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SIMULATION OF RUIN PROBABILITIES FOR RISK PROCESSES OF MARKOVIAN TYPE

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Abstract

We consider a generalisation of the classical risk model, where consecutive claims may be dependent according to a Markovian structure represented by a copula function for the joint distribution function of the claims. For various marginal claim size distributions and copula functions ruin probabilities are simulated via Monte Carlo and an importance sampling technique for variance reduction is developed.

1 Introduction

In classical risk theory the surplus process $U(t)$ of an insurance company is described by $U(t) = u + ct - \sum_{i=1}^{N_P(t)} X_i$, where $u > 0$ is the initial capital, c is the constant premium income density and t denotes time. The claim number process $N_P(t)$ is a homogeneous Poisson process with intensity λ and the claim amounts $X_1, X_2, \ldots$ are iid random variables with common distribution function F, where F has a finite mean μ. Let us define $T = \min\{t : t \geq 0 \text{ and } U(t) < 0\}$ as the time when ruin occurs (i.e. the surplus becomes negative for the first time) with the understanding that $T = \infty$ if ruin does not occur. Furthermore let $\psi(u) = Pr(T < \infty)$ denote the probability of ruin considered as a function of the initial surplus u. The usual net profit condition $c > \lambda\mu$ ensures that $\lim_{u\to\infty} \psi(u) = 0$. If there exists an $R > 0$ such that

$$\int_0^\infty e^{Rx}(1 - F(x))dx = \frac{c}{\lambda}$$

then for all $u \geq 0$ the inequality $\psi(u) \leq e^{-Ru}$ holds and if moreover $\int_0^\infty x\, e^{Rx}(1 - F(x))dx < \infty$, then

$$\lim_{u\to\infty} e^{Ru}\psi(u) = C, \tag{1}$$

where $C < \infty$ is a constant. Equation (1) is called the Cramer-Lundberg approximation and R is the so-called Lundberg exponent (see e.g. GRANDELL [9]).

In many situations the assumption of independence among different claims is too restrictive (e.g. the insurance of natural events) and it is of particular interest to obtain results similar to (1) for dependent scenarios (see ALBRECHER [1] for a discussion of several types of dependency structures relevant in risk theory). GERBER [8] and PROMISLOW [15] consider dependencies between annual gains of an insurance company according to a moving average as well as to an autoregressive model and under some assumptions they derive an approximation in the spirit of (1), where R is a function of the underlying

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dependency structure.
NYRHINEN [13] recently showed by means of large deviation techniques that for a general dependency structure the appropriate definition of the Lundberg exponent is

$$R = \sup\{t | c(t) \le 0\} \in [0, \infty], \tag{2}$$

where

$$c_n(t) = \frac{\log E(e^{tS_n})}{n} \quad \text{and} \quad c(t) = \limsup_{n \to \infty} c_n(t)$$

in the sense that then (under several conditions)

$$\lim_{u \to \infty} u^{-1} \log \psi(u) = -R \tag{3}$$

(see [13] for details). Here the risk process is rewritten as a random walk $S_n = u + \sum_{i=1}^{n} Y_i$, where $Y_i = c(T_i - T_{i-1}) - X_i$ and T_k is the time of occurrence of the kth claim.
However, for a given dependence structure it is usually very intricate to check the validity of the assumptions underlying (3) and then to calculate the Lundberg exponent according to (2). Whenever analytical or numerical results are not available, simulation can help to get information about the behavior of the process of interest. Especially the development of fast computers has made stochastic simulation a popular experimental tool also in risk theory (see for example ASMUSSEN [2] for a survey on the subject).

The aim of this paper is to use Monte Carlo simulation to gain more insight into the sensitivity of the Lundberg exponent on the degree of dependence among subsequent claims X_i. We simulate the risk process for a Markovian dependence structure, but in a more general framework than Gerber and Promislow in that our approach allows for arbitrary marginal distributions. For that purpose we use copulas for combining the risks. The notion of copulas has recently been identified to be a powerful tool for modelling dependence structures in insurance and finance (cf. [4], [6] and [7]). For a general introduction to copulas we refer to NELSEN [12] and techniques of determining the appropriate copula function for given data are surveyed e.g. in JOE [10].
In Section 2 we describe in more detail the underlying dependency model that we use. In Section 3 we present the simulation technique and we develop an appropriate importance sampling technique suitable for the dependent scenario in order to reduce the variance of our Monte Carlo simulations. Section 4 gives the simulation results for various underlying copula functions and these results are discussed in Section 5.

2 The Dependency Risk Model

We consider a dependence structure of Markovian first-order type among subsequent claims X_i expressed by a one-parameter bivariate copula function, i.e. the joint distribution function $H(x_i, x_{i+1})$ of two successive claims is given by

$$H(x_i, x_{i+1}) = C_\theta(F(x_i), F(x_{i+1})),$$

where $C_\theta(u, v)$ denotes a bivariate copula function with dependency measure θ and F is the marginal distribution function of the claims (the marginal distributions of the claims are assumed to be identical). An attractive feature of the copula representation of dependence is that the dependence structure is fully characterized by the copula (and thus does not depend on the marginals) and, moreover, is invariant under increasing and continuous transformations of the marginals.

In order to compare the impact of the choice of the copula on the ruin probability for various copula functions, we need a dependence measure that is independent of the choice of the marginal distributions. For a bivariate copula C Spearman's rank correlation coefficient

$$\rho_{X,Y} = 12 \iint_{[0,1]^2} [C(u,v) - uv]\, du\, dv \tag{4}$$

for two random variables X and Y fulfills this requirement ($\rho_{X,Y}$ is also called grade correlation coefficient as it is identical to Pearson's product-moment correlation coefficient for the grades $F(X)$ and $F(Y)$ of X and Y, respectively; see NELSEN [12] for details). $\rho_{X,Y}$ is a measure of concordance from which it follows that $-1 \leq \rho_{X,Y} \leq 1$, $\rho_{X,X} = 1$ and, if X and Y are independent, $\rho_{X,Y} = 0$ (as the copula of two independent random variables is $C(u,v) = uv$). Furthermore, if C_1 and C_2 are two copulas such that $C_1 \prec_c C_2$, then $\rho_{X,Y}^{(C_1)} < \rho_{X,Y}^{(C_2)}$, where $\prec_c$ denotes the concordance ordering (i.e. $C_1 \prec_c C_2$ iff $C_1(u,v) \leq C_2(u,v)$ for all u, v in [0,1]). Techniques for statistical estimation of $\rho_{X,Y}$ from given data can be found in TJØSTHEIM [17].

In our investigations we will focus on positive dependence (i.e. $\rho \geq 0$) and to the following copula functions, which all interpolate between independence $F(x_i, x_{i+1}) = F(x_i)F(x_{i+1})$ (i.e. $\rho = 0$) and the Fréchet upper bound $F(x_i, x_{i+1}) = \min_{j \in \{i,i+1\}} F(x_j)$ (e.g. $\rho = 1$). Moreover all of the following copulas are absolutely continuous and symmetric in the two arguments, have support on all of $[0,1]^2$ and an increasing dependence parameter as the dependence increases (in the sense that they are increasing in $\prec_c$):

A) Gaussian copula:

$$C_\theta^{Ga}(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\theta^2)^{1/2}} \exp\left\{ \frac{-(s^2 - 2\theta st + t^2)}{2(1-\theta^2)} \right\} dsdt, \tag{5}$$

where $0 \leq \theta < 1$ and Φ is the univariate standard normal distribution function. This copula function is also called Normal Copula. Note that for normal marginal distribution functions, $C_\theta^{Ga}(\Phi(x), \Phi(y))$ is the standard bivariate normal distribution with correlation coefficient θ.

