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Unbalanced selection: the challenge of maintaining a social polymorphism when a supergene is selfish

Alireza G. Tafreshi¹, Sarah P. Otto¹ and Michel Chapuisat^{2*}

1. Department of Zoology & Biodiversity Research Centre, University of British Columbia, Vancouver, BC V6T 1Z4, Canada

2. Department of Ecology and Evolution, University of Lausanne, 1015 Lausanne, Switzerland

* Author for correspondence: Michel.Chapuisat@unil.ch, +41 21 692 41 78

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1 **Abstract**

2 Supergenes often have multiple phenotypic effects, including unexpected detrimental ones, because
3 recombination suppression maintains associations among co-adapted alleles but also allows the
4 accumulation of recessive deleterious mutations and selfish genetic elements. Yet, supergenes often persist
5 over long evolutionary periods. How are such polymorphisms maintained in the face of selection, drive
6 and drift? We present a population genetic model that investigates the conditions necessary for a stable
7 polymorphic equilibrium when one of the supergene haplotypes is a selfish genetic element. The model
8 fits the characteristics of the Alpine silver ant, *Formica selysi*, in which a large supergene underlies colony
9 social organization and one haplotype distorts Mendelian transmission by killing progeny that did not
10 inherit it. The model shows that such maternal-effect killing strongly limits the maintenance of the social
11 polymorphism. Under random mating, transmission ratio distortion prevents rare single-queen colonies
12 from invading populations of multiple-queen colonies, regardless of the fitness of each genotype. A stable
13 polymorphic equilibrium can, however, be reached when high rates of assortative mating are combined
14 with large fitness differences among supergene genotypes. The model reveals that the persistence of the
15 social polymorphism is non-trivial and expected to occur only under restrictive conditions that deserve
16 further empirical investigation.

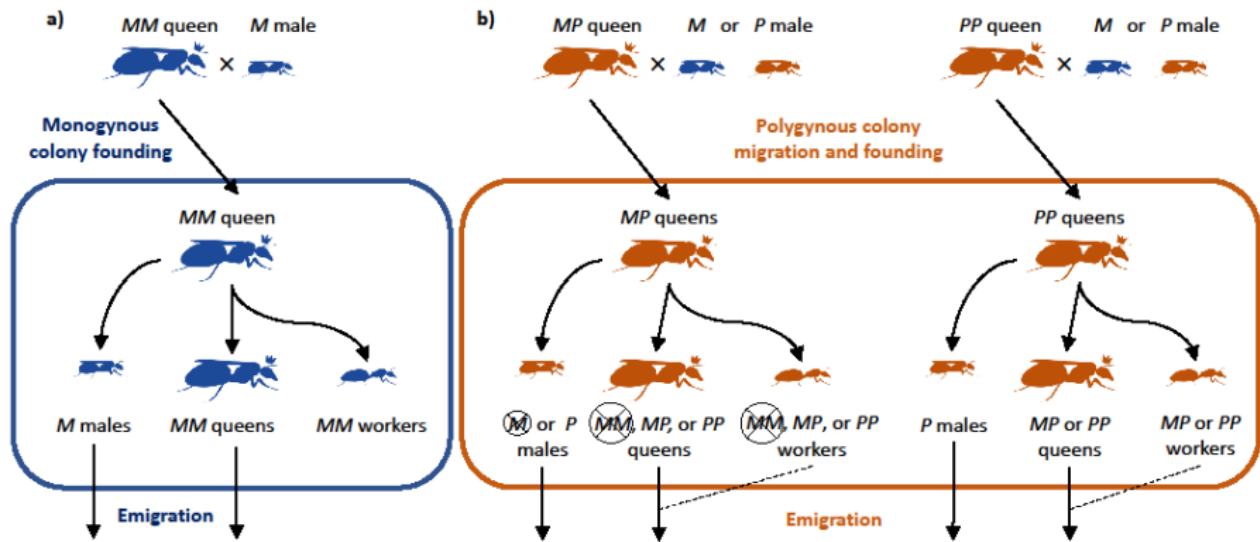
17 **Introduction**

18 Supergenes – large non-recombining genomic regions – underlie some of the more striking
19 polymorphisms in nature [1, 2]. They commonly affect multiple traits of the phenotype, controlled by
20 linked alleles, of which some are co-adapted [3, 4]. Supergenes are also prone to accumulate recessive
21 deleterious mutations [2, 5] and selfish genetic elements that distort the laws of Mendelian inheritance
22 [6-8]. Indeed, the lack of recombination hinders the purging of deleterious elements and allows the
23 accumulation of selfish genetic elements that favour their own transmission, such as toxin-antidote
24 elements [9]. Yet, supergenes often persist over long evolutionary periods [10]. How are such
25 polymorphisms balanced in the face of selection, drive, and drift? Fundamental mechanisms leading to
26 balanced polymorphisms at supergenes include various forms of negative frequency-dependent
27 selection, temporally or spatially-varying selection, and overdominance [2, 5]. Because supergenes have
28 complex effects, understanding their evolutionary trajectory remains a substantial challenge.

29 A supergene usually arises when recombination is suppressed, often as a result of an inversion, so that a
30 group of neighbouring genes becomes inherited as a single Mendelian element. The long-term fate of the
31 novel haplotype will depend on drift and on the combined selective effects over multiple supergene
32 elements. A mutant non-recombining haplotype will spread if it has captured adaptive combinations of
33 alleles [the supergene hypothesis; reviewed in 4]. Non-recombining haplotypes can also spread selfishly,
34 by being transmitted to a disproportionate number of adult offspring. Indeed, gene drive arises when a
35 driver locus (typically with toxic “killer” effects on the product of a responder locus) becomes tightly
36 linked to an insensitive allele at the responder locus, so that the selfish genetic elements is not suicidal
37 [6, 9]. Such selfish genetic elements tend to be located in non-recombining regions, like supergenes [8,
38 11]. Counteracting these advantages, non-recombining haplotypes are predicted to accumulate
39 deleterious mutations [5]. Hence, in many cases homozygous lethality prevents a positively selected or
40 driven haplotype from reaching fixation [1, 2]. As supergenes influence multiple traits at once, which
41 often have confounding or antagonistic effects, formal modelling is needed to understand their
42 evolutionary dynamics.

43 Here, we present a model examining the conditions for the long-term persistence of a polymorphism in a
44 supergene controlling alternative forms of social organisation in ants, in which one haplotype is a
45 transmission ratio distorter. The model is designed to fit the properties of the Alpine silver ant, *Formica*
46 *selysi*. This species is polymorphic for colony social organisation: within the same populations, it forms
47 monogynous colonies, which have a single reproductive queen, and polygynous colonies, in which

48 multiple queens share offspring production [12-14]. A genome-wide association study coupled with
 49 linkage maps has identified that a large supergene with two highly differentiated non-recombining
 50 haplotypes, *Sm* and *Sp*, underlies this social polymorphism [15]. We will simplify the notation slightly
 51 and refer to the *Sm* haplotype as *M* and the *Sp* haplotype as *P*. Mature (= large-sized, several years old)
 52 monogynous colonies consist of individuals carrying exclusively the *M* haplotype: all females (queens
 53 and workers) have the supergene genotype *MM*, while all males have the haplotype *M* (females are
 54 diploid and males haploid in ants; Figure 1a). In contrast, polygynous colonies consist of female ants
 55 carrying at least one copy of the *P* haplotype, i.e. having the supergene genotype *PP* or *MP*, and
 56 producing only *P* males (Figure 1b). Monogynous colonies are established by *MM* queens
 57 independently, without the help of workers, whereas polygynous colonies are founded by *MP* or *PP*
 58 queens, and possibly *MM* queens mated to *P* males, either independently or accompanied by workers
 59 from their natal colony (Figure 1).



60 **Figure 1.** Social and genetic system of *F. selysi*. (a) Mature monogynous colonies contain a single *MM*
 61 queen mated with an *M* male. The queen produces *M* males (haploid, from unfertilized eggs), as well as
 62 *MM* queens and *MM* workers (diploid, from fertilized eggs). The offspring (males and queens) fly out of
 63 the colony for mating, and queens establish colonies independently. (b) Mature polygynous colonies
 64 contain multiple *MP* or *PP* queens mated with either *M* or *P* males. The offspring (queens and males)
 65 also fly out of the colony for mating. *MP* and *PP* queens (and possibly *MM* queens mated to *P* males)
 66 may establish colonies independently, or, for polygynous queens, with the help of workers from their
 67 natal colony (dashed line). The *P* haplotype acts as a maternal effect killer, so that all offspring of *MP*
 68 queens that do not inherit the *P* haplotype die during development. As a result, *M* males and *MM*
 69 females are never produced by polygynous colonies.

