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Valuing life over the life cycle ☆

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ABSTRACT

Adjusting the valuation of life along the (i) person-specific (age, health, wealth) and (ii) mortality risk-specific (beneficial or detrimental, temporary or permanent changes) dimensions is relevant in prioritizing healthcare interventions. These adjustments are provided by solving a life cycle model of consumption, leisure and health choices and the associated Hicksian variations for mortality changes. The calibrated model yields plausible Values of Life Year between 154K$ and 200K$ and Values of Statistical Life close to 6.0M$. The willingness to pay (WTP) and to accept (WTA) compensation are equal and symmetric for one-shot beneficial and detrimental changes in mortality risk. However, permanent, and expected longevity changes are both associated with larger willingness for gains, relative to losses, and larger WTA than WTP. Ageing lowers both variations via falling resources and health, lower marginal continuation utility of living and decreasing longevity returns of changes in mortality.

1. Introduction

The COVID-19 pandemic has been associated with considerable economic and personal tolls. Indeed, both the preventive (e.g. lock-downs, testing and vaccination, quarantine for infected and contact persons) as well as curative measures (e.g. postponement of elective surgeries and reallocation of resources towards dedicated acute and intensive care units and COVID-related research) have led to substantial macroeconomic, public finance and individual costs.

Two of the motivations often invoked to justify these interventions have been (i) the collective duty to protect society's most vulnerable members, and (ii) the consequences of pandemic-driven excess demand for medical care. In particular, both the elderly, and individuals with pre-existing medical conditions were considered to be more at risk of developing life-threatening complications and had to be shielded from COVID-19 infection and prioritized in intervention (e.g. Briggs et al., 2021; Viscusi, 2020; Brotherhood et al., 2020). Moreover, the allocation of scarce medical resources (e.g. access to intensive care and ventilators) in situations of excess demand for life support raised the specter of uncomfortable medical triage decisions between saving one person against another.

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These considerations highlight the fundamental questions of (i) how to value longevity in general and how to adjust this value to account for (ii) the personal characteristics such as age, health, labor market and financial statuses, as well as (iii) the characteristics of the changes in death risk (e.g. magnitude, beneficial vs detrimental, permanent vs temporary, longevity mean vs variance). Indeed, the substantial costs to society of COVID-19 measures should be contrasted with the presumably large economic value of those lives saved by intervention.[1] Moreover, the reallocation of such consequential financial and medical resources to the pandemic raises the issue of the long-term arbitrage of addressing a single illness at the potential expense of others. Put more bluntly, the delicate question of *which lives* should be prioritized – contemporary COVID-19 infected vs other current or future illnesses, young vs old, healthy vs unhealthy, rich vs poor – was brutally unearthed by the pandemic.

Addressing the first question of life value measurement involves proxying the (non-marketed) value of longevity through a theoretical (shadow) price. A natural candidate is the marginal rate of substitution (MRS) between additional life/mortality and wealth which, at the optimum, will capture the relative price of longevity. A second related alternative is the maximal willingness to pay (WTP) or the minimal willingness to accept compensation (WTA) for changes in life expectancy. The Value of a Statistical Life (VSL) is an infra-marginal approximation to the MRS that sums the willingness across agents to calculate an aggregate WTP or WTA to save *someone*, i.e. an unidentified (statistical) member of the community. Personalized life values can be assessed from the market value of an agent's foregone net revenues such as in the Human Capital (HK) value. Despite its usefulness in wrongful death litigation, the HK value is arguably less relevant for non-working (e.g. retired or disabled) agents, and therefore imperfectly applicable for society's more vulnerable members. Identified values can alternatively be recovered from the agent-specific MRS, WTP and WTA. An extreme example, potentially useful in both litigation and terminal care decisions, is a person's two Gunpoint (GPV) values: her willingness to pay to prevent and to receive compensation to accept imminent and certain death which gauges a specific person's willingness to save or lose her *own* life.

Secondly, adjusting identified life values for personal characteristics involves charting how ageing processes (e.g. the life cycles of wages, morbidity and mortality risks, and finite biological longevity bounds), quality of life (e.g. health status, mix between market activities such as consumption and non-market ones, such as leisure) and disposable resources (financial wealth, labor income) affect an agent's shadow price of longevity. Third, since life values are to be inferred from changes in death risk exposure, the distributional characteristics of these changes are relevant. Indeed, whether the changes correspond to small or large, temporary or permanent increases or decreases in mortality risk and whether those changes affect the mean and/or the variance of longevity will alter the individual and societal willingness measures, and therefore the degree of substitution between personalized lives. For example, how do we compare the possibly large contemporary beneficial gains of intervention on the survival outcomes of currently infected persons versus the possibly small, but long-term detrimental increases in the risk of dying of agents whose interventions have been postponed is certainly relevant to both groups and to society as a whole.

This paper addresses the three issues of life valuations (i) methodology, as well as characterization along the (ii) person-specific and (iii) mortality risk-specific dimensions.[2] Its main research questions are to what extent (i) exogenous ageing processes determine the life cycle (LC) of both endogenous decisions and their associated utility of living, (ii) this continuation utility is altered by changes in longevity risk, (iii) these welfare benefits and costs translate into nominal life valuation metrics, and (iv) the agent's personal characteristics, as well as the distribution of changes in mortality risk affect this endogenous life valuation process.

To address these questions, I adopt the Revealed Preference viewpoint and resort to a flexible LC model where the agents optimally choose investment in their health, labor/leisure, and consumption/savings in an environment where increasing exogenous wages peak at mid-life and decrease afterwards, whereas mortality and morbidity risks exposures both exogenously increase in age. Flexibility is achieved by appending intra-temporal substitution between consumption and leisure, as well as bequest motives to a generalized recursive preferences framework (Epstein and Zin, 1989, 1991; Weil, 1990, henceforth EZW). This specification disentangles inter-temporal substitution (EIS) from risk aversion (RRA) and admits non-indifference to the early or late timing of the resolution of uncertainty, allowing for rich nonlinear effects of mortality risks on welfare that are abstracted from in standard expected-utility settings.[3]

The identification of the life cycle of life values is made possible through the continuation (indirect) utility, i.e. the forward-looking welfare of continued living along the optimal path. More precisely, I calculate the Hicksian Equivalent (EV), and Compensating (CV) Variations for both beneficial, and detrimental, permanent and temporary, marginal and infra-marginal changes in death risk exposure using the indirect utility from the agent's optimal health investment, leisure, and consumption choices. The two Hicksian variations directly identify the agent-specific MRS, as well as four other life valuations: the WTP (resp. WTA) to attain (resp. forego) beneficial changes, as well as to prevent (resp. accept) detrimental changes in mortality. Importantly, all rates of substitution between longevity and resources directly encompass quality of life concerns by incorporating the shadow value of the human (i.e. health) capital, as well as the leisure utility in the Hicksian values.[4]

[1] For example, Hammitt (2020) relies on VSL estimates (discussed below) to conjecture that the U.S. population could be willing to spend 5% of GDP to reduce COVID-19 death by 100,000. Hall et al. (2020) find that agents would be willing to pay between 28% and 41% of annual consumption to avoid the increase in death exposition associated with COVID-19. See also Viscusi (2020), Brotherhood et al. (2020) and Bloom et al. (2022) for evidence and discussion.

[2] The expression 'life valuation' will be relied generically throughout the paper to refer to willingness to pay or receive compensation for infra- or marginal changes in longevity, as well as in the polar case of a whole life where the change involves all remaining longevity.

[3] See Eeckhoudt and Hammitt (2004), Bleichrodt and Pinto (2005), Córdoba and Ripoll (2017) and Hugonnier et al. (2022) for discussion of EZW preferences in life valuation settings.

[4] Murphy and Topel (2006) also advocate a similar leisure- and health-based perspective on quality of life considerations. Note however that, unlike our specification, the multiplicative functional form for health benefits they resort to implies that the latter is absent from VSL calculations.

A Revealed Preference perspective is again relied upon to compute monetary life value estimates. The model is first calibrated at realistic values to reproduce observed LC's for both exogenous (mortality, morbidity and wages) and endogenous variables (health, wealth, work, labor income). Since no unique data set regroups all these variables, I combine Panel Study of Income Dynamics (PSID), American Time Use Survey (ATUS), and U.S. Life Tables under a common time frame, and under the usual assumption that the data sets are representative of the same set of agents. The calibrated model parameters are next incorporated with the model's dynamics in the closed-form solutions to evaluate the life cycles of the Hicksian Variational and Willingness measures over marginal and infra-marginal changes in death risk. The Hicksian measures can then be adapted to calculate the model-implied life cycle of life values such as the value of a life year (VOLY), the VSL and the MRS, as well as the two Gunpoint values.

An often overlooked caveat of the life valuation analysis is that changes in the death probability affect both the mean and the variance of longevity.[5] Indeed, the agent is willing to pay or receive compensation for either beneficial or detrimental changes that *jointly* alter both the desirable life expectancy and the undesirable longevity risk. Fortunately, the closed-form solutions allow to disentangle the two effects via a mean-preserving calculation of the utilitarian cost of mortality *risk exposure*. More precisely, the Deterministic Longevity Value (DLV) can be computed as the WTP/WTA to eliminate all longevity risk and be assured to live a person's age-adjusted remaining life expectancy. It provides a useful complement to the more traditional life values in gauging the cost of not knowing the timing of death.

The three key takeaways from this paper are the following. First, the model offers a tractable framework to identify the contributing channels to life valuations of preferences, socio-economic gradients, ageing, as well as death risk characteristics. Indeed, all the welfare effects of changes in mortality are subsumed in the marginal continuation utility of net total wealth i.e. the sum of financial wealth, net present value of wages (permanent income), and the shadow value of the health capital. At realistic EIS, an increase in death risk induces heavier discounting of future welfare and higher marginal propensity to consume (MPC) which decrease the marginal value of net total wealth. The associated welfare loss warrants positive willingness for both beneficial and detrimental increments in death risk exposure, consistent with the preference for life axiom.[6] Moreover, the elasticities of all life values with respect to financial wages, permanent income and health capital simplify to the shares of these variables in the agent's net total wealth.

Second, the effects are symmetric in the case of one-shot changes in death risk exposure, but are asymmetric for permanent changes, as well as for changes in expected longevity. In particular, one-shot mortality changes are valued indiscriminately whether they correspond to survival gains or losses, and whether from a WTP or WTA perspective. In contrast, the WTP/WTA to attain/forego permanent survival gains are larger than those to prevent/accept permanent losses of equivalent magnitude. They are also consistent with endowment-type effects associated with loss aversion whereby longevity selling prices (WTA) are larger than buying prices (WTP). Similar asymmetries arise when the changes are expressed in expected longevity (instead of survival) increments, regardless of whether they are one-shot or permanent.

Third, ageing is associated with (i) lower WTP/WTA per given change in death intensity, but (ii) higher willingness per given change in expected longevity. Indeed, the combined influence of falling wages, increased morbidity and mortality risks exposures and eroding remaining horizon imply falling net total wealth. Moreover, increasing mortality risks induces lower marginal (and therefore continuation) utility, although the mortality effects are dampened by age. Finally, the longevity returns of changes in survival fall in age, i.e. elders require much larger mortality changes to attain a given change in expected longevity. The combination of the three factors induces a lower willingness for changes in survival risk, but a higher willingness for expected longevity changes for older agents. This is evidenced in the predicted VSL for a one-shot mortality change of 1.0e−03 which falls from 11.0 M$ at age 25 to 3.45 M$ at age 65. Similarly, the WTP to avoid certain imminent death falls from 1.75 M$ at 25 to 1.15 M$ at 65, whereas the WTA to accept death is unsurprisingly higher and falls from 4.13 M$ at 25 to 1.92 M$ at 65. Conversely, the WTP/WTA associated with changes in expected longevity increase in age, although the effects of ageing are weaker. The WTP per additional life-year through one-shot changes thus increases from 211 K$ at age 25 to 220 K$ at age 65. One interesting non-monotone exception to ageing effects is the hump-shaped DLV. The willingness to eliminate all mortality risk and be assured of living the remaining expected longevity is lower for both young agents (endowed with large *expected* longevity) and elders (endowed with large *realized* longevity). The WTP (resp. WTA) is 344 K$ (resp. 388 K$) at 25, peaks at 503 K$ (resp. 605 K$) at age 52 and falls to 450 K$ (resp. 542 K$) at 65, consistent with peaking mid-life demand for longevity-risk insurance such as annuities.

