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8 **5.1 Age estimation of living persons: a coherent approach to inference and**
9 **decision**

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16 **Abstract**

17 Evidence interpretation is a fundamental aspect of forensic science: it is basically a problem of inference and decision.
18 Forensic age estimation is no exception to this reality. Evidence related to the biological development of an individual is
19 often relevant from a legal perspective, such as when examining the probability that a person is younger or older than a
20 given age threshold, for instance the age of majority. Provided that uncertainty in forensic evidence should be measured
21 by means of probability, the Bayesian approach represents the ideal solution for both inference and decision. This
22 contribution aims to illustrate how this perspective operates in age estimation from both theoretical and operational points
23 of view.

24

25 **Keywords**

26 Bayesian approach

27 Forensic Age estimation of living persons

28 Inductive inference

29 Normative decision theory

30 Forensic evidence evaluation and interpretation

31 Subjective probability

32 Forensic inference and decision

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35 Glossary

36
37 **Subjective probability:** the probability of an event is interpreted as the expression of a degree of belief in that particular
38 event. Subjective probabilities are personal and conditional on the individual's experience and knowledge. As all
39 probabilities, subjective probabilities take values in the range between 0 and 1.

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41 **Frequentist probability:** The frequentist probability is defined as the limit of the relative frequency of a target event that
42 has occurred in a large number of trials if it is conceivable that the same experiment may be repeated under identical
43 conditions a very large number of times.

44
45 **Utility function:** the utility measures on some numerical scale the desirability of decision consequences $C_{ij} = c(d_i, \theta_j)$
46 that take place when a decision d_i is taken and θ_j turns out to be the true state of nature. Whenever a '0-1' scale is used,
47 a value equal to 0 and 1 is assigned to the least and to the most desirable consequences, respectively. All the remaining
48 consequences are assigned a value within this interval with the sole constraint of coherence: if a consequence is more
49 desirable than another, it must have a greater utility, and vice versa.

50
51 **Loss function:** the loss measures on some numerical scale the undesirability of decision consequences $C_{ij} = c(d_i, \theta_j)$
52 that take place when a decision d_i is taken and θ_j turns out to be the true state of nature. Whenever a '0-1' scale is used,
53 a value equal to 1 and 0 is assigned to the least and to the most desirable consequences, respectively. All the remaining
54 consequences are assigned a value within this interval with the sole constraint of coherence: if a consequence is less
55 desirable than another, it must have a greater loss, and vice versa. Note that that the loss function is obtained as the
56 difference between the utility of the best consequence under the state of nature at hand and the utility for the consequence
57 of interest. Stated otherwise, the loss measures the penalty for choosing a non-optimal decision,

58
59 **Maximizing expected utility:** the desirability of alternative decisions is measured by their corresponding expected utility
60 which is obtained by combining utilities $u(C_{ij})$ associated with the consequences of decisions C_{ij} and probabilities for
61 states of nature $Pr(\theta_j)$ as $\bar{u}(d_i) = \sum_{j=1}^n u(C_{ij}) \times Pr(\theta_j), i = 1, \dots, m$. A standard decision rule instructs one to select
62 the action which maximizes the expected utility.

63
64 **Minimizing expected loss:** the undesirability of alternative decisions is measured by their corresponding expected loss
65 which is obtained by combining losses $l(C_{ij})$ associated with the consequences of decisions C_{ij} and probabilities for states
66 of nature $Pr(\theta_j)$ as $\bar{l}(d_i) = \sum_{j=1}^n l(C_{ij}) \times Pr(\theta_j), i = 1, \dots, m$. A standard decision rule instructs one to select the action
67 which minimizes the expected loss.

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71 **1. Introduction**

72 Nowadays age estimation of living persons is a recognized discipline in the forensic panorama. Age is a fundamental
73 piece of information in our society for the exercise of personal rights and duties. Thus, faced with persons unable or
74 unwilling to declare their age, judicial or administrative authorities often request an expert opinion. Such request have
75 been on the rise recently, since the number of individuals of questioned age has increased, due to the tremendous tide of
76 migration movements, ease of world travel, but also due to the professionalization of criminal organizations involved in
77 human smuggling or trafficking (Law et al., 2010). As highlighted by Schmeling et al. (2007), a question to be answered
78 in case of age diagnostics for living persons mostly concerns the probability of a person being younger or older than a
79 legally relevant age threshold, such as the age of majority. In other cases, a point estimate of the real age can be of interest.
80 In any case, the two pieces of information are strictly related, since the former logically depends on the latter.

81 Although European Asylum Support Office (EASO, 2018) recommends a holistic perspective for age estimation in living
82 individuals (including psychological assessments), we believe that scientific evidence-based anthropological/medico-
83 legal methods should provide the basis of an age estimate. The Study Group on Forensic Age Diagnostic (AGFAD) has
84 provided, particularly in Europe, a strong contribution to this field, by publishing a set of recommendations mainly
85 focused on operational perspectives (Schmeling et al., 2008). These recommendations include the choice of reference
86 studies, the examination steps that ideally ought to be performed during a medico-legal age estimation appraisal, and the
87 structure of expert reports. Similar recommendations were published by the Forensic Anthropology Society of Europe
88 (Cunha et al., 2009).

89 An evaluation of multiple items of evidence is highly recommended in order to increase the accuracy of the age estimate
90 (Schmeling et al., 2003, Schmeling et al., 2008, Bassed, 2012). However, in the early stages of applied forensic age
91 estimation , the domain suffered from a lack of adequate (statistical) methods that would allow to comprehensively
92 evaluate the age-related evidence (Ritz-Timme et al., 2000). The aim of this contribution is to illustrate that forensic age
93 estimation is a problem of inference and decision, and should not be considered only from a statistical perspective.
94 Provided that uncertainty is unavoidable and should be measured by probability, the Bayesian paradigm represents a
95 formidable tool to combine different sources of information that are at the disposal to the different actors involved in the
96 legal disputes regarding a person's age.