70 The supergene controlling social organization in *F. selysi* is very ancient, being shared by multiple
 71 *Formica* lineages separated by 20-40 MY of independent evolution [16]. The two haplotypes have
 72 accumulated substantial sequence divergence and differ by several inversions [15, 16]. Field data in *F.*

73 *selysi* suggest that the proportion of monogynous and polygynous colonies tends to be stable across
74 years, remaining close to 50% in one well-studied population [12, 14], while varying between patches of
75 mosaic habitat within populations [17]. Moreover, many populations contain both types of colonies [18].
76 Overall, the comparative genomic and population genetic data show the signature of strong and
77 persistent balancing selection, consistent with an evolutionarily stable polymorphism.

78 Genetic and behavioral data indicate ongoing gene flow between the monogynous and polygynous social
79 forms of *F. selysi*. First, the two social forms show no or minimal signs of genetic differentiation across
80 most of their genome, outside of the supergene [15, 19]. Second, monogynous and polygynous colonies
81 are found in close spatial proximity within the same populations [12, 17, 18]. Third, virgin queens and
82 males fly to mating sites on neighbouring trees, where they mate [12, 20]. Queens and males from
83 alternative social forms readily mate in choice experiments [21], as well as in field mating swarms [20].
84 In mature field colonies, the polygynous queens had frequently mated with males of monogynous origin,
85 whereas the monogynous queens had only mated with males of monogynous origin, which suggests
86 unidirectional gene flow from the monogynous to the polygynous social form [19]. However, recent
87 analyses of mating swarms and incipient colonies revealed that *MM* females do mate with *P* males [20,
88 22]. These queens might establish colonies that do not reach a mature size or that become polygynous,
89 given that in the field, we never detected a mature colony headed by a single monogynous queen mated
90 with a *P* male [15, 19].

91 Mating in Alpine silver ants is non-random with respect to the social form of queens and males [15, 19,
92 20]. The sampling and genotyping of ants from mature field colonies revealed that queens from
93 monogynous colonies had mated exclusively with *M* males ($n = 69$ matings), while queens from
94 polygynous colonies had mated with both *P* and *M* males, the latter accounting for 23% of the matings,
95 for both *MP* and *PP* queens ($n = 190$ matings) [19]. While matings outside of the colony have been well
96 documented [12, 20], evidence for within-colony mating is only indirect [19]. In monogynous colonies,
97 mates were not significantly related to queens, suggesting that within-colony mating is rare or absent in
98 this social form. In polygynous colonies, mates were slightly related to queens, suggesting that some of
99 the queens had mated with nestmate males within or near the natal colony [19]. Queens occasionally
100 mate with multiple males [12, 19]. Local mate availability or mate preferences, as well as sperm
101 discrimination or fertility differences after mating, may influence the probability of each cross. After
102 mating, both types of queens are able to establish colonies independently [22]. In addition, polygynous
103 queens may establish colonies with workers from their natal colony (colony budding). Overall, due to

104 pronounced differences in life-history, including dispersal, mating and colony founding [13, 20], queens
105 from alternative social forms are likely to greatly differ in fitness.

106 A striking feature of the genetic system underlying this social polymorphism is that the *P* haplotype acts
107 as a maternal-effect killer, causing the early death of any offspring of *MP* mothers that do not bear the *P*
108 haplotype [8; Figure 1b]. While *MP* females commonly mate with *M* males and produce *MP* and *MM*
109 eggs in Mendelian proportions (no meiotic drive), *MM* or *M* eggs from *MP* mothers fail to hatch [8, 19].
110 Thus, *MP* females never produce adult *MM* daughters nor *M* sons (Figure 1b). In short, the *P* haplotype
111 causes complete gene drive, distorting Mendelian transmission in such a way that all offspring produced
112 by polygynous colonies carry a *P* haplotype. Furthermore, substantial fitness differences have been
113 detected among females carrying alternative genotypes, in both the lab and the field (Blacher, De
114 Gasperin, Grasso, Sarton-Lohéac, Alleman and Chapuisat, unpublished results).

115 In this article, we show that the transmission ratio distortion induced by maternal-effect killing strongly
116 limits the maintenance of the social polymorphism. We develop a population genetic model that
117 incorporates the known aspects of the genetic, social, and mating systems of *F. selysi*. Using this model,
118 we determine the conditions necessary to reach a stable polymorphic equilibrium. Our model reveals
119 that, under many conditions, classical forms of balancing selection, including when selfish genetic
120 elements are balanced by strong counterselection in homozygotes, fail to stabilize the social
121 polymorphism. In other examples of transmission ratio distortion (e.g., tailless in mice), an element that
122 is driven (e.g., the *t* allele) spreads when rare but is prevented from fixing by the sterility of *tt* males
123 [23]. With *F. selysi*, however, maternal-effect killing prevents *MP* females from producing the *MM*
124 females that are needed to establish monogynous colonies. Thus, while low fitness of the driven
125 haplotype *P* can ensure that *M* spreads when rare, the result is generally an equilibrium consisting of
126 only *PP* and *MP* females in polygynous colonies, and the social polymorphism is lost. Even if maternal-
127 effect killing is not complete and the occasional *M* son or *MM* daughter is produced, they would
128 generally fail to establish new monogynous colonies, because most of their matings would be with
129 polygynous individuals, whose offspring are of the polygynous type. Hence, when mating is random,
130 maternal effect killing leads to the extinction of monogynous colonies regardless of how strong selection
131 is against homozygotes. Overall, as we will show, the gene drive caused by maternal-effect killing
132 destabilizes the genetic polymorphism and precludes the maintenance of a social polymorphism with
133 both polygynous and monogynous colonies when mating is random and fitnesses are frequency-

134 independent. The goal of this paper is to determine what exactly could account for the maintenance of
135 the supergene polymorphism and both social forms.

136 Here we briefly outline the approach that we take, section by section. We start by constructing a general
137 model that can be used to explore all scenarios described below. We then apply this model to different
138 forms of reproduction, starting with random mating, to determine the conditions under which the social
139 polymorphism observed in *F. selysi* would be stable:

- 140 1) Random Mating: We first considered the case of a randomly mating population, allowing for
141 arbitrarily strong natural selection, finding that the social polymorphism is never stable.
- 142 2) Variation in Fertility: We then include fertility differences between mating pairs, again finding
143 that the social polymorphism is never stable.
- 144 3) Sexual Selection: Next, we incorporate sexual selection, using the fixed relative preference
145 scheme of Kirkpatrick [24]. This model assumes that all queens will be mated and that their
146 preferences determine the relative frequency of mating with *M* and *P* males. Again, we show that
147 a social polymorphism is not maintained except when sexual selection is so extreme that it
148 results in assortative mating.
- 149 4) Assortative Mating: Finally, we model assortative mating, finding that partial (but not complete)
150 assortative mating can maintain the social polymorphism if combined with strong enough
151 selection.

152 Together, these models demonstrate how challenging it is to maintain the *F. selysi* social polymorphism
153 because of the selfish supergene drive of the *P* haplotype and clarify the conditions required to account
154 for the persistence of both monogynous and polygynous colonies.

155 **Model**

156 Motivated by *F. selysi*, we develop a population genetic model to investigate the conditions necessary
157 for a stable polymorphic equilibrium when one haplotype is a maternal-effect killer. This model with
158 non-overlapping generations and male-haploidy follows the dynamics of genotypes at a supergene
159 controlling social organisation among reproductive females (= queens) and among males separately
160 (Table 1). Because workers do not reproduce (Figure 1), we do not incorporate them within the
161 population dynamics, except indirectly via the fitness of queens and males. We census at the adult stage,
162 among surviving ants, who then mate and reproduce, followed by selection among their offspring (see
163 Appendix Table for definitions of all parameters).

Table 1. Genotype frequencies among adult queens and males.
Social form refers to the social organization of the colony of origin.

Supergene genotype	Sex	Social form	Frequency
<i>MM</i>	Queen	Monogynous	X_{MM}
<i>MP</i>	Queen	Polygynous	X_{MP}
<i>PP</i>	Queen	Polygynous	X_{PP}
<i>M</i>	Male	Monogynous	Y_M
<i>P</i>	Male	Polygynous	Y_P

164 To model the production of males, which is not dependent on mating, we let $O_{a|ij}^\sigma$ represent the
 165 production rate of surviving male offspring of genotype a produced by a queen of genotype ij . We
 166 assume Mendelian segregation but allow the number and survival of the male offspring, $V_{a|ij}^\sigma$, to depend
 167 on its mother's genotype, enabling us to incorporate maternal effects, including maternal-effect killing,
 168 and effects of colony type on fitness. For example, *MP* queens at frequency X_{MP} produce *M* and *P* males
 169 at rates proportional to:

$$170 \quad O_{M|MP}^\sigma = \frac{1}{2} X_{MP} V_{M|MP}^\sigma \quad (1a)$$

$$171 \quad O_{P|MP}^\sigma = \frac{1}{2} X_{MP} V_{P|MP}^\sigma \quad (1b)$$

172 If there is complete maternal-effect killing of all offspring that did not inherit the *P* haplotype from their
 173 *MP* mother, then $V_{M|MP}^\sigma = 0$. If colonies are unable to recover the resources invested in lost males, $V_{P|MP}^\sigma$
 174 is expected to be near one (all else being equal). If colonies are able to fully recover the energy invested
 175 in lost embryos and use it to produce *P* males, then $V_{P|MP}^\sigma$ may be elevated relative to other male
 176 fitnesses.