A comparative statics exercise reveals how the results are sensitive to the model assumptions. Increasing the bequest motive produces two opposing forces with respect to life values: an increase in disposable net total wealth to meet intended bequests and an attenuation in the utility cost of dying. The latter is dominant with a reduction in all life value measures. Similarly, removing non-marketed leisure activities increases disposable resources, but reduces the quality of life; the latter is again the dominant force with decreases in all life values. Abstracting from health unambiguously lowers the values of life by omitting to account for quality of healthy living considerations. Finally, the more restrictive Expected Utility (EU) and Constant Relative Risk Aversion (CRRA) paradigm without leisure, health or bequests lowers the life values compared to the complex interactions made possible by EZW preferences.

This paper is primarily related to quantitative, life cycle approaches to life valuations initiated by Conley (1976), Shepard and Zeckhauser (1984) and Rosen (1988).[7] (Córdoba and Ripoll, 2017) also consider a LC framework of consumption and leisure choices

[5] The binomial Bernoulli processes, which are typically used for mortality risk, restrict all the distributional moments to depend on the same parameter governing the probability of occurrence.

[6] See Weinstein et al. (1980, Assumptions 1 and 2) for characterization and discussion of preference for life over death.

[7] See Hugonnier et al. (2021) for a more thorough review of the life valuation literature.

featuring EZW preferences to analyze the value of changes in mortality exposure. They emphasize the importance of non-linearities in death probabilities made possible by recursive preferences to generate rich predictions regarding WTP's. However, they integrate neither endogenous human capital choices, nor morbidity shocks in their analysis of life valuations. Hugonnier et al. (2013, 2022) do incorporate stochastic human capital considerations in a continuous-time model with similar recursive preferences. However, they neither explicitly focus on life cycle implications, nor integrate the leisure choices, nor the role of bequests in life values. Martin and Pindyck (2021) rely on a VSL perspective in a dynamic model to study the WTP to prevent welfare losses accruing from independent catastrophic events linked to loss of life and/or loss of consumption. Their modeling approach is very different; they resort to EU/CRRA preferences, abstract from health, leisure and life cycle considerations and compute societal WTP accounting for both alive and dead agents, rather than from the living agents exclusively. Importantly, none of these papers provide full characterization of Hicksian willingness measures for beneficial and detrimental, one-shot and permanent changes in longevity, and none explicitly focus on the LC trajectories for life values.[8]

The rest of the paper is organized as follows. After presenting a simplified version to illustrate the main intuition in Section 2, I outline the full LC model and its solution in Section 3. Section 4 presents the theoretical implications for the Hicksian measures. The empirical strategy and the life cycle for the agent's resources and welfare metrics as well as the empirical Hicksian willingness measures is reported in Section 5. Discussion and concluding remarks are regrouped in Section 6.

2. A simplified life-cycle model of life valuation

2.1. Two-period model

Before introducing the full model in Section 3, it is useful to consider a simplified version to highlight the key elements of the life cycle and main differences with static approaches to life valuation. For that purpose, assume the agent lives for at most $T = 2$ periods and faces a probability $\exp(-\lambda^m) \in (0, 1)$ of surviving from period t to $t+1$; has VNM/CRRA preferences with utility at death normalized to zero; has access to net total wealth N_t composed of financial wealth, and human wealth associated with wages, and health capital. The agent chooses consumption (c_t, c_{t+1}) to solve:

$$\begin{aligned} V(N_t, \lambda^m) &= \max \frac{c_t^{1-\gamma}}{1-\gamma} + \exp(-\rho)\mathrm{E}_t \frac{c_{t+1}^{1-\gamma}}{1-\gamma} \\ &= \max \frac{c_t^{1-\gamma}}{1-\gamma} + \exp(-\rho-\lambda^m)\frac{c_{t+1}^{1-\gamma}}{1-\gamma} \end{aligned} \tag{1}$$

$$\text{subject to: } c_t + \exp(-r)c_{t+1} \leq N_t,$$

with positive discount ρ, interest r rates, and risk aversion γ. A standard argument in Appendix B.1 establishes that the solution to this problem is:

$$\begin{aligned} c_t &= \omega(\lambda^m)N_t, \quad c_{t+1} = [1-\omega(\lambda^m)]\exp(r)N_t \\ V(N_t, \lambda^m) &= \frac{\left[\psi(\lambda^m)N_t\right]^{1-\gamma}}{1-\gamma}, \quad \text{where} \\ \omega(\lambda^m) &= \left\{1 + \exp\left[\frac{-\rho-\lambda^m+(1-\gamma)r}{\gamma}\right]\right\}^{-1} \\ \psi(\lambda^m) &= \omega(\lambda^m)^{\frac{\gamma}{\gamma-1}}. \end{aligned} \tag{2}$$

This simple example illustrates three key takeaways. First, a stochastic setting where the agent is exposed to mortality is equivalent to a deterministic problem with additional discounting at rate $(\rho + \lambda^m)$. Second, the heavier discounting from mortality risk λ^m unconditionally increases the marginal propensity to consume $\omega(\lambda^m)$ in (2); the higher MPC induces a detrimental effect on marginal value $\psi(\lambda^m)$ (and therefore welfare V) only when $\gamma \in (0, 1)$. Otherwise, high risk aversion ($\gamma > 1$, consistent with empirical asset pricing literature) leads to the well-known paradox that death (utility of zero) is preferable to life (negative utility).[9] Third, the willingness to pay $v(N_t, \lambda^m, \Delta)$ to prevent an increase $\Delta > 0$ in death risk λ^m is obtained by solving the Hicksian indifference equation:

$$\begin{aligned} &V(N_t - v, \lambda^m) = V(N_t, \lambda^m + \Delta), \\ \Longrightarrow\ &v(\lambda^m, N_t, \Delta) = \left[1 - \frac{\psi(\lambda^m+\Delta)}{\psi(\lambda^m)}\right]N_t, = \left[1 - \left(\frac{\omega(\lambda^m+\Delta)}{\omega(\lambda^m)}\right)^{\frac{\gamma}{\gamma-1}}\right]N_t. \end{aligned} \tag{3}$$

The WTP is proportional to net total wealth N_t, and depends primarily on changes in marginal values $\psi(\lambda^m)$ and MPC $\omega(\lambda^m)$ that are induced by discounting changes Δ. The willingness (3) is positive at low RRA and can be relied upon to compute the MRS (i.e. $\partial v/\partial \Delta\mid_{\Delta=0}$), the VSL (i.e. $v(N, \lambda^m, \Delta)/\Delta$) or Gunpoint values (i.e. $\lim_{\Delta\to\infty} v(\lambda^m, N_t, \Delta)$).

[8] One exception is Hammitt and Treich (2007) who provide a characterization along the beneficial/detrimental dimensions of both WTP and WTA measures. However, their analysis is static, does not integrate endogenous LC decisions, nor permanent vs temporary changes, and is mainly focused on the role of personalization (i.s. statistical vs identified) of life values.

[9] See Shepard and Zeckhauser (1984) and Rosen (1988) for discussion and Becker et al. (2005) and Hall and Jones (2007) for solutions to this paradox.

2.2. Comparison with the static model

A standard static approach to life valuations (e.g. Hammitt, 2020; Andersson and Treich, 2011) is to impose $\rho = 0$ and postulate (rather than solve) a welfare function:

$$V(\lambda^m, N) = \exp(-\lambda^m)u^a(\lambda^m, N) + [1 - \exp(-\lambda^m)]u^m(\lambda^m, N)$$

where $u^s(\lambda^m, N)$ are the states- $s = a, m$, and mortality-dependent felicity for alive and dead agents. In the particular case of CRRA utility and proportional bequests:

$$u^a(\lambda^m, N) = \frac{[\tilde{\psi}(\lambda^m)N]^{1-\gamma}}{1-\gamma}, \quad u^m(\lambda^m, N) = \frac{[b\tilde{\psi}(\lambda^m)N]^{1-\gamma}}{1-\gamma},$$

for $b \in (0, 1)$ and a generic, mortality-dependent marginal value $\tilde{\psi}(\lambda^m)$, the corresponding welfare and WTP for Δ are:

$$V(\lambda^m, N) = \frac{[\tilde{\psi}(\lambda^m)N]^{1-\gamma}}{1-\gamma} \left\{\exp(-\lambda^m) + [1 - \exp(-\lambda^m)]b^{1-\gamma}\right\}$$
$$\implies v(\lambda^m, N, \Delta) = \left\{1 - \frac{\tilde{\psi}(\lambda^m + \Delta)}{\tilde{\psi}(\lambda^m)} \left[\frac{b^{1-\gamma} + \exp(-\lambda^m - \Delta)[1 - b^{1-\gamma}]}{b^{1-\gamma} + \exp(-\lambda^m)[1 - b^{1-\gamma}]}\right]^{\frac{1}{1-\gamma}}\right\} N.$$

In the absence of bequest motives ($b^{1-\gamma} = 0$) the static model simplifies to:

$$V(\lambda^m, N) = \frac{[\tilde{\psi}(\lambda^m)N]^{1-\gamma}}{1-\gamma} \exp(-\lambda^m),$$
$$\implies v(\lambda^m, N, \Delta) = \left\{1 - \frac{\tilde{\psi}(\lambda^m + \Delta)}{\tilde{\psi}(\lambda^m)} \exp\left[\frac{-\Delta}{1-\gamma}\right]\right\} N. \quad (4)$$

Whereas the static, and life-cycle life valuation models are isomorphic, several key differences can be noted. First, unlike its static counterpart $\tilde{\psi}(\lambda^m)$ in (4), the life cycle model (2) and (3) fully integrates the discounting effect of mortality λ^m on both the MPC $\omega(\lambda^m)$ and the marginal value $\psi(\lambda^m)$. Second, the crucial role of risk aversion γ cannot be identified without further characterization of the marginal value $\tilde{\psi}(\lambda^m)$. Third, incorrectly assuming mortality-independent marginal value $\tilde{\psi} \perp \lambda^m$ in (4) leads to:

$$V(\lambda^m, N) = \frac{[\tilde{\psi} N]^{1-\gamma}}{1-\gamma} \exp(-\lambda^m),$$
$$\implies v(\lambda^m, N, \Delta) = \left\{1 - \exp\left[\frac{-\Delta}{1-\gamma}\right]\right\} N$$

which only accounts for the discounting effect and produces a biased representation of both welfare and the willingness to pay. Indeed, the latter is (i) independent of baseline risk λ^m, and (ii) counter-intuitively associated with infinite negative GPV when $\Delta = \infty$ and $\gamma > 1$.

