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Essays on monetary policy, sovereign debt and banking

Perazzi Elena

UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT D'ÉCONOMIE

ESSAYS ON MONETARY POLICY, SOVEREIGN DEBT AND BANKING

THÈSE DE DOCTORAT

présentée à la

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de l'Université de Lausanne

pour l'obtention du grade de
Docteure ès Sciences Économiques, mention « Économie politique »

par

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Directeur de thèse
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Introduction

The four chapters of this thesis contribute to topics and debates that arose or attracted renewed interest following the major crises of recent years. The first two chapters are related to the sovereign debt crisis that hit the Euro area in 2009-2012. In the first part of 2010 leading European nations implemented a series of financial measures in support of countries that were unable to repay or refinance their debt. Such measures included the establishment of financial assistance programs such as the European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM), which granted bailout loans to Greece, Ireland and Portugal. Starting in 2010, the European Central Bank took a series of "unconventional monetary policy" measures to curb the excessive volatility in sovereign debt markets. Such measures culminated in the Draghi speech of July 2012, in which he pledged to do "whatever it takes" to preserve the monetary union. Although this last step was widely credited with changing the course of events, many economists remained convinced that the crisis uncovered the fundamental fragility of the Euro project and of monetary unions in general. Some, for example Krugman,[1] argued that a country able to conduct its own monetary policy would never have experienced a similar debt crisis.

In the first chapter of this thesis I study the effects on financial stability of programs such as the EFSF and the ESM, namely bailout funds with a predetermined lending capacity, designed to protect several countries in a monetary union from a sovereign default. I build a model to show that such programs can generate financial contagion in sovereign debt markets and that, by inducing countries to borrow more (as a moral hazard effect) they can increase the frequency of debt crises and defaults.

In the second chapter, Philippe Bacchetta, Eric van Wincoop and I conduct a counterfactual thought experiment: we ask whether a country similar to one in the Euro periphery could have escaped the debt crisis if it was able to conduct its own monetary policy, rather than belonging to a monetary union. We find that although an independent monetary policy could in principle avoid the crisis, doing so involves in general several years of sustained inflation, the cost of which could in many cases be higher than the cost of a sovereign default.

[1] see chapter 2 references.

The topics of the third and fourth chapter are related to reforms of the banking sector that were proposed after the financial crisis of 2007-2009. As Cochrane puts it[2], *"at its core, our financial crisis was a systemic run. The run started in the shadow banking system of overnight repurchase agreements, asset-backed securities and investment banks. Arguably, it was about to spread to the large commercial banks when the Treasury Department and the Federal Reserve Board stepped in with a blanket debt guarantee and TARP (Troubled Asset Relief Program) recapitalization. But the basic economic structure of our financial crisis was the same as that of the panics and runs on demand deposits that we have seen many times before".*
This narrative of the financial crisis lead many economists to embrace the idea that we should prevent banks from financing loans and other risky investments through run-prone contracts such as deposits. "Narrow banking" proposals advocate the separation of current banks into one entity (the "narrow bank") issuing deposits and investing in very safe assets, and one entity dedicated to lending and other risky activities, which would be financed by long-term debt or equity.

One, rather extreme, version of this kind of proposal is the "sovereign money" proposal, which advocates that the function of issuing and managing deposits be entirely taken by the central bank. Recently, the "Vollgeld initiative" in Switzerland resulted in a referendum on sovereign money, which the Swiss people voted down in June 2018. The proponents of the referendum referred to a recent paper by Benes and Kumhof[3] to argue that this reform would bring huge benefits to the economy. In the third chapter of this thesis, Philippe Bacchetta and I develop a simple model to evaluate the effect this reform would have on an economy with similar characteristics as Switzerland. Contrary to the paper by Benes and Kumhof, we find that the reform advocated by the Vollgeld initiative would have negative welfare effects, and some slightly modified versions of it would be essentially welfare-neutral. We also find that the reform would have virtually no impact on bank lending. This result is consistent with the vast majority of the literature, which regards the banks' lending function as independent of the deposit function. However, this result is dependent on the fact that we restrict bank lending to be of the same (short) duration as the banks' liabilities, thus abstracting from the fact that maturity transformation is one of the specialties of banks.

In the fourth chapter I model banks as entities that borrow short-term and lend long-term, and are thus exposed to interest rate risk arising from monetary policy. It turns out that deposits are a special form of financing, as the interest the banks need to pay on deposits is "sticky": it changes less than one-for-one with the policy interest rate set by the central bank. Thanks to this property, banks that borrow in the form of deposits are less exposed to the risk of monetary policy than banks that borrow in the form of bonds. I show that, thanks to deposits, after an interest rate increase banks cut their lending significantly less than they would if they borrowed fully in the form of bonds. This shows one downside of narrow banking proposals.

[2]see chapter 4 references.
[3]see chapter 3 references.

Chapter 1

Sovereign Debt in an Economic Union: a Model of Bailouts and Contagion

Abstract We propose a model to account for two stylized facts about sovereign yields in the Euro Area: their convergence after 2000 and subsequent divergence after 2008, and the contagion among yields of the Euro periphery after the financial crisis of 2007-2008.

Two borrowing countries share a bailout guarantee by an institution with limited lending capacity. Contagion occurs as one country needing (or being perceived as close to needing) a bailout diminishes the prospects of another country receiving a bailout as well. We investigate how the availability of bailouts affects the probability of a debt crisis and find the following: i) a bailout available with (near) certainty has the good features of a lender of last resort; ii) uncertainty arises as the bailout resources are shared between countries, which may dramatically increase the probability of a crisis; iii) a small country has the highest potential for moral hazard: it can disproportionately increase its debt beyond its ability to repay, and effectively take exclusive control of the shared bailout resources.

1.1 Introduction

Sovereign yields for the Euro Area were remarkably low and stable in the years 2001-2008 and subsequently diverged in many countries (Greece, Ireland, Italy, Portugal, Spain) in a way that many find excessive relative to the change in country fundamentals (for example, DeGrauwe and Ji (2013)). A lot of the discussion has focused on the "contagion" experienced by sovereign spreads of the Euro periphery after the financial crisis of 2007-2008 (for example, Constancio (2011))[1].

In this paper, we propose a mechanism that can contribute to explain the "convergence" of the Euro Area (EA) spreads in 2000-2007 and the subsequent "excessive" divergence, and provides a novel channel for contagion.

The main idea is that since the start of the EA, government bond markets started pricing in an implicit bailout guarantee from the strong countries in the union, presumably on the grounds that the default of a country within the union would have severe spillovers over the union as a whole. Thanks to this guarantee, the spread of a country in the EA is necessarily lower than that of an otherwise identical stand-alone country. However, since the union does not have an unlimited bailout capacity, the spread dynamics are affected by the joint dynamics between the fundamentals of the country, the fundamentals of other countries in the area (that might also be in need of a bailout, thus exhausting the resources for bailout of the union), and, possibly, the overall bailout capacity of the EA, that might also become weaker in a downturn.

Our model involves two borrowing countries and a pool of resources of fixed size M that we call "the insurance", which can be used to bail out a country that would otherwise default. The "insurance" is similar to financial assistance programs in the Eurozone, such as the EFSF and the ESM, that were established after the start of the sovereign debt crisis as a firewall for Eurozone members in financial difficulty, with a pre-specified maximum lending capacity.

If the amount M is insufficient to prevent default, there is no bailout and the resources of the insurance cannot be used.[2] If both Country 1 and Country 2 need a bailout, they have to share the amount M, provided that this is enough to avoid both defaults. Possibly, M could represent the spillover cost that the provider of the insurance would incur in case of a default. For example we could think the provider of the insurance as another country whose banking system is heavily exposed to the two borrower countries. Alternatively, the fixed limit M could be due to the limited resources of the insurer.

[1] Anecdotal evidence mentioned by Constancio, Vice President of the European Central Bank, includes the following: *"When the sovereign crisis became more severe again and Moodys downgraded Portugal on July 5, 2011, it cited among other factors developments in Greece. Moody's believed that contagion from a default of Greece made it more likely that Portugal would require a second round of official financing. The downgrade of Portugal and, above all, the continuing fears of a Greek default apparently triggered a sell-off in Spanish and Italian government bonds. There had not been adverse data releases concerning the Spanish and Italian economies or budgetary situations around that time. By July 18, 2011 Italian government bond yields had increased by almost 100 basis points, while Spanish ones had increased by more than 80 basis points."*

[2] A bailout can sometimes coexist with partial default. For example, in 2011 Greece received a bailout loan and negotiated a 50% haircut on debt owed to private banks. This possibility will be considered in a future extension of the model.

In our model, a bailout may happen even in the absence of an ex-ante contract between the borrower countries and the provider of the insurance. The latter will ex-post offer a bailout in order to avoid the spillover costs from a default. This justifies why the markets were assuming an implicit guarantee even in times when no official mechanisms to avoid sovereign defaults were in place.

We build a dynamic, infinite horizon model, based on the exactly solvable dynamic model for sovereign debt by Collard, Habib and Rochet (2015), henceforth CHR, which we calibrate to replicate realistic values for the Debt-to-GDP ratio and the probability of default of a country.

The introduction of the insurance, and the fact that the insurance is shared by two countries, complicate this model, so that we need to resort to numerical methods for the solution.

We first obtain a solution using a reduced-form approach in which we effectively reduce the problem to a one-country problem: we posit that in each period, conditional on the insurance not having been depleted before, one country can, if it is otherwise unable to repay its debt, access the insurance with probability p. In other words, $1 - p$ is the probability that the insurance gets depleted by "the other country" in each period, conditional on no previous bailouts.

We then obtain a full two-country solution, which relies on the assumption that both countries have the objective of maximizing their borrowing in every period.

We use this model to analyze a classical question about bailouts: what is the impact of the insurance on the frequency of crises? Does the insurance have a stabilizing effects on debt markets or is it a source of moral hazard?

The moral hazard problem has been discussed extensively in the literature devoted to the international bailouts in Latin America and East Asia in the 90's (e.g. Calomiris (1998), Jeanne and Zettelmeyer (2001)). The coexistence of the pernicious effects associated with moral hazard and the beneficial effects associated with a lender of last resort have been studied extensively in connection with bank bailouts (see e.g. Goodhart and Huang (1999) and Cordella and Levy Yeyati (2003)).

In regard to this question our reduced-form approach is very useful to provide intuition about the situations in which the positive or the negative effects prevail. On the one hand, if the insurance is available with a high degree of certainty (we need $p > 95\%$ when the insurance limit is 50% of one country's income), it has the classical beneficial effects of a lender of last resort: even if a country takes maximum advantage of the insurance to increase its level of borrowing and its debt, the incidence of crises decreases. A country's ability to repay depends crucially on how much it can re-borrow from the markets when its debt is due. With the insurance markets lend more, and this is especially so when income is low, since in this case the insurance is in relative terms higher. The decrease in the incidence of crises happens entirely through the higher re-borrowing ability from the markets at low income, and the insurance resources hardly ever need to be used.

On the other hand, when the insurance is more uncertain (i.e. for lower p) the moral hazard problem comes back with a vengeance.

First, for any p, a country can in general borrow more from the markets relative to a world without insurance. But if next period the insurance has been depleted, the borrowing capacity from the market is lower again, which makes debt repayment more difficult. Second, if the insurance limit is bigger than the current country's borrowing capacity from the market (roughly 40% of its current income), and if the probability p that the insurance is available next period is in an intermediate range, the country can increase its borrowing capacity by essentially promising to use the insurance to repay when the latter is available, and default otherwise. In this case the probability of a crisis can rise to enormous levels. For example, for $M = 50\%$ of the country's current income and p between 70% and 90% the probability of a crisis can rise to over 70%. [3]

The full two-country solution uncovers additional phenomena. Even if both countries try to exploit the insurance and borrow as much as they can, our results indicate that in vast regions of the income space only one country is able to significantly increase its borrowing level thanks to the insurance. One of the situations in which this occurs is when the income of one country is smaller than twice the insurance limit, while at the same time the income of the other country is bigger than roughly three times the insurance limit. In this case the smaller country can disproportionately increase its debt beyond its ability to repay, and effectively take exclusive control of the shared bailout resources. We call this the "small country effect". This might shed some light on the puzzle of how a country such as Greece can become systemic despite its small size.

This paper is organized as follows. Section 2 presents the structure of the model and reviews the solution of the baseline 1-country model by CHR. Section 3 solves the shared insurance problem through a reduced-form approach, and in this context explores the question of the impact of the insurance on the incidence of crises. Section 4 derives a full two-country solution of the shared insurance problem. Section 5 concludes.

Related literature

Part of the existing literature on sovereign spreads in EA emphasizes the role of a global risk aversion factor. With the financial crisis balance sheet-constrained investors developed a stronger preference for assets perceived as "safe havens" (the "flight-to-quality" effect). This behavior not only benefited sovereign securities as an asset class, but also introduced a higher degree of differentiation among sovereign securities. For example, an IMF study by Caceres, Guzzo and

[3] Policy might be able to mitigate this problem. For example, Jeanne and Zettelmeyer (2001) advocate making bailouts conditional onborrowing limits or other government policies. Forms of ex-ante conditionality might however be difficult to enforce, as the bailouts in the EA show.

Segoviano (2010) finds that the flight-to-quality effect had a significant impact lowering German and, to some extent, other core Euro Area sovereign spreads during what they call “the financial crisis build-up”, between July 2007 and September 2008. However, this factor does not help explain the convergence of sovereign spreads in the years 2000-2007.

The role of the “flight-to-quality” effect, as well as that of the “flight-to-liquidity” effect, as drivers of sovereign spreads in EA, is also documented by Beber, Brandt and Kavajecz (2009).

“Behavioral” explanations for the spreads behavior in the period 2000-2007 include the “convergence trading” hypothesis (e.g. Arghirou and Kontalakis (2010)): according to this view, markets in this period were pricing in the expectation of a convergence between the economies in the European core and the economies in the periphery, while disregarding other possibilities.

DeGrauwe and Ji (2013) argue that the “disconnect” between spreads and fundamentals (in particular, Debt-to-GDP ratio) after 2008 would be evidence of multiple equilibria in the government bond markets. According to this view, member countries of monetary unions, that issue debt in a currency they do not control, may more easily fall victims of self-fulfilling prophecies. The mechanism would be similar to the one in the seminal paper by Calvo (1988): if investors start worrying that the Debt-to-GDP ratio of a country is excessive, or that a country might have a liquidity problem, they require a higher spread, which worsens the Debt-to-GDP ratio and exacerbates the liquidity problem.

Benzoni, Collin-Dufresne, Goldstein and Helwege (2014) develop a model of contagion between sovereign spreads, where the source of contagion is the existence of an underlying state of the economy affecting all the countries in the EA, about which agents are uncertain and have to update their beliefs.

Tirole (2014) is a recent reference about the optimality of bailouts and other forms of country solidarity, directly inspired by the Eurozone crisis.

1.2 Model

Our full model comprises two countries that share an “insurance” with fixed capacity M.

The baseline 1-country model

The 1-country model (with no insurance) is closely related to CHR. A country is an endowment small open economy (SOE) that borrows from financial markets at each date t. Only non-contingent 1-period debt is available. A debt contract at time t prescribes that for a given borrowing level B_t the country needs to repay D_{t+1} at $t+1$. We can express $D_{t+1} = (1 + r_f + x(B_t))B$, where r_f is the risk-free interest rate, which is exogenous and constant, and $x(B_t)$ is the spread. In order to repay its debt, at $t+1$ the country can use at most a fraction

ϕ of its current income Y_{t+1}, plus the amount that the market is willing to lend again at $t+1$. The country decides to repay whenever it can (no strategic default). In default, markets stop lending and the country repays a fraction ϕ of its income. Formally, repayment at time $t+1$ is

$$R_{t+1} = \begin{cases} D_{t+1} = (1 + r_f + x_t)B_t & \text{if } \phi Y_{t+1} + B^*_{t+1} > D_{t+1} \\ \phi Y_{t+1} & \text{otherwise} \end{cases} \tag{1.1}$$

where B^*_{t+1} is the maximum amount the market is willing to lend at $t+1$.
Income Y_t follows the stochastic process

$$Y_t = g_t Y_{t-1} \tag{1.2}$$

where g_t is i.i.d. and lognormally distributed, $g_t \sim LN(\mu_g, \sigma_g)$. Lenders are risk-neutral[4].

The government's objective

While many equilibrium relations, such as the spread as a function of the borrowing level, can be obtained without modeling the government's borrowing choices, when needed we make the extreme assumption that the government has the objective to maximize its borrowing level in every period. We denote a government with this objective by "profligate". In Section 2.3 we will briefly discuss the motivations for this assumption and hint at possible microfoundations.

Full model: two countries and a shared insurance

In the absence of the insurance, each country is described by the baseline model above. We add an insurer that can bail out one or both countries at repayment time $t+1$ if they are unable to repay by using the maximum attainable primary surplus ϕY_{t+1} plus the maximum they can re-borrow from the markets B^{mkt}_{t+1}. However, the insurance can contribute a maximum amount M. If M is not enough to avoid *both* defaults, there is no bail out and default occurs.

1.2.1 Baseline one-country model: brief review and calibration.

This section contains a brief review of some results in CHR, followed by our calibration. More analytical details can be found in Appendix A.

Lenders' risk neutrality implies the following equilibrium condition between B_t and D_{t+1}

$$E_0[R_{t+1}(D_{t+1})] = (1 + r_f)B_t \tag{1.3}$$

where R_{t+1} is debt repayment, defined in (1.1).

Thanks to the stationarity of the distribution of growth factor g, the spread at any time t is a function (independent of t) of borrowing level as a fraction of Y_t:

$$x_t = x(b_t) \qquad \text{with } \; b_t \equiv \frac{B_t}{Y_t}. \tag{1.4}$$

[4] Risk neutrality could be justified by the hypothesis that the SOE is uncorrelated with the rest of the world, so its risk could be fully diversified away. We make this assumption because we specifically want to focus on a model of the spreads based on default risk, rather than on risk aversion.

Intuitively, since the level of income has no effect on the distribution of future growth, a change in the level of income should allow one country to proportionally change its level of borrowing, without any effect on the probability of default. The same intuition applies to the borrowing capacity and the maximum debt:

$$B_t^* = b^* Y_t \tag{1.5}$$

$$D_{t+1}^* = d^* Y_t \tag{1.6}$$

with the maximum Debt-to-Income ratio d^* related to b^* by $d^* = (1 + r_f + x(b^*))b^*$. Notice that D_{t+1}, the debt due at $t+1$, is determined at time t as a function of B_t, therefore it is natural to express it as a fraction of time-t income.

The borrowing capacity is a crucial ingredient to determine the default probability and hence the spreads: in order to know if the country will default next period, we need to know what is the maximum amount it will be able to re-borrow. Equation (1.1) implies that default happens when

$$Y_{t+1} < \frac{D_{t+1} - B_{t+1}^*}{\phi} \tag{1.7}$$

Using the proportionality between borrowing capacity and income (1.5) and the definition of the growth factor (2.8), and expressing the face value of the debt D_{t+1} in terms of time-t income $D_{t+1} \equiv d_{t+1} Y_t$, (1.7) can be translated into a condition on growth between t and $t+1$, i.e. given d_{t+1} default happens when

$$g_{t+1} < \frac{d_{t+1}}{\phi + b^*} \tag{1.8}$$

The borrowing capacity increases with the mean of (the lognormal distribution of) the growth factor, μ_g, and decreases with its volatility σ_g, reflecting the higher probability of falling below the default threshold when volatility increases. As discussed by CHR, the prediction that default probability increases with income volatility is an advantage of this model over models of strategic default. In models of strategic default, pioneered by Eaton and Gersovitz (1982), a country repays to avoid the punishment of being excluded from the financial markets. Since volatility increases the option value of borrowing in the future, these models predict that more volatile countries default less often, a prediction at odds with empirical evidence[5].

Baseline parameter choice

We use $\mu_g = 0.02$ (2% average growth), $\sigma_g = 0.1$ (10% volatility), $\phi = 0.1$ (10% maximum primary surplus), and $r_f = 0.04$. With these inputs, the borrowing capacity b^* is 39.8% of income, with

[5] For example Catao and Kapur (2004) look at a set of 26 developing countries over the period 1970-2001 and find that all serial defaulters are highly volatile economies, with average output volatility about twice the sample average. Strategic default models are at odds with other stylized facts as well. First, default occurs by and large in bad times. For example, Reinhart-Rogoff (2009) document the link between sovereign defaults and financial crises. If default was purely a strategic decision, it would happen a lot also in good times. Second, strategic default, together with the absence of bankruptcy courts for sovereigns, would naturally imply zero recovery rates. This is clearly not in line with the stylized fact that default generates lengthy renegotiations (as documented for example by Benjamin-Wright (2008)) and that recovery rates are typically significantly higher than zero.

corresponding Debt-to-GDP ratio $d^* = 43.4\%$, and the probability of default if the country borrows the maximum is 5.8%.[6]

We set the income volatility to 10%. A volatility of this magnitude is necessary to obtain a sizeable and realistic default probability. Although this value seems very high, what matters from the point of view of debt repayment is the distribution of the primary surplus, whose maximum attainable value is a constant fraction of income in this model. A 10% volatility seems much more realistic when referring to the primary surplus.

We set the maximum primary surplus to 10%. Eichengreen and Panizza (2014) document that very rarely the primary surplus of highly indebted countries exceeded 5% for an extended time, but also report several cases in which it was in excess of 10% for short periods.

As documented by Cohen and Villemot (2012), most models in the literature, especially models of strategic default, predict minuscule Debt-to-GDP ratios (often below 10%) when calibrated to reproduce realistic default probabilities. This model, while predicting a maximum Debt-to-GDP ratio a bit lower than those observed in the real world, does a lot better in this respect.

1.2.2 The government's objective: discussion

Why the assumption of "profligate" governments, that maximize their borrowing in every period?

Empirical evidence supports the view that many countries borrow "because they can", rather than for the more sound economic reason of smoothing consumption. A government with a consumption smoothing objective would borrow anticyclically. Gavin and Perotti (1997) showed the strong procyclicality of fiscal policy in Latin America. Furthermore, Talvi and Végh(2000) showed that a procyclical fiscal policy is the norm, rather than the exception, not only in developing countries, but also in industrial countries, with the exception of G7 economies. (see also Levy-Yeyati (2009), and Kaminsky, Reinhart and Végh (2005)).

A vast literature provides explanations to fiscal procyclicality and to the high debt levels accumulated by many governments in terms of political imperfections. In Habib and Rochet (2013) governments have short horizons because re-election is uncertain, so they discount the future at a higher rate than households. If the discount factor is low enough, maximum front-loading of government consumption occurs. Talvi and Végh (2000) build a model in which, due to the political pressure of competing interest groups, running a high primary surplus is very costly or impossible.

The model by CHR naturally produces procyclical borrowing since income shocks are permanent.

[6] Benjamin and Wright (2009) estimate an average default rate across countries of 4.4% for the period 1989-2006. Cohen and Valadier (2011) document probabilities of default around 7% over the period 1970-2011.

In addition, the assumption of a maximum primary surplus points to the limited power of governments over fiscal policy. Along the lines of Talvi and Végh (2000) one could further imagine that it be less costly for the government to raise a substantial primary surplus when the alternative is default than for debt reduction purposes. In this scenario one country starting with close to maximum debt could find itself trapped in the highest debt region for long time, able to raise surpluses only just so as to (temporarily) avert default, thus effectively becoming a “profligate” country.

1.3 Full model with insurance: a reduced-form approach

Our 2-country model with insurance is considerably more complicated than the 1-country model reviewed in Section 2.1. First, since the insurance limit M is constant (hence the insurance as a fraction of income $m_t \equiv \frac{M}{Y_t}$ is time-dependent), the main quantities of interest, such as the borrowing capacity from the market B^{mkt} and the spread, are not proportional to income any more. Intuitively, when the insurance is large relative to income $B^{mkt} >> b^*Y$. In contrast, when the insurance is small relative to income $B^{mkt} \simeq b^*Y$. (b^*Y would be the borrowing capacity in a world without insurance). Second, spread and maximum borrowing capacity for one country depend on details of the other country. For example, will the other country deplete the insurance? What is the probability of this happening?

With the assumption that both countries are “profligate”, a full recursive solution to the 2-country problem is feasible. The full solution will be the topic of Section 4. In this Section, we focus on a simple approach that effectively reduces the 2-country problem to a 1-country problem.

We posit that in each period the probability that one country has access to the insurance, conditional on the insurance not having been used before, is a constant p. In other words, the conditional probability that “the other country” depletes the insurance is $1 - p$ every period. p, which is exogenous in this context, may represent the certainty equivalent of a stochastic variable.

Despite the disadvantage of treating p as an exogenous parameter, the simplicity of this approach is very useful to build intuition about the results.

In Section 3.1 we obtain the borrowing capacity as a function of income and the spread as a function of the amount borrowed. The results show how the model can generate something qualitatively similar to the stylized facts of spreads in the Euro Area. In Section 3.2 we obtain results about how the insurance affects the probability of debt crises, focusing on the “profligate country” scenario.

1.3.1 Spreads and borrowing capacity

Time-t investors know that when repayment time comes, at $t+1$, with probability p the country will benefit from increased borrowing capacity from the markets like at time t (in addition to the option of using the insurance right away), and with probability $1-p$ there will not be any insurance and the country will only be able to re-borrow a fraction b^* of its income.

If the first case (insurance still available at $t+1$) the country fully repays its debt D if its income is above a threshold value $Y_p^{(def)}(D)$ (with the subscript to highlight its dependence on p). $Y_p^{(def)}(D)$ is determined by the condition that at this income level the maximum attainable primary surplus plus the maximum that can be borrowed either from the market or from the bailout fund is exactly equal to the face value of the debt[7]:

$$\phi Y_p^{(def)}(D) + max(M, B_p^{(mkt)}(Y_p^{(def)}(D))) = D \tag{1.9}$$

In the second case (insurance depleted or insufficient at $t+1$) the threshold value above which there is full repayment, which we call $Y_0^{(def)}(D)$, is

$$Y_0^{(def)}(D) = \frac{D}{\phi + b*} \tag{1.10}$$

(remember that if the insurance is depleted the borrowing capacity from the market is a fraction b^* of income). The borrowing capacity, that by lenders' risk neutrality is the maximum present discounted value of the expected repayment, can therefore be written as

$$\begin{aligned} B_p^{(mkt)}(Y_t) = & \quad \frac{1}{1+r_f} max_D \left\{ p \left(P_p^{def}(E_t[\phi Y_{t+1}|Y_{t+1} < Y_p^{(def)}(D)] + (1-P_p^{def})D \right) \right. \\ + & \quad \left. (1-p) \left(P_0^{def} E_t[\phi Y_{t+1}|Y_{t+1} < Y_0^{(def)}(D)] + (1-P_0^{def})D \right) \right\} \end{aligned} \tag{1.11}$$

with

$$P_p^{def} \equiv Prob(Y_{t+1} < Y_p^{(def)}(D)) \tag{1.12}$$

$$P_0^{def} \equiv Prob(Y_{t+1} < Y_0^{(def)}(D)) \tag{1.13}$$

Our problem is to find $B_p^{(mkt)}$, which appears on the LHS of (1.11) and, through (1.9), also on the RHS.

[7] We assume that when a country accepts a bailout, it cannot at the same time borrow from the market. Debt contracted with institutional lenders in a bailout event is typically senior to market debt. As the marginal lenders, market investors would stop lending to the country if the latter's total borrowing were to exceed its borrowing capacity b^*Y. Therefore a country cannot increase its repaying ability by combining the lending capacity of the bailout agency with the willingness to lend of the markets.

This can be solved with recursive methods. It is straightforward to verify that the operator defined by (1.11) satisfies the Blackwell sufficiency conditions hence is a contraction, which ensures the existence and uniqueness of a solution for $B_p^{(mkt)}(Y_t)$.

Once we have numerically solved the problem in (1.9)-(1.11) and obtained $B_p^{(mkt)}(Y)$ and $Y_p^{(def)}(D)$ it is easy to solve for the spread $x(B)$ for any borrowing level B. The equilibrium relationship between B and $D(B)$ is

$$\begin{aligned} B = \quad & \frac{1}{1+r_f}\left\{p\left(P_p^{def}(E_t[\phi Y_{t+1}|Y_{t+1} < Y_p^{(def)}(D)] + (1-P_p^{def})D\right)\right. \\ + \quad & \left.(1-p)\left(P_0^{def}E_t[\phi Y_{t+1}|Y_{t+1} < Y_0^{(def)}(D)] + (1-P_0^{def})D\right)\right\} \end{aligned} \tag{1.14}$$

with P_p^{def} and P_0^{def} given by (1.12) and (1.13), respectively. $x(B)$ is then found through the relationship $D(B) = (1 + r_f + x(B))B$.

Results

Figure 1.1 shows the resulting borrowing capacity as a function of income for different values of p, for a reference insurance limit $M = 0.5$. Figure 1.2 shows the spreads as a function of borrowing level for different values of p, for $M = 50\%$ of income. Tables in Appendix B show the numerical values for the borrowing capacity, the maximum debt level and the spread for a wide range of values for p, when M is equal to 50% and 25% of income.

The probability p that the insurance is depleted every period is the channel through which contagion operates. The borrowing capacity of one country changes very significantly when p changes even by a few percentage points, all fundamentals of the country unchanged.

For example, with insurance limit equal to 50% of income, the borrowing capacity is 51% of income with $p = 0.9$, 55% of income when $p = 0.95$ and 64% of income when $p = 1$. Also, assuming that the country borrows to the maximum, the spread changes dramatically with p: over 8% for $p = 0.9$, around 4.5% for $p = 0.95$ and less than 1.5% for $p = 1$.

A "structural break" in the behavior of spreads

DeGrauwe and Ji (2013) show that the relationship between spreads and Debt-to-GDP ratio in the Euro Area experienced what they denote as a "structural break" around 2008. In particular they show that the positive relationship between spreads and Debt-to-GDP ratio, as well as the negative relationship between spreads and GDP growth rate, was very strong after the crisis but insignificant before the crisis, and that this occurred also for some countries for which the change in fundamentals was modest in this period.

It is not hard to see how such a phenomenon can occur in our model. Imagine that initially the Economic Union is experiencing calm times so that p is very close to 1. One country is borrowing say between 50% and 60% of its income, so that, as we see from Figure 1.2 (solid line) its spread is very close to 0 and is insensitive to changes in borrowing (as long as the latter stays below 60%). Now imagine that p unexpectedly falls to 95% without any changes in the fundamentals of our country of reference, which continues to borrow between 50% and 60% of its income. Now the spread experienced by this country is described by the dot-dashed line in Figure 1.2. The spread is now high (between 2 and 4.5 percentage points) and very sensitive to the borrowed amount, and default occurs if the country needs to re-borrow more than 55%. The two ingredients that produce this result are the sensitivity of the borrowing capacity to p and the high non-linearity of spreads for borrowing levels close to their borrowing capacity.

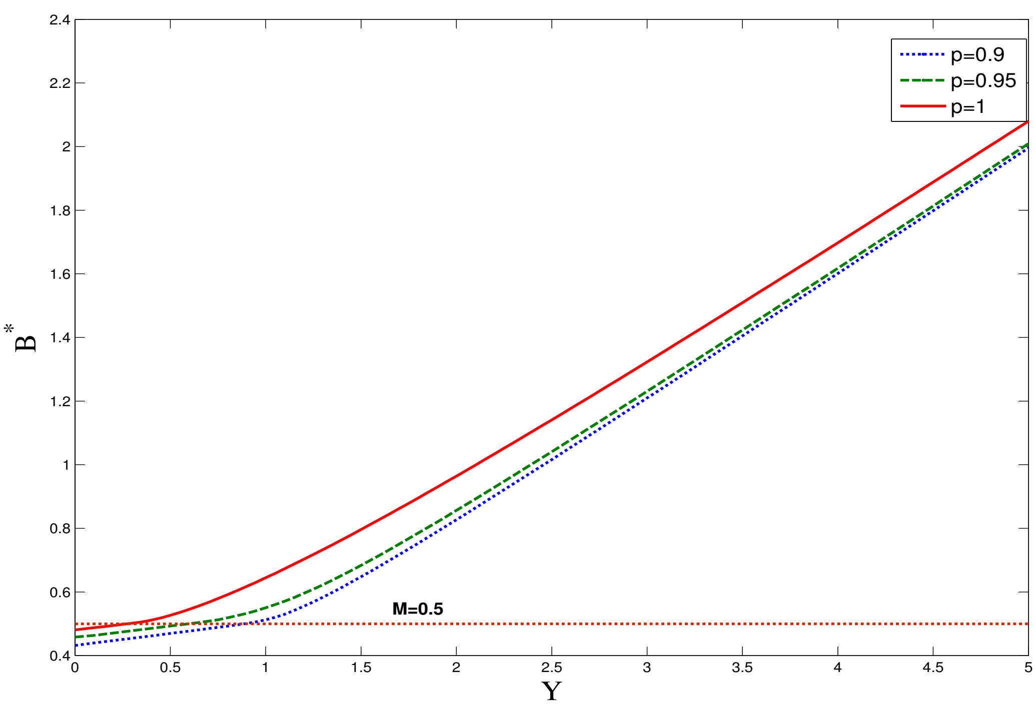

Figure 1.1: **Borrowing capacity for different levels of p. $M = 0.5$.**

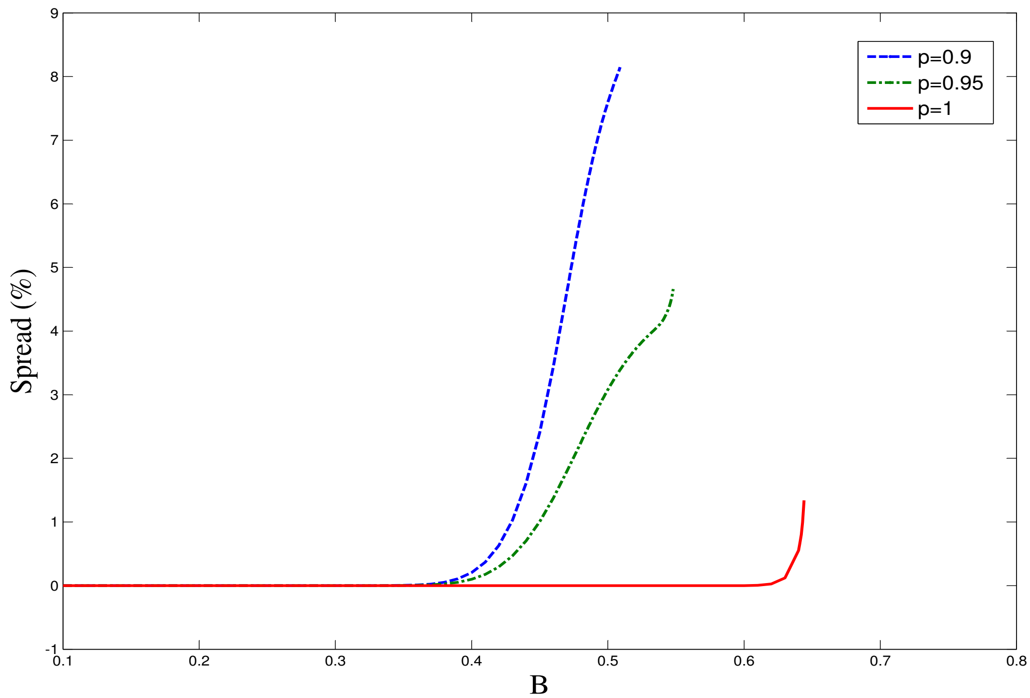

Figure 1.2: **Spread as a function of borrowing level for different levels of p. Borrowing level is expressed as a fraction of income and M is 50% of income.**

1.3.2 Debt accumulation and debt crises

In this section we explore how the insurance affects borrowing, debt levels, and, most importantly, the incidence of debt crises and defaults in our model. We define as "debt crisis" the situation in which a country is unable to repay by re-borrowing from the market, and needs a bailout. A default event is a debt crisis in which there is no available bailout or the bailout resources are insufficient.

We work within the reduced-form framework previously described in this section, in which every period the insurance is available to one country with constant probability p, conditional of not having been depleted before. Conditional on the insurance being available, a crisis occurs if income falls below the threshold Y_p^{cr}, which is such that the maximum primary surplus plus the maximum that can be re-borrowed from the markets at this income is just enough for full debt repayment:

$$\phi Y_p^{cr}(D) + B_p^{mkt}(Y_p^{cr}(D)) = D \tag{1.15}$$

Default occurs if income falls below the threshold Y_p^{def}, which solves (1.9). Conditional on the insurance not being available, crises and defaults coincide, and happen if income falls below the threshold Y_0^{def} in (1.10). Crises and defaults coincide (hence $Y_p^{def} = Y_p^{cr}$) also when

$$M < B_p^{mkt}(Y_p^{def}(D)) = B_p^{mkt}(Y_p^{cr}(D)) \tag{1.16}$$

i.e. when, at the income level where the borrowing capacity from the market is insufficient to repay, the maximum funding ability of the insurance is even lower. This appears clearly from the comparison between (3.39) and (1.9).

