

1 **Estimates of individual fracture compliances along**
2 **boreholes from full-waveform sonic log data**

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9 **Key Points:**

- 10 • Fracture compliance is estimated from arrival time and amplitude differences of
11 the critically refracted P-wave.
12 • The method is applied to FWS data acquired along a borehole penetrating mod-
13 erately fractured crystalline rocks.
14 • The inferred fracture compliance profile highlights the most permeable fractures
15 of the borehole.

16 **Abstract**

17 An important seismic attenuation mechanism in fractured environments is related
 18 to energy conversion into reflected and transmitted waves at fracture interfaces. Using
 19 full-waveform sonic (FWS) log data, we show that it is possible to quantify transmis-
 20 sion losses across a set of fractures from time delays and amplitude differences of the crit-
 21 ically refracted P-wave as compared to intact sections along the borehole. In the pres-
 22 ence of fractures, the transmission coefficient associated with a given fracture is obtained
 23 by combining information on transmission losses from multiple receivers and source po-
 24 sitions into a linear system of equations for all fractures intersecting the borehole. Frac-
 25 ture compliance is computed from the inferred transmission coefficient based on a lin-
 26 ear slip model. For validation, we use synthetic FWS log data obtained from numer-
 27 ical simulations of wave propagation in a water-filled borehole surrounded by low-permeability
 28 rocks with discrete fractures. The methodology is then applied to field data acquired along
 29 boreholes penetrating multiple fractures in a granodioritic host rock. We show that our
 30 estimations of mechanical compliance are consistent with previously reported values, which
 31 were estimated for individual fractures intersecting one of the boreholes with a related
 32 method valid only for isolated single fractures. Comparison between our estimates of frac-
 33 ture compliance and transmissivity profiles from previous hydraulic characterizations of
 34 the fractures suggests that the proposed method may also allow to locate the most per-
 35 meable fractures along a borehole, which, in turn, opens the perspective of enhancing
 36 the design and effectiveness of subsequent hydraulic testing and fracturing experiments.

37 **1 Introduction**

38 Fractures are ubiquitous in upper crustal rocks. As potential fluid pathways, their
 39 presence can strongly influence many geological processes (e.g., earthquakes, volcanic ac-
 40 tivity, hydrothermal flow) as well as associated human activities (e.g., development of
 41 hydrocarbon and geothermal reservoirs, CO₂ and nuclear waste storage, construction of
 42 tunnels and dams) (e.g. Gudmundsson, 2011). For this reason, the use of remote-sensing-
 43 type geophysical techniques for fracture detection and characterization is of great inter-
 44 est. In particular, the well known influence of fractures on the travel times and ampli-
 45 tudes of seismic waves makes associated techniques valuable tools for fracture network
 46 imaging, characterization, and monitoring (e.g. National Research Council, 1996; Liu
 47 & Martinez, 2013). Furthermore, several authors point to the existence of an interde-
 48 pendence between the mechanical (e.g., compliance) and hydraulic (e.g., transmissivity)
 49 properties of fractures, which, in turn, has motivated the development of methods for
 50 predicting the hydraulic response of fractures from seismic signatures (Pyrak-Nolte &
 51 Morris, 2000; Bakku et al., 2013).

52 The seismic signature of fractured rocks is defined by the relation between the char-
 53 acteristic size and spacing of fractures and the prevailing seismic wavelengths. When the
 54 wavelength is much larger than the fractures, the characteristics of seismic wave prop-
 55 agation through the fractured medium are described by an effective stiffness tensor, which,
 56 in general, can be anisotropic (e.g. Gurevich, 2003; Hudson, 1980; Schoenberg & Douma,
 57 1988) with complex-valued and frequency-dependent elements (e.g. Chapman, 2003; Ru-
 58 bino et al., 2013). In this context, methods based on amplitude and/or velocity varia-
 59 tions with offset and/or azimuth are typically used to characterize the effective mechan-
 60 ical behavior of sets of fractures (e.g. Lubbe & Worthington, 2006; Fang et al., 2016; Bar-
 61 bosa, Köpke, et al., 2020). When the wavelength becomes comparable to the distance
 62 between fractures and their characteristic length, fractures are seismically characterized
 63 as discrete features (Schoenberg & Douma, 1988). The detection and analysis of these
 64 macro-fractures is particularly relevant as they tend to dominate the hydraulic behav-
 65 ior of the associated subsurface volume (Bakku et al., 2013). Theoretical and experimen-
 66 tal works have shown that the scattering of seismic wavefields due to the presence of iso-

67 lated fractures can significantly affect their amplitudes and velocities (e.g. Morris et al.,
68 1964; Pyrak-Nolte et al., 1990b; Worthington & Hudson, 2000; Minato & Ghose, 2016),
69 which, in turn, allows for mechanical characterization of the fractures.

70 In spite of the potential of seismic data to provide hydraulic information on frac-
71 tures, field-scale studies providing evidence and insights on this relation are still scarce
72 and, so far, most studies on the hydromechanical behavior of fractures using seismic meth-
73 ods have been performed on numerical samples (e.g., Pyrak-Nolte & Morris, 2000; Kang
74 et al., 2016; Pyrak-Nolte & Nolte, 2016). Arguably, the main reason for this is that the
75 hydromechanical interdependence, which is not known a priori, depends on complex ge-
76 ometrical aspects of the fractures (e.g., spatial correlation of the fracture aperture dis-
77 tribution) that can only be known with sufficient accuracy for synthetic samples. This
78 problem can be potentially alleviated by studying fractures intersecting a borehole as
79 they can be mechanically and hydraulically characterized under well-controlled condi-
80 tions. The hydraulic characterization of fractures typically consists of estimating effective
81 transmissivity, storativity, and/or diffusivity through either conventional transient
82 tests, such as, for example, constant head, constant flow, slug, pulse, and recovery tests
83 (e.g., Dutler et al., 2019; Brixel et al., 2020; Krietsch et al., 2020) or periodic pumping
84 tests (e.g., Y. Cheng & Renner, 2018). On the other hand, various borehole geophys-
85 ical methods are commonly employed for fracture mechanical and geometrical character-
86 ization including, for example, geophysical logging, vertical seismic profiling (VSP), and
87 ground penetrating radar (GPR) surveys (e.g., National Research Council, 1996; Doetsch
88 et al., 2020; Shakas et al., 2020). As an example, following a long-wavelength approach,
89 Prioul et al. (2007) and Prioul and Jocker (2009) quantified the mechanical effects of nat-
90 ural and drilling-induced fractures along a borehole using conventional full-waveform sonic
91 (FWS) data. Using P- and S-wave slownesses, the authors inferred a single effective com-
92 pliance value characteristic of each fracture type. However, even under well-known and
93 well-controlled experimental conditions, conventional interpretation methods often do
94 not allow for estimating the compliances of individual fractures. Thus, a necessary first
95 step to improve our understanding of the hydromechanical behavior of fractures is the
96 development of new seismic methods, which, exploiting the advantages of common bore-
97 hole setups, allow for estimating the compliances of individual fractures.

98 Following this idea, Bakku et al. (2013) presented a method to estimate fracture
99 compliance, aperture, and length using the amplitude ratio of the pressure due to a fracture-
100 related tube wave and the corresponding incident P-wave. They reported compliance es-
101 timations for a single fracture intersecting a borehole using amplitudes recorded during
102 a VSP experiment. Also using VSP data, Hunziker et al. (2020) developed a Bayesian
103 Markov chain Monte Carlo (MCMC) full-waveform inversion algorithm to simultaneously
104 infer hydraulic (i.e., aperture) and mechanical (i.e., compliance) characteristics of indi-
105 vidual fractures from tube wave signals. The authors considered a subsection contain-
106 ing 3 fractures of a VSP dataset acquired along a borehole at the Grimsel Test Site (GTS)
107 in Switzerland and estimated fracture compliance and hydraulic aperture as well as the
108 elastic moduli of the background rock. The results showed that, due to the close spac-
109 ing, the hydraulic apertures of individual fractures could not be determined, which hin-
110 dered a quantitative analysis of their hydromechanical responses. Barbosa et al. (2019)
111 proposed a method to estimate the mechanical compliance of isolated fractures intersec-
112 ting a borehole using P-wave velocity changes and transmission losses inferred from FWS
113 log data. In the given context, transmission losses refer to the conversion of the incident
114 wave into reflected and transmitted waves at a given fracture. Using the spectral ratio
115 and phase difference techniques to estimate attenuation and velocity, respectively, in an
116 interval between two receivers bounding a fracture, they reported compliance estimates
117 for 5 isolated fractures intersecting a borehole at the GTS. Interestingly, the highest com-
118 pliance values were associated with fractures for which strong tube waves were excited
119 in the VSP experiment reported by Hunziker et al. (2020). This correlation was attributed
120 to the expected scaling between hydraulic and mechanical properties of fractures, accord-

121 ing to which hydraulically open fractures tend to be more compliant (Pyrak-Nolte & Nolte,
122 2016).

