Supplementary material
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Unbalanced selection: the challenge of maintaining a social polymorphism when a supergene is selfish

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Appendix: Table of parameters

Symbol	Meaning
i,j,k	Individual social alleles carried by the parents
a,b	Individual social alleles carried by the offspring
X_{ij}	Frequency of queens with *ij* genotype (X'_{ij} in next generation)
Y_k	Frequency of males with *k* genotype (Y'_k in next generation)
$O^{♀}_{ab\|ij\times k}$	Production rate of surviving *ab* queen offspring from *ij* queen and *k* male parents
$O^{♂}_{a\|ij}$	Production rate of surviving *a* male offspring from *ij* queens
$R_{ij\times k}$	Frequency of crosses between queens of genotype *ij* and males of genotype *k*
$V^{♀}_{ab\|ij\times k}$	Viability of *ab* queen offspring from *ij* queen and *k* male parents
$V^{♂}_{a\|ij}$	Viability of male offspring with genotype *a* produced by *ij* queens
$f_{ij\times k}$	Offspring fertility of an *ij* queen and *k* male parents
$\alpha_{ij\times k}$	Fixed relative-preference of an *ij* queen for *k* males
m_i	Frequency of assortative mating where queens with social type *i* mate exclusively with *i* males
$\sum_{ij,k}$	Sum over all possible mating pairs
$\sum_{ab,ij,k}$	Sum over all possible mating pairs and queen offspring
$\sum_{ij}$	Sum over all queens
$\sum_{a,ij}$	Sum over all queens and male offspring

Summary of model

This model follows the dynamics of two non-recombining (supergene) regions, M and P.

M represents the monogynous allele and P represents the polygynous allele. Females (queens and workers) are diploid: MM, MP, PP, and males are haploid: M and P. The MM females and M males live in monogynous colonies with one queen and the MP and PP females and P males live in polygynous colonies with multiple queens. Monogynous and Polygynous colonies are found in close proximity within the same populations.

Queen generation: census -> mating -> queen production (Mendelian segregation) -> selection (based on parent and offspring genotype)

Male generation: census -> male production (Mendelian segregation) -> selection (based on mother and offspring genotype)

Frequency of queens is pi, where i is:
1 = MM
2 = MP
3 = PP

Frequency of males is hj, where j is:
1 = M
2 = P

We will account for the selection, brood size and viability of queens produced by queen and male mating pairs, by Vqijk, where i, j, and k are:

i is the queen genotype in the mating pair
j is the male genotype in the mating pair
k is the queen genotype produced by mating pair, where k is:

1 = MM
2 = MP
3 = PP

We will account for viability of males produced by a queen by Vmij, where i and j are:
i is the queen genotype
j is the male offspring genotype

We will account for the probability of mating between a queen and a male by the function: Rij[pi, hj]' where i and j are:
i is the queen genotype in the mating pair
j is the male genotype in the mating pair
Notice that the input to the mating function will be the frequency of each mate.
For example R12[p1, h2] is the probability that an MM queen mates with a MP queen, where the input of the function is the frequency of MM queens and frequency of MP queens.

Recursions and conditions

Queens and males mate.
Mated queens produce next generation queens and males.

Recursion equations for frequency of each genotype in the next generation:

```
In[•]:= assumptions = {1 ≥ Vm11 > 0, 1 ≥ Vm21 > 0, 1 ≥ Vm22 > 0, 1 ≥ Vm32 > 0,
    1 ≥ Vq111 > 0, 1 ≥ Vq211 > 0, 1 ≥ Vq122 > 0, 1 ≥ Vq212 > 0, 1 ≥ Vq222 > 0,
    1 ≥ Vq312 > 0, 1 ≥ Vq223 > 0, 1 ≥ Vq323 > 0, 1 ≥ p1 ≥ 0, 1 ≥ p2 ≥ 0, 1 ≥ h1 ≥ 0,
    1 ≥ f11 > 0, 1 ≥ f12 > 0, 1 ≥ f21 > 0, 1 ≥ f22 > 0, 1 ≥ f31 > 0, 1 ≥ f32 > 0};
d1n = R11[p1, h1] Vq111 + R21[p2, h1] Vq211 1/2;
d2n =
  R12[p1, h2] Vq122 + R21[p2, h1] Vq212 1/2 + R22[p2, h2] Vq222 1/2 + R31[p3, h1] Vq312;
d3n = R22[p2, h2] Vq223 1/2 + R32[p3, h2] Vq323;
b1n = p1 Vm11 + p2 Vm21 1/2;
b2n = p3 Vm32 + p2 Vm22 1/2;

dT = d1n + d2n + d3n;
bT = b1n + b2n;
```

Frequency set for queen genotypes:

In[•]:=
```
recursionsF = {d1n/dT, d2n/dT, d3n/dT} /. p3 → 1 - p1 - p2 /. h2 → 1 - h1 // Factor
```

Out[•]=
```
{(-2 h1 p1 Vq111 - 2 m1 p1 Vq111 + 2 h1 m1 p1 Vq111 - h1 p2 Vq211 + h1 m2 p2 Vq211) /
  (-2 h1 p1 Vq111 - 2 m1 p1 Vq111 + 2 h1 m1 p1 Vq111 - 2 p1 Vq122 + 2 h1 p1 Vq122 +
    2 m1 p1 Vq122 - 2 h1 m1 p1 Vq122 - h1 p2 Vq211 + h1 m2 p2 Vq211 - h1 p2 Vq212 +
    h1 m2 p2 Vq212 - p2 Vq222 + h1 p2 Vq222 - h1 m2 p2 Vq222 - p2 Vq223 + h1 p2 Vq223 -
    h1 m2 p2 Vq223 - 2 h1 Vq312 + 2 h1 m2 Vq312 + 2 h1 p1 Vq312 - 2 h1 m2 p1 Vq312 +
    2 h1 p2 Vq312 - 2 h1 m2 p2 Vq312 - 2 Vq323 + 2 h1 Vq323 - 2 h1 m2 Vq323 + 2 p1 Vq323 -
    2 h1 p1 Vq323 + 2 h1 m2 p1 Vq323 + 2 p2 Vq323 - 2 h1 p2 Vq323 + 2 h1 m2 p2 Vq323),
 (2 p1 Vq122 - 2 h1 p1 Vq122 - 2 m1 p1 Vq122 + 2 h1 m1 p1 Vq122 + h1 p2 Vq212 -
    h1 m2 p2 Vq212 + p2 Vq222 - h1 p2 Vq222 + h1 m2 p2 Vq222 + 2 h1 Vq312 -
    2 h1 m2 Vq312 - 2 h1 p1 Vq312 + 2 h1 m2 p1 Vq312 - 2 h1 p2 Vq312 + 2 h1 m2 p2 Vq312) /
  (2 h1 p1 Vq111 + 2 m1 p1 Vq111 - 2 h1 m1 p1 Vq111 + 2 p1 Vq122 - 2 h1 p1 Vq122 -
    2 m1 p1 Vq122 + 2 h1 m1 p1 Vq122 + h1 p2 Vq211 - h1 m2 p2 Vq211 + h1 p2 Vq212 -
    h1 m2 p2 Vq212 + p2 Vq222 - h1 p2 Vq222 + h1 m2 p2 Vq222 + p2 Vq223 - h1 p2 Vq223 +
    h1 m2 p2 Vq223 + 2 h1 Vq312 - 2 h1 m2 Vq312 - 2 h1 p1 Vq312 + 2 h1 m2 p1 Vq312 -
    2 h1 p2 Vq312 + 2 h1 m2 p2 Vq312 + 2 Vq323 - 2 h1 Vq323 + 2 h1 m2 Vq323 - 2 p1 Vq323 +
    2 h1 p1 Vq323 - 2 h1 m2 p1 Vq323 - 2 p2 Vq323 + 2 h1 p2 Vq323 - 2 h1 m2 p2 Vq323),
 ((1 - h1 + h1 m2) (p2 Vq223 + 2 Vq323 - 2 p1 Vq323 - 2 p2 Vq323)) /
  (2 h1 p1 Vq111 + 2 m1 p1 Vq111 - 2 h1 m1 p1 Vq111 + 2 p1 Vq122 - 2 h1 p1 Vq122 -
    2 m1 p1 Vq122 + 2 h1 m1 p1 Vq122 + h1 p2 Vq211 - h1 m2 p2 Vq211 + h1 p2 Vq212 -
    h1 m2 p2 Vq212 + p2 Vq222 - h1 p2 Vq222 + h1 m2 p2 Vq222 + p2 Vq223 - h1 p2 Vq223 +
    h1 m2 p2 Vq223 + 2 h1 Vq312 - 2 h1 m2 Vq312 - 2 h1 p1 Vq312 + 2 h1 m2 p1 Vq312 -
    2 h1 p2 Vq312 + 2 h1 m2 p2 Vq312 + 2 Vq323 - 2 h1 Vq323 + 2 h1 m2 Vq323 - 2 p1 Vq323 +
    2 h1 p1 Vq323 - 2 h1 m2 p1 Vq323 - 2 p2 Vq323 + 2 h1 p2 Vq323 - 2 h1 m2 p2 Vq323)}
```

Frequency set for male genotypes:

In[•]:=
```
recursionsM = {b1n/bT, b2n/bT} /. p3 → 1 - p1 - p2 /. h2 → 1 - h1 // Factor
```

Out[•]=
```
{(2 p1 Vm11 + p2 Vm21) / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32),
 (p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)}
```

The following substitution describes maternal-effect killing:

In[•]:=
```
maternaleffect = {Vq211 → 0, Vm21 → 0};
```

Random mating

```
R11[p1_, h1_] = p1 h1;
R12[p1_, h2_] = p1 h2;
R21[p2_, h1_] = p2 h1;
R22[p2_, h2_] = p2 h2;
R31[p3_, h1_] = p3 h1;
R32[p3_, h2_] = p3 h2;
```

Stability of M-fixed and polygynous edge

M fixed

The following calculates the local stability matrix around the equilibrium where M is fixed:

```
| D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
| D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
| D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
      p1 → 1 /. p2 → 0 /. h1 → 1 // Simplify;
MatrixForm[
 %]
```

$$\begin{pmatrix} \frac{Vq312}{Vq111} & -\frac{Vq212-2\,Vq312}{2\,Vq111} & \frac{Vq122}{Vq111} \\ -\frac{Vq312}{Vq111} & \frac{Vq212-2\,Vq312}{2\,Vq111} & -\frac{Vq122}{Vq111} \\ \frac{Vm32}{Vm11} & -\frac{Vm22-2\,Vm32}{2\,Vm11} & 0 \end{pmatrix}$$

The eigenvalues of this matrix are determined by the roots of the following characteristic polynomial:

```
charpoly1 = Factor[Det[% - λ IdentityMatrix[3]]] // Factor
```

$$-\frac{\lambda\left(-Vm22\,Vq122-Vm11\,Vq212\,\lambda+2\,Vm11\,Vq111\,\lambda^2\right)}{2\,Vm11\,Vq111}$$

Dropping out the λ=0 root and writing as an upwards facing parabola:

```
charpoly1 = Collect[charpoly1 / (-λ), λ, Factor]
```

$$-\frac{Vm22\,Vq122}{2\,Vm11\,Vq111}-\frac{Vq212\,\lambda}{2\,Vq111}+\lambda^2$$

```
Solve[charpoly1 == 0, λ]
```

$$\left\{\left\{\lambda \to \frac{1}{4}\left(\frac{\text{Vq212}}{\text{Vq111}} - \frac{\sqrt{8\,\text{Vm22}\,\text{Vq111}\,\text{Vq122} + \text{Vm11}\,\text{Vq212}^2}}{\sqrt{\text{Vm11}}\,\text{Vq111}}\right)\right\},\ \left\{\lambda \to \frac{1}{4}\left(\frac{\text{Vq212}}{\text{Vq111}} + \frac{\sqrt{8\,\text{Vm22}\,\text{Vq111}\,\text{Vq122} + \text{Vm11}\,\text{Vq212}^2}}{\sqrt{\text{Vm11}}\,\text{Vq111}}\right)\right\}\right\}$$

The roots are real (positive term inside radical), and the root that is larger in magnitude is the second one, which adds together two positive terms (rather than subtracting one from the other).

The equilibrium is unstable if this larger root is greater than 1:

$$\frac{1}{4}\left(\frac{\text{Vq212}}{\text{Vq111}} + \frac{\sqrt{8\,\text{Vm22}\,\text{Vq111}\,\text{Vq122}+\text{Vm11}\,\text{Vq212}^2}}{\sqrt{\text{Vm11}}\,\text{Vq111}}\right) > 1$$

$$\frac{\sqrt{8\,\text{Vm22}\,\text{Vq111}\,\text{Vq122}+\text{Vm11}\,\text{Vq212}^2}}{\sqrt{\text{Vm11}}\,\text{Vq111}} > 4 - \frac{\text{Vq212}}{\text{Vq111}}$$

squaring both sides and simplifying gives that the following must be positive for instability:

$$\text{Factor}\left[\left(\frac{\sqrt{8\,\text{Vm22}\,\text{Vq111}\,\text{Vq122} + \text{Vm11}\,\text{Vq212}^2}}{\sqrt{\text{Vm11}}\,\text{Vq111}}\right)^2 - \left(4 - \frac{\text{Vq212}}{\text{Vq111}}\right)^2\right]$$

$$-\frac{8\,(2\,\text{Vm11}\,\text{Vq111} - \text{Vm22}\,\text{Vq122} - \text{Vm11}\,\text{Vq212})}{\text{Vm11}\,\text{Vq111}}$$

Therefore M-Fixed is unstable when:

2 Vq111 Vm11 < Vm22 Vq122+Vm11 Vq212

Note that a stability analysis assumes that the frequency of the rare P haplotype is so small that a Taylor series can be performed with respect to that frequency (in the stability matrix above), implicitly assuming that the frequency is much smaller than any of the fitnesses.

Polygynous edge

The possible edge equilibria occur when:

```
recursionsF == {0, p2, 1 - p2} /. p1 → 0 /. h1 → 0 /. maternaleffect
```

$$\left\{0,\ \frac{\text{p2 Vq222}}{\text{p2 Vq222} + \text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}},\ \frac{\text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}}{\text{p2 Vq222} + \text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}}\right\} == \{0,\ \text{p2},\ 1 - \text{p2}\}$$

```
Solve[%, p2]
edgeequil = p2 /. %;
```

$$\left\{\{p2 \to 0\}, \left\{p2 \to \frac{Vq222 - 2\,Vq323}{Vq222 + Vq223 - 2\,Vq323}\right\}\right\}$$

Looking at stability when MM queens are rare:

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
    p1 → 0 /. h1 → 0 /. maternaleffect // Simplify;
MatrixForm[%]
Factor[Det[% - λ IdentityMatrix[3]]];
```

$$\begin{pmatrix} \frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2} & \frac{(p2\,(Vq212-2\,Vq312)+2\,Vq312)\,(p2\,(Vq223-2\,Vq323)+2\,Vq323)}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2} & \frac{2\,(2\,Vq122\,Vq323+p2\,(Vq122}{(p2\,(Vq222+Vq2}} \\ 0 & 0 & \overline{p2\,Vm22+2} \\ 0 & 0 & \end{pmatrix}$$

The term $\frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2}$ represents the internal stability of the equilibria along the polygynous edge with respect to changes in p2 (from D[recursionsF[[2]], p2]). Evaluating this at the two possible edge equilibria we get:

```
2 Vq222 Vq323
(p2 (Vq222 + Vq223 - 2 Vq323) + 2 Vq323)^2 /. p2 -> edgeequil
```

$$\left\{\frac{Vq222}{2\,Vq323}, \frac{2\,Vq323}{Vq222}\right\}$$

Thus, if $Vq222 > 2\,Vq323$ then the p2=0 equilibrium is unstable and the $p2 = \frac{Vq222-2\,Vq323}{Vq222+Vq223-2\,Vq323}$ equilibrium is stable along the polygynous edge (and vice versa). Thus, $\frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2}$ will be less than one at the stable equilibrium, whichever one it is.

The spread of the rare M haplotype is determined by the 2x2 external stability matrix obtained by deleting the first row and column, the eigenvalues of which are both 0. Thus, the M haplotype will never spread when rare. In fact, the rare MM and M genotypes will both disappear in a single generation (the meaning of both eigenvalues being equal to zero), because of maternal-effect killing.

Polygynous edge - relaxed maternal effect killing

Furthermore, by the small parameter theorem of Karlin and MacGregor (1972 TPB), the leading eigenvalues will be similar if we allow some small number of surviving M and MM offspring of MP queens (i.e., Vq211→ϵ and Vm21→ϵ).

We'll show why this result holds in the following section, which assumes small values for Vq211 and

Vm21.

Looking at stability when MM queens are rare:

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
      p1 → 0 /. h1 → 0 /. Vq211 → ϵ /. Vm21 → ϵ // Simplify;
MatrixForm[
 %]
```

$$\begin{pmatrix} \frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2} & \frac{4\,Vq312\,Vq323+2\,p2\,(Vq223\,Vq312+(Vq212-4\,Vq312)\,Vq323)+p2^2\,(-2\,Vq223\,Vq312+Vq212\,(Vq2}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2} \\ \frac{2\,Vm32\,\epsilon}{(2\,Vm32+p2\,(Vm22-2\,Vm32+\epsilon))^2} & 0 \\ 0 & \frac{p2\,\epsilon}{p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323} \end{pmatrix}$$

Now MP queens can produce M sons (the {2,1} element), and we have to consider the full matrix to determine stability. Note that the leading eigenvalue when $\epsilon = 0$ is the element in the {1,1} position, which is less than one at the stable equilibrium along the polygynous edge (see previous section).

Here we quickly calculate the leading eigenvalue when there is some small survival rate from maternal-effect killing (ϵ small but near zero).

```
charpoly2 = Factor[Det[% - λ IdentityMatrix[3]]];
```

We can represent the leading eigenvalue as a power series in ϵ ($\lambda = \lambda 0 + \lambda 1 * \epsilon + \lambda 2 * \epsilon^2 + \ldots$). Then we take the Taylor series of the characteristic polynomial in ϵ. Then to find the eigenvalues, we note that each order term in the Taylor series must equal zero (for the characteristic polynomial to equal zero at that root). Starting with the largest order term:

```
Factor[Normal[Series[charpoly2 /. λ → λ0 + λ1 * ϵ + λ2 * ϵ^2, {ϵ, 0, 0}]]]
```

$$-\left(\left(\lambda 0^2\,\left(-2\,Vq222\,Vq323+p2^2\,Vq222^2\,\lambda 0+2\,p2^2\,Vq222\,Vq223\,\lambda 0+p2^2\,Vq223^2\,\lambda 0+4\,p2\,Vq222\,Vq323\,\lambda 0-4\,p2^2\,Vq222\,Vq323\,\lambda 0+4\,p2\,Vq223\,Vq323\,\lambda 0-4\,p2^2\,Vq223\,Vq323\,\lambda 0+4\,Vq323^2\,\lambda 0-8\,p2\,Vq323^2\,\lambda 0+4\,p2^2\,Vq323^2\,\lambda 0\right)\right)\Big/(p2\,Vq222+p2\,Vq223+2\,Vq323-2\,p2\,Vq323)^2\right)$$

This to leading order, the eigenvalues are the same as we found in the previous section (two zeros and $\frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2}$):

```
Simplify[Solve[% == 0, λ0]]
```

$$\left\{\{\lambda 0 \to 0\},\ \{\lambda 0 \to 0\},\ \left\{\lambda 0 \to \frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2}\right\}\right\}$$

The leading eigenvalue is near $\frac{2\,Vq222\,Vq323}{(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)^2}$ and its next order term is:

```
Factor[Normal[Series[charpoly2 /. λ → λ0 + λ1 * ϵ + λ2 * ϵ^2, {ϵ, 0, 1}]] /.
  {λ0 → (2 Vq222 Vq323)/((p2 (Vq222 + Vq223 - 2 Vq323) + 2 Vq323)^2)}]
```

$$-\left(\left(4\,\text{Vq222}\,\text{Vq323}\,\epsilon\left(-\text{p2}^2\,\text{Vm32}\,\text{Vq212}\,\text{Vq223}-2\,\text{p2}\,\text{Vm32}\,\text{Vq223}\,\text{Vq312}+2\,\text{p2}^2\,\text{Vm32}\,\text{Vq223}\,\text{Vq312}-2\,\text{p2}\,\text{Vm32}\,\text{Vq212}\,\text{Vq323}+2\,\text{p2}^2\,\text{Vm32}\,\text{Vq212}\,\text{Vq323}-4\,\text{Vm32}\,\text{Vq312}\,\text{Vq323}+8\,\text{p2}\,\text{Vm32}\,\text{Vq312}\,\text{Vq323}-4\,\text{p2}^2\,\text{Vm32}\,\text{Vq312}\,\text{Vq323}+\text{p2}^2\,\text{Vm22}^2\,\text{Vq222}\,\text{Vq323}\,\lambda1+4\,\text{p2}\,\text{Vm22}\,\text{Vm32}\,\text{Vq222}\,\text{Vq323}\,\lambda1-4\,\text{p2}^2\,\text{Vm22}\,\text{Vm32}\,\text{Vq222}\,\text{Vq323}\,\lambda1+4\,\text{Vm32}^2\,\text{Vq222}\,\text{Vq323}\,\lambda1-8\,\text{p2}\,\text{Vm32}^2\,\text{Vq222}\,\text{Vq323}\,\lambda1+4\,\text{p2}^2\,\text{Vm32}^2\,\text{Vq222}\,\text{Vq323}\,\lambda1\right)\right)\Big/\left(\left(\text{p2}\,\text{Vm22}+2\,\text{Vm32}-2\,\text{p2}\,\text{Vm32}\right)^2\left(\text{p2}\,\text{Vq222}+\text{p2}\,\text{Vq223}+2\,\text{Vq323}-2\,\text{p2}\,\text{Vq323}\right)^4\right)\right)$$

```
Simplify[Solve[% == 0, λ1]]
```

$$\left\{\left\{\lambda1\to\frac{\text{Vm32}\left(\text{p2}\left(\text{Vq212}-2\,\text{Vq312}\right)+2\,\text{Vq312}\right)\left(\text{p2}\left(\text{Vq223}-2\,\text{Vq323}\right)+2\,\text{Vq323}\right)}{\left(\text{p2}\left(\text{Vm22}-2\,\text{Vm32}\right)+2\,\text{Vm32}\right)^2\text{Vq222}\,\text{Vq323}}\right\}\right\}$$

Thus the leading eigenvalue can be written as

$$\lambda=\lambda0+\lambda1*\epsilon+\lambda2*\epsilon^2=\frac{2\,\text{Vq222}\,\text{Vq323}}{\left(\text{p2}\left(\text{Vq222}+\text{Vq223}-2\,\text{Vq323}\right)+2\,\text{Vq323}\right)^2}+\epsilon\,\frac{\text{Vm32}\left(\text{p2}\left(\text{Vq212}-2\,\text{Vq312}\right)+2\,\text{Vq312}\right)\left(\text{p2}\left(\text{Vq223}-2\,\text{Vq323}\right)+2\,\text{Vq323}\right)}{\left(\text{p2}\left(\text{Vm22}-2\,\text{Vm32}\right)+2\,\text{Vm32}\right)^2\text{Vq222}\,\text{Vq323}}+\lambda2*\epsilon^2$$

But note that because $\frac{2\,\text{Vq222}\,\text{Vq323}}{\left(\text{p2}\left(\text{Vq222}+\text{Vq223}-2\,\text{Vq323}\right)+2\,\text{Vq323}\right)^2}<1$ at the stable equilibrium along the polygynous edge then it will remain less than one for a small enough value of ϵ. Furthermore, the two eigenvalues at 0 will remain small (or order ϵ) and so M will not spread when rare.

