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Data Envelopment Analysis (DEA)

**A pedagogical guide for decision makers
in the public sector**

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1 INTRODUCTION

This guide introduces Data Envelopment Analysis (DEA), a performance measurement technique, in such a way as to be appropriate to decision makers with little or no background in economics and operational research. The use of mathematics is kept to a minimum. This guide therefore adopts a strong practical approach in order to allow decision makers to conduct their own efficiency analysis and to easily interpret results.

DEA helps decision makers for the following reasons:

- By calculating an efficiency score, it indicates if a firm is efficient or has capacity for improvement.
- By setting target values for input and output, it calculates how much input must be decreased or output increased in order to become efficient.
- By identifying the nature of returns to scale, it indicates if a firm has to decrease or increase its scale (or size) in order to minimize the average cost.
- By identifying a set of benchmarks, it specifies which other firms' processes need to be analysed in order to improve its own practices.

After this introduction, Chapter 2 presents the essentials about DEA, alongside a case study to intuitively understand its application. Chapter 3 introduces *Win4DEAP*, a software package that conducts efficiency analysis based on DEA methodology. Chapter 4 is dedicated to more demanding readers interested in the methodical background of DEA. Four advanced topics of DEA (adjustment to the environment; preferences; sensitivity analysis; time series data) are presented in Chapter 5. Finally, Chapter 6 shows how to program the Solver in *Microsoft Excel* © in order to run a basic DEA efficiency analysis.

2 BASICS OF DEA

2.1 AN EFFICIENCY MEASUREMENT METHOD

DEA is used to measure the performance of firms or entities (called Decision-Making Units –DMUs–) which convert multiple inputs into multiple outputs. Firm efficiency is defined as the ratio of the sum of its weighted outputs to the sum of its weighted inputs (Thanassoulis *et al.*, 2008, p. 264). DEA is suitable for the use of both private sector firms and public sector organizations (and even for entities such as regions, countries, etc.)¹. It was formulated in Charnes *et al.* (1978, 1981) in order to evaluate a US federal government program in the education system called ‘Program Follow Through’. The use of DEA then spread to other public organizations (hospitals, aged-care facilities, social service units, unemployment offices, police forces, army units, prisons, waste management services, power plants, public transportation companies, forestry companies, libraries, museums, theatres, etc.) and to the private sector (banks, insurance companies, retail stores, etc.).

Each firm’s efficiency score is calculated relative to an efficiency frontier. Firms located on the efficiency frontier have an efficiency score of 1 (or 100%). Firms operating beneath the frontier have an efficiency score inferior to 1 (or 100%) and hence have the capacity to improve future performance. Note that no firm can be located above the efficiency frontier because they cannot have an efficiency score greater than 100%. Firms located on the frontier serve as benchmarks –or peers– to inefficient firms. These benchmarks (i.e. real firms with real data) are associated with best practices. DEA is therefore a powerful benchmarking technique.

¹ To keep it simple and to make this pedagogical guide more understandable, the term ‘firm’ is used in a generic way.

2.2 CASE STUDY 1

To better understand the mechanics behind DEA, this section develops a simple practical case study. It includes only one input and one output, although DEA can handle multiple inputs and multiple outputs.

Five register offices (A to E) produce one output (total number of documents, such as marriage or birth certificates) with one input (number of full-time equivalent public servants)². The data are listed in Table 1. For example, two public servants work in Register Office A. They produce one document (during a certain period of time).

TABLE 1:
Case study 1 – Five register offices produce documents with public servants.

Register Office	Input	Output
	Public servants (x)	Documents (y)
A	2	1
B	3	4
C	5	5
D	4	3
E	6	7

a) Case study 1 – Two basic DEA models

Two basic models are used in DEA, leading to the identification of two different frontiers:

- The first model assumes constant returns to scale technology (CRS model). This is appropriate when all firms are operating at an optimal scale. However, note that this is quite an ambitious assumption. To operate at an optimal scale, firms should evolve in a perfectly competitive environment, which is seldom the case. The CRS model calculates an efficiency score called constant returns to scale technical efficiency (CRSTE).
- The second model assumes variable returns to scale technology (VRS model). This is appropriate when firms are

² Note that DEA can handle more outputs and inputs. In order to represent this example in a two-dimensional graph, we consider a total of two outputs and inputs of two (one output, one input; no variable representing the quality of the variables).

not operating at an optimal scale. This is usually the case when firms face imperfect competition, government regulations, etc. The VRS model calculates an efficiency score called variable returns to scale technical efficiency (VRSTE).

Comparison between the two models reveals the source of inefficiency. Constant returns to scale technical efficiency corresponds to the global measure of firm performance. It is composed by a 'pure' technical efficiency measure (captured by the variable returns to scale technical efficiency score) and a scale efficiency measure (SE). Section 4.2 demonstrates how these three notions (CRSTE, VRSTE and SE) relate to each other.

b) Case study 1 – Input or output orientation

A DEA model can be input or output oriented:

- In an input orientation, DEA minimizes input for a given level of output; in other words, it indicates how much a firm can decrease its input for a given level of output.
- In an output orientation, DEA maximizes output for a given level of input; in other words, it indicates how much a firm can increase its output for a given level of input.

The efficiency frontier will be different in a CRS or a VRS model (see Section 4.2). However, within each model, the frontier will not be affected by an input or an output orientation. For example, the efficiency frontier under VRS will be exactly the same in an input or an output orientation. Firms located on the frontier in an input orientation will also be on the frontier in an output orientation.

In a CRS model, technical efficiency scores have the same values in an input or an output orientation. But these values will be different according to the model's orientation when VRS is assumed. However, Coelli and Perelman (1996, 1999) note that, in many instances, the choice of orientation has only a minor influence upon the technical efficiency scores calculated in a VRS model.

Choosing between an input or an output orientation

The model's orientation should be selected according to which variables (inputs or outputs) the decision maker has most control over. For example, a school principal will probably have more control over his teaching staff (input) than over the number of pupils (output). An input orientation will be more appropriate in this case.

In the public sector, but sometimes also in the private, a given level of input can be granted and secured to a firm. In this case, the decision maker may want to maximize the output (and therefore choose an output orientation). Alternatively, if the decision maker's task is to produce a given level of output (e.g. a quota) with the minimum input, he will opt for an input orientation.

If the decision maker is not facing any constraints and has control of both input and output, the model's orientation will depend on his objectives. Does he need to cut costs (input orientation) or does he want to maximize production (output orientation)?

c) Case study 1 – CRS efficient frontier

Figure 1 represents the efficient frontier assuming constant returns to scale technology (CRS efficient frontier). The CRS efficient frontier starts at the origin and runs through Register Office B. Register Office B happens to be the observation with the steepest slope, or the highest productivity ratio, among all register offices ($4 / 3 = 1.33$, meaning that one public servant produces 1.33 documents). Register Office B is on the frontier; it is 100% efficient. Register Offices A, C, D and E are beneath the frontier. Their respective efficiency scores are less than 100%. DEA assumes that the production possibility set is bounded by the frontier. This actually implies that DEA calculates relative and not absolute efficiency scores. Although firms on the efficient frontier are granted a 100% efficiency score, it is likely that they could further improve their productivity.

FIGURE 1:

Case study 1 – Register offices beneath the efficient frontier have the capacity to improve performance.

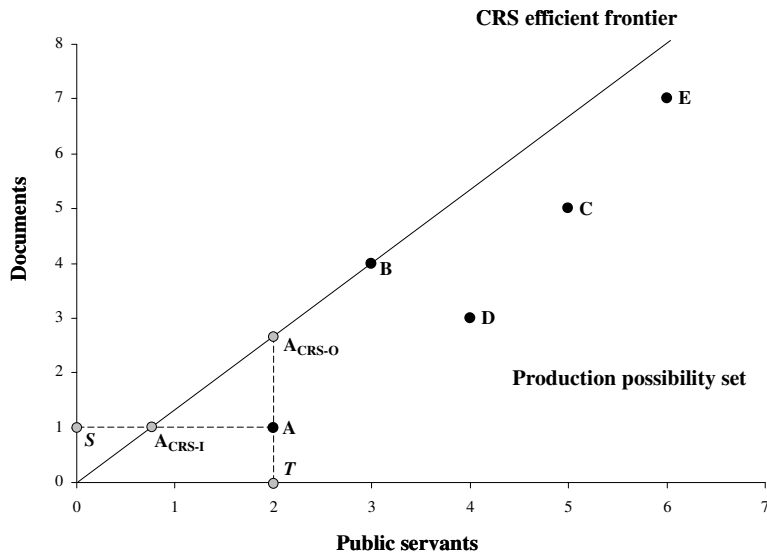


Figure 1 also illustrates how DEA measures efficiency scores. The example of Register Office A is described below:

- In an input orientation, A's efficiency score is equal to the distance SA_{CRS-I} divided by the distance SA . A_{CRS-I} is the projection of point A on the efficient frontier (assuming constant returns to scale –CRS– and an input orientation –I). Note that one can easily calculate efficiency scores using a ruler and measuring the distances on the graph. A's score is 37.5%. This means that Register Office A could reduce the number of public servants employed (input) by 62.5% ($100 - 37.5$) and still be able to produce the same number of documents (one).
- In an output orientation, A's efficiency score is equal to the distance TA divided by the distance TA_{CRS-O} . A_{CRS-O} is the projection of point A on the efficient frontier (assuming

constant returns to scale –CRS– and an output orientation –O–). A's score is 37.5%, as in an input orientation³. This means that Register Office A could increase its production of documents (output) by 62.5% (100 - 37.5) whilst holding the number of public servants constant at two.

d) Case study 1 – VRS efficient frontier

Figure 2 represents the efficient frontier assuming variable returns to scale technology (VRS efficient frontier). The VRS efficient frontier is formed by enveloping all the observations. Register Offices A, B and E are on the frontier. They are 100% efficient. Register Offices C and D are beneath the frontier. Their respective efficiency scores are inferior to 100%. DEA assumes that the production possibility set is bounded by the frontier. Again, this implies that DEA calculates relative and not absolute efficiency scores. Although firms on the efficient frontier are granted a 100% efficiency score, it is likely that they could further improve their productivity.

³ Note that the efficiency scores in a CRS model are always the same for an input or an output orientation.

FIGURE 2:

Case study 1 – For the same level of input, Register Office D could improve its output up to the projected values of point D_{VRS-O} (i.e. from 3 to 5 documents).

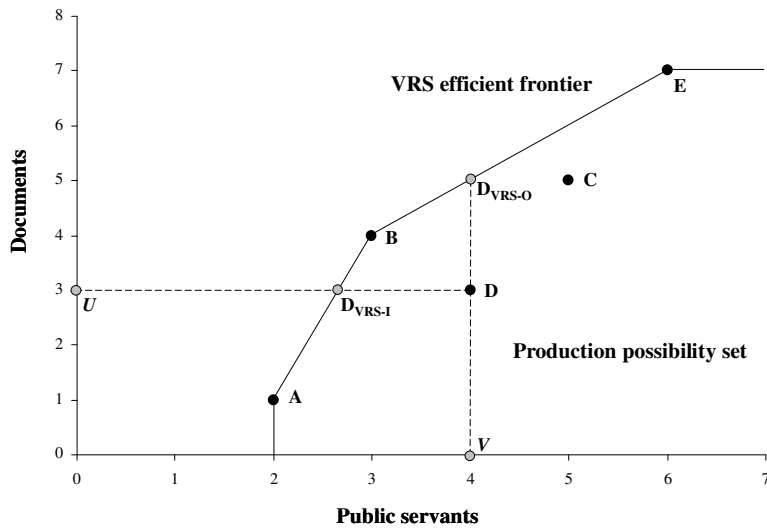


Figure 2 also illustrates how DEA measures efficiency scores. The example of Register office D is described below:

- In an input orientation, D's efficiency score is equal to the distance UD_{VRS-I} divided by the distance UD . D_{VRS-I} is the projection of point D on the efficient frontier (assuming variable returns to scale –VRS– and an input orientation –I–). Note that one can easily calculate efficiency scores using a ruler and measuring the distances on the graph. D's score is 66.7%. This means that Register Office D could reduce the number of public servants employed (input) by 33.3% (100 - 66.7) and still be able to produce the same number of documents (three).
- In an output orientation, D's efficiency score is equal to the distance VD divided by the distance VD_{VRS-O} . D_{VRS-O} is the

projection of point D on the efficient frontier (assuming variable returns to scale –VRS– and an output orientation –O–). D's score is 60%⁴. This means that Register Office D could increase its production of documents (output) by 40% (100 - 60) whilst holding the number of public servants constant at four.

