0(H|E,I) = \frac{\Pr_0(H,E|I)}{\Pr_0(E|I)} = \frac{\Pr_0(E|H,I)\Pr_0(H|I)}{\Pr_0(E|I)}, \tag{1}$$

where letter I denotes the background knowledge.[2] Afterwards, at time t_1, it is discovered that E is true. Denote the degree of belief at time t_1 by $\Pr_1(\cdot)$: what is the degree of belief in the truth of H at time t_1, $\Pr_1(H|I)$? If it has been learned at time t_1 that E is true, then the knowledge that E is true has become part of the background knowledge I at time t_1; therefore, the overall degree of belief in H at time t_1 is equal to the degree of belief in H, conditional on E, at time t_0:

$$\Pr_1(H|I) = \Pr_0(H|E,I). \tag{2}$$

This updating process is called *Simple conditioning principle* where the probability of H at time t_1 is assessed as the posterior probability (conditional on E) of H at time t_0.

The Simple conditioning principle is however not applicable whenever there is uncertainty about the truthfulness of E. The four possible scenarios that can be derived from the combination of two compatible propositions H and E and their negation (H, E; $H, \bar{E}$; $\bar{H}, E$ and $\bar{H}, \bar{E}$), cannot be reduced to H, E and $\bar{H}, E$ if there is uncertainty about E. The rule for updating probabilities taking into account uncertain evidence can be derived from the *Symmetry Principle* here below:

$$\frac{\Pr_1(H,E|I)}{\Pr_0(H,E|I)} = \frac{\Pr_1(E|I)}{\Pr_0(E|I)}. \tag{3}$$

This is a direct consequence of the logical assumption that one's belief on hypothesis H does not change at time t_1 if conditioned on the same evidence E and background knowledge I available at time t_0, so that $\Pr_1(H|E,I) = \Pr_0(H|E,I)$. It is sufficient to reformulate the latter equality as $\Pr_1(H,E|I)/\Pr_1(E|I) = \Pr_0(H,E|I)/\Pr_0(E|I)$ and a simple algebraic manipulation to obtain Equation (3). Note that the letter I will be omitted in what follows for the sake of simplicity.

From (3) the rule for calculating the probabilities of interest, e.g. $\Pr_1(H,E|I)$, in presence of uncertain evidence is as follows:

$$\Pr_1(H,E) = \Pr_0(H,E) \times \frac{\Pr_1(E)}{\Pr_0(E)}. \tag{4}$$

This allows one to take into account the uncertainty about the truthness of event E and therefore of the change in the scientist's state of information between time t_0 and time t_1.

[2] The background information available to a decision-maker and present in the notation of the a priori probabilities on the hypotheses, $\Pr_0(H|I)$ differs from that available to the forensic scientist who is in charge of the assessment of $\Pr_0(E|H,I)$. To facilitate mathematical notation, a single letter I is used to characterize both kind of background knowledge. For a detailed discussion of this aspect, see Aitken and Nordgaard (2017).

Clearly, the Symmetry Principle is also valid for $\bar{H}$ and $\bar{E}$, and the same property in (3) can be invoked to update all involved probabilities whenever faced with uncertain evidence. The probability of hypothesis H at time t_1 can then be obtained in a straightforward manner as:

$$\begin{aligned}\mathrm{Pr}_1(H) &= \mathrm{Pr}_1(H,E) + \mathrm{Pr}_1(H,\bar{E}) \\ &= \mathrm{Pr}_0(H,E)\frac{\mathrm{Pr}_1(E)}{\mathrm{Pr}_0(E)} + \mathrm{Pr}_0(H,\bar{E})\frac{\mathrm{Pr}_1(\bar{E})}{\mathrm{Pr}_0(\bar{E})} \\ &= \mathrm{Pr}_0(H|E)\mathrm{Pr}_1(E) + \mathrm{Pr}_0(H|\bar{E})\mathrm{Pr}_1(\bar{E}).\end{aligned} \tag{5}$$

Note that if evidence E is considered as true (i.e. unequivocal evidence), then $\mathrm{Pr}_1(E) = 1$ and $\mathrm{Pr}_1(\bar{E}) = 0$. As a consequence, (5) reduces to Bayes' theorem as in (1) and

$$\mathrm{Pr}_1(H) = \mathrm{Pr}_0(H|E) = \frac{\mathrm{Pr}_0(H,E)}{\mathrm{Pr}_0(E)} = \frac{\mathrm{Pr}_0(E|H)\mathrm{Pr}_0(H)}{\mathrm{Pr}_0(E)}.$$

In this way, it is implicitly assumed that the observation of evidence about an event is equivalent to the observation of the event itself. A distinction must however be made between evidence E (e.g. the positivity or negativity characterizing a given analytical procedure) and a reported testimony R about the evidence E. Consider a reported testimony R on the event E: the role of the forensic scientist is to assess the value of such scientific reported evidence R. As supported by the European Network of Forensic Science Institutes, through its guidelines for the evaluative report (ENFSI, 2015),[3] the scientist have to report the value of evidence in a probabilistic format called *Bayes factor*.[4] The Bayes factor, BF for short, is the primary element in Bayesian methodology for comparing hypotheses and it represents the logical measure for hypothesis confirmation (Taroni et al. 2021). It is defined as the ratio between the posterior odds and the prior odds and it measures the change produced by new evidence in the odds when going from the prior to the posterior distribution in favour of one proposition to another.

