COMMUNICATIONS PHYSICS

ARTICLE

https://doi.org/10.1038/s42005-019-0144-6 OPEN

Extreme risk induced by communities in interdependent networks

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Networks in nature not only depend on each other but also have internal community structures, such as infrastructure networks with links within and across geographic regions. The communities play an important role when the networks undergo localized failures in specific regions, for instance when natural disasters or economic sanctions disrupt a local community region and consequently influence the whole system. How a disruption in one community propagates through the entire system is a crucial, but still open, question. Here we find that the community structure embeds extreme risk: weakening the community strength could abruptly drive the system to a precarious state. Examining the business-flight network among cities as a proxy for the world economy, we find this real coupled system evolving towards the extreme vulnerable phase due to ongoing globalization. This shows the community risk indeed exists in real world networks and deserves more attention from the scientific community.

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Interdependent networks are important since many real work systems have interactions at different levels, and such complexity often leads to new and rich phase transition behaviors not present in single layer networks[1–3]. Although much research has focused on the robustness to failure in multilayer interdependent networks[1,4–12], these studies have assumed that networks are unstructured. This is in sharp contrast to real-world networks that have an internal community structure[6,13–16]. For example, transportation networks have more connections within urban regions than between urban regions. A group of countries usually have more economic ties within the group than with countries outside the group. A problem of particular interest is determining how these complex systems with a rich community structure behave under localized disruption. Such community structure has been studied on single layer networks[17] to understand the impact of removing intercommunity links, and rich phase transition has been found when the number of communities changes. Random attack on intercommunity links on interdependent networks have also been found[18,19] to have even more complex transitions and scaling relations. Yet, certain attacks are localized in specific communities, like natural disasters and economic sanctions. The underlying mechanism controlling how such localized disruption in one community of a complex system disseminates throughout the entire system has not been understood, as well as the conditions that the network becomes vulnerable to abrupt collapse. Analytically, it is challenging to study since the network system with both interdependency and communities structure is highly complex.

Here, we present a generalized framework of interdependent multilayer network with community structure[1,6] based on generating functions, and study cascading processes that occur across the entire network initiated by random damage disruption in a single community. Such framework can be generalized to any layers of interdependent networks with arbitrary number of communities. Our analysis reveals that such system has rich phase transition behaviors that are much more complicated than interdependent networks without community structures. In particular, we find both theoretically and empirically that network robustness changes abruptly from safe to vulnerable as the strength of the community changes. The safe region of the system is characterized by the lack of phase transition phenomenon, i.e. the system does not disintegrate even with one whole community fully removed from the system. The vulnerable region is when phase transition is present, i.e. removing enough fraction of one community will disintegrate the whole system. Employing the business-flight network among cities of North America, Asia, and Europe as an example with strong community structure, we confirmed the presence of such risk in this interdependent system. More strikingly, this system is evolving towards the transition point from the safe to the vulnerable phase due to ongoing globalization.

Results

Theoretical analysis. We consider an interdependent multilayer network with a community structure. In a simple network consisting of two network layers A and B, each layer has the same number of communities m in which A_i and B_i are of size N_i, $i = 1,2, \ldots..m$. Every node in A_i has exactly one interdependent node in B_i, and vice versa [see Fig. 1a]. We define the generating function of A_i as

$$Q^{iA}(x) = \sum_{k=0}^{\infty} p_{iA}(k)x^k, \qquad (1)$$

where $p_{iA}(k)$ is the degree distribution of nodes in community i of layer A. We define the community structure using the distribution

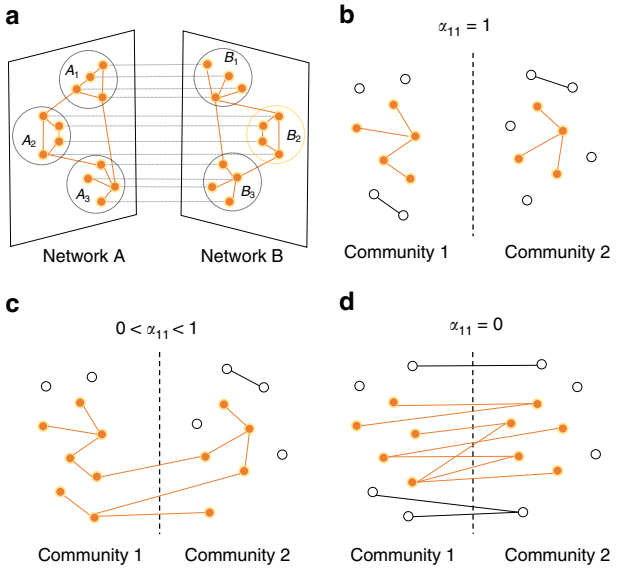

Fig. 1 Illustration of the giant components of the communities. The set of connected (orange) nodes denotes the giant component of the communities. **a** The nodes in networks A and B have one-to-one interdependence. The communities of each network also has one-to-one interdependence as well, meaning the interdependent nodes of community A_1's nodes form community B_1 in network B. **b** The giant components for $\alpha_{11} = 1$. α_{11} is the average fraction of links per node in community A_1 connected to nodes within the same community in A. The two communities are disconnected and each of them has a giant component. **c** The giant component for $0 < \alpha_{11} < 1$. The two giant components in both communities are connected through intercommunity links. **d** The giant component for $\alpha_{11} = 0$. The giant components are connected only though inter links between the two communities

of both intra- and intercommunity links[14,16]. For a community A_i within network layer A and has average degree $\langle K_{iA}\rangle$, we use $\langle k_{ijA}\rangle$ to denote the average number of intercommunity links per node in community A_i that connects to community A_j. We further let $\alpha_{ij;} = \langle k_{ijA}\rangle/\langle K_{iA}\rangle$ to be the fraction cross community links in A_i. These parameters can also be defined in terms of stub that is a link with one end from a node and the other end not connected yet. Then in the network formation process one make α_{ii} fraction of stubs in community A_i to be connected to other stubs in community A_i, and connect α_{ij} fraction of stubs in community A_i to stubs in community A_j. β_{ii} and β_{ij} are defined similarly for community B.

The generating functions for communities A_i and B_i are $G^{iA}(\xi_1, \xi_2, \ldots, \xi_m) = \sum P_{iA}(k_1, k_2, \ldots, k_m)\xi_1^{k_1}\ldots\xi_m^{k_m}$ and $G^{iB}(\zeta_1, \zeta_2, \ldots, \zeta_m) = \sum P_{iB}(k_1, k_2, \ldots, k_m)\zeta_1^{k_1}\ldots\zeta_m^{k_m}$, respectively[20]. $P_{iA}(k_1, k_2, \ldots, k_m)$ is the probability of finding a node in community A_i with k_1 links connecting to nodes in community 1, k_2 links connecting to nodes in community 2,…, and k_m links connecting nodes to community m. $P_{iB}(k_1, k_2, \ldots, k_m)$ is defined analogously for network layer B. These generating functions can be expressed by substituting x in Eq. (1) with $(\xi_1, \xi_2, \ldots, \xi_m)$ (derivation provided in Methods),

$$\begin{aligned} G^{iA}(\xi_1, \xi_2, \ldots, \xi_m) &= Q^{iA}\left(\sum_{j=1}^{m} \alpha_{ij}\xi_j\right), \\ G^{iB}(\zeta_1, \zeta_2, \ldots, \zeta_m) &= Q^{iB}\left(\sum_{j=1}^{m} \beta_{ij}\zeta_j\right). \end{aligned} \qquad (2)$$

We next attack community A in one of the network layers (say A_i) by initially removing a fraction of $1-p_i$ nodes in community A_i. In interdependent networks, it is usually assumed, based on percolation theory, that nodes become non-functional if they become disconnected to the network giant component[1,5,21–25]. That is all nodes outside the giant component in network A are removed, and so are their interdependent nodes in network B. This cascading process continues until no more nodes can be removed from the system. We finally obtain the mutually connected giant component of the remaining functional nodes.

As we remove nodes and links during the cascading process, the entire network breaks down into the components connected through inter- and intra-community links. The components in community A_i comprises the set of A_i nodes belonging to a single percolation component of the entire network. The largest of these components is the giant component of A_i, and only nodes of the giant component continue to function. Figure 1b–d shows the various types of giant component in multicommunity networks. Note that some nodes are not linked to nodes in the same community but to nodes in the other community.

At the end of the cascading process, the remaining size of A_i and B_i is

$$\mu_i^{\infty} = p_i g_i^{\infty} h_i^{\infty}, \quad (3)$$

where $g_i^{\infty} = 1 - G^{iA}(f_1^{\infty}, f_2^{\infty}, ..., f_m^{\infty})$ and $h_i^{\infty} = 1 - G^{iB}(f_1^{\infty}, f_2^{\infty}, ..., f_m^{\infty})$ are the giant component sizes of community i in network layers A and B, respectively. Here f_i^{∞} is the probability that a node in A_i along a randomly selected link is non-functional (i.e. not connected to the giant component at the steady state). It satisfies the self-consistent equation (see Methods for details).

$$f_i^{\infty} = 1 - p_i(1 - G_1^{iA}(f_1^{\infty}, ..., f_m^{\infty}))(1 - G_1^{iB}(f_1^{\infty}, ..., f_m^{\infty})), \quad (4)$$

where $G_1^{iA}(\xi_1, \xi_2, ..., \xi_m)$ is the generating function for the branching process $G_1^{iA}(\xi_1, \xi_2, ..., \xi_m) = \frac{\partial_{\xi_i} G^{iA}(\xi_1, \xi_2, ..., \xi_m)}{\partial_{\xi_i} G^{iA}(\xi_1, \xi_2, ..., \xi_m)|_{\xi_i \equiv 1}}$. $G_1^{iB}(\zeta_1, \zeta_2, ..., \zeta_m)$ is defined analogously for community B_i. For example, for a pair of Erdös–Renyi networks[26], Eq. (3) reduces to a simple form, the derivation of which is provided in Supplementary Note 1. The internal structure of our network model is a generalization of a model[1] in which networks have no internal community structure.

To demonstrate critical phenomena in our network model, we consider a simple case of two equally sized communities that are symmetrical in each layer of the network[27], i.e., $m = 2$. Without loss of generality, we attack community 1 at the initial stage, i.e., $p_1 < 1$ and $p_2 = 1$. We also set $\alpha_{11} = \alpha_{22}$, $\beta_{11} = \beta_{22}$, $\langle K_A \rangle \equiv \langle K_{1A} \rangle = \langle K_{2A} \rangle$, and $\langle K_B \rangle \equiv \langle K_{1B} \rangle = \langle K_{2B} \rangle$. Here the set of parameters α_{11} and β_{11} is sufficient to describe the community structure. A key parameter that quantifies the robustness of the system is the size of the critical value p_{1c} that describes the threshold below which the entire system disintegrates with no remaining functional giant component. Thus the smaller the p_{1c} value, the less vulnerable the network, implying that when $p_{1c} = 0$ the network is perfectly robust.

Our first main finding is that a stronger community structure does not always increase the robustness of the interdependent networks, a phenomenon significantly different from the one found in single networks. In single networks, the stronger community structure always increases robustness as shown in Fig. 2a (see Supplementary Note 2 for derivation). Differently, Fig. 2b shows the critical point p_{1c} against the community strength α_{11} of network layer A when the community strength β_{11} of network layer B is fixed. The average degree is fixed at $\langle K_A \rangle = \langle K_B \rangle = 4$ in both network layers. When the community structure is strong in layer B—which occurs when $\beta_{11} > 0.436$—there is a monotonic dependence of p_{1c} on α_{11}, but when the community structure in B is relatively weak (i.e., when $\beta_{11} < 0.436$) the behavior of p_{1c} is non-monotonic.

In order to better understand this, we draw the contour of p_{1c} with respect to the changes of community structures strength in Fig. 2c. Figure 2c shows that when $\beta_{11} < 0.436$ the contour of p_{1c} is the bulging of the equipotential line, specified by the constant value of p_{1c}, cuts the horizontal line characterized by constant β_{11} value twice, resulting in non-monotonous changes as the parameter α_{11} changes. Besides this, we find that that system robustness falls into one of the three regions (see Fig. 2c): "vulnerable" $(p_{1c} > p_{1c}^r)$, "robust" $(p_{1c} < p_{1c}^r)$, and "safe" $(p_{1c} = 0$, which means that even after the removal of all nodes in a community, there still exist a giant component of the system) regions with the increase of community structure strength in both two network layers A and B. Here p_{1c}^r is the critical point of the corresponding interdependent networks without any community structures ($\alpha_{11} = \beta_{11} = 0.5$).

We also note that the contour lines intersect at two interesting symmetrical points: (α_{11}, β_{11}) equals to (0.436, 1) and (1, 0.436) (see Fig. 2c and Supplementary Note 3 for deviation), which implies p_{1c} changes abruptly at these two unusual points. $\alpha_{11} = 1$ or $\beta_{11} = 1$ means network A or B has two disconnected (localized) communities. This is a good approximation of real-world network segmentations which could be either geographical or political/economic imposed by embargo/sanctions. For instance, the world-wide business-flight network, which we examine below, has $\beta_{11} = 0.98$. We find that when one network layer has disconnected communities, as the community strength in another network layer weakens, the critical point suddenly jumps from 0 to a finite number, which is our second main finding. This abrupt jump of the critical point value is a first-order phase transition in which a small change in community strength dramatically increases structural risk (see Fig. 2d and Supplementary Note 4 for derivation).

Empirical implications. An example of a multilayered community structure network is the network of global cities. We examine the data from the system of North American, European, and Asian cities—three different communities—in which transportation and business connections among them define two layers of network in the system. As expected, there are more connections among cities located on the same continent than that among cities located on different continents[28]. We collect business and flight data for 145 North American cities, 158 Asian cities, and 334 European cities and for companies across 21 major industrial sectors in 2010 and 2013 (see Supplementary Note 10 for data description)[29,30]. We use the data to construct an interdependent network of business and flight connections among these cities, where cities are nodes and business and flight connections are links between the cities. In the business network layer (Fig. 3a), a connectivity link between two cities is formed when at least 10 pairs of companies have business connections with each other. In the flight network layer (Fig. 3b), a link is formed when there are at least 200,000 passenger trips between the two cities annually. Simulations demonstrate that the results are not sensitive to these two threshold values (see Supplementary Fig. 5).

The business and transportation network layers are interdependent because businessmen must travel to conduct their business, and airport of one city also dependents on the companies of this city. Usually, it is not easy to obtain the interdependency relationships on important infrastructure networks, thus we begin by assuming a single interdependency link[1]

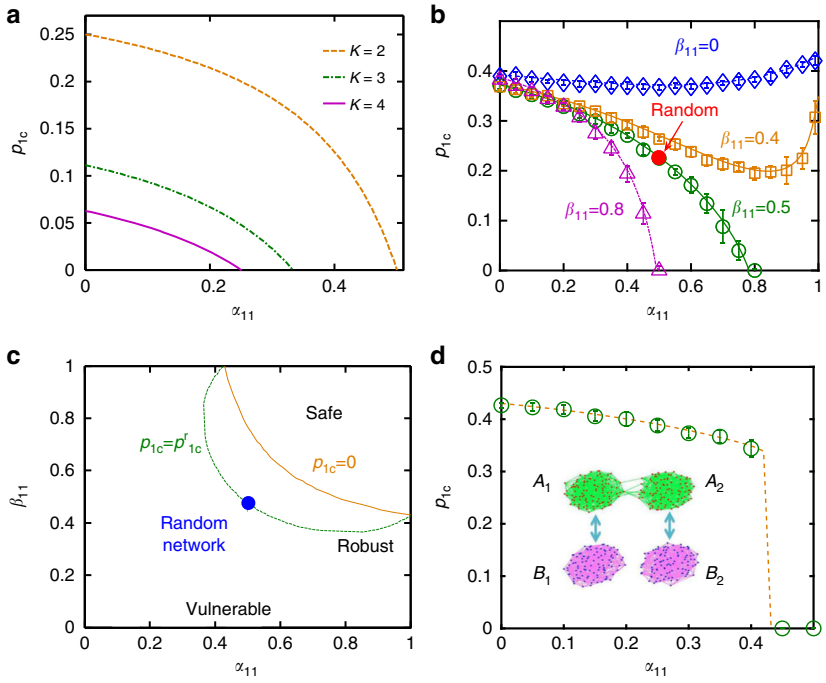

Fig. 2 Critical behavior of network(s) with community structure. **a** Critical transition point p_{1c} of a single layer random network subjecting to community attack is plotted against its community strength α_{11}. As the positive community structure strengthens with increasing α_{11}, p_{1c} decreases monotonically before reaching 0, after which point the two communities both can have a giant component even when the other community is removed from the network. **b** Critical transition point p_{1c} of interdependent networks subjecting to community attack is plotted against the community strength α_{11} of layer *A* for different β_{11} values of layer *B*. For large β_{11} values (0.5 and 0.8), the behavior is similar to single networks with monotonic decrease of critical threshold p_{1c} value. Whereas for small β_{11}(0 or 0.4), the change of p_{1c} is non-monotonic and does not reach the safe state of $p_{1c}=0$. The solid red circle represents random failures of an unstructured random network system which is equivalent to $\alpha_{11}=\beta_{11}=0.5$. **c** Phase diagram of p_{1c} with respect to the changes in community structures described by α_{11} and β_{11} in interdependent networks. The upper right region is the safe region, in which even if we remove a whole community, there still exist a giant component in the whole system. The robust region between boundary $p_{1c}=0$ and $p_{1c}=p'_{1c}$ represents a phase that is more robust than unstructured networks without community structures. The vulnerable region below the boundary $p_{1c}=p'_{1c}$ represents a phase that is more vulnerable than unstructured networks. In **b** and **c**, the two interdependent networks are Erdös–Rényi (ER) networks and both have two communities with equal size and equal average degree 4. **d** When one layer of the networks is completely localized ($\beta_{11}=1$), extreme community structure risk exists when the community strengths α_{11} decreases, as p_{1c} changes discontinuously from 0 to 0.34. In the system, $K=4$ for two equally sized communities. The network has a total of $N_c=10{,}000$ nodes, with each community having 5000 nodes. Each data point is an average of 10 simulations and the standard deviation is presented by the error bar

between business and transportation in the same city. We then assume the typical failure mechanism for interdependent networks in which the failure of one node leads to the failure of its interdependent node.