123 In this work, we propose a new method to estimate fracture compliance profiles along
124 a borehole from FWS data. The method is a generalization of the one of Barbosa et al.
125 (2019) for individual, well-separated fractures. Another important and potentially error-
126 prone aspect of the method of Barbosa et al. (2019) is the need to correct attenuation
127 estimates for mechanisms other than transmission losses, such as, for example, geomet-
128 rical spreading, in order to isolate the effects of fractures. To alleviate this problem, we
129 propose to infer P-wave transmission coefficients associated with the presence of frac-
130 tures from the amplitude ratios and time delays obtained when comparing the critically
131 refracted P-wave signal in fractured and intact sections along the borehole. The assump-
132 tions behind this method are that the amplitude decay due to geometrical spreading is
133 similar in both cases, which implies that the corresponding corrections of the overall am-
134 plitude decay are no longer necessary and that the intact sections provide the necessary
135 information on the embedding background medium of the fractured sections. Moreover,
136 an independent estimate of the transmission coefficient of a set of fractures is obtained
137 for each receiver. Then, by exploiting the corresponding redundancy of transmission co-
138 efficient information obtained from multiple receivers, we relax the single-fracture assump-
139 tion of Barbosa et al. (2019) and estimate effective mechanical compliance for sets of frac-
140 tures intersecting the borehole. The newly proposed methodology is validated using syn-
141 thetic FWS log data representative of a section of an open borehole intersecting multi-
142 ple fractures. We then proceed to apply the method to FWS datasets from two bore-
143 holes at the GTS, which penetrate a granodioritic rock mass intersected by multiple frac-
144 tures. We also compare the estimated compliance profile with corresponding transmis-
145 sivity estimates from hydraulic tests to explore the potential of the former with regard
146 to identifying the most permeable sections along a borehole.

147 2 Methodology

148 For the following, we shall consider a FWS experiment, in which a monopole source
149 creates a pressure perturbation in the center of a water-filled borehole. The associated
150 wavefields consisting mainly of critically refracted P- and S-waves, and Stoneley waves,
151 which are commonly referred to as tube waves (e.g. Toksoz et al., 1983; Haldorsen et al.,
152 2006; Durán et al., 2018), are sampled by pressure sensors along the borehole. In this
153 work, we consider the critically refracted P-wave in general and its amplitude and ve-
154 locity characteristic in intact and fractured rocks in particular.

155 The amplitude spectrum of the critically refracted P-wave can be modelled as (Sun
156 et al., 2000)

$$157 \quad A(\omega, r) = F(\omega, r_s, r)G(\omega, r_s, r) \exp\left(-\frac{\omega}{2}Q_p^{-1}\Delta t_r\right), \quad (1)$$

158 with ω denoting the angular frequency; r_s and r are the depth coordinate of the source
159 and the receiver, respectively; Q_p^{-1} and Δt_r are the effective attenuation (without ge-
160 ometrical spreading contribution) and P-wave travel time between the source and the
161 receiver, respectively; G is the geometrical spreading function; and F is a function ac-
162 counting for source and receiver spectra and corresponding coupling terms, which are
163 associated with signal transmission losses at the borehole wall.

164 2.1 Compliance estimation from transmission losses

165 The mechanical compliance of fractures is estimated from the P-wave transmission
166 coefficient T associated with them, which, in turn, can be expressed as (Barbosa et al.,
167 2019)

$$168 \quad T = e^{i(k_p^b - k_p^{eff})\Delta r}, \quad (2)$$

169 where k_p^b and k_p^{eff} denote the P-wavenumbers of the intact background rock and the frac-
 170 tured section, respectively, and Δr is the propagating distance. Following Barbosa et al.
 171 (2019), the functions G and F in Eq. 1 are expected to be similar for intact and frac-
 172 tured rock. In this scenario, we can approximate the transmission coefficient of the frac-
 173 tured section in Eq. 2 as

$$174 \quad T = e^{-\ln[A^b/A^{eff}] + i\omega\delta t}, \quad (3)$$

175 with $A^b(\omega, r)$ and $A^{eff}(\omega, r)$ being the amplitude spectra of the critically refracted P-
 176 wave after propagating through intact and fractured rock, respectively, and δt the P-wave
 177 arrival time difference between propagation in intact and fractured rock. We compute
 178 δt at the nominal source frequency and use the sign convention for which $\delta t > 0$ in the
 179 presence of compliant fractures. The ratio $\ln[A^b/A^{eff}]$ is a typical indicator of ampli-
 180 tude decay (e.g. C. H. Cheng et al., 1982; Molyneux & Schmitt, 2000; Milani et al., 2015).
 181 An advantage of using Eq. 3 instead of Eq. 2, as in Barbosa et al. (2019), to compute
 182 T is that we avoid the estimation of phase velocity and attenuation corrections due to,
 183 for example, geometrical spreading.

184 For a critically refracted P-wave that propagates across a set of aligned fractures
 185 having individual transmission coefficients t_i , the magnitude of the bulk transmission co-
 186 efficient of the fracture set T given by Eq. 3 can be written as (Pyrak-Nolte et al., 1990a)

$$187 \quad T = \prod_i^N t_i, \quad (4)$$

188 where N is the number of fractures located in the interval between the source and the
 189 receiver, which can be known in advance, for example, from core analyses and/or tele-
 190 viewer data. Note that for each source-receiver position, we obtain a single transmission
 191 coefficient T representative of the set of fractures located between the source and the re-
 192 ceiver. In order to estimate the N individual transmission coefficients t_i , we solve a sys-
 193 tem of equations, in which the number of equations N_T is at most the number of source
 194 locations times the number of receivers

$$195 \quad \begin{cases} a_{1,1} \log(t_1) + a_{1,2} \log(t_2) + \dots + a_{1,N} \log(t_N) = \log(T_1) \\ a_{2,1} \log(t_1) + a_{2,2} \log(t_2) + \dots + a_{2,N} \log(t_N) = \log(T_2) \\ \vdots \\ a_{N_T,1} \log(t_1) + a_{N_T,2} \log(t_2) + \dots + a_{N_T,N} \log(t_N) = \log(T_{N_T}) \end{cases} \quad (5)$$

196 In Eq. 5, the matrix coefficients $a_{i,j}$ are equal to 1 if t_i contributes to T_j and 0 other-
 197 wise. Again, the contribution of the i -th fracture to the overall T_j can be determined from
 198 core analyses and televiewer data for given positions of the source and the receivers. Each
 199 row, or equation, in the linear system of Eq. 5 represents a source-receiver combination
 200 along the borehole for which a transmission coefficient can be computed. That is, a re-
 201 ceiver's position, at which

- 202 1- there is at least one fracture between the source and the receiver;
- 203 2- the refracted P-wave arrives later in fractured than intact sections, as we deal
 204 with fractures that are more compliant than the embedding medium;
- 205 3- the closest fracture below and above the receiver is located at a distance larger
 206 than $3/2$ and $1/2$ of the P-wavelength, respectively (note that we assume that the source
 207 is always below the receivers) to reduce effects of other wave modes (please refer to Sec-
 208 tion 3.1 of the manuscript for more details on this condition);
- 209 4- there are no damage-related features (e.g., S3-type shear zone) between the source
 210 and the receiver except for fractures.

211 The number of rows in the system of Eqs. 5 is thus given by the number of source
 212 locations times the number of receivers meeting the above-mentioned conditions. Finally,
 213 by solving the linear system of Eqs. 5, we can thus estimate the individual t_i values for
 214 all the observed fractures.

Table 1. Physical properties of background rock, fractures, and fluid.

Physical property	Background	Fracture 1	Fracture 2	Fracture 3
Dry frame bulk modulus K_m [GPa]	33	0.056	0.056	0.056
Dry frame shear modulus μ_m [GPa]	29	0.033	0.033	0.033
Porosity ϕ [-]	0.004	0.1	0.5	0.75
Permeability κ [mD]	0.5	100	100	100
Solid grain bulk modulus K_s [GPa]	37	37	37	37
Solid grain density ρ_s [Kg/m ³]	2730	2730	2730	2730
Fluid bulk modulus K_f [GPa]	2.25	2.25	2.25	2.25
Fluid density ρ_f [Kg/m ³]	1000	1000	1000	1000
Fluid viscosity η [Pa s]	0.001	0.001	0.001	0.001
Aperture h [mm]	–	5	5	10

215 Lastly, we assume a linear slip model to represent the seismic effect of fractures,
 216 which are conceptualized as planes of weakness that produce a seismic displacement jump
 217 proportional to their mechanical compliance. In this scenario, the compliance of a frac-
 218 ture Z_{N_i} can be estimated from the transmission coefficient t_i as (Schoenberg, 1980)

$$219 \quad Z_{N_i} = \frac{(1 - t_i)}{it_i} \frac{2}{\omega I_b}, \quad (6)$$

220 where $I_b = \rho_b v_p^b$ is the impedance of the intact background rock. Note that Eq. 6 is
 221 strictly valid for normal P-wave incidence, in which case Z_{N_i} corresponds to the so-called
 222 normal compliance of the fracture.

223 Barbosa, Caspari, et al. (2020) compared the results of the method given by Eqs.
 224 3 to 6 with those obtained using the methodology presented in Barbosa et al. (2019) to
 225 show that both methods provide similar results for the case of an isolated single frac-
 226 ture. In the following, we apply the newly proposed method to synthetic and observed
 227 FWS data acquired across multiple, closely spaced fractures.

228 3 Results

229 3.1 Synthetic data set

230 To validate the proposed methodology of fracture compliance estimation, we re-
 231 produce a FWS experiment under open borehole conditions by performing numerical sim-
 232 ulations of seismic wave propagation based on Biot's (1962) dynamic equations for a ro-
 233 tationally symmetric medium (Sidler et al., 2013b, 2013a). In Fig. 1, we show examples
 234 of the numerically recorded pressure fields for an intact (Fig. 1a) and a fractured for-
 235 mation (Fig. 1b). We have considered a single source position at 0.5 m depth and, for
 236 illustration purposes, pressure recordings are sampled every 5 mm along the borehole.
 237 In general, however, pressure sensors in a FWS tool are spaced at ~ 1 ft intervals. The
 238 nominal source frequency is $f = \omega/2\pi = 20$ kHz. The chosen material properties rep-
 239 resenting a low-porosity crystalline background rock and the highly compliant, porous,
 240 and permeable fractures are given in Table 1. The solid grain and fluid properties are
 241 assumed to be the same for the fractures and the embedding background (Table 1). Fig.
 242 1b clearly shows that, when a critically refracted P-wave travelling along the borehole
 243 hits a fracture, it creates tube waves and other reflected and transmitted wave modes,
 244 which, in turn, manifests itself in a more complex pressure field compared to the non-
 245 fractured medium (Fig. 1a).