To summarize, the solution to the life cycle model yields a more complete characterization of higher discounting effect of mortality on both the MPC $\omega(\lambda^m)$ and the marginal value $\psi(\lambda^m)$. The full model presented next generalizes the simplified LC version along several dimensions. First, I consider a flexible T-periods horizon with exogenous time variation in wages w_t, morbidity λ_t^h and mortality λ_t^m. Second, I allow for a large spectrum for Δ, including beneficial, detrimental, one-shot and permanent changes in death intensity λ_t^m. Third, endogenous investment I_t in health H_t, under exogenous morbidity is appended. Fourth, I fully characterize the role of capitalized wages and health in net total wealth N_t. Fifth, both consumption/leisure substitution in felicity $u(c_t, \ell_t)$, as well as a bequest motive are added. Finally, replacing VNM with Non-Expected utility provides richer non-linear effects of mortality λ^m and disentangles the incidence of risk aversion and inter-temporal substitution.

3. General model

3.1. Economic environment

Horizon. Let $t \in [0, \min(T^m, T)]$ denote discrete time, also mapping to age since entering adulthood, where T^m is the stochastic timing of death following a Poisson binomial process with exogenous, age-increasing intensity λ_t^m, and T is the maximal biological longevity.[10] Life valuations will be established through additive one-shot and permanent changes $\Delta_t \in [-\lambda_t^m, \infty]$ in death intensity λ_t^m occurring at age t.[11] The effects of Δ_t on the age-t one-period p_t and k−periods ahead $P_{t,t+k}$ survival probabilities, as well as expected longevity L_t are summarized in Table 1.

[10] The assumption of exogenous mortality is required to attain closed-form solutions in the current framework with endogenous health investment. As shown by Hugonnier et al. (2013), the agent problem with health-dependent mortality otherwise becomes non-separable, and requires approximate (instead of exact) solution methods in dynamic settings. See Ehrlich and Becker (1972), Ehrlich (2000) and Liu and Neilson (2006) for other applications with endogenous investment in individual safety and self-insurance.

[11] The literature stresses the importance of distinguishing between transient vs permanent, level vs proportional changes in mortality. See Rosen (1988), Liu and Neilson (2006), Nielsen et al. (2010), Aldy and Smyth (2014), Jones-Lee et al. (2015) and Hammitt and Tunçel (2015) in particular.

Table 1
Effects of one-shot and permanent changes in λ^m.

Base λ_t^m	Change $\lambda_t^m + \Delta_t$	
	One-shot: $\Delta_t = \mathbb{1}_t \Delta$	Permanent: $\Delta_t = \Delta, \forall t$
$p_t = \exp(-\lambda_t^m)$	$\nabla p_t = [\exp(-\Delta) - 1] p_t$	$\nabla p_t = [\exp(-\Delta) - 1] p_t$
$P_{t,t+k} = \exp\left(-\sum_{\tau=0}^{k-1} \lambda_{t+\tau}^m\right)$	$\nabla P_{t,t+k} = [\exp(-\Delta) - 1] P_{t,t+k}$	$\nabla P_{t,t+k} = [\exp(-\Delta k) - 1] P_{t,t+k}$
$L_t = \sum_{k=1}^{T-t} P_{t,t+k}$	$\nabla L_t = [\exp(-\Delta) - 1] L_t$	$\nabla L_t = \sum_{k=1}^{T-t} [\exp(-\Delta k) - 1] P_{t,t+k}$

Notes: The changes $\nabla x_t \equiv x_t(\lambda_t^m + \Delta_t) - x_t(\lambda_t^m)$ are for variables $x_t \in \{p_t, P_{t,t+k}, L_t\}$.

Hence, one-shot changes to the death intensity λ_t^m imply proportional changes in both one-period ∇p_t and k-periods $\nabla P_{t,t+k}$ survival probabilities, as well as in longevity ∇L_t. Unsurprisingly, permanent changes imply much more potent effects on the latter two relative to one-shot changes; a target change in longevity ∇L_t will therefore require much lower permanent than one-shot changes to be attained.

Financial and health capital dynamics. Let W_t denote the agent's financial wealth, c_t her consumption and $\ell_t \in [0,1]$ her leisure. The agent faces the following financial constraints:

$$W_{t+1} = \left[W_t + Y_t - M_t - c_t\right] R, \tag{5a}$$

$$Y_t = y + w_t(1 - \ell_t), \tag{5b}$$

$$M_t = i + I_t - BH_t. \tag{5c}$$

The budget constraint assumes a constant risk-free rate $R = \exp(r)$ in (5a). Income Y_t in (5b) is net wages w_t over work time $n_t = (1 - \ell_t)$, plus constant revenues y (e.g. private pension, social security). Medical expenses M_t in (5c) include a constant level i (e.g. health insurance) plus the endogenous expenses that increase in chosen level of investment I_t, but decrease in the agent's health H_t.[12]

Next, I assume the following dynamics for health capital:

$$H_{t+1} = A I_t^\alpha H_t^{1-\alpha} + (1 - \delta - \epsilon_{t+1}^h \phi) H_t, \tag{6a}$$

where the stochastic morbidity schock follows:

$$\epsilon_{t+1}^h = \begin{cases} 0 & \text{with prob. } \exp(-\lambda_t^h), \\ 1 & \text{with prob. } 1 - \exp(-\lambda_t^h). \end{cases} \tag{6b}$$

The Grossman (1972) and Ehrlich and Chuma (1990) demand for health function (6a) is similar to Palacios (2015), Hugonnier et al. (2013, 2020, 2022) and assumes Cobb–Douglas technology in (I_t, H_t). Stochastic morbidity ϵ_{t+1}^h is appended to gross investment in (6b) where illness is also Poisson binomial, occurs at age-increasing rate λ_t^h, and induces additional depreciation $\phi \in (0, 1-\delta)$ to the health stock. The dual effects of I_t on expenses (5c) and health dynamics (6a) imply that the health investment is more comprehensive than only medical expenses in capturing the monetary value of *all* preventive and curative, market and non-market care (e.g. healthy habits, informal care) affecting health H_t.

We close our discussion of financial constraints and health dynamics by following Rosen (1988), Murphy and Topel (2006) and Hugonnier et al. (2013, 2022) in assuming perfect and complete financial markets. In particular, (i) health shocks in ϵ_{t+1}^h in (6b) can be fully insured against at actuarially-fair premia,[13] (ii) a claim to any net income stream can be sold at no additional costs in exchange for its capitalized value, and (iii) borrowing capacity is only subject to the budget constraint, and no additional restrictions. Admittedly, the perfection and completeness assumptions are often at odds with real-life financial markets. Indeed, agents may be unable to fully insure at actuarially-fair prices and hence remain exposed to idiosyncratic net income shocks, such as wages, unemployment and medical expenses. Moreover, not all income stream (e.g. public pension claims) can be capitalized. Finally, borrowing capacity is often restricted by both asset (loan-to-value, LTV) and income (debt-to-income, DTI) limitations, in addition to the budget constraint. Nonetheless, perfect markets are often assumed in Life Cycle, Asset Pricing and Human Capital settings as a necessary tradeoff to attain analytical solutions. Indeed, closed-form solutions otherwise become notoriously challenging, and

[12] Notice that the budget constraint (5) can equivalently be rewritten as:

$$W_{t+1} = \left\{W_t + [y + w_t(1-\ell_t) - BS_t] - (\tilde{i} + I_t) - c_t\right\} R.$$

where $S_t \equiv H^+ - H_t \geq 0$ is a sickness index for some upper bound on health $H^+ \equiv \sup(H_t)$ and where $\tilde{i} \equiv i - BH^+$ is fixed medical expenses. The unhealthy agent suffers from sick-leave penalties BS_t on labor income that can be mitigated by investing I_t in her own health (see Hugonnier et al., 2013, 2022, for a similar interpretation).

[13] Markets are *ex-ante* incomplete since only morbidity λ_t^h (e.g. health insurance), and not mortality λ_t^m (e.g. life insurance) can be insured at fair prices. Nevertheless, as mentioned in Section 2, *ex-post* completeness obtains since mortality exposure can equivalently be recast as heavier discounting of future utility flows. See Hugonnier et al. (2013) for discussion.

require numerical approaches that face their own tradeoffs between realism and computability.[14] Importantly, as will be seen in Section 3.3, assuming perfect and complete financial markets *does* allow for closed-form solutions in a fairly complex setting, with tractable implications for life valuation purposes.

3.2. Agent's problem and preferences

Agent's problem. I resort to Epstein and Zin (1989, 1991) and Weil (1990) non-expected utility to represent the agent's preferences. In addition to adding a bequest motive, I append separate iso-elastic preferences over consumption and leisure to the EZW framework.[15] More precisely the agent's problem can be written as:

$$V_t = V_t(W_t, H_t) = \max_{\{c_t, \ell_t, I_t\}} \left[(1-\beta)u(c_t, \ell_t)^{1-\varepsilon} + \beta \mathrm{CE}_t(V_{t+1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{7a}$$

where the felicity function is:

$$u(c_t, \ell_t) = \left[\theta c_t^{1-\sigma} + (1-\theta)\ell_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{7b}$$

the certainty-equivalent (CE) of the continuation utility is:

$$\mathrm{CE}_t(V_{t+1}) = \left[p_t \mathrm{E}_t V_{t+1}^{1-\gamma} + \left(1-p_t\right) E_t V_{t+1}^{m\ 1-\gamma}\right]^{\frac{1}{1-\gamma}},$$

the utility at death is:

$$V_{t+1}^m = \tilde{b}[W_{t+1} + V_{t+1}^H(H_{t+1})], \quad \tilde{b} \equiv b^{1/(1-\gamma)} \tag{7c}$$

and the shadow value of health capital V_{t+1}^H is:

$$V_{t+1}^H = \eta_{t+1} H_{t+1}. \tag{7d}$$

The dynamic problem is subject to financial (5) and health dynamics (6) and where initial wealth and health (W_0, H_0) are taken as given.

First, $\beta \in (0,1)$ is a subjective discount factor in the agent's problem (7a), whereas $1/\varepsilon > 0$ measures the elasticity of *inter*-temporal substitution (EIS) between current felicity and the CE of future utility. Second, the parameter $\theta \in (0,1)$ in the felicity (7b) measures the consumption share whereas $1/\sigma > 0$ is the elasticity of *intra*-temporal substitution (EIaS) between consumption and leisure. Third, $\gamma > 0$ is the agent's relative risk aversion that is disentangled from the EIS, unlike in the VNM case which imposes $\varepsilon \equiv \gamma$. Fourth, V_{t+1}^m is the warm-glow utility at death provided by bequests, with b (resp. $\tilde{b}$) capturing the bequest motive (resp. share).[16] The specification in (7c) and (7d) allows for perfect substitution between financial wealth W_{t+1} and the endogenous value of the health capital $V_{t+1}^H = \eta_{t+1} H_{t+1}$. The latter can be interpreted as more sense of fulfillment, or less suffering for healthier agents in the last period of life, as well as through the lenses of intra-family informal long-term care. Indeed Lemma 2 below establishes that V_{t+1}^H captures the shadow value of the optimal (comprehensive) medical expenses net benefits of better health $BH_t - I_t$, and that the Tobin's-q η_{t+1} is falling in age under the increased exposure to morbidity risk λ_{t+1}^h. The linear indifference curves at death $W_{t+1} = \tilde{b}^{-1}\bar{V}^m - \eta_{t+1} H_{t+1}$ indicate that (i) unhealthy and/or (ii) older agents facing higher sickness risks must bequeath higher financial wealth W_{t+1} to compensate for the cost of more informal care provided by children, siblings and spouse in the last periods of life in order to attain a given level of utility at death $\bar{V}^m$.[17]

Properties. The agent's problem with health-independent preferences and health-dependent medical expenses is iso-morphic to one where utility depends on H_t and health expenditures M_t are not separately health-dependent. Indeed, we can follow Hugonnier et al. (2013, 2022) in setting $\hat{c}_t \equiv c_t - BH_t$, substitute in budget constraint (5a) and felicity (7b) to obtain that the agent selects $\hat{c}_t$ and ℓ_t to solve (7a) with health-dependent felicity $u(\hat{c}_t + BH_t, \ell_t)$ and equivalent wealth dynamics replaced by:

$$W_{t+1} = \left[W_t + Y_t - i - I_t - \hat{c}_t\right] R.$$

Moreover, three elements concur to induce an age-dependent allocation $\{c_t, I_t, \ell_t\}_{t=0}^T$: (i) longevity is bounded above by T, (ii) both risk exposures to death λ_t^m and illness λ_t^h increase with age, and (iii) wages w_t are subject to exogenous time variation. As will become apparent shortly, the induced time variation in the associated continuation utility $V_t(W_t, H_t)$ will generate age-dependency in the life valuation measures calculated from the welfare function.

