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Review of Economic Dynamics 6 (2003) 672–684

Review of Economic Dynamics

www.elsevier.com/locate/red

Recursive utility, endogenous growth, and the welfare cost of volatility

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Received 13 October 1999; revised 3 April 2002

Abstract

This paper proposes a measure of the welfare cost of volatility derived from a stochastic endogenous growth model extended to the case of a recursive utility function which disentangles risk aversion from intertemporal elasticity of substitution. The measure of the welfare cost of volatility takes into account not only the direct effect of volatility on expected utility but also the link between volatility and growth. It thus encompasses a direct welfare cost of fluctuations and a welfare cost due to the endogeneity of the consumption. We obtain a closed form solution for these two costs and show that the total welfare cost of volatility increases with both the risk aversion and the intertemporal elasticity of substitution. For plausible values of the agent's preference parameters, the cost of volatility may be greater than measures based on an exogenous process for consumption. However, when applied to the US economy, our measure shows little differences compared with the one derived under the assumption that the consumption process is exogenous. Yet, we show that this may not be the case for more volatile economies.

JEL classification: E32; E60

Keywords: Welfare cost of fluctuations; Recursive utility; Endogenous growth; Risk

1. Introduction

This paper revisits the following question: would we benefit, and by how much, from reducing all macroeconomic shocks and surprises? Lucas (1987) has provided a well-

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1094-2025/03/$ – see front matter
doi:10.1016/S1094-2025(03)00016-4

known response to this question: whenever it could be possible to eliminate fluctuations in consumption, the gain in terms of welfare would be negligible compared with what could be achieved with more growth. Thus economists should look for ways to attain higher growth rates rather than for economic policies to reduce fluctuations in consumption.

As this position is in conflict with both the actual practice of short-term economic policies and the textbook view for which, following Musgrave, stabilization is one of the three goals of political economy, many economists have proposed other measures of the welfare cost of fluctuations. One way of challenging Lucas's conclusion is to relax some of the assumptions under which he performed his computation. For example, Imrohoroglu (1989) and Atkeson and Phelan (1994) measure the welfare cost of fluctuations for heterogeneous agents in the absence of a perfect insurance market. Preferences of the representative agent have also been reconsidered. Campbell and Cochrane (1995) investigate the case of habit formation while Obstfeld (1994a), Tallarini (1997), and Dolmas (1998) use a recursive utility function. Dolmas (1998) also considers the case of rank-dependent preferences. These studies generally lead to a slightly larger cost of fluctuations but still too small to refute Lucas's conclusion (Dolmas (1998) is an exception). However, even if most of them depart from Lucas's assumption of consumption transitory shocks, they share the hypothesis of an exogenous process for consumption. Otrok (2001) revisits the welfare cost of fluctuations in a complete business cycle model in which consumption is endogenous for various kinds of preferences. His conclusion is that the cost of business cycles is not much larger than Lucas's estimate.

This paper explores the idea that the link between growth and volatility might result into a larger cost of fluctuations. Since in an endogenous growth model, reducing the source of consumption fluctuations modifies the consumption/saving trade-off and thus the growth trend of the entire economy, removing volatility may both smooth consumption and increase growth. The issue of the relationship between volatility and growth has recently been revisited (Ramey and Ramey, 1995; Jones et al., 1999) and Barlevy (2000) considers its consequence for the evaluation of the cost of fluctuations in a model allowing for diminishing returns to investment. Since we use a model which only encompasses a very simple AK technology, we get an analytical resolution which allows us to split up the total welfare cost between a direct welfare cost (i.e., a cost directly resulting from the fluctuations in consumption) and a cost due to the growth endogeneity.[1] As it will be explained further in the paper, the recursive utility function, which disentangles the risk aversion from the intertemporal elasticity of substitution, allows for cases where the consumption/saving trade-off leads to a rather large welfare cost. Nevertheless for reasonable values of the parameters, empirical investigation assesses that, for the United States, the likelihood of a high cost of volatility remains very small. Yet, we show that this may not be the case for more volatile economies.