177 To model the production of queens, we let $O_{ab|ij \times k}^\varnothing$ represent the production rate of surviving queen
 178 offspring of genotype ab produced from matings between a queen of genotype ij and a male of genotype
 179 k . As described below, $O_{ab|ij \times k}^\varnothing$ incorporates both the reproductive mode and several components of
 180 fitness, including the rate at which different crosses occur, fertility differences, offspring number, and
 181 the survival rate of daughters, $V_{ab|ij \times k}^\varnothing$ (Table 2). We assume that the zygotes are produced in a

182 Mendelian fashion but allow complete maternal-effect killing, eliminating MM offspring of MP mothers
 183 ($V_{MM|MP \times M}^{\circ} = 0$).

Table 2. Production of queens

Mating	Frequency	Viability of ab offspring	Proportion of MM offspring	Proportion of MP offspring	Proportion of PP offspring
$MM \times M$	$R_{MM \times M}$	$V_{ab MM \times M}^{\circ}$	1	0	0
$MM \times P$	$R_{MM \times P}$	$V_{ab MM \times P}^{\circ}$	0	1	0
$MP \times M$	$R_{MP \times M}$	$V_{ab MP \times M}^{\circ}$	1/2	1/2	0
$MP \times P$	$R_{MP \times P}$	$V_{ab MP \times P}^{\circ}$	0	1/2	1/2
$PP \times M$	$R_{PP \times M}$	$V_{ab PP \times M}^{\circ}$	0	1	0
$PP \times P$	$R_{PP \times P}$	$V_{ab PP \times P}^{\circ}$	0	0	1

184 To model mating, we introduce an arbitrary reproductive function, $R_{ij \times k}$, that describes the frequency of
 185 crosses between queens of genotype ij and males of genotype k . This function depends on the
 186 reproductive mode considered in each section of the results:

- 187 1) Random Mating: When mating is random, $ij \times k$ crosses occur at a frequency of $R_{ij \times k} = X_{ij} Y_k$.
- 188 2) Variation in Fertility: If there are fertility differences among mating pairs ($f_{ij \times k}$), but mating is
 189 otherwise random, we set $R_{ij \times k} = f_{ij \times k} X_{ij} Y_k$, where $f_{ij \times k}$ is measured relative to the mean
 190 fertility in each generation.
- 191 3) Sexual Selection: Using the fixed relative preference scheme of Kirkpatrick [24], females of
 192 genotype ij mate with each type of male k in proportion to her mating preference ($\alpha_{ij \times k}$), leading
 193 to $R_{ij \times k} = X_{ij} \frac{\alpha_{ij \times k} Y_k}{\alpha_{ij \times P} Y_P + \alpha_{ij \times M} Y_M}$, where females are assumed to be able to remate after rejecting a
 194 mate. If mating preferences are costly, this is assumed to be incorporated into the female's
 195 viability ($V_{ab|ij \times k}^{\circ}$), regardless of the composition of the male population (a constitutive cost of
 196 sexual selection).

197 4) Assortative Mating: Finally, we consider assortative mating by social type with $R_{MM \times M} =$
 198 $(1 - m_M) X_{MM} Y_M + m_M X_{MM}$, where a proportion m_M of all MM queen mating is by assortment
 199 with males from their own social form regardless of the frequency of M males (see equation (9)
 200 for additional crosses).

201 Importantly, except with assortative mating, the frequency of matings involving rare genotypes is
 202 proportional to the frequency of the rare female times the frequency of the rare male (e.g., proportional
 203 to $X_{MM} Y_M$ when both MM females and M males are rare).

204 Overall, the production of MP queens from matings between MP queens and P males, for example, is
 205 proportional to:

$$206 \quad O_{MP|MP \times P}^{\circ} = \frac{1}{2} R_{MP \times P} V_{MP|MP \times P}^{\circ} \quad (2)$$

207 Finally, we normalize the frequencies in each sex to obtain the genotype frequencies in the next
 208 generation among queens and males, respectively:

$$209 \quad X'_{ab} = \frac{\sum_{ij,k} O_{ab|ij \times k}^{\circ}}{\sum_{ab,ij,k} O_{ab|ij \times k}^{\circ}} \quad (3a)$$

$$210 \quad Y'_a = \frac{\sum_{ij} O_{a|ij}^{\sigma}}{\sum_{a,ij} O_{a|ij}^{\sigma}} \quad (3b)$$

211 where the sums in the numerator are taken over all maternal (ij) and, for female offspring, paternal
 212 genotypes (k), and the sums in the denominator are also taken over all offspring genotypes (ab for
 213 female and a for male offspring). Given this normalization, all fitnesses need only be measured relative
 214 to one another within a sex, so we consider $V_{a|ij}^{\sigma}$ and $V_{ab|ij \times k}^{\circ}$ to vary between 0 and 1 in our numerical
 215 analysis.

216 *F. selysi* queens may differ in how they form new colonies. Queens from monogynous colonies fly away
 217 from their natal colony and establish novel colonies independently, while queens from polygynous
 218 colonies may also establish novel colonies with the help of workers, by walking away from their natal
 219 colony [colony budding; 12, 25]. Furthermore, polygynous colonies are larger than monogynous
 220 colonies, in line with their greater number of queens and longer colony lifespan [13]. Queens of
 221 monogynous origin may, however, be more successful at establishing new colonies independently,

222 compared to queens of polygynous origin [21, 26]. We do not explicitly model colony dynamics but
 223 include any fitness differences in colony success through the fitness terms, $V_{a|ij}^\sigma$ and $V_{ab|ij \times k}^\varnothing$, which also
 224 allows for the possibility that a queen's fitness depends on her parents' genotypes through their effect on
 225 the colony type and the genotypes of siblings. All of the recursion equations and the full analyses are
 226 detailed in the supplemental *Mathematica* file.

227 **Maintenance of the social polymorphism**

228 Below, we analytically search for conditions under which both monogynous and polygynous colonies
 229 will increase when rare, indicating that there is a protected polymorphism. We assume that maternal-
 230 effect killing is complete [8], with $V_{MM|MP \times M}^\varnothing = V_{M|MP}^\sigma = 0$, although we discuss the effect of allowing
 231 some M male and MM female offspring to be produced by MP mothers. We supplement this analysis
 232 with numerical searches, drawing parameters at random (as specified below) and determining
 233 numerically all equilibria and their stability properties. These numerical searches were used to confirm
 234 the analytical results and to determine whether stable internal polymorphisms could occur even when the
 235 polymorphism was not protected (i.e., when monogynous and polygynous colonies could not both
 236 increase when rare).

237 *1) Random Mating* – We start by considering the case of random mating, $R_{ij \times k} = X_{ij} Y_k$. With complete
 238 maternal-effect killing, there are three ways that the social polymorphism could be lost from the system
 239 at equilibrium: (a) MM queens and M males are fixed, (b) PP queens and P males are fixed, and (c) PP
 240 and MP queens co-exist alongside P males. The first of these equilibria consists only of monogynous
 241 colonies, while the other two consist only of polygynous colonies, falling along the polygynous edge
 242 defined by the absence of MM queens and M males ($X_{MM} = Y_M = 0$; recall that maternal-effect killing
 243 prevents monogynous genotypes from being produced by the MP queens).

244 A local stability analysis (supplementary material) shows that, under random mating, monogynous
 245 fixation with MM queens and M males is unstable to the introduction of the P haplotype when:

$$246 \quad V_{MM|MM \times M}^\varnothing V_{M|MM}^\sigma < \frac{V_{MP|MM \times P}^\varnothing V_{P|MP}^\sigma + V_{MP|MP \times M}^\varnothing V_{M|MM}^\sigma}{2} \quad (4)$$

247 This result shows that polygynous colonies can spread when rare if the P haplotype confers a higher
 248 average fitness, calculated as the average fitness of offspring queens that bear the rare P haplotype when
 249 the P was inherited from the father (first term in the fraction) or the mother (second term), in each case
 250 multiplied by the fitness of the father.