As a first step, one should be able to ensure the correct distribution of probabilities for the four possible scenarios once quantifying $\mathrm{Pr}_1(H|I)$, in accordance with the Symmetry Principle. Following the same line of reasoning as in (3), the probability $\mathrm{Pr}_1(H,E,R)$ can be obtained as:

$$\begin{aligned}\mathrm{Pr}_1(H,E,R) &= \mathrm{Pr}_0(H,E,R) \times \frac{\mathrm{Pr}_1(R)}{\mathrm{Pr}_0(R)} \\ &= \mathrm{Pr}_0(H|E)\mathrm{Pr}_0(E|R)\mathrm{Pr}_1(R) \\ &= \mathrm{Pr}_0(H)\mathrm{Pr}_0(E|H)\mathrm{Pr}_0(R|E)\frac{\mathrm{Pr}_1(R)}{\mathrm{Pr}_0(R)},\end{aligned}$$

where $\mathrm{Pr}_0(R|E)$ represents the probability to detect correctly a feature when the feature does exist and $\mathrm{Pr}_0(R|\bar{E})$ represents the probability to detect a given feature of interest when that feature does not exist. This can be defined as a 'false association'. In a DNA evidence scenario, this represents a false declaration for a correspondence between profiles coming from a recovered biological stain and that of a person of interest. Probability $\mathrm{Pr}_0(R)$ at time t_0 is obtained through extension of conversation as

[3] The report is available at http://enfsi.eu/documents/forensic-guidelines/.

[4] Note that the ENFSI guidelines make reference to a *Likelihood ratio* (LR) rather than to a *Bayes factor* (BF). It is worth noting that while in the ENFSI context the two terms are equivalent, this is not valid in general. In evaluative settings, when the competing hypotheses are simple (the term 'simple' means that there is only one possible value for the null and the alternative hypothesis), the Bayes factor reduces to the likelihood ratio of, say H_p to H_d and depends only upon the sample data. When composite hypotheses (i.e. there are more possible values for at least one of the tested hypotheses) are compared, the BF does not reduce to the LR. In this second scenario, the value of the evidence is not given by the data alone, but are weighted by the prior distributions.

$$\Pr_0(R) = \Pr_0(R|E)\Pr_0(E) + \Pr_0(R|\bar{E})\Pr_0(\bar{E}), \tag{6}$$

where

- $\Pr_0(E) = \Pr_0(E|H)\Pr_0(H) + \Pr_0(E|\bar{H})\Pr_0(\bar{H})$ and
- $\Pr_0(\bar{E}) = \Pr_0(\bar{E}|H)\Pr_0(H) + \Pr_0(\bar{E}|\bar{H})\Pr_0(\bar{H})$.

The probabilities of all other scenarios can be obtained analogously. The probability of proposition H given equivocal testimony R becomes

$$\begin{aligned}\Pr_1(H) &= \Pr_1(H,E,R) + \Pr_1(H,E,\bar{R}) + \Pr_1(H,\bar{E},R) + \Pr_1(H,\bar{E},\bar{R}) \\ &= \frac{\Pr_1(R)}{\Pr_0(R)}\Pr_0(H)\{\Pr_0(E|H)[\Pr_0(R|E) - \Pr_0(R|\bar{E})] + \Pr_0(R|\bar{E})\} \\ &\quad + \frac{\Pr_1(\bar{R})}{\Pr_0(\bar{R})}\Pr_0(H)\{\Pr_0(E|H)[\Pr_0(\bar{R}|E) - \Pr_0(\bar{R}|\bar{E})] + \Pr_0(\bar{R}|\bar{E})\}.\end{aligned} \tag{7}$$

The posterior probability of the negation of H, $\bar{H}$, can be obtained analogously. The Bayes factor can therefore be obtained as the ratio between the posterior odds and the prior odds, and after some manipulation, it takes the following form:

$$\mathrm{BF} = \frac{\begin{aligned}&\frac{\Pr_1(R)}{\Pr_0(R)}\{\Pr(E|H)[\Pr(R|E) - \Pr(R|\bar{E})] + \Pr(R|\bar{E})\} \\ &\quad + \frac{\Pr_1(\bar{R})}{\Pr_0(\bar{R})}\{\Pr(E|H)[\Pr(\bar{R}|E) - \Pr(\bar{R}|\bar{E})] + \Pr(R|\bar{E})\}\end{aligned}}{\begin{aligned}&\frac{\Pr_1(R)}{\Pr_0(R)}\{\Pr(E|\bar{H})[\Pr(R|E) - \Pr(R|\bar{E})] + \Pr(R|\bar{E})\} \\ &\quad + \frac{\Pr_1(\bar{R})}{\Pr_0(\bar{R})}\{\Pr(E|\bar{H})[\Pr(\bar{R}|E) - \Pr(\bar{R}|\bar{E})] + \Pr(R|\bar{E})\}\end{aligned}}. \tag{8}$$

See Taroni, Garbolino, and Bozza (2020) for an extensive presentation. Note that the expression for the BF in (8) simplifies the equation presented in Schum (1994) when the testimony is unequivocal, i.e. $\Pr_1(R) = 1$.

The application of (8) in practice can however represent a tedious task. A probabilistic graphical model is therefore proposed in Section 3 and its implementation is illustrated in Section 4.