B) Frank copula:

$$C_\vartheta^{F}(u,v) = -\frac{1}{\vartheta} \log\left(1 - \frac{(1-e^{-\vartheta u})(1-e^{-\vartheta v})}{(1-e^{-\vartheta})}\right), \tag{6}$$

where $0 \leq \vartheta \leq \infty$ and we have independence for $\vartheta \to 0$ and comonotonic dependence for $\vartheta = \infty$.

C) Gumbel copula:

$$C_\beta^{Gu}(u,v) = \exp\left[-\left\{ (-\log u)^\beta + (-\log v)^\beta \right\}^{\frac{1}{\beta}} \right], \tag{7}$$

where $1 \leq \beta < \infty$ is the parameter that controls the amount of dependence between U and V; $\beta = 1$ gives independence and the limit of C_β^{Gu} for $\beta \to \infty$ leads to comonotonic dependence. This copula, unlike the Gaussian and the Frank copula, has upper tail dependence (a definition which relates to the amount of dependence in the upper-quadrant tail of a bivariate distribution, cf.[10]).

We now show the desirable property that for this choice of copulas the dependence of the stationary first order Markov chain decreases with lag.

Proposition 1 *Let $X_1, X_2, \ldots$ be a stationary first-order Markov chain with underlying copula function given by (5),(6) or (7), and denote the bivariate distribution for X_i, X_j by F_{ij}. In addition, let $F^{(2)}(x_1, x_2) = F(x_1)F(x_2)$. Then for all $m \geq 2$ and $\rho_{X,Y} \geq 0$*

$$F^{(2)} \prec_c \cdots \prec_c F_{1m} \prec_c \cdots \prec_c F_{13} \prec_c F_{12} \tag{8}$$

Proof: According to Theorem 8.3 of [10] it suffices to show that the transition distribution $H(x_t|x_{t-1}) = C_{2|1}(F(x_t)|F(x_{t-1}))$ is stochastically increasing (SI), that is

$$Pr(X_t > x_t | X_{t-1} = x_{t-1}) \text{ is increasing in } \; x_{t-1} \; \forall \, x_t.$$

Here $C_{2|1}(u, v)$ denotes the conditional distribution function of u given v. Defining a bivariate "more regression dependent" ordering $\prec_{SI}$ among bivariate distributions, one can show that the copulas (A),(B) and (C) above are increasing in $\prec_{SI}$ with respect to their dependence parameter (see [10]). But from Theorem 2.11 in [10] it then follows that $C_{2|1}(u, v)$ is SI and as F is a monotone function, we thus have $H(x_t|x_{t-1})$ is SI. □

3 Simulation technique

3.1 Monte Carlo Simulation

The risk process $U(t) = u + ct - \sum_{i=1}^{N_P(t)} X_i$ can be simulated by randomly drawing sample paths according to the homogeneous Poisson process $N_P(t)$ for the claim arrivals and according to the given marginal distributions and copula structure of the claims X_i, each sample path starting at $U(0) = u$. By counting the trajectories that lead to ruin and dividing this number by the total number N of simulated trajectories, we get an unbiased estimator for the ruin probability $\psi(u)$

$$\hat{\psi}(u) = \frac{1}{N} \sum_{i=1}^{N} 1_A(W_i), \tag{9}$$

where A is the set of all trajectories W_i that lead to ruin. As we only consider cases where $\psi(u) < 1$ it will frequently happen that $U(t) \to \infty$ as $t \to \infty$ without U ever becoming negative. It is therefore necessary to stop the process at some time (for a number of risk models it is possible to circumvent this problem by relating the surplus process to another process which leads to ruin with certainty (cf. [2]), a method which is not applicable in our case). This is done at time T_{st} for suitably large T_{st}, which will of course lead to a downward bias in the estimate of $\psi(u)$, but for T_{st} large enough this bias becomes negligible (in our simulations we have increased T_{st} until practically no difference in the value of (9) was observable by further increasing T_{st}). Instead of considering a finite time T_{st} one could employ an upper surplus barrier as a stopping criterion for trajectories not leading to ruin (i.e. fix a number $\bar{u} > u$ and stop every realization at $T = \min\{T_y, \inf\{U(t) \geq \bar{u}\}\}$, where T_y is the time of ruin). If $U(T) < 0$, a ruin is recorded, otherwise a nonruin is recorded. Again, if $\bar{u}$ is chosen large enough, the downward bias becomes negligible. However, the first technique is more suitable for our purposes as will become clear in the next section.

For the Markovian structure of first order among consecutive claims expressed through the bivariate copula $C(u, v)$, recursive simulation using univariate conditional densities is appropriate. That is, given the claim $X_i = x_i$, we have the following algorithm for simulating X_{i+1} (where $F(x_i) = u$):

1. Generate a uniformly distributed random variable t on (0,1)
2. Define $v = c_u^{(-1)}(t)$, where $c_u^{(-1)}$ is the quasi-inverse of c_u and

$$c_u(v) = \mathbf{P}[V \leq v | U = u] = \lim_{\Delta u \to 0} \frac{C(u + \Delta u, v) - C(u, v)}{\Delta u} = \frac{\partial}{\partial u} C(u, v) \tag{10}$$

3. Take $X_{i+1} = F^{-1}(v)$.

Thus, starting with $U(0) = u$ we simulate exponentially distributed inter-occurrence times T_k (which is done by the well-known inversion method) and increase the surplus by cT_k, followed by a claim which reduces the surplus (where the claim is generated by the above algorithm). This procedure is repeated until ruin has occurred or the stopping criterion for non-ruin is fulfilled. After N simulation runs we get the estimate (9).

For the generation of uniformly distributed random variables there are a lot of efficient algorithms available in the literature. We use an improved version of a so-called Minimal Standard generator which is based on a multiplicative congruential algorithm (see e.g. Press et al. [14]).

Section 4 gives the simulation results for exponential, normal, Pareto and gamma marginal distributions of claim sizes, respectively. In order to assess the differences for ruin probabilities for given dependency parameters among various marginal distributions and copula structures, the value for the mean and the value for the variance are taken constant for the first three of these marginal distributions of the claim sizes. Concretely we chose mean $\mu = 10$ and variance $\sigma^2 = 100$, where the value of σ^2 is determined by the exponential distribution. For the simulation of the risk process with gamma-distributed claim sizes (given in Section 4.3), we chose the parameters such that again $\mu = 10$, but $\sigma^2 = 50$. In this way some insight on the impact of the coefficient of variation CoV=$\frac{\sqrt{Var(X)}}{E(X)}$ of the claim size distribution on the ruin probability can be gained.
In all the simulations we have $\lambda = 15$ and $c = 210$. For the stopping time we chose $T_{st} = 600$, which is the time, when on average 9000 claims have occurred. If one observes sample paths of the surplus process, it turns out that this choice of T_{st} is very generous and practically no difference in the final value of $\psi(u)$ is caused by this truncation.