251 Along the polygynous edge, the system equilibrates at either the fixation of PP queens and P males or an
 252 equilibrium with a mixture of MP and PP queens and P males, where the equilibrium frequency of MP
 253 queens is:

$$254 \quad \hat{X}_{MP} = \frac{V_{MP|MP \times P}^{\circ} - 2 V_{PP|PP \times P}^{\circ}}{V_{MP|MP \times P}^{\circ} + V_{PP|MP \times P}^{\circ} - 2 V_{PP|PP \times P}^{\circ}} \quad (5)$$

255 Note that equilibrium (5) is valid ($0 < \hat{X}_{MP} < 1$) only when MP queen viability is more than twice that
 256 of PP queens ($V_{MP|MP \times P}^{\circ} > 2 V_{PP|PP \times P}^{\circ}$). Of the two equilibria along the polygynous edge (\hat{X}_{MP} equal to
 257 0 or to (5)), only one is ever stable. Specifically, polygynous colonies consisting of only PP queens are
 258 unstable to the introduction of MP queens if:

$$259 \quad V_{PP|PP \times P}^{\circ} < \frac{V_{MP|MP \times P}^{\circ}}{2}, \quad (6)$$

260 in which case the equilibrium with only MP and PP queens (5) is stable. Conversely, if condition (6)
 261 fails to hold, then the only stable equilibrium along the polygynous edge is the PP/P fixation point. Note
 262 that maintaining both MP and PP queens in polygynous-only colonies requires very strong heterozygous
 263 advantage, with MP queens more than twice as fit as PP queens (condition 6). This requirement for
 264 strong selection emerges because, when mated with P males, only half of the daughters of MP queens
 265 are MP , whereas all daughters of PP queens are PP .

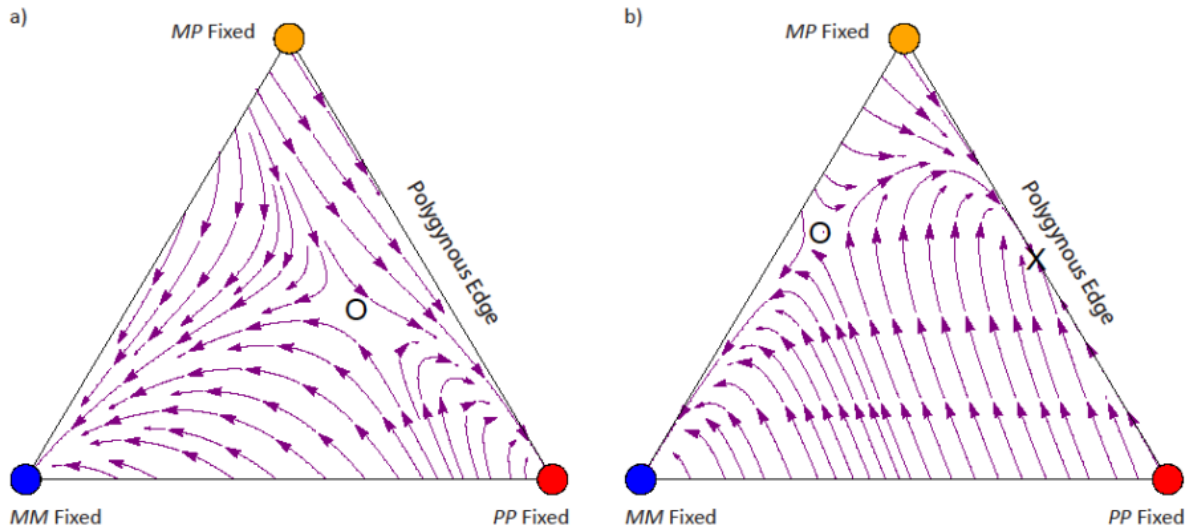
266 We next considered the stability of these polygynous equilibria to the introduction of monogynous
 267 genotypes (MM queens and M males), under random mating. We find that MM queens and M males
 268 never spread when rare, from either the P -fixed or MP/PP equilibria. This is because almost all matings
 269 involving a rare M male are with the common PP or MP females, which do not produce MM daughters
 270 because of maternal-effect killing. Similarly, the vast majority of matings involving the rare MM queen
 271 are with the common P male, which also do not generate MM daughters. Thus, the MM genotype rapidly

272 disappears from the population of queens, taking along with it the ability to produce M males. Even if
273 maternal-effect killing is strong but not complete, such that both $V_{MM|MP \times M}^{\circ}$ and $V_{M|MP}^{\sigma}$ are small but not
274 zero, the monogynous genotypes do not invade under biologically reasonable conditions (with
275 exceptions only in the extreme case where the fitness of M males is so much higher than P males that
276 rare M males become common following a single bout of selection; see supplementary material). This
277 result, that monogynous colonies cannot establish when rare, holds whether or not colonies with MP
278 queens are able to recover the energy invested in lost embryos (i.e., whether or not $V_{P|MP}^{\sigma}$ is elevated due
279 to recovered resources following maternal-effect killing).

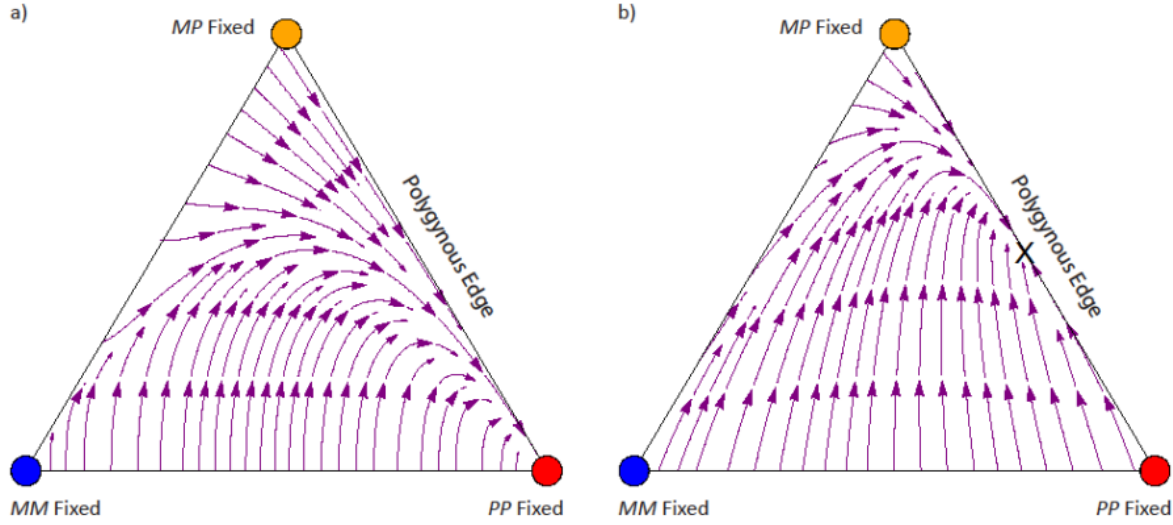
280 In short, if MP queens are much fitter than PP queens (condition 6 holds), then a genetic polymorphism
281 with both PP and MP queens can result, but the system consists of only polygynous colonies. In
282 particular, no matter how strong heterozygous advantage might be, a social polymorphism cannot be
283 maintained. Polygynous colonies can spread when rare if condition (4) holds, and the system converges
284 to one of the polygynous equilibria (with P fixed or with MP/PP queens), whereas monogynous colonies
285 cannot generally spread when rare. This result is in stark contrast to other systems, where a
286 polymorphism can be stably maintained by heterozygote advantage [27] or a balance between meiotic
287 drive and homozygous sterility/lethality [23].

288 We next illustrate the dynamics of the *F. selysi* system numerically. Because the frequency of males is
289 determined entirely by the genotype frequencies in their mothers (equation 3b), we initialize the male
290 frequencies according to equation (3b) using the current frequency of females, allowing us to closely
291 approximate the dynamics using only genotype frequencies among queens. We then visualize these
292 dynamics using ternary stream plots (Figures 2 and 3), where each corner of the triangle represents the
293 fixation of a queen genotype, the opposite edge corresponds to the absence of that genotype, and the
294 distance from the edge to the corner represents the frequency of that queen genotype. Arrows on the
295 stream plot represent the direction of the change in queen frequencies from different initial positions.
296 Figure 2 illustrates a case where the M -fixed equilibrium is stable along with either the P -fixed
297 equilibrium (panel a) or the MP/PP equilibrium (5) on the polygynous edge (panel b). In such cases, a
298 polymorphic internal equilibrium with all social types can exist but is unstable. Figure 3 illustrates a case
299 where the M fixed is unstable, in which case the only stable equilibrium is on the polygynous edge,
300 either the P -fixed equilibrium (panel a) or the MP/PP equilibrium (5) (panel b), and there is no
301 equilibrium with both monogynous and polygynous colonies.