3 Bayesian networks

3.1 Preliminaries

It is well known that Bayesian networks, BNs for short, have been developed in the field of artificial intelligence as a framework that assists researchers and practitioners in applying the theory of probability to inference problems. Since the late 1980s, BNs have also attracted researchers in forensic science (Aitken and Gammerman 1989), and this tendency has considerably intensified throughout the past decade (see, e.g., Dawid and Mortera (2021)). Bayesian networks can roughly be defined as a pictorial representation of the probabilistic dependencies and influences (represented by arcs) among variables (represented by nodes) deemed to be relevant for a particular probabilistic inferential problem and it is the basis for the probability propagation algorithm that allows exact variable probability updating in such a structure.

BNs are a combination of graph theory, which is used to provide a qualitative model structure, and probability theory, which is used to characterize the nature and strength of the relationships that reign within a model. More formally, a BN covers the following elements:

- a finite collection of random variables that are represented by nodes. Each of these nodes has either a finite set of mutually exclusive states or may represent a continuous measurement;
- a set of directed arcs that connect pairs of nodes;
- the set of variables and the set of directed arcs are combined in such a way that a directed acyclic graph is obtained, that is, a graph where no loops are permitted[5];
- node probability tables are associated with each variable of the network. The probability table of a variable, say A, that receives entering arcs from variables $B_1, \ldots, B_n$ contains conditional probabilities $\Pr(A|B_1, \ldots, B_n)$, whereas a variable A with no entering arcs from other variables contains unconditional[6] probabilities $\Pr(A)$. It is assumed that personal degrees of belief can be assigned to these states.

The actual state of a variable may not be known with certainty. For example, there may be uncertainty about the truth or otherwise of the proposition according to which, for example, a crime stain has been left by the person of interest. Within a BN, such a proposition is conceptualized in terms of a Boolean node, whose states represent the truth and the falsity of that proposition. The degree of belief maintained in each of these states is expressed numerically, that is, in terms of probabilities. These probabilities are organized in that node's probability table as described above. Note that the arcs in a BN represent probabilistic relationships that correspond to a property that a modeller assumes to hold within the context of an inferential problem at hand. If a network is properly constructed, then a directed arc from a node B to a node A signifies that variable B (the ancestor, or parent node) has a direct influence on variable A (the descendant, or child node).

The key task operated by Bayesian networks consists of the processing of newly acquired information; that is, calculating the conditional probabilities of the states of the nodes in the network one is interested in (e.g. a hypothesis node) given that the states of some other nodes (e.g. evidence node) have been observed. An extensive presentation of Bayesian networks in forensic science applications can be found in Taroni et al. (2014).

3.2 A model for equivocal evidence

Consider, as starting point, the structural example involving three variable nodes in a medical diagnosis case or in a DNA evidence case, where one is interested in using probability assignments concerning the methodological accuracy (quantified through the sensitivity and the specificity of the analytical method) and the expert's objectivity (quantified through correct and false attributions such as being able to detect a positive results when a positive result should be detected, and avoid to declare a false positive or a false correspondence). A detailed example of DNA evidence evaluation is illustrated in Thompson, Taroni, and Aitken (2003).

This scenario can be illustrated through cascaded inferential steps. Consider the serial connection $H \rightarrow E \rightarrow R$ in Fig. 1, where node H with states $\{h, \bar{h}\}$ (e.g. 'the patient is affected by a given disease', h and 'the patient is not affected by a given disease', $\bar{h}$), node E with states $\{e, \bar{e}\}$ (e.g. 'there is a positive result', e and 'there is a negative result', $\bar{e}$), and node R with states $\{r, \bar{r}\}$

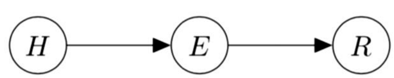

Figure 1. A serial connection for the scenario involving unequivocal evidence with nodes H (the hypothesis of interest), E (the theoretical evidence) and R (the reported evidence).

[5] 'Hereafter, only directed graphs that do not contain cycles and that are connected are considered. A cycle is said to exist if a node is an ancestor, and hence descendant, of itself, and a graph is connected if there exist at least one path between every two nodes. A connected directed graph with no cycles is called a *directed acyclic graph* (DAG)' (see Taroni et al. (2014) at p. 47).

[6] The term 'unconditional' refers here only to the absence of an explicit conditioning on other variables (nodes) in a network. Strictly speaking, a probability of the kind $\Pr(A)$ should be considered as conditional because there is always contextual knowledge, generally denoted by the letter I, which is used to associate a value to $\Pr(A)$. This implies that $\Pr(A)$ should be written more correctly as $\Pr(A|I)$, though I is omitted to reduce notational burden.

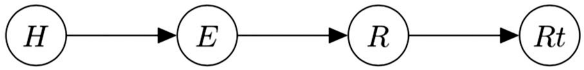

Figure 2. A serial connection for the scenario involving equivocal evidence with nodes *H* (the hypothesis of interest), *E* (the theoretical evidence), *R* (the reported evidence) and *Rt* (the truthfulness of the reported evidence; it represents the factor that allows one to update the measure of the state of uncertainty about proposition *H*, in accordance to the Symmetry principle).

(e.g. 'a positive result is reported', r and 'a negative result is reported', $\bar{r}$) represent the main propositions of interest, the theoretical evidence (that is unknown to the user) and the reported evidence, respectively.

This is a well-known scenario, and the BF can be calculated rather straightforwardly according to the developments proposed by Schum (1994).