For all copula structures and marginal distributions $\psi(u)$ is simulated for the values $u = 85, 130, 175, 220, 265$ and $\rho = 0, \ldots, 0.6$ in steps of 0.1 (ρ=0 corresponds to the independent case). The idea behind this choice of values for u is that in principle we want to estimate $\hat{\psi}(u)$ for large u to get an idea about a possible existence of a Lundberg exponent. However, since most of the following simulations are done by Crude Monte Carlo, the magnitude of the probability to be simulated must not be lower than some threshold, which depends on the number N of simulation paths, in order to ensure a good estimate (for risk models with independent increments some sophisticated rare events simulation techniques have been developed to circumvent this problem, see e.g. Asmussen [2] and Asmussen and Binswanger [3]).
Using (4) we can for each copula function determine the values of the dependence parameter that correspond to the value of Spearman's ρ:

Spearman's ρ	θ (Gaussian copula)	β (Gumbel copula)	ϑ (Frank copula)
0	0	1	0
0.1	0.105	1.07	0.60
0.2	0.209	1.16	1.22
0.3	0.313	1.26	1.88
0.4	0.416	1.38	2.61
0.5	0.518	1.54	3.45
0.6	0.618	1.75	4.47

These values have been inserted in (5),(6) and (7), respectively to determine the quasi-inverse of the conditional density function (10) of the corresponding copula needed for the generation of the claim sizes.

Each simulation is carried out by 10 independent simulation runs with 3000 sample paths each, i.e. N=30000. These 10 estimators are then used to calculate the final estimate $\hat{\psi}(u)$ and the empirical variance s^2 of $\hat{\psi}(u)$. The values of the estimates are given together with their 95% confidence interval corresponding to s^2. In addition the simulated data are depicted in a diagram with logarithmic scale, where datapoints belonging to the same value of ρ are connected by straight lines. The values of $R(\rho)$, which are calculated for each ρ by common least-squares regression of $\log \hat{\psi}(u)$ against u, are depicted in a separate figure and the corresponding regression line of R against ρ is drawn and given analytically in the figure.

3.2 Importance sampling in a dependent scenario

Usually the probability of ruin is a small number and thus the direct estimation by relative frequency is rather inefficient in the sense that too many observation runs must be undertaken to obtain a satisfying estimate. For a closely related risk model it is possible to develop an importance sampling technique which considerably improves this situation. Namely we rewrite the risk process as a random walk $S_n = u + \sum_{i=1}^{n} Y_i$ (see Section 1), where S_n denotes the surplus after the nth claim and $Y_i = c(T_i - T_{i-1}) - X_i$ (recall that T_k is the occurrence time of the kth claim) and consider a Markovian dependence structure according to a given copula function among the consecutive random variables Y_i instead of X_i. Let us denote the distribution function of Y_i by F_c (where the subscript c indicates that it is a compound distribution) and let furthermore $T = \inf\{n : u + S_n < 0\}$ be the time of ruin. Then the ruin probability until claim M is defined by $\psi(u, M) = P(T \leq M)$. In the case where the Y_i are independent we have

$$\psi(u, M) = \sum_{n=1}^{M} \int_{(T=n)} dH(y_1, \ldots, y_n) = \sum_{n=1}^{M} \int_{(T=n)} dF_c(y_1) \cdots dF_c(y_n)$$

(where H denotes the joint distribution function of $Y_1, \ldots, Y_n$) or briefly

$$\psi(u, M) = \int 1_{\{T \leq M\}} dF_c(y_1) \cdots dF_c(y_T). \tag{11}$$

Importance sampling now suggests to replace the true distribution F_c (with density f_c) by a simulation distribution G (with density g), where $F_c << G$ and G is chosen in a way to reduce the variance of the simulation. As shown in COTTRELL ET AL. [5] the smallest variance is obtained, if about half of the trajectories lead to ruin before the Mth claim (at this point it becomes apparent why for our purposes it is preferable to truncate the risk process at some time T_{st} instead of a barrier $\bar{u}$ as discussed in Section 3.1).

In the case of independent Y_i the importance sampling technique is well-known (see e.g. LEHTONEN AND NYRHINEN [11]): Simulate the random walk with distribution G instead of F_c and let $Z_1, Z_2, \ldots, Z_T$ denote the sequence of independent G-distributed random variables generated in the j-th single replication in the simulation. Then the correction factor for the j-th replication is

$$Y_G^{(j)} = \prod_{i=1}^{T} \frac{dF_c}{dG}(Z_i)\, 1_{\{T \leq M\}} = \prod_{i=1}^{T} \frac{f(z_i)}{g(z_i)}\, 1_{\{T \leq M\}}, \tag{12}$$

where z_i is the outcome of the simulation of Z_i, and the importance sampling estimator $\hat{\psi}(u, M)$ is now defined by

$$\hat{\psi}(u, M) = \sum_{j=1}^{N} \frac{Y_G^{(j)}}{N},$$

where N is the number of replications. It is straightforward to show that this is an unbiased estimator of $\psi(u, M)$.

We now want to adapt this technique to our situation of dependent Y_i according to a Markovian model of first order. Similar to (11) we can write

$$\begin{aligned}\psi(u, M) &= \int 1_{\{T \leq M\}} dH(y_1, \ldots, y_T) = \\ &= \int 1_{\{T \leq M\}} f_c(y_1) f_c(y_2|y_1) f_c(y_3|y_1, y_2) \cdots f_c(y_T|y_1, \ldots, y_{T-1}) dy_1 \cdots dy_T\end{aligned}$$

and due to the first order Markovian structure this is equal to

$$\psi(u, M) = \int 1_{\{T \leq M\}} f_c(y_1) f_c(y_2|y_1) f_c(y_3|y_2) \cdots f_c(y_T|y_{T-1}) dy_1 \cdots dy_T.$$

It is assumed throughout this section that $0 < f_c(y) < \infty$ $(\forall y \in \mathbf{R})$ to avoid complications with the conditional densities.

If now F_c is replaced by G, this leads for each replication j to a correction factor

$$ {}^{d}Y_G^{(j)} = \frac{f_c(z_1)\prod_{i=2}^{T} f_c(z_i|z_{i-1})}{g(z_1)\prod_{i=2}^{T} g(z_i|z_{i-1})}\, 1_{\{T\le M\}}, \tag{13} $$

where the z_i denote the outcome of the simulation of the random variable Z_i. The importance sampling estimator in the dependent case is then given by

$$ \hat{\psi}(u,M) = \sum_{j=1}^{N} \frac{{}^{d}Y_G^{(j)}}{N}. \tag{14} $$

Since

$$ \begin{aligned} \mathbf{E}\left({}^{d}Y_G^{(j)} \right) &= \int \left[\frac{f_c(y_1)\prod_{i=2}^{T} f_c(y_i|y_{i-1})}{g(y_1)\prod_{i=2}^{T} g(y_i|y_{i-1})}\, 1_{\{T\le M\}} \right] g(y_1) \prod_{i=2}^{T} g(y_i|y_{i-1}) dy_1 \cdot\cdot dy_T \\ &= \int 1_{\{T\le M\}} f_c(y_1) \prod_{i=2}^{T} f_c(y_i|y_{i-1}) dy_1 \cdots dy_T \\ &= \psi(u,M), \end{aligned} $$

we have $\mathbf{E}\big(\hat{\psi}(u,M)\big) = \psi(u,M)$, i.e. the estimator (14) is unbiased.
Again, if M is chosen suitably large, (14) will be a good approximation for the ruin probability $\psi(u)$. In order to implement the above algorithm we see from (13) that we need to calculate the conditional densities $f_c(y_i|y_{i-1})$ according to the given copula structure and the (compound) marginal distributions.

Section 4.1 gives the results of this simulation technique. Due to the complexity of the calculations of the conditional densities involved, we confined the application of importance sampling in this paper to the case of exponential marginals for the claim sizes and the bivariate copulas of Gumbel and Frank. For the ease of computation, the simulation distribution G is chosen to be of the same type as F_c, but with the parameters changed such that about half of the trajectories lead to ruin. We use an adaptive algorithm to find appropriate values for the parameters. The simulations have also been done by ordinary Monte Carlo (which is often called "Crude Monte Carlo" (CMC)) for the same choice of parameters (and dependence among Y_i) to allow a comparison of the empirical variances of these two simulation algorithms. As can be seen in Tables 4.4 and 4.5, the outperformance of importance sampling (IS) over CMC is striking (on average the empirical variance is reduced by a factor of 10).