This paper is organized as follows. Section 2 proposes a straightforward macroeconomic model of endogenous growth under uncertainty extended to the case of a recursive utility function (Smith, 1996). It shows that the growth rate of the economy may decrease with

[1] Our paper may also be viewed as an extension of the idea stressed by Obstfeld (1994b)—who takes into account *both* the reduction of volatility and the induced growth when evaluating the effect of international risk-sharing through capital markets liberalization—to the welfare cost of volatility measurement.

uncertainty. Such a result is also valid in the special case of an additive utility function; the new point is that the size of the effect of volatility on the economy growth rate increases with both the risk aversion and the intertemporal elasticity of substitution. In Section 3, we derive a measure of the total welfare cost of volatility and propose a decomposition of this total cost into a direct cost and a cost due to the trend. Section 4 shows that when evaluating this cost, agents' preferences really do matter: for the USA only very large intertemporal elasticity of substitution and risk aversion give rise to a significant cost while more plausible values for the former parameter confirm Lucas's conclusion. Section 5 concludes the paper.

2. Volatility and growth

2.1. The underlying macroeconomic model

2.1.1. The recursive utility function

The model is in continuous time. The representative agent maximizes a recursive utility function as defined by Weil (1990), Epstein and Zin (1991), and Svensson (1989):

$$\begin{aligned}(1-\gamma)U(t) = \Big[&c(t)^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}t \\ &+ \mathrm{e}^{-\delta\,\mathrm{d}t}\big((1-\gamma)E_t U(t+\mathrm{d}t)\big)^{(\varepsilon-1)/((1-\gamma)\varepsilon)}\Big]^{(1-\gamma)\varepsilon/(\varepsilon-1)}\\ &\text{with } \gamma \neq 1 \text{ and } \varepsilon > 0,\ \varepsilon \neq 1, \qquad (1)\end{aligned}$$

$$\begin{aligned}U(t) = \Big[c(t)^{(\varepsilon-1)/\varepsilon}\mathrm{d}t + \mathrm{e}^{-\delta\,\mathrm{d}t}\big(\mathrm{e}^{E_t(\ln U(t+\mathrm{d}t))}\big)^{(\varepsilon-1)/\varepsilon}\Big]^{\varepsilon/(\varepsilon-1)}\\ \text{with } \gamma = 1 \text{ and } \varepsilon > 0,\ \varepsilon \neq 1, \qquad (2)\end{aligned}$$

where γ is the relative risk aversion coefficient, ε is the intertemporal elasticity of substitution and δ is the time preference rate. $1/\varepsilon$ may also be understood as a measure of the resistance to intertemporal substitution (sometimes called fluctuations aversion).

2.1.2. Technology and volatility

The technology is AK. Technological shocks continuously perturb the production process. Over the period $(t, t+\mathrm{d}t)$ the flow of output is:

$$F(K)\,\mathrm{d}t = K[A\,\mathrm{d}t + \sigma\,\mathrm{d}z], \qquad (3)$$

where $\mathrm{d}z$ is the increment of a standard Wiener process ($\mathrm{d}z = \eta(t)\sqrt{\mathrm{d}t}$; $\eta(t) \sim N(0,1)$). Equation (3) asserts that the flow of output accumulated over the period $(t+\mathrm{d}t)$ consists into two components: a deterministic component $(AK\,\mathrm{d}t)$, and a stochastic component $(K\sigma\,\mathrm{d}z)$ reflecting the random influences that impact on the production. The stochastic term $\sigma\,\mathrm{d}z$ may be referred to as a productivity shock and assumed to be temporally independent, normally distributed with zero mean and variance $\sigma^2\,\mathrm{d}t$. Thus, as far as productivity is concerned, shocks are neither correlated nor persistent. This leads to a stochastic capital accumulation equation:

$$\mathrm{d}K(t) = [AK(t) - C(t)]\,\mathrm{d}t + K(t)\sigma\,\mathrm{d}z(t). \qquad (4)$$

It is clear from (4) that as soon as consumption does not exactly compensate for productivity shocks, these shocks will have persistent effects on the whole economy through capital accumulation.

2.2. The optimal growth rate of consumption and capital

Maximizing (1) under (4) gives the optimal stochastic level of consumption:[2]

$$C^* = \left(\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\frac{\sigma^2}{2}\right)K \tag{5}$$

and we note $c(\sigma)$ the propensity to consume the current wealth (capital).