302 Finally, we verified the above results numerically, confirming that a social polymorphism was never
 303 stable using a numerical search. To conduct this search, we randomly drew one million sets of
 304 parameters (all chosen uniformly between 0 and 1, other than the maternal-effect killing parameters
 305 maintained at $V_{MM|MP \times M}^{\circ} = V_{M|MP}^{\sigma} = 0$). For each parameter set, we numerically calculated all equilibria
 306 and determined their stability properties, confirming that there was never a stable internal equilibrium
 307 under random mating with both monogynous and polygynous colonies (see supplementary material for
 308 all numerical results).



309 **Figure 2.** Stream plots show the dynamics of *F. selysi* queens when individuals in monogynous colonies
 310 are most fit. The fixation of monogynous colonies (blue) is then stable to the spread of the *P* supergene
 311 (condition 4 does not hold because *MM* queens and *M* males are sufficiently fit). (a) When heterozygous
 312 females are low enough in fitness, the system evolves towards fixation of either *MM* females (blue) or
 313 *PP* females (red) ($V_{MP|ij \times k}^{\circ} = 1/5$). (b) When heterozygous females are intermediate in fitness, the
 314 system can evolve towards fixation of either *MM* females (blue) or a polymorphism with both *MP* and
 315 *PP* polygynous queens (marked by an \times) ($V_{MP|ij \times k}^{\circ} = 3/5$). The open circle marks an unstable
 316 equilibrium point. Other parameters: $V_{MM|MP \times M}^{\circ} = V_{M|MP}^{\sigma} = 0$ (complete maternal-effect killing),
 317 $V_{MM|MM \times M}^{\circ} = 1$, and otherwise $V_{a|ij}^{\sigma} = V_{ab|ij \times k}^{\circ} = 1/5$.



318 **Figure 3.** Stream plots show the dynamics of *F. selysi* queens when individuals in monogynous colonies
 319 are not more fit than individuals in polygynous colonies. The fixation of monogynous colonies (blue) is
 320 then unstable (condition 4 holds). (a) There are no fitness differences except those caused by maternal-
 321 effect killing, in which case *PP*-fixed is the only stable equilibrium (red). (b) There is heterozygous
 322 advantage, such that condition 6 holds, and *MP/PP* with only polygynous colonies is the only stable
 323 equilibrium (marked by an \times on plot; $V_{MP|ij \times k}^{\varnothing} = 3/5$). No internal equilibrium point exists in these
 324 cases. Other parameters: $V_{MM|MP \times M}^{\varnothing} = V_{M|MP}^{\sigma} = 0$ (complete maternal-effect killing) and otherwise
 325 $V_{a|ij}^{\sigma} = V_{ab|ij \times k}^{\varnothing} = 1/5$.

326 2) Variation in Fertility – Adding fertility differences alters equations (4) – (6) but does not qualitatively
 327 affect the outcome. In particular, there is never a case where both monogynous and polygynous
 328 haplotypes can spread when rare, except for biologically extreme male fitness differences. For example,
 329 with fertility differences ($R_{ij \times k} = f_{ij \times k} X_{ij} Y_k$), condition (4) determining when a monogynous social
 330 system is unstable to the introduction of polygyny becomes:

$$331 \quad V_{MM|MM \times M}^{\varnothing} f_{MM \times M} V_{M|MM}^{\sigma} < \frac{V_{MP|MM \times P}^{\varnothing} f_{MM \times P} V_{P|MP}^{\sigma} + V_{MP|MP \times M}^{\varnothing} f_{MP \times M} V_{M|MM}^{\sigma}}{2} \quad (7)$$

332 Similarly, condition (6) determining when polygynous-only colonies will evolve towards an equilibrium
 333 with both *MP/PP* becomes:

$$334 \quad V_{PP|PP \times P}^{\varnothing} f_{PP \times P} < \frac{V_{MP|MP \times P}^{\varnothing} f_{MP \times P}}{2} \quad (8)$$

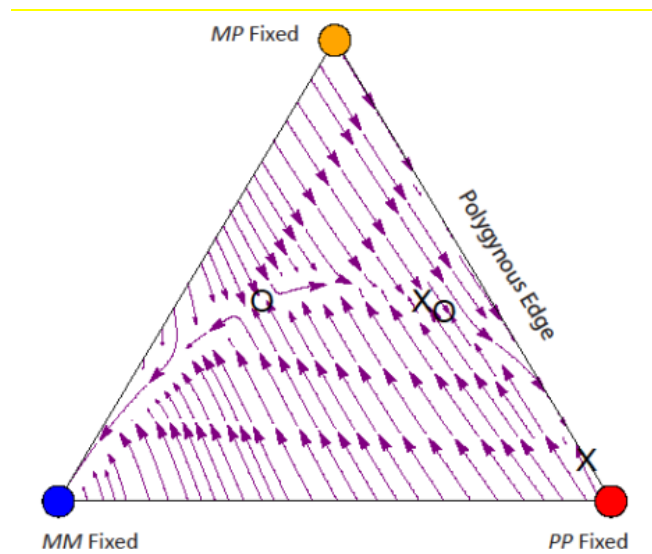
335 Again, however, monogynous colonies can never spread when rare within a system consisting only of
 336 polygynous colonies. Furthermore, a numerical search across one million randomly drawn parameter
 337 sets failed to find any case of a stable social polymorphism.

338 Therefore, we have shown that, under a broad range of conditions including arbitrary fertility
 339 differences, there is not a protected polymorphism where both monogynous and polygynous forms can
 340 spread when rare. Consequently, we would not expect a social polymorphism to be maintained under
 341 random mating, with or without fertility differences, because of the strong constraints imposed by
 342 maternal-effect killing. Graphically, the problem is that the polygynous edge, consisting of only *PP* and
 343 *MP* queens in Figures 2 and 3, is always strongly absorbing because neither rare *M* males nor rare *MM*
 344 queens produce *MM* daughters: rare *M* males mate predominantly with common *MP/PP* queens whose
 345 daughters are never *MM* because of maternal-effect killing, and *MM* queens mate predominantly with
 346 common *P* males whose daughters are never *MM* by the rules of inheritance.

347 3) Sexual Selection – The same conclusions regarding invasion of rare *M* and *P* haplotypes hold true
 348 with sexual selection using the fixed relative preference scheme of Kirkpatrick [24], where $R_{ij \times k} =$
 349 $X_{ij} \frac{\alpha_{ij \times k} Y_k}{\alpha_{ij \times P} Y_P + \alpha_{ij \times M} Y_M}$. In this case, invasion analysis gives the same result as (7), setting $f_{MM \times M} =$
 350 $f_{MP \times M} = 1$ and $f_{MM \times P} = \frac{\alpha_{MM \times P}}{\alpha_{MM \times M}}$ (because *P* males are rare and the Y_P term in the denominator of $R_{ij \times k}$
 351 is negligible to leading order in the analysis). Similarly, equation (8) determining which equilibrium is
 352 stable along the polygynous edge continues to hold, where now $f_{PP \times P} = f_{MP \times P} = 1$ given that only *P*
 353 males are present along this edge. Most importantly, polygynous-only colonies are always stable to the
 354 introduction of *MM* queens and *M* males, regardless of the values of $\alpha_{ij \times k}$, when the mating scheme is
 355 proportional to the product of the frequency of both mates.

356 In the cases discussed thus far (random mating, fertility differences, and sexual selection with fixed
 357 relative preferences), monogynous colonies can never invade when very rare. However, with very strong
 358 sexual preferences, *MM* queens can so strongly prefer *M* males that monogynous colonies can be
 359 maintained, once they rise to a sufficiently high frequency. In this case, a stable social polymorphism
 360 may exist under extreme preference differences (see supplementary material for specific description of
 361 parameter conditions required). Figure 4 illustrates one example, where *MM* queens prefer to mate with
 362 *M* males almost 100 times more than *P* males, allowing *M* to spread from intermediate frequency even
 363 though it cannot spread when rare. Importantly, a numerical search indicates that the degree of

364 preference of queens for males of their own social form must differ between monogynous and
 365 polygynous queens in order to stabilize a social polymorphism. When monogynous and polygynous
 366 queens had the same degree of preference for males belonging to their own social form, a numerical
 367 search of one million random parameter sets found no stable social polymorphism. Note that with sexual
 368 selection and fixed relative mating preferences [24], if *MM* queens have a strong preference for *M*
 369 males, then mating becomes effectively assortative, because *MM* females search until finding an *M*
 370 mate.