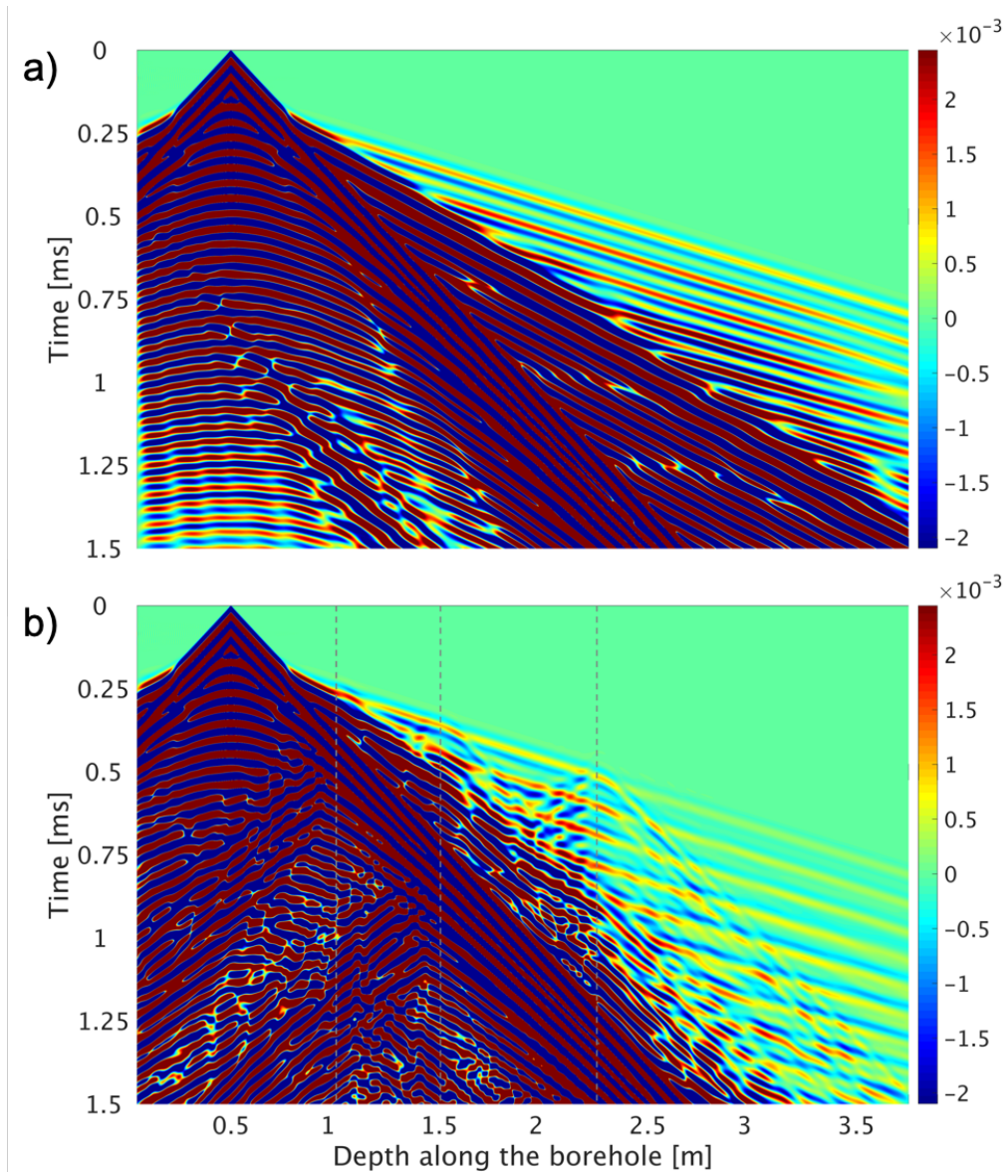


Figure 1. Numerical FWS-type simulation of fluid pressure recordings at the center of a numerical water-filled borehole as functions of time and depth. The source is located at 0.5 m depth and emits a compact pulse with a nominal center frequency of 20 kHz. The borehole is surrounded by (a) intact and (b) fractured crystalline rocks. Vertical dashed lines in (b) illustrate the location of the fractures. For illustration purposes, the traces are normalized with respect to the overall maximum pressure recorded in each experiment.

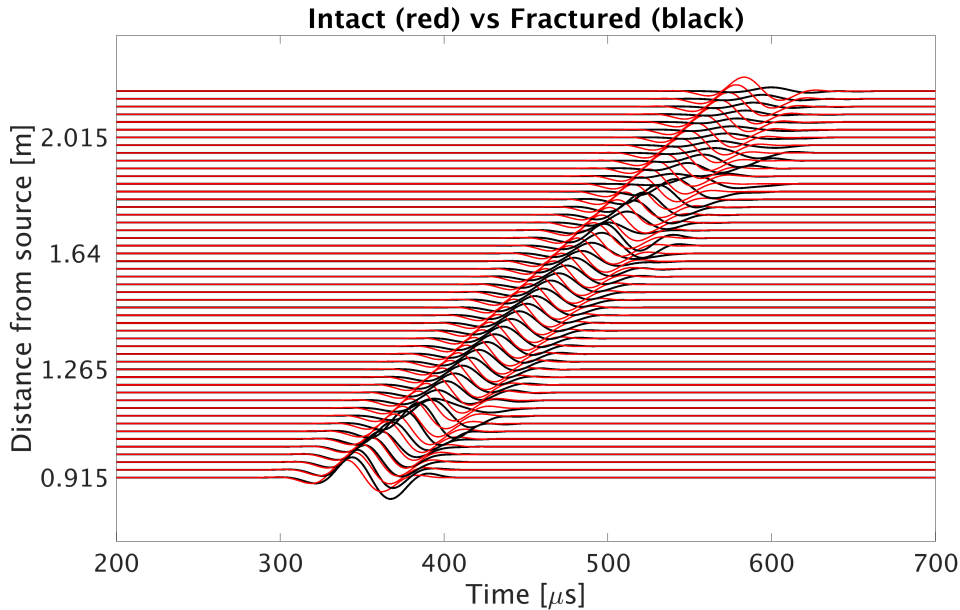


Figure 2. First-arriving P-wave extracted from the fluid pressure synthetic traces shown in Fig. 1. Red and black lines correspond to the experiments on intact and fractured rock, respectively. Depth increment between consecutive traces is 2.5 cm. The distances of the fractures from the source are 0.5 m, 1 m, and 1.75 m. Note that the amplitude and arrival time differences observed for the first traces are mainly associated with the fracture located at 0.5 m from the source, which is not visible in the considered depth range of 3 ft to 7 ft from the source.

246 Fig. 2 illustrates in detail the changes in the waveform of the first-arriving P-wave.
 247 We applied a time-windowing processing to isolate the first-arriving P-wave signal from
 248 the recordings shown in Fig. 1 as in Barbosa et al. (2019). In the case of an intact for-
 249 mation, we observe a smooth amplitude decay as the receiver distance to the source in-
 250 creases (red traces in Fig. 2), which is due to geometrical spreading. The black traces
 251 in Fig. 2, which correspond to a formation with fractures located at distances of 0.5 m,
 252 1 m, and 1.75 m from the source, exhibit a more complex behavior, particularly those
 253 in close proximity to the fractures. The significant amplitude decays and time delays ob-
 254 served for longer offsets in the fractured case, relative to the intact rock scenario, are mainly
 255 associated with transmission losses across the fractures. We recall that by quantifying
 256 the time delays and amplitude decays, such as those observed in Fig. 2, we can estimate
 257 the associated transmission coefficient (Eq. 3) and ultimately the fracture compliance
 258 (Eq. 6).

259 Fig. 3 shows the transmission coefficients computed using Eq. 3 for 5 different source
 260 positions. The first source position corresponds to the case shown in Figs. 1 and 2 (Pos1
 261 in Fig. 3), while the consecutive cases (Pos2 to Pos5) are obtained by a stepwise increase
 262 of the source depth by 30 cm. The first to fourth columns in Fig. 3 show $\ln[A^b/A^{eff}]$,
 263 δt , and the absolute value and phase of T (Eq. 3), respectively. Note that for the syn-
 264 thetic case, we can use very small receiver spacings and apply the method to each re-
 265 ceiver. The receiver spacing considered in Fig. 3 is 2.5 cm, which is much smaller than
 266 the typical spacings of the order of 1 ft or ~ 30 cm in FWS tools. The reason for this choice
 267 is to better illustrate the behavior of the numerical estimates as a function of the dis-
 268 tance between receivers and fractures. We observe that the estimated quantities exhibit
 269 a fluctuating behavior in the proximity of the fractures, which are denoted by solid red,

270 green, and magenta lines in Fig. 3. This is related to the extraction of the first-arriving
 271 P-wave from the recorded traces (Fig. 2). Close to the fractures, the amplitude of the
 272 first-arriving P-wave is directly affected by other scattered wavefields at the fracture (e.g.,
 273 tube waves), which, in turn, has a negative impact on the estimation of the transmis-
 274 sion coefficient. Similar effects have also been described by Barbosa et al. (2019), who
 275 demonstrated that, a larger distance between the receivers and the fracture results in
 276 more robust fracture compliance estimates. We have found that when the receiver is lo-
 277 cated at distances of at least 1/2 and 3/2 times the dominant wavelength from a deeper
 278 or a shallower fracture, respectively, the fluctuations of the estimated values decrease sig-
 279 nificantly. These noisy receiver offsets with regard to the fractures are denoted in Fig.
 280 3 by grey areas. Following this criterion, we only use information from receivers outside
 281 the grey areas for the estimation of fracture compliance. Furthermore, in order to pro-
 282 duce results that are comparable to a real data case, the numerical dataset used to solve
 283 Eq. 5 is composed only of the records of 5 receivers with offsets to the source ranging
 284 from 3 ft to 7 ft, whose positions are denoted by red circles in Fig. 3.