4 Simulation Results

In the sequel all values are rounded to their last digit.

4.1 Exponential marginals

TABLE 4.1: SIMULATED RUIN PROBABILITIES FOR EXPONENTIALLY DISTRIBUTED CLAIMS AND GUMBEL'S COPULA (USING IMPORTANCE SAMPLING)

F∼Exp(0.1), GumC, c=210, λ=15, N=30000					
$\rho \backslash u$	85	130	175	220	265
0	(6.39±0.02) E-02	(1.83±0.17) E-02	(4.63±0.51) E-03	(1.20±0.53) E-03	(4.00±2.00) E-04
0.1	(8.99±0.41) E-02	(3.23±0.23) E-02	(1.07±0.11) E-02	(3.37±0.51) E-03	(1.30±0.42) E-03
0.2	(1.34±0.06) E-01	(5.47±0.21) E-02	(2.33±0.13) E-02	(9.37±1.06) E-03	(4.20±1.00) E-03
0.3	(1.83±0.04) E-01	(8.99±0.21) E-02	(4.10±0.15) E-02	(2.13±0.20) E-02	(1.04±0.08) E-02
0.4	(2.34±0.03) E-01	(1.36±0.03) E-01	(7.50±0.19) E-02	(4.29±0.15) E-02	(2.44±0.12) E-02
0.5	(3.13±0.03) E-01	(1.99±0.04) E-01	(1.26±0.04) E-01	(7.96±0.30) E-02	(5.07±0.24) E-02
0.6	(3.87±0.06) E-01	(2.76±0.07) E-01	(1.98±0.04) E-01	(1.44±0.03) E-01	(1.03±0.03) E-01

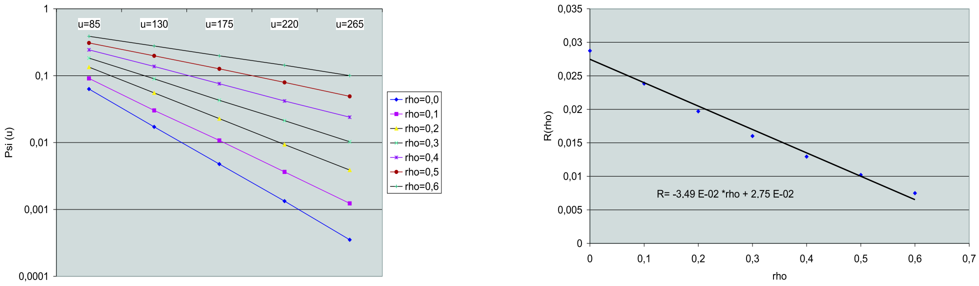

FIGURE 4.1: F∼EXP(0.1), GUMBEL'S COPULA

TABLE 4.2: SIMULATED RUIN PROBABILITIES FOR EXPONENTIALLY DISTRIBUTED CLAIMS AND FRANK'S COPULA (USING IMPORTANCE SAMPLING)

F∼Exp(0.1), FraC, c=210, λ=15, N=30000					
$\rho \backslash u$	85	130	175	220	265
0	(6.21±0.08) E-02	(1.75±0.05) E-02	(4.79±0.17) E-03	(1.35±0.05) E-03	(3.44±0.18) E-04
0.1	(9.18±0.11) E-02	(3.12±0.05) E-02	(9.99±0.25) E-03	(3.35±0.10) E-03	(1.12±0.07) E-03
0.2	(1.25±0.02) E-01	(5.11±0.13) E-02	(2.02±0.07) E-02	(7.87±0.33) E-03	(3.13±0.16) E-03
0.3	(1.67±0.02) E-01	(7.96±0.19) E-02	(3.66±0.17) E-02	(1.70±0.06) E-02	(8.00±0.25) E-03
0.4	(2.17±0.03) E-01	(1.19±0.02) E-01	(6.34±0.02) E-02	(3.42±0.18) E-02	(1.87±0.12) E-02
0.5	(2.81±0.02) E-01	(1.71±0.03) E-01	(1.05±0.02) E-01	(6.47±0.35) E-02	(3.94±0.25) E-02
0.6	(3.49±0.04) E-01	(2.44±0.04) E-01	(1.66±0.04) E-01	(1.14±0.04) E-01	(7.75±0.28) E-02

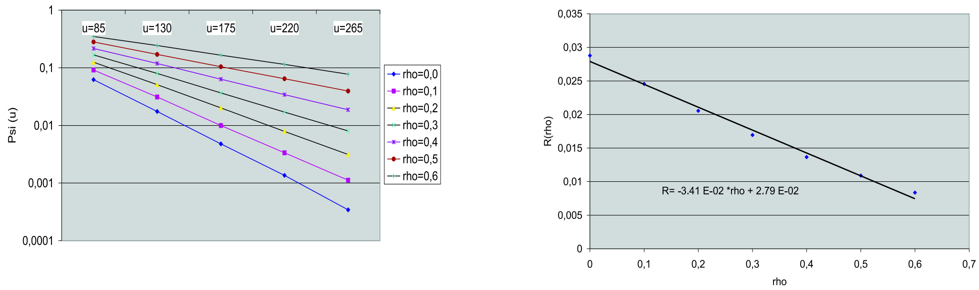

FIGURE 4.2: F∼EXP(0.1), FRANK'S COPULA

Table 4.3: Simulated ruin probabilities for exponentially distributed claims and Normal copula

F∼Exp(0.1), NorC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(6.35±0.31) E-02	(1.74±0.17) E-02	(5.23±0.79) E-03	(1.03±0.48) E-03	(1.67±1.03) E-04
0.1	(9.47±0.24) E-02	(3.30±0.27) E-02	(1.12±0.16) E-02	(4.37±0.47) E-03	(1.27±0.42) E-03
0.2	(1.38±0.03) E-01	(5.64±0.23) E-02	(2.48±0.15) E-02	(1.04±0.11) E-02	(3.93±0.47) E-03
0.3	(1.81±0.04) E-01	(9.27±0.27) E-02	(4.56±0.17) E-02	(2.36±0.17) E-02	(1.24±0.14) E-02
0.4	(2.41±0.05) E-01	(1.42±0.05) E-01	(7.99±0.30) E-02	(4.70±0.17) E-02	(2.62±0.15) E-02
0.5	(3.04±0.04) E-01	(1.96±0.05) E-01	(1.27±0.03) E-01	(8.72±0.29) E-02	(5.61±0.16) E-02
0.6	(3.69±0.04) E-01	(2.67±0.05) E-01	(1.96±0.04) E-01	(1.41±0.03) E-01	(1.04±0.03) E-01

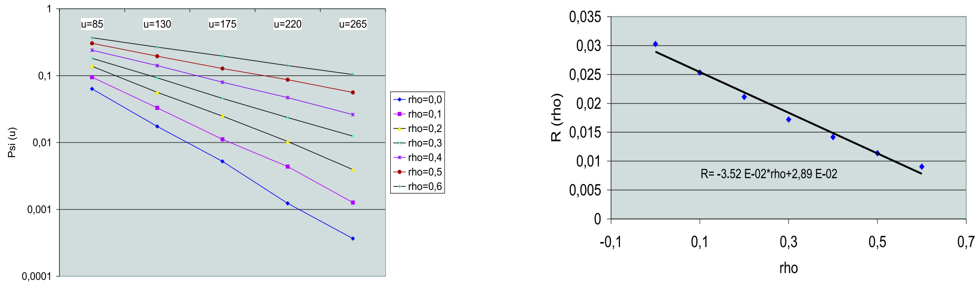

Figure 4.3: F∼Exp(0.1), Normal Copula

Table 4.4: Comparison of empirical variances of $\hat{\psi}(u)$ for Crude Monte Carlo (CMC) and Importance Sampling (IS) for Gumbel's copula