371 **Figure 4.** Stream plot showing the dynamics of *F. selysi* queens with sexual selection such that queens
 372 exhibit fixed relative mating preferences [24]. Here we illustrate a case when populations composed of
 373 only monogynous colonies or only polygynous colonies are both stable (condition 4 holds). The open
 374 circles mark unstable equilibrium points and the \times 's mark stable equilibrium points. Parameters:
 375 $\frac{f_{MM \times M}}{f_{MM \times P}} = 96$ (an extremely strong preference), $\frac{f_{MP \times P}}{f_{MP \times M}} = \frac{f_{PP \times P}}{f_{PP \times M}} = 1.47$, $V_{MM|ij \times k}^{\circ} = 0.46$, $V_{MP|ij \times k}^{\circ} =$
 376 0.69 , $V_{PP|ij \times k}^{\circ} = 0.33$, $V_{MM|MP \times M}^{\circ} = V_{M|MP}^{\circ} = 0$ (complete maternal-effect killing), and $V_{a|ij}^{\sigma} = 1$ (males
 377 have the same viability).

378 4) Assortative Mating – Given the above results, we conjectured that a key feature necessary for both
 379 social haplotypes to spread when rare and maintain a social polymorphism is assortative mating, such
 380 that rare individuals can find and mate with similarly rare partners belonging to the same social form. A
 381 variety of mating schemes can produce assortative mating, for example when queens actively seek their
 382 preferred mate or when mating occurs near or within colonies.

383 We thus introduce assortative mating by social form, allowing mating among members of the same
 384 colony type. Specifically, we assume monogynous queens mate exclusively with *M* males at some

385 frequency, m_M , and otherwise mate randomly, while polygynous queens mate with P males at some
 386 frequency, m_P , and otherwise mate randomly. We allow the two social morphs to mate assortatively at
 387 different rates because of empirical results indicating that monogynous queens have more often mated
 388 assortatively than polygynous queens [19]. With assortative mating by social form, we have:

$$\begin{aligned}
 389 \quad R_{MM \times M} &= (1 - m_M) X_{MM} Y_M + m_M X_{MM} & (9) \\
 390 \quad R_{MM \times P} &= (1 - m_M) X_{MM} Y_P \\
 391 \quad R_{MP \times M} &= (1 - m_P) X_{MP} Y_M \\
 392 \quad R_{MP \times P} &= (1 - m_P) X_{MP} Y_P + m_P X_{MP} \\
 393 \quad R_{PP \times M} &= (1 - m_P) X_{PP} Y_M \\
 394 \quad R_{PP \times P} &= (1 - m_P) X_{PP} Y_P + m_P X_{PP}
 \end{aligned}$$

395 In the absence of MM female or M males, we again have two possible equilibria on the polygynous
 396 edge, with only P males present and either PP queens or a PP/MP polymorphism, given by equation (5)
 397 (details of the stability analyses are given in the supplementary material). On this edge, the MP/PP
 398 equilibrium is stable if (6) holds, and otherwise the P -fixed equilibrium is stable.

399 With assortative mating, it is now possible, however, for the equilibria on the polygynous edge to be
 400 unstable to the introduction of MM queens and M males. Specifically, when haplotype P is fixed,
 401 monogynous colonies spread when rare if:

$$402 \quad V_{PP|PP \times P}^{\circ} < m_M V_{MM|MM \times M}^{\circ} \quad (12a)$$

403
 404 Alternatively, if polygynous colonies consist of PP and MP queens at equilibrium (equation 5), then
 405 monogynous colonies spread when rare if:

$$406 \quad \frac{V_{MP|MP \times P}^{\circ}}{2} < m_M V_{MM|MM \times M}^{\circ} \quad (12b)$$

407 Notice that the relative magnitudes of the left-hand sides of (12) determine which equilibrium is stable
 408 along the polygynous edge (equation 6). Whichever value is larger determines the composition of
 409 polygynous colonies against which rare monogynous colonies must compete. In either case, the key
 410 feature is that rare MM queens are able to find and mate with rare M males, regardless of how rare they

411 are. Only this feature allows monogynous colonies to escape the strongly absorbing polygynous edge
 412 caused by maternal effect killing and to establish when rare.

413 In addition, under certain conditions, polygynous colonies can also spread within a fully monogynous
 414 population. Analyzing the invasion of P into a population fixed on M involves solving a cubic equation
 415 for the leading eigenvalue (see supplementary material). When there is complete assortment ($m_M =$
 416 $m_P = 1$), this cubic equation factors, allowing us to determine that P will spread if either of these
 417 conditions is met:

$$418 \quad V_{PP|PP \times P}^{\circ} > V_{MM|MM \times M}^{\circ} \quad (13a)$$

$$419 \quad \frac{V_{MP|MP \times P}^{\circ}}{2} > V_{MM|MM \times M}^{\circ} \quad (13b)$$

420 These conditions for the spread of P when rare with complete assortative mating contradict the
 421 conditions for the spread of M when rare. Effectively, with complete assortative mating, the system
 422 quickly approaches two isolated sub-populations consisting of $\{MM, M\}$ and either $\{PP, P\}$ or
 423 $\{PP, MP, P\}$ individuals (whichever polygynous set is more fit). Without genetic exchange between these
 424 sub-populations, either the monogynous sub-population $\{MM, M\}$ is more fit and fixes or is less fit and
 425 disappears. Either way, a social polymorphism is not possible with complete assortative mating by social
 426 form. This was confirmed in a numerical search across one million randomly chosen parameter sets.

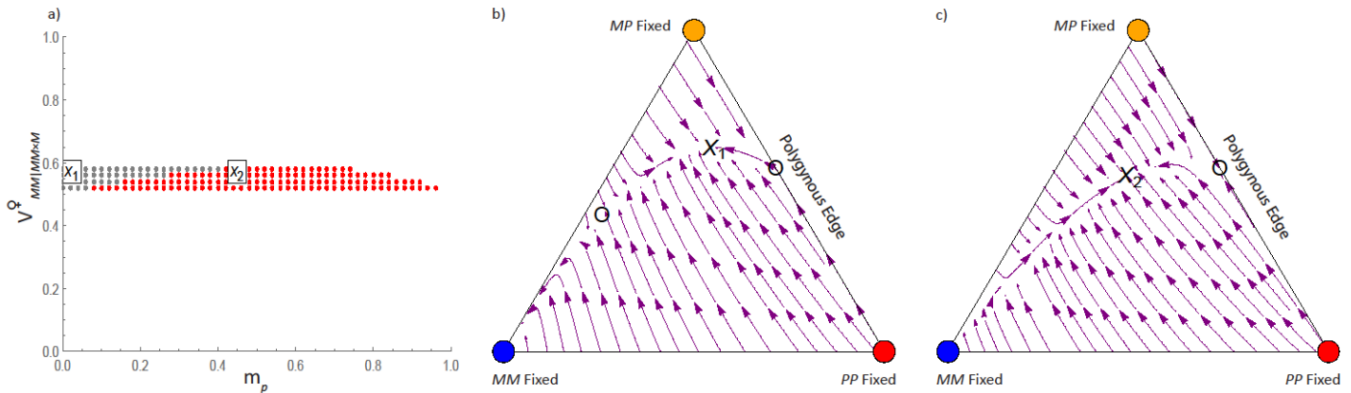
427 Thus, with either random mating ($m_M = m_P = 0$) or with complete assortative mating by social form
 428 ($m_M = m_P = 1$), a stable social polymorphism does not exist. With an intermediate rate of assortative
 429 mating by social form, however, a social polymorphism can result (Figure 5). For example, if the P -
 430 fixed equilibrium is stable along the polygynous edge, it is now possible for M to spread when rare if
 431 assortative mating is sufficiently high in monogynous colonies, as long as MM queens are fitter than PP
 432 queens, and for P to spread when rare if MP queens are fitter than MM queens. Both social haplotypes
 433 can invade when rare even if selection is weak (all $V_{a|ij}^{\circ}$ and $V_{ab|ij \times k}^{\circ}$ near 1, except for $V_{MM|MP \times M}^{\circ} =$
 434 $V_{M|MP}^{\circ} = 0$ due to maternal-effect killing), as long as:

$$435 \quad (1 - m_P)^2 < (1 - m_M) < V_{MM|MM \times M}^{\circ} - V_{PP|PP \times P}^{\circ} \quad (14)$$

436 This result requires that $V_{MM|MM \times M}^{\circ} > V_{PP|PP \times P}^{\circ}$ and that monogynous queens mate assortatively at a
 437 very high rate (but not exclusively). Note that the more that polygynous queens mate assortatively (the
 438 higher m_p , including $m_p=1$), the easier this condition is to satisfy.