In numerical simulations, we first construct three interdependent networks comprising only cities in North America vs Asia, Europe vs Asia, and Europe vs North America. Secondly, altering the business network *A* but fixing the flight network *B* (β_{11} is close to 1), we tune down community strength α_{11} of business network by rewiring some of its links while keeping their node degrees unchanged (see Supplementary Note 5–8 for the technical details). This approach enables us to assess the system robustness, i.e. how p_{1c} depends on α_{11}, and for what range of parameter values does the system reside in the safe phase or the vulnerable phase. As shown in Fig. 3c–e, we see a phenomena similar to Fig. 2d that p_{1c} changes abruptly from 0 to a finite value, signaling the emergence of the community structure risk in the system. As the trend of globalization continues, we expect the community structure in the business network to become weaker. Indeed this was the case when comparing the value of community strength α_{11} value between year 2010 and 2013 in Fig. 3 for each pair of communities North America–Asia, Europe–Asia, and Europe–North America. Our results from Fig. 3c–e show that as globalization drives global economic system to be more integrated, the North American and European economies exhibit larger robustness than the Asian economy. Finally, we construct an interdependent network of business and flight connections comprising all cities in North America, Europe, and Asia and we find that the overall system is still shifting towards the abrupt change through removing nodes within the Europe community (Fig. 3f). Our results and analysis are based on communities divided according to different continents, and use them to serve as proxies to represent the coarse structure of the world economy. Using our mathematical framework, we can estimate the risk towards the unstable region given the trend from 2010 to 2013. In essence, the world economy, according to this mechanism, had been likely heading towards the unstable phase. In Supplementary Note 9 we discuss some additional weaker forms of interdependency between two network layers characterized by probabilistic interdependency links.

Discussion

In summary, we have studied the robustness of multilayer interdependent networks under community attacks, characteristic of many real system constrained by geographic locations. Through our developed framework, we find that such a system

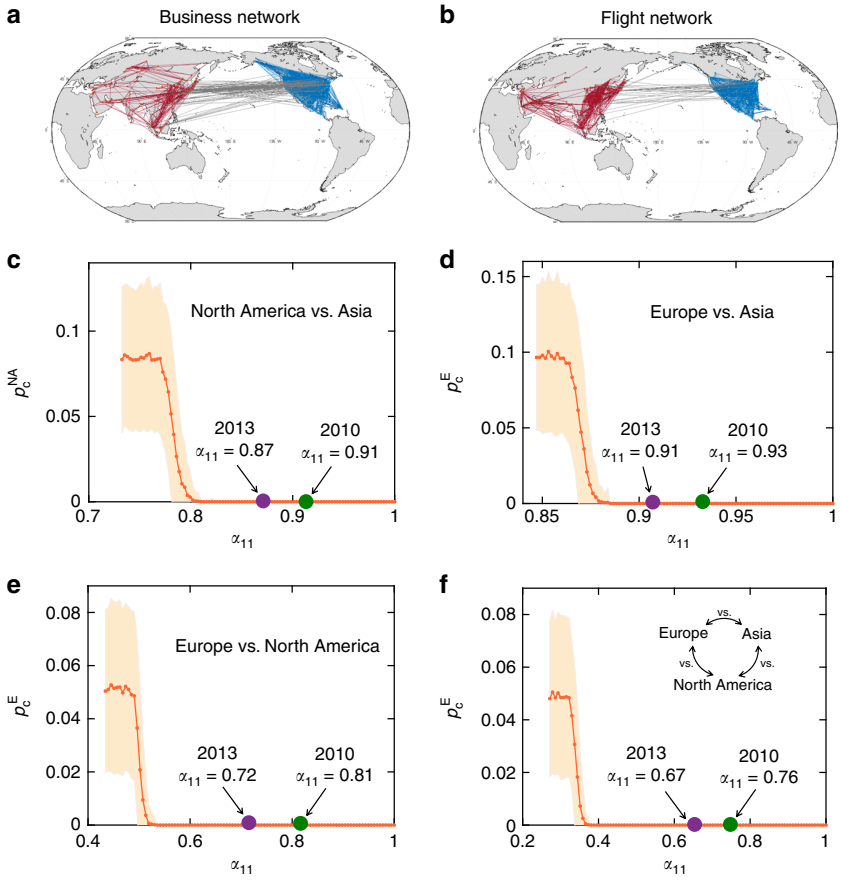

Fig. 3 Abrupt change in systemic risks with respect to community strengths. The parameters are in Table 1 and Supplementary Tables 1 and 2. **a** Firm and **b** flight networks between Asia and North America. The red nodes represent cities in Asia, and the blue nodes represent the cities in the North America. The links represent air travel activities in the flight network and business activities in the firm network. We consider the Asian cities as one community and the North America cities as another community in both networks. **c** Empirical study of the structural risks shows the same abrupt change. The two communities are cities in North America and Asia, and the two layers of networks are business interaction networks and flight networks. The critical transition point p_c^{NA} (removing nodes within the North America community) is plotted against community structure strength α_{11} of the business layer for the business-flight networks. The original value of α_{11} is the empirical value of the system in 2010 and is marked on the horizontal axis (green dot), while the new value of α_{11} is measured in 2013. Each data point is an average of 1000 simulations and the shaded region is the bounded by the standard deviation. The result shows that the whole system is shifting towards the abrupt change, where there will be extreme systemic risk. The decrease in α_{11} value is possibly due to the effect of globalization, which is weakening the community structure. **d**, **e** Similar to **c** but the results is on the two communities of Europe vs. Asia and North America. From 2010 (green dot) to 2013 (purple dot), the system has shifted from the safe phase to the critical point p_c^{E} (removing nodes within the Europe community) close to the vulnerable state. **f** Similar trend for the systemic risks can be observed in an interdependent network involving all of three communities, and the node removal is done on the Europe community. For all of the simulations in **c**–**f**, we only show the attack on one community, as the attack on the other communities does not have the abrupt change on the systemic resilience, i.e. no p_c is found.

exhibits much more complex behaviors than the ones without clear community structures. Depending on the respective community strengths of the different network layers, the whole system may reside in one of the three different regions of phase diagram, with different phase transition behaviors. In particular, we find that the system embeds extreme risk when one of the layers has strong community structure. In contrary to a single network which is always more robust with enhanced community structure, interdependent networks showing strong community structure sometimes makes the system more vulnerable. In cases of extremely strong community structure with very sparse intercommunity in one layer of network, a small change in the other layer of network's community strength could induce abrupt changes in total systemic resilience if one community is under attack. This new finding adds to the growing knowledge of resilience of interdependent networks with community structure, in particular attacks on specific communities that is representative of realistic events like natural disaster and economic embargo. From the global business-flight networks taken as a proxy for the world economy, we observe that, as globalization weakens the community structure, the entire network is approaching a state in which there is a potential for abrupt disruptions in certain communities. This also leads to the finding that Asia strongly depends on North America and Europe economies but not the other way round. Our result is indicative for a broad range of systems which have community structures defined by geographic location and physical infrastructure. While we mainly focused on the analytical results of two communities on two-layer interdependent networks, our adopted mathematical framework allows for more complex network structures, and their phase transition behaviors could be very interesting for further studies. On the empirical side, our methods can be similarly extended to study other real interdependent networks with community structures, to give better understanding of their dynamical

Table 1 Information of the communities in terms of continents

Name	North America	Asia
N	145	158
$\langle K_{iA}\rangle$ of business	4.33	1.06
$\alpha_{ii}(\alpha_{ii}^r)$ of business	0.91(0.79)	0.67(0.21)
$\langle K_{iB}\rangle$ of flight	8.77	3.20
$\beta_{ii}(\beta_{ii}^r)$ of flight	0.98(0.72)	0.95(0.28)
Threshold	Business:10	Flight:200000

Community size, average degree, and community structure index are indicated $\alpha_{ii}^r = K_{iA}N_i/(K_{1A}N_1 + K_{2A}N_2)$ is the index if the given network has no community structure and α_{ii} is the actual community structure index. β_{ii} and β_{ii}^r are the same for network B

behaviors for risk assessment, which then leads to the development of efficient risk mitigation strategies in those systems.

Methods

Multivariate generating function. In this subsection, we harness multivariate generating function to formalize the cascading process of interdependent networks with multiple communities[20,30,31]. We first construct the multivariate generating function of the interdependent networks with communities. For conciseness, we illustrate the generating function associated with the bi-community structure (a network with two communities). The result can be easily generalized to account for multicommunity systems. For bi-community system, the probability normalization requires that $\alpha_{12} = 1 - \alpha_{11}$ and $\alpha_{21} = 1 - \alpha_{22}$ and the inter-link identity requires that $N_1\cdot\langle K_{1A}\rangle\cdot\alpha_{12} = N_2\cdot\langle K_{2A}\rangle\cdot\alpha_{21}$. Therefore one can describe the structure of network A by one parameter α_{11}, because the other three parameters can be calculated from the above three relations. By the definition of α_{11}, the probability of finding a node in community 1 with intra-degree k_1 and inter-degree k_2 should be $\binom{k_1+k_2}{k_1}\alpha_{11}^{k_1}(1-\alpha_{11})^{k_2}$. Such node with degree $k_1 + k_2$ has a prior probability $p_{iA}(k_1 + k_2)$. Thus one can write the generating function of community 1 in terms of ξ_1 and ξ_2:

$$\begin{aligned} G^{1A}(\xi_1,\xi_2) &= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\binom{k_1+k_2}{k_1}\alpha_{11}^{k_1}(1-\alpha_{11})^{k_2}p_{1A}(k_1+k_2)\xi_1^{k_1}\xi_2^{k_2} \\ &= \sum_{k_1+k_2=0}^{\infty} p_{1A}(k_1+k_2)(\alpha_{11}\xi_1+(1-\alpha_{11})\xi_2)^{k_1+k_2} \\ &= \sum_{k=0}^{\infty} p_{1A}(k)(\alpha_{11}\xi_1+(1-\alpha_{11})\xi_2)^{k} \\ &= \sum_{k=0}^{\infty} p_{1A}(k)(\alpha_{11}\xi_1+\alpha_{12}\xi_2)^{k} \\ &= = Q^{1A}\left(\sum_j \alpha_{1j}\xi_j\right) \end{aligned} \quad . \tag{5}$$

The prefactor of $\xi_1^{k_1}\xi_2^{k_2}$ represents the probability of finding a node in community 1 of network A with intra-degree k_1 and inter-degree k_2. By a similar derivation, one can write out the generating function for community i in multicommunity network A and B as

$$\begin{aligned} G^{iA}(\xi_1,\xi_2,..,\xi_m) &= Q^{iA}\left(\sum_j \alpha_{ij}\xi_j\right), \\ G^{iB}(\zeta_1,\zeta_2,..,\zeta_m) &= Q^{iB}\left(\sum_j \beta_{ij}\zeta_j\right). \end{aligned} \tag{6}$$

Percolation equation. Here we show the percolation equation in the form of generating functions[32]. Initially a fraction $(1 - p_i)$ of nodes from community i are randomly removed. The initial removal is followed by a cascading process of percolation failures of the rest nodes. Recall a single complex network with generating function $G(x)$ and branching process $G_1(x) = G'(x)/G'(1)$ (ref. [33]), it is known that after a random removal of $1 - p$ nodes, the generating function of the degree distribution of the remaining nodes can be written by $G^{\text{remain}}(x) = G(1 - p(1 - x))$. At step n, say the size of the giant component of community i in layer A and B due to percolation failures are g_i^n and h_i^n. Analogously, we introduce u_i^n and v_i^n as the probability of finding a non-functional node by following a link of a randomly chosen node in community i of network A and B at step n after the initial removing of $1-p_i$ fraction of nodes from each community, respectively. We define f_i^n by

$$f_i^n = 1 - p_i(1 - u_i^n)(1 - v_i^n). \tag{7}$$

The physical meaning of f_i^n is the probability of finding a node along a randomly selected link in community i of the original network to be non-functional at step n. So the fraction of nodes that are still functional at step $n + 1$ is

$$g_i^{n+1} = 1 - G^{iA}(f_1^n, f_2^n, ..., f_m^n), \tag{8}$$

or equivalently

$$\begin{aligned} g_i^{n+1} = 1 - G^{iA}&(1 - p_1(1 - u_1^n)(1 - v_1^n), 1 - p_2(1 - u_2^n)(1 - v_2^n), ..., \\ &1 - p_m(1 - u_m^n)(1 - v_m^n)). \end{aligned} \tag{9}$$

u_i^n should obey a transcendental equation[3]

$$\begin{aligned} u_i^n = G_1^{iA}&(1 - p_1(1 - u_1^n)(1 - v_1^n), 1 - p_2(1 - u_2^n)(1 - v_2^n), ..., \\ &1 - p_m(1 - u_m^n)(1 - v_m^n)), \end{aligned} \tag{10}$$

in which $G_1^{iA}(u_1^n, u_2^n, ..., u_m^n)$ is the generating function associated with the branching process:

$$\begin{aligned} G_1^{iA}(\xi_1,\xi_2,..,\xi_m) &= \frac{\partial_{\xi_i}Q^{iA}\left(\sum_j \alpha_{ij}\xi_j\right)}{\partial_{\xi_i}Q^{iA}\left(\sum_j \alpha_{ij}\xi_j\right)|_{\xi_1=\xi_2=...=\xi_m=1}} \\ &= \sum_{k_1,k_2,\cdots,k_m}\left[\frac{P_{iA}(k_1,k_2,\cdots k_m)k_i}{\langle k_i\rangle}(\alpha_{ii}\xi_i)^{k_i-1}\prod_{j=1,\cdots,m}^{j\neq i}(\alpha_{ij}\xi_j)^{k_j}\right]. \end{aligned} \tag{11}$$

Similarly, the fraction of functional nodes h_i^{n+1} in community i in network B at step $n,+1$ is

$$\begin{aligned} h_i^{n+1} = 1 - G^{iB}&(1 - p_1(1 - u_1^n)(1 - v_1^n), 1 - p_2(1 - u_2^n)(1 - v_2^n), ..., \\ &1 - p_m(1 - u_m^n)(1 - v_m^n)), \end{aligned} \tag{12}$$

in which v_i^n is the probability of finding functional node by following a link of a randomly chosen node in community i of network B. It satisfies the following transcendental equation:

$$\begin{aligned} v_i^n = G_1^{iB}&(1 - p_1(1 - u_1^n)(1 - v_1^n), 1 - p_2(1 - u_2^n)(1 - v_2^n), ..., \\ &1 - p_m(1 - u_m^n)(1 - v_m^n)), \end{aligned} \tag{13}$$

where $G_1^{iB}(v_1^n, v_2^n, ..., v_m^n)$ is the generating function related to the branching process:

$$\begin{aligned} G_1^{iB}(\zeta_1,\zeta_2,..,\zeta_m) &= \frac{\partial_{\zeta_i}Q^{iB}\left(\sum_j \beta_{ij}\zeta_j\right)}{\partial_{\zeta_i}Q^{iB}\left(\sum_j \beta_{ij}\zeta_j\right)|_{\zeta_1=\zeta_2=...=\zeta_m=1}} \\ &= \sum_{k_1,k_2,\cdots,k_m}\left[\frac{P_{iB}(k_1,k_2,\cdots k_m)k_i}{\langle k_i\rangle}(\beta_{ii}\zeta_i)^{k_i-1}\prod_{j=1,\cdots,m}^{j\neq i}(\beta_{ij}\zeta_j)^{k_j}\right] \end{aligned} \tag{14}$$

At equilibrium when no more nodes can be removed due to percolation and interdependency failures, the above important parameters u_i^∞ and v_i^∞ should obey the following self-consistent equations:

$$\begin{aligned} u_i^\infty &= G_1^{iA}(1 - p_1(1 - u_1^\infty)(1 - v_1^\infty), 1 - p_2(1 - u_2^\infty)(1 - v_2^\infty), ..., 1 - p_m(1 - u_m^\infty)(1 - v_m^\infty)), \\ v_i^\infty &= G_1^{iB}(1 - p_1(1 - u_1^\infty)(1 - v_1^\infty), 1 - p_2(1 - u_2^\infty)(1 - v_2^\infty), ..., 1 - p_m(1 - u_m^\infty)(1 - v_m^\infty)), \end{aligned} \tag{15}$$

and the remaining size of i'th community μ_i^∞ becomes

$$\begin{aligned} \mu_i^\infty = p_i g_i^\infty h_i^\infty = p_i\big[1 - G^{iA}&\big(1 - p_1(1 - u_1^\infty)(1 - v_1^\infty), \\ &1 - p_2(1 - u_2^\infty)(1 - v_2^\infty), ..., 1 - p_m(1 - u_m^\infty)(1 - v_m^\infty)\big)\big] \\ \cdot\big[1 - G^{iB}&\big(1 - p_1(1 - u_1^\infty)(1 - v_1^\infty), \\ &1 - p_2(1 - u_2^\infty)(1 - v_2^\infty), ..., 1 - p_m(1 - u_m^\infty)(1 - v_m^\infty)\big)\big], \end{aligned} \tag{16}$$

$$i = 1, 2, ..., m$$

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Received: 13 September 2018 Accepted: 11 March 2019
Published online: 08 May 2019

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Acknowledgements

This work is supported by the National Nature Science Foundation of China (Nos. U1711265, 61773412, 61771499, and 91438101 and 61473321), Guangzhou Science and Technology Project (No. 201804010473), Three Big Constructions-Supercomputing Application Cultivation Projects sponsored by National Supercomputer Center in Guangzhou, and in part by Academic Research Fund (AcRF) R-144-000-415-114 National University of Singapore. Boris Podobnik received the support from Slovenian Research Agency (ARRS) via Project J5-8236 and from the University of Rijeka. Céline Rozenblat received the support from FP7 FET Multiplex.

Author contributions

J.S., R.Z., and L.F performed experiments and numerical modeling described in this manuscript. C.M., X.M., H.E.S., and B.P. discussed and analyzed the results. C.R. provided the data. Y.H. provided basic idea and supervised the project.

Additional information

Supplementary information accompanies this paper at https://doi.org/10.1038/s42005-019-0144-6.

Competing interests: The authors declare no competing interests.