How to interpret efficiency scores according to the DEA model's output or input orientation

Register Office C has an efficiency score of 75% in the CRS model. It will get the same efficiency score in an output or in an input-oriented model under the constant returns to scale assumption. However:

- In the input-oriented model, the capacity to improve input (i.e. a reduction) by 25% (100 - 75) is calculated using the original input value of 5 public servants. The DEA model calculates a projected value of 3.75. The 25% improvement is then calculated according to the **original** value: $[(5 - 3.75) / 5] \times 100 = 25$. From a practical point of view, the capacity to improve input by 25% means that the Register Office should reduce all of its inputs by 25% in order to become efficient.
- In the output-oriented model, the capacity to improve output (i.e. an augmentation) by 25% (100 - 75) is calculated using the projected output value. Register Office C has an original output value of 5 documents. The DEA model calculates a projected value of 6.67 documents. The 25% improvement is calculated according to the **projected** value: $[(6.67 - 5) / 6.667] \times 100 = 25$. From a practical point of view, the capacity to improve output by 25% means that the Register Office should augment all of its outputs by 25% in order to become efficient.

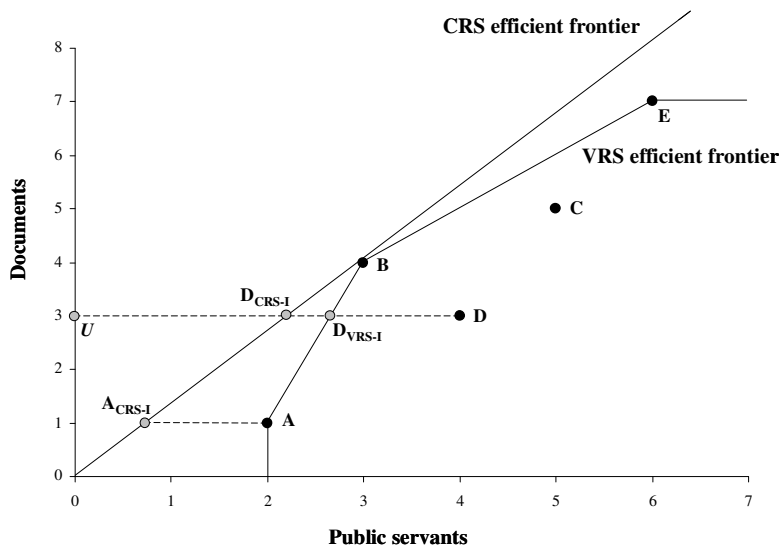
⁴ Note that the efficiency scores in a VRS model are different for an input or an output orientation.

e) Case study 1 – CRS, VRS and scale efficiency

Figure 3 represents the CRS and the VRS efficient frontiers on the same graph. Register Office B is CRS and VRS efficient, as it is located on both frontiers. Register Offices A and E are efficient under the variable returns to scale assumption but inefficient under the constant returns to scale assumption. Finally, Register Offices D and C are both CRS and VRS inefficient; they are located neither on the CRS nor on the VRS frontiers.

FIGURE 3:

Case study 1 – Register Offices A and E are VRS efficient but CRS inefficient.



The gap observed between the CRS and the VRS frontiers is due to a problem of scale. For example, Register Office A is VRS efficient. To become CRS efficient, Register Office A should modify its scale (or size). Only by operating at point A_{CRS-I} would Register Office A be as productive as Register Office B, which is the only CRS efficient Register Office.

Some Register Offices (D and C) are not even located on the VRS frontier. These Register Offices not only have a scale problem but are also poorly managed. For example, Register Office D should move to point D_{VRS-I} located on the VRS frontier in order to become VRS efficient (i.e. to eliminate the inefficiency attributable to poor management). Furthermore, Register Office D should move from point D_{VRS-I} to point D_{CRS-I} located on the CRS frontier in order to become CRS efficient (i.e. to eliminate the inefficiency attributable to a problem of scale).

As a result, the CRS efficiency (also called 'total' efficiency) can be decomposed into two components: the VRS efficiency (also called 'pure' efficiency) and the scale efficiency. The following ratios represent these three types of efficiency for Register Office D (input orientation).

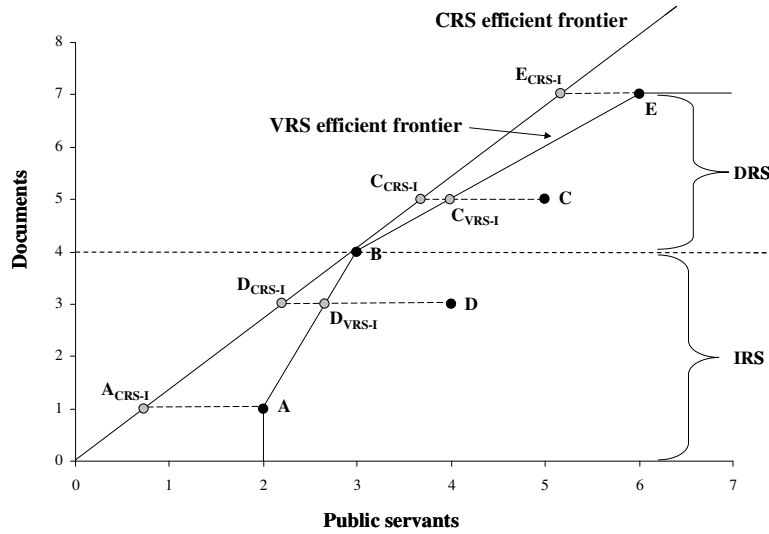
Technical efficiency of D under CRS	Technical efficiency of D under VRS	Scale efficiency of D
$TE_{CRS} = \frac{UD_{CRS-I}}{UD} = 56.3\%$	$TE_{VRS} = \frac{UD_{VRS-I}}{UD} = 66.7\%$	$SE = \frac{UD_{CRS-I}}{UD_{VRS-I}} = 84.4\%$

f) Case study 1 – Nature of returns to scale

The nature of returns to scale of register offices not located on the CRS frontier (in other words, scale inefficient) has to be identified. Figure 4 represents the CRS efficient points A_{CRS-I} and E_{CRS-I} of Register Offices A and E (which are CRS inefficient but VRS efficient). It also represents the CRS efficient points D_{CRS-I} and C_{CRS-I} and the VRS efficient points D_{VRS-I} and C_{VRS-I} of Register Offices D and C (which are CRS and VRS inefficient).

FIGURE 4:

Case study 1 – Register Offices A and D face increasing returns to scale –IRS– (economies of scale); C and E face decreasing returns to scale –DRS– (diseconomies of scale).



To identify the nature of returns to scale, one has to focus on the slope of the VRS efficient points A, D_{VRS-I}, B, C_{VRS-I} and E (or productivity). Three situations can occur:

- A register office is located both on the CRS and the VRS efficient frontiers (such as point B). Register Office B has the highest productivity of all VRS efficient points ($4 / 3 = 1.33$). It is facing constant returns to scale. Such a firm reaches its optimal size (or efficient scale)⁵. It is operating at a point where the scale (or size) has no impact on productivity. This situation occurs when the average inputs consumption is minimized and does not vary with output. In a situation of constant returns to

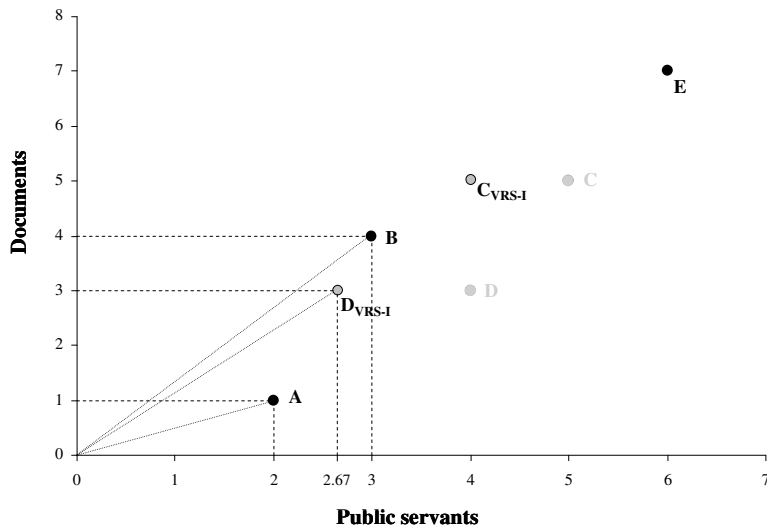
⁵ In the economic context, a firm operates at the optimal size (or efficient scale) when it minimizes its average cost. In the context of DEA, we can measure efficiency in physical or in monetary terms. Because cost and price information is not always available or appropriate, the use of technical efficiency is often preferred. As this latter measure is based on physical terms, we prefer to use the expression of average inputs consumption instead of average cost.

scale, an increase in output of 1 percent requires a proportionate increase in input (i.e. 1 percent).

- A register office (or the projected point of a register office) is located at a point where the scale (or the size) has a positive impact on productivity. Points A and D_{VRS-I} are in such a position (see Figure 5). The productivity of A ($1 / 2 = 0.5$) is inferior to the productivity of D_{VRS-I} ($3 / 2.67 = 1.12$). The ratio of productivity is increasing with the scale. This situation occurs until point B, which has a productivity of 1.33. Register Offices A and D are therefore facing increasing returns to scale (IRS) –or economies of scale–. In this situation, the average inputs consumption declines whilst output rises. Register Offices A and D have not yet reached their optimal size (or efficient scale). To improve their scale efficiency, they have to expand their output. In a situation of economies of scale, a variation in output of 1 percent results in a variation in input of less than 1 percent. Hence, an increase in output results in a reduction of the average inputs consumption.

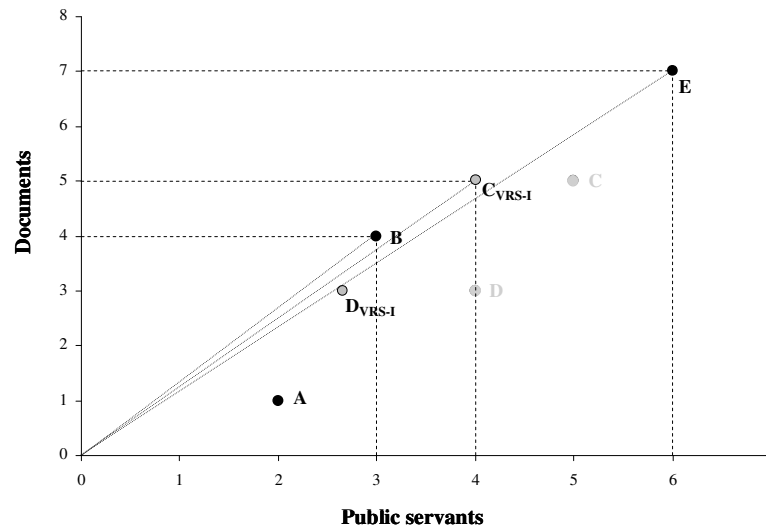
FIGURE 5:

Case study 1 – The ratio of productivity is increasing with the scale.



- A register office (or the projected point of a register office) is located at a point where the scale (or the size) has a negative impact on productivity. Points C_{VRS-I} and E are in such a position (see Figure 6). The productivity of C_{VRS-I} ($5 / 4 = 1.25$) is superior to the productivity of E ($7 / 6 = 1.17$). The ratio of productivity is decreasing with the scale. This situation occurs from point B, which has a productivity of 1.33. Register Offices C and E are therefore facing decreasing returns to scale (DRS) –or diseconomies of scale–. In this situation, the average inputs consumption rises whilst output rises. Register Offices C and E have exceeded their optimal size (or efficient scale). To improve their scale efficiency, they have to reduce their output. In a situation of diseconomies of scale, a variation in output of 1 percent results in a variation in input of more than 1 percent. Hence, a decrease in output results in a reduction of the average inputs consumption.

FIGURE 6:
Case study 1 – The ratio of productivity is decreasing with the scale.



The specific cases of the five Register offices are described below (see Figure 4):

- Register Office A is located on the VRS frontier but not on the CRS frontier. Its inefficiency is due to an inappropriate scale. A is facing increasing returns to scale. A variation in output of 1 percent results in a variation in input of less than 1 percent.
- Register Office D is neither located on the CRS nor on the VRS frontier. Its inefficiency is due to an inappropriate scale and to poor management. D is facing increasing returns to scale. A variation in output of 1 percent results in a variation in input of less than 1 percent.
- Register Office B is located both on the CRS and on the VRS frontier. It has no inefficiency at all. B is facing constant returns to scale. A variation in output of 1 percent results in a variation in input of 1 percent.

- Register Office C is neither located on the CRS nor on the VRS frontier. Its inefficiency is due to an inappropriate scale and to poor management. C is facing decreasing returns to scale. A variation in output of 1 percent results in a variation in input of more than 1 percent.
- Register office E is located on the VRS frontier (but not on the CRS frontier). Its inefficiency is due to an inappropriate scale. E is evolving in a situation of decreasing returns to scale. A variation in output of 1 percent results in a variation in input of more than 1 percent.

g) Case study 1 – Peers (or benchmarks)

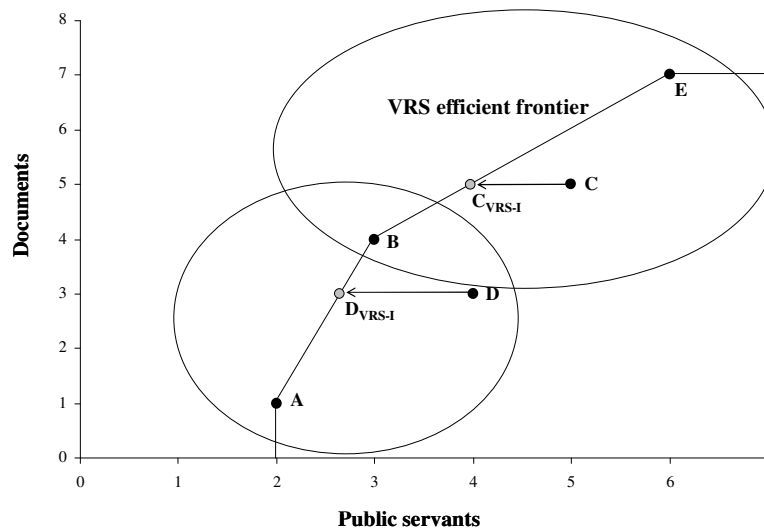
DEA identifies, for each inefficient firm, the closest efficient firms located on the frontier. These efficient firms are called peers or benchmarks. If inefficient firms want to improve their performance, they have to look at the best practices developed by their respective peers.