Social polymorphism numerical search

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] // Simplify
eqn2 = Factor[recursionsF[[2]] - p2] // Simplify
eqn3 = Factor[recursionsM[[1]] - h1] // Simplify
```

```
findequil[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = {p1, p2, h1} /.
   NSolve[{(-p1 (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323) +
          h1 (p2 Vq211 - 2 p1^2 (Vq111 - Vq122 - Vq312 + Vq323) + p1 (2 Vq111 - 2 Vq312 + p2
                (-Vq211 - Vq212 + Vq222 + Vq223 + 2 Vq312 - 2 Vq323) + 2 Vq323))) /
        (p2 Vq222 + p2 Vq223 + 2 p1 (Vq122 - Vq323) + 2 Vq323 - 2 p2 Vq323 +
          h1 (2 (Vq312 - Vq323) + 2 p1 (Vq111 - Vq122 - Vq312 + Vq323) +
             p2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323))) == 0,
      (2 p1 (Vq122 - p2 Vq122 + p2 Vq323) + p2 (Vq222 - p2 Vq222 - p2 Vq223 - 2 Vq323 +
             2 p2 Vq323) + h1 (2 Vq312 + p2^2 (-Vq211 - Vq212 + Vq222 + Vq223 + 2
                Vq312 - 2 Vq323) + p2 (Vq212 - Vq222 - 4 Vq312 + 2 Vq323) -
             2 p1 (Vq122 + Vq312 + p2 (Vq111 - Vq122 - Vq312 + Vq323)))) /
        (p2 Vq222 + p2 Vq223 + 2 p1 (Vq122 - Vq323) + 2 Vq323 - 2 p2 Vq323 +
          h1 (2 (Vq312 - Vq323) + 2 p1 (Vq111 - Vq122 - Vq312 + Vq323) +
             p2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323))) == 0,
      (-2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32)) /
        (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32) == 0}, {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

```
D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] //
D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1]
 Simplify
```

```
Clear[internstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, p1, p2, h1}] =
  Eigenvalues[{{-(((2 h1 p1 Vq111 + h1 p2 Vq211) (2 h1 Vq111 + 2 (1 - h1) Vq122 -
                2 h1 Vq312 - 2 (1 - h1) Vq323)) / (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 +
             h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + (1 - h1) p2 Vq223 +
             2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) + (2 h1 Vq111) /
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
          (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323),
      -(((2 h1 p1 Vq111 + h1 p2 Vq211) (h1 Vq211 + h1 Vq212 + (1 - h1) Vq222 +
                (1 - h1) Vq223 - 2 h1 Vq312 - 2 (1 - h1) Vq323)) /
```

```
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
            (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) +
    (h1 Vq211) / (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2
         Vq222 + (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323),
  - (((2 h1 p1 Vq111 + h1 p2 Vq211) (2 p1 Vq111 - 2 p1 Vq122 + p2 Vq211 + p2 Vq212 -
              p2 Vq222 - p2 Vq223 + 2 (1 - p1 - p2) Vq312 - 2 (1 - p1 - p2) Vq323)) /
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
            (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) +
    (2 p1 Vq111 + p2 Vq211) / (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 +
        h1 p2 Vq212 + (1 - h1) p2 Vq222 + (1 - h1) p2 Vq223 +
        2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)},
 {- (((2 (1 - h1) p1 Vq122 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + 2 h1 (1 - p1 - p2) Vq312)
            (2 h1 Vq111 + 2 (1 - h1) Vq122 - 2 h1 Vq312 - 2 (1 - h1) Vq323)) /
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
            (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) +
    (2 (1 - h1) Vq122 - 2 h1 Vq312) / (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 +
        h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + (1 - h1) p2 Vq223 +
        2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323),
  - (((2 (1 - h1) p1 Vq122 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + 2 h1 (1 - p1 - p2) Vq312)
            (h1 Vq211 + h1 Vq212 + (1 - h1) Vq222 +
              (1 - h1) Vq223 - 2 h1 Vq312 - 2 (1 - h1) Vq323)) /
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
            (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) +
    (h1 Vq212 + (1 - h1) Vq222 - 2 h1 Vq312) / (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 +
        h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + (1 - h1) p2 Vq223 +
        2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323),
  - (((2 (1 - h1) p1 Vq122 + h1 p2 Vq212 + (1 - h1) p2 Vq222 + 2 h1 (1 - p1 - p2) Vq312)
            (2 p1 Vq111 - 2 p1 Vq122 + p2 Vq211 + p2 Vq212 - p2 Vq222 -
              p2 Vq223 + 2 (1 - p1 - p2) Vq312 - 2 (1 - p1 - p2) Vq323)) /
        (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
            (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)^2) +
    (- 2 p1 Vq122 + p2 Vq212 - p2 Vq222 + 2 (1 - p1 - p2) Vq312) /
     (2 h1 p1 Vq111 + 2 (1 - h1) p1 Vq122 + h1 p2 Vq211 + h1 p2 Vq212 + (1 - h1) p2 Vq222 +
        (1 - h1) p2 Vq223 + 2 h1 (1 - p1 - p2) Vq312 + 2 (1 - h1) (1 - p1 - p2) Vq323)},
 {- (((2 p1 Vm11 + p2 Vm21) (2 Vm11 - 2 Vm32)) /
        (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)^2) +
    2 Vm11 / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32),
  - (((2 p1 Vm11 + p2 Vm21) (Vm21 + Vm22 - 2 Vm32)) /
        (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)^2) +
```

```
                                  Vm21
 ----------------------------------------------------------, 0}}]
 2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-10);
```

```
Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search:

We numerically search stable polymorphic equilibria by running random variations of our parameter set.

```
SeedRandom[465 234]
Clear[tab, tab1]
For[j = 1; stabpoly = {}, j ≤ 1 000 000, j++,
 tab = N[Table[Round[Random[] * 10^10] / 10^10, {i, 1, 12}], 60];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 If[
  testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  testPol,
  Print[tab1] && AppendTo[stabpoly, tab1]];
 If[Mod[j, 5000] == 0, Print[j]];
 If[Mod[j, 100 000] == 0, Print[stabpoly]];
]
```

Simplex stream diagrams

We create streamplot diagrams for the following scenarios:

PP fixed and M fixed stable together

```
E11[p1_, h1_] := p1 h1
E12[p1_, h2_] := p1 h2
E21[p2_, h1_] := p2 h1
E22[p2_, h2_] := p2 h2
E31[p3_, h1_] := p3 h1
E32[p3_, h2_] := p3 h2
```

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame0.18079491958209112`,0.3841016083581789`*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog, {Text[Style["O", Large],
     {-0.180794919 + (1 - 0.384101608 - 0.180794919), 0.384101608}]}];
  (*first coordinate = p3-p1, second coordinate = p2*)

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0.0, 1.0}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → Sqrt[3] / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]
```

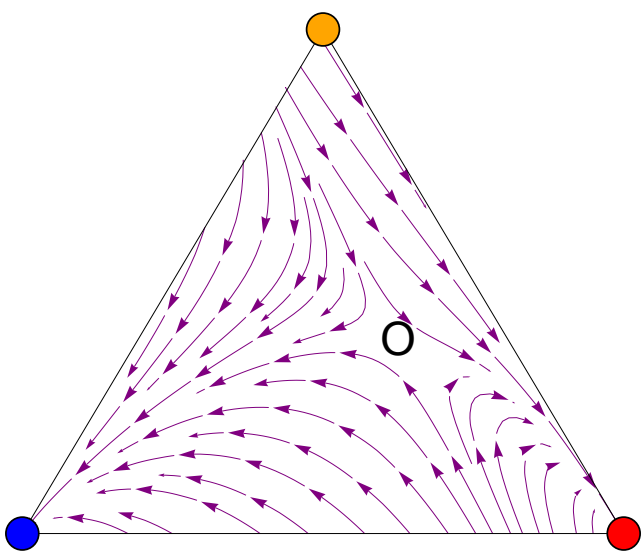
```
Solve[recursionsM[[1]] - h1 == 0, h1]
```

$$\left\{\left\{h1 \to \frac{2\ p1\ Vm11 + p2\ Vm21}{2\ p1\ Vm11 + p2\ Vm21 + p2\ Vm22 + 2\ Vm32 - 2\ p1\ Vm32 - 2\ p2\ Vm32}\right\}\right\}$$

```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2, recursionsF[[3]] - (1 - p1 - p2)} /.
                    maternaleffect /. h2 → 1 - h1 /. h1 → (2 p1 Vm11 + p2 Vm21) /
                     (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) /.
                 Vm11 → 1 / 5 /. Vm21 → 0 /. Vm22 → 1 / 5 /. Vm32 → 1 / 5 /.
            Vq111 → 1 /. Vq212 → 1 / 5 /. Vq323 -> 1 / 5 /. Vq223 → 1 / 5 /.
      Vq312 -> 1 / 5 /. Vq222 -> 1 / 5 /. Vq122 -> 1 /
      5 // Factor
```

$$\left\{-\frac{p1\left(2-10\,p1+8\,p1^2-p2-p1\,p2\right)}{2+8\,p1^2-p2-p1\,p2},\ -\frac{8\,p1-8\,p1^2-2\,p2-6\,p1\,p2-16\,p1^2\,p2+p2^2+2\,p1\,p2^2}{2\left(-2-8\,p1^2+p2+p1\,p2\right)},\ \frac{-4\,p1-12\,p1^2+16\,p1^3+2\,p2+4\,p1\,p2+14\,p1^2\,p2-p2^2-2\,p1\,p2^2}{2\left(2+8\,p1^2-p2-p1\,p2\right)}\right\}$$

```
vars = {p1[t], p2[t], p3[t]};
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



Where open circle is an unstable polymorphic equilibrium.

Coordinates are found below through findequil and determined to be unstable through sieve3:

{Vq111,Vq122,Vq211,Vq212,Vq222,Vq223,Vq312,Vq323,Vm11,Vm21,Vm22,Vm32}

```
findequil[{1, 1 / 5, 0, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
{{-0.395081, 11.9016, 0.0798013}, {1., 0, 1.},
 {0.180795, 0.384102, 0.22377}, {0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

```
sieve3[{1, 1 / 5, 0, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
```

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}, {1., 0, 1.}}
```

MP/PP fixed and M fixed stable together

```
E11[p1_, h1_] := p1 h1
E12[p1_, h2_] := p1 h2
E21[p2_, h1_] := p2 h1
E22[p2_, h2_] := p2 h2
E31[p3_, h1_] := p3 h1
E32[p3_, h2_] := p3 h2
```

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

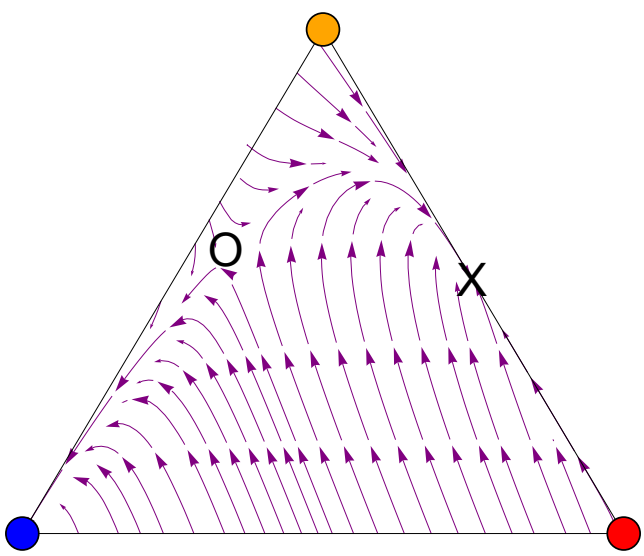
```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog,
   {Text[Style["O", Large], {-0.379919 + (1 - 0.379919 - 0.5590219), 0.5590219}]}];
  AppendTo[myEpilog, {Text[Style["X", Large], {-0 + (1 - 0 - 0.5), 0.5}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0.0, 1.0}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → Sqrt[3] / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]

Solve[recursionsM[[1]] - h1 == 0, h1]
```

$$\left\{\left\{h1 \to \frac{2\ p1\ Vm11 + p2\ Vm21}{2\ p1\ Vm11 + p2\ Vm21 + p2\ Vm22 + 2\ Vm32 - 2\ p1\ Vm32 - 2\ p2\ Vm32}\right\}\right\}$$

```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2, recursionsF[[3]] - (1 - p1 - p2)} /.
                maternaleffect /. h2 → 1 - h1 /. h1 → (2 p1 Vm11 + p2 Vm21) /
                 (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) /.
            Vm11 → 1 / 5 /. Vm21 → 0 /. Vm22 → 1 / 5 /. Vm32 → 1 / 5 /.
        Vq111 → 1 /. Vq212 → 3 / 5 /. Vq323 → 1 / 5 /. Vq223 → 1 / 5 /.
    Vq312 → 3 / 5 /. Vq222 → 3 / 5 /. Vq122 → 3 /
    5 // Factor
```

$$\left\{-\frac{p1\left(-2+2\,p1-p2+7\,p1\,p2+p2^2\right)}{-2-8\,p1-p2+7\,p1\,p2+p2^2},\ -\frac{24\,p1-24\,p1^2+2\,p2-34\,p1\,p2-5\,p2^2+14\,p1\,p2^2+2\,p2^3}{2\left(-2-8\,p1-p2+7\,p1\,p2+p2^2\right)},\ \frac{20\,p1-20\,p1^2+2\,p2-36\,p1\,p2+14\,p1^2\,p2-5\,p2^2+16\,p1\,p2^2+2\,p2^3}{2\left(-2-8\,p1-p2+7\,p1\,p2+p2^2\right)}\right\}$$

```
vars = {p1[t], p2[t], p3[t]};
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



Where open circle is an unstable polymorphic equilibrium.

Coordinates are found below through findequil and determined to be unstable through sieve3:

{Vq111,Vq122,Vq211,Vq212,Vq222,Vq223,Vq312,Vq323,Vm11,Vm21,Vm22,Vm32}

```
findequil[{1, 3 / 5, 0, 3 / 5, 3 / 5, 1 / 5, 3 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
```

```
{{1., 0, 1.}, {0.379918, 0.559022, 0.527306},
 {0.207801, -1.50639, 0.118527}, {0, 0.5, 0}, {0, 0.5, 0}, {0, 0, 0}}
```

```
sieve3[{1, 3 / 5, 0, 3 / 5, 3 / 5, 1 / 5, 3 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
```

```
{{0, 0.5, 0}, {0, 0.5, 0}, {1., 0, 1.}}
```

PP fixed only

```
E11[p1_, h1_] := p1 h1
E12[p1_, h2_] := p1 h2
E21[p2_, h1_] := p2 h1
E22[p2_, h2_] := p2 h2
E31[p3_, h1_] := p3 h1
E32[p3_, h2_] := p3 h2
```

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0.0, 1.0}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → √3 / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]
```

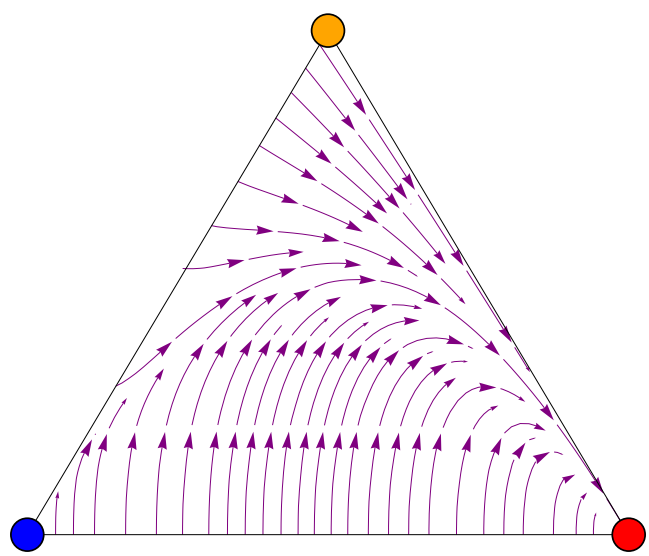
```
Solve[recursionsM[[1]] - h1 == 0, h1]
```

$$\left\{\left\{h1 \to \frac{2\ p1\ Vm11 + p2\ Vm21}{2\ p1\ Vm11 + p2\ Vm21 + p2\ Vm22 + 2\ Vm32 - 2\ p1\ Vm32 - 2\ p2\ Vm32}\right\}\right\}$$

```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2, recursionsF[[3]] - (1 - p1 - p2)} /.
                  maternaleffect /. h2 → 1 - h1 /. h1 → (2 p1 Vm11 + p2 Vm21) /
                   (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) /.
              Vm11 → 1 / 5 /. Vm21 → 0 /. Vm22 → 1 / 5 /. Vm32 → 1 / 5 /.
          Vq111 → 1 / 5 /. Vq212 -> 1 / 5 /. Vq323 -> 1 / 5 /. Vq223 → 1 / 5 /.
     Vq312 -> 1 / 5 /. Vq222 -> 1 / 5 /. Vq122 -> 1 /
     5 // Factor
```

$$\left\{-\frac{p1\,(-2+2\,p1+p2+p1\,p2)}{-2+p2+p1\,p2},\ -\frac{8\,p1-8\,p1^2-2\,p2-6\,p1\,p2+p2^2+2\,p1\,p2^2}{2\,(-2+p2+p1\,p2)},\ \frac{4\,p1-4\,p1^2-2\,p2-4\,p1\,p2+2\,p1^2\,p2+p2^2+2\,p1\,p2^2}{2\,(-2+p2+p1\,p2)}\right\}$$

```
vars = {p1[t], p2[t], p3[t]};

args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];

Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



MP/PP fixed only

```
E11[p1_, h1_] := p1 h1
E12[p1_, h2_] := p1 h2
E21[p2_, h1_] := p2 h1
E22[p2_, h2_] := p2 h2
E31[p3_, h1_] := p3 h1
E32[p3_, h2_] := p3 h2
```

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog, {Text[Style["X", Large], {-0 + (1 - 0 - 0.5), 0.5}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0.0, 1.0}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → √3 / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]
```

```
Solve[recursionsM[[1]] - h1 == 0, h1]
```

$$\left\{\left\{h1 \to \frac{2\,p1\,Vm11 + p2\,Vm21}{2\,p1\,Vm11 + p2\,Vm21 + p2\,Vm22 + 2\,Vm32 - 2\,p1\,Vm32 - 2\,p2\,Vm32}\right\}\right\}$$

```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2, recursionsF[[3]] - (1 - p1 - p2)} /.
                maternaleffect /. h2 → 1 - h1 /. h1 → (2 p1 Vm11 + p2 Vm21) /
                 (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) /.
            Vm11 → 1 / 5 /. Vm21 → 0 /. Vm22 → 1 / 5 /. Vm32 → 1 / 5 /.
        Vq111 → 1 / 5 /. Vq212 → 3 / 5 /. Vq323 -> 1 / 5 /. Vq223 → 1 / 5 /.
    Vq312 → 3 / 5 /. Vq222 → 3 / 5 /. Vq122 → 3 /
    5 // Factor
```

$$\left\{-\frac{p1\left(-2 - 6\,p1 + 8\,p1^2 - p2 + 7\,p1\,p2 + p2^2\right)}{-2 - 8\,p1 + 8\,p1^2 - p2 + 7\,p1\,p2 + p2^2},\right.$$

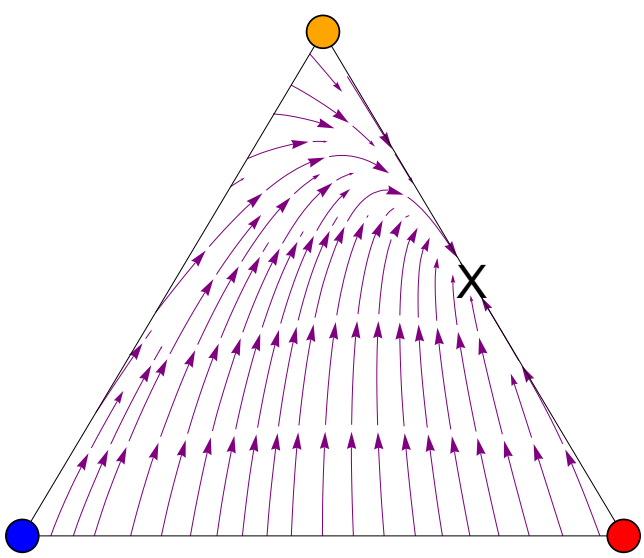
$$-\frac{24\,p1 - 24\,p1^2 + 2\,p2 - 34\,p1\,p2 + 16\,p1^2\,p2 - 5\,p2^2 + 14\,p1\,p2^2 + 2\,p2^3}{2\left(-2 - 8\,p1 + 8\,p1^2 - p2 + 7\,p1\,p2 + p2^2\right)},$$

$$\left.\frac{20\,p1 - 36\,p1^2 + 16\,p1^3 + 2\,p2 - 36\,p1\,p2 + 30\,p1^2\,p2 - 5\,p2^2 + 16\,p1\,p2^2 + 2\,p2^3}{2\left(-2 - 8\,p1 + 8\,p1^2 - p2 + 7\,p1\,p2 + p2^2\right)}\right\}$$

```
sieve3[{1 / 5, 3 / 5, 0, 3 / 5, 3 / 5, 1 / 5, 3 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
```

{{0, 0.5, 0}, {0, 0.5, 0}}

```
vars = {p1[t], p2[t], p3[t]};
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



There is one stable equilibrium with only MP/PP polygynous queens.

```
sieve3[{1 / 5, 3 / 5, 0, 3 / 5, 3 / 5, 1 / 5, 3 / 5, 1 / 5, 1 / 5, 0, 1 / 5, 1 / 5}]
```

{{0, 0.5, 0}, {0, 0.5, 0}}

Random mating with variation in fertility

```
R11[p1_, h1_] = p1 h1 f11;
R12[p1_, h2_] = p1 h2 f12;
R21[p2_, h1_] = p2 h1 f21;
R22[p2_, h2_] = p2 h2 f22;
R31[p3_, h1_] = p3 h1 f31;
R32[p3_, h2_] = p3 h2 f32;
```

Stability of M fixed and polygynous edge

M fixed

Stability, jacobian matrix, analysis of M-fixed:

```
| D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
| D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
| D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
    p1 → 1 /. p2 → 0 /. h1 → 1 // Simplify;
MatrixForm[
 %]
```

$$\begin{pmatrix} \frac{f31\ Vq312}{f11\ Vq111} & -\frac{f21\ Vq212-2\ f31\ Vq312}{2\ f11\ Vq111} & \frac{f12\ Vq122}{f11\ Vq111} \\ -\frac{f31\ Vq312}{f11\ Vq111} & \frac{f21\ Vq212-2\ f31\ Vq312}{2\ f11\ Vq111} & -\frac{f12\ Vq122}{f11\ Vq111} \\ \frac{Vm32}{Vm11} & -\frac{Vm22-2\ Vm32}{2\ Vm11} & 0 \end{pmatrix}$$

```
charpoly1 = Factor[Det[% - λ IdentityMatrix[3]]] // Factor
```

$$-\frac{\lambda\left(-f12\ Vm22\ Vq122 - f21\ Vm11\ Vq212\ \lambda + 2\ f11\ Vm11\ Vq111\ \lambda^2\right)}{2\ f11\ Vm11\ Vq111}$$

Dropping out the λ=0 root and writing as an upwards facing parabola:

```
charpoly1 = Collect[charpoly1 / (-λ), λ, Factor]
```

$$-\frac{f12\ Vm22\ Vq122}{2\ f11\ Vm11\ Vq111} - \frac{f21\ Vq212\ \lambda}{2\ f11\ Vq111} + \lambda^2$$

```
Solve[charpoly1 == 0, λ]
```

$$\left\{\left\{\lambda \to \frac{1}{4}\left(\frac{f21\ Vq212}{f11\ Vq111} - \frac{\sqrt{8\ f11\ f12\ Vm22\ Vq111\ Vq122 + f21^2\ Vm11\ Vq212^2}}{f11\ \sqrt{Vm11}\ Vq111}\right)\right\},\right.$$
$$\left.\left\{\lambda \to \frac{1}{4}\left(\frac{f21\ Vq212}{f11\ Vq111} + \frac{\sqrt{8\ f11\ f12\ Vm22\ Vq111\ Vq122 + f21^2\ Vm11\ Vq212^2}}{f11\ \sqrt{Vm11}\ Vq111}\right)\right\}\right\}$$

The roots are real (positive term inside radical), and the root that is larger in magnitude is the second one, which adds together two positive terms (rather than subtracting one from the other).

The equilibrium is unstable if this larger root is greater than 1:

$$\left\{\lambda \to \frac{1}{4}\left(\frac{f21\ Vq212}{f11\ Vq111} + \frac{\sqrt{8\ f11\ f12\ Vm22\ Vq111\ Vq122 + f21^2\ Vm11\ Vq212^2}}{f11\ \sqrt{Vm11}\ Vq111}\right) > 1\right.$$

$$\frac{\sqrt{8\ f11\ f12\ Vm22\ Vq111\ Vq122 + f21^2\ Vm11\ Vq212^2}}{f11\ \sqrt{Vm11}\ Vq111} > 4 - \frac{f21\ Vq212}{f11\ Vq111}$$

squaring both sides and simplifying gives that the following must be positive for instability:

$$\mathbf{Factor}\left[\left(\frac{\sqrt{8\ f11\ f12\ Vm22\ Vq111\ Vq122 + f21^2\ Vm11\ Vq212^2}}{f11\ \sqrt{Vm11}\ Vq111}\right)^2 - \left(4 - \frac{f21\ Vq212}{f11\ Vq111}\right)^2\right]$$
$$-\frac{8\ (2\ f11\ Vm11\ Vq111 - f12\ Vm22\ Vq122 - f21\ Vm11\ Vq212)}{f11\ Vm11\ Vq111}$$

Therefore M-Fixed is unstable when:

2 Vq111 f11 Vm11 < f12 Vm22 Vq122+f21 Vm11 Vq212

Note that a stability analysis assumes that the frequency of the rare P haplotype is so small that a Taylor series can be performed with respect to that frequency (in the stability matrix above), implicitly assuming that the frequency is much smaller than any of the fitnesses.

