Investment Decisions, Choice Value and Preemption when Demand is Uncertain

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Abstract

We study the investment decision problem of a duopoly with price competition on a market of finite size driven by stochastic taste shocks. Each player has the choice between two technologies: a large unit and a small one. We prove that different equilibria may exist depending on the parameters' values: simultaneous investment equilibrium in the small unit or two mixed strategy equilibria, one in which each firm invests in the large unit with a strictly positive intensity and another one in which each firm invests with a strictly positive intensity in the small unit and in the large unit. The inaction regions where both technologies provide expected net payoffs that are too similar do not survive the introduction of preemption. Moreover, we prove that for some values of the demand, the preemption equilibrium is more efficient than the joint adoption equilibrium.

1 Introduction

When a firm contemplates the possibility to undertake an investment, it has to consider all the technologies that are available on the market. More precisely, the capacity choice in a competitive market of finite size is crucial. Indeed, even if an investor alone preferred to invest in a large unit, the presence of competitors who generate positive profit could make him invest in his least preferred technology, namely, the small unit. This question arises for each investment. Should an airline company invest in a huge plane or in a medium one? When an electric investment is needed, is it better to invest in a large unit with huge capacity or in a smaller one given that many actors on the market face the same dilemma? In addition to all these strategic factors, there is uncertainty on the future cash-flow generated by each investment. This paper proposes to study the investment decision of a firm in a competitive framework when it has the choice between different technologies to produce the same output. We consider a duopoly model with price competition on a market of finite size driven by stochastic taste shocks. We prove that the possibility of preemption makes investment earlier than what would be optimal. Moreover, the

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choice the investor faces is reduced since he may invest in the small unit whereas the large one would be more profitable for him.

Without uncertainty on the future cash-flow, models of adoption of a new technology have been developed by Reinganum [24] and Fudenberg and Tirole [8], for instance. Reinganum studies the diffusion of a new technology in a duopoly when the players have precommitment to adoption dates and she shows that the outcome is a "diffusion" equilibrium (firms adopt a different dates). Fudenberg and Tirole model the adoption of a new technology as a timing game and show that two equilibria could emerge: a preemption equilibrium in which the payoffs of the two players are equalized and a joint adoption equilibrium. The introduction of an "option value" by Arrow and Fisher [1] and Henry [13] has considerably influenced investment under uncertainty. For the last decade, competition has been introduced in these models. In this case, the classic result saying that investment has to be triggered at a higher threshold than the one at which the expected net present value of the project is equal to zero is no longer valid. The fear of preemption indeed accelerates investment. In such a strategic framework and under income uncertainty, Lambrecht and Perraudin [19] show that under incomplete information on the rivals' costs, a preemption equilibrium may exist where the threshold at which the first mover invests lies between the threshold at wich the expected net present value equals zero and the non-strategic one. Another extension has been proposed by Pawlina and Kort [23] in a duopoly model in which the investment costs of each firm are different and when demand is uncertain. In their setting where the second mover's payoff is lower than the leader's one, three equilibria might emerge as the cost difference increases: a preemption equilibrium, a sequential equilibrium (when the firm with the highest cost has no incentive to become the leader) and a simultaneous equilibrium (one of the firms adopts a payoff that does not optimize its payoff unconditionally). Kulatilaka and Perotti [18] study the role of the cost advantage of the new technology.

A large body of the literature extend these results to sequential investments by the same firm. On the one hand, Besanko and Doraszelski [7] present a model of capacity accumulation in an oligopolistic industry where the players have identical marginal costs. In their model, the outcome of the investment and depreciation processes are uncertain. In the case of quantity competition, the equilibrium converges to equal-sized firms, whereas in the case of price competition, the equilibrium composition results in an asymmetric structure in which the more irreversible the investment, the fiercer the preemption race. On the other hand, under uncertain demand, Boyer et al. [4] study a model of Bertrand competition on a market of finite size in which each firm can undertake sequential investments. They show that for some parameters' values that are empirically relevant, the equilibrium timing consists in a joint investment (and therefore preemption is not an equilibrium). Other aspects of sequential investments have been studied by Grenadier [12] or Grenadier and Weiss [11], for instance.

On the contrary, the introduction of a choice between technologies is still quite limited.

Décamps et al. [5] and Bobtcheff and Villeneuve [3] propose a model where a firm has the choice between two technologies under output price uncertainty in Décamps et al. and under both input price and output price uncertainty in Bobtcheff and Villeneuve. They prove that when the expected payoffs generated by each technology are close to each other (even if they are high), the investor prefers to wait in order to avoid investing in a technology that will appear later to be the least profitable one. The presence of the two technologies creates a choice value (in addition to the time value due to uncertainty) that results in the existence of inaction regions. Huisman and Kort [15] also consider the possibility to invest in different technologies but the time at which each technology will be available is uncertain. There are two sources of uncertainty: a technological uncertainty modeled by a Poisson arrival for discoveries and an economic uncertainty on the demand. Two firms can take the decision to invest in technology 1 or in technology 2 that is more profitable and whose arrival follows a Poisson process with parameter λ. If the two technologies were available at the same time, firms would never invest in technology 1. Three equilibria may arise as λ increases: a preemption equilibrium, an attrition equilibrium and an equilibrium where both firms wait to invest until technology 2 arrives. Weeds [29] consider a model of R&D competition. There is no technology choice but the two firms have the opportunity to invest in competing research projects. Research is competitive as far as the first firm to be successful in implementing the project eliminates all possible profit for the other. Here also there are two uncertainty sources: a technological uncertainty and an economic uncertainty on the patent. The resulting equilibrium is either sequential or simultaneous.

In this paper, we study the investment decision of a firm in a competitive framework when it has the choice between two technologies: either a large unit or a small one to produce the same output. Demand for the output depends on the consumers' total willingness to pay that is assumed to be stochastic. Moreover, firms compete in price meaning that no one has interest to produce too many output units. Does the choice value survive such a competitive framework? This paper gathers together two different features of the literature on investment under uncertainty: first of all, the setting in which firms take their decision is a competitive one, and then each firm is confronted with a choice at the moment it takes its decision. The competitive framework is close to Boyer et al. [4]. However, we assume each firm only invests once in the small or in the large unit unlike Boyer et al. who authorize sequential investment in the small unit exclusively. With our setting, we highlight the capacity choice firms face. Moreover, we allow for a broader ranking for the sunk costs. Unlike Huisman and Kort [15], no technology strictly dominates the other when both are available at the same time. Whereas they focus on the imminent arrival of a new technology, we prefer to study choice in a competitive setting.

We prove that when one of the firms is constrained to invest second, two equilibria exist depending on the demand: a sequential investment equilibrium in the large unit for the first mover and then in the small unit for the second mover or a simultaneous investment equilibrium in the small unit for the two players. Depending on the parameters' values, one or two inaction regions exist in which no one invests but rather observes the evolution of the demand to chose the technology later. These inaction regions reflect the choice value introduced in Bobtcheff and Villeneuve [3]. When there is no constraint on the investment's order any more, the inaction regions disappear. Indeed, the fear of being preempted makes players invest earlier. They no longer take the time to ensure they invest in the technology that will turn out to be the most profitable in the future. Moreover, it may happen that firms invest in their least preferred technology, namely, in the technology that generates the lowest payoff in case of a first mover. In fact, three equilibria may arise depending on the demand and on the sunk costs: a simultaneous investment equilibrium in which each firm invests in the small unit, and two mixed strategy equilibria, one in which each firm invests in the large unit with a strictly positive intensity and another one in which each firm invests with a strictly positive intensity in the small and in the large unit. When the large unit s not too expensive relative to the small one and when demand is not too high neither, the preemption equilibrium is more efficient than the joint adoption equilibrium. When the cost advantage of the small unit is very large, the optimal solution that consists in investing jointly in the small unit is replicated under preemption.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the case of a natural leader and a natural follower. In section 4, we derive the optimal solution and the cooperative allocation. Section 5 is devoted to the situation under preemption and section 6 concludes.

2 The model

Time is continuous and indexed by $t \geq 0$. At any date t, the demand side of the market is described by a price inelastic unit demand:

$$D_t(P) = \begin{cases} 0 & \text{if } P > P_t, \\ \in [0,1] & \text{if } P = P_t, \\ 1 & \text{if } P < P_t, \end{cases}$$

where the total willingness to pay P_t for the commodity produced by the firms is subject to aggregate demand shocks described by a geometric Brownian motion

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_t.$$

$P_0 = p$, μ and σ are positive constants and $\{W_t\}_{t\geq 0}$ is a standard Brownian motion.

We suppose there are two firms. Both firms are risk neutral and discount future revenues and costs at a constant risk-free rate $r > \mu$. Variable costs are normalized to zero. Investment is irreversible and takes place in a lumpy way. The setting is thus close to the one described in Boyer et al. [4] except that here, each firm has the choice between two technologies:

- technology 1 has a capacity of 1 and its sunk cost is equal to I_1,

- technology 2 has a capacity of 2 and its sunk cost is equal to I_2, with $I_2 > I_1$.

Moreover, we allow for a broader ranking of the sunk costs. We assume each capacity unit allows a firm to cover half of the market. Furthermore, we suppose that once the firm has invested in one technology, it cannot make a second investment.

Concerning competition, within each instant $[t, t + dt)$, the timing of the game is the following: (i) first, each firm chooses how many units of capacity to invest in, given the realization of P_t; (ii) next, each firm quotes a price given its new capacity level and that of its rival; (iii) last, consumers choose from which firm to purchase. Production and transfers take place.

We denote $\pi(n_i, n_j, p)$ the instantaneous expected profit flow of firm i when it holds n_i unit whereas its rival holds n_j units and when demand is equal to p. As the market can be covered with two units, $\pi(n_i, n_j, p)$ is defined for $(n_i, n_j) \in \{0, 1, 2\}^2$ and the values taken by π are given in the following lemma. Note that to make the reading of this paper more flowing, all the proofs are relegated to the Appendix.

Lemma 1 *For each $p \in [0, \infty)$, the instantaneous expected profits are equal to:*

- $\pi(1, 0, p) = p/2$ *and* $\pi(2, 0, p) = p$,
- $\pi(1, 1, p) = p/2$,
- $\pi(2, 2, p) = 0$,
- $\pi(2, 1, p) = p/2$ *and* $\pi(1, 2, p) = p/4$.

As Boyer et al. [4], we focus on Markov perfect equilibria (MPE) in which firms' investment and pricing decisions depend only on the current value of the consumers' reserve price p and the firms' capital stock measured in units of capacity (n_i, n_j). At each period, firms play an equilibrium of the static Bertrand-Edgeworth pricing game given their current capacities.

3 Benchmark

As a benchmark, we study the case where there are a natural leader (L) and a natural follower (F). The natural leader invests first and the natural follower may enter the market only once investment occurred. Moreover, according to Lemma 1, once the leader has made his capacity choice, the follower does not have the choice any more. He always invests in technology 1. In the next subsections, we solve this problem by backward induction, focusing first on the behavior of the follower.

3.1 Follower's strategy

The follower's strategy depends on the leader's choice.

3.1.1 The leader has invested in technology 1

We are here in the classic setting of optimal stopping time models. Let us first compute the expected net discounted profit of the follower when he is going to invest in technology 1, given that the leader has invested in technology 1 and that demand is equal to p

$$\Phi_{11}^{F}(p) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}\pi(1,1,P_t)\,dt|P_0 = p\right] - I_1 = \frac{\pi(1,1,p)}{r-\mu} - I_1. \tag{1}$$

The option value created by such a possible investment has the following expression

$$V_{11}^{F}(p) = \sup_{\tau}\mathbb{E}\left[e^{-r\tau}\Phi_{11}^{F}(P_\tau)|P_0 = p\right]. \tag{2}$$

This problem can be easily solved[1], and recalling that $\pi(1,1,p) = p/2$, the solution is

$$V_{11}^{F}(p) = \begin{cases} \left(\frac{p}{p_{11}^{F*}}\right)^{\beta}\left(\frac{p_{11}^{F*}}{2(r-\mu)} - I_1\right) & \text{if } p \leq p_{11}^{F*}, \\ \frac{p}{2(r-\mu)} - I_1 & \text{if } p > p_{11}^{F*}, \end{cases}$$

where

$$p_{11}^{F*} = \frac{\beta}{\beta-1}2(r-\mu)I_1, \tag{3}$$

and β is the positive root of the second order equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0. \tag{4}$$

The investment strategy of the follower is to invest in technology 1 as soon as p crosses the threshold p_{11}^{F*}. If $p < p_{11}^{F*}$, he prefers to wait and see the evolution of demand. As $\beta > 1$, $p_{11}^{F*} > 2(r-\mu)I_1$. Indeed the follower values the information he can collect on the demand and prefers to delay investment: this is a classic result of the real option theory.

3.1.2 The leader has invested in technology 2

The problem is very similar in the case where the leader has invested in technology 2. The expected net discounted profit of the follower when he is going to invest in technology 1, given that the leader has invested in technology 2 and demand is equal to p is equal to

$$\Phi_{12}^{F}(p) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}\pi(1,2,P_t)\,dt|P_0 = p\right] - I_1 = \frac{\pi(1,2,p)}{r-\mu} - I_1. \tag{5}$$

The option value created by the follower's investment is equal to

$$V_{12}^{F}(p) = \sup_{\tau}\mathbb{E}\left[e^{-r\tau}\Phi_{12}^{F}(P_\tau)|P_0 = p\right]. \tag{6}$$

As for the previous case, this option value is easily computed

$$V_{12}^{F}(p) = \begin{cases} \left(\frac{p}{p_{12}^{F*}}\right)^{\beta}\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right) & \text{if } p \leq p_{12}^{F*}, \\ \frac{p}{4(r-\mu)} - I_1 & \text{if } p > p_{12}^{F*}, \end{cases}$$

[1] See for example Dixit and Pindyck, Chapter V [6].

where

$$p_{12}^{F*} = \frac{\beta}{\beta - 1} 4 (r - \mu) I_1. \tag{7}$$

Here, investment in technology 1 is triggered as soon as p reaches the threshold p_{12}^{F*}. Note that $p_{12}^{F*} = 2p_{11}^{F*}$. The follower invests earlier if the leader has invested in technology 1 than if he has invested in technology 2. Indeed, when the leader invests in technology 2, the follower's net profit is twice less than if the leader had invested in technology 1. On Figure 1, we present the two option values depending on the leader's choice when $\mu = 0.01$, $r = 0.06$, $\sigma = 0.1$ (β is thus equal to 3), $I_1 = 50$ and $I_2 = 96$. These values will be taken in each illustration if nothing else is mentioned.

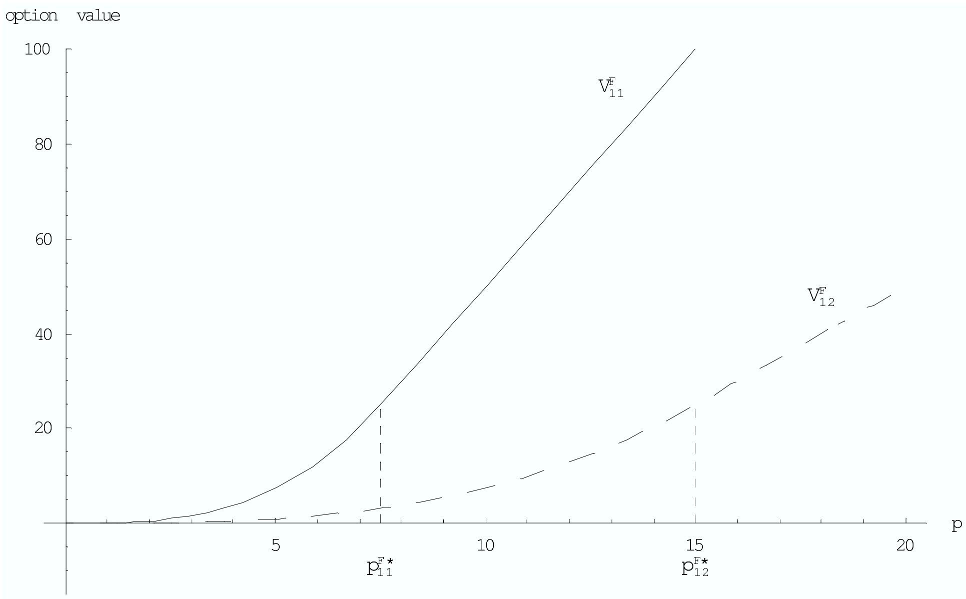

Figure 1: Option values $V_{11}^F(p)$ and $V_{12}^F(p)$.

$V_{11}^F(p)$ is greater than $V_{12}^F(p)$ whatever the demand value p. The follower prefers that the leader invests in technology 1. We now turn to the analysis of the leader's strategy.

3.2 Leader's investment decision: two auxiliary problems

The leader has the choice between the two technologies. In a first step, we consider two auxiliary problems when the leader does not have this choice.

3.2.1 Leader's investment decision in technology 1

The leader's profit flow depends on whether or not the follower has already invested. However, in the case of an investment in technology 1, we have seen in Lemma 1 that $\pi(1, 0, p) = \pi(1, 1, p) = p/2$. Therefore, the leader's instantaneous profit flow is not modified by the follower's investment.

The expected net discounted profit of the leader is thus straightforward to compute and equals

$$\Phi_1^L(p) = \mathbb{E}\left[\int_0^{+\infty} e^{-rt}\frac{P_t}{2}dt|P_0 = p\right] - I_1 = \frac{p}{2(r-\mu)} - I_1. \tag{8}$$

The option value of the leader is as usual equal to

$$V_1^L(p) = \sup_{\tau}\mathbb{E}\left[e^{-r\tau}\Phi_1^L(P_\tau)|P_0 = p\right]. \tag{9}$$

Computation leads to

$$V_1^L(p) = \begin{cases} \left(\frac{p}{p_1^{L*}}\right)^{\beta}\left(\frac{p_1^{L*}}{2(r-\mu)} - I_1\right) & \text{if } p \leq p_1^{L*}, \\ \frac{p}{2(r-\mu)} - I_1 & \text{if } p > p_1^{L*}, \end{cases}$$

where

$$p_1^{L*} = \frac{\beta}{\beta-1}2(r-\mu)I_1. \tag{10}$$

Note that $p_1^{L*} = p_{11}^{F*}$. If technology 1 were the unique available technology to the leader, the leader and the follower would invest at the same time, when demand reaches the threshold $p_1^{L*} = p_{11}^{F*}$. We now turn to the investment decision in technology 2.