285 To assess the validity of the estimated transmission coefficients, we show in Fig.
 286 3 the analytical approximation of the magnitude and phase of the transmission coeffi-
 287 cient obtained using Eq. 4 (black dashed line). To this end, we first compute the indi-
 288 vidual fracture compliances Z_{N_i} using its common definition for very thin layers (Schoenberg,
 289 1980), which is given by the ratio between the aperture h and the undrained P-wave mod-
 290 ulus of the material filling the fracture. The latter is computed following Gassmann (1951).
 291 Using the Z_{N_i} values and Eq. 6, we compute the individual t_i coefficients that enter into
 292 Eq. 4 to obtain the bulk transmission coefficient of the set of fractures T . We observe
 293 that, in general, the numerical results are consistent with the analytical prediction out-
 294 side the grey areas. In spite of this overall good agreement, there are some discrepan-
 295 cies between the analytical and numerical transmission coefficients, which can be due to:
 296 (i) errors associated with the extraction of the first arriving P-wave (Eq. 1) for the in-
 297 tact and fractured cases (Fig. 1); (ii) small differences in the functions F and G for in-
 298 tact and fractured formations (relevant for the derivation of Eq. 3); (iii) the analytical
 299 approximation of the transmission losses, which, for example, does not account for the
 300 hydraulic communication between the fractures and the borehole fluid that allows for
 301 the tube wave generation when the incident wave compresses the fracture; (iv) the lim-
 302 itations of Eq. 4 to reproduce the effective transmission coefficient T across multiple frac-
 303 tures (e.g., in the presence of fracture interaction effects); and (v) the use of the linear
 304 slip model to estimate the seismic response of relatively thick fractures at sonic frequen-
 305 cies (for numerical reasons, the apertures of the fractures range from 5 mm to 1 cm, which
 306 can be considered as an extreme scenario of realistic fracture apertures). We therefore
 307 consider the analytical transmission coefficient T as a guideline, rather than an exact ref-
 308 erence, for the assessment of numerical results.

309 We consider the numerically estimated transmission coefficient T_j for each receiver
 310 at the 5 different source positions shown in Fig. 3 and then use Eq. 5 to find the com-
 311 pliance values Z_{N_i} for the three fractures present in the numerical model. As mentioned
 312 before, signals from receivers located in the grey areas of Fig. 3 are not considered in
 313 the analysis. Fig. 4 shows a comparison between the analytical approximation of the com-
 314 pliance of the fractures and the ones obtained with our method applied to numerical FWS
 315 data. Note that, in Fig. 4, we plot the real component of the numerically estimated com-
 316 pliance values whereas the analytical ones are real-valued. We refer to the latter as $Z_N(\text{HiF})$
 317 as it corresponds to the high-frequency (HiF) limit of the poroelastic response of a sin-
 318 gle infinitely long fracture (Barbosa et al., 2017). Overall, the agreement is very good
 319 for the three fractures, not only with regard to the absolute values, but also in terms of
 320 the general trend of the values. This suggests that the differences in transmission coef-
 321 ficients observed in Fig. 3 can be considered to have a relatively small impact on the com-
 322 pliance estimates. The difference between numerical and analytical results is most sig-
 323 nificant for the stiffest fracture located at 1 m depth (overestimated by $\sim 40\%$) and de-

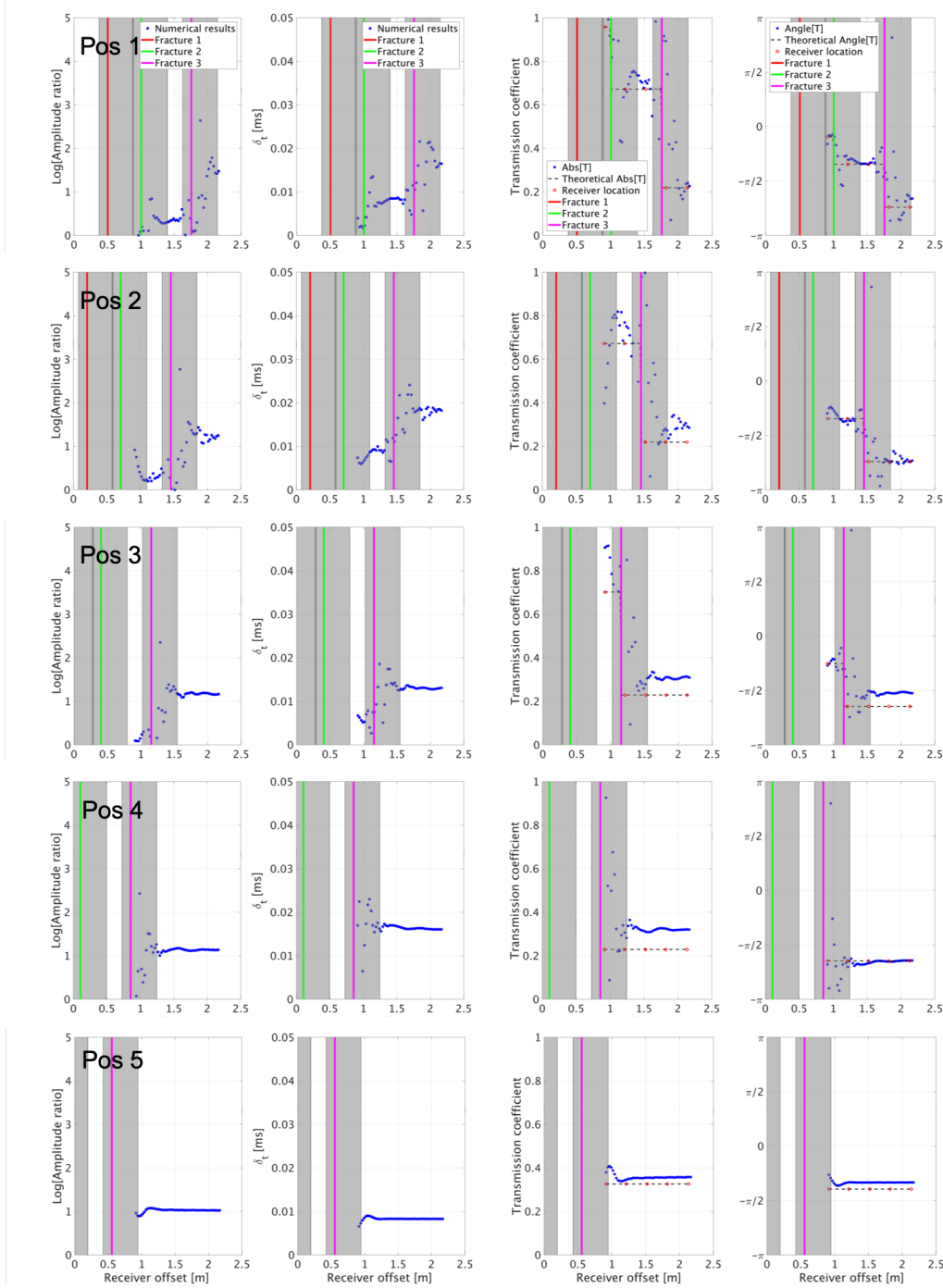


Figure 3. Numerically obtained $\ln[A^b/A^{eff}]$, δt , $|T|$, and phase of T (blue dots) for 5 source positions across a fractured formation. Fracture locations are denoted by the red, green, and magenta vertical lines, which are surrounded by grey areas characterizing by the presence of noisy traces. The magnitude and phase of the theoretical transmission coefficient (Eq. 4) is denoted by the black dashed line. Red circles illustrate the position of 5 receivers located from 3 ft to 7 ft away from the source.

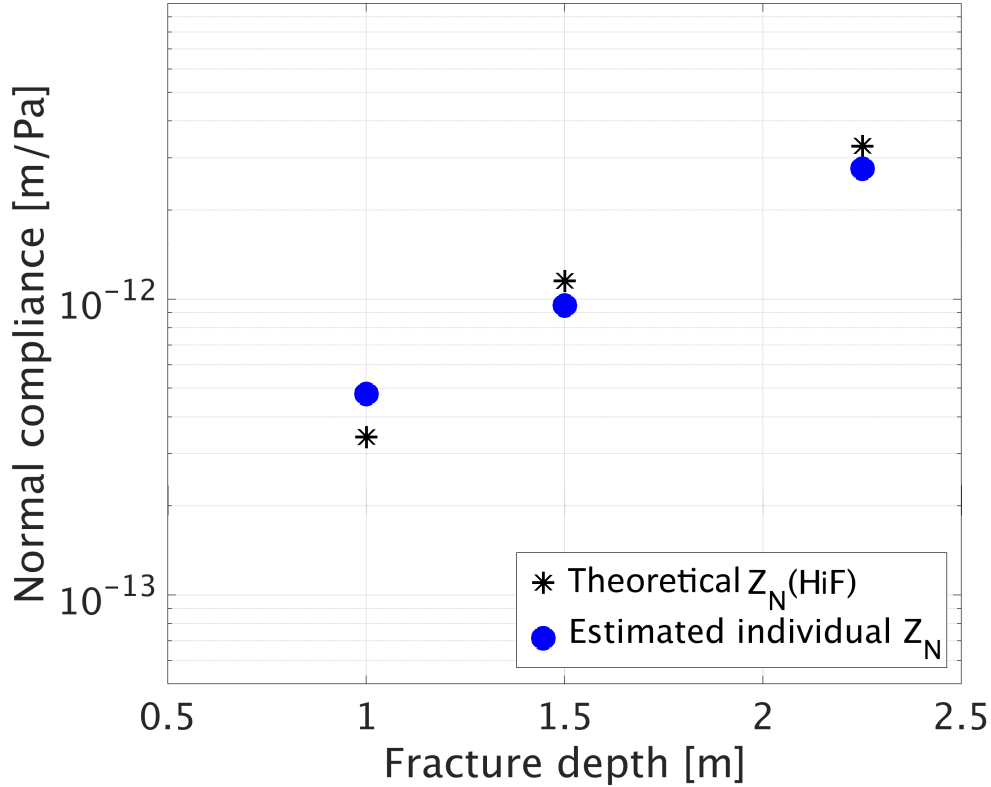


Figure 4. Numerical (dots) and analytical (stars) estimations of the normal compliance of the three fractures present in the numerical model.