Debt accumulation

The borrowing capacity of a country unambiguously increases in the presence of the insurance. The country can borrow more at t because in expectation it will be able to repay more at $t+1$: it might use the insurance resources in case of need, or otherwise, if the insurance is still available for the future, it will be able to re-borrow more from the markets. However, the spread for a given borrowing level is lower in the presence of the insurance. The effect on debt accumulation depends therefore on the behavior of the country: a prudent country that refrains from increasing its spending tends to accumulate less debt thanks to the lower spreads, whereas a "profligate" country takes advantage of the insurance to borrow to the maximum might accumulate more debt. Here we fully focus our attention on the "profligate" type of country.

We normalize initial income Y_0 to 1 and consider two values for the insurance limit M: a high value 0.5 and a low value 0.25. "High" and "low" are meant in relation to the current borrowing capacity from the market in the absence of insurance, which would be around 0.39.

Figure 1.3 (for the case $M = 0.5$ on the left panel and for $M = 0.25$ on the right panel) show how the borrowing level and the debt of a profligate country increases in the presence of the insurance, as a function of the probability p that the insurance will be available next period.

We see that the maximum debt that a country can undertake is increasing in p, and in particular it is always higher than in the case without insurance. Although in principle the insurance could lower the default probability (and hence the spread) so much that even while borrowing more a country could contract less debt, we find the result that debt increases with insurance robust to any reasonable parameter change.

Crises and Defaults

Now that we have established that profligate countries borrow more and accumulate more debt in the presence of the insurance, we turn to the question we are most interested in: how does the insurance affect the probability of a debt crisis? From the point of view of time t, when our profligate country accumulates maximum debt D^*, the probability of a crisis at $t+1$ is

$$P_{cr} = pP(Y_{t+1} < Y_p^{cr}(D^*)) + (1-p)P(Y_{t+1} < Y_0^{def}(D^*)) \tag{1.17}$$

and the probability of default is

$$P_{def} = pP(Y_{t+1} < Y_p^{def}(D^*)) + (1-p)P(Y_{t+1} < Y_0^{def}(D^*)) \tag{1.18}$$

The difference between the two is the probability of avoiding default thanks to the insurance.

Figure 1.4 (for $M = 0.5Y$ on the left panel and for $M = 0.25Y$ on the right panel) shows the probability of crises alongside the probability of default. Notice that with the low value of M the probability of default is equal to the probability of a crisis. Indeed, with $M = 0.25$ condition (1.16) is satisfied at every p: even for $p = 0$ it is $B_p^{mkt}(Y_p^{cr}(D^*)) = 0.347$, bigger than M.

We find that when the probability p that the insurance is available next period is high, the insurance reduces the probability of crises, which is noteworthy given that the profligate country takes maximum advantage of the insurance to borrow to the maximum. This happens for $p \gtrsim 95.5\%$ in case $M = 0.5$, and for $p \gtrsim 52.5\%$ for $M = 0.25$. On the other hand, for lower values of p the probability of crises and even of defaults increases relative to the case of no insurance, and can reach very high values when the insurance is high.

To understand the result with high p, consider the extreme case in which the insurance is available with certainty, i.e. $p = 1$. A country can take more debt at t thanks to the insurance, but it will be able to re-borrow more at $t+1$ to repay this debt, and in particular it will be able to re-borrow more (relative to the world without insurance) when its income is low. This increase in market borrowing capacity at low income is what reduces the probability of crises.

Paradoxically, for $p = 1$ the country can borrow more, increase its debt and be able to repay more often thanks to the insurance, but will hardly ever need the insurance (unless the insurance is very big relative to the country's income). Knowing that the insurance will be there for sure next period, markets are always willing to lend at least $\frac{M}{1+r_f}$. Moreover, with our baseline parameters, for $p = 1$ $B^{mkt}(Y) > M$ for $Y \gtrsim 0.55M$. A bailout occurs only if the borrowing capacity is not enough to repay and M is enough to fully repay, so it occurs only if income is small compared to the size of the insurance, $Y < 0.55M$, and debt is in the narrow range $\phi Y + M/(1+r_f) < D < \phi Y + M$.

If $Y_t >> 0.55M$ the insurer really becomes a lender of last resort that stabilizes the markets without (almost) ever needing to intervene.

Two effects contribute to increasing the probability of crises and defaults for p lower than 1. First, the country can take more debt at t thanks to the insurance, but as p falls below 1 there is increasing probability $(1-p)$ that the insurance ceases to be available at $t+1$, in which case its borrowing capacity is lower again, thus the likelihood of default is much higher. [8]

[8] To identify this effect from numerical results, we rewrite (1.17) as

$$P_{cr} = pP(cr|A) + (1-p)P(cr|NA) \tag{1.19}$$

A second effect occurs for $M \gtrsim 0.4$. For $M = 0.5$ this occurs in the range of p between 0.75 and 0.9 (see Figure 1.4, left panel). In this range there is true moral hazard: by borrowing to its maximum capacity, the country essentially promises to use the insurance to repay whenever the insurance is available, and default when the insurance is not available. As we see from Figure 1.3, around $p = 0.7$ debt increases much faster in p than the borrowing capacity from the market, so that in the range between $p = 0.75$ and $p = 0.9$ debt is very high relative to borrowing capacity, which makes it difficult to repay by re-borrowing from the market. At the same time, M is bigger than the borrowing capacity from the market. This implies that (almost) the only chance for the country to repay is to use the insurance when available. For example, consider the case of $p = 0.75$. A country can take a debt equal to around 58% of its income but its borrowing capacity next period will be only around 45% of its income, if the latter does not change too much and the insurance is still available. Next period, even if the insurance is available, a crisis occurs if $Y < 1.08$. However, given the insurance $M = 0.5$, even in a crisis the country can still repay if its income is in the interval $0.83 < Y < 1.08$, which happens with 70% probability.

The probability of a crisis rises to enormous levels in this region (the peak is over 75% in case $M = 0.5$), as does the probability of default (around 25%). Even the probability of a crisis conditional on the insurance being available peaks at over 70%.

The main message of this section is that, while a certain or almost certain insurance reduces the incidence of crises, the sharing mechanism introduces uncertainty and can have counterproductive effects. While bailouts are meant to avoid defaults, we have shown that an uncertain insurance can increase the incidence of both crises and defaults.

where $P(cr|A)$ and $P(cr|NA)$, shown in Figure 1.5 for $M = 0.5$ (left panel) and $M = 0.25$ (right panel), are the probabilities of a crisis next period conditional on the insurance being available and not available, respectively. We can check from the numerical values reported in Appendix B that the second term on the RHS of (1.19), $(1-p)P(cr|NA)$, is decreasing in p, (for any p, in case $M = 0.25$, and for $p > 0.7$, in case $M = 0.5$). This occurs despite the fact that $P(cr|NA)$ is (weakly) increasing in p.

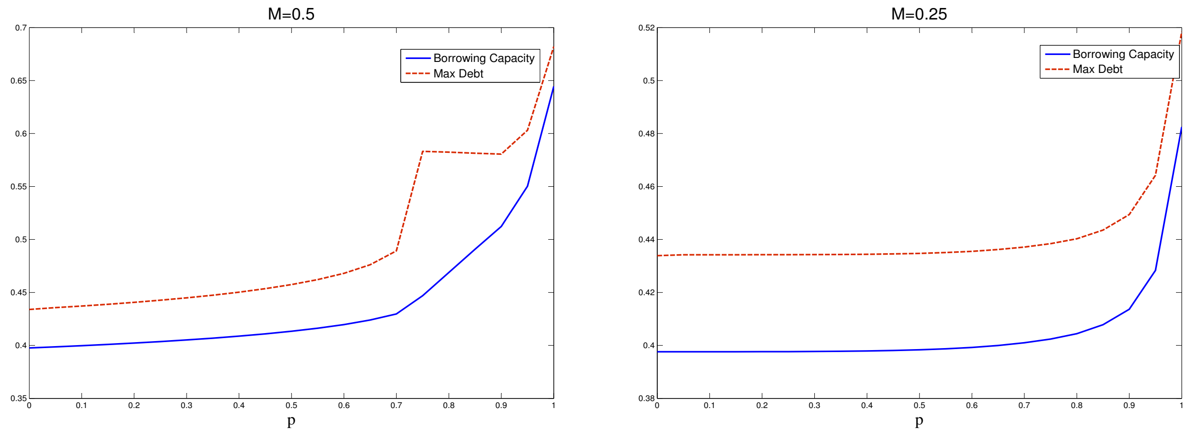

Figure 1.3: **Borrowing capacity and debt accumulation as a function of p, for M equal to 50% of income (left) and 25% of income (right).**

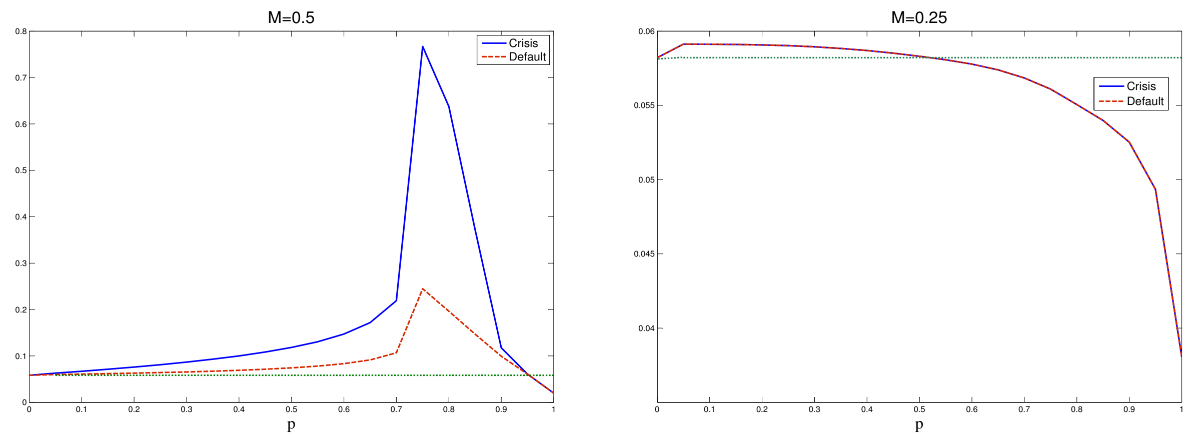

Figure 1.4: **Probability of crisis and default as a function of p, for M equal to 50% of income (left) and 25% of income (right). The dotted line is the probability of default in case of no insurance.**

1.4 Two profligate countries: full solution

In this section we solve the full "two-profligate-country" problem: each country borrows to its maximum capacity. With the reduced form approach of section 3, one country's borrowing capacity depended on p, an exogenous parameterization of the other country's fundamentals and choices. In the full two-country problem, the two countries solve a joint problem. Their borrowing

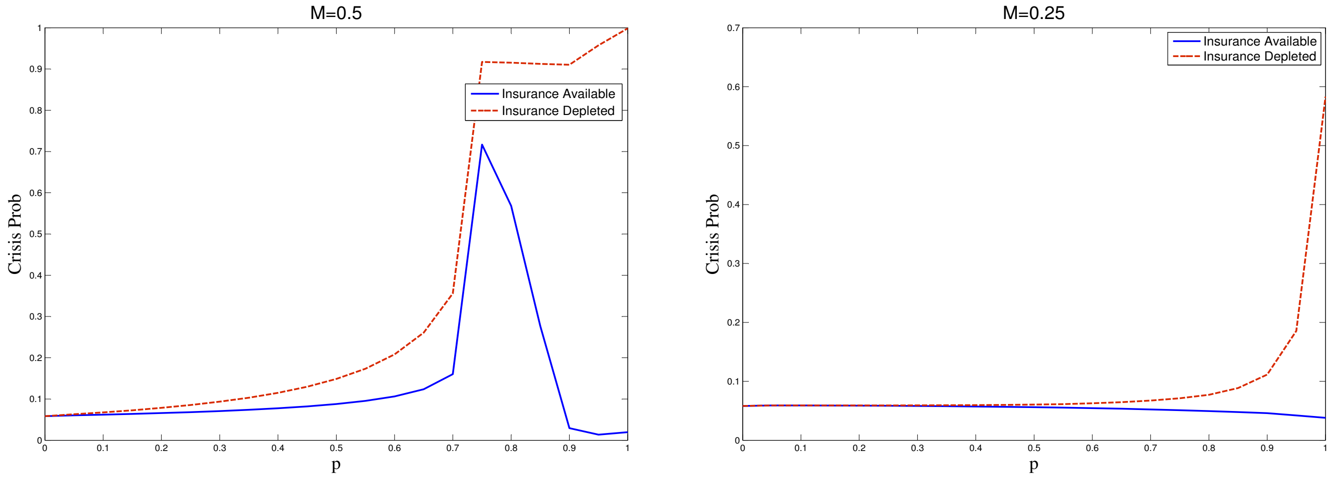

Figure 1.5: **Probability of crisis as a function of p, for M equal to 50% of income (left) and 25% of income (right), conditional on the insurance being available or not.**

capacities solve the system

$$B^{*(1)}\left(Y_t^{(1)}, Y_t^{(t)}\right) = \frac{1}{1+r_f} max_{D^{(1)}}\{P_{def}^{(1)} E_t[\phi Y_{t+1}^{(1)} | def^{(1)}] + D^{(1)}(1 - P_{def}^{(1)})]\} \quad (1.20)$$

$$B^{*(2)}\left(Y_t^{(1)}, Y_t^{(t)}\right) = \frac{1}{1+r_f} max_{D^{(t)}}\{P_{def}^{(t)} E_t[\phi Y_{t+1}^{(t)} | def^{(t)}] + D^{(t)}(1 - P_{def}^{(t)})+]\} \quad (1.21)$$

This is a joint fixed point problem, since the probability of default $P_{def}^{(i)}$ for country $i = 1, 2$ also involves the arg max of the other country's problem, $D^{*(-i)}$, and both borrowing capacities $B^{*(i)}(Y_{t+1}^{(1)}, Y_{t+1}^{(t)})$. Indeed, whether or not one country defaults next period depends on how much it will be able to re-borrow, which also depends on whether the insurance is still available (i.e. on whether the other country is able to repay), which in turn depends on the debt contracted by the other country and how much the other country can re-borrow. More details about this maximization problem are given in Appendix A.

We use the normalization $M = 0.5$. The solution can be obtained in a recursive way similarly to what described in Section 3. Numerical results for $B^{*(1)}$ and $B^{*(2)}$ for a grid of values of $[Y^{(1)}, Y^{(t)}]$ can be found in Appendix B.[9] Here we would like to briefly discuss two features that emerge from the solution: the advantage of the "leader" country toward the "follower" country, and the advantage of the "small" country toward the "big" country.

[9]Notice that results for a different value of M can easily be inferred from our results. We have indeed

$$\frac{B_{\lambda M}^{*(i)}(\lambda Y_i, \lambda Y_{-i})}{\lambda Y_i} = \frac{B_M^{*(i)}(Y_i, Y_{-i})}{Y_i} \quad (1.22)$$

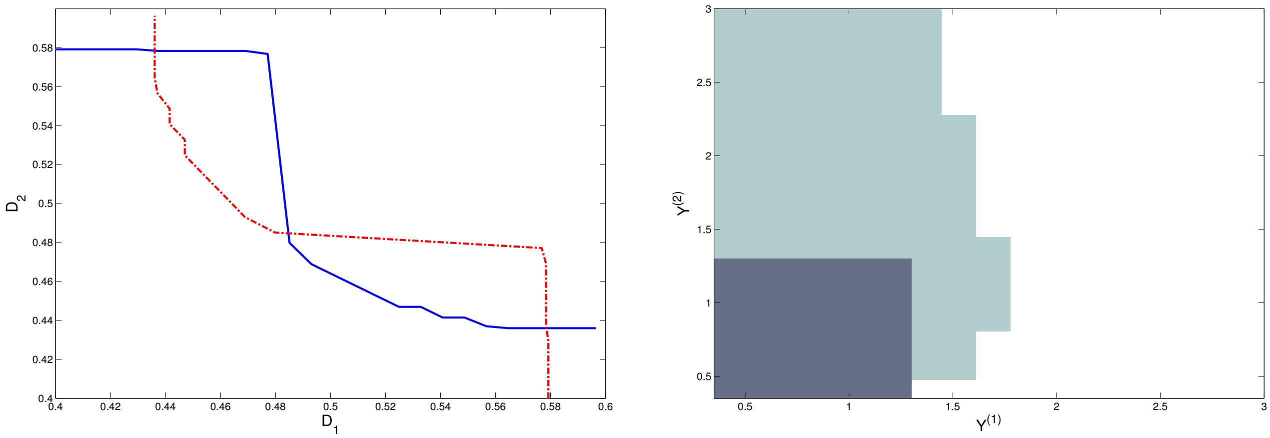

Figure 1.6: **Left panel: Best response function for Country 1 and Country 2, when $Y_1 = Y_2 = 1$. Right panel: in dark blue, the multiple-equilibrium region at first order in the recursion; in light blue, the region where $B^{*(1)}(y, y') > B^{*(2)}(y', y)$ (final solution).**

The advantage of the "leader" vs the "follower". The two countries simultaneously choose the amount of debt they issue. When solving the system (1.20)-(1.21), the first finding is that there are in general multiple Nash equilibria.

At first order in the recursion, we use $B^{*(i)}(Y_i, Y_{-i}) = b^* Y_i$ for the borrowing capacity entering the RHS of (1.20)-(1.21), i.e. the borrowing capacity at $t+1$. Notice that this would be the correct borrowing capacity if the insurance was not available any more at $t = 1$, i.e. if the insurance lasted only one period.

We find that for Y_1 and Y_2 equal to or smaller than 1.2 (240% the insurance size) there are 3 values of $\{D_1, D_2\}$ such that D_1 is the best response of Country 1 to Country 2 issuing D_2 and D_2 is the best response of Country 2 to Country 1 issuing D_1. See for example the best response plot in Figure 6, referring to the case $Y_1 = Y_2 = 1$. [10] In the latter case, in addition to a "symmetric" equilibrium in which the two countries issue the same debt, there are two "asymmetric" equilibria in which one country is able to issue more debt and the other less debt than in the symmetric equilibrium. When $Y_1 = Y_2$, the two asymmetric equilibria, the "top left" and the "bottom right", are identical, except that the roles of the two countries are switched. For $Y_1 \neq Y_2$, the "top left" ("bottom right") equilibrium is identical (after switching Country 1 $\leftrightarrow$ Country 2) to the "bottom right" ("top left") equilibrium for $Y_1' = Y_2$ and $Y_2' = Y_1$. Therefore, for every point in the joint income space $\{Y_1, Y_2\}$, we can limit ourselves to considering one "asymmetric" equilibrium, for example the "bottom-right", thus using the convention that Country 1 is the one that, everything

[10] The blue line (solid) represents the debt chosen by Country 2 (on the vertical axis) as a best response to the debt chosen by Country 1 (on the horizontal axis). The red line (dot-dashed) represents the debt chosen by Country 1 (on the horizontal axis) as a best response to the debt chosen by Country 2 (on the vertical axis).

else equal, can borrow more:

$$B^{*(1)}(y, y') \geq B^{*(2)}(y', y) \tag{1.23}$$

We call Country 1 the "leader" and Country 2 the "follower" [11].

In the region where multiple equilibria exist, we focus our attention on the "asymmetric" equilibrium with a leader and a follower [12].

When proceeding to the next orders in the recursion, the leader has a twofold advantage: in addition to being the most aggressive in the borrowing game, it can also borrow more because its borrowing ability next period is higher. Indeed, at each order n in the recursion, next period's borrowing capacity, entering the RHS of (1.20)-(1.21)), is the one obtained at order $n-1$, which gives at each order a higher advantage to the "leader". The right panel of Figure 6 shows the region in the joint income space where (1.23) holds with strict inequality at the fixed point in the recursion, and compares it with the smaller area in which an asymmetric equilibrium exists where (1.23) holds with strict inequality at first order.

The advantage of the small country. The second pattern that we see in the result is the advantage (in terms of borrowing capacity) of a small country over a big country. First, looking at the tables in Appendix B, we notice that the advantage of the leader tends to fade out as the leader's income increases. When the leader's income is smaller than 0.8 (160% the insurance size), all the benefits of the insurance, in terms of increased borrowing capacity, belong to the leader. The follower's borrowing capacity is unaffected by the insurance. When the leader's income is between 0.8 and 1, the follower's borrowing capacity is only modestly affected by the insurance, by at most a few percentage points. In contrast, in the region where the follower's income is below 0.8 and the leader's income is above 1.2, or where the follower's income is between 0.8 and 1 and the leader's income is above 1.5, all the benefits of the insurance shift to the follower, and it is the leader whose borrowing capacity falls to the same level as in the absence of insurance.

To illustrate the mechanism, consider the extreme case in which Country 2, the follower, has 0 income and Country 1 has positive income. For any debt level chosen by Country 1, the best response of Country 2 is to issue debt M. Indeed, even if D_1 is high, there is still some probability, maybe small, that Country 1 is able to repay without the help of the insurance, in which case Country 1 can avoid default by transfering the full amount of the insurance to its creditors. This gives a positive borrowing capacity to Country 1. Given that Country 1 has no ability to repay without the help of the insurance, this is clearly the strategy that maximizes its borrowing ability.

[11] We stress again that this is a simultaneous-move game, so the "leader" and the "follower " in this game are not to be confused with the homonymous concepts in a Stackelberg type of game.

[12] Asymmetric initial conditions, or asymmetric shocks at one point in time, may create a situation in which one country (Country 1 in our convention) needs to borrow more aggressively to be able to repay. In this situation Country 2 does not have an incentive to compete to be the "leader" since a default of Country 1 would deplete the shared insurance. Once Country 1 is the "leader" in one period, the very high debt it accumulates very likely requires it to be the leader in the following period, and so on.

On the other hand Country 1, despite being the leader, has lost any protection from the insurance given that Country 2 will for sure deplete it next period. As a result, Country 1's borrowing capacity is reduced to b^*Y_2 (as in a world without insurance).

In other words, the ability of the small country to engage in moral hazard takes away any protection from the big country. This is one mechanism by which a small country such as Greece may have been able to borrow a lot more than bigger countries in the EA and almost taken full control of the bailout resources of the union.

The "small country effect" results in a non-monotonic and non-smooth behavior of the borrowing surface, as we see in Figure 7, representing sections of the surface for one country in which the income of "the other" country is kept constant, and Figure 8, representing sections of the surface for one country in which its own income is kept constant.

Consider as an example the case in which the follower's income is $Y^{(t)} = 0.4$. Given the follower's small income relative to the insurance, the leader starts losing its advantage as a leader for $Y^{(1)} > 1.2$, resulting in a loss of borrowing capacity even as its income increases (see Figure 7, left panel, solid line). Correspondingly, at the same point $[Y^{(1)} = 1.2, Y^{(t)} = 0.4]$ the follower's borrowing capacity jumps up (Figure 8, right panel, solid line).

Similarly, consider the dash-dotted lines in the right panel of Figure 7 and left panel of Figure 8, showing the sections of the leader's and follower's surface for $Y^{(1)} = 1.4$. As the follower's income grows beyond 0.6, its borrowing capacity decreases and correspondingly the leader's income suddenly increases. Here, the follower is losing its small country advantage, and the leader is regaining its leader advantage.

1.4.1 Probability of a crisis: Results

Figure 9 gives a visual representation of the probability of a crisis involving the leader (on the left) or the follower (on the right). Numerical values can be found in Appendix B.

In the region where both incomes are high (relative to the insurance), the crisis probability for both countries is around or below the crisis probability without insurance. In particular, in the regions in black in Figure 9, the crisis probability is between 4% and 5.3%, significantly smaller than the probability of a crisis without insurance for a profligate country (which is 5.8%). As in the discussion in Section 3, these are the areas where the insurance is unlikely to be depleted next period, and the insurance induces the markets to lend more, especially in a downturn. However, when a country's income is very big relative to the insurance, so that the latter becomes irrelevant, its crisis probability is essentially the same as without insurance (areas in dark blue).

"Moral hazard" regions with crisis probability above 25% occur mostly when at least one income is smaller than the insurance size. When both incomes are smaller than the insurance, it is the leader that can take control of the bailout resources, therefore there is a high probability of a crisis involving the leader. In this case, since the insurance will almost surely be depleted by the leader, the follower's borrowing capacity and crisis probability are unchanged relative to a world without insurance. Conversely, in the regions where the follower's income is smaller than the insurance and and the leader's income is bigger than 1.3 (260% the insurance size), it is the follower that can engage in moral hazard, and the leader's crisis probability is the same as without the insurance.

In the region in which the income of Country 1 is between 1 and 1.8 (between 2 and 3.6 times the insurance limit), and the income of Country 2 is smaller than 1.5 (3 times the insurance limit), we observe many different small spots of lighter or darker color. This is the region in which big shifts in borrowing capacity occur between leader and follower, the new effects discussed in this section. It is intuitive that sudden shifts in borrowing capacity can lead one country that borrowed a lot in one period unable to re-borrow enough the next period, or the other way around.

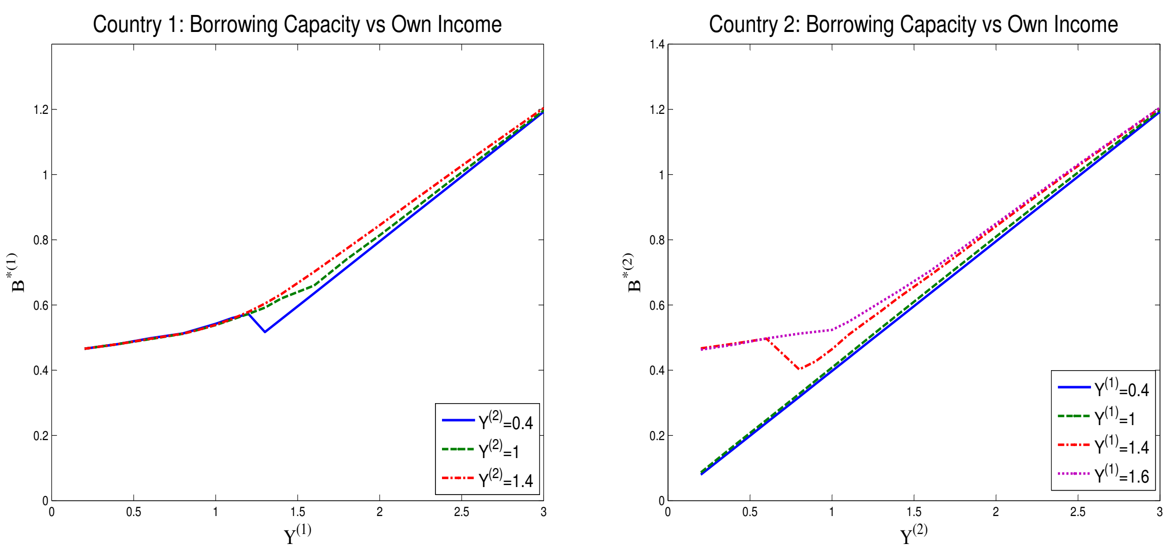

Figure 1.7: **Left (right) panel: sections of the leader's (follower's) borrowing capacity surface as its own income moves and the other country's income is fixed.**

1.5 Conclusion

In this paper we have suggested that the availability of bailouts in the Euro Area can explain some broad features of sovereign yields before and after the financial crisis. We have extended the model of CHR to include two countries and a bailout fund with fixed size resources. We have obtained a solution using two different approaches that may be thought as complementary: a "reduced form

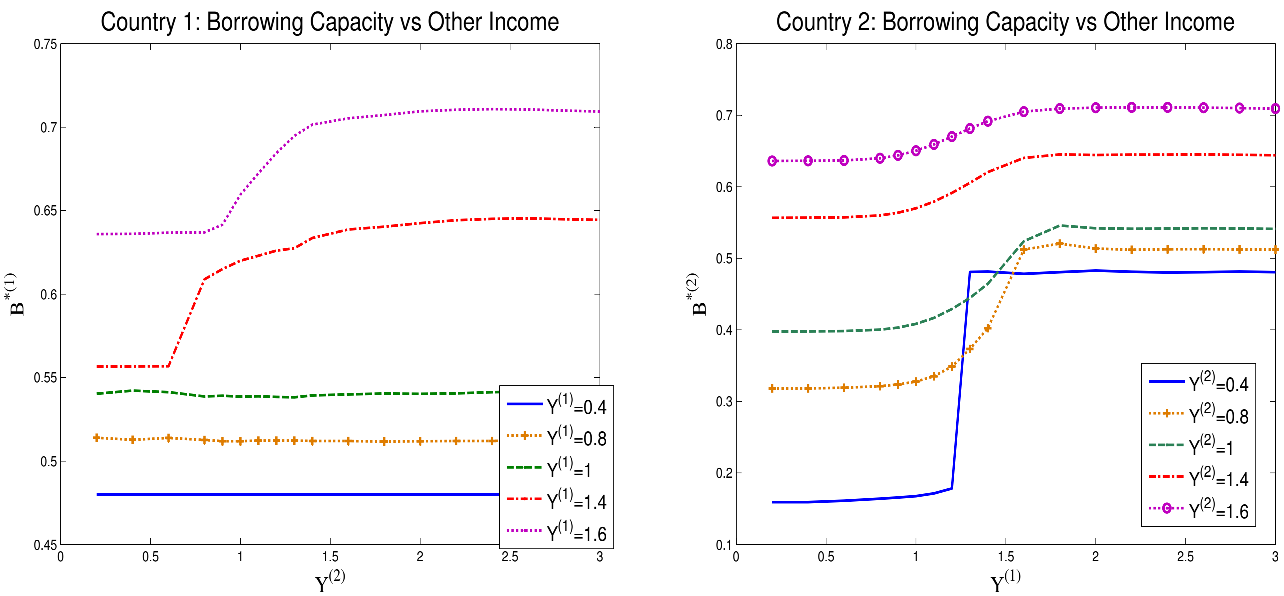

Figure 1.8: **Left (right) panel: sections of the leader's (follower's) borrowing capacity surface as its own income is fixed and the other country's income moves.**

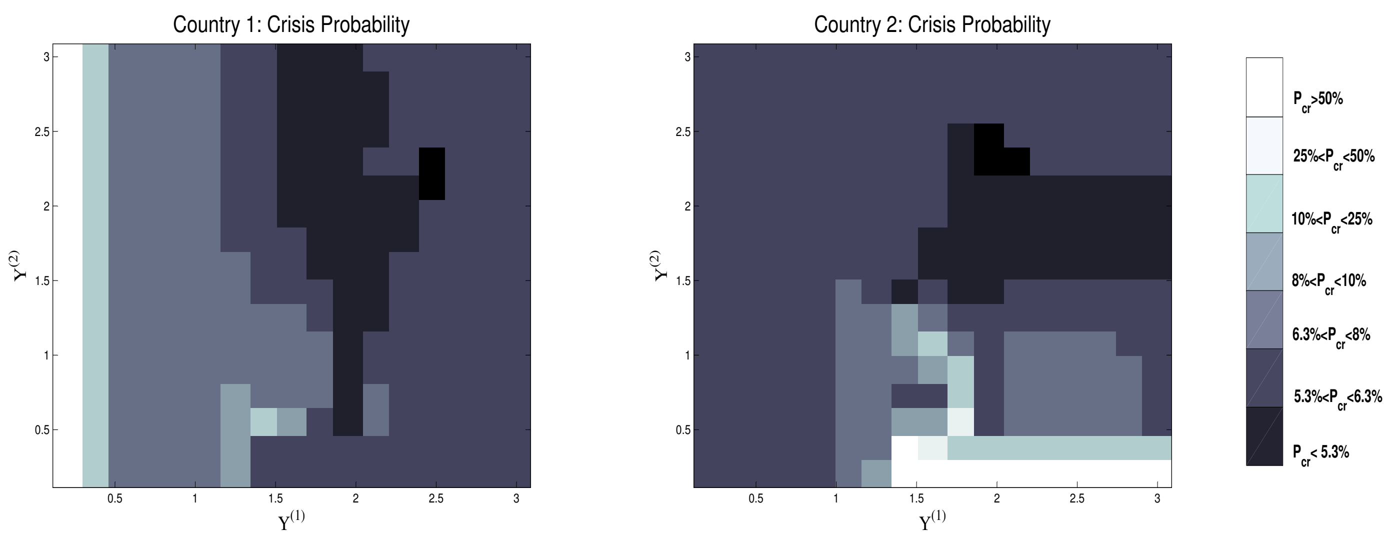

Figure 1.9: **Probability of a crisis over the next period for the leader (left) and the follower (right)**.

approach", which is more appropriate when one of the borrowing countries is "profligate" whereas the other country (or countries) tries to keep its crisis probability stable, and a full 2-country approach relying on the assumption that both countries are profligate.

We have also discussed how a shared pool of bailout resources has ambiguous effects on the probability of a crisis. A country in a union where bailouts are available has two possible lenders, the markets and the "institutional" lenders. We have discovered that the interaction between the two lenders can lead to a lower probability of crisis: markets can lend more in a downturn because of the "buffer" provided by the bailout resources, so that the latter are actually rarely used. However, big crisis probabilities arise as a result of the competition between countries to exploit the insurance, especially when the insurance resources are big relative to one country's borrowing ability.

A final remark: after ECB's President Draghi "whatever it takes" speech in the summer of 2012, sovereign spreads in the Euro periphery decreased dramatically, without any need for the ECB to actually intervene. Many saw these events as confirmation that the preceding high-spread period was a "bad equilibrium"[13]. In the framework of this paper we could instead interpret Draghi's speech as causing a revision of beliefs toward a higher value of the insurance limit M, thus as a change in fundamentals. However, in this paper we only considered the case when M is fixed and known with certainty. The case of an uncertain M could be a worthwhile extension.

Appendix A

1. Model by CHR: derivation of main results

Since by assumption lenders are risk neutral, we find B_t^*, the borrowing capacity at time t, as

$$(1+r_f)B_t^* = Max_{D_{t+1}} E_t[R_{t+1}(D_{t+1})] \tag{1.24}$$

We want to show that the borrowing capacity B_t^* is proportional to income Y_t:

$$B_t^* = b^* Y_t \tag{1.25}$$

If this is true, then, as per (1.8), default happens when $g_{t+1} < \frac{d_{t+1}}{\phi+b^*}$. Using the assumption that the distribution of the growth factor is stationary, (1.24) can be written as

$$(1+r_f)b^* Y_t = Max_d \left[\int_0^{\frac{d}{\phi+b^*}} \phi g Y_t \; pdf(g) \; dg + dY_t \times P\left(g > \frac{d}{\phi+b^*}\right) \right] \tag{1.26}$$

or, dividing the LHS and RHS by Y_t,

$$(1+r_f)b^* = Max_d \left[\int_0^{\frac{d}{\phi+b^*}} \phi g \; pdf(g) \; dg + d \times P\left(g > \frac{d}{\phi+b^*}\right) \right] \tag{1.27}$$

[13]Draghi himself proposed this view in his speech: "... the assessment of the Governing Council is that we are in a situation now where you have large parts of the Euro Area in what we call a 'bad equilibrium' , namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios."