97 **2. Uncertainty and inference in forensic age estimation**

98 In forensic age estimation the forensic experts needs to translate their scientific findings into legal information by
99 evaluating and interpreting the evidence (Sironi et al., 2017). Typically, a judicial authority needs to decide whether a
100 person is an adult (e.g. 18 years old or more) or a minor, within the meaning of the law. In order to make a decision, the
101 authority must obtain information about the chronological age of the individual, i.e., the quantity that measures – in years,
102 months and days – the time since the person was born (EASO, 2018). However, age-related and developmental evidence

103 collected by the forensic expert consists of data on the individual's biological age, i.e., the developmental step reached
104 by the individual reflected by achieving a given specific physical attribute at the time of the examination (Hackman et
105 al., 2010). This biological (scientific) information has to be correctly used in order (1) to infer the chronological age, i.e.,
106 the information needed by the mandating authority, and (2) for the authority to make a decision about the matter (i.e., in
107 most cases whether the person in question shall be declared a minor or an adult).

108 Although physical development is a continuous biological process, it is generally described in categorical steps due to the
109 extreme difficulty in appreciating globally all changes which occur during aging in a continuous scale (Lucy et al., 2002).

110 A body of age-related evidence consists therefore in the detection (by an expert) of the developmental steps reached by
111 specific physical attributes (or age indicators) in an examined individual constituting the biological age. It is recognized
112 that the developmental steps are universally reached in their totality and in the same sequence, but the chronology of the
113 developmental phases varies considerably between individuals (Boldsen et al., 2002, Cameron and Jones, 2010). This is
114 mainly due to the influence of a large panoply of individual and environmental factors (i.e., social context or ethnic
115 origin), which may affect the developmental process (Kemkes-Grottenthaler, 2002, Cameron and Jones, 2010) with
116 varying extent for each biological system, such as the skeletal or dental systems (Schmelting et al., 2005). Interpretation
117 of age-related evidence must be provided by assessing the uncertainty regarding the relationship between biological age
118 (i.e. the scientific information) and chronological age (i.e. the legal information). In the presence of available data
119 collected from subjects whose age is known, the uncertainty about such relationships could be modeled by means of
120 appropriate statistical models. However, it must be added that an *ad-hoc* statistical model is not the end of the matter, as
121 highlighted by Taroni and Biedermann (2014, p. 3948) recalling a statement of I. W. Evett:

122 “[...] Statistics concentrates primarily on data, whereas the retrospective meaning of an observation
123 relies on the more general concept of inference which focuses on the notion of uncertainty. [...]”

124 In forensic science, the decision-maker seeks to evaluate a hypothesis or a feature of interest in the light of scientific
125 findings (Taroni and Biedermann, 2015). This is typically an inductive line of reasoning; it is said to be ampliative, since
126 the conveyed conclusions contain elements that are not present in the premises. One's knowledge is extended by inference
127 throughout data (findings, observations) acquisition. Such amplification naturally implies uncertainty: the key elements
128 that allows one to move from an initial belief about the feature of interest (i.e. the age of a given individual) to an updated
129 belief is data acquisition (so often called ‘evidence’), which is generally incomplete, imprecise and rarely conclusive
130 (Schum, 1994). This is also the case in forensic age estimation: the age-related evidence concerning the biological age is
131 incomplete by nature, since it informs us about a given moment in time of a continuous evolutionary process. Moreover,
132 the conclusions in the legal process refer to the chronological age. However, the relationship between biological and

133 chronological age is affected by many sources of uncertainty. It is therefore fundamental to handle such uncertainty in a
134 logical way, in order to provide meaningful findings to the mandating authority.

135 Uncertainty can be qualified and quantified by means of probability. In fact, probability is the standard measure for
136 uncertainty (Lindley, 1991). Probability is defined as the measure of one’s degree of belief on a given event or statement
137 (Taroni et al., 2001). Such an interpretation is generally referred to as subjective (in the meaning of personal, and not
138 arbitrary) and plays an important role in the forensic context (Berger et al., 2011, Biedermann, 2015, Taroni et al., 2015,
139 Biedermann et al., 2017b). An ideal solution to handle the uncertainty in inductive reasoning is provided by the Bayesian
140 paradigm (Taroni and Biedermann, 2014), that formalizes the general approach for thinking about evidence (Robertson
141 and Vignaux, 1998). The usefulness of Bayesian perspective for forensic evidence interpretation has been recognized
142 (Aitken and Taroni, 2004, Comittee of forensic experts, 2011, ENFSI, 2015, Robertson et al., 2016) to the point that Evett
143 (2015, p. 10) states that “The nature of forensic science is now firmly founded in the Bayesian paradigm [...]”.

144 **3. Bayesian perspective in forensic age estimation**

145 The Bayesian paradigm states that all uncertainties characterizing an issue of interest must be described by means of
146 probabilities or probability distributions. Probabilities are interpreted as a conditional measure of uncertainty associated
147 with the unknown feature of interest (e.g., the chronological age) given the available information. The learning process
148 about the feature of interest is described as the modification of the uncertainty in the light of new information, the scientific
149 findings; the Bayes theorem explains how this should be done, formalizing the common notion ‘learning from experience’
150 (Jeffreys, 1961). The algebraic expression of the Bayes theorem depends on the nature of the involved variables. Suppose
151 we have a set \mathbf{E} of categorical age-related evidence and that we consider the chronological age to be the realization of a
152 continuous variable A . A standard application of the Bayes' theorem allows one to obtain the posterior distribution of the
153 chronological age as

$$154 \quad f_A(a|\mathbf{E}, I) = \frac{Pr(\mathbf{E}|a, I) \times f_A(a|I)}{\int Pr(\mathbf{E}|a, I) \times f_A(a|I) da}, \quad (1)$$

155 where I indicates the background information related to the examined person, such as the sex and the ethnic origin and
156 any other relevant pieces of information. A discussion on the meaning and the role of background information can be
157 found in Aitken and Nordgaard (2018). The formulae state that the updated belief, i.e., the posterior probability
158 distribution on the age $f_A(a|\mathbf{E}, I)$ results from a normalized combination of the initial beliefs about the age, represented
159 by the prior probability distribution $f_A(a|I)$ and the information originating from the available evidence, quantified as
160 $Pr(\mathbf{E}|a, I)$.

162 Note that analytically solving Bayesian models can be a tedious and time-consuming procedure, which is unsuitable for
163 daily practice. Nonetheless, specific statistical tools exist in order to simplify this aspect, especially so-called Bayesian

164 Networks (BNs). BNs are probabilistic graphical models that present the dual advantage of graphically describing the
165 relationship between variables describing the inferential model, as well as of directly providing automatic computations
166 following the rule of the probability theory (Taroni and Biedermann, 2013). BNs are widely used in forensic science
167 (Taroni et al., 2014), including age estimation (Sironi et al., 2016). They can be easily extended to take into account the
168 decisional aspect (Taroni et al., 2014).