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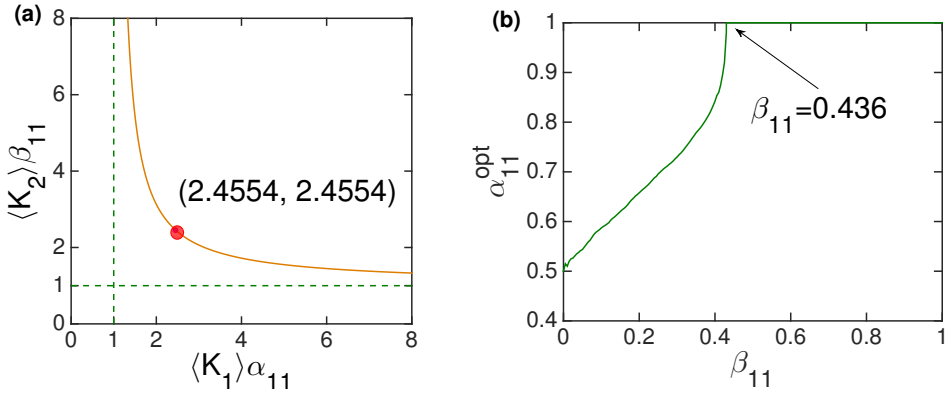
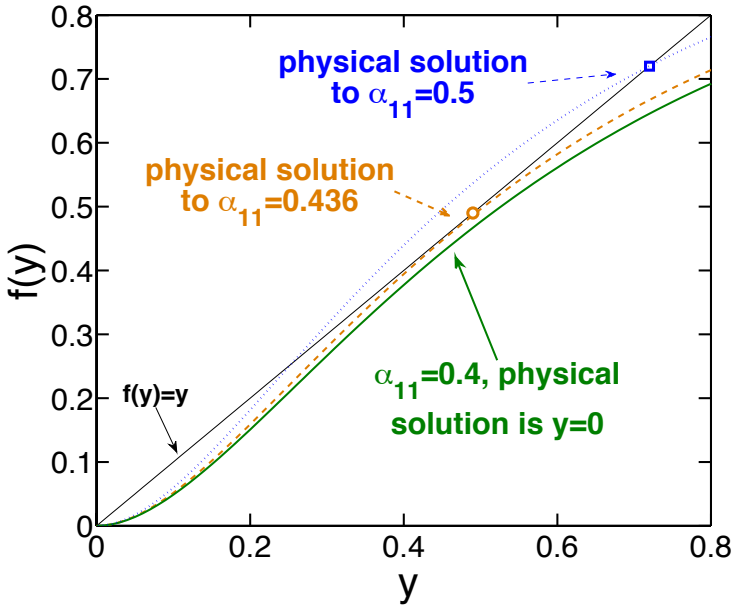
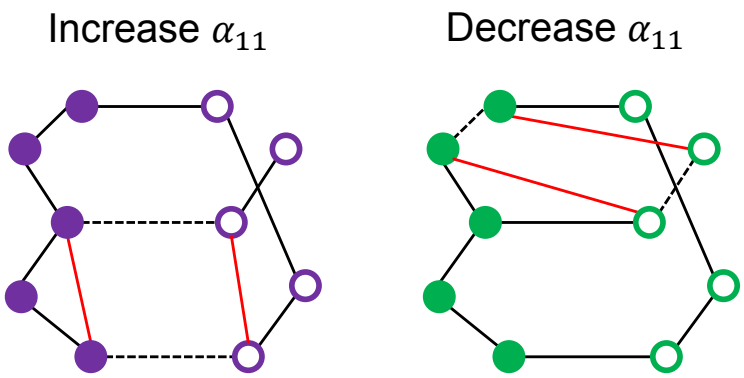
Supplementary Figures



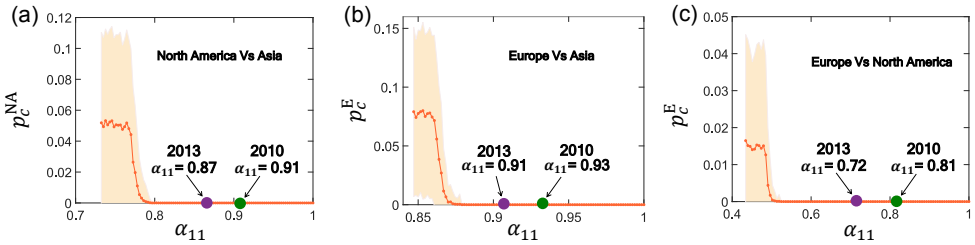
Supplementary Figure 1. Criticality of interdependent networks with communities. Here $\langle K_{iA}\rangle = \langle K_{iB}\rangle = K_i$, $i = 1, 2$ and $N_1 = N_2$. Here $\langle K_1\rangle = \langle K_2\rangle = 4$. (a) Universal boundary between Safe region and Robust region in terms of $(\langle K_1\rangle\alpha_{11}, \langle K_2\rangle\beta_{11})$; Dashed are their asymptotic lines; Under this boundary, when we remove a full community, the other community will be fragmented. Above the boundary, when we remove a full community, the other community can survival with a mutual giant component at the equilibrium state. (b) Optimum α_{11}^{opt} vs β_{11}.The optimized α_{11}^{opt} means that it can maximize the percolation robustness (minimizing p_{1c}). When $\beta_{11} < 0.436$, α_{11}^{opt} increases from 0.5 to 1 as β_{11} increases. When $\beta_{11} \geq 0.436$, α_{11}^{opt} remains 1.



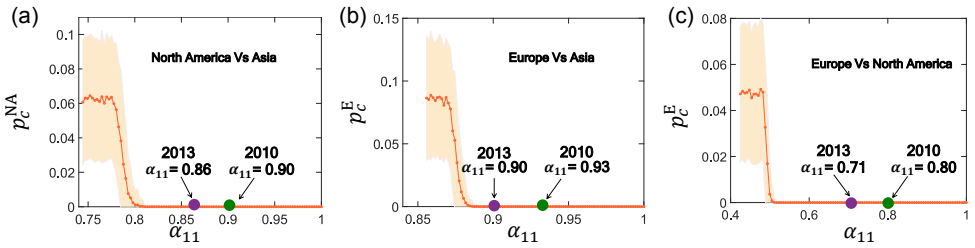
Supplementary Figure 2. Illustration of the physical solution of Eq. 5 at $\langle K_{2A}\rangle = \langle K_{2B}\rangle = 4$. $f(y) = [1-\exp(-\alpha_{11}\langle K_{2A}\rangle y)][1-\exp(-\langle K_{2B}\rangle y)]$ is the right hand side of Eq. 5; $f(y) = y$ is the left hand side of Eq. 5. The three curves correspond to three different α_{11}'s. Physical solutions (markers in the figure) are the intersections of these curves with $f(y) = y$. $\alpha_{11} = 0.436$ is the critical point. If $\alpha_{11} > 0.436$, there exists two nontrivial solutions (the higher one is the physical solution), community 2 survives; If $\alpha_{11} < 0.436$, there is no nontrivial solution, community 2 collapses, which leads to finite transition point p_{1c}.



Supplementary Figure 3. Illustration of the rules of varying α_{11}. Closed circles denote community 1 and open circles denote community 2. To increase α_{11}, each time one intra link from each community is randomly selected (denoted by the dashed lines). Then they are replaced by two inter-links denoted by the thick (red online) lines. The process repeats until α_{11} reaches the target value. To decrease α_{11}. Each time two inter links between these two communities are selected (denoted by the dashed lines). They are then replaced by two intra links (denoted by thick lines (red online)) connecting nodes of the same community. The above process repeats until α_{11} reaches expected value. This way can keep the original network structure as much as possible. Note that, if the two communities are of different sizes or average degrees, the maximum α_{11} can be 1, but the minimum α_{11} may not reach 0.



Supplementary Figure 4. Abrupt change of p_{1c} vs α_{11} under $Q = 0.9$. Threshold weights: business network is 10 and flights network is 200000 which are same with Fig.3 in the main text. We only present the attack on the first community which is North America of (a), Europe for (b) and (c). For the attack on the second community in respective plots, the system does not disintegrate with when the whole of second community is removed, leading to $p_{2c} = 0$ regardless of α_{11} values, and it is not plotted here.



Supplementary Figure 5. Same simulation as the previous figure but with different Weights threshold for network construction: Business network is 5 and flights network is 200000. We also can see the abrupt changes of the system robustness.

Supplementary Tables

Name	Europe	Asia
N	334	158
$\langle K_{iA} \rangle$ of Business	3.97	1.28
$\alpha_{ii}(\alpha_{ii}^r)$ of Business	0.93(0.87)	0.74(0.13)
$\langle K_{iB} \rangle$ of Flight	1.97	3.43
$\beta_{ii}(\beta_{ii}^r)$ of Flight	0.91(0.55)	0.83(0.45)
Threshold	Business:10 Flight:200000	

Supplementary Table 1. Information of the communities in terms of continents (Europe vs Asia). The parameters are defined analogously as Tab.1 in main text.

Name	Europe	North America
N	334	145
$\langle K_{iA} \rangle$ of Business	4.59	5.97
$\alpha_{ii}(\alpha_{ii}^r)$ of Business	0.81(0.64)	0.66(0.36)
$\langle K_{iB} \rangle$ of Flight	2.00	9.10
$\beta_{ii}(\beta_{ii}^r)$ of Flight	0.89(0.34)	0.95(0.66)
Threshold	Business:10 Flight:200000	

Supplementary Table 2. Information of the communities in terms of continents (Europe vs North America). The parameters are defined analogously as Tab.1 in main text.

Supplementary Note 1 Interdependent Erdős-Rényi networks

Here we show the percolation theory for Erdős-Rényi (ER) networks. For simplicity we assume $N_1 = N_2$. The generating functions for ER networks with two equal communities can be analytically expressed by the average

degrees:

$$G^{iA}(\xi_1,\xi_2)=G_1^{iA}(\xi_1,\xi_2)=\exp[\alpha_{i1}\langle K_{iA}\rangle(\xi_1-1)+\alpha_{i2}\langle K_{iA}\rangle(\xi_2-1)],$$

$$G^{iB}(\zeta_1,\zeta_2)=G_1^{iB}[\zeta_1,\zeta_2)=\exp(\beta_{i1}\langle K_{iB}\rangle(\zeta_1-1)+\beta_{i2}\langle K_{iB}\rangle(\zeta_2-1)].$$

By substituting the above equations into Eq.(14) in the main text, the self-consistent equations reduce to

$$\begin{aligned}
u_1^\infty &= \exp[-\alpha_{11}p_1\langle K_{1A}\rangle(u_1^\infty-1)(v_1^\infty-1)-(1-\alpha_{11})p_2\langle K_{1A}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
u_2^\infty &= \exp[-(1-\alpha_{22})p_1\langle K_{2A}\rangle(u_1^\infty-1)(v_1^\infty-1)-\alpha_{22}p_2\langle K_{2A}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
v_1^\infty &= \exp[-\beta_{11}p_1\langle K_{1B}\rangle(u_1^\infty-1)(v_1^\infty-1)-(1-\beta_{11})p_2\langle K_{1B}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
v_2^\infty &= \exp[-(1-\beta_{22})p_1\langle K_{2B}\rangle(u_1^\infty-1)(v_1^\infty-1)-\beta_{22}p_2\langle K_{2B}\rangle(u_2^\infty-1)(v_2^\infty-1)],
\end{aligned} \tag{1}$$

and the final size of the network becomes

$$\begin{aligned}
\mu_i^\infty =& p_i(1-G^{iA}(1-p_1(1-u_1^\infty)(1-v_1^\infty),1-p_2(1-u_2^\infty)(1-v_2^\infty)))\\
&\cdot(1-G^{iB}(1-p_1(1-u_1^\infty)(1-v_1^\infty),1-p_2(1-u_2^\infty)(1-v_2^\infty)))=p_i(1-u_i^\infty)(1-v_i^\infty).
\end{aligned}$$

The relevant variable is μ_i^∞, therefore substituting Eq. 2 into the above equation yields

$$\begin{aligned}
\mu_1^\infty &= p_1\{1-\exp[-\alpha_{11}\langle K_{1A}\rangle\mu_1^\infty-(1-\alpha_{11})\langle K_{1A}\rangle\mu_2^\infty]\}\{1-\exp(-\beta_{11}\langle K_{1B}\rangle\mu_1^\infty-(1-\beta_{11})\langle K_{1B}\rangle\mu_2^\infty]\},\\
\mu_2^\infty &= p_2\{1-\exp(-(1-\alpha_{11})\langle K_{2A}\rangle\mu_1^\infty-\alpha_{11}\langle K_{2A}\rangle\mu_2^\infty]\}\{1-\exp(-(1-\beta_{11})\langle K_{2B}\rangle\mu_1^\infty-\beta_{11}\langle K_{2B}\rangle\mu_2^\infty]\}.
\end{aligned} \tag{2}$$

Because each layer of networks have two equal communities, thus we can use the fact that $\alpha_{11}=\alpha_{22}$ and $\beta_{11}=\beta_{22}$,$N_1=N_2$, $\langle K_{1A}\rangle=\langle K_{2A}\rangle=K_A$ and $\langle K_{1B}\rangle=\langle K_{2B}\rangle=K_B$. For unbiased attack, $p_1=p_2=p$, the above expressions for μ_1^∞ and μ_2^∞ can be written as the following form:

$$\mu_1^\infty=\mu_2^\infty=p[1-\exp(-K_A\mu_1^\infty)][1-\exp(-K_B\mu_1^\infty)].$$

One can see this equation is independent of the community structure, which implies that the percolation failures of interdependent ER networks with two equal communities is independent of the community structure.

Supplementary Note 2 Single networks with community structure

Our percolation model can be applied to single networks with community structure by removing terms that are associated with network B. The self consistency equations are simplified to

$$u_i^\infty=G_1^{iA}(1-p_1(1-u_1^\infty),1-p_2(1-u_2^\infty),...,1-p_m(1-u_m^\infty))$$

and the remaining size of i'th community μ_i^∞ becomes

$$\mu_i^\infty = p_i g_i^\infty = p_i[1 - G^{iA}(1 - p_1(1 - u_1^\infty), 1 - p_2(1 - u_2^\infty), ..., 1 - p_m(1 - u_m^\infty))],$$

with $i = 1, 2, ..., m$. If we consider ER networks, the above self consistency equations can be written as

$$u_1 = \exp[\alpha_{11} p_1 \langle K_{1A}\rangle (u_1 - 1) + (1 - \alpha_{11}) p_2 \langle K_{1A}\rangle (u_2 - 1)],$$

$$u_2 = \exp[(1 - \alpha_{22}) p_1 \langle K_{2A}\rangle (u_1 - 1) + \alpha_{22} p_2 \langle K_{2A}\rangle (u_2 - 1)],$$

and the remaining giant component sizes reduce to

$$\mu_1^\infty = p_1\{1 - \exp[-\alpha_{11}\langle K_{1A}\rangle \mu_1^\infty - (1 - \alpha_{11})\langle K_{1A}\rangle \mu_2^\infty]\},$$

$$\mu_2^\infty = p_2\{1 - \exp(-(1 - \alpha_{11})\langle K_{2A}\rangle \mu_1^\infty - \alpha_{11}\langle K_{2A}\rangle \mu_2^\infty]\}.$$

with $i = 1, 2, ..., m$, corresponding to the m communities in the network A. For community attack one has $p_1 = p$ and $p_2 = 1$. For community attack, it can be shown that the transition point p_{1c} satisfies

$$p_{1c} = \frac{\langle k_{22}\rangle - 1}{\langle k_{11}\rangle\langle k_{22}\rangle - \langle k_{12}\rangle\langle k_{21}\rangle - \langle k_{11}\rangle}.$$

We can see that, if $p_{1c} = 0$, it implies that $\langle k_{22}\rangle \leq 1$. This allow us got $a_{22} \leq \frac{1}{K_{2A}}$.

Supplementary Note 3 Safe and robust boundaries on ER network systems

For community attack, say we only attack community 1, i.e. $p_1 = p$ and $p_2 = 1$. Now we consider the safe-unsafe transition. At the transition point, the complete failure of community 1 leads to the failure of community 2. Within the safe region, the removal of community 1 does not lead to the failure of community 2. In other words, whether or not community 2 can self survive determines the safe-unsafe transition boundary. Thus we set $\mu_1^\infty = 0$, the second self-consistent equation becomes

$$\mu_2^\infty = (1 - e^{-\alpha_{11}\langle K_{2A}\rangle \mu_2^\infty})(1 - e^{-\beta_{11}\langle K_{2B}\rangle \mu_2^\infty}).$$

We have $\langle k_{22A}\rangle = \alpha_{11}\langle K_{2A}\rangle$ and $\langle k_{22B}\rangle = \beta_{11}\langle K_{2B}\rangle$, and let $x = \mu_2^\infty$, thus

$$x = (1 - e^{-\langle k_{22A}\rangle x})(1 - e^{-\langle k_{22B}\rangle x}).$$

$x = 0$ is the solution to the critical point. If the system is at transition from safe (which has nonzero solution of x) to unsafe region (only zero solution of x exists), one expects the derivatives of the two sides of the above equation should equal:

$$1 = \partial_x \left[(1 - e^{-\langle k_{22A} \rangle x})(1 - e^{-\langle k_{22B} \rangle x}) \right].$$

By solving the above equations, one can obtain the universal boundary between safe and unsafe regions. The physical meaning of the above equations are as follow: If community 2 can survive from the full failures of community 1, the intra-links in community 2 can prevent the system from spontaneous collapse. The symmetric solution of the above equations is $\langle k_{22A} \rangle = \langle k_{22B} \rangle = 2.4554$, which is the critical average degree (two layers have equal average degree) for an unstructured interdependent network to be stable[1].

To obtain robust-vulnerable boundary, we let $p_1 = p_{1c}^r$ and $p_2 = 1$ in Eq. 2. p_{1c}^r is the critical transition point for a unstructured network under random attack. For conciseness, let $x = \mu_1^\infty$ and $y = \mu_2^\infty$. To find solution to Eq. 2 graphically, one can plot the two equations in terms of two curves $x(y)$ and $y(x)$, the cross points of which are the solutions. At the critical point $p_1 = p_{1c}^r$ and $p_2 = 1$, we have nontrivial (nonzero) solutions to x and y. When $p_1 < p_{1c}^r$, the system's giant component size drops to 0, or equivalently we have only trivial solutions $x = 0$ and $y = 0$. Therefore we should expect that at the critical point the two curves are tangential to each other, i.e. $\frac{\partial x}{\partial y}\frac{\partial y}{\partial x} = 1$[2]:

$$\begin{aligned} \frac{\partial x}{\partial y}\frac{\partial y}{\partial x} =& \frac{\partial}{\partial y}[-p_{1c}^r(1 - e^{-\alpha_{11} x \langle K_{1A} \rangle - (1-\alpha_{11}) y \langle K_{1A} \rangle})(1 - e^{-\beta_{11} x \langle K_{1B} \rangle - (1-\beta_{11}) y \langle K_{1B} \rangle})] \\ & \cdot \frac{\partial}{\partial x}[-(1 - e^{-\alpha_{11} y \langle K_{2A} \rangle - (1-\alpha_{11}) x \langle K_{2A} \rangle})(1 - e^{-\beta_{11} y \langle K_{2B} \rangle - (1-\beta_{11}) x \langle K_{2B} \rangle})] = 1. \end{aligned} \tag{3}$$

Therefore we have 3 equations with 4 variables x, y, α_{11} and β_{11}. By eliminating x and y from the above 3 equations, we obtain an equation that describes the robust-vulnerable boundary curve in terms of α_{11} and β_{11}. The boundary curve is numerically solved and shown in Supplementary Figure 1.