324 creases for the more compliant fractures ($\sim 16\%$). Note that, given that the grey zones
 325 of the fractures located at 1 m and 1.5 m depth overlap, it is not possible to obtain in-
 326 dependent information about the shallower fracture. In spite of this, the overdetermined
 327 nature of Eqs. 5, and the fact that the other two fractures can be observed independently,
 328 allows us to get reasonable compliance estimates for all the fractures. Although not shown
 329 here, we have also computed the imaginary components of the numerically inferred Z_N
 330 values. We found that the imaginary components are approximately one order-of-magnitude
 331 smaller than real components. That is, they are not negligible as predicted by the real-
 332 valued analytical solution $Z_N(\text{HiF})$. The reconciliation of the differences between the an-
 333 alytical and numerical compliance estimates follows the same reasoning as the one out-
 334 lined above for the transmission coefficients (Fig. 3). In the following, we only show the
 335 real component of the estimated compliance.

336 Given that the FWS field data considered in this work are characterized by con-
 337 secutive source positions separated by ~ 2 ft, it is important to also show a case in which
 338 individual compliances cannot be obtained with our method. This occurs when two or
 339 more fractures cannot be observed independently in spite of the multiple source and re-
 340 ceiver positions considered. Fig. 5 provides an example of such a scenario, in which the
 341 same three fractures described in Table 1 are located at distances of 0.45 m, 0.5 m, and
 342 0.55 m from the source (Figs. 5a and b). If consecutive source positions always include
 343 the three fractures between the source and the receivers, then it is not possible to ob-
 344 tain individual transmission coefficients t_i using the estimated T_j -values. That is, Eq.
 345 5 becomes underdetermined. As a consequence, for this subset of fractures, our method

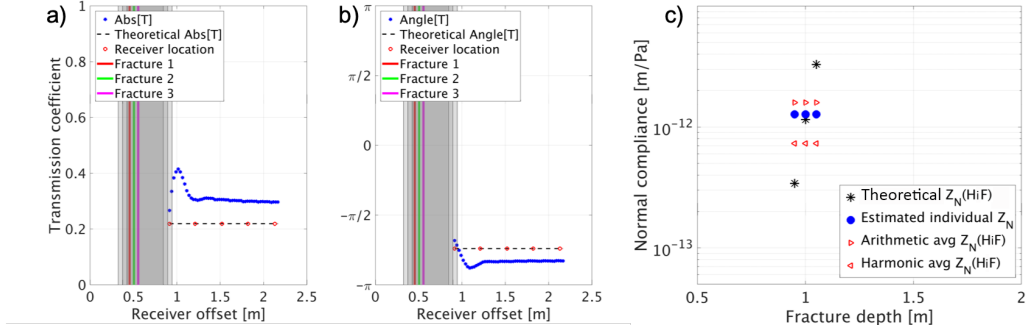


Figure 5. Numerically estimated (a) absolute value and (b) phase of the transmission coefficient T associated with the set of closely spaced fractures. Fracture locations are denoted by the red, green, and magenta vertical lines. (c) Numerically estimated (dots) and analytical (stars) compliance values for the three fractures. Triangles denote the arithmetic and harmonic averages of the analytical individual compliances.

346 only provides an estimate of the effective compliance. We therefore assume that all frac-
 347 tures in the subset are associated with the same transmission coefficient t_i to solve the
 348 system of Eqs. 5. Fig. 5c shows the result of this procedure, in which a single effective
 349 compliance value representing the overall effect of the fracture set is obtained. Note that
 350 this estimate falls between the arithmetic and harmonic averages of the individual compli-
 351 ance values.

352 3.2 Field data set

353 We acquired FWS logs in two boreholes at the GTS, which were drilled as part of
 354 a decameter-scale, in situ stimulation and circulation (ISC) experiment (e.g., Amann et
 355 al., 2018). The two boreholes considered here were dedicated to stimulation-type injec-
 356 tions and are referred to as INJ1 and INJ2 (Fig. 6). They penetrate moderately frac-
 357 tured crystalline rocks dissected by major shear zones, which are classified as being of
 358 S1- and S3-type in Fig. 6 (Wenning et al., 2018; Krietsch et al., 2018; Brixel et al., 2020;
 359 Doetsch et al., 2020). The S3-type shear zones are associated with lamprophyre dikes.
 360 The host rock, referred to as Grimsel granodiorite, exhibits strong foliation due to aligned
 361 grains of biotite as well as bands of mylonite (Majer et al., 1990) and shows no signs of
 362 pervasive weathering. The more recent stages of brittle deformation are manifested by
 363 the presence of macroscopic fractures as well as micro-fractures.

364 The FWS data were acquired using a MSI 2SAA-1000-F modular multi-frequency
 365 sonic logging tool comprising a monopole source separated 91.4 cm (3 ft) from an array
 366 of five receivers spaced at 30.48 cm (1 ft) intervals. Note that this configuration is simi-
 367 lar to the one illustrated by the red circles in Fig. 3. We considered nominal source fre-
 368 quencies of 15 kHz and 25 kHz. An optimal signal-to-noise ratio for the data was achieved
 369 by performing multiple static measurements, in which 15 traces were stacked at each sta-
 370 tionary tool position. The temporal sampling rate was 4 μ s.

371 A summary of the source positions, at which static measurements were acquired
 372 at the two boreholes, is given in Table 2. We have followed the characterization of the
 373 features intersecting the borehole (i. e., location, orientation, spacing, and width) per-
 374 formed by Krietsch et al. (2018) and Dutler et al. (2019) using acoustic (ATV) and opti-
 375 cal televiewer (OTV) logs. In particular, we use this information to locate (i) the frac-

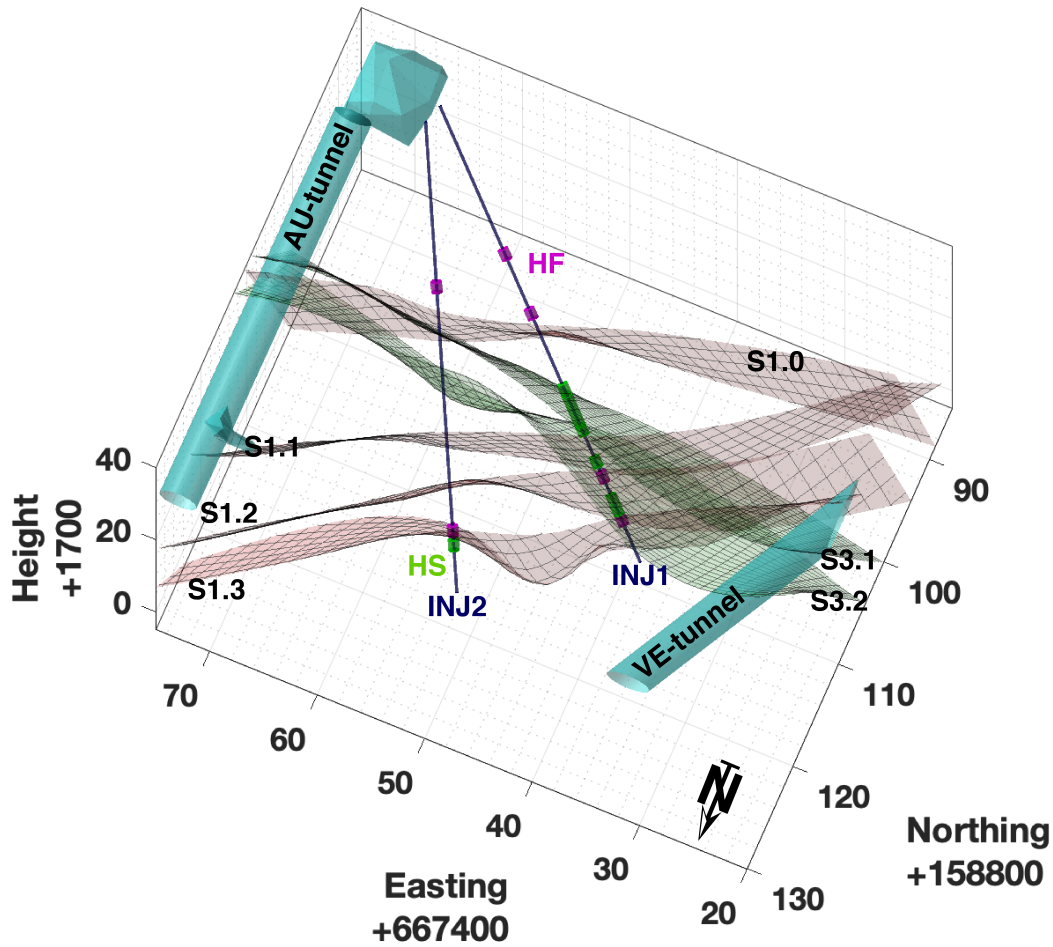


Figure 6. Locations of the considered boreholes INJ1 and INJ2 at the GTS. The two boreholes intersect six major sub-vertical ductile shear zones. Four of these shear zones follow a NE-SW strike, classified as being of S1-type, and two an E-W strike, classified as being of S3-type. Between the two S3-type shear zones the rock mass is highly fractured. The parts of the boreholes used in the hydraulic stimulation and circulation experiments performed prior to our measurements are illustrated as magenta and green intervals, respectively. HF and HS refers to hydrofracturing and hydroshearing experiments, respectively.