Notice that any dependence on time has disappeared, so the b^* that solves (1.27) must indeed be constant. By defining $d' \equiv \frac{d}{\phi+b^*}$ we obtain

$$(1+r_f)b^* = Max_{d'}\left[\int_0^{d'} \phi g\ pdf(g)\ dg + d'(\phi+b^*) \times P(g > d')\right] \tag{1.28}$$

hence b^* can be found as the solution of the fixed-point equation

$$b^* = g(b^*) \tag{1.29}$$

where

$$g(b^*) = \frac{Max_{d'}\left[\int_0^{d'} \phi g\ pdf(g)\ dg + d'(\phi+b^*) \times P(g > d')\right]}{1+r_f} \tag{1.30}$$

For a given borrowing level $B_t = b_t Y_t$, debt is $D_{t+1} = d_{t+1}Y_t$ where d'_{t+1} solves

$$(1+r_f)b_t = \left[\int_0^{d'_{t+1}} \phi g\ pdf(g)\ dg + d'_{t+1}(\phi+b^*) \times P(g > d'_{t+1})\right] \tag{1.31}$$

and correspondingly the required debt repayment (as a fraction of Y_t) $d_{t+1} = (\phi+b^*)d'_{t+1}$. The spread x_t is easily obtained by solving

$$d_{t+1} = (1+r_f+x_t)b_t \tag{1.32}$$

2. Full solution with two profligate countries.

The borrowing capacity of the two country solves the system (1.20)-(1.21), that we repeat here for convenience

$$\begin{aligned} B^{*(1)}\left(Y_t^{(1)}, Y_t^{(t)}\right) &= \frac{1}{1+r_f} max_{D_1}\{P_{def}^{(1)} E_t[\phi Y_{t+1}|def^{(1)}] + D_1(1-P_{def}^{(1)})]\} && (1.33) \\ B^{*(2)}\left(Y_t^{(t)}, Y_t^{(1)}\right) &= \frac{1}{1+r_f} max_{D_2}\{P_{def}^{(t)} E_t[\phi Y_{t+1}|def^{(t)}] + D_2(1-P_{def}^{(t)})+]\} && (1.34) \end{aligned}$$

Given the debt issued by Country 2 D_2^* (the arg max of (1.34)), Country 1 fully repays in one of the following cases

$$\begin{aligned} &\text{Case1}: && Y_{t+1}^{(1)} > \tfrac{D^{(1)}-M}{\phi} \quad \& \quad Y_{t+1}^{(t)} > \tfrac{D^{*(2)}}{\phi+b^*} \\ &\text{Case2}: && Y^{(1)} + Y^{(t)} > \tfrac{D^{(1)}+D^{*(2)}-M}{\phi} \\ &\text{Case3}: && \left(\phi Y_{t+1}^{(1)} + B^{*(1)}(Y_{t+1}^{(1)}, Y_{t+1}^{(t)})) > D^{(1)}\right) \quad \& \quad \left(\phi Y_{t+1}^{(t)} + B^{*(2)}(Y_{t+1}^{(t)}, Y_{t+1}^{(1)})) > D^{*(2)}\right) \\ &\text{Case4}: && Y_{t+1}^{(1)} > \tfrac{D^{(1)}}{\phi+b^*} \end{aligned}$$

In Case 1, Country 1 needs the insurance to repay, but Country 2 can repay by borrowing from the markets (notice that since Country 1 depletes the insurance, Country 2's borrowing capacity is reduced to $b^*Y_{t+1}^{(t)}$). In Case 2, bothe countries can repay by sharing the insurance. In Case 3, both countries are able to repay by borrowing from the markets (both have a higher borrowing capacity from the markets since the insurance will continue to be available in the future). In Case 4, Country 1 is able to repay by borrowing from the markets, even if Country 1 depletes the insurance. It is thus

$$P_{def}^{(1)} = 1 - P\left(\text{Case1} \;\; or \text{ Case2 } or \text{ Case3 } or \text{ Case4}\right) \tag{1.35}$$

As discussed in Section 5, the default probability for Country 1 involves the borrowing capacities of both countries and the argument of the maximization problem of Country 2, $D^{*(2)}$. By simply switching (1) $\leftrightarrow$ (2) we can obtain $P_{def}^{(t)}$.

Appendix B

The following tables contain all the numerical results discussed in Section 3 and 4. Table 1 and Table 2 contain results for Section 3. We report borrowing capacity, max debt, spread, crisis and default probability for a set of values of p, the probability that the insurance is available next period.

Table 3, 4, 5 and 6 contain all the numerical results discussed in Section 4. In particular, for a grid of values of the income of the two countries, Table 4 and 5 report the leader's and follower's borrowing surface, respectively; Table 5 and 6 report the crisis probability for the leader and follower, respectively.

A. Numerical results for Section 3

p	B^*	D^*	x	P_{cr}	P_{def}	$P(cr\|A)$	$P(cr\|NA)$
0	0.398	0.434	0.051	0.058	0.058	0.058	0.058
0.05	0.399	0.436	0.053	0.063	0.060	0.061	0.063
0.1	0.400	0.437	0.054	0.067	0.061	0.062	0.068
0.15	0.401	0.439	0.055	0.071	0.062	0.064	0.073
0.2	0.402	0.441	0.056	0.076	0.063	0.066	0.079
0.25	0.404	0.443	0.057	0.081	0.064	0.068	0.085
0.3	0.405	0.445	0.058	0.087	0.066	0.071	0.094
0.35	0.407	0.447	0.060	0.093	0.067	0.074	0.103
0.4	0.409	0.450	0.061	0.100	0.069	0.078	0.115
0.45	0.411	0.454	0.064	0.108	0.071	0.082	0.130
0.5	0.413	0.457	0.066	0.118	0.074	0.088	0.149
0.55	0.416	0.462	0.070	0.131	0.078	0.095	0.174
0.6	0.420	0.468	0.075	0.147	0.083	0.106	0.208
0.65	0.424	0.476	0.083	0.172	0.091	0.124	0.261
0.7	0.430	0.489	0.099	0.219	0.107	0.160	0.356
0.75	0.447	0.583	0.265	0.767	0.245	0.717	0.917
0.8	0.469	0.582	0.202	0.638	0.196	0.568	0.915
0.85	0.491	0.581	0.145	0.372	0.147	0.277	0.913
0.9	0.512	0.581	0.094	0.118	0.099	0.030	0.910
0.95	0.550	0.603	0.056	0.061	0.061	0.014	0.958
1	0.644	0.682	0.019	0.020	0.020	0.020	0.998

Table 1.1: Initial income $Y = 1$, $M = 0.5$.

p	B^*	D^*	x	P_{cr}	P_{def}	P^1_{cr}	P^2_{cr}
0	0.398	0.434	0.051	0.058	0.058	0.058	0.058
0.05	0.398	0.434	0.052	0.059	0.059	0.059	0.059
0.1	0.398	0.434	0.052	0.059	0.059	0.059	0.059
0.15	0.398	0.434	0.052	0.059	0.059	0.059	0.059
0.2	0.398	0.434	0.052	0.059	0.059	0.059	0.059
0.25	0.398	0.434	0.052	0.059	0.059	0.059	0.059
0.3	0.398	0.434	0.052	0.059	0.059	0.058	0.059
0.35	0.398	0.434	0.052	0.059	0.059	0.058	0.059
0.4	0.398	0.434	0.052	0.059	0.059	0.057	0.060
0.45	0.398	0.435	0.051	0.059	0.059	0.057	0.060
0.5	0.398	0.435	0.052	0.058	0.058	0.056	0.061
0.55	0.399	0.435	0.051	0.058	0.058	0.055	0.061
0.6	0.399	0.436	0.051	0.058	0.058	0.055	0.063
0.65	0.400	0.436	0.050	0.057	0.057	0.054	0.065
0.7	0.401	0.437	0.050	0.057	0.057	0.052	0.067
0.75	0.402	0.438	0.049	0.056	0.056	0.051	0.071
0.8	0.404	0.440	0.049	0.055	0.055	0.050	0.077
0.85	0.408	0.444	0.048	0.054	0.054	0.048	0.089
0.9	0.414	0.449	0.047	0.053	0.053	0.046	0.111
0.95	0.428	0.464	0.044	0.049	0.049	0.042	0.186
1	0.483	0.518	0.034	0.038	0.038	0.038	0.583

Table 1.2: Initial income $Y = 1$, $M = 0.25$.

B. Numerical results for Section 4

Leader's borrowing surface

Y^1 \ Y^2	0.4	0.6	0.8	0.9	1	1.1	1.2	1.3	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0.4	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480
0.6	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
0.8	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512
0.9	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526
1	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540
1.1	0.557	0.557	0.557	0.557	0.557	0.557	0.557	0.557	0.559	0.560	0.560	0.561	0.561	0.561	0.561	0.562	0.562
1.2	0.568	0.569	0.569	0.574	0.574	0.574	0.575	0.579	0.581	0.583	0.584	0.584	0.584	0.585	0.585	0.586	0.586
1.3	0.517	0.517	0.582	0.594	0.594	0.594	0.594	0.606	0.606	0.609	0.610	0.612	0.612	0.613	0.614	0.614	0.614
1.4	0.557	0.557	0.611	0.612	0.612	0.615	0.624	0.631	0.636	0.638	0.640	0.642	0.643	0.643	0.644	0.644	0.644
1.6	0.636	0.637	0.637	0.640	0.655	0.676	0.687	0.696	0.701	0.704	0.707	0.708	0.709	0.709	0.710	0.710	0.710
1.8	0.716	0.718	0.730	0.734	0.741	0.751	0.761	0.767	0.771	0.776	0.778	0.779	0.779	0.780	0.780	0.780	0.780
2	0.795	0.797	0.806	0.812	0.820	0.829	0.837	0.843	0.846	0.850	0.851	0.852	0.852	0.852	0.852	0.852	0.852
2.2	0.875	0.878	0.887	0.892	0.899	0.907	0.913	0.918	0.922	0.925	0.926	0.926	0.926	0.926	0.926	0.926	0.926
2.4	0.954	0.957	0.965	0.970	0.977	0.984	0.990	0.994	0.997	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2.6	1.034	1.036	1.043	1.048	1.054	1.060	1.065	1.070	1.073	1.076	1.076	1.076	1.076	1.076	1.076	1.076	1.076
2.8	1.113	1.115	1.120	1.125	1.130	1.136	1.141	1.145	1.148	1.151	1.152	1.152	1.152	1.152	1.152	1.152	1.152
3	1.193	1.194	1.199	1.203	1.208	1.213	1.218	1.222	1.224	1.227	1.227	1.227	1.227	1.227	1.227	1.227	1.227

Follower's borrowing surface

Y^2 \ Y^1	0.4	0.6	0.8	0.9	1	1.1	1.2	1.3	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0.4	0.159	0.161	0.163	0.165	0.167	0.171	0.178	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480
0.6	0.239	0.240	0.243	0.245	0.249	0.254	0.265	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
0.8	0.318	0.320	0.323	0.325	0.330	0.337	0.351	0.375	0.404	0.512	0.512	0.512	0.512	0.512	0.512	0.512	0.512
0.9	0.358	0.359	0.362	0.365	0.370	0.378	0.392	0.410	0.429	0.526	0.526	0.526	0.526	0.526	0.526	0.526	0.526
1	0.398	0.399	0.402	0.405	0.410	0.419	0.431	0.446	0.464	0.522	0.540	0.540	0.540	0.540	0.540	0.540	0.540
1.1	0.438	0.439	0.442	0.445	0.451	0.459	0.472	0.487	0.507	0.542	0.560	0.561	0.561	0.561	0.561	0.561	0.561
1.2	0.477	0.478	0.481	0.485	0.491	0.500	0.513	0.529	0.544	0.568	0.584	0.584	0.584	0.585	0.585	0.586	0.586
1.3	0.517	0.518	0.521	0.526	0.531	0.540	0.553	0.563	0.577	0.602	0.610	0.612	0.612	0.613	0.614	0.614	0.614
1.4	0.557	0.558	0.561	0.565	0.571	0.580	0.591	0.603	0.614	0.638	0.640	0.642	0.643	0.643	0.644	0.644	0.644
1.6	0.636	0.637	0.641	0.645	0.651	0.659	0.669	0.679	0.689	0.704	0.707	0.708	0.709	0.709	0.710	0.710	0.710
1.8	0.716	0.717	0.721	0.725	0.731	0.739	0.748	0.757	0.765	0.776	0.778	0.779	0.779	0.780	0.780	0.780	0.780
2	0.795	0.796	0.801	0.805	0.811	0.818	0.826	0.834	0.841	0.850	0.851	0.852	0.852	0.852	0.852	0.852	0.852
2.2	0.875	0.876	0.880	0.884	0.890	0.897	0.904	0.911	0.916	0.925	0.926	0.926	0.926	0.926	0.926	0.926	0.926
2.4	0.954	0.955	0.960	0.964	0.970	0.976	0.982	0.988	0.997	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2.6	1.033	1.035	1.039	1.043	1.049	1.055	1.061	1.066	1.073	1.076	1.076	1.076	1.076	1.076	1.076	1.076	1.076
2.8	1.113	1.114	1.119	1.123	1.128	1.134	1.139	1.143	1.148	1.151	1.152	1.152	1.152	1.152	1.152	1.152	1.152
3	1.193	1.194	1.198	1.202	1.207	1.212	1.217	1.220	1.224	1.227	1.227	1.227	1.227	1.227	1.227	1.227	1.227

Leader's crisis probability

Y^1 \ Y^2	0.4	0.6	0.8	0.9	1	1.1	1.2	1.3	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0.4	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880
0.6	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171
0.8	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.063
0.9	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065
1	0.072	0.072	0.072	0.072	0.072	0.071	0.072	0.072	0.070	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069
1.1	0.078	0.078	0.078	0.076	0.074	0.073	0.073	0.073	0.068	0.066	0.064	0.064	0.063	0.063	0.063	0.063	0.063
1.2	0.096	0.093	0.090	0.083	0.078	0.075	0.077	0.066	0.063	0.059	0.058	0.057	0.056	0.055	0.055	0.055	0.054
1.3	0.061	0.060	0.105	0.078	0.078	0.078	0.079	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058
1.4	0.061	0.060	0.086	0.074	0.074	0.074	0.066	0.060	0.054	0.052	0.052	0.049	0.047	0.049	0.048	0.049	0.048
1.6	0.061	0.059	0.054	0.067	0.078	0.064	0.058	0.052	0.049	0.049	0.049	0.047	0.048	0.047	0.047	0.047	0.047
1.8	0.061	0.059	0.040	0.041	0.038	0.045	0.048	0.049	0.049	0.049	0.049	0.048	0.048	0.050	0.050	0.047	0.050
2	0.060	0.059	0.066	0.064	0.061	0.055	0.052	0.051	0.051	0.050	0.051	0.050	0.052	0.051	0.051	0.050	0.053
2.2	0.060	0.058	0.057	0.056	0.056	0.055	0.053	0.053	0.052	0.051	0.050	0.050	0.053	0.052	0.052	0.052	0.053
2.4	0.059	0.058	0.059	0.058	0.056	0.055	0.055	0.054	0.053	0.053	0.052	0.052	0.052	0.052	0.052	0.052	0.053
2.6	0.059	0.058	0.057	0.056	0.056	0.055	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.055	0.055	0.055	0.055
2.8	0.059	0.058	0.057	0.057	0.056	0.055	0.055	0.055	0.054	0.053	0.055	0.055	0.056	0.055	0.055	0.055	0.055
3	0.059	0.057	0.057	0.057	0.057	0.057	0.057	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055

Follower's crisis probability

Y^2 \ Y^1	0.4	0.6	0.8	0.9	1	1.1	1.2	1.3	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0.4	0.059	0.059	0.059	0.059	0.059	0.064	0.082	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880	0.880
0.6	0.059	0.059	0.059	0.059	0.059	0.064	0.078	0.662	0.460	0.214	0.173	0.177	0.175	0.176	0.175	0.175	0.174
0.8	0.059	0.059	0.059	0.059	0.059	0.063	0.074	0.095	0.083	0.263	0.056	0.073	0.065	0.065	0.065	0.065	0.060
0.9	0.059	0.059	0.059	0.059	0.059	0.063	0.066	0.062	0.062	0.201	0.055	0.072	0.064	0.065	0.064	0.065	0.060
1	0.059	0.059	0.059	0.059	0.059	0.063	0.066	0.070	0.086	0.117	0.055	0.070	0.063	0.065	0.063	0.065	0.060
1.1	0.059	0.059	0.059	0.059	0.059	0.063	0.067	0.080	0.118	0.068	0.054	0.063	0.062	0.063	0.062	0.062	0.058
1.2	0.059	0.059	0.059	0.059	0.059	0.063	0.073	0.089	0.071	0.058	0.053	0.061	0.055	0.057	0.056	0.056	0.053
1.3	0.059	0.059	0.059	0.059	0.059	0.063	0.059	0.041	0.057	0.052	0.049	0.053	0.054	0.054	0.054	0.054	0.054
1.4	0.059	0.059	0.059	0.059	0.059	0.058	0.056	0.053	0.050	0.047	0.049	0.050	0.050	0.050	0.050	0.050	0.051
1.6	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.054	0.052	0.047	0.048	0.044	0.045	0.045	0.045	0.045	0.045
1.8	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.053	0.052	0.049	0.048	0.047	0.046	0.047	0.047	0.047	0.047
2	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.054	0.054	0.049	0.048	0.051	0.051	0.051	0.052	0.052	0.052
2.2	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.054	0.053	0.050	0.052	0.052	0.054	0.054	0.054	0.054	0.054
2.4	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.054	0.053	0.052	0.052	0.052	0.053	0.054	0.054	0.054	0.053
2.6	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.055	0.054	0.054	0.054	0.053	0.053	0.054	0.054	0.054	0.053
2.8	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.055	0.055	0.054	0.054	0.053	0.054	0.054	0.054	0.054	0.054
3	0.059	0.059	0.059	0.059	0.059	0.057	0.056	0.056	0.055	0.055	0.054	0.056	0.056	0.056	0.057	0.057	0.057

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Chapter 2

Self-Fulfilling Debt Crises: What Can Monetary Policy Do?

Abstract This chapter examines the potential for monetary policy to avoid self-fulfilling sovereign debt crises. We combine a version of the slow-moving debt crisis model proposed by Lorenzoni and Werning (2014) with a standard New Keynesian model. Monetary policy could preclude a debt crisis through raising inflation and output and lowering the real interest rate. These reduce the real value of outstanding debt and the cost of new borrowing, and increase tax revenues and seigniorage. We determine the optimal path of inflation required to avoid a self-fulfilling debt crisis. Stronger price rigidity implies more sustained inflation.

2.1 Introduction

A popular explanation for sovereign debt crises is self-fulfilling sentiments. If market participants believe that sovereign default of a country is more likely, they demand higher spreads, which raises the debt burden and therefore indeed makes eventual default more likely.[1] This view of self-fulfilling beliefs is consistent with the evidence that the surge in sovereign bond spreads in

[0]**This chapter was written in collaboration with Philippe Bacchetta and Eric van Wincoop**

[1]For discussions of self-fulfilling crises in sovereign debt models see for example Aguiar et al. (2013), Calvo (1988), Camous and Cooper (2014), Cohen and Villemot (2015), Conesa and Kehoe (2015), Corsetti and Dedola (2014), de Grauwe (2011), de Grauwe and Ji (2013), Gros (2012), Jeanne (2012), Jeanne and Wang (2013), Krugman (2013), Lorenzoni and Werning (2014), and Miller and Zhang (2012). Ayres et al. (2015) show that multiple equilibria arise in sovereign debt models when the government chooses current debt, as opposed to debt at maturity. Even if the government chooses debt at maturity, they show that there are still multiple equilibria when lenders move first (choose an interest rate at which they are willing to lend).

Europe during 2010-2011 was disconnected from debt ratios and other macroeconomic fundamentals (e.g., de Grauwe and Ji, 2013).[2] It has also been suggested as an explanation for the Argentine crisis of 1998-2002 (Ayres et al., 2015). Recently a debate has developed about what role the central bank may play in avoiding such self-fulfilling debt crises. The central bank has additional tools to support the fiscal authority, either in the form of standard inflation policy or by providing liquidity. Some have argued that the US, Japan, UK and others have avoided such crises altogether because they have their own currency and monetary policy.[3]

In this paper we examine what central banks can do to avert self-fulfilling debt crises. We pose this as a general question, without focusing on a specific country or historical episode. For a realistic analysis, we allow for long-term debt, nominal rigidities and dynamics leading to slow-moving debt crises. We do so by combining a standard New Keynesian (NK) model with the "slow moving" debt crisis framework proposed by Lorenzoni and Werning (2014, henceforth LW). The LW model is in the spirit of Calvo (1988), but the latter is a two-period model with one-period bonds, while the LW model is dynamic and has long-term bonds. As is standard in sovereign debt models with self-fulfilling equilibria, there is a region of debt in which there are multiple equilibria. This exposes the country to self-fulfilling beliefs that lead to default premia, debt accumulation and possible default. We refer to this debt region as the "multiplicity region". The NK block of the model allows us to consider realistic monetary policy. The central bank can reduce the real value of original debt and of distributed coupons through inflation. It can also reduce the real cost of borrowing by reducing the (riskfree) real interest rate. Finally, expansionary monetary policy delivers seigniorage revenue and can raise the primary surplus by raising output. This all helps to slow down the accumulation of debt.

The objective of our investigation is to examine what optimal monetary policy looks like under the constraint that it avoids a self-fulfilling crisis. The central bank is capable of avoiding self-fulfilling default by acting aggressively enough, but it may not always be optimal to do so. There is a tradeoff between the cost of monetary policy, particularly inflation, versus the cost of outright default.[4] A monetary policy aimed at avoiding default is not credible when the cost of the policy is larger than that of outright default. Rather than making judgements about this tradeoff, our aim here is simply to document the cost of the monetary policy that avoids self-fulfilling default. We particularly focus on inflation under such a policy. We examine what the central bank can do when hit by default expectations at an initial time $t = 0$. The factors leading to such a crisis, in particular the initial level of debt, and the future path of fiscal policies, are taken as given by the

[2] This view was held by the ECB President Draghi himself: "... the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." (press conference, September 6, 2012).

[3] See for example de Grauwe (2011), de Grauwe and Ji (2013), Jeanne (2012) and Krugman (2013).

[4] This tradeoff is standard in the literature, e.g. see Aguiar et al. (2013), Camous and Cooper (2014), or Corsetti and Dedola (2016).

central bank. In this context our numerical analysis determines the optimal inflation path that is required to avoid a default. This deviation should be unexpected, i.e., we determine the required level of surprise inflation.

The required level of surprise inflation depends on the distance between the initial level of debt, B_0, and the maximum level of debt that avoids multiplicity; or equivalently, the minimum debt level in the multiplicity zone, B_{low}. When B_0 is close to B_{low}, the required level of additional inflation is obviously modest. However, we find that the required level of inflation can easily be large. Consider for example our benchmark parameterization that generates a multiplicity region ranging from 80 to 150 percent of GDP. If we take the debt ratio in the middle of this range, i.e., 115 of GDP, the price level ultimately has to increase by a factor 5 and the additional level of inflation should be higher than 20% for 4 years and over 10% for 8 years.

The potentially high level of inflation needed to avoid self-fulfilling crises has an intuitive explanation. Consider the above example for our benchmark case. If the central bank could raise the price level right away, without delay, the price level would only need to rise by 42 percent to reduce debt from 115 to 80 percent of GDP. In reality though, inflation is more gradual, both because of price stickiness and because it is optimal from a welfare point of view to have more gradual inflation.[5] However, this will ultimately lead to a much larger increase in the price level. The reason is that inflation becomes less effective over time because the interest rate for newly issued debt incorporates inflation expectations. For a realistic maturity of government debt, inflation gradually loses power in reducing debt. Hilscher et al. (2014) take this point seriously in estimating the impact of unexpected inflation in reducing outstanding debt and estimate this impact to be limited.

We devote significant space to the question of how robust this result is. We consider changes to all the parameters of both the LW and NK components of the model and find that the result is quite robust. We also consider results based exclusively on the LW part of the model. This monetary version of the LW model leads to a condition on inflation and real interest rates over time that needs to be satisfied in order to avoid the multiplicity region. It holds independent of how monetary policy affects inflation and real interest rates and is therefore independent of the NK portion of the model. Regarding the LW part of the model, the only key parameter is the maturity of the debt. All the other parameters can have an impact on the multiplicity region, but do not significantly affect the results if we keep B_0/B_{low} fixed.

We consider the standard case where in normal times it is optimal for the central bank to commit to zero inflation. But if initial inflation is already large, the additional surprise inflation needed to avoid a self-fulfilling crisis would be even more costly given the convex cost of inflation. We also

[5] Since inflation operates gradually it is not equivalent to outright default. It is true that both inflation and outright default reduce the real value of the debt, but gradual inflation also affects relative prices and intertemporal decisions by households and firms.

abstract from liquidity or rollover crises, such as Cole and Kehoe (2000). As Bocola and Dovis (2015) point out, we often see a substantial shortening of the maturity structure under steep inflation, which leads to additional problems in terms of exposure to rollover crises that we abstract from.

This is not the first paper to analyze the impact of monetary policy in a self-fulfilling debt crisis environment. The main difference is that previous work focuses on more qualitative questions using more stylized frameworks. It typically does not consider standard interest rate policies and considers mainly two-period models with one period bonds, flexible prices, constant real interest rates and a constant output gap. The role of monetary policy was first analyzed by Calvo (1988), who examined the trade-off between outright default and debt deflation. Corsetti and Dedola (2014) extend the Calvo model to allow for both fundamental and self-fulfilling default. They show that with optimal monetary policy debt crises can still happen, but for larger levels of debt. They also show that a crisis can be avoided if government debt can be replaced by risk-free central bank debt that is convertible into cash. Reis (2013) and Jeanne (2012) also develop stylized two-period models with multiple equilibria to illustrate ways in which the central bank can act to avoid the bad equilibrium.

Some papers consider more dynamic models, but still assume flexible prices and one-period bonds.[6] Camous and Cooper (2014) use a dynamic overlapping-generation model with strategic default. They show that the central bank can avoid self-fulfilling default if they commit to a policy where inflation depends on the state (productivity, interest rate, sunspot). Aguiar et al. (2013) consider a dynamic model to analyze the vulnerability to self-fulfilling rollover crises, depending on the aversion of the central bank to inflation. Although a rollover crisis occurs suddenly, it is assumed that there is a grace period to repay the debt, allowing the central bank time to reduce the real value of the debt through inflation. They find that only for intermediate levels of the cost of inflation do debt crises occur under a narrower range of debt values.

The rest of the paper is organized as follows. Section 2 presents the slow-moving debt crisis model based on LW. It starts with a real version of the model and then presents its extension to a monetary environment. Subsequently, it analyzes the various channels of monetary policy in this framework. Section 3 describes the New Keynesian part of the model, discusses results under optimal policy and considers sensitivity analysis and extensions. Section 4 provides results that do not rely on the New Keynesian part of the model. Section 5 considers alternative policies. Section 6 concludes. Some of the technical details are left to the Appendix, while additional algebraic

[6]Nuño and Thomas (2015) analyze the role of monetary policy with long-term debt in the context of a dynamic Eaton and Gersovitz (1981) model (with a unique equilibrium) and find that debt deflation is not optimal. There are also recent models that examine the impact of monetary policy in the presence of long-term government bonds, but they do not allow for the possibility of sovereign default. For example, Leeper and Zhou (2013) analyze optimal monetary (and fiscal) policy with flexible prices, while Bhattarai et al. (2013) consider a New Keynesian environment at ZLB. Sheedy (2014) and Gomes et al. (2016) examine monetary policy with long-term private sector bonds.

details and results can be found in a separate online Appendix.

2.2 A Model of Slow-Moving Self-Fulfilling Debt Crisis

In this section we present a dynamic sovereign debt crisis model based on LW. We first describe the basic structure of the model in a real environment. We then extend the model to a monetary environment and discuss the impact of monetary policy on the existence of self-fulfilling debt crises. We focus on the dynamics of asset prices and debt for given interest rates and goods prices. The latter will be determined in a New Keynesian model that we describe in Section 3.

2.2.1 A Real Model

We consider a simplified version of the LW model. As in the applications considered by LW, there is a key date T at which uncertainty about future primary surpluses is resolved and the government makes a decision to default or not.[7] Default occurs at time T if the present value of future primary surpluses is insufficient to repay the debt. We assume that default does not happen prior to date T as there is always a possibility of large primarily surpluses from T onward. In one version of their model LW assume that T is known to all agents, while in another they assume that it is unknown and arrives each period with a certain probability. We adopt the former assumption. In the online Appendix we analyze an extension where T is uncertain. While this significantly complicates the analysis, it does not alter the key findings.[8]

The only simplification we adopt relative to LW concerns the process of the primary surplus. For now we assume that the primary surplus s_t is constant at $\overline{s}$ between periods 0 and $T-1$. LW assume a fiscal rule whereby the surplus is a function of debt. Not surprisingly, they find that the range of debt where a country is vulnerable to self-fulfilling crises narrows if the fiscal surplus is more responsive to debt. Very responsive fiscal policy could in principle eliminate the concern about self-fulfilling debt crises. In this paper, however, we take vulnerability to self-fulfilling debt crises as given in the absence of monetary policy action. We therefore abstract from such strong stabilizing fiscal policy. However, we will consider an extension where the primary surplus depends on output and is pro-cyclical as this provides an additional avenue through which monetary policy can be effective.

A second assumption concerns the primary surplus value starting at date T. Let $\tilde{s}$ denote the maximum potential primary surplus that the government is able to achieve, which becomes known

[7]One can for example think of countries that have been hit by a shock that adversely affected their primary surpluses, which is followed by a period of uncertainty about whether and how much the government is able to restore primary surpluses through higher taxation or reduced spending.

[8]Notice that uncertainty about T implies uncertainty about the range of the multiplicity zone.

at time T and is constant from thereon. LW assume that it is drawn from a log normal distribution. Instead we assume that it is drawn from a binary distribution, which simplifies the algebra and the presentation. It can take on only two values: s_{low} with probability ψ and s_{high} with probability $1-\psi$. When the present discounted value of $\tilde{s}$ is at least as large as what the government owes on debt, there is no default at time T and the actual surplus is just sufficient to satisfy the budget constraint (generally below $\tilde{s}$). We assume that s_{high} is big enough such that this is always the case when $\tilde{s} = s_{high}$.[9] When $\tilde{s} = s_{low}$ and its present value is insufficient to repay the debt, the government defaults.

A key feature of the model is the presence of long-term debt. As usual in the literature, assume that bonds pay coupons (measured in goods) that depreciate at a rate of $1-\delta$ over time: κ, $(1-\delta)\kappa$, $(1-\delta)^2\kappa$, and so on.[10] A smaller δ therefore implies a longer maturity of debt. This facilitates aggregation as a bond issued at $t-s$ corresponds to $(1-\delta)^s$ bonds issued at time t. We can then define all outstanding bonds in terms of the equivalent of newly issued bonds. We define b_t as debt measured in terms of the equivalent of newly issued bonds at $t-1$ on which the first coupon is due at time t. As in LW, we take δ as given. It is associated with tradeoffs that are not explicitly modeled, and we do not allow the government to change the maturity to avoid default.

Let Q_t be the price of a government bond. At time t the value of government debt is $Q_t b_{t+1}$. In the absence of default the return on the government bond from t to $t+1$ is

$$r_t^g = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{2.1}$$

If there is default at time T, bond holders are able to recover a proportion $\zeta < 1$ of the present discounted value s^{pdv} of the primary surpluses s_{low}.[11] In that case the gross return on the government bond is

$$r_{T-1}^g = \frac{\zeta s^{pdv}}{Q_{T-1} b_T} \tag{2.2}$$

Government debt evolves according to

$$Q_t b_{t+1} = r_{t-1}^g Q_{t-1} b_t - s_t \tag{2.3}$$

In the absence of default this may also be written as $Q_t b_{t+1} = ((1-\delta)Q_t + \kappa)b_t - s_t$. The initial stock of debt b_0 is given.

We assume that investors also have access to a short-term bond with a gross real interest rate r_t. The only shocks in the model occur at time 0 (self-fulfilling shock to expectations) and time T (value of $\tilde{s}$). In other periods the following risk-free arbitrage condition holds (for $t \geq 0$ and

[9] See online Appendix for details.

[10] See for example Hatchondo and Martinez (2009).

[11] One can think of ζ as the outcome of a bargaining process between the government (representing taxpayers) and bondholders. Since governments rarely default on all their debt, we assume $\zeta > 0$.

$t \neq T-1$):

$$r_t = \frac{(1-\delta)Q_{t+1} + \kappa}{Q_t} \tag{2.4}$$

For now we assume, as in LW, a constant interest rate, $r_t = r$. In that case $s^{pdv} = rs_{low}/(r-1)$ is the present discounted value of s_{low}. There is no default at time T if s^{pdv} covers current and future debt service at T, which is $((1-\delta)Q_T + \kappa)b_T$. Since there is no default after time T, Q_T is the risk-free price, equal to the present discounted value of future coupons. For convenience it is assumed that $\kappa = r - 1 + \delta$, so that (2.4) implies that $Q_T = 1$. This means that there is no default as long as $s^{pdv} \geq rb_T$, or if

$$b_T \leq \frac{1}{r-1} s_{low} \equiv \tilde{b} \tag{2.5}$$

When $b_T > \tilde{b}$, the government partially defaults on debt, with investors seizing a fraction ζ of the present value s^{pdv}.

This framework may lead to multiple equilibria and to a slow-moving debt crisis, as described in LW. The existence of multiple equilibria can be seen graphically from the intersection of two schedules, as illustrated in Figure 1. The first schedule, labeled "pricing schedule", is a consistency relationship between price and outstanding debt at $T-1$, in view of the default decision that may be taken at T. This is given by:

$$Q_{T-1} = 1 \qquad \text{if } \mathrm{b_T} \leq \tilde{\mathrm{b}} \tag{2.6}$$

$$= \psi \frac{\zeta s^{pdv}}{rb_T} + (1-\psi) \qquad \text{if } \mathrm{b_T} > \tilde{\mathrm{b}} \tag{2.7}$$

When $b_T \leq \tilde{b}$, the arbitrage condition (2.4) also applies to $t = T-1$, implying $Q_{T-1} = 1$. When b_T is just above $\tilde{b}$, there is a discrete drop of the price because only a fraction ζ of primary surpluses can be recovered by bond holders in case of default. For larger values of debt, Q_{T-1} will be even lower as the primary surpluses have to be shared among more bonds.

The second schedule is the "debt accumulation schedule," which expresses the amount of debt that accumulates through time $T-1$ as a function of prices between 0 and $T-1$. Every price Q_t between 0 and $T-1$ can be expressed as a function of Q_{T-1} by integrating (2.4) backwards from $T-1$ to 0:

$$Q_t - 1 = \left(\frac{1-\delta}{r}\right)^{T-1-t} (Q_{T-1} - 1) \tag{2.8}$$

Substituting in (2.3) and integrating the government budget constraint forward from 0 to $T-1$, we get (see Appendix A):

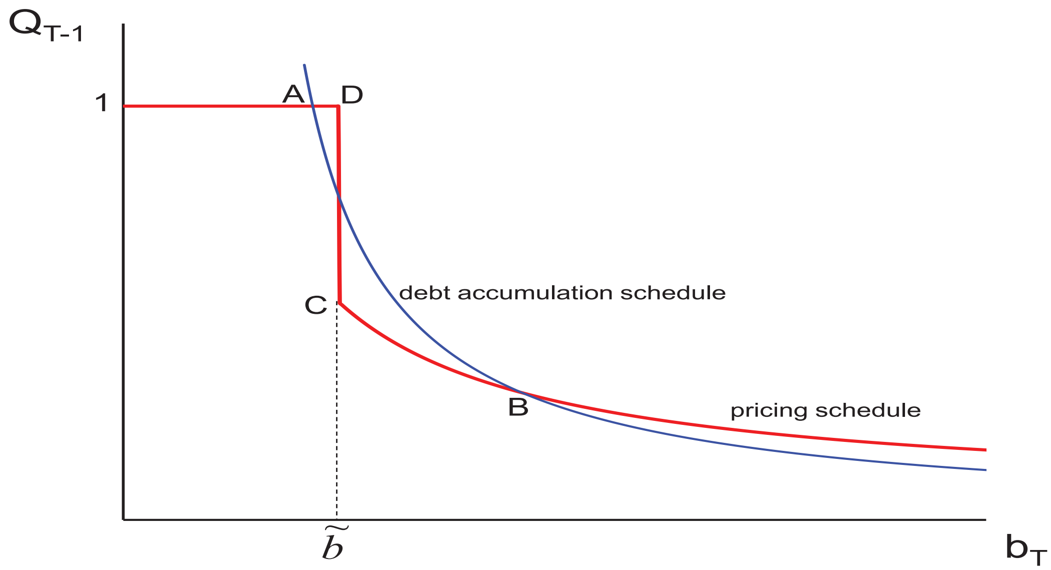
$$b_T = (1-\delta)^T b_0 + \frac{\chi^\kappa \kappa b_0 - \chi^s \overline{s}}{Q_{T-1}} \tag{2.9}$$

where

$$\chi^\kappa = r^{T-1} + (1-\delta)r^{T-2} + (1-\delta)^2 r^{T-3} + ... + (1-\delta)^{T-1}$$

$$\chi^s = 1 + r + r^2 + ... + r^{T-1}$$

Figure 1 Multiple Equilibria Lorenzoni-Werning Model



The numerator $\chi^{\kappa}\kappa b_0 - \chi^s\overline{s}$ in (2.9) corresponds to the accumulated new borrowing between 0 and T. $\chi^{\kappa}\kappa b_0$ represents the debt service on the initial debt and $\chi^s\overline{s}$ is the time T value of all surpluses (or deficits) between 0 and T. We assume that $\chi^{\kappa}\kappa b_0 - \chi^s\overline{s}$ is positive, i.e., the primary surplus is insufficient to pay the coupons on the initial debt and the government needs to issue new debt. In this case a lower price of debt makes financing more difficult and may contribute to a debt crisis. This implies that the debt accumulation schedule (2.9) gives a negative relationship between b_T and Q_{T-1}. When Q_{T-1} is lower, asset prices from 0 to $T-2$ are also lower. This implies a higher yield on newly issued debt, reflecting a premium for possible default at time T. These default premia lead to a more rapid accumulation of debt and therefore a higher b_T at $T-1$.