Polygynous edge

The possible edge equilibria occur when:

```
recursionsF == {0, p2, 1 - p2} /. p1 → 0 /. h1 → 0 /. maternaleffect
```

$$\left\{0, \frac{f22\ p2\ Vq222}{f22\ p2\ Vq222 + f22\ p2\ Vq223 + 2\ f32\ (1 - p2)\ Vq323},\right.$$
$$\left.\frac{f22\ p2\ Vq223 + 2\ f32\ (1 - p2)\ Vq323}{f22\ p2\ Vq222 + f22\ p2\ Vq223 + 2\ f32\ (1 - p2)\ Vq323}\right\} == \{0, p2, 1 - p2\}$$

```
Solve[%, p2]
edgeequil = p2 /. %;
```

$$\left\{\{p2 \to 0\}, \left\{p2 \to \frac{f22\ Vq222 - 2\ f32\ Vq323}{f22\ Vq222 + f22\ Vq223 - 2\ f32\ Vq323}\right\}\right\}$$

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
      p1 → 0 /. h1 → 0 /. Vq211 → 0 /. Vm21 → 0 // Simplify;
MatrixForm[%]
Factor[Det[% - λ IdentityMatrix[3]]];
quad2 = Collect[%, λ, Factor]
```

$$\begin{pmatrix} \frac{2\ f22\ f32\ Vq222\ Vq323}{(f22\ p2\ (Vq222+Vq223)-2\ f32\ (-1+p2)\ Vq323)^2} & \frac{(f21\ p2\ Vq212-2\ f31\ (-1+p2)\ Vq312)\ (f22\ p2\ Vq223-2\ f32\ (-1+p2)\ Vq323)}{(f22\ p2\ (Vq222+Vq223)-2\ f32\ (-1+p2)\ Vq323)^2} & \frac{2\ (f22\ f}{} \\ 0 & 0 & \\ 0 & 0 & \end{pmatrix}$$

$$\frac{2\ f22\ f32\ Vq222\ Vq323\ \lambda^2}{(f22\ p2\ Vq222 + f22\ p2\ Vq223 + 2\ f32\ Vq323 - 2\ f32\ p2\ Vq323)^2} - \lambda^3$$

The term $\frac{2\ f22\ f32\ Vq222\ Vq323}{(f22\ p2\ (Vq222+Vq223)-2\ f32\ (-1+p2)\ Vq323)^2}$ represents the internal stability of the equilibria along the polygynous edge with respect to changes in p2 (from D[recursionsF[[2]], p2]). Evaluating this at the two possible edge equilibria we get:

$$\frac{2\ f22\ f32\ Vq222\ Vq323}{(f22\ p2\ (Vq222 + Vq223) - 2\ f32\ (-1 + p2)\ Vq323)^2}\ /.\ p2 \to edgeequil\ //\ Simplify$$

$$\left\{\frac{f22\ Vq222}{2\ f32\ Vq323}, \frac{2\ f32\ Vq323}{f22\ Vq222}\right\}$$

Thus, if $f22\ Vq222 > 2\ f32\ Vq323$ then the p2=0 equilibrium is unstable and the $p2 = \frac{f22\ Vq222-2\ f32\ Vq323}{f22\ Vq222+f22\ Vq223-2\ f32\ Vq323}$ equilibrium is stable along the polygynous edge (and vice versa). Thus, $\frac{2\ f22\ f32\ Vq222\ Vq323}{(f22\ p2\ (Vq222+Vq223)-2\ f32\ (-1+p2)\ Vq323)^2}$ will be less than one at the stable equilibrium, whichever one it is.

The spread of the rare M haplotype is determined by the 2x2 external stability matrix obtained by deleting the first row and column, the eigenvalues of which are both 0. Thus, the M haplotype will never spread when rare. In fact, the rare MM and M genotypes will both disappear in a single generation (the meaning of both eigenvalues being equal to zero), because of maternal-effect killing.

Social polymorphism numerical search

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] // Simplify

eqn2 = Factor[recursionsF[[2]] - p2] // Simplify

eqn3 = Factor[recursionsM[[1]] - h1] // Simplify

findequil[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323,
    Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}] =
  {p1, p2, h1} /. NSolve[{(-2 f11 h1 (-1 + p1) p1 Vq111 + 2 f12 (-1 + h1) p1^2 Vq122 +
          f21 h1 p2 Vq211 - f21 h1 p1 p2 Vq211 - f21 h1 p1 p2 Vq212 - f22 p1 p2 Vq222 +
          f22 h1 p1 p2 Vq222 - f22 p1 p2 Vq223 + f22 h1 p1 p2 Vq223 - 2 f31 h1 p1 Vq312 +
          2 f31 h1 p1^2 Vq312 + 2 f31 h1 p1 p2 Vq312 - 2 f32 p1 Vq323 + 2 f32 h1 p1 Vq323 +
          2 f32 p1^2 Vq323 - 2 f32 h1 p1^2 Vq323 + 2 f32 p1 p2 Vq323 - 2 f32 h1 p1 p2 Vq323) /
        (2 f11 h1 p1 Vq111 - 2 f12 (-1 + h1) p1 Vq122 + f21 h1 p2 Vq211 + f21 h1 p2 Vq212 +
          f22 p2 Vq222 - f22 h1 p2 Vq222 + f22 p2 Vq223 - f22 h1 p2 Vq223 + 2 f31 h1 Vq312 -
          2 f31 h1 p1 Vq312 - 2 f31 h1 p2 Vq312 + 2 f32 Vq323 - 2 f32 h1 Vq323 -
          2 f32 p1 Vq323 + 2 f32 h1 p1 Vq323 - 2 f32 p2 Vq323 + 2 f32 h1 p2 Vq323) == 0,
      (-2 f11 h1 p1 p2 Vq111 + 2 f12 (-1 + h1) p1 (-1 + p2) Vq122 - f21 h1 p2^2 Vq211 +
          f21 h1 p2 Vq212 - f21 h1 p2^2 Vq212 + f22 p2 Vq222 - f22 h1 p2 Vq222 -
          f22 p2^2 Vq222 + f22 h1 p2^2 Vq222 - f22 p2^2 Vq223 + f22 h1 p2^2 Vq223 +
          2 f31 h1 Vq312 - 2 f31 h1 p1 Vq312 - 4 f31 h1 p2 Vq312 + 2 f31 h1 p1 p2 Vq312 +
          2 f31 h1 p2^2 Vq312 - 2 f32 p2 Vq323 + 2 f32 h1 p2 Vq323 + 2 f32 p1 p2 Vq323 -
          2 f32 h1 p1 p2 Vq323 + 2 f32 p2^2 Vq323 - 2 f32 h1 p2^2 Vq323) /
        (2 f11 h1 p1 Vq111 - 2 f12 (-1 + h1) p1 Vq122 + f21 h1 p2 Vq211 + f21 h1 p2 Vq212 +
          f22 p2 Vq222 - f22 h1 p2 Vq222 + f22 p2 Vq223 - f22 h1 p2 Vq223 + 2 f31 h1 Vq312 -
          2 f31 h1 p1 Vq312 - 2 f31 h1 p2 Vq312 + 2 f32 Vq323 - 2 f32 h1 Vq323 -
          2 f32 p1 Vq323 + 2 f32 h1 p1 Vq323 - 2 f32 p2 Vq323 + 2 f32 h1 p2 Vq323) == 0,
      (-2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32)) /
        (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32) == 0}, {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

D[recursionsF[[1]], p1]	D[recursionsF[[1]], p2]	D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1]	D[recursionsF[[2]], p2]	D[recursionsF[[2]], h1]
D[recursionsM[[1]], p1]	D[recursionsM[[1]], p2]	D[recursionsM[[1]], h1]

```
Clear[checkstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_, Vm11_,
   Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323,
    Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32, p1, p2, h1}] =
  Eigenvalues[{{-(((2 f11 h1 p1 Vq111 + f21 h1 p2 Vq211)
              (2 f11 h1 Vq111 + 2 f12 (1 - h1) Vq122 - 2 f31 h1 Vq312 - 2 f32 (1 - h1) Vq323)) /
            (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
               f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
               2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
        (2 f11 h1 Vq111) / (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
            f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
            2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323),
      -(((2 f11 h1 p1 Vq111 + f21 h1 p2 Vq211) (f21 h1 Vq211 + f21 h1 Vq212 + f22
                (1 - h1) Vq222 + f22 (1 - h1) Vq223 - 2 f31 h1 Vq312 - 2 f32 (1 - h1) Vq323)) /
            (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
               f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
               2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
        (f21 h1 Vq211) / (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
            f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
            2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323),
      -(((2 f11 h1 p1 Vq111 + f21 h1 p2 Vq211) (2 f11 p1 Vq111 - 2 f12 p1 Vq122 +
                f21 p2 Vq211 + f21 p2 Vq212 - f22 p2 Vq222 - f22 p2 Vq223 +
                2 f31 (1 - p1 - p2) Vq312 - 2 f32 (1 - p1 - p2) Vq323)) /
            (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
               f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
               2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
        (2 f11 p1 Vq111 + f21 p2 Vq211) / (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 +
            f21 h1 p2 Vq211 + f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
            2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)},
     {-(((2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 +
                2 f31 h1 (1 - p1 - p2) Vq312)
              (2 f11 h1 Vq111 + 2 f12 (1 - h1) Vq122 - 2 f31 h1 Vq312 - 2 f32 (1 - h1) Vq323)) /
            (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
               f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
               2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
        (2 f12 (1 - h1) Vq122 - 2 f31 h1 Vq312) / (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 +
            f21 h1 p2 Vq211 + f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
            2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323),
```

```
- (((2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 +
          2 f31 h1 (1 - p1 - p2) Vq312) (f21 h1 Vq211 + f21 h1 Vq212 + f22 (1 - h1) Vq222 +
          f22 (1 - h1) Vq223 - 2 f31 h1 Vq312 - 2 f32 (1 - h1) Vq323)) /
      (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
         f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
         2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
  (f21 h1 Vq212 + f22 (1 - h1) Vq222 - 2 f31 h1 Vq312) /
   (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
     f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
     2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323),
- (((2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 +
          2 f31 h1 (1 - p1 - p2) Vq312)
        (2 f11 p1 Vq111 - 2 f12 p1 Vq122 + f21 p2 Vq211 + f21 p2 Vq212 - f22 p2 Vq222 -
          f22 p2 Vq223 + 2 f31 (1 - p1 - p2) Vq312 - 2 f32 (1 - p1 - p2) Vq323)) /
      (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
         f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
         2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)^2) +
  (-2 f12 p1 Vq122 + f21 p2 Vq212 - f22 p2 Vq222 + 2 f31 (1 - p1 - p2) Vq312) /
   (2 f11 h1 p1 Vq111 + 2 f12 (1 - h1) p1 Vq122 + f21 h1 p2 Vq211 +
     f21 h1 p2 Vq212 + f22 (1 - h1) p2 Vq222 + f22 (1 - h1) p2 Vq223 +
     2 f31 h1 (1 - p1 - p2) Vq312 + 2 f32 (1 - h1) (1 - p1 - p2) Vq323)},
{- (2 p1 Vm11 + p2 Vm21) (2 Vm11 - 2 Vm32) / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)^2 +
   2 Vm11 / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32),
 - (2 p1 Vm11 + p2 Vm21) (Vm21 + Vm22 - 2 Vm32) / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)^2 +
   Vm21 / (2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32), 0}}]
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-6);
```

```
Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21,
    Vm22, Vm32, f11, f12, f21, f22, f31, f32}] = Block[{}, For[i = 1; write = {},
    i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
           Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
             Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32,
            Chop[eq[[i, 1]]], Chop[eq[[i, 2]]], Chop[eq[[i, 3]]]}]]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21,
    Vm22, Vm32, f11, f12, f21, f22, f31, f32}] = Block[{}, For[i = 1; write = {},
    i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
           Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
             Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32,
            Chop[eq[[i, 1]]], Chop[eq[[i, 2]]], Chop[eq[[i, 3]]]}]]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search:

```
SeedRandom[847 362];
Clear[tab, tab1]
For[j = 1; stabpoly = {}, j ≤ 1 000 000, j++,
 tab = N[Table[Round[Random[] * 10^10] / 10^10, {i, 1, 18}], 60];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 If[
  testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  testPol,
  Print[tab1] && AppendTo[stabpoly, tab1]];
 If[Mod[j, 5000] == 0, Print[j]];
 If[Mod[j, 100 000] == 0, Print[stabpoly]];
]
```

Fixed relative preference scheme of Kirkpatrick

the fixed relative preference scheme of Kirkpatrick :

$$R_{i\,j\times k} = X_{i\,j}\,\frac{\alpha_{i\,j\times k}\,Y_k}{\alpha_{i\,j\times \mathrm{P}}\,Y_{\mathrm{P}}+\alpha_{i\,j\times \mathrm{M}}\,Y_{\mathrm{M}}}$$

$$\mathbf{R11[p1\_,\ h1\_] = p1}\ \frac{\mathbf{f11\ h1}}{\mathbf{f11\ h1 + f12\ (1 - h1)}};$$

$$\mathbf{R12[p1\_,\ h2\_] = p1}\ \frac{\mathbf{f12\ h2}}{\mathbf{f11\ (1 - h2) + f12\ h2}};$$

$$\mathbf{R21[p2\_,\ h1\_] = p2}\ \frac{\mathbf{f21\ h1}}{\mathbf{f21\ h1 + f22\ (1 - h1)}};$$

$$\mathbf{R22[p2\_,\ h2\_] = p2}\ \frac{\mathbf{f22\ h2}}{\mathbf{f21\ (1 - h2) + f22\ h2}};$$

$$\mathbf{R31[p3\_,\ h1\_] = p3}\ \frac{\mathbf{f31\ h1}}{\mathbf{f31\ h1 + f32\ (1 - h1)}};$$

$$\mathbf{R32[p3\_,\ h2\_] = p3}\ \frac{\mathbf{f32\ h2}}{\mathbf{f31\ (1 - h2) + f32\ h2}};$$

Stability of M fixed and polygynous edge

M fixed

Stability, jacobian matrix, analysis of M-fixed:

```
D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1]
D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] /.
D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1]
      p1 → 1 /. p2 → 0 /. h1 → 1 /. maternaleffect // Simplify;
MatrixForm[
 %]
```

$$\begin{pmatrix} \frac{Vq212-2\,Vq312}{2\,Vq111} & -\frac{f12\,Vq122}{f11\,Vq111} & -\frac{Vq312}{Vq111} \\ -\frac{Vm22-2\,Vm32}{2\,Vm11} & 0 & \frac{Vm32}{Vm11} \\ -\frac{Vq212-2\,Vq312}{2\,Vq111} & \frac{f12\,Vq122}{f11\,Vq111} & \frac{Vq312}{Vq111} \end{pmatrix}$$

```
charpoly1 = Factor[Det[% - λ IdentityMatrix[3]]] // Factor
```

$$-\frac{\lambda\left(-f12\,Vm22\,Vq122-f11\,Vm11\,Vq212\,\lambda+2\,f11\,Vm11\,Vq111\,\lambda^2\right)}{2\,f11\,Vm11\,Vq111}$$

Dropping out the λ=0 root and writing as an upwards facing parabola:

```
charpoly1 = Collect[charpoly1 / (-λ), λ, Factor]
```

$$-\frac{f12\,Vm22\,Vq122}{2\,f11\,Vm11\,Vq111}-\frac{Vq212\,\lambda}{2\,Vq111}+\lambda^2$$

```
Solve[charpoly1 == 0, λ]
```

$$\left\{\left\{\lambda\to\frac{1}{4}\left(\frac{Vq212}{Vq111}-\frac{\sqrt{8\,f12\,Vm22\,Vq111\,Vq122+f11\,Vm11\,Vq212^2}}{\sqrt{f11}\,\sqrt{Vm11}\,Vq111}\right)\right\},\right.$$
$$\left.\left\{\lambda\to\frac{1}{4}\left(\frac{Vq212}{Vq111}+\frac{\sqrt{8\,f12\,Vm22\,Vq111\,Vq122+f11\,Vm11\,Vq212^2}}{\sqrt{f11}\,\sqrt{Vm11}\,Vq111}\right)\right\}\right\}$$

The roots are real (positive term inside radical), and the root that is larger in magnitude is the second one, which adds together two positive terms (rather than subtracting one from the other).

The equilibrium is unstable if this larger root is greater than 1:

$$\left\{\lambda\to\frac{1}{4}\left(\frac{f21\,Vq212}{f11\,Vq111}+\frac{\sqrt{8\,f11\,f12\,Vm22\,Vq111\,Vq122+f21^2\,Vm11\,Vq212^2}}{f11\,\sqrt{Vm11}\,Vq111}\right)>1\right.$$

$$\frac{\sqrt{8\,f11\,f12\,Vm22\,Vq111\,Vq122+f21^2\,Vm11\,Vq212^2}}{f11\,\sqrt{Vm11}\,Vq111}>4-\frac{f21\,Vq212}{f11\,Vq111}$$

squaring both sides and simplifying gives that the following must be positive for instability:

$$\text{Factor}\left[\left(\frac{\sqrt{8\,\text{f11 f12 Vm22 Vq111 Vq122} + \text{f21}^2\,\text{Vm11 Vq212}^2}}{\text{f11}\,\sqrt{\text{Vm11}}\,\text{Vq111}}\right)^2 - \left(4 - \frac{\text{f21 Vq212}}{\text{f11 Vq111}}\right)^2\right]$$

$$-\frac{8\,(2\,\text{f11 Vm11 Vq111} - \text{f12 Vm22 Vq122} - \text{f21 Vm11 Vq212})}{\text{f11 Vm11 Vq111}}$$

Therefore M-Fixed is unstable when:

2 Vq111 f11 Vm11 < f12 Vm22 Vq122+f21 Vm11 Vq212

Note that a stability analysis assumes that the frequency of the rare P haplotype is so small that a Taylor series can be performed with respect to that frequency (in the stability matrix above), implicitly assuming that the frequency is much smaller than any of the fitnesses.

Polygynous edge

The possible edge equilibria are:

```
recursionsF == {0, p2, 1 - p2} /. p1 → 0 /. h1 → 0 /. maternaleffect
```

$$\left\{0,\ \frac{\text{p2 Vq222}}{\text{p2 Vq222} + \text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}},\ \frac{\text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}}{\text{p2 Vq222} + \text{p2 Vq223} + 2\,(1 - \text{p2})\,\text{Vq323}}\right\} == \{0,\ \text{p2},\ 1 - \text{p2}\}$$

```
Solve[%, p2]
edgeequil = p2 /. %;
```

$$\left\{\{\text{p2} \to 0\},\ \left\{\text{p2} \to \frac{\text{Vq222} - 2\,\text{Vq323}}{\text{Vq222} + \text{Vq223} - 2\,\text{Vq323}}\right\}\right\}$$

Note that when there is a discontinuity when h1 ==0. As soon as h1 is present at any frequency the probability of p1 mating with h1 goes form being 0 to 1.

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
      maternaleffect /. p1 → 0 /. h1 → 0 // Simplify;
MatrixForm[%]
Factor[Det[% - λ IdentityMatrix[3]]];
quad2 = Collect[%, λ, Factor];
```

$$\begin{pmatrix} \frac{2\,\text{Vq222 Vq323}}{(\text{p2}\,(\text{Vq222}+\text{Vq223}-2\,\text{Vq323})+2\,\text{Vq323})^2} & \frac{\text{f21 f32 p2}\,(2\,(\text{Vq212}-\text{Vq222})\,\text{Vq323}+\text{p2}\,(\text{Vq212 Vq223}-2\,\text{Vq212 Vq323}+2\,\text{Vq222 Vq323}))-2\,\text{f22}}{\text{f22 f32}\,(\text{p2}\,(\text{Vq222}+\text{Vq223}-2\,\text{Vq}} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The term $\frac{2\,\text{f22 f32 Vq222 Vq323}}{(\text{f22 p2}\,(\text{Vq222}+\text{Vq223})-2\,\text{f32}\,(-1+\text{p2})\,\text{Vq323})^2}$ represents the internal stability of the equilibria

along the polygynous edge with respect to changes in p2 (from D[recursionsF[[2]], p2]). Evaluating this at the two possible edge equilibria we get:

```
2 f22 f32 Vq222 Vq323
---------------------------------------------------------- /. p2 -> edgeequil // Simplify
(f22 p2 (Vq222 + Vq223) - 2 f32 (-1 + p2) Vq323)^2
```

$$\left\{\frac{f22\ Vq222}{2\ f32\ Vq323},\ \frac{2\ f22\ f32\ Vq222\ (Vq222+Vq223-2\ Vq323)^2\ Vq323}{(f22\ (Vq222+Vq223)\ (Vq222-2\ Vq323)+2\ f32\ Vq223\ Vq323)^2}\right\}$$

Thus, if $f22\ Vq222 > 2\ f32\ Vq323$ then the p2=0 equilibrium is unstable and the $p2 = \frac{f22\ Vq222-2\ f32\ Vq323}{f22\ Vq222+f22\ Vq223-2\ f32\ Vq323}$ equilibrium is stable along the polygynous edge (and vice versa). Thus, $\frac{2\ f22\ f32\ Vq222\ Vq323}{(f22\ p2\ (Vq222+Vq223)-2\ f32\ (-1+p2)\ Vq323)^2}$ will be less than one at the stable equilibrium, whichever one it is.

The spread of the rare M haplotype is determined by the 2x2 external stability matrix obtained by deleting the first row and column, the eigenvalues of which are both 0. Thus, the M haplotype will never spread when rare. In fact, the rare MM and M genotypes will both disappear in a single generation (the meaning of both eigenvalues being equal to zero), because of maternal-effect killing.

Approximate dynamics when M is rare but allowing extreme preference differences (P-fixed)

Lets assume we are near P-fixed and ask what would happen for MM females to not disappear when rare. To do this, we assume that MM, MP, and M are rare and determine when MM in the next generation won't be negligible:

```
Normal[
  Series[recursionsF /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /. p1 → ϵ * ϵp1 /.
    p2 → ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1
```

$$\left\{0,\ \frac{2\ f31\ Vq312\ \epsilon h+2\ f32\ Vq122\ \epsilon p1+f32\ Vq222\ \epsilon p2}{2\ f32\ Vq323},\ 1+\frac{-2\ f31\ Vq312\ \epsilon h-2\ f32\ Vq122\ \epsilon p1-f32\ Vq222\ \epsilon p2}{2\ f32\ Vq323}\right\}$$

```
Normal[
  Series[recursionsM /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /. p1 → ϵ * ϵp1 /.
    p2 → ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1
```

$$\left\{\frac{Vm11\ \epsilon p1}{Vm32},\ 1-\frac{Vm11\ \epsilon p1}{Vm32}\right\}$$

The first term of the female recursions indicates that ϵp1 is zero in the next generation, which means loss of MM females and then subsequently the loss of M males. We need the first element to not be zero in order to allow the M haplotype to persist, so we ask what fitness regime would compensate for the rarity of the M haplotype and yet allow that first element to be large:

```
Normal[Series[recursionsF[[1]] /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /.
    p1 → ϵ * ϵp1 /. p2 → ϵ * ϵp2, {ϵ, 0, 2}]] /. ϵ → 1
```

$$\frac{f11\ Vq111\ \epsilon h\ \epsilon p1}{f12\ Vq323}$$

So if $\frac{f11\ Vq111}{f12\ Vq323}$ is of order $1/\epsilon$ then rare MM females will be maintained at a rare state (but not disappear).

Approximate dynamics when M is rare but allowing extreme preference differences (MP/PP-edge)

Lets assume we are near MP/PP edge where p2 = $\frac{Vq222-2\ Vq323}{Vq222+Vq223-2\ Vq323}$ and ask what would happen for MM females to not disappear when rare. To do this, we determine when MM in the next generation won't be negligible:

```
Simplify[Normal[
  Series[recursionsF /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /. p1 → ϵ * ϵp1 /.
    p2 → (Vq222 - 2 Vq323)/(Vq222 + Vq223 - 2 Vq323) + ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1]
```

$$\left\{0,\ \frac{1}{f22\ f32\ Vq222\ (Vq222+Vq223-2\ Vq323)^2}\left(f21\ f32\ Vq223\ (Vq212-2\ Vq323)\ (Vq222-2\ Vq323)\ \epsilon h + f22\left(2\ f31\ Vq223\ (Vq223\ Vq312+(Vq222-2\ Vq323)\ Vq323)\ \epsilon h + f32\ (Vq222+Vq223-2\ Vq323)\left(Vq222^2+2\ Vq222\ Vq323\ (-1+\epsilon p1+\epsilon p2)+2\ (Vq122\ Vq223\ \epsilon p1+Vq323\ (Vq223\ \epsilon p2-2\ Vq323\ (\epsilon p1+\epsilon p2)))\right)\right)\right),\ \frac{1}{f22\ f32\ Vq222\ (Vq222+Vq223-2\ Vq323)^2}\ (f21\ f32\ Vq223\ (Vq212-2\ Vq323)\ (-Vq222+2\ Vq323)\ \epsilon h + f22\ (-2\ f31\ Vq223\ (Vq223\ Vq312+(Vq222-2\ Vq323)\ Vq323)\ \epsilon h + f32\ (Vq222+Vq223-2\ Vq323)\ (-2\ Vq122\ Vq223\ \epsilon p1+Vq222\ (Vq223-2\ Vq323\ (\epsilon p1+\epsilon p2))+2\ Vq323\ (-Vq223\ \epsilon p2+2\ Vq323\ (\epsilon p1+\epsilon p2)))))\right\}$$

```
Normal[
  Series[recursionsM /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /. p1 → ϵ * ϵp1 /.
    p2 → (Vq222 - 2 Vq323)/(Vq222 + Vq223 - 2 Vq323) + ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1
```

$$\left\{\frac{2\ Vm11\ (Vq222+Vq223-2\ Vq323)\ \epsilon p1}{Vm22\ Vq222+2\ Vm32\ Vq223-2\ Vm22\ Vq323},\ 1-\frac{2\ (Vm11\ Vq222\ \epsilon p1+Vm11\ Vq223\ \epsilon p1-2\ Vm11\ Vq323\ \epsilon p1)}{Vm22\ Vq222+2\ Vm32\ Vq223-2\ Vm22\ Vq323}\right\}$$

The first term of the female recursions indicates that ϵp1 is zero in the next generation, which means loss of MM females and then subsequently the loss of M males. We need the first element to not be zero in order to allow the M haplotype to persist, so we ask what fitness regime would compensate for the rarity of the M haplotype and yet allow that first element to be large:

```
Normal[Series[
   recursionsF[[1]] /. maternaleffect /. h1 → ϵ * ϵh /. h2 → 1 - ϵ * ϵh /. p1 → ϵ * ϵp1 /.
    p2 → (Vq222 - 2 Vq323)/(Vq222 + Vq223 - 2 Vq323) + ϵ * ϵp2, {ϵ, 0, 2}]] /. ϵ → 1
```

$$\frac{2\ \mathrm{f11}\ \mathrm{Vq111}\ \epsilon\mathrm{h}\ \epsilon\mathrm{p1}}{\mathrm{f12}\ \mathrm{Vq222}}$$

So if $\frac{2\ \mathrm{f11}\ \mathrm{Vq111}}{\mathrm{f12}\ \mathrm{Vq222}}$ is of order $1/\epsilon$ then rare MM females will be maintained at a rare state (but not disappear).

Approximate dynamics when P is rare but allowing extreme preference differences

Lets assume we are near M-fixed and ask what would allow PP females to not disappear when rare (i.e., when would mating preferences be strong enough that they induce nearly assortative mating for a given frequency of the P haplotype).