The common growth rate of consumption, capital, and production is then:

$$\frac{\mathrm{d}K}{K} = \frac{\mathrm{d}C^*}{C^*} = \left[\varepsilon(A-\delta) + (1-\varepsilon)\gamma\sigma^2/2\right]\mathrm{d}t + \sigma\,\mathrm{d}z = \mu(\sigma)\,\mathrm{d}t + \sigma\,\mathrm{d}z. \tag{6}$$

Thus, the optimal consumption follows a geometric Brownian process, that is, it increases according to a deterministic trend $\mu(\sigma)$ per unit of time continuously perturbed by shocks. For our purpose, the interesting results in (6) are that (i) the consumption process exhibits a unit root (the exact discretization of (6) is

$$\ln C_t^* - \ln C_{t-1}^* = \left[\varepsilon(A-\delta) + (1-\varepsilon)\gamma\sigma^2/2 - \sigma^2/2\right] + \sigma\eta_t,$$

where $\eta_t \sim \mathit{NIID}(0,1)$), and (ii) the deterministic trend of the consumption process depends on each structural parameter and also on the size of the uncertainty (σ). It is straightforward to show that all other macroeconomic variables (capital stock and production) grow at this same rate.

Ignoring the volatility, Eq. (5) reduces to the relationship between capital and consumption which may be derived from a standard deterministic AK model: as suggested by intuition, the optimal propensity to consume wealth increases with the time preference rate whereas it decreases with the intertemporal elasticity of substitution, and the risk aversion plays no role.

As far as the effect of volatility is concerned, one may consider the optimal propensity to consume current wealth or equivalently the consumption deterministic trend. The effect of an increase in uncertainty on the latter depends on both the risk-aversion and the intertemporal elasticity of substitution; it is merely:

$$\frac{\partial\mu(\sigma)}{\partial\sigma^2} = -\frac{\partial c(\sigma)}{\partial\sigma^2} = (1-\varepsilon)\frac{\gamma}{2}. \tag{7}$$

When the representative consumer is not too fluctuations-averse ($\varepsilon > 1$), more uncertainty (i.e., a rise of volatility) increases the current marginal propensity to consume current wealth. To escape future uncertainty, she chooses to consume more today and accepts the counterpart of less consumption tomorrow. Following Weil (1990) one may notice that an increase in the volatility reduces the certainty equivalent return on savings ($A - \gamma\sigma^2/2$); the way this reduction affects the consumption/saving trade-off

[2] Cf. Appendix A.

depends on the relative strength of the income and substitution effects. Obviously, for a large intertemporal elasticity of substitution, the latter dominates, leading the agent to increase her current consumption. Thus, more uncertainty reduces the deterministic trend in consumption. Whereas the direction of the volatility effect is governed by the intertemporal elasticity of substitution, its size also depends on the risk aversion: the higher the risk aversion, the larger the effect. Of course, when consumers are reluctant to accept intertemporal substitution ($\varepsilon < 1$), more uncertainty urges them to reduce their current consumption.

As a benchmark, it may be useful to recall what one would have got using the standard time-additive utility function. Since the risk aversion is then the inverse of the intertemporal elasticity of substitution, formula (7) reduces to $\partial\mu/\delta\sigma^2 = (1-\varepsilon)/(2\varepsilon) = (1-\gamma)/2$ and one cannot identify the reason why volatility would increase consumption (small risk aversion or large intertemporal elasticity of substitution?). Furthermore, the higher the risk aversion, the smaller the current consumption, whereas with a recursive utility this is only true when the intertemporal elasticity of substitution is less than unity (see above).

3. The welfare cost of volatility

Following Lucas's seminal monograph, the welfare cost of fluctuations is usually expressed in terms of percentage of consumption the agent is ready to give up *at all dates* to join the deterministic world. But such a measure may be no longer very informative when the agent chooses her propensity to consume. Since the trend in the consumption process may differ as a result of the change in the agent's propensity to consume when reaching the deterministic world, the percentage of loss in consumption varies with respect to time. In fact, for some sets of preference parameters, the agent's consumption may even be higher, after some time spent in the deterministic world, than the one she would have expected at this same time in the stochastic world. Barlevy (2000) chooses to express the welfare cost of fluctuations in terms of initial consumption. This has the advantage of facilitating comparisons with previous measures of the welfare cost of fluctuations. But it may be misleading as well, since an identical cut in consumption may be followed by various consumption trends, and one would wrongly interpret a same initial cut as the fact that the welfare cost is the same. That is the reason why we compute the welfare cost of fluctuations in terms of percentage of the initial capital the agent is ready to give up to join the certain world.