Figure 1 shows these two schedules and illustrates the multiplicity of equilibria. There are two stable equilibria, represented by points A and B. At point A, $Q_{T-1} = 1$. The bond price is then equal to 1 at all times. This is the "good" equilibrium in which there is no default. At point B, $Q_{T-1} < 1$. This is the "bad" equilibrium. Asset prices starting at time 0 are less than 1 in anticipation of possible default at time T. Intuitively, when agents believe that default is likely, they demand default premia (implying lower asset prices), leading to a more rapid accumulation of debt, which in a self-fulfilling way indeed makes default more likely.

In the bad equilibrium there is a slow-moving debt crisis. As can be seen from (2.8), using $Q_{T-1} < 1$, the asset price instantaneously drops at time 0 and then continues to drop all the way to $T-1$. Correspondingly, default premia gradually rise over time. Such a slow-moving crisis occurs only for intermediate levels of debt. When b_0 is sufficiently low, the debt accumulation schedule is further to the left, crossing below point C, and only the good equilibrium exists. When b_0 is sufficiently high, the debt accumulation schedule is further to the right, crossing above point D, and only a bad equilibrium exists. In that case default is unavoidable when $\tilde{s} = s_{low}$. There is therefore an intermediate region for b_0 under which there are multiple equilibria, which we refer to as the multiplicity region.

2.2.2 A Monetary Model: The Impact of Monetary Policy

In this section we start addressing the central question of this paper, i.e. what the central bank can do to prevent a debt crisis of the type described above. We extend the model to a monetary economy, in which the central bank can set the interest rate and affect the goods price level P_t. For simplicity of exposition, in this section we only consider the case of a cashless economy. In Section 3.5 we will introduce money supply. We will show that the ability of the central bank to collect seigniorage, while being one of the available channels to avoid default, is of limited quantitative importance.

R_t is the gross nominal interest rate and $r_t = R_t P_t / P_{t+1}$ the gross real interest rate in the economy. The coupons on government debt are defined in nominal terms. On the other hand, we assume that primary surpluses before T, and their distribution after T, are known in real terms. The number of bonds outstanding at time $t-1$ is B_t and B_0 is given. We define $b_t = B_t/P_t$. The arbitrage equation with no default remains (2.4), while the government budget constraint for $t \neq T$ becomes

$$Q_t B_{t+1} = ((1-\delta)Q_t + \kappa)B_t - s_t P_t \tag{2.10}$$

where s_t is the real primary surplus, $s_t P_t$ the nominal surplus.

At time T the real obligation of the government to bond holders is $[(1-\delta)Q_T + \kappa]b_T$. The no-default condition is $b_T \leq \tilde{b}$, with the latter now defined as

$$\tilde{b} = \frac{s^{pdv}}{(1-\delta)Q_T + \kappa} \tag{2.11}$$

where

$$s^{pdv} = \left[1 + \frac{1}{r_T} + \frac{1}{r_T r_{T+1}} + ...\right] s_{low} \tag{2.12}$$

and Q_T is equal to the present discounted value of coupons:

$$Q_T = \frac{\kappa}{R_T} + \frac{(1-\delta)\kappa}{R_T R_{T+1}} + \frac{(1-\delta)^2\kappa}{R_T R_{T+1} R_{T+2}} + ... \tag{2.13}$$

In analogy to the real model, the new pricing schedule becomes

$$
\begin{aligned}
Q_{T-1} &= \frac{(1-\delta)Q_T+\kappa}{R_{T-1}} && \text{if } b_T \leq \tilde{b} && (2.14)\\
&= \psi\frac{\zeta s^{pdv}}{R_{T-1}b_T}+(1-\psi)\frac{(1-\delta)Q_T+\kappa}{R_{T-1}} && \text{if } b_T > \tilde{b} && (2.15)
\end{aligned}
$$

The new pricing schedule implies a relationship between Q_{T-1} and b_T that has the same shape as in the real model, but is now impacted by monetary policy through real and nominal interest rates and inflation. In particular, the pricing schedule is affected by inflation and interest rates after T (*ex-post policy*). Inflation after T lowers Q_T. Lower real interest rates after T raise the time-T value of future surpluses, s^{pdv}. Both these effects contribute to raising the default threshold $\tilde{b}$, with the effect of shifting the pricing schedule to the right. Raising s^{pdv} also increases the recovery in case of default, which moves the decreasing branch of the pricing schedule up.

The debt accumulation schedule now becomes (see Appendix A):

$$
b_T = (1-\delta)^T\frac{B_0}{P_T}+\frac{P_{T-1}}{P_T}\frac{\chi^\kappa\kappa B_0/P_0-\chi^s\bar{s}}{Q_{T-1}} \tag{2.16}
$$

where

$$
\begin{aligned}
\chi^\kappa &= \left[r_{T-2}...r_1r_0+(1-\delta)r_{T-2}...r_1\frac{P_0}{P_1}+(1-\delta)^2r_{T-2}...r_2\frac{P_0}{P_2}+...+(1-\delta)^{T-1}\frac{P_0}{P_{T-1}}\right]\\
\chi^s &= 1+r_{T-2}+r_{T-2}r_{T-3}+...+r_{T-2}...r_1r_0
\end{aligned}
$$

As in the real model, the schedule implies a negative relationship between Q_{T-1} and b_T. Monetary policy affects this schedule through its impact on interest rates and inflation before T (*ex-ante policy*). Clearly inflation reduces the real value of the debt outstanding at time 0. This is captured by term $(1-\delta)^T\frac{B_0}{P_T}$ on the RHS of (2.16) and by the term proportional to χ^κ. In addition lower real rates can reduce the cost of new borrowing (χ^κ decreases), and decrease the time-T (absolute) value of the surpluses between 0 and T (χ^s decreases), which is beneficial to the government if it is running deficits. All these effects decrease debt accumulation, hence shift the corresponding schedule to the left.

By shifting the two schedules, monetary policy can affect the existence of self-fulfilling debt crises: in terms of Figure 1, the crisis equilibrium is avoided when the debt accumulation schedule goes through point C or below, so that the two schedules intersect only once, in the good equilibrium. This is the case when

$$
\frac{\chi^\kappa\kappa B_0/P_0-\chi^s\bar{s}}{s^{pdv}-((1-\delta)Q_T+\kappa)(1-\delta)^TB_0/P_T}r_{T-1}\leq 1-\psi(1-\zeta) \tag{2.17}
$$

Note that point C itself is not on the price schedule as its lower section starts for $b_t > \tilde{b}$. The crisis equilibrium is therefore avoided even if this condition holds as an equality, which corresponds to point C. At point C, $Q_{T-1} < 1$. All prices from 0 to $T-1$ will then be less than one, implying rising default premia that lead to an accumulation of debt. (2.17) gives a condition for what the central bank needs to do to counteract these rising default premia and avoid default. This condition is key and applies no matter what specific model we assume that relates interest rates, prices and output. We will refer to this as the *default avoidance condition*. Notice that, since the Lagrange multiplier associated with (2.17) is positive, the optimal policy that averts default is always such that this condition holds as an equality.

In Section 3.5 we will discuss two more channels, besides inflation and low real rates, through which monetary policy can help. First, through an expansionary monetary policy the central bank can collect extra revenue, seigniorage, that can help avoiding default. The last channel is through output. If we allow the primary surplus to be pro-cyclical, expansionary monetary policy that raises output will raise primary surpluses, which again helps avert default.

2.3 A New Keynesian Model

The default avoidance condition (2.17) depends on interest rates, prices and output. We now consider a specific New Keynesian model that determines prices and output given interest rates that will be controlled by the central bank. The model is used to examine the policies needed to eliminate the default equilibrium. More precisely, we consider the optimal monetary policy that satisfies both the default avoidance condition and the zero lower bound constraint on nominal interest rates.

2.3.1 Model Description

We consider a standard NK model based on Gali (2008, ch. 3), with three extensions suggested by Woodford (2003): i) habit formation; ii) price indexation; iii) lagged response in price adjustment. These extensions are standard in the monetary DSGE literature and are introduced to generate more realistic responses to monetary shocks. The main effect of these extensions is to generate a delayed impact of a monetary policy shock on output and inflation, leading to the humped-shaped response seen in the data.

Households

With habit formation, households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - \eta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{2.18}$$

where total consumption C_t is

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.19}$$

and N_t is labor. Habit persistence, measured by η, is a common feature in NK models to generate a delayed response of expenditure and output.

The budget constraint is

$$\begin{aligned} & P_t C_t + D_{t+1} + Q_t B_{t+1} = \\ & W_t N_t + \Pi_t + R_{t-1} D_t + R^g_{t-1} Q_{t-1} B_t - T_t \end{aligned} \tag{2.20}$$

Here D_{t+1} are holdings of one-period bonds that are in zero net supply. P_t is the standard aggregate price level and W_t is the wage level. Π_t are firms profits distributed to households and T_t are lump-sum taxes. We will abstract from government consumption, so that the primary surplus is $P_t s_t = T_t$.

The first-order conditions with respect to D_{t+1} and B_{t+1} are

$$\widetilde{C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{2.21}$$

$$\widetilde{C}_t = \beta E_t R^g_t \frac{P_t}{P_{t+1}} \widetilde{C}_{t+1} \tag{2.22}$$

where

$$\widetilde{C}_t \equiv (C_t - \eta C_{t-1})^{-\sigma} - \eta \beta E_t (C_{t+1} - \eta C_t)^{-\sigma}$$

The combination of (2.21) and (2.22) gives the arbitrage equations (2.4), (2.14), and (2.15). This is because government default, which lowers the return on government bonds, does not affect consumption due to Ricardian equivalence.

Firms

There is a continuum of firms on the interval $[0, 1]$, producing differentiated goods. The production function of firm i is

$$Y_t(i) = A N_t(i)^{1-\alpha} \tag{2.23}$$

We follow Woodford (2003) by assuming firm-specific labor.

Calvo price setting is assumed, with a fraction $1 - \theta$ of firms re-optimizing their price each period. In addition, it is assumed that re-optimization at time t is based on information from date $t - d$. This feature, adopted by Woodford (2003), is in the spirit of the model of information delays of Mankiw and Reis (2002). It has the effect of a delayed impact of a monetary policy shock on inflation, consistent with the data.[12] Analogous to Christiano et al. (2005), Smets and Wouters

[12] This feature can also be justified in terms of a delay by which newly chosen prices go into effect.

(2003) and many others, we also adopt an inflation indexation feature in order to generate more persistence of inflation. Firms that do not re-optimize follow the simple indexation rule

$$ln(P_t(i)) = ln(P_{t-1}(i)) + \gamma\pi_{t-1} \tag{2.24}$$

where $\pi_{t-1} = ln\ P_{t-1} - ln\ P_{t-2}$ is aggregate inflation one period ago.

Linearized system

Let c_t, y_t and y_t^n denote logs of consumption, output and the natural rate of output. Using $c_t = y_t$, and defining $x_t = y_t - y_t^n$ as the output gap, log-linearization of the Euler equation (2.21) gives the dynamic IS equation

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(i_t - E_t\pi_{t+1} - r^n) \tag{2.25}$$

where

$$\tilde{x}_t = x_t - \eta x_{t-1} - \beta\eta E_t(x_{t+1} - \eta x_t) \tag{2.26}$$

Here $i_t = ln(R_t)$ will be referred to as the nominal interest rate and $r^n = -ln(\beta)$ is the natural rate of interest. The latter uses our assumption of constant productivity, which implies a constant natural rate of output.

Leaving the algebra to the online Appendix, we find the following Phillips curve:

$$\pi_t = \gamma\pi_{t-1} + \beta E_{t-d}(\pi_{t+1} - \gamma\pi_t) + E_{t-d}(\omega_1 x_t + \omega_2\tilde{x}_t) \tag{2.27}$$

where

$$\omega_1 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{\phi+\alpha}{1-\alpha+(\alpha+\phi)\varepsilon} \tag{2.28}$$

$$\omega_2 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1-\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}\frac{\sigma}{(1-\eta\beta)(1-\eta)} \tag{2.29}$$

Monetary Policy

We follow most of the literature by using a quadratic approximation of utility. Conditional on avoiding the default equilibrium, the central bank then minimizes the following objective function:

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\} \tag{2.30}$$

where ν, μ_x and μ_π a function of model parameters (see the Appendix B for the derivation). The central bank chooses the optimal path of nominal interest rates over $H > T$ periods. After that, we assume an interest rate rule as in Clarida et al. (1999):

$$i_t - \bar{\imath} = \rho(i_{t-1} - \bar{\imath}) + (1-\rho)(\psi_\pi E_t\pi_{t+1} + \psi_y x_t) \tag{2.31}$$

where $\bar{\imath} = -ln(\beta)$ is the steady state nominal interest rate. We will choose H to be large. Interest rates between time T and H involve ex-post-policy.[13]Appendix C shows how to solve for the time series of $[\pi_t, x_t, i_t]$ given the policy choice of i_t for $0 \leq t < H$ and the adoption of the policy rule (2.31) for $t \geq H$.

Optimal policy is chosen conditional on two types of constraints. The first is the ZLB constraint that $i_t \geq 0$ for all periods. The second is the default avoidance condition (2.17) as an equality.[14] Using the NK Phillips curve (2.27), the dynamic IS equation (2.25), and the policy rule (2.31) after time H, we solve for the path of inflation and output gap conditional on the set of H interest rates chosen. We then minimize the welfare cost (2.30) over the H interest rates subject to $i_t \geq 0$ and the default avoidance condition.

2.3.2 Calibration

We consider one period to be a quarter and normalize the constant productivity A such that the natural rate of output is equal to 1 annually (0.25 per quarter). The other parameters are listed in Table 1. The left panel shows the parameters from the LW model, while the right panel lists the parameters that pertain to the New Keynesian part of the model.

Table 1 Calibration

Lorenzoni-Werning parameters		New Keynesian parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	$\sigma = 1$	elasticity of intertemporal subsitution
$\delta = 0.05$	coupon depreciation rate	$\phi = 1$	Frisch elasiticity
$\kappa = 0.06$	coupon	$\varepsilon = 6$	demand elasticity
$T = 20$	quarters before default decision	$\alpha = 0.33$	capital share
$\zeta = 0.5$	recovery rate	$\theta = 0.66$	Calvo pricing parameter
$\psi = 0.95$	probability low surplus state	$\eta = 0.65$	habit parameter
$s_{low} = 0.02$	low state surplus	$\gamma = 1$	indexation parameter
$\bar{s} = -0.01$	surplus before T	$d = 2$	lag in price adjustment
		$\rho = 0.8$	persistence in interest rate rule
		$\psi_\pi = 1.5$	inflation parameter in interest rule
		$\psi_y = 0.1$	output parameter in interest rule

A key parameter is δ. In the benchmark parameterization we set it equal to 0.05, which implies a

[13]Since H will be large, the precise policy rule after H does not have much effect on the results.

[14]In the good equilibrium $i_t \geq 0$ is the only constraint and the optimal policy implies $i_t = \bar{\imath}$ each period, delivering zero inflation and a zero output gap. However, conditional on a sunspot that could trigger a default equilibrium condition (2.17) becomes an additional constraint.

government debt duration of 4.2 years. This is typical in the data. For example, OECD estimates of the Macauley duration in 2010 are 4.0 in the US and 4.4 for the average of the five European countries that experienced a sovereign debt crisis (Greece, Italy, Spain, Portugal and Ireland). The coupon is determined such that $\kappa = 1/\beta - 1 + \delta$.

The other LW parameters, β, T and the fiscal surplus parameters, have an impact on the "multiplicity range". The range of B_0 for which there are multiple equilibria under passive monetary policy ($i_t = \bar{\imath}$) is $[B_{low}, B_{high}]$, where[15]

$$B_{low} = \frac{\beta}{1-\beta} \frac{(\psi\zeta + 1 - \psi)\beta^T s_{low} + (1-\beta^T)\bar{s}}{1-(1-\zeta)(1-\delta)^T \beta^T \psi} \tag{2.32}$$

$$B_{high} = \frac{\beta}{1-\beta} \left(\beta^T s_{low} + (1-\beta^T)\bar{s}\right) \tag{2.33}$$

This range may be wide or narrow, dependent on the chosen parameters. For example, when $\zeta \to 1$, the range narrows to zero. We set $\beta = 0.99$ (4% annual natural real rate of interest), $T = 20$ (uncertainty resolved in 5 years), $\bar{s} = -0.01$ (4% annual primary deficit), $s_{low} = 0.02$, $\zeta = 0.5$ and $\psi = 0.95$. This gives a multiplicity range of $[0.79, 1.46]$, so that a country is subject to multiple equilibria when debt is in the range of 79 to 146 percent of GDP. Although we are by no means aiming to calibrate to a particular historical episode, we note that during the Eurozone crisis the range of debt of periphery countries varied from 62% in Spain to 148% in Greece.

The New Keynesian parameters are standard in the literature. The first 5 parameters correspond exactly to those in Gali (2008). The habit formation parameter, the indexation parameter and the parameters in the interest rate rule are all the same as in Christiano et al. (2005). We take $d = 2$ from Woodford (2003, p. 218-219), which also corresponds closely to Rotemberg and Woodford (1997). This set of parameters implies a response to a small monetary policy shock under the Taylor rule that is similar to the empirical VAR results reported by Christiano et al. (2005). The response of output, inflation and interest rates to such a shock are shown in the online Appendix. The levels of output and inflation at their peak are similar to what is found in the data. Both the output and inflation response are humped shaped, although the peak response occurs a bit earlier than in the data.

We also show in some detail what the optimal policy would be if there was no inflation inertia, i.e. $d = 0$. The situation we are considering, with the central bank embarking on a policy of massive inflation to avoid a debt crisis, represents an exceptional occurrence, in which it is not implausible that inflation would react with less inertia than in normal times. Even if this is not the case, showing the comparison between $d = 2$ and $d = 0$ is still useful as it allows us to disentangle the impact of inflation inertia on the optimal policy.

[15] These values lead to equilibria at points C and D in Figure 1.

2.3.3 Results under Benchmark Parameterization

Figure 2 shows the dynamics of inflation under optimal policy under the benchmark parameterization for $H = 40$ (and $d = 2$), and compares to the case with $d = 0$. The results are shown for various levels of B_0. The optimal path for inflation is hump shaped in both cases $d = 2$ and $d = 0$. Optimal inflation gradually rises because the welfare cost (2.30) depends on the change in inflation, and in the case $d = 2$ also due to inflation rigidity. Eventually optimal inflation decreases as it becomes less effective over time when the original debt depreciates and is replaced by new debt that incorporates inflation expectations. For $d = 2$, when $B_0 = B_{middle} = 1.12$, which is in the middle of the range of debt levels giving rise to multiple equilibria, the maximum inflation rate reaches 23.8%. Inflation is over 20% for 4 years, over 10% for 8 years and the price level ultimately increases by a factor 5.3. Inflation needed to avoid default gets much higher for higher debt levels. When B_0 reaches the upper bound B_{high} for multiple equilibria, the maximum inflation rate is close to 47% and ultimately the price level increases by a factor 25! Only when B_0 is very close to the lower bound for multiplicity, as illustrated for $B_0 = 0.8$, is little inflation needed.

By comparison, for $d = 0$, at $B_0 = 1.12$ the maximum inflation rate is 19%, and inflation is over 15% for almost 4 years and above 10% for 7 years. The price level ultimately increases by a factor 4.2. At $B_0 = B_{high}$, inflation peaks at 36% and the final price level increases by a factor 14.

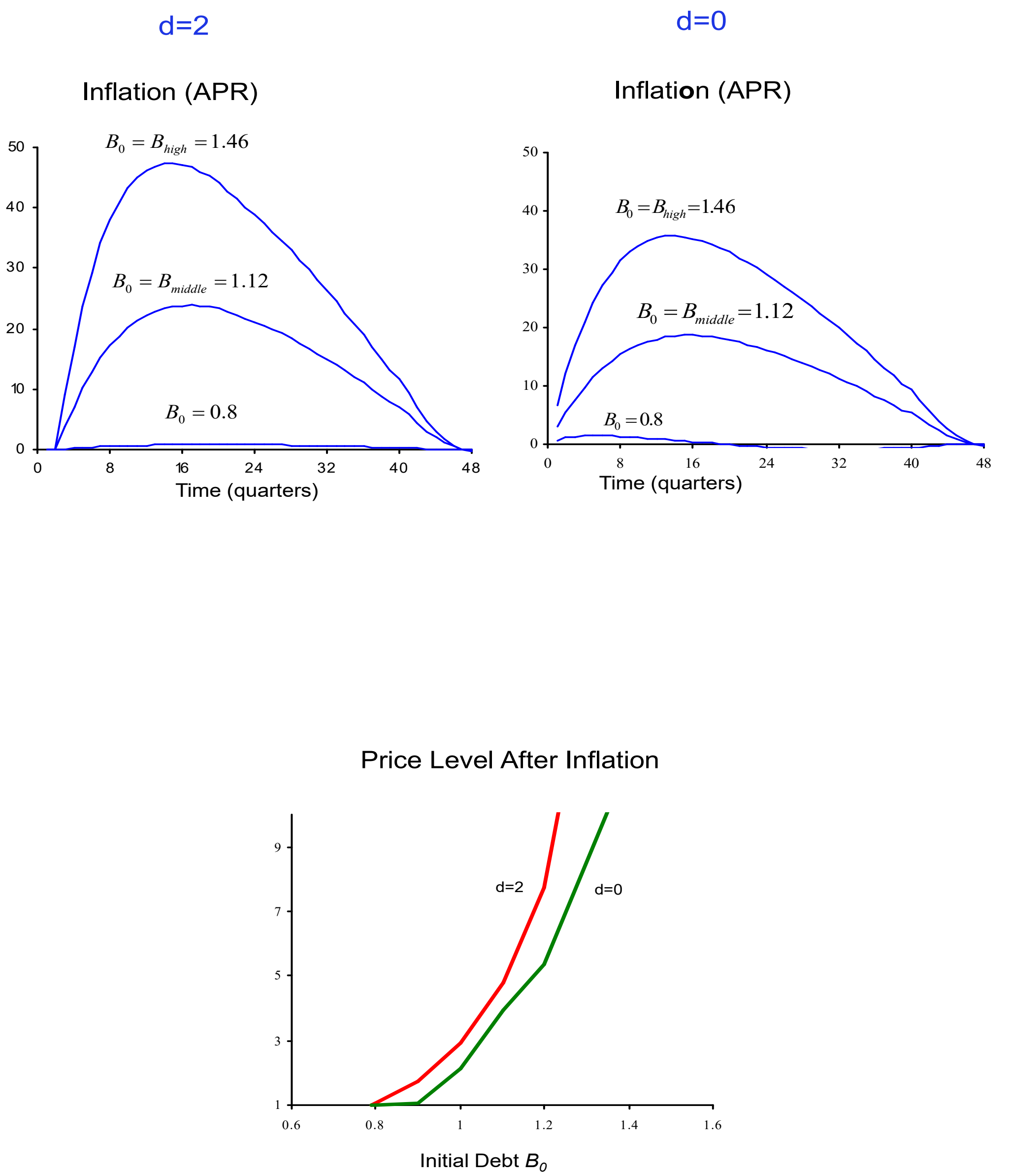
In order to understand why so much inflation is needed, it is useful to first consider a rather extreme experiment where all of the increase in prices happens right away in the first quarter. This cannot happen in the NK model, so assume that prices are perfectly flexible, the real interest rate is constant at $1/\beta$ and the output gap remains zero. When $B_0 = B_{middle} = 1.12$, the price level would need to rise by 42%. This is needed to lower debt so that we are no longer in the multiplicity range.

In reality inflation will be spread out over a period of time, both because sticky prices imply a gradual change in prices and because it is optimal from a welfare perspective not to have the increase in the price level happen all at once. However, such a delay increases the ultimate increase in the price level that is needed. As the time zero debt depreciates, inflation quickly becomes less effective as it only helps to reduce the real value of coupons on the original time zero debt. The interest rate on new debt incorporates the higher inflation expectations. More inflation is therefore needed to avoid the default equilibrium.

2.3.4 Sensitivity Analysis

We now consider changes to both the LW and NK parameters. Changes in the LW parameters change the multiplicity region $[B_{low}, B_{high}]$. As discussed in section 3.3, the relative distance between the initial debt level B_0 and the lower bound of the multiplicity range B_{low} dramatically

Figure 2 Benchmark NK Model: Inflation Needed to Avoid Default



affects the inflation path. The objective of this section is instead to examine how the required inflation depends on model parameters, keeping B_0/B_{low} fixed.

Table 2 shows the maximum inflation level and the final price level when changing one parameter at a time, while keeping $B_0/B_{low} = 1.42$ (the same as for the middle of the multiplicity range in the benchmark parameterization). Table 2 also shows how a variation in each LW parameter

Table 2 Sensitivity Analysis				
Parameters	B_{low}	B_{high}	maximum inflation	price level after inflation
Benchmark	0.79	1.46	23.8	5.3
Lorenzoni-Werning Parameters				
$T = 10$	1.15	1.71	25.8	6.3
$T = 30$	0.56	1.22	21.4	4.5
$\delta = 1/40$	0.89	1.46	15.1	3.0
$\delta = 1/10$	0.71	1.46	37.5	12.0
$\bar{s} = -0.02$	0.58	1.28	23.2	5.1
$\bar{s} = 0$	1.00	1.64	23.9	5.3
$s_{low} = 0.01$	0.28	0.63	23.2	5.1
$s_{low} = 0.03$	1.27	2.25	23.7	5.3
$\zeta = 0.3$	0.46	1.46	26.4	6.1
$\zeta = 0.7$	1.08	1.46	21.3	4.5
$\psi = 0.7$	0.98	1.46	22.0	4.7
$\psi = 1$	0.75	1.46	24.0	5.4
$\beta = 0.98$	0.61	1.17	26.7	6.7
$\beta = 0.995$	0.90	1.62	21.8	4.6
New Keynesian Parameters				
$\theta = 0.5$	0.79	1.46	23.7	5.0
$\theta = 0.8$	0.79	1.46	22.9	5.5
$\eta = 0$	0.79	1.46	22.3	4.9
$\eta = 0.8$	0.79	1.46	23.8	5.3
$\epsilon = 4$	0.79	1.46	24.1	5.3
$\epsilon = 8$	0.79	1.46	23.2	5.2
$d = 0$	0.79	1.46	19.6	4.2
$d = 4$	0.79	1.46	29.6	7.2
$\sigma = 2$	0.79	1.46	22.7	5.0
$\sigma = 0.5$	0.79	1.46	24.4	5.4
$\phi = 2$	0.79	1.46	23.6	5.3
$\phi = 0.5$	0.79	1.46	23.7	5.2
$\gamma = 0.5$	0.79	1.46	19.5	2.0
$\gamma = 0$	0.79	1.46	26.4	1.7
$\gamma = \eta = d = 0$	0.79	1.46	23.0	1.7

affects the multiplicity region. Figure 3 shows how each of the LW affects the inflation path. We see that the only LW parameter that substantially affects the results is δ. A lower debt depreciation δ, which implies a longer maturity of debt, implies lower inflation. The reason is that inflation is effective for a longer period of time as the time 0 debt depreciates more slowly. But even when $\delta = 0.025$, so that the duration is 7.2 years, optimal inflation is still above 10% for 6.5 years and the price level ultimately triples.

Figure 3 Sensitivity Analysis LW Parameters (B_0/B_{low}=1.42)

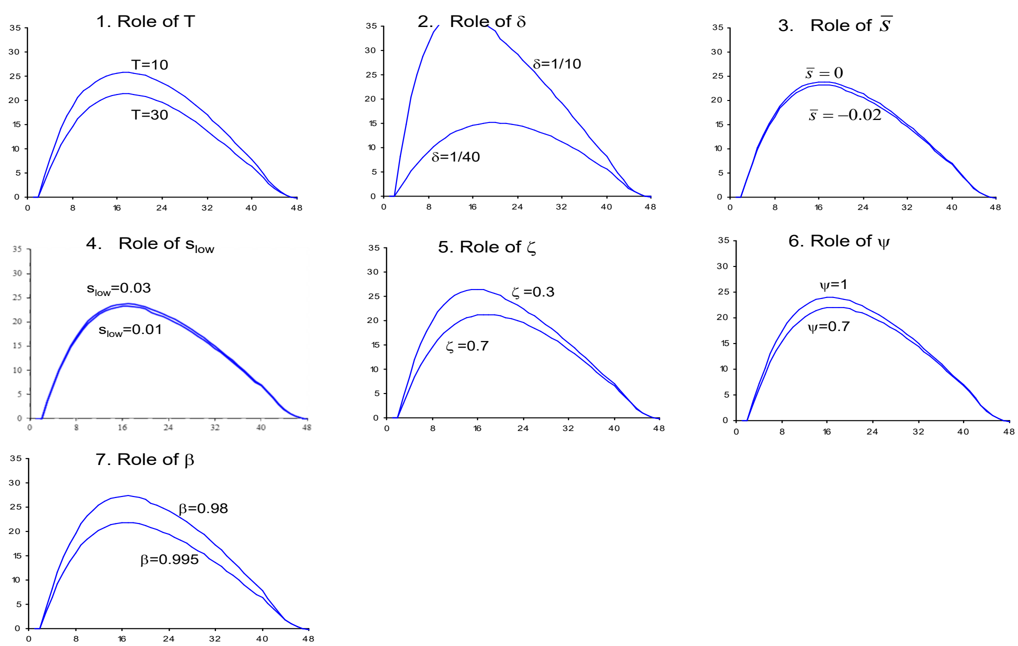
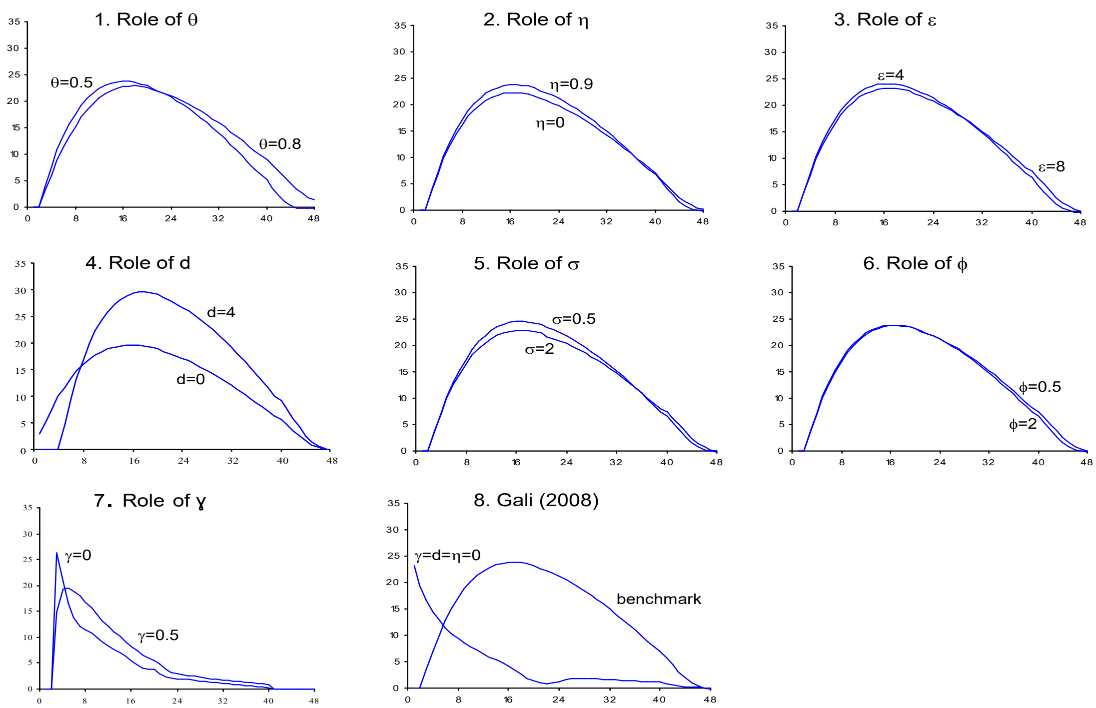


Figure 4 shows analogous results for the NK parameters. Most parameters again have remarkably little effect. There are two cases where parameter values do matter. One is the lag in price adjustment d, which has already been discussed in Figure 2. The other parameter that significantly affects results is γ. When $\gamma = 0$ (no price indexation), it is optimal to concentrate inflation as early as possible since only the inflation level, and not inflation changes, matters for the central bank objective function (2.30). In this case the price level ultimately rises by a relatively more modest factor 1.7.

In the last chart we show $\gamma = d = \eta = 0$ as in the Gali (2008) textbook model and compare the results to the benchmark parameterization. This case combines the features of the $d = 0$ and $\gamma = 0$ cases discussed above. All the inflation comes upfront, both because there is no delay in price adjustment and because, with no price indexation, there is no rationale for inflation smoothing. Inflation starts at 23% (APR) in the first quarter, and the ultimate increase in the price level is very similar to the $\gamma = 0$ case seen above, only 66%. However, this case appears to be of little

Figure 4 Sensitivity Analysis NK Parameters (B_0/B_{low}=1.42)



practical relevance. The default avoidance condition is met to a large extent by an unrealistic steep drop in the real interest rate, from 4% (APR) to -19% in the first quarter, while output rises immediately by 25% in just the first quarter.[16] Even for small monetary shocks it is well known that these parameters lead to an unrealistic dynamic response of output and inflation to monetary shocks.

2.3.5 Extensions

We consider two extensions: seigniorage and pro-cyclical primary surplus.

Seigniorage

We consider a consolidated government whose budget constraint is

$$Q_t B_{t+1}^p = ((1-\delta)Q_t + \kappa)B_t^p - [M_t - M_{t-1}] - s_t P_t \tag{2.34}$$

[16] In the case $\gamma = 0$, where all the other parameters are at the benchmark level, including $d = 2$, the output increase in the first quarter is 20%.

where B_t^p is government debt held by the general public. It is $B_t^p = B_t - B_t^c$, where B_t^c is the debt held by the central bank. The consolidated government can reduce the debt of the private sector by issuing monetary liabilities $M_t - M_{t-1}$. Money demand is generated by a transaction cost $f(M_t, Y_t^n)$, where $Y_t^n = P_t Y_t$ is nominal GDP and $\partial f/\partial M \leq 0$. This transaction cost should be subtracted from the RHS of (2.20).

Let $\widetilde{m}$ represent accumulated seigniorage between 0 and $T-1$:

$$\widetilde{m} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}} + r_{T-2}\frac{M_{T-2} - M_{T-3}}{P_{T-2}} + ... + r_0 r_1 ... r_{T-2}\frac{M_0 - M_{-1}}{P_0} \tag{2.35}$$

Similarly, let m^{pdv} denote the present discounted value of seigniorage revenues starting at date T:

$$m^{pdv} = \frac{M_T - M_{T-1}}{P_T} + \frac{1}{r_T}\frac{M_{T+1} - M_T}{P_{T+1}} + \frac{1}{r_T r_{T+1}}\frac{M_{T+2} - M_{T+1}}{P_{T+2}} + ... \tag{2.36}$$

As derived in the online Appendix, if we take seigniorage into acount the default avoidance condition becomes

$$\frac{\chi^\kappa \kappa B_0^p/P_0 - \chi^s \bar{s} - \widetilde{m}}{s^{pdv} + m^{pdv} - ((1-\delta)Q_T + \kappa)(1-\delta)^T B_0^p/P_T} r_{T-1} \leq \psi \frac{min\{0, \zeta s^{pdv} + m^{pdv}\}}{s^{pdv} + m^{pdv}} + 1 - \psi \tag{2.37}$$

In order to derive a relationship between money supply and interest rates, we use a convenient form of the transaction cost $f(M_t, Y_t^n)$ that gives rise to a standard specification for money demand when $i_t > 0$ $(m_t = ln(M_t))$[17]:

$$m_t = \alpha_m + p_t + y_t - \alpha_i i_t \tag{2.38}$$

When i_t is close to zero, money demand reaches the satiation level $\alpha_m + p_t + y_t$. We limit ourselves to conventional monetary policy, where the money supply does not go beyond the satiation level. Section 5 makes some comments on unconventional monetary policy.