3.2.2 Leader's investment decision in technology 2

In this case, the leader's instantaneous profit flow is modified by the follower's investment. Indeed, before the follower's entry the leader was able to cover the whole market. From the follower's entry, the leader only serves half of the market. Therefore, the expected net discounted profit of the leader if $p < p_{12}^{F*}$, that is if the follower has not entered yet, is equal to

$$\Phi_2^L(p) = \mathbb{E}\left[\int_0^{\tau_{12}^{F*}} e^{-rt}P_t dt + \int_{\tau_{12}^{F*}}^{+\infty}\frac{e^{-rt}P_t}{2}dt|P_0^p = p\right] - I_2, \tag{11}$$

where $\tau_{12}^{F*} = \inf\left\{t|P_t = p_{12}^{F*}\right\}$. If $p \geq p_{12}^{F*}$, there is a simultaneous investment by the two players and the expected net discounted profit of the leader is equal to

$$\Phi_2^L(p) = \mathbb{E}\left[\int_0^{+\infty}\frac{e^{-rt}P_t}{2}dt|P_0 = p\right] - I_2. \tag{12}$$

We obtain that (see the Appendix)

$$\Phi_2^L(p) = \begin{cases} \frac{p}{r-\mu} - \frac{p_{12}^{F*}}{2(r-\mu)}\left(\frac{p}{p_{12}^{F*}}\right)^{\beta} - I_2 & \text{if } p \leq p_{12}^{F*}, \\ \frac{p}{2(r-\mu)} - I_2 & \text{if } p > p_{12}^{F*}. \end{cases}$$

If $p \leq p_{12}^{F*}$, when the leader invests, he is alone on the market, meaning that he serves the whole market. The term $-\frac{p_{12}^{F*}}{2(r-\mu)}\left(\frac{p}{p_{12}^{F*}}\right)^{\beta}$ represents the loss in the leader's profit induced by the potential entry of the follower. On the contrary, if $p \geq p_{12}^{F*}$, both the leader and the follower enter at the same time.

As usual, the option value of the investment in the second technology has the following expression

$$V_2^L(p) = \sup_{\tau} \mathbb{E}\left[e^{-r\tau}\Phi_2^L(P_\tau)\,|P_0 = p\right], \tag{13}$$

and is analytically equal to

$$V_2^L(p) = \begin{cases} \left(\frac{p}{p_2^{L*}}\right)^{\beta} \Phi_2^L\left(p_2^{L*}\right) & \text{if } p \leq p_2^{L*}, \\ \Phi_2^L(p) & \text{if } p > p_2^{L*}, \end{cases}$$

where

$$p_2^{L*} = \frac{\beta}{\beta - 1}(r - \mu) I_2. \tag{14}$$

Depending on the ranking of I_2 relative to $2I_1$ and $4I_1$, p_2^{L*} may be greater or smaller than p_1^{L*} and p_{12}^{F*}. The leader's strategy to invest in each technology is well characterized. What happens in the case where the leader has indeed the choice between the two technologies?

3.3 Leader's investment decision when he has the choice

If the leader has the choice between the two technologies, the stopping time problem he faces is:

$$V^L(p) = \sup_{\tau} \mathbb{E}\left[e^{-r\tau}\max\left(\Phi_1^L(P_\tau), \Phi_2^L(P_\tau)\right)|P_0 = p\right]. \tag{15}$$

Indeed, while the investment has not been undertaken, the leader still has the choice between the two technologies. This kind of problems has been deeply studied by Décamps et al. [5] and we remind their main results. First of all, as we do not want one technology to dominate strictly the other, we put some restrictions on the parameters' values.

Lemma 2 *Under the assumption*

$$A1 : I_2 < \left(1 + 2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}\right) I_1, \tag{16}$$

there exist two thresholds $\widetilde{p}$ and $\widetilde{\widetilde{p}}$, such that

- $\forall p \in [0, \widetilde{p}[$, $\Phi_1^L(p) > \Phi_2^L(p)$,
- $\forall p \in \left]\widetilde{p}, \widetilde{\widetilde{p}}\right[$, $\Phi_1^L(p) < \Phi_2^L(p)$,
- $\forall p \in \left]\widetilde{\widetilde{p}}, +\infty\right[$, $\Phi_1^L(p) > \Phi_2^L(p)$.

On Figure 2, we represent the net profit functions in both cases, $\Phi_1^L(p)$ and $\Phi_2^L(p)$.

If Assumption $A1$ did not hold, the leader would have no choice, since technology 1 would be the preferred one and both the leader and the follower would invest simultaneously at p_1^{L*}. Therefore, from now on, we suppose that Assumption $A1$ holds. Technology 1 is thus preferred when demand is low or high enough. In between, there exists a range of demand values such that technology 2 is preferred. As $\beta > 1$, Assumption $A1$ implies that $I_2 \leq 3I_1$. With Assumption $A1$, a preliminary ranking of different investment thresholds holds: $p_{12}^{F*} > p_2^{L*}$ and $p_{12}^{F*} > p_{11}^{F*} = p_1^{L*}$. The ranking of p_2^{L*} relative to p_{11}^{F*} and p_1^{L*} depends on the ranking of I_2 relative to $2I_1$.

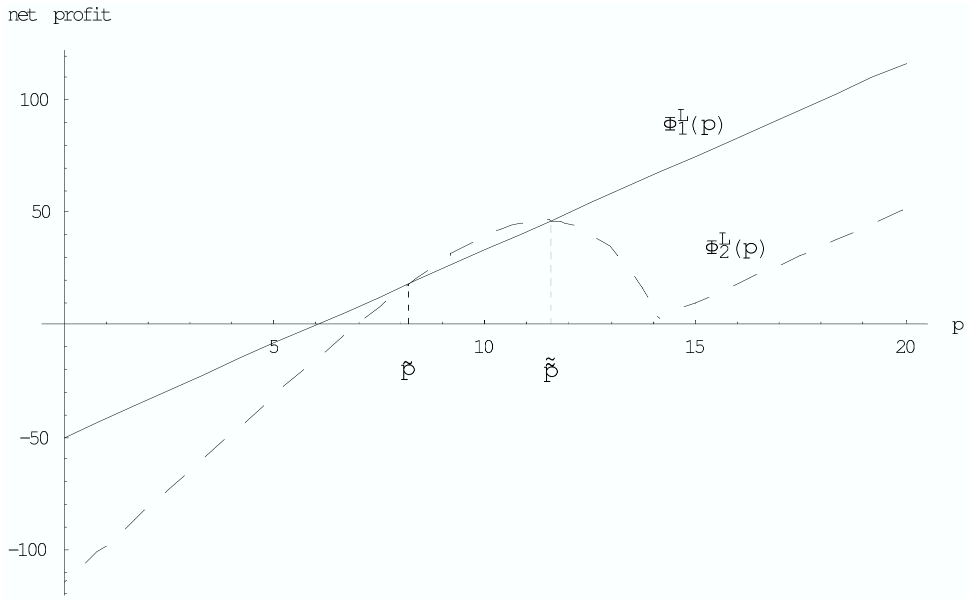

Figure 2: Net profits $\Phi_1^L(p)$ and $\Phi_2^L(p)$.

Lemma 3 *Under Assumption A1, if $I_2 \geq 2I_1$ then $\beta > 2$.*

$\beta > 2$ is equivalent to $r > 2\mu + \sigma^2$. This implies that either the volatility of the consumers' willingness to pay σ^2 or the growth rate μ is low or that the discount rate r is high.

Simplifying function $\Phi_1^L(p) - \Phi_2^L(p)$ implies that $\widetilde{p}$ (respectively $\widetilde{\widetilde{p}}$) is the lowest (respectively the highest) root of

$$\frac{2\beta I_1}{\beta - 1}\left(\left(\frac{p}{p_{12}^{F*}}\right)^{\beta} - \frac{p}{p_{12}^{F*}}\right) + I_2 - I_1 = 0. \tag{17}$$

This expression is strictly positive for $p < \widetilde{p}$ and $p > \widetilde{\widetilde{p}}$ and is strictly negative for $p \in \left]\widetilde{p}, \widetilde{\widetilde{p}}\right[$. We impose a second assumption, namely,

$$A2 : \Phi_1^L(\widetilde{p}) = \Phi_2^L(\widetilde{p}) > 0. \tag{18}$$

Assumption $A2$ means that once technology 2 is preferred, its expected net profit is strictly positive. If it is satisfied, then $\widetilde{p} > 2(r - \mu) I_1$. Using Equation (17), this leads to

$$\frac{I_2}{I_1} > 2 - \left(\frac{\beta - 1}{2\beta}\right)^{\beta - 1}. \tag{19}$$

Let us introduce the three sets

$$E^L = \left\{p \geq 0 | V^L(p) = \max\left(\Phi_1^L(p), \Phi_2^L(p)\right)\right\},$$

$$E_1^L = \left\{p \geq 0 | V^L(p) = \Phi_1^L(p)\right\}, \text{ and } E_2^L = \left\{p \geq 0 | V^L(p) = \Phi_2^L(p)\right\}.$$

E^L is the "exercise region" (also called "investment region") of the leader when he can choose between the two technologies, E_1^L is the exercise region of the leader when technology 1 is the preferred one and E_2^L is the exercise region of the leader when technology 2 is the preferred one. According to Décamps et al. [5], we know E^L is the disjoint union of the two exercise sets E_1^L and E_2^L. They show the following theorem[2] that we apply to our setting: *The indifference points $\widetilde{p}$ and $\widetilde{\widetilde{p}}$ do not belong to the exercise region E^L.* Therefore, the leader's investment region in technology 1, E_1^L, can also be decomposed into two disjoint sets depending on the ranking of the current demand value p relative to the two indifference thresholds $\widetilde{p}$ and $\widetilde{\widetilde{p}}$:

$$E_1^L = \underline{E}_1^L \cup \overline{E}_1^L,$$

where $\underline{E}_1^L = \left\{0 \leq p < \widetilde{p} | V^L(p) = \Phi_1^L(p)\right\}$ and $\overline{E}_1^L = \left\{p > \widetilde{\widetilde{p}} | V^L(p) = \Phi_1^L(p)\right\}$. Concerning the leader's exercise region in technology 2, we have that $E_2^L \subset \left]\widetilde{p}, \widetilde{\widetilde{p}}\right[$. We will see in the next subsections that depending on the values taken by the parameters of the model, the exercise region for investment in technology 1, $\underline{E}_1^L$, does not always exist (is empty). Lemma 4 gives a first result on the exercise region $\overline{E}_1^L$.

Lemma 4 *The exercise region $\overline{E}_1^L$ is never empty.*

When demand is high enough, whatever the parameters' values, the leader is going to invest in technology 1 and $\overline{E}_1^L$ is never empty. Let us now focus on the other exercise region in technology 1, $\underline{E}_1^L$.

3.3.1 The case of two exercise regions: $\underline{E}_1^L$ is empty

In this part, we study the case where $\underline{E}_1^L$ is empty. Let us consider

$$A3 : \Phi_1^L\left(p_1^{L*}\right) < V_2^L\left(p_1^{L*}\right). \tag{20}$$

Assumption $A3$ implies that, at the threshold p_1^{L*}, the option value of investing in technology 2 is strictly greater than the expected discounted net profit of investing in technology 1. The leader prefers not to invest immediately in technology 1 and to keep alive the option to invest in technology 2.

Lemma 5 *Under Assumption $A3$, $\underline{E}_1^L = \emptyset$ and $E_2^L \neq \emptyset$.*

Under Assumption $A3$, there are two exercise regions, E_2^L and $\overline{E}_1^L$. Lemma 6 specifies Assumption $A3$ for different parameters' values.

Lemma 6 *The existence of $\underline{E}_1^L$ depends on I_1, I_2 and β as it is summarized in Table 1.*

It is interesting to note that $\underline{E}_1^L$ may be empty when $I_2 \geq 2I_1$ or $I_2 < 2I_1$. Note that when $I_2 < 2I_1$ and $\beta > 2$, $\underline{E}_1^L$ is always empty (Assumption $A3$ is always satisfied).

[2] See proposition 2.2 p.431 [5].

	$I_2 \geq 2I_1$	$I_2 < 2I_1$
$\beta \leq 2$	not possible	$\underline{E}_1^L$ is empty iff $\frac{I_2}{I_1} < \frac{1}{\beta-1}\left(2\beta - 1 - \beta\left(\frac{1}{2}\right)^{\beta-1}\right)$
$\beta > 2$	$\underline{E}_1^L$ is empty iff $\frac{1}{2} + \beta\left(\frac{1}{2}\right)^{\beta} < \left(\frac{2I_1}{I_2}\right)^{\beta-1}$	$\underline{E}_1^L$ is empty

Table 1: Conditions under which $\underline{E}_1^L$ is empty (Lemma 6).

1. When technology 2 is quite expensive relative to technology 1 ($I_2 \geq 2I_1$) and since Assumption $A1$ holds, we know that $\beta > 2$ (or, equivalently $r > 2\mu + \sigma^2$), implying that the volatility of the consumers' willingness to pay or the growth rate μ is low. Therefore, the time period during which the leader serves the whole market in case of an investment in technology 2 is long and he is better off waiting to invest in technology 2. However, it is only the case if I_2 is low enough (Assumption $A3$). Indeed, if I_2 were really high, investing in technology 2 would not be so profitable relative to technology 1 and thus the leader would want to invest soon in technology 1, implying that $\underline{E}_1^L$ would not be empty.

2. When $I_2 < 2I_1$ and $\beta \geq 2$ (or equivalently $r \geq 2\mu + \sigma^2$), the leader is better off not investing in technology 1 when p is low. Indeed, not only is technology 2 very favorable relative to technology 1, but $\beta \geq 2$ implying that the time period during which he will serve all the market will be long. Therefore, $\underline{E}_1^L$ is empty. But when $\beta < 2$, this time period may be shorter and an additional assumption (Assumption $A3$) is required in order I_2 not to be too high.

Under Assumption $A3$ and according to Décamps et al. [5], we know that the leader's option value is equal to:

$$V^L(p) = \begin{cases} B_2 p^\beta & \text{if } p \leq p_2^{L*}, \\ \Phi_2^L(p) & \text{if } p_2^{L*} < p \leq p_3, \\ Ap^\alpha + Bp^\beta & \text{if } p_3 < p \leq p_4, \\ \Phi_1^L(p) & \text{if } p > p_4, \end{cases}$$

where

$$B_2 = \left(\frac{1}{p_2^{L*}}\right)^\beta \Phi_2^L\left(p_2^{L*}\right),$$

and p_3, p_4, A and B can be numerically obtained thanks to the value matching and smooth pasting conditions at p_3 and p_4. It is represented on Figure 3.

The leader invests in technology 2 if $p \in \left[p_2^{L*}, p_3\right]$, he invests in technology 1 if $p \geq p_4$. The indifference point $\widetilde{\widetilde{p}}$ does not belong to any exercise region as we already mentioned: $\widetilde{\widetilde{p}} \in [p_3, p_4]$. Between these two exercise regions $\left[p_2^{L*}, p_3\right]$ and $[p_4, +\infty[$, the leader faces an inaction region in

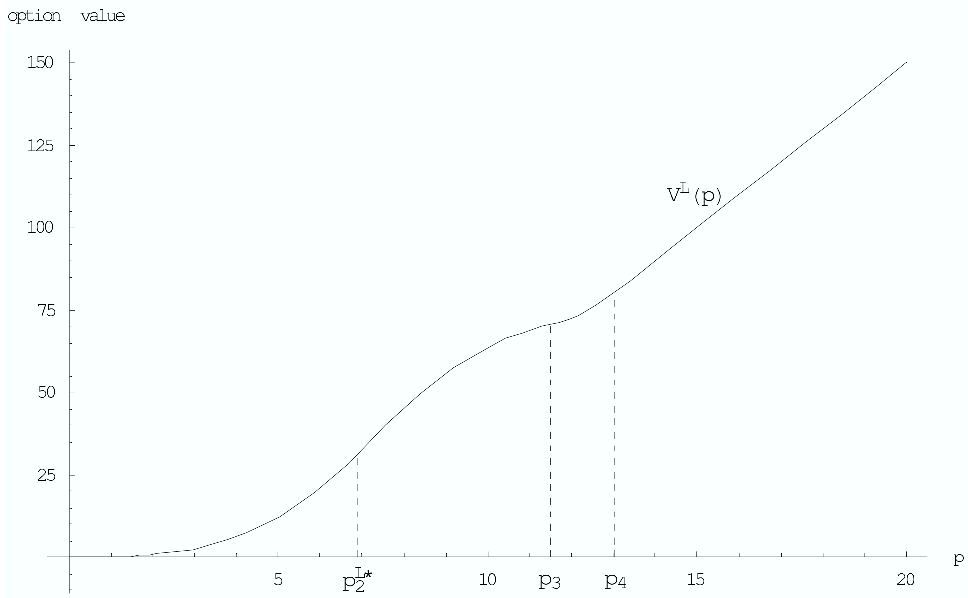

Figure 3: Option value $V^L(p)$.

which he prefers to wait rather than to invest in one of the two technologies as we see on Figure 4.

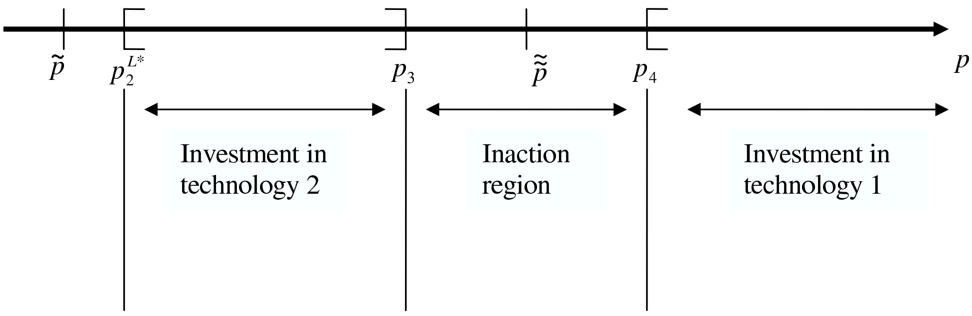

Figure 4: Leader's strategy with two exercise regions.