169 **4.1 Posterior probability distribution on the chronological age**

170 Posterior density is the target outcome of the inductive process: it encapsulates all available information related to the
171 specific case, ranging from the collected evidence, the available background information up to the specific expert
172 knowledge at a given time. It is therefore possible to quantify and combine all sources of uncertainty about the target
173 quantity (i.e., the chronological age) in a rational way, as clearly requested in the age estimation domain (EASO, 2018,
174 Malmqvist et al., 2018).

175 The posterior distribution can provide point and interval estimates of the chronological age. Furthermore, it can be used
176 to inform us about the probability of competing propositions such as whether the examined individual is older or younger
177 than a specific threshold. For instance, the probability that an individual is 18 years or older can be obtained by the
178 integration of the *posterior* density function $f_A(a|\mathbf{E}, I)$ over the age-space of interest (Thevissen et al., 2010, Sironi et al.,
179 2016):

$$180 \Pr(\theta_1|\mathbf{E}, I) = \int_{\theta_1} f_A(a|\mathbf{E}, I) da = \int_{\theta_1} \frac{\Pr(\mathbf{E}|a, I) \times f_A(a|I)}{\int \Pr(\mathbf{E}|a, I) \times f_A(a|I) da} da \quad (2)$$

181 where θ_1 is the interval that covers the age space equal to and greater than 18 years of age. The (posterior) probability
182 that the individual is younger than 18 years, θ_2 , can be computed analogously.

184 **4.2. Prior probability distribution on the chronological age**

185 The prior probability distribution should reflect prior beliefs on the chronological age of the examined individual before
186 the evaluation of the collected age-related evidence. Though strategies suitable for formalizing prior beliefs in terms of a
187 probability distribution have been proposed both in statistical and forensic literature (O'Hagan et al., 2006, Taroni et al.,
188 2010, Sironi et al., 2017, Bolstad and Curran, 2017), the elicitation of a prior in age estimation can be a challenging task,
189 since there is generally little initial information at the expert's disposal (Schmeling et al., 2003). From a Bayesian
190 perspective this must not be felt to be an insurmountable drawback, as the prior probability distribution "[...] reflects
191 [one's] belief about the subject matter, conditioned as these will presumably be by [one's] available background evidence
192 [...]" (Howson, 2002, p. 53). The expert should be able to qualify or quantify their personal belief on the age of the
193 examined person based on preliminary information on the case at hand.

194 In the framework suggested by the AGFAD, the prior probability distribution could be assigned by the expert based on
195 information collected during an interview or the physical examination of the person under investigation, since it is
196 generally acknowledged that data gathered in this initial step of the examination sequence should not be used for the
197 effective age estimation (Schmeling et al., 2006a, Schmeling et al., 2011). Due to the lack of available information, many
198 authors have proposed adopting so-called non-informative or vague prior probability distributions, such as a uniform
199 distribution over a given age range, according to which all possible age values are considered, *a priori*, equally likely
200 (Braga et al., 2005, Thevissen et al., 2010, Cameriere et al., 2016, Bleka et al., 2018). This way of thinking should result
201 in posterior conclusions that will be minimally dependent on prior distribution. This is not intrinsically wrong, but
202 information-less priors may be misleading as such priors actually do not exist (Howson, 2002). The introduction of a
203 uniform distribution over a given age range is far from being without information, as it conveys the belief that all ages in
204 the chosen interval are considered equally likely, whilst those outside the range are considered as not possible (Sironi et
205 al., 2017). The elicitation of such a uniform distribution may be reasonable in some scenarios (Sironi et al., 2018b), when
206 the age interval is chosen based on the available knowledge (Sironi et al., 2018b, Konigsberg et al., 2019). Other choices
207 of probability distributions over the chronological age are clearly possible. For instance, in case of forensic age estimation
208 of a young adult, a probability distribution centered around the legal threshold may be preferred. Asymmetrical
209 distribution has been proposed in the scientific literature (Sironi et al., 2016), nonetheless Konigsberg et al. (2019) argued
210 that it would be more beneficial to rather assign a symmetrical one, as the normal or the Laplace distribution, located
211 around the mentioned legal threshold, which leads to an equal support of the competing intervals under consideration
212 (here named θ_1 – a given person is aged 18 years or older; and θ_2 – a given person is younger than 18 years) (Konigsberg
213 et al., 2019).

214 Examiner may be concerned about the sensitivity of the posterior distribution to alternative prior distributions that fit just
215 as well the prior beliefs. The sensitivity analysis is a powerful tool for investigating the robustness of the posterior
216 inference on prior assignments (Sironi and Taroni, 2015, Sironi et al., 2015, Sironi et al., 2018a, Sironi et al., 2018b,
217 Konigsberg et al., 2019).

218 **4.3. Likelihood function**

219 The likelihood function models the relationship between the age-related evidence (i.e., the biological age) and the
220 chronological age. The choice of standard statistical models (such as classical regression models) is generally unfeasible.
221 It must be acknowledged that in the age estimation scenario, there are generally multiple items of evidence available to
222 authorities, and that the quantified variables do not necessarily have identical scales of measurement. A statistical model
223 must clearly provide a coherent assessment of the uncertainty about the relationship between the biological and the
224 chronological age, but must also be capable of dealing with multiple items of evidence.

225 Several statistical methods have been proposed in the literature for age estimation purposes, although, they generally
226 present some drawbacks, such as the unsuitability of being employed for the evaluation of multiple items of evidence
227 having different scales of measurement (Thevissen et al., 2010, Hillewig et al., 2013, Sironi et al., 2018a) or operational
228 limitations (Braga et al., 2005).

229 The assumption of conditional independence between pieces of evidence given the age has often been retained (Corradi
230 et al., 2013a, Corradi et al., 2013b, Gelbrich et al., 2015, Fieuws et al., 2015, Tangmose et al., 2015, Bleka et al., 2018).
231 If this assumption were reliable, the likelihood function could be obtained as the product of individual likelihoods for
232 each considered item of evidence, when n items of evidence are evaluated:

$$233 \quad \Pr(\mathbf{E}|a, I) = \prod_{k=1}^n \Pr(E_k|a, I). \quad (3)$$

234 However, this assumption does not seem to meet the biological reality, since the development of a single part of the body
235 is rarely independent from the others (Boldsen et al., 2002). The consequence may be that the estimated posterior density
236 is too narrow compared to what it should be (Fieuws et al., 2015). In this perspective, Boldsen et al. (2002) have suggested
237 a statistical procedure for correcting the posterior interval estimates, and Fieuws et al. (2015) have extended the procedure
238 in order to allow one to correct directly the posterior density. The assumption of conditional independence may be
239 strengthened by considering other pieces of information provided by the examined person. Examples of such information
240 may be the knowledge of diseases or lower socio-economic status of the examined person during development, which
241 may affect the skeletal and dental development to a different degree (Schmeling et al., 2005, Schmeling et al., 2006b).