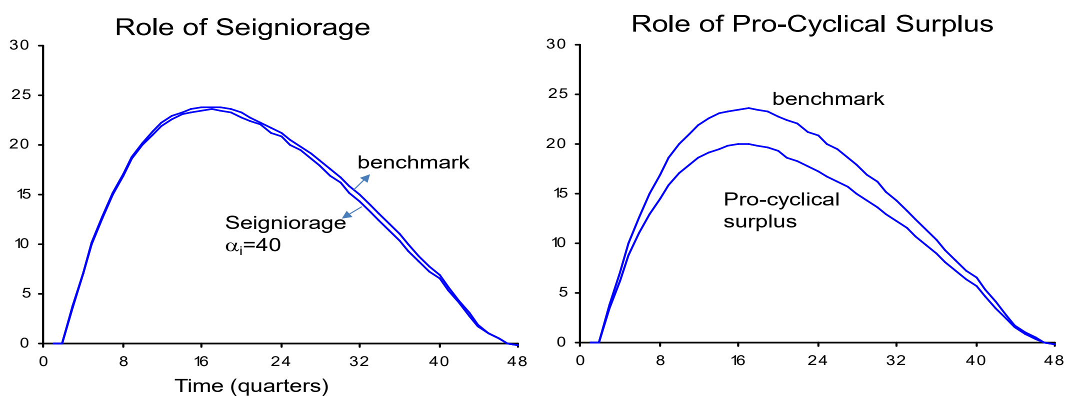
Seigniorage revenue depends on the semi-elasticity α_i of money demand. Seigniorage is larger for lower values of α_i since that leads to a smaller drop in real money demand when inflation rises. We set $\alpha_i = 40$.[18] The left chart of Figure 5 compares the optimal inflation path with seigniorage to that in the benchmark without seigniorage, assuming $B_0 = B_{middle}$.[19] The effect of seigniorage is clearly negligible. This result is consistent with Reis (2013) and Hilscher et al. (2014). As Reis (2013) puts it, "In spite of the mystique behind the central bank's balance sheet, its resource constraint bounds the dividends it can distribute by the present value of seigniorage, which is a modest share of GDP".

[17] The transaction cost $f(M_t, Y_t^n) = \alpha_0 + M_t\left(ln\left(\frac{M_t}{P_t Y_t}\right) - 1 - \alpha_m\right)/\alpha_i$ gives rise to money demand (2.38). This function applies for values of M_t where $\partial f/\partial M \leq 0$. Once the derivative becomes zero, we reach a satiation level and we assume that the transaction cost remains constant for larger M_t.

[18] Estimates of α_i vary a lot, from as low as 6 in Ireland (2009) to as high as 60 in Bilson (1978). Lucas (2000) finds a value of 28 when translated to a quarterly frequency. Engel and West (2005) review many estimates that also fall in this range.

[19] We calibrate α_m to the U.S., such that the satiation level of money corresponds to the monetary base just prior to its sharp rise in the Fall of 2008 when interest rates approached the ZLB. At that time the velocity of the monetary base was 17. This gives $\alpha_m = -1.45$. The velocity is $4P_t Y_t/M_t$ as output needs to be annualized, which is equal to $4e^{-\alpha_m}$ at the satiation level.

Figure 5 Role of Seigniorage and Pro-cyclical Surplus*



*The charts show the inflation rate over time under optimal monetary policy. The left chart compares the benchmark parameterization (cashless economy) to the extension with seigniorage in which α_i=40. The right chart compares the benchmark parameterization to the case where the primary surplus is pro-cyclical with λ=0.1.

Pro-cyclical surplus

In this case, nominal rigidities give the central bank control over the accumulation of debt through the level of output that affects the primary surplus. From 0 through $T-1$ assume that we have

$$s_t = \bar{s} + \lambda(y_t - \bar{y}) \tag{2.39}$$

where $\bar{y}$ is steady-state output. We similarly assume that s_{low} is pro-cyclical: $s_{low} = \bar{s}_{low} + \lambda(y_t - \bar{y})$. We set the value of the cyclical parameter of the fiscal surplus to $\lambda = 0.1$, in line with empirical estimates.[20]

With this additional effect from an output increase, the right chart of Figure 5 shows that the optimal inflation decreases slightly, assuming again $B_0 = B_{middle}$. But the effect is again limited. The maximum inflation rate is reduced from 23.8% in the benchmark to 19.9%. The increase in the price level after inflation is reduced from 5.3 under the benchmark to 4.0, which remains large. Optimal policy now gives more emphasis to raising output, leading to a first quarter output increase that is 10% (APR), pushing the boundary of what is plausible.[21]

[20]Note that since $\bar{Y} = 0.25$ for quarterly GDP, the specification implies that $\Delta s = 0.4\Delta Y$. This is consistent for example with estimates by Girouard and André (2005) for the OECD.

[21]It may appear surprising that output increases more than in the benchmark, while inflation increases less. Output increase relative to the benchmark is temporary (5 quarters) and is made possible by an initially more aggressive monetary policy. This increases surplus and implies that less inflation is needed as debt accumulates more slowly.

2.4 Beyond the NK Model

So far we have cast our analysis in the context of a specific NK model, combined with the LW model. While we have done sensitivity analysis with respect to the various NK parameters, we have not considered alternative versions of the NK block of the model. It is not hard to criticize the specific model we have chosen. We have assumed a particular form of price stickiness (Calvo pricing). One could consider alternatives, such as Taylor price setting or menu costs. We have also abstracted from many features that would complicate the structure, but perhaps make it more realistic, such as investment and wage rigidities. Finally, the dynamic IS equation relies on an intertemporal consumption Euler equation that has recently been criticized in the context of the debate about forward guidance.[22]

In this section we therefore take an alternative approach by considering what paths of real interest rates and inflation are consistent with the default avoidance condition. The advantage of this approach is that we do not need to take any stand on the underlying model that maps monetary policy (interest rate decisions) into inflation and real interest rates. Before presenting the results, we first discuss the constraint imposed by the standard consumption Euler equation in the NK model and how it may have affected the results. In particular, we argue that it limits the extent to which the central bank is capable of lowering real interest rates in a sustained way. After that we consider specific paths of real interest rates and inflation that satisfy the default avoidance condition. This relies only on the monetary version of the LW block of the model.

2.4.1 Euler Equation

The needed inflation may be smaller when lower real interest rates, by lowering the costs of borrowing, help to avoid the default equilibrium. But the consumption Euler equation may constrain the ability of the central bank to reduce real interest rates in a sustained way. In order to see why, abstract from habit formation for a moment ($\eta = 0$). The dynamic IS equation, which comes from the intertemporal consumption Euler, can then be solved as

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0(r_t - r^n) \tag{2.40}$$

This precludes a large and sustained drop in the real interest rate as it would imply an enormous and unrealistic immediate change in output at time zero, especially with $\sigma = 1$ as often assumed.

The same point applies when we introduce habit formation, in which case (2.40) becomes (See online Appendix)

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0\left(1 - (\beta\eta)^{t+1}\right)(r_t - r^n) \tag{2.41}$$

[22]E.g. see Del Negro et al. (2015) or McKay et al. (2015)

Removing the expectation operator and the r^n for convenience, for the benchmark parameterization ($\sigma = 1$, $\eta = 0.65$), we have

$$x_0 = -0.36r_0 - 0.58r_1 - 0.73r_2 - 0.83r_3 - 0.89r_4 - 0.93r_5 - 0.95r_6 - 0.97r_7 - 0.98r_8 - ... \quad (2.42)$$

Subsequent coefficients are very close to -1. For the path of real interest rates under optimal policy this implies $x_0 = 0.0157$. This translates into an immediate increase in output of 6.3% on an annualized basis, which is already pushing the boundaries of what is plausible. The real interest rate quickly returns to steady state after dropping from 4% (APR) to 0% for the first two quarters under the benchmark parameterization. This limits the ability of the central bank to satisfy the default avoidance condition.

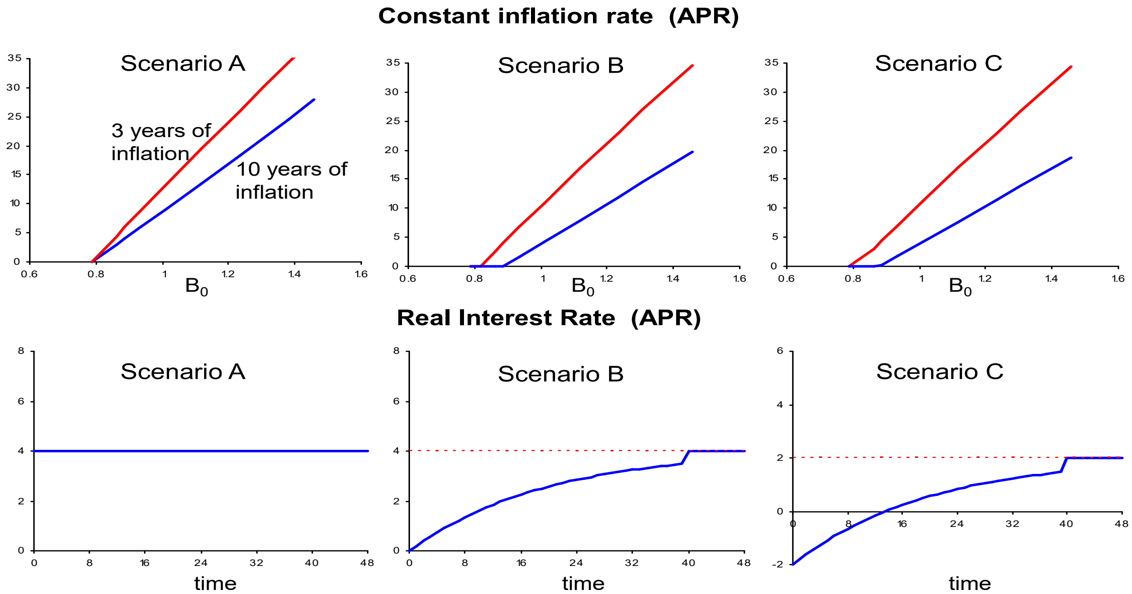
Related to this, NK models have been found to deliver unrealistically large effects of output and consumption to changes in future interest rates, which Del Negro et al. (2015) have dubbed the forward guidance puzzle. McKay et al. (2015) also argue that it is not realistic that consumption today responds equally to an announced interest rate cut in the far future as to an announced interest rate cut today. Indeed, (2.40) shows that real interest rate changes at any future date have the same effect on current output (and consumption) as a current real interest rate change. With habit formation future interest rate changes have an even larger effect than current changes. In order to rectify that problem, models have been proposed leading to a reduced effect of real interest rates on consumption and output today when the expected changes occur further into the future. McKay et al. (2015) do so in the context of a model with idiosyncratic risk and borrowing constraints. Del Negro et al. (2015) do so by introducing finite lives through a positive probability of death. In the context of our model, such alternatives allow for a larger drop in future real interest rates without generating unrealistic implications for current output and consumption. We provide an illustration below.

2.4.2 Some Results

Figure 6 considers three scenarios for the real interest rate path, shown in the bottom charts. The top charts show the corresponding inflation needed to satisfy the default avoidance condition. The results are shown as a function of initial debt. We consider both a constant inflation rate over 10 years and a constant inflation rate over 3 years. The latter is represented by the higher line as a higher inflation rate is needed when inflation is limited to three years. The motivation behind this case is that we have seen that inflation is most effective when it occurs soon after the time zero shock, before most of the original debt has to be rolled over and replaced with new debt that incorporates the higher inflation expectations.

The first scenario for real interest rates (scenario A) assumes that the real interest rate simply remains constant and equal to the natural rate. In this case all the burden is on inflation to reach

Figure 6 Constant Inflation Needed to Avoid Default*



*Three scenarios for the real interest rate path are shown in the bottom three charts. The corresponding top three charts show the constant inflation that is needed to avoid default. Both a constant inflation rate for 3 years (12 quarters) and a constant inflation rate of 10 years (40 quarters) are reported as a function of the initial debt at time 0.

the default avoidance condition. In scenario B the annualized real interest rate immediately drops in period 0 from 4% (the natural rate) to 0. After that we assume that the gap between real interest rate and the natural rate is multiplied by 0.95 each quarter and we close the remaining gap entirely after 40 quarters. As can be seen, this delivers a very large and persistent drop in the real interest rate. Scenario C is similar to scenario B, but the level of the real interest rate is always 2% (APR) lower than in scenario B. So we start from a 2% real interest rate and it drops right away to -2% and then very gradually, over a period of 10 years, returns to 2%. For this case we set the natural rate to 2% by assuming $\beta = 0.995$, while at the same time we change $\bar{s}$ and s_{low} to keep the multiplicity region unchanged ($\bar{s}$ is lowered from -0.01 to -0.010787 and s_{low} is lowered from 0.02 to 0.00922).

The sharp and persistent drop in real interest rates in scenarios B and C is very large in historical context. Consider for example the 1970s, a decade of significant monetary expansion in many countries leading to steep inflation rates. Italy experienced inflation rates between 10 and 20% for most of the decade. This was the result of extensive money financing of fiscal deficits. The real interest rate in Italy dropped, but no more than in our scenarios B and C.[23]

[23] When subtracting expected inflation from nominal interest rates, using either survey data or econometrics, the real interest rate dropped from slightly above zero to about -4% after the first oil price shock, then returned to zero after a couple of years before dropping somewhat again after the second oil price shock (e.g. see Atkinson and Chouraqui, 1985). Moreover, many other factors, specifically the oil price shocks, obviously played a role as well,

The drop in real interest rates in scenarios B and C in Figure 6 would lead to an implausible rise in output by more than 30% within a year under the consumption Euler equation of the model. However, output would respond less under alternative modeling approaches mentioned above where future real interest rate changes have a smaller effect on output today. To illustrate this, we have implemented the dynamic IS equation from Castelnuovo and Nisticò (2010) who, like Del Negro et al. (2015), assume a positive probability of death. They also allow for habit formation as in our model. Their estimated parameters imply

$$x_0 = -0.14 \sum_{t=0}^{\infty} E_0 \Theta^t (r_t - r^n) \tag{2.43}$$

with $\Theta = 0.8$. The weight on the real interest rate 2 years from now is then only a fraction 0.16 of the weight on the current real interest rate. Even with the large drop in real interest rates in scenarios B and C, output then rises only a modest 2.4% (APR) during the first quarter.

However, Figure 6 shows that still large inflation is needed, even with the large sustained drop in real interest rates. Scenarios B and C deliver very similar results for inflation. When inflation is spread over 10 years, annual inflation needs to be anywhere from 0 to 20%, dependent on where we are in the multiplicity region. Correspondingly, the price level needs to increase by a factor between 1 and 6.8. This is a bit less than without the drop in real interest rates, but unless we are near the lower range of debt in the multiplicity region it remains the case that very large and sustained inflation is needed to avoid default.

The lowest ultimate increase in the price level is achieved when the real interest rate drops as in scenarios B and C, while at the same time inflation is limited to the first 3 years, when it is most effective. Annual inflation then varies between 0 to 34% per year. Correspondingly, the price level rises by a factor between 1 and 2.7. Even in this case, unless debt is near the lower end of the multiplicity range, the inflation cost of such policies is generally very substantial.

Of course these results are not entirely model free as we still rely on the LW part of the model. But the LW model matters mainly in generating a certain multiplicity range and in the duration of the government debt. For a realistic assumption about duration and a broad range of multiplicity for debt it is generally difficult to avoid the default equilibrium through monetary policy other than by generating very steep inflation. It is hard to see how this result would change by changing aspects of the LW model as the intuition behind our findings (section 3.3) does not depend on details of the LW framework.

leading to a decline in real interest rates also in countries with a more modest monetary policy.

2.5 Discussion of Alternative Policies

In the NK model, we have examined the role of optimal interest rate policies. But other policies are often mentioned in the context of sovereign debt crises. In this section, we examine three of these policies: i) an interest rate ceiling; ii) quantitative easing at the ZLB; iii) sterilized purchase of debt.

2.5.1 Interest Rate Ceiling

Some, including Calvo (1988), have argued that the bad equilibrium may be avoided if the government commits to an interest rate ceiling. LW counter that if the government refuses to pay more than a certain (real) interest rate, and the market is unwilling to lend at that interest rate, the government would be forced to significantly cut spending or raise taxes without delay. They consider this not to be credible. LW argue that in reality the government will have to do another auction at a price that the market is willing to pay (the bad equilibrium). While they consider a real model without a central bank, alternatively one could imagine the central bank committing to buy government debt at a low (default free) real interest rate. But as we have already seen, this will make little difference. Seigniorage revenue is very small in reality. Ultimately the private sector will need to absorb new debt unless the central bank is able to slow down debt accumulation through the inflationary policies that we have already considered.

2.5.2 Quantitative Easing

When considering the role of seigniorage in section 3.5.1, we assumed that monetary expansions do not go beyond the satiation level. But one can consider much larger monetary expansions that go well beyond the satiation level, where we reach the ZLB. We examine such policies in an earlier draft of this paper, Bacchetta et al. (2015). Such a large monetary expansion can for example result from the central bank buying back a lot of government debt or providing liquidity support to the government that obviates the need for new government borrowing. In both cases government debt to the private sector is reduced and replaced by monetary liabilities. However, the large monetary expansion will be unwound when the economy exits the ZLB. Therefore, the present discounted value of seigniorage does not change and large asset purchases have no impact.[24]

[24] The only case we find where there could be an impact is when the natural rate of interest stays at zero or negative for a sustained period of time.

2.5.3 Sterilized Debt Purchases

The central bank could potentially sterilize the purchase of government debt by the sale of other securities. There are two ways in which one can imagine the sterilized purchase of government debt by the central bank, which does not increase monetary liabilities and is therefore not inflationary. One case is where the central bank has assets other than government debt on its balance sheet, such as for example gold or foreign exchange reserves. The central bank can purchase government debt, sell gold or reserves, and keep the money supply unchanged. This typically is not a solution though, as central banks' other assets are small compared to government debt.[25] More generally, this illustrates that we should look at government debt as a net concept, subtracting any assets (other than government bonds) that either the central bank or the government itself has on its balance sheet. When this net government debt is in the multiplicity region, the economy becomes subject to self-fulfilling debt crises.

The second case of sterilization applies to a monetary union, where the central bank intervenes in a crisis in its periphery. In the context of the Eurozone it would involve the ECB intervening in a self-fulfilling crisis in its periphery. In that case the central bank can be quite effective. For example, the ECB could buy periphery debt at a low interest rate and sell German debt, without a change in monetary liabilities. The threat alone of doing so is sufficient, which is, in our view, what happened under the OMT policy in the summer of 2012 and the famous Draghi statement "to do whatever it takes". This threat was credible as such an intervention would not overwhelm the ECB.[26] This explains why sovereign spreads quickly fell due to the change in policy. But such a policy applies to a periphery and would not work if the ECB aimed to avoid a self-fulfilling sovereign debt crisis across the entire Eurozone.

2.6 Conclusion

Several recent contributions have derived analytical conditions under which the central bank can avoid a self-fulfilling sovereign debt crisis. Extreme central bank intervention, generating extraordinary inflation, would surely avoid a sovereign debt crisis. In order to have a better quantitative assessment of what a successful intervention would require, we have adopted a dynamic model with many realistic elements. We introduced a New Keynesian model with nominal rigidities in which monetary policy has realistic effects on output and inflation. We introduced long-term bonds in a slow-moving debt crises model and calibrated the debt maturity to what is

[25]There may however be special cases. For example, Switzerland has FX reserves that are larger than government debt.

[26]For example, in 2010 the sum of all the periphery country government deficits together (Greece, Ireland, Portugal, Spain, Italy) amounted to 13% of the ECB balance sheet. And a self-fulfilling default can be avoided even if only a portion of these financing needs are covered by the ECB.

observed in many industrialized countries. Overall our conclusion is that, unless debt is close to the bottom of the interval where multiple equilibria occur, monetary policy leads to very high inflation for a sustained period of time.

To determine the desirability of asking the central bank to deflate the government debt, a more public finance approach would be needed as typically done in the literature. The cost of inflation should be weighted against the cost of outright default or the cost of fiscal austerity. Such an analysis goes beyong the scope of this paper. Nevertheless, the high unanticipated inflation rates required when initial debt level is in the middle or high range of the multiplicity region appear highly implausible in developed economies. First, there is currently a strong preference for low inflation rates. Second, even in previous decades with higher inflation rates, the use of inflation to reduce the debt burden has been limited. Indeed, Reinhart and Sbrancia (2015) find that in the post WWII era, particularly 1945-1980, public debt reductions in industrialized countries have been achieved mostly through financial repression as opposed to inflation surprises. To the extent that debt reduction has been partly achieved through inflation surprises, this often has gone hand in hand with financial repression. The extensive use of financial repression tools is a reflection of the difficulty of achieving debt reduction through inflation surprises alone.

Several extensions are worthwhile considering for future work. We have focused on a closed economy. In an open economy monetary policy also affects the exchange rate, which affects relative prices and output. Related to that, one can also consider the case where a large share of the debt is held by foreigners or is denominated in a foreign currency. Both might make inflationary central bank policy an even less attractive option. Default is more appealing to the extent that the debt is held by foreigners, while in the case of foreign currency denominated debt a depreciation increases the value of debt denominated in domestic currency. Finally, we have only considered one type of self-fulfilling debt crises, associated with the interaction between sovereign spreads and debt. It would be of interest to also consider rollover crises or even a combination of both types of crises. This also provides an opportunity to consider the endogenous maturity of sovereign debt. As pointed out in the introduction, high inflation can reduce the maturity of government debt, which can lead to exposure to rollover crises.

Appendix

A. *Derivation of the debt accumulation schedule*

We derive the debt accumulation schedule in the monetary cashless economy in Section 2.2. We first derive a relationship between Q_0 and Q_{T-1}. Integrating forward the one-period arbitrage equation (2.4) from $t = 1$ to $t = T-1$, we have:

$$Q_0 = A^{\kappa}\kappa + A^{Q} Q_{T-1} \tag{2.44}$$

where

$$A^{\kappa} = \frac{1}{R_0} + \frac{1-\delta}{R_0 R_1} + \frac{(1-\delta)^2}{R_0 R_1 R_2} + ... + \frac{(1-\delta)^{T-2}}{R_0 R_1 R_2 ... R_{T-2}} \tag{2.45}$$

$$A^{Q} = \frac{(1-\delta)^{T-1}}{R_0 R_1 R_2 ... R_{T-2}} \tag{2.46}$$

Next consider the government budget constraint (2.34):

$$Q_t B_{t+1} = ((1-\delta)Q_t + \kappa)B_t - s_t P_t \tag{2.47}$$

At $t = 0$ this is:

$$\frac{Q_0 B_1}{P_0} = \left((1-\delta)Q_0 + \kappa\right) b_0 - \overline{s} \tag{2.48}$$

For $1 < t < T$

$$\frac{Q_t B_{t+1}}{P_t} = r_{t-1} \frac{Q_{t-1} B_t}{P_{t-1}} - \overline{s} \tag{2.49}$$

Using equations (2.49) and (2.48) and integrating forward, we obtain

$$\frac{Q_{T-1} B_T}{P_{T-1}} = r_{T-2} ... r_1 r_0 \frac{Q_0 B_1}{P_0} - \overline{s}(1 + r_{T-2} + r_{T-2} r_{T-1} + ... + r_{T-2} r_{T-1} ... r_1) \tag{2.50}$$

Combining equation (2.50) with (2.48) and (2.44), we obtain:

$$\begin{aligned} \frac{Q_{T-1} B_T}{P_{T-1}} = & \; r_{T-2} ... r_1 r_0 (1-\delta) b_0 Q_0 + r_{T-2} ... r_1 r_0 \kappa b_0 \\ & - \overline{s}(1 + r_{T-2} + r_{T-2} r_{T-3} + ... + r_{T-2} ... r_1 r_0) \end{aligned} \tag{2.51}$$

Using equations (2.44)-(2.46), we can rewrite equation (2.51) as

$$\begin{aligned} \frac{Q_{T-1} B_T}{P_{T-1}} = & \; \frac{P_0}{P_{T-1}} (1-\delta)^T b_0 Q_{T-1} \\ & + r_{T-2} ... r_1 r_0 \left[1 + \frac{1-\delta}{R_0} + \frac{(1-\delta)^2}{R_0 R_1} + ... + \frac{(1-\delta)^{T-1}}{R_0 R_1 R_2 ... R_{T-2}}\right] \kappa b_0 \\ & - \overline{s}(1 + r_{T-2} + r_{T-2} r_{T-3} + ... + r_{T-2} ... r_1 r_0) \end{aligned} \tag{2.52}$$

This yields (2.16).

B. *Objective function of the Central Bank*

The Central Bank wants to rule out the bad equilibrium while minimizing welfare losses. Welfare is equal to

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \tag{2.53}$$

With the specific functional form that we have used, a second order approximation of U_t is:

$$U_t = -0.5(cU_c)\left\{\mu_x(x_t - \nu x_{t-1})^2 + \left[\frac{1}{\varepsilon} + \frac{\alpha+\phi}{1-\alpha}\right](1-\beta\eta) var_i \hat{y}_t(i)\right\} \tag{2.54}$$

(see Woodford(2003)), where

$$\mu_x = \frac{1-\beta\eta}{1+\beta\nu^2}\left[\frac{\alpha+\phi}{1-\alpha} + \frac{\sigma}{(1-\beta\eta)(1-\eta)}(1+\beta\eta^2)\right] \tag{2.55}$$

and ν is defined as the smaller root of the quadratic equation

$$\eta\frac{\sigma}{(1-\beta\eta)(1-\eta)}(1+\beta\nu^2) = \left[\frac{\alpha+\phi}{1-\alpha} + \frac{\sigma}{(1-\beta\eta)(1-\eta)}(1+\beta\eta^2)\right]\nu \tag{2.56}$$

Welfare expression (2.54) assumes that there is a subsidy to firms to offset the inefficiency caused by monopolistic competition.

We now derive an expression for $var_i \hat{y}_t(i)$, the cross sectional variance of output across firms. The demand for the goods of firm i is

$$ln(y_t(i)) = ln(Y_t) - \varepsilon(p_t(i) - p_t) \tag{2.57}$$

where $p_t(i)$ is the log price of the goods produced by firm i. Hence it is

$$var_i \hat{y}_t(i) = \varepsilon^2 var_i p_t(i) \tag{2.58}$$

Defining

$$\Delta_t = var_i p_t(i) \tag{2.59}$$

it can be shown (see Woodford(2003)) that

$$\Delta_t = \theta\Delta_{t-1} + \frac{\theta}{1-\theta}(\pi_t - \gamma\pi_{t-1})^2 \tag{2.60}$$

which implies

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)} \sum_{t=0}^{\infty} \beta^t (\pi_t - \gamma\pi_{t-1})^2 \tag{2.61}$$

Using (2.57)-(2.61) welfare becomes

$$-0.5(cU_c)\sum_{t=0}^{\infty} \beta^t \left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\} \tag{2.62}$$

with

$$\mu_\pi = \left[\frac{1}{\varepsilon} + \frac{\alpha+\phi}{1-\alpha}\right] \frac{\theta\varepsilon^2(1-\beta\eta)}{(1-\theta)(1-\theta\beta)} \tag{2.63}$$

Starting from a steady state, a marginal change in consumption by dc that lasts 4 quarters raises welfare by

$$\left(1+(1-\eta)\beta+(1-\eta)\beta^2+(1-\eta)\beta^3-\eta\beta^4\right)(cU_c)(dc/c) \tag{2.64}$$

We can then express the welfare loss in terms of one year's consumption, or output, as

$$\frac{dc}{c} = -\frac{0.5}{1+(1-\eta)\beta+(1-\eta)\beta^2+(1-\eta)\beta^3-\eta\beta^4} \sum_{t=0}^{\infty} \beta^t \left\{\mu_x(x_t-\nu x_{t-1})^2 + \mu_\pi(\pi_t-\gamma\pi_{t-1})^2\right\} \tag{2.65}$$

C. *Solution method for the optimal policy*

We outline the solution method for the optimal policy. The central bank chooses the interest rate i_t for $t=[0,H-1]$. At time H the Central Bank resumes the Taylor rule (2.31). The first step is to compute the sequences of output gap x_t, inflation π_t and interest rate i_t implied by the choice of $\{i_1, i_2, .., i_{H-1}\}$. Once we have these sequences, we can compute the objective function (2.30). The optimal policy is the choice of $\{i_1, i_2, .., i_{H-1}\}$ that maximizes the objective function.

From time H on, the economy is described by the system of 3 equations

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(i_t - \pi_{t+1} - r^n) \tag{2.66}$$

$$\pi_t = \gamma\pi_{t-1} + \beta\pi_{t+1} - \beta\gamma\pi_t + (\omega_1 x_t + \omega_2\tilde{x}_t) \tag{2.67}$$

$$i_t - \bar{\imath} = \rho(i_{t-1} - \bar{\imath}) + (1-\rho)(\psi_\pi\pi_{t+1} + \psi_y x_t) \tag{2.68}$$

We omitted all expectations as the only shock in the economy after time 0 is the shock to the primary surplus at time T, which does not affect output, inflation or interest rates. $\tilde{x}_t$ is defined in (2.26) and ω_1 and ω_2 are defined in (2.28) and (2.29). We can write this system as

$$M_1 v_{t+1} + M_2 v_t = 0 \tag{2.69}$$

where

$$v_t = \begin{pmatrix} i_{t-1} \\ \pi_t \\ \pi_{t-1} \\ x_{t+1} \\ x_t \\ x_{t-1} \end{pmatrix} \tag{2.70}$$

$$M_1 = \begin{pmatrix} -1 & (1-\rho)\psi_\pi & 0 & 0 & 0 & 0 \\ -\frac{1-\beta\eta}{\sigma} & \frac{1-\beta\eta}{\sigma} & 0 & -\beta\eta & \beta\eta+1+\beta\eta^2 & 0 \\ 0 & \beta & -1-\gamma\beta & 0 & -\beta\eta\omega_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{2.71}$$

and

$$M_2 = \begin{pmatrix} \rho & 0 & 0 & 0 & (1-\rho)\psi_y & 0 \\ 0 & 0 & 0 & 0 & -\eta-1-\beta\eta^2 & \eta \\ 0 & 0 & \gamma & 0 & \omega_1+\omega_2+\omega_2\beta\eta^2 & -\omega_2\eta \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.72}$$

(2.69) implies

$$v_{t+1} = Mv_t \tag{2.73}$$

with $M = -M_1^{-1}M_2$. It can be verified that the matrix M has 3 non-explosive eigenvalues, equal to the number of pre-determined variables in v_t. v_t at time H needs to be a linear combination of the 3 eigenvectors corresponding to the non-explosive eigenvalues, which implies a relationship between the non-pre-determined and the pre-determined variables at time H.

$$\begin{pmatrix} \pi_H \\ x_{H+1} \\ x_H \end{pmatrix} = G \begin{pmatrix} i_{H-1} \\ \pi_{H-1} \\ x_{H-1} \end{pmatrix} \tag{2.74}$$

Between time 0 and time $H-1$, call $\{\bar{i}_0, ..., \bar{i}_{H-1}\}$ the sequence of interest rates chosen by the central bank. The economy is described by the following system of $2H$ equations:

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(\bar{i}_t - \pi_{t+1} - r^n) \tag{2.75}$$

$$\pi_t = 0 \tag{2.76}$$

for $t < d$ and

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}(\bar{i}_t - \pi_{t+1} - r^n) \tag{2.77}$$

$$\pi_t = \gamma\pi_{t-1} + \beta\pi_{t+1} - \beta\gamma\pi_t + (\omega_1 x_t + \omega_2\tilde{x}_t) \tag{2.78}$$

for $d \le t \le H-1$. After substituting (2.74) into (2.77)-(2.78), (2.75)-(2.78) becomes a linear system of 2H equations in 2H variables. After obtaining $[\pi_t, x_t]$ for $0 \le t \le H-1$, we can obtain $[\pi_t, x_t, i_t]$ for $t \ge H$ by using (2.73).

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Chapter 3

Sovereign Money Reforms and Welfare

Abstract A monetary reform was voted by the Swiss people in June 2018. The Sovereign Money Initiative proposes that all sight deposits should be controlled by the Swiss National Bank (SNB) and that the SNB could distribute its additional resources. While a sovereign money reform would clearly affect the structure of the banking sector, it would also have macroeconomic implications, in particular because it would transfer resources from banks to the central bank. The objective of this paper is to analyze these macroeconomic implications using a simple infinite-horizon open-economy model calibrated to the Swiss economy. While we consider several policy experiments, we find that there is a key trade-off between a reduction in distortionary labor taxes and an increase in the opportunity cost of holding money. However, in the proposed Swiss reform it is this latter cost that dominates and we find that the reform unambiguously would lower welfare.

3.1 Introduction

The recent financial crisis generated renewed interest in reforming the financial or the monetary system. A bold proposal of monetary reform was recently presented in Switzerland in the form of a political initiative voted at the Federal level. The Swiss people voted on this initiative, often called the Vollgeld initiative, in June 2018. The proposal was to have *sovereign money*, where all bank notes and scriptural money included in M1 would be issued by the Swiss National Bank (SNB). All

[0]**This chapter was written in collaboration with Philippe Bacchetta**

sight deposits of commercial banks would be removed from their balance sheets and would be directly invested at the central bank. In addition, the central bank would be allowed to distribute the increase in its balance sheet to the public or to the government.[1] While Swiss voters voted this proposal down, interest for this type of monetary reform is still alive and goes beyond Switzerland. The government of Iceland advanced a similar proposal in 2016. The economic commentator Martin Wolf of the Financial Times wrote in June 2018 that the Vollgeld proposal would be a "credible test" for a "possible better future for the world's most perilous industry"[2].

A sovereign money reform would clearly affect the structure of the banking sector, but it would also has macroeconomic implications, in particular because it would transfer resources from banks to the central bank. The objective of this paper is to analyze these macroeconomic implications.

This type of reform is related to proposals of full reserve requirements or of "narrow banking", where banks are only allowed to invest in very safe and liquid assets, while the function of originating loans would be performed by institutions financed by long-term debt or equity. Among these proposals, one specific brand is the "Chicago Plan", originally initiated by Irvin Fisher and other Chicago economists in 1933, and recently revived by Benes and Kumhof (2012) (hereafter BK). In addition to avoiding runs on deposits, advocates of these reforms see another benefit which would be to cut banks' ability to collect rents from deposits, and give back the full benefits of money creation (seigniorage) where they belong, i.e., to the central bank.[3]

The objective of this paper paper is to analyze the costs and benefits of the proposed reform by taking a macroeconomic perspective and by using a very simple framework. So far, the type of monetary reform considered in Switzerland has not been formally analyzed in the literature. Advocates of the reform often refer to the BK paper, who show that a Chicago plan could significantly increase the economy's output. However, BK's analysis is not adequate for the Swiss monetary reform, because the experiment is different and their closed-economy model introduces mechanisms that are not present in an open-economy like Switzerland. Our analysis focuses on the main features of the reform proposed for Switzerland and calibrates the model on the Swiss economy.

A major issue is to understand the implications of resource allocation implied by the reform. BK shows that one of the benefits of narrow banking is to enable the government to reduce distortionary labor taxation, which increases output and should increase welfare. However, the increase in government resources is significant only if interest rates on deposits at the central bank are low. In the Swiss initiative they would be zero. But lower interest rates on deposits increase the opportunity cost of holding money. Consequently, the monetary reform might imply a trade-off

[1] For further details on this initiative see the SNB webpage or Bacchetta (2018).

[2] see "Why the Swiss should vote for Vollgeld", Financial Times, 5 June 2018.

[3] Another purported benefit is to better control economic fluctuations – the view being that the banks' ability to create money, i.e., to create their own funding, allows them to excessively expand credit during booms and contract it during contractions.

between lower distortionary labor taxes and a higher opportunity cost of money.[4] We analyze the outcome of this trade-off by carefully calibrating labor elasticity and the interest elasticity of money demand. In this context, we find that a sovereign monetary reform as proposed in Switzerland unambiguously decreases welfare.

We develop a deterministic infinite-horizon model with households, firms, banks and the government/central bank. Households work in firms, consume and save in two different instruments: an international asset in perfectly elastic supply yielding a real interest rate r^*, or domestic bank deposits, giving an interest rate r^d typically lower than r^* due to banks' market power. The reason why households save a positive amount in the form of bank deposits is that the latter have money properties: deposit (money) holdings reduce the transaction cost from consumption. We model the transaction cost following Schmitt-Grohe and Uribe (2004). [5]

For exogenous reasons, which have been identified in the literature as informational frictions, a fraction of firms' capital needs to be financed by bank loans. Banks collect deposits from households and grant loans to firms. Although deposits are a source of financing for bank loans, bank's marginal financing cost is r^*, as banks need to borrow at rate r^* to finance loans in excess of the deposit supply, or lend at rate r^* to invest deposits in excess of the loan demand. This is in contrast with the approach of BK, who model all bank liabilities as a single asset yielding an interest rate lower than the policy rate. Moreover, BK consider a closed economy model with an endogenous equilibrium interest rate. The government finances an exogenous and fixed consumption g through a tax proportional to labor income and a tax on dividends.