376 tures for which compliance will be estimated and (ii) the intact zones of the boreholes
 377 from where the reference signal is obtained (Brixel et al., 2020).

378 Figs. 7 and 8 show examples of the extracted first-arriving P wave at the 5 receivers
 379 and for 4 different source positions along INJ1 and INJ2, respectively. Red dashed curves
 380 illustrate the reference signals that are used to compute the time delays and amplitude
 381 ratios (Eq. 3). For each receiver, the reference signals are obtained by averaging the signals
 382 recorded in those positions where no fractures or other visible heterogeneities were
 383 observed in the OTV and ATV images between the source and the corresponding receiver.
 384 Note that we assign a single reference signal to the intact sections of the borehole fol-

Table 2. Source positions along boreholes INJ1 and INJ2.

	INJ1	INJ2
Source depth range [m]	23.77–42.67	3.04–43.28
Spatial sampling rate [m]	0.60	0.60

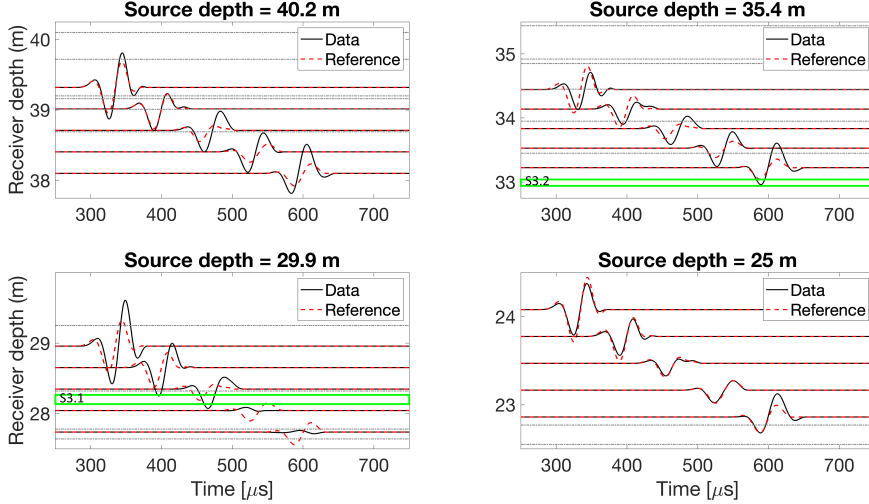


Figure 7. Examples of extracted P-wave first-arrivals at 5 receivers for 4 different source positions along the INJ1 borehole. Red dashed traces correspond to the reference signals used to compute transmission coefficients (Eq. 3). The nominal source frequency of the FWS source is 25 kHz. Grey lines and green rectangles denote fracture and S3-type shear zone locations, respectively.

385 lowing the results of Barbosa et al. (2019), which showed that P-wave velocities in intact
 386 sections of INJ2 exhibit little variation. When velocities measured in intact rock exhibit
 387 strong fluctuation along the borehole, the reference signal should be extracted as close
 388 as possible to the fractured section in which fracture-related time delays and amplitude
 389 decays are estimated. Moreover, our method assumes that scattering effects are exclusively
 390 related to the presence of fractures and not to abrupt changes in lithological facies along
 391 the borehole. For this reason, sections containing S3-type of shear zones have been removed
 392 from the analysis. Figs. 7 and 8 illustrate the significant impact of the S3-type shear zones
 393 on both the amplitude and arrival time of the recorded signal.

394 Using the extracted P-wave first-arrivals (Figs. 7 and 8), we applied the method
 395 given by Eqs. 3 to 6 to obtain the fracture compliance estimates along INJ1 and INJ2,
 396 respectively. It is important to remark that only signals recorded beyond the noisy receiver
 397 offset affected by other scattered wavefields in the vicinity of the fractures (as illustrated
 398 in Fig. 3 by grey areas) are considered for compliance estimation. In the following, we provide
 399 a discussion of the results and their implications.

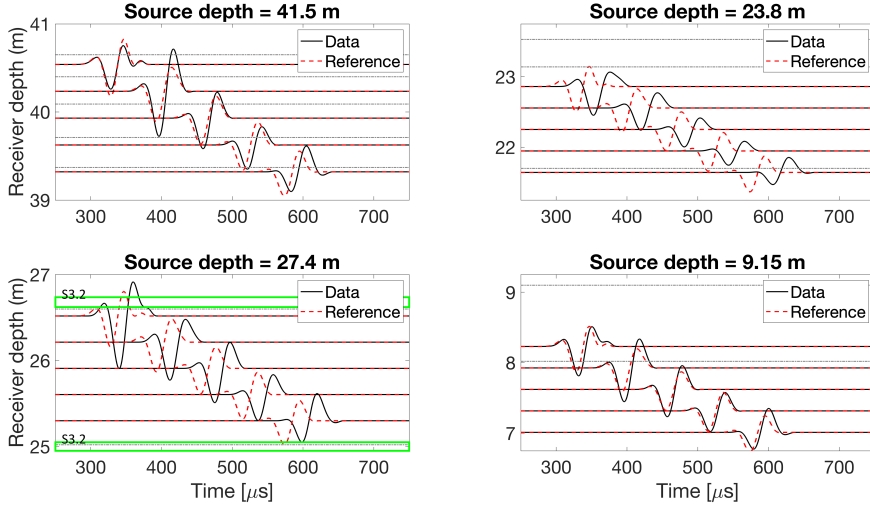


Figure 8. Examples of extracted P-wave first-arrivals at 5 receivers for 4 different source positions along the INJ2 borehole. Red dashed traces correspond to the reference signals used to compute transmission coefficients (Eq. 3). The nominal source frequency of the FWS source is 25 kHz. Grey lines and green rectangles denote fracture and S3-type shear zone locations, respectively.

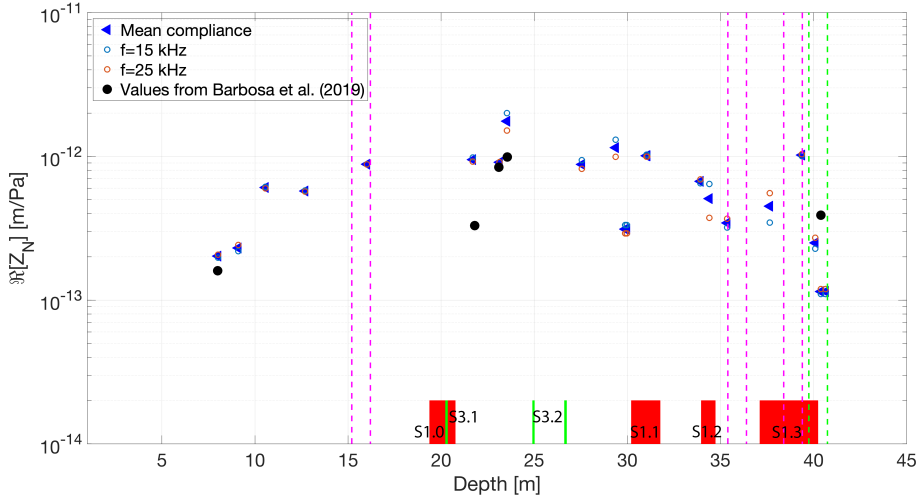


Figure 9. Real component of the compliance of fractures intersecting the INJ2 borehole. Blue triangles and black circles denote the compliance estimates from this work and from Barbosa et al. (2019), respectively, which both correspond to the arithmetic mean of values obtained for 15 kHz (blue circle) and 25 kHz (red circle). Red and green markers denote S1- and S3-type shear zones, respectively. Magenta and green dashed lines denote borehole intervals where hydrofracturing and hydroshearing experiments, respectively, were carried out after the measurements of Barbosa et al. (2019)

400 **3.2.1 Relation with previous estimates**

401 Let us first consider Fig. 9, which presents results for INJ2 (blue triangles). For
 402 comparison, we plot the results obtained by Barbosa et al. (2019) for 5 isolated fractures

403 intersecting the same borehole (black circles), which, in the following, are referred to as
 404 the old estimates. Both sets of compliance estimates correspond to the arithmetic mean
 405 of the values obtained for nominal source center frequencies of 15 kHz and 25 kHz, which
 406 are denoted by blue and red circles in Fig. 9, respectively. Overall, the agreement be-
 407 tween the old and new estimates is good in terms of magnitude and depth variation. We
 408 observe a general increase in fracture compliance towards the S3-type shear zones, as de-
 409 noted by green markers in Fig. 9, located at ~ 20 m to ~ 27 m borehole depth. As men-
 410 tioned before, an advantage of the new method over the one presented in Barbosa et al.
 411 (2019) is that fractures do not need to be isolated, which allows us to obtain a profile
 412 of fracture compliance along the borehole depth.

413 In spite of the overall consistency, discrepancies in the magnitude of old and new
 414 compliance estimates are noticeable for some fractures (e.g., fractures at 21.8 m and 40.4 m
 415 depth). The methodological aspects that contribute to the observed differences include
 416 the following:

417 - With the old method, the estimated attenuation has to be corrected for intrin-
 418 sic background attenuation and geometrical spreading in order to obtain the transmis-
 419 sion coefficient associated with a given fracture. The latter implies assuming or deter-
 420 mining the function describing the amplitude decay of head waves travelling along a bore-
 421 hole (Barbosa et al., 2019). With the new method, these corrections are assumed to be
 422 the same for intact and fractured sections, thus, simplifying the procedure to obtain the
 423 transmission coefficient.