A slightly different situation is the following where for some reasons the scientist is doubtful about the evidence (e.g. she/he is uncertain about the result of the test). Imagine for example that she/he is, e.g. 70% sure that the test is positive: there is just a 0.7 probability associated with the positivity of the reported result of the test. The task is to integrate this probabilistic judgment for an equivocal evidence into the model.[7]

In order to calculate the probability $\Pr_1(H)$ of the proposition of interest (e.g. the patient is affected by a given disease) given equivocal evidence (e.g. the scientist is uncertain about the result of the test), one should take into account the ratios $\Pr_1(R)/\Pr_0(R)$ and $\Pr_1(\bar{R})/\Pr_0(\bar{R})$ that allows one to update the measure of the state of uncertainty about the hypothesis of interest H, in accordance to the Symmetry Principle and Equation (7).

To take into account this additional source of uncertainty, there is the need to extend the serial connection $H \to E \to R$ in Fig. 1 by adding an extra variable node, say Rt, that leads to the serial connection in Fig. 2: $H \to E \to R \to Rt$. This is a Boolean node with states *true* and *false* defined by the factors $\Pr_1(R)/\Pr_0(R)$ and $\Pr_1(\bar{R})/\Pr_0(\bar{R})$ when the corresponding state of R is *positive* ($R = r$), and by the factors $\Pr_1(\bar{R})/\Pr_0(\bar{R})$ and $\Pr_1(R)/\Pr_0(R)$ when the corresponding state of R is *negative* ($R = \bar{r}$) describing one's uncertainty on the reported evidence. Note that values associated with the states of this node can be set equal to 1 and 0, respectively, when there's no uncertainty about the reported evidence (see Section 2).

The probabilistic graphical model in Fig. 2 must be further developed in order to allow for the calculation of the BF. The proposed Bayesian network is depicted in Fig. 3. Three function[8] nodes (represented by a double-border hexagon) have been included to allow the user to compute the Bayes factor as reported in Equation (8): (*i*) Prior odds, quantifying the ratio between the prior probabilities $\Pr_0(H)/\Pr_0(\bar{H})$; (*ii*) Posterior odds, quantifying the ratio between the posterior probabilities $\Pr_1(H)/\Pr_1(\bar{H})$; and (*iii*) BF, quantifying the ratio between the posterior odds and the prior odds. Prior (time t_0) and posterior (time t_1) probabilities of proposition H ($\bar{H}$) are modelled by nodes $H_{0\text{n(d)}}$ and $H_{1\text{n(d)}}$, where subscripts n(d) stand for numerator and denominator, respectively. The dynamic[9] node T_H (depicted by means of a dashed-border node) is included to allow the user to fix the prior probabilities related to the proposition of interest H at time t_0.

Some examples will be illustrated in Section 4.

4 Examples

4.1 Case scenario 1: the medical diagnosis example

Consider first the scenario, already introduced in subsection 3.2, where it is of interest to quantify the probability that a patient is affected by a certain disease, and the available knowledge includes the result of a clinical test. Suppose that the prevalence of the disease in the relevant population is known to be 0.1. Using the previous notation, $\Pr_0(H = h|I) = 0.1$. Probabilities

[7] Some solutions alternative to Jeffrey's conditionalization approach have been proposed, involving either algebraic and graphical modeling (see, e.g. Duda, Hart, and Nilsson (1976), Pearl (1988) and Korb and Nicholson (2004)).
[8] A function node handles numerical functions within a Bayesian Network.
[9] A dynamic node is time-based (temporal) related; it handles a time dynamic with arcs connecting adjacent time sequences and capture the fact that time flows forward.

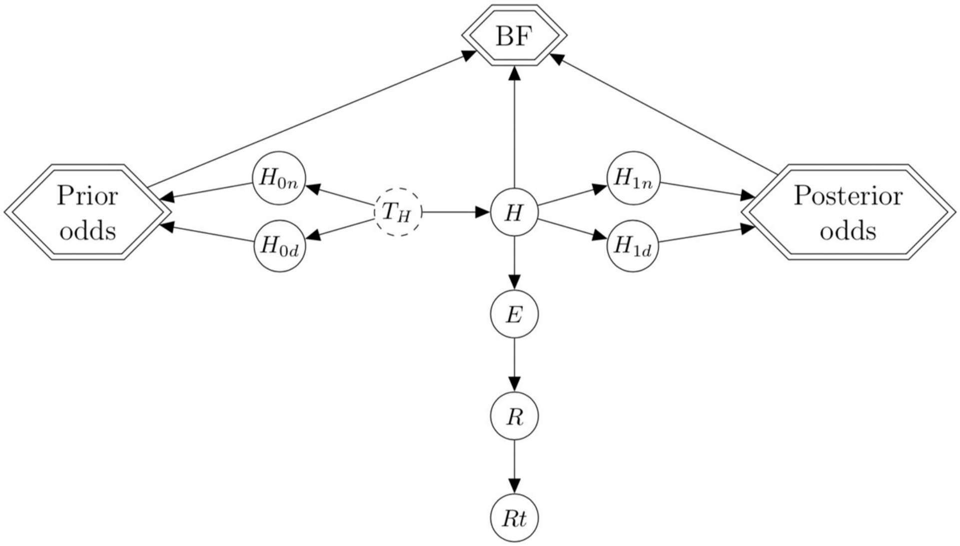

Figure 3. A Bayesian network for a scenario involving equivocal evidence with nodes H (the hypothesis of interest), E (the theoretical evidence), R (the reported evidence) and R_t (the truthfulness of the reported evidence). Function nodes (represented by a double-border hexagon) quantify prior odds, posterior odds and the Bayes factor (BF). Prior (time t_0) and posterior (time t_1) probabilities of proposition H ($\bar{H}$) are modelled by nodes $H_{0n(d)}$ and $H_{1n(d)}$, where subscripts n(d) stand for numerator and denominator, respectively. The dynamic node T_H (depicted by means of a dashed-border node) is included to allow the user to fix the prior probabilities related to the propositions of interest H and $\bar{H}$ at time t_0.