3.1. Deriving a measure of the total welfare cost of volatility

Measuring the cost of volatility in this model requires the evaluation of the expected lifetime utility associated with the optimal consumption path. It is straightforward that this lifetime utility may be evaluated as

$$V^*[K(0);\sigma] = \left[\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2\right]^{(1-\gamma)/(1-\varepsilon)} \frac{K(0)^{1-\gamma}}{1-\gamma}. \tag{8}$$

Definition. [3] The total welfare cost of volatility is defined as the percentage of capital the representative agent is ready to give up at period zero to be as well off in a certain world as she is in a stochastic one:

$$V^*\big[K(0);\sigma\big] = V^*\big[(1-k)K(0);0\big]. \tag{9}$$

Using (8) the total cost of volatility may be written:

$$k = 1 - \left[\frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{\varepsilon\delta + (1-\varepsilon)A}\right]^{1/(1-\varepsilon)}$$
$$\text{if } \gamma > 0 \text{ and } \lim_{\varepsilon\to 1} k = 1 - \mathrm{e}^{-\gamma\sigma^2/(2\delta)}. \tag{10}$$

3.2. *Splitting up the total welfare cost of volatility*

To split up the total welfare cost of fluctuations we will consider three different economies. To the two economies considered above (the stochastic and the deterministic ones), in which the agent is free to choose her saving rate, we add what we call the constrained deterministic economy whereby *constrained* stands for the fact that the agent's propensity to consume is kept equal to the one she had chosen in the stochastic economy. The growth rate of this constrained economy is thus equal to the one expected in the stochastic economy.

Let us now break down the switch from the stochastic economy to the deterministic economy into two successive stages.

The first stage consists in joining the constrained deterministic economy, and we may wonder how much capital the agent is ready to give up to achieve this step. As the two economies only differ in their nature (stochastic/deterministic) and not in their growth trend, one may refer to this cost as to the welfare cost of fluctuations in the narrow sense or as the *direct welfare cost of volatility*. In the following, this cost is noted k_{F}.

The second stage consists in switching from the constrained deterministic economy to the optimal one. The task is now to determine how much remaining capital the agent is ready to give up to have the opportunity to choose her propensity to consume. Since the two economies are deterministic, and differ only in their growth trend, one will refer to this cost as to the *trend-related welfare cost of volatility* which is noted k_{T}.

These costs are built such that: $(1-k) = (1-k_{\mathrm{F}})(1-k_{\mathrm{T}})$.

3.2.1. *The direct welfare cost of volatility*

The intertemporal lifetime utility of the agent in the constrained deterministic economy ($\overline{V}$) is computed using Eq. (1) with consumption growing at the same rate as in the

[3] The definition we used to compute this welfare cost is the compensating variation. One may check that the equivalent variation measure (that is the percentage the agent requires to be as well off under uncertainty as it is in the deterministic environment, noted *keq* below) is always greater than the compensating variation measure of the welfare cost of volatility:

$$keq = \left[\frac{\varepsilon\delta + (1-\varepsilon)(A - \gamma\sigma^2/2)}{\varepsilon\delta + (1-\varepsilon)A}\right]^{1/(\varepsilon-1)} - 1 > k \quad \text{and} \quad \lim_{\varepsilon\to 1} keq = \mathrm{e}^{\gamma\sigma^2/(2\delta)} - 1 > 1 - \mathrm{e}^{-\gamma\sigma^2/(2\delta)}.$$

stochastic economy. Note that this intertemporal utility is no longer the value of the maximized agent's program since her consumption choice is constrained. One may calculate:

$$\overline{V}[K(0);\sigma] = \left[\frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{(\varepsilon\delta + (1-\varepsilon)A + ((1-\varepsilon)^2/\varepsilon)\gamma\sigma^2/2)^{-\varepsilon/(1-\varepsilon)}}\right]^{1-\gamma} \frac{K(0)^{1-\gamma}}{1-\gamma}.$$