Under the CRS assumption, Register Office B is the only firm located on the efficient frontier. Hence it is identified as the peer for all other inefficient register offices.

Figure 7 illustrates the peers under the VRS assumption. Three Register Offices (A, B and E) are located on the efficient frontier. Two Register Offices (C and D) are inefficient. Register Office C has two assigned peers: B and E. C_{VRS-I} , the projected point of C on the VRS frontier, lies between these two benchmarks. Register Office D also has two assigned peers: A and B. D_{VRS-I} , the projected point of D on the VRS frontier, lies between these two benchmarks.

FIGURE 7:

Case study 1 – Register Offices A and B are peers of Register Office D; Register Offices B and E are peers of Register Office C.



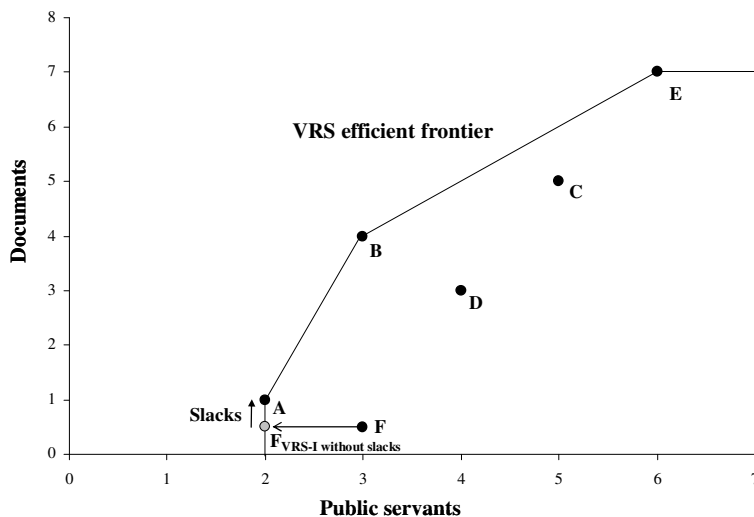
h) Case study 1 – Slacks

Particular positions located on the frontier are inefficient. Assume there is an additional register office in our sample, F. It produces 0.5 document with two public servants. Figure 8 illustrates the efficient frontier under VRS. Register Office F is not located on the frontier. In order to become efficient, it has first to move to point $F_{VRS-I \text{ without slacks}}$. At this location, Register Office F should have an efficiency score of 100%, as it is located on the frontier. But Register Office A, next to him on the frontier, is also 100% efficient. The difference between F and A is striking. With the same number of inputs (two public servants), F produces 0.5 document and A produces one document (i.e. 0.5 more than F). Therefore point $F_{VRS-I \text{ without slacks}}$ cannot be considered as 100% efficient, because it produces less output with the same amount of input than another register office (A). To get a 100% efficiency score, point $F_{VRS-I \text{ without slacks}}$ has to move further up to point A. This additional improvement needed for a firm to become efficient is called a slack.

Indeed, every point located on the sections of the frontier which run parallel to either the x or the y axes has to be adjusted for slacks. DEA is designed to take slacks into account.

FIGURE 8:

Case study 1 – DEA adjusts the projected values of inefficient firms to take slacks into account.



2.3 MULTIPLE OUTPUTS AND INPUTS

DEA allows multiple outputs and multiple inputs to be taken into account. For example, a shirt company uses machines, workers and tissue (three inputs) in order to produce T-shirts, pants and underwear (three outputs). DEA can account for all of these variables and even more. As a result, DEA goes far beyond the calculation of single productivity ratios such as, for example, the number of T-shirts produced per worker (one output divided by one input).

However, the total number of outputs and inputs being considered is not limitless from a practical point of view. It depends on the number of firms in the data set. If the number of firms is smaller than, roughly

speaking, three times the sum of the total number of inputs and outputs, it is highly probable that several firms, if not all, will obtain a 100% score⁶. For example, a dataset containing 21 shirt companies allows a total of seven outputs and inputs to be dealt with (21 divided by 3). As Cooper *et al.* (2006, p. 106) point out, “if the number of DMUs (n) is less than the combined number of inputs and outputs ($m + s$), a large portion of the DMUs will be identified as efficient and efficiency discrimination among DMU is questionable due to an inadequate number of degrees of freedom. (...). Hence, it is desirable that n exceeds $m + s$ by several times. A rough rule of numbs in the envelopment model is to choose n (= the number of DMUs) equal to or greater than $\max \{m \times s, 3 \times (m + s)\}$.”

DEA measures firm efficiency based on multiple outputs and multiple inputs. If Shirt Company A produces a lot of T-shirts but only a few pants and underwear, DEA will automatically attribute a high weighting to the T-shirts variable in order to emphasize this strength. As a result, DEA ‘automatically’ optimizes the weighting of each variable in order to present each firm in the best possible light.

The particularity of DEA is that weights assigned to outputs and inputs are not decided by users. Moreover, it does not use a common set of weights for all firms. Instead, a different set of weights is calculated by a linear optimization procedure.

Unfortunately, DEA does not work with negative or zero values for inputs and outputs. However, zero values can be substituted with very low values such as 0.01.

It is also noted that each DMU must have the same number of inputs and outputs in order to be compared, otherwise DEA cannot be applied.

A distinction has to be made between variables which are under the control of management (discretionary variables) and variables which are not (non-discretionary or environmental variables). Ideally, a DEA model will exclusively include discretionary variables although some

⁶ *The higher the number of inputs and outputs that are taken into consideration for a given number of firms, the more probable it is that each firm will be the best producer of at least one of the outputs. Therefore, all firms could obtain a 100% efficiency score.*

DEA models can also accommodate non-discretionary. In a second step, efficiency scores can be adjusted to account for environmental variables (i.e. such variables influence the efficiency of a firm but are not a traditional input and are not under the control of the manager).

Moreover, variables should reflect both quantitative and qualitative characteristics of firms' resources and services. Although it may not be easy to identify and to convert qualitative characteristics into numbers, it is desirable to include such variables in the model in order to appropriately benchmark firms.

2.4 TYPES OF EFFICIENCY

The notion of efficiency refers to an optimal situation; the maximum output for a given level of input or the minimum input for a given level of output. Subject to data availability, several types of efficiency can be measured:

- Technical efficiency, in which both outputs and inputs are measured in physical terms⁷.
- Cost efficiency: identical to technical efficiency, except that cost (or price) information about inputs is added to the model.
- Revenue efficiency: identical to technical efficiency, except that price information about outputs is added to the model.
- Profit efficiency: identical to technical efficiency, except that cost information about inputs and price information about outputs are added to the model.

⁷ This pedagogical guide will focus on the measurement of technical efficiency for two main reasons: first, firms in the public sector are often not responsible for the age pyramid of their employees; therefore taking into account the wages of the employees (which often grow higher alongside seniority) would unfairly alter efficiency of a firm with a greater proportion of senior employees; second, firms in the public sector do not often produce commercial goods or services with a set price.

Technical efficiency is a global measure of firm performance. However, it does not indicate the source of inefficiency. This source could be twofold:

- First, the firm could be poorly managed and operated.
- Second, it could be penalised for not operating at the right scale.

Technical efficiency can be decomposed into a 'pure' technical efficiency measure and a scale efficiency measure to reflect these two sources of inefficiency⁸.

2.5 MANAGERIAL IMPLICATIONS

DEA is a benchmarking technique. The efficiency scores provide information about a firm's capacity to improve output or input. In this sense, DEA offers strong support to decision making. To conduct an efficiency analysis and to interpret results often raises practical questions. The following list of frequently asked questions offers some advice.

- **Is it advisable to involve the managers of the firms to be benchmarked in the efficiency analysis from the beginning of the process?**

Yes, it is, and for two main reasons. First, managers know the processes of their firms and the data available. Therefore they are the right persons to pertinently identify which inputs and outputs have to be integrated into the analysis. Second, managers involved from the beginning of the process are more likely to accept the results of the analysis (rather than to reject them) if they have been involved in the process.

⁸ *The firm's management team will definitely be held responsible for the 'pure' technical efficiency score. In a situation where it does not have the discretionary power to modify the firm's size, it will likely not be accountable for the scale efficiency score. However, especially in the private sector, one has the choice of the scale at which it operates: the management team can easily downsize the firm and, with some efforts, upsize it also.*

- **How should one respond to managers who claim that their firms are different from others, and therefore cannot be compared to them?**

Sometimes, inefficiencies can be explained by indisputable environmental variables. But sometimes they cannot. Managers often justify the low efficiency scores of their firms by arguing that their situations are different compared to the situations of the other firms. They claim to be a 'special case' (and therefore it is acceptable to be inefficient). Actually, the majority of firms could possibly claim to be different as most possess a specificity that others do not have. However, it is likely that the difference of one firm will be compensated by the difference of another. More generally, it is up to the managers to prove that they really face a hostile environment. If they cannot prove it, management measures have to be taken to improve efficiency.

- **Assume that a firm obtains an efficiency score of 86.3%. Does this number have to be strictly applied?**

Not really, it should be interpreted more as an order of magnitude. This order of magnitude informs managers that they have to increase their outputs or to decrease their inputs in order to become more efficient. But one should not focus too strictly on the capacity for 13.7% improvement. Such a number could be interpreted by practitioners as too 'accurate' and may offend their sensibilities. Therefore it is better to consider efficiency scores more as more of an objective basis to hold an open discussion about the way to improve firm efficiency rather than a number to be strictly applied.

- **A firm faces increasing returns to scale. It has economies of scale. What does that concretely mean from a managerial point of view?**

Such a firm has not yet reached its optimal size. In order to reduce its average cost (or its average inputs consumption), it has to increase its size. Practically, this could be done either by internal growth (i.e. producing more output) or by merging

with another firm which is also facing increasing returns to scale. If, for some reason, managers cannot influence the scale of a firm, they should not be held accountable for this source of inefficiency.

- **A firm faces decreasing returns to scale. It has diseconomies of scale. What does that concretely mean from a managerial point of view?**

Such a firm is already oversized, having exceeded its optimal size. In order to reduce its average cost (or its average inputs consumption), it has to decrease its size. Practically, this could be done either by internal decay (i.e. producing less output) or by splitting the firm into two separate businesses. Note that some of the production could be transferred to a firm facing increasing returns to scale. If, for some reason, managers cannot influence the scale of a firm, they should not be held accountable for this source of inefficiency.

- **Is efficiency the only criteria to assess a firm's performance?**

Not necessarily. Basically, the assessment of a firm's performance will depend on the management objectives. Other criteria such as effectiveness or equity are often considered alongside efficiency. If this is the case, the overall performance should be balanced with the various criteria.

- **One firm obtains a score of 100% but all the others in the dataset obtain much lower scores (for example, starting at 40% or lower). Is this realistic?**

It could be realistic, but the gap appears to be important. In such a case, data have to be carefully checked, and especially data of the efficient firm. If a data problem is not identified, such results mean that the efficient firm is likely to have completely different processes than the other firms. It should therefore be absolutely presented as a best practice model. However, even if they are realistic, such results are likely to be rejected by managers whose firms have low efficiency scores.

These managers are likely to be discouraged because it is obviously unrealistic for them to improve their firm's efficiency by 60% (or more) in the short run. Therefore it is better to exclude the efficient firm from the sample and to run a new model.

- **Almost all the firms obtain an efficiency score of 100%. Does that mean that all of them are really efficient?**

Yes, it could mean that all the firms are efficient. Such results would be great! But they are unlikely. Here, the total number of inputs and outputs is probably too high compared to the number of firms in the dataset. In this case, one variable has to be excluded and a new model has to be run. If the number of firms obtaining a 100% score decreases, it indicates that the number of variables was too high compared to the number of firms. If not, all the firms are just efficient and must be congratulated.

- **The model does not show any results. What does that mean?**

Data has to be checked. This could happen when data with a value of zero are in the set. Zeros have to be substituted by a very small number (0.01).

Exercise 1

The following multiple choice questions test one's knowledge on the basics of DEA. Only one answer is correct. Solutions are listed at the end of this exercise.