[14] Notably state space dimensionality, parametric dependency of results, local optima, computing time, among others.

[15] See also (Córdoba and Ripoll, 2017; Kraft et al., 2022) for discussion of bequests and van Binsbergen et al. (2012) for inclusion of leisure utility in EZW preferences contexts.

[16] See also (Kraft et al., 2022) for discussion on the importance of accounting for risk aversion in the identification of the bequest motive in an EZW setting.

[17] See Groneck (2017), Cremer and Roeder (2017) and Jones et al. (2020) for evidence and theory on the positive links between expected financial bequests and amount of informal LTC provided by heirs.

3.3. Optimal allocation

Overview. To alleviate notation I omit time subscripts for contemporary variables, use prime (′) for next-period variables, and rely on t subscripts to emphasize feedback rules calculated at time t whenever appropriate. The method used to solve the agent's problem (7) subject to financial (5) and health (6) constraints involves three steps that are described in greater details in Appendix A. I first resort to standard static optimization to solve for the optimal mix between leisure ℓ and consumption c and recover total expenses $\tilde{c} \equiv c + w\ell$ and felicity $u(c, \ell)$ in terms of $\tilde{c}$ (Lemma 1). Second, separability between health-related and financial decisions is invoked to solve for the optimal health investment I and shadow value V^H (Lemma 2). Third, under perfect markets, the endogenous shadow value of health and the exogenous net present value of wages V^w are both added to financial wealth W to obtain net total wealth $N = W + V^w + V^H$. The equivalent problem of maximizing utility over total expenses $\tilde{c}$ subject to dynamics for N is then solved (Theorem 1).[18]

3.3.1. Optimal labor-consumption choices

A standard argument, applicable in our setting, establishes the well-known a-temporal condition equalizing the marginal rate of substitution between optimal leisure and consumption to wages to obtain the following intermediate result:

Lemma 1 (*Consumption-Leisure*). *The optimal total expenses $\tilde{c} \equiv c + w\ell$ and felicity $u(c, \ell)$ are given by:*

$$\tilde{c} = c\mu(w) \tag{8}$$

$$u(c, \ell) = \tilde{c}\nu(w), \tag{9}$$

where the wage-dependent loadings are:

$$\mu(w) \equiv 1 + \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\sigma}} w^{1-\frac{1}{\sigma}} \geq 0 \tag{10a}$$

$$\nu(w) \equiv \left[\theta\mu(w)^{\sigma}\right]^{\frac{1}{1-\sigma}} \geq 0. \tag{10b}$$

Properties. First, the variable $\mu(w) - 1 = w\ell/c$ represents the optimal leisure to consumption ratio, with $\mu(w)^{-1} = c/\tilde{c}$ capturing the consumption share of total expenditures $\tilde{c} \equiv c + w\ell$. As is well known, substitution effects outweigh income effects at high elasticity of intra-temporal substitution $1/\sigma > 1$; falling wages after mid-life are thus associated with an increasing role of leisure (resp. decreasing role of consumption) in total expenditures.[19] Second, the marginal felicity out of total expenses $\tilde{c}$:

$$\frac{\partial u(c,\ell)}{\partial \tilde{c}} = \nu(w) = \left[\theta^{\frac{1}{\sigma}} + (1-\theta)^{\frac{1}{\sigma}} w^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{1-\sigma}}$$

is unconditionally decreasing in wages. Substituting (10a) in (10b) also reveals that the contribution of consumption to felicity:

$$\frac{c}{u(c,\ell)} = [\mu(w)\nu(w)]^{-1} = \left[\theta + (1-\theta)^{\frac{1}{\sigma}} (\theta w)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}$$

is unconditionally increasing in wages. Hence falling wages after mid-life are also associated with (i) an increasing marginal felicity of (total) expenses $\tilde{c}$, as well as (ii) a decreasing role of consumption (and increasing role of leisure) in optimal felicity. Observe finally that all life cycle dynamics stem from those in the wage rate w, and that neither the morbidity λ^h nor mortality λ^m risks, nor remaining lifespan to maximal longevity $T - t$ has any effect on the optimal leisure/consumption mix.

3.3.2. Health investment and shadow value

I next characterize the optimal solution for the dynamic health investment decision I, and the associated value of the health stock V^H. Under separability, the shadow value of health is obtained by maximizing net benefits:

$$V^H(H) = \max_{I} (BH - I) + \exp(-r)\mathrm{E}V^H(H'), \tag{11}$$

subject to health dynamics (6). The solution to this problem is characterized as follows:

Lemma 2. *The optimal investment and corresponding value of human capital solving (11) are:*

$$I = \kappa_t(\lambda^h)H, \tag{12}$$

$$V^H = \eta_t(\lambda^h)H, \tag{13}$$

where the loadings $\{\kappa_t, \eta_t\}_{t=1}^T$ satisfy the following recursion:

$$\kappa_t(\lambda^h) = \left[\eta_{t+1}(\lambda^h)R^{-1}\alpha A\right]^{\frac{1}{1-\alpha}} \tag{14a}$$

[18] See Appendix B for proofs, as well as Appendix B.3 for verification and confirmation of the equivalence between the separable and direct solution approaches.

[19] Low elasticity $1/\sigma < 1$ induces opposite patterns, whereas $1/\sigma = 1$ leads to exact cancellation of income and substitution effects, resulting in constant (μ, ν) and age-independent leisure to consumption ratios.

$$\eta_t(\lambda^h) = B - \kappa_t(\lambda^h) + \eta_{t+1}(\lambda^h) R^{-1} \left\{ A\kappa_t(\lambda^h)^\alpha + (1-\delta) - [1-\exp(-\lambda^h)]\phi \right\} \tag{14b}$$

with terminal values $(\kappa_T, \eta_T) = (0, B)$.

Properties. First, the age-dependent feedback rule κ_t in (14a) crucially determines optimal health decumulation. Indeed, substituting optimal investment (12) in the health dynamics (6a) reveals that, despite being transitory, the morbidity shocks ϵ^h affect the rate of change:

$$(H' - H)/H = A\kappa_t^\alpha - \left(\delta + \epsilon^{h\prime}\phi\right),$$

and therefore have permanent effects on the optimal time path of health. This implication is consistent with the interpretation of ϵ^h as chronic shocks, with long-lasting consequences and age-increasing likelihood of occurrence λ^h, such as dementia, heart diseases, diabetes, and cancer (Center for Disease Control and Prevention, 2023). Second, the loading η_t in (13) corresponds to a shadow price, i.e. marginal and average Tobin's-q of the health capital H. Both κ_t in (14a) and η_t in (14b) are increasing in the health benefits B and investment productivity A, but are decreasing in depreciation δ, as well as in the intensity λ^h and consequences ϕ of morbidity. For the same reasons, age-increasing morbidity risk λ^h accelerates both the fall in H, as well as lowers its shadow value η_t.[20] Observe finally that neither mortality λ^m, nor preferences play any role in the optimal health policy and value.

3.3.3. Total expenses and value function

Next, under perfect financial markets, a claim to the exogenous net revenues stream can be sold in exchange for:

$$V^w(w) = (w + y - i) + \exp(-r)V^w(w') \tag{15}$$

over the period $t = 0, \dots T$. At time 0, this net permanent income value can be added to the value of the health capital V^H and to financial wealth W to recover net total wealth N. The agent then solves for optimal total expenses $\tilde{c}$ in the equivalent problem:

$$V = \max_{\tilde{c}} \left\{ (1-\beta)(v\tilde{c})^{1-\epsilon} + \beta \left[p(\lambda^m) V'^{1-\gamma} + \left[1 - p(\lambda^m)\right] b\, N'^{1-\gamma} \right]^{\frac{1-\epsilon}{1-\gamma}} \right\}^{\frac{1}{1-\epsilon}} \tag{16a}$$

subject to

$$\begin{aligned} N' &= [N - \tilde{c}]\, R, \text{ and} \\ N_0 &\equiv W_0 + V_0^w(w) + V_0^H(H_0), \text{ given} \end{aligned} \tag{16b}$$

where $p(\lambda^m)$ is the one-period ahead survival probability in Table 1. The solution to this problem is given as follows:

Theorem 1. *The optimal total expenditures and corresponding value function solving (19) are:*

$$\tilde{c} = \omega_t(\lambda^m, w) N \tag{17}$$

$$V = \psi_t(\lambda^m, w) N \tag{18}$$

where the loadings $\{\omega_t, \psi_t\}_{t=1}^T$ *satisfy the following recursion:*

$$\beta_t(\lambda^m, w) = \beta \left\{ p(\lambda^m)\psi_{t+1}(\lambda^m, w)^{1-\gamma} + \left[1 - p(\lambda^m)\right] \tilde{b}^{1-\gamma} \right\}^{\frac{1-\epsilon}{1-\gamma}}, \tag{19a}$$

$$\omega_t(\lambda^m, w) = \frac{(1-\beta)^{\frac{1}{\epsilon}} v(w)^{\frac{1-\epsilon}{\epsilon}}}{(1-\beta)^{\frac{1}{\epsilon}} v(w)^{\frac{1-\epsilon}{\epsilon}} + \beta_t(\lambda^m, w)^{\frac{1}{\epsilon}} R^{\frac{1-\epsilon}{\epsilon}}} \tag{19b}$$

$$\psi_t(\lambda^m, w) = \left\{ \chi_t^v(\lambda^m, w) v(w)^{1-\epsilon} + \chi_t^R(\lambda^m, w) R^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}} \tag{19c}$$

where

$$\begin{aligned} \chi_t^v(\lambda^m, w) &= (1-\beta)\omega_t(\lambda^m, w)^{1-\epsilon}, \\ \chi_t^R(\lambda^m, w) &= \beta_t(\lambda^m, w) \left[1 - \omega_t(\lambda^m, w)\right]^{1-\epsilon} \end{aligned} \tag{19d}$$

with terminal value $\beta_T(\lambda^m) = \beta b^{\frac{1-\epsilon}{1-\gamma}}$ *and where* (μ, v) *are given in* (10)*, and where net total wealth is:*

$$N = N_t(W, V_t^w(w), H) = W + V_t^w(w) + \eta_t(\lambda^h) H \tag{20}$$

and $\tilde{b} \equiv b^{1/(1-\gamma)}$ *is the share of bequeathed net total wealth.*

[20] The model can be generalized to allow for age-increasing consequences of sickness ϕ_t without qualitative changes. Indeed, using the Hicks approximation $x \approx 1 - \exp(-x)$ in (14b) reveals that

$$\eta_t \approx B - \kappa_t + \eta_{t+1} R^{-1} \left\{ A\kappa_t^\alpha + (1-\delta) - \lambda_t^h \phi_t \right\}$$

i.e. age-increasing consequences ϕ_t amplify the effects of age-increasing morbidity risk λ_t^h via the term $\lambda_t^h \phi_t$ that lowers the Tobin's-q η_t.

Table 2
Main channels for continuation utility.