$\Pr_0(E=e|H=h,I)$ and $\Pr_0(E=\bar{e}|H=\bar{h},I)$ stand for the sensitivity and the specificity of the test, respectively. Suppose that the sensitivity of the test is set equal to 0.95, while the specificity is 0.99. Suppose also that the laboratory always yields a positive result when the target feature exists, and that never yields a positive result when the target feature does not exist, so that $\Pr_0(R=r|E=e)=1$ and $\Pr_0(R=r|E=\bar{e})=0$.

If the evidence is taken as unequivocal, the calculation of the posterior probability of the proposition of interest H is a trivial problem, as it only requires a simple application of Bayes' theorem. However, imagine the analyst from the laboratory is uncertain about the result of the test, and she/he's only 70% certain that the test is positive, i.e. $\Pr_1(R=r)=0.7$. To initialize the BN, it is necessary to assess the values to be associated with the states of the node Rt. As a first step, one needs to calculate the marginal probability $\Pr_0(R)$ in (6). Note that, being $\Pr_0(R|E)=1$ and $\Pr_0(R|\bar{E})=0$, $\Pr_0(R)$ simplifies to $\Pr_0(E)$, that can be easily obtained by extension of conversation as:

$$\begin{aligned}\Pr_0(E) &= \Pr_0(E|H)\Pr_0(H)+\Pr_0(E|\bar{H})\Pr_0(\bar{H})\\ &= 0.95\times 0.1+0.01\times 0.9=0.104.\end{aligned}$$

To each state of the node Rt there are therefore associated the following values if the corresponding state of node R is *positive*:

- $\Pr_1(R)/\Pr_0(R)=0.7/0.104=6.731$, and
- $\Pr_1(\bar{R})/\Pr_0(\bar{R})=0.3/0.896=0.335$.

Conversely, if the corresponding state of node R is *negative,* the same values are associated, but in reverse order. The uncertainty related to the reported evidence is expressed in a ratio format.[10]

[10] The ratio values are automatically normalized by the BN software.

Table 1. Definition of nodes used in the Bayesian networks shown in Figs 2 and 3 and their associated probabilities

Node	Definition	States	State of parent's node	Node probability table	
				Case 1	Case 2
H	Proposition of interest	h		0.1	0.1
		$\bar{h}$		0.9	0.9
E	Theoretical Evidence	e	$H = h$	0.95	0.95
		$\bar{e}$	$H = h$	0.05	0.05
		e	$H = \bar{h}$	0.01	0.01
		$\bar{e}$	$H = \bar{h}$	0.99	0.99
R	Reported Evidence	r	$E = e$	1	1
		$\bar{r}$	$E = e$	0	0
		r	$E = \bar{e}$	0	0.04
		$\bar{r}$	$E = \bar{e}$	1	0.96
Rt	$Pr_1(R)/Pr_0(R)$	*true*	$R = r$	6.731	5.005
		false	$R = r$	0.335	0.348
		true	$R = \bar{r}$	0.335	0.348
		false	$R = \bar{r}$	6.731	5.005

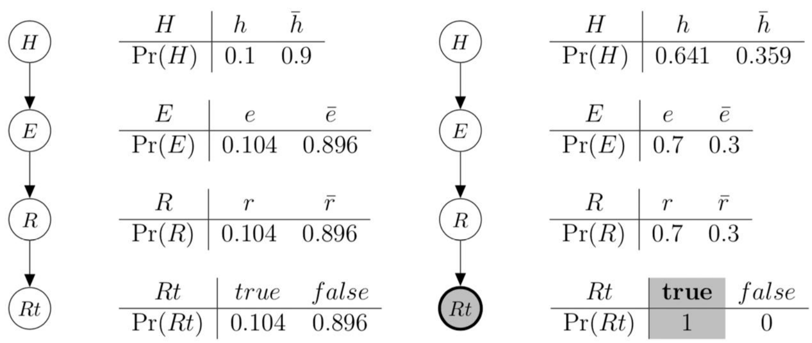

Figure 4. Extended representation of the Bayesian network for medical diagnosis in presence of equivocal evidence (Fig. 2): (left) initialized state with marginal probabilities, (right) state after instantiations of the node *Rt* (indicated in bold over a grey shaded area). Node definitions and probability tables are given in Table 1.

Probability tables associated to the nodes of the proposed graphical model are displayed in Table 1.

The posterior probability of hypothesis H given the reported (equivocal) evidence R (the node Rt is instantiated and set equal to *true*) equals 0.641 (see Fig. 4). The same result can be obtained, through Jeffrey's conditionalization, by using Equation (7). Posterior probabilities reported in Table 1 of Taroni, Garbolino, and Bozza (2020) can be easily reproduced through the four-node serial connection proposed in this article. It must however be observed that in those examples, while the probability $Pr_0(R = r|E = e)$ to detect a feature when that feature does exist was taken equal to 1, as in the current example, the probability of a false association, that is the probability to detect a feature when that feature does not exist, was set equal to 0.04, i.e. $Pr_0(R = r|E = e) = 0.04$.