439 *Complete assortative mating in monogynous queens*

440 As queens heading mature monogynous colonies have exclusively mated with M males, while
 441 polygynous queens have mostly, but not exclusively, mated with P males [19], we first focus
 442 numerically on cases where $m_M = 1$ and explore the conditions needed for the maintenance of a
 443 polymorphism. With complete assortative mating ($m_M = 1$) and when males are assumed equally fit, a
 444 stable social polymorphism is possible, but only when there is strong heterozygous advantage, as
 445 illustrated in Figure 5. For some parameters, there are two separate stable equilibria (grey dots in Figure
 446 5a), one is a stable social polymorphism and the other is fixed for the M haplotype. These cases tend to
 447 be found when polygynous queens often mate randomly (low m_p). Such systems are, however, prone to
 448 losing social polymorphism. For example, the colonization of new environments by monogynous queens
 449 could result in the loss of polygynous colonies, and such populations would then resist invasion by rare
 450 P haplotypes. When polygynous queens mate assortatively at a higher rate (intermediate m_p), the P
 451 haplotype is more likely to spread when rare, leading to cases where the only stable polymorphism
 452 involves a social polymorphism (red dots in Figure 5a), which better resist loss of the polygynous type.

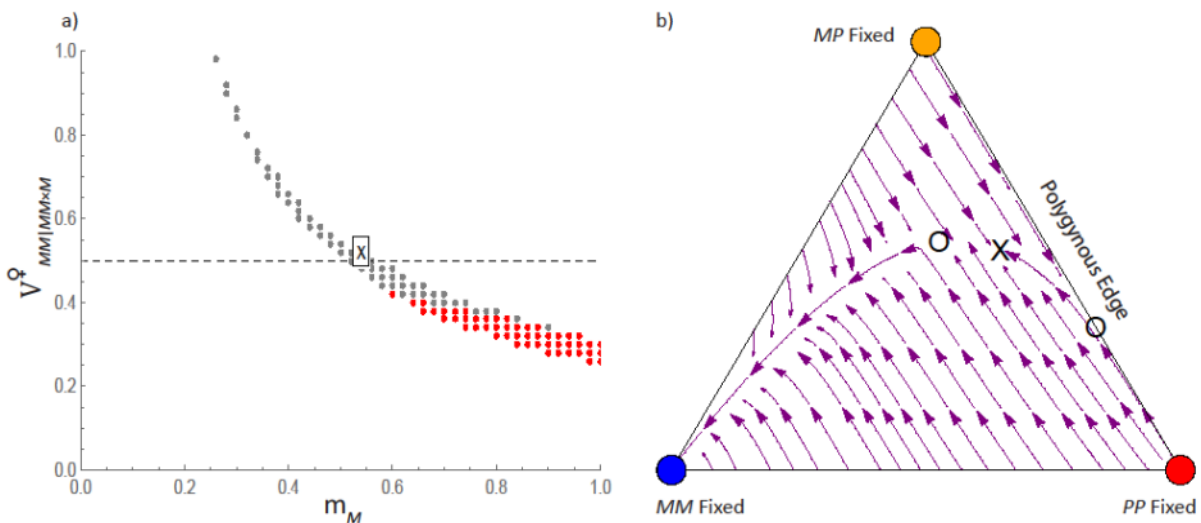


453 **Figure 5.** The maintenance of social polymorphism is possible with assortative mating by social form,
 454 here illustrated with complete assortative mating in monogynous queens and partial assortative mating in
 455 polygynous queens. Here, MM queens only mate with M males ($m_M = 1$). Panel (a) shows the range of
 456 assortative mating in polygynous colonies (m_p) and the viability of monogynous queens ($V_{MM|MM \times M}^{\circ}$)
 457 for which a stable social polymorphism persists when $V_{MP|ij \times k}^{\circ} = 1$ and $V_{PP|PP \times P}^{\circ} = 0.3$. Grey dots: both
 458 a social polymorphism and M -fixed equilibria are stable. Red dots: only the social polymorphism is

459 stable. Panels (b) and (c) show stream plots for the parameter sets indicated in panel a (panel b at x_1 ,
 460 panel c at x_2). Open circles mark unstable equilibrium points, while \times_1 and \times_2 indicate stable internal
 461 equilibrium points. Other parameters: $V_{MM|MP \times M}^{\ominus} = V_{M|MP}^{\ominus} = 0$ (maternal-effect killing) and otherwise
 462 $V_{a|ij}^{\ominus} = V_{ab|ij \times k}^{\ominus} = 1$.

463 *Incomplete assortative mating in monogynous queens*

464 Recent field work has shown that monogynous queens sampled from mating swarms do mate with P
 465 males [20], with $m_M = \sim 0.65$, even though this cross is not observed in mature monogynous colonies
 466 [15, 19]. We thus relax the assumption that $m_M = 1$ and explore the conditions under which a
 467 polymorphism can be maintained. In the numerical cases explored, as the rate of assortative mating by
 468 monogynous queens declines, the M haplotype must be increasingly fit for a social polymorphism to be
 469 stably maintained (recall that it is assortative mating that protects the M haplotype from being eliminated
 470 by maternal effect killing in MP daughters produced when MM queens mate with P males). For
 471 example, with the parameters in Figure 6a, there must be heterozygote advantage with MM queens less
 472 fit than MP queens under intermediate to high rates of assortment (m_M above 0.6), but there must be
 473 directional selection favoring MM queens under low to intermediate rates of assortment (m_M between
 474 0.2 and 0.6), with no stable polymorphism possible for weaker assortment. Again, for each set of
 475 parameters that ensures a stable social polymorphism, the fixation of the M haplotype may (grey points
 476 in Figure 6a, stream plot in Figure 6b) or may not (red points in Figure 6a) be a separate stable
 477 equilibrium (see supplementary material).



478 **Figure 6.** Partial assortative mating by both social forms allows for the maintenance of social
 479 polymorphism. Panel (a) shows the range of assortative mating by social form in monogynous colonies

480 (m_M) and the viability of monogynous queens ($V_{MM|MM \times M}^\circ$) for which a social polymorphism persists.
 481 Above the dashed line corresponds to directional selection favouring the M haplotype in females, while
 482 below the dashed line there is heterozygous advantage. Grey dots: both a social polymorphism and M -
 483 fixed equilibria are stable. Red dots: only the social polymorphism is stable. Panel (b) shows the stream
 484 plot for a parameter set (marked by an \times in panel a) that permits a stable internal equilibrium denoted by
 485 an \times in panel b ($m_M = 0.54$, $V_{MM|MM \times M}^\circ = 0.52$). Open circles mark unstable equilibrium points. Other
 486 parameters are as in Figure 5 except $V_{MP|ij \times k}^\circ = 0.5$, $V_{PP|ij \times k}^\circ = 0.2$, and $m_P = 0.5$.

487 In summary we find that, under the right balance of parameters (as illustrated in Figures 5 and 6),
 488 assortative mating of monogynous queens allows for the spread of monogynous colonies when rare. For
 489 example, if selection is weak (all $V_{a|ij}^\sigma$ and $V_{ab|ij \times k}^\circ$ near 1, except for $V_{MM|MP \times M}^\circ = V_{M|MP}^\sigma = 0$ due to
 490 maternal-effect killing), for a social polymorphism to be maintained, this balance requires that both
 491 monogynous and polygynous queens mostly, but not always, mate assortatively, in such a way that
 492 equation 14 is satisfied. Although constitutive costs of assortative mating are incorporated in the
 493 viability terms, we have not incorporated frequency-dependent costs of assortative mating. If costs rise
 494 with the rarity of similar mates, we expect that it would be harder to maintain a social polymorphism
 495 because rare genotypes would pay that cost and be less likely to spread (particularly, rare monogynous
 496 types would be less able to spread near the polygynous edge).

497 The results above do not change substantially when assortative mating depends on genotype, rather than
 498 on social type, with MP queens mating equally with M and P males (except that the equivalent to
 499 condition (5) for a social polymorphism under weak selection is never satisfied; see supplementary
 500 material). While assortative mating, by either social form or genotype, does allow the maintenance of a
 501 social polymorphism (Figures 5 and 6), the conditions for stability are restrictive, requiring a delicate
 502 balance of selection among genotypes depending on the strength of assortment (e.g., Figure 6a).

503 Discussion

504 Selfish genetic elements favouring their own transmission are ubiquitous across the tree of life, yet
 505 explaining their dynamics and observed frequencies in natural systems is challenging [6, 28]. In short,
 506 when a genetic element drives, what prevents it from reaching fixation and becoming undetectable due
 507 to the lack of polymorphism? Maternal-effect post-segregational killers have been detected in ants [8],
 508 beetles [29], nematodes [30] and many bacteria, including endosymbionts [6, 9]. In such systems, a
 509 maternally-expressed toxin linked to a zygotically-expressed antidote causes the death of progeny that
 510 did not inherit the element [9, 31]. Simple population genetic models show that these selfish genetic

511 elements are expected to increase in frequency and that high fitness costs of the elements are needed to
512 prevent their fixation [28, 32]. Supergenes controlling complex phenotypes are prone to harbour selfish
513 genetic elements, yet the effect of drive on their evolutionary dynamics had not been modeled so far.