In the baseline model we assume that there is a continuum of banks in monopolistic competition and that all bank profits are distributed to domestic households. Monopolistic competition requires some degree of differentiation between the services sold by different banks. With respect to deposits, in the presence of transportation costs, differentiation could be provided by bank location, as in the model of Salop (1979). With respect to loans, differentiation could arise from the fact that different banks specialize in monitoring different types of firms, and from the fact that banks accumulate informational capital with respect to the firms with which they already have a relationship, so that switching bank would entail a cost for a firm.

With monopolistic competition there is no interplay between the deposit and lending functions of the bank: the banks' optimization problem yields two independent equations for the deposit rate $r^d(i)$ and the loan rate $r^l(i)$ offered by bank i. Separating the two functions in two entities, a "narrow bank" managing deposits and the payment system and a lending institution, would therefore have no effect on the decisions of the two entities. In particular, it would have no effect

[4]Another way to look at it is that "seigniorage" is essentially a tax on deposits. Since deposits are in the model proportional to consumption, a tax on deposits is a progressive tax, which is also distortionary.

[5]Feenstra (1986) demonstrates the functional equivalence between the money-in-utility-function (MIU) approach to generate money demand and the transaction cost (TC) approach, that we adopt. Wang and Yip (1992) establish a qualitative equivalence between the TC approach, the MIU approach and the cash-in-advance (CIA) approach.

on the volume of lending. We also consider a model with N banks in Cournot competition, and in this case we do find a degree of interplay between the two functions of the banks. However, we find that the impact is only significant with a very small number banks. With a realistic number of banks the effects of the reform are almost identical whether we model banks as being in monopolistic competition or Cournot competition. Thus the only effect of the reform would stem from the fact that the central bank may have a different objective function from commercial banks and might use the profits to offset other (distortionary) taxes, as discussed below.

Besides analyzing the case proposed by the Swiss initiative, we will also consider alternative scenarios. We consider three different policies. "Policy 1" is the policy which is the closest to the proposal of the Vollgeld initiative.[6] In this case deposits would yield zero interest and the government/central bank would collect the entire rebate the additional seigniorage to the public in lump-sum fashion. Our "Policy 2" assumes that additional seigniorage is used to lower the (distortionary) labor income tax. This is one of the channels through which the switch to sovereign money would benefit the economy according to BK. Finally our "Policy 3" is the case in which the central bank chooses r^d after the reform with the objective of maximizing welfare, and adjusts the labor income tax in order to be able to finance its consumption.

If "Policy 1" is adopted after the reform, welfare decreases regardless of the parameter choice, and the welfare loss is increasing in the elasticity of labor supply and in the interest semi-elasticity of money demand. If "Policy 2" is adopted, we find that the reform is essentially welfare-neutral. If "Policy 3" is adopted, we find that the central bank, in order not to increase the opportunity cost of holding money, optimally chooses an interest rate on deposits close to the risk-free rate (around 3.5% for a risk-fre rate of 4%), and in this case welfare increases between 20 and 40 bps, depending on the elasticity of labor supply. We only find a higher welfare increase if we assume that banks are owned by foreigners. In this case the reform would also have the effect of redistributing the portion of bank dividends coming from deposits from bank owners to domestic households. In this case welfare could increase by around 1.2% if "Policy 2" is adopted and by around 1.7% if "Policy 3" is adopted. If the Vollgeld policy is adopted, welfare would decrease despite the redistribution.

The paper is structured as follows. Section 2 presents the basic model and the objective of its agents: households, firms and banks. Section 3 analyzes the impact of the reform. Section 4 consider several extension of the baseline analysis. Section 5 concludes.

[6] Notice that the text of the initiative is not very precise and people also rely on the arguments made by the initiative committee.

3.2 Model

We consider a small open economy model with households, firms, and banks. The world price level is assumed constant and equal to one and purchasing power parity is assumed to hold. Therefore the price level is equal to the nominal exchange rate: $P_t = S_t$. The world real interest rate is also constant at r^* and uncovered interest rate parity holds. Thus we have $(1 + i_{t+1}) = (1 + r^*)(1 + \pi^e_{t+1})$, where i_{t+1} is the nominal interest rate on the domestic safe asset and π^e_{t+1} is the expected inflation rate.

Since the objective of our analysis is to examine the impact of sovereign money reforms in the long run, we focus on deterministic steady states; hence $\pi^e_{t+1} = \pi_{t+1}$. We will assume that the central bank can set inflation at its target level $\overline{\pi} \geq 0$.[7] We will describe the model in real terms.

3.2.1 Households

Households work in firms, consume and save, either in a safe asset A giving nominal interest rate i_t or in bank deposits. Due to the costs of managing deposits and to banks' market power, discussed in section 2.3, bank deposits give a lower interest rate $i^d_t < i_t$, but households still hold them because they reduce the transaction costs from consumption.[8] The real interest rate on deposits is r^d_t. The safe asset can be either in domestic or in foreign currency as both are perfect substitutes.

The representative household derives utility from consumption and disutility from working. We assume separable CRRA preferences so that the household's periodic flow utility is given by

$$u(c, h) = log(c) - \frac{h^{1+\gamma}}{1+\gamma} \qquad \gamma > 1$$

where c is consumption and h denotes labor supply. The household's expected lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{3.1}$$

where we assume $\beta(1 + r^*) = 1$ for stationarity. Expressed in real terms, the households budget constraint is

$$\begin{aligned} & (1 - \tau_h)w_t h_t + (1 + r^*)a_{t-1} + (1 + r^d_{t-1})d_{t-1} + (1 - \tau^b)\Pi^b_t \\ = \quad & c_t(1 + s_t) + d_t + a_t + t_t \end{aligned} \tag{3.2}$$

where w_t is the real wage, a_t are real holdings of the safe asset, d_t are real deposits, Π^b_t are real bank dividends, t_t is a lump-sum transfer, and τ^h and τ^b are labor income and dividend tax rates.

[7] The objective of our analysis is to determine the impact sovereign monetary reform for a given monetary policy. It would be of interest to examine how such a reform could affect optimal fiscal and monetary policies, but this goes beyond the scope of this paper.

[8] We do not introduce bank notes, as they would play a minor role in the analysis.

In our benchmark case, we assume that households own the banks and receive all bank profits, but we will also consider the case where banks are not held by households. To consume c_t, households incur transactions costs $c_t s_t$, that can be reduced by holding deposits.[9]

Households maximize their utility subject to (3.2). First-order conditions are standard and are described in the Appendix. Below we will assume a specific form for the transactions cost, similar to Schmitt-Grohé and Uribe (2004). Denoting money velocity as $x_t \equiv c_t/d_t$, we assume that transactions costs function is:

$$s(x) = Ax_t + \frac{B}{x_t} - 2\sqrt{AB} \tag{3.3}$$

where A and B are constant. As can be easily obtained from the Euler equations shown in Appendix, this specification implies that money velocity is

$$x_t = \sqrt{\frac{B(1+r^*) + r^* - r_t^d}{A(1+r^*)}} \tag{3.4}$$

hence the demand for deposits is

$$d_t = c_t \sqrt{\frac{A(1+r^*)}{B(1+r^*) + r^* - r_t^d}} \tag{3.5}$$

In the benchmark model, we assume monopolistic competition in the deposits market, so that deposits are distributed across banks. We assume that deposits from different banks provide slightly different liquidity services. In this case, $s(x)$ depends a bundle of deposits from different institutions:

$$d_t \equiv \left(\int (d_t(j))^{1-\frac{1}{\epsilon^d}} dj \right)^{\frac{\epsilon^d}{\epsilon^d - 1}} \tag{3.6}$$

where ϵ^d is the elasticity of substitution between deposits at different banks. Notice that in principle every deposit institution j offers a different interest rate $i_t^d(j)$, corresponding to a real rate $r_t^d(j)$. We can obtain the demand equation

$$d_t(j) = \left(\frac{r^* - r_t^d(j)}{r^* - r_t^d} \right)^{-\epsilon^d} d_t \tag{3.7}$$

where

$$r^* - r_t^d \equiv \left(\int (r^* - r_t^d(j))^{1-\epsilon^d} dj \right)^{\frac{1}{1-\epsilon^d}} \tag{3.8}$$

3.2.2 Firms

There is a representative firm with Cobb-Douglas production function

$$y_t = z k_t^{\alpha} h_t^{1-\alpha} \tag{3.9}$$

[9]Modeling money demand by the reduction in transaction or liquidity costs can be found for example in Feenstra (1986), Rebelo and Vegh (1996) or Schmitt-Grohé and Uribe (2004)

where k_t is capital. Firms are not constrained, but a fraction φ of capital can only be financed by banks (e.g., for the financing of working capital), so that $\varphi k_t = l_t$, where l_t are the real loans that the firm obtains from the bank in period t. The remaining fraction $1-\varphi$ is financed by issuing bonds at interest rate r^*.

In the benchmark model, we assume monopolistic competition in the loan market, so that, similarly to deposits, loans are a bundle of loans from different banks

$$l_t \equiv \left(\int di (l_t(i))^{1-\frac{1}{\epsilon^l}} \right)^{\frac{\epsilon^l}{\epsilon^l - 1}} \tag{3.10}$$

where ϵ^l is the elasticity of substitution for loans from different banks and the index i denotes a bank. We obtain that loan demand is

$$l_t(i) = \left(\frac{r_t^l(i)}{r_t^l} \right)^{-\epsilon^l} \tag{3.11}$$

where $r_t^l(i)$ is the loan interest rate charged by bank i and

$$r_t^l = \left(\int di (r_t^l(i))^{1-\epsilon^l} \right)^{\frac{1}{1-\epsilon^l}} \tag{3.12}$$

In equilibrium all banks choose the same rate r_t^l. Given the constraint $\varphi k_t = l_t$, the real cost of a unit of capital is $r_t^K = \varphi r_t^l + (1-\varphi) r^*$. From the first order conditions of the firm we easily obtain

$$k_t = \left(\frac{z\alpha}{r_t^K} \right)^{\frac{1}{1-\alpha}} h_t \tag{3.13}$$

and (with competitive labor markets)

$$w_t = (1-\alpha) z \left(\frac{z\alpha}{r_t^K} \right)^{\frac{\alpha}{1-\alpha}} \tag{3.14}$$

.

3.2.3 Banks

In our benchmark specification, we assume that there is is a continuum of banks with monopolistic competition in the deposit market and in the loans market. We will examine other market structures in Section 5. The aggregate banks balance sheet is $l_t + b_t^b + m_t = d_t + a_t^b$, where b_t^b are bonds held by the banks, m_t are required reserves held at the central bank and a_t^b are other bank liabilities. b_t^b and a_t^b yield an interest rate r^*, whereas reserves yield an interest rate r_t^r determined by the central bank. Banks hold reserves in proportion ϕ_t of deposits: $m_t = \phi_t d_t$. The reform sets $\phi_t = 1$ and $i_t^r = 0$.

Loans are provided with cost c^l at interest rate $r_t^l(j)$ for bank j. Deposits are provided with cost c^d at interest rate $i_t^d(j)$. Profits of bank j are

$$\Pi_t^b(j) = (1+r_{t-1}^l(j)-c^l)l_{t-1}(j)+(1+r^*)(b_{t-1}^b(j)-a_{t-1}^b(j))+(1+r_{t-1}^r)m_{t-1}(j)-(1+r_{t-1}^d(j)+c^d)d_{t-1}(j) \tag{3.15}$$

Using the bank balance sheet and the reserve ratio, this can be rewritten as:

$$\Pi_t^b(j) = [(1-\phi)r^* + \phi r_t^r - (r_{t-1}^d(j) + c^d)]d_{t-1}(j) + [r_{t-1}^l(j) - c^l - r^*]l_{t-1}(j) \tag{3.16}$$

In equilibrium all profit-maximizing banks choose the same deposit rate[10]

$$r_t^d(j) = r_t^d = r^* - (c^d + \phi(r^* - r_t^r))\frac{\epsilon^d}{\epsilon^d - 1} \tag{3.17}$$

and loan rate

$$r_t^l(j) = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c^l) \tag{3.18}$$

3.2.4 Central bank

The central bank issues the monetary base m_t and holds assets b_t^c. Assets bear an interest rate r^* and it does not matter if they are domestic or foreign assets as they are assumed to be perfect substitutes. Central bank profits are $(r^* - r_t^r)m_{t-1}$ and are distributed each period to the government. The growth in monetary base is determined by the inflation target and money market equilibrium is simply given by $m_t = \phi_t d_t$.

3.2.5 Government

The government needs to fund a constant exogenous real expenditure g. The government receives central bank profits, levies taxes on labor income at rate τ^h and on bank profits at rate τ^d (firm profits are 0), and possibly imposes a lump sum tax or transfer t_t. It pays interest r^* on its debt b_t^g. The government budget constraint is:

$$\tau^h w_t h_t + \tau^d \Pi_t^b + t_t + (r^* - r_{t-1}^r)\phi_{t-1}d_{t-1} + b_t^g = g + (1+r^*)b_{t-1}^g \tag{3.19}$$

We assume that the government keeps its real debt b^g constant, so that (3.19) becomes

$$\tau^h w_t h_t + \tau^d \Pi_t^b + t_t + (r^* - r_{t-1}^r)\phi_{t-1}d_{t-1} = g + r^* b^g \tag{3.20}$$

3.2.6 Steady State

In this section we summarize the equations that determine the steady state variables. Given the exogenous real government debt b^g, the expenditure g and the reserve rate r^r (chosen by the

[10]We impose however a zero-lower-bound condition on the nominal depsit rate $0 \leq i_t^d \equiv r_t^d + \pi_{t+1}^e$

central bank), the government chooses the tax rates τ^h and τ^d and the transfers t to satisfy

$$\tau^h wh + \tau^b \Pi^b + (r^* - r^r)\phi d = g + r^* b^g + t \tag{3.21}$$

with $d = \frac{c}{x}$. Banks all choose the deposit rate and the loan rate

$$r^d = r^* - (c^d + \phi(r^* - r^r))\frac{\epsilon^d}{\epsilon^d - 1} \tag{3.22}$$

(this holds provided that $i^d = r^d + \pi > 0$), and

$$r^l = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c^l) \tag{3.23}$$

Given r^d, velocity x is given by (3.4). Bank profits are

$$\Pi^b = ((1-\phi)r^* + \phi r_t^r - (r^d + c^d))d + (r^l - c^l - r^*)l \tag{3.24}$$

Loans are $l = \varphi k$ and k is

$$k = \left(\frac{z\alpha}{r^K}\right)^{\frac{1}{1-\alpha}} h \tag{3.25}$$

Labor is related to consumption by the tradeoff condition

$$h^\gamma = \frac{w(1-\tau^h)}{c(1 + 2Ax - 2\sqrt{AB})} \tag{3.26}$$

which is the steady state version of (3.43), where the wage is

$$w = (1-\alpha)z\left(\frac{z\alpha}{r^K}\right)^{\frac{\alpha}{1-\alpha}} \tag{3.27}$$

and $r^k = \varphi r^l + (1-\varphi)r^*$. Inserting (3.26) and (3.27) in the steady-state version of (3.2) we obtain the equation that determines steady-state consumption c:

$$c^{1+\frac{1}{\gamma}}(1 + s(x) + \frac{(r^* - r^d)}{x}) - c^{\frac{1}{\gamma}}\mathcal{R} = \frac{w^{1+\frac{1}{\gamma}}(1-\tau^h)^{1+\frac{1}{\gamma}}}{(1 + s(x) + xs'(x))^{\frac{1}{\gamma}}} \tag{3.28}$$

where $\mathcal{R} = (r^* a_{TOT} + \Pi^b + t)$ and where a_{TOT} represents the total household savings, $a_{TOT} \equiv a + d$. Finally, welfare can be obtained as $W = log(c) - h^{1+\gamma}/(1+\gamma)$.

Government/central bank policy can affect the steady state variables through its choice of the reserve ratio ϕ, tax rates τ^h and τ^d, and transfers t, subject to its budget constraint (3.21). Notice that the loan rate (3.23) is not affected by the parameters set by the government. However, capital (3.25) and loan demand $l = \varphi k$ can change as they proportional to labor, which is affected by tax rates and by the deposit rate.

3.2.7 Calibration

The model needs to be calibrated for the numerical analysis in the next section. Using Swiss data from 1980 to 2013, we estimate the A and B in the transaction cost (3.3), following the the same approach as Schmitt-Grohé and Uribe (2004), and the reserve ratio ϕ. For the parameters of the transaction cost we obtain $A = 0.0279$, $B = 0.0241$. With a spread $r^* - r^d = 2\%$, the implied elasticity of money demand is -0.114, in line with the elasticity estimated by Bacchetta (2018) and Benati (2016). For the reserve ratio, estimated as the average value over time of $(Monetary\ Base - Bank\ Notes)/(M1 - Bank\ Notes)$, we obtain ϕ=0.08. For the other parameters, we use the following benchmark values: $r^* = 4\%$; $r^d = 2\%$ and $c^d = 0.25\%$; $r^l = 5\%$ and $c^l = 0.25\%$; inverse elasticity of labor supply: $\gamma = 1$; $\tau^h = \tau^d = 25\%$.

3.3 Sovereign Money Reforms

3.3.1 Policy Scenarios

The reforms we consider have two dimensions. The first dimension is the transfer of bank deposits to the central bank, and the setting of the nominal deposit rate $i^d = 0$. In the model this is equivalent to keeping deposits in banks but imposing a full reserve requirement, i.e. $\phi = 1$, and zero nominal interest on reserves $i^r = 0$. In this case (3.22) shows that banks would want to set the real deposit rate lower than the real rate on reserves (which would imply $i^d < i^r$), but they cannot because of the zero-lower-bound on nominal deposit rates. Hence it would be $i^d = 0$. However we also consider a policy in which the central bank sets the deposit rate to a positive level.

The second dimension is the fiscal implication of the reform. Overall, a sovereign money reform increases seigniorage and thus government income. Since we assume a constant debt level, the government can then satisfy its budget constraint (3.21) either by increasing transfers t, or by decreasing the tax rates τ^h and/or τ^d.

Since ours is an open-economy model, after the reform capital would adjust immediately and the economy would be at the new steady state. We evaluate the impact of the reform by comparing welfare in the steady state before and after the reform. We consider three variants of sovereign money reforms.

- *Policy 1: Vollgeld*: The extra seigniorage goes directly to increase transfers. This is the policy envisioned by the Vollgeld reform, and can be thought as a form of helicopter drop. In that case the tax rates τ^h and τ^d actually have to increase as a result of this policy: since bank profits and the corresponding tax revenues $\tau^d \Pi^b$ decrease after the reform, if tax rates are not increased the government does not have sufficient resources to fund its expenditure and debt service. We assume that the government decides to increase τ^h and leaves τ^d unchanged.

- *Policy 2: Lower taxes*: The increased seigniorage is offset by lower tax rates. We assume that the government decides to decrease τ^h (the most distortionary tax) while leaving τ^d unchanged.

- *Policy 3: Lower taxes and positive interest on reserves*: The government/central bank is allowed to set an interest rate for deposits different from zero after the reform, and uses the seigniorage revenues to lower τ^h. In particular the government chooses τ^h and i^d (or equivalently, given an inflation level, r^d) to maximize welfare $W = log(c) + h^{1+\gamma}/(1+\gamma)$, where, given the problem's first order conditions, consumption c is given by the solution of (3.28) and labor h is given by (3.26), subject to the government budget constraint (3.21). This is an especially simple kind of static Ramsey problem in which the government picks an optimal taxation as a combination of labor and seigniorage taxes.

Below are the numerical results about the welfare impact of these three policies. We consider our benchmark model, but we also examine where households and bankers are separate agents and households do not receive bank profits. In the latter case, the reform brings a clear extra benefit to households, as they get lower tax rates at the expense of bankers' profits.

We also compare the impact of the different versions of the reform with the impact of an hypothetical reform that could bring perfect competition in the deposit and loan market, i.e. that would result in $r^d = r^* - c^d$ and $r^l = r^* + c^l$. Imperfect competition is a key distortion in the economy that affects not only deposits but also the loan market, and we think it would be interesting to compare the benefits of easing this distortion with the benefits or costs of a sovereign money reform, which has an impact on other distortions. Notice that improving competition in the deposit market would bring the deposit rate closer to the risk-free rate, which goes in the opposite direction relative to the sovereign money proposal, at least in the Vollgeld version.

3.3.2 Numerical results

For each policy after the reform, we show the changes in consumption, labor and welfare (in consumption terms) relative to their pre-reform values. We also show the labor income tax rate (25% before the reform) and the deposit rate (2% before the reform). We first compute the results for zero steady state inflation, $\overline{\pi} = 0$. In this case the nominal and real interest rates coincide.

Policy 1 (*Vollgeld*) is unambiguously bad for the economy. Labor taxes need to increase because, with lower bank profits, revenues from profit taxes are lower. So the distortion from labor taxation increases. At the same time, with $r^d = 0$, money holdings decrease and the transaction cost of

Table 1: Benchmark				
	Policy 1	Policy 2	Policy 3	Perfect Competition
Consumption	-1.41%	+0.52%	+0.65%	+3.21%
Labor	-0.99%	+0.81%	+0.45%	+2.60%
Welfare (in c terms)	-0.67%	-0.11%	+0.30%	+1.14%
τ^h	25.74%	22.91%	25.13%	24.80%
r^d	0	0	3.46%	3.75%

consumption increases. In conclusion Policy 1 exacerbates two economic distortions: the labor wedge and the suboptimal money holdings associated with low interest on deposits.

Policy 2 (*Lower Taxes*) entails a consumption increase but a welfare decrease of 11 bps in consumption terms. Higher seigniorage revenues allow the government to decrease labor income taxes, which makes people work more. However, for the same reasons as for Policy 1, the transaction cost of consumption increases. In conclusion, one distortion in the economy, the labor wedge, is reduced, but the one associated with money is exacerbated.

Policy 3, in which the government optimizes the composition of the revenues between labor taxes and seigniorage, allows for a modest reduction of labor income taxes and an increase of the interest rate on deposits (from 2% to 3.46%). This policy entails a welfare increase of 30 bps.

With perfect competition, the deposit rate would be very close to the that of Policy 3. However, with perfect competition a major positive effect on the economy would come from the reduction in the loan rate (from our baseline value of 5% to $r^* + c^l = 4.25\%$). This would induce an increase in the capital stock, hence an increase in wages, that would encourage labor and increase production and consumption.

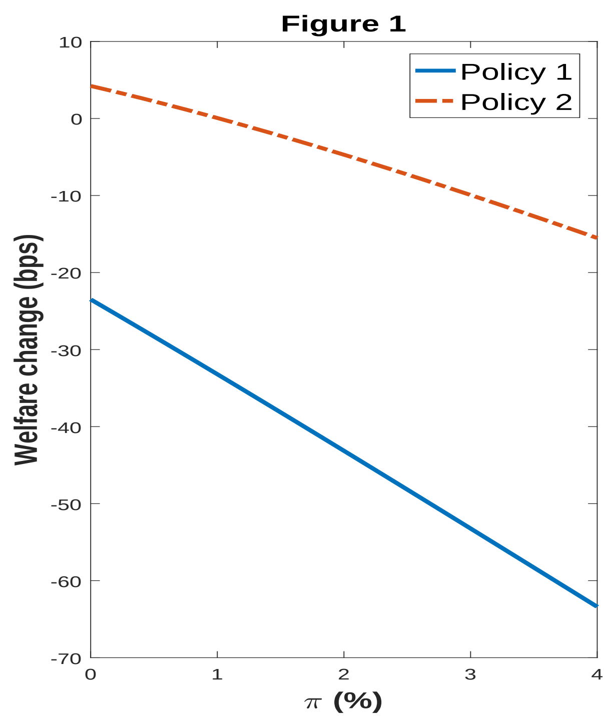
The following table presents the results where bank profits are distributed to foreign rather than domestic households.

Table 2: Banks owned by foreigners				
	Policy 1	Policy 2	Policy 3	Perfect Competition
Consumption	-1.33%	+1.53%	+1.72%	+4.76%
Labor	-1.48%	+0.35%	0.05%	+0.37%
Welfare (in c terms)	-0.16%	+1.23%	+1.67%	+4.35%
τ^h	26.05%	22.49%	24.69%	25.33%
r^d	0	0	3.53%	3.75%

With all 3 policies the reform looks better than in the benchmark case, when domestic households share the dividends. The difference lies essentially in the redistribution from foreigners, who lose the profits from deposits, to households, who receive extra transfers from the government, or whose labor taxes are reduced. With the Vollgeld policy, however, due to the fact that both distortions in the economy are exacerbated, welfare still decreases.

3.3.3 The Impact of Inflation

With steady state inflation $\overline{\pi}$, the reform (with Policy 1 and 2) sets $r^d = -\overline{\pi}$. With positive inflation, the welfare consequences of the reform with Policy 1 and 2 are worse than with zero inflation, since the opportunity cost of holding money increases. The results for Policy 3 show that the optimal real interest on deposits is positive, hence the more negative the real interest the farther we are from optimality. Figure 1 plots the welfare effects of the reform with Policy 1 and 2 as a function of $\overline{\pi}$.



Policy 3, which sets an optimal deposit rate in real terms, is inflation neutral, at least as long as the zero-lower-bound on nominal rates is not reached.

3.4 Robustness and Extensions

It is interesting to see how results change when we deviate from our benchmark analysis. In this section, we examine the impact of another utility function frequently used in the literature, GHH (see Greenwood, Hercowitz and Huffman (1988)), of a different banking market structure, and different parameter choices, affecting the labor supply elasticity and the interest semi-elasticity of money demand.

3.4.1 GHH utility Function

The household flow utility is

$$u(c, h) = \frac{1}{1-\sigma}(c - \theta h^{\nu})^{1-\sigma} \tag{3.29}$$

While most papers[11] calibrate the parameters, in particular the parameter ν, to the Frisch elasticity, the most relevant quantity for our experiment is the uncompensated elasticity of labor supply

$$\epsilon^{unc} \equiv \frac{w}{h}\frac{\partial h}{\partial w} \tag{3.30}$$

Indeed, as seen in the previous sections, after the reform we have a permanent change in the labor tax rate, the sign and magnitude of which depend on the policy choice. For the utility (3.29), with the transaction cost specification (3.3), the labor supply is

$$h^{ghh} = \left(\frac{w(1-\tau^l)}{\theta\nu(1 + 2Av - 2\sqrt{AB})}\right)^{\frac{1}{\nu-1}} \tag{3.31}$$

hence

$$\epsilon^{unc} = \frac{1}{\nu - 1} \tag{3.32}$$

We pick $\nu = 5$ which implies $\epsilon^{unc} = -0.25$.[12] . We further pick $\sigma = 5$ and calibrate $\theta = 118$ (the latter value is chosen so that the pre-reform average working time is 1/5 of total time, as in Hnatkovska, Lahiri and Vegh (2016)). The results, shown in Table 3, are qualitatively similar to those obtained with the separable utility function (4.2).

Table 3: GHH preferences

	Policy 1	Policy 2	Policy 3
Consumption	-1.14%	+0.02%	+0.67%
Labor	-0.52%	+0.25%	+0.35%
Welfare (in c terms)	-0.82%	-0.26%	+0.43%
τ^h	25.50%	23.16%	25.03%
r^d	0	0	3.61%

[11] see for example Hnatkovska, Lahiri and Vegh (2016), Neumeyer and Perri (2005) or Mendoza (1991)

[12] For example the review paper by Evers, De Mooji and van Vuuren (2008) find an uncompensated elasticity of 0.1 for men and 0.5 for women. This is consistent with the numbers reported by E. Saez on his lectures in Public Economics, which can be found at
https : //eml.berkeley.edu/ saez/course/Labortaxes/laborsupply/laborsupply_slides.pdf.

3.4.2 Different Banking Market Structure: Cournot Competition

Before the reform
N banks choose deposit and loan quantities L^i, with $i = 1, ..., N$, and deposit quantities D_i to maximize profits, which, in real terms, are

$$\Pi_t^b(j) = ((1-\phi)r^* + \phi r_{t-1}^r - (r_{t-1}^d + c^d))d_{t-1}(j) + (r_{t-1}^l - c^l - r^*)l_{t-1}(j) \tag{3.33}$$

The only difference relative to (3.16) is that the deposit and loan rate are the same for every bank. Deposits D appearing in the transaction cost are the sum of deposits from individual banks $d_t = \Sigma_{j=1}^N d_t^j$. Similarly, for loans $l_t = \Sigma_{l=1}^N l_t^j$. The deposit rate is determined by the aggregate deposit quantity: using the deposit demand (4.12) we obtain

$$r_t^d = B(1+r^*) + r^* - A(1+r^*)\left(\frac{c}{\Sigma_{j=1}^N d_t(j)}\right)^2 \tag{3.34}$$

The aggregate loan supply $l_t = \Sigma_{j=1}^N l_t(j)$ determines capital $k_t = \frac{l_t}{\varphi}$ which in turn determines r_t^K(see (3.13)).

The novelty in the case of Cournot competition is that an individual bank has an impact on aggregate quantities. In particular, an individual bank's decision to increase its loan quantity $l_t(j)$ has the effect of lowering r_t^K, which has a positive effect on wages (see (3.14)) and consumption. With higher consumption, the demand for deposits is higher, or, given an aggregate deposit supply $d_t = \Sigma d_t(j)$, the deposit rate r_t^d is lower, which increases bank profits. In sum, an individual bank has an incentive to increase its loan supply (or lower the loan rate) in order to increase its profits from deposits. Clearly, the higher the number of banks, the less an individual bank internalizes this effect.

After the reform After the reform bank profits come only from loans:

$$\Pi_t^b(j) = (r_t^l - c^l - r^*)l_{t-1}(j) \tag{3.35}$$

Banks lose the incentive to give more loans in order to increase the profits from deposits. hence we can expect the loan rate to be higher after the reform than before the reform, and we expect the difference between the pre-reform and the after-reform loan rate to be stronger for lower N.

The Appendix shows how the loan rate (before and after the reform) and the deposit rate (before the reform) depend on the number of banks N. Our results indicate that a deposit rate of 2% before the reform (given $r^* = 4\%$) is consistent with $N = 10$. With $N = 10$, however, the difference between the pre-reform and the after-reform loan rate is only 7 bps, and the effects of the reform on labor, consumption, welfare, the tax rate and the deposit rate in the case with Cournot competition are almost indistinguishable from those with monopolistic competition.

3.4.3 Other parameter choices

The parameters that have a sizeable impact on the results are those that affect the elasticity of labor supply and the interest semi-elasticity of money demand.

In this section we first look at the results obtained with different values of γ (inverse elasticity of labor supply): results for $\gamma = 0.5$ are shown in Table 4 and those for $\gamma = 2$ are shown in Table 5. In Table 6 we look at the results obtained with the parameters A and B of the transaction cost used by Schmitt-Grohé and Uribe (2004), calibrated to US data. The latter parameters imply a much lower semi-elasticity of money demand (-0.05, compared to -0.11 in our benchmark case, for $r^* - r^d = 2\%$).

When the elasticity of labor supply is higher, labor taxes are more distortionary. Hence Policy 2, which leads to lower taxes, has a better potential to improve welfare, and the opposite for the Vollgeld policy, which leads to a tax increase.

When the semi-elasticity of money demand is lower, as in the US case, the friction associated with money is less important, so both Policy 1 and Policy 2, which lower the interest on deposit, have lower welfare costs.

Table 4: $\gamma = 0.5$

	Policy 1	Policy 2	Policy 3
Consumption	-2.11%	+0.87%	+0.88%
Labor	-1.44%	+1.15%	+0.68%
Welfare (in c terms)	-1.13%	+0.06%	+0.40%
τ^h	26.07%	22.82%	24.88%
r^d	0	0	3.24%

Table 5: $\gamma = 2$

	Policy 1	Policy 2	Policy 3
Consumption	-1.26%	+0.20%	+0.46%
Labor	-0.65%	+0.51%	+0.27%
Welfare (in c terms)	-0.72%	-0.24%	+0.23%
τ^h	25.85%	22.99%	25.32%
r^d	0	0	3.61%

Table 6: $A = 0.0111$**,** $B = 0.07524$			
	Policy 1	Policy 2	Policy 3
Consumption	-0.60%	+0.31%	+0.26%
Labor	-0.49%	+0.37%	+0.23%
Welfare (in c terms)	-0.23%	+0.03%	+0.09%
τ^h	25.33%	23.99%	24.87%
r^d	0	0	3.62%

3.5 Conclusion

This paper has proposed a simple framework to analyze sovereign money proposals. We find that a reform along the lines of the Vollgeld proposal would entail welfare losses. In order for a sovereign money reform to generate even modest welfare gains, as obtained with our "Policy 3", the central bank should essentially reward deposits with an interest close to the risk-free rate, rather than setting the interest on deposits to 0, as advocated by the Vollgeld reform.

It is obvious that some of the issues in the debate surrounding this proposed reform cannot be addressed by the current simple model and that richer extensions would be required. For example, in our model there are no shocks, hence there is no need for liquidity insurance and maturity transformation, the traditional rationales for the coexistence of deposits and lending. We implicitly take the view expressed by Cochrane (2014) that, with modern financial technology, this is not a function that banks are uniquely able to fulfill.

Also, we disregard the possibility of bank runs, one of the arguments in favor of the reform. As discussed by Bacchetta (2018), although the financial crisis can be viewed in the perspective of runs, there was no run on deposits. Even in the case of Northern Rock, the run was on short-run liabilities not included in M1, which would not be touched by the proposed reforms. The possibility of a run on deposits is greatly limited by the existence of deposit insurance. If we view runs as a self-fulfilling "bad equilibrium", deposit insurance rules them out altogether.

Finally, we point out that in this model there is no mechanism by which the co-existence of the deposit and loan functions within banks amplifies economic fluctuations. On the other hand, such mechanism is not clearly identified in the existing literature either, not even by BK, who seem to take for granted the connection between banks' money creating ability and economic fluctuations. Bacchetta (2018) documents that in Switzerland the correlation between credit and M1 is insignificant, whereas it should be strongly positive if the above intuition reflected economic reality.

Appendix

A. Household FOCs

FOC with respect to consumption

$$\frac{1}{c_t} = \lambda_t(1 + s(x_t) + x_t s'(x_t)) \tag{3.36}$$

Specialized to the case of the transaction cost in the form (3.3), (3.36) becomes

$$\frac{1}{c_t} = \lambda_t(1 + 2Ax_t - 2\sqrt{AB}) \tag{3.37}$$

FOC with respect to hours worked

$$h_t^{\gamma} = \lambda_t W_t(1 - \tau^h) \tag{3.38}$$

FOC with respect to d_{t+1}

$$\lambda_t(1 - Ax_t^2 + B) = \lambda_{t+1}\frac{1 + i_t^d}{1 + \pi_{t+1}} \tag{3.39}$$

FOC with respect to a_{t+1}

$$\lambda_t = \lambda_{t+1}\frac{1 + i_t}{1 + \pi_{t+1}} \tag{3.40}$$

(3.37), (3.38), (3.39)and (3.40) imply the two Euler equations

$$\frac{1}{c_t(1 - Ax_t^2 + B)} 1 + 2Ax_t - 2\sqrt{AB} = \beta(1 + r^d)\frac{1}{c_{t+1}(1 + 2Ax_{t+1} - 2\sqrt{AB})} \tag{3.41}$$

$$\frac{1}{c_t(1 + 2Ax_t - 2\sqrt{AB})} = \beta(1 + r)\frac{1}{c_{t+1}(1 + 2Ax_{t+1} - 2\sqrt{AB})} \tag{3.42}$$

and the labor/leisure tradeoff condition

$$h_t^{\gamma} = \frac{W_t(1 - \tau^l)}{c_t(1 + 2Ax_t - 2\sqrt{AB})} \tag{3.43}$$

B. Deposit and loan rates with Cournot competition

In the table below, r^l "before" is the loan rate chosen by the banks before reform, when banks perform both the deposit and the loan functions, and r^l "after" is the loan rate chosen after the reform, when banks do not manage deposits. Clearly r^d refers to the deposit rate chosen by the banks before the reform, since after the reform the deposit rate is set to 0 by the central bank. As in the monopolistic competition case, we assume that the cost of managing loans and deposits, c^l and c^d respectively, are equal to 0.25%.

Table 7: Cournot Competition

N	r^l Before	r^l After	r^d
1	18.72%	19.17%	0%
2	8.43%	8.55%	0%
3	6.66%	6.76%	0%
4	5.94%	5.02%	0%
5	5.54%	5.62%	0%
6	5.29%	5.39%	0.22%
7	5.13%	5.22%	0.89%
8	5.02%	5.11%	1.35%
9	4.93%	5.01%	1.79%
10	4.85%	4.92%	1.99%
15	4.65%	4.67%	2.66%
20	4.55%	4.57%	2.94%

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Chapter 4

On the Special Role of Deposits for Long-Term Lending

Abstract I build a general equilibrium model to show that deposits are a special form of financing, that makes banks more suitable to extend long-term loans when confronted with the risks of monetary policy. In the model, banks borrow short-term and lend long-term, are subject to a minimum equity requirement consistent with Basel III, and face a financial friction: they cannot raise equity on the market. Consistent with the ”bank-capital channel” of monetary policy, when the risk-free rate increases, the value of the banks' assets and equity are eroded, and banks deleverage by cutting their lending. I show that, thanks to a combination of banks' market power in the deposit market and of the money-like properties of deposits, the profits on deposits are strongly countercyclical, and reduce by about one third the contraction of lending at high interest rates due to the bank capital channel. Amid current proposals for narrow banking, this effect provides a rationale for the coexistence of lending and deposit-taking activities in current commercial banks.