$\rho\backslash u$	u=85		u= 130		u=175		u=220		u=265	
	CMC	IS	CMC	IS	CMC	IS	CMC	IS	CMC	IS
0	1.18E-05	5.50E-06	7.40E-06	1.97E-07	6.77E-07	2.89E-08	7.38E-07	1.48E-08	1.07E-07	1.53E-09
0.1	4.52E-05	6.25E-06	1.37E-05	9.33E-07	3.30E-06	3.54E-07	6.77E-07	1.25E-07	4.77E-07	8.51E-09
0.2	1.14E-04	1.28E-05	1.10E-05	4.61E-06	4.40E-06	9.93E-07	2.94E-06	4.46E-07	2.63E-06	9.00E-08
0.3	5.76E-05	9.68E-06	1.10E-05	3.47E-06	6.14E-06	3.05E-06	1.08E-05	1.89E-06	1.75E-06	8.68E-07
0.4	2.20E-05	5.11E-05	2.76E-05	1.11E-05	9.78E-06	1.54E-05	5.93E-06	1.45E-06	3.79E-06	1.35E-06
0.5	3.31E-05	3.21E-05	4.65E-05	2.63E-05	4.82E-05	1.65E-05	2.22E-05	9.79E-06	1.51E-05	4.46E-06
0.6	1.23E-04	7.52E-05	1.29E-04	6.94E-05	4.35E-05	2.41E-05	2.31E-05	4.42E-05	2.40E-05	2.54E-05

Table 4.5: Comparison of empirical variances of $\hat{\psi}(u)$ for Crude Monte Carlo (CMC) and Importance Sampling (IS) for Frank's copula

$\rho\backslash u$	u=85		u= 130		u=175		u=220		u=265	
	CMC	IS	CMC	IS	CMC	IS	CMC	IS	CMC	IS
0	1.77E-05	1.75E-06	3.73E-06	6.52E-07	6.49E-07	7.69E-08	4.62E-07	8.08E-09	1.56E-07	8.76E-10
0.1	1.59E-05	3.54E-06	6.60E-06	8.52E-07	4.02E-06	1.66E-07	1.89E-06	2.62E-08	4.99E-07	1.38E-08
0.2	3.06E-05	1.07E-05	1.05E-05	4.82E-06	3.21E-06	1.30E-06	1.43E-06	2.83E-07	4.89E-07	7.29E-08
0.3	3.26E-05	5.84E-06	1.57E-05	8.63E-06	5.63E-06	7.23E-06	5.25E-06	1.03E-06	4.31E-06	1.65E-07
0.4	2.08E-05	2.73E-05	2.32E-05	6.53E-06	1.97E-05	5.90E-06	9.49E-06	7.97E-06	2.03E-06	3.67E-06
0.5	4.79E-05	1.29E-05	3.93E-05	2.12E-05	2.04E-05	1.02E-05	2.94E-05	3.91E-05	5.80E-06	1.60E-05
0.6	3.99E-05	4.75E-05	3.15E-05	5.77E-05	2.53E-05	4.84E-05	3.60E-05	5.12E-05	2.80E-05	1.98E-05

4.2 Normal marginals

Table 4.6: Simulated ruin probabilities for normally distributed claims and Gumbel's copula

F∼N(10,100), GumC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(8.44±0.29) E-02	(2.39±0.27) E-02	(7.37±0.82) E-03	(2.23±0.46) E-03	(5.33±1.65) E-04
0.1	(1.02±0.03) E-01	(3.58±0.13) E-02	(1.18±0.10) E-02	(3.63±0.72) E-03	(1.70±0.48) E-03
0.2	(1.28±0.02) E-01	(5.14±0.25) E-02	(1.97±0.15) E-02	(8.00±1.33) E-03	(3.73±0.50) E-03
0.3	(1.53±0.03) E-01	(6.96±0.20) E-02	(3.52±0.15) E-02	(1.53±0.08) E-02	(7.20±0.80) E-03
0.4	(1.86±0.04) E-01	(9.67±0.34) E-02	(4.75±0.19) E-02	(2.68±0.18) E-02	(1.37±0.12) E-02
0.5	(2.28±0.03) E-01	(1.31±0.04) E-01	(7.72±0.11) E-02	(4.60±0.15) E-02	(2.69±0.17) E-02
0.6	(2.70±0.04) E-01	(1.76±0.03) E-01	(1.15±0.04) E-01	(7.75±0.21) E-02	(5.20±0.27) E-02

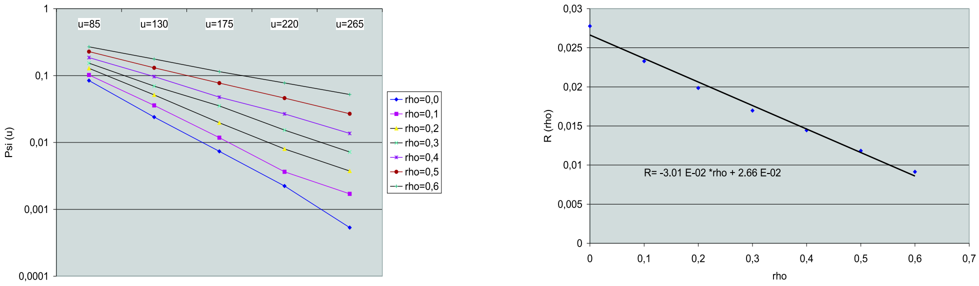

FIGURE 4.4: F∼N(10,100), GUMBEL'S COPULA

TABLE 4.7: SIMULATED RUIN PROBABILITIES FOR NORMALLY DISTRIBUTED CLAIMS AND FRANK'S COPULA

F∼N(10,100), FraC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(8.43±0.33) E-02	(2.60±0.18) E-02	(7.00±0.84) E-03	(2.23±0.55) E-03	(9.00±4.03) E-04
0.1	(9.60±0.32) E-02	(3.12±0.17) E-02	(1.07±0.08) E-02	(3.63±0.73) E-03	(9.33±3.17) E-04
0.2	(1.16±0.04) E-01	(3.96±0.15) E-02	(1.49±0.17) E-02	(6.20±1.17) E-03	(2.27±0.44) E-03
0.3	(1.34±0.02) E-01	(5.37±0.23) E-02	(2.02±0.16) E-02	(7.57±1.12) E-03	(3.10±0.37) E-03
0.4	(1.62±0.04) E-01	(7.10±0.28) E-02	(3.17±0.21) E-02	(1.37±0.12) E-02	(6.23±0.84) E-03
0.5	(1.95±0.01) E-01	(9.21±0.27) E-02	(4.73±0.21) E-02	(2.44±0.13) E-02	(1.12±0.14) E-02
0.6	(2.34±0.04) E-01	(1.29±0.04) E-01	(6.88±0.22) E-02	(4.00±0.16) E-02	(2.04±0.13) E-02