The simultaneous presence of the two technologies creates a choice value for the leader that results in an inaction region. On $]p_3, p_4[$, the leader prefers to keep this choice value alive than to get an immediate profit flow. Therefore, in the case of a unique inaction region, two equilibria may arise: a sequential equilibrium (the leader invests in technology 2 and the follower invests in technology 1 at p_{12}^{F*}) when $p \in \left[p_2^{L*}, p_3\right]$ and a simultaneous investment equilibrium $\forall p \geq p_4$.

3.3.2 The case of three exercise regions: $\underline{E}_1^L$ is not empty

To ensure the existence of $\underline{E}_1^L$, we need to assume that

$$A4 : \Phi_1^L\left(p_1^{L*}\right) \geq V_2^L\left(p_1^{L*}\right) \text{ and } p_1^{L*} < \widetilde{p}. \tag{21}$$

Lemma 7 *Under Assumption A4, $\underline{E}_1^L$ and E_2^L are not empty.*

This implies that at p_1^{L*}, the leader prefers to invest immediately in technology 1 rather than to wait to invest in technology 2. This is a sufficient condition for $\underline{E}_1^L$ not to be empty. Lemma 8 specifies Assumption $A4$.

Lemma 8 *$\underline{E}_1^L$ is not empty if and only if $\frac{1}{2}+\beta\left(\frac{1}{2}\right)^{\beta} \geq \left(\frac{2I_1}{I_2}\right)^{\beta-1}$ and $\beta > 2$.*

First, this lemma implies that $\underline{E}_1^L$ is not empty only if $I_2 > 2I_1$. Moreover, it is optimal for the leader to invest in technology 1 for low values of the demand p, if and only if Assumption $A4$ is satisfied. Indeed, in this case, although the leader only covers half of the market with an investment in this technology, the low sunk cost imposed by Assumption $A4$ induces him to invest early, when technology 1 is the preferred one. Under Assumption $A4$, we are able to characterize the shape of the solution. This is a succession of three disjoint exercise regions: $\underline{E}_1^L$, E_2^L and $\overline{E}_1^L$.

$$V^L(p) = \begin{cases} B_1 p^{\beta} & \text{if } p \leq p_1^{L*}, \\ \Phi_1^L(p) & \text{if } p_1^{L*} \leq p \leq \widehat{p}_2, \\ \underline{A}p^{\alpha} + \underline{B}p^{\beta} & \text{if } \widehat{p}_2 \leq p \leq \widehat{p}_3, \\ \Phi_2^L(p) & \text{if } \widehat{p}_3 \leq p \leq \widehat{p}_4, \\ \overline{A}p^{\alpha} + \overline{B}p^{\beta} & \text{if } \widehat{p}_4 \leq p \leq \widehat{p}_5, \\ \Phi_1^L(p) & \text{if } p \geq \widehat{p}_5, \end{cases}$$

where

$$B_1 = \left(\frac{1}{p_1^{L*}}\right)^{\beta} \Phi_1^L\left(p_1^{L*}\right),$$

and $\widehat{p}_2$, $\widehat{p}_3$, $\widehat{p}_4$, $\widehat{p}_5$, $\underline{A}$, $\underline{B}$, $\overline{A}$ and $\overline{B}$ can be numerically obtained thanks to the value matching and smooth pasting conditions at $\widehat{p}_2$, $\widehat{p}_3$, $\widehat{p}_4$ and $\widehat{p}_5$ as we see on Figure 5.

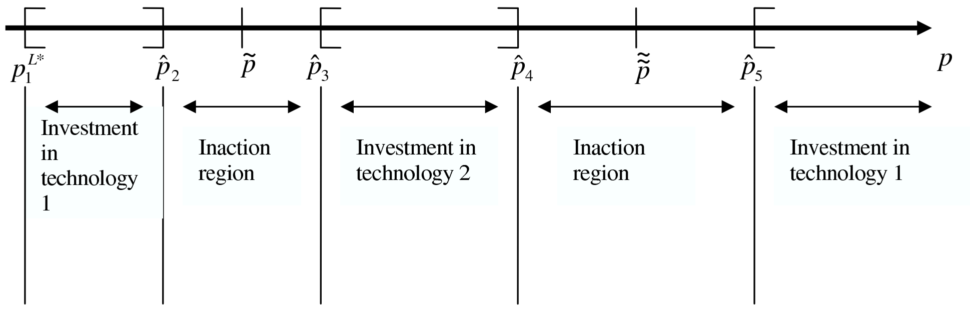

Figure 5: Leader's strategy with three exercise regions.

Figure 6 represents the option value $V^L(p)$ when $\mu = 0.01$, $\sigma = 0.015$, $r = 0.07$, $I_1 = 50$ and $I_2 = 115$.

In this case with three exercise regions, when $p \in \left[p_1^{L*}, \widehat{p}_2\right] \bigcup [\widehat{p}_5, +\infty[$, the equilibrium is a simultaneous investment in technology 1 and when $p \in [\widehat{p}_3, \widehat{p}_4,]$, the equilibrium is a sequential investment.

We have carefully described the benchmark case when there are a natural leader and a natural follower. The leader's optimal strategy is characterized by a succession of exercise regions and

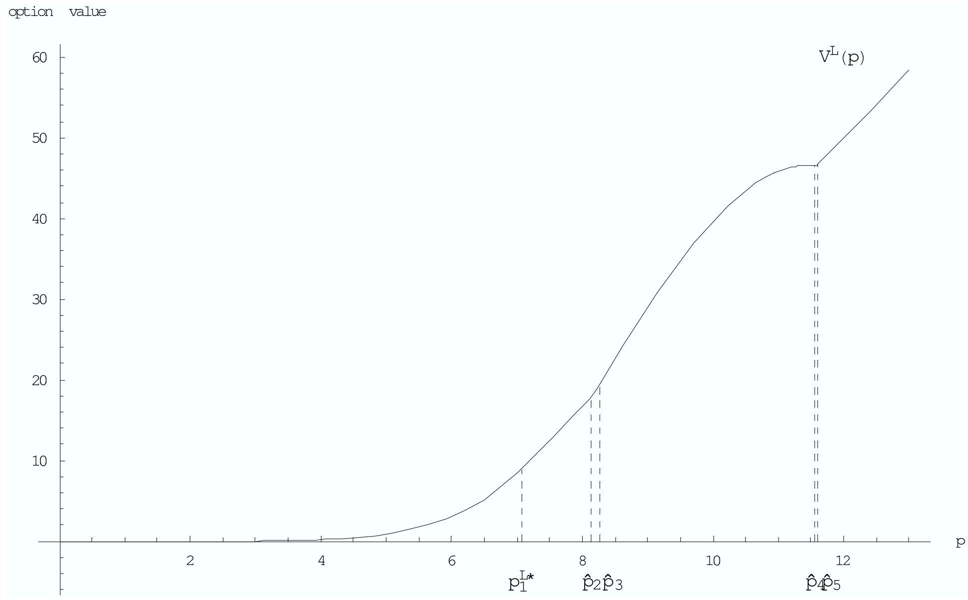

Figure 6: Option value $V^L(p)$.

inaction regions as demand increases. Once more, the two inaction regions characterize the leader's choice value. When one technology becomes the preferred one (around $\widetilde{p}$ and $\widetilde{\widetilde{p}}$), an inaction region appears that illustrates the choice value the leader wants to keep alive instead of getting an immediate profit flow. Let us have a look at the socially optimal solution of this problem in the next section.

4 Optimal solution and cooperative allocation

4.1 Optimal solution

In this section, we first look at the optimal solution. This comes down to studying the total payoff

$$S(p) = \Phi^L(p) + \Phi^F(p) + CS(p),$$

where $CS(p)$ denotes the consumer surplus and to maximizing it. In the case of a simultaneous investment, the payoffs of the leader and of the follower are the same. Using the results of Lemma 1, the instantaneous consumer surplus is equal to:

- 0 if the total number of capacity units equals 1 or 2,
- $p/4$ if the total capacity equals 3,
- p if the total capacity equals 4.

Three strategies have to be taken into account:

1. simultaneous investment in technology 1 by each player,

2. sequential investment (technology 2 then technology 1),

3. only one firm invests in technology 2 and the other abstains from investing.

In the case of a sequential investment, the instantaneous total surplus, p, is not modified by the entry of the second player. But an additional sunk cost equal to I_1 is added. Therefore, it is more efficient for the second player not to invest. In the case of a simultaneous investment, the instantaneous total surplus is also equal to p. Let us denote $S_{11}(p)$ (respectively $S_{20}(p)$) the total surplus in the case of a simultaneous investment (respectively in the case of a unique investment in technology 2):

$$S_{11}(p) = \frac{p}{r-\mu} - 2I_1 \text{ and } S_{20}(p) = \frac{p}{r-\mu} - I_2.$$

The optimal solution satisfies:

$$V_{opt}(p) = \sup_{\tau} \mathbb{E}\left[e^{-r\tau} \max\left\{S_{11}(P_\tau), S_{20}(P_\tau)\right\} | P_0 = p\right], \text{ or equivalently} \tag{22}$$

$$V_{opt}(p) = \begin{cases} \sup_\tau \mathbb{E}\left[e^{-r\tau} S_{11}(P_\tau) | P_0 = p\right] & \text{if } 2I_1 \leq I_2, \\ \sup_\tau \mathbb{E}\left[e^{-r\tau} S_{20}(P_\tau) | P_0 = p\right] & \text{if } 2I_1 > I_2. \end{cases}$$

Such an option value is straightforward to compute. In the case where $2I_1 \leq I_2$, then

$$V_{opt}(p) = \begin{cases} \left(\frac{p}{p_1^{L*}}\right)^\beta \left(\frac{p_1^{L*}}{r-\mu} - 2I_1\right) & \text{if } p \leq p_1^{L*}, \\ \frac{p}{r-\mu} - 2I_1 & \text{if } p > p_1^{L*}. \end{cases}$$

On the contrary, in the case where $2I_1 > I_2$,

$$V_{opt}(p) = \begin{cases} \left(\frac{p}{p_2^{L*}}\right)^\beta \left(\frac{p_2^{L*}}{r-\mu} - I_2\right) & \text{if } p \leq p_2^{L*}, \\ \frac{p}{r-\mu} - I_2 & \text{if } p > p_2^{L*}. \end{cases}$$

Proposition 1 *At the optimum, two cases may happen:*

1. *If $2I_1 \leq I_2$, the two players simultaneously invest in technology 1 as soon as demand reaches the trigger p_1^{L*};*

2. *If $I_2 < 2I_1$, one player invests in technology 2 as soon as demand is greater than or equal to p_2^{L*}. The other player refrains from investing.*

If technology 1 is quite favorable relative to technology 2 ($2I_1 \leq I_2$), then the optimal investment that consists in a simultaneous investment in technology 1 is triggered at the threshold p_1^{L*}. In the other case where $I_2 < 2I_1$, it is optimal that only one player moves and invests in technology 2. Note that the threshold at which the unique investment is triggered, p_2^{L*}, is the same than the one that triggers the sequential investment of the leader in technology 2 and of the follower in technology 1. This result is classic and replicates the conclusions of Leahy [20]. An investor does not take into account the potential investments of competitors to trigger his investment. The result holds in this setting where the demand function is very simple: the consumer surplus is equal to zero since two capacity units have been built. In this analysis of the optimal solution, firms' preferences do not matter any more. Indeed, a joint investment in technology 1 is optimal even in a region where technology 2 is the preferred one and similarly in the case of a unique investment in technology 2. Let us now turn to the case of the cooperative equilibrium.

4.2 Cooperative allocation

The outcome of the cooperative allocation is obtained by maximizing the sum of the two expected profits (the one of the leader and the one of the follower):

$$\Phi^{L+F}(p) = \Phi^{L}(p) + \Phi^{F}(p).$$

Three cases have also to be taken into account: simultaneous investment, sequential investment and unique investment in technology 2. In the case of a sequential investment, as soon as the two players are active on the market, the instantaneous total profit flow is equal to $3p/4$. When the first mover is still alone on the market, the instantaneous profit flow is equal to p. Therefore, not only does the entry of the second mover induces a sunk cost equal to I_1, but it also decreases the total instantaneous profit flow. His entry is thus not profitable. The cooperative allocation consists either in a simultaneous investment or in a unique investment in technology 2. In these two cases, the sum of the two expected profits is equal to the total surplus $S(p)$ since the consumer surplus is equal to zero. The cooperative allocation is thus equal to the optimal solution and consists in a simultaneous investment if $2I_1 \leq I_2$ and in a unique investment in technology 2 if $I_2 < 2I_1$.

Proposition 2 *The cooperative allocation replicates the optimal solution.*

In the two cases (simultaneous investment and unique investment), the number of installed capacity units equals the market size and the consumer surplus is null. Therefore the two allocations (optimal and cooperative) turn out to be the same. What happens when preemption is at play?

5 Choice under preemption

Before going deeply into the model, we list all possible equilibria that are likely to emerge. We first focus on pure strategy equilibria. Since the instantaneous expected profit is equal to zero in case of a simultaneous investment in technology 2, this case will never happen. But an equilibrium involving a simultaneous investment in technology 1 may exist: in this case, the two firms equally share the demand. Concerning the mixed strategy equilibria, the unique possibility for a firm to preempt its rival is to invest in technology 2[3]. By doing so, the first mover

[3]Note that no player has interest to preempt its rival with one unit. Indeed, once a player has invested in technology 1, it does not change the strategy of the other player relative to an investment in technology 1, since in this case, they equally share the market.

delays the other player's entry and serves the whole market while he is alone. This preemption equilibrium may happen only if there is a gain by being the first to invest, i.e. if $\Phi_2^L(p) \geq V_{12}^F(p)$ (and $\Phi_2^L(p) \geq \Phi_1^L(p)$ since an investment in technology 2 should be preferred). Therefore the threshold at which each player is indifferent between between investing first and second plays a key role. This threshold $\overline{\overline{p}}$ is defined by $\Phi_2^L(\overline{\overline{p}}) = V_{12}^F(\overline{\overline{p}})$. In this mixed strategy equilibrium, the intensity with which each player invests in technology 2 is determined so that each player is indifferent between being the leader and being the follower[4] given that at least one player is going to undertake an investment. Furthermore, this equilibrium only arises if the follower's payoff is greater than the payoff generated by a simultaneous investment in technology 1. If this is not the case, a mixed strategy equilibrium involving investment in strategies 1 and 2 has to be determined. Therefore, the threshold $\overline{\overline{\overline{p}}}$ such that $\Phi_1^L\left(\overline{\overline{\overline{p}}}\right) = V_{12}^F\left(\overline{\overline{\overline{p}}}\right)$ is also very crucial.

To sum up three cases may happen:

1. *"Simultaneous investment equilibrium"*: a simultaneous investment in technology 1 when $p \geq p_1^*$,

2. *"Mixed strategy equilibrium* $\{2,0\}$ *(MSE_{20})"*: both firms invest in technology 2 with a strictly positive intensity. In this case, there exist two outcomes. Either one player immediately invest in technology 2 and the other waits and invest in technology 1 when $p = p_{12}^{F*}$, or the two players simultaneously invest in technology 2,

3. *"Mixed strategy equilibrium* $\{2,1,0\}$ *(MSE_{210})"*: both firms invest in technology 1 with a strictly positive intensity and in technology 2 with a strictly positive intensity. In this case there exist four outcomes: one player immediately invest in technology 2 and the other waits and invest in technology 1 when $p = p_{12}^{F*}$, or the two players simultaneously invest in technology 2, or the two players simultaneously invest in technology 1, or lastly one player immediately invest in technology 1 and the other immediately invest in technology 2.

According to the previous discussion, the ranking of the different thresholds $\widetilde{p}$, $\overline{\overline{p}}$, $\overline{\overline{\overline{p}}}$, p_1^*, $\widetilde{\widetilde{p}}$ and p_{12}^{F*} is highly determinant to find the players' strategies. We prove in the following lemma that four possible cases are possible.

Lemma 9 *There are four possible rankings for the thresholds:*

1. $\widetilde{p} < \overline{\overline{p}} < \overline{\overline{\overline{p}}} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$,

2. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < \widetilde{p} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$,

3. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$,

4. $\overline{\overline{\overline{p}}} < p_1^* < \overline{\overline{p}} < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$.

[4] See Fudenberg and Tirole [8] and Boyer et al. [4] for the definition of an intensity and the precise description of Markov strategies and payoffs (see Appendix A p.323 [4]).

When $\underline{E}_1^L$ is empty, the first three rankings are possible, whereas when $\underline{E}_1^L$ is not empty, the last two rankings are possible.

Note first that the last two cases lead to the same equilibrium, therefore we merge them into a case called Case C characterized by $\overline{\overline{\overline{p}}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$. Before describing the outcomes in the different cases, we propose to summarize the conditions under which each case may happen on Table 2. Furthermore, on Figure 7, we propose a phase diagram where each case is characterized by a set in the space $\{\beta, I_2/I_1\}$[5]

	$I_2 \geq 2I_1$ (and thus $\beta > 2$)	$I_2 < 2I_1$ and $\beta \leq 2$	$I_2 < 2I_1$ and $\beta > 2$
Case A	not possible	$\widetilde{p} < \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)I_1 - I_2\right]$	$\widetilde{p} < \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)I_1 - I_2\right]$
Case B	$\widetilde{p} \geq \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)I_1 - I_2\right]$ $\frac{I_2}{I_1} < 1 + \frac{\beta}{\beta-1}\left(1 - \left(\frac{1}{2}\right)^{\beta-1}\right)$	$\widetilde{p} \geq \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)I_1 - I_2\right]$ Assumption $A3$	$\widetilde{p} \geq \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)I_1 - I_2\right]$
Case C	$\frac{I_2}{I_1} \geq 1 + \frac{\beta}{\beta-1}\left(1 - \left(\frac{1}{2}\right)^{\beta-1}\right)$	not possible	not possible

Table 2: Conditions under which each case may occur.