1. What is the main purpose of DEA?
 - a) DEA measures firms' effectiveness
 - b) DEA measures firms' efficiency
 - c) DEA measures firms' profit
 - d) DEA measures firms' productivity

2. A dataset includes information about input quantity, input cost and output quantity. Which type of efficiency cannot be measured?
 - a) Technical efficiency
 - b) Cost efficiency
 - c) Revenue efficiency
 - d) Scale efficiency
3. 'Pure' technical efficiency reflects:
 - a) A global measure of firm performance
 - b) The efficiency of a firm operating at an incorrect scale
 - c) A measure of profit efficiency
 - d) The efficiency of a poorly managed firm
4. Firm A is inefficient. Who is its peer(s)?
 - a) One or several firms whose efficiency scores are worse than firm A's efficiency
 - b) One or several firms whose efficiency scores are better than firm A's efficiency, but which are not located on the efficiency frontier
 - c) Any firm located on the efficiency frontier
 - d) One or several specific firms (i.e. a subgroup of efficient DMUs) located on the efficiency frontier
5. A firm is producing laptops. Which input reflects quality?
 - a) The number of FTE employees with a Master's degree
 - b) Total number of FTE employees, disregarding their educational background
 - c) Total number of square meters of the factory
 - d) Energy consumption
6. A firm has diseconomies of scale. How can the management team improve its efficiency?
 - a) By merging with another firm
 - b) By producing more output
 - c) By producing less output
 - d) By producing the same amount of output

7. A manager plans to measure efficiency using three inputs and two outputs. What is the minimum number of firms that should be included in the dataset?
- a) 10
 - b) 6
 - c) 15
 - d) It does not matter

Correct answers: 1b; 2c; 3d; 4d; 5a; 6c; 7c.

3 DEA SOFTWARE

3.1 EXISTING SOFTWARE

The user-friendly software packages of DEA incorporate intuitive graphical user interfaces and automatic calculation of efficiency scores. Some of them are compatible with *Microsoft Excel* ©. For a survey of DEA software packages, one can refer to Barr (2004). Today, several software packages have been developed:

- Free packages include *DEAP* (Timothy Coelli, Coelli Economic Consulting Services) and *Win4DEAP* (Michel Deslièrres, University of Moncton), *Benchmarking package in R* (Peter Bogetoft, Copenhagen Business School, and Lars Otto, University of Copenhagen), *Efficiency Measurement System* (Holger Scheel, University of Dortmund) or *DEA Solver Online* (Andreas Kleine and Günter Winterholer, University of Hohenheim).
- Commercial packages include *DEAFrontier*⁹ (Joe Zhu, Worcester Polytechnic Institute), *DEA-Solver PRO*¹⁰ (Saitech, Inc.), *PIM-DEA* (Ali Emrouznejad, Aston Business School) or *Frontier Analyst* (Banxia Software Ltd).

This section focuses on the ‘twin’ DEA software packages *DEAP/Win4DEAP*¹¹. These packages centre on the basics of DEA, are simple to use and are stable over time. They are freely available¹² and come with data files as examples. As *Win4DEAP* is the Windows based interface of *DEAP* (which is a DOS program), the current section refers only to *Win4DEAP*. All screenshots and icons presented in this section

⁹ Zhu (2003) includes an earlier version of DEAFrontier, DEA Excel Solver, on a CD-ROM. This software works only under Excel © 97, 2000 and 2003. It deals with an unlimited number of DMUs and is available at little cost.

¹⁰ Cooper et al. (2006) include a CD-ROM with a DEA-Solver version limited at 50 DMUs. It is available at little cost.

¹¹ As DEAP is a DOS program, a user friendly Windows interface has been developed for it (Win4DEAP). These ‘twin’ software packages have to be both downloaded and extracted to the same folder. Win4DEAP cannot work without DEAP.

¹² DEAP Version 2.1: <http://www.uq.edu.au/economics/cepa/deap.htm>
 Win4DEAP Version 1.1.3: http://www8.umoncton.ca/umcm-deslièrres_michel/dea/install.html

and coming from *DEAP* or *Win4DEAP* are reproduced by permission of Timothy Coelli and Michel Deslièrres.

3.2 CASE STUDY 2

The use of *Win4DEAP* is illustrated by a case study including a sample of 15 primary schools (see Table 2 below).

The data used in this case study are fictitious (but are very similar to real ones). 15 schools produce one output (number of pupils) with three inputs (number of full-time equivalent teachers, number of full-time administrative staff and number of computers –used as a proxy for technology investment). For example, School # 8 educates 512 pupils with 28.6 teachers, 1.3 administrative staff and 26 computers.

TABLE 2:

Case study 2 – On average, each school has 393.6 pupils, 23.2 teachers, 1.3 administrative staff and 21.6 computers.

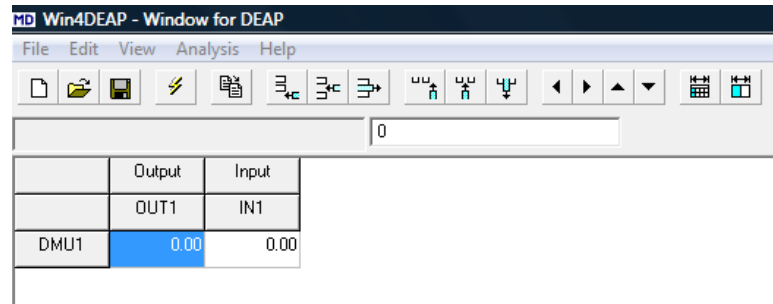
School	Input			Output
	FTE teachers	FTE adm. staff	Computers	Pupils
1	40.2	2.0	37.0	602.0
2	18.1	1.1	17.0	269.0
3	42.5	2.1	41.0	648.0
4	11.0	0.8	10.0	188.0
5	24.8	1.3	22.0	420.0
6	21.1	1.3	19.0	374.0
7	13.5	1.0	13.0	247.0
8	28.6	1.3	26.0	512.0
9	23.5	1.3	22.0	411.0
10	15.9	1.0	15.0	285.0
11	23.2	1.3	22.0	397.0
12	26.0	1.4	25.0	466.0
13	11.1	0.8	11.0	198.0
14	28.8	1.6	26.0	530.0
15	19.7	1.3	18.0	357.0

a) Case study 2 – Building a spreadsheet in *Win4DEAP*

Win4DEAP is launched by clicking the MD icon (**MD**). Firms (called decision-making units or DMUs) are listed in the rows and variables (outputs and inputs) in the columns. The opening spreadsheet contains one decision-making unit (DMU1), one output (OUT1) and one input (IN1) by default (see Figure 9).

FIGURE 9:

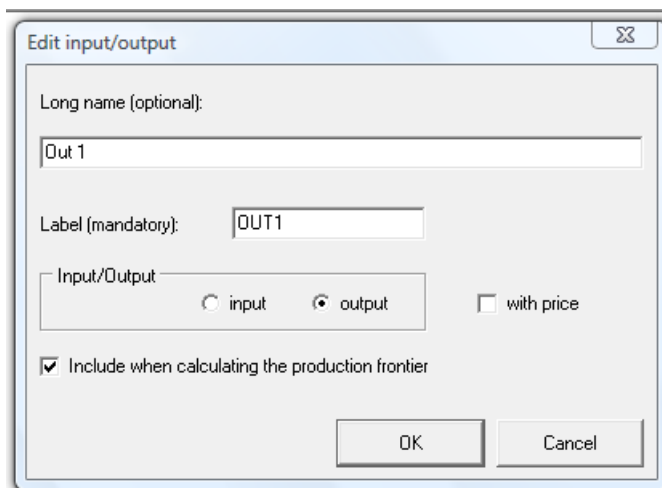
Case study 2 – The opening spreadsheet contains one DMU, one output and one input.











	Output	Input
	OUT1	IN1
DMU1	0.00	0.00

To edit and name firms, outputs and inputs, the user has to click the DMU1 (**DMU1**), OUT1 (**OUT1**) and IN1 (**IN1**) icons, respectively. The window reproduced in Figure 10 allows the user to (1) assign a long name and a label (maximum of eight characters) to any variable and (2) select the nature of the variables (either ‘input’ or ‘output’). Finally, the user has to select the ‘with price’ option if he intends to measure cost, revenue or profit efficiency (i.e. a ‘price’ column will be added to the selected variable in the spreadsheet).

FIGURE 10:
Case study 2 – Input and output editing.



The icons   enable the user to add firms (DMUs). The icons   enable the user to add variables (inputs or outputs). The icons  and  are used to delete any existing DMUs or variables. Finally, the following icons   allow the user to reverse the order of appearance of DMUs (rows) or variables (columns).

How to import *Microsoft Excel* © data into *Win4DEAP*

Note that data can be imported from an *Excel* © file into *Win4DEAP* by following these steps:

- Save the *Microsoft Excel* © data (only numbers, no names of DMUs or variables should be included) into the CSV format (Comma delimited).
- In *Win4DEAP*, first select the 'File' menu, then the 'Import' option and finally the 'New data set' application.
- Select the CSV file and open it.
- The data is now presented in the *Win4DEAP* spreadsheet, which still has to be configured (DMUs and variables have to be named and variables must be defined as inputs or outputs).

Exercise 2

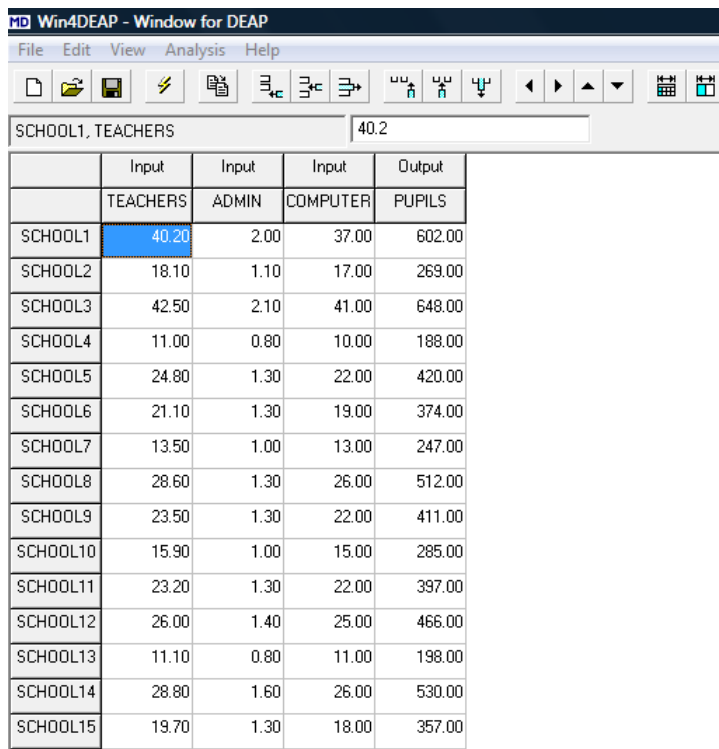
The objective of this exercise is to correctly calibrate a spreadsheet in *Win4DEAP*.

Tasks

- a) Prepare a spreadsheet in *Win4DEAP* including 15 DMUs, 3 inputs and one output. Name the DMUs 'School 1' to 'School 15'. The first input is 'FTE teachers', the second 'FTE administrative staff' and the third 'Number of computers'. The output corresponds to the number of pupils.
- b) Feed the data appearing in Table 2 into the spreadsheet.
- c) Save the file, preferably into the same folder containing *DEAP/Win4DEAP* (menu 'File', option 'Save as').

Answer: The spreadsheet should be similar to the one represented in Figure 11.


FIGURE 11:
Case study 2 – A ready-to-use spreadsheet in *Win4DEAP*.