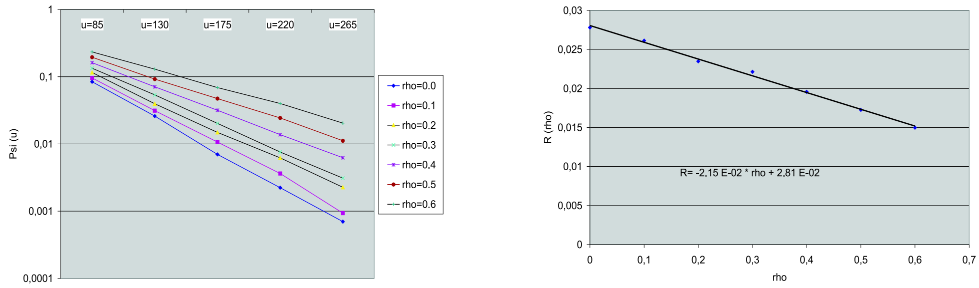

FIGURE 4.5: F∼N(10,100), FRANK'S COPULA

TABLE 4.8: SIMULATED RUIN PROBABILITIES FOR NORMALLY DISTRIBUTED CLAIMS AND NORMAL COPULA

F∼N(10,100), NorC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(8.42±0.19) E-02	(2.63±0.17) E-02	(7.07±0.73) E-03	(2.23±0.38) E-03	(7.67±2.27) E-04
0.1	(9.75±0.21) E-02	(3.15±0.19) E-02	(1.06±0.12) E-02	(3.77±0.42) E-03	(1.07±0.30) E-03
0.2	(1.15±0.03) E-01	(4.30±0.19) E-02	(1.55±0.12) E-02	(5.17±0.79) E-03	(2.17±0.45) E-03
0.3	(1.40±0.04) E-01	(5.60±0.26) E-02	(2.26±0.14) E-02	(8.67±0.79) E-03	(3.53±0.60) E-03
0.4	(1.63±0.03) E-01	(7.50±0.37) E-02	(3.30±0.22) E-02	(1.66±0.13) E-02	(6.80±0.93) E-03
0.5	(2.02±0.04) E-01	(1.04±0.03) E-01	(5.44±0.28) E-02	(2.72±0.22) E-02	(1.27±0.13) E-02
0.6	(2.47±0.05) E-01	(1.40±0.02) E-01	(8.04±0.23) E-02	(4.64±0.23) E-02	(2.81±0.17) E-02

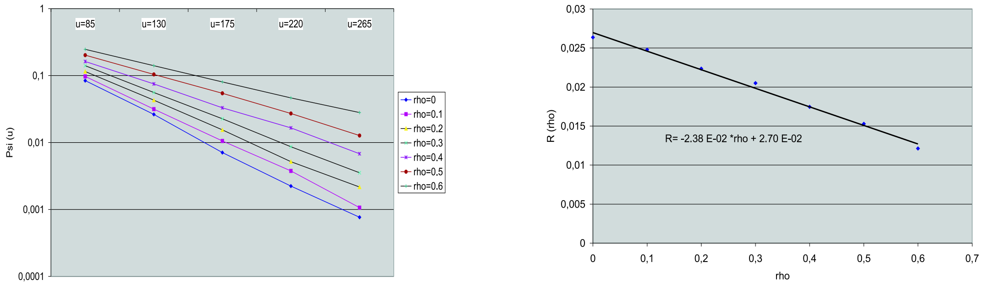

FIGURE 4.6: F∼N(10,100), NORMAL COPULA

4.3 Gamma marginals

TABLE 4.9: SIMULATED RUIN PROBABILITIES FOR GAMMA DISTRIBUTED CLAIMS AND GUMBEL'S COPULA

F∼ $\Gamma(2,5)$, GumC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(2.57±0.13) E-02	(4.70±0.71) E-03	(7.33±3.17) E-04	(1.00±0.83) E-04	(3.33±0.91) E-05
0.1	(3.95±0.32) E-02	(8.03±0.96) E-03	(2.10±0.44) E-03	(4.00±2.23) E-04	(1.67±1.39) E-04
0.2	(5.30±0.25) E-02	(1.35±0.12) E-02	(4.30±0.53) E-03	(1.43±0.53) E-03	(4.67±1.65) E-04
0.3	(7.07±0.22) E-02	(2.54±0.16) E-02	(9.00±1.15) E-03	(3.80±0.66) E-03	(1.47±0.23) E-03
0.4	(8.77±0.28) E-02	(3.71±0.21) E-02	(1.74±0.07) E-02	(8.37±0.98) E-03	(4.17±0.43) E-03
0.5	(1.18±0.03) E-01	(5.98±0.30) E-02	(2.97±0.15) E-02	(1.74±0.21) E-02	(8.87±1.14) E-03
0.6	(1.50±0.03) E-01	(8.72±0.22) E-02	(5.21±0.21) E-02	(3.57±0.22) E-02	(2.08±0.13) E-02

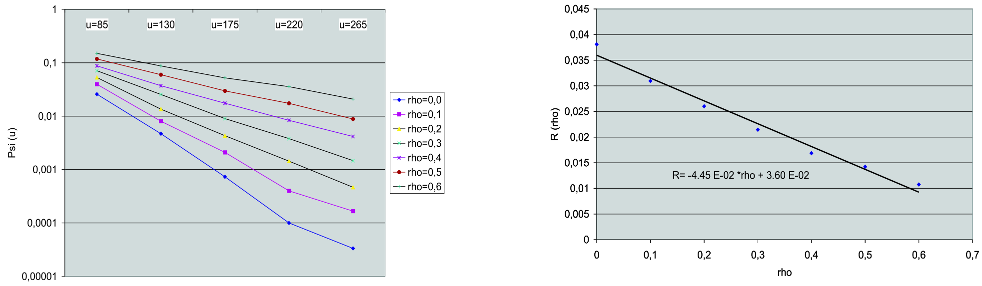

FIGURE 4.7: F∼ $\Gamma(2,5)$, GUMBEL COPULA

TABLE 4.10: SIMULATED RUIN PROBABILITIES FOR GAMMA DISTRIBUTED CLAIMS AND FRANK'S COPULA

F∼ $\Gamma(2,5)$, FraC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(2.78±0.19) E-02	(4.90±0.55) E-03	(6.67±2.61) E-04	(1.00±0.95) E-04	(3.33±6.20) E-05
0.1	(3.26±0.14) E-02	(6.07±0.70) E-03	(1.00±0.38) E-03	(2.00±1.37) E-04	(6.67±6.20) E-05
0.2	(3.65±0.20) E-02	(8.27±0.75) E-03	(1.43±0.37) E-03	(4.00±1.37) E-04	(1.00±1.32) E-04
0.3	(4.84±0.24) E-02	(1.08±0.12) E-02	(2.37±0.66) E-03	(7.00±1.95) E-04	(1.67±1.03) E-04
0.4	(5.79±0.22) E-02	(1.53±0.17) E-02	(4.57±0.77) E-03	(1.13±0.37) E-03	(3.67±1.95) E-04
0.5	(7.62±0.21) E-02	(2.34±0.14) E-02	(8.37±0.97) E-03	(2.37±0.55) E-03	(7.00±1.72) E-04
0.6	(9.57±0.32) E-02	(3.56±0.22) E-02	(1.43±0.12) E-02	(4.70±0.65) E-03	(1.93±0.32) E-03