242 When considering a single item of evidence, the use of regression models specifically developed for the treatment of
243 categorical dependent variables is beneficial. Notably, models from both *probit* and *logit* families have been used
244 (Konigsberg (2015) and references therein). Specifically, *unrestricted cumulative* models (Sironi and Taroni, 2015, Sironi
245 et al., 2015, Sironi et al., 2018a, Konigsberg et al., 2019) and *continuation ratio* models (Fieuws et al., 2015, Tangmose
246 et al., 2015, Sironi and Taroni, 2015, Sironi et al., 2016, Bleka et al., 2018) have been proposed. Note that the models
247 employed by Sironi and Taroni (2015), Sironi et al. (2016) and Bleka et al. (2018) are sometimes also referred to as
248 *stopping ratio models* (Konigsberg et al., 2019). *Proportional-odds* (or *restricted cumulative*) models have also been
249 employed (Bleka et al., 2018). These models are based on assumptions that do not meet the biological reality of the
250 developmental process, thus they are not appropriate in this field (Boldsen et al., 2002). As pointed out by Konigsberg et
251 al. (2019), there is little practical difference between the *logit* or *probit* models: traditionally *logit* models were preferred
252 because of computational ease compared to the *probit*, but nowadays this is no longer a rational argument, considering
253 the availability of statistical software (Konigsberg, 2015). Moreover, it may be felt that models from the *probit* family
254 may be easier to implement for practitioners, since they refer to normal distribution rather than to logistic distribution,
255

256 which may be less intuitive (Myers et al., 2002). According to Konigsberg et al. (2019), this aspect is particularly relevant
257 in the forensic field, since scientific results originating from the selected models should be presented to an audience with
258 a limited scientific background. Furthermore, Konigsberg (2015) highlighted that univariate *probit* models can be
259 extended to consider multiple variables by using a multivariate normal distribution, whilst this task would be more
260 complex with logistic distribution.

261 An attractive feature of the cumulative models is that some consecutive stages can be collapsed into a single one without
262 affecting the estimation of the parameters of the curves of the other stages (Konigsberg et al., 2008). This is particularly
263 interesting in case of age estimation from physical attributes for which development is described by means of several
264 categorical steps. However, these models may generate overlapping regression curves that may lead to inconsistencies
265 associated with a generic developmental stage (Konigsberg and Herrmann, 2002). Sironi and Taroni (2015) showed that
266 different regression models may lead to different quantifications of the posterior probabilities on a given age cohort (such
267 as θ_1 or θ_2). Bleka et al. (2018) adopted the regression model that provided the best fit with the available data. Note that
268 in order to avoid inconsistencies on the value of age, the variable ‘age’ can be transformed into a logarithmic (Konigsberg
269 et al., 2008) or exponential scale (Bleka et al., 2018). Non-parametric models (such as those based on the Kernel
270 distribution) may also be employed (Lucy et al., 2002).

271 The main drawback affecting the above models, is that for their operational implementation, there is an urgent need for
272 structured data samples, that unfortunately are unlikely to be available (Konigsberg, 2015). Nonetheless, the lack of
273 adequate data should not be considered as an insurmountable impasse. For instance, the guidelines for evaluative reporting
274 in forensic science published by the European Network of Forensic Science Institutes (ENFSI, 2015) states that:

275 “[...] Relevant and appropriate published data will be used wherever possible. If appropriate
276 published data are not available then data from unpublished sources may be used. Regardless of the
277 existence of sources (published or not) of numerical data, personal data such as experience in similar
278 cases and peer consultations may be used, provided that the forensic practitioner can justify the use
279 of such data. [...]” (ENFSI, 2015, p. 15).

280 Furthermore,

281 “[...] likelihood [...] can be informed by subjective probabilities using expert knowledge. [...] Such
282 personal probability assignment is not arbitrary or speculative, but is based on a body of knowledge
283 that should be available for auditing and disclosure.” (ENFSI, 2015, p. 16)

284 Note that the guidelines focus on the assignment of the probabilities that represent the ingredients of the likelihood ratio
285 (i.e., the ratio between the probability of the evidence given two competing propositions), but these statements can be
286 extended in order to choose an appropriate statistical model that is capable of taking into account in a coherent way the
287 uncertain relationship between biological and chronological age. An extensive discussion about the role of the subjectivist

288 approach to the elicitation of probabilities in forensic science can be found in Taroni et al. (2001), Berger et al. (2011),
289 Biedermann et al. (2017b), Taroni et al. (2018).

290 **5. Bayesian inference from an operational perspective**

291 The regression models discussed above represent the ideal approach to forensic age estimation. However, because of the
292 lack of reference data, their application in practice may be problematic. In case of lacking data (see above), forensic
293 practitioners may feel more comfortable in assigning probabilities in the form of relative frequencies rather than
294 probability distributions. The use of relative frequencies to inform subjective probabilities has recently been discussed by
295 Taroni et al. (2018). However, such frequencies cannot logically be assigned for each age of a continuous scale. Since the
296 mandating authority usually asks whether or not an age threshold was exceeded, it may be sensible to consider two
297 alternative propositions in the form:

- 298 • θ_1 : the examined person is aged 18 years or older;
- 299 • θ_2 : the examined person is younger than 18 years of age;

300 Note that in this paper the notation θ_j is used in two different meanings. In Section 4, θ_1 and θ_2 represent a subset of the
301 support of the continuous variable “age”, whilst in the current one it represents a discrete event. Note also that the age
302 limits considered as reasonable in the case at hand should be expressed directly in the propositions, whilst in the
303 framework discussed in section 4, these limits are implicitly defined by the choice of the probability distributions of
304 interest. The Bayes’ theorem can therefore be formulated as follows:

$$305 \Pr(\theta_j | \mathbf{E}, I) = \frac{\Pr(\mathbf{E} | \theta_j, I) \times \Pr(\theta_j | I)}{\sum_{j=1}^2 \Pr(\mathbf{E} | \theta_j, I) \times \Pr(\theta_j | I)} \quad (4)$$