As we see on Figure 7, when β is high (or the volatility is low), the boundaries of the different regions only depend on the cost advantage I_2/I_1: for a low cost advantage of technology 1, this is Case A that will emerge, whereas Case C will emerge when technology 2 is very expensive relative to technology 1. However, when β is low, the boundaries are highly dependent on the volatility. Indeed, recall that when β is low, this means that σ^2 is high (if μ and r are given). In this case, this is only Case A or B that emerge. We are going to describe carefully Case A and then give rapidly the outcome for Cases B and C.

1. **if $\mathbf{p} \in \left[\mathbf{0}, \overline{\mathbf{p}}\right]$:**

   Nothing happens since it is in the interest of nobody to preempt its rival and any of the thresholds p_1^* or p_{12}^{L*} is crossed,

2. **if $\mathbf{p} \in \left[\overline{\overline{\mathbf{p}}}, \mathbf{p_1^*}\right[$:**

   In this case, $\Phi_2^L(p) \geq V_{12}^F(p)$ and $\Phi_2^L(p) \geq \Phi_1^L(p)$. Therefore, each player has an incentive to preempt its rival with two units. Moreover, as $p < p_1^*$, an investment in technology 1 is not optimal. MPE_{210} is thus the unique possible equilibrium. Let us determine the

[5]Cases A and B happen only when $\underline{E}_1^L$ is empty. Concerning Case C, when Assumption $A3$ is satisfied this means that $\underline{E}_1^L$ is empty, whereas when Assumption $A4$ is satisfied, then $\underline{E}_1^L$ is not empty.

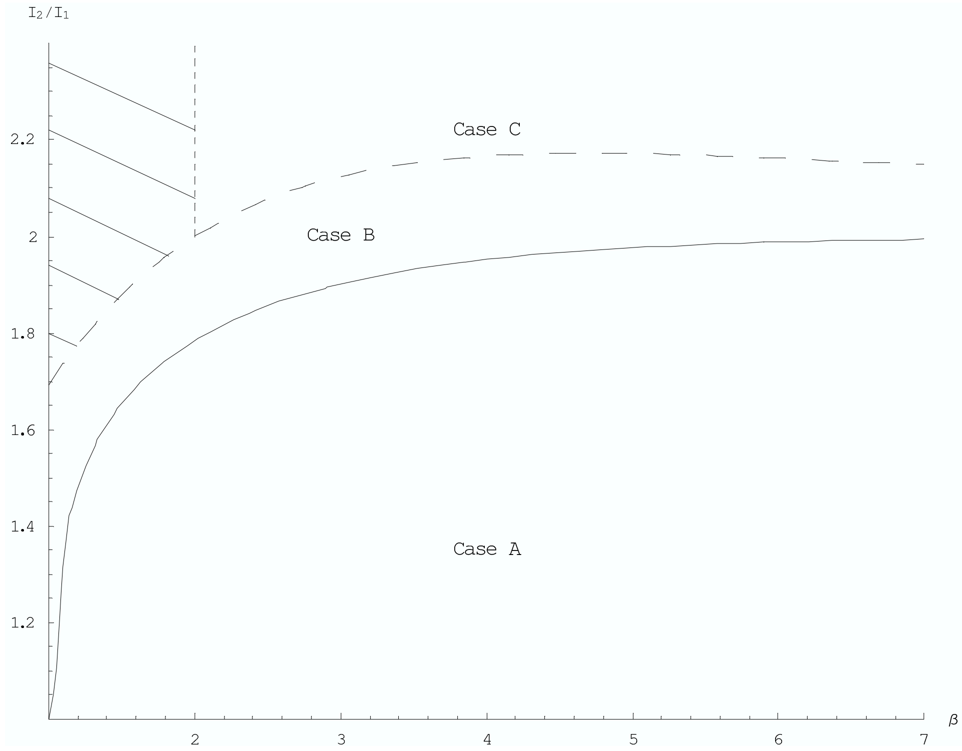

Figure 7: Description of the different cases in function of the parameters.

intensity s with which each player invests in technology 2 such that players are indifferent between investing as a leader or being the follower. Denoting u_2 and $u_\emptyset$ the net payoffs generated by an investment in technology 2 and the best response to this investment, s is determined such that $u_2 = u_\emptyset$, where

$$u_\emptyset = V_{12}^F(p) = \left(\frac{p}{p_{12}^{F*}}\right)^\beta \left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right), \tag{23}$$

and

$$u_2 = (1-s)\,\Phi_2^L(p) - sI_2. \tag{24}$$

Therefore

$$s(p) = \frac{\Phi_2^L(p) - \left(\frac{p}{p_{12}^{F*}}\right)^\beta \left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right)}{\Phi_2^L(p) + I_2}. \tag{25}$$

In MSE_{20}, each player invests in technology 2 with the strictly positive intensity $s(p)$ that is represented on Figure 8.

The equilibrium payoff is $u_\emptyset = u_2 = V_{12}^F(p)$. At $p = \overline{\overline{p}}$, $s(p) = 0$ and there is no mistake: one player invests in technology 2 and the other invests in technology 1 when $p = p_{12}^{F*}$. But, as soon as $p > \overline{\overline{p}}$, the intensity with which each player invests in technology 2 is increasing with demand meaning that competition becomes fiercer. The mistake probability (the probability that the two players invest in technology 2), $\frac{s^2(p)}{1-(1-s(p))^2}$, also increases. Indeed,

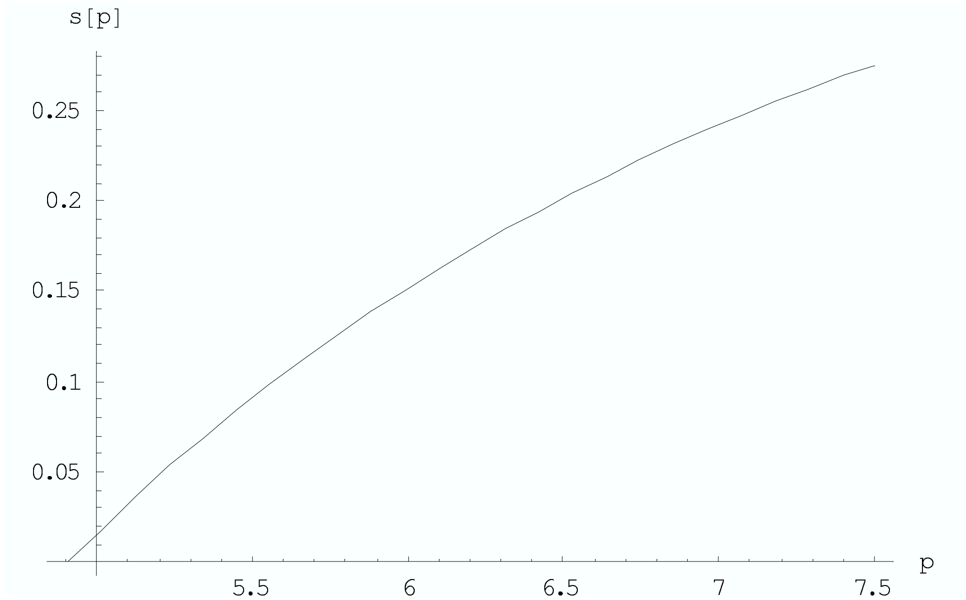

Figure 8: $s(p)$ when $p \in \left[\overline{\overline{p}}, p_1^*\right[$ in Case A.

as p increases, the difference between the leader's and the follower's payoff increases and each player has more reason to preempt its rival.

3. **if** $\mathbf{p} \in \left[\mathbf{p_1^*}, \tilde{\tilde{\mathbf{p}}}\right[$:

   The description of the possible MPE outcomes is a bit more involved, since an investment in technology 1 may be optimal. Indeed, on this interval, $u_2 = u_\emptyset = V_{12}^F(p) < \Phi_1^L(p)$ (since $\overline{\overline{\overline{p}}} < p_1^*$) and MSE_{20} is not an equilibrium anymore since an investment in technology 1 is preferred. In addition to a simultaneous investment equilibrium in technology 1, we thus have to consider a mixed strategy equilibrium involving the three strategies: $\{\emptyset, 1, 2\}$ (this equilibrium will be denoted "mixed strategy equilibrium $\{0, 1, 2\}$", MSE_{210}). The intensities t_1 and t_2 with which each player invests in technology 1 and technology 2 are defined such that $u_2 = u_1 = u_\emptyset$ where

$$u_\emptyset = V_{12}^F(p), \tag{26}$$

$$u_1 = \frac{t_1\left(\frac{p}{2(r-\mu)} - I_1\right) + t_2\left(\frac{p}{4(r-\mu)} - I_1\right)}{t_1 + t_2}, \tag{27}$$

$$u_2 = (1 - t_1 - t_2)\,\Phi_2^L(p) + t_1\left(\frac{p}{2(r-\mu)} - I_2\right) - t_2 I_2. \tag{28}$$

The resolution of this system gives the values of $t_1(p)$ and $t_2(p)$.

$$t_1(p) = \frac{\left(\Phi_2^L(p) - V_{12}^F(p)\right)\left(\frac{p}{4(r-\mu)} - I_1 - V_{12}^F(p)\right)}{\left(V_{12}^F(p) - \frac{p}{4(r-\mu)} + I_1\right)\left(\frac{p}{2(r-\mu)} - I_2\right) - \Phi_2^L(p)\frac{p}{4(r-\mu)} - \left(\frac{p}{2(r-\mu)} - I_1 - V_{12}^F(p)\right) I_2}, \tag{29}$$

$$t_2(p) = \frac{\frac{p}{2(r-\mu)} - I_1 - V_{12}^F(p)}{V_{12}^F(p) - \frac{p}{4(r-\mu)} + I_1} t_1(p) \tag{30}$$

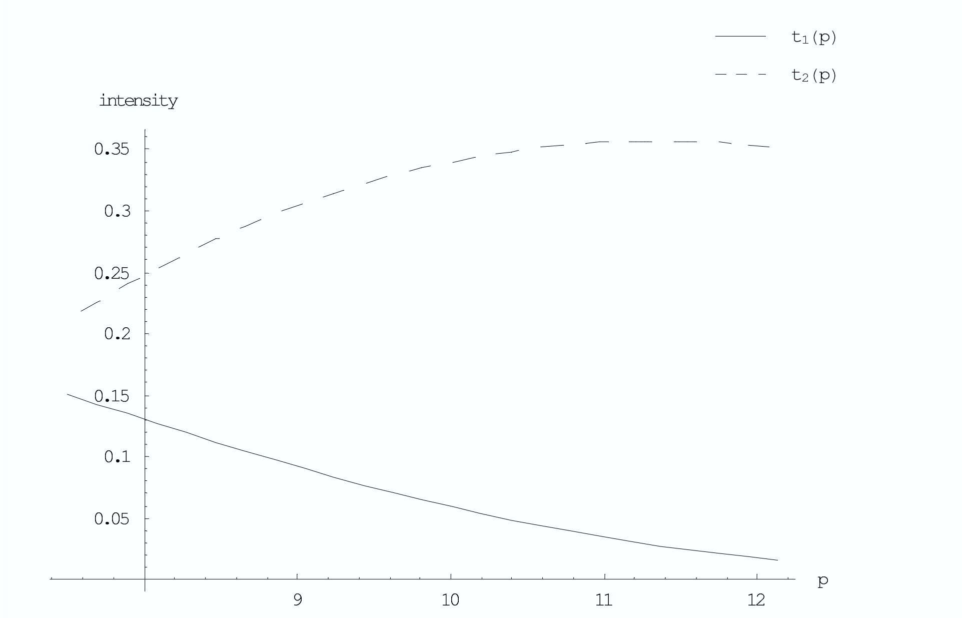

Figure 9: $t_1(p)$ and $t_2(p)$ when $p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$ in Case A.

Function t_1 decreases with p and function t_2 first increases and then decreases with p. As demand increases, each player invests with a higher intensity in technology 2 and with a lower intensity in technology 1. The difference between the leader's and the follower's payoff indeed increases as demand increases, each player invests thus with a higher intensity in technology 2 in order to preempt its rival. However, at the same time, the mistake probability, that is equal to $\frac{t_2^2}{(2-t_2)t_2+t_1^2}$, also increases. Therefore, when p begins to be very high, the increases in t_2 decreases and t_2 even begins to decrease: players are afraid of any mistake. Note that the probability with which each player invests in technology 1 and in technology 2 (q_1 and q_2) are easily computed from the intensities:

$$q_1(p) = \frac{t_1(p)(t_1(p) + t_2(p))}{(2 - t_2(p))\, t_2(p) + t_1^2(p)}, \text{ and} \tag{31}$$

$$q_2(p) = \frac{t_2(p)}{(2 - t_2(p))\, t_2(p) + t_1^2(p)}. \tag{32}$$

To sum-up, on interval $\left[p_1^*, \widetilde{\widetilde{p}}\right[$, two equilibria coexist: the simultaneous investment equilibrium and MSE_{210}.

4. **if $\mathbf{p} \in \left[\widetilde{\widetilde{\mathbf{p}}}, +\infty\right[$:**

Once $p \geq \widetilde{\widetilde{p}}$, there is no equilibrium involving an investment in technology 2 any more, and the only equilibrium is the simultaneous investment one.

Before summarizing the results into a proposition, let us explain why the inaction region does not exist any more contrary to the case without preemption. Suppose such a region $\left]\widetilde{\widetilde{p}} - \varepsilon_1, \widetilde{\widetilde{p}} + \varepsilon_2\right[$ exists around $\widetilde{\widetilde{p}}$. On $\left]\widetilde{\widetilde{p}} - \varepsilon_1, \widetilde{\widetilde{p}}\right[$, MSE_{20} exists because no firm wants to take the risk of being inactive and thus preempted. Thus, nobody remains inactive. At $p = \widetilde{\widetilde{p}}$, this is the simultaneous investment in technology 1 that occurs since the payoff generated by an investment in technology 1, $\Phi_1^L\left(\widetilde{\widetilde{p}}\right)$, is greater than any other payoff ($\Phi_2^L\left(\widetilde{\widetilde{p}}\right)$, $V_{12}^F\left(\widetilde{\widetilde{p}}\right)$, $\frac{\widetilde{\widetilde{p}}}{4(r-\mu)} - I_2$,$-I_2$). At $p = \widetilde{\widetilde{p}} + \varepsilon_2$, the simultaneous investment exists by assumption. As $\widetilde{\widetilde{p}} > p_1^*$, the simultaneous investment is a trigger strategy in this region where technology 1 is the preferred one, and $\forall p \geq \widetilde{\widetilde{p}}$, a simultaneous investment in technology 1 is optimal. Therefore, the inaction region does not exist.

We are thus able to state the following proposition.

Proposition 3 *There exist two MPE outcomes:*

- *In the first one, nobody invests when $p < \overline{\overline{p}}$. When $p \in \left[\overline{\overline{p}}, p_1^*\right[$, each firm invests in technology 2 with an intensity $s(p)$. When $p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$, each firm invests in technology 1 with an intensity $t_1(p)$ and in technology 2 with an intensity $t_2(p)$. When $p \geq \widetilde{\widetilde{p}}$, each firm simultaneously invests in technology 1.*

- *In the second one, nobody invests when $p < \overline{\overline{p}}$ either. When $p \in \left[\overline{\overline{p}}, p_1^*\right[$, each firm invests in technology 2 with an intensity $s(p)$. When $p \geq p_1^*$, each firm simultaneously invests in technology 1.*

On Figure 10, we describe the two equilibria.

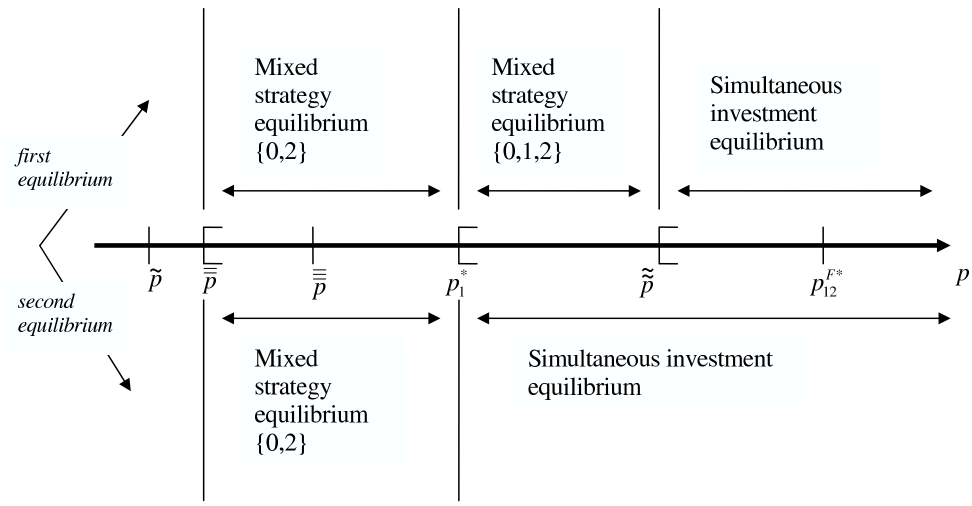

Figure 10: The two equilibria in Case A.

Let us briefly comment Proposition 3. The first point is to compare $\overline{\overline{p}}$ with p_2^{L*} in order to know if investment is triggered earlier than without preemption. To do so, we compute $f\left(p_2^{L*}\right)$[6] and we determine its sign.

$$
\begin{aligned}
f\left(p_2^{L*}\right) &= \Phi_2^L\left(p_2^{L*}\right) - \left(\frac{p_2^{L*}}{p_{12}^{F*}}\right)^\beta \left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right), \\
&> \left(\frac{p_2^{L*}}{p_1^*}\right)^\beta \left(\frac{p_1^*}{2(r-\mu)} - I_1\right) - \left(\frac{p_2^{L*}}{p_{12}^{F*}}\right)^\beta \left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right) \text{ (Assumption } A3\text{)}, \\
&= \left(\frac{I_2}{2I_1}\right)^\beta \frac{I_1}{\beta-1}\left(1-\left(\frac{1}{2}\right)^\beta\right), \\
&\geq 0.
\end{aligned}
$$

Therefore, $\overline{\overline{p}} < p_2^{L*}$ and investment is triggered earlier than in the benchmark case without preemption. This is the classic "rent dissipation result" already obtained by Fudenberg and Tirole [8]. The fear of being preempted makes investment occur earlier. The rents of both players are lower than in the case of a natural leader. Moreover, we have proved that the inaction region does not survive to the introduction of preemption. Indeed, investors do not take the time any more to be sure of their decisions. The fear of losing a market share is too high relative to the gain of waiting to invest in the technology that will turn out to be the most profitable.