The screenshot shows the Win4DEAP software window. The title bar reads 'Win4DEAP - Window for DEAP'. The menu bar includes 'File', 'Edit', 'View', 'Analysis', and 'Help'. The toolbar contains various icons for file operations and analysis. Below the toolbar, there is a text box containing 'SCHOOL1, TEACHERS' and a numerical input field with '40.2'. The main area displays a spreadsheet with the following data:

	Input	Input	Input	Output
	TEACHERS	ADMIN	COMPUTER	PUPILS
SCHOOL1	40.20	2.00	37.00	602.00
SCHOOL2	18.10	1.10	17.00	269.00
SCHOOL3	42.50	2.10	41.00	648.00
SCHOOL4	11.00	0.80	10.00	188.00
SCHOOL5	24.80	1.30	22.00	420.00
SCHOOL6	21.10	1.30	19.00	374.00
SCHOOL7	13.50	1.00	13.00	247.00
SCHOOL8	28.60	1.30	26.00	512.00
SCHOOL9	23.50	1.30	22.00	411.00
SCHOOL10	15.90	1.00	15.00	285.00
SCHOOL11	23.20	1.30	22.00	397.00
SCHOOL12	26.00	1.40	25.00	466.00
SCHOOL13	11.10	0.80	11.00	198.00
SCHOOL14	28.80	1.60	26.00	530.00
SCHOOL15	19.70	1.30	18.00	357.00

b) Case study 2 – Running a DEA model

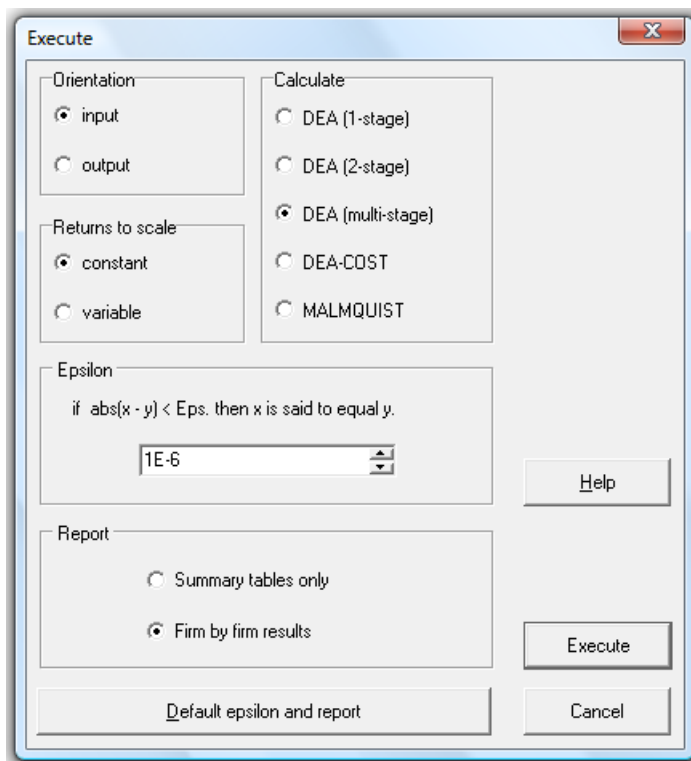
To run a DEA model, the user has to click the 'lightning' icon () . The window represented in Figure 12 will then appear. This window allows a calibration of the model following steps 1 to 4 described below:

1. Select an input or an output orientation (Orientation box).
2. Select the assumption about returns to scale (Returns to scale box).
By ticking 'constant', one assumes constant returns to scale (CRS);
by ticking 'variable', one assumes variable returns to scale (VRS).

If one cannot be certain about the fact that firms are operating at an optimal scale, running a VRS model is recommended.

3. Select a model (Calculate box). Three main options are available:
 - To calculate technical efficiency (TE) or technical (CRS), 'pure' (VRS) and scale efficiency (SE), tick 'DEA (multi-stage)'. Options 'DEA (1-stage)', 'DEA (2-stage)' and 'DEA (multi-stage)' correspond to different treatments of slacks. Following Coelli (1998), the multi-stage treatment is recommended.
 - To calculate cost, revenue or profit efficiency, tick 'DEA-COST'. For this option, cost and/or price information about variables must be available and added to the spreadsheet.
 - To calculate technical and scale efficiency when panel data are available, tick 'MALMQUIST'. See Section 5.4 to learn more about this.
4. Choose how to display the results: only summarized or reported firm by firm (Report box).
5. Click 'Execute' to run the model.

FIGURE 12:
Case study 2 – *Win4DEAP's* cockpit.



Exercise 3

The objective of this exercise is to run a DEA model in *Win4DEAP* based on the schools case study. The following information is available:

- Schools are confronted with budget restrictions;
- The school system is heavily regulated;
- An obligatory school by school report is expected.

Prerequisites

Exercise 2

Tasks

- Open the schools data spreadsheet (i.e. the calibrated spreadsheet in exercise 2)
- Calibrate the model
- Execute the model

Answer: The model should be similar to the one represented in Figure 13.

FIGURE 13:

Case study 2 – An input oriented model calibrated for VRS.

The screenshot shows a software dialog box titled "Execute". It contains the following settings:

- Orientation:** input, output
- Returns to scale:** constant, variable
- Epsilon:** if $\text{abs}(x - y) < \text{Eps}$, then x is said to equal y . The value is set to $1\text{E-}6$.
- Calculate:** DEA (1-stage), DEA (2-stage), DEA (multi-stage), DEA-COST, MALMQUIST
- Report:** Summary tables only, Firm by firm results

Buttons: Help, Execute, Cancel, and a button at the bottom labeled "Default epsilon and report".

c) Case study 2 – Interpreting results

After executing the selected model, a short notice appears with information about Timothy Coelli, the developer of *DEAP*. Results are displayed after closing this window. It is recommendable for first time users to take some time navigating through the results file in order to become familiar with it. Some results tables are commented on in this section. Table 3 contains a list of abbreviations with the main acronyms used in the results file.

TABLE 3:

Case study 2 – A table of abbreviations to help with reading the results file.

Acronym	Full name
DEA	Data Envelopment Analysis
CRS	Constant Returns to Scale
VRS	Variable Returns to Scale
TE	Technical Efficiency
CRSTE	Constant Returns to Scale Technical Efficiency
VRSTE	Variable Returns to Scale Technical Efficiency
SE	Scale Efficiency
IRS	Increasing Returns to Scale
DRS	Decreasing Returns to Scale

Figure 14 represents the first table to be commented on. It is an extract of the results file and features an efficiency summary. The first column contains the 15 schools (listed 1 to 15). The second one displays the constant returns to scale technical efficiency scores (CRSTE)¹³. This 'total' efficiency score is decomposed into a 'pure' technical efficiency measure (variable returns to scale technical efficiency –VRSTE– in the third column) and a scale efficiency measure (scale efficiency –SE– in the fourth column). The last column indicates the nature of returns to scale (IRS, DRS or a dash):

¹³ Note that if you had run a constant returns to scale model instead of a variable returns to scale one, you would have obtained only one type of efficiency score in your results file (technical efficiency –TE). Technical efficiency scores are strictly equal to constant returns to scale technical efficiency scores obtained in the CRSTE column of your variable returns to scale model.

- Firms associated with IRS are facing increasing returns to scale (economies of scale).
- Firms associated with DRS are facing decreasing returns to scale (diseconomies of scale).
- Firms associated with a dash are facing constant returns to scale; they are operating at an optimal scale.

On average, schools efficiency scores are:

- 94% for CRSTE; overall, schools could reduce their inputs by 6% whilst educating the same number of pupils.
- 97.5% for VRSTE; a better school organization would be able to reduce input consumption by 2.5%.
- 96.4% for SE; in adjusting their scale, schools could reduce their inputs by 3.6%.

FIGURE 14:

Case study 2 – Technical efficiency (CRSTE) is decomposed into 'pure' technical efficiency (VRSTE) and scale efficiency (SE).

EFFICIENCY SUMMARY:

firm	crste	vrste	scale	
1	0.827	0.951	0.869	drs
2	0.808	0.838	0.964	irs
3	0.842	1.000	0.842	drs
4	0.929	1.000	0.929	irs
5	0.943	0.962	0.981	irs
6	0.966	0.984	0.981	irs
7	0.994	1.000	0.994	irs
8	1.000	1.000	1.000	-
9	0.951	0.963	0.987	irs
10	0.974	0.995	0.978	irs
11	0.930	0.943	0.986	irs
12	0.978	0.984	0.994	irs
13	0.969	1.000	0.969	irs
14	1.000	1.000	1.000	-
15	0.985	0.998	0.987	irs
mean	0.940	0.975	0.964	

Note: crste = technical efficiency from CRS DEA
 vrste = technical efficiency from VRS DEA
 scale = scale efficiency = crste/vrste

Note also that all subsequent tables refer to VRS results

All subsequent tables displayed in the results file refer to the VRSTE scores. These tables contain the following information:

- The number of the DMU under review ('Results for firm').
- The technical efficiency score ('Technical efficiency'), corresponding to the VRSTE when a VRS model has been run or to the CRSTE when a CRS model has been run.
- The scale efficiency score ('Scale efficiency'); note that the SE is mentioned only when a VRS model has been run.
- The lines of the matrix represent the outputs and the inputs of the model ('output 1', 'output 2', etc., 'input 1', 'input 2', etc.).
- The first column of the matrix recalls the original values of the variables' outputs and inputs ('original values').

- The second column of the matrix represents the movement an inefficient DMU has to take in order to be located on the frontier ('radial movement').
- The third column of the matrix is the additional movement a DMU located on a segment of the frontier running parallel to the axis has to take in order to become efficient ('slack movement').
- The fourth column of the matrix lists the values of the variables which enable the DMU to be efficient ('projected value'); these projected values take into account both the radial and the slack movements.
- Finally, the listing of peers is mentioned. Each peer is identified by a number and has an associated weight ('lambda weight') representing the relative importance of the peer.

As illustrations, three individual school tables are specifically commented on below: School # 1 (Figure 15), # 2 (Figure 16) and # 3 (Figure 17).

School # 1 (Figure 15) has a 'pure' efficiency score of 95.1% and a scale efficiency score of 86.9%. It is facing decreasing returns to scale (DRS). By improving the operation of the school, 4.9% (100 - 95.1) of inputs could be saved. By adjusting the school to its optimal size, 13.1% (100 - 86.9) of inputs could be saved.

The 'original value' column contains the original values of the school's variables: School # 1 educates 602 pupils with 40.2 teachers, 2 administrative staff and 37 computers. However, School # 1 could 'produce' the same quantity of output with fewer inputs: 37.186 teachers instead of 40.2; 1.902 administrative staff instead of 2; 35.185 computers instead of 37 (see the 'projected value' column). The decreases in inputs 2 and 3 are equal to 4.9 % of the original values: $(- 0.098 / 2) \times 100$ for input 2 and $(- 1.815 / 37) \times 100$ for input 3¹⁴. The case of input 1 is slightly different: to become efficient, it has to

¹⁴ In a VRS model, the improvement in variables (decrease in inputs or increase in outputs) is calculated according to the VRS technical efficiency score (only). In a CRS model, it is calculated according to the CRS technical efficiency score, or TE score, including not only the pure efficiency but also the scale efficiency.

decrease not only by 4.9% (minus 1.972 from the 'radial movement' column) but also by an additional 1.042 (from the 'slack movement' column). Overall School # 1 has to decrease its first input by minus 3.014 $[(- 1.972) + (- 1.042)]$ to become efficient. This represents 7.5 % $[(- 3.014 / 40.2) \times 100]$.

To improve its efficiency, School # 1 has to analyse the practice of Schools # 3, # 14 and # 8, which are identified as its peers. To be a peer (or a benchmark), a firm must have a 'pure' efficiency score of 100%. The lambda weight associated with each peer corresponds to its relative importance among the peer group. Ideally, School # 1 should analyse best practice from a composite school formed by 61.2% of School # 3, 37.3% of School # 14 and 1.4% of School # 8. As such a 'virtual' school does not exist. School # 1 should concentrate its best practice analysis on the peer associated with the highest lambda value (i.e. School # 3).

FIGURE 15:
Case study 2 – School # 1 results table.

Results for firm: 1					
Technical efficiency = 0.951					
Scale efficiency = 0.869 (drs)					
PROJECTION SUMMARY:					
variable		original	radial	slack	projected
		value	movement	movement	value
output	1	602.000	0.000	0.000	602.000
input	1	40.200	-1.972	-1.042	37.186
input	2	2.000	-0.098	0.000	1.902
input	3	37.000	-1.815	0.000	35.185
LISTING OF PEERS:					
peer		lambda	weight		
3		0.612			
14		0.373			
8		0.014			

School # 2 (Figure 16) has a 'pure' efficiency score of 83.8% and a scale efficiency score of 96.4%. It is facing increasing returns to scale (IRS). By improving the operation of the school, 16.2% (100 - 83.8) of inputs could be saved. By adjusting the school to its optimal size, 3.6% (100 - 96.4) of inputs could be saved.

The 'original value' column contains the original values of the school's variables: School # 2 educates 269 pupils with 18.1 teachers, 1.1 administrative staff and 17 computers. However, School # 2 could

'produce' the same quantity of output with fewer inputs: 15.163 teachers instead of 18.1, 0.922 administrative staff instead of 1.1; 14.242 computers instead of 17 (see the 'projected value' column). The decreases in inputs 1, 2 and 3 are equal to 16.2% of the original values ('radial movement' column). No slack movement is identified.

To improve its efficiency, School # 2 has to refer to Schools # 13, # 4, # 14 and # 8, which are identified as its peers.

FIGURE 16:
Case study 2 – School # 2 results table.

Results for firm: 2					
Technical efficiency = 0.838					
Scale efficiency = 0.964 (irs)					
PROJECTION SUMMARY:					
variable		original value	radial movement	slack movement	projected value
output	1	269.000	0.000	0.000	269.000
input	1	18.100	-2.937	0.000	15.163
input	2	1.100	-0.178	0.000	0.922
input	3	17.000	-2.758	0.000	14.242
LISTING OF PEERS:					
peer	lambda	weight			
13	0.506				
4	0.261				
14	0.016				
8	0.218				

School # 3 (Figure 17) has a 'pure' efficiency score of 100% and a scale efficiency score of 84.2%. It is facing decreasing returns to scale (DRS). This school is well managed. It cannot improve its 'pure' efficiency. The only capacity for improvement lies in a scale adjustment: 15.8% (100 - 84.2) of inputs could be saved.

The 'original value' column contains the original values of the school's variables: School # 3 educates 648 pupils with 42.5 teachers, 2.1 administrative staff and 41 computers. These values are equal to the projected ones ('pure' efficiency = 100%).