The percentage of capital the representative agent is ready to give up to join the constrained deterministic economy is then obtained comparing the lifetime utilities $V^*[K(0);\sigma]$ and $\overline{V}[(1-k_F)K(0);0]$. The direct welfare cost is then such that

$$V^*\big[K(0);\sigma\big] = \overline{V}\big[(1-k_{\mathrm{F}})K(0);0\big], \tag{11}$$

that is

$$k_{\mathrm{F}} = 1 - \left[\frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{\varepsilon\delta + (1-\varepsilon)A + ((1-\varepsilon)^2/\varepsilon)\gamma\sigma^2/2}\right]^{\varepsilon/(1-\varepsilon)} \quad \text{and} \tag{12}$$

$$\lim_{\varepsilon\to 1} k_{\mathrm{F}} = 1 - \mathrm{e}^{-\gamma\sigma^2/(2\gamma)}, \tag{13}$$

which is always positive. Since consumption grows at the same rate in both economies, this direct welfare cost of volatility is the initial (as well as permanent) cut in the agent's consumption. This is the measure derived by Obstfeld (1994a).

3.2.2. The trend-related welfare cost of volatility

Once the agent has reached the constrained deterministic economy, the percentage of capital the representative agent is ready to give up to choose her consumption path is obtained by comparing the corresponding lifetimes utilities $\overline{V}[K(0);0]$ and $V^*[(1-k_{\mathrm{T}})K(0);0]$:

$$\overline{V}\big[K(0);0\big] = V^*\big[(1-k_{\mathrm{T}})K(0);0\big], \tag{14}$$

that is

$$k_{\mathrm{T}} = 1 - \left[\frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{(\varepsilon\delta + (1-\varepsilon)A)^{1/(1-\varepsilon)}}\right] \times \left(\varepsilon\delta + (1-\varepsilon)A + \frac{(1-\varepsilon)^2}{\varepsilon}\gamma\sigma^2/2\right)^{\varepsilon/(1-\varepsilon)} \quad \text{and} \tag{15}$$

$$\lim_{\varepsilon\to 1} k_{\mathrm{F}} = 0. \tag{16}$$

This cost is always positive since the agent switches from a constrained consumption path to an optimal one. In the special case where the intertemporal elasticity of substitution is equal to one, it is null (the constrained path is then the optimal one).

4. Is the welfare cost of volatility negligible?

The point is now to evaluate the three costs defined above in order first, to appraise whether the total welfare cost of volatility may be significant, and second, to know how much the endogeneity of the consumption/saving trade-off matters when evaluating this total welfare cost. For sake of comparison with former proposed measures, our evaluations are conducted on US data for a large range of preference parameters. To calibrate the model, we use econometrics performed by Obstfeld (1994) on annual data (1950–1990) which gives the volatility and the deterministic trend of the nondurable goods and services consumption: $\sigma = 0.0112$ and $\mu = 0.0185$. For each preference parameters set, we compute $A = \mu/\varepsilon + \delta + (\varepsilon - 1)/(2\varepsilon)\gamma\sigma^2$ such that the theoretical model for the stochastic economy (Eq. (6)) matches the actual consumption trend and volatility.[4]

Table 1
Welfare cost of volatility for the US economy

ε	γ				
	1	2	5	10	20
			Total welfare cost k		
0.1	0.03	0.07	0.17	*0.35*	0.71
0.5	0.16	*0.33*	0.81	1.64	3.28
1	*0.31*	0.63	1.56	3.09	6.08
2	0.58	1.16	2.85	5.67	11.0
5	1.22	2.46	5.72	13.5	33.9
			Direct welfare cost k_F		
0.1	0.03	0.07	0.17	0.34	0.69
0.5	0.16	0.33	0.81	1.63	3.26
1	0.31	0.63	1.56	3.09	6.08
2	0.58	1.16	2.85	5.59	10.7
5	1.19	2.36	5.72	10.9	19.8
			Trend-related welfare cost k_T		
0.1	0.00	0.00	0.00	0.00	0.02
0.5	0.00	0.00	0.00	0.00	0.03
1	0	0	0	0	0
2	0.00	0.00	0.02	0.09	0.34
5	0.02	0.1	0.65	2.94	17.6