Note that these two equilibria happen when $\underline{E}_1^L$ is empty and when $I_2 < 2I_1$ associated with Assumption $A5$. In this case, technology 2 is quite cheap relative to technology 1. Therefore, technology 2 is relatively favorable and players want to take advantage of it. There is thus a range $\left[\overline{\overline{p}}, p_1^*\right[$ of fierce competition where each player wants to preempt his rival with two capacity units. However, as soon as demand is high enough, two cases emerge: either a simultaneous investment in technology 1 or MSE_{210} that involves the three strategies (investing in technology 1, investing in technology 2 and waiting). MSE_{210} is more competitive than the simultaneous investment equilibrium since, in expected value, the number of capacity units invested by each player is higher in the case of MSE_{210}. Indeed, $\forall p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$, $q_1(p) + q_2(p)\,2 = \frac{t_2(p)(2+t_1(p))+t_1^2(p)}{(2-t_2(p))t_2(p)+t_1^2(p)} > 1$ (see Figure 11).

When demand is higher ($p \geq \widetilde{\widetilde{p}}$), the simultaneous investment in technology 1 is the unique equilibrium. In the case of a simultaneous investment equilibrium, for demand values belonging to $\left[p_1^*, \widetilde{\widetilde{p}}\right[$, players invest in technology 1 whereas technology 2 is the preferred one. So, not only is the choice value equal to zero since nobody is willing to wait to invest in the technology that will turn out to be the most profitable, but players even invest in the technology that is the least profitable (if they were the first mover) at the time they take their decision. Let us now turn to the case where $\overline{\overline{p}} < \widetilde{p}$.

We now compare the two equilibria from a social point of view. They only differ when $p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$. Therefore, we compute the difference in surplus between the joint adoption equilibrium,

[6] Function f is defined in the Appendix: $f(p) = \Phi_2^L(p) - \left(\frac{p}{p_{12}^{F*}}\right)^\beta \Phi_{12}^{F*}(p)$. $f(p) < 0\ \forall p < \overline{\overline{p}}$ and $f(p) > 0$ $\forall p > \overline{\overline{p}}$

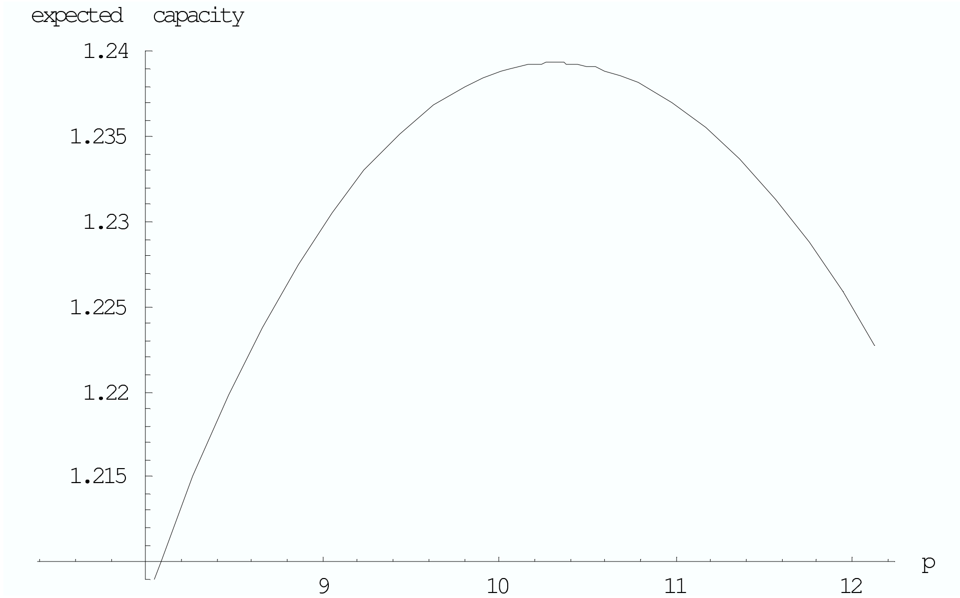

Figure 11: Expected capacity in the case of MSE_{210}.

$S_{11}(p)$, and the mixed strategy equilibrium MSE_{210}, $S_{210}(p)$[7]:

$$\begin{aligned} S_{11} - S_{210} &= \frac{p}{r-\mu} - 2I_1 - \left[\frac{p}{r-\mu} - \frac{2t_2\,(1-t_1-t_2)}{(2-t_2)\,t_2+t_1^2}\left(\left(\frac{p}{p_{12}^{F*}}\right)^\beta I_1 + I_2\right)\right. \\ &- \left.\frac{t_1^2}{(2-t_2)\,t_2+t_1^2}2I_1 - \frac{t_2^2}{(2-t_2)\,t_2+t_1^2}2I_2 - \frac{2t_1t_2}{(2-t_2)\,t_2+t_1^2}\,(I_1+I_2)\right] \\ &= \frac{2t_2}{(2-t_2)\,t_2+t_1^2}\left[\left(\left(\frac{p}{p_{12}^{F*}}\right)^\beta (1-t_1-t_2) - (2-t_1-t_2)\right) I_1 + I_2\right]. \end{aligned}$$

Note first that Case A never occurs when $I_2 > 2I_1$, therefore the sign of $S_{11}(p) - S_{210}(p)$ is not straightforward to compute. But the analysis of the difference allows us to state the following proposition:

Proposition 4 *If $\frac{I_2}{I_1} < \frac{2^{-\beta}+2^{2+\beta}-\beta-5}{32^\beta-\beta-3}$, then as p increases from p_1^* to $\widetilde{\widetilde{p}}$, the mixed strategy equilibrium is first more efficient than the joint adoption equilibrium and then this is the joint adoption equilibrium that is more efficient than the mixed strategy equilibrium.*

On the contrary, if $\frac{I_2}{I_1} \geq \frac{2^{-\beta}+2^{2+\beta}-\beta-5}{32^\beta-\beta-3}$, then $\forall p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$ the joint adoption equilibrium is more efficient than the mixed strategy equilibrium.

This proposition is interesting since for some parameters' values, the mixed strategy equilibrium is more efficient than the joint adoption one, and this is result is new. Indeed, Fudenberg and Tirole [8] find that the joint adoption equilibrium is always more efficient than the diffusion adoption equilibrium and this is not the case here. How to explain this? The preemption equilibrium is more efficient than the simultaneous equilibrium when p is quite low and when

[7]In this expression, in order to be clear we omit to specify the argument in p of the different functions.

$\frac{I_2}{I_1} < \frac{2^{-\beta}+2^{2+\beta}-\beta-5}{32^{\beta}-\beta-3}$, meaning that technology 2 is not too expensive relative to technology 1. Therefore, when one player preempts the other, the total sunk cost equals I_2 to which is added the discounted sunk cost of investing later in technology 1 by the follower. It is thus quite advantageous compared to the sunk cost in case of a simultaneous investment in technology 1. Moreover, as it holds when p is close to p_1^*, the probability that the two players simultaneously invest in technology 2 is small.

In fact this happens when technology 2 is quite cheap relative to technology 1, therefore when there is a mistake, the loss is not too high. On Figure 12, we have represented the boundary $\frac{I_2}{I_1} < \frac{2^{-\beta}+2^{2+\beta}-\beta-5}{32^{\beta}-\beta-3}$.

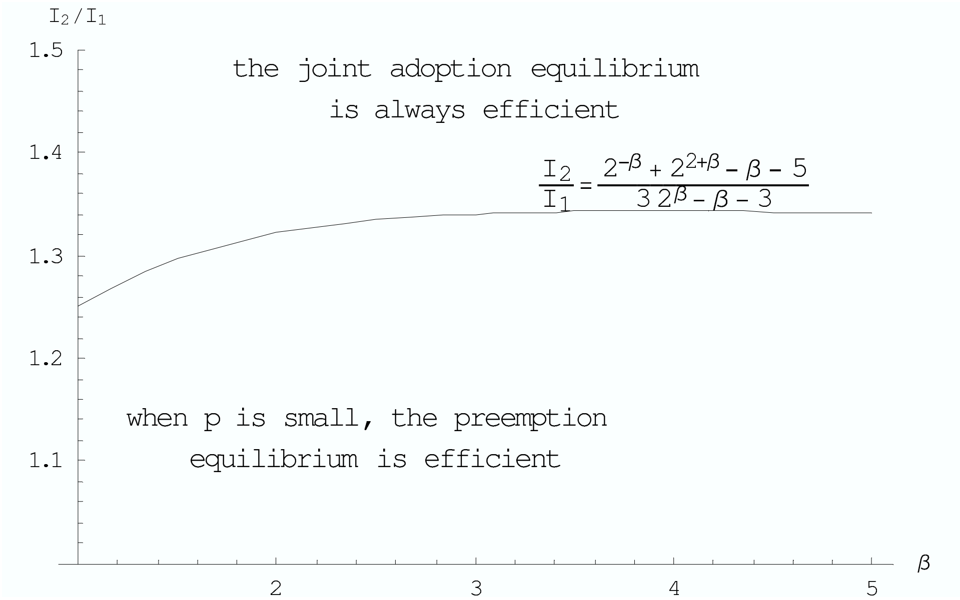

Figure 12: Description of the boundary of Proposition 4.

Once more, the parameter that plays the most important role is the cost advantage I_2/I_1 and not the volatility (that plays through β when r and μ are constant). Moreover, in the case of preemption the cost paid by the second mover comes later and that is why in this case preemption is favorable from a social view point since it allows to delay the cost.

We present rapidly the results in Case C (Case B is very similar to Case A and is therefore relegated to the Appendix).

Proposition 5 *There exist two MPE outcomes:*

- *In the first one, nobody invests when $p < p_1^*$. When $p \in [p_1^*, \widetilde{p}[$, each firm simultaneously invests in technology 1. When $p \in \left]\widetilde{p}, \widetilde{\widetilde{p}}\right[$, each firm invests in technology 1 with an intensity $t_1(p)$ and in technology 2 with an intensity $t_2(p)$. When $p \geq \widetilde{\widetilde{p}}$, each firm simultaneously invests in technology 1.*

- *In the second one, nobody invests when $p < p_1^*$ either. But as soon as $p \geq p_1^*$, each firm* *simultaneously invests in technology 1.*

The second equilibrium replicates the socially optimal outcome.

Both equilibria are represented on Figure 13.

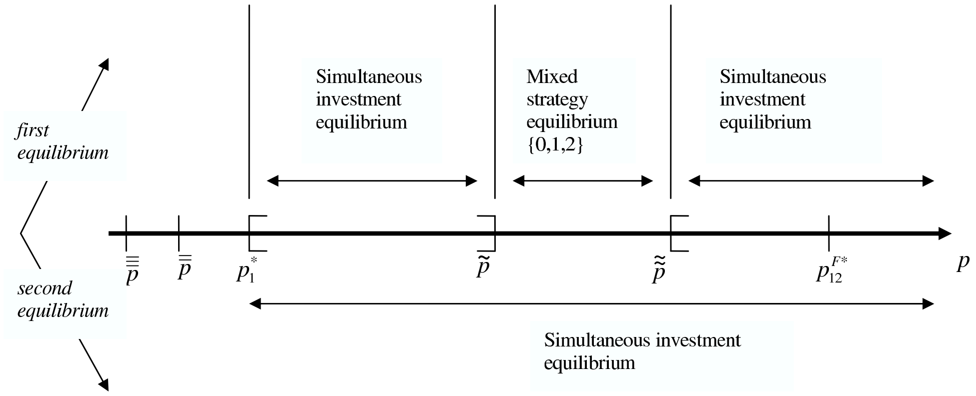

Figure 13: The two equilibria in Case C.

This case is different from Case A. In the first equilibrium, there is a succession of simultaneous investment equilibrium and MSE_{210} as the values taken by the demand increases. As soon as technology 2 is the preferred one (in case of a leader), this is MSE_{210} that occurs. In fact, in this case, as technology 2 is expensive relative to technology 1 ($I_2 \geq 2I_1$), players are reluctant to invest in it, although $\beta > 2$ implies that the time period during which the one who has preempted is alone is long. However, players dread any mistakes. Therefore, MSE_{20} does not exist any more. Indeed, as soon as technology 2 is the preferred one, this is MSE_{210} that arises, taken into account the possibility of investing in technology 1 and 2. On the contrary, in the second result, as soon as the threshold p_1^* is crossed, the equilibrium is to invest in technology 1 even if technology 2 is preferred. There is no investment in technology 2 any more. We retrieve the optimal result. Any choice is eliminated: as technology 2 is quite expensive, firms do not want to take the risk of making any mistake and they behave as if they did not have the choice any more.

6 Concluding remarks

This paper analyzes the investment strategy of a duopoly with price competition on a market of finite size driven by stochastic taste shocks. Each firm has the choice between two technologies: a large unit and a small unit.

We first study the case where one player is constrained to invest second. We find that, depending on the parameters' values, there are one or two inaction region(s). In these regions, the first mover does not invest in any technology whereas without choice, he would have immediately invested in one of the two. The leader prefers to wait and see which technology turns out to be the most profitable to invest later in it. The inaction regions reveal the existence of a choice value for the leader. In this case, two types of equilibria may exist: a sequential investment (large unit for the first mover and then the small unit for the follower) or a simultaneous investment in the small unit.

When no constraint on the investment's order holds any more, the inaction regions disappear. The fear of being preempted indeed makes firms invest earlier. Depending on the parameters' values, three types of equilibria exist: a simultaneous investment equilibrium in which both firms invest in the small unit and two mixed strategy equilibria, one in which each firm invests in the small unit with a strictly positive intensity and in the large unit with a strictly positive intensity, and another one in which each firm invests in the large unit with a strictly positive intensity. In this case, when technology 2 is quite cheap, the mixed strategy equilibrium is more efficient than the joint adoption equilibrium when p is not too high. This is quite new and is due to the fact that the cost paid by the second mover is delayed. When the cost advantage of the small unit increases, the second mixed strategy equilibrium becomes less likely and then disappears totally. In fact, in the mixed strategy equilibrium, there is always a probability that players simultaneously invest in the large unit. Thus, when the cost of the large unit is very high, firms prefer to invest simultaneously in the small unit even if the expected net discounted profit generated by an investment as first mover in the large unit is greater. In this case, not only is the choice value equal to zero (since the inaction regions disappear), but the choice firms face is also reduced. In this case, the simultaneous investment equilibrium replicates the socially optimal outcome.

Table of notations

P_t Total willingness to pay for the commodity

I_1 Sunk cost of technology 1

I_2 Sunk cost of technology 2

σ^2 Volatility

r Discount rate

μ Drift

β Positive root of $1/2\sigma^2\beta(\beta-1)+\mu\beta-r=0$

p_{11}^{F*} Threshold of the demand at which the follower invests in technology 1 given that the leader has invested in technology 1

p_{12}^{F*} Threshold of the demand at which the follower invests in technology 1 given that the leader has invested in technology 2

p_1^{L*} Threshold of the demand at which the leader invests in technology 1 given that the follower will invest in technology 1 when $p=p_{12}^{F*}$

p_2^{L*} Threshold of the demand at which the leader invests in technology 2 given that the follower will invest in technology 1 when $p=p_{12}^{F*}$

$\widetilde{p}$ Demand value such that $\Phi_1^L(\widetilde{p})=\Phi_2^L(\widetilde{p})$

$\widetilde{\widetilde{p}}$ Demand value such that $\Phi_1^L\left(\widetilde{\widetilde{p}}\right)=\Phi_2^L\left(\widetilde{\widetilde{p}}\right)$ with $\widetilde{p}<\widetilde{\widetilde{p}}$

p_3 Upper threshold of the demand of E_2^L for the leader when $\underline{E}_1^L$ is empty

p_4 Lower threshold of the demand of $\overline{E}_1^L$ for the leader when $\underline{E}_1^L$ is empty

$\widehat{p}_2$ Upper threshold of the demand of $\underline{E}_1^L$ for the leader when $\underline{E}_1^L$ is not empty

$\widehat{p}_3$ Lower threshold of the demand of E_2^L for the leader when $\underline{E}_1^L$ is not empty

$\widehat{p}_4$ Upper threshold of the demand of E_2^L for the leader when $\underline{E}_1^L$ is not empty

$\widehat{p}_5$ Lower threshold of the demand of $\overline{E}_1^L$ for the leader when $\underline{E}_1^L$ is not empty

$\overline{\overline{p}}$ Threshold of the demand such that each player is indifferent between an immediate investment in technology 2 and an investment, as a follower, in technology 1 at p_{12}^{F*}: $\Phi_2^L\left(\overline{\overline{p}}\right)=V_{12}^F\left(\overline{\overline{p}}\right)$

$\overline{\overline{\overline{p}}}$ Threshold of the demand such that each player is indifferent between an immediate investment in technology 1 and an investment, as a follower, in technology 1 at p_{12}^{F*}: $\Phi_1^L\left(\overline{\overline{\overline{p}}}\right)=V_{12}^F\left(\overline{\overline{\overline{p}}}\right)$

E^L Exercise region

E_1^L Exercise region when technology 1 is the preferred one

E_2^L Exercise region when technology 2 is the preferred one

$\underline{E}_1^L$	Exercise region when technology 1 is the preferred one and when the demand value is less than $\widetilde{p}$
$\overline{E}_1^L$	Exercise region when technology 1 is the preferred one and when the demand value is greater than $\widetilde{\widetilde{p}}$
MSE_{20}	Mixed strategy equilibrium involving the two strategies: $\{\emptyset, 2\}$
MSE_{210}	Mixed strategy equilibrium involving the three strategies: $\{\emptyset, 1, 2\}$
$\pi\left(n_i, n_j, p\right)$	Instantaneous profit flow of firm i when it holds n_i units whereas its leader holds n_j units and when demand equals p
$CS\left(p\right)$	Consumer surplus
$S\left(p\right)$	Total surplus
$S_{11}\left(p\right)$	Total surplus in case of a simultaneous investment in technology 1
$S_{20}\left(p\right)$	Total surplus in case of a unique investment in technology 2
$s\left(p\right)$	Intensity with which each player invests in technology 2 in MSE_{20}
$t_1\left(p\right)$	Intensity with which each player invests in technology 1 in MSE_{210}
$t_2\left(p\right)$	Intensity with which each player invests in technology 2 in MSE_{210}
$\Phi_{11}^F\left(p\right)$	Expected discounted profit of the follower when he invests in technology 1 given that the leader has invested in technology 1 and that demand is equal to p
$\Phi_{12}^F\left(p\right)$	Expected discounted profit of the follower when he invests in technology 1 given that the leader has invested in technology 2 and that demand is equal to p
$\Phi_1^L\left(p\right)$	Expected discounted profit of the leader when he invests in technology 1 given that the follower will invest in technology 1 when $p = p_{11}^{F*}$ and that demand is equal to p
$\Phi_2^L\left(p\right)$	Expected discounted profit of the leader when he invests in technology 2 given that the follower will invest in technology 1 when $p = p_{12}^{F*}$ and that demand is equal to p
$\Phi^{L+F}\left(p\right)$	Join profit of the leader and of the follower
$V_{11}^F\left(p\right)$	Option value of an investment of the follower in technology 1 given that the leader has invested in technology 1 and that demand is equal to p
$V_{12}^F\left(p\right)$	Option value of an investment of the follower in technology 1 given that the leader has invested in technology 2 and that demand is equal to p
$V_1^L\left(p\right)$	Option value of an investment of the leader in technology 1 given that the follower will invest in technology 1 when $p = p_{11}^{F*}$ and that demand is equal to p
$V_2^L\left(p\right)$	Option value of an investment of the leader in technology 2 given that the follower will invest in technology 1 when $p = p_{11}^{F*}$ and that demand is equal to p
$V^L\left(p\right)$	Option value of an investment of the leader in one of the two technologies given that the follower will invest later and that demand is equal to p

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A Proof of Lemma 1

The proof of the shape of the instantaneous profit flows has been done in Boyer et al. [4]. We recall their result.