4.1 Introduction

Since the Great Financial Crisis of 2007-2009, amid discussions about how to reform the banking system, narrow banking proposals have received renewed interest. In a nutshell, what the different versions of narrow banking proposals have in common is the idea that the two main functions of current banks, deposit-taking and lending activities, should be separated in two different institutions. While the discussion has focused on the advantages of such proposals in terms of financial stability, in particular to avoid bank runs (see for example Cochrane (2004)), little has been said about the possible disadvantages.

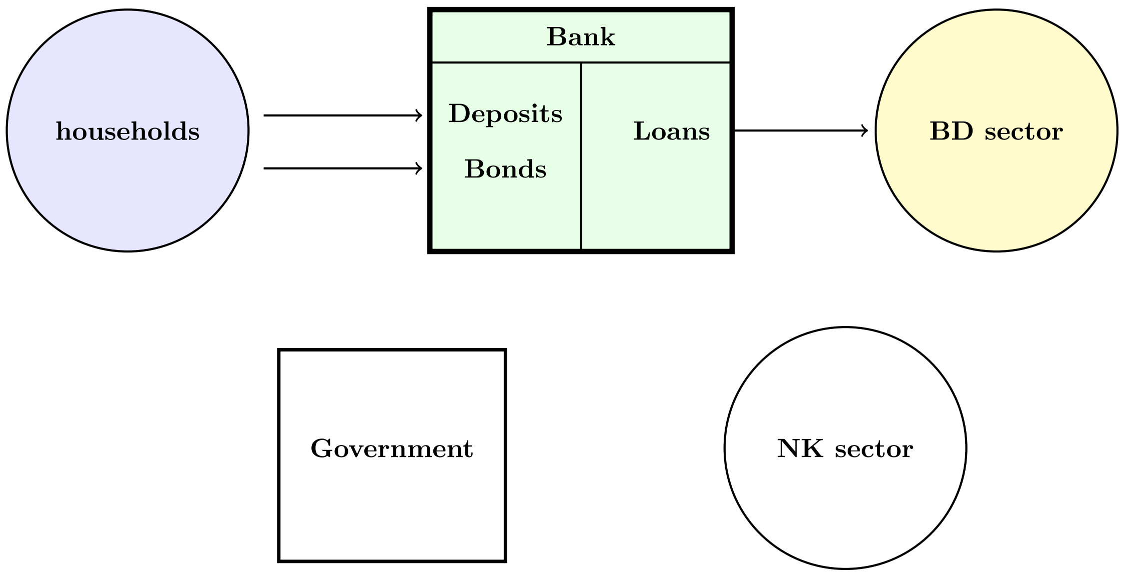
What, if any, are the synergies between the deposit function and the lending function? This question is also important in the discussion about the transmission mechanism of monetary policy. While there is a literature about the "lending channel" (see Bernanke and Blinder (1988) and Kashyap and Stein (1994)), positing that, by affecting the supply of reserves and hence of deposits, monetary policy shifts the supply of loans, the more recent literature has internalized the Romer and Romer (2000) critique, which argues that if banks can switch without frictions to non-reservable forms of funding, deposit supply should not affect loan supply.

In this paper I argue instead that deposits are a special form of funding for banks that engage in maturity transformation: deposits do affect the supply of loans, as they provide banks with a natural hedge against the interest rate risk of monetary policy. I rely on the empirical results of Drechsel, Savov and Schnabl (2017), henceforth DSS. They show empirically that the spread between the Fed Fund rate and the deposit rate is increasing in the Fed Fund rate: for a percentage point increase in the Fed Fund rate, the interest rate on "core deposits" (defined as checking + saving + small time deposits) increases on average by only 40 bps, while at the same time deposit demand decreases by around 3%. While DSS focus on the fact that deposit demand decreases when the policy rate increases, which in their view amplifies the contraction in lending at high interest rates, I read the data from a different viewpoint and argue that *profits on deposits* are the important quantity. Profits on deposits, the product of the deposit spread and the quantity of deposits, strongly increase after a policy rate increase: I estimate that when the policy rate is 6%, such profits almost triple relative to when the policy rate is 2%.

For a bank that borrows short-term and lends long-term, an interest rate increase results in an erosion of equity. To the extent that banks face a friction in raising new equity on the market, and that they are subject to an equity constraint (for economic or regulatory reasons), an erosion of bank equity leads to a contraction in bank lending. This is the basis of the "bank-capital channel" of monetary policy, see van den Heuvel (2003), henceforth vdH. The objective of my model is to show that profits on deposits, which increase after a policy rate increase, significantly mitigate the contraction in bank lending due to the bank-capital channel.

The agents in the model and the credit flow in the economy are shown in Figure 1. The central agents in the model are in the top row of the figure. In addition to banks, the other important agents are households, that generate deposit demand, and firms in the "Bank-Dependent (BD) sector" (which need to borrow from banks), that generate loan demand. The agents in the bottom row, the government and firms in the "New Keynesian (NK) sector", allow the embedding of the model in General Equilibrium: the government sets the policy rate, the central stochastic quantity in the economy, following a Taylor rule; firms in the NK sector generate the New Keynesian Phillips Curve (NKPC).

Figure 1



Section 2 of the paper focuses on households and banks in their deposit-taking function. These are the agents that determine deposit demand and supply. Households allocate their savings among three assets: cash, deposits and an asset paying the policy rate. Cash and deposits are "money-like" assets, as they reduce a transaction cost of consumption, as in Schmitt-Grohé and Uribe (2004), and, as in DSS, are imperfect substitutes. Cash always pays 0 interest. Deposits are issued by banks, each of which is a monopolist in its own county and sets the deposit rate to maximize profits. I show that this setup results in deposit spreads increasing in the policy rate. The intuition is that, since cash always gives 0 interest, as the policy rate increases, for a given deposit spread households choose to hold more deposits and less cash, so the bank increases its market power in the market for "money-like" assets. Deposit demand decreases in the policy rate for two reasons. First, as the deposit spread decreases at higher policy rates, households choose to hold less liquid assets and more assets paying the policy rate. Second, as the policy rate increases, as a general equilibrium effect household consumption decreases, which in turn decreases the demand for money-like assets. Combining the effect of the deposit spread, increasing in the policy rate, and of deposit demand, decreasing in the policy rate, I obtain the aforementioned result that that profits on deposits almost triple when the policy rate goes from 2% to 6%.

Section 3 of the paper focuses on the bank's problem in its loan-extending function. While a monopolist in the deposit market, each bank is in monopolistic competition in the loan market. As in vdH, the bank is risk-neutral and its objective is to maximize the present discounted value of future dividends, subject to an equity requirement constraint consistent with the Basel III Accord requirement. The financial friction in the model is that the bank cannot raise equity on the market. It can retain profits, but this is expensive as accounting profits, i.e. operating income minus interest payments and write-offs on loans, are taxed at a constant tax rate τ. The bank is prevented from extending new loans or distributing dividends as long as it violates the equity requirement (which stipulates that bank's equity be at least 8% of the value of the assets). Monetary policy can erode the value of the existing bank assets and affect future loan profitability, which can make the bank violate the equity constraint or increase the probability of a future violation, thus affecting the willingness of the bank to extend new loans. My objective is to quantify by how much the profits on deposits can mitigate the "bank capital channel", i.e. the contraction in bank lending at high policy rates due to balance sheet effects.

I solve the problem of the bank with a value-function-iteration method. The value function of the bank has a kink for equity equal to the minimum equity requirement value, and it is highly non-linear for equity-to-assets ratios lower than 10% above the equity requirement. This makes linear methods unsuitable to solve the problem. I discretize the state space, comprised of equity, outstanding loans and interest rates, making the grid denser in the region of high non-linearity. I consider values of interest rates from 2% to 6%. 4% is the natural rate, corresponding to the household discount factor. The interest rate follows a discretized stochastic Taylor rule.

The impact of the "bank-capital channel" can be evaluated by comparing the amount of new lending in the model with the financial friction (the inability to raise equity in the market) to new lending without the financial friction. Without the financial friction, deposits do not have any impact on lending, as the bank, by raising equity, is always able to undertake all profitable lending opportunities. With the financial friction this is not the case, and deposits can have an impact. The impact of the financial friction is very significant at high interest rates. If deposits are set to 0, when the interest rate is 5% new lending is on average 9.5% lower than in the model without the financial friction, and when the interest rate is 6% new lending is on average 24.5% lower. With deposits and deposit rates optimally chosen by banks, the reduction in average new lending, relative to the case without friction, is 6.5% at 5% interest rate and 16.5% at 6% interest rate. Thus deposits reduce the contraction in new lending by close to one third. Notice that, when comparing the results with and without deposits, I add to the model with deposits a fixed cost equal to the average value of the profits they generate. This cost could represent a tax or an operating cost. So the impact of deposits reported above is due exclusively to the countercyclical nature of those profits.

I also show that when equity-to asset-ratio is in the lowest percentile values, for every initial value of the interest rate, an interest rate increase produces a much more persistent contraction in lending if deposits are 0, relative to when deposits are optimally chosen by banks. On the one hand, bank profits on deposits, increasing in the risk-free interest rate, significantly contribute to rebuilding equity and thus restoring the bank's ability to lend after a contractionary monetary shock. On the other hand, in the absence of deposits, the bank cuts lending in the early periods after the shock also as a way to rebuild equity. This is because, due to the downward-sloping loan demand faced by the bank, if the bank lends less it is able to get loans for a price that is lower than their accounting value. However, this implies that the bank forgoes more profitable lending opportunities, which slows down the rebuilding of equity over time.

In the Appendix I describe in detail the two firm sectors, that correspond to two alternative technologies for the production of the consumption good. In particular, the "NK sector", similar to firms in the basic New-Keynesian model in Gali (2008), is useful to generate the NKPC. To make the embedding of the bank model in general equilibrium simpler, I assume that the household disutility of labor and the production function in the "New-Keynesian" sector are linear in labor. I show that this implies that the Euler equation, the New-Keynesian Phillips curve and the Taylor rule form a system of three equations in three variables, consumption, inflation and the policy interest rate, independent of banks (except for the choice of the deposit rate) and of production in the bank-dependent sector. The intuition for why this happens is that, with linear labor disutility and linear production function in the New-Keynesian sector, households would be able to adjust their labor supply and thus production in the New-Keynesian sector to compensate for fluctuations in the production of the bank-dependent sector. Although with these assumption the model would be unsuitable to evaluate the impact of the banking sector on the macroeconomy, it is calibrated to generate a realistic response of consumption and inflation to monetary shocks, very similar to that of the basic New-Keynesian model in Gali (2008), and is thus suitable to analyze the effect of the macroeconomic environment on the bank's problem, which is the focus of this paper.

To sum up, the contribution of this paper is threefold: first, I identify a new mechanism by which deposits affects banks' loan extension and the transmission of monetary policy. Second, I quantify the impact of deposits by embedding this mechanism in a bank model inspired by the partial equilibrium model of vdH. Third, I embed the bank model in general equilibrium, which allows me to take into account the effect of changes in the macroeconomic environment, in particular changes in aggregate consumption and inflation, on the bank's lending problem.

Literature Review

This paper contributes to the literature on the role of bank lending in the transmission of monetary policy, often called "the credit channel" of monetary policy. This literature has two objectives. The first objective is to understand whether and through which channels a shift in monetary policy has an impact bank loan supply (or demand), independently of the traditional interest rate channel. The second objective is to understand the extent to which the shift in bank loan supply (or demand) affects economic outcomes.

A vast part of this literature (see, for example, Gertler and Gilchrist (1993), Bernanke and Gertler (1995), Bernanke, Gertler and Gilchrist (1999)) focuses on the so-called "broad credit channel", or"financial accelerator": an interest rate increase causes a deterioration of borrower firms' balance sheet, which in turn causes an increase in the external finance premium and a decrease in loan demand over and above the decrease due purely to the risk-free interest rate increase.

Another part of the literature (see, for example, Bernanke and Blinder (1988), Kashyap and Stein (1994)) focuses on the "narrow credit channel", or "lending channel": a decrease in central bank reserves forces banks to issue fewer reservable deposits. Assuming that deposits are the main source of funding for bank loans, lower deposit issuance would result in lower loan issuance. As previously discussed, this line of reasoning has been criticized by Romer and Romer (2000).

Other branches of the literature find links between monetary policy and bank loan supply through different channels: for example the previously mentioned "bank capital channel" (van den Heuvel (2003), Adrian and Shin (2010a)) and the "risk-taking channel" (Borio and Zhu (2012) and Adrian and Shin (2010b)), according to which monetary policy may influence banks' perception of risk or attitude toward risk.

This paper is closer to the "lending channel" literature in that it focuses on the role of deposits and on the effect of monetary policy on loan supply, and to the "bank capital channel" literature, in that it argues that deposits have an impact on this specific channel.

In contrast to the lending channel literature, however, this paper argues that deposits have a mitigating, rather than amplifying, effect on loan supply, the main difference being that instead of focusing on the quantity of deposits, that decreases after an interest rate increase, I focus on deposit profits, that increase after an interest rate increase.

Two recent papers emphasizing that an increase an interest rate has a positive effect on banks' interest rate margins and hence banks' profits on new business are Brunnermeier and Koby (2018) and Di Tella and Kurlat (2017).

4.2 The Economics of Deposits

4.2.1 Empirical Evidence

DSS present detailed empirical evidence on the relationship between deposit quantities and the Fed fund rate, and between deposit spreads and the Fed Fund rate. They estimate that an increase of 100 bps in the Fed Fund rate leads on average to a 61 bps increase in the deposit spread. The increase in the spread is shown to be clearly correlated to banks market power: within the same bank, branches in high-concentration areas increase their spread by less than branches in low-concentration areas.

They also estimate the semi-elasticity of deposits with respect to the deposit spread to be -5.3. Thus, an increase of 100 bps in the Fed Fund rate, which is estimated to raise the deposit spread by 61 bps, would induce a decrease in the quantity of deposits of around 320 bps.

A clear implication, which is not drawn by DSS, is that profits on deposits increase after a Fed fund increase. For example, an increase of the Fed Fund rate from 4% to 5% represents a 25% increase in the Fed Fund rate rate, which also translates in a 25% increase in the deposit spread if the relationship between the latter and the Fed Fund rate is approximately linear. Despite the 3% decrease in the quantity of deposits, profits on deposits (equal to the product of deposit spread and deposit quantity) would increase by over 20%. Only for extremely high values of the Fed Fund rate (over 30%!) the decrease in the deposit quantity would be more important than the increase in the spread.

4.2.2 Model: Households and Deposit Demand

Households consume, work and save in three different nominal assets: an asset A_t (government or bank bonds), in bank deposits D_t or in cash M_t. Each of the three saving instruments pays a different interest rate: cash pays zero interest, deposits pay an interest i_t^d and the asset A pays the policy rate i_t. The policy rate follows a stochastic process and is set by the government, as we will see later. Households maximize

$$U = \Sigma_{t=0}^{\infty} \beta^t E_0[u(c_t, h_t)] \tag{4.1}$$

where c_t is consumption and h_t hours worked. The intra-period utility function is separable in consumption and labor

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - F(h) \tag{4.2}$$

[1] subject to the budget constraint

$$\begin{aligned} & W_t h_t + (1 + i_{t-1})A_{t-1} + (1 + i_{t-1}^d)D_{t-1} + M_{t-1} + P_t \Pi_t \\ = \; & P_t c_t (1 + \chi_t(x_t)) + D_t + A_t + M_t \end{aligned} \tag{4.3}$$

where W_t is the (nominal) wage, Π_t are real profits from firms and banks and P_t is the price level of the consumption good. Households face a proportional transaction cost of consumption χ_t, which depends on the ratio x_t between nominal consumption and *liquidity* l_t, a bundle of cash M_t and bank deposits D_t:

$$x_t \equiv \frac{P_t c_t}{l_t} \tag{4.4}$$

$$l_t \equiv \left(\delta M_t^{\frac{\epsilon-1}{\epsilon}} + D_t^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4.5}$$

[1] In order to embed the model in General Equilibrium in a simple way, I use a linear disutility of labor $F(h) \propto h$. This will be explained in detail in Appendix B1. It has no impact on the demand for deposits that is the focus of this section.

x_t can be interpreted as "liquidity velocity",

As in DSS, δ measures the liquidity of cash relative to deposits and ϵ is the elasticity of substitution between cash and deposits. Appendix A1 contains all the first-order conditions of the household problem.

The choice among the three assets

From the Euler equations with respect to cash and deposits, I obtain that the ratio between cash and deposit holdings is a function of the deposit spread relative to the policy rate, $\frac{i_t - i_t^d}{i_t}$, and is independent of the transaction cost $\chi(x)$

$$\frac{M_t}{D_t} = \left(\delta \ \frac{i_t - i_t^d}{i_t}\right)^{\epsilon} \tag{4.6}$$

This also implies that liquidity can be written as

$$l_t = f D_t \tag{4.7}$$

with

$$f \equiv \left(1 + \delta \left(\delta \frac{i_t - i_t^d}{i_t}\right)^{\epsilon - 1}\right)^{\frac{\epsilon}{\epsilon - 1}} \tag{4.8}$$

From the Euler equations with respect to deposits and the risk-free asset we can obtain the equation

$$f^{\frac{1}{\epsilon}} x_t^2 \chi'(x_t) = \frac{i_t - i_t^d}{1 + i_t} \tag{4.9}$$

which allows us to find the demand for liquidity $l_t = \frac{P_t c_t}{x_t}$ and the demand for deposits $D_t = f^{-1} l_t$ as a function of i_t and i_t^d, for a given level of nominal consumption $P_t c_t$. The meaning of (4.9) is simple: for a given level of consumption, liquidity holdings are chosen so that the marginal benefits in terms of reduction of the transaction cost (LHS) equate the marginal cost, in terms of forgone interest (RHS).

I now specialize the transaction cost to the form used by Schmitt-Grohé and Uribe (2004)

$$\chi(x) = ax + \frac{b}{x} - 2\sqrt{ab} \tag{4.10}$$

This function has a minimum for $x = \sqrt{\frac{b}{a}}$, which represents the satiation level of liquidity.[2] Inserting (4.10) in (4.9) we obtain

$$x = \sqrt{\frac{f^{-\frac{1}{\epsilon}}(i_t - i_t^d) + b(1 + i_t)}{a(1 + i_t)}} \tag{4.11}$$

[2]Taken literally, this tansaction cost increases for liquidity holdings bigger than the satiation level. This has no consequence because liquidity holdings bigger than satiation level will never be chosen.

from which it is easy to obtain the deposit demand

$$D_t = \frac{P_t c_t}{f}\sqrt{\frac{a(1+i_t)}{f^{-\frac{1}{\epsilon}}(i_t - i_t^d) + b(1+i_t)}} \tag{4.12}$$

4.2.3 Deposit Rate Determination

At time t a monopolist bank issues issues deposits D_t and sets the deposit rate i_t^d. The deposit demand as a function of the deposit rate is (4.12). The objective of the bank is to maximize the profits it will realize next period, which are, in real terms

$$\Pi_{t+1} = (i_i - i_t^d)\frac{D_t}{P_{t+1}} \tag{4.13}$$

taking the policy rate i_t, consumption c_t and inflation $\frac{P_{t+1}}{P_t}$ as given. The demand for deposit D_t is given by (4.12). Appendix A2 shows the first-order condition for i_t^d. There I also show that for $i_t << b$ the optimal spread is

$$i_t - i_t^d = \left(\frac{1}{(\epsilon-1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} i_t \tag{4.14}$$

whereas for $i_t >> b$

$$i_t - i_t^d = \left(\frac{1}{2(\epsilon-1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} i_t \tag{4.15}$$

4.2.4 Consumption and Inflation Determination

A full description of the economics of deposits, including the quantity of deposits and the banks' profits on deposit as a function of the policy rate, requires knowledge of aggregate consumption, affecting the demand for deposits and hence the banks' profits on deposits, and inflation, also affecting the profits on deposits.

The full model including government and firms results in a system of three equations: the Euler equation, the Taylor rule, which describes how the government sets the policy rate, and the New-Keynesian Phillips Curve (NKPC), obtained from firms in the "NK sector", plus an equation describing the stochastic process followed by a "monetary shock". The solution of this system gives consumption c_t, the policy rate i_t and inflation π_t as a function of the shock v_t, or, equivalently, consumption and inflation as a function of the policy rate i_t. Details about firms in the NK sector and about the derivation of the NKPC will be given in Appendix B1.

Using the velocity equation (4.11) and the deposit rate $i^d = i^d(i_t)$, obtained as a function of the policy rate as the solution of (4.56), the Euler equation (4.53) can be written in log-linear form around the steady state

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}((1+p)(i_t - r_n) - \mathbb{E}_t[\pi_{t+1}]) + \frac{p}{\sigma}E_t[i_{t+1} - r_n] \tag{4.16}$$

$r_n \equiv \beta^{-1} - 1$ is the steady state value of the policy rate, $\hat{c} \equiv \frac{c_t - c^{SS}}{c^{SS}}$ (c^{SS} is steady-state consumption) and

$$p \equiv \frac{2a\overline{x'}}{(1 + 2a\bar{x} - \sqrt{ab})} \tag{4.17}$$

where $\bar{x}$ is the value of velocity (4.11) at $i_t = r_n$ and $\bar{x'}$ is the derivative of velocity with respect to i_t at $i_t = r_n$.
The policy rate i_t is set according to the Taylor rule

$$i_t = r_n + \phi_\pi \pi_t + \phi_c c_t + v_t \tag{4.18}$$

The monetary shock v_t follows an auto-regressive process

$$v_{t+1} = \rho_v v_t + \epsilon^v_{t+1} \tag{4.19}$$

Finally, the NKPC is

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{c}_t \tag{4.20}$$

The slope of the Phillips curve Λ, as seen in Appendix B1, is related to parameters of the NK firms, in particular to the degree of price stickiness faced by these firms.

The system (4.16), (4.18), (4.19) and (4.20) implies that the inflation rate and consumption (in log-deviation from steady state) are proportional to the policy rate i_t (in deviation from the steady state value r_n):

$$\begin{aligned} \hat{c}_t &= b_c(i_t - r_n) \\ \pi_t &= b_\pi(i_t - r_n) \end{aligned}$$

where

$$\begin{aligned} b_c &= -\frac{1 + p(1-\rho_v)}{\sigma(1-\rho_v) - \rho_v \frac{\Lambda}{1-\beta\rho_v}} \\ b_\pi &= -\frac{1 + p(1-\rho_v)}{\sigma(1-\rho_v)\frac{1-\beta\rho_v}{\Lambda} - \rho_v} \end{aligned}$$

4.2.5 Economics of Deposits: Calibration and Results

The parameters that are relevant for the economics of deposits are shown in Table 1. On the left-hand side of the table we find the parameters related to household preferences and money demand. The parameters a and b appearing in the transaction cost have been calibrated to the US economy by Schmitt-Grohé and Uribe (2004), using quarterly data from 1960 to 2000[3] . The elasticity of substitution between cash and deposits, ϵ_L, and the liquidity of cash relative to deposits δ are such that for a one percent increase in the policy rate, the increase in the deposit rate ranges from 35 bps (when the policy rate is close to zero) to 55 bps (when the policy rate is high). On the right-hand side of the table we find the parameters of the Taylor rule and the New-Keynesian Phillips curve. These are close to the parameters in Gali (1983). The slope of the NKPC, as we see in (4.69) and (4.70), is essentially related to the parameter α of the production function of NK firms, that I take equal to 0 for reasons explained in Appendix B1, and the price stickiness Calvo parameter θ, that I take equal to 0.75, implying that prices are reset once per year. With these parameters, 1% increase in the policy rate results in a decrease in consumption of 0.68% and a decrease in inflation of 0.87%.

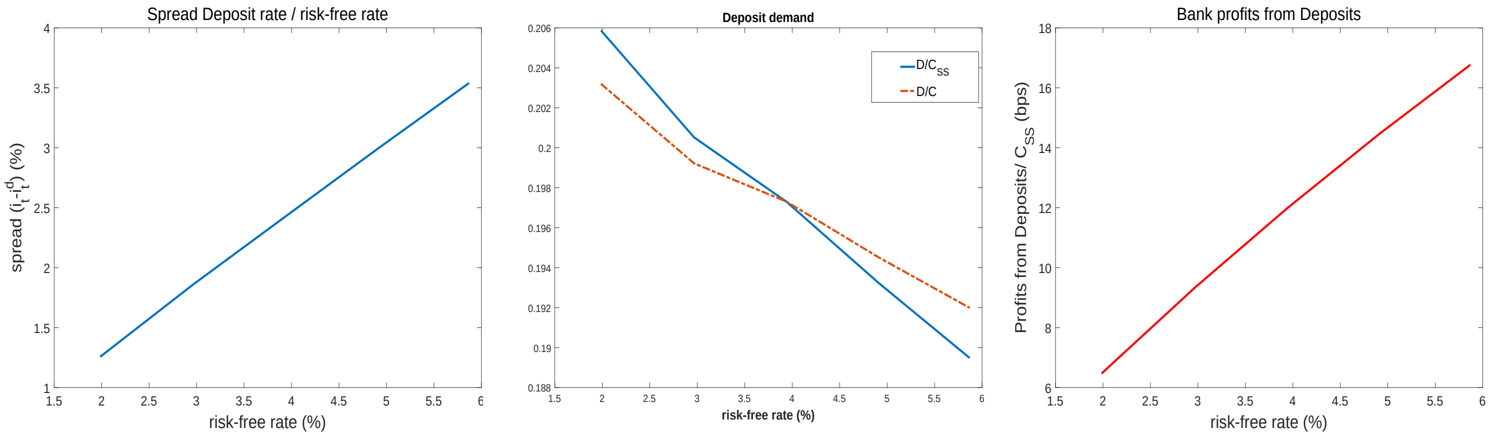
Table 1 Calibration: Economics of Deposits

Household & Money Demand Parameters		Taylor rule and NKPC parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	ρ_v=0.6	persistence of monetary policy shock
$\sigma = 1.5$	elasticity of intertemporal substitution	ϕ_c=0.5/4	Taylor rule parameter
$\delta = 1.05$	liquidity of cash relative to deposits	$\phi_\pi = 1.5$	Taylor rule parameter
$\epsilon_L = 3$	elast. of subst. cash/deposits	$\Lambda = 0.086$	slope of the NKPC
$a = 0.0111 \times 4$	transaction cost parameter		
$b = 0.07524/4$	transaction cost parameter		

The three panels of Figure 1 show the main results related to deposits. The panel on the left shows that the deposit spread, $i - i^d$, grows from 127 bps when $i = 2\%$ to 365 bps when $i = 6\%$. The spread grows approximately linearly in this range, by about 60 bps for every 1% increase in the policy rate. The center panel shows that the quantity of deposits is decreasing in the policy rate. For a 1% increase in the policy rate, the quantity of deposits decreases by 2.86% in this range. It decreases through two channels: first, since the deposit spread increases, households prefer to allocate more of their savings to the asset A, and they lower their holdings of deposits as a fraction of consumption (see the dashed line in the center panel). Second, household consumption decreases after a contractionary shock, so households need to hold less deposits. The solid line in the center panel shows that the quantity of deposits as a fraction of steady state consumption decreases even more. All in all, however, when $i = 6\%$ the quantity of deposits is only 11% smaller than when $i = 2\%$, while the spread increases by almost a factor 3. The right panel shows that profits on deposits at $i = 6\%$ are higher than at $i = 2\%$ by a factor 2.6.

[3]To transform the parameters a and b estimated by Schmitt-Grohé and Uribe (2004), which are appropriate when annual consumption is used, into the corresponding parameters that are appropriate when using quarterly consumption, I multiply their value of a by 4 and divide their value of b by 4.

Figure 1: Deposits



4.3 The Banks' Problem

In this section I outline the decision problem of the bank. There is a continuum of banks of measure 1, each of which borrows in the form of deposits and bonds and extends loans. The key friction is that banks cannot raise equity on the market, they can only build equity by retaining profits. Moreover, banks need to satisfy a minimum equity requirement, as described below.

Some elements of the structure of the bank's problem, notably the financial friction and the equity requirement, are similar to vdH. The objective of vdH was to evaluate the impact of the friction by comparing the lending behavior of the firm facing the friction to the behavior of the "unconstrained bank", i.e. the bank that can raise equity on the market or that holds enough equity that the constraint does not currently bind and will never bind in the future. In contrast, the main purpose of this paper is to evaluate the impact of deposits on the lending behavior of the constrained bank. I will compare the lending behavior of the constrained bank that optimally chooses deposits to that of the constrained bank that borrows fully in bonds, against the benchmark of the unconstrained bank. The latter is able to exploit all profitable opportunities and its lending is not affected by deposits.

I assume that each bank is a monopolist in the deposit market, and is in monopolistic competition with other banks in the loan market. We can imagine for example that each bank operates in a county, and regulation (or high transportation costs) prevent households in one county from holding deposits in another county. Firms however are allowed to take loans from banks outside their county. A bank's balance sheet is

Assets	Liabilities
$P_t L_t$	D_t
	B_t
	$P_t E_t$

The following subsections describe in detail the components of the balance sheet.

4.3.1 Loans

Loans are long-term and risky real assets. A unit of loan granted at time t (i.e. one unit of good transferred from the bank to a firm) demands the following payments from the borrower in real terms:

$$\begin{aligned}
(\bar{\delta} + \rho^{(t)}) &\qquad \text{at} \quad t+1 \\
(\bar{\delta} + \rho^{(t)})(1-\bar{\delta}) &\qquad \text{at} \quad t+2 \\
(\bar{\delta} + \rho^{(t)})(1-\bar{\delta})^2 &\qquad \text{at} \quad t+3 \\
\dots\dots & \\
(\bar{\delta} + \rho^{(t)})(1-\bar{\delta})^{n-1} &\qquad \text{at} \quad t+n
\end{aligned}$$

In other words, every period the borrower is required to repay a fraction $\bar{\delta}$ of the outstanding principal, and a real interest rate $\rho^{(t)}$, where the superscript indicates that the (fixed) rate has been decided at time t. The contractual loan duration is therefore $\frac{1}{\bar{\delta}}$. In nominal terms, a borrower receiving $P_t L$ at time t is required to pay $P_s(\bar{\delta} + \rho^{(t)})(1-\bar{\delta})^{s-t} L$ in each subsequent period s.

However, loans carry default risk: a (stochastic) fraction ω_t of the outstanding loans defaults at each time t. Hence, a unit of loan granted at time t results in the following actual payments

$$\begin{aligned}
(\bar{\delta} + \rho^{(t)})(1-\omega_{t+1}) &\qquad \text{at} \quad t+1 \\
(\bar{\delta} + \rho^{(t)})(1-\omega_{t+2})(1-\bar{\delta}-\omega_{t+1}) &\qquad \text{at} \quad t+2 \\
(\bar{\delta} + \rho^{(t)})(1-\omega_{t+3})(1-\bar{\delta}-\omega_{t+2})(1-\bar{\delta}-\omega_{t+1}) &\qquad \text{at} \quad t+3 \\
\dots\dots & \\
(\bar{\delta} + \rho^{(t)})(1-\omega_{t+n})(1-\bar{\delta}-\omega_{t+n-1})\dots(1-\bar{\delta}-\omega_{t+2})(1-\delta-\omega_{t+1}) &\qquad \text{at} \quad t+n
\end{aligned} \tag{4.21}$$

I take the shocks ω to be i.i.d (and independent of the other shock in the economy, the monetary shock).

To facilitate aggregation between loans issued in different periods, I define *standardized loans*, with constant interest rate $\bar{\rho}$. A unit of loan granted at time t at interest ρ_t is equivalent to $\frac{\bar{\delta}+\rho^{(t)}}{\bar{\delta}+\bar{\rho}}$

standardized loans. Equivalently, we can think of a standardized loan as a security bought by the bank at time at time t at price

$$\mathcal{P}_t = \frac{\bar{\bar{\delta}} + \bar{\rho}}{\bar{\bar{\delta}} + \rho^{(t)}} \tag{4.22}$$

I call L_t the outstanding standardized loans at time t (a state variable), and N_t the number of new standardized loans granted at time t (a decision variable). In the following, I will always refer to standardized loans simply as “loans”.

Loans evolve according to

$$L_{t+1} = (1 - \bar{\bar{\delta}} - \omega_{t+1})(L_t + N_t) \tag{4.23}$$

Loan demand comes from firms in the *BD sector*. These firms view loans from different banks are imperfect substitutes, which implies that banks are in monopolistic competition in the loan market. The production function of firms in the BD sector, described in detail in Appendix B2, generates the loan demand curve

$$N_t^i = (\zeta\nu)^{\frac{1}{1-\nu}} (\bar{\rho} + \bar{\bar{\delta}})^{-\frac{1}{1-\nu}} \left(\mathcal{P}^{(t)i}\right)^{\epsilon_B - 1} \left(\mathcal{P}^{(t)M}\right)^{\frac{1}{1-\nu} - \epsilon_B} \tag{4.24}$$

Here $\mathcal{P}^{(t)i}$ is the loan price obtained by bank i at time t, ζ and ν are parameters of the BD firms production function, $\epsilon_B > 0$ is the elasticity of substitution of loans from different banks, and $\mathcal{P}^{(t)M}$ is the “market loan price” at time t (details in Appendix B2). The demand for loans of bank i is thus upward sloping in the loan price paid by the bank.

4.3.2 Liabilities: Deposits and Bonds

Deposits D_t are one-period securities issued at time t, yielding an interest i_t^d chosen by the bank at time t. The downward sloping demand curve (taking households consumption as given) is given by (4.12).

Bonds B_t are one-period securities yielding the policy rate i_t, set by the government.[4]

4.3.3 Equity

At the beginning of period t, before the bank makes new loan, deposit, and dividend decisions, the bank's equity is equal to the value of the outstanding loans, plus the cashflows coming from assets and liabilities

$$E_t = V_t^L L_t + CF_t \tag{4.25}$$

[4] I assume that the bank ends its activity when $E_t < 0$, and the bonds are guaranteed by the government. This guarantees that the bank can borrow at the risk-free rate.

All terms in (4.25) are in real terms. I take the value of a loan, used by the regulator to compute the accounting vaue of the bank's equity, to be equal to the price that an unconstrained bank would pay for an identical loan

$$V_t^L = \frac{\bar{\delta} + \bar{\rho}}{\bar{\delta} + \rho_t^U} \tag{4.26}$$

Although in this model there is no secondary market for loans, (4.26) is in the spirit of "mark-to-market accounting", which is the current accounting standard.[5] The unconstrained loan rate ρ_t^U is obtained by solving the problem of an unconstrained bank. As we will see, ρ_t^U is an (increasing) function depending only on i_t. V_t^L is therefore a decreasing function of i_t.

Cashflows CF_t include cashflows from assets at time t, i.e. coupons and principal repayments, the repayment of liabilities, taxes T_t and fixed costs $c_F < 0$

$$CF_t = (\bar{\delta} + \bar{\rho})(1 - \omega_t)(L_{t-1} + N_{t-1}) - (1 + i_{t-1})\frac{B_{t-1}}{P_t} - (1 + i_{t-1}^d)\frac{D_{t-1}}{P_t} + c_F - \frac{T_t}{P_t} \tag{4.27}$$

where B_{t-1} and D_{t-1} are the bonds and deposits issued at time $t-1$, due at t. Taxes T_t are a fraction τ of the taxable base TB_t

$$T_t = \tau \times TB_t = \tau \times [P_t(\bar{\rho}(1-\omega_t)(L_{t-1}+N_{t-1}) + c_F - \omega_t(L_{t-1}+N_{t-1})) - i_{t-1}B_{t-1} - i_{t-1}^d D_{t-1}] \tag{4.28}$$

The bank then makes a decision about the dividends Div_t to distribute in period t, the new loans N_t to purchase at price $\mathcal{P}_t(N_t)$ and the liabilities B_t and D_t. In order to honor the previous-period liabilities and carry out the decision about Div_t and N_t, B_t and D_t need to satisfy

$$\begin{aligned} D_t + B_t &= P_t(Div_t + \mathcal{P}_t(N_t)N_t - CF_t) \\ &= P_t(Div_t + \mathcal{P}_t(N_t)N_t - E_t + V_t^L L_t) \end{aligned} \tag{4.29}$$

so that at the end of the period the bank's equity, equal to the value of the asets minus the liabilities, can be written as

$$E_t' = V_t^L(L_t + N_t) - B_t - D_t = E_t - Div_t + (V_t^L - \mathcal{P}_t(N_t))N_t \tag{4.30}$$

Interestingly, (4.30) shows that reducing N_t is a way to immediately rebuild equity: since $\mathcal{P}_t(N_t)$ is upward sloping, by reducing the supply of new loans the bank can purchase the new loans at a price which is lower than their accounting value V_t^L.