Item	Channels	
	Variable	Interpretation
Preferences:		
- consumption, leisure: θ, σ	$v(w) \to \psi_t(\lambda^m, w)$,	marginal felicity/value
- total expenditures dyn.: $\beta, \gamma, b, \epsilon$	$\psi_t(\lambda^m, w)$	marginal value
Risks:		
- morbidity: λ^h	$\eta_t \to V^H(H) \to N$	Tobin's-q, net total wealth
- mortality: λ^m	$\psi_t(\lambda^m, w)$	marginal value
Health:		
- technology: $A, B, \alpha, \delta, \phi$	$\eta_t \to V^H(H) \to N$	Tobin's-q, net total wealth
- level: H	$V^H(H) = \eta_t H \to N$	value health
Resources:		
- wages: w	$v(w) \to \psi_t(\lambda^m, w)$,	marginal felicity/value
	$V^w(w) \to N$	permanent income
- fin. wealth: W	N	net total wealth

Notes: Main channels for continuation utility $V_t = \psi_t(\lambda^m, w)N$ in Theorem 1, where $N = W + V^w(w) + V^H(H)$ is net total wealth.

Properties. The continuation utility function (18) naturally inherits the properties of the original problem (7). First, the term β_t in (19a) is the endogenous discount factor applied to the utility from future wealth N', and is a Certainty Equivalent (with CRRA parameter γ) in the next-period marginal continuation utility ψ_{t+1} if alive, and in the bequeathed share of wealth $\tilde{b}$ if dead. Second, $\omega_t \in (0, 1)$ in (19b) is the marginal (and average) propensity (MPC) to consume $\tilde{c}$ out of net total wealth N. Third, the term ψ_t in (19c) is the marginal (and average) continuation utility of net total wealth N, and can be rearranged as:

$$\psi_t(\lambda^m, w) = \frac{\partial V}{\partial N} = \left\{ (1 - \beta) \left[\frac{\partial u(c, \ell)}{\partial N} \right]^{1-\epsilon} + \beta_t(\lambda^m, w) \left[\frac{\partial N'}{\partial N} \right]^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}}.$$

It is a CES (with inverse EIS parameter ϵ) in the twin benefits of current net worth N: current marginal felicity $\partial u(c, \ell)/\partial N$ and future marginal wealth $\partial N'/\partial N$.

Separability. The continuation utility (18) displays separability along several dimensions that are summarized in Table 2. First, all the effects of mortality λ^m and agent's preferences parameters $(\theta, \sigma, \gamma, \epsilon, \beta, b)$ are exclusively encompassed in the marginal utility ψ_t in (19c) via the recursion (19). This marginal utility is also dependent on wages w (via marginal felicity v), but is independent of both morbidity λ^h and of health dynamics parameters $(A, B, \alpha, \delta, \phi)$. Conversely, all resources are regrouped in net total wealth N in (20) which is both mortality- and preferences-independent, but encompasses the effects of morbidity and health dynamics via the Tobin's-q term η_t calculated from (14). Second, net total wealth is linearly additive in wealth W, the permanent income of exogenous wages $V_t^w(w)$ and the shadow value of health $\eta_t H$. It follows directly that (i) the willingness to pay and to accept compensation for changes in λ^m is much simplified and can be interpreted through the lenses of changes in marginal utility ψ_t only, and (ii) the willingness measures presented below will also be linearly separable in wealth, capitalized wages and health.

3.3.4. Mortality and ageing

The effects of mortality λ^m and of ageing t on both marginal ψ_t and level of continuation utility V_t, as well as on net total wealth N_t can be analytically summarized in the following result.

Proposition 1 (*Mortality and Ageing Effects*). *Assume that the both the inverse elasticities of intra- and inter-temporal substitution $\sigma, \epsilon \in (0, 1)$.*

1. *If the model's parameters are such that the following conditions hold:*

$$\tilde{b} \leq \psi_t(\lambda^m, w), \quad \forall t. \tag{21a}$$

$$v(w) \leq R, \tag{21b}$$

Then, increases in death intensity λ^m:

   (a) are detrimental, i.e. lower $\psi_t(\lambda^m, w)$ and V;
   (b) are more detrimental for permanent, than one-shot changes;
   (c) are less detrimental for elders;

2. *If, in addition the following condition holds:*

$$\left[\frac{\beta_t (R-1)^{\epsilon}}{1-\beta} \right]^{\frac{1}{1-\epsilon}} \leq \frac{v(w)}{R}, \tag{21c}$$

then, net total wealth N_t:

(a) decumulates;
(b) decumulates faster for elders.

The complete proof and discussion are given in Appendix B.2. Condition (21a) is consistent with preference for life and ensures that the marginal utility of net total wealth when alive in the next period ψ_{t+1} outweighs that from dying and leaving bequests $\tilde{b}$. Conditions (21b) and (21c) respectively state that the marginal felicity $v(w)$ in (10b) and the marginal propensity to save $1-\omega_t$ are both lower than the gross interest rate $R = \exp(r)$. The assumption of inverse EIaS and EIS $\sigma, \epsilon \in (0,1)$ and all three conditions will be verified and confirmed below.

Heuristically, whenever condition (21a) holds, one-shot increases in λ^m are associated with lower survival $p(\lambda^m)$ and heavier discounting of future flows, i.e. lower β_t in (19a), thereby increasing the MPC ω_t in (19b).[21] Condition (21b) ensures that the marginal felicity $v(w)$ is lower than R; the mortality-induced decline in β_t and associated increase in ω_t both increase the loading χ_t^v and decrease the loading χ_t^R and are therefore detrimental to marginal utility ψ_t in (19c). Since a given N_t is independent from λ^m, total welfare V_t falls. Permanent changes account for this lower ψ_{t+1} and amplify the decline in discount factor in (19a), and corresponding detrimental effects. Elders face lower wages (and higher $v(w)$ if $\sigma \in (0,1)$) and are thus less exposed to this welfare loss induced by condition (21b). Moreover, the insufficient savings rate from condition (21c) entails falling net total wealth. Lower discount β_t and higher marginal felicity $v(w)$ accelerate this decumulation for elders.

4. Hicksian life valuation measures

4.1. Equivalent and compensating variations

The implications of the life cycle model for life valuations are derived by relying on the Hicksian Equivalent and Compensating Variations (e.g. Hanemann, 1991; Varian, 1984, p. 264) to calculate the maximal willingness to pay and to accept compensation to attain/forego beneficial (resp. prevent/accept detrimental) changes Δ in mortality λ^m along the optimal life cycle path corresponding to the (indirect) continuation utility V_t.[22]

Proposition 2 (*Equivalent and Compensating Variations*)**.** *Consider a change of magnitude $\Delta \in [-\lambda^m, \infty]$ in base death risk exposure λ^m occurring at age t. Then, given the continuation utility V_t in (18), the Equivalent (v_t^e) and Compensating (v_t^c) Variations along the optimal path solving the indifference conditions:*

$$V_t(\lambda^m, N - v_t^e) = V_t(\lambda^m + \Delta, N) \tag{22a}$$

$$V_t(\lambda^m + \Delta, N - v_t^c) = V_t(\lambda^m, N) \tag{22b}$$

are given by:

$$v_t^k(\Delta, \lambda^m, N) = \Psi_t^k(\Delta, \lambda^m) N, \quad \text{where} \tag{23a}$$

$$\Psi_t^k(\Delta, \lambda^m) = \begin{cases} \left[1 - \frac{\psi_t(\lambda^m+\Delta)}{\psi_t(\lambda^m)}\right], & k = e \\ \left[1 - \frac{\psi_t(\lambda^m)}{\psi_t(\lambda^m+\Delta)}\right], & k = c \end{cases} \tag{23b}$$

where ψ_t is marginal utility in (19), and N is net total wealth in (20).

The proof follows directly by substituting continuation utility (18) in the Hicksian variations (22) and is therefore omitted.

Links between variational and willingness measures. The links between the two Hicksian variation measures and the WTP's, and WTA's can be deducted from (22) and (23) and are summarized in Table 3.

The Equivalent Variation (22a) takes current exposure λ^m as status-quo to calculate maximal $WTP_t = v_t^e$ to prevent detrimental change $\Delta > 0$ or minimal $WTA_t = -v_t^e$ to forego beneficial changes $\Delta < 0$. The Compensating Variation (22b) instead takes altered exposure $\lambda^m + \Delta$ as status-quo and computes the $WTP_t = v_t^c > 0$ to attain beneficial change $\Delta < 0$ and $WTA_t = -v_t^c$ to accept detrimental changes $\Delta > 0$.

Role of wealth, permanent income and health in life valuations. The Hicksian equivalent and compensating values v_t^k in Proposition 2, and consequently the two willingness measures to pay/accept compensation in Table 3 are all proportional to $N_t = W_t + V_t^w + \eta_t H_t$ in (20), which is additive in the individual wealth W_t, permanent income $V_t^w(w)$ in (15) and shadow value of health $V_t^H = \eta_t H_t$ in (13). It follows directly that the elasticities of all willingness with respect to the individual's financial wealth, permanent income, and health capital simplify to the shares of these components in net total wealth N_t.

[21] One-shot increases in λ^m abstract from the effects on future $\psi_{t+1}(\lambda^m, w)$ in (19a); these effects are re-instated in the case of permanent changes. Equivalence between higher mortality and heavier discounting is standard in Perpetual Youth models (e.g. Blanchard, 1985). The associated increase in the MPC is referred to as an optimal 'Live Fast and Die Young' strategy by Hugonnier et al. (2022).

[22] The explicit dependence of the marginal value $\psi_t(\lambda^m, w)$ on wages w is henceforth omitted to simplify notation.

Table 3
Links between Hicksian Variations and Willingness Measures.

	Beneficial change $\Delta < 0$	Detrimental change $\Delta > 0$
$WTP_t(\Delta, \lambda^m, N)$	$v_t^c(\Delta, \lambda^m, N)$ (attain)	$v_t^e(\Delta, \lambda^m, N)$ (prevent)
$WTA_t(\Delta, \lambda^m, N)$	$-v_t^e(\Delta, \lambda^m, N)$ (forego)	$-v_t^c(\Delta, \lambda^m, N)$ (accept)

4.2. Related life valuations

In addition to the marginal rate of substitution (corresponding to the polar case $\Delta \to 0$) presented in Appendix C.1, the Hicksian variations (23) and associated WTP's and WTA's are readily adaptable to several life valuations found in the literature.

4.2.1. Value of a life year (VOLY)

For each decrease/increase in remaining longevity $\nabla L_t \in \pm\{6, 12, 18, \dots\}$ months, the corresponding set of $\Delta_t = \Delta(\nabla L_t)$ is obtained by inverting the expressions in Table 1. In the case of $\nabla L_t = \pm 12$ months, the WTP/WTA thus capture the willingness over beneficial/detrimental one-shot/permanent changes corresponding to one life-year in life expectancy.[23] This VOLY accounts for the current and future health- and morbidity-related conditions in which this change occurs for several reasons. First, the reliance on the forward-looking continuation utility $V_t(\lambda^m, N)$ explicitly incorporates the welfare from both market (i.e. consumption) and non-market (i.e. leisure) optimal choices. Second, the net total wealth $N_t = W + V_t^w(w) + \eta_t H$ in (16b) explicitly incorporates the agent's health status H, in addition to financial wealth and permanent income. Third, the shadow price of health η_t in (14b) is corrected for age-varying exposure λ^h and consequence ϕ of morbidity.[24]

4.2.2. Value of statistical life (VSL)

The empirical VSL commonly resorted in the literature can be equivalently interpreted as an aggregate willingness over a one-shot change that saves one unidentified (i.e. statistical) person in a group of size n, as well as an infra-marginal approximation over change $\Delta = n^{-1}$ to the MRS (C.5).[25] The closed-form expression for the VSL in our setting is very tractable under the assumption of age and preferences homogeneity:

Corollary 1 (*VSL*)**.** *Assume that agents $i = 1, 2, \dots, n$ share the same age t (and therefore death and morbidity risk exposure) and preferences, but differ in wealth W_i, wages w_i and health H_i (and therefore net total wealth N_i). Then the VSL defined as the collective willingness measures $k = e, c$ for a one-shot change $\Delta = n^{-1}$ is:*

$$
\begin{aligned}
VSL_t^k(\Delta, \lambda^m) &= \sum_{i=1}^{n} v_{is}^k(\Delta, \lambda^m, N_i) = \Psi_t^k(\Delta, \lambda^m)\, n\, \bar{N} \\
&= \frac{v_t^k\left(\Delta, \lambda^m, \bar{N}\right)}{\Delta}, \\
\textit{where} \quad \bar{N} &\equiv n^{-1} \sum_{i=1}^{n} N_i = \bar{W} + V^w(\bar{w}) + \eta_t \bar{H}
\end{aligned}
\tag{24}
$$

is the population average of net total wealth.