The first three expressions for the instantaneous expected profit flows are immediate.

When one firm has one capacity unit, for instance firm i, and the other, firm j, two capacity units, it is necessary to find a mixed strategy equilibrium since no pure strategy equilibrium exists. Indeed, in this case, a strategy profile is a vector of prices $\left(p^i, p^j\right) \in [0,p] \times [0,p]$. For any strategy profile $\left(p^i, p^j\right)$, firms' payoff are equal to

$$\pi^i\left(p^i, p^j\right) = \begin{cases} p^i & \text{if } p^i < p^j, \\ \frac{p^i}{2} & \text{if } p^i \geq p^j, \end{cases}$$

and

$$\pi^j\left(p^j, p^i\right) = \begin{cases} \frac{p^j}{2} & \text{if } p^j \leq p^i, \\ 0 & \text{if } p^j > p^i. \end{cases}$$

As the game of these two equations do not have any pure strategy equilibrium, we look for a mixed strategy equilibrium described by a pair of cumulative distribution functions $\left(F^i, F^j\right)$ over $[0,p]$. Note first that any $p^i < p/2$ is a strictly dominated strategy for i because i can secure a payoff of $p/2$ by charging $p^i = p$. Hence, necessarily supp $\left(F^i\right) \subset [p/2, p]$, so that one must have supp $\left(F^j\right) \subset [p/2, p]$ as well. Define

$$F^i\left(p^i\right) = \begin{cases} 0 & \text{if } p^i \in [0, p/2], \\ 1 - \frac{1}{2p^i} & \text{if } p^i \in [p/2, p[, \\ 1 & \text{if } p^i = p \end{cases}$$

and

$$F^j\left(p^j\right) = \begin{cases} 0 & \text{if } p^j \in [0, p/2], \\ 2 - \frac{1}{p^j} & \text{if } p^j \in]p/2, p]. \end{cases}$$

It follows that $\pi\left(p^j, F^i\right) = p^j\left(1 - F^i\left(p^j\right)\right)\frac{p}{2} = \frac{p}{4}$ for all $p^j \in [p/2, p)$. Moreover for $p^j = p$, we have $\pi^j\left(p, F^i\right) = \mathbb{P}^{F^i}\left(p^i = p\right)\frac{p}{2} = \frac{p}{4}$. Hence, given the strategy F^i of firm i, firm j is indifferent between all possible prices in supp $\left(F^j\right) = [p/2, p]$. Similarly, for all $p^i \in [p/2, p]$, we have $\pi^i\left(p^i, F^j\right) = p^i\left(1 - F^j\left(p^i\right)\right)p + p^i F^j\left(p^i\right)p/2 = p/2$. Hence, given the strategy F^j of firm j, firm i is indifferent between all possible prices in supp $\left(F^i\right) = [p/2, p]$. It follows that $\left(F^i, F^j\right)$ is a mixed strategy equilibrium of the static pricing game with corresponding profits $(p/2, p/4)$. The proof that this equilibrium is unique is standard and therefore omitted.

Regardless the leader's choice, it is never optimal for the follower to invest in technology 2 since $\pi\left(1, n_j, p\right) \geq \pi\left(2, n_j, p\right), \forall n_j \in \{1, 2\}$ and $I_2 \geq I_1$. $\square$

B Computation of the leader's payoff

Suppose the leader has to invest in technology 1. His net discounted payoff is equal to:

$$\Phi_2^L(p) = \mathbb{E}\left[\int_0^{\tau_{12}^{F*}} e^{-rt} p_t^p dt + \int_{\tau_{12}^{F*}}^{+\infty} \frac{e^{-rt} p_t^p}{2} dt | p_0^p = p\right] - I_2,$$

where $\tau_{12}^{F*} = \inf\left\{t | p_t = p_{12}^{F*}\right\}$.

$$\begin{aligned}
\Phi_2^L(p) &= \mathbb{E}\left[\int_0^{\tau_{12}^{F*}} e^{-rt} p_t^p dt + \int_{\tau_{12}^{F*}}^{+\infty} \frac{e^{-rt} p_t^p}{2} dt | p_0^p = p\right] - I_2, \\
&= \mathbb{E}\left[\int_0^{+\infty} e^{-rt} p_t^p dt - \int_{\tau_{12}^{F*}}^{+\infty} \frac{e^{-rt} p_t^p}{2} dt | p_0^p = p\right] - I_2, \\
&= \frac{p}{r-\mu} - \mathbb{E}\left[\int_{\tau_{12}^{F*}}^{+\infty} \frac{e^{-rt} p_t^p}{2} dt | p_0^p = p\right] - I_2, \\
&= \frac{p}{r-\mu} - \mathbb{E}\left[\frac{e^{-r\tau_{12}^{F*}} p_{\tau_{12}^{F*}}^p}{2(r-\mu)} dt | p_0^p = p\right] - I_2, \\
&= \frac{p}{r-\mu} - \frac{p_{12}^{F*}}{2(r-\mu)} \left(\frac{p}{p_{12}^{F*}}\right)^\beta - I_2.
\end{aligned}$$

The last equality holds because $\mathbb{E}\left[e^{-r\tau_{12}^{F*}} p_{\tau_{12}^{F*}}^p dt | p_0^p = p\right] = \left(p/p_{12}^{F*}\right)^\beta p_{12}^{F*}$.

C Proof of Lemma 2

Let us study function $h(p) = \Phi_1^L(p) - \Phi_2^L(p)$. First of all, note that if $p > p_{12}^{F*}$, then $h(p) = I_2 - I_1 > 0$ and the leader always prefers technology 1. Therefore, we assume $p < p_{12}^{F*}$.

In this case, $h(p) = \frac{1}{2(r-\mu)}\left[p_{12}^{F*}\left(\frac{p}{p_{12}^{F*}}\right)^\beta - p\right] + I_2 - I_1$ and $h'(p) = \frac{1}{2(r-\mu)}\left[\beta\left(\frac{p}{p_{12}^{F*}}\right)^{\beta-1} - 1\right]$. Therefore, h is an increasing function as soon as $p \geq p_{12}^{F*}\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$. $h(0)$ and $h\left(p_{12}^{F*}\right)$ are strictly positive. Moreover, as $\beta > 1$, $p_{12}^{F*}\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} < p_{12}^{F*}$. Therefore, there are two indifference points if and only if $h\left(p_{12}^{F*}\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}\right) < 0$, or $I_2 < \left(1 + 2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}\right) I_1$. □

D Proof of Lemma 3

Under Assumption $A1$, if $I_2 \geq 2I_1$ then $\beta < 2^{\beta-1}$. Let us study function $x \mapsto k(x) = x - 2^{x-1}$. $k'(x) > 0$ if $x < x^* = \frac{\ln 2 - \ln(\ln 2)}{\ln 2} = 1.52$. Moreover $k(x^*) > 0$ and $k(2) = 0$. Therefore, $k(\beta) < 0 \Rightarrow \beta > 2$. □

E Proof of Lemma 4

Suppose that $\overline{E}_1^L = \emptyset$. Then, $\forall p > \tilde{\tilde{p}}, V(p) > \Phi_1^L(p) > \Phi_2^L(p)$. at the same time, $\forall p \geq p_1^{L*}, V_1^L(p) = \Phi_1^L(p)$. But, when $p \to +\infty, V^L(p) = V_1^L(p)$. This leads to $\lim_{p\to+\infty} \Phi_1^L(p) > \lim_{p\to+\infty} \Phi_1^L(p)$, a contradiction. □

F Proof of Lemma 5

We first prove that Assumption $A3$ implies that $\underline{E}_1^L = \emptyset$. Suppose that Assumption $A3$ is satisfied and that $\underline{E}_1^L \neq \emptyset$. According to Décamps et al. [5], $\underline{E}_1^L$ is of the form $\left[p_1^{L*}, p_2\right[$. Therefore, $V^L\left(p_1^{L*}\right) = \Phi_1^L\left(p_1^{L*}\right) = V_1^L\left(p_1^{L*}\right) \geq V_2^L\left(p_1^{L*}\right)$, and this is in contradiction with Assumption $A3$.

Now, we focus on E_2^L. First note that without assuming that Assumption $A3$ is satisfied, we have that if $\widetilde{p} < p_1^{L*} < \widetilde{\widetilde{p}}$, then $E_2^L \neq \emptyset$. Suppose indeed this is not the case. According to Villeneuve [?], we know that in this case, $V^L(p) = V_1^L(p)$. In particular, $V^L\left(p_1^{L*}\right) = \Phi_1^L\left(p_1^{L*}\right) < \Phi_2^L\left(p_1^{L*}\right)$. But at the same time, $V^L\left(p_1^{L*}\right) \geq V_2^L\left(p_1^{L*}\right) \geq \Phi_2^L\left(p_1^{l*}\right)$ and this leads to a contradiction.

We moreover assume that Assumption $A3$ is satisfied. Suppose that $E_2^L = \emptyset$. Then, $V^L(p) = V_1^L(p)$ and in particular, Assumption $A3$ implies that $\Phi_1^L\left(p_1^{L*}\right) = V^L\left(p_1^{L*}\right) < V_2^{L*}\left(p_1^{L*}\right)$ what is not possible. Therefore, we have a contradiction and $E_2^L \neq \emptyset$. □

G Proof of Lemma 6

Suppose first that $I_2 \geq 2I_1$. According to Lemma 3, this implies that $p_{11}^{F*} = p_1^{L*} \leq p_2^{L*} < p_{12}^{F*}$. Therefore, $\Phi_1^L\left(p_1^{L*}\right) = \frac{1}{\beta-1}I_1$ and $V_2^L\left(p_1^{L*}\right) = \frac{I_2}{\beta-1}\left[\left(\frac{2I_1}{I_2}\right)^\beta - \beta\left(\frac{1}{2}\right)^\beta\frac{2I_1}{I_2}\right]$. Assumption $A3$ reduces to $\frac{1}{2} + \beta\left(\frac{1}{2}\right)^\beta < \left(\frac{2I_1}{I_2}\right)^{\beta-1}$.

Similarly, $\Phi_2^L\left(p_2^{L*}\right) = \frac{I_2}{\beta-1} - \frac{2\beta I_1}{\beta-1}\left(\frac{I_2}{4I_1}\right)^\beta$ and $V_1^L\left(p_2^{L*}\right) = \frac{\beta}{2(\beta-1)}I_2 - I_1$. Assumption $A4$ reduces to $\frac{\beta}{\beta-1}\left(\frac{1}{2}\right)^\beta\left(\frac{I_2}{2I_1}\right)^\beta + \frac{\beta-2}{2(\beta-1)}\frac{I_2}{2I_1} - \frac{1}{2} \leq 0$. As $I_2 \geq 2I_1$, $\beta > 2$ and therefore $\frac{\beta-2}{2(\beta-1)} < 0$. Thus,

$$\begin{aligned}
\frac{\beta}{\beta-1}\left(\frac{1}{2}\right)^\beta\left(\frac{I_2}{2I_1}\right)^\beta + \frac{\beta-2}{2(\beta-1)}\frac{I_2}{2I_1} - \frac{1}{2} \quad &< \quad \frac{\beta}{\beta-1}\left(\frac{1}{2}\right)^\beta\left(\frac{I_2}{2I_1}\right)^\beta + \frac{\beta-2}{2(\beta-1)} - \frac{1}{2}, \\
&= \quad \frac{\beta}{\beta-1}\left(\frac{1}{2}\right)^\beta\left(\frac{I_2}{2I_1}\right)^\beta - \frac{1}{2(\beta-1)}, \\
&< \quad \frac{\beta}{\beta-1}\left(\frac{1}{2}\right)^\beta\frac{I_2}{2I_1}\frac{1}{1/2+\beta(1/2)^\beta} - \frac{1}{2(\beta-1)}, \\
&= \quad \frac{\beta\frac{I_1}{I_2} - 2^{\beta-1} - \beta}{2(\beta-1)\left(2^{\beta-1}+\beta\right)},
\end{aligned}$$

where the last but one inequality holds since Assumption $A3$ is satisfied. As Assumption $A1$ is also satisfied, $\frac{I_2}{I_1} < 1 + 2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} < 1 + \frac{2^{\beta-1}}{\beta}$. Therefore, $\beta\frac{I_1}{I_2} - 2^{\beta-1} - \beta < 0$ and Assumption $A4$ is satisfied as soon as Assumptions $A1$ and $A3$ are satisfied.

Suppose now that $I_2 < 2I_1$. In this case, Assumption $A3$ implies that $\frac{I_2}{I_1} < \frac{1}{\beta-1}\left(2\beta - 1 - \beta\left(\frac{1}{2}\right)^{\beta-1}\right)$. At the same time, not only do we have $\frac{I_2}{I_1} < 2$, but Assumption $A1$, $\frac{I_2}{I_1} < 1 + 2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$, has also to be satisfied. Which of the two constraints is the most difficult to satisfy depending on the values taken by β?

We first prove that $1+2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}<2 \Leftrightarrow \beta<2$. Indeed, $F(x)=1+2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}-2$ and $F'(x)=\frac{2(x\ln x-(x-1))}{(x-1)^2}\left(\frac{1}{x}\right)^{\frac{2-x}{1-x}}$ is positive for $x\geq 1$. As $F(1)<0$ and $\lim_{x\to+\infty}F(x)=1$, there exists a unique x such that $F(x)=0$ and this happens when $x=2$.

Therefore, we have to distinguish the two cases:

First case $\beta\geq 2$: $\underline{E}_1^L$ is not empty if and only if $\frac{I_2}{I_1}<\frac{1}{\beta-1}\left(2\beta-1-\beta\left(\frac{1}{2}\right)^{\beta-1}\right)$ and $\frac{I_2}{I_1}<2$. If $2\leq\frac{1}{\beta-1}\left(2\beta-1-\beta\left(\frac{1}{2}\right)^{\beta-1}\right)$, or $1-\beta\left(\frac{1}{2}\right)^{\beta-1}\geq 0$, this is always satisfied. $\forall\beta>2$, $1-\beta\left(\frac{1}{2}\right)^{\beta-1}>0$. Therefore both equalities are satisfied and $\underline{E}_1^L$ is empty. In the case where $I_2<2I_1$, Assumption $A4$ reduces to $\left(1+\frac{\beta}{2^{\beta-1}}\right)\left(\frac{I_2}{2I_1}\right)^{\beta-1}\leq 2$. As $I_2<2I_1$, $\left(1+\frac{\beta}{2^{\beta-1}}\right)\left(\frac{I_2}{2I_1}\right)^{\beta-1}<1+\frac{\beta}{2^{\beta-1}}\leq 2$ since $\beta\geq 2$. Therefore, in this case, Assumption $A4$ is satisfied.

Second case $\beta<2$: $\underline{E}_1^L$ is empty if and only if $\frac{I_2}{I_1}<\frac{1}{\beta-1}\left(2\beta-1-\beta\left(\frac{1}{2}\right)^{\beta-1}\right)$ and $\frac{I_2}{I_1}<1+2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$. As this is not always the case that $\frac{1}{\beta-1}\left(2\beta-1-\beta\left(\frac{1}{2}\right)^{\beta-1}\right)<1+2\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$, $\underline{E}_1^L$ may be not empty. As in the previous case, Assumption $A4$ reduces to $\left(1+\frac{\beta}{2^{\beta-1}}\right)\left(\frac{I_2}{2I_1}\right)^{\beta-1}\leq 2$. As $\frac{2^\beta}{\beta+2^{\beta-1}}\geq\left[\frac{1}{2(\beta-1)}\left(2\beta-1-\beta\left(\frac{1}{2}\right)^{\beta-1}\right)\right]^{\beta-1}$, for $\beta\in[1,2[$, as soon as Assumption $A3$ is satisfied, Assumption $A4$ is also satisfied. □

H Proof of Lemma 7

Suppose first that Assumption $A4$ is satisfied and that $E_2^L=\emptyset$. According to Villeneuve [?], $V^L(p)=V_1^L(p)$. Therefore, $\forall p\in\left]\widetilde{p},\widetilde{\widetilde{p}}\right[$, $V^L(p)=\Phi_1^L(p)<\Phi_2^L(p)\leq V^L(p)$. There is a contradiction and $E_2^L\neq\emptyset$.