The three welfare costs are such that $(1 - k) = (1 - k_F)(1 - k_T)$. The feasibility condition is met for the parameters. Time preference rate $\delta = 2\%$. Reading indications: For a risk aversion equal to 10 and an intertemporal elasticity of substitution equal to 2, the representative agent accepts to give up 5.67% of her initial capital (upper part of Table 1) to meet the deterministic economy. Had the trend remained constant after the switch into the deterministic economy, the cost would have been 5.59% of her initial capital (middle part of Table 1). Having the opportunity to leave the constrained deterministic economy to optimally choose her level of savings, the agent accepts to give up 0.09% of her initial capital (bottom part of Table 1).

[4] Another way for evaluating the welfare cost of volatility would be to keep the same value for A (the productivity parameter in the production function) whatever the set of preference parameters. One drawback of this alternative solution is that, for most sets of preference parameters, the resulting consumption path in the stochastic economy (the only one for which data are available) is completely different from what is observed. That is why we choose to recalibrate the model (calculate a new A) for each set of parameters, using the actual trend and volatility in consumption as the benchmark. However, the figures one would have obtained using this alternative possibility to evaluate the welfare cost of fluctuations would not have been dramatically different from the one reported in Table 1.

Table 1 gives the values for the total welfare cost of volatility in terms of initial capital (Eq. (10)) and its decomposition into two parts: the direct welfare cost of volatility (Eq. (12)) and the trend-related welfare cost (Eq. (15)).

The total welfare cost of volatility (k) for different values of the intertemporal elasticity of substitution and of the risk aversion is presented in the upper part of Table 1. This cost computed in the special case of the time-additive utility function is in italics: it is then small, since the representative agent accepts to pay about 0.3 percent of her capital to live in a deterministic economy, whatever her risk aversion. Relaxing the constraint imposed by the time additivity, the cost of volatility rises with both the risk aversion and the intertemporal elasticity of substitution. Two effects which apply in the same direction are combined: the effect due to the deterministic trend endogeneity (bottom part of Table 1), and the direct effect of fluctuations (middle part of Table 1). The welfare cost of volatility may then be large: for instance, if the representative agent has a high risk aversion (say 20) and a high intertemporal elasticity of substitution (say 5), she accepts to pay more than 30 percent of her initial capital to join the deterministic economy! In this case, the approximate decomposition shows that a non-negligible part ($k_{\mathrm{T}} = 17.6$ percent) of this cost is trend related. Nevertheless, for more plausible (but still high) values of the preference parameters—a relative risk aversion coefficient equals to 5 and an intertemporal elasticity of substitution equals to 2—the total welfare cost reduces to less than 3 percent and quite nothing is due to the trend-related welfare cost ($k_{\mathrm{T}} = 0.02$ percent).

To sum up, as far as the US economy is considered, the total welfare cost of volatility remains negligible and the trend-related welfare cost of volatility does not matter when both the intertemporal elasticity of substitution and the risk aversion parameters have plausible values. This result comes from the fact that, as shown in Table 2, the growth rate of the entire economy would be barely modified by a shift from the stochastic to the deterministic economy. Of course this may not be the case for economies where volatility is much higher (e.g., developing countries). For those countries, not only the standard welfare cost of fluctuations would be much higher than the one for the United Sates (see Pallage and Robe (2003)), but the trend related welfare cost of volatility would matter as well. For example, using Pallage and Robe's estimations of μ and σ for the Algerian economy ($\mu = 0.0214$ and $\sigma = 0.0636$, see Table 1 in Pallage and Robe) and applying the same experiment as the one reported in Table 1 for the US economy we find that

Table 2
Change in the growth rate for the US economy, $(1-\varepsilon)\gamma\sigma^2/2$

ε	γ				
	1	2	5	10	20
0.1	−0.01	−0.01	−0.03	−0.06	−0.11
0.5	0.00	−0.01	−0.02	−0.03	−0.06
1	0.00	0.00	0.00	0.00	0.00
2	0.01	0.01	0.03	0.06	0.13
5	0.03	0.05	0.13	0.25	0.50