306 It is worth emphasizing that not only the evidence, but also the chronological age is categorized as a discrete variable.
307 This formulation is rarely addressed in the literature (Lucy, 2010). However, from an operational perspective, the task of
308 the forensic expert is potentially simplified, since it will be limited to the assessment of a probability of discrete events
309 (the exceeding or vice versa of an age threshold). These probabilities can be assigned relying on available relative
310 frequencies of the developmental evidence in the given cohort. Such relative frequencies can sometimes be extrapolated
311 from reference studies, or from unpublished data (ENFSI, 2015).
312

313 It must be clarified that this does not amount to equating a conditional probability that according to our view represents a
314 degree of belief, with relative frequency that represents a normalized count of a given quantity. From a frequentist point
315 of view, the probability can indeed be defined as a limiting value of relative frequency, assuming a large repetition of the
316 event under identical conditions is feasible. This is generally not the case for forensic evidence (Lucy, 2010, Curran,
317 2013), and a subjectivist approach is strongly encouraged (Lindley, 1991). Nevertheless, a personal degree of belief can
318 be informed by relative frequencies, which is even recommended, when data are available (Taroni et al., 2018).

319 Notably, while the expert needs to assign the probability of the evidence \mathbf{E} based on the observation of multiple age
320 indicators, data collected simultaneously from multiple age indicators are infrequently available in forensic literature
321 (Schmelting et al., 2016). However, if the assumption of conditional independence between different pieces of evidence
322 is feasible, the conditional probability $Pr(\mathbf{E}|\theta_j, I)$ can be simplified as follows:

$$323 \quad Pr(\mathbf{E}|\theta_j, I) = \prod_{k=1}^n Pr(E_k|\theta_j, I) \quad (5)$$

324 Then, conditional probabilities for each type of evidence E_k , $Pr(E_k|\theta_j, I)$, can be derived from relative frequencies
325 accessed from available databases or published reference studies.

326 Two aspects of the use of relative frequencies to elicit conditional probabilities in Eq. (5) need to be considered. Firstly,
327 the frequency of a specific complex developmental pattern is very low, since the number of observable patterns can be
328 very large. Thus, the posterior inference may be highly sensitive to such assignment. Secondly, the relative frequencies
329 are logically influenced by the structure of the reference sample.

330 Several studies report the probability of being at least 18 years old given the observed developmental stage (Liversidge
331 and Marsden, 2010, Mincer et al., 1993). However, such probabilities refer to the proposition given the evidence (i.e.,
332 $Pr(\theta_1|\mathbf{E}, I)$), and not to the evidence given the proposition (i.e., $Pr(\mathbf{E}|\theta_1, I)$). Equating these two probabilities would
333 amount to a transposition of conditionals (Evet, 1995). Note that the probability $Pr(\theta_1|\mathbf{E}, I)$, whenever available, does
334 not incorporate the prior knowledge about age.

336 **6. Normative approach to decision in age estimation**

337 The problem of evidence interpretation concerns both inference and decision. The importance of a rational approach to
338 decision-making for questions by the different actors of the legal process has gained an increasing attention in forensic
339 literature (Taroni et al., 2005, Taroni et al., 2010, Gittelsohn, 2013, Biedermann et al., 2016, Biedermann et al., 2018). In
340 age estimation, the expert is asked to make several decisions, including the choice of an appropriate method for assessing
341 the developmental process, or the identification of the developmental status reached by a physical attribute. Nonetheless,
342 the more relevant decision in age estimation concerns indubitably the fact that a person is adult or minor.

343 Note that a clear distinction is to be made between deciding that an individual being evaluated is younger or older than
344 18 years of age, and the decision to declare him or her minor or adult within the meaning of the law. Such decision in age
345 estimation will be taken, at a given moment, by one of the participants of the legal or administrative procedure
346 (Biedermann et al., 2017a). The decision maker may be either the scientist or the mandating authority, depending on the
347 framework of the circumstances of a specific case. However, the main focus of this paper is not a discussion about *who*
348 is entitled to make this kind of decision in forensic age estimation framework, but rather *how* such a decision problem
349 should be tackled (see Biedermann et al. (2008) for a wide discussion on this aspect in forensic science). Therefore, the

350 current discussion focuses on the general aspects of the normative approach to making decision in forensic age estimation.
 351 From a general point of view, this decision problem can be resumed on evaluating “what is the minimum degree of
 352 probability $Pr(\theta_1|E,I)$ to be required for accepting that the adulthood of the examined person is established?”
 353 (Biedermann et al., 2017a). A study conducted by Polo Grillo et al. (2002) on 47 judicial cases involving immigrants
 354 lacking valid ID documents illustrated how decision-makers expressed appreciation for the quantification of uncertainty
 355 in terms of probability in age estimation cases. The study pointed out that the judges felt “confident” in declaring an
 356 individual to be an adult if the reported probability that he or she is 18 years or older was greater than 0.70. However, the
 357 empirical nature of this study must be emphasized, because it is generated by the observation of the behavior of the
 358 decision-maker not using a specific decisional criterion. For this reason, it cannot serve as a framework to support a
 359 rational decision in specific casework. For this reason, we endorse a normative approach to decision-making, which can
 360 be integrated into the Bayesian framework. The normative approach to decision-making has only recently been explored
 361 in forensic sciences (Taroni et al., 2005, Biedermann et al., 2008, Gittelsohn et al., 2013, Gittelsohn et al., 2014, Biedermann
 362 et al., 2016, Gittelsohn et al., 2016, Biedermann et al., 2017a). More details about the decision theory and its application
 363 can be found, among others, in DeGroot (1970), Lindley (2006) and Berger (2010).