```
Simplify[
 Normal[Series[recursionsF /. maternaleffect /. h1 → 1 - ϵ * ϵh /. h2 → ϵ * ϵh /. p1 →
       1 - ϵ * ϵp2 - ϵ * ϵp3 /. p2 → ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1]
```

$$\left\{1 - \frac{2\ \mathrm{f12}\ \mathrm{Vq122}\ \epsilon\mathrm{h} + \mathrm{f11}\ \mathrm{Vq212}\ \epsilon\mathrm{p2} + 2\ \mathrm{f11}\ \mathrm{Vq312}\ \epsilon\mathrm{p3}}{2\ \mathrm{f11}\ \mathrm{Vq111}},\ \frac{2\ \mathrm{f12}\ \mathrm{Vq122}\ \epsilon\mathrm{h} + \mathrm{f11}\ \mathrm{Vq212}\ \epsilon\mathrm{p2} + 2\ \mathrm{f11}\ \mathrm{Vq312}\ \epsilon\mathrm{p3}}{2\ \mathrm{f11}\ \mathrm{Vq111}},\ 0\right\}$$

```
Simplify[
 Normal[Series[recursionsM /. maternaleffect /. h1 → 1 - ϵ * ϵh /. h2 → ϵ * ϵh /. p1 →
       1 - ϵ * ϵp2 - ϵ * ϵp3 /. p2 → ϵ * ϵp2, {ϵ, 0, 1}]] /. ϵ → 1]
```

$$\left\{1 - \frac{\mathrm{Vm22}\ \epsilon\mathrm{p2} + 2\ \mathrm{Vm32}\ \epsilon\mathrm{p3}}{2\ \mathrm{Vm11}},\ \frac{\mathrm{Vm22}\ \epsilon\mathrm{p2} + 2\ \mathrm{Vm32}\ \epsilon\mathrm{p3}}{2\ \mathrm{Vm11}}\right\}$$

The last term of the female recursions indicates that ϵp3 is zero in the next generation, which means loss of PP females and then subsequently the loss of M males. We need the first element to not be zero in order to allow the M haplotype to persist, so we ask what fitness regime would compensate for the rarity of the M haplotype and yet allow that first element to be large:

```
Expand[
 Normal[Series[recursionsF[[3]] /. maternaleffect /. h1 → 1 - ϵ * ϵh /. h2 → ϵ * ϵh /.
      p1 → 1 - ϵ * ϵp2 - ϵ * ϵp3 /. p2 → ϵ * ϵp2, {ϵ, 0, 2}]] /. ϵ → 1]
```

$$\frac{f22\ Vq223\ \epsilon h\ \epsilon p2}{2\ f21\ Vq111} + \frac{f32\ Vq323\ \epsilon h\ \epsilon p3}{f31\ Vq111}$$

So if $\frac{f22\ Vq223}{2\ f21\ Vq111} + \frac{f32\ Vq323}{f31\ Vq111}$ is of order $1/\epsilon$ then rare PP females will be maintained despite being the rare homozygote.

Social polymorphism numerical search

Numerical search: same preference for all queens

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] // Simplify

eqn2 = Factor[recursionsF[[2]] - p2] // Simplify

eqn3 = Factor[recursionsM[[1]] - h1] // Simplify

findequil[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
    Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323,
    Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}] =
  {p1, p2, h1} /. NSolve[{(f12 (-1 + h1) (f22 (-1 + h1) p1
                (- f31 h1 (p2 (Vq222 + Vq223 - 2 Vq312) + 2 p1 (Vq122 - Vq312) + 2 Vq312) + f32
                    (-1 + h1) (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323)) +
              f21 h1 (f31 h1 (-p2 Vq211 + 2 p1^2 (Vq122 - Vq312) + p1
                      (p2 (Vq211 + Vq212 - 2 Vq312) + 2 Vq312)) - f32 (-1 + h1) (-p2 Vq211 +
                     2 p1^2 (Vq122 - Vq323) + p1 (p2 (Vq211 + Vq212 - 2 Vq323) + 2 Vq323)))) -
            f11 h1 (f22 (-1 + h1) p1 (- f31 h1 (2 (-1 + p1) Vq111 + p2 (Vq222 +
                        Vq223 - 2 Vq312) - 2 (-1 + p1) Vq312) + f32 (-1 + h1)
                    (2 (-1 + p1) Vq111 + p2 (Vq222 + Vq223 - 2 Vq323) - 2 (-1 + p1) Vq323)) +
              f21 h1 (f31 h1 (-p2 Vq211 + 2 p1^2 (Vq111 - Vq312) + p1
                      (-2 Vq111 + p2 (Vq211 + Vq212 - 2 Vq312) + 2 Vq312)) -
                  f32 (-1 + h1) (-p2 Vq211 + 2 p1^2 (Vq111 - Vq323) +
                     p1 (-2 Vq111 + p2 (Vq211 + Vq212 - 2 Vq323) + 2 Vq323)))) /
          (f11 h1 (f21 h1 (f31 h1 (p2 (Vq211 + Vq212 - 2 Vq312) + 2 p1 (Vq111 - Vq312) +
                     2 Vq312) - f32 (-1 + h1) (p2 (Vq211 + Vq212 - 2 Vq323) +
```

```
                2 p1 (Vq111 - Vq323) + 2 Vq323)) + f22 (-1 + h1) (-f31 h1
            (p2 (Vq222 + Vq223 - 2 Vq312) + 2 p1 (Vq111 - Vq312) + 2 Vq312) + f32 (-1 +
              h1) (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323))) -
    f12 (-1 + h1) (f21 h1 (f31 h1 (p2 (Vq211 + Vq212 - 2 Vq312) + 2 p1
                (Vq122 - Vq312) + 2 Vq312) - f32 (-1 + h1)
            (p2 (Vq211 + Vq212 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323)) +
        f22 (-1 + h1) (-f31 h1 (p2 (Vq222 + Vq223 - 2 Vq312) +
                2 p1 (Vq122 - Vq312) + 2 Vq312) + f32 (-1 + h1)
            (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323)))) == 0,
(-f11 h1 (f21 h1 (f31 h1 (-p2 (Vq212 - 4 Vq312) + p2^2 (Vq211 + Vq212 - 2 Vq312) -
                2 Vq312 + 2 p1 (p2 (Vq111 - Vq312) + Vq312)) - f32 (-1 + h1) p2
            (-Vq212 + p2 (Vq211 + Vq212 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323)) +
        f22 (-1 + h1) (-f31 h1 (-p2 (Vq222 - 4 Vq312) + p2^2 (Vq222 + Vq223 - 2 Vq312) -
                2 Vq312 + 2 p1 (p2 (Vq111 - Vq312) + Vq312)) + f32
            (-1 + h1) p2 ((-1 + p2) Vq222 + p2 Vq223 + 2 p1 (Vq111 - Vq323) +
                2 Vq323 - 2 p2 Vq323)) + f12 (-1 + h1)
      (f21 h1 (f31 h1 (-p2 (Vq212 - 4 Vq312) + p2^2 (Vq211 + Vq212 - 2 Vq312) +
                2 p1 (-1 + p2) (Vq122 - Vq312) - 2 Vq312) - f32 (-1 + h1)
            (p2 (-Vq212 + p2 (Vq211 + Vq212 - 2 Vq323) + 2 Vq323) +
                2 p1 ((-1 + p2) Vq122 - p2 Vq323)) + f22 (-1 + h1)
          (f31 h1 (p2 (Vq222 - 4 Vq312) - p2^2 (Vq222 + Vq223 - 2 Vq312) -
                2 p1 (-1 + p2) (Vq122 - Vq312) + 2 Vq312) + f32 (-1 + h1)
            (p2 ((-1 + p2) Vq222 + p2 Vq223 + 2 Vq323 - 2 p2 Vq323) +
                2 p1 ((-1 + p2) Vq122 - p2 Vq323)))) /
  (f11 h1 (f21 h1 (f31 h1 (p2 (Vq211 + Vq212 - 2 Vq312) + 2 p1 (Vq111 - Vq312) +
                2 Vq312) - f32 (-1 + h1) (p2 (Vq211 + Vq212 - 2 Vq323) +
                2 p1 (Vq111 - Vq323) + 2 Vq323)) + f22 (-1 + h1) (-f31 h1
            (p2 (Vq222 + Vq223 - 2 Vq312) + 2 p1 (Vq111 - Vq312) + 2 Vq312) + f32 (-1 +
              h1) (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323))) -
    f12 (-1 + h1) (f21 h1 (f31 h1 (p2 (Vq211 + Vq212 - 2 Vq312) + 2 p1
                (Vq122 - Vq312) + 2 Vq312) - f32 (-1 + h1)
            (p2 (Vq211 + Vq212 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323)) +
        f22 (-1 + h1) (-f31 h1 (p2 (Vq222 + Vq223 - 2 Vq312) +
                2 p1 (Vq122 - Vq312) + 2 Vq312) + f32 (-1 + h1)
            (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323)))) == 0,
 -2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32)
 ------------------------------------------------------------------------------------------ ==
                p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32
 0}, {p1,
p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

D[recursionsF[[1]], p1]	D[recursionsF[[1]], p2]	D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1]	D[recursionsF[[2]], p2]	D[recursionsF[[2]], h1]
D[recursionsM[[1]], p1]	D[recursionsM[[1]], p2]	D[recursionsM[[1]], h1]

```
Clear[checkstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_, Vm11_,
   Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323,
    Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32, p1, p2, h1}] =
  Eigenvalues[{{-(((2 f11 h1 p1 Vq111/(f12 (1 - h1) + f11 h1) + f21 h1 p2 Vq211/(f22 (1 - h1) + f21 h1))
          (2 f11 h1 Vq111/(f12 (1 - h1) + f11 h1) +
            2 f12 (1 - h1) Vq122/(f12 (1 - h1) + f11 h1) - 2 f31 h1 Vq312/(f32 (1 - h1) + f31 h1) -
            2 f32 (1 - h1) Vq323/(f32 (1 - h1) + f31 h1)))/
        (2 f11 h1 p1 Vq111/(f12 (1 - h1) + f11 h1) + 2 f12 (1 - h1) p1 Vq122/(f12 (1 - h1) + f11 h1) +
          f21 h1 p2 Vq211/(f22 (1 - h1) + f21 h1) +
          f21 h1 p2 Vq212/(f22 (1 - h1) + f21 h1) + f22 (1 - h1) p2 Vq222/(f22 (1 - h1) + f21 h1) +
          f22 (1 - h1) p2 Vq223/(f22 (1 - h1) + f21 h1) +
          2 f31 h1 (1 - p1 - p2) Vq312/(f32 (1 - h1) + f31 h1) +
          2 f32 (1 - h1) (1 - p1 - p2) Vq323/(f32 (1 - h1) + f31 h1))^2)) +
      (2 f11 h1 Vq111)/((f12 (1 - h1) + f11 h1) (2 f11 h1 p1 Vq111/(f12 (1 - h1) + f11 h1) +
          2 f12 (1 - h1) p1 Vq122/(f12 (1 - h1) + f11 h1) + f21 h1 p2 Vq211/(f22 (1 - h1) + f21 h1) +
          f21 h1 p2 Vq212/(f22 (1 - h1) + f21 h1) + f22 (1 - h1) p2 Vq222/(f22 (1 - h1) + f21 h1) +
          f22 (1 - h1) p2 Vq223/(f22 (1 - h1) + f21 h1) +
          2 f31 h1 (1 - p1 - p2) Vq312/(f32 (1 - h1) + f31 h1) +
          2 f32 (1 - h1) (1 - p1 - p2) Vq323/(f32 (1 - h1) + f31 h1))),
     -(((2 f11 h1 p1 Vq111/(f12 (1 - h1) + f11 h1) + f21 h1 p2 Vq211/(f22 (1 - h1) + f21 h1))
          (f21 h1 Vq211/(f22 (1 - h1) + f21 h1) +
            f21 h1 Vq212/(f22 (1 - h1) + f21 h1) + f22 (1 - h1) Vq222/(f22 (1 - h1) + f21 h1) +
            f22 (1 - h1) Vq223/(f22 (1 - h1) + f21 h1) -
            2 f31 h1 Vq312/(f32 (1 - h1) + f31 h1) - 2 f32 (1 - h1) Vq323/(f32 (1 - h1) + f31 h1)))/
        (2 f11 h1 p1 Vq111/(f12 (1 - h1) + f11 h1) + 2 f12 (1 - h1) p1 Vq122/(f12 (1 - h1) + f11 h1) +
          f21 h1 p2 Vq211/(f22 (1 - h1) + f21 h1) +
          f21 h1 p2 Vq212/(f22 (1 - h1) + f21 h1) + f22 (1 - h1) p2 Vq222/(f22 (1 - h1) + f21 h1) +
          f22 (1 - h1) p2 Vq223/(f22 (1 - h1) + f21 h1) +
```

$$\frac{2\,f31\,h1\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}+\frac{2\,f32\,(1-h1)\,(1-p1-p2)\,Vq323}{f32\,(1-h1)+f31\,h1}\Bigg)^2\Bigg)+$$

$$(f21\,h1\,Vq211)\Big/\Bigg((f22\,(1-h1)+f21\,h1)\Bigg(\frac{2\,f11\,h1\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}+$$

$$\frac{2\,f12\,(1-h1)\,p1\,Vq122}{f12\,(1-h1)+f11\,h1}+\frac{f21\,h1\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}+$$

$$\frac{f21\,h1\,p2\,Vq212}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq222}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq223}{f22\,(1-h1)+f21\,h1}+$$

$$\frac{2\,f31\,h1\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}+\frac{2\,f32\,(1-h1)\,(1-p1-p2)\,Vq323}{f32\,(1-h1)+f31\,h1}\Bigg)\Bigg),$$

$$-\Bigg(\Bigg(\Bigg(\frac{2\,f11\,h1\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}+\frac{f21\,h1\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}\Bigg)\Bigg(-\frac{2\,f11\,(f11-f12)\,h1\,p1\,Vq111}{(f12\,(1-h1)+f11\,h1)^2}+$$

$$\frac{2\,f11\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}-\frac{2\,(f11-f12)\,f12\,(1-h1)\,p1\,Vq122}{(f12\,(1-h1)+f11\,h1)^2}-$$

$$\frac{2\,f12\,p1\,Vq122}{f12\,(1-h1)+f11\,h1}-\frac{f21\,(f21-f22)\,h1\,p2\,Vq211}{(f22\,(1-h1)+f21\,h1)^2}+$$

$$\frac{f21\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}-\frac{f21\,(f21-f22)\,h1\,p2\,Vq212}{(f22\,(1-h1)+f21\,h1)^2}+\frac{f21\,p2\,Vq212}{f22\,(1-h1)+f21\,h1}-$$

$$\frac{(f21-f22)\,f22\,(1-h1)\,p2\,Vq222}{(f22\,(1-h1)+f21\,h1)^2}-\frac{f22\,p2\,Vq222}{f22\,(1-h1)+f21\,h1}-$$

$$\frac{(f21-f22)\,f22\,(1-h1)\,p2\,Vq223}{(f22\,(1-h1)+f21\,h1)^2}-\frac{f22\,p2\,Vq223}{f22\,(1-h1)+f21\,h1}-$$

$$\frac{2\,f31\,(f31-f32)\,h1\,(1-p1-p2)\,Vq312}{(f32\,(1-h1)+f31\,h1)^2}+\frac{2\,f31\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}-$$

$$\frac{2\,(f31-f32)\,f32\,(1-h1)\,(1-p1-p2)\,Vq323}{(f32\,(1-h1)+f31\,h1)^2}-\frac{2\,f32\,(1-p1-p2)\,Vq323}{f32\,(1-h1)+f31\,h1}\Bigg)\Bigg)\Big/$$

$$\Bigg(\frac{2\,f11\,h1\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}+\frac{2\,f12\,(1-h1)\,p1\,Vq122}{f12\,(1-h1)+f11\,h1}+\frac{f21\,h1\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}+$$

$$\frac{f21\,h1\,p2\,Vq212}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq222}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq223}{f22\,(1-h1)+f21\,h1}+$$

$$\frac{2\,f31\,h1\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}+\frac{2\,f32\,(1-h1)\,(1-p1-p2)\,Vq323}{f32\,(1-h1)+f31\,h1}\Bigg)^2\Bigg)+$$

$$\Bigg(-\frac{2\,f11\,(f11-f12)\,h1\,p1\,Vq111}{(f12\,(1-h1)+f11\,h1)^2}+\frac{2\,f11\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}-$$

$$\frac{f21\,(f21-f22)\,h1\,p2\,Vq211}{(f22\,(1-h1)+f21\,h1)^2}+\frac{f21\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}\Bigg)\Big/$$

$$\Bigg(\frac{2\,f11\,h1\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}+\frac{2\,f12\,(1-h1)\,p1\,Vq122}{f12\,(1-h1)+f11\,h1}+\frac{f21\,h1\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}+$$

$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)\Bigg\},$$

$$\Bigg\{-\left(\left(\left(\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\right.\right.\right.$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}\right)\left(\frac{2\ f11\ h1\ Vq111}{f12\ (1-h1)+f11\ h1}+\right.$$

$$\left.\left.\frac{2\ f12\ (1-h1)\ Vq122}{f12\ (1-h1)+f11\ h1}-\frac{2\ f31\ h1\ Vq312}{f32\ (1-h1)+f31\ h1}-\frac{2\ f32\ (1-h1)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)\right)\Bigg/$$

$$\left(\frac{2\ f11\ h1\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}+\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}+\right.$$

$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)^2\Bigg)+$$

$$\left(\frac{2\ f12\ (1-h1)\ Vq122}{f12\ (1-h1)+f11\ h1}-\frac{2\ f31\ h1\ Vq312}{f32\ (1-h1)+f31\ h1}\right)\Bigg/\left(\frac{2\ f11\ h1\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}+\right.$$

$$\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}+$$

$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right),$$

$$-\left(\left(\left(\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\right.\right.\right.$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}\right)$$

$$\left(\frac{f21\ h1\ Vq211}{f22\ (1-h1)+f21\ h1}+\frac{f21\ h1\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ Vq222}{f22\ (1-h1)+f21\ h1}+\right.$$

$$\left.\left.\frac{f22\ (1-h1)\ Vq223}{f22\ (1-h1)+f21\ h1}-\frac{2\ f31\ h1\ Vq312}{f32\ (1-h1)+f31\ h1}-\frac{2\ f32\ (1-h1)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)\right)\Bigg/$$

$$\left(\frac{2\ f11\ h1\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}+\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}+\right.$$

$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$

$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)^2\Bigg)+$$

$$\left(\frac{f21\ h1\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ Vq222}{f22\ (1-h1)+f21\ h1}-\frac{2\ f31\ h1\ Vq312}{f32\ (1-h1)+f31\ h1}\right)\Big/$$
$$\left(\frac{2\ f11\ h1\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}+\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}+\right.$$
$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$
$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right),$$
$$-\left(\left(\left(\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\right.\right.\right.$$
$$\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}\right)\left(-\frac{2\ f11\ (f11-f12)\ h1\ p1\ Vq111}{(f12\ (1-h1)+f11\ h1)^2}+\right.$$
$$\frac{2\ f11\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}-\frac{2\ (f11-f12)\ f12\ (1-h1)\ p1\ Vq122}{(f12\ (1-h1)+f11\ h1)^2}-$$
$$\frac{2\ f12\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}-\frac{f21\ (f21-f22)\ h1\ p2\ Vq211}{(f22\ (1-h1)+f21\ h1)^2}+$$
$$\frac{f21\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}-\frac{f21\ (f21-f22)\ h1\ p2\ Vq212}{(f22\ (1-h1)+f21\ h1)^2}+\frac{f21\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}-$$
$$\frac{(f21-f22)\ f22\ (1-h1)\ p2\ Vq222}{(f22\ (1-h1)+f21\ h1)^2}-\frac{f22\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}-$$
$$\frac{(f21-f22)\ f22\ (1-h1)\ p2\ Vq223}{(f22\ (1-h1)+f21\ h1)^2}-\frac{f22\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}-$$
$$\frac{2\ f31\ (f31-f32)\ h1\ (1-p1-p2)\ Vq312}{(f32\ (1-h1)+f31\ h1)^2}+\frac{2\ f31\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}-$$
$$\left.\left.\frac{2\ (f31-f32)\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{(f32\ (1-h1)+f31\ h1)^2}-\frac{2\ f32\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)\right)\Big/$$
$$\left(\frac{2\ f11\ h1\ p1\ Vq111}{f12\ (1-h1)+f11\ h1}+\frac{2\ f12\ (1-h1)\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}+\frac{f21\ h1\ p2\ Vq211}{f22\ (1-h1)+f21\ h1}+\right.$$
$$\frac{f21\ h1\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}+\frac{f22\ (1-h1)\ p2\ Vq223}{f22\ (1-h1)+f21\ h1}+$$
$$\left.\left.\frac{2\ f31\ h1\ (1-p1-p2)\ Vq312}{f32\ (1-h1)+f31\ h1}+\frac{2\ f32\ (1-h1)\ (1-p1-p2)\ Vq323}{f32\ (1-h1)+f31\ h1}\right)^2\right)+$$
$$\left(-\frac{2\ (f11-f12)\ f12\ (1-h1)\ p1\ Vq122}{(f12\ (1-h1)+f11\ h1)^2}-\frac{2\ f12\ p1\ Vq122}{f12\ (1-h1)+f11\ h1}-\right.$$
$$\frac{f21\ (f21-f22)\ h1\ p2\ Vq212}{(f22\ (1-h1)+f21\ h1)^2}+\frac{f21\ p2\ Vq212}{f22\ (1-h1)+f21\ h1}-$$
$$\frac{(f21-f22)\ f22\ (1-h1)\ p2\ Vq222}{(f22\ (1-h1)+f21\ h1)^2}-\frac{f22\ p2\ Vq222}{f22\ (1-h1)+f21\ h1}-$$

$$\frac{2\,f31\,(f31-f32)\,h1\,(1-p1-p2)\,Vq312}{(f32\,(1-h1)+f31\,h1)^2}+\frac{2\,f31\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}\Bigg)\Bigg/$$
$$\Bigg(\frac{2\,f11\,h1\,p1\,Vq111}{f12\,(1-h1)+f11\,h1}+\frac{2\,f12\,(1-h1)\,p1\,Vq122}{f12\,(1-h1)+f11\,h1}+\frac{f21\,h1\,p2\,Vq211}{f22\,(1-h1)+f21\,h1}+$$
$$\frac{f21\,h1\,p2\,Vq212}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq222}{f22\,(1-h1)+f21\,h1}+\frac{f22\,(1-h1)\,p2\,Vq223}{f22\,(1-h1)+f21\,h1}+$$
$$\frac{2\,f31\,h1\,(1-p1-p2)\,Vq312}{f32\,(1-h1)+f31\,h1}+\frac{2\,f32\,(1-h1)\,(1-p1-p2)\,Vq323}{f32\,(1-h1)+f31\,h1}\Bigg)\Bigg\},$$
$$\Bigg\{-\frac{(2\,p1\,Vm11+p2\,Vm21)\,(2\,Vm11-2\,Vm32)}{(2\,p1\,Vm11+p2\,Vm21+p2\,Vm22+2\,Vm32-2\,p1\,Vm32-2\,p2\,Vm32)^2}+$$
$$\frac{2\,Vm11}{2\,p1\,Vm11+p2\,Vm21+p2\,Vm22+2\,Vm32-2\,p1\,Vm32-2\,p2\,Vm32},$$
$$-\frac{(2\,p1\,Vm11+p2\,Vm21)\,(Vm21+Vm22-2\,Vm32)}{(2\,p1\,Vm11+p2\,Vm21+p2\,Vm22+2\,Vm32-2\,p1\,Vm32-2\,p2\,Vm32)^2}+$$
$$\frac{Vm21}{2\,p1\,Vm11+p2\,Vm21+p2\,Vm22+2\,Vm32-2\,p1\,Vm32-2\,p2\,Vm32},\ 0\Bigg\}\Bigg\}\Bigg]$$

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-10);
```

```
Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21,
    Vm22, Vm32, f11, f12, f21, f22, f31, f32}] = Block[{}, For[i = 1; write = {},
    i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32,
            Chop[eq[[i, 1]]], Chop[eq[[i, 2]]], Chop[eq[[i, 3]]]}]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21,
    Vm22, Vm32, f11, f12, f21, f22, f31, f32}] = Block[{}, For[i = 1; write = {},
    i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
           Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
             Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32,
            Chop[eq[[i, 1]]], Chop[eq[[i, 2]]], Chop[eq[[i, 3]]]}]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   write = DeleteDuplicates[write];
   Sort[write]
  ]
```