Readings indications: For a risk aversion equal to 5 and an intertemporal elasticity of substitution equal to 2, the annual growth rate is 0.03 points higher in the optimal determinist economy than in the constrained and stochastic economies.

with a risk aversion equal to 5 and an intertemporal elasticity of substitution equal to 2, the representative agent in the Algerian economy would be ready to give up as much as 70% of her wealth to join a deterministic world where she could choose the saving rate according to her preferences ($k = 70\%$). This cost splits-up as follows: the agent would give up 58% of her wealth to join the constrained deterministic world ($k_{\mathrm{F}} = 58\%$) and 30% of the remaining capital to be able to choose her saving rate in the deterministic world ($k_{\mathrm{T}} = 30\%$). The economy would then exhibit a growth rate higher by one percent than the one in the stochastic economy.

5. Conclusion

In this paper, we have proposed a measure of the total welfare cost of volatility. This total cost is derived from a stochastic endogenous growth model, and it is computed as the percentage of capital the representative agent is ready to give up to join a deterministic economy. Since we consider a whole economy, and not solely the consumption process, the total welfare cost of volatility we obtain has two components: a direct welfare cost of volatility and a trend-related one. The former is the welfare cost of fluctuations first put forward by Obstfeld (1994a) when the consumption process exhibits a unit root; in our whole economy framework it is computed as the percentage of capital the representative agent would accept to give up to join the deterministic economy while being constrained to keep the saving rate she had chosen in the stochastic world. The trend-related welfare cost of volatility is then computed as the percentage of her remaining capital stock the representative agent is ready to give up to get the opportunity to choose her saving rate. This trend-related cost is linked to the *cost of reducing growth* considered by Lucas (1987). The recursive utility function we used to model the representative agent's preferences allows us to show that the total cost of volatility increases with both the risk aversion and the intertemporal elasticity of substitution.

Calibrating our stochastic endogenous growth model to match the actual consumption process for the United States, we show that for plausible values of the preference parameters, the endogeneity of the consumption process does not matter when computing the welfare cost of volatility. As far as policy-making is concerned, our calibrations suggest that even if the reduction of volatility may induce growth, the volatility of the US economy is too small for economic policies aimed at smoothing further fluctuations to affect growth in a significant way. However, as illustrated above with the Algerian example, this may not be the case for more volatile economies.

The wish to obtain analytical formula for the total welfare cost of volatility and its decomposition prevented us from incorporating some important features: there is no labor, the shocks are uncorrelated and agents' heterogeneity is ignored.. Introducing a recursive utility function in the model recently proposed by Jones et al. (2000), which cleverly studies the business cycle properties of a stochastic endogenous growth model with human and capital formation, as well as leisure in the utility function, might be a good starting point to go further in the exploration of the welfare cost of volatility in an endogenous growth model.

Acknowledgments

We thank Gadi Barlevy, Roland Benabou, Roger Farmer, Saleh Nsouli, Franck Portier, Philippe Weil, Oved Yosha, and the referees of the Review of Economic Dynamics for their comments on previous versions of this paper. All remaining errors are the authors'. The views expressed in this paper are those of the authors and do not necessarily reflect those of the International Monetary Fund.

Appendix A. Identifying the optimal consumption path[5]

The Bellman function associated with the program is

$$(1-\gamma)V(t) = \operatorname*{Max}_{\{c(t)\}}\Big[c(t)^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}t + \mathrm{e}^{-\delta\,\mathrm{d}t}\big((1-\gamma)E_t V(t+\mathrm{d}t)\big)^{(\varepsilon-1)/((1-\gamma)\varepsilon)}\Big]^{(1-\gamma)\varepsilon/(\varepsilon-1)}. \quad (16)$$

By analogy with the standard time-additive deterministic program, one can guess that the value function is

$$V(t) = B^{(1-\gamma)/(1-\varepsilon)}\frac{K(t)^{1-\gamma}}{1-\gamma},$$

where B is a constant to be calculated, and that the optimal consumption at t is a linear function of the current wealth; that is $C(t) = DK(t)$, where D is a constant to be calculated.