We suppose now that Assumption $A4$ is satisfied and that $\underline{E}_1^L=\emptyset$. Thus, $\forall p<\widetilde{p}, V^L(p)>\Phi_L^1(p)$. In particular, $V^L\left(p_1^{L*}\right)>\Phi_L^1\left(p_1^{L*}\right)\geq V_2^L\left(p_1^{L*}\right)$ (because of Assumption $A4$). According to Øksendal [22],

$$\begin{aligned} V^L\left(p_1^{L*}\right) &= \mathbb{E}\left[e^{-r\tau_E}\max\left(\Phi_1^L\left(P_{\tau_E}\right),\Phi_2^L\left(P_{\tau_E}\right)|P_0=p_1^{L*}\right)\right] \\ &= \mathbb{E}\left[e^{-r\tau_E}\Phi_2^L\left(P_{\tau_E}\right)|P_0=p_1^{L*}\right] \text{ since } p_1^{L*}<\widetilde{p},\underline{E}_1^L=\emptyset \text{ and } E_2^L\neq\emptyset, \\ &\leq V_2^L\left(p_1^{L*}\right), \end{aligned}$$

where $\tau_E=\inf\left\{t>0|P_t\in E^L\right\}$. Therefore, there is a contradiction and $\underline{E}_1^L\neq\emptyset$. □

I Proof of Lemma 8

First of all, the condition that $p_1^{L*}<\widetilde{p}$ is equivalent to $\Phi_1^L\left(p_1^{L*}\right)-\Phi_2^L\left(p_1^{L*}\right)>0$ and $\Phi_1^{L'}\left(p_1^{L*}\right)-\Phi_2^{L'}\left(p_1^{L*}\right)<0$. The last inequality is equivalent to $\beta>2$. The first condition reduces to

$\frac{I_2}{I_1} > 1 + \frac{\beta}{\beta-1}\left(1-\left(\frac{1}{2}\right)^{\beta-1}\right)$. Associated to $\beta > 2$, this leads to $I_2 > 2I_1$. When $I_2 > 2I_1$, $\Phi_1^L\left(p_1^{L*}\right) \geq V_2^L\left(p_1^{L*}\right)$ becomes $\frac{1}{2} + \beta\left(\frac{1}{2}\right)^{\beta} \geq \left(\frac{2I_1}{I_2}\right)^{\beta-1}$, or $\frac{I_2}{I_1} \geq 2\left(\frac{1}{2} + \beta\left(\frac{1}{2}\right)^{\beta}\right)^{-\frac{1}{\beta-1}}$. As $\forall\beta > 2$, $1 + \frac{\beta}{\beta-1}\left(1-\left(\frac{1}{2}\right)^{\beta-1}\right) < 2\left(\frac{1}{2} + \beta\left(\frac{1}{2}\right)^{\beta}\right)^{-\frac{1}{\beta-1}}$, if $\Phi_1^L\left(p_1^{L*}\right) \geq V_2^L\left(p_1^{L*}\right)$, then $p_1^{L*} < \widetilde{p}$. □

J Proof of Lemma 9

To prove this result, we consider two cases: when $\underline{E}_1^L$ is empty and when $\underline{E}_1^L$ is not empty.

First case: $\underline{E}_1^L$ is empty.

Recall that in section 3, we proved that $\underline{E}_1^L$ is empty if $I_2 \geq 2I_1$ and if Assumption $A3$ is satisfied, or if $I_2 < 2I_1$ and $\beta < 2$ if Assumption $A3$ is satisfied, or lastly if $I_2 < 2I_1$ and $\beta \geq 2$. From the previous discussion, six thresholds are of particular interest: $\widetilde{p}$, $\overline{\overline{p}}$, $\overline{\overline{\overline{p}}}$, p_1^*, $\widetilde{\widetilde{p}}$ and p_{12}^{F*}.

Lemma 10 *When $\underline{E}_1^L$ is empty, three different rankings are possible*

1. $\widetilde{p} < \overline{\overline{p}} < \overline{\overline{\overline{p}}} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$
2. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < \widetilde{p} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$
3. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$

Proof:

Before comparing the thresholds, let us briefly focus on $\overline{\overline{p}}$ and $\overline{\overline{\overline{p}}}$.

Definition of $\overline{\overline{p}}$: Recall that $\overline{\overline{p}}$ is such that the two players are indifferent between investing immediately in technology 2 and waiting to invest optimally in technology 1 at p_{12}^{F*}:

$$\Phi_2^L\left(\overline{\overline{p}}\right) = V_{12}^{F*}\left(\overline{\overline{p}}\right).$$

Lemma 11 *The threshold for which each player is indifferent between being the leader and the follower, $\overline{\overline{p}}$, is lower than p_{12}^{F*}.*

Proof: Suppose by contradiction that $\overline{\overline{p}} \geq p_{12}^{F*}$ and $\Phi_2^L\left(\overline{\overline{p}}\right) = V_{12}^{F*}\left(\overline{\overline{p}}\right)$. This implies that $\frac{\overline{\overline{p}}}{2(r-\mu)} - I_2 = \frac{\overline{\overline{p}}}{4(r-\mu)} - I_1$ and $\overline{\overline{p}} = 4(r-\mu)(I_2 - I_1)$. As $\overline{\overline{p}} \geq p_{12}^{F*}$, it implies that $\frac{I_2}{I_1} \geq \frac{2\beta-1}{\beta-1}$. Two cases may then occur:

- either $I_2 < 2I_1$. In this case, the inequality $\frac{I_2}{I_1} \geq \frac{2\beta-1}{\beta-1}$ cannot hold since $\frac{2\beta-1}{\beta-1} > 2$, and we have a contradiction,
- or $I_2 \geq 2I_1$. In this case, as $\underline{E}^L$ is empty and depending on Lemma 6,
$$\frac{I_2}{I_1} < \frac{1}{\beta-1}\left(2\beta - 1 - \beta\left(1/2\right)^{\beta-1}\right).$$
Therefore, the inequality $\frac{I_2}{I_1} \geq \frac{2\beta-1}{\beta-1}$ cannot hold and we have a contradiction. □

Therefore, we necessary have that $\overline{\overline{p}} < p_{12}^{F*}$ and $\overline{\overline{p}}$ is defined by:

$$\Phi_2^L\left(\overline{\overline{p}}\right) = \left(\frac{\overline{\overline{p}}}{p_{12}^{F*}}\right)^{\beta} \Phi_{12}^F\left(p_{12}^{F*}\right).$$

Let us define function f by:

$$f\left(p\right) = \Phi_2^L\left(p\right) - \left(\frac{p}{p_{12}^{F*}}\right)^{\beta} \Phi_{12}^F\left(p_{12}^{F*}\right).$$

For $p \leq p_{12}^{F*}\left(\frac{4}{2\beta+1}\right)^{\frac{1}{\beta-1}}$, f is an increasing function. Moreover, $f\left(0\right) = -I_2$ and $f\left(p_{12}^{F*}\right) = \frac{2\beta-1}{\beta-1}I_1 - I_2 > 0$, according to the proof of Lemma 11. Therefore, if $p < \overline{\overline{p}}$, then $\Phi_2^L\left(p\right) < \left(\frac{p}{p_{12}^{F*}}\right)^{\beta}\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right)$ and if $p > \overline{\overline{p}}$, then $\Phi_2^L\left(p\right) > \left(\frac{p}{p_{12}^{F*}}\right)^{\beta}\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right)$. Rearranging the terms when $f\left(\overline{\overline{p}}\right) = 0$ implies that $\overline{\overline{p}}$ is the solution of

$$\frac{4p}{p_{12}^{F*}} - \frac{\beta-1}{\beta}\frac{I_2}{I_1} - \frac{2\beta+1}{\beta}\left(\frac{p}{p_{12}^{F*}}\right)^{\beta} = 0.$$

This expression is strictly negative for $p < \overline{\overline{p}}$ and strictly negative for $p > \overline{\overline{p}}$.

Definition of $\overline{\overline{\overline{p}}}$***:*** $\overline{\overline{\overline{p}}}$ is such that the payoff of the second mover in the preemption equilibrium is equal to the payoff in the case of a simultaneous investment in technology 1. Let us therefore function g:

$$g\left(p\right) = \Phi_1^L\left(p\right) - \left(\frac{p}{p_{12}^{F*}}\right)^{\beta} \Phi_{12}^F\left(p_{12}^{F*}\right).$$

For $p \leq p_{12}^{F*}$, g is an increasing function. $g\left(0\right) = -I_1$ and $g\left(p_{12}^{F*}\right) = \frac{p_{12}^{F*}}{4(r-\mu)} > 0$. Therefore there exists a unique $\overline{\overline{\overline{p}}}$ such that $\Phi_1^L\left(\overline{\overline{\overline{p}}}\right) = \left(\frac{\overline{\overline{\overline{p}}}}{p_{12}^{F*}}\right)^{\beta}\Phi_{12}^F\left(p_{12}^{F*}\right)$. Moreover, if $p < \overline{\overline{\overline{p}}}$, then $\Phi_1^L\left(p\right) < \left(\frac{p}{p_{12}^{F*}}\right)^{\beta}\Phi_{12}^F\left(p_{12}^{F*}\right)$, and if $p > \overline{\overline{\overline{p}}}$, then $\Phi_1^L\left(p\right) > \left(\frac{p}{p_{12}^{F*}}\right)^{\beta}\Phi_{12}^F\left(p_{12}^{F*}\right)$.

****Ranking of*** $\overline{\overline{p}}$ ***relative to*** $\overline{\overline{\overline{p}}}$**:**

Lemma 12 *We have the following results concerning the ranking of the thresholds:*

- $\overline{\overline{p}} > \widetilde{p} \Rightarrow \overline{\overline{\overline{p}}} > \overline{\overline{p}} > \widetilde{p}$,
- $\overline{\overline{p}} < \widetilde{p} \Rightarrow \overline{\overline{\overline{p}}} < \overline{\overline{p}} < \widetilde{p}$.

Proof: Suppose it does not hold: $\overline{\overline{\overline{p}}} < \overline{\overline{p}}$ and $\overline{\overline{p}} > \widetilde{p}$. There exists $p_0 \in \left]\overline{\overline{\overline{p}}}, \overline{\overline{p}}\right[$ such that $p_0 > \widetilde{p}$. Therefore, $\Phi_2^L\left(p_0\right) > \Phi_1^L\left(p_0\right)$ and $\Phi_2^L\left(p_0\right) - \left(\frac{p_0}{p_{12}^{F*}}\right)^{\beta}\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right) > \Phi_1^L\left(p_0\right) -$

$\left(\frac{p_0}{p_{12}^{F*}}\right)^{\beta}\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right)$. The left hand side is strictly negative since $p_0 < \overline{\overline{p}}$ and the right hand side is strictly positive since $p_0 > \overline{\overline{\overline{p}}}$. This leads to a contradiction. Therefore, if $\overline{\overline{p}} > \widetilde{p}$, then $\overline{\overline{\overline{p}}} > \overline{\overline{p}}$. A symmetric proof can be obtained if $\overline{\overline{p}} < \widetilde{p}$. □

According to this lemma, the point now is to compare $\overline{\overline{p}}$ and $\widetilde{p}$.

Ranking of $\overline{\overline{p}}$ relative to $\widetilde{p}$: Recall that $\widetilde{p}$ and $\widetilde{\widetilde{p}}$ are solutions of

$$\frac{2\beta I_1}{\beta-1}\left(\left(\frac{p}{p_{12}^{F*}}\right)^{\beta} - \frac{p}{p_{12}^{F*}}\right) + I_2 - I_1 = 0.$$

Similarly, $\overline{\overline{p}}$ is solution of

$$\frac{4p}{p_{12}^{F*}} - \frac{\beta-1}{\beta}\frac{I_2}{I_1} = \frac{2\beta+1}{\beta}\left(\frac{p}{p_{12}^{F*}}\right)^{\beta}.$$

Therefore, $\overline{\overline{p}} > \widetilde{p}$ if and only if

$$A6 : \widetilde{p} < \frac{2\,(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I1 - I2\right]. \tag{33}$$

Moreover we have the following result:

Lemma 13 *If $I_2 \geq 2I_1$, then $\overline{\overline{p}} < \widetilde{p}$ and $\overline{\overline{\overline{p}}} < \overline{\overline{p}}$. On the contrary, if $I_2 < 2I_1$, we can have $\overline{\overline{p}} < \widetilde{p}$ and $\overline{\overline{\overline{p}}} < \overline{\overline{p}}$ or $\overline{\overline{p}} > \widetilde{p}$ and $\overline{\overline{\overline{p}}} > \overline{\overline{p}}$.*

Proof: Suppose that $I_2 \geq 2I_1$ and $\widetilde{p} < \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I1 - I2\right]$. It follows that

$$\begin{aligned}
\widetilde{p} &< \frac{2\,(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I1 - I2\right], \\
&\leq \frac{2\,(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I1 - 2I1\right] \ \text{(because } I_2 \geq 2I_1\text{)}, \\
&= 2\,(r-\mu)\,I_1.
\end{aligned}$$

But this is in contradiction with Assumption $A2$ according to which $\widetilde{p} > 2\,(r-\mu)\,I_1$. Therefore, we have a contradiction and when $I_2 \geq 2I_1$, $\widetilde{p} > \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I1 - I2\right]$ and $\overline{\overline{p}} < \widetilde{p}$. □

Ranking of $\widetilde{\widetilde{p}}$ relative to p_{12}^{F*}: In order to compare p_{12}^{F*} with $\widetilde{\widetilde{p}}$, we compute:

$$\begin{aligned}
\Phi_1^L\left(p_{12}^{F*}\right) - \Phi_2^L\left(p_{12}^{F*}\right) &= \frac{p_{12}^{F*}}{2\,(r-\mu)} - I_1 - \left(\frac{p_{12}^{F*}}{2\,(r-\mu)} - I_2\right), \\
&= I_2 - I_1, \\
&> 0.
\end{aligned}$$

Therefore, $p_{12}^{F*} > \widetilde{\widetilde{p}}$.

Ranking of p_1^* ***relative to*** $\widetilde{p}$**:** This comparison is necessary in the case where $\widetilde{p} > \overline{\overline{p}}$ (what can happen whenever $I_2 \geq 2I_1$ or $I_2 < 2I_1$).

If $I_2 < 2I_1$, then $p_2^{L*} < p_1^* < p_{12}^{F*}$. Therefore, Assumption $A3$, $\Phi_1^L(p_1^*) < V_2^L(p_1^*)$ reduces to $\Phi_1^L(p_1^*) < \Phi_2^L(p_1^*)$. And this last inequality is equivalent to $p_1^* > \widetilde{p}$.

If $I_2 \geq 2I_1$, then $\beta > 2$ and $\Phi_1^L(p_1^*) - \Phi_2^L(p_1^*) = I_1\left(-\frac{\beta}{\beta-1}\left(1-\left(\frac{1}{2}\right)^{\beta-1}\right)+\frac{I_2}{I_1}-1\right)$. This is negative if and only if

$$A7 : \frac{I_2}{I_1} < 1 + \frac{\beta}{\beta-1}\left(1-\left(\frac{1}{2}\right)^{\beta-1}\right). \tag{34}$$

This inequality is not always satisfied under Assumptions $A1$ and $A3$, therefore, when $I_2 \geq 2I_1$, both cases may happen: $p_1^* > \widetilde{p}$ or $p_1^* < \widetilde{p}$.

Ranking of p_1^* ***relative to*** $\overline{\overline{\overline{p}}}$**:** Let us compute:

$$\begin{aligned} g\left(p_1^{L*}\right) &= \frac{p_1^*}{2(r-\mu)} - I_1 - \left(\frac{p_1^*}{p_{12}^{F*}}\right)^\beta\left(\frac{p_{12}^{F*}}{4(r-\mu)} - I_1\right), \\ &= \frac{I_1}{\beta-1}\left(1-\left(\frac{1}{2}\right)^\beta\right), \\ &> 0. \end{aligned}$$

Therefore, $p_1^* > \overline{\overline{\overline{p}}}$.

Ranking of p_1^* ***relative to*** $\overline{\overline{p}}$**:** This comparison is meaningful only when $\overline{\overline{p}} > \widetilde{p}$ (what can only be the case if $I_2 < 2I_1$). Let us compute:

$$\begin{aligned} \frac{4p_1^*}{p_{12}^{F*}} - \frac{\beta-1}{\beta}\frac{I_2}{I_1} - \frac{2\beta+1}{\beta}\left(\frac{p_1^*}{p_{12}^{F*}}\right)^\beta &= \frac{1}{\beta}\left[2\beta-(\beta-1)\frac{I_2}{I_1}-(2\beta+1)\left(\frac{1}{2}\right)^\beta\right], \\ &\geq \frac{1}{\beta}\left[1-\left(\frac{1}{2}\right)^\beta\right] \text{ (Assumption } A2 \text{ and } I_2 < 2I_1), \\ &> 0 \text{ (because } \beta \geq 1). \end{aligned}$$

Therefore, $\overline{\overline{p}} < p_1^*$.

We have proven that three rankings were possible when $\underline{E}^L$ is empty:

1. $\widetilde{p} < \overline{\overline{p}} < \overline{\overline{\overline{p}}} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$,
2. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < \widetilde{p} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$,
3. $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$. □

The proof of this Lemma highlights the importance of two assumptions

$$A5 : \widetilde{p} < \frac{2\,(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right], \tag{35}$$

that ensures that $\overline{\overline{p}} > \widetilde{p}$ and

$$A6 : \frac{I_2}{I_1} < 1 + \frac{\beta}{\beta-1}\left(1-\left(\frac{1}{2}\right)^{\beta-1}\right), \tag{36}$$

that ensures that $p_1^* > \widetilde{p}$. The following lemma allows to give another expression for Assumption $A5$.