4.2 Case scenario 2: the forensic evaluative example

Consider the following scenario as a general example of a forensic scientist's problem of potential equivocal evidence. An investigator collects a stain on a crime scene, believing that it may contain blood coming from the criminal. The forensic scientist performs a presumptive test for haemoglobin. There are different types of presumptive tests for haemoglobin (e.g. Bluestar, Hexagon OBTI). Here, just for the sake of illustration, the immunochromatographic Hexagon OBTI test is considered because it is used in practice by some European forensic laboratory. The test can produce a positive ($E = e$) or a negative ($E = \bar{e}$) results. The BN in Fig. 3 can therefore be implemented to calculate the BF in (8) for this scenario.

Consider the same values used in subsection 4.1 for nodes H, E, and $R|E$, while the probability of a false association is set equal to 0.04, i.e. $\Pr(R = r|E = \bar{e}) = 0.04$. Again, there is uncertainty about the result of the test, and the analyst is only 70% certain that the result is positive. Probability $\Pr_0(R)$ in (6) can be obtained as

$$\Pr_0(R) = 0.104 + 0.04 \times 0.896 = 0.139,$$

and

- $\Pr_1(R)/\Pr_0(R) = 0.7/0.139 = 5.005;$
- $\Pr_1(\bar{R})/\Pr_0(\bar{R}) = 0.3/0.860 = 0.348.$

Suppose that a positive result is reported ($R = r$) (though there's uncertainty about this result). Once the node Rt is instantiated (i.e. node Rt is set equal to *true*), the BN provides the value of the Bayes factor, that is equal to 8.25 (see Fig. 5). One can also easily verify that the posterior probability of H has lowered to 0.4782. This result shows the great inferential importance played by false positive values. In fact, rare events (such as the probability of false association) can have more force than non-rare events if one is able to 'capture the nature of the interaction between event rareness and the credibility of our sources of evidence' (Schum (2009) p. 225).

It must be underlined that to implement the BN that is depicted in Fig. 3 and illustrated in its initial and instantiated state in Fig. 5, the user is asked to make some off-diagrams computations to obtain the marginal probabilities $\Pr_0(R)$ in (6) and therefore the values associated to the states of node Rt (i.e. $\Pr_1(R)/\Pr_0(R)$ and $\Pr_1(\bar{R})/\Pr_0(\bar{R})$). To avoid such off-diagrams computations, an alternative BN can be implemented, such as the one depicted in Fig. 6, where there are added (*i*) an additional node U to allow the user to assign a value to $\Pr_1(R)$ and (*ii*) a sub-BN (denominated M) to calculate automatically the marginal probability $\Pr_0(R)$ (for $R = r$ and $R = \bar{r}$) in (6) informing node Rt. The extended representation of the sub-BN M is illustrated in Fig. 7. It contains a function node $\Pr_0(R)$ that collects the marginal probabilities of node R from the serial connection $H \to E \to R$ that is depicted in Fig. 1. The definition of the additional node U, its states and the update of the state of parent's nodes of node Rt are presented in Table 2. The extended representation of the BN depicted in Fig. 6 is illustrated in Fig. 8.

4.3 Equivocal evidence in a different forensic domain

The topic of the evaluation of equivocal evidence has gained very poor attention in forensic science applications. Caseworks where analysts need to deal with uncertain evidence are not infrequent. The problem of age estimation of living person where the chronological age is estimated from the observed degree of maturity of some selected physical attributes represents one example. Dental examination, which is typically focused on the observation of the third molar mineralization stage, is performed by medicolegal specialists who also collect information on the skeletal development through a radiographic examination of the left hand as well as by an assessment of the ossification status of the medial epiphysis of the clavicle by means of a computerised tomography (CT) scan. The task for an examiner consists of assessing the degree of maturity observed during the examination of each physical attribute. This assessment is usually performed by classifying the degree of maturity observed in categorical developmental stages or by means of a referenced atlas, although this latter may be seen as a categorical classification in which each reference corresponds to a developmental stage. The choice of a categorical assessment follows from the difficulty or even the practical impossibility of evaluating the degree of maturity on a continuous scale. Different classification criteria may exist for a given physical attribute and, usually, the key elements for assigning a given developmental stage are defined by means of a visual or descriptive manner. For the sake of simplicity, consider a case example where it is examined only the development of the medial clavicle. Suppose that the examiner observes in the CT scan image a union of the medial clavicular epiphysis which is partially compatible with the