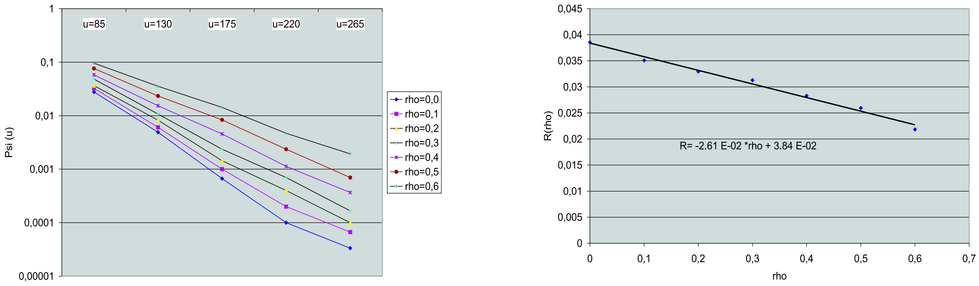

FIGURE 4.8: F∼ $\Gamma(2,5)$, FRANK COPULA

TABLE 4.11: SIMULATED RUIN PROBABILITIES FOR GAMMA DISTRIBUTED CLAIMS AND NORMAL COPULA

F∼ $\Gamma(2,5)$, NorC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(2.30±0.16) E-02	(4.27±0.59) E-03	(5.67±2.93) E-04	(1.00±1.32) E-04	(3.33±6.20) E-05
0.1	(3.15±0.17) E-02	(5.97±0.81) E-03	(1.43±0.37) E-03	(3.00±1.95) E-04	(6.67±8.26) E-05
0.2	(3.93±0.23) E-02	(9.60±0.94) E-03	(2.27±0.41) E-03	(5.67±1.55) E-04	(2.00±1.37) E-04
0.3	(5.02±0.26) E-02	(1.36±0.11) E-02	(3.27±0.76) E-03	(9.67±4.18) E-04	(1.67±1.03) E-04
0.4	(6.80±0.24) E-02	(2.06±0.12) E-02	(6.37±0.91) E-03	(1.97±0.67) E-03	(5.33±2.30) E-04
0.5	(8.70±0.21) E-02	(3.16±0.16) E-02	(1.16±0.13) E-02	(4.07±0.37) E-03	(1.60±0.39) E-03
0.6	(1.13±0.02) E-01	(4.99±0.17) E-02	(2.19±0.22) E-02	(1.03±0.13) E-02	(4.00±0.65) E-03

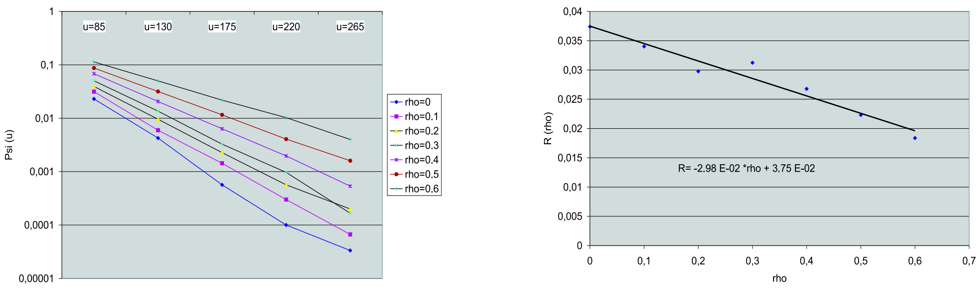

FIGURE 4.9: F∼ $\Gamma(2,5)$, NORMAL COPULA

4.4 Pareto marginals

TABLE 4.12: SIMULATED RUIN PROBABILITIES FOR PARETO DISTRIBUTED CLAIMS AND GUMBEL'S COPULA

F∼ Par(2.41,5.85), GumC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(5.02±0.30) E-02	(2.43±0.45) E-02	(1.34±0.25) E-02	(8.60±1.67) E-03	(6.43±1.22) E-03
0.1	(6.67±0.39) E-02	(3.31±0.62) E-02	(2.14±0.40) E-02	(1.40±0.27) E-02	(1.04±0.20) E-02
0.2	(8.08±0.48) E-02	(4.59±0.27) E-02	(3.21±0.60) E-02	(2.19±0.41) E-02	(1.64±0.10) E-02
0.3	(9.38±0.55) E-02	(5.71±0.34) E-02	(3.99±0.24) E-02	(2.80±0.52) E-02	(2.23±0.13) E-02
0.4	(1.11±0.07) E-01	(7.25±0.43) E-02	(5.22±0.31) E-02	(3.83±0.23) E-02	(2.98±0.18) E-02
0.5	(1.29±0.08) E-01	(8.95±0.53) E-02	(6.71±0.40) E-02	(5.16±0.30) E-02	(4.21±0.25) E-02
0.6	(1.50±0.09) E-01	(1.10±0.02) E-01	(8.53±0.50) E-02	(7.17±0.42) E-02	(5.71±0.11) E-02

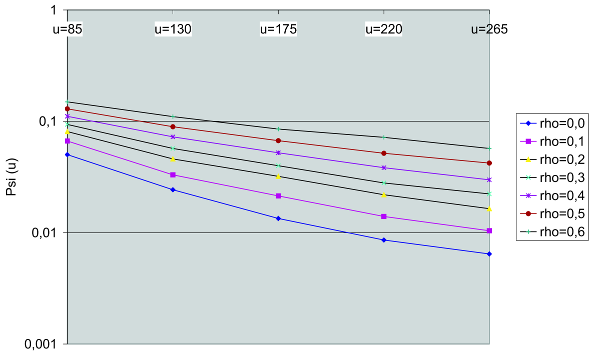

FIGURE 4.10: F∼ PAR(2.41,5.85), GUMBEL COPULA

TABLE 4.13: SIMULATED RUIN PROBABILITIES FOR PARETO DISTRIBUTED CLAIMS AND FRANK'S COPULA

F∼ Par(2.41,5.85), FraC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(5.43±0.18) E-02	(2.33±0.11) E-02	(1.30±0.12) E-02	(9.10±1.33) E-03	(5.93±0.84) E-03
0.1	(5.46±0.33) E-02	(2.47±0.24) E-02	(1.43±0.15) E-02	(8.47±0.92) E-03	(6.13±1.08) E-03
0.2	(5.70±0.25) E-02	(2.62±0.14) E-02	(1.59±0.10) E-02	(9.50±0.83) E-03	(7.40±1.24) E-03
0.3	(6.39±0.27) E-02	(3.17±0.20) E-02	(1.55±0.08) E-02	(1.03±0.10) E-02	(6.43±0.55) E-03
0.4	(6.92±0.22) E-02	(3.03±0.23) E-02	(1.69±0.10) E-02	(9.80±1.29) E-03	(7.10±0.90) E-03
0.5	(8.10±0.29) E-02	(3.76±0.17) E-02	(1.92±0.10) E-02	(1.13±0.15) E-02	(8.10±0.92) E-03
0.6	(9.13±0.45) E-02	(4.42±0.23) E-02	(2.41±0.19) E-02	(1.39±0.10) E-02	(9.10±0.83) E-03