Lemma 14 *Assumption* $A5$ *is equivalent to*

$$\frac{1}{2\beta-1}\left(2\beta+1-\frac{I_2}{I_1}\right) > \frac{1}{\beta-1}\left[\frac{\beta-1}{2\beta\,(2\beta-1)}\left(2\beta+1-\frac{I_2}{I_1}\right)\right]^{\beta} + 1 \tag{37}$$

Proof:

Suppose first that $\widetilde{p} < \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right] < \widetilde{\widetilde{p}}$. This implies that $\Phi_2^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right) > \Phi_1^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right)$. Straightforward computations lead to

$$\frac{1}{2\beta-1}\left(2\beta+1-\frac{I_2}{I_1}\right) > \frac{1}{\beta-1}\left[\frac{\beta-1}{2\beta\,(2\beta-1)}\left(2\beta+1-\frac{I_2}{I_1}\right)\right]^{\beta} + 1.$$

Suppose now that $\widetilde{\widetilde{p}} < \frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]$. This implies that $\Phi_1^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right) > \Phi_2^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right)$ and $\Phi_1'^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right) > \Phi_2'^L\left(\frac{2(r-\mu)}{2\beta-1}\left[(2\beta+1)\,I_1 - I_2\right]\right)$. The first condition leads to

$$\frac{1}{2\beta-1}\left(2\beta+1-\frac{I_2}{I_1}\right) > \frac{1}{\beta-1}\left[\frac{\beta-1}{2\beta\,(2\beta-1)}\left(2\beta+1-\frac{I_2}{I_1}\right)\right]^{\beta} + 1,$$

and the second one to

$$\frac{I_2}{I_1} < 2\beta+1-\frac{2\beta\,(2\beta-1)}{\beta-1}\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$$

The second condition on the first order derivatives implies that $\frac{I_2}{I_1} < 2\beta+1-\frac{2\beta(2\beta-1)}{\beta-1}\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}$ that is not satisfied since the right hand side is negative for the values of β that we consider. Therefore this case cannot happen. $\square$

Moreover, in the case where $I_2 < 2I_1$, Assumption $A6$ is equivalent to Assumption $A3$.

Second case: $\underline{E}_1^L$ is not empty.

In this case, if there is a natural leader, he waits until p_1^* to invest in technology 1. We prove in the Appendix that two rankings are possible.

Lemma 15 *When* $\underline{E}_1^L$ *is not empty, two rankings are possible*

1. *if* $f\left(p_1^*\right) > 0$, *then* $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$,

2. *if* $f\left(p_1^*\right) < 0$, *then* $\overline{\overline{\overline{p}}} < p_1^* < \overline{\overline{p}} < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$.

Proof:

First of all, the proofs of $\widetilde{\widetilde{p}} < p_{12}^{F*}$ and of $\overline{\overline{\overline{p}}} < p_1^*$ does not take into account the emptiness (or not) of $\underline{E}_1^L$. Moreover, as $\underline{E}_1^L$ is not empty and because of the shape of the option value V^L, it is immediate that $p_1^* < \widetilde{p}$. Therefore, $\overline{\overline{\overline{p}}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$. The point is to rank $\overline{\overline{p}}$.

***Ranking of p_1^* relative to $\overline{\overline{p}}$:** It is given by the sign of $f\left(p_1^*\right)$:

$$\begin{aligned} f\left(p_1^*\right) &= \Phi_2^L\left(p_1^*\right) - \left(\frac{p_1^*}{p_{12}^{F*}}\right)^\beta \left(\frac{p_{12}^{F*}}{4\left(r-\mu\right)} - I_1\right), \\ &= \frac{2\beta - \left(1/2\right)^\beta\left(2\beta+1\right)}{\beta - 1} I_1 - I_2, \\ &> \frac{2\beta}{\beta-1} I_1 - I_2, . \end{aligned}$$

The sign of $f\left(p_1^*\right)$ is not clear since $I_2 \geq 2I_1$. All may happen. However, we know that $\overline{\overline{p}} < \widetilde{p}$ (thanks to the result of the previous section). Therefore two rankings are possible when $\underline{E}_1^L$ is not empty (when Assumption $A4$ holds):

- if $f\left(p_1^*\right) > 0$, then $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < p_1^* < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$,
- if $f\left(p_1^*\right) < 0$, then $\overline{\overline{\overline{p}}} < p_1^* < \overline{\overline{p}} < \widetilde{p} < \widetilde{\widetilde{p}} < p_{12}^{F*}$. □

All these steps complete the proof of Lemma 9. □

K Analysis of the equilibria in Case B

This may be the case when $I_2 \geq 2I_1$ or $I_2 < 2I_1$ if and only if Assumptions $A3$ and $A6$ are satisfied and Assumption $A5$ is not satisfied. If $I_2 \geq 2I_1$, then Assumption $A6$ implies Assumption $A3$. If $I_2 < 2I_1$, Assumptions $A3$ and $A6$ are equivalent. If, in addition, $\beta > 2$, Assumption $A3$ is automatically satisfied ($\underline{E}_1^L$ is never empty) and the unique condition is that Assumption $A5$ is not satisfied. This implies that technology 2 is still quite cheap relative to technology 1. In this case, $\overline{\overline{\overline{p}}} < \overline{\overline{p}} < \widetilde{p} < p_1^* < \widetilde{\widetilde{p}} < p_{12}^{F*}$. A very similar analysis than in Case A allows to state the following proposition.

Proposition 6 *There exist two MPE outcomes:*

- *In the first one, nobody invests when $p < \widetilde{p}$. When $p \in [\widetilde{p}, p_1^*[$, each firm invests in technology 2 with an intensity $s\left(p\right)$. When $p \in \left[p_1^*, \widetilde{\widetilde{p}}\right[$, each firm invests in technology 1 with an intensity $t_1\left(p\right)$ and in technology 2 with an intensity $t_2\left(p\right)$. When $p \geq \widetilde{\widetilde{p}}$, each firm simultaneously invests in technology 1.*

- *In the second one, nobody invests when $p < \widetilde{p}$ either. When $p \in [\widetilde{p}, p_1^*[$, each firm invests in technology 2 with an intensity $s(p)$. When $p \geq p_1^*$, each firm simultaneously invests in technology 1.*

On Figure 14, we describe the two types of equilibria.

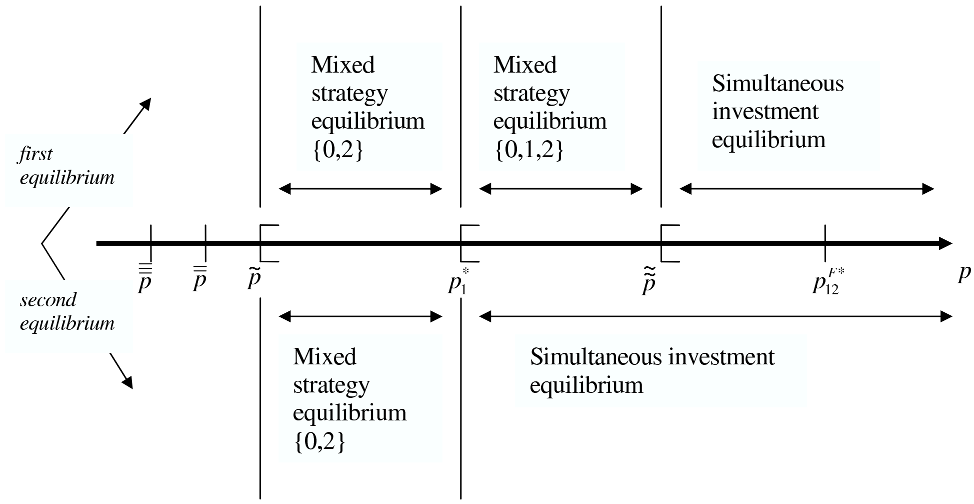

Figure 14: The two equilibria in Case B.

This situation is very close to Case A. The only difference arises from the first investment threshold that is equal to $\widetilde{p}$ instead of $\overline{\overline{p}}$. At $p = \overline{\overline{p}}$, technology 1 is still preferred and therefore investing in technology 2 is not an equilibrium any more. Moreover, nobody will invest in technology 1 at this state of demand since it is less than the investment threshold p_1^*. In fact, when $\overline{\overline{p}} < \widetilde{p} < p_1^*$, technology 2 is more expensive than in the previous situation. Therefore, the region $\left[\widetilde{p}, \widetilde{\widetilde{p}}\right]$ tends to be smaller than in the previous case and preemption is feared as soon as technology 2 becomes profitable (i.e., p crosses $\widetilde{p}$).

L Proof of Proposition 4

We are going to prove this proposition in three steps. We first compute the difference in surplus at $p = p_1^*$, then we evaluate it at $p = \widetilde{\widetilde{p}}$ and finally, in between we evaluate the first derivative.

1. Computation of $S_{11} - S_{210}$ at $p = p_1^*$:

$$\begin{aligned} S_{11} - S_{210} &= \frac{2t_2}{(2-t_2)\,t_2 + t_1^2}\left[\left(\left(\frac{p}{p_{12}^{F*}}\right)^{\beta}(1-t_1-t_2) - (2-t_1-t_2)\right) I_1 + I_2\right] \\ &= 2t_2\left[I_2 - I_1 - \left(1 - \left(\frac{p}{p_{12}F*}\right)^{\beta}\right)(1-t_1-t_2)\, I_1\right]. \end{aligned}$$

   As t_2 is a probability, it is positive, it is therefore sufficient to study the sign of $I_2 - I_1 - \left(1 - \left(\frac{p}{p_{12}F*}\right)^{\beta}\right)(1-t_1-t_2)\, I_1$. And at $p = p_1^*$, it is positive if and only if $\frac{I_2}{I_1} < \frac{2^{-\beta}+2^{2+\beta}-\beta-5}{32^{\beta}-\beta-3}$.

2. Computation of $S_{11} - S_{210}$ at $p = \tilde{\tilde{p}}$:

   Once more, it is sufficient to evaluate $I_2 - I_1 - \left(1 - \left(\frac{p}{p_{12}F*}\right)^{\beta}\right)(1 - t_1 - t_2)\, I_1$ at $p = \tilde{\tilde{p}}$. The difficulty here relies on the fact that we do not have an analytical expression for $\tilde{\tilde{p}}$, therefore we use numerical simulations. On Figure 15, we see that the surplus is positive for every value of $\frac{I_2}{I_1}$ and of β.

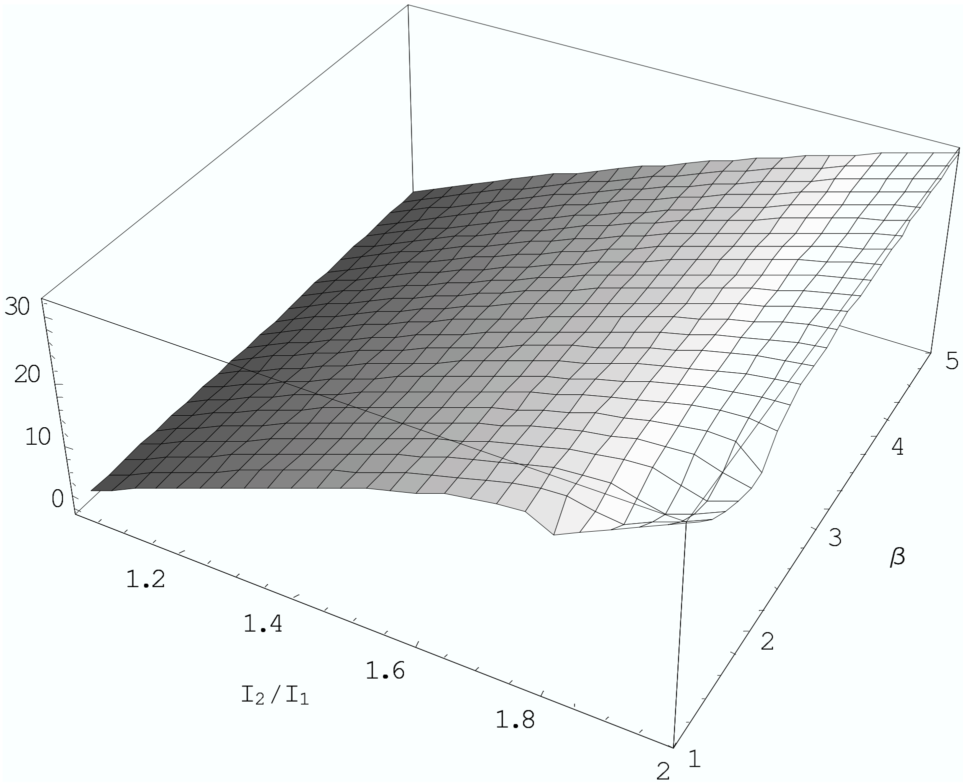

Figure 15: Difference in surplus evaluated et $p = \tilde{\tilde{p}}$.

3. Analysis of the sign of the first derivative:

   The last step is to evaluate the first derivative of the function. Once more, it is very difficult to sign expression, therefore we have simulated this derivative for different values of β on Figure 16.

   The maximum between 0 and the derivative of function $S_{11} - S_{210}$ is represented on Figure 3 for different values of β. We see that for a given I_2/I_1 and a given β, $S_{11} - S_{210}$ is either decreasing and then increasing or increasing and then decreasing as p increases. As at $\tilde{\tilde{p}}$, the difference in surplus is always positive, we just have to check that $S_{11} - S_{210}$ decreases only when it is already negative at $p = p_1^*$. The two boundaries are represented on Figure 17 and we see that this is indeed the case. Thus, we have proves that $S_{11} - S_{210}$ is either negative and then positive or positive as p increases.

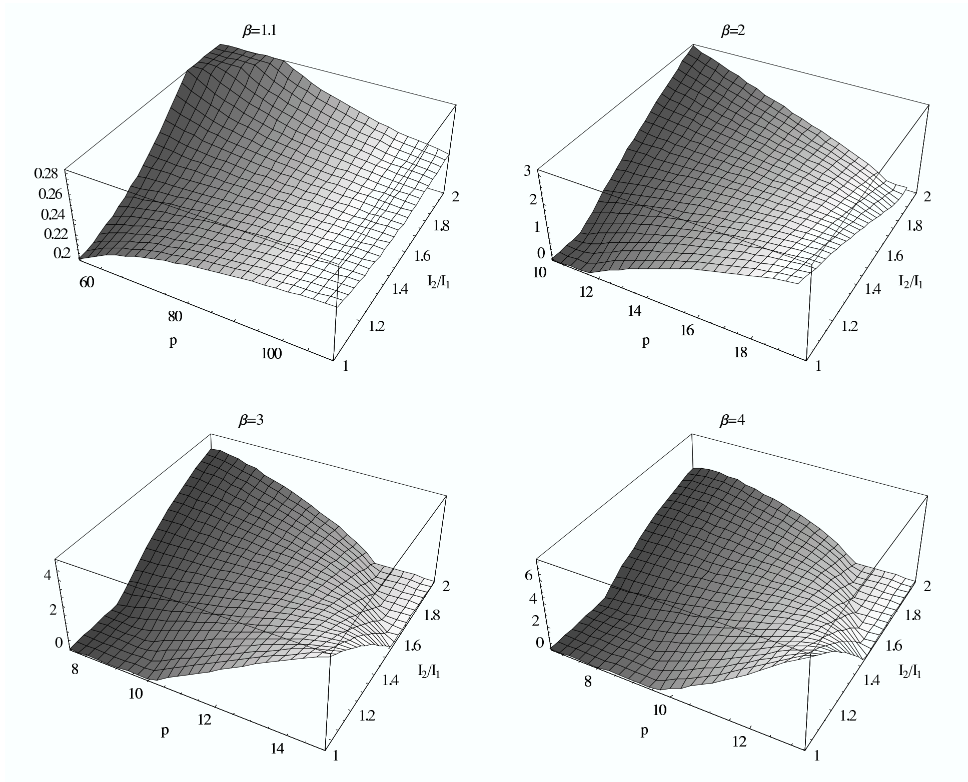

Figure 16: Analysis of the derivative of the difference in surplus for different values of β.

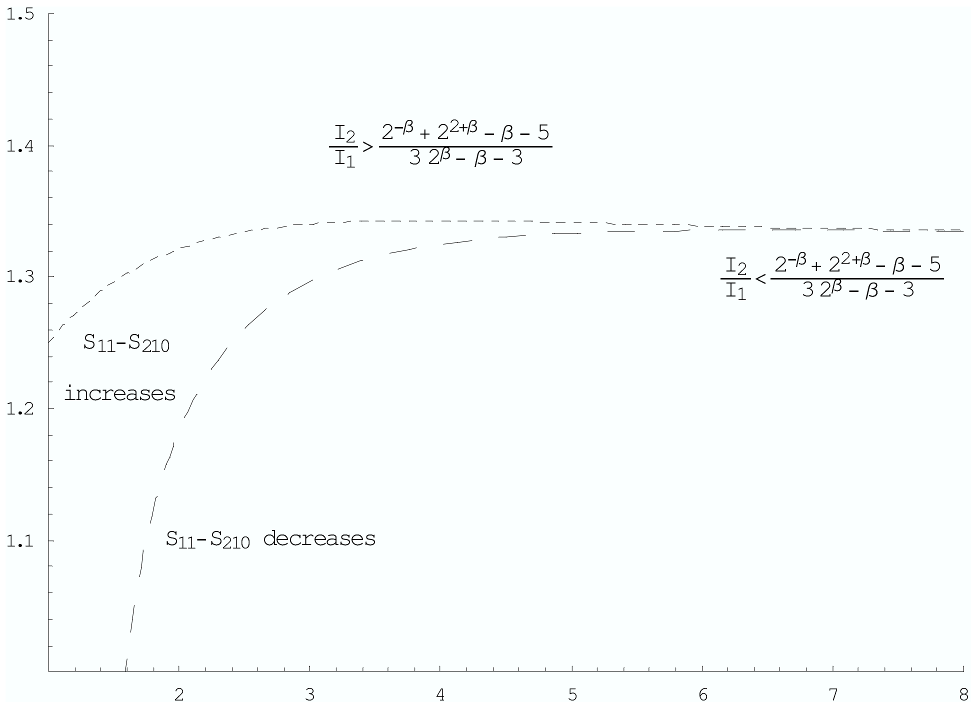

Figure 17: Analysis of the derivative of the difference in surplus for different values of β.