Supplementary Note 4 Discontinuous transition from safe to unsafe region

Consider community attack (to community 1) and network B being localized, one has $p_1 < 1, p_2 = 1$ and $\beta_{11} = 1$. We look for the transition point p_{1c} when the whole system fails. Eqs. 2 reduce to

$$\begin{aligned} x &= p_1[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x - (1-\alpha_{11})\langle K_{1A}\rangle y)][1 - \exp(-\langle K_{1B}\rangle x)], \\ y &= [1 - \exp(-(1-\alpha_{11})\langle K_{2A}\rangle x - \alpha_{11}\langle K_{2A}\rangle y)][1 - \exp(-\langle K_{2B}\rangle y)], \end{aligned} \tag{4}$$

in which we use x and y to denote μ_1^∞ and μ_2^∞ respectively, for conciseness' sake. When community 1 completely fails, one has $x = 0$ and the second equation above further reduces to

$$y = [1 - \exp(-\alpha_{11}\langle K_{2A}\rangle y)][1 - \exp(-\langle K_{2B}\rangle y)]. \tag{5}$$

If a nontrivial solution exists in the above equation, i.e. there exists an solution with $y > 0$, community 2 can survive from the failures of community 1, thus transition point p_{1c} is indefinite (does not exist); otherwise, if $y = 0$ is the only solution to Eq. 5, p_{1c} is finite. By tuning community structure index α_{11}, Eq. 5 may or may not have a nontrivial solution. Therefore the transition from finite p_{1c} to indefinite p_{1c} is denoted by α_{11c}. If $\alpha_{11} < \alpha_{11c}$, the above equation has no nontrivial solution, community 2 fails; If $\alpha_{11} > \alpha_{11c}$, the nontrivial solution leads to the survival of community 2. When $\langle K_{2A}\rangle = \langle K_{2B}\rangle$, the critical $\alpha_{11c} = 0.436$. Illustrated in Supplementary Figure 2, one can visualize how the nontrivial solution emerges with α_{11} increases. At safe-unsafe boundary, the failures of the two communities occur simultaneously. So we assume $y = 0$, Eq. 4 becomes

$$x = p_1[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)], \tag{6}$$

critical p_{1c} requires that the derivatives of the two sides in the above equation should equal, therefore one has

$$\begin{aligned} x &= p_{1c}[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)], \\ 1 &= p_{1c}\frac{\partial}{\partial x}\{[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)]\}, \end{aligned}$$

The above p_{1c} is finite. If $\langle K_{1A}\rangle = \langle K_{1B}\rangle = 4$ and $\alpha_{11} = \alpha_{11c}$, $p_{1c} = 0.34$. Eq. 6 is similar to Eq. 5 except there is a prefactor p_1. From Supplementary Figure 2 one can tell that with sufficiently small p_1, one can always eliminate nontrivial solution. So if $\alpha_{11} < \alpha_{11c}$, Eq. 5 has no nontrivial solutions, while with sufficiently small but

finite p_1 one can make Eq. 6's solution trivial, thus p_{1c} is finite; If $\alpha_{11} > \alpha_{11c}$, the second equation of Eq. 4 with $x = 0$ always has nontrivial solutions, therefore p_{1c} becomes indefinite; If $\alpha_{11} = \alpha_{11c}$, one finds $p_{1c} = 0.34$. The change of α_{11} leads to an abrupt change of the transition point p_{1c} from finiteness to indefiniteness. However if $\beta_{11} \neq 1$, we no longer have Eq. 5. The equations for y also involves x, α_{11} can continuously drops to 0 before nontrivial solution appears.

Supplementary Note 5 Simulations

In this subsection we describe simulation details. To generate networks with community structure, we first generate a Poisson degree distribution for each network. Next we assign each node with a probability of being connected by a randomly chosen link. The probability is proportional to the node's degree. Last we loop over all links and assign each of them with two ends according to nodes' probabilities. For higher probability nodes, they are more likely to be connected by the links. The actual number of links they are connected are proportional to their preassigned degrees. Hence the final network recovers the degree distribution. In order to obtain community structure, we use the same protocol for both intra and inter links. To control the community structure, the number of intra and inter links are evaluated in advance. We generate random networks with size $N_1 = N_2 = 10,000$. To calculate giant component size after random or community attacks, we perform at least 10 realizations for each parameter set and take the average.

To find the giant components in our interdependent system, we first look for the largest percolation cluster in the networks without considering the community structure. We later categorize the nodes in the cluster by their associated communities. The giant component in each community is therefore obtained. Specifically, if the communities are disconnected (localized), we look for the largest cluster in each disconnected communities. Therefore we allow multiple components in our final giant component. As in the real system, localized community can be self supportive without relying on external resources.

Supplementary Note 6 Find the p_{ic} by simulations

Binary searching algorithm was employed to calculate the critical point p_{ic} for both numerical and simulation results. At the beginning, we setup $p_{ic}^0 = 0$ and $p_{ic}^1 = 1$ and $p_{ic} = \frac{p_{ic}^0 + p_{ic}^1}{2}$. If $\mu^\infty(p_{ic}) > c$, $p_{ic}^1 = p_{ic}$, otherwise, $p_{ic}^0 = p_{ic}$. By the iterations, the above algorithm allows us to find the critical point p_{ic}. It is well-know that, binary searching algorithm convergences to the fix point exponentially. It implies that the critical point can be found efficiently. For the numerical and simulation code, we setup $c = 10^{-8}$ and 0.01 respectively.

Supplementary Note 7 How to change community strength α_{11}

Supplementary Figure 3 shows a typical case of how we vary community structure to interdependent networks. After the reconnecting the links, the nodes' degrees preserve and we also make the reconnected network as close as possible to the original one.

Supplementary Note 8 Empirical studies

To manually change community structure of an empirical network, we use the same protocol described in Supplementary Note 7 to regenerate the links in the network. To control the community structure, we evaluate in advance the number of inter-links and intra-links so that the average degree and community index would be obeyed in the following process. Lastly, we assign two nodes to each link to form the network. We do so according to the link property. If the link is an inter-link, we assign nodes of different communities to it; while if the link is an intra-link of community 1, we restrict the selection of nodes to community 1 during the process. To minimize statistical error during the manipulation of empirical data, we perform 100 independent simulations by randomly carrying out the above process, and take the average of any measurement we need.

Supplementary Note 9 Partial dependency case

Here we have also empirically studied the partial interdependent case [2]. For the North America-Asia Firm-Flight interdependent networks, we setup the interdependent ratio between the nodes in two layer to be $Q < 1$, which

is more realistic. It means that a failed node in one layer leads to the failure of the same node in other layer with probability Q. We also detect the abrupt change of p_{1c} when $Q = 0.9$ shown in Supplementary Figure 4.

Supplementary Note 10 Details of the 21 different categories for business networks

1. AGRICULTURE, FORESTRY AND FISHING
Crop and animal production, hunting and related service activities
Growing of non-perennial crops
Growing of cereals (except rice), leguminous crops and oil seeds
Growing of rice
Growing of vegetables and melons, roots and tubers
Growing of sugar cane
Growing of tobacco
Growing of fibre crops
Growing of other non-perennial crops
Growing of perennial crops
Growing of grapes
Growing of tropical and subtropical fruits
Growing of citrus fruits
Growing of pome fruits and stone fruits
Growing of other tree and bush fruits and nuts
Growing of oleaginous fruits
Growing of beverage crops
Growing of spices, aromatic, drug and pharmaceutical crops
Growing of other perennial crops
Plant propagation
Animal production
Raising of dairy cattle
Raising of other cattle and buffaloes
Raising of horses and other equines
Raising of camels and camelids
Raising of sheep and goats
Raising of swine/pigs
Raising of poultry
Raising of other animals
Mixed farming
Support activities to agriculture and post-harvest crop activities
Support activities for crop production
Support activities for animal production
Post-harvest crop activities
Seed processing for propagation
Hunting, trapping and related service activities
Forestry and logging
Silviculture and other forestry activities
Logging
Gathering of wild growing non-wood products
Support services to forestry
Fishing and aquaculture

Fishing
Marine fishing
Freshwater fishing
Aquaculture
Marine aquaculture
Freshwater aquaculture

2. MINING AND QUARRYING

Mining of coal and lignite
Mining of hard coal
Mining of lignite
Extraction of crude petroleum and natural gas
Extraction of crude petroleum
Extraction of natural gas
Mining of metal ores
Mining of iron ores
Mining of non-ferrous metal ores
Mining of uranium and thorium ores
Mining of other non-ferrous metal ores
Other mining and quarrying
Quarrying of stone, sand and clay
Quarrying of ornamental and building stone, limestone, gypsum, chalk and slate
Operation of gravel and sand pits; mining of clays and kaolin
Mining and quarrying n.e.c.
Mining of chemical and fertiliser minerals
Extraction of peat
Extraction of salt
Other mining and quarrying n.e.c.
Mining support service activities
Support activities for petroleum and natural gas extraction
Support activities for other mining and quarrying

3. MANUFACTURING

Manufacture of food products
Processing and preserving of meat and production of meat products
Processing and preserving of meat
Processing and preserving of poultry meat
Production of meat and poultry meat products
Processing and preserving of fish, crustaceans and molluscs
Processing and preserving of fruit and vegetables
Processing and preserving of potatoes
Manufacture of fruit and vegetable juice
Other processing and preserving of fruit and vegetables
Manufacture of vegetable and animal oils and fats
Manufacture of oils and fats
Manufacture of margarine and similar edible fats
Manufacture of dairy products
Operation of dairies and cheese making
Manufacture of ice cream
Manufacture of grain mill products, starches and starch products
Manufacture of grain mill products
Manufacture of starches and starch products

Manufacture of bakery and farinaceous products
Manufacture of bread; manufacture of fresh pastry goods and cakes
Manufacture of rusks and biscuits; manufacture of preserved pastry goods and cakes
Manufacture of macaroni, noodles, couscous and similar farinaceous products
Manufacture of other food products
Manufacture of sugar
Manufacture of cocoa, chocolate and sugar confectionery
Processing of tea and coffee
Manufacture of condiments and seasonings
Manufacture of prepared meals and dishes
Manufacture of homogenised food preparations and dietetic food
Manufacture of other food products n.e.c.
Manufacture of prepared animal feeds
Manufacture of prepared feeds for farm animals
Manufacture of prepared pet foods
Manufacture of beverages
Distilling, rectifying and blending of spirits
Manufacture of wine from grape
Manufacture of cider and other fruit wines
Manufacture of other non-distilled fermented beverages
Manufacture of beer
Manufacture of malt
Manufacture of soft drinks; production of mineral waters and other bottled waters
Manufacture of tobacco products
Manufacture of textiles
Preparation and spinning of textile fibres
Weaving of textiles
Finishing of textiles
Manufacture of other textiles
Manufacture of knitted and crocheted fabrics
Manufacture of made-up textile articles, except apparel
Manufacture of carpets and rugs
Manufacture of cordage, rope, twine and netting
Manufacture of non-wovens and articles made from non-wovens, except apparel
Manufacture of other technical and industrial textiles
Manufacture of other textiles n.e.c.
Manufacture of wearing apparel
Manufacture of wearing apparel, except fur apparel
Manufacture of leather clothes
Manufacture of workwear
Manufacture of other outerwear
Manufacture of underwear
Manufacture of other wearing apparel and accessories
Manufacture of articles of fur
Manufacture of knitted and crocheted apparel
Manufacture of knitted and crocheted hosiery
Manufacture of other knitted and crocheted apparel
Manufacture of leather and related products
Tanning and dressing of leather; manufacture of luggage, handbags, saddlery and harness
Tanning and dressing of leather; dressing and dyeing of fur
Manufacture of luggage, handbags and the like, saddlery and harness
Manufacture of footwear

Manufacture of wood and of products of wood and cork, except furniture
Sawmilling and planing of wood
Manufacture of products of wood, cork, straw and plaiting materials
Manufacture of veneer sheets and wood-based panels
Manufacture of assembled parquet floors
Manufacture of other builders' carpentry and joinery
Manufacture of wooden containers
Manufacture of other products of wood
Manufacture of paper and paper products
Manufacture of pulp, paper and paperboard
Manufacture of pulp
Manufacture of paper and paperboard
Manufacture of articles of paper and paperboard
Manufacture of corrugated paper and paperboard and of containers of paper and paperboard
Manufacture of household and sanitary goods and of toilet requisites
Manufacture of paper stationery
Manufacture of wallpaper
Manufacture of other articles of paper and paperboard
Printing and reproduction of recorded media
Printing and service activities related to printing
Printing of newspapers
Other printing
Pre-press and pre-media services
Binding and related services
Reproduction of recorded media
Manufacture of coke and refined petroleum products
Manufacture of coke oven products
Manufacture of refined petroleum products
Manufacture of chemicals and chemical products
Manufacture of basic chemicals, fertilisers and nitrogen compounds
Manufacture of industrial gases
Manufacture of dyes and pigments
Manufacture of other inorganic basic chemicals
Manufacture of paints, varnishes and similar coatings, printing ink and mastics
Manufacture of soap and detergents, cleaning and polishing preparations
Manufacture of perfumes and toilet preparations
Manufacture of other chemical products
Manufacture of explosives
Manufacture of glues
Manufacture of essential oils
Manufacture of other chemical products n.e.c.
Manufacture of man-made fibres
Manufacture of basic pharmaceutical products and pharmaceutical preparations
Manufacture of basic pharmaceutical products
Manufacture of pharmaceutical preparations
Manufacture of rubber and plastic products
Manufacture of rubber products
Manufacture of rubber tyres and tubes; retreading and rebuilding of rubber tyres
Manufacture of other rubber products
Manufacture of plastic products
Manufacture of plastic plates, sheets, tubes and profiles
Manufacture of plastic packing goods

Manufacture of builders
Manufacture of other plastic products
Manufacture of other non-metallic mineral products
Manufacture of glass and glass products
Manufacture of flat glass
Shaping and processing of flat glass
Manufacture of hollow glass
Manufacture of glass fibres
Manufacture and processing of other glass, including technical glassware
Manufacture of refractory products
Manufacture of clay building materials
Manufacture of ceramic tiles and flags
Manufacture of bricks, tiles and construction products, in baked clay
Manufacture of other porcelain and ceramic products
Manufacture of ceramic household and ornamental articles
Manufacture of ceramic sanitary fixtures
Manufacture of ceramic insulators and insulating fittings
Manufacture of other technical ceramic products
Manufacture of other ceramic products
Manufacture of cement, lime and plaster
Manufacture of cement
Manufacture of lime and plaster
Manufacture of articles of concrete, cement and plaster
Manufacture of concrete products for construction purposes
Manufacture of plaster products for construction purposes
Manufacture of ready-mixed concrete
Manufacture of mortars
Manufacture of fibre cement
Manufacture of other articles of concrete, plaster and cement
Cutting, shaping and finishing of stone
Manufacture of abrasive products and non-metallic mineral products n.e.c.
Production of abrasive products
Manufacture of other non-metallic mineral products n.e.c.
Manufacture of basic metals
Manufacture of basic iron and steel and of ferro-alloys
Manufacture of basic iron and steel and of ferro-alloys
Manufacture of tubes, pipes, hollow profiles and related fittings, of steel
Manufacture of other products of first processing of steel
Cold drawing of bars
Cold rolling of narrow strip
Cold forming or folding
Cold drawing of wire
Manufacture of basic precious and other non-ferrous metals
Precious metals production
Aluminium production
Lead, zinc and tin production
Copper production
Other non-ferrous metal production
Processing of nuclear fuel
Casting of metals
Casting of iron
Casting of steel

Casting of light metals
Casting of other non-ferrous metals
Manufacture of fabricated metal products, except machinery and equipment
Manufacture of structural metal products
Manufacture of metal structures and parts of structures
Manufacture of doors and windows of metal
Manufacture of tanks, reservoirs and containers of metal
Manufacture of central heating radiators and boilers
Manufacture of other tanks, reservoirs and containers of metal
Manufacture of steam generators, except central heating hot water boilers
Manufacture of weapons and ammunition
Forging, pressing, stamping and roll-forming of metal; powder metallurgy
Treatment and coating of metals; machining
Treatment and coating of metals
Machining
Manufacture of cutlery, tools and general hardware
Manufacture of cutlery
Manufacture of locks and hinges
Manufacture of tools
Manufacture of other fabricated metal products
Manufacture of steel drums and similar containers
Manufacture of light metal packaging
Manufacture of wire products, chain and springs
Manufacture of fasteners and screw machine products
Manufacture of other fabricated metal products n.e.c.
Manufacture of computer, electronic and optical products
Manufacture of electronic components and boards
Manufacture of electronic components
Manufacture of loaded electronic boards
Manufacture of computers and peripheral equipment
Manufacture of communication equipment
Manufacture of consumer electronics
Manufacture of instruments and appliances for measuring, testing and navigation
Manufacture of watches and clocks
Manufacture of irradiation, electromedical and electrotherapeutic equipment
Manufacture of optical instruments and photographic equipment
Manufacture of magnetic and optical media
Manufacture of electrical equipment
Manufacture of electric motors, generators, transformers and electricity distribution
Manufacture of electric motors, generators and transformers
Manufacture of electricity distribution and control apparatus
Manufacture of batteries and accumulators
Manufacture of wiring and wiring devices
Manufacture of fibre optic cables
Manufacture of other electronic and electric wires and cables
Manufacture of wiring devices
Manufacture of electric lighting equipment
Manufacture of domestic appliances
Manufacture of electric domestic appliances
Manufacture of non-electric domestic appliances
Manufacture of other electrical equipment
Manufacture of machinery and equipment n.e.c.