Calculating $E_t[V(K(t+\mathrm{d}t))]$:

$$E_t\big[V\big(K(t+\mathrm{d}t)\big)\big] - E_t\big[V\big(K(t)\big)\big] = E_t[\mathrm{d}V] = B^{(1-\gamma)/(1-\varepsilon)}\frac{E_t[K(t+\mathrm{d}t)^{1-\gamma}]}{1-\gamma} - B^{(1-\gamma)/(1-\varepsilon)}\frac{E_t[K(t)^{1-\gamma}]}{1-\gamma}. \quad (17)$$

Moreover, applying Itô's lemma, one calculates:

$$E_t[\mathrm{d}V] = \frac{\partial V}{\partial K}E[\mathrm{d}K] + \frac{1}{2}\frac{\partial^2 V}{\partial K^2}E[\mathrm{d}K]^2, \quad (18)$$

where $E[\mathrm{d}K] = (A-D)K(t)\,\mathrm{d}t$ and $E[\mathrm{d}K]^2 = \sigma^2 K(t)^2\,\mathrm{d}t$ when neglecting power of $\mathrm{d}t$ superior to one.

From (17) and (18) one calculates:

$$E_t\big[V\big(K(t+\mathrm{d}t)\big)\big] = B^{(1-\gamma)/(1-\varepsilon)}\bigg[\bigg(A - D - \frac{1}{2}\sigma^2\bigg)\mathrm{d}t + \frac{1}{1-\gamma}\bigg]K(t)^{1-\gamma}. \quad (19)$$

Substituting (19) into (16), the Bellman equation can be rewritten:

[5] Cf. Smith (1996).

$$
\begin{aligned}
&B^{(1-\gamma)/(1-\varepsilon)}K(t)^{1-\gamma}\\
&\quad=\operatorname*{Max}_{\{D\}}\biggl[D^{(\varepsilon-1)/\varepsilon}K^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}t+\mathrm{e}^{-\delta\,\mathrm{d}t}\biggl((1-\gamma)^{(\varepsilon-1)/((1-\gamma)\varepsilon)}B^{-1/\varepsilon}K^{(\varepsilon-1)/\varepsilon}\\
&\qquad\times\biggl[(A-D-\gamma\sigma^2/2)\,\mathrm{d}t+\frac{1}{1-\gamma}\biggr]^{(\varepsilon-1)/((1-\gamma)\varepsilon)}\biggr)\biggr]^{(1-\gamma)\varepsilon/(\varepsilon-1)};
\end{aligned}
\tag{20}
$$

using the fact that $\lim_{x\to 0}(1+x)^y = 1+xy$ and $\lim_{x\to 0} e^x = (1+x)$, leads to

$$
\begin{aligned}
B^{(1-\gamma)/(1-\varepsilon)}K(t)^{1-\gamma}&=\operatorname*{Max}_{\{D\}}\biggl[D^{(\varepsilon-1)/\varepsilon}K^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}t+B^{-1/\varepsilon}K^{(\varepsilon-1)/\varepsilon}\\
&\qquad\times\biggl(\frac{\varepsilon-1}{\varepsilon}(A-D-\gamma\sigma^2/2)\,\mathrm{d}t-\delta\,\mathrm{d}t+1\biggr)\biggr].
\end{aligned}
\tag{21}
$$

The optimization shows that D, the optimal propensity to consume current wealth is equal to B. B is then identified by replacing D in (20):

$$
B=D=\varepsilon\biggl(\delta-\frac{\varepsilon-1}{\varepsilon}\bigl(A-\gamma\sigma^2/2\bigr)\biggr).
$$

The value function is then

$$
V\bigl(K(t)\bigr)=\biggl[\varepsilon\biggl(\delta-\frac{\varepsilon-1}{\varepsilon}\bigl(A-\gamma\sigma^2/2\bigr)\biggr)\biggr]^{(1-\gamma)/(1-\varepsilon)}\frac{K(t)^{1-\gamma}}{1-\gamma}.
$$

The feasibility condition requires $C^* > 0$, which may be rewritten:

$$
\varepsilon\biggl(\delta-\frac{\varepsilon-1}{\varepsilon}\bigl(A-\gamma\sigma^2/2\bigr)\biggr)>0\quad\Leftrightarrow\quad\delta>\frac{\varepsilon-1}{\varepsilon}\bigl(A-\gamma\sigma^2/2\bigr).
$$

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