Classify evaluates what kind of equilibria are stable (P1=P-fixed, P2=PP/MP edge, M=M-fixed,B=both M and P haplotypes). The classifier gives the number of stable equilibria, the classes of those equilibria (should sum to the number), and then the three fertility advantages to the rare types (near P1, P2, or M equilibria).

```
Clear[classify]
classify[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_, Vq312_, Vq323_,
   Vm11_, Vm21_, Vm22_, Vm32_, f11_, f12_, f21_, f22_, f31_, f32_}] :=
 classify[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323,
    Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}] =
  Block[{myeq = sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
        Vq323, Vm11, Vm21, Vm22, Vm32, f11, f12, f21, f22, f31, f32}]},
   write = {Length[myeq]};
   If[Length[Select[myeq, (#[[1]] == 0 && #[[2]] == 0 && #[[3]] == 0) &]] > 0,
    write = Append[write, "P1"]];
   If[Length[Select[myeq, (#[[1]] == 0 && #[[2]] > 0 && #[[3]] == 0) &]] > 0,
    write = Append[write, "P2"]];
   If[Length[Select[myeq, (#[[1]] == 1 && #[[2]] == 0 && #[[3]] == 1) &]] > 0,
    write = Append[write, "M"]];
   If[Length[Select[myeq, (#[[1]] > 0 && #[[2]] > 0 && #[[3]] > 0) &]] > 0,
    write = Append[write, "B"]];
   write = Append[write, {f11 Vq111/(f12 Vq323), 2 f11 Vq111/(f12 Vq222), f22 Vq223/(2 f21 Vq111) + f32 Vq323/(f31 Vq111)}]
  ]
```

Numerical search: (1 million complete no results)

Male preference is constant among queens:
(near zero values for preference are the same as assortative mating which is looked at in the following section, therefore the parameter space looked at here is between 10^(-4) and 1 for all parameters)

```
SeedRandom[25 434]
Clear[tab]
For[j = 1;
 polytabsMalePrefCons = {},
 j ≤ 1 000 000, j ++,
 tab = N[Table[Round[Random[Real, {10^(-4), 1}] * 10^10] / 10^10, {i, 1, 18}], 60];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 tab1[[15 ;; 16]] = tab1[[13 ;; 14]];
 (*pref for m males and p males constant*)
 tab1[[17 ;; 18]] = tab1[[13 ;; 14]];
 (*pref for m males and p males constant*)
 If[testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  (*True if stable social polymorphism exists*)
  testPol,
  AppendTo[polytabsMalePrefCons, tab1] && Print[classify[tab1]]];
 If[Mod[j, 5000] == 0, Print[j]]]
```

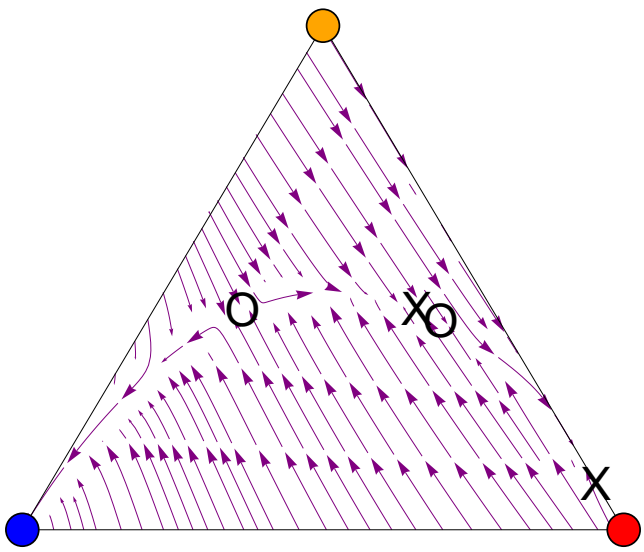
Simplex stream diagram

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)

  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog,
   {Text[Style["X", Large], {-0.125 + (1 - 0.125 - 0.4345), .4345}]}];
  AppendTo[myEpilog, {Text[Style["X", Large], {-0 + (1 - 0 - 0.0886), 0.0886}]}];

  AppendTo[myEpilog,
   {Text[Style["O", Large], {-0.415 + (1 - 0.415 - 0.4341), .4341}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0.096 + (1 - 0.096 - 0.412), .412}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0.0, 1.0}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → √3 / 2,
   Frame → False,
   StreamPoints → Fine,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]
```

```
funcs = {d1n/dT - p1, d2n/dT - p2, d3n/dT - p3} /. maternaleffect /. h2 → 1 - h1 /.
    h1 → (2 p1 Vm11 + p2 Vm21)/(2 p1 Vm11 + p2 Vm21 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32) /.
    Vq111 -> 0.459 /. Vq122 -> 0.690 /. Vq211 -> 0 /. Vq212 ->
    0.690 /. Vq222 -> 0.690 /. Vq223 -> 0.329 /. Vq312 -> 0.690 /.
    Vq323 -> 0.329 /. Vm11 → 1 /. Vm21 -> 0 /. Vm22 → 1 /. Vm32 → 1 /.
    f11 -> 0.5 /. f12 -> 0.0052 /. f21 -> 0.245 /. f22 -> 0.361 /.
    f31 -> 0.245 /. f32 -> 0.361 // Simplify;

vars = {p1[t], p2[t], p3[t]};

args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];

Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



```
findequil[{0.459`, 0.69`, 0, 0.69`, 0.69`, 0.329`, 0.69`,
  0.329`, 1, 0, 1, 1, 0.5`, 0.0052`, 0.245`, 0.361`, 0.245`, 0.361`}]
```

```
{{1., 0, 1.}, {1., 0, 1.}, {0.415413, 0.434145, 0.530589},
 {0.415413, 0.434145, 0.530589}, {0.415413, 0.434145, 0.530589},
 {0, 0, 0}, {0, 0.0886427, 0}, {0, 0.0886427, 0}, {0, 0.0886427, 0},
 {0, 0.0886427, 0}, {0.124968, 0.43456, 0.159659},
 {0.0963242, 0.411999, 0.121315}, {0.0198361, -0.276211, 0.0174291}}
```

```
sieve3[{0.459`, 0.69`, 0, 0.69`, 0.69`, 0.329`, 0.69`,
  0.329`, 1, 0, 1, 1, 0.5`, 0.0052`, 0.245`, 0.361`, 0.245`, 0.361`}]
```

```
{{0, 0.0886427, 0}, {0, 0.0886427, 0}, {0, 0.0886427, 0},
 {0.124968, 0.43456, 0.159659}, {1., 0, 1.}, {1., 0, 1.}}
```

Assortative mating by social form

Social form mating involves every queen mating with males from their respective colony form: MM queens mating with M males and MP,PP queens mating with P males

In[∘]:=
```
R11[p1_, h1_] = (1 - m1) p1 h1 + m1 p1;
R12[p1_, h2_] = (1 - m1) p1 h2 ;
R21[p2_, h1_] = (1 - m2) p2 h1;
R22[p2_, h2_] = (1 - m2) p2 h2 + m2 p2;
R31[p3_, h1_] = (1 - m2) p3 h1;
R32[p3_, h2_] = (1 - m2) p3 h2 + m2 p3;
```

Stability of M fixed and polygynous edge

M fixed

Stability analysis of MM:

In[∘]:=
```
| D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
| D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
| D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
    p1 → 1 /. p2 → 0 /. h1 → 1 // Simplify;
MatrixForm[
 %]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{Vq312-m2\,Vq312+m2\,Vq323}{Vq111} & \frac{(-1+m2)\,Vq212+2\,Vq312-m2\,(Vq222+Vq223+2\,Vq312-2\,Vq323)}{2\,Vq111} & \frac{Vq122-m1\,Vq122}{Vq111} \\ \frac{(-1+m2)\,Vq312}{Vq111} & \frac{Vq212-m2\,Vq212+m2\,Vq222-2\,Vq312+2\,m2\,Vq312}{2\,Vq111} & \frac{(-1+m1)\,Vq122}{Vq111} \\ \frac{Vm32}{Vm11} & -\frac{Vm22-2\,Vm32}{2\,Vm11} & 0 \end{pmatrix}$$

In[∘]:=
```
charpoly = Collect[-Det[% - λ IdentityMatrix[3]], λ, Factor]
```

Out[∘]=

$$\frac{(-1+m1)\,m2\,Vq122\,(Vm32\,Vq223-Vm22\,Vq323)}{2\,Vm11\,Vq111^2}+\frac{1}{2\,Vm11\,Vq111^2}\left(-Vm22\,Vq111\,Vq122+m1\,Vm22\,Vq111\,Vq122-m2\,Vm11\,Vq223\,Vq312+m2^2\,Vm11\,Vq223\,Vq312+m2\,Vm11\,Vq212\,Vq323-m2^2\,Vm11\,Vq212\,Vq323+m2^2\,Vm11\,Vq222\,Vq323\right)\lambda+\frac{(-Vq212+m2\,Vq212-m2\,Vq222-2\,m2\,Vq323)\,\lambda^2}{2\,Vq111}+\lambda^3$$

Given that the leading term is $+\lambda^3$, if the intercept at $\lambda = 1$ is negative then we are guaranteed invasion (the largest root must be greater than one):

In[∘]:=
```
interceptMM = Factor[charpoly /. λ → 1]
```

Out[∘]=
$$\frac{1}{2\,\text{Vm11}\,\text{Vq111}^2}\Big(2\,\text{Vm11}\,\text{Vq111}^2 - \text{Vm22}\,\text{Vq111}\,\text{Vq122} + \text{m1}\,\text{Vm22}\,\text{Vq111}\,\text{Vq122} - \text{Vm11}\,\text{Vq111}\,\text{Vq212} + \text{m2}\,\text{Vm11}\,\text{Vq111}\,\text{Vq212} - \text{m2}\,\text{Vm11}\,\text{Vq111}\,\text{Vq222} - \text{m2}\,\text{Vm32}\,\text{Vq122}\,\text{Vq223} + \text{m1}\,\text{m2}\,\text{Vm32}\,\text{Vq122}\,\text{Vq223} - \text{m2}\,\text{Vm11}\,\text{Vq223}\,\text{Vq312} + \text{m2}^2\,\text{Vm11}\,\text{Vq223}\,\text{Vq312} - 2\,\text{m2}\,\text{Vm11}\,\text{Vq111}\,\text{Vq323} + \text{m2}\,\text{Vm22}\,\text{Vq122}\,\text{Vq323} - \text{m1}\,\text{m2}\,\text{Vm22}\,\text{Vq122}\,\text{Vq323} + \text{m2}\,\text{Vm11}\,\text{Vq212}\,\text{Vq323} - \text{m2}^2\,\text{Vm11}\,\text{Vq212}\,\text{Vq323} + \text{m2}^2\,\text{Vm11}\,\text{Vq222}\,\text{Vq323}\Big)$$

Polygynous edge

Confirming that we have an equilibrium for the males with the P haplotype fixed:

```
recursionsM == {0, 1} /. p1 → 0 /. h1 → 0 /. maternaleffect // Factor
```

```
True
```

The possible edge equilibria are:

```
recursionsF == {0, p2, 1 - p2} /. p1 → 0 /. h1 → 0 /. maternaleffect
```

$$\{0, (((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq222}) / (((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq222} + ((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq223} + 2\,((1-\text{m2})\,(1-\text{p2}) + \text{m2}\,(1-\text{p2}))\,\text{Vq323}),\ (((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq223} + 2\,((1-\text{m2})\,(1-\text{p2}) + \text{m2}\,(1-\text{p2}))\,\text{Vq323}) / (((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq222} + ((1-\text{m2})\,\text{p2} + \text{m2}\,\text{p2})\,\text{Vq223} + 2\,((1-\text{m2})\,(1-\text{p2}) + \text{m2}\,(1-\text{p2}))\,\text{Vq323})\} == \{0, \text{p2}, 1-\text{p2}\}$$

```
Solve[%, p2]
edgeequil = p2 /. %;
```

$$\left\{\{\text{p2} \to 0\}, \left\{\text{p2} \to \frac{\text{Vq222} - 2\,\text{Vq323}}{\text{Vq222} + \text{Vq223} - 2\,\text{Vq323}}\right\}\right\}$$

Stability analysis of PP/MP:

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
      p1 → 0 /. h1 → 0 /. Vq211 → 0 /. Vm21 → 0 // Simplify;
MatrixForm[%]
```

$$\begin{pmatrix} \frac{2\,\text{Vq222}\,\text{Vq323}}{(\text{p2}\,(\text{Vq222}+\text{Vq223}-2\,\text{Vq323})+2\,\text{Vq323})^2} & -\frac{(-1+\text{m2})\,(\text{p2}\,(\text{Vq212}-2\,\text{Vq312})+2\,\text{Vq312})\,(\text{p2}\,(\text{Vq223}-2\,\text{Vq323})+2\,\text{Vq323})}{(\text{p2}\,(\text{Vq222}+\text{Vq223}-2\,\text{Vq323})+2\,\text{Vq323})^2} & \frac{2\,(2\,\text{Vq122}\,\text{Vq323}}{} \\ 0 & 0 & \\ 0 & 0 & \end{pmatrix}$$

```
Factor[Det[% - λ IdentityMatrix[3]]]
```

$$-\frac{1}{(p2\ Vq222 + p2\ Vq223 + 2\ Vq323 - 2\ p2\ Vq323)^3}$$
$$\lambda\ (-2\ m1\ Vq111 + p2\ Vq222\ \lambda + p2\ Vq223\ \lambda + 2\ Vq323\ \lambda - 2\ p2\ Vq323\ \lambda)$$
$$\left(-2\ Vq222\ Vq323 + p2^2\ Vq222^2\ \lambda + 2\ p2^2\ Vq222\ Vq223\ \lambda + p2^2\ Vq223^2\ \lambda + 4\ p2\ Vq222\ Vq323\ \lambda - 4\ p2^2\ Vq222\ Vq323\ \lambda + 4\ p2\ Vq223\ Vq323\ \lambda - 4\ p2^2\ Vq223\ Vq323\ \lambda + 4\ Vq323^2\ \lambda - 8\ p2\ Vq323^2\ \lambda + 4\ p2^2\ Vq323^2\ \lambda\right)$$

Notice how all the m2's cancel out, which reflects the fact that the internal stability properties of the MP/PP versus P-fixed equilibria do not depend on the rate of assortment (because they are all mating with P males at this equilibrium).

Stability of the PP/MP equilibrium along the edge is determined by the first element in the matrix:

```
edgeStab =
 D[recursionsF[[2]], p2] /. p1 → 0 /. h1 → 0 /. Vq211 → 0 /. Vm21 → 0 /. p2 → 0 // Simplify
```

$$\frac{Vq222}{2\ Vq323}$$

Thus, if $\frac{Vq222}{2\ Vq323} < 1$ then p2 = 0 is stable and otherwise p2 $= \frac{Vq222-2\ Vq323}{Vq222+Vq223-2\ Vq323}$ is stable, as before.

Stability of the PP/MP equilibrium to introduction of MM/M is determined by the last element in the matrix:

```
eigenEDGE = (2 m1 Vq111)/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 Vq323);
```

At the two possible polygynous edge equilibrium, the eigenvalue equals:

```
eigenEDGE /. p2 -> {0, (Vq222 - 2 Vq323)/(Vq222 + Vq223 - 2 Vq323)} // Factor
```

$$\left\{\frac{m1\ Vq111}{Vq323}, \frac{2\ m1\ Vq111}{Vq222}\right\}$$

Thus if m1 is large enough, the M haplotype could potentially invade.

Complete assortative mating analytical analysis: social fixation points cannot both be invaded

Below we show that if complete assortative mating then both fixation points cannot be unstable at the same time. Numerical search confirmed that no stable polymorphic equilibrium is found when both possible fixation points cannot be unstable.

The two equilibrium on the polygynous edge:

```
edgeequil
```

$$\left\{0, \frac{Vq222 - 2\,Vq323}{Vq222 + Vq223 - 2\,Vq323}\right\}$$

MP/PP equilibrium is stable if edgeStab is above one, P-fixed is stable otherwise.

```
edgeStab
```

$$\frac{Vq222}{2\,Vq323}$$

MM queens can invade at P-fixed when first element of eigenEDGE is greater than one. MM queens can invade MP/PP equilibrium when second element is greater than 1.

```
eigenEDGE /. p2 -> edgeequil /. m1 → 1 // Factor
```

$$\left\{\frac{Vq111}{Vq323}, \frac{2\,Vq111}{Vq222}\right\}$$

When roots of charpoly are greater than 1 then M-fixed can be invaded.

```
charpoly
```

$$\frac{(-1 + m1)\,m2\,Vq122\,(Vm32\,Vq223 - Vm22\,Vq323)}{2\,Vm11\,Vq111^2} + \frac{1}{2\,Vm11\,Vq111^2}\left(-Vm22\,Vq111\,Vq122 + m1\,Vm22\,Vq111\,Vq122 - m2\,Vm11\,Vq223\,Vq312 + m2^2\,Vm11\,Vq223\,Vq312 + m2\,Vm11\,Vq212\,Vq323 - m2^2\,Vm11\,Vq212\,Vq323 + m2^2\,Vm11\,Vq222\,Vq323\right)\lambda + \frac{(-Vq212 + m2\,Vq212 - m2\,Vq222 - 2\,m2\,Vq323)\,\lambda^2}{2\,Vq111} + \lambda^3$$

When there is only assortative mating in both social forms charpoly becomes:

```
charpoly /. m1 → 1 /. m2 → 1 // Factor
```

$$\frac{\lambda\,(-Vq323 + Vq111\,\lambda)\,(-Vq222 + 2\,Vq111\,\lambda)}{2\,Vq111^2}$$

```
Solve[% == 0, λ]
```

$$\left\{\{\lambda \to 0\}, \left\{\lambda \to \frac{Vq222}{2\,Vq111}\right\}, \left\{\lambda \to \frac{Vq323}{Vq111}\right\}\right\}$$

Therefore one of the above roots of charpoly needs to be greater than 1.

If P Fixed is stable on the polygynous edge when $\frac{Vq222}{2\,Vq323}<1$. MM can invade when $\frac{Vq111}{Vq323}>1$. Note that both inequalities ensure that the roots of charpoly are smaller than 1.

If MP/PP equilibrium is stable on the polygynous edge then $\frac{Vq222}{2\,Vq323}>1$, MM can invade when $\frac{Vq111}{Vq323} > 1$. Note that both inequalities ensure that the roots of charpoly are smaller than 1.

Therefore when there is complete social form assortative mating both possible fixation points at M-fixed and polygynous edge cannot invade.
Numerical search confirms that when complete social form assortative mating no stable social polymorphism exists.

Weak selection analysis

Invasion of M along the polygynous edge:

A quick check of what happens with weak selection. We know that the polygynous edge must then go to the P-fixed equilibrium (because strong MP advantage is needed to go to the PP/MP point, $\frac{Vq222}{2\ Vq323} > 1$):

```
eigenEDGE
```

$$\frac{2\ m1\ Vq111}{p2\ (Vq222 + Vq223 - 2\ Vq323) + 2\ Vq323}$$

```
weaksel = {Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 -> 1 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 -> 1 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq312 -> 1 + sq312 ϵ, Vq223 → 1 + sq223 ϵ, Vq323 → 1 + sq323 ϵ};
```

```
Factor[Normal[Series[eigenEDGE /. weaksel /. p2 → 0, {ϵ, 0, 1}]]]
```

```
m1 (1 + sq111 ϵ - sq323 ϵ)
```

The above will be less than one as long as m1 is substantially smaller one. If, however, m1 is nearly one then we repeat the Taylor series making this assumption:

```
weaksel2 = {m1 → 1 - sm1 * ϵ, Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 -> 1 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 -> 1 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq312 -> 1 + sq312 ϵ, Vq223 → 1 + sq223 ϵ, Vq323 → 1 + sq323 ϵ};
```

Then

```
Factor[Normal[Series[eigenEDGE /. weaksel2 /. p2 → 0, {ϵ, 0, 1}]]] // Simplify
```

```
1 - sm1 ϵ + sq111 ϵ - sq323 ϵ
```

So it would be possible with weak selection and almost complete assortment for M to invade if:

sm - sq111 + sq323 < 0
(1-m1) - (Vq111-1) + (Vq323-1) <0
1-m1 - Vq111 + Vq323 < 0
1 - Vq111 + Vq323 < m1

Because m1 must be less than one, this requires Vq111>Vq323.

Invasion of P when the M haplotype is fixed:

We require λ>1 where:

```
Factor[Normal[Series[charpoly /. λ → λ0 + λ1 ϵ /. weaksel, {ϵ, 0, 0}]]]
```

$$\frac{1}{2}\lambda 0\left(-1+m1+m2^2-\lambda 0-2\,m2\,\lambda 0+2\,\lambda 0^2\right)$$

To leading order, the eigenvalue equals:

```
Simplify[Solve[% == 0, λ0]]
```

$$\left\{\{\lambda 0\to 0\},\left\{\lambda 0\to\frac{1}{4}\left(1+2\,m2-\sqrt{9-8\,m1+4\,m2-4\,m2^2}\right)\right\},\left\{\lambda 0\to\frac{1}{4}\left(1+2\,m2+\sqrt{9-8\,m1+4\,m2-4\,m2^2}\right)\right\}\right\}$$

The largest of which is $\frac{1}{4}\left(1+2\,m2+\sqrt{9-8\,m1+4\,m2-4\,m2^2}\right)$. This eigenvalue must be real (examining the term inside the radical) and so the P haplotype can invade if:

$$\frac{1}{4}\left(1+2\,m2+\sqrt{9-8\,m1+4\,m2-4\,m2^2}\right)>1$$

$\sqrt{9-8\,m1+4\,m2-4\,m2^2}>4-(1+2\,m2)$ squaring both sides (note that the right hand side is positive)

$$8\left(1-m1-(1-m2)^2\right)>0$$

So $1-m1>(1-m2)^2$. This is not possible if m1 is too large (e.g., m1=1) unless m2 is also near one.

If we then take the Taylor series to the next order (introducing ϵ terms), it will only cause a small perturbation to the above eigenvalue.

Social polymorphism numerical search

Confirming analytical results for complete assortment (m1=1, m2=1)

```
In[•]:= compAbs = {m1 → 1, m2 → 1};
```

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
In[•]:= eqn1 = Factor[recursionsF[[1]] - p1] /. compAbs // Simplify
In[•]:= eqn2 = Factor[recursionsF[[2]] - p2] /. compAbs // Simplify
In[•]:= eqn3 = Factor[recursionsM[[1]] - h1] /. compAbs // Simplify
```

In[•]:=

```
findequil[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = {p1, p2, h1} /.
   NSolve[{-(p1 (2 (-1 + p1) Vq111 + p2 (Vq222 + Vq223 - 2 Vq323) - 2 (-1 + p1) Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323) == 0,
     (p2 (-2 p1 Vq111 + Vq222 - p2 Vq222 - p2 Vq223 - 2 Vq323 + 2 p1 Vq323 + 2 p2 Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323) == 0,
     (-2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32))/(p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32) ==
      0}, {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

In[•]:=

D[recursionsF[[1]], p1]	D[recursionsF[[1]], p2]	D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1]	D[recursionsF[[2]], p2]	D[recursionsF[[2]], h1]
D[recursionsM[[1]], p1]	D[recursionsM[[1]], p2]	D[recursionsM[[1]], h1]

```
/.
  compAbs // Simplify
```

In[•]:=

```
Clear[checkstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, p1, p2, h1}] =
  Eigenvalues[{{(2 Vq111 (p2 (Vq222 + Vq223 - 2 Vq323) + 2 Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323)^2,
     -(2 p1 Vq111 (Vq222 + Vq223 - 2 Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323)^2, 0},
    {-(2 p2 Vq222 (Vq111 - Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323)^2,
     (2 Vq222 (p1 (Vq111 - Vq323) + Vq323))/(p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq111 - Vq323) + 2 Vq323)^2, 0},
    {(2 (p2 Vm11 (Vm22 - 2 Vm32) + 2 Vm11 Vm32 + p2 Vm21 Vm32))/(p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32)^2,
     (2 Vm21 Vm32 - 2 p1 (Vm11 (Vm22 - 2 Vm32) + Vm21 Vm32))/(p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32)^2, 0}}]
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
In[•]:= cutoff = 10^(-10);
```

```
In[•]:= Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
In[•]:= Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   write = DeleteDuplicates[write];
   Sort[write]
  ]
```

Numerical search:

```
SeedRandom[842 949];
Clear[tab, tab1]
For[j = 1; stabpoly = {}, j ≤ 1 000 000, j++,
 tab = N[Table[Round[Random[] * 10^10] / 10^10, {i, 1, 12}], 60];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 If[
  testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  testPol,
  Print[tab1] && AppendTo[stabpoly, tab1]];
 If[Mod[j, 5000] == 0, Print[j]];
]
```

Simplex stream diagrams

m1=1, m2=0 and stable social polymorphism

In[•]:=
```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
In[•]:= PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog,
   {Text[Style["X1", Large], {-0.128 + (1 - 0.128 - 0.635), .635}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0.529 + (1 - 0.529 - 0.426), .426}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0 + (1 - 0 - 0.5714), 0.5714}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0, 1}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → Sqrt[3] / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]

In[•]:= paramSet2 = {h2 → 1 - h1, h1 → - (2 p1)/(-2 + p2), Vm11 -> 1, Vq211 → 0,
   Vm21 -> 0, Vm22 -> 1, Vm32 -> 1, Vq111 → 0.57, Vq212 -> 1, Vq323 → .3,
   Vq223 → .3, Vq312 -> 1, Vq222 → 1, Vq122 -> 1, m1 → 0.99999, m2 → 0};
```

In[•]:=
```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2,
    recursionsF[[3]] - (1 - p1 - p2)} /. paramSet2 // Factor
```