364 A problem of decision can be described in terms of three principal components (Table 1):

- 365 • A collection of states of nature, denoted $\theta_1, \theta_2, \dots, \theta_n$, that represent the events of interest in the decision-making
 366 process and about which the decision-maker is uncertain. Assuming that probability is the standard measure of
 367 uncertainty, the uncertainty about these events can be quantified in a collection of probabilities $Pr(\theta_1|\cdot)$,
 368 $Pr(\theta_2|\cdot)$, ..., $Pr(\theta_n|\cdot)$ which are conditioned by all available knowledge in the given as $\sum_{j=1}^n Pr(\theta_j|\cdot) = 1$.
 369 In age estimation, the events of interest can be stated as the two competing propositions θ_1 and θ_2 . To each state
 370 of nature a probability will be assigned, namely $Pr(\theta_1|E,I)$ and $Pr(\theta_2|E,I)$ as in Eqs 2 and 4 (see section 4
 371 and 5).
- 372 • A collection of decisions (or actions), denoted d_1, d_2, \dots, d_m . Decisions must be exhaustive and mutually
 373 exclusive, that is, the collection must cover all possible decisions and the decision-maker can choose one, and
 374 only one, decision among all the available ones. In the scenario discussed in this Chapter, the available decisions
 375 are d_1 : to declare the examined person as adult, and d_2 : to declare the examined person as minor.
- 376 • A collection of consequences that result from the combination of the states of nature and the available decisions:
 377 the choice of a decision d_i when θ_j is the true state of nature leads to a consequence $C(d_i; \theta_j)$, denoted as C_{ij} .

Table 1

Decision matrix for the given age estimation scenario. d_i with $i = 1,2$ denotes the available decisions, θ_j with $j = 1,2$ denotes the events of interests, and C_{ij} denotes the possible decision consequences.

Decision	States of nature	
	θ_1 : aged 18 years or older	θ_2 : younger than 18 years of age

d_1 : adult	C_{11} : correct declaration as adult	C_{12} : false declaration as adult
d_2 : minor	C_{21} : false declaration as minor	C_{22} : correct declaration as minor

378
379 Decision consequences are characterized by a different level of desirability or undesirability. In the age estimation
380 scenario, the more favorable consequence is when the examined person is correctly declared as being adult or minor (C_{11}
381 and C_{22}), whilst the worst consequence is when a person who is actually younger than 18 years of age, is declared an
382 adult (C_{12}). This type of error is generally referred to as ethically unacceptable (Garamendi et al., 2005). The consequence
383 of declaring a person who actually is older than 18 years of age (C_{21}) to be a minor is generally favorable to the individual,
384 but adverse to the society, because of the additional social expenses that could have been avoided. For this reason, the
385 undesirability of consequence C_{21} can be considered to be intermediary between those referred to the best and the worst
386 consequence. This amounts to the following preference ordering:

$$C_{11} \sim C_{22} \succ C_{21} \succ C_{12}$$

387 where \succ denotes the preference of the decision-maker for one consequence over another, whilst \sim denotes the indifference
388 on the desirability of two or more consequences.

389 The desirability of a given consequence C_{ij} can be measured by a numerical value, by means of a function called *utility*
390 *function*, denoted as $u(d_i; \theta_j) = u(C_{ij})$. Analogously, it is possible to express preferences among decision consequences
391 by means of a *loss function*, denoted as $l(d_i; \theta_j) = l(C_{ij})$, that quantifies on a numerical scale the undesirability of
392 decision consequences. Note that utilities and losses are conceptually and mathematically connected (Berger, 2010),
393 though in the current scenario the more intuitive reasoning would be in terms of losses.

394 Various strategies can be implemented to build an appropriate loss function (Lindley, 1985, Koller and Friedman, 2009,
395 Berger, 2010), with the sole constraint being that the loss function must correctly reflect the preference ordering. This
396 implies that if $l(\cdot)$ is a loss function and one consequence is felt as less desirable than another one, i.e., $C_{12} < C_{22}$, then
397 $l(C_{21}) < l(C_{12})$. The highest loss value, according to the unit scale, will be assigned to the more adverse outcome (i.e.,
398 C_{12}), while the smallest will be assigned to the less adverse or more favorable outcome (i.e., C_{11} and C_{22}).

399 As far as the practical and controversial issue of building the loss function is concerned, both the utility and the loss
400 functions are invariant to linear transformations, so that any particular choice of the unit scale is theoretically allowed
401 (Berger, 2010). A convenient choice is the '0-1' scale, where the minimum and the maximum of the loss scale are fixed
402 at 0 and 1, respectively (Table 2). This implies that a loss equal to 0 is assigned to C_{11} and C_{22} (the most favored outcomes),
403 that is $l(C_{21}) = l(C_{12}) = 0$, and a loss equal to 1 is assigned to C_{12} (the worst outcome), that is $l(C_{12}) = 1$. In this way
404 the construction of a loss function for the age estimation scenario is greatly simplified as the only remaining value that
405 needs to be assigned is the loss associated with the intermediate consequence C_{21} .
406

407 A feasible strategy to assign the loss associated with C_{21} refers to a context where one player (i.e., the decision-maker) is
 408 asked to choose between a sure event (in this case, the outcome corresponding to the intermediate consequence whose
 409 loss needs to be assigned), and a gamble where the worst outcome takes place with probability p and the best outcome
 410 takes place with probability $1 - p$ (see Lindley (1985) or (Berger, 2010) for a formal approach). The loss of the
 411 intermediate consequence C_{21} can be set to be equal to the probability p that makes one indifferent between the sure event
 412 and the gamble. Note that this assignment has to be made under the constraint imposed by the personal scale of desirability
 413 of the possible consequence: if $C_{11} \sim C_{22} \succ C_{21} \succ C_{12}$, then $l(C_{11}) = l(C_{22}) > l(C_{21}) > l(C_{12})$, thus $l(C_{21})$ has to take a
 414 value between 0 and 1 (excluded).

Table 2

The '0-1' loss function for the age estimation scenario, where d_1 and d_2 denote the available decisions, θ_1 and θ_2 denote the events of interest (as illustrated in Table 1), and $l(C_{21})$ denotes the loss associated with the intermediate consequence C_{21} .

	θ_1 : aged 18 years or older	θ_2 : younger than 18 years of age
d_1 : adult	0	1
d_2 : minor	$l(C_{21})$	0

416
 417 The undesirability of available decisions can be measured by their corresponding (posterior) expected losses:

$$\bar{l}(d_i | \cdot) = \sum_{j=1}^n l(C_{ij}) \times Pr(\theta_j | \mathbf{E}, I) \quad (6)$$

419 where $Pr(\theta_j | \mathbf{E}, I)$ are the posterior probabilities of the states of nature and $l(C_{ij})$ the losses for the consequences of
 420 interest. The optimal decision, also called Bayes decision, is the decision that minimizes the Bayesian expected losses,
 421 formally:

$$arg \min_i \bar{l}(d_i | \cdot) = arg \min_i \sum_{j=1}^n l(C_{ij}) \times Pr(\theta_j | \cdot) \quad (7)$$

424 In this way both the undesirability of the consequences (in terms of losses) and the uncertainty of the state of nature (in
 425 terms of probabilities) are considered related to the decision.