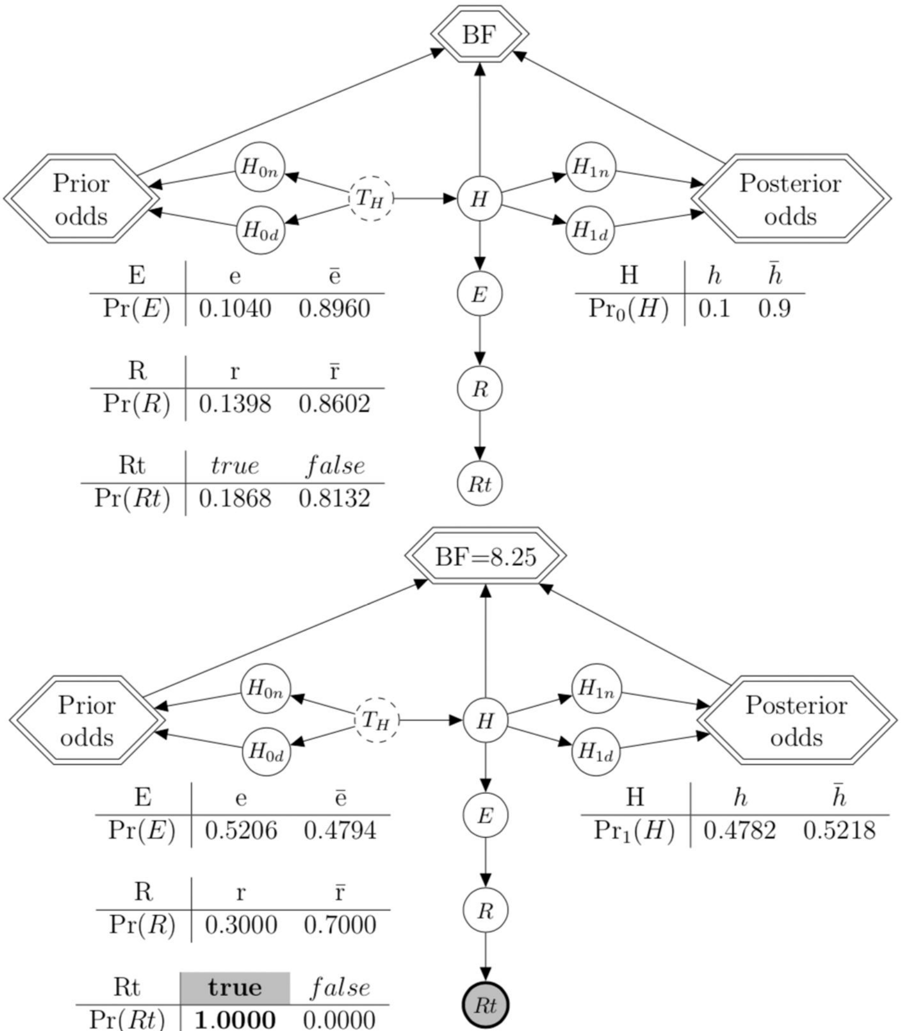

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Figure 5. Extended representation of the Bayesian network for the forensic scenario (Fig. 3): (top) initialized state with marginal probabilities, (bottom) state after instantiations of the node *Rt* indicated in bold over grey shaded area. Node definitions are given in Table 1.

ossification status defined in the 3rd stage of a traditional four-stage classification. The examiner will then classify their observation in this specific stage. The final step in the age estimation process consists of using the assessed degree of maturity to infer the chronological age of an individual or to compare competing hypotheses concerning the chronological age of the examined individual (e.g. a recurrent question of interest is whether an individual is adult or minor). The rationale behind the application of the Bayesian approach for age estimation is very intuitive: initial beliefs about some given propositions (which may concern, for example, the chronological age of a subject) are updated into posterior beliefs given the observation of given evidential elements (e.g. the developmental stages of the physical attribute belonging to that subject). This procedure is formally described by Bayes' theorem, as in Equation (1); several papers described such a methodology (see, e.g., Sironi, Bozza, and Taroni (2020)). However, the examinations of scan images do not necessarily deliver a clear attribution of the developmental stage, and a probabilistic assignment may be preferable to a categorical one. In such a context, Jeffrey's conditionalization allows one to take into account equivocal stage attribution.

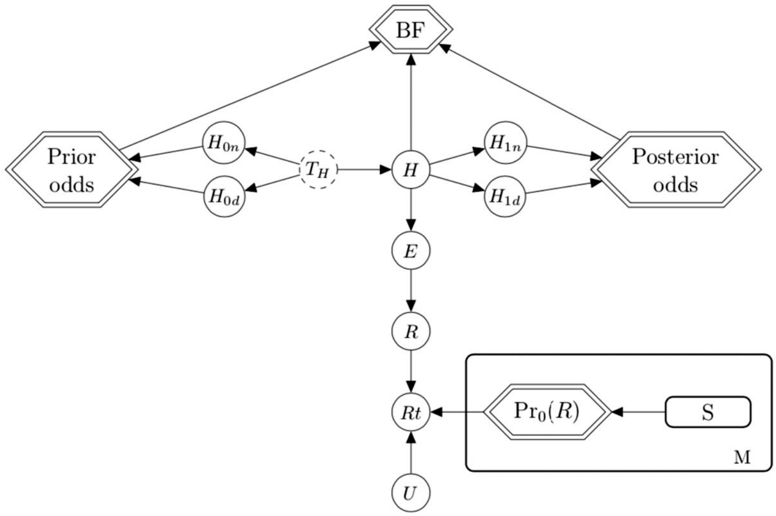

Figure 6. A Bayesian network for a scenario involving equivocal evidence including (*i*) an additional node *U* (expressing degrees of belief $\mathrm{Pr}_1(R)$ upon the reported evidence *R*,) and (*ii*) a sub-BN M to incorporate the calculation of the marginal probabilities $\mathrm{Pr}_0(R)$ in (6) informing node *Rt*. Node definitions are given in Tables 1 and 2.