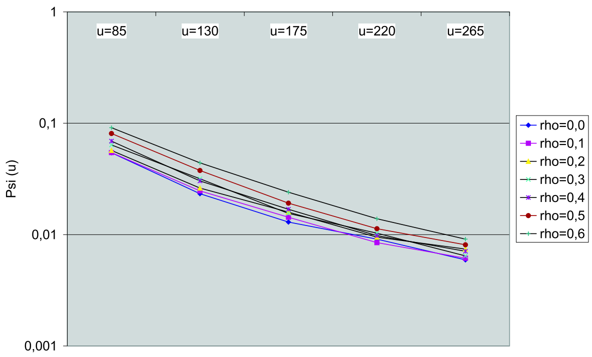

FIGURE 4.11: F∼ PAR(2.41,5.85), FRANK COPULA

TABLE 4.14: SIMULATED RUIN PROBABILITIES FOR PARETO DISTRIBUTED CLAIMS AND NORMAL COPULA

F∼ Par(2.41,5.85), NorC, c=210, λ=15, N=30000					
$\rho\backslash u$	85	130	175	220	265
0.0	(5.12±0.33) E-02	(2.45±0.14) E-02	(1.43±0.14) E-02	(9.23±1.27) E-03	(5.77±0.61) E-03
0.1	(5.78±0.23) E-02	(2.69±0.09) E-02	(1.52±0.12) E-02	(8.97±1.59) E-03	(5.90±0.69) E-03
0.2	(6.25±0.29) E-02	(3.07±0.16) E-02	(1.78±0.18) E-02	(1.10±0.12) E-02	(8.67±0.67) E-03
0.3	(6.83±0.35) E-02	(3.43±0.19) E-02	(2.02±0.17) E-02	(1.24±0.12) E-02	(8.10±0.86) E-03
0.4	(8.31±0.29) E-02	(4.14±0.30) E-02	(2.49±0.14) E-02	(1.53±0.18) E-02	(1.03±0.13) E-02
0.5	(9.71±0.53) E-02	(5.23±0.24) E-02	(3.21±0.21) E-02	(2.16±0.16) E-02	(1.46±0.10) E-02
0.6	(1.15±0.03) E-01	(6.60±0.30) E-02	(4.56±0.31) E-02	(3.06±0.21) E-02	(2.20±0.08) E-02

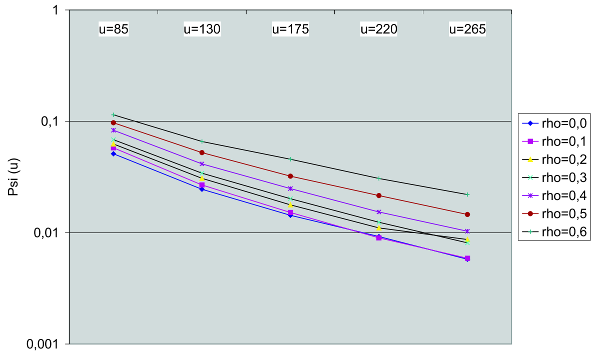

FIGURE 4.12: F∼ PAR(2.41,5.85), NORMAL COPULA

5 Discussion

The simulation results of Section 4 clearly indicate, that for surplus processes with light-tailed marginal distributions for the claim sizes a Lundberg exponent still exists, if dependence of the above kind is introduced into the model (note that the diagrams are drawn on logarithmic scale for the ψ-axis). If the degree of (positive) dependence increases, the numerical value of the Lundberg exponent decreases. Moreover, the simulated data suggest that the Lundberg exponent is a linear function of Spearman's ρ for the range of ρ under consideration, which in turn indicates that ρ is a suitable measure for the degree of dependence also in this respect (by calculating the corresponding F-statistic, it turns out that the hypothesis of a linear relationship between ρ and R holds on a 99%-significance level for each of the above regression lines).

On comparison of the ruin probabilities of the corresponding processes for fixed ρ among the different copula structures, the simulation results show that, whereas Frank's copula and the Gaussian copula yield relatively similar results, the impact of ρ on ψ is much more pronounced in the case of the Gumbel copula, for any simulated marginal distribution of the claim sizes. Apart from the numerical values of the ruin probabilities this can also be seen by comparing the slopes of the regression lines $R = R(\rho)$ (for the light-tailed distributions). Due to the upper tail dependence property of the Gumbel Copula this was somewhat to be expected. Altogether these results indicate that the knowledge of ρ and the marginal distributions alone can lead to quite different results for the ruin probability depending on the copula used.
Note once again that the estimates of Table 4.1 and 4.2 stand somehow separate in that they refer to a model where the consecutive r.v. $c(T_i - T_{i-1}) - X_i$ instead of the claims X_i alone are dependent according to the given copula structure.
Tables 4.9, 4.10 and 4.11 show that a reduction of the coefficient of variation of the claim size distribution leads to a considerable reduction of the ruin probability for any of the investigated copula structures. Since for each estimate we have $N = 30000$ simulation runs, the accuracy of probability estimates of order 10^{-4} for CMC is rather poor, which is reflected by the confidence intervals in the last rows of these three tables. We have nevertheless included these values in the tables to allow comparisons with the other claim size distributions (for $\rho = 0$ the exact value of $\psi(u)$ in Tables 4.9, 4.10 and 4.11 can be calculated by standard techniques (see e.g. SIEGL AND TICHY [16]) and it turns out that

the first two significant digits of the corresponding estimates obtained in Table 4.9 are actually accurate).

Of course, in case of Pareto distributed claim sizes, the notion of a Lundberg exponent is meaningless, but information on the behavior of the ruin probability for various dependence structures of a representative of heavy-tailed claim size distributions is of independent interest. From Tables 4.12, 4.13 and 4.14 it follows that the difference of ruin probabilities for fixed ρ among the copulas is more pronounced than for light-tailed claim size distributions.

Finally, as can be seen in Tables 4.4 and 4.5, the importance sampling technique for a dependent scenario developed in this paper is a considerable improvement to the CMC-algorithm in terms of variances. In principle it can be applied to any risk process of this type whenever it is possible to calculate the conditional densities involved.

References

[1] H. Albrecher. Dependent risks and ruin probabilities in insurance. *IIASA Interim Report*, IR-98-072, 1998.

[2] S. Asmussen. *Ruin probabilities.* World Scientific, Singapore, 2000.

[3] S. Asmussen and K. Binswanger. Simulation of ruin probabilities for subexponential claims. *Astin Bull.*, 27(2):297–318, 1997.

[4] E. Bouye, A. Nikeghbali, G. Riboulet, and T. Roncalli. Copulas for finance. *Value at Risk Working Papers*, 2000.

[5] M. Cottrell, J. Fort, and G. Malgouyres. Large deviations and rare events in the study of stochastic algorithms. *IEEE Transactions on Automatic Control*, 28(11):907–920, 1983.

[6] P. Embrechts, A. McNeil, and D. Straumann. Correlation and dependence in risk management: Properties and pitfalls. *RiskLab Research Paper, ETH Zrich*, 1999.

[7] E. Frees and E. Valdez. Understanding relationships using copulas. *North American Actuarial Journal*, 2(1), 1998.

[8] H. U. Gerber. Ruin theory in the linear model. *Insurance: Mathematics and Economics*, 1:177–184, 1982.

[9] J. Grandell. *Aspects of Risk Theory.* Springer, New York, 1992.

[10] H. Joe. *Multivariate Models and Dependence concepts.* Chapman & Hall, London, 1997.

[11] T. Lehtonen and H. Nyrhinen. Simulating level-crossing probabilities by importance sampling. *Adv. Appl. Prob.*, 24:858–874, 1992.

[12] R. Nelsen. *An Introduction to copulas.* Lecture notes in statistics 139. Springer, Berlin, 1999.

[13] H. Nyrhinen. Rough descriptions of ruin for a general class of surplus processes. *Adv. Appl. Prob.*, 30:1008–1026, 1998.

[14] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. *Numerical Recipes in C.* Cambridge University Press, Cambridge, 1992.

[15] S. D. Promislow. The probability of ruin in a process with dependent increments. *Insurance: Mathematics and Economics*, 10:99–107, 1991.

[16] T. Siegl and R. F. Tichy. Lösungsmethoden eines Risikomodells bei exponentiell fallender Schadensverteilung. *Mitteilungen der schweizer Vereinigung der Versicherungsmathematiker*, 1:85–118, 1996.

[17] D. Tjostheim. Measures of dependence and tests of independence. *Statistics*, 28:249–284, 1996.

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