The proof obtains directly by exploiting the homogeneity and additive separability properties of the Hicksian variations (23), and is therefore omitted.

Corollary 1 reveals that the VSL is the infra-marginal approximation $v_t^k(\Delta, \lambda^m, \bar{N})/\Delta$ to the theoretical MRS $\partial v_t^k(\Delta, \lambda^m, \bar{N})/\partial \Delta$ in (C.4) and (C.5), where all are evaluated at the population average net total wealth, and therefore mean financial wealth, permanent income, and health levels. Equivalently, the VSL measured by the aggregate willingness is equal to the mean VSL, where the latter is the willingness at mean wealth, wages and health divided by common change Δ, and therefore independent of person-specific characteristics.[26]

[23] See Hall et al. (2020) for a VOLY analysis related to COVID-19.

[24] This interpretation differs from the value of a Quality-Adjusted Life Year (QALY) in which health status is associated with a scale between 0 (death) and 1 (perfect health). A value of one QALY corresponds to the willingness for one-year change in life expectancy in *perfect* health condition. Moreover, it differs from empirical VSL-based estimates of the value of a life-year (VSLY) which typically adjust a VSL by an arbitrary quality-adjusted life expectancy measures (e.g. Kniesner and Viscusi, 2019, p. 14). See Herrera-Araujo et al. (2020) for discussion and criticism of this approach.

[25] See Rosen (1988) and Aldy and Smyth (2014) for reliance on one-shot changes for the VSL in LC models, and Eeckhoudt and Hammitt (2004), Murphy and Topel (2006), Bellavance et al. (2009), Andersson and Treich (2011) and Hugonnier et al. (2022) for additional theoretical and empirical considerations for VSL.

[26] See also Hugonnier et al. (2022) for a similar interpretation.

4.2.3. Gunpoint Value of Life (GPV)

The Gunpoint value is the maximal WTP to prevent and WTA to accept certain death $\nabla L_t = -L_t$ at the end of current period t, corresponding to the polar case $\Delta \to \infty$.[27] Relying on the WTP in Proposition 2 reveals the following result:

Corollary 2 (*GPV*). *The maximal willingness to pay to prevent (GPV_t^p), or willingness to accept compensation (GPV_t^a) for instantaneous, certain death are:*

$$\begin{aligned} GPV_t^p &= \left[1 - \frac{\psi_t(\infty)}{\psi_t(\lambda^m)}\right] N \\ GPV_t^a &= \left[\frac{\psi_t(\lambda^m)}{\psi_t(\infty)} - 1\right] N \end{aligned} \tag{25}$$

where the marginal utility of total wealth $\psi_t(\lambda^m)$ solves the recursion (19), and where $\psi_t(\infty)$ solves:

$$\beta_t(\infty) = \beta \tilde{b}^{1-\epsilon} \geq 0, \tag{26a}$$

$$\omega_t(\infty) = \frac{(1-\beta)^{\frac{1}{\epsilon}} v_t(w)^{\frac{1-\epsilon}{\epsilon}}}{(1-\beta)^{\frac{1}{\epsilon}} v_t(w)^{\frac{1-\epsilon}{\epsilon}} + \beta_t(\infty)^{\frac{1}{\epsilon}} R^{\frac{1-\epsilon}{\epsilon}}} \in [0,1], \tag{26b}$$

$$\psi_t(\infty) = \left\{(1-\beta)\left[v_t(w)\omega_t(\infty)\right]^{1-\epsilon} + \beta_t(\infty)\left[\left(1-\omega_t(\infty)\right) R\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}} \geq 0. \tag{26c}$$

The proof follows directly by setting $\Delta = \infty$ resulting in survival $p(\lambda^m) = 0, \forall s$ in the expression (19a) to obtain $\beta_t(\infty)$. Next, we can substitute the latter in (19b) and (19c) to obtain $\omega_t(\infty)$ and $\psi_t(\infty)$.

Eqs. (25), and (26) show that the two Gunpoint measures are unconditionally finite for positive bequests motives $\tilde{b} > 0$. Both Gunpoint values are useful in ex-ante instances where death is a certain outcome under a specific action or inaction, such as in terminal care decisions, or in ex-post instances where death has occurred, such as in wrongful death litigation.[28]

4.2.4. Deterministic Longevity Value (DLV)

The Poisson binomial mortality process in Table 1 relies on a single variable λ^m to capture all longevity moments, including: (i) exposure to mortality risk and (ii) expected longevity. Consequently, all life values are calculated with respect to changes Δ affecting both; e.g. the WTP to pay for beneficial change $\Delta < 0$ reflects composite welfare gains stemming from both (i) a reduced exposure to longevity uncertainty *as well as* (ii) an increase in expected longevity. In the spirit of the literature on the utilitarian costs of uncertainty, it is possible to disentangle the welfare cost of exposure to mortality risk via aversion to mean-preserving spreads for risk-averse agents.[29]

Definition 1 (*DLV*). The Deterministic Longevity Value is the maximal willingness to pay (DLV_t^p) to attain, or to accept compensation (DLV_t^a) to forego the opportunity to eliminate all mortality risk λ^m and live current expected longevity L_t in Table 1 with certainty.

The DLV is computed as follows:

1. fix maximal remaining horizon $T - t$ to the age-dependent expected remaining longevity L_t;
2. set changes to eliminate all current and future death risk exposure $\Delta = -\lambda_t^m, \forall t$;
3. recursively calculate the WTP to attain and the WTA to forego Δ starting at the modified maximal longevity L_t.

The two Deterministic Longevity Value (DLV) thus recovered gauge the welfare costs of death *risk exposure* through the monetary value of transforming a dynamic problem with a finite, stochastic horizon into a finite certain one where the agent is assured to live exactly L_t remaining periods and to die afterwards. In the spirit of Epstein et al. (2014) and Caliendo et al. (2016), the DLV can be also interpreted to a 'timing premium' capturing the value of early, rather than late, resolution of lifespan uncertainty.[30]

5. Empirical strategy

The previous discussion has shown that all the relevant information for life valuation purposes can be recovered from the indirect utility V_t in (18), as well as the associated Hicksian variations v_t^k in (23). To identify these variables, our Revealed-Preference

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[27] Since death is an absorbing state, computing the GPV via one-shot or permanent changes yields identical results.

[28] See Jones-Lee (1974), Cook and Graham (1977), Weinstein et al. (1980), Eeckhoudt and Hammitt (2004) and Hugonnier et al. (2022) for GPV-related definitions, applications and discussion.

[29] Examples of WTP to eliminate risk while keeping mean values include Barro (2009), Lucas (1987, ch. 3) for consumption, Schlee and Smith (2019), Luttmer and Samwick (2018) for policy and Caliendo et al. (2016) for retirement age uncertainty. Edwards (2009) studies the welfare cost of aggregate *business cycles* fluctuations in mortality risk, but not the cost of person-specific death uncertainty.

[30] See also Jones-Lee (1974, pp. 841–842) and Eeckhoudt and Hammitt (2004) for analysis of WTP to eliminate death risk in a two-period model without controlling for expected longevity.

Table 4
Descriptive statistics.

Variable	Symbol	Mean	Median	Std	Min	Max
			a. PSID			
Age	t	46.53	45.00	16.34	21.00	100.00
Wealth (K$)	W_t	243.47	185.73	164.79	−3.26	544.01
Health	H_t	−0.06	−0.01	0.52	−2.62	0.59
Sick	λ_t^h	0.04	0.03	0.04	0.01	0.43
			b. ATUS			
Age	t	49.72	48.00	16.40	21.00	85.00
Hours	$1-\ell_t$	36.09	38.93	5.53	21.56	40.33
Income ($)	$w_t(1-\ell_t)$	692.95	743.76	126.83	399.52	822.16
Wages ($)	w_t	17.87	18.58	2.02	10.82	19.84

Notes: a. PSID. Wealth: net financial and residential. Health: Score function from panel multinomial probit on self-reported polytomous health status. Sickness: marginal probability of reporting worst health outcome from panel multinomial probit on self-reported polytomous health status. b. ATUS. Hours: spent working, per week. Income: salaried income per week. Wages: per hour.

empirical strategy relies on a calibration of the model's deep parameters to generate predicted life cycles of a subset of key variables for which observable counterparts exist. Unfortunately, no unique data set can be found for the variables to be matched. I therefore combine several well-known databases over a common period under the assumption that they are representative of a common set of US agents. The theoretical optimal rules and associated optimal dynamics provide predictions for the life cycles of variables such as wealth, leisure, income, and health. Since these depend on age only, so must their observable counterparts. Consequently, observed life cycles must be computed accounting for the relevant socio-economic variables to recover pure age-dependent effects.

5.1. Data

Health, morbidity and mortality sources. I rely on Panel Study on Income Dynamics (PSID) for health-related data. More specifically, I use a panel ordered probit, with random effects over the unbalanced household data for the period 2003–2019 to regress the household head's polytomous self-reported health status on socio-demographics,[31] as well as on a fractional polynomial in age. The associated score function is scaled upwards to guarantee positive observed values consistent with the Cobb–Douglas technology in (6a), and is evaluated by age to represent the health variable H_t. The imputed marginal probability of being in the worst health state by age, is used as proxy to recover the sickness intensity $\lambda_t^h = -\ln(\Pr_t[\text{Poor health}])$. Finally, the Life Tables of the United States (Arias and Xu, 2020, Tab. 1) report age-specific one-year survival probabilities p_t, from which the intensity is directly recovered as $\lambda_t^m = -\ln(p_t)$

Financial and labor market sources. The PSID data is also resorted to for wealth proxied by the net financial and residential wealth of agents. Again, net worth is regressed on socio-economic variables[32] as well as a fractional polynomial in age, and accounting for random effects. The fitted variable by age corresponds to our wealth variable W_t. Labor market variables are taken from the American Time Use Survey (ATUS) for the 2003–2019 period. Controlling for random effects, sex, occupation and industrial sector, as well as year dummies, wages, hours and income are again regressed on a fractional polynomial from which the fitted values by age are recovered. Summary statistics for the variables $w_t, 1-\ell_t, w_t(1-\ell_t)$ are provided in Table 4.

Exogenous ageing processes. The three exogenous ageing variables are plotted in Fig. 1. The sickness intensities λ_t^h in panel a grow exponentially from 1.42% at age 30 to 15.28% at age 80. Panel b reports the death intensities λ_t^m that also display an exponential growth 0.23% at 30 to 4.45% at 80, consistent with Gompertz law. Finally, the fitted hourly wages in panel c are hump-shaped, starting from 16.94$ at 30, peaking at 19.72$ at 46, and falling to 14.45$ at 80.