Manufacture of general-purpose machinery
Manufacture of engines and turbines, except aircraft, vehicle and cycle engines
Manufacture of fluid power equipment
Manufacture of other pumps and compressors
Manufacture of other taps and valves
Manufacture of bearings, gears, gearing and driving elements
Manufacture of other general-purpose machinery
Manufacture of ovens, furnaces and furnace burners
Manufacture of lifting and handling equipment
Manufacture of office machinery and equipment
Manufacture of power-driven hand tools
Manufacture of non-domestic cooling and ventilation equipment
Manufacture of other general-purpose machinery n.e.c.
Manufacture of agricultural and forestry machinery
Manufacture of metal forming machinery and machine tools
Manufacture of metal forming machinery
Manufacture of other machine tools
Manufacture of other special-purpose machinery
Manufacture of machinery for metallurgy
Manufacture of machinery for mining, quarrying and construction
Manufacture of machinery for food, beverage and tobacco processing
Manufacture of machinery for textile, apparel and leather production
Manufacture of machinery for paper and paperboard production
Manufacture of plastics and rubber machinery
Manufacture of other special-purpose machinery n.e.c.
Manufacture of motor vehicles, trailers and semi-trailers
Manufacture of motor vehicles
Manufacture of bodies (coachwork) for motor vehicles
Manufacture of parts and accessories for motor vehicles
Manufacture of electrical and electronic equipment for motor vehicles
Manufacture of other parts and accessories for motor vehicles
Manufacture of other transport equipment
Building of ships and boats
Building of ships and floating structures
Building of pleasure and sporting boats
Manufacture of railway locomotives and rolling stock
Manufacture of air and spacecraft and related machinery
Manufacture of military fighting vehicles
Manufacture of transport equipment n.e.c.
Manufacture of motorcycles
Manufacture of bicycles and invalid carriages
Manufacture of other transport equipment n.e.c.
Manufacture of furniture
Manufacture of office and shop furniture
Manufacture of kitchen furniture
Manufacture of mattresses
Manufacture of other furniture
Other manufacturing
Manufacture of jewellery, bijouterie and related articles
Striking of coins
Manufacture of jewellery and related articles
Manufacture of imitation jewellery and related articles

Manufacture of musical instruments
Manufacture of sports goods
Manufacture of games and toys
Manufacture of medical and dental instruments and supplies
Manufacturing n.e.c.
Manufacture of brooms and brushes
Other manufacturing n.e.c.
Repair and installation of machinery and equipment
Repair of fabricated metal products, machinery and equipment
Repair of fabricated metal products
Repair of machinery
Repair of electronic and optical equipment
Repair of electrical equipment
Repair and maintenance of ships and boats
Repair and maintenance of aircraft and spacecraft
Repair and maintenance of other transport equipment
Repair of other equipment
Installation of industrial machinery and equipment

4. ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY

Electricity, gas, steam and air conditioning supply
Electric power generation, transmission and distribution
Production of electricity
Transmission of electricity
Distribution of electricity
Trade of electricity
Manufacture of gas; distribution of gaseous fuels through mains
Manufacture of gas
Distribution of gaseous fuels through mains
Trade of gas through mains
Steam and air conditioning supply

5. WATER SUPPLY; WASTE MANAGEMENT AND REMEDIATION ACTIVITIES

Water collection, treatment and supply
Sewerage
Waste collection, treatment and disposal activities; materials recovery
Waste collection
Collection of non-hazardous waste
Collection of hazardous waste
Waste treatment and disposal
Treatment and disposal of non-hazardous waste
Treatment and disposal of hazardous waste
Materials recovery
Dismantling of wrecks
Recovery of sorted materials
Remediation activities and other waste management services

6. CONSTRUCTION

Construction of buildings
Development of building projects
Construction of residential and non-residential buildings
Civil engineering

Construction of roads and railways
Construction of roads and motorways
Construction of railways and underground railways
Construction of bridges and tunnels
Construction of utility projects
Construction of utility projects for fluids
Construction of utility projects for electricity and telecommunications
Construction of other civil engineering projects
Construction of water projects
Construction of other civil engineering projects n.e.c.
Specialised construction activities
Demolition and site preparation
Demolition
Site preparation
Test drilling and boring
Electrical, plumbing and other construction installation activities
Electrical installation
Plumbing, heat and air-conditioning installation
Other construction installation
Building completion and finishing
Plastering
Joinery installation
Floor and wall covering
Painting and glazing
Other building completion and finishing
Other specialised construction activities
Roofing activities
Other specialised construction activities n.e.c.

7. WHOLESALE AND RETAIL TRADE; REPAIR OF MOTOR VEHICLES

Wholesale and retail trade and repair of motor vehicles and motorcycles
Sale of motor vehicles
Sale of cars and light motor vehicles
Sale of other motor vehicles
Maintenance and repair of motor vehicles
Sale of motor vehicle parts and accessories
Wholesale trade of motor vehicle parts and accessories
Retail trade of motor vehicle parts and accessories
Sale, maintenance and repair of motorcycles and related parts and accessories
Wholesale trade, except of motor vehicles and motorcycles
Wholesale on a fee or contract basis
Agents involved in the sale of agricultural raw materials, live animals, textile raw materials
Agents involved in the sale of fuels, ores, metals and industrial chemicals
Agents involved in the sale of timber and building materials
Agents involved in the sale of machinery, industrial equipment, ships and aircraft
Agents involved in the sale of furniture, household goods, hardware and ironmongery
Agents involved in the sale of textiles, clothing, fur, footwear and leather goods
Agents involved in the sale of food, beverages and tobacco
Agents specialised in the sale of other particular products
Agents involved in the sale of a variety of goods
Wholesale of agricultural raw materials and live animals
Wholesale of grain, unmanufactured tobacco, seeds and animal feeds

Wholesale of flowers and plants
Wholesale of live animals
Wholesale of hides, skins and leather
Wholesale of food, beverages and tobacco
Wholesale of fruit and vegetables
Wholesale of meat and meat products
Wholesale of dairy products, eggs and edible oils and fats
Wholesale of beverages
Wholesale of tobacco products
Wholesale of sugar and chocolate and sugar confectionery
Wholesale of coffee, tea, cocoa and spices
Wholesale of other food, including fish, crustaceans and molluscs
Non-specialised wholesale of food, beverages and tobacco
Wholesale of household goods
Wholesale of textiles
Wholesale of clothing and footwear
Wholesale of electrical household appliances
Wholesale of china and glassware and cleaning materials
Wholesale of perfume and cosmetics
Wholesale of pharmaceutical goods
Wholesale of furniture, carpets and lighting equipment
Wholesale of watches and jewellery
Wholesale of other household goods
Wholesale of information and communication equipment
Wholesale of computers, computer peripheral equipment and software
Wholesale of electronic and telecommunications equipment and parts
Wholesale of other machinery, equipment and supplies
Wholesale of agricultural machinery, equipment and supplies
Wholesale of machine tools
Wholesale of mining, construction and civil engineering machinery
Wholesale of machinery for the textile industry and of sewing and knitting machines
Wholesale of office furniture
Wholesale of other office machinery and equipment
Wholesale of other machinery and equipment
Other specialised wholesale
Wholesale of solid, liquid and gaseous fuels and related products
Wholesale of metals and metal ores
Wholesale of wood, construction materials and sanitary equipment
Wholesale of hardware, plumbing and heating equipment and supplies
Wholesale of chemical products
Wholesale of other intermediate products
Wholesale of waste and scrap
Non-specialised wholesale trade
Retail trade, except of motor vehicles and motorcycles
Retail sale in non-specialised stores
Retail sale in non-specialised stores with food, beverages or tobacco predominating
Other retail sale in non-specialised stores
Retail sale of food, beverages and tobacco in specialised stores
Retail sale of fruit and vegetables in specialised stores
Retail sale of meat and meat products in specialised stores
Retail sale of fish, crustaceans and molluscs in specialised stores
Retail sale of bread, cakes, flour confectionery and sugar confectionery in specialised stores

Retail sale of beverages in specialised stores
Retail sale of tobacco products in specialised stores
Other retail sale of food in specialised stores
Retail sale of automotive fuel in specialised stores
Retail sale of information and communication equipment in specialised stores
Retail sale of computers, peripheral units and software in specialised stores
Retail sale of telecommunications equipment in specialised stores
Retail sale of audio and video equipment in specialised stores
Retail sale of other household equipment in specialised stores
Retail sale of textiles in specialised stores
Retail sale of hardware, paints and glass in specialised stores
Retail sale of carpets, rugs, wall and floor coverings in specialised stores
Retail sale of electrical household appliances in specialised stores
Retail sale of furniture, lighting equipment and other household articles in specialised stores
Retail sale of cultural and recreation goods in specialised stores
Retail sale of books in specialised stores
Retail sale of newspapers and stationery in specialised stores
Retail sale of music and video recordings in specialised stores
Retail sale of sporting equipment in specialised stores
Retail sale of games and toys in specialised stores
Retail sale of other goods in specialised stores
Retail sale of clothing in specialised stores
Retail sale of footwear and leather goods in specialised stores
Dispensing chemist in specialised stores
Retail sale of medical and orthopaedic goods in specialised stores
Retail sale of cosmetic and toilet articles in specialised stores
Retail sale of flowers, plants, seeds, pet animals and pet food in specialised stores
Retail sale of watches and jewellery in specialised stores
Other retail sale of new goods in specialised stores
Retail sale of second-hand goods in stores
Retail sale via stalls and markets
Retail sale via stalls and markets of food, beverages and tobacco products
Retail sale via stalls and markets of textiles, clothing and footwear
Retail sale via stalls and markets of other goods
Retail trade not in stores, stalls or markets
Retail sale via mail order houses or via Internet
Other retail sale not in stores, stalls or markets

8. TRANSPORTATION AND STORAGE

Land transport and transport via pipelines
Passenger rail transport, interurban
Freight rail transport
Other passenger land transport
Urban and suburban passenger land transport
Taxi operation
Other passenger land transport n.e.c.
Freight transport by road and removal services
Freight transport by road
Removal services
Transport via pipeline
Water transport
Sea and coastal freight water transport

Inland freight water transport
Air transport
Passenger air transport
Freight air transport and space transport
Freight air transport
Space transport
Warehousing and support activities for transportation
Warehousing and storage
Support activities for transportation
Service activities incidental to land transportation
Service activities incidental to water transportation
Service activities incidental to air transportation
Cargo handling
Other transportation support activities
Postal and courier activities
Postal activities under universal service obligation
Other postal and courier activities

9. ACCOMMODATION AND FOOD SERVICE ACTIVITIES

Accommodation
Hotels and similar accommodation
Holiday and other short-stay accommodation
Camping grounds, recreational vehicle parks and trailer parks
Other accommodation
Food and beverage service activities
Restaurants and mobile food service activities
Event catering and other food service activities
Event catering activities
Other food service activities
Beverage serving activities

10. INFORMATION AND COMMUNICATION

Publishing activities
Publishing of books, periodicals and other publishing activities
Book publishing
Publishing of directories and mailing lists
Publishing of newspapers
Publishing of journals and periodicals
Other publishing activities
Software publishing
Publishing of computer games
Other software publishing
Motion picture, video and television programme production and music publishing activities
Motion picture, video and television programme activities
Motion picture, video and television programme production activities
Motion picture, video and television programme post-production activities
Motion picture, video and television programme distribution activities
Motion picture projection activities
Sound recording and music publishing activities
Programming and broadcasting activities
Radio broadcasting
Television programming and broadcasting activities

Telecommunications
Wired telecommunications activities
Wireless telecommunications activities
Satellite telecommunications activities
Other telecommunications activities
Computer programming, consultancy and related activities
Computer programming activities
Computer consultancy activities
Computer facilities management activities
Other information technology and computer service activities
Information service activities
Data processing, hosting and related activities; web portals
Data processing, hosting and related activities
Web portals
Other information service activities
News agency activities
Other information service activities n.e.c.

11. FINANCIAL AND INSURANCE ACTIVITIES

Financial service activities, except insurance and pension funding
Monetary intermediation
Central banking
Other monetary intermediation
Activities of holding companies
Trusts, funds and similar financial entities
Other financial service activities, except insurance and pension funding
Financial leasing
Other credit granting
Other financial service activities, except insurance and pension funding n.e.c.
Insurance, reinsurance and pension funding, except compulsory social security
Insurance
Life insurance
Non-life insurance
Reinsurance
Pension funding
Activities auxiliary to financial services and insurance activities
Activities auxiliary to financial services, except insurance and pension funding
Administration of financial markets
Security and commodity contracts brokerage
Other activities auxiliary to financial services, except insurance and pension funding
Activities auxiliary to insurance and pension funding
Risk and damage evaluation
Activities of insurance agents and brokers
Other activities auxiliary to insurance and pension funding
Fund management activities

12. REAL ESTATE ACTIVITIES

Real estate activities
Buying and selling of own real estate
Renting and operating of own or leased real estate
Real estate activities on a fee or contract basis
Real estate agencies

Management of real estate on a fee or contract basis

13. PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES

Legal and accounting activities
Legal activities
Accounting, bookkeeping and auditing activities; tax consultancy
Activities of head offices; management consultancy activities
Activities of head offices
Management consultancy activities
Public relations and communication activities
Business and other management consultancy activities
Architectural and engineering activities; technical testing and analysis
Architectural and engineering activities and related technical consultancy
Architectural activities
Engineering activities and related technical consultancy
Technical testing and analysis
Scientific research and development
Research and experimental development on natural sciences and engineering
Research and experimental development on biotechnology
Other research and experimental development on natural sciences and engineering
Research and experimental development on social sciences and humanities
Advertising and market research
Advertising
Advertising agencies
Media representation
Market research and public opinion polling
Other professional, scientific and technical activities
Specialised design activities
Photographic activities
Translation and interpretation activities
Other professional, scientific and technical activities n.e.c.
Veterinary activities

14. ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES

Rental and leasing activities
Renting and leasing of motor vehicles
Renting and leasing of cars and light motor vehicles
Renting and leasing of trucks
Renting and leasing of personal and household goods
Renting and leasing of recreational and sports goods
Renting of video tapes and disks
Renting and leasing of other personal and household goods
Renting and leasing of other machinery, equipment and tangible goods
Renting and leasing of agricultural machinery and equipment
Renting and leasing of construction and civil engineering machinery and equipment
Renting and leasing of office machinery and equipment (including computers)
Renting and leasing of water transport equipment
Renting and leasing of air transport equipment
Renting and leasing of other machinery, equipment and tangible goods n.e.c.
Leasing of intellectual property and similar products, except copyrighted works
Employment activities
Activities of employment placement agencies

Temporary employment agency activities
Other human resources provision
Travel agency, tour operator and other reservation service and related activities
Travel agency and tour operator activities
Travel agency activities
Tour operator activities
Other reservation service and related activities
Security and investigation activities
Private security activities
Security systems service activities
Investigation activities
Services to buildings and landscape activities
Combined facilities support activities
Cleaning activities
General cleaning of buildings
Other building and industrial cleaning activities
Other cleaning activities
Landscape service activities
Office administrative, office support and other business support activities
Office administrative and support activities
Combined office administrative service activities
Photocopying, document preparation and other specialised office support activities
Activities of call centres
Organisation of conventions and trade shows
Business support service activities n.e.c.
Activities of collection agencies and credit bureaus
Packaging activities
Other business support service activities n.e.c.

15. PUBLIC ADMINISTRATION AND DEFENCE; COMPULSORY SOCIAL SECURITY

Public administration and defence; compulsory social security
Administration of the State and the economic and social policy of the community
General public administration activities
Regulation of the activities of providing health care and other social services
Regulation of and contribution to more efficient operation of businesses
Provision of services to the community as a whole
Foreign affairs
Defence activities
Justice and judicial activities
Public order and safety activities
Fire service activities
Compulsory social security activities

16. EDUCATION

Education
Pre-primary education
Pre-primary education
Primary education
Primary education
Secondary education
General secondary education
Technical and vocational secondary education

Higher education
Post-secondary non-tertiary education
Tertiary education
Other education
Sports and recreation education
Cultural education
Driving school activities
Other education n.e.c.
Educational support activities

17. HUMAN HEALTH AND SOCIAL WORK ACTIVITIES

Human health activities
Hospital activities
Medical and dental practice activities
General medical practice activities
Specialist medical practice activities
Dental practice activities
Other human health activities
Residential care activities
Residential nursing care activities
Residential care activities for mental retardation, mental health and substance abuse
Residential care activities for the elderly and disabled
Other residential care activities
Social work activities without accommodation
Social work activities without accommodation for the elderly and disabled
Other social work activities without accommodation
Child day-care activities
Other social work activities without accommodation n.e.c.

18. ARTS, ENTERTAINMENT AND RECREATION

Creative, arts and entertainment activities
Performing arts
Support activities to performing arts
Artistic creation
Operation of arts facilities
Libraries, archives, museums and other cultural activities
Library and archives activities
Museums activities
Operation of historical sites and buildings and similar visitor attractions
Botanical and zoological gardens and nature reserves activities
Gambling and betting activities
Sports activities and amusement and recreation activities
Sports activities
Operation of sports facilities
Activities of sports clubs
Fitness facilities
Other sports activities
Amusement and recreation activities
Activities of amusement parks and theme parks
Other amusement and recreation activities

19. OTHER SERVICE ACTIVITIES

Activities of membership organisations
Activities of business, employers and professional membership organisations
Activities of business and employers membership organisations
Activities of professional membership organisations
Activities of trade unions
Activities of other membership organisations
Activities of religious organisations
Activities of political organisations
Activities of other membership organisations n.e.c.
Repair of computers and personal and household goods
Repair of computers and communication equipment
Repair of computers and peripheral equipment
Repair of communication equipment
Repair of personal and household goods
Repair of consumer electronics
Repair of household appliances and home and garden equipment
Repair of footwear and leather goods
Repair of furniture and home furnishings
Repair of watches, clocks and jewellery
Repair of other personal and household goods
Other personal service activities
Washing and (dry-)cleaning of textile and fur products
Hairdressing and other beauty treatment
Funeral and related activities
Physical well-being activities
Other personal service activities n.e.c.