As School # 3 is purely efficient, it acts as its own peer.

FIGURE 17:
Case study 2 – School # 3 results table.

Results for firm:		3			
Technical efficiency		= 1.000			
Scale efficiency		= 0.842 (drs)			
PROJECTION SUMMARY:					
variable		original value	radial movement	slack movement	projected value
output	1	648.000	0.000	0.000	648.000
input	1	42.500	0.000	0.000	42.500
input	2	2.100	0.000	0.000	2.100
input	3	41.000	0.000	0.000	41.000
LISTING OF PEERS:					
peer		lambda	weight		
3		1.000			

Exercise 4

The objective of this exercise is to interpret DEA results. Figure 18 displays results for one of the 15 schools. It has been truncated in order to hide the VRS technical efficiency score.

Tasks

Answer the following questions:

- a) The variable returns to scale technical efficiency score has been removed from the table. Find a way to calculate it.

Answer: For example, input 1 can be reduced by 0.864 (radial movement). This represents 3.7% $[(-0.864 / 23.5) \times 100]$. Therefore VRSTE corresponds to 96.3% $(100 - 3.7)$. You would have obtained the same result if you based your calculations on input 2. If you based your calculations on input 3, be careful not to take into account the slack movement.

- b) Assume that the 'pure' efficiency score is equal to 96.3%. What is the main feature in need for improvement: the school's management or the school's scale?

Answer: The school's management has the capacity to improve efficiency by 3.7% $(100 - 96.3)$. Modifying the school's scale could improve efficiency by 1.3% $(100 - 98.7)$. Therefore the main feature in need of improvement is school management.

- c) Assume that the school has only time to analyze best practice from one of its peers. Which one should it select?

Answer: School # 14. Among the three peers listed (13, 14 and 8), School # 14 is associated with the highest weight (41.7%).

- d) How much must the school reduce input 3 in order to be located on the efficiency frontier?

Answer: - 1.184 [(- 0.809) + (- 0.375)].

FIGURE 18:

Case study 2 – An efficiency table helps a firm to make decisions based on objective information.

Scale efficiency = 0.987 (irs)

PROJECTION SUMMARY:

variable		original value	radial movement	slack movement	projected value
output	1	411.000	0.000	0.000	411.000
input	1	23.500	-0.864	0.000	22.636
input	2	1.300	-0.048	0.000	1.252
input	3	22.000	-0.809	-0.375	20.817

LISTING OF PEERS:

peer	lambda weight
13	0.346
14	0.417
8	0.238

4 DEA IN THE BLACK BOX

This section describes the two principal DEA models: the constant returns to scale model (Charnes *et al.*, 1978) and the variable returns to scale model (Banker *et al.*, 1984). DEA is based on the earlier work of Dantzig (1951) and Farrell (1957), whose approach adopted an input orientation. Zhu and Cook (2008), Cooper *et al.* (2007) or Coelli *et al.* (2005) provide a comprehensive treatment of the methodology. By 2007, Emrouznejad *et al.* (2008) identified more than 4000 research articles about DEA published in scientific journals or books.

DEA is a non-parametric method. Unlike parametric methods (such as ordinary least square, maximum likelihood estimation or stochastic frontier analysis), inputs and outputs are used to compute, using linear programming methods, a hull to represent the efficiency frontier. As a result, a non-parametric method does not require specification of a functional form.

4.1 CONSTANT RETURNS TO SCALE

Charnes *et al.* (1978) propose a model assuming constant returns to scale (CRS model)¹⁵. It is appropriate when all firms operate at the optimal scale. Efficiency is defined by Charnes *et al.* (1978, p. 430) as “the maximum of a ratio of weighted outputs to weighted inputs subject that the similar ratios for every DMU be less or equal to unity”. The following notation is adopted, as in Johnes (2004):

$$TE_k = \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \quad (1)$$

¹⁵ This model is also known as the Charnes, Cooper & Rhodes model (CCR model).

Where:

TE_k is the technical efficiency of firm k using m inputs to produce s outputs;

y_{rk} is the quantity of output r produced by firm k ;

x_{ik} is the quantity of input i consumed by firm k ;

u_r is the weight of output r ;

v_i is the weight of input i .

n is the number of firms to be evaluated

s is the number of outputs

m is the number of inputs

The technical efficiency of firm k is maximized under two constraints. First, the weights applied to outputs and inputs of firm k cannot generate an efficiency score greater than 1 when applied to each firm in the dataset (equation # 3). Second, the weights on the outputs and on the inputs are strictly positive (equation # 4). The following linear programming problem has to be solved for each firm:

$$\text{Maximize } \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \quad (2)$$

$$\text{Subject to } \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad j = 1, \dots, n \quad (3)$$

$$u_r, v_i > 0 \quad \forall r = 1, \dots, s; i = 1, \dots, m \quad (4)$$

This linear programming problem can be dealt following two different approaches. In the first one, the weighted sums of outputs are maximized holding inputs constant (output-oriented model). In the second one, the weighted sums of inputs are minimized holding outputs

constant (input-oriented model)¹⁶. The primal equations for each model, known as the multiplier form, are given below:

CRS output-oriented model Primal equation	CRS input-oriented model Primal equation
Minimize $\sum_{i=1}^m v_i x_{ik}$	Maximize $\sum_{r=1}^s u_r y_{rk}$
Subject to	Subject to
$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0 \quad j = 1, \dots, n$	$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0 \quad j = 1, \dots, n$
$\sum_{r=1}^s u_r y_{rk} = 1$	$\sum_{i=1}^m v_i x_{ik} = 1$
$u_r, v_i > 0 \quad \forall r = 1, \dots, s; i = 1, \dots, m$	$u_r, v_i > 0 \quad \forall r = 1, \dots, s; i = 1, \dots, m$

Using the duality in linear programming, an equivalent form, known as the envelopment form, can be derived from this problem. It is often preferable to solve the computation using the envelopment form because it contains only $s+m$ constraints rather than $n+1$ constraints in the multiplier form.

CRS output-oriented model Dual equation	CRS input-oriented model Dual equation
Maximize ϕ_k	Minimize θ_k
Subject to	Subject to
$\phi_k y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} \leq 0 \quad r = 1, \dots, s$	$y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} \leq 0 \quad r = 1, \dots, s$
$x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} \geq 0 \quad i = 1, \dots, m$	$\theta_k x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} \geq 0 \quad i = 1, \dots, m$
$\lambda_j \geq 0 \quad \forall j = 1, \dots, n$	$\lambda_j \geq 0 \quad \forall j = 1, \dots, n$

Where:

$1/\phi_k$ and θ_k represent the technical efficiency of firm k

λ_j represents the associated weighting of outputs and inputs of firm j

¹⁶ Note that the input and output orientations refer to the dual equations of each model (and not to the primal ones).

Every firm located on the sections' envelope running parallel to the axes has to be adjusted for output and input slacks. However, the preceding formulation does not integrate the role of slacks in measuring efficiency. Considering output slacks, s_r , and input slacks, s_i , the above equations become:

CRS output-oriented model Dual equation with slacks	CRS input-oriented model Dual equation with slacks
Maximize $\phi_k + \mathcal{E} \sum_{r=1}^s s_r + \mathcal{E} \sum_{i=1}^m s_i$ (21)	Minimize $\theta_k - \mathcal{E} \sum_{r=1}^s s_r - \mathcal{E} \sum_{i=1}^m s_i$ (25)
Subject to	Subject to
$\phi_k y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} + s_r = 0 \quad r = 1, \dots, s$ (22)	$y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} + s_r = 0 \quad r = 1, \dots, s$ (26)
$x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} - s_i = 0 \quad i = 1, \dots, m$ (23)	$\theta_k x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} - s_i = 0 \quad i = 1, \dots, m$ (27)
$\lambda_j, s_r, s_i \geq 0 \quad \forall j = 1, \dots, n; r = 1, \dots, s; i = 1, \dots, m$ (24)	$\lambda_j, s_r, s_i \geq 0 \quad \forall j = 1, \dots, n; r = 1, \dots, s; i = 1, \dots, m$ (28)

Here, \mathcal{E} is a non-Archimedean value defined to be smaller than any positive real number. \mathcal{E} is greater than 0. The firm k is efficient only if:

- the efficiency score $TE_k = \left(\frac{1}{\phi_k} \right) = 1$ (or $TE_k = \theta_k = 1$);
- and the slacks $s_r, s_i = 0, \forall_r = 1, \dots, s$ and $i = 1, \dots, m$.

For an in-depth analysis on the treatment of slacks, and especially the multi-stage methodology, see Coelli (1998).

4.2 VARIABLE RETURNS TO SCALE

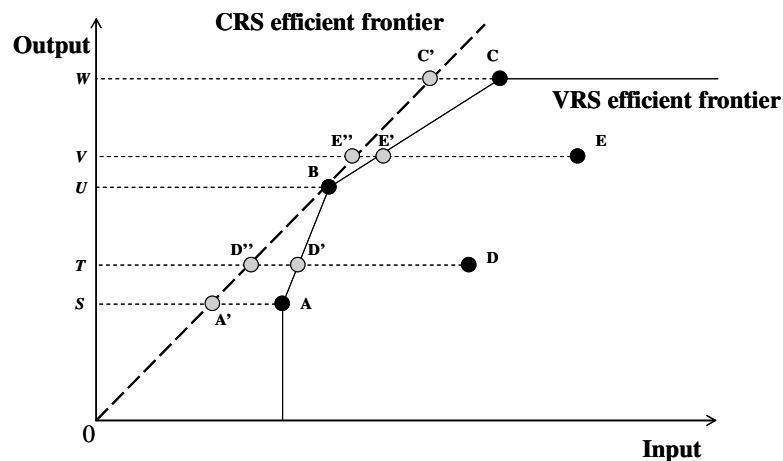
Banker *et al.* (1984) propose a model assuming variable returns to scale (VRS model)¹⁷. It is appropriate when all firms do not operate at optimal scale. As Coelli *et al.* (2005, p. 172) point out, “the use of the CRS specification when not all firms are operating at the optimal scale, results in measures of TE that are confounded by scale efficiencies (SE). The use of the VRS specification permits the calculation of TE devoid of these SE effects”. The CRS model can be modified by relaxing the constant returns to scale assumption. A measure of return to scale for firm k is added in the primal equation (or the convexity constraint

$$\sum_{j=1}^n \lambda_j = 1 \text{ in the dual equations).}$$

Figure 19 represents the CRS efficiency frontier (the dashed line) and the VRS efficiency frontier (the solid line) on the same graph to illustrate a simple example with one output and one input. Only one firm, B, is located on both frontiers. A and C are 100% efficient under the VRS assumption, but inefficient under the CRS assumption. D and E are inefficient under both specifications.

¹⁷ This model is also known as the Banker, Charnes & Cooper model (BCC model).

FIGURE 19:
Constant versus variable returns to scale.



The specific situation of firm D is commented on in detail below:

- Firm D is inefficient under VRS and CRS. In order to become VRS efficient, it has to move to point D'. The input-oriented VRS technical inefficiency of point D is the distance DD'. In order to become CRS efficient, firm D has to move further toward point D''. The input-oriented CRS technical inefficiency of point D is the distance DD''. The distance between D' and D'' corresponds to scale inefficiency. The ratio efficiency measures, bounded by zero and one, are as follows:

Technical efficiency of D under CRS	Technical efficiency of D under VRS	Scale efficiency of D
$TE_{CRS} = \frac{TD''}{TD}$	$TE_{VRS} = \frac{TD'}{TD}$	$SE = \frac{TD''}{TD'}$

Exercise 5

The objective of this exercise is (1) to comment in detail on the situation of firms E, A, B and C represented in Figure 19 (as above for firm D) and (2) to provide ratios of TE_{CRS} , TE_{VRS} and SE for firms E, A, B and C.