Longevity returns to changes in survival Δ. Given death risk intensity λ_t^m, I next evaluate the marginal rates of transformation between changes in survival Δ_t and changes in longevity ∇L_t. More precisely, I invert ∇L_t in Table 1 to calculate both one-shot (panel a), and permanent (panel b) changes in survival $\Delta_t = \Delta(\nabla L_t)$ required to attain a specific change in expected longevity $\nabla L_t \in \pm\{6, 12, 18\}$ months, plotted by age in Fig. 2. First, the required changes in survival are unsurprisingly increasing in longevity changes, and much larger if delivered as one-shot, rather than continuously over the remaining horizon. Second, ageing significantly augments the required change Δ_t in order to attain a given ∇L_t. Equivalently, the longevity return of a given change Δ_t in death intensity exponentially declines as the agent becomes older and is faced with increasing death risk exposure (Fig. 1.b).

[31] More specifically, I use sex, gender, race, education and year dummies, as well as financial wealth as regressors.
[32] I use year, sex, gender, race, and education dummies, as well as self-reported health as regressors.

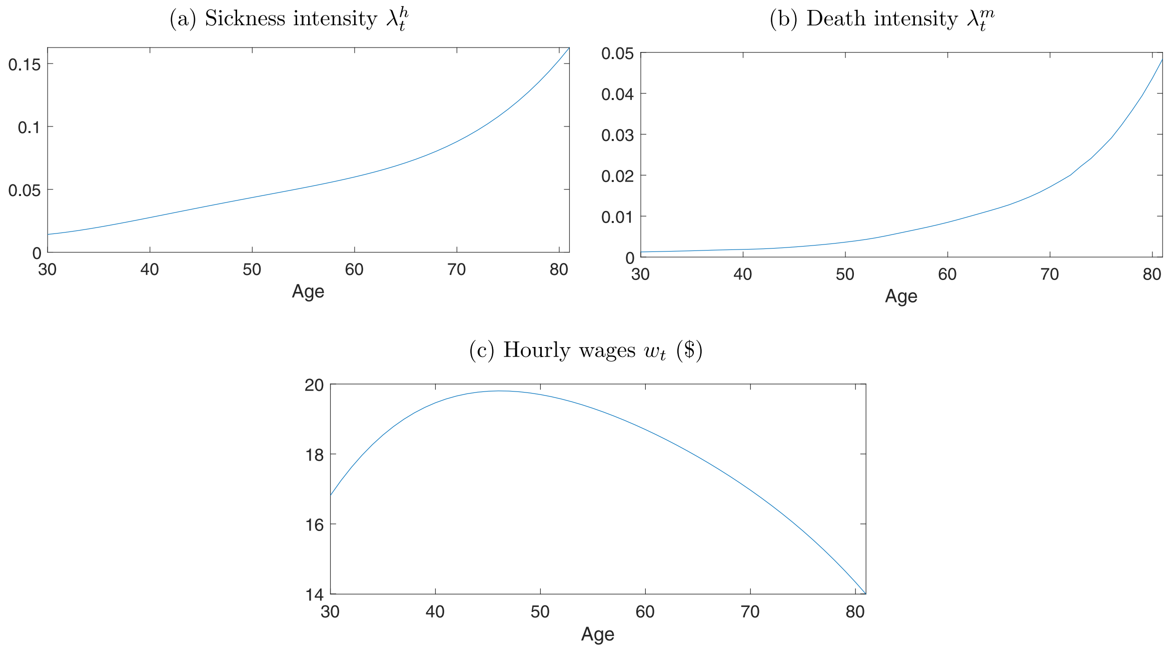

Fig. 1. Exogenous ageing processes.
Notes: PSID and author's calculations from the longitudinal Probit estimation. (b) Arias and Xu (2020) and author's calculations. (c) ATUS and author's calculations.

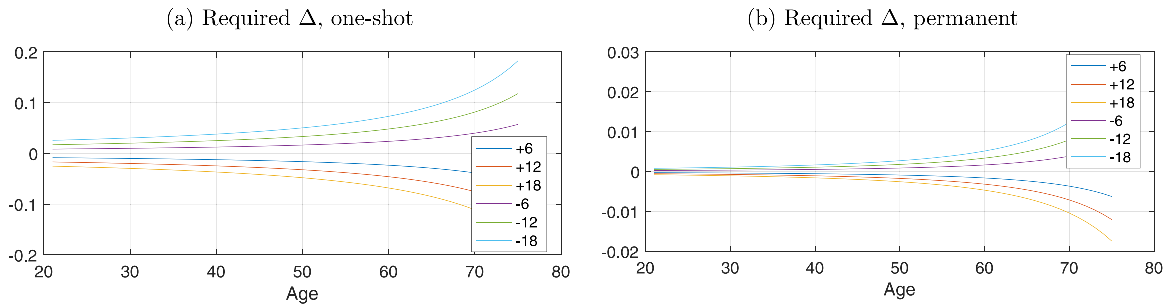

Fig. 2. Rates of transformation between survival Δ and longevity ∇L_t.
Notes: Required change Δ in base mortality λ^m to attain change in longevity $\nabla L_t \in \pm\{6, 12, 18\}$ mo., by age t. (a) One-shot: $\Delta_t = \mathbb{1}_t \Delta$, (b) Permanent: $\Delta_t = \Delta, \forall t$.

Endogenous variables. The endogenous variables that the model aims to replicate are plotted in blue in Fig. 3 — the fitted values in red are discussed below. First, the health proxy variable in panel a displays a continuous decline that accelerates after 70.[33] Second, panel b shows the accumulation of financial wealth up to mid-70's and slow de-cumulation afterwards. Third, the weekly hours in panel c slowly increase up to mid-life, before falling rapidly afterwards. The weekly income in panel d is also hump-shaped, peaking at mid-life, and falling thereafter.

5.2. Calibration strategy and fit adequacy

Calibrated parameters. The model's deep parameters are calibrated so as to match the joint lifetime dynamics of health, wealth and labor market variables along the optimal path with their observable counterparts. The details are provided in Appendix D. In particular, the key parameters are chosen to minimize the optimally-weighted distance between observed and predicted life cycles for health H_t, financial wealth W_t, hours $n_t = (1 - \ell_t)$, and labor income Y_t in Fig. 3, using the identification moments in Table D.1.

The calibrated parameters are reported in Table 5. First, in panel a, all nominal variables expressed in dollars are scaled by a factor of 10^{-3}. The risk-free discount rate is set at 5%. The parameters $m = i - y$, and B are obtained by regressing income net of

[33] See also Hosseini et al. (2022) for additional evidence and discussion of declining health captured by poor health proxies (frailty) that increase over the life cycle.

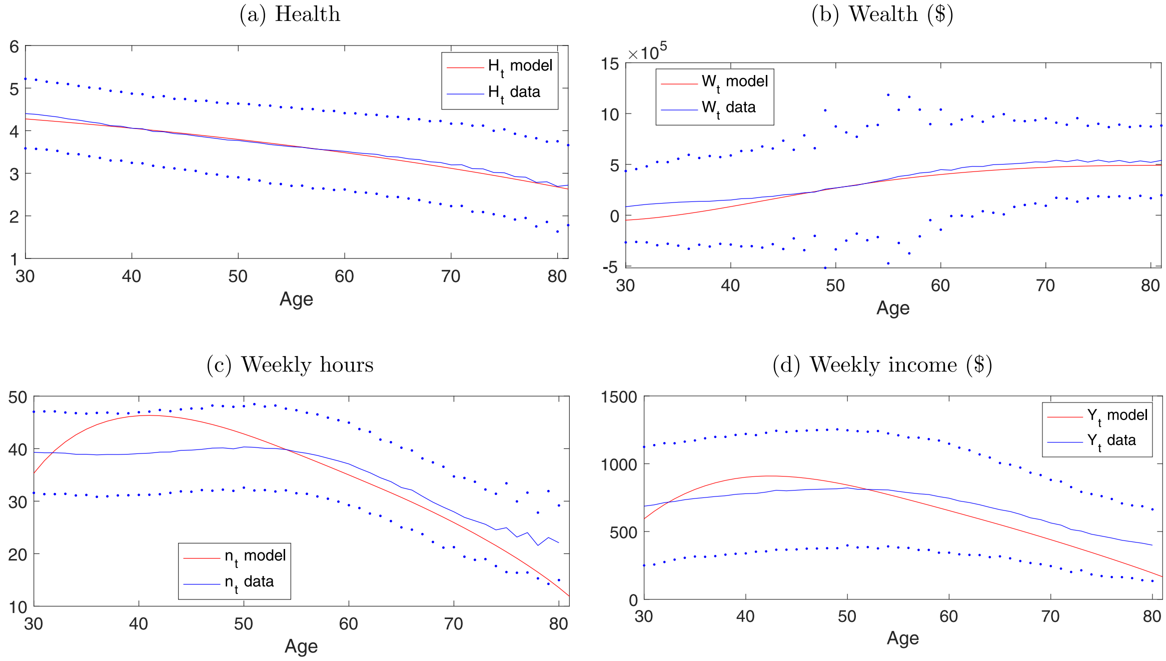

Fig. 3. Data and fitted variables. Notes: Calculated along optimal path at calibrated parameters.

Table 5
Calibrated parameters.

Parameter	Value	Parameter	Value
a. Scaling and budget constraint (5)			
scale	0.001	r	0.0500
B	13.3762	$m = i - y$	−34.7817
b. Health (6)			
A	0.0080	α	0.7500
δ	0.0175	ϕ	0.0350
c. Preferences (7)			
θ	0.4500	σ	0.8164
γ	3.5382	ε	0.5009
b	2.6e+07	ρ	0.0438

medical expenses (both from PSID data) on a constant and the score function proxy for health level H. Second, by exploiting the separation properties between health-related and financial decisions, the health production parameters in panel b are separately chosen to reproduce the observed life cycle dynamics of health via its predicted optimal path (D.1). The parameters are in the same range as in similar models of health dynamics (Hugonnier et al., 2013, 2022). They are indicative of significant diminishing returns to investment ($\alpha = 0.75$), non-negligible depreciation ($\delta = 1.75\%$), and consequential additional depreciation through illness ($\phi = 3.50\%$). Third, the preference parameters in panel c are obtained by minimizing an optimally-weighted sum of squares of residuals between observed and predicted LC's for wealth, hours and income. The consumption share $\theta = 0.45$, is set at a realistic value, and $\sigma = 0.8164$ is indicative of intra-temporal substitutability $1/\sigma = 1.2249$ between consumption and leisure. Similarly, the parameter $\varepsilon = 0.5009$ shows high elasticity of inter-temporal substitution $1/\varepsilon = 1.9964$ between current consumption and the certainty equivalent of future utility.[34] Importantly, both $\sigma, \varepsilon \in (0, 1)$ are consistent with the restriction in Proposition 1. Next, the risk aversion $\gamma = 3.5382$ and discount rate $\rho = 0.0438$ are both set at realistic values, with $\gamma > \varepsilon$ consistent with preference for early (rather than late) resolution of uncertainty. Finally, the bequest parameter b is indicative of low bequests intentions $\tilde{b} \equiv b^{1/(1-\gamma)} = 0.0012$.[35]

[34] High EIS (i.e. larger than 1.0) has been identified by several strands of the literature. First, the long-run consumption risks literature advocates large EIS to generate sensible results (e.g. Bansal and Yaron, 2004; Yang, 2016). Second, DSGE models with EZW preferences and labor supply considerations similar to ours have also estimated large EIS (e.g. van Binsbergen et al., 2012, Tab. 2). Third, EIS larger than 1.0 has also been identified in cross-sectional households estimations. For example, Calvet et al. (2021) structurally estimate EZW preference parameters using a cross-section of Swedish households. They identify considerable heterogeneity with a mean (s.d.) values for risk aversion of 5.24 (0.47), for time preference of 6.2% (6.0%) and EIS of 0.99 (0.96), with 40% of households exhibiting EIS values larger than one. Finally, high EIS has also been found in the experimental economics literature (Andersen et al., 2018).