4.3.4 Equity Requirement

The equity requirement constraint can be expressed in two statements

[5] The Financial Accounting Standard 157 (FAS157) introduced by the Financial Accounting Standard Board (FASB) in 2006, established that assets should be valued in agreement with their "exit price", replacing the previously used "historical-cost" accounting.

- If, at the beginning of period t, it is $E_t < \gamma V_t^L L_t$, then it must be $N_t = Div_t = 0$.
- Otherwise, N_t and Div_t must be such that at the end of the period it is $E'_t \geq \gamma V_t^L (L_t + N_t)$.

4.3.5 Bank's objective

The bank is a risk-neutral entity whose value function is given by the present discounted value of future dividends. It can be written in recursive form

$$V_t(E_t, L_t, i_t) = max_{\{N_t, Div_t, D_t\}} Div_t + \frac{1}{1+i_t}\mathbb{E}_t[V_{t+1}(E_{t+1}, L_{t+1}, i_{t+1})] \tag{4.31}$$

subject to

- The laws of motion of loans (4.23);
- The law of motion of equity

  $$E_{t+1} = V_{t+1}^L(i_{t+1})(1 - \delta - \omega_{t+1})(L_t + N_t) + CF_{t+1} \tag{4.32}$$

  with

  $$\begin{aligned} CF_{t+1} &= (\bar{\delta} + (1-\tau)\bar{\rho})(1 - \omega_{t+1})(L_t + N_t) - (1 + (1-\tau)i_t)\frac{B_t + D_t}{P_{t+1}} \\ &+ (1-\tau)(i_t - i_t^d)\frac{D_t}{P_{t+1}} + (1-\tau)c_F + \tau\omega_{t+1}(L_t + N_t) \end{aligned} \tag{4.33}$$

  As shown in (4.29) the sum $B_t + D_t$ can be written in terms of the decision variables Div_t and N_t. Hence time-$t+1$-equity depends on time-t state variables, time-t decision variables (Div_t, N_t and D_t) time-$t+1$ shocks.
- The deposit demand function (4.12), which also depends on aggregate consumption c_t, an endogenous macroeconomic variable;
- The equity requirement constraint;
- The loan demand function (4.24).

Notice that the decision about deposits D_t (deposits) only affects the bank through its contribution to next period's cashflows (4.33) and hence equity (4.32). Since $B_t + D_t$, as already pointed out, can be expressed in terms of the decision variables N_t and Div_t, deposits affect the bank's problem only through the contribution to next period's cashflows

$$(1-\tau)(i_t - i_t^d)\frac{D_t}{P_{t+1}} \tag{4.34}$$

This shows that despite the complexity of the bank's decision problem, the choice about deposits is a static one, and the deposit rate (and hence deposit quantity) chosen by the bank are, for each value of the policy rate, those obtained in section 2.

Notice also that, with perfect competition in the lending market, and given banks' risk neutrality, only unconstrained banks would lend. Constrained banks would have no incentive to lend, given that they would make no profits in expectation, and that they would risk violating the equity requirement, with the consequence of not being able to distribute dividends.

4.3.6 The Unconstrained Value Function

The value function and policy functions of the unconstrained bank are the benchmark against which we can compare the policy functions of the constrained bank. As in vdH, the value function of the unconstrained bank has the form

$$V(E_t, L_t, i_t) = a_0(i_t) + E_t + a_L(i_t)L_t \tag{4.35}$$

which can be verified by inserting (4.35) in (4.31), and maximizing with respect to N_t. By doing this, we can find the functional form of $a_0(i_t)$ and $a_L(i_t)$, and the policy function $N(i_t)$, which is only a function of i_t. Equations for these quantities can be found in Appendix A3.

Finally, for the unconstrained bank, it is always the case that

$$Div_t = E_t - \gamma V_t^L(L_t + N_t) + (V_t^L - \mathcal{P}_t)N_t \tag{4.36}$$

so that it is $E_t' = \gamma V_t^L(L_t + N_t)$. There is no point in keeping equity in the bank in excess of the equity requirement. Equity can be raised without frictions next period, and this is more attractive than keeping profits in the firm due to taxes.

4.4 Calibration

I take one period to be a quarter and normalize consumption to be 10 each quarter in steady state, i.e. when the nominal interest rate is equal to the natural rate, taken to be equal to 4% in annual terms. The parameter values are summarized in Table 2. The (discrete) shock distributions are summarized in Table 3. I discretize the interest rate process i_t: I use the 5 values $[2\%, 3\%, 4\%, 5\%, 6\%]$. The transition matrix for the nominal interest rate is consistent with the solution of the system (4.16), (4.18), (4.20), namely it is $\mathbb{E}_t[i_{t+1} - r_n] = \rho_v(i_t - r_n)$, and corresponds to a volatility of the monetary shock ϵ^v of about 0.57%.

The mean of ω (annualized) is 1.3%, close to the historic US average for commercial, industrial and consumer loans. The value of the repayment rate $\bar{\delta}$, 0.25% annualized, implies a loan maturity of 4 years.

Without deposits the fixed cost c_F is calibrated so that the average "market-to-book" ratio of the bank's equity $q \equiv V/E$, where V is the bank's value function, is 1.15, close to an average value for

Table 2: Bank & BD sector parameters

Parameter	Description
$\gamma = 0.08$	equity requirement
$\tau = 0.35$	corporate tax
$\epsilon_B = 30$	elast.of substit. bank loans
$\nu = 0.33$	capital share BD sector
$\bar{\rho} = 0.08$	stand. loans coupon rate (annual)
$\zeta = 0.22$	aggregate productivity BD sector
$c_F^{dep} = -0.046$	fixed cost with deposits
$c_F^{no\ dep} = -0.119$	fixed cost without deposits

banks in the US.[6] In the case with deposits, I add an extra fixed cost equal to the average value of the bank's profits on deposits. I choose to do this because I want to isolate the effect of the countercyclical nature of the bank's profits on deposits, rather than their average value. The additional fixed cost could be interpreted either as an operating cost of managing deposits or as a tax.

Table 3: Shocks

Shock	Values (Annualized)	Probabilities
ω	$[-0.0052,\ 0.0016,\ 0.0168,\ 0.04]$	$[0.25,\ 0.25,\ 0.25,\ 0.25]$
Variable	Values	Transition Matrix
i	$[0.02,\ 0.03,\ 0.04,\ 0.05,\ 0.06]$	$\begin{bmatrix} 0.3 & 0.6 & 0.1 & 0 & 0 \\ 0.05 & 0.5 & 0.45 & 0 & 0 \\ 0 & 0.16 & 0.68 & 0.16 & 0 \\ 0 & 0 & 0.45 & 0.5 & 0.05 \\ 0 & 0 & 0.1 & 0.6 & 0.3 \end{bmatrix}$

[6] For example https://www.investopedia.com/ask/answers/040815/what-average-pricetobook-ratio-bank.asp reports that, as January 2015, the average market-to-book ratio for US banks was 1.1.

4.5 Results

4.5.1 The Unconstrained Bank

The numerical value of the value function and policy function N^U (where the superscript emphasizes that it is the policy function for the unconstrained bank) are in Table 4, for the case in which the bank optimally chooses deposits, and in the case in which deposits are 0. Analytic formulas for a_0 and a_L, as well as for the maximization problem that determines N^U, can be found in Appendix A3. Notice that deposits affect the term a_0 of the value function, but not the policy function N^U. The intuition is that the ability to raise equity on the market allows the bank to undertake all profitable lending opportunities, regardless of the profits it makes in its deposit activity. Table 4 also shows the loan rate ρ^U of the unconstrained bank, related to N^U by (4.75).

Table 4: Unconstrained Bank

With Deposits	$i = 2\%$	$i = 3\%$	$i = 4\%$	$i = 5\%$	$i = 6\%$
$N^U(i)$	0.879	0.874	0.869	0.863	0.858
$a_0(i)$	-0.0423	-0.0158	0.0093	0.0331	0.0557
$a_L(i)$	0.0342	0.0340	0.0339	0.0338	0.0336
$\rho^U(i)$	6.97%	7.10%	7.23%	7.36%	7.48%
Without Deposits	$i = 2\%$	$i = 3\%$	$i = 4\%$	$i = 5\%$	$i = 6\%$
$a_0(i)$	0.0102	0.0096	0.0090	0.0084	0.0078

4.5.2 The Constrained Bank: Moments

The bank's problem in the constrained case can be solved with value function iteration methods. As also found by vdH, the value function is highly non-linear, epecially when equity is close to the equity requirement value $E = \gamma V^L L$, which implies that linearization techniques are unsuitable for this problem. I discretize the state variable L and E, in addition to i as discussed in the calibration section, making the grid denser in the region of high non-linearity.

The main moments from the simulation of the solved model are shown in Table 5. The left side of the table shows the moments for the problem without deposits (in which it is simply $D_t = 0$ for every t). The left side shows the moments for the full problem with deposits. With deposits, the average number of new loans N and the stock of all outstanding loans L increase , and their standard deviations decrease. Equity as a fraction of assets, both at the beginning and at the end of the period, decrease, but their standard deviations decrease. All autocorrelations decrease, reflecting a higher tendency of the main variables to mean-revert. However, the effect of deposits seems very modest if we just look at these moments.

The effect of deposits is however quite sizeable in periods of high interest rates and when equity is low. Notice that, with the interest rate transition matrix shown in Table 2, which is consistent with a persistence of the monetary policy shock $\rho_v = 0.6$, the two extreme values of interest rate (2% and 6%) occur with a probability of only 1.5% each, so even if deposits have a big effect on lending for $i = 6\%$, this has low impact on averages.

Table 5: Simulated Moments

Without Deposits				With Deposits			
variable	mean	std	autocorr	variable	mean	std	autocorr
E (frac of L)	0.260	0.0581	0.64	E (frac of L)	0.2531	0.053	0.57
L	11.80	0.70	0.98	L	11.90	0.57	0.96
N	0.83	0.083	0.62	N	0.84	0.070	0.48
E' (frac of L)	0.179	0.036	0.66	E' (frac of L)	0.171	0.035	0.58

Table 6: New Loans by Interest Rate

i	Probability	N^U	N without Deposits	N with Deposits
2%	1.5%	0.8788	0.8762	0.8716
3%	20%	0.8736	0.8658	0.8602
4%	57%	0.8684	0.8413	0.8434
5%	20%	0.8633	0.7815	0.8068
6%	1.5%	0.8583	0.6496	0.7163

Figure 2: New Loans

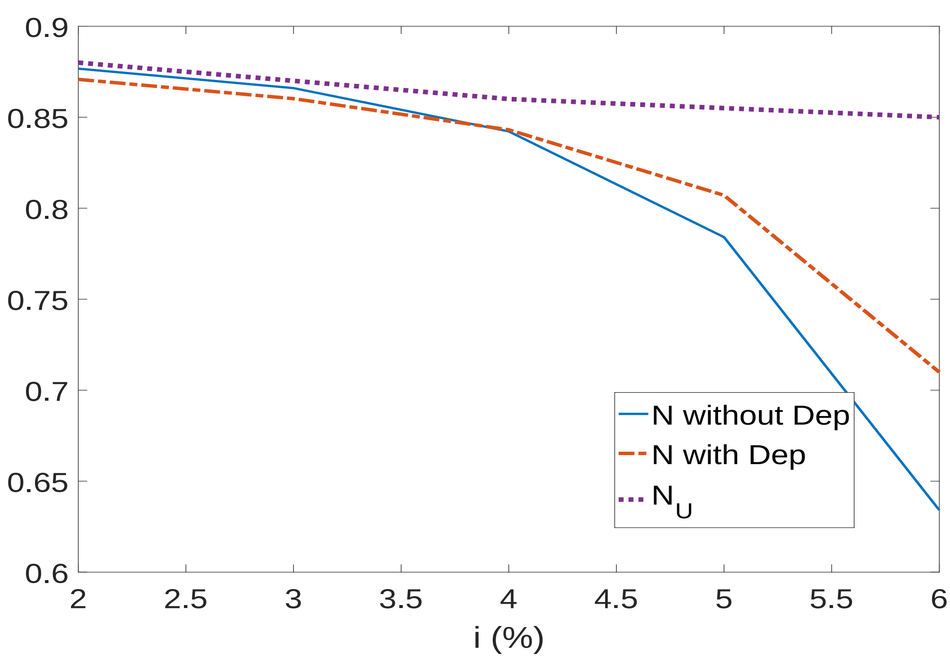


Table 6 and Figure 2 show average new lending for each interest rate value, comparing the unconstrained lending N^U with the constrained lending with and without deposits. We see that, lending above 4% and especially at 6% is dramatically affected by the friction on equity. At 5%, if deposits are 0 new lending is about 9.5% lower than in the unconstrained case, and with deposits the reduction in new loans is only 6.5%. At 6%, if deposits are 0 new lending is about 24.5% lower than in the unconstrained case, and with deposits the reduction in new loans is 16.5%.

4.5.3 The Constrained Bank Problem: Impulse Response Functions at Low Equity

To see the effect of deposits for low equity, I look at impulse-response functions after a contractionary monetary policy shock, using a value of beginning-of-period equity before the shock equal to 15.5% of assets. This low value of beginning-of-period equity occurs about 4% of the time. If the contractionary shock occurs at $t = \bar{t}$, the impulse-response function for new lending is defined as

$$IRF_{\bar{t}}^{N} = E_{\bar{t}}[N_s | i_{\bar{t}} = i_0 + 0.01] - E_{\bar{t}}[N_s | i_{\bar{t}} = i_0], \quad for\ s \geq \bar{t} \tag{4.37}$$

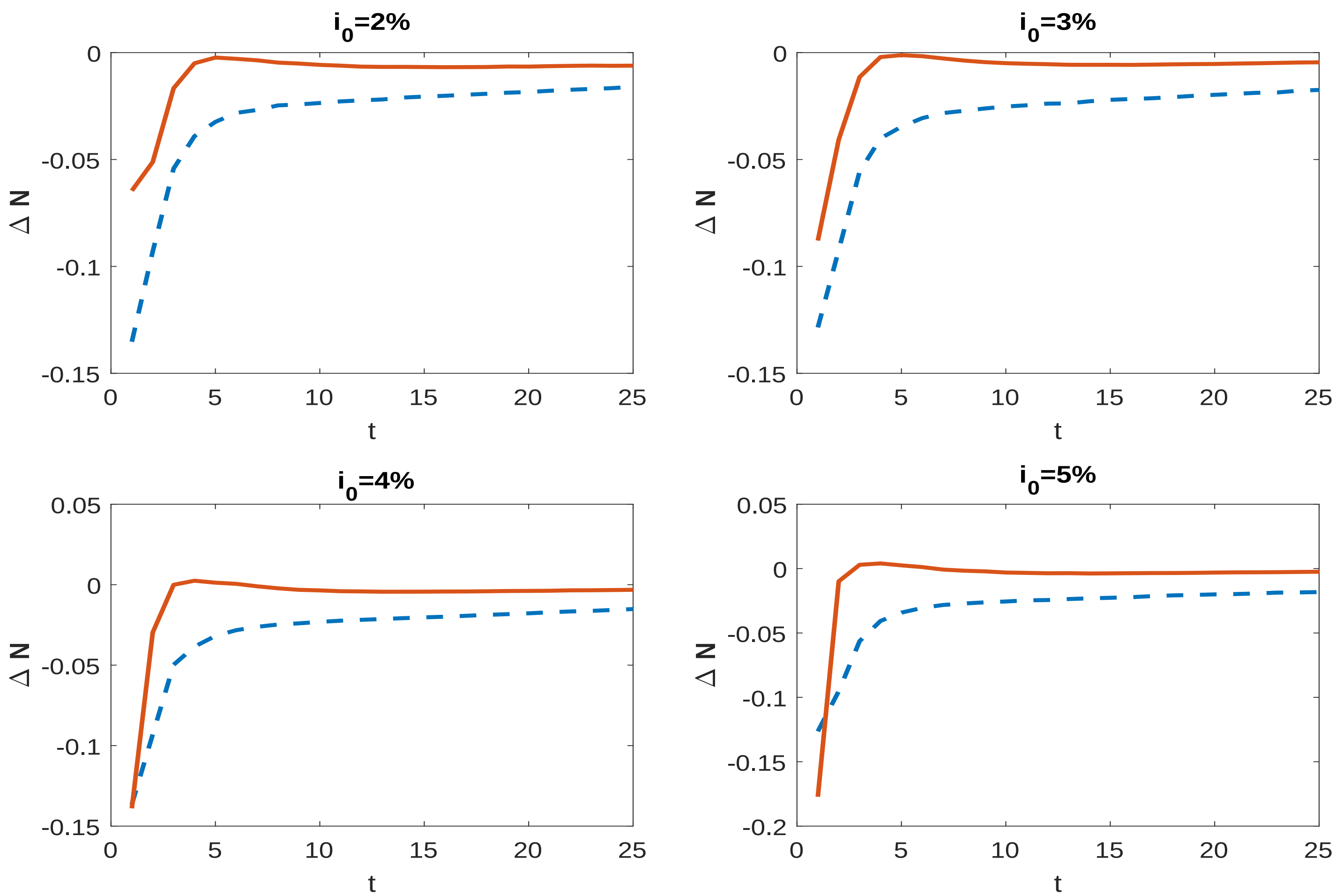
Notice that the non-linearity of the model implies that the impulse response functions cannot be calculated by setting all shocks after the initial one to 0, rather they must be obtained by averaging over all possible future paths.

Figure 3 plots the impulse-response functions (4.37) for $i_0 = [2\%, 3\%, 4\%, 5\%]$, in the model with deposits (solid line) and without deposits (dashed line). For all values of the interest rate we notice how the shock has much more persistent effects without deposits. Two effects contribute to this. First, in the case with deposits, the increased profits on deposits after a contractionary shock immediately contribute to rebuilding equity. Second, in the absence of deposits the bank starts cutting new lending also as a way to rebuild equity quickly. However, this means foregoing more profitable lending opportunities, with the effect of slowing down the rebuilding of equity in later periods.

4.6 Conclusion

The results in this paper show that, thanks to their money-like properties, deposits are a special form of financing. Households want to hold them in spite of the low interest rate they yield because they provide transaction services. As the policy rate increases, households prefer to hold deposits than cash, which gives 0 interest. Hence banks optimally decide to increase the spread between the deposit rate and the policy rate. As a result, banks are less exposed to interest rate

Figure 3: IRFs



swings when they finance themselves at least in part through deposits.[7] This makes them more willing to extend long-term loans. This result contradicts the commonly held view, inspired by Romer and Romer (1990), that deposits and any shocks to them do not affect the supply of credit, if the bank can easily switch to alternative forms of financing.

[7]This was also remarked by Drechsler, Savov and Schnabl (2017b).

Appendix A

A1: Household FOCs and asset allocation choice

FOC with respect to c_t

$$c_t^{-\sigma} = \lambda_t P_t(1 + \chi(x_t) + x_t \chi'(x_t))) \tag{4.38}$$

which becomes, specialized to the transaction cost (4.10)

$$c_t^{-\sigma} = \lambda_t P_t(1 + 2ax_t - 2\sqrt{ab}) \tag{4.39}$$

FOC with respect to h_t

$$F'(h_t) = \lambda_t W_t \tag{4.40}$$

FOC with respect to A_t

$$\lambda_t = \beta \lambda_{t+1}(1 + i_t) \tag{4.41}$$

FOC with respect to M_t

$$\lambda_t \left(1 - x_t^2 \chi'(x_t) \frac{\partial l_t}{\partial M_t}\right) = \beta \lambda_{t+1} \tag{4.42}$$

which becomes, specialized to the transaction cost (4.10)

$$\lambda_t \left(1 - (ax_t^2 - b) \frac{\partial l_t}{\partial M_t}\right) = \beta \lambda_{t+1} \tag{4.43}$$

Using the definition of liquidity (4.5), (4.42) can be written as

$$\lambda_t \left(1 - x_t^2 \chi'(x_t) \delta \left(\frac{M_t}{l_t}\right)^{-\frac{1}{\epsilon}}\right) = \beta \lambda_{t+1} \tag{4.44}$$

FOC with respect to D_t

$$\lambda_t \left(1 - x_t^2 \chi'(x_t) \frac{\partial l_t}{\partial D_t}\right) = \beta \lambda_{t+1}(1 + i_t^d) \tag{4.45}$$

which becomes, specialized to the transaction cost (4.10)

$$\lambda_t \left(1 - (ax_t^2 - b) \frac{\partial l_t}{\partial D_t}\right) = \beta \lambda_{t+1}(1 + i_t^d) \tag{4.46}$$

Using (4.5), (4.45) can be written as

$$\lambda_t \left(1 - x_t^2 \chi'(x_t) \left(\frac{D_t}{l_t}\right)^{-\frac{1}{\epsilon}}\right) = \beta \lambda_{t+1}(1 + i_t^d) \tag{4.47}$$

Combining (4.41), (4.44) and (4.47) we get

$$\frac{1}{\delta} \left(\frac{D_t}{M_t}\right)^{-\frac{1}{\epsilon}} = \frac{i_t - i_t^d}{i_t} \equiv s_t \tag{4.48}$$

so

$$M_t = (s_t\delta)^\epsilon D_t \tag{4.49}$$

Hence

$$l_t = D_t(1+\delta(\delta s_t)^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}} = D_t f_t \tag{4.50}$$

with $f_t \equiv (1+\delta(\delta s_t)^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}}$. Combining (4.41), (4.47) and (4.50) we get

$$f^{\frac{1}{\epsilon}} x_t^2 \chi'(x_t) = \frac{i_t - i_t^d}{1+i_t} \tag{4.51}$$

which implies, with transaction cost (4.10),

$$D_t = \frac{P_t c_t}{f_t}\sqrt{\frac{a(1+i_t)}{f^{-\frac{1}{\epsilon}}(i_t - i_t^d) + b(1+i_t)}} \tag{4.52}$$

Finally, the three Euler equations, with respect to A, D and M, respectively, are

$$\frac{c_t^{-\sigma}}{P_t(1+2ax_t - 2\sqrt{ab})} = \beta(1+i_t)E_t\left[\frac{c_{t+1}^{-\sigma}}{P_{t+1}(1+2ax_{t+1} - 2\sqrt{ab})}\right] \tag{4.53}$$

$$\frac{c_t^{-\sigma}(1-f^{\frac{1}{\epsilon}}(ax_t^2 - b))}{P_t(1+2ax_t - 2\sqrt{ab})} = \beta(1+i_t^d)E_t\left[\frac{c_{t+1}^{-\sigma}}{P_{t+1}(1+2ax_{t+1} - 2\sqrt{ab})}\right] \tag{4.54}$$

and

$$\frac{c_t^{-\sigma}(1-\delta((\delta s_t)^{1-\epsilon}+\delta)^{\frac{1}{\epsilon-1}}(ax_t^2 - b))}{P_t(1+2ax_t - 2\sqrt{ab})} = \beta E_t\left[\frac{c_{t+1}^{-\sigma}}{P_{t+1}(1+2ax_{t+1} - 2\sqrt{ab})}\right] \tag{4.55}$$

A2: The choice of the deposit rate

The monopolist bank maximizes (4.13) subject to the deposit demand (4.12). The first order condition, written in terms of the relative spread $s_t \equiv \frac{i_t - i_t^d}{i_t}$ is

$$\left(1 - \frac{\epsilon\delta^\epsilon s_t^{\epsilon-1}}{1+\delta^\epsilon s_t^{\epsilon-1}}\right) + \frac{i_t}{2((1+\delta^\epsilon s_t^{\epsilon-1})^{-\frac{1}{\epsilon-1}} s_t i_t + b(1+i_t))}\left(\frac{\delta^\epsilon s_t^\epsilon}{(1+\delta^\epsilon s_t^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}}} - \frac{s_t}{(1+\delta^\epsilon s_t^{\epsilon-1})^{\frac{1}{\epsilon-1}}}\right) = 0 \tag{4.56}$$

This condition implicitly defines s_t (or i_t^d) as a function of i_t. For small i_t, meaning $i_t << b$ we can neglect the second term on the LHS of (4.56), which can then be written as

$$\left(1 - \frac{\epsilon\delta^\epsilon s_t^{\epsilon-1}}{1+\delta^\epsilon s_t^{\epsilon-1}}\right) = 0 \tag{4.57}$$

Hence, for $i_t << b$ it is

$$s_t = \frac{i_t - i_t^d}{i_t} = \left(\frac{1}{(\epsilon-1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} \tag{4.58}$$

For high i_t, meaning $i_t >> b$ then (4.56) can be approximated as

$$\left(1 - \frac{\epsilon\delta^\epsilon s_t^{\epsilon-1}}{1+\delta^\epsilon s_t^{\epsilon-1}}\right) + \frac{1}{2}\left(\frac{\delta^\epsilon s_t^{\epsilon-1}}{(1+\delta^\epsilon s_t^{\epsilon-1})} - 1\right) = 0 \tag{4.59}$$

wich implies

$$s_t = \frac{i_t - i_t^d}{i_t} = \left(\frac{1}{2(\epsilon-1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} \tag{4.60}$$

A3: The Unconstrained Value Function

Inserting (4.35) in (4.31), and using the law of motion of loans (4.23), the law of motion of equity (4.32) and the choice of dividends of the unconstrained bank (4.36), I find that the vector a_L (each component of which correspond to a value of i) satisfies

$$\begin{aligned} a_L &= (1 - DF \times M)^{-1}\left[-\gamma V_L + (1-\bar{\delta}-\bar{\omega})DF \times M \times V_L + (\bar{\delta}+\tau\bar{\omega})DF_v\right. \\ &+ \left.(1-\tau)(1-\bar{\omega})\bar{\rho}DF_v - (1-\gamma)DF \times (V_L + (1-\tau)(V_L * i))\right] \end{aligned} \tag{4.61}$$

where DF is the square diagonal matrix $n \times n$ (if the vector i has n elements) with the n discount factors $\frac{1}{1+i}$ on the diagonal, DF_v is an n-component vector equal to the diagonal of DF, M is the transition matrix for the risk-free rate i (see Table 3), $\bar{\omega}$ is the average value of the shock ω, V_L is the n-component vector of loan values (see (4.26)) for each value of i. I denote by $a * b$ the element-by element product of two vectors a and b, so that for example $V_L * i$ is the element-by-element product of V_L and i.

N^U is the n-component vector that maximizes

$$\begin{aligned} &N^U * [(1-\gamma)V_L + (1-\bar{\delta}-\bar{\omega})DF \times M \times a_L + (1-\bar{\delta}-\bar{\omega})DF \times M \times V_L \\ &-(1-\gamma)DF \times ((1-(1-\tau)i) * V_L) + ((1-\tau)(1-\bar{\omega})\bar{\rho} + \bar{\delta} + \tau\bar{\omega})M \times (1+\pi)] \\ &-\mathcal{P}^U N^U \end{aligned} \tag{4.62}$$

where π is the n-component inflation vector and $\mathcal{P}^U$ is the n-component price vector. Finally

$$\begin{aligned} a_0 &= (1 - DF \times M)^{-1}[\tau DF \times (V_L * i * N^U) - \mathcal{P}^U * N^U \\ &+ (1-\bar{\delta}-\bar{\omega})(DF \times M \times a_L) * N^U + DF \times ((1-\tau)(i-i^d) * D \\ &+ \bar{\delta} * N^U + (1-\bar{\omega})\bar{\rho}N^U + (1-\tau)c^F + \tau\bar{\omega}N^U)] \end{aligned} \tag{4.63}$$

Notice that a_0 is the only quantity which depends on deposits D.

Appendix B: The Macroeconomic Environment

There are two firm sectors in the economy, corresponding to two technologies for the production of the consumption good. One sector, the *NK sector*, is typical of a standard New-Keynesian model and generates the NKPC. Notice that NK firms use only labor as factor of production and do not need to borrow (either from households or from banks). The other sector, the *BD sector* is comprised of firms working on long-term projects, that need to borrow from banks. BD firms therefore generate loan demand.

B1: The NK sector

As in the basic NK model (e.g. Gali(2008)), there are intermediate good producers and final good producers. Intermediate good producers are a continuum of firms indexed by $i \in [0, 1]$. Each firm i produces a different variety of intermediate good, using only labor as input, according to the production function

$$Y_t(i) = aN_t(i)^{1-\alpha} \tag{4.64}$$

where a is a represents the (constant) level of technology and N_t is labor at time t. I take $\alpha = 0$. This choice, together with the choice of linear labor disutility (or Frisch elasticity $\phi = 0$) results in a New-Keynesian Phillips curve that is independent of the banking sector, as shown below.

Each firm produces a differentiated good and is a price setter for that good. However, following Calvo (1983), each firm is able to reset its price only with probability $1 - \theta$ in any given period. This each period a fraction θ of firms keeps their price unchanged.

Final good producers are perfectly competitive firms taking the different varieties of intermediate product as input. The production function for the final good is

$$Y_t^{NK} = \left(\int di \ \ y_{iy}^{\frac{\epsilon_G - 1}{\epsilon_G}} \right)^{\frac{\epsilon_G}{\epsilon_G - 1}} \tag{4.65}$$

where the superscript indicates that this is the final production of the NK sector and ϵ_G is the elasticity of substitution between different varieties of intermediate goods. The final good producer's problem is

$$max_{Y_t, \ y_{j,t}, \ i \in [0,1]} P_t Y_t - \int_0^1 di \ p_{it} y_{it} \tag{4.66}$$

where y_{jt} is the demanded quantity of the intermediate good j. From the first-order conditions, the demand for the intermediate good of variety i is

$$y_{it} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon_G} Y_t \tag{4.67}$$

Moreover, from the zero-profit condition, the price for the final good is given by $P_t = \left(\int_0^1 p_{it}^{1-\epsilon_G} di \right)^{\frac{1}{1-\epsilon_G}}$

As in Gali (2008), the fraction $1 - \theta$ of intermediate-good producers who can reset their price at time t need to solve the intertemporal problem of choosing the price that maximizes the present value of profits from the current period to the next period they will be able to reset their price, discounted with the household discount factor $Q_{t,t+k} = \beta^k \lambda_{t+k}/\lambda_t$. Gali (2008) shows that this problem leads to the New-Keynesian Phillips curve, which in its log-linear form reads

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{mc}_t \tag{4.68}$$

where $\hat{mc}$ is the log-deviation of the marginal cost from steady state, and

$$\Lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta \tag{4.69}$$

$$\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon_G} \tag{4.70}$$

Again, Gali (2008) shows that marginal cost can be written as

$$mc_t = \sigma c_t + (\phi + \alpha)n_t - log(a) - log(1-\alpha) \tag{4.71}$$

Notice that, if both ϕ and α are equal to 0, i.e. if both the production function and the disutility of labor are linear in labor, marginal cost (4.71) is independent of labor. Since I assume constant productivity a, we have

$$\hat{mc} = \sigma \hat{c} \tag{4.72}$$

and the New-Keynesian Phillips curve is

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{c}_t \tag{4.73}$$

where $\hat{c}$ is the log-deviation of consumption from steady state.

B2: The BD sector

Each period there is a measure 1 of potential new long-term projects, which need to be financed by bank loans. The aggregate production function of the projects that start at t is

$$Y^{BD}_{New} = \zeta \tilde{L}_t^{\nu} \tag{4.74}$$

with $\tilde{L} = \left(\int_0^1 di (L^{(i)})^{\frac{\epsilon_B - 1}{\epsilon_B}}\right)^{\frac{\epsilon_B}{\epsilon_B - 1}}$, where $L^{(i)}$ are the loans granted by bank i and ϵ_B is the elasticity of substitution of loans from different banks. One loan denotes here one unit of good lent to the firm at the initial time, resulting in subsequent payments of the form (4.21). The subscript in $Y^{BD}_{New}(t)$ serves to highlight that this is the production function of the new projects starting at t, which generate a demand for new loans.

The aggregate production function (4.74) generates the following demand for loans from bank i

$$L^i = (\zeta\nu)^{\frac{1}{1-\nu}}(\rho_i + \bar{\delta})^{-\epsilon_B}(\rho_M + \bar{\delta})^{\epsilon_B - \frac{1}{1-\nu}} \tag{4.75}$$

with

$$\rho_M = \left(\int di \rho_i^{1-\epsilon_B} \right)^{\frac{1}{1-\epsilon_B}} \tag{4.76}$$

In terms of standardized loans, where bank i makes an initial payment $\mathcal{P}^i$ to the firm and the loan rate is a constant $\bar{\rho}$, (standardized) loan demand is

$$N^i = (\zeta\nu)^{\frac{1}{1-\nu}} (\bar{\rho} + \bar{\delta})^{-\frac{1}{1-\nu}} \left(\mathcal{P}^i\right)^{\epsilon_B - 1} \left(\mathcal{P}^M\right)^{\frac{1}{1-\nu} - \epsilon_B} \tag{4.77}$$

where the market price $\mathcal{P}^M$ is related to ρ_M in (4.76) by $\mathcal{P}^M = \frac{\bar{\delta}+\bar{\rho}}{\bar{\delta}+\rho_M}$. The notation N^i for the demand of standardized loans reflects the notation in section 3 and highlights the fact that this is a demand for *new loans*, a decision variable for the bank.

Below is a possible microfoundation of (4.74).

Microfoundation Each potential new project, indexed by k, needs a fixed size of initial real investment. If financed, project k starts at t with a new investment $\bar{L} = \int di L_{(k)}^{(i)}$, where $L_{(k)}^{(i)}$ is the loan granted by bank i to firm k. If project k receives financing at time t, its production at $t+1$ is $z_k \bar{L}$, and in any given subsequent period s it is $z_k \bar{L}(1-\bar{\delta})^{t-s}$, reflecting the depreciation rate $\bar{\delta}$ of the input capital. The productivity z_k of project k is the product of an idiosyncratic factor z_k^0, and a factor reflecting the degree of loan diversification across banks.

$$z_k = z_k^0 \left(\frac{\tilde{L}_{(k)}}{\bar{L}} \right)^{\nu} \tag{4.78}$$

where $\tilde{L}_{(k)} = \left(\int_0^1 di (L_{(k)}^{(i)})^{\frac{\epsilon_B - 1}{\epsilon_B}} \right)^{\frac{\epsilon_B}{\epsilon_B - 1}}$. A firm's problem is to choose whether or not to take a loan, and if they do, to choose the loan composition across banks. Formally, the firm's optimization problem is the following[8]

$$max_{L_i,\ i\ \in[0,1]} z_k \bar{L} - \int di (\rho_i + \bar{\delta}) L_{(k)}^{(i)} \tag{4.79}$$

such that $\int di L_{(k)}^{(i)}$ is either 0 or $\bar{L}$ and z_k is given by (4.78). All firms who choose to take a loan will choose

$L_{(k)}^{(i)} \propto (\rho_i + \bar{\delta})^{-\epsilon_B}$, so that $\tilde{L}_{(k)} \equiv \tilde{L}_0$ is common for all firms that take a loan (while $\tilde{L}_{(k)} = 0$ for those that do not take a loan).

The time-invariant distribution of the productivity factor z^0 across different BD firms is $m(z^0)$, with support on an interval $[0,\ \bar{z}]$, so that

$$\int_0^{\bar{z}} dz^0\ m(z^0) = 1 \tag{4.80}$$

[8]Notice that the choice that maximizes the first period's profits also maximizes next period profits, since each period both production and the payment that the firm owes to the banks is just rescaled by a factor $(1-\bar{\delta})$. This is thanks to the fact that the inverse duration of the loans matches the depreciation rate of capital $\bar{\delta}$. With this loan, the firm is fully protected from interest rate risk.

If a fraction f of project is financed, i.e. all those with $z^0 > z^*$, such that $\int_{z^*}^{\bar{z}} m(z^0)dz^0 = f$, the average productivity factor is a decreasing function of the total amount of loans granted by banks

$$mean(z) = \frac{\int_{z^*}^{\bar{z}} z^0 m(z^0)dz}{\int_{z^*}^{\bar{z}} m(z^0)dz} = cf^{-\xi}, \quad \xi > 0 \tag{4.81}$$

$$Y_{New}^{BD} = \zeta f^{1-\xi}(\tilde{L}_0)^{\nu} \tag{4.82}$$

with $\zeta = c\bar{L}^{1-\nu}$, which coincides with (4.74) in the special case $\nu = 1 - \xi$ since it is

$$\tilde{L} = \int dk \tilde{L}_{(k)} = f\tilde{L}_0$$

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