Answer for firm E:

Firm E is inefficient under VRS and CRS. In order to become CRS efficient, it has to move toward point E''. The input-oriented CRS technical inefficiency of point E is the distance EE''. In order to become VRS efficient, it has to move to point E'. The input-oriented VRS technical inefficiency of point E is the distance EE'. The difference between these two distances, i.e. the distance E'E'', corresponds to scale inefficiency. The ratio efficiency measures, bounded by zero and one, are as follows:

<i>Technical efficiency of E under CRS</i>	<i>Technical efficiency of E under VRS</i>	<i>Scale efficiency of E</i>
$TE_{CRS} = \frac{VE''}{VE}$	$TE_{VRS} = \frac{VE'}{VE}$	$SE = \frac{VE''}{VE'}$

Answer for firm A:

Firm A is efficient under VRS but inefficient under CRS. In order to become CRS efficient, it has to move toward point A'. The input-oriented CRS technical inefficiency of point A is the distance AA'; this also corresponds to scale inefficiency. The ratio efficiency measures, bounded by zero and one, are as follows:

<i>Technical efficiency of A under CRS</i>	<i>Technical efficiency of A under VRS</i>	<i>Scale efficiency of A</i>
$TE_{CRS} = \frac{SA'}{SA}$	$TE_{VRS} = \frac{SA}{SA} = 1$	$SE = \frac{SA'}{SA}$

Answer for firm B:

Firm B is efficient both under VRS and CRS. It is operating at the optimal scale. The ratio efficiency measures, bounded by zero and one, are as follows:

Technical efficiency of B under CRS	Technical efficiency of B under VRS	Scale efficiency of B
$TE_{CRS} = \frac{UB}{UB} = 1$	$TE_{VRS} = \frac{UB}{UB} = 1$	$SE = \frac{UB}{UB} = 1$

Answer for firm C:

Firm C is efficient under VRS but inefficient under CRS. In order to become CRS efficient, it has to move toward point C'. The input-oriented CRS technical inefficiency of point C is the distance CC'; this also corresponds to scale inefficiency. The ratio efficiency measures, bounded by zero and one, are as follows:

Technical efficiency of C under CRS	Technical efficiency of C under VRS	Scale efficiency of C
$TE_{CRS} = \frac{WC'}{WC}$	$TE_{VRS} = \frac{WC}{WC} = 1$	$SE = \frac{WC'}{WC}$

Knowing TE under CRS and TE under VRS, the scale efficiency is easily calculated. As $TE_{k,CRS} = TE_{k,VRS} \times SE_k$, the scale efficiency is obtained through the division of TE under CRS by TE under VRS:

$$SE_k = \frac{TE_{k,CRS}}{TE_{k,VRS}}.$$

The linear programming problem to be solved under VRS includes a measure of returns to scale on the variables axis, c_k , for the firm k . The primal equations are as follows:

VRS output-oriented model	VRS input-oriented model
Primal equation	Primal equation
Minimize $\sum_{i=1}^m v_i x_{ik} - c_k$ (29)	Maximize $\sum_{r=1}^s u_r y_{rk} + c_k$ (33)
Subject to	Subject to
$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} - c_k \geq 0 \quad j=1, \dots, n$ (30)	$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} - c_k \geq 0 \quad j=1, \dots, n$ (34)
$\sum_{r=1}^s u_r y_{rk} = 1$ (31)	$\sum_{i=1}^m v_i x_{ik} = 1$ (35)
$u_r, v_i > 0 \quad \forall r=1, \dots, s; i=1, \dots, m$ (32)	$u_r, v_i > 0 \quad \forall r=1, \dots, s; i=1, \dots, m$ (36)

The dual linear programming models are presented hereafter.

VRS output-oriented model	VRS input-oriented model
Dual equation	Dual equation
Maximize ϕ_k (37)	Minimize θ_k (42)
Subject to	Subject to
$\phi_k y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} \leq 0 \quad r=1, \dots, s$ (38)	$y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} \leq 0 \quad r=1, \dots, s$ (43)
$x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} \geq 0 \quad i=1, \dots, m$ (39)	$\theta_k x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} \geq 0 \quad i=1, \dots, m$ (44)
$\sum_{j=1}^n \lambda_j = 1$ (40)	$\sum_{j=1}^n \lambda_j = 1$ (45)
$\lambda_j \geq 0 \quad \forall j=1, \dots, n$ (41)	$\lambda_j \geq 0 \quad \forall j=1, \dots, n$ (46)

When slacks are added into the model, the dual linear programming equations become:

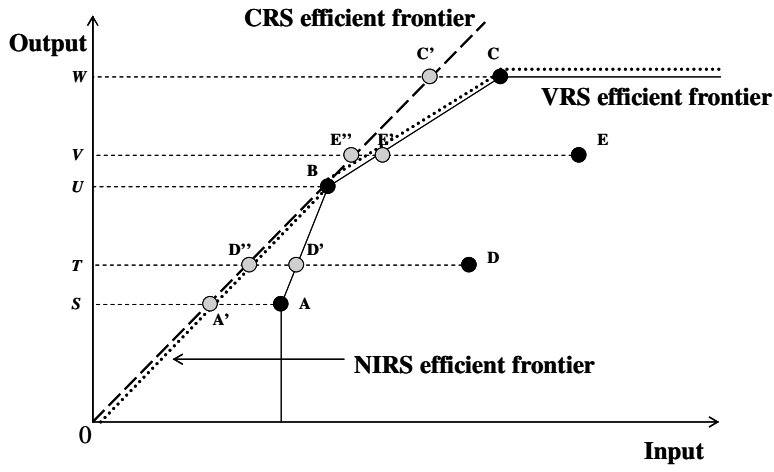
VRS output-oriented model Dual equation with slacks	VRS input-oriented model Dual equation with slacks
Maximize $\phi_k + \varepsilon \sum_{r=1}^s s_r + \varepsilon \sum_{i=1}^m s_i$ (47)	Minimize $\theta_k - \varepsilon \sum_{r=1}^s s_r - \varepsilon \sum_{i=1}^m s_i$ (52)
Subject to	Subject to
$\phi_k y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} + s_r = 0 \quad r = 1, \dots, s$ (48)	$y_{rk} - \sum_{j=1}^n \lambda_j y_{rj} + s_r = 0 \quad r = 1, \dots, s$ (53)
$x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} - s_i = 0 \quad i = 1, \dots, m$ (49)	$\theta_k x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} - s_i = 0 \quad i = 1, \dots, m$ (54)
$\sum_{j=1}^n \lambda_j = 1$ (50)	$\sum_{j=1}^n \lambda_j = 1$ (55)
$\lambda_j, s_r, s_i \geq 0 \quad \forall j = 1, \dots, n; r = 1, \dots, s; i = 1, \dots, m$ (51)	$\lambda_j, s_r, s_i \geq 0 \quad \forall j = 1, \dots, n; r = 1, \dots, s; i = 1, \dots, m$ (56)

A further step has to be taken in order to identify the nature of the returns to scale. This relates to another model, the non-increasing returns to scale model (NIRS), derived from the VRS model in which the

$\sum_{j=1}^n \lambda_j = 1$ restriction is substituted by the $\sum_{j=1}^n \lambda_j \leq 1$ constraint

(Coelli et al., 2005). In Figure 20, the NIRS efficiency frontier has been added (the dotted line). It corresponds to the CRS frontier from the origin to point B followed by the VRS frontier from point B. The nature of the scale inefficiencies for each firm can be determined by comparing technical efficiency scores under NIRS and VRS. If NIRS TE \neq VRS TE (as for firms A and D), increasing returns to scale apply. If NIRS TE = VRS TE (but \neq CRS TE) (as for firms E and C), decreasing returns to scale apply. Finally, if NIRS TE = VRS TE = CRS TE, as for firm B, constant returns to scale apply.

FIGURE 20:
The nature of returns to scale is identified by comparing a NIRS and a VRS model.



5 EXTENSIONS OF DEA

In this section, a selection of four extensions of DEA is shortly introduced: adjusting for the environment, preferences (weight restrictions), sensitivity analysis and time series data. For a broader overview of the major developments in DEA, see Cook and Seiford (2008). For an up-to-date review of DEA, readers will refer to Cooper *et al.* (2011).

5.1 ADJUSTING FOR THE ENVIRONMENT

Environmental variables influence the efficiency of firms but are not under the control of the management team. In DEA, several methods accommodate such variables. Those include the Charnes *et al.* (1981) approach, the categorical model (Banker & Morey, 1986a) or the non-discretionary variable model derived by Banker and Morey (1986b) (which indeed includes the environmental variable directly into the DEA model).

The most convincing of these methods, however, is the two-stage method, the advantages of which are described in Coelli *et al.* (2005, pp. 194-195) or in Pastor (2002, p. 899). The two-stage method combines a DEA model and a regression analysis. In the first stage, a traditional DEA model is conducted. This model includes only discretionary inputs and outputs. In the second stage, the efficiency scores are regressed against the environmental (i.e. non-discretionary or exogenous) variables. Tobit regression is often used in the second stage. However, recent studies have shown that ordinary least squares regression is sufficient to model the efficiency scores (Hoff, 2007) or even more appropriate than Tobit (McDonald, 2009).

The coefficients of the environmental variables, estimated by the regression, are used to model the efficiency scores to correspond to an identical condition of environment (e.g. usually the average condition). Simar and Wilson (2007, p. 32) provide a selection of studies using the two-stage method. Among those are applications in education

(Chakraborty *et al.*, 2001, McMillan & Datta, 1998, McCarty & Yaisawarng, 1993), hospitals (Burgess & Wilson, 1998), defence (Barros, 2004), police (Carrington *et al.*, 1997), farming (Binam *et al.*, 2003) or banking (O'Donnell & van der Westhuizen, 2002). Sueyoshi *et al.* (2010) and Sibiano and Agasisti (2012) provide more recent applications in the sector of manufacturing sector and education.

5.2 PREFERENCES

For different reasons (e.g. the weights assigned to variables by DEA are considered unrealistic for some firms; the management team may wish to give priority to certain variables; etc.), preferences about the relative importance of individual inputs and outputs can be set by the decision maker. This is done by placing weight restrictions onto outputs and inputs (also called multiplier restrictions). Cooper *et al.* (2011) and Thanassoulis *et al.* (2004) provide a review of models regarding the use of weights restrictions. An earlier review can be found in Allen *et al.* (1997). Generally, the imposition of weight restrictions worsens efficiency scores. Three main approaches are identified to accommodate preferences:

- Dyson and Thanassoulis (1988) propose an approach which imposes absolute upper and lower bounds on input and output weights. This technique is applied in Roll *et al.* (1991) to highway maintenance units or in Liu (2009) to garbage clearance units.
- Charnes *et al.* (1990) develop the cone-ratio method. This approach imposes a set of linear restrictions that define a convex cone, corresponding to an 'admissible' region of realistic weight restrictions. See Brockett *et al.* (1997) for an application to banks.
- Thompson *et al.* (1986, 1990) propose the assurance region method. This approach is actually a special case of the cone ratio. It imposes constraints on the relative magnitude of the weights. For example, a constraint on the ratio of weights for

input 1 and input 2 can be included, such as the following:

$$L_{1,2} \leq \frac{V_2}{V_1} \leq U_{1,2}, \text{ where } L_{1,2} \text{ and } U_{1,2} \text{ are lower and upper}$$

bounds for the ratio of the weight of input 2 (V_2) to the weight of input 1 (V_1). As a result, the assurance region method limits the ‘region’ of weights to a restricted area by prohibiting large differences in the value of those weights. An application of this model is provided by Sarica and Or (2007) in the assessment of power plants.

5.3 SENSITIVITY ANALYSIS

Cooper *et al.* (2006, p.271) mention that the term ‘sensitivity’ corresponds to stability or robustness. For Zhu (2003, p. 217), “the calculated frontiers of DEA models are stable if the frontier DMUs that determine the DEA frontier remain on the frontier after particular data perturbations are made”. Sensitivity analysis aims to identify the impact on firm efficiency when certain parameters are modified in the model.

The first way to test the sensitivity of DEA results consists in adding or extracting firms to DEA models. Dusansky and Wilson (1994, 1995) and Wilson (1993, 1995) provide different approaches to deal with this concern. The approach of Pastor *et al.* (1999) allows users to identify the observations which considerably affect the efficiency of the remaining firms. It also determines the statistical significance of efficiency variations which are due to the inclusion of a given firm in the sample.

Another way to test the sensitivity of DEA results consists in modifying the values of outputs and inputs. They focus on the maximum data variations a given firm can endure, whilst maintaining its efficiency status. Approaches include data perturbation of:

- a single variable of an efficient firm (Charnes *et al.*, 1985), data of other firms remaining fixed;

- simultaneous proportional data perturbation of all outputs and inputs of an efficient firm (Charnes & Neralic, 1992), data of other firms remaining fixed;
- simultaneous data perturbation of an efficient firm in a situation where outputs and inputs can be modified individually (Seiford & Zhu, 1998a, or Neralic & Wendell, 2004), data of other firms remaining fixed;
- simultaneous proportional data perturbation of all outputs and inputs of all firms (Seiford & Zhu, 1998b).

For further review of sensitivity analysis, readers can refer to Zhu (2001).

5.4 TIME SERIES DATA

In DEA, panel data are considered using two methods: window analysis and the Malmquist index.

Window analysis, introduced by Charnes *et al.* (1985), examines the changes in the efficiency scores of a set of firms over time. A 'window' of time periods is chosen for each firm. The same firm is treated as if it represented a different firm in every time period. In this sense, window analysis can also be considered as a sensitivity analysis method. For instance, a model including n firms with annual data and a chosen 'window' of t years will result in $n \times t$ units to be evaluated. For each firm, t different efficiency scores will be measured. The 'window' is then shifted by one period (one year in our example) and the efficiency analysis is repeated. Yue (1992) provides a didactical application of window analysis. Other applications include Yang and Chang (2009), Avkiran (2004) or Webb (2003).

The Malmquist total factor productivity index was first introduced by Malmquist (1953) before being further developed in the frame of DEA. It is used to measure the change in productivity over time. The Malmquist index decomposes this productivity change into two components:

- The first one is called 'catch-up'. This captures the change in technical efficiency over time.
- The second one is called 'frontier-shift'. This captures the change in technology which occurs over time (i.e. the movement of efficiency frontiers over time).