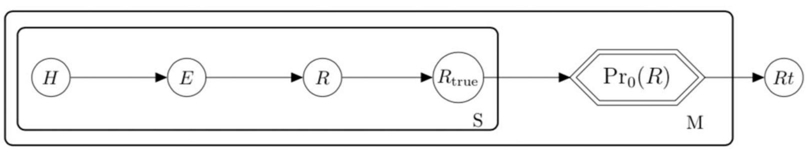

Figure 7. Extended representation of the sub-network M appearing in collapsed form in Fig. 6. The serial connection $H \rightarrow E \rightarrow R$ has been presented in Fig. 1. Node R_{true} is a logical (true/false) node that allows to extract the marginal probability $\mathrm{Pr}_0(R)$ informing node *Rt*.

Table 2. Definition of nodes used in the Bayesian network shown in Fig. 6 and their associated probabilities. Nodes *H*, *E* and *R* have been defined in Table 1

Node	Definition	States	State of parent's nodes
U	Degrees of belief about R ($\mathrm{Pr}_1(R)$)	$u = 0, 0.1, \ldots, 0.9, 1$	
Rt	$\mathrm{Pr}_1(R)/\mathrm{Pr}_0(R)$	*true*	$R = r,\ U = u,\ \mathrm{Pr}_0(R)$
		false	$R = r,\ U = u,\ \mathrm{Pr}_0(R)$
		true	$R = \bar{r},\ U = u,\ \mathrm{Pr}_0(R)$
		false	$R = \bar{r},\ U = u,\ \mathrm{Pr}_0(R)$

5. Conclusion

The use of the BF as a metric to assess the probative value of forensic traces is largely supported by operational standards and recommendations in different forensic disciplines. BF also satisfies a list of logical desiderata. However, the calculation of a BF falls shortly into operational impasses and probabilistic graphical models may represent a valuable tool capable of assisting practitioners in the different steps of their probabilistic reasoning.

A practical example is the calculation of the BF for equivocal testimony, as discussed by Taroni, Garbolino, and Bozza (2020). Starting from the observation that there may be uncertainty about reporting, the authors have adopted an extended operational perspective and proposed a cascaded inference taking into account this further source of uncertainty for Bayesian

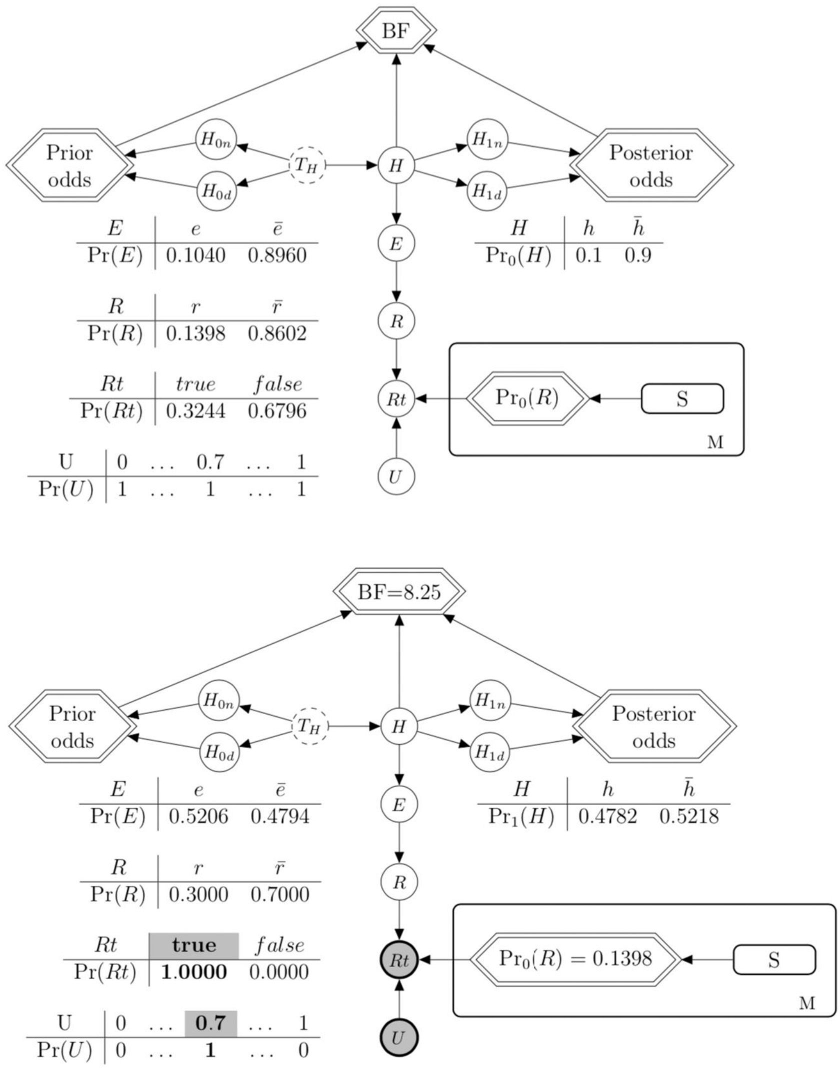

Figure 8. Extended representation of the Bayesian network for the forensic scenario (Fig. 6): (top) initialized state with marginal probabilities, (bottom) state after instantiations of node *Rt* and node *U* (indicated in bold over grey shaded area). Node definitions are given in Tables 1 and 2.

learning. These ideas have been further developed in this article, where a Bayesian network for evidence evaluation in the presence of equivocal testimony is delivered.

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Conflicts of interest

The paper is not currently under consideration for another journal. There are no problems of duplication with other work. There are no financial or other relationships which could give rise to conflicts of interest.

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