Out[•]=
$$\left\{-\frac{1.\ p1\left(0.385704-1.3857\ p1+1.\ p1^2-0.692849\ p2+1.40714\ p1\ p2+0.249998\ p2^2\right)}{-0.428569-1.38571\ p1+1.\ p1^2-0.285713\ p2+1.40714\ p1\ p2+0.249998\ p2^2},\right.$$
$$-\left(\left(1.\left(5.71434\ p1-5.71434\ p1^2+1.14286\ p2-11.2572\ p1\ p2+4.00002\ p1^2\ p2-2.57143\ p2^2+5.62858\ p1\ p2^2+1.\ p2^3\right)\right)\Big/\left(-1.71429-5.54288\ p1+4.00002\ p1^2-1.14286\ p2+5.62858\ p1\ p2+1.\ p2^2\right)\right),$$
$$\left(1.\left(1.81428\ p1-2.81428\ p1^2+1.\ p1^3+0.285713\ p2-3.50713\ p1\ p2+2.40714\ p1^2\ p2-0.642853\ p2^2+1.65714\ p1\ p2^2+0.249998\ p2^3\right)\right)\Big/\left(-0.428569-1.38571\ p1+1.\ p1^2-0.285713\ p2+1.40714\ p1\ p2+0.249998\ p2^2\right)\Big\}$$

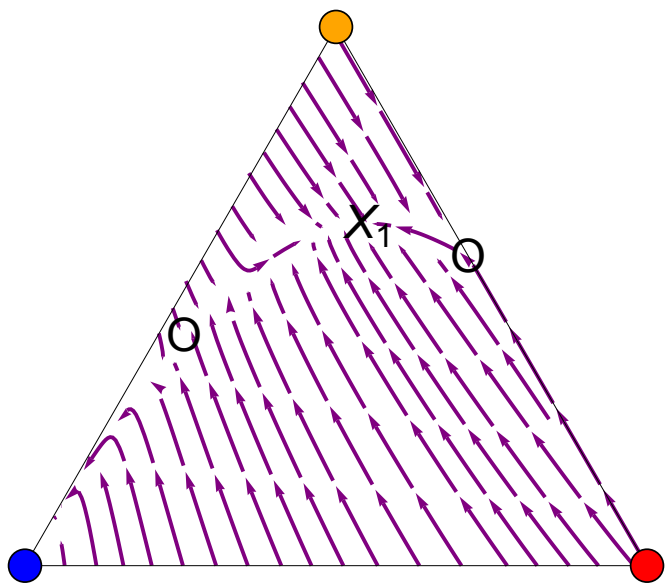
In[•]:=
```
vars = {p1[t], p2[t], p3[t]};
```

In[•]:=
```
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
```

In[•]:=
```
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```

Join: Heads List and If at positions 1 and 2 are expected to be the same.

Out[•]=



Using findequil and sieve3 from the contour diagrams section:

```
findequil[{.57, .3, 0}]
{{1., 0, 1.}, {1., 0, 1.}, {1., 0, 1.}, {0.529295, 0.42634, 0.672693},
 {0.127848, 0.634884, 0.187307}, {0, 0.571429, 0}, {0, 0.571429, 0}, {0, 0, 0}}
sieve3[{.57, .3, 0}]
{{0.127848, 0.634884, 0.187307}, {1., 0, 1.}, {1., 0, 1.}, {1., 0, 1.}}
```

m1=1, m2=.45 and stable social polymorphism

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog,
   {Text[Style["X2", Large], {-0.247 + (1 - 0.247 - 0.548), .548}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0 + (1 - 0 - 0.5714), 0.5714}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0, 1}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → Sqrt[3] / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]
```

```
recursionsM[[1]] /. maternaleffect // Factor
```

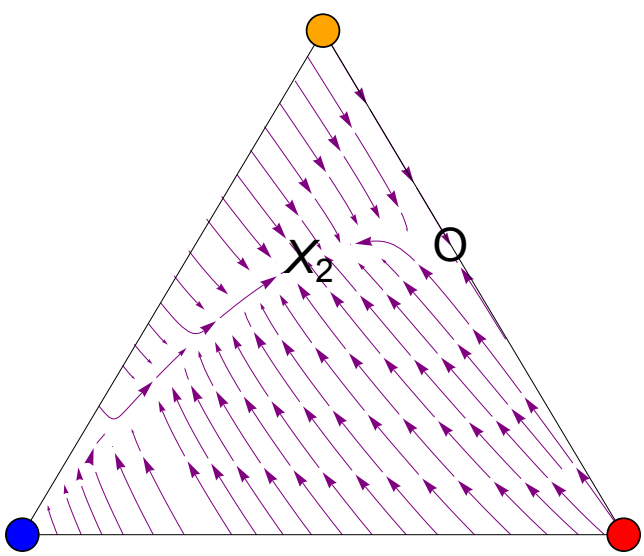
$$\frac{2\ p1\ Vm11}{2\ p1\ Vm11 + p2\ Vm22 + 2\ Vm32 - 2\ p1\ Vm32 - 2\ p2\ Vm32}$$

```
paramSet2 = {Vm11 → 1, Vq211 → 0, Vm21 -> 0, Vm22 → 1, Vm32 → 1, Vq111 → 0.57, Vq212 -> 1,
   Vq323 → .3, Vq223 → .3, Vq312 -> 1, Vq222 → 1, Vq122 -> 1, m1 → .9999999, m2 → 0.45};
```

```
funcs =
 {recursionsF[[1]] - p1, recursionsF[[2]] - p2, recursionsF[[3]] - (1 - p1 - p2)} /. {h2 →
      1 - h1, h1 → (2 p1 Vm11)/(2 p1 Vm11 + p2 Vm22 + 2 Vm32 - 2 p1 Vm32 - 2 p2 Vm32)} /. paramSet2 // Factor
```

$$\left\{-\frac{1.\ p1\ (0.701298 - 1.7013\ p1 + 1.\ p1^2 - 1.25974\ p2 + 1.56493\ p1\ p2 + 0.454545\ p2^2)}{-0.779221 - 1.7013\ p1 + 1.\ p1^2 - 0.51948\ p2 + 1.56493\ p1\ p2 + 0.454545\ p2^2},\right.$$
$$-((1.\ (3.14286\ p1 - 3.14286\ p1^2 + 1.14286\ p2 - 6.88571\ p1\ p2 + 2.2\ p1^2\ p2 - 2.57143\ p2^2 + 3.44286\ p1\ p2^2 + 1.\ p2^3)) / (-1.71429 - 3.74286\ p1 + 2.2\ p1^2 - 1.14286\ p2 + 3.44286\ p1\ p2 + 1.\ p2^2)),$$
$$(1.\ (2.12987\ p1 - 3.12987\ p1^2 + 1.\ p1^3 + 0.51948\ p2 - 4.38961\ p1\ p2 + 2.56493\ p1^2\ p2 - 1.16883\ p2^2 + 2.01948\ p1\ p2^2 + 0.454545\ p2^3)) / \left.(-0.779221 - 1.7013\ p1 + 1.\ p1^2 - 0.51948\ p2 + 1.56493\ p1\ p2 + 0.454545\ p2^2)\right\}$$

```
vars = {p1[t], p2[t], p3[t]};
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



Using findequil and sieve3 from the contour diagrams section:

```
findequil[{.57, .3, .45}]
{{1.50006, -0.452933, 1.22308}, {1.50006, -0.452933, 1.22308},
 {1.50006, -0.452933, 1.22308}, {1., 0, 1.}, {0, 0.571429, 0},
 {0, 0.571429, 0}, {0.247251, 0.547965, 0.340558}, {0, 0, 0}}
sieve3[{.57, .3, .45}]
{{0.247251, 0.547965, 0.340558}}
```

m1=1/2, m2=1/2 and stable social polymorphism (directional selection)

```
FrameToEpilogS3[] := Module[{myEpilog, labelPos, myFrame},
   labelPos = {{-1.15, -0.05}, {1.15, -0.05}, {0.02, 1.05}};
   myFrame = Line[{{-1, 0}, {1, 0}, {0, 1}, {-1, 0}}];
   myEpilog = {myFrame}
  ];
```

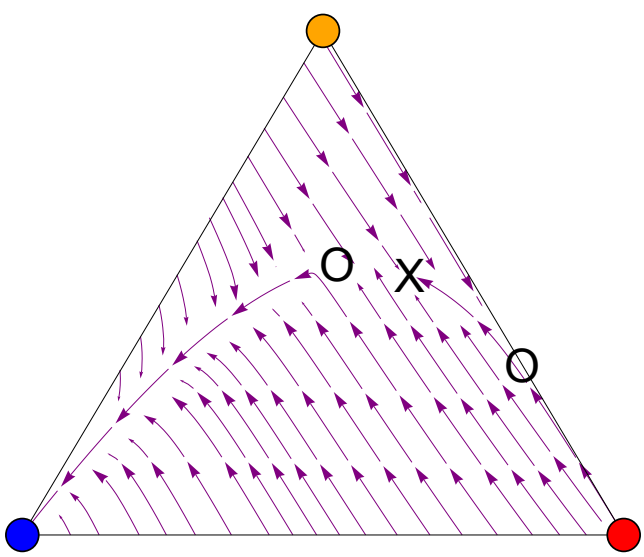
Define a function that graphs stream fields for frequencies of the three queen genotypes. *PlotS3Field* takes as input the dynamical equations to be solved, *funcs*, the variables to solve for, *vars*, and the color of the stream fields, *col*, and returns a 2-simplex with a stream field that corresponds to the dynamical equations.

```
PlotS3Field[funcs_, vars_, col_] := Module[{thisEq, p, π, myEpilog, plotOpts},
  myEpilog = FrameToEpilogS3[]; (*draw the frame using the function above*)

  thisEq = If[Abs[p] + π > 1, (*define the equations for drawing the streamplot*)
    {0, 0},
    {funcs[[3]] - funcs[[1]], funcs[[2]]} /.
     {vars[[3]] → (1 - π + p) / 2, vars[[2]] → π, vars[[1]] → (1 - π - p) / 2}
   ];

  (*add colored points to the frame*)
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{-1, 0}]}}];
  AppendTo[myEpilog, {{PointSize[0.05], RGBColor["Black"], Point[{1, 0}]}}];

  AppendTo[myEpilog, {Text[Style["X", Large], {-0.098 + (1 - 0.098 - 0.512), .512}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0.208 + (1 - 0.208 - 0.533), .533}]}];
  AppendTo[myEpilog, {Text[Style["O", Large], {-0 + (1 - 0 - 0.3333), 0.3333}]}];

  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Orange"], Point[{0, 1}]}}];
  AppendTo[myEpilog, {PointSize[0.045], RGBColor["Blue"], Point[{-1, 0}]}];
  AppendTo[myEpilog, {{PointSize[0.045], RGBColor["Red"], Point[{1, 0}]}}];

  StreamPlot[thisEq, {p, -1.0, 1.0},
   {π, 0, 1}, (*draw the streamplot on the 2-simplex*)
   AspectRatio → √3 / 2,
   Frame → False,
   StreamPoints → Automatic,
   StreamScale → 0.1,
   StreamColorFunction → (col &),
   StreamStyle → Thick,
   Epilog → myEpilog (*draw the stream plot on the 2-simplex*)]]

paramSet2 = {h2 → 1 - h1, h1 → - 2 p1 / (-2 + p2), Vm11 -> 1, Vq211 → 0,
   Vm21 -> 0, Vm22 -> 1, Vm32 -> 1, Vq111 → 0.52, Vq212 -> 1 / 2, Vq323 → .2,
   Vq223 → .2, Vq312 -> 1 / 2, Vq222 → 1 / 2, Vq122 -> 1 / 2, m1 → 0.54, m2 → 0.5};
```

```
funcs = {recursionsF[[1]] - p1, recursionsF[[2]] - p2,
    recursionsF[[3]] - (1 - p1 - p2)} /. paramSet2 // Factor
```

$$\left\{-\frac{1.\ p1\left(0.573864-1.57386\ p1+1.\ p1^2-1.35227\ p2+2.52415\ p1\ p2+0.53267\ p2^2\right)}{-1.42045-3.27273\ p1+1.\ p1^2-0.355114\ p2+2.52415\ p1\ p2+0.53267\ p2^2},\right.$$
$$-\left(\left(1.\left(6.4\ p1-6.4\ p1^2+0.666667\ p2-11.0107\ p1\ p2+1.87733\ p1^2\ p2-2.33333\ p2^2+4.73867\ p1\ p2^2+1.\ p2^3\right)\right)\Big/\left(-2.66667-6.144\ p1+1.87733\ p1^2-0.666667\ p2+4.73867\ p1\ p2+1.\ p2^2\right)\right),$$
$$\left(1.\left(-2.22045\times10^{-16}+3.98295\ p1-4.98295\ p1^2+1.\ p1^3+0.355114\ p2-7.21733\ p1\ p2+3.52415\ p1^2\ p2-1.2429\ p2^2+3.05682\ p1\ p2^2+0.53267\ p2^3\right)\right)\Big/\left.\left(-1.42045-3.27273\ p1+1.\ p1^2-0.355114\ p2+2.52415\ p1\ p2+0.53267\ p2^2\right)\right\}$$

```
vars = {p1[t], p2[t], p3[t]};
args = Flatten[Thread[{p1, p2, p3} → #] & /@ {Flatten[vars]}];
Show[PlotS3Field[(funcs /. args), vars, Purple]]
```



Using findequil and sieve3 from the contour diagrams section:

```
findequil[{0.52, .2, .54}]
{{1., 0, 1.}, {0.20771, 0.533428, 0.283259},
 {0.0984741, 0.517606, 0.132858}, {0, 0.333333, 0}, {0, 0.333333, 0}, {0, 0, 0}}
sieve3[{0.52, .2, .54}]
{{0.0984741, 0.517606, 0.132858}, {1., 0, 1.}}
```

Contour diagrams

Numerical sampling for contour (MP fitness=1, m1=1)

In[•]:=
```
PPfiteq = {Vq223 → VqPP, Vq323 → VqPP};
MMfiteq = {Vq111 → VqMM, Vq211 → 0};
MPfiteq = {Vq122 → VqMP, Vq212 → VqMP, Vq222 → VqMP, Vq312 → VqMP};
mfiteq = {Vm11 → Vm, Vm21 → 0, Vm22 → Vm, Vm32 → Vm};
```

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

In[•]:=
```
eqn1 =
 Factor[recursionsF[[1]] - p1] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /. VqMP → 1 /.
   m1 → 1 // Simplify
```

In[•]:=
```
eqn2 =
 Factor[recursionsF[[2]] - p2] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /. VqMP → 1 /.
   m1 → 1 // Simplify
```

In[•]:=
```
eqn3 =
 Factor[recursionsM[[1]] - h1] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /. VqMP → 1 /.
   m1 → 1 // Simplify
```

In[•]:=
```
findequil[{VqMM_, VqPP_, m2_}] := findequil[{VqMM, VqPP, m2}] = {p1, p2, h1} /.
   NSolve[{(p1 (p2 - h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
            2 (-1 + p1) (VqMM - VqPP) - p2 VqPP)) /
         (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
           p2 (-1 + VqPP) - 2 (p1 (VqMM - VqPP) + VqPP)) == 0,
       (-h1 (-1 + m2) (-2 + 2 p1 (1 + p2 (-1 + VqPP)) - 2 p2 (-2 + VqPP) + p2^2 (-2 + VqPP)) +
           p2 (-1 + p2 + 2 p1 (VqMM - VqPP) + 2 VqPP - p2 VqPP)) /
         (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
           p2 (-1 + VqPP) - 2 (p1 (VqMM - VqPP) + VqPP)) == 0,
       -h1 - (2 p1)/(-2 + p2) == 0},
     {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

```
In[•]:= | D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
        | D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
        | D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
          mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /. VqMP → 1 /. m1 → 1 // Simplify
```

```
In[•]:= Clear[checkstab];
checkstab[{VqMM_, VqPP_, m2_, p1_, p2_, h1_}] :=
 checkstab[{VqMM, VqPP, m2, p1, p2, h1}] = Eigenvalues[
   {{-((2 VqMM (h1 (-1 + m2) (2 + p2 (-2 + VqPP) - 2 VqPP) + p2 (-1 + VqPP) - 2 VqPP)) /
         (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
            p2 (-1 + VqPP) - 2 (p1 (VqMM - VqPP) + VqPP))^2),
      (2 p1 VqMM (-1 + h1 (-1 + m2) (-2 + VqPP) + VqPP)) /
       (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
          p2 (-1 + VqPP) - 2 (p1 (VqMM - VqPP) + VqPP))^2,
      (2 (-1 + m2) p1 VqMM (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP)) /
       (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) + p2 (-1 + VqPP) -
          2 (p1 (VqMM - VqPP) + VqPP))^2}, {(2 (h1^2 (-1 + m2)^2 p2 VqPP +
           p2 (-VqMM + VqPP) - 2 h1 (-1 + m2) ((-1 + p2) VqMM - p2 VqPP))) /
       (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) + p2 (-1 + VqPP) -
          2 (p1 (VqMM - VqPP) + VqPP))^2, (2 ((1 + h1 (-1 + m2))^2 VqPP +
           p1 (VqMM + 2 h1 (-1 + m2) VqMM - (1 + h1 (-1 + m2))^2 VqPP))) /
       (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) +
          p2 (-1 + VqPP) - 2 (p1 (VqMM - VqPP) + VqPP))^2, ((-1 + m2)
         (4 p1 (-1 + p2) VqMM + 4 p1^2 (VqMM - VqPP) - 4 p1 (-2 + p2) VqPP - (-2 + p2)^2 VqPP)) /
       (h1 (-1 + m2) (2 + p2 (-2 + VqPP) + 2 p1 (-1 + VqPP) - 2 VqPP) + p2 (-1 + VqPP) -
          2 (p1 (VqMM - VqPP) + VqPP))^2}, {-2/(-2 + p2), 2 p1/(-2 + p2)^2, 0}}]
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
In[•]:= cutoff = 10^(-10);
```

In[•]:=
```
Clear[sieve]
sieve[{VqMM_, VqPP_, m2_}] := sieve[{VqMM, VqPP, m2}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{VqMM, VqPP, m2}]]), i++,
    If[Length[test = Cases[eq[[i]],
          x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) && Abs[Im[x]] < cutoff)]] == 3 &&
      (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) && (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠
        {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{VqMM, VqPP, m2, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

In[•]:=
```
Clear[sieve3]
sieve3[{VqMM_, VqPP_, m2_}] := sieve3[{VqMM, VqPP, m2}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{VqMM, VqPP, m2}]]), i++,
    s3p1 = If[Chop[eq[[i, 1]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 1]]] == 0, 1 - 10^(-10), Chop[eq[[i, 1]]]]];
    s3p2 = If[Chop[eq[[i, 2]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 2]]] == 0, 1 - 10^(-10), Chop[eq[[i, 2]]]]];
    s3h1 = If[Chop[eq[[i, 3]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 3]]] == 0, 1 - 10^(-10), Chop[eq[[i, 3]]]]];
    If[Length[test = Cases[eq[[i]],
          x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) && Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{VqMM, VqPP, m2, s3p1, s3p2, s3h1}]]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search: (VqPP set as 30/100)

```
MySearchGrid = {};
GridDim = 50;
SetVqPP = 30 / 100;
Do[
 Do[
  If[i == 0 && n == 0,
   AppendTo[MySearchGrid, {10^(-10) / GridDim, SetVqPP, 10^(-10) / GridDim}],
   If[i == 0,
    AppendTo[MySearchGrid, {10^(-10) / GridDim, SetVqPP, n / GridDim}],
    If[n == 0,
     AppendTo[MySearchGrid, {i / GridDim, SetVqPP, 10^(-10) / GridDim}],
     AppendTo[MySearchGrid, {i / GridDim, SetVqPP, n / GridDim}]]]],
  {n, 0, GridDim}],
 {i, 0, GridDim}]
MySearchGrid;
MySearchGrid = N[MySearchGrid, 100];
```

```
Clear[tab]
For[j = 1;
 searchsize = Length[MySearchGrid];
 stabpolyMMandPP = {};
 stabpolyMMandMP = {};
 stab000 = {};
 stab101 = {};
 stab010 = {};
 stabonlypoly = {};
 Count1 = 0;
 Count2 = 0;
 Count3 = 0;
 Count4 = 0;
 Count5 = 0;
 Count6 = 0, j ≤ searchsize, j++,
 tab1 = MySearchGrid[[j]];
 {myeq = sieve3[tab1]};
 If[
  test000 = (Length[Select[myeq, (#[[1]] == 0 && #[[2]] == 0 && #[[3]] == 0) &]] > 0);
  (*True if P-fixed is stable*)
  test101 = (Length[Select[myeq,
        (Chop[(#[[1]]) - 1] == 0 & × #[[2]] == 0 && Chop[(#[[3]]) - 1] == 0) &]] > 0);
  (*True if M-fixed is stable*)
  test010 = (Length[Select[myeq, (#[[1]] == 0 && #[[2]] > 0 && #[[3]] == 0) &]] > 0);
  (*True if MP/PP/P equilibrium is stable*)
  testPol = (Length[Select[myeq, (#[[1]] > 0 && #[[2]] > 0 && #[[3]] > 0) &]] > 0);
  (*True if stable social polymorphism exists*)
  testPol,
  If[test101,
   If[test000, Count1++ && AppendTo[stabpolyMMandPP, tab1],
    If[test010, Count5++ && AppendTo[stabpolyMMandMP, tab1],
     Count2++ && AppendTo[stab101, tab1]]],
   If[test000, Count3++ && AppendTo[stab000, tab1],
    If[test010, Count6++ && AppendTo[stab010, tab1],
     Count4++ && AppendTo[stabonlypoly, tab1]]]]];
 If[Mod[j, 500] == 0, Print[j]];
]
Print[Style[Count1, Red], Style[Count2, Blue], Style[Count3, Green],
 Style[Count4, Brown], Style[Count5, Yellow], Style[Count6, Orange]]
```

500

NSolve: Infinite solution set has dimension at least 1. Returning intersection of solutions with

$$\frac{171802\,h1}{178835} - \frac{113492\,p1}{178835} - \frac{121484\,p2}{178835} == 1.$$

```
1000
```

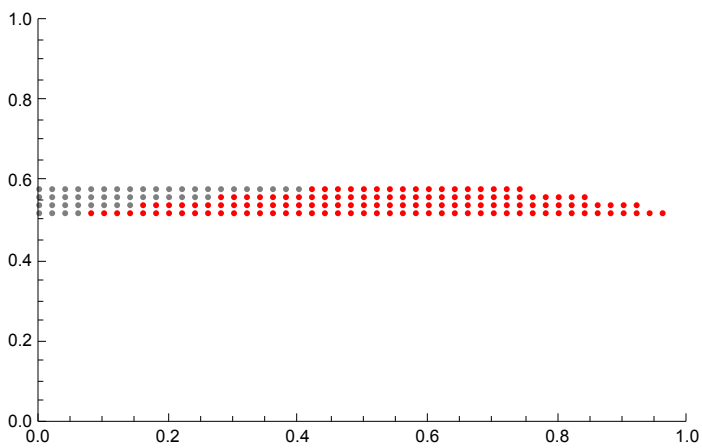
NSolve: Infinite solution set has dimension at least 1. Returning intersection of solutions with

$$\frac{171802\,h1}{178835} - \frac{113492\,p1}{178835} - \frac{121484\,p2}{178835} == 1.$$

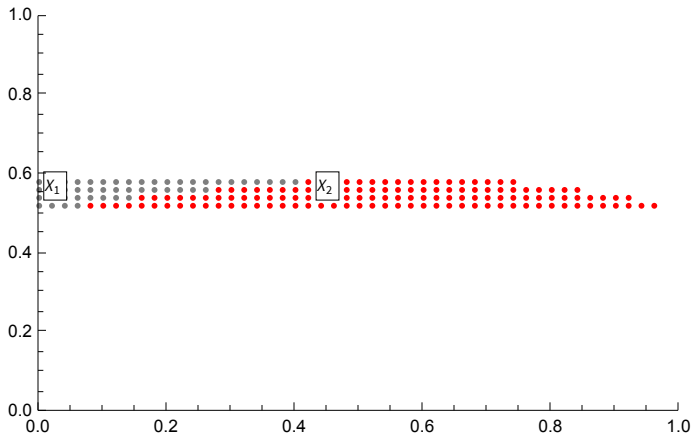
```
1500

2000

2500

047013000
```

```
PPFixed = {};
MMFixed = {};
MPeq = {};
polyfix = {};
bothMPpoly = {};
bothPPpoly = {};
polySets =
  {stab000, stab101, stab010, stabonlypoly, stabpolyMMandMP, stabpolyMMandPP};
polycoordinate = {PPFixed, MMFixed, MPeq, polyfix, bothMPpoly, bothPPpoly};
Do[
 Do[
  AppendTo[polycoordinate[[i]], Delete[polySets[[i]][[n]], 2]],
  {n, 1, Length[polySets[[i]]]}],
 {i, 1, 6}]
polycoordinate = polycoordinate /. {} → {{0, 0}};
polycoordinate = polycoordinate /. {x_, y_} → {y, x};
```

```
m1mp1contour = ListPlot[polycoordinate, PlotRange → {{0, 1}, {0, 1}},
  PlotStyle → {White, Gray, White, Red, White, White}]
```



```
Show[m1mp1contour,
 Graphics[Text[Framed[Style["X1", "Text", Small], FrameMargins → Tiny,
    FrameStyle → Thin], {0.025, 0.57}, Background → White]],
 Graphics[Text[Framed[Style["X2", "Text", Small], FrameMargins → Tiny,
    FrameStyle → Thin], {.45, 0.57}, Background → White]]]
```



Finding the equilibria and stable polymorphism from the parameter set of X1:

```
In[•]:= findequil[{.57, .3, 0}]
Out[•]= {{1., 0, 1.}, {1., 0, 1.}, {0.529295, 0.42634, 0.672693},
  {0.127848, 0.634884, 0.187307}, {0, 0.571429, 0}, {0, 0.571429, 0}, {0, 0, 0}}

In[•]:= sieve3[{.57, .3, 0}]
Out[•]= {{0.127848, 0.634884, 0.187307}, {1., 0, 1.}, {1., 0, 1.}}
```