Readers will refer to Färe *et al.* (2011) and Tone (2004) for actual reviews. Applications of the Malmquist index can be found in Coelli and Prasada Rao (2005) and Behera *et al.* (2011).

6 DEA WITH MICROSOFT EXCEL © SOLVER

6.1 MICROSOFT EXCEL © SOLVER

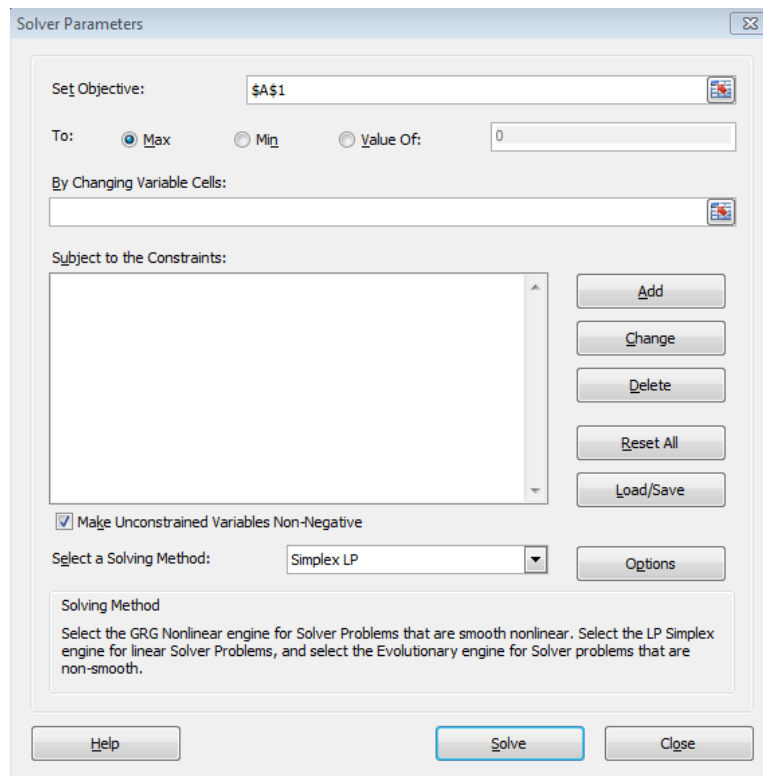
Excel © Solver is a tool used to find the best way to do something, in other words to optimize an objective. Instructions on loading *Excel* © Solver are easily found on the Internet¹⁸.

Excel © Solver allows users to solve optimization problems. An optimization model is composed of three elements: the target cell, the changing cells and the constraints. These three elements correspond to the parameters to be defined in *Excel* © Solver (see Figure 21).

- The target cell ('Set objective') corresponds to the objective. It has to be either minimized or maximized.
- The changing variable cells are the cells which can be altered in order to optimize the target cell.
- The constraints (one or several) correspond to restrictions placed on the changing cells.

¹⁸ In Microsoft Excel © 2010, the Solver has to be loaded by clicking the File button, then the Excel Options and finally the Add-Ins button. In the Manage box, Excel Add-ins has to be selected before clicking the Go button. In the Add-Ins box, the Solver Add-in has to be selected. Finally, the OK button has to be clicked. Once the Solver is loaded, it is located in the Analysis group on the Data tab.

FIGURE 21:
Three parameters have to be defined in *Excel* Solver.



6.2 PROGRAMMING A CRS MODEL

Consider five register offices (A to E) producing two outputs (birth and marriage certificates) with one input (full-time equivalent public servant). The data are listed in Table 4. For example, one full-time equivalent (FTE) public servant works in Register Office A. He produces one birth and six marriage certificates during a certain period of time.

TABLE 4:

Five Register offices produce birth and marriage certificates using public servants.

Register Office	Input	Output	
	Public servant (x)	Birth (y ₁)	Marriage (y ₂)
A	1	1	6
B	1	3	8
C	1	4	3
D	1	5	6
E	1	6	2

The use of *Excel* © Solver is illustrated with the following CRS model.

CRS input-oriented model

Primal equation

$$\text{Maximize } \sum_{r=1}^s u_r y_{rk}$$

$$\text{Subject to } \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0 \quad j = 1, \dots, n$$

$$\sum_{i=1}^m v_i x_{ik} = 1$$

$$u_r, v_i > 0 \quad \forall r = 1, \dots, s; i = 1, \dots, m$$

In this model, the objective is to maximize the weighted sum of outputs of firm k . Two constraints have to be considered. First, the weighted sum of inputs minus the weighted sum of outputs of firm j has to be greater than or equal to zero. Second, the weighted sum of inputs of firm k has to be equal to one.

Users have to prepare an *Excel* © spreadsheet, such as the one appearing in Figure 22. This is divided into two parts:

- The first part comprises rows 2 and 3. This section enables users to successively calculate the efficiency of the five register

offices (one at a time). To do this, data of each register office have to be entered successively in cells B2 to D2 (dark grey cells). Figure 22 already contains data on Register office C. The two outputs and one input of Register Office C are assigned weights in cells B3 to D3 (light grey cells). A value of one has been assigned to all of them in the spreadsheet. These values will be precisely modified by *Excel* © Solver in order to maximize the register offices' efficiency scores. Cell E2 contains the weighted sum of outputs for Register Office C. The formula associated with cell E2 is $(B2*\$B\$3) + (C2*\$C\$3)$. Cell F2 contains the weighted sum of the input for Register Office C. The formula associated with cell F2 is $(D2*\$D\$3)$. Finally, cell G2 contains the efficiency score of Register office C as a percentage (light grey cell). The formula associated with cell G2 is $(E2/F2)*100$. Note that the score of 700% appearing in the spreadsheet is calculated using weighted values of 1 and without any constraints. In other words, this score has not yet been optimized under varying constraints.

- The second part comprises rows 6 to 10. It contains the data for register offices A to E (output 1 = column B, output 2 = column C, input = column D, weighted sum of outputs = column E, weighted sum of the input = column F). The same formulae as above apply to the weighted sums of outputs and the input. An additional column, G, is added in the spreadsheet. It is a working column which will be used by *Excel* © Solver. Column G contains the weighted sum of the input minus the weighted sum of outputs to adequately reflect

the
$$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0$$
 constraint. The formula

associated with cell G6 is $F6 - D6$, the formula associated with cell G7 is $F7 - D7$, etc.

FIGURE 22:
An *Excel* © spreadsheet ready to use with *Excel* © Solver.

	A	B	C	D	E	F	G
1		Output 1 Birth	Output 2 Marriage	Input 1 Public servant	Weighted output	Weighted input	Efficiency (%)
2	Register office	4	3	1	7.00	1.00	700.00
3	Weight	1.00	1.00	1.00			
4							
5		Output 1 Birth	Output 2 Marriage	Input 1 Public servant	Weighted output	Weighted input	Weighted input minus Weighted output
6	Register office A	1	6	1	7.00	1.00	-6.00
7	Register office B	3	8	1	11.00	1.00	-10.00
8	Register office C	4	3	1	7.00	1.00	-6.00
9	Register office D	5	6	1	11.00	1.00	-10.00
10	Register office E	6	2	1	8.00	1.00	-7.00

Once the spreadsheet is ready, the parameters of *Excel* © Solver have to be specified in the following way:

- The objective is to maximize the weighted sum of outputs of

Register Office k ($\sum_{r=1}^s u_r y_{rk}$). In the objective parameter, cell

\$E\$2 has to be specified. The Max option has to be ticked.

- To optimize the objective, the changing variable cells have to be specified. They correspond to the weights associated with outputs and inputs. In the changing variable cells parameter, cells \$B\$3 to \$D\$3 (\$B3:\$D3) have to be specified.

- Finally, the restrictions placed on the changing cells have to be introduced as constraints. A constraint is added by clicking the Add button. In the Add Constraint box, three parameters have to be specified: the cell reference, the sign of the constraint (\leq , $=$ or \geq) and the value of the constraint. The first constraint of

the CRS model ($\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0$) is therefore

specified as follows: \$G\$6:\$G\$10 \geq 0 (where the cell reference is \$G\$6:\$G\$10, the sign is \geq and the constraint is 0). The

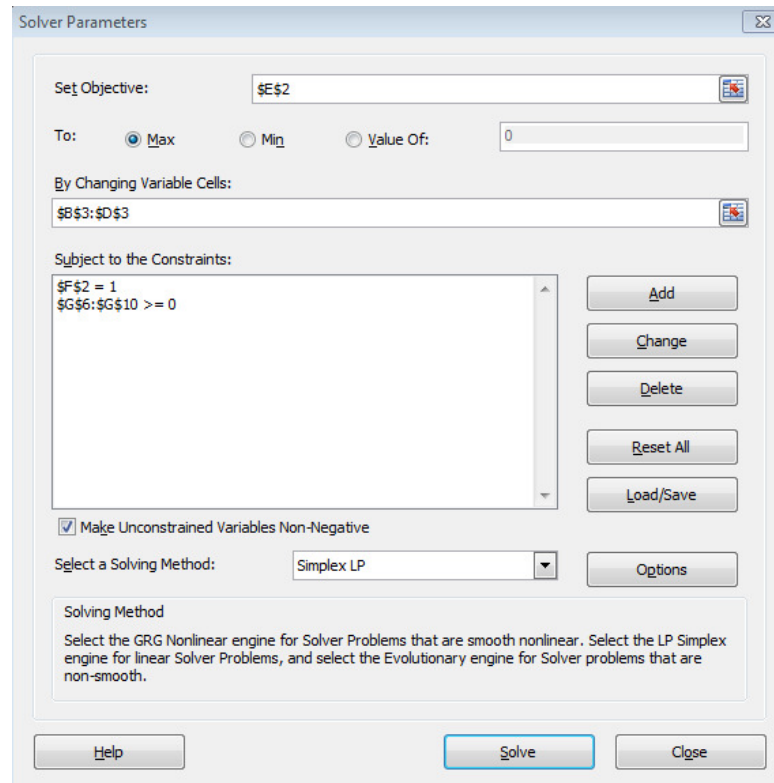
second constraint ($\sum_{i=1}^m v_i x_{ij} = 1$) is specified as follows:

\$F\$2=1 (where the cell reference is \$F\$2, the sign is $=$ and the

constraint is 1). Note that this constraint means that the given level of input is kept constant.

Figure 23 represents the Solver Parameters defined above.

FIGURE 23:
The Solver parameters are specified.



Finally, a Simplex LP solving method has to be selected and the 'Make Unconstrained Variables Non-Negative box' has to be ticked. This indicates that a linear model with non-negative variables is appropriate (and therefore the third and last 'constraint' $u_r, v_i > 0$ is taken into account).

The Solve button should be clicked in order to execute *Excel* © Solver. *Excel* © Solver will search every feasible solution to determine the solution which has the best target cell value. Register Office C obtains an efficiency score of 73.08% (cell G2). This score is obtained using weights of 0.15, 0.04 and 1 assigned to output 1, output 2 and input 1, respectively (cells B3, C3 and D3). A Solver Results box appears after solving the model. Before solving the model again for the other register offices, 'Restore Originals Values' has to be ticked before clicking the OK button.

To measure the efficiency of Register Office A (for example), it is necessary to replace the values of cells B3 to D3 (which currently refer to Register Office C) with the values of cells B6 to D6 (which refer to Register Office A). Solving the model will calculate an efficiency score of 75% for Register Office A.

Exercise 6

The objective of this exercise consists in programming the following CRS model using *Microsoft Excel* © Solver. The same data as above (see Table 4) have to be used. Note that this CRS model is equivalent to the one developed above. Instead of maximizing the weighted sum of outputs, however, it minimizes the weighted sum of the input.

$$\begin{aligned} & \text{Minimize} && \sum_{i=1}^m v_i x_{ik} \\ & \text{Subject to} && \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0 \quad j = 1, \dots, n \\ & && \sum_{r=1}^s u_r y_{rk} = 1 \\ & && u_r, v_i > 0 \quad \forall r = 1, \dots, s; i = 1, \dots, m \end{aligned}$$

Tasks

a) Prepare an *Excel* © spreadsheet adapted to the use of *Excel* © Solver.

b) Which cell has to be optimized?

Answer: The target cell to be optimized is \$F\$2

c) In this case, do you maximize output or minimize input?

Answer: Input minimization (the Min option has to be ticked in the Solver)

d) Which equation of the CRS model is optimized by the Solver?

Answer: $\sum_{i=1}^m v_i x_{ik}$. This equation corresponds to the minimization of the weighted sum of the input.

e) Which variables can be changed in the optimization process?

Answer: The weights associated with output 1, output 2 and input 3 can be changed (cells B3, C3 and D3)

f) Which constraint in the *Excel* © Solver corresponds to the

equation $\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0$ in the CRS model?

Answer: \$G\$6:\$G\$10 >= 0

g) Which constraint in the *Excel* © Solver corresponds to the equation

$\sum_{r=1}^s u_r y_{rk} = 1$ in the CRS model?

Answer: \$E\$2=1. This constraint means that the weighted sum of outputs must equal one (e.g. the given levels of outputs are kept constant).

h) Solve the CRS model for each register office. What are the efficiency scores?

Answer: A=75, B=100, C=73.08, D=100 and E=100

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