20. ACTIVITIES OF HOUSEHOLDS; UNDIFFERENTIATED GOODS AND SERVICE
Activities of households as employers of domestic personnel
Undifferentiated goods- and services-producing activities of private households for own use
Undifferentiated service-producing activities of private households for own use

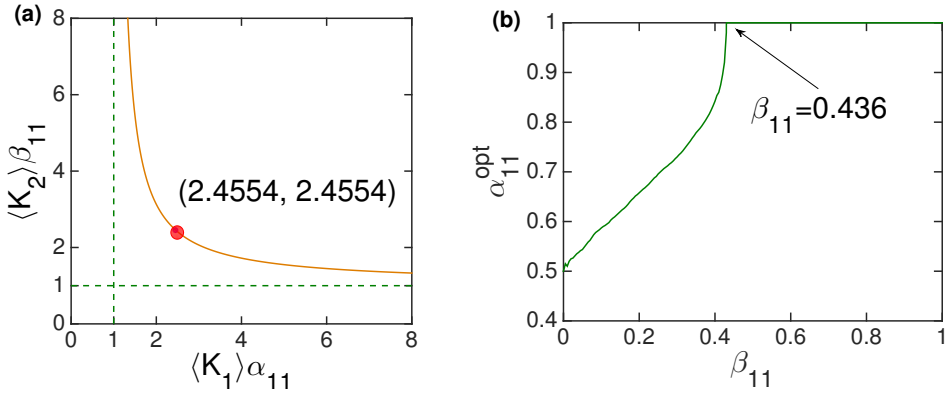
21. ACTIVITIES OF EXTRATERRITORIAL ORGANISATIONS AND BODIES
Activities of extraterritorial organisations and bodies

Supplementary References

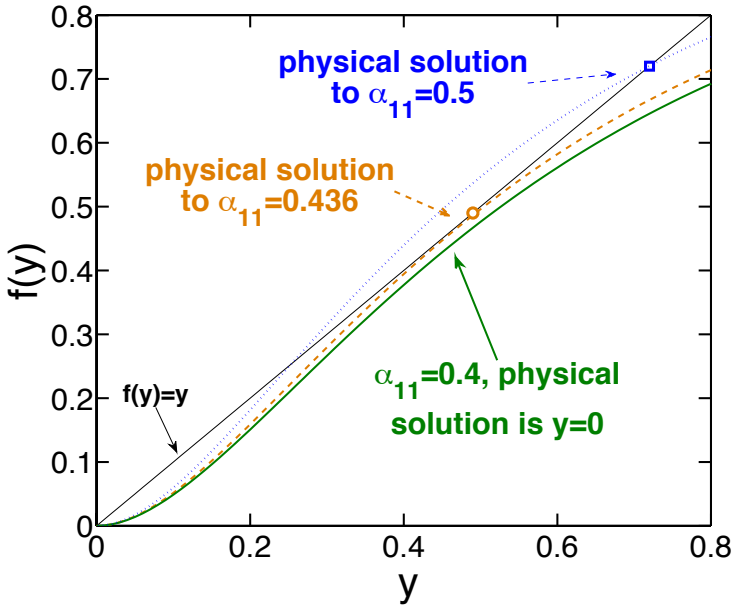
[1] Sergey V Buldyrev, Roni Parshani, Gerald Paul, H Eugene Stanley, and Shlomo Havlin. Catastrophic cascade of failures in interdependent networks. *Nature*, 464:1025–8, 2010.

[2] R. Parshani, Sergey V. Buldyrev, and Shlomo Havlin. Interdependent networks: Reducing the coupling strength leads to a change from a first to second order percolation transition. *Phys. Rev. Lett.*, 105(048701), 2010.

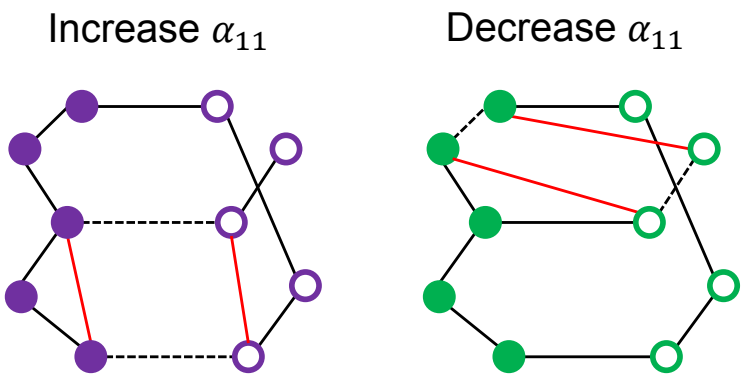
Supplementary Figures



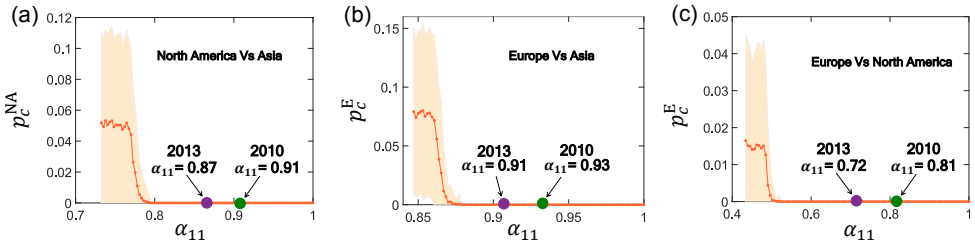
Supplementary Figure 1. Criticality of interdependent networks with communities. Here $\langle K_{iA}\rangle = \langle K_{iB}\rangle = K_i$, $i = 1, 2$ and $N_1 = N_2$. Here $\langle K_1\rangle = \langle K_2\rangle = 4$. (a) Universal boundary between Safe region and Robust region in terms of $(\langle K_1\rangle\alpha_{11}, \langle K_2\rangle\beta_{11})$; Dashed are their asymptotic lines; Under this boundary, when we remove a full community, the other community will be fragmented. Above the boundary, when we remove a full community, the other community can survival with a mutual giant component at the equilibrium state. (b) Optimum α_{11}^{opt} vs β_{11}.The optimized α_{11}^{opt} means that it can maximize the percolation robustness (minimizing p_{1c}). When $\beta_{11} < 0.436$, α_{11}^{opt} increases from 0.5 to 1 as β_{11} increases. When $\beta_{11} \geq 0.436$, α_{11}^{opt} remains 1.



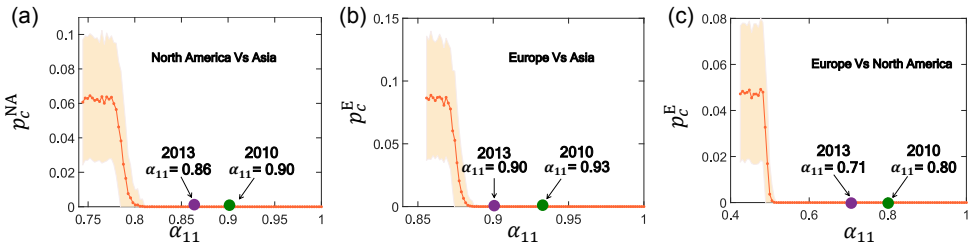
Supplementary Figure 2. Illustration of the physical solution of Eq. 5 at $\langle K_{2A}\rangle = \langle K_{2B}\rangle = 4$. $f(y) = [1-\exp(-\alpha_{11}\langle K_{2A}\rangle y)][1-\exp(-\langle K_{2B}\rangle y)]$ is the right hand side of Eq. 5; $f(y) = y$ is the left hand side of Eq. 5. The three curves correspond to three different α_{11}'s. Physical solutions (markers in the figure) are the intersections of these curves with $f(y) = y$. $\alpha_{11} = 0.436$ is the critical point. If $\alpha_{11} > 0.436$, there exists two nontrivial solutions (the higher one is the physical solution), community 2 survives; If $\alpha_{11} < 0.436$, there is no nontrivial solution, community 2 collapses, which leads to finite transition point p_{1c}.



Supplementary Figure 3. Illustration of the rules of varying α_{11}. Closed circles denote community 1 and open circles denote community 2. To increase α_{11}, each time one intra link from each community is randomly selected (denoted by the dashed lines). Then they are replaced by two inter-links denoted by the thick (red online) lines. The process repeats until α_{11} reaches the target value. To decrease α_{11}. Each time two inter links between these two communities are selected (denoted by the dashed lines). They are then replaced by two intra links (denoted by thick lines (red online)) connecting nodes of the same community. The above process repeats until α_{11} reaches expected value. This way can keep the original network structure as much as possible. Note that, if the two communities are of different sizes or average degrees, the maximum α_{11} can be 1, but the minimum α_{11} may not reach 0.



Supplementary Figure 4. Abrupt change of p_{1c} vs α_{11} under $Q = 0.9$. Threshold weights: business network is 10 and flights network is 200000 which are same with Fig.3 in the main text. We only present the attack on the first community which is North America of (a), Europe for (b) and (c). For the attack on the second community in respective plots, the system does not disintegrate with when the whole of second community is removed, leading to $p_{2c} = 0$ regardless of α_{11} values, and it is not plotted here.



Supplementary Figure 5. Same simulation as the previous figure but with different Weights threshold for network construction: Business network is 5 and flights network is 200000. We also can see the abrupt changes of the system robustness.

Supplementary Tables

Name	Europe	Asia
N	334	158
$\langle K_{iA} \rangle$ of Business	3.97	1.28
$\alpha_{ii}(\alpha_{ii}^r)$ of Business	0.93(0.87)	0.74(0.13)
$\langle K_{iB} \rangle$ of Flight	1.97	3.43
$\beta_{ii}(\beta_{ii}^r)$ of Flight	0.91(0.55)	0.83(0.45)
Threshold	Business:10 Flight:200000	

Supplementary Table 1. Information of the communities in terms of continents (Europe vs Asia). The parameters are defined analogously as Tab.1 in main text.

Name	Europe	North America
N	334	145
$\langle K_{iA} \rangle$ of Business	4.59	5.97
$\alpha_{ii}(\alpha_{ii}^r)$ of Business	0.81(0.64)	0.66(0.36)
$\langle K_{iB} \rangle$ of Flight	2.00	9.10
$\beta_{ii}(\beta_{ii}^r)$ of Flight	0.89(0.34)	0.95(0.66)
Threshold	Business:10 Flight:200000	

Supplementary Table 2. Information of the communities in terms of continents (Europe vs North America). The parameters are defined analogously as Tab.1 in main text.

Supplementary Note 1 Interdependent Erdős-Rényi networks

Here we show the percolation theory for Erdős-Rényi (ER) networks. For simplicity we assume $N_1 = N_2$. The generating functions for ER networks with two equal communities can be analytically expressed by the average

degrees:

$$G^{iA}(\xi_1,\xi_2)=G_1^{iA}(\xi_1,\xi_2)=\exp[\alpha_{i1}\langle K_{iA}\rangle(\xi_1-1)+\alpha_{i2}\langle K_{iA}\rangle(\xi_2-1)],$$

$$G^{iB}(\zeta_1,\zeta_2)=G_1^{iB}[\zeta_1,\zeta_2)=\exp(\beta_{i1}\langle K_{iB}\rangle(\zeta_1-1)+\beta_{i2}\langle K_{iB}\rangle(\zeta_2-1)].$$

By substituting the above equations into Eq.(14) in the main text, the self-consistent equations reduce to

$$\begin{aligned}
u_1^\infty &= \exp[-\alpha_{11}p_1\langle K_{1A}\rangle(u_1^\infty-1)(v_1^\infty-1)-(1-\alpha_{11})p_2\langle K_{1A}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
u_2^\infty &= \exp[-(1-\alpha_{22})p_1\langle K_{2A}\rangle(u_1^\infty-1)(v_1^\infty-1)-\alpha_{22}p_2\langle K_{2A}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
v_1^\infty &= \exp[-\beta_{11}p_1\langle K_{1B}\rangle(u_1^\infty-1)(v_1^\infty-1)-(1-\beta_{11})p_2\langle K_{1B}\rangle(u_2^\infty-1)(v_2^\infty-1)],\\
v_2^\infty &= \exp[-(1-\beta_{22})p_1\langle K_{2B}\rangle(u_1^\infty-1)(v_1^\infty-1)-\beta_{22}p_2\langle K_{2B}\rangle(u_2^\infty-1)(v_2^\infty-1)],
\end{aligned} \tag{1}$$

and the final size of the network becomes

$$\begin{aligned}
\mu_i^\infty =& p_i(1-G^{iA}(1-p_1(1-u_1^\infty)(1-v_1^\infty),1-p_2(1-u_2^\infty)(1-v_2^\infty)))\\
&\cdot(1-G^{iB}(1-p_1(1-u_1^\infty)(1-v_1^\infty),1-p_2(1-u_2^\infty)(1-v_2^\infty)))=p_i(1-u_i^\infty)(1-v_i^\infty).
\end{aligned}$$

The relevant variable is μ_i^∞, therefore substituting Eq. 2 into the above equation yields

$$\begin{aligned}
\mu_1^\infty &= p_1\{1-\exp[-\alpha_{11}\langle K_{1A}\rangle\mu_1^\infty-(1-\alpha_{11})\langle K_{1A}\rangle\mu_2^\infty]\}\{1-\exp(-\beta_{11}\langle K_{1B}\rangle\mu_1^\infty-(1-\beta_{11})\langle K_{1B}\rangle\mu_2^\infty]\},\\
\mu_2^\infty &= p_2\{1-\exp(-(1-\alpha_{11})\langle K_{2A}\rangle\mu_1^\infty-\alpha_{11}\langle K_{2A}\rangle\mu_2^\infty]\}\{1-\exp(-(1-\beta_{11})\langle K_{2B}\rangle\mu_1^\infty-\beta_{11}\langle K_{2B}\rangle\mu_2^\infty]\}.
\end{aligned} \tag{2}$$

Because each layer of networks have two equal communities, thus we can use the fact that $\alpha_{11}=\alpha_{22}$ and $\beta_{11}=\beta_{22}$,$N_1=N_2$, $\langle K_{1A}\rangle=\langle K_{2A}\rangle=K_A$ and $\langle K_{1B}\rangle=\langle K_{2B}\rangle=K_B$. For unbiased attack, $p_1=p_2=p$, the above expressions for μ_1^∞ and μ_2^∞ can be written as the following form:

$$\mu_1^\infty=\mu_2^\infty=p[1-\exp(-K_A\mu_1^\infty)][1-\exp(-K_B\mu_1^\infty)].$$

One can see this equation is independent of the community structure, which implies that the percolation failures of interdependent ER networks with two equal communities is independent of the community structure.

Supplementary Note 2 Single networks with community structure

Our percolation model can be applied to single networks with community structure by removing terms that are associated with network B. The self consistency equations are simplified to

$$u_i^\infty=G_1^{iA}(1-p_1(1-u_1^\infty),1-p_2(1-u_2^\infty),...,1-p_m(1-u_m^\infty))$$

and the remaining size of i'th community μ_i^∞ becomes

$$\mu_i^\infty = p_i g_i^\infty = p_i[1 - G^{iA}(1 - p_1(1 - u_1^\infty), 1 - p_2(1 - u_2^\infty), ..., 1 - p_m(1 - u_m^\infty))],$$

with $i = 1, 2, ..., m$. If we consider ER networks, the above self consistency equations can be written as

$$u_1 = \exp[\alpha_{11} p_1 \langle K_{1A}\rangle (u_1 - 1) + (1 - \alpha_{11}) p_2 \langle K_{1A}\rangle (u_2 - 1)],$$

$$u_2 = \exp[(1 - \alpha_{22}) p_1 \langle K_{2A}\rangle (u_1 - 1) + \alpha_{22} p_2 \langle K_{2A}\rangle (u_2 - 1)],$$

and the remaining giant component sizes reduce to

$$\mu_1^\infty = p_1\{1 - \exp[-\alpha_{11}\langle K_{1A}\rangle \mu_1^\infty - (1 - \alpha_{11})\langle K_{1A}\rangle \mu_2^\infty]\},$$

$$\mu_2^\infty = p_2\{1 - \exp(-(1 - \alpha_{11})\langle K_{2A}\rangle \mu_1^\infty - \alpha_{11}\langle K_{2A}\rangle \mu_2^\infty]\}.$$

with $i = 1, 2, ..., m$, corresponding to the m communities in the network A. For community attack one has $p_1 = p$ and $p_2 = 1$. For community attack, it can be shown that the transition point p_{1c} satisfies

$$p_{1c} = \frac{\langle k_{22}\rangle - 1}{\langle k_{11}\rangle\langle k_{22}\rangle - \langle k_{12}\rangle\langle k_{21}\rangle - \langle k_{11}\rangle}.$$

We can see that, if $p_{1c} = 0$, it implies that $\langle k_{22}\rangle \leq 1$. This allow us got $a_{22} \leq \frac{1}{K_{2A}}$.

Supplementary Note 3 Safe and robust boundaries on ER network systems

For community attack, say we only attack community 1, i.e. $p_1 = p$ and $p_2 = 1$. Now we consider the safe-unsafe transition. At the transition point, the complete failure of community 1 leads to the failure of community 2. Within the safe region, the removal of community 1 does not lead to the failure of community 2. In other words, whether or not community 2 can self survive determines the safe-unsafe transition boundary. Thus we set $\mu_1^\infty = 0$, the second self-consistent equation becomes

$$\mu_2^\infty = (1 - e^{-\alpha_{11}\langle K_{2A}\rangle \mu_2^\infty})(1 - e^{-\beta_{11}\langle K_{2B}\rangle \mu_2^\infty}).$$

We have $\langle k_{22A}\rangle = \alpha_{11}\langle K_{2A}\rangle$ and $\langle k_{22B}\rangle = \beta_{11}\langle K_{2B}\rangle$, and let $x = \mu_2^\infty$, thus

$$x = (1 - e^{-\langle k_{22A}\rangle x})(1 - e^{-\langle k_{22B}\rangle x}).$$

$x = 0$ is the solution to the critical point. If the system is at transition from safe (which has nonzero solution of x) to unsafe region (only zero solution of x exists), one expects the derivatives of the two sides of the above equation should equal:

$$1 = \partial_x \left[(1 - e^{-\langle k_{22A} \rangle x})(1 - e^{-\langle k_{22B} \rangle x}) \right].$$

By solving the above equations, one can obtain the universal boundary between safe and unsafe regions. The physical meaning of the above equations are as follow: If community 2 can survive from the full failures of community 1, the intra-links in community 2 can prevent the system from spontaneous collapse. The symmetric solution of the above equations is $\langle k_{22A} \rangle = \langle k_{22B} \rangle = 2.4554$, which is the critical average degree (two layers have equal average degree) for an unstructured interdependent network to be stable[1].

To obtain robust-vulnerable boundary, we let $p_1 = p_{1c}^r$ and $p_2 = 1$ in Eq. 2. p_{1c}^r is the critical transition point for a unstructured network under random attack. For conciseness, let $x = \mu_1^\infty$ and $y = \mu_2^\infty$. To find solution to Eq. 2 graphically, one can plot the two equations in terms of two curves $x(y)$ and $y(x)$, the cross points of which are the solutions. At the critical point $p_1 = p_{1c}^r$ and $p_2 = 1$, we have nontrivial (nonzero) solutions to x and y. When $p_1 < p_{1c}^r$, the system's giant component size drops to 0, or equivalently we have only trivial solutions $x = 0$ and $y = 0$. Therefore we should expect that at the critical point the two curves are tangential to each other, i.e. $\frac{\partial x}{\partial y}\frac{\partial y}{\partial x} = 1$[2]:

$$\begin{aligned} \frac{\partial x}{\partial y}\frac{\partial y}{\partial x} =& \frac{\partial}{\partial y}[-p_{1c}^r(1 - e^{-\alpha_{11} x \langle K_{1A} \rangle - (1-\alpha_{11}) y \langle K_{1A} \rangle})(1 - e^{-\beta_{11} x \langle K_{1B} \rangle - (1-\beta_{11}) y \langle K_{1B} \rangle})] \\ & \cdot \frac{\partial}{\partial x}[-(1 - e^{-\alpha_{11} y \langle K_{2A} \rangle - (1-\alpha_{11}) x \langle K_{2A} \rangle})(1 - e^{-\beta_{11} y \langle K_{2B} \rangle - (1-\beta_{11}) x \langle K_{2B} \rangle})] = 1. \end{aligned} \tag{3}$$

Therefore we have 3 equations with 4 variables x, y, α_{11} and β_{11}. By eliminating x and y from the above 3 equations, we obtain an equation that describes the robust-vulnerable boundary curve in terms of α_{11} and β_{11}. The boundary curve is numerically solved and shown in Supplementary Figure 1.