Finding the equilibria and stable polymorphism from the parameter set of X2:

```
In[•]:= findequil[{.57, .3, .45}]
Out[•]= {{1.50006, -0.452933, 1.22308}, {1.50006, -0.452933, 1.22308}, {1., 0, 1.},
  {1., 0, 1.}, {0, 0.571429, 0}, {0.247251, 0.547965, 0.340558}, {0, 0, 0}, {0, 0, 0}}

In[•]:= sieve3[{.57, .3, .45}]
Out[•]= {{0.247251, 0.547965, 0.340558}}
```

Numerical sampling for contour (MP fitness=1/2, m2=1/2)

```
PPfiteq = {Vq223 → VqPP, Vq323 → VqPP};
MMfiteq = {Vq111 → VqMM, Vq211 → 0};
MPfiteq = {Vq122 → VqMP, Vq212 → VqMP, Vq222 → VqMP, Vq312 → VqMP};
mfiteq = {Vm11 → Vm, Vm21 → 0, Vm22 → Vm, Vm32 → Vm};
```

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /.
    VqMP → 1 / 2 /. m2 → 1 / 2 // Simplify

eqn2 = Factor[recursionsF[[2]] - p2] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /.
    VqMP → 1 / 2 /. m2 → 1 / 2 // Simplify

eqn3 = Factor[recursionsM[[1]] - h1] /. mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /.
    VqMP → 1 / 2 /. m2 → 1 / 2 // Simplify


findequil[{VqMM_, VqPP_, m1_}] := findequil[{VqMM, VqPP, m1}] =
  {p1, p2, h1} /. NSolve[{- ((p1 (p2 - 4 m1 VqMM + p1 (2 + m1 (-2 + 4 VqMM) - 4 VqPP) +
                4 VqPP - 2 p2 VqPP + h1 (1 - 4 VqMM + 4 m1 VqMM + p2 (-1 + VqPP) -
                    2 VqPP + p1 (-3 + m1 (2 - 4 VqMM) + 4 VqMM + 2 VqPP)))) /
            (p2 + p1 (2 + m1 (-2 + 4 VqMM) - 4 VqPP) + 4 VqPP - 2 p2 VqPP +
              h1 (1 + p2 (-1 + VqPP) - 2 VqPP + p1 (-3 + m1 (2 - 4 VqMM) + 4 VqMM + 2 VqPP)))) == 0,
      (2 p1 (-1 + m1 + p2 - m1 p2 + 2 m1 p2 VqMM - 2 p2 VqPP) + p2 (-1 + p2 + 4 VqPP - 2 p2 VqPP) +
          h1 (-1 - 2 p2 (-1 + VqPP) + p2^2 (-1 + VqPP) +
              p1 (3 + m1 (-2 + p2 (2 - 4 VqMM)) + p2 (-3 + 4 VqMM + 2 VqPP))) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
          h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP)) == 0,
      - (2 p1 + h1 (-2 + p2)) / (-2 + p2) == 0},
    {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

```
| D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
| D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
| D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
        mfiteq /. MMfiteq /. PPfiteq /. MPfiteq /. VqMP → 1 / 2 /. m2 → 1 / 2 // Simplify
```

```
Clear[checkstab];
checkstab[{VqMM_, VqPP_, m1_, p1_, p2_, h1_}] :=
 checkstab[{VqMM, VqPP, m1, p1, p2, h1}] = Eigenvalues[
   {{-((4 (h1 (-1 + m1) - m1) VqMM (p2 + h1 (1 + p2 (-1 + VqPP) - 2 VqPP) + 4 VqPP - 2 p2 VqPP)) /
         (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
            h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP))^2),
      (4 (h1 (-1 + m1) - m1) p1 VqMM (1 + h1 (-1 + VqPP) - 2 VqPP)) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
           h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP))^2,
      (4 p1 VqMM (p2 + p1 (2 - 4 VqPP) + 4 VqPP - 2 p2 VqPP +
            m1 (-1 + (-2 + p2) VqPP + p1 (-1 + 2 VqPP)))) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
           h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP))^2},
     {(8 VqPP + h1^2 (4 (-1 + m1 + p2 - m1 p2) VqMM + (4 + 2 m1 (-2 + p2) - p2) VqPP) -
          4 m1 (p2 VqMM + 2 VqPP - p2 VqPP) +
          2 h1 (-2 p2 VqMM - 6 VqPP + p2 VqPP + m1 (-2 VqMM + 4 p2 VqMM + 6 VqPP - 3 p2 VqPP))) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
           h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP))^2,
      (4 (m1 p1 (VqMM - VqPP) + VqPP) - 2 h1 (2 VqPP + p1 (-2 VqMM + 4 m1 VqMM + VqPP -
               3 m1 VqPP)) + h1^2 (VqPP + p1 (4 (-1 + m1) VqMM + VqPP - 2 m1 VqPP))) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
           h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) + 2 VqPP - p2 VqPP))^2,
      (-8 p1^2 (VqMM - VqPP) + (-2 + p2)^2 VqPP - 2 p1 (2 p2 VqMM + 6 VqPP - 3 p2 VqPP) +
          2 m1 p1 (2 (1 + p1) VqMM - (-2 + 2 p1 + p2) VqPP)) /
        (-p2 - 4 VqPP + 2 p2 VqPP + p1 (-2 + 2 m1 - 4 m1 VqMM + 4 VqPP) +
           h1 (-1 + p2 + p1 (3 - 2 m1 - 4 VqMM + 4 m1 VqMM - 2 VqPP) +
              2 VqPP - p2 VqPP))^2}, {-2/(-2 + p2), 2 p1/(-2 + p2)^2, 0}}]
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-10);
```

```
Clear[sieve]
sieve[{VqMM_, VqPP_, m1_}] := sieve[{VqMM, VqPP, m1}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{VqMM, VqPP, m1}]]), i++,
    If[Length[test = Cases[eq[[i]],
          x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) && Abs[Im[x]] < cutoff)]] == 3 &&
      (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) && (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠
        {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{VqMM, VqPP, m1, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{VqMM_, VqPP_, m1_}] := sieve3[{VqMM, VqPP, m1}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{VqMM, VqPP, m1}]]), i++,
    s3p1 = If[Chop[eq[[i, 1]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 1]]] == 0, 1 - 10^(-10), Chop[eq[[i, 1]]]]];
    s3p2 = If[Chop[eq[[i, 2]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 2]]] == 0, 1 - 10^(-10), Chop[eq[[i, 2]]]]];
    s3h1 = If[Chop[eq[[i, 3]]] == 0, 10^(-10),
      If[Chop[1 - eq[[i, 3]]] == 0, 1 - 10^(-10), Chop[eq[[i, 3]]]]];
    If[Length[test = Cases[eq[[i]],
          x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) && Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{VqMM, VqPP, m1, s3p1, s3p2, s3h1}]]] < 1),
     write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search: (VqPP set as 20/100)

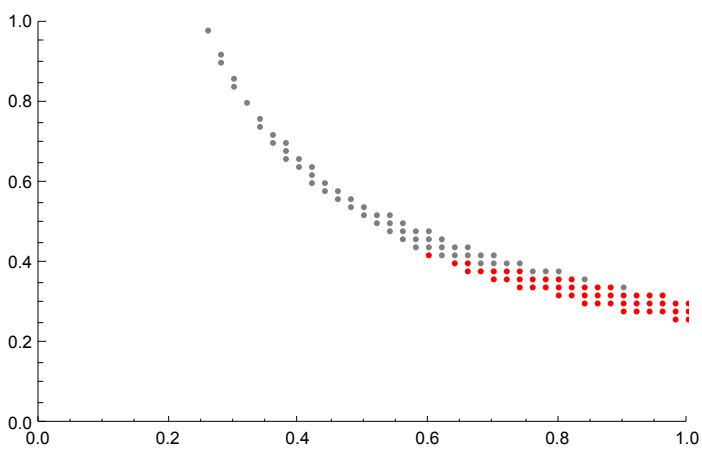
```
MySearchGrid = {};
GridDim = 50;
SetVqPP = 2 / 10;
Do[
 Do[
  If[i == 0 && n == 0,
   AppendTo[MySearchGrid, {10^(-10) / GridDim, SetVqPP, 10^(-10) / GridDim}],
   If[i == 0,
    AppendTo[MySearchGrid, {10^(-10) / GridDim, SetVqPP, n / GridDim}],
    If[n == 0,
     AppendTo[MySearchGrid, {i / GridDim, SetVqPP, 10^(-10) / GridDim}],
     AppendTo[MySearchGrid, {i / GridDim, SetVqPP, n / GridDim}]]]],
  {n, 0, GridDim}],
 {i, 0, GridDim}]
MySearchGrid;
MySearchGrid = N[MySearchGrid, 60];
```

```
Clear[tab]
For[j = 1;
 searchsize = Length[MySearchGrid];
 stabpolyMMandPP = {};
 stabpolyMMandMP = {};
 stab000 = {};
 stab101 = {};
 stab010 = {};
 stabonlypoly = {};
 Count1 = 0;
 Count2 = 0;
 Count3 = 0;
 Count4 = 0;
 Count5 = 0;
 Count6 = 0, j ≤ searchsize, j++,
 tab1 = MySearchGrid[[j]];
 {myeq = sieve3[tab1]};
 If[
  test000 = (Length[Select[myeq, (#[[1]] == 0 && #[[2]] == 0 && #[[3]] == 0) &]] > 0);
  (*True if P-fixed is stable*)
  test101 = (Length[Select[myeq,
        (Chop[(#[[1]]) - 1] == 0 & × #[[2]] == 0 && Chop[(#[[3]]) - 1] == 0) &]] > 0);
  (*True if M-fixed is stable*)
  test010 = (Length[Select[myeq, (#[[1]] == 0 && #[[2]] > 0 && #[[3]] == 0) &]] > 0);
  (*True if MP/PP/P equilibrium is stable*)
  testPol = (Length[Select[myeq, (#[[1]] > 0 && #[[2]] > 0 && #[[3]] > 0) &]] > 0);
  (*True if stable social polymorphism exists*)
  testPol,
  If[test101,
   If[test000, Count1++ && AppendTo[stabpolyMMandPP, tab1],
    If[test010, Count5++ && AppendTo[stabpolyMMandMP, tab1],
     Count2++ && AppendTo[stab101, tab1]]],
   If[test000, Count3++ && AppendTo[stab000, tab1],
    If[test010, Count6++ && AppendTo[stab010, tab1],
     Count4++ && AppendTo[stabonlypoly, tab1]]]]];
 If[Mod[j, 500] == 0, Print[j]];
]
Print[Style[Count1, Red], Style[Count2, Blue], Style[Count3, Green],
 Style[Count4, Brown], Style[Count5, Yellow], Style[Count6, Orange]]
```

```
500
1000
1500
2000
2500
05804900
```

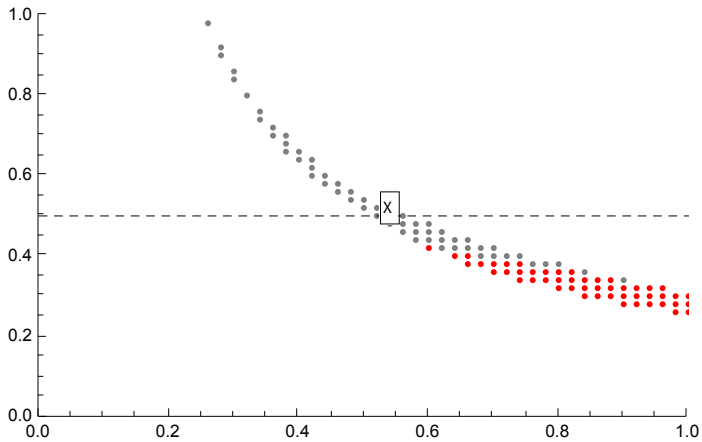
```
PPFixed = {};
MMFixed = {};
MPeq = {};
polyfix = {};
bothMPpoly = {};
bothPPpoly = {};
polySets =
  {stab000, stab101, stab010, stabonlypoly, stabpolyMMandMP, stabpolyMMandPP};
polycoordinate = {PPFixed, MMFixed, MPeq, polyfix, bothMPpoly, bothPPpoly};
Do[
 Do[
  AppendTo[polycoordinate[[i]], Delete[polySets[[i]][[n]], 2]],
  {n, 1, Length[polySets[[i]]]}],
 {i, 1, 6}]
polycoordinate = polycoordinate /. {} → {{0, 0}};
polycoordinate = polycoordinate /. {x_, y_} → {y, x};
```

```
dirSelGraph = ListPlot[polycoordinate, PlotRange → {{0, 1}, {0, 1}},
  PlotStyle → {White, Gray, White, Red, White, White}]
```




```
halfLine = Plot[0.5, {x, 0, 1}, PlotStyle → {Dashed, Black, Thin}];
```

```
Show[dirSelGraph, halfLine,
 Graphics[Text[Framed[Style["X", "Text", Small], FrameMargins -> Small,
    FrameStyle → Thin], {0.54, 0.52}, Background → White]]]
```



Finding the equilibria and stable polymorphism from the parameter set of X1:

```
findequil[{0.52, .2, .54}]
```

```
{{1., 0, 1.}, {0.20771, 0.533428, 0.283259},
 {0.0984741, 0.517606, 0.132858}, {0, 0.333333, 0}, {0, 0.333333, 0}, {0, 0, 0}}
```

```
sieve3[{0.52, .2, .54}]
```

```
{{0.0984741, 0.517606, 0.132858}, {1., 0, 1.}}
```

Assortative mating by genotype

Genotype based mating involves MM queens mating only with M males, PP queens mating only with P males, and MP queens mating equally with M and P males.

```
R11[p1_, h1_] = (1 - m) p1 h1 + m p1;
R12[p1_, h2_] = (1 - m) p1 h2;
R21[p2_, h1_] = (1 - m) p2 h1 + m p2/2;
R22[p2_, h2_] = (1 - m) p2 h2 + m p2/2;
R31[p3_, h1_] = (1 - m) p3 h1;
R32[p3_, h2_] = (1 - m) p3 h2 + m p3;
```

Stability of M fixed and polygynous edge

M-Fixed

Stability, jacobian matrix, analysis of M-fixed:

```
| D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] |
| D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | /.
| D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] |
    p1 → 1 /. p2 → 0 /. h1 → 1 // Simplify
MatrixForm[
 %]
```

$$\left\{\left\{\frac{Vq312 - m\,Vq312 + m\,Vq323}{Vq111}, \frac{(-2+m)\,Vq212 + 4\,Vq312 - m\,(Vq222 + Vq223 + 4\,Vq312 - 4\,Vq323)}{4\,Vq111}, \frac{Vq122 - m\,Vq122}{Vq111}\right\}, \left\{\frac{(-1+m)\,Vq312}{Vq111}, \frac{2\,Vq212 - m\,Vq212 + m\,Vq222 - 4\,Vq312 + 4\,m\,Vq312}{4\,Vq111}, \frac{(-1+m)\,Vq122}{Vq111}\right\}, \left\{\frac{Vm32}{Vm11}, -\frac{Vm22 - 2\,Vm32}{2\,Vm11}, 0\right\}\right\}$$

$$\begin{pmatrix} \frac{Vq312-m\,Vq312+m\,Vq323}{Vq111} & \frac{(-2+m)\,Vq212+4\,Vq312-m\,(Vq222+Vq223+4\,Vq312-4\,Vq323)}{4\,Vq111} & \frac{Vq122-m\,Vq122}{Vq111} \\ \frac{(-1+m)\,Vq312}{Vq111} & \frac{2\,Vq212-m\,Vq212+m\,Vq222-4\,Vq312+4\,m\,Vq312}{4\,Vq111} & \frac{(-1+m)\,Vq122}{Vq111} \\ \frac{Vm32}{Vm11} & -\frac{Vm22-2\,Vm32}{2\,Vm11} & 0 \end{pmatrix}$$

```
charpoly = Collect[-Det[% - λ IdentityMatrix[3]], λ, Factor]
```

$$\frac{(-1+m)\,m\,Vq122\,(Vm32\,Vq223 - 2\,Vm22\,Vq323)}{4\,Vm11\,Vq111^2} + \frac{1}{4\,Vm11\,Vq111^2}\left(-2\,Vm22\,Vq111\,Vq122 + 2\,m\,Vm22\,Vq111\,Vq122 - m\,Vm11\,Vq223\,Vq312 + m^2\,Vm11\,Vq223\,Vq312 + 2\,m\,Vm11\,Vq212\,Vq323 - m^2\,Vm11\,Vq212\,Vq323 + m^2\,Vm11\,Vq222\,Vq323\right)\lambda + \frac{(-2\,Vq212 + m\,Vq212 - m\,Vq222 - 4\,m\,Vq323)\,\lambda^2}{4\,Vq111} + \lambda^3$$

Polygynous edge

On the polygynous edge, there are two possible equilibria, P-fixed and MP/PP equilibrium:

```
recursionsF == {0, p2, 1 - p2} /. p1 → 0 /. h1 → 0 /. maternaleffect // Factor
```

$$\left\{0, \frac{p2\,(m\,Vq212 + 2\,Vq222 - m\,Vq222)}{m\,p2\,Vq212 + 2\,p2\,Vq222 - m\,p2\,Vq222 + 2\,p2\,Vq223 - m\,p2\,Vq223 + 4\,Vq323 - 4\,p2\,Vq323}, \frac{-2\,p2\,Vq223 + m\,p2\,Vq223 - 4\,Vq323 + 4\,p2\,Vq323}{-m\,p2\,Vq212 - 2\,p2\,Vq222 + m\,p2\,Vq222 - 2\,p2\,Vq223 + m\,p2\,Vq223 - 4\,Vq323 + 4\,p2\,Vq323}\right\} == \{0, p2, 1 - p2\}$$

```
Solve[%, p2]
edgeequil = p2 /. %;
```

$$\left\{\{p2 \to 0\}, \left\{p2 \to \frac{m\,Vq212 + 2\,Vq222 - m\,Vq222 - 4\,Vq323}{m\,Vq212 + 2\,Vq222 - m\,Vq222 + 2\,Vq223 - m\,Vq223 - 4\,Vq323}\right\}\right\}$$

Stability analysis of PP/MP:

```
| D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] | D[recursionsF[[2]], p1] |
| D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1] | D[recursionsM[[1]], p1] | /.
| D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1] | D[recursionsF[[1]], p1] |
      p1 → 0 /. h1 → 0 /. Vq211 → 0 /. Vm21 → 0 // Simplify;
MatrixForm[%]
Factor[Det[% - λ IdentityMatrix[3]]]
```

$$\begin{pmatrix} \frac{4\,(m\,(Vq212-Vq222)+2\,Vq222)\,Vq323}{(m\,p2\,(Vq212-Vq222-Vq223)+2\,(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323))^2} & \frac{4\,(-1+m)\,(-4\,Vq312\,Vq323+p2\,((-2+m)\,Vq223\,Vq312+(-(2+m)}{(m\,p2\,(V} \\ 0 & \\ 0 & \end{pmatrix}$$

$$-\Big(\Big(\lambda\,(-4\,m\,Vq111 + m\,p2\,Vq212\,\lambda + 2\,p2\,Vq222\,\lambda - m\,p2\,Vq222\,\lambda + 2\,p2\,Vq223\,\lambda - m\,p2\,Vq223\,\lambda + 4\,Vq323\,\lambda - 4\,p2\,Vq323\,\lambda)\,(-4\,m\,Vq212\,Vq323 - 8\,Vq222\,Vq323 + 4\,m\,Vq222\,Vq323 + m^2\,p2^2\,Vq212^2\,\lambda + 4\,m\,p2^2\,Vq212\,Vq222\,\lambda - 2\,m^2\,p2^2\,Vq212\,Vq222\,\lambda + 4\,p2^2\,Vq222^2\,\lambda - 4\,m\,p2^2\,Vq222^2\,\lambda + m^2\,p2^2\,Vq222^2\,\lambda + 4\,m\,p2^2\,Vq212\,Vq223\,\lambda - 2\,m^2\,p2^2\,Vq212\,Vq223\,\lambda + 8\,p2^2\,Vq222\,Vq223\,\lambda - 8\,m\,p2^2\,Vq222\,Vq223\,\lambda + 2\,m^2\,p2^2\,Vq222\,Vq223\,\lambda + 4\,p2^2\,Vq223^2\,\lambda - 4\,m\,p2^2\,Vq223^2\,\lambda + m^2\,p2^2\,Vq223^2\,\lambda + 8\,m\,p2\,Vq212\,Vq323\,\lambda - 8\,m\,p2^2\,Vq212\,Vq323\,\lambda + 16\,p2\,Vq222\,Vq323\,\lambda - 8\,m\,p2\,Vq222\,Vq323\,\lambda - 16\,p2^2\,Vq222\,Vq323\,\lambda + 8\,m\,p2^2\,Vq222\,Vq323\,\lambda + 16\,p2\,Vq223\,Vq323\,\lambda - 8\,m\,p2\,Vq223\,Vq323\,\lambda - 16\,p2^2\,Vq223\,Vq323\,\lambda + 8\,m\,p2^2\,Vq223\,Vq323\,\lambda + 16\,Vq323^2\,\lambda - 32\,p2\,Vq323^2\,\lambda + 16\,p2^2\,Vq323^2\,\lambda)\Big) \Big/ (m\,p2\,Vq212 + 2\,p2\,Vq222 - m\,p2\,Vq222 + 2\,p2\,Vq223 - m\,p2\,Vq223 + 4\,Vq323 - 4\,p2\,Vq323)^3\Big)$$

Stability of the PP/MP equilibrium along the edge is determined by the first element in the matrix:

```
edgeStab =
 D[recursionsF[[2]], p2] /. p1 → 0 /. h1 → 0 /. maternaleffect /. p2 → 0 // Simplify
```

$$\frac{m\,Vq212 + 2\,Vq222 - m\,Vq222}{4\,Vq323}$$

Thus, if $\frac{m\,Vq212+2\,Vq222-m\,Vq222}{4\,Vq323} < 1$ then p2 = 0 is stable and otherwise

$p2 = \frac{m\,Vq212+2\,Vq222-m\,Vq222-4\,Vq323}{m\,Vq212+2\,Vq222-m\,Vq222+2\,Vq223-m\,Vq223-4\,Vq323}$ is stable.

Stability of the PP/MP equilibrium to introduction of MM/M is determined by the last element in the matrix:

```
eigenEDGE =
 D[recursionsF[[1]], p1] /. p1 → 0 /. h1 → 0 /. Vq211 → 0 /. Vm21 → 0 // Simplify
```

$$\frac{4\,m\,Vq111}{m\,p2\,(Vq212-Vq222-Vq223)+2\,(p2\,(Vq222+Vq223-2\,Vq323)+2\,Vq323)}$$

Complete assortative mating by genotype: social fixation points cannot both be invaded

Below we show that if there is complete assortative mating then M-fixed and the polygynous edge cannot both be invaded.

We note the two possible equilibria on the polygynous edge:

```
edgeequil
```

$$\left\{0,\ \frac{m\,Vq212+2\,Vq222-m\,Vq222-4\,Vq323}{m\,Vq212+2\,Vq222-m\,Vq222+2\,Vq223-m\,Vq223-4\,Vq323}\right\}$$

If edgeStab is less than 1 then the P-fixed equilibrium is stable along the polygynous edge. If not, then the MP/PP equilibrium is stable.

```
edgeStab
```

$$\frac{m\,Vq212+2\,Vq222-m\,Vq222}{4\,Vq323}$$

If P-Fixed is stable on the polygynous edge then if the first element of eigenEDGE is greater than 1, MM queens invade.
If MP/PP equilibrium is stable on the polygynous edge then if the second element of eigenEDGE is greater than 1 then MM queens invade.

```
eigenEDGE /. p2 -> edgeequil // Factor
```

$$\left\{\frac{m\,Vq111}{Vq323},\ \frac{4\,m\,Vq111}{m\,Vq212+2\,Vq222-m\,Vq222}\right\}$$

To investigate if it is possible for both M-fixed and polygynous edge to be unstable with complete assortative mating we assume m=1.

The largest eigenvalue of M-fixed jacobian matrix needs to be greater than 1 for M-fixed to be unstable.