424 - The old method relies on comparing the signals recorded at two receivers. Barbosa
 425 et al. (2019) used a tool containing 3 receivers spaced at 1 ft intervals, but only the es-
 426 timates related to the first and last receivers, spaced at 2 ft, were considered to estimate
 427 compliance. With the new method, a reference signal, representing the propagation across
 428 intact background rock, is compared with the signals recorded at intervals containing frac-
 429 tures. In the new data set, the tool consisted of 5 receivers. As a consequence, for each
 430 source position, up to five signals were used to constrain fracture compliance as opposed
 431 to only one with the old method.

432 - The new method combines information about the transmission coefficient asso-
 433 ciated with different fractures or sets of fractures to provide individual fracture compli-
 434 ance estimates. Correspondingly, the overall solution of the system of Eqs. 5 is deter-
 435 mined through a least squares procedure. With the old method, one collects informa-
 436 tion on the individual fractures only, which, in turn, is expected to increase its accuracy
 437 as compared with a least squares solution for a set of fractures. Indeed, although not shown
 438 for brevity, using numerical simulations, we found that when the new method is applied
 439 in a single fracture scenario, compliance estimates are slightly closer to the analytical
 440 values than for sets of fractures. It is important to mention, however, that part of this
 441 behavior could be associated to the correctness of the analytical approximation of the
 442 fracture compliance (as discussed in Section 3.1).

443 Finally, it is worth mentioning that the time elapsed between the old and new mea-
 444 surements is approximately 2 years. During this period, several hydraulic shearing (HS)
 445 and hydraulic fracturing (HF) experiments were carried out in INJ1 and INJ2 as part
 446 of the ISC project (Dutler et al., 2019; Krietsch et al., 2020). The location of these ex-
 447 periments is denoted by the magenta (HF) and green (HS) dashed lines in Fig. 9. For
 448 this reason, although the FWS signals used in both methods are similar, the environ-
 449 ment through which they propagate may have undergone some changes. These changes
 450 could be particularly relevant for the data acquired around ~ 40 m depth. This region
 451 was affected by HS experiments (Krietsch et al., 2020) as well as by steeply inclined hy-
 452 drofractures created by the later experiments of Dutler et al. (2019) and where our re-
 453 sults exhibit the largest discrepancies between the old and new estimates.

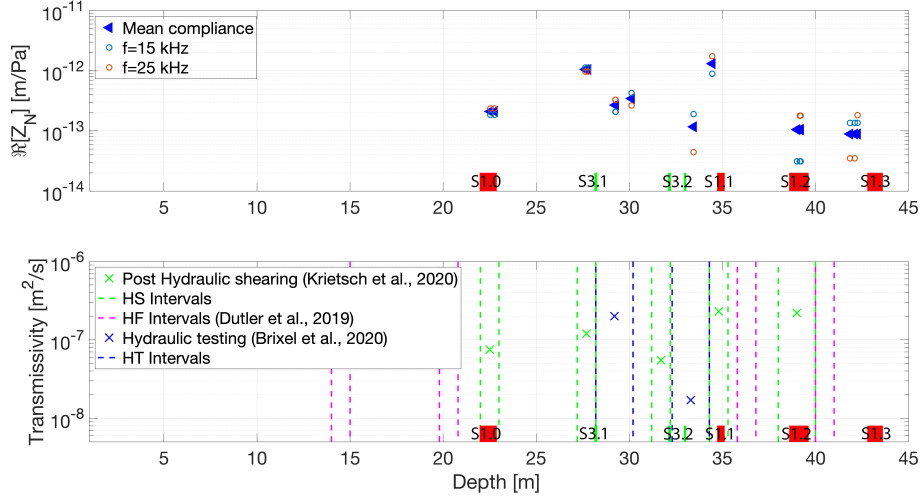


Figure 10. a) Real component of the compliance of fractures intersecting the INJ1 borehole (blue triangles). Values correspond to the arithmetic mean of values obtained for 15 kHz (blue circle) and 25 kHz (red circle). Red and green markers denote the depths of S1 and S3 shear zones, respectively. b) Green and blue crosses denote transmissivity values obtained by Krietsch et al. (2020) and Brixel et al. (2020), respectively. Blue, magenta, and green dashed lines denote borehole intervals used in the experiments of Brixel et al. (2020), Dutler et al. (2019), and Krietsch et al. (2020), respectively.

454 **3.2.2 Relation between hydraulic and mechanical fracture behavior**

455 Fig. 10a shows the compliance estimates for the INJ1 borehole. The depth range
 456 of the data set is shorter than for INJ2 (Table 2) and mainly covers the S1-type (red dots)
 457 and S3-type (green dots) shear zones. It is important to mention that, for this borehole,
 458 there are no previous compliance estimates. However, transmissivity estimates from hy-
 459 draulic tests have been reported (Krietsch et al., 2020; Brixel et al., 2020), which, in the
 460 context of this work, are useful to qualitatively analyze the hydromechanical coupling
 461 of the fractures. The hydraulic tests performed by Brixel et al. (2020) determined the
 462 transmissivity of both the non-fractured and fractured sections to explore the hydraulic
 463 behavior of the shear zones as a function of fracture density. The transmissivity values
 464 obtained for intervals containing fractures range from 10^{-12} m²/s to 10^{-6} m²/s. Here,
 465 we consider only transmissivity values obtained for short testing intervals of 1 to 2 m length,
 466 referred to as high-resolution tests, for which the length of the support volume sampled
 467 by the hydraulic test remained shorter than 5 m, as inferred from the lack of response
 468 detected in adjacent observation boreholes. Like this, both transmissivity and compli-
 469 ance values are mainly representative of the immediate vicinity of the borehole. In ad-
 470 dition, we do not show values obtained from intervals that could have been affected by
 471 the HF or HS experiments of Krietsch et al. (2020) and Dutler et al. (2019), which were
 472 performed after the measurements of Brixel et al. (2020).

473 Overall, we observe that the fracture compliance estimates tend to follow the trans-
 474 missivity profile (Fig. 10), that is, higher transmissivities are, in general, associated with
 475 more compliant fractures. Interestingly, fracture compliance estimates from FWS data
 476 are generally associated with the unrelaxed or high-frequency limit, in which there is no
 477 hydraulic communication between the fracture and the embedding background. Thus,
 478 normal-to-the-fracture permeability has limited impact on the estimated compliance. More-

479 over, it has been shown that the scattering of seismic waves from fractures is also not
 480 directly influenced by the permeability along the fracture in the case of transversely isotropic
 481 fractures (e.g., Nakagawa & Schoenberg, 2007). Lastly, the good agreement between the
 482 estimated compliance values and the analytical solution for planar thin-layers shown in
 483 Section 3.1 imply that the impact of tube wave energy conversion at the fracture, which
 484 can be related to the permeability of fractures (e.g., Bakku et al., 2013), on the estimated
 485 compliance is small. Thus, while transmission losses across fractures are highly depen-
 486 dent on fracture compliance, this theoretical evidence points to a lack of sensitivity with
 487 regard to fracture permeability. Nevertheless, as pointed out by several authors (e.g., Zim-
 488 merman & Main, 2004; Pyrak-Nolte & Nolte, 2016), the internal structure of fractures
 489 controls both their capacity to let fluid to flow and their deformation upon loading. This,
 490 in turn, is expected to manifest itself as an interdependence between the mechanical and
 491 hydraulic behavior of fractures. The observed correlation between the hydraulic response
 492 and the compliance of fractures in Fig. 10 provide further evidence to such hydromechan-
 493 ical coupling of fractures and suggests that compliance profiles inferred from FWS data
 494 could be used for identifying the most permeable sections along a borehole.

495 A closer inspection of the results shown in Fig. 10 indicates that the fractures in
 496 the interval between 38 m and 40 m depth seem to be less compliant than expected. This
 497 could be related to the large number of fractures present in the interval. Indeed, this in-
 498 terval contains 5 fractures according to the OTV and ATV images. However, after ap-
 499 plying the criterion for the minimum receiver offset to the fractures, and due to the pro-
 500 ximity between some of the fractures, we could only get an effective compliance estimate
 501 for a subset of three fractures from the data and the other two fractures could not be
 502 characterized at all (Fig. 10a). Furthermore, Dutler et al. (2019) performed HF exper-
 503 iments (denoted by the magenta dashed lines in Fig. 10b), which resulted in long hydrofrac-
 504 tures quasi-parallel to the borehole axis that intersect most of the fractures of this in-
 505 terval. Thus, the unexpected hydromechanical relation could also be associated with the
 506 effects of the hydraulic connectivity between the pre-existing fractures and the newly-
 507 created hydrofractures.

508 It is important to mention that, even though we only show two values from the work
 509 of Brixel et al. (2020), they also found the highest transmissivity values next to S3.1 and
 510 S3.2 shear zones, for which volumetric fracture density is the largest. Furthermore, Brixel
 511 et al. (2020) observed either no correlation or a weakly negative correlation between trans-
 512 missivity and fracture density in the interval between the S1.1 and S1.2 fracture zones,
 513 which was attributed to asymmetrical variations in fracture density next to discrete faults.
 514 Interestingly, this interval of weak correlation also exhibits an unexpected hydromechan-
 515 ical correlation as the one we described above (Fig. 10).