Supplementary Note 4 Discontinuous transition from safe to unsafe region

Consider community attack (to community 1) and network B being localized, one has $p_1 < 1, p_2 = 1$ and $\beta_{11} = 1$. We look for the transition point p_{1c} when the whole system fails. Eqs. 2 reduce to

$$\begin{aligned} x &= p_1[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x - (1-\alpha_{11})\langle K_{1A}\rangle y)][1 - \exp(-\langle K_{1B}\rangle x)], \\ y &= [1 - \exp(-(1-\alpha_{11})\langle K_{2A}\rangle x - \alpha_{11}\langle K_{2A}\rangle y)][1 - \exp(-\langle K_{2B}\rangle y)], \end{aligned} \tag{4}$$

in which we use x and y to denote μ_1^∞ and μ_2^∞ respectively, for conciseness' sake. When community 1 completely fails, one has $x = 0$ and the second equation above further reduces to

$$y = [1 - \exp(-\alpha_{11}\langle K_{2A}\rangle y)][1 - \exp(-\langle K_{2B}\rangle y)]. \tag{5}$$

If a nontrivial solution exists in the above equation, i.e. there exists an solution with $y > 0$, community 2 can survive from the failures of community 1, thus transition point p_{1c} is indefinite (does not exist); otherwise, if $y = 0$ is the only solution to Eq. 5, p_{1c} is finite. By tuning community structure index α_{11}, Eq. 5 may or may not have a nontrivial solution. Therefore the transition from finite p_{1c} to indefinite p_{1c} is denoted by α_{11c}. If $\alpha_{11} < \alpha_{11c}$, the above equation has no nontrivial solution, community 2 fails; If $\alpha_{11} > \alpha_{11c}$, the nontrivial solution leads to the survival of community 2. When $\langle K_{2A}\rangle = \langle K_{2B}\rangle$, the critical $\alpha_{11c} = 0.436$. Illustrated in Supplementary Figure 2, one can visualize how the nontrivial solution emerges with α_{11} increases. At safe-unsafe boundary, the failures of the two communities occur simultaneously. So we assume $y = 0$, Eq. 4 becomes

$$x = p_1[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)], \tag{6}$$

critical p_{1c} requires that the derivatives of the two sides in the above equation should equal, therefore one has

$$\begin{aligned} x &= p_{1c}[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)], \\ 1 &= p_{1c}\frac{\partial}{\partial x}\{[1 - \exp(-\alpha_{11}\langle K_{1A}\rangle x)][1 - \exp(-\langle K_{1B}\rangle x)]\}, \end{aligned}$$

The above p_{1c} is finite. If $\langle K_{1A}\rangle = \langle K_{1B}\rangle = 4$ and $\alpha_{11} = \alpha_{11c}$, $p_{1c} = 0.34$. Eq. 6 is similar to Eq. 5 except there is a prefactor p_1. From Supplementary Figure 2 one can tell that with sufficiently small p_1, one can always eliminate nontrivial solution. So if $\alpha_{11} < \alpha_{11c}$, Eq. 5 has no nontrivial solutions, while with sufficiently small but

finite p_1 one can make Eq. 6's solution trivial, thus p_{1c} is finite; If $\alpha_{11} > \alpha_{11c}$, the second equation of Eq. 4 with $x = 0$ always has nontrivial solutions, therefore p_{1c} becomes indefinite; If $\alpha_{11} = \alpha_{11c}$, one finds $p_{1c} = 0.34$. The change of α_{11} leads to an abrupt change of the transition point p_{1c} from finiteness to indefiniteness. However if $\beta_{11} \neq 1$, we no longer have Eq. 5. The equations for y also involves x, α_{11} can continuously drops to 0 before nontrivial solution appears.

Supplementary Note 5 Simulations

In this subsection we describe simulation details. To generate networks with community structure, we first generate a Poisson degree distribution for each network. Next we assign each node with a probability of being connected by a randomly chosen link. The probability is proportional to the node's degree. Last we loop over all links and assign each of them with two ends according to nodes' probabilities. For higher probability nodes, they are more likely to be connected by the links. The actual number of links they are connected are proportional to their preassigned degrees. Hence the final network recovers the degree distribution. In order to obtain community structure, we use the same protocol for both intra and inter links. To control the community structure, the number of intra and inter links are evaluated in advance. We generate random networks with size $N_1 = N_2 = 10,000$. To calculate giant component size after random or community attacks, we perform at least 10 realizations for each parameter set and take the average.

To find the giant components in our interdependent system, we first look for the largest percolation cluster in the networks without considering the community structure. We later categorize the nodes in the cluster by their associated communities. The giant component in each community is therefore obtained. Specifically, if the communities are disconnected (localized), we look for the largest cluster in each disconnected communities. Therefore we allow multiple components in our final giant component. As in the real system, localized community can be self supportive without relying on external resources.

Supplementary Note 6 Find the p_{ic} by simulations

Binary searching algorithm was employed to calculate the critical point p_{ic} for both numerical and simulation results. At the beginning, we setup $p_{ic}^0 = 0$ and $p_{ic}^1 = 1$ and $p_{ic} = \frac{p_{ic}^0 + p_{ic}^1}{2}$. If $\mu^\infty(p_{ic}) > c$, $p_{ic}^1 = p_{ic}$, otherwise, $p_{ic}^0 = p_{ic}$. By the iterations, the above algorithm allows us to find the critical point p_{ic}. It is well-know that, binary searching algorithm convergences to the fix point exponentially. It implies that the critical point can be found efficiently. For the numerical and simulation code, we setup $c = 10^{-8}$ and 0.01 respectively.

Supplementary Note 7 How to change community strength α_{11}

Supplementary Figure 3 shows a typical case of how we vary community structure to interdependent networks. After the reconnecting the links, the nodes' degrees preserve and we also make the reconnected network as close as possible to the original one.

Supplementary Note 8 Empirical studies

To manually change community structure of an empirical network, we use the same protocol described in Supplementary Note 7 to regenerate the links in the network. To control the community structure, we evaluate in advance the number of inter-links and intra-links so that the average degree and community index would be obeyed in the following process. Lastly, we assign two nodes to each link to form the network. We do so according to the link property. If the link is an inter-link, we assign nodes of different communities to it; while if the link is an intra-link of community 1, we restrict the selection of nodes to community 1 during the process. To minimize statistical error during the manipulation of empirical data, we perform 100 independent simulations by randomly carrying out the above process, and take the average of any measurement we need.

Supplementary Note 9 Partial dependency case

Here we have also empirically studied the partial interdependent case [2]. For the North America-Asia Firm-Flight interdependent networks, we setup the interdependent ratio between the nodes in two layer to be $Q < 1$, which

is more realistic. It means that a failed node in one layer leads to the failure of the same node in other layer with probability Q. We also detect the abrupt change of p_{1c} when $Q = 0.9$ shown in Supplementary Figure 4.

Supplementary Note 10 Details of the 21 different categories for business networks

1. AGRICULTURE, FORESTRY AND FISHING
Crop and animal production, hunting and related service activities
Growing of non-perennial crops
Growing of cereals (except rice), leguminous crops and oil seeds
Growing of rice
Growing of vegetables and melons, roots and tubers
Growing of sugar cane
Growing of tobacco
Growing of fibre crops
Growing of other non-perennial crops
Growing of perennial crops
Growing of grapes
Growing of tropical and subtropical fruits
Growing of citrus fruits
Growing of pome fruits and stone fruits
Growing of other tree and bush fruits and nuts
Growing of oleaginous fruits
Growing of beverage crops
Growing of spices, aromatic, drug and pharmaceutical crops
Growing of other perennial crops
Plant propagation
Animal production
Raising of dairy cattle
Raising of other cattle and buffaloes
Raising of horses and other equines
Raising of camels and camelids
Raising of sheep and goats
Raising of swine/pigs
Raising of poultry
Raising of other animals
Mixed farming
Support activities to agriculture and post-harvest crop activities
Support activities for crop production
Support activities for animal production
Post-harvest crop activities
Seed processing for propagation
Hunting, trapping and related service activities
Forestry and logging
Silviculture and other forestry activities
Logging
Gathering of wild growing non-wood products
Support services to forestry
Fishing and aquaculture

Fishing
Marine fishing
Freshwater fishing
Aquaculture
Marine aquaculture
Freshwater aquaculture

2. MINING AND QUARRYING

Mining of coal and lignite
Mining of hard coal
Mining of lignite
Extraction of crude petroleum and natural gas
Extraction of crude petroleum
Extraction of natural gas
Mining of metal ores
Mining of iron ores
Mining of non-ferrous metal ores
Mining of uranium and thorium ores
Mining of other non-ferrous metal ores
Other mining and quarrying
Quarrying of stone, sand and clay
Quarrying of ornamental and building stone, limestone, gypsum, chalk and slate
Operation of gravel and sand pits; mining of clays and kaolin
Mining and quarrying n.e.c.
Mining of chemical and fertiliser minerals
Extraction of peat
Extraction of salt
Other mining and quarrying n.e.c.
Mining support service activities
Support activities for petroleum and natural gas extraction
Support activities for other mining and quarrying

3. MANUFACTURING

Manufacture of food products
Processing and preserving of meat and production of meat products
Processing and preserving of meat
Processing and preserving of poultry meat
Production of meat and poultry meat products
Processing and preserving of fish, crustaceans and molluscs
Processing and preserving of fruit and vegetables
Processing and preserving of potatoes
Manufacture of fruit and vegetable juice
Other processing and preserving of fruit and vegetables
Manufacture of vegetable and animal oils and fats
Manufacture of oils and fats
Manufacture of margarine and similar edible fats
Manufacture of dairy products
Operation of dairies and cheese making
Manufacture of ice cream
Manufacture of grain mill products, starches and starch products
Manufacture of grain mill products
Manufacture of starches and starch products

Manufacture of bakery and farinaceous products
Manufacture of bread; manufacture of fresh pastry goods and cakes
Manufacture of rusks and biscuits; manufacture of preserved pastry goods and cakes
Manufacture of macaroni, noodles, couscous and similar farinaceous products
Manufacture of other food products
Manufacture of sugar
Manufacture of cocoa, chocolate and sugar confectionery
Processing of tea and coffee
Manufacture of condiments and seasonings
Manufacture of prepared meals and dishes
Manufacture of homogenised food preparations and dietetic food
Manufacture of other food products n.e.c.
Manufacture of prepared animal feeds
Manufacture of prepared feeds for farm animals
Manufacture of prepared pet foods
Manufacture of beverages
Distilling, rectifying and blending of spirits
Manufacture of wine from grape
Manufacture of cider and other fruit wines
Manufacture of other non-distilled fermented beverages
Manufacture of beer
Manufacture of malt
Manufacture of soft drinks; production of mineral waters and other bottled waters
Manufacture of tobacco products
Manufacture of textiles
Preparation and spinning of textile fibres
Weaving of textiles
Finishing of textiles
Manufacture of other textiles
Manufacture of knitted and crocheted fabrics
Manufacture of made-up textile articles, except apparel
Manufacture of carpets and rugs
Manufacture of cordage, rope, twine and netting
Manufacture of non-wovens and articles made from non-wovens, except apparel
Manufacture of other technical and industrial textiles
Manufacture of other textiles n.e.c.
Manufacture of wearing apparel
Manufacture of wearing apparel, except fur apparel
Manufacture of leather clothes
Manufacture of workwear
Manufacture of other outerwear
Manufacture of underwear
Manufacture of other wearing apparel and accessories
Manufacture of articles of fur
Manufacture of knitted and crocheted apparel
Manufacture of knitted and crocheted hosiery
Manufacture of other knitted and crocheted apparel
Manufacture of leather and related products
Tanning and dressing of leather; manufacture of luggage, handbags, saddlery and harness
Tanning and dressing of leather; dressing and dyeing of fur
Manufacture of luggage, handbags and the like, saddlery and harness
Manufacture of footwear

Manufacture of wood and of products of wood and cork, except furniture
Sawmilling and planing of wood
Manufacture of products of wood, cork, straw and plaiting materials
Manufacture of veneer sheets and wood-based panels
Manufacture of assembled parquet floors
Manufacture of other builders' carpentry and joinery
Manufacture of wooden containers
Manufacture of other products of wood
Manufacture of paper and paper products
Manufacture of pulp, paper and paperboard
Manufacture of pulp
Manufacture of paper and paperboard
Manufacture of articles of paper and paperboard
Manufacture of corrugated paper and paperboard and of containers of paper and paperboard
Manufacture of household and sanitary goods and of toilet requisites
Manufacture of paper stationery
Manufacture of wallpaper
Manufacture of other articles of paper and paperboard
Printing and reproduction of recorded media
Printing and service activities related to printing
Printing of newspapers
Other printing
Pre-press and pre-media services
Binding and related services
Reproduction of recorded media
Manufacture of coke and refined petroleum products
Manufacture of coke oven products
Manufacture of refined petroleum products
Manufacture of chemicals and chemical products
Manufacture of basic chemicals, fertilisers and nitrogen compounds
Manufacture of industrial gases
Manufacture of dyes and pigments
Manufacture of other inorganic basic chemicals
Manufacture of paints, varnishes and similar coatings, printing ink and mastics
Manufacture of soap and detergents, cleaning and polishing preparations
Manufacture of perfumes and toilet preparations
Manufacture of other chemical products
Manufacture of explosives
Manufacture of glues
Manufacture of essential oils
Manufacture of other chemical products n.e.c.
Manufacture of man-made fibres
Manufacture of basic pharmaceutical products and pharmaceutical preparations
Manufacture of basic pharmaceutical products
Manufacture of pharmaceutical preparations
Manufacture of rubber and plastic products
Manufacture of rubber products
Manufacture of rubber tyres and tubes; retreading and rebuilding of rubber tyres
Manufacture of other rubber products
Manufacture of plastic products
Manufacture of plastic plates, sheets, tubes and profiles
Manufacture of plastic packing goods

Manufacture of builders
Manufacture of other plastic products
Manufacture of other non-metallic mineral products
Manufacture of glass and glass products
Manufacture of flat glass
Shaping and processing of flat glass
Manufacture of hollow glass
Manufacture of glass fibres
Manufacture and processing of other glass, including technical glassware
Manufacture of refractory products
Manufacture of clay building materials
Manufacture of ceramic tiles and flags
Manufacture of bricks, tiles and construction products, in baked clay
Manufacture of other porcelain and ceramic products
Manufacture of ceramic household and ornamental articles
Manufacture of ceramic sanitary fixtures
Manufacture of ceramic insulators and insulating fittings
Manufacture of other technical ceramic products
Manufacture of other ceramic products
Manufacture of cement, lime and plaster
Manufacture of cement
Manufacture of lime and plaster
Manufacture of articles of concrete, cement and plaster
Manufacture of concrete products for construction purposes
Manufacture of plaster products for construction purposes
Manufacture of ready-mixed concrete
Manufacture of mortars
Manufacture of fibre cement
Manufacture of other articles of concrete, plaster and cement
Cutting, shaping and finishing of stone
Manufacture of abrasive products and non-metallic mineral products n.e.c.
Production of abrasive products
Manufacture of other non-metallic mineral products n.e.c.
Manufacture of basic metals
Manufacture of basic iron and steel and of ferro-alloys
Manufacture of basic iron and steel and of ferro-alloys
Manufacture of tubes, pipes, hollow profiles and related fittings, of steel
Manufacture of other products of first processing of steel
Cold drawing of bars
Cold rolling of narrow strip
Cold forming or folding
Cold drawing of wire
Manufacture of basic precious and other non-ferrous metals
Precious metals production
Aluminium production
Lead, zinc and tin production
Copper production
Other non-ferrous metal production
Processing of nuclear fuel
Casting of metals
Casting of iron
Casting of steel

Casting of light metals
Casting of other non-ferrous metals
Manufacture of fabricated metal products, except machinery and equipment
Manufacture of structural metal products
Manufacture of metal structures and parts of structures
Manufacture of doors and windows of metal
Manufacture of tanks, reservoirs and containers of metal
Manufacture of central heating radiators and boilers
Manufacture of other tanks, reservoirs and containers of metal
Manufacture of steam generators, except central heating hot water boilers
Manufacture of weapons and ammunition
Forging, pressing, stamping and roll-forming of metal; powder metallurgy
Treatment and coating of metals; machining
Treatment and coating of metals
Machining
Manufacture of cutlery, tools and general hardware
Manufacture of cutlery
Manufacture of locks and hinges
Manufacture of tools
Manufacture of other fabricated metal products
Manufacture of steel drums and similar containers
Manufacture of light metal packaging
Manufacture of wire products, chain and springs
Manufacture of fasteners and screw machine products
Manufacture of other fabricated metal products n.e.c.
Manufacture of computer, electronic and optical products
Manufacture of electronic components and boards
Manufacture of electronic components
Manufacture of loaded electronic boards
Manufacture of computers and peripheral equipment
Manufacture of communication equipment
Manufacture of consumer electronics
Manufacture of instruments and appliances for measuring, testing and navigation
Manufacture of watches and clocks
Manufacture of irradiation, electromedical and electrotherapeutic equipment
Manufacture of optical instruments and photographic equipment
Manufacture of magnetic and optical media
Manufacture of electrical equipment
Manufacture of electric motors, generators, transformers and electricity distribution
Manufacture of electric motors, generators and transformers
Manufacture of electricity distribution and control apparatus
Manufacture of batteries and accumulators
Manufacture of wiring and wiring devices
Manufacture of fibre optic cables
Manufacture of other electronic and electric wires and cables
Manufacture of wiring devices
Manufacture of electric lighting equipment
Manufacture of domestic appliances
Manufacture of electric domestic appliances
Manufacture of non-electric domestic appliances
Manufacture of other electrical equipment
Manufacture of machinery and equipment n.e.c.