427 It is thus possible to quantify the expected loss for the two available decisions in the age estimation scenario. The
 428 (posterior) expected loss for the decision d_1 (i.e., to formally declare the person an adult) can be quantified as

$$\bar{l}(d_1 | \mathbf{E}, I) = \sum_{j=1}^2 l(C_{1j}) \times Pr(\theta_j | \mathbf{E}, I) = \underbrace{l(C_{11})}_0 \times Pr(\theta_1 | \mathbf{E}, I) + \underbrace{l(C_{12})}_1 \times Pr(\theta_2 | \mathbf{E}, I), \quad (8)$$

430 while the (posterior) expected loss of decision d_2 (i.e., to formally declare the person a minor) can be quantified
 431 analogously as

$$\bar{l}(d_2|\mathbf{E}, I) = \sum_{j=1}^2 l(C_{2j}) \times Pr(\theta_j|\mathbf{E}, I) = l(C_{21}) \times Pr(\theta_1|\mathbf{E}, I) + \frac{l(C_{22})}{0} \times Pr(\theta_2|\mathbf{E}, I), \quad (9)$$

434
 435 According to the Bayesian decision criterion in Eq. (7), that prescribes making the decision having the lower expected
 436 loss, the decision d_1 is therefore preferable to the decision d_2 when $\bar{l}(d_1|\mathbf{E}, I) < \bar{l}(d_2|\mathbf{E}, I)$. Writing expected losses
 437 $\bar{l}(d_i|\mathbf{E}, I)$ in full length, and eliminating the terms in Eqs. (8) and (9) involving zero losses, leads to the following
 438 expression

$$l(C_{12}) \times Pr(\theta_2|\mathbf{E}, I) < l(C_{21}) \times Pr(\theta_1|\mathbf{E}, I), \quad (10)$$

440
 441 or equivalently to

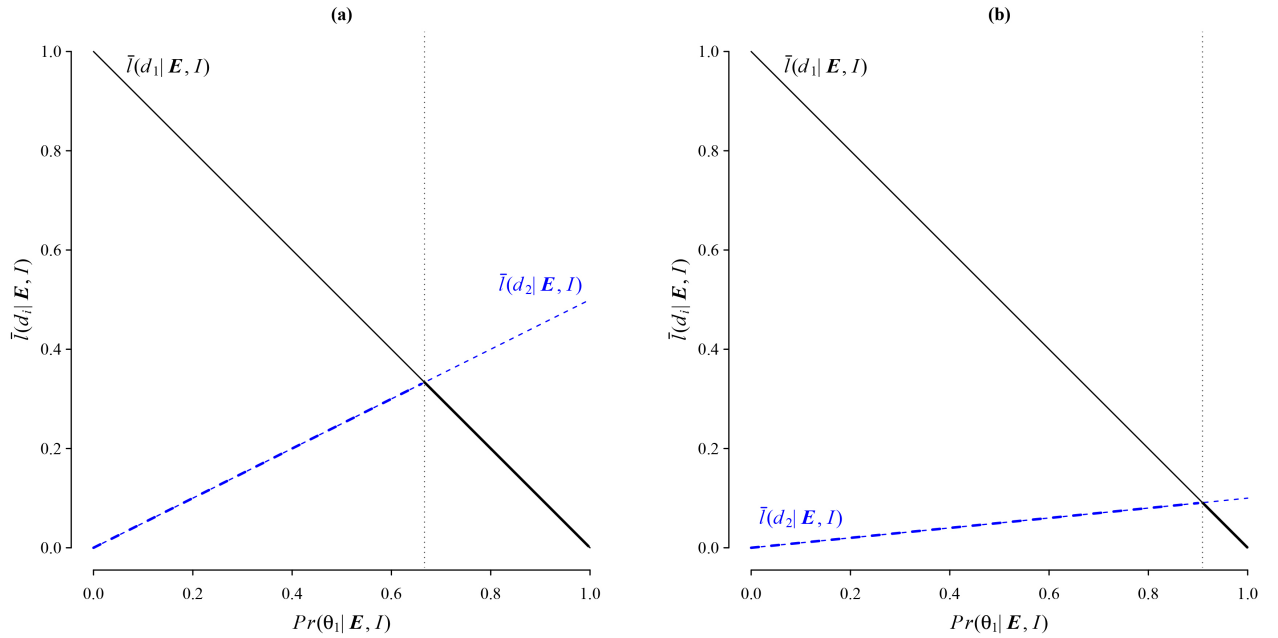
$$\frac{Pr(\theta_2|\mathbf{E}, I)}{Pr(\theta_1|\mathbf{E}, I)} < \frac{l(C_{21})}{l(C_{12})}. \quad (11)$$

443
 444 Eq. (11) states that the decision d_1 to declare the examined individual an adult is preferable if, and only if, the posterior
 445 odds in favor of θ_2 are smaller than the loss associated to C_{21} , which corresponds to wrongly declaring an individual who
 446 has effectively exceeded the age of 18 years as a minor. Note that $l(C_{12}) = 1$ and $Pr(\theta_2|\mathbf{E}, I) = 1 - Pr(\theta_1|\mathbf{E}, I)$, thus,
 447 the Eq. (11) can be rearranged as follow:

$$Pr(\theta_1|\mathbf{E}, I) > \frac{1}{1 + l(C_{21})}. \quad (12)$$

449
 450 That is, the minimum degree of probability $Pr(\theta_1|\mathbf{E}, I)$ that is required to decide about the adulthood of the examined
 451 person is given by the ratio $1/[1 + l(C_{21})]$. A more intuitive way to interpret the principle of minimizing the expected
 452 loss in the current scenario is provided in Figure 1, that shows the (posterior) expected losses in Eqs. (8) and (9) as a
 453 function of the posterior probability of θ_1 , $Pr(\theta_1|\mathbf{E}, I)$, with $l(C_{21}) = 0.50$ (Figure 1a) and $l(C_{21}) = 0.10$ (Figure 1b).