```
charpoly /. m → 1 // Factor
```

$$\frac{\lambda\,(-\text{Vq323}+\text{Vq111}\,\lambda)\,(-\text{Vq212}-\text{Vq222}+4\,\text{Vq111}\,\lambda)}{4\,\text{Vq111}^2}$$

```
Solve[% == 0, λ]
```

$$\left\{\{\lambda\to 0\},\ \left\{\lambda\to\frac{\text{Vq212}+\text{Vq222}}{4\,\text{Vq111}}\right\},\ \left\{\lambda\to\frac{\text{Vq323}}{\text{Vq111}}\right\}\right\}$$

Therefore one of the above roots of charpoly needs to be greater than 1.

P Fixed is stable along the polygynous edge when $\frac{\text{Vq212}+\text{Vq222}}{4\,\text{Vq323}}<1$ and MM queens can invade when $\frac{\text{Vq111}}{\text{Vq323}}>1$. Therefore $\frac{\text{Vq212}+\text{Vq222}}{4\,\text{Vq111}}<1$. Note that the latter two inequalities ensure that the roots of charpoly are smaller than 1.

If MP/PP equilibrium is stable on the polygynous edge then $\frac{\text{Vq212}+\text{Vq222}}{4\,\text{Vq323}}>1$, MM can invade when $\frac{4\,\text{Vq111}}{\text{Vq212}+\text{Vq222}}>1$. Therefore Vq111>Vq323. Note that the latter two inequalities ensure that the roots of charpoly are smaller than 1.

Therefore when there is complete genotype-based assortative mating both possible fixation points at M-fixed and polygynous edge cannot invade.
Numerical search confirms that when complete assortative mating by genotype no stable social polymorphism exists.

Weak selection analysis: social fixation points cannot both be invaded

Here we investigate if it is possible for M-fixed and polygynous edge to be invaded if there is weak selection.

We know that the polygynous edge must then go to the P-fixed equilibrium (because strong MP advantage is needed to go to the PP/MP point):

```
weaksel = {Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 -> 1 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 -> 1 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq312 -> 1 + sq312 ϵ, Vq223 → 1 + sq223 ϵ, Vq323 → 1 + sq323 ϵ};
```

```
Factor[Normal[Series[eigenEDGE /. λ → 1 + λ1 ϵ /. weaksel /. p2 → 0, {ϵ, 0, 1}]]]
```

$$m\,(1+\text{sq111}\,\epsilon-\text{sq323}\,\epsilon)$$

For invasion of P-fixed the above expression must be greater than 1.

If selection is weak (ϵ small), this cannot be accomplished unless m is very nearly one and $\frac{m\,\text{Vq111}}{\text{Vq323}}>1$,

so we add the assumption that m is near one:

```
weaksel = {m → 1 - sm * ϵ, Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 -> 1 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 -> 1 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq312 -> 1 + sq312 ϵ, Vq223 → 1 + sq223 ϵ, Vq323 → 1 + sq323 ϵ};
```

In addition we require $\lambda > 1$ where:

```
Factor[Normal[Series[charpoly /. λ → 1 + λ1 ϵ /. weaksel, {ϵ, 0, 1}]]]
```

$\frac{1}{2} \epsilon \, (\text{sq111} - \text{sq323} + \lambda 1)$

```
λ1 /. Flatten[Solve[% == 0, λ1]] // Simplify
```

$-\text{sq111} + \text{sq323}$

But this indicates that invasion (λ>1 so λ0>0) will only occur if Vq323 > Vq111, which contradicts the requirement above that $\frac{\text{m Vq111}}{\text{Vq323}} > 1$, so invasion of M-fixed and the polygynous edge, with weak selective differences, is not possible, even with a mixture of assortative and random mating.

We check if MP queens have almost as many offspring when mated with M males as otherwise, then that suggests that there is some sort of resource competition for the lost MM and M offspring. If so, then Vm22 and Vq212 would be nearer two due to the maternal effect killing with resources shifted to these offspring. Otherwise assuming weak selection, we get:

```
weaksel = {Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 → 2 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 → 2 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq223 → 1 + sq223 ϵ, Vq312 -> 1 + sq312 ϵ, Vq323 → 1 + sq323 ϵ};
```

```
Factor[Normal[Series[eigenEDGE /. λ → 1 + λ1 ϵ /. weaksel /. p2 → 0, {ϵ, 0, 1}]]]
```

$\text{m} \, (1 + \text{sq111} \, \epsilon - \text{sq323} \, \epsilon)$

If selection is weak (ϵ small), this cannot be accomplished unless m is very nearly one and $\frac{\text{m Vq111}}{\text{Vq323}} > 1$, so let's add the assumption that m is near one:

```
weaksel = {m → 1 - sm * ϵ, Vm11 -> 1 + sm11 ϵ, Vm21 -> 0, Vm22 → 2 + sm22 ϵ,
   Vm32 -> 1 + sm32 ϵ, Vq111 → 1 + sq111 ϵ, Vq122 → 1 + sq122 ϵ, Vq212 → 2 + sq212 ϵ,
   Vq222 → 1 + sq222 ϵ, Vq223 → 1 + sq223 ϵ, Vq312 -> 1 + sq312 ϵ, Vq323 → 1 + sq323 ϵ};
```

In addition we require λ>1 where:

```
Factor[Normal[Series[charpoly /. λ → 1 + λ1 ϵ /. weaksel, {ϵ, 0, 1}]]]
```

$-\frac{1}{4} \epsilon \, (\text{sm} - \text{sq111} + \text{sq323} - \lambda 1)$

```
λ1 /. Flatten[Solve[% == 0, λ1]] // Simplify
```

$\text{sm} - \text{sq111} + \text{sq323}$

which is the equivalent of $1 - \frac{\text{m Vq111}}{\text{Vq323}}$:

```
Factor[Normal[Series[1 - m Vq111/Vq323 /. λ → 1 + λ1 ϵ /. weaksel, {ϵ, 0, 1}]]]
```

(sm - sq111 + sq323) ϵ

Invasion (λ>1 so λ0>0) will thus occur if $\frac{m\,Vq111}{Vq323} < 1$, but this contradicts the requirement above that $\frac{m\,Vq111}{Vq323} > 1$, so invasion of both M-fixed and the polygynous edge, with weak selective differences, is not possible, even with mixture of assortative and random mating and complete compensation.

Social polymorphism numerical search

Confirming analytical results for complete assortative mating by genotype (m=1)

We have shown that M-fixed and the polygynous edge cannot both be invaded when m=1. Here we confirm that no stable social polymorphism can be found when m=1.

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] /. m → 1 // Simplify
eqn2 = Factor[recursionsF[[2]] - p2] /. m → 1 // Simplify
eqn3 = Factor[recursionsM[[1]] - h1] /. m → 1 // Simplify

findequil[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = {p1, p2, h1} /.
   NSolve[{(p2 Vq211 + p1 (4 Vq111 - p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) - 4 Vq323) -
          4 p1^2 (Vq111 - Vq323)) /
        (p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323) == 0,
      - ((p2 (-Vq212 - Vq222 + p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) +
             4 p1 (Vq111 - Vq323) + 4 Vq323)) / (p2
            (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323)) == 0,
      (-2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32))/
        (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32) ==
       0}, {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

```
D[recursionsF[[1]], p1] | D[recursionsF[[1]], p2] | D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1] | D[recursionsF[[2]], p2] | D[recursionsF[[2]], h1] /. m → 1 //
D[recursionsM[[1]], p1] | D[recursionsM[[1]], p2] | D[recursionsM[[1]], h1]
 Simplify
```

```
Clear[checkstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, p1, p2, h1}] = Eigenvalues[
   {{4 (4 Vq111 Vq323 + p2 (Vq111 (Vq212 + Vq222 + Vq223 - 4 Vq323) + Vq211 Vq323)) / (p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323)^2,
     - 4 (-Vq211 Vq323 + p1 (Vq111 (Vq212 + Vq222 + Vq223 - 4 Vq323) + Vq211 Vq323)) / (p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323)^2, 0},
    {- 4 p2 (Vq212 + Vq222) (Vq111 - Vq323) / (p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323)^2,
     4 (Vq212 + Vq222) (p1 (Vq111 - Vq323) + Vq323) / (p2 (Vq211 + Vq212 + Vq222 + Vq223 - 4 Vq323) + 4 p1 (Vq111 - Vq323) + 4 Vq323)^2, 0},
    {2 (2 Vm11 Vm32 + p2 (Vm11 (Vm22 - 2 Vm32) + Vm21 Vm32)) / (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32)^2,
     2 Vm21 Vm32 - 2 p1 (Vm11 (Vm22 - 2 Vm32) + Vm21 Vm32) / (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32)^2, 0}}]
```

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-10);
```

```
Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_,
   Vq222_, Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search:

```
SeedRandom[324 356]
Clear[tab, tab1]
For[j = 1; stabpoly = {}, j ≤ 1 000 000, j++,
 tab = N[Table[Round[Random[] * 10^10] / 10^10, {i, 1, 12}], 60];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 If[
  testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  testPol,
  Print[tab1] && AppendTo[stabpoly, tab1]];
 If[Mod[j, 5000] == 0, Print[j]];
 If[Mod[j, 10 000] == 0, Print[stabpoly]];
]
```

Confirming analytical results for weak selection

Find equilibria:

We create the function “findequil” which numerically searches equilibrium values:

```
eqn1 = Factor[recursionsF[[1]] - p1] // Simplify
```

```
eqn2 = Factor[recursionsF[[2]] - p2] // Simplify
```

```
eqn3 = Factor[recursionsM[[1]] - h1] // Simplify
```

```
findequil[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, m_}] :=
 findequil[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312, Vq323, Vm11,
    Vm21, Vm22, Vm32, m}] = {p1, p2, h1} /. NSolve[{(m (4 p1^2 (Vq111 - Vq122) -
              p2 Vq211 + p1 (-4 Vq111 + p2 (Vq211 + Vq212 - Vq222 - Vq223))) +
          2 p1 (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323) - 2 h1 (-1 + m)
           (-p2 Vq211 + 2 p1^2 (Vq111 - Vq122 - Vq312 + Vq323) + p1 (-2 Vq111 + 2 (Vq312 -
                    Vq323) + p2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323)))) /
        (m (-4 p1 (Vq111 - Vq122) + p2 (-Vq211 - Vq212 + Vq222 + Vq223)) -
          2 (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323) +
          2 h1 (-1 + m) (2 (Vq312 - Vq323) + 2 p1 (Vq111 - Vq122 - Vq312 + Vq323) +
              p2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323))) == 0,
      (m (4 p1 (p2 (Vq111 - Vq122) + Vq122) + p2 (-Vq212 + Vq222 + p2
                 (Vq211 + Vq212 - Vq222 - Vq223))) + 2 (p2 ((-1 + p2) Vq222 + p2
                 Vq223 + 2 Vq323 - 2 p2 Vq323) + 2 p1 ((-1 + p2) Vq122 - p2 Vq323)) -
          2 h1 (-1 + m) (-2 Vq312 + p2 (-Vq212 + Vq222 + 4 Vq312 - 2 Vq323) +
              p2^2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323) +
              2 p1 (Vq122 + Vq312 + p2 (Vq111 - Vq122 - Vq312 + Vq323)))) /
        (m (-4 p1 (Vq111 - Vq122) + p2 (-Vq211 - Vq212 + Vq222 + Vq223)) -
          2 (p2 (Vq222 + Vq223 - 2 Vq323) + 2 p1 (Vq122 - Vq323) + 2 Vq323) +
          2 h1 (-1 + m) (2 (Vq312 - Vq323) + 2 p1 (Vq111 - Vq122 - Vq312 + Vq323) +
              p2 (Vq211 + Vq212 - Vq222 - Vq223 - 2 Vq312 + 2 Vq323))) == 0,
      (-2 h1 Vm32 + 2 p1 (Vm11 - h1 Vm11 + h1 Vm32) + p2 (Vm21 - h1 Vm21 - h1 Vm22 + 2 h1 Vm32)) /
        (p2 (Vm21 + Vm22 - 2 Vm32) + 2 p1 (Vm11 - Vm32) + 2 Vm32) == 0}, {p1, p2, h1}]
```

Check stability:

We create the function “checkstab” which checks the stability of each equilibrium point.

D[recursionsF[[1]], p1]	D[recursionsF[[1]], p2]	D[recursionsF[[1]], h1]
D[recursionsF[[2]], p1]	D[recursionsF[[2]], p2]	D[recursionsF[[2]], h1]
D[recursionsM[[1]], p1]	D[recursionsM[[1]], p2]	D[recursionsM[[1]], h1]

```
Clear[checkstab];
checkstab[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_, Vq223_,
   Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, m_, p1_, p2_, h1_}] :=
 checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, m, p1, p2, h1}] =
  Eigenvalues[{{-(((2 (h1 (1 - m) p1 + m p1) Vq111 + (h1 (1 - m) p2 + m p2/2) Vq211)
            (2 (h1 (1 - m) + m) Vq111 + 2 (1 - h1) (1 - m) Vq122 -
              2 h1 (1 - m) Vq312 + 2 (-(1 - h1) (1 - m) - m) Vq323))/
```

$$\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+2\,(1-h1)\,(1-m)\,p1\,Vq122+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)\right.$$
$$Vq211+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq212+\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq222+$$
$$\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq223+2\,h1\,(1-m)\,(1-p1-p2)\,Vq312+$$
$$\left.\left.2\,((1-h1)\,(1-m)\,(1-p1-p2)+m\,(1-p1-p2))\,Vq323\right)^2\right)+$$
$$(2\,(h1\,(1-m)+m)\,Vq111)\Big/\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+2\,(1-h1)\,(1-m)\,p1\,Vq122+\right.$$
$$\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq211+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq212+$$
$$\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq222+\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq223+2\,h1\,(1-m)$$
$$\left.(1-p1-p2)\,Vq312+2\,((1-h1)\,(1-m)\,(1-p1-p2)+m\,(1-p1-p2))\,Vq323\right),$$
$$-\left(\left(\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq211\right)\left(\left(h1\,(1-m)+\frac{m}{2}\right)\right.\right.\right.$$
$$Vq211+\left(h1\,(1-m)+\frac{m}{2}\right)Vq212+\left((1-h1)\,(1-m)+\frac{m}{2}\right)Vq222+\left((1-h1)\right.$$
$$\left.\left.\left.(1-m)+\frac{m}{2}\right)Vq223-2\,h1\,(1-m)\,Vq312+2\,(-(1-h1)\,(1-m)-m)\,Vq323\right)\right)\Big/$$
$$\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+2\,(1-h1)\,(1-m)\,p1\,Vq122+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)\right.$$
$$Vq211+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq212+\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq222+$$
$$\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq223+2\,h1\,(1-m)\,(1-p1-p2)\,Vq312+$$
$$\left.\left.2\,((1-h1)\,(1-m)\,(1-p1-p2)+m\,(1-p1-p2))\,Vq323\right)^2\right)+$$
$$\left(\left(h1\,(1-m)+\frac{m}{2}\right)Vq211\right)\Big/\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+\right.$$
$$2\,(1-h1)\,(1-m)\,p1\,Vq122+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq211+$$
$$\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq212+\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq222+$$
$$\left((1-h1)\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq223+2\,h1\,(1-m)\,(1-p1-p2)\,Vq312+$$
$$\left.2\,((1-h1)\,(1-m)\,(1-p1-p2)+m\,(1-p1-p2))\,Vq323\right),$$
$$(2\,(1-m)\,p1\,Vq111+(1-m)\,p2\,Vq211)\Big/\left(2\,(h1\,(1-m)\,p1+m\,p1)\,Vq111+\right.$$
$$2\,(1-h1)\,(1-m)\,p1\,Vq122+\left(h1\,(1-m)\,p2+\frac{m\,p2}{2}\right)Vq211+$$

$$\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq222+$$
$$\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq223+2\ h1\ (1-m)\ (1-p1-p2)\ Vq312+$$
$$2\ ((1-h1)\ (1-m)\ (1-p1-p2)+m\ (1-p1-p2))\ Vq323\Big)-$$
$$\Bigg(\Bigg(2\ (h1\ (1-m)\ p1+m\ p1)\ Vq111+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq211\Bigg)\ (2\ (1-m)\ p1\ Vq111-$$
$$2\ (1-m)\ p1\ Vq122+(1-m)\ p2\ Vq211+(1-m)\ p2\ Vq212-(1-m)\ p2\ Vq222-$$
$$(1-m)\ p2\ Vq223+2\ (1-m)\ (1-p1-p2)\ Vq312-2\ (1-m)\ (1-p1-p2)\ Vq323)\Bigg)\Big/$$
$$\Bigg(2\ (h1\ (1-m)\ p1+m\ p1)\ Vq111+2\ (1-h1)\ (1-m)\ p1\ Vq122+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right)$$
$$Vq211+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq222+$$
$$\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq223+2\ h1\ (1-m)\ (1-p1-p2)\ Vq312+$$
$$2\ ((1-h1)\ (1-m)\ (1-p1-p2)+m\ (1-p1-p2))\ Vq323\Bigg)^2\Bigg\},$$
$$\Bigg\{-\Bigg(\Bigg(\Bigg(2\ (1-h1)\ (1-m)\ p1\ Vq122+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\Bigg((1-h1)\ (1-m)\ p2+$$
$$\frac{m\ p2}{2}\Bigg) Vq222+2\ h1\ (1-m)\ (1-p1-p2)\ Vq312\Bigg)\ (2\ (h1\ (1-m)+m)\ Vq111+$$
$$2\ (1-h1)\ (1-m)\ Vq122-2\ h1\ (1-m)\ Vq312+2\ (-(1-h1)\ (1-m)-m)\ Vq323)\Bigg)\Big/$$
$$\Bigg(2\ (h1\ (1-m)\ p1+m\ p1)\ Vq111+2\ (1-h1)\ (1-m)\ p1\ Vq122+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right)$$
$$Vq211+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq222+$$
$$\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq223+2\ h1\ (1-m)\ (1-p1-p2)\ Vq312+$$
$$2\ ((1-h1)\ (1-m)\ (1-p1-p2)+m\ (1-p1-p2))\ Vq323\Bigg)^2\Bigg)+$$
$$(2\ (1-h1)\ (1-m)\ Vq122-2\ h1\ (1-m)\ Vq312)\Big/\ \Bigg(2\ (h1\ (1-m)\ p1+m\ p1)\ Vq111+$$
$$2\ (1-h1)\ (1-m)\ p1\ Vq122+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq211+$$
$$\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq222+$$
$$\left((1-h1)\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq223+2\ h1\ (1-m)\ (1-p1-p2)\ Vq312+$$
$$2\ ((1-h1)\ (1-m)\ (1-p1-p2)+m\ (1-p1-p2))\ Vq323\Bigg),$$
$$-\Bigg(\Bigg(\Bigg(2\ (1-h1)\ (1-m)\ p1\ Vq122+\left(h1\ (1-m)\ p2+\frac{m\ p2}{2}\right) Vq212+\Bigg((1-h1)\ (1-m)\ p2+$$

$$\frac{m\,p2}{2}\Big)\,Vq222 + 2\,h1\,(1-m)\,(1-p1-p2)\,Vq312\Big)\,\Big(\Big(h1\,(1-m) + \frac{m}{2}\Big)\,Vq211 +$$
$$\Big(h1\,(1-m) + \frac{m}{2}\Big)\,Vq212 + \Big((1-h1)\,(1-m) + \frac{m}{2}\Big)\,Vq222 + \Big((1-h1)\,(1-m) + \frac{m}{2}\Big)$$
$$Vq223 - 2\,h1\,(1-m)\,Vq312 + 2\,(-(1-h1)\,(1-m) - m)\,Vq323\Big)\Big)\Big/$$
$$\Big(2\,(h1\,(1-m)\,p1 + m\,p1)\,Vq111 + 2\,(1-h1)\,(1-m)\,p1\,Vq122 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)$$
$$Vq211 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq212 + \Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq222 +$$
$$\Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq223 + 2\,h1\,(1-m)\,(1-p1-p2)\,Vq312 +$$
$$2\,((1-h1)\,(1-m)\,(1-p1-p2) + m\,(1-p1-p2))\,Vq323\Big)^2\Big) +$$
$$\Big(\Big(h1\,(1-m) + \frac{m}{2}\Big)\,Vq212 + \Big((1-h1)\,(1-m) + \frac{m}{2}\Big)\,Vq222 - 2\,h1\,(1-m)\,Vq312\Big)\Big/$$
$$\Big(2\,(h1\,(1-m)\,p1 + m\,p1)\,Vq111 + 2\,(1-h1)\,(1-m)\,p1\,Vq122 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)$$
$$Vq211 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq212 + \Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq222 +$$
$$\Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq223 + 2\,h1\,(1-m)\,(1-p1-p2)\,Vq312 +$$
$$2\,((1-h1)\,(1-m)\,(1-p1-p2) + m\,(1-p1-p2))\,Vq323\Big),$$
$$(-2\,(1-m)\,p1\,Vq122 + (1-m)\,p2\,Vq212 - (1-m)\,p2\,Vq222 + 2\,(1-m)\,(1-p1-p2)\,Vq312)\Big/$$
$$\Big(2\,(h1\,(1-m)\,p1 + m\,p1)\,Vq111 + 2\,(1-h1)\,(1-m)\,p1\,Vq122 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)$$
$$Vq211 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq212 + \Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq222 +$$
$$\Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq223 + 2\,h1\,(1-m)\,(1-p1-p2)\,Vq312 +$$
$$2\,((1-h1)\,(1-m)\,(1-p1-p2) + m\,(1-p1-p2))\,Vq323\Big) -$$
$$\Big(\Big(2\,(1-h1)\,(1-m)\,p1\,Vq122 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq212 +$$
$$\Big((1-h1)\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)\,Vq222 + 2\,h1\,(1-m)\,(1-p1-p2)\,Vq312\Big)$$
$$(2\,(1-m)\,p1\,Vq111 - 2\,(1-m)\,p1\,Vq122 + (1-m)\,p2\,Vq211 +$$
$$(1-m)\,p2\,Vq212 - (1-m)\,p2\,Vq222 - (1-m)\,p2\,Vq223 +$$
$$2\,(1-m)\,(1-p1-p2)\,Vq312 - 2\,(1-m)\,(1-p1-p2)\,Vq323)\Big)\Big/$$
$$\Big(2\,(h1\,(1-m)\,p1 + m\,p1)\,Vq111 + 2\,(1-h1)\,(1-m)\,p1\,Vq122 + \Big(h1\,(1-m)\,p2 + \frac{m\,p2}{2}\Big)$$

$$\text{Vq211} + \left(\text{h1}\ (1-\text{m})\ \text{p2} + \frac{\text{m p2}}{2}\right) \text{Vq212} + \left((1-\text{h1})\ (1-\text{m})\ \text{p2} + \frac{\text{m p2}}{2}\right) \text{Vq222} +$$
$$\left((1-\text{h1})\ (1-\text{m})\ \text{p2} + \frac{\text{m p2}}{2}\right) \text{Vq223} + 2\ \text{h1}\ (1-\text{m})\ (1-\text{p1}-\text{p2})\ \text{Vq312} +$$
$$2\ ((1-\text{h1})\ (1-\text{m})\ (1-\text{p1}-\text{p2}) + \text{m}\ (1-\text{p1}-\text{p2}))\ \text{Vq323}\Big)^2\Big\},$$
$$\Big\{-\Big(((2\ \text{p1 Vm11} + \text{p2 Vm21})\ (2\ \text{Vm11} - 2\ \text{Vm32})) \Big/ (2\ \text{p1 Vm11} + \text{p2 Vm21} +$$
$$\text{p2 Vm22} + 2\ \text{Vm32} - 2\ \text{p1 Vm32} - 2\ \text{p2 Vm32})^2\Big) +$$
$$\frac{2\ \text{Vm11}}{2\ \text{p1 Vm11} + \text{p2 Vm21} + \text{p2 Vm22} + 2\ \text{Vm32} - 2\ \text{p1 Vm32} - 2\ \text{p2 Vm32}},$$
$$-\Big(((2\ \text{p1 Vm11} + \text{p2 Vm21})\ (\text{Vm21} + \text{Vm22} - 2\ \text{Vm32})) \Big/$$
$$(2\ \text{p1 Vm11} + \text{p2 Vm21} + \text{p2 Vm22} + 2\ \text{Vm32} - 2\ \text{p1 Vm32} - 2\ \text{p2 Vm32})^2\Big) +$$
$$\frac{\text{Vm21}}{2\ \text{p1 Vm11} + \text{p2 Vm21} + \text{p2 Vm22} + 2\ \text{Vm32} - 2\ \text{p1 Vm32} - 2\ \text{p2 Vm32}}, 0\Big\}\Big\}\Big]$$

Sieve:

Sieve finds equilibria, checks stability, and keeps only the stable polymorphic equilibrium.

```
cutoff = 10^(-10);
```

```
Clear[sieve]
sieve[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, m_}] :=
 sieve[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, m}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32, m}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 && (Chop[eq[[i]], 10^-4] ≠ {0, 0, 0}) &&
      (Chop[eq[[i]] - {1, 0, 1}, 10^-4] ≠ {0, 0, 0}) && (eq[[i, 1]] + eq[[i, 2]] ≤ 1) &&
      (MemberQ[Chop[eq[[i]], 10^-4], 0] == False) &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
            Vq323, Vm11, Vm21, Vm22, Vm32, m, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
            Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Sieve3 shows all stable equilibria.

```
Clear[sieve3]
sieve3[{Vq111_, Vq122_, Vq211_, Vq212_, Vq222_,
   Vq223_, Vq312_, Vq323_, Vm11_, Vm21_, Vm22_, Vm32_, m_}] :=
 sieve3[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
    Vq323, Vm11, Vm21, Vm22, Vm32, m}] = Block[{},
   For[i = 1; write = {}, i ≤ (max = Length[eq = findequil[{Vq111, Vq122, Vq211,
            Vq212, Vq222, Vq223, Vq312, Vq323, Vm11, Vm21, Vm22, Vm32, m}]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff ≤ Re[x] ≤ 1 + cutoff) &&
              Abs[Im[x]] < cutoff)]] == 3 &&
      (Max[Abs[checkstab[{Vq111, Vq122, Vq211, Vq212, Vq222, Vq223, Vq312,
             Vq323, Vm11, Vm21, Vm22, Vm32, m, Chop[eq[[i, 1]]], Chop[eq[[i, 2]]],
             Chop[eq[[i, 3]]]}]]] < 1), write = Append[write, Chop[eq[[i]]]]]];
   Sort[write]
  ]
```

Numerical search:

```
SeedRandom[324 356]
Clear[tab, tab1]
For[j = 1; stabpoly = {}, j ≤ 1 000 000, j++,
 tab = N[Table[Round[Random[Real, {.95, 1}] * 10^10] / 10^10, {i, 1, 12}], 60];
 AppendTo[tab, N[Round[Random[Real, {0, 1}] * 10^10] / 10^10, 60]];
 tab1 = ReplacePart[tab, {3 → 0, 10 → 0}];
 If[
  testPol = Reduce[sieve[tab1] == {}, j == j] == False;
  testPol,
  Print[tab1] && AppendTo[stabpoly, tab1]];
 If[Mod[j, 5000] == 0, Print[j]];
 If[Mod[j, 10 000] == 0, Print[stabpoly]];
]
```