516 Brixel et al. (2020) also provide transmissivity estimates from high-resolution hy-
 517 draulic tests in INJ2. However, only three of the corresponding borehole intervals were
 518 not potentially affected by the tests performed by Dutler et al. (2019); Krietsch et al.
 519 (2020). Two of these intervals correspond to intervals for which we did not obtained com-
 520 pliance values. For this reason, we do not show a hydromechanical comparison for INJ2.
 521 Nevertheless, Brixel et al. (2020) mentioned that no pressure pulse could be induced for
 522 the fracture located at a distance of ~ 1.5 m from the S3.2 fracture zone (Fig. 9). This
 523 is an indication of the presence of a highly permeable fracture, for which they could only
 524 assign a lower-bound transmissivity value of 10^{-5} m²/s based on the upper limit of the
 525 test apparatus. Fig. 9 shows that this highly transmissive fracture is associated with the
 526 highest fracture compliance observed along INJ2.

527 *3.2.3 Relation between INJ1 and INJ2 estimates*

528 Finally, by comparing Figs. 9 and 10a, we observe that the two sets of compliance
 529 estimates are consistent with each other. Compliance values across the S1- and S3-type

530 shear zones mainly oscillate between $1\text{e-}13$ m/Pa and $1\text{e-}12$ m/Pa. More importantly,
 531 the overall variation of the compliance across the shear zones is similar, with increasing
 532 values towards the S3.1 shear zone in both boreholes. Similarly, compliance estimates
 533 tend to increase close to the S1.1 shear zone, which intersects INJ1 and INJ2 at ~ 35 m
 534 and ~ 31 m depth, respectively.

535 From the analysis of transmissivity estimates across the different shear zones in-
 536 tersecting INJ1 and INJ2, Brixel et al. (2020) pointed out that fracture permeability tends
 537 to increase from the host rock to the core of the shear zones. This behaviour was attributed
 538 to possible differences in aperture, length, orientation, and degree of interconnectivity
 539 of fractures in different regions of the borehole. Overall, our compliance estimates re-
 540 produce the same trend for both boreholes, showing lower compliance estimates towards
 541 the intact host rock.

542 Wenning et al. (2018) describe the S3-type shear zones as “mature” brittle faults
 543 with a pronounced damage zone (DZ) and altered fault core composition. As a conse-
 544 quence, lower velocities (associated with the presence of compliant fractures in the DZ)
 545 and higher transmissivities were predicted by Wenning et al. (2018) in the vicinity of S3-
 546 type shear zones. Our compliance estimates are consistent with this conceptual model
 547 in both boreholes. Doetsch et al. (2020) provided a seismic characterization of the same
 548 rock volume prior to hydraulic stimulations and found that the S3-type shear zones are
 549 characterized by reduced seismic wave velocities with respect to the host rock. They ar-
 550 gue that a higher fracture density could be the controlling factor associated with the ve-
 551 locity decrease. Our results show that decreasing velocities could not only be associated
 552 with variations in fracture density but also in fracture compliance, which was found to
 553 vary by an order-of-magnitude along a ~ 30 m borehole section. This further suggests
 554 that compliance estimates from FWS data can provide valuable insight prior to hydraulic
 555 stimulation experiments. Doetsch et al. (2020) also pointed out that the elevated frac-
 556 ture density at ~ 43 m depth in INJ1 does not manifest itself with a clear decrease in seis-
 557 mic velocity. Here, the lack of correlation between high fracture density and low veloc-
 558 ity zones in the 3D seismic volume obtained by Doetsch et al. (2020), could be explained
 559 by the low compliance exhibited by the fractures close to the S1.3 shear zone.

560 Finally, it is important to remark that Eq. 6 is strictly valid only when consider-
 561 ing the transmission coefficient at normal P-wave incidence. Barbosa et al. (2019) has
 562 shown that when fractures are not normal to the borehole axis, an underestimation of
 563 the transmission coefficient and, consequently, an overestimation of the fracture compli-
 564 ance is expected. Corresponding errors are expected to be negligible except for high an-
 565 gle fractures (greater than 60°) acutely intersecting the borehole. OTV and ATV im-
 566 ages reported in previous works for INJ1 and INJ2 boreholes (e.g., Dutler et al., 2019;
 567 Barbosa et al., 2019; Krietsch et al., 2020) showed that most fractures are not perpen-
 568 dicular to the axis of the borehole. Furthermore, the orientations of INJ1 and INJ2 ren-
 569 der the intersection angles between the fractures and the borehole axes different for both
 570 boreholes. In spite of this, estimates from the two boreholes are remarkably consistent,
 571 which, in turn, implies that the corresponding errors are not contributing significantly
 572 to the observed compliance variation along the boreholes.

573 4 Discussion

574 Unlike the method of Barbosa et al. (2019) for single fractures, the newly proposed
 575 approach allows for characterizing a set of fractures located in the interval between the
 576 source and the receivers of the sonic tool. This makes it particularly useful when deal-
 577 ing with conventional non-static FWS data from heavily fractured borehole sections. In
 578 the context of FWS experiments, wavelengths are expected to be smaller than the length
 579 of the fractures but can be comparable or larger than the distance between adjacent frac-
 580 tures. Thus, the proposed methodology can be considered as a hybrid between effective

581 medium (e.g., Prioul et al., 2007; Prioul & Jocker, 2009) and discrete approaches (e.g.,
 582 Barbosa et al., 2019). In some cases, the separation between fractures allows for a dis-
 583 crete treatment, in which individual fracture compliances are estimated, whereas in other
 584 situations, the fractures are so close together that their joint effects can only be treated
 585 in an effective way. This relates to the system of equations solved to find the transmis-
 586 sion coefficients associated with each fracture (Eqs. 5). When the system of equations
 587 relating the observed transmission coefficient with the individual coefficients of each frac-
 588 ture becomes overdetermined, we can obtain an independent compliance value for each
 589 fracture intersecting the borehole. In some cases, the system of equations becomes un-
 590 derdetermined because a subset of two or more fractures are always observed together.
 591 In this case, fractures are assumed to have the same compliance and an effective com-
 592 pliance for the subset is obtained. Using numerical modelling, we have shown that this
 593 effective compliance lies between the arithmetic and harmonic averages of their true com-
 594 pliances.

595 When comparing the two cases analyzed in Section 3.1, we observe that the dif-
 596 ferences between the analytical and numerical estimates of the transmission coefficients
 597 are similar, regardless of the distance between fractures. One important point of reduc-
 598 ing the distance between fractures is that, in general, the compliance of one fracture, and
 599 the corresponding effects on wave propagation, can be affected by the elastic interaction
 600 with adjacent fractures, which, in turn, could impact on the estimated transmission losses.
 601 In this sense, Cai and Zhao (2000) studied wave propagation across multiple parallel frac-
 602 tures and analyzed the effects of interactions between multiple wave reflections and trans-
 603 missions on the Eq. 4. They argue that Eq. 4 may not be applicable if the effects of mul-
 604 tiple reflections are significant due to the close spacing of the fractures. The limit of va-
 605 lidity proposed by the authors is given by the ratio between the fracture spacing and the
 606 prevailing wavelength. When this ratio is much lower than 1, the effective transmission
 607 coefficient depends on the distance between fractures and Eq. 4 is not strictly valid any-
 608 more. For the smallest spacing considered in this work, the ratio is ~ 0.2 , for which we
 609 still obtain expected values for the real component of Z_N (Fig. 5). This means that any
 610 prevailing fracture interaction effects do not affect significantly the real component of
 611 our compliance estimates.

612 It is important to note that the proposed methodology provides frequency-dependent
 613 and complex-valued fracture compliances. Complex-valued fracture compliance can oc-
 614 cur in fluid-saturated environments, for example, when the so-called wave-induced fluid
 615 flow (WIFF) between the fracture and the embedding background produces sufficient
 616 seismic energy dissipation. For a low-permeability background and at sonic frequencies,
 617 however, WIFF effects are expected to be negligible. Although the imaginary part may
 618 still be non-negligible, we have not analyzed this component in detail as there are no an-
 619 alytical solutions for the frequency-dependent compliance of a fracture intersecting a bore-
 620 hole. A first-order approximation corresponds to the thin-layer model considered in this
 621 work. In this case, the imaginary component is expected to be negligible for the petro-
 622 physical properties considered. Conversely, our results showed that the imaginary com-
 623 ponent is one order-of-magnitude smaller than the real component but it is not negli-
 624 gible even in a single-fracture scenario. This implies that the imaginary component of
 625 the compliance can be more affected by the assumptions of our method, as described in
 626 the analysis of Fig. 3. In this context, it is interesting to note that, the impact of elas-
 627 tic stress interaction due to the presence of multiple fractures on the imaginary compo-
 628 nent of the compliance remains unexplored.

629 5 Conclusions

630 We have presented a new methodology for the quasi-continuous estimation of frac-
 631 ture compliance along a borehole based on the time delays and amplitude decays experi-
 632 enced by the critically refracted P-wave. By quantifying them, we can compute the trans-

633 mission coefficient associated with a given individual fracture, which is then used to es-
 634 timate fracture compliance. We have validated the method using numerical simulations,
 635 for which the estimated fracture compliances were found to be in good agreement with
 636 corresponding analytical approximations.

637 We then applied the method to FWS data acquired along two boreholes penetrat-
 638 ing moderately fractured granodioritic rock. The required reference values were estimated
 639 from the sections of the borehole that did not exhibit any visible fractures or other me-
 640 chanical damage in OTV and ATV images. Our estimates of mechanical normal com-
 641 pliance are consistent with previously reported values for the same site. Interestingly,
 642 even though the estimated compliance values are representative of the vicinity of the bore-
 643 hole, compliance profiles from two boreholes intersecting the same shear zones exhibit
 644 comparable values. Finally, comparison between the compliance profiles obtained from
 645 FWS log data and transmissivity values from hydraulic experiments suggests that the
 646 former can be a valuable tool for identifying the most permeable fractures of a borehole.

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