454



455

456 **Fig. 1:** Expected losses $\bar{l}(d_i|E, I)$, with $i = 1, 2$, for different values of $Pr(\theta_1|E, I)$ with $l(C_{21}) = \mathbf{0.50}$ (a) and
 457 $l(C_{21}) = \mathbf{0.10}$ (b). The dotted vertical line indicates the threshold value of $Pr(\theta_1|E, I)$ that inverses the preferability
 458 of the decision, in this case $Pr(\theta_1|E, I) = \mathbf{0.66}$ (a) and $Pr(\theta_1|E, I) = \mathbf{0.90}$

459

460 In Figure 1, the optimal decision is the one for which the values of the corresponding expected losses attain the minimum:
 461 these values are highlighted in bold. Given $l(C_{21}) = 0.50$, Figure 1a shows that the optimal choice is d_1 if, and only if,
 462 $Pr(\theta_1|E, I) > 1/[1 + 0.50] = \mathbf{0.66}$, otherwise it is d_2 . In the second case (Figure 1b), given $l(C_{21}) = 0.01$, the optimal
 463 choice is d_1 if, and only if, $Pr(\theta_1|E, I) > 1/[1 + 0.01] = \mathbf{0.90}$

464 Let us now consider the problem of choosing a loss function. In the current scenario, the problem is confined to the choice
 465 of a meaningful value for $l(C_{21})$. Note that, as pointed out by Biedermann et al. (2016, p. 34), the necessary comparison
 466 implied by Eq. (11) “[...] is essentially qualitative and reduces to a single factor, call it x for simplicity, that states how
 467 much greater one loss value is compared to the other.” Given the actual preference ordering, one can define

468

$$l(C_{12}) = xl(C_{21}), \quad \text{for } x > 1 \quad (13)$$

469
 470 The practitioner needs to specify how much worse he considers it to wrongly declare an individual who has not effectively
 471 exceeded the age of 18 years an adult, with respect to the opposite, that is, to wrongly declare an individual who is
 472 effectively older than 18 years a minor. Then, being $l(C_{12})$ set equal to 1 because of the choice of a ‘0-1’ unit scale, the
 473 loss associated with C_{21} can be immediately obtained as $l(C_{21}) = 1/x$.

474 For instance if the decision-maker feels that an erroneous declaration about a person being an adult is two times worse
 475 than an erroneous declaration of the person being a minor, then, $x = 2$ and $l_{21}=0.50$; in case the former would be
 476 considered 10 times worse than the latter, then $x = 10$ and $l_{21}=0.10$. Recalls the example provided by the study of Polo
 477 Grillo et al. (2002) and suppose that the evidence evaluation lead to a value of the posterior probability on θ_1 of 0.70, i.e.,

478 $Pr(\theta_1|E, I) = 0.70$. In this case, from Eq. (12), the rational decision is d_1 (declare the examined person as adult) if and
479 only if $l(C_{21}) > 0.42$ (approximately). Thus, if the decision-maker believes that a “false adult” is only two time worse than
480 a “false minor”, then the rational decision is to declare the examined person as adult (d_1). If instead, he or she feels that
481 it is 10 times worse, then the rational decision is to declare the examined person as minor (d_2). Note that the choice of
482 the value of x (as well as any quantifications of the loss value) is subjective and it is based on the personal belief and the
483 personal knowledge of the decision-maker. Elements that can support such choice are the framework of the case (criminal
484 versus asylum cases) or the law system in force in a given country. Analogously to the assignment of subjective
485 probability, these subjective losses are a formalization of the personal belief of the decision-maker, which is informed by
486 all the pieces of information available by the decision-maker. Subjective losses are thus perfectly compatible with the
487 forensic and legal framework, provided that the choice made is coherent and can be justified (Taroni et al., 2010,
488 Biedermann et al., 2016).

489 A sensitivity analysis is strongly suggested also in this case. Different loss assignments, as well as different prior
490 assignments, will give rise to different expected losses. This must not be considered to be a weakness of the endorsed
491 Bayesian criterion for making decisions. Different prior probabilities, as well as different losses, might fit as well a given
492 degree of belief or a given preference structure, and therefore different expected losses might be entirely justifiable.

493 **7. Discussion and conclusion**

494 The interpretation of scientific evidence is an inferential (inductive) task and thus naturally involves uncertainty. The
495 specific case of age-related evidence in the forensic age estimation framework does not represent an exception to this
496 statement. Assuming that probability is the measure of quantifying uncertainty and that the Bayesian approach provides
497 a logical framework to the problem of induction, the expert plays a central role in the choice of (i) the prior probability
498 distribution on the chronological age, (ii) the statistical model to handle uncertainty about the evidence, and (iii) the
499 relevant database to inform the likelihood function.

500 It has already been pointed out that the expert must often deal with the absence of adequate databases. This is not
501 surprising, as the collection of the biological age from multiple age indicators for the same individual is extremely
502 difficult, especially in populations generally involved in age estimation procedures. However, the lack of information
503 must not be equated with the impossibility of implementing a probabilistic model. The lack of background data is a
504 common problem shared in various forensic disciplines, at the point that the ENFSI (2015) considers this possibility in
505 its guidelines. In such a context, the role of the expert becomes even more relevant, since the probabilities of the events
506 of interest (i.e., the probability of dental or skeletal evidence given the chronological age) can reasonably be assigned
507 based on a personal (and justified) body of knowledge, by using unpublished data or data published in reference studies
508 (ENFSI, 2015). In this perspective, reference studies including the parameters estimated through regression models

509 applied to unpublished datasets can be extremely useful: such data can be implemented by the expert to quantify likelihood
510 function (Konigsberg, 2015, Konigsberg et al., 2019).

511 As far as the uncertainty about the chronological age is concerned, an operationally valid alternative to the introduction
512 of a probability density function over a continuous random variable (i.e., age) would be to consider specific age cohorts
513 in the form of competing propositions. This allows us to facilitate the quantification of uncertainty regarding the age in
514 the likely situation of scarce information, as probabilities must be assigned to two discrete events rather than over the
515 entire span of the continuous age range. In this particular case, Taroni et al. (2018) recommend to gather available
516 information (e.g., from the literature) in the form of relative frequencies to inform one's personal beliefs.

517 In recent years, the normative decisional approach has been promoted in forensic sciences (Taroni et al., 2005, Taroni et
518 al., 2010, Gittelsohn, 2013, Biedermann et al., 2016, Biedermann et al., 2018), since it offers a structured way of thinking
519 taking advantage of quantitative data, knowledge and experience. This paper illustrates how to use this approach for
520 forensic age estimation in living persons.

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524

525

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