514 We have modeled the dynamics of a supergene controlling colony social organization in the Alpine
515 silver ant, *F. selysi*, exploring a broad range of fitness functions and mating regimes and incorporating
516 the transmission ratio distortion caused by maternal-effect killing by the *P* haplotype [8]. A key finding
517 of the model is that maternal-effect killing creates such an imbalance in the selective forces acting that it
518 is challenging to account for the long-term persistence of both social forms, i.e. monogynous colonies
519 with a single queen and polygynous colonies with multiple queens. Indeed, there is no stable
520 polymorphic equilibrium in any model with constant fitnesses and mating regimes that did not generate
521 assortative mating. Essentially, maternal-effect killing prevents rare monogynous colonies from
522 invading populations of polygynous colonies whenever rare *MM* queens mate predominantly with the
523 common type of male (*P*), producing *MP* daughters that do not regenerate *MM* queens or *M* males.

524 For both social forms to be protected from loss when rare (i.e., for there to be no stable equilibrium with
525 only one social form), we find that there must be strong assortative mating, so that *MM* queens
526 frequently mate with *M* males even when both are rare, with the right balance of fitnesses involving
527 strong selective differences among genotypes (e.g., Figures 5 and 6) or very high, but not complete,
528 assortative mating by both social types (equation 14). Alternatively, there may be a stable social
529 polymorphism alongside stable equilibria with only one social haplotype, which we observed with
530 assortative mating (including with sexual selection when fixed-relative preferences were so strong that
531 assortment arose). The latter cases are, however, more prone to losing social polymorphism, if by chance
532 one social form goes locally extinct. Whether the restrictive conditions allowing a social polymorphism
533 are satisfied in Alpine silver ants remains an open question, as discussed below.

534 In many supergenes, one haplotype is a homozygous lethal, which selects for disassortative mating [33].
535 The Alpine silver ant system is unusual, as both homozygotes are viable. Moreover, maternal effect
536 killing by the *P* haplotype may select for assortative mating by *MM* queens for *M* males and *MP* queens
537 for *P* males, respectively. In line with that prediction, effective mating in Alpine silver ants appears to be
538 strongly assortative: when analysing queens heading mature field colonies, all *MM* queens had mated
539 with *M* males, while *MP* and *PP* queens had mated mostly with *P* males [77%; 15, 19]. The proximate
540 mechanisms underlying this pattern of assortative mating remain unclear. In mate choice experiments in

541 captivity, queens and males mated randomly with respect to social form [21], but the experimental
542 conditions may have interfered with natural mate choice. In the field, queens and males from each social
543 form fly away from their natal colony to join mating swarms, where *MM* queens mate mostly
544 assortatively ($m_M \sim 0.65$, based on mating swarm data in [20]). Queens in monogynous colonies do
545 not mate with members of the same colony [19]. Hence, the assortative mating pattern observed in the
546 field is consistent with a major condition for stability in the model, but we do not know whether and
547 how *MM* queens actually seek out and mate preferentially with *M* males when both are rare, as required
548 to maintain the social polymorphism. Assortative mating appears easier to achieve within the
549 polygynous social form, as *PP* or *MP* queens as well as *P* males may mate within or close to their natal
550 colony.

551 Regarding the strength and direction of selection, it is extremely difficult to obtain precise estimates of
552 fitness, given that silver ant queens are long-lived and iteroparous [13]. Moreover, the two social forms
553 differ in morphology, sex allocation, colony size and other life-history traits, which hampers fitness
554 comparisons [13, 34, 35]. Queens from alternative social forms may well differ in fitness, as they
555 typically use alternative modes of dispersal and colony founding [independent colony founding versus
556 budding; 25]. Queens from monogynous colonies disperse on the wing and found colonies
557 independently, while queens from polygynous colonies have the additional options of staying in their
558 natal colony, seeking adoption in other colonies, founding colonies with the help of workers or in
559 association with other queens [36]. Queens from monogynous colonies are more fertile and more
560 successful at independent colony founding in harsh ecological conditions than queens of polygynous
561 origin [21, 26, 37]. The accumulation of deleterious mutations in the non-recombining supergene can
562 also cause fitness differences among genotypes. In particular, deleterious mutations that primarily affect
563 females can accumulate in the *P* haplotype yet be masked in *MP* polygynous queens (by contrast,
564 mutations in monogynous colonies are never masked in *MM* queens). Recent studies in the field and
565 laboratory revealed that *PP* females have lower survival, fertility, and fecundity than *MP* females
566 (Blacher, De Gasperin, Grasso, Sarton-Lohéac, Alleman and Chapuisat, unpublished results). Overall,
567 life-history and dispersal differences between social forms, combined with a recessive genetic load that
568 can accumulate in the *P* haplotype, might thus create fitness differences between supergene genotypes
569 that contribute to stabilizing the polymorphism (e.g., as seen in Figure 5).

570 While general in many respects, our model does not investigate all forms of frequency- or density-
571 dependent selection. Frequency-dependent selection may arise from antagonistic interactions [38]. For

572 example, if one social form is more aggressive than the other, it may be favoured when rare, and
573 disfavoured when common. Spatially-varying selection due to habitat heterogeneity may also stabilize
574 the polymorphism [17, 39]. In particular, monogynous queens may be better at dispersing and colonizing
575 novel or patchy habitats, while polygynous queens may outperform monogynous ones in old, continuous
576 and saturated habitats, due to their larger colony size and possibility of establishing novel colonies by
577 budding [13, 17, 25]. Such ecological niche differentiation could maintain both types, just as it can
578 maintain different species, but it would not explain why mating between social forms persists, as the
579 continued gene flow between the types prevents further genetic adaptation of each social form to its
580 specialized ecological niche.

581 Supergenes, such as found in Alpine silver ants, typically affect multiple morphological, physiological
582 and behavioral traits [1], and over time they tend to accumulate recessive deleterious mutations [5] and
583 selfish genetic elements [8]. The *P* haplotype of *F. selysi*, which is derived and differs by several
584 inversion from the ancestral *M* haplotype, causes selfish post-segregational killing [8, 16]. The
585 supergene that controls social organization in the fire ant *Solenopsis invicta* also causes unusual patterns
586 of segregation distortion and genotype-specific mortality [7, 40, 41]. In mice, the *t*-haplotype not only
587 distorts segregation, it also affects sperm competition and increases dispersal propensity [23, 42, 43].
588 The *Segregation Distorter* supergene in fruit flies has diverse effects on fitness and sex ratio [44]. In
589 many empirical cases, the driven or positively selected supergene haplotype is a homozygous lethal but
590 has lower frequencies than expected by simple models [45]. Although the details of each system differ,
591 our model shows that explaining the long-term maintenance of selfish supergenes is not trivial. Overall,
592 the drive induced by maternal-effect killing in Alpine silver ants destabilizes the polymorphism and
593 prevents rare monogynous colonies from invading populations fixed for polygynous colonies. Under the
594 broad range of models considered here, a stable polymorphic equilibrium can only be reached under a
595 regime of strong but not complete assortative mating, combined with the right balance of fitnesses for
596 alternative supergene genotypes.

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603 **Supplementary material**

- 604 • *Mathematica* file with all main analyses
- 605 • Numerical searches and local stability analyses

606 **Authors' Contributions**

607 A.G., S.O. and M.C. conceived the study. A.G. and S.O. developed the model. A.G. performed the
608 numerical analysis, analysed the results and drew the figures. A.G., S.O. and M.C. interpreted the results
609 and wrote the manuscript. All authors approved the final version.

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Appendix: Table of parameters

Symbol	Meaning
i,j,k	Individual social alleles carried by the parents
a,b	Individual social alleles carried by the offspring
X_{ij}	Frequency of queens with ij genotype (X'_{ij} in next generation)
Y_k	Frequency of males with k genotype (Y'_k in next generation)
$O_{ab ij \times k}^{\varnothing}$	Production rate of surviving ab queen offspring from ij queen and k male parents
$O_{a ij}^{\sigma}$	Production rate of surviving a male offspring from ij queens
$R_{ij \times k}$	Frequency of crosses between queens of genotype ij and males of genotype k
$V_{ab ij \times k}^{\varnothing}$	Viability of ab queen offspring from ij queen and k male parents
$V_{a ij}^{\sigma}$	Viability of male offspring with genotype a produced by ij queens
$f_{ij \times k}$	Offspring fertility of an ij queen and k male parents
$\alpha_{ij \times k}$	Fixed relative-preference of an ij queen for k males
m_i	Frequency of assortative mating where queens with social type i mate exclusively with i males
$\sum_{ij,k}$	Sum over all possible mating pairs
$\sum_{ab,ij,k}$	Sum over all possible mating pairs and queen offspring
\sum_{ij}	Sum over all queens
$\sum_{a,ij}$	Sum over all queens and male offspring