Manufacture of general-purpose machinery
Manufacture of engines and turbines, except aircraft, vehicle and cycle engines
Manufacture of fluid power equipment
Manufacture of other pumps and compressors
Manufacture of other taps and valves
Manufacture of bearings, gears, gearing and driving elements
Manufacture of other general-purpose machinery
Manufacture of ovens, furnaces and furnace burners
Manufacture of lifting and handling equipment
Manufacture of office machinery and equipment
Manufacture of power-driven hand tools
Manufacture of non-domestic cooling and ventilation equipment
Manufacture of other general-purpose machinery n.e.c.
Manufacture of agricultural and forestry machinery
Manufacture of metal forming machinery and machine tools
Manufacture of metal forming machinery
Manufacture of other machine tools
Manufacture of other special-purpose machinery
Manufacture of machinery for metallurgy
Manufacture of machinery for mining, quarrying and construction
Manufacture of machinery for food, beverage and tobacco processing
Manufacture of machinery for textile, apparel and leather production
Manufacture of machinery for paper and paperboard production
Manufacture of plastics and rubber machinery
Manufacture of other special-purpose machinery n.e.c.
Manufacture of motor vehicles, trailers and semi-trailers
Manufacture of motor vehicles
Manufacture of bodies (coachwork) for motor vehicles
Manufacture of parts and accessories for motor vehicles
Manufacture of electrical and electronic equipment for motor vehicles
Manufacture of other parts and accessories for motor vehicles
Manufacture of other transport equipment
Building of ships and boats
Building of ships and floating structures
Building of pleasure and sporting boats
Manufacture of railway locomotives and rolling stock
Manufacture of air and spacecraft and related machinery
Manufacture of military fighting vehicles
Manufacture of transport equipment n.e.c.
Manufacture of motorcycles
Manufacture of bicycles and invalid carriages
Manufacture of other transport equipment n.e.c.
Manufacture of furniture
Manufacture of office and shop furniture
Manufacture of kitchen furniture
Manufacture of mattresses
Manufacture of other furniture
Other manufacturing
Manufacture of jewellery, bijouterie and related articles
Striking of coins
Manufacture of jewellery and related articles
Manufacture of imitation jewellery and related articles

Manufacture of musical instruments
Manufacture of sports goods
Manufacture of games and toys
Manufacture of medical and dental instruments and supplies
Manufacturing n.e.c.
Manufacture of brooms and brushes
Other manufacturing n.e.c.
Repair and installation of machinery and equipment
Repair of fabricated metal products, machinery and equipment
Repair of fabricated metal products
Repair of machinery
Repair of electronic and optical equipment
Repair of electrical equipment
Repair and maintenance of ships and boats
Repair and maintenance of aircraft and spacecraft
Repair and maintenance of other transport equipment
Repair of other equipment
Installation of industrial machinery and equipment

4. ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY

Electricity, gas, steam and air conditioning supply
Electric power generation, transmission and distribution
Production of electricity
Transmission of electricity
Distribution of electricity
Trade of electricity
Manufacture of gas; distribution of gaseous fuels through mains
Manufacture of gas
Distribution of gaseous fuels through mains
Trade of gas through mains
Steam and air conditioning supply

5. WATER SUPPLY; WASTE MANAGEMENT AND REMEDIATION ACTIVITIES

Water collection, treatment and supply
Sewerage
Waste collection, treatment and disposal activities; materials recovery
Waste collection
Collection of non-hazardous waste
Collection of hazardous waste
Waste treatment and disposal
Treatment and disposal of non-hazardous waste
Treatment and disposal of hazardous waste
Materials recovery
Dismantling of wrecks
Recovery of sorted materials
Remediation activities and other waste management services

6. CONSTRUCTION

Construction of buildings
Development of building projects
Construction of residential and non-residential buildings
Civil engineering

Construction of roads and railways
Construction of roads and motorways
Construction of railways and underground railways
Construction of bridges and tunnels
Construction of utility projects
Construction of utility projects for fluids
Construction of utility projects for electricity and telecommunications
Construction of other civil engineering projects
Construction of water projects
Construction of other civil engineering projects n.e.c.
Specialised construction activities
Demolition and site preparation
Demolition
Site preparation
Test drilling and boring
Electrical, plumbing and other construction installation activities
Electrical installation
Plumbing, heat and air-conditioning installation
Other construction installation
Building completion and finishing
Plastering
Joinery installation
Floor and wall covering
Painting and glazing
Other building completion and finishing
Other specialised construction activities
Roofing activities
Other specialised construction activities n.e.c.

7. WHOLESALE AND RETAIL TRADE; REPAIR OF MOTOR VEHICLES

Wholesale and retail trade and repair of motor vehicles and motorcycles
Sale of motor vehicles
Sale of cars and light motor vehicles
Sale of other motor vehicles
Maintenance and repair of motor vehicles
Sale of motor vehicle parts and accessories
Wholesale trade of motor vehicle parts and accessories
Retail trade of motor vehicle parts and accessories
Sale, maintenance and repair of motorcycles and related parts and accessories
Wholesale trade, except of motor vehicles and motorcycles
Wholesale on a fee or contract basis
Agents involved in the sale of agricultural raw materials, live animals, textile raw materials
Agents involved in the sale of fuels, ores, metals and industrial chemicals
Agents involved in the sale of timber and building materials
Agents involved in the sale of machinery, industrial equipment, ships and aircraft
Agents involved in the sale of furniture, household goods, hardware and ironmongery
Agents involved in the sale of textiles, clothing, fur, footwear and leather goods
Agents involved in the sale of food, beverages and tobacco
Agents specialised in the sale of other particular products
Agents involved in the sale of a variety of goods
Wholesale of agricultural raw materials and live animals
Wholesale of grain, unmanufactured tobacco, seeds and animal feeds

Wholesale of flowers and plants
Wholesale of live animals
Wholesale of hides, skins and leather
Wholesale of food, beverages and tobacco
Wholesale of fruit and vegetables
Wholesale of meat and meat products
Wholesale of dairy products, eggs and edible oils and fats
Wholesale of beverages
Wholesale of tobacco products
Wholesale of sugar and chocolate and sugar confectionery
Wholesale of coffee, tea, cocoa and spices
Wholesale of other food, including fish, crustaceans and molluscs
Non-specialised wholesale of food, beverages and tobacco
Wholesale of household goods
Wholesale of textiles
Wholesale of clothing and footwear
Wholesale of electrical household appliances
Wholesale of china and glassware and cleaning materials
Wholesale of perfume and cosmetics
Wholesale of pharmaceutical goods
Wholesale of furniture, carpets and lighting equipment
Wholesale of watches and jewellery
Wholesale of other household goods
Wholesale of information and communication equipment
Wholesale of computers, computer peripheral equipment and software
Wholesale of electronic and telecommunications equipment and parts
Wholesale of other machinery, equipment and supplies
Wholesale of agricultural machinery, equipment and supplies
Wholesale of machine tools
Wholesale of mining, construction and civil engineering machinery
Wholesale of machinery for the textile industry and of sewing and knitting machines
Wholesale of office furniture
Wholesale of other office machinery and equipment
Wholesale of other machinery and equipment
Other specialised wholesale
Wholesale of solid, liquid and gaseous fuels and related products
Wholesale of metals and metal ores
Wholesale of wood, construction materials and sanitary equipment
Wholesale of hardware, plumbing and heating equipment and supplies
Wholesale of chemical products
Wholesale of other intermediate products
Wholesale of waste and scrap
Non-specialised wholesale trade
Retail trade, except of motor vehicles and motorcycles
Retail sale in non-specialised stores
Retail sale in non-specialised stores with food, beverages or tobacco predominating
Other retail sale in non-specialised stores
Retail sale of food, beverages and tobacco in specialised stores
Retail sale of fruit and vegetables in specialised stores
Retail sale of meat and meat products in specialised stores
Retail sale of fish, crustaceans and molluscs in specialised stores
Retail sale of bread, cakes, flour confectionery and sugar confectionery in specialised stores

Retail sale of beverages in specialised stores
Retail sale of tobacco products in specialised stores
Other retail sale of food in specialised stores
Retail sale of automotive fuel in specialised stores
Retail sale of information and communication equipment in specialised stores
Retail sale of computers, peripheral units and software in specialised stores
Retail sale of telecommunications equipment in specialised stores
Retail sale of audio and video equipment in specialised stores
Retail sale of other household equipment in specialised stores
Retail sale of textiles in specialised stores
Retail sale of hardware, paints and glass in specialised stores
Retail sale of carpets, rugs, wall and floor coverings in specialised stores
Retail sale of electrical household appliances in specialised stores
Retail sale of furniture, lighting equipment and other household articles in specialised stores
Retail sale of cultural and recreation goods in specialised stores
Retail sale of books in specialised stores
Retail sale of newspapers and stationery in specialised stores
Retail sale of music and video recordings in specialised stores
Retail sale of sporting equipment in specialised stores
Retail sale of games and toys in specialised stores
Retail sale of other goods in specialised stores
Retail sale of clothing in specialised stores
Retail sale of footwear and leather goods in specialised stores
Dispensing chemist in specialised stores
Retail sale of medical and orthopaedic goods in specialised stores
Retail sale of cosmetic and toilet articles in specialised stores
Retail sale of flowers, plants, seeds, pet animals and pet food in specialised stores
Retail sale of watches and jewellery in specialised stores
Other retail sale of new goods in specialised stores
Retail sale of second-hand goods in stores
Retail sale via stalls and markets
Retail sale via stalls and markets of food, beverages and tobacco products
Retail sale via stalls and markets of textiles, clothing and footwear
Retail sale via stalls and markets of other goods
Retail trade not in stores, stalls or markets
Retail sale via mail order houses or via Internet
Other retail sale not in stores, stalls or markets

8. TRANSPORTATION AND STORAGE

Land transport and transport via pipelines
Passenger rail transport, interurban
Freight rail transport
Other passenger land transport
Urban and suburban passenger land transport
Taxi operation
Other passenger land transport n.e.c.
Freight transport by road and removal services
Freight transport by road
Removal services
Transport via pipeline
Water transport
Sea and coastal freight water transport

Inland freight water transport
Air transport
Passenger air transport
Freight air transport and space transport
Freight air transport
Space transport
Warehousing and support activities for transportation
Warehousing and storage
Support activities for transportation
Service activities incidental to land transportation
Service activities incidental to water transportation
Service activities incidental to air transportation
Cargo handling
Other transportation support activities
Postal and courier activities
Postal activities under universal service obligation
Other postal and courier activities

9. ACCOMMODATION AND FOOD SERVICE ACTIVITIES

Accommodation
Hotels and similar accommodation
Holiday and other short-stay accommodation
Camping grounds, recreational vehicle parks and trailer parks
Other accommodation
Food and beverage service activities
Restaurants and mobile food service activities
Event catering and other food service activities
Event catering activities
Other food service activities
Beverage serving activities

10. INFORMATION AND COMMUNICATION

Publishing activities
Publishing of books, periodicals and other publishing activities
Book publishing
Publishing of directories and mailing lists
Publishing of newspapers
Publishing of journals and periodicals
Other publishing activities
Software publishing
Publishing of computer games
Other software publishing
Motion picture, video and television programme production and music publishing activities
Motion picture, video and television programme activities
Motion picture, video and television programme production activities
Motion picture, video and television programme post-production activities
Motion picture, video and television programme distribution activities
Motion picture projection activities
Sound recording and music publishing activities
Programming and broadcasting activities
Radio broadcasting
Television programming and broadcasting activities

Telecommunications
Wired telecommunications activities
Wireless telecommunications activities
Satellite telecommunications activities
Other telecommunications activities
Computer programming, consultancy and related activities
Computer programming activities
Computer consultancy activities
Computer facilities management activities
Other information technology and computer service activities
Information service activities
Data processing, hosting and related activities; web portals
Data processing, hosting and related activities
Web portals
Other information service activities
News agency activities
Other information service activities n.e.c.

11. FINANCIAL AND INSURANCE ACTIVITIES

Financial service activities, except insurance and pension funding
Monetary intermediation
Central banking
Other monetary intermediation
Activities of holding companies
Trusts, funds and similar financial entities
Other financial service activities, except insurance and pension funding
Financial leasing
Other credit granting
Other financial service activities, except insurance and pension funding n.e.c.
Insurance, reinsurance and pension funding, except compulsory social security
Insurance
Life insurance
Non-life insurance
Reinsurance
Pension funding
Activities auxiliary to financial services and insurance activities
Activities auxiliary to financial services, except insurance and pension funding
Administration of financial markets
Security and commodity contracts brokerage
Other activities auxiliary to financial services, except insurance and pension funding
Activities auxiliary to insurance and pension funding
Risk and damage evaluation
Activities of insurance agents and brokers
Other activities auxiliary to insurance and pension funding
Fund management activities

12. REAL ESTATE ACTIVITIES

Real estate activities
Buying and selling of own real estate
Renting and operating of own or leased real estate
Real estate activities on a fee or contract basis
Real estate agencies

Management of real estate on a fee or contract basis

13. PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES

Legal and accounting activities
Legal activities
Accounting, bookkeeping and auditing activities; tax consultancy
Activities of head offices; management consultancy activities
Activities of head offices
Management consultancy activities
Public relations and communication activities
Business and other management consultancy activities
Architectural and engineering activities; technical testing and analysis
Architectural and engineering activities and related technical consultancy
Architectural activities
Engineering activities and related technical consultancy
Technical testing and analysis
Scientific research and development
Research and experimental development on natural sciences and engineering
Research and experimental development on biotechnology
Other research and experimental development on natural sciences and engineering
Research and experimental development on social sciences and humanities
Advertising and market research
Advertising
Advertising agencies
Media representation
Market research and public opinion polling
Other professional, scientific and technical activities
Specialised design activities
Photographic activities
Translation and interpretation activities
Other professional, scientific and technical activities n.e.c.
Veterinary activities

14. ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES

Rental and leasing activities
Renting and leasing of motor vehicles
Renting and leasing of cars and light motor vehicles
Renting and leasing of trucks
Renting and leasing of personal and household goods
Renting and leasing of recreational and sports goods
Renting of video tapes and disks
Renting and leasing of other personal and household goods
Renting and leasing of other machinery, equipment and tangible goods
Renting and leasing of agricultural machinery and equipment
Renting and leasing of construction and civil engineering machinery and equipment
Renting and leasing of office machinery and equipment (including computers)
Renting and leasing of water transport equipment
Renting and leasing of air transport equipment
Renting and leasing of other machinery, equipment and tangible goods n.e.c.
Leasing of intellectual property and similar products, except copyrighted works
Employment activities
Activities of employment placement agencies

Temporary employment agency activities
Other human resources provision
Travel agency, tour operator and other reservation service and related activities
Travel agency and tour operator activities
Travel agency activities
Tour operator activities
Other reservation service and related activities
Security and investigation activities
Private security activities
Security systems service activities
Investigation activities
Services to buildings and landscape activities
Combined facilities support activities
Cleaning activities
General cleaning of buildings
Other building and industrial cleaning activities
Other cleaning activities
Landscape service activities
Office administrative, office support and other business support activities
Office administrative and support activities
Combined office administrative service activities
Photocopying, document preparation and other specialised office support activities
Activities of call centres
Organisation of conventions and trade shows
Business support service activities n.e.c.
Activities of collection agencies and credit bureaus
Packaging activities
Other business support service activities n.e.c.

15. PUBLIC ADMINISTRATION AND DEFENCE; COMPULSORY SOCIAL SECURITY

Public administration and defence; compulsory social security
Administration of the State and the economic and social policy of the community
General public administration activities
Regulation of the activities of providing health care and other social services
Regulation of and contribution to more efficient operation of businesses
Provision of services to the community as a whole
Foreign affairs
Defence activities
Justice and judicial activities
Public order and safety activities
Fire service activities
Compulsory social security activities

16. EDUCATION

Education
Pre-primary education
Pre-primary education
Primary education
Primary education
Secondary education
General secondary education
Technical and vocational secondary education

Higher education
Post-secondary non-tertiary education
Tertiary education
Other education
Sports and recreation education
Cultural education
Driving school activities
Other education n.e.c.
Educational support activities

17. HUMAN HEALTH AND SOCIAL WORK ACTIVITIES

Human health activities
Hospital activities
Medical and dental practice activities
General medical practice activities
Specialist medical practice activities
Dental practice activities
Other human health activities
Residential care activities
Residential nursing care activities
Residential care activities for mental retardation, mental health and substance abuse
Residential care activities for the elderly and disabled
Other residential care activities
Social work activities without accommodation
Social work activities without accommodation for the elderly and disabled
Other social work activities without accommodation
Child day-care activities
Other social work activities without accommodation n.e.c.

18. ARTS, ENTERTAINMENT AND RECREATION

Creative, arts and entertainment activities
Performing arts
Support activities to performing arts
Artistic creation
Operation of arts facilities
Libraries, archives, museums and other cultural activities
Library and archives activities
Museums activities
Operation of historical sites and buildings and similar visitor attractions
Botanical and zoological gardens and nature reserves activities
Gambling and betting activities
Sports activities and amusement and recreation activities
Sports activities
Operation of sports facilities
Activities of sports clubs
Fitness facilities
Other sports activities
Amusement and recreation activities
Activities of amusement parks and theme parks
Other amusement and recreation activities

19. OTHER SERVICE ACTIVITIES

Activities of membership organisations
Activities of business, employers and professional membership organisations
Activities of business and employers membership organisations
Activities of professional membership organisations
Activities of trade unions
Activities of other membership organisations
Activities of religious organisations
Activities of political organisations
Activities of other membership organisations n.e.c.
Repair of computers and personal and household goods
Repair of computers and communication equipment
Repair of computers and peripheral equipment
Repair of communication equipment
Repair of personal and household goods
Repair of consumer electronics
Repair of household appliances and home and garden equipment
Repair of footwear and leather goods
Repair of furniture and home furnishings
Repair of watches, clocks and jewellery
Repair of other personal and household goods
Other personal service activities
Washing and (dry-)cleaning of textile and fur products
Hairdressing and other beauty treatment
Funeral and related activities
Physical well-being activities
Other personal service activities n.e.c.

20. ACTIVITIES OF HOUSEHOLDS; UNDIFFERENTIATED GOODS AND SERVICE
Activities of households as employers of domestic personnel
Undifferentiated goods- and services-producing activities of private households for own use
Undifferentiated service-producing activities of private households for own use

21. ACTIVITIES OF EXTRATERRITORIAL ORGANISATIONS AND BODIES
Activities of extraterritorial organisations and bodies

Supplementary References

[1] Sergey V Buldyrev, Roni Parshani, Gerald Paul, H Eugene Stanley, and Shlomo Havlin. Catastrophic cascade of failures in interdependent networks. *Nature*, 464:1025–8, 2010.

[2] R. Parshani, Sergey V. Buldyrev, and Shlomo Havlin. Interdependent networks: Reducing the coupling strength leads to a change from a first to second order percolation transition. *Phys. Rev. Lett.*, 105(048701), 2010.