[35] The agent expects to bequest on average $\tilde{b}\mathrm{E}(N_t) = 3,279$\$; the effects of larger bequests are discussed below.

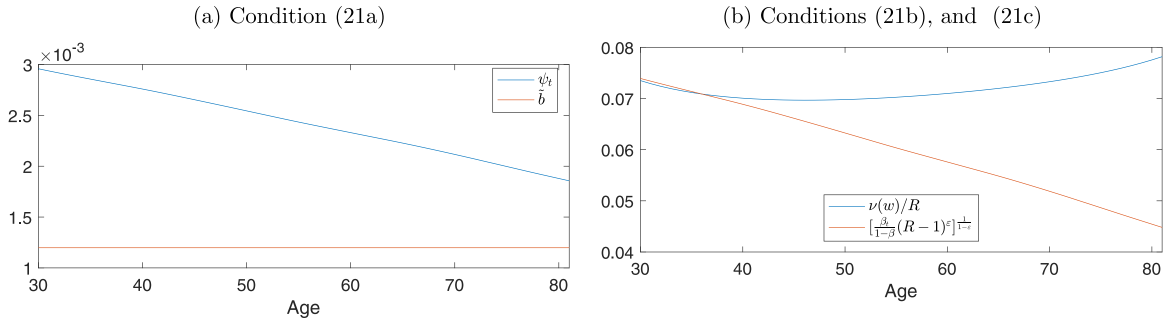

Fig. 4. Theoretical restrictions in Proposition 1.
Notes: Conditions (21a) and (21b) for detrimental effects of mortality (claim 1) and (21c) for net total wealth decumulation (claim 2) in Proposition 1.

In-sample fit adequacy. Contrasting the observed (in blue) and predicted (in red) life cycles in Fig. 3 reveals that the model performs reasonably well in reproducing in-sample data. First, both the level, and rate of decline in health are precisely matched in panel a. Similarly, the level, accumulation and de-cumulation phases of financial wealth are precisely predicted in panel b. The model performance is arguably less stellar for labor market outcomes in panels c, and d, yet both levels and extent of declines are correctly predicted and within confidence bounds.[36] Importantly, the model fit should also be assessed out-of-sample. Indeed, none of the life values discussed next were taken into account when calibrating the structural parameters. As will become clear shortly, the model performance from that out-of-sample perspective is remarkable as well.

5.3. Theoretical restrictions, net total wealth and welfare

Fig. 4 plots the theoretical restrictions in Proposition 1. Panel a shows that the marginal welfare ψ_t (in blue) is decreasing, and always above the bequest share $\tilde{b}$ (in red), consistent with restriction (21a). Panel b plots marginal felicity $\nu(w)$ divided by R (in blue); the ratio is well below 1.0, in accord with restriction (21b). It follows that the parameters are consistent with preference for life (Proposition 1, claim 1): increases in mortality induce heavier discounting of future utility, higher MPC, and are detrimental for welfare, particularly for permanent changes, although less so for elders. Moreover, panel b shows that, except for young agents, $[\beta_t/(1-\beta)(R-1)^{\epsilon}]^{1/(1-\epsilon)}$ (in red) is lower than $\nu(w)/R$ consistent with restriction (21c). It follows that net total wealth N_t is expected to be decreasing for most of the life cycle (Proposition 1, claim 2).

Fig. 5 reports the net total wealth, as well as the welfare. First, panel a plots the predicted net total wealth N_t levels and specific components calculated from the recursion (D.2) in Appendix D. The level is considerably higher than financial wealth, confirming the importance of non-financial assets. Moreover, the fall in both the permanent income V_t^w and shadow value of health V_t^H dominates the accumulation phase of financial wealth W_t, resulting in a near-continuous decline in total resources N_t throughout the life cycle. Both the decline and accelerating rate of decumulation are consistent with the theoretical predictions (Proposition 1, claim 2).

Second, as was mentioned earlier, the elasticities of all life values with respect to the individual characteristics simplify to their relative shares of N_t plotted in panel b. Ageing lowers the shares of the permanent income V_t^w/N_t and of health V_t^H/N_t, whereas those of financial wealth W_t/N_t increase. Consequently, the willingness are more responsive to changes in the permanent income and of health for younger agents, and gradually become more responsive to financial wealth for older agents. The elasticities levels and life-cycle patterns compare advantageously to the life values elasticities reported in the literature.[37] Third, the declining marginal utility ψ_t in Fig. 4.a combined with the drop in net total wealth N_t in Fig. 5.a jointly lead to an accelerating decline in the continuation utility $V_t = \psi_t N_t$ in Fig. 5.c. Importantly, the benchmark metric against which changes in longevity are valued is therefore falling throughout the life cycle under the combined influences of diminishing marginal value and levels of net total wealth.

5.4. WTP/WTA for changes in death intensity Δ

The two upper panels of Fig. 6 report the Hicksian WTP (lines) and WTA (dots) for changes Δ_t in death intensity λ_t^m, where the Δ_t are converted to the relative changes in survival probability $\nabla p_t/p_t = \exp(-\Delta_t) - 1$ (see Table 1) reported on the horizontal axis. The willingness correspond to changes in survival occurring at ages $t = 25$ (in blue), $t = 45$ (in black), and $t = 65$ (in red), distinguishing between one-shot $\Delta_t = \mathbb{1}_t \Delta$ (panel a) and permanent changes $\Delta_t = \Delta, \forall t$ (panel b).

[36] The excessive responsiveness of predicted hours worked to the changes in wages (Fig. 1.c) is likely related to real-life labor market frictions, such as statutory number of hours per week, that are abstracted from in the model.

[37] The evidence surveyed in Viscusi and Masterman (2017b), Masterman and Viscusi (2018) and Alberini and Ščasný (2021) indicate a VSL income elasticity between 0.5 and 0.8. Aldy and Smyth (2014, Tab. 4 and 5) report realized income elasticities for the VSL falling from 0.82 at 30 to 0.55 at 45, and to 0.15 at 80.

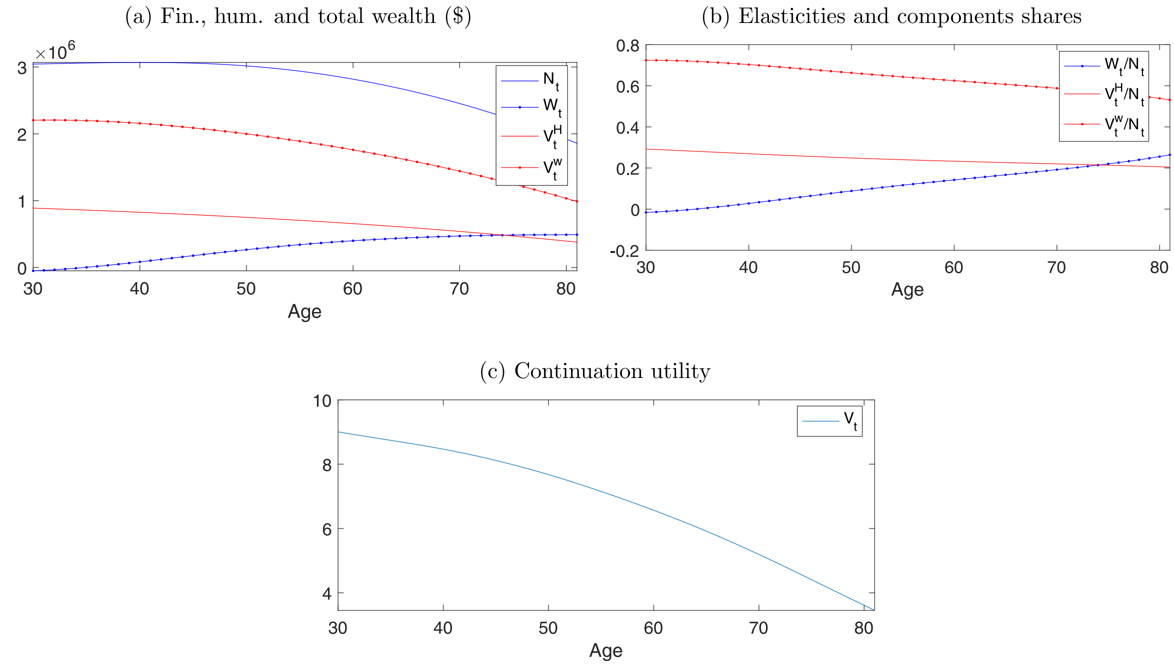

Fig. 5. Net total wealth and welfare.
Notes: (a) Financial (W_t), Human (V_t^w, V_t^H) and Net total wealth (N_t). (b) Elasticities of life values correspond to component shares of net total wealth $N_t = W_t + V_t^w + V_t^H$. (c) Continuation utility level ($V_t = \psi_t N_t$) . Calculated along optimal path at calibrated parameters.

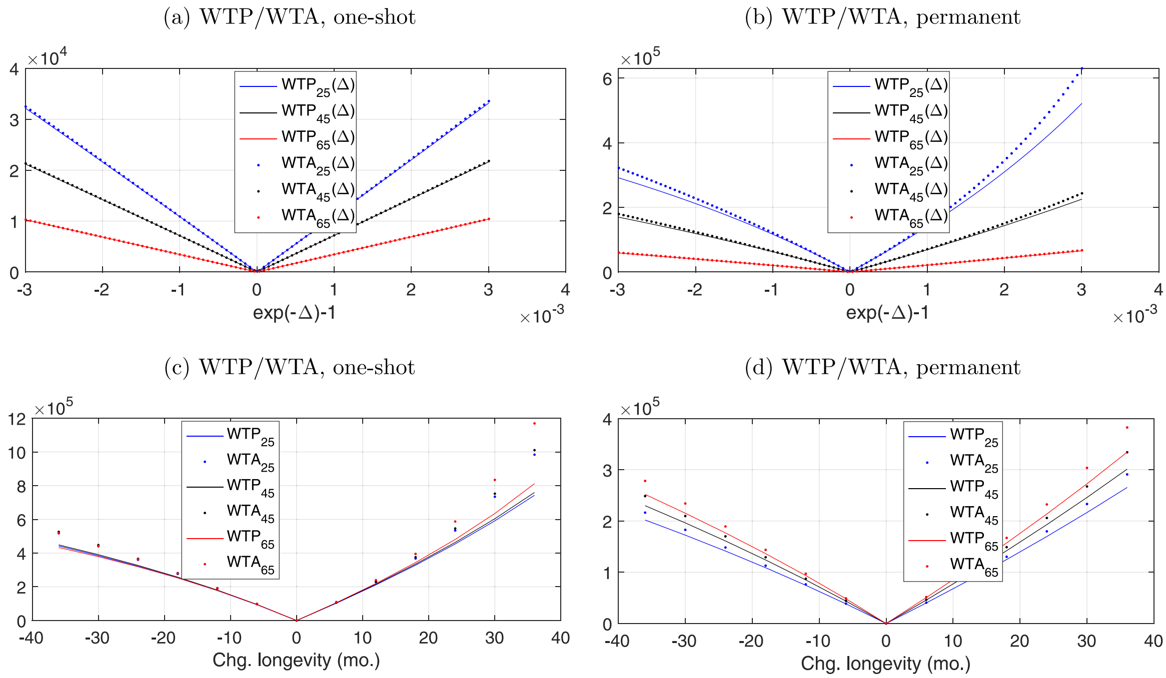

Fig. 6. Willingness by relative change $\nabla p_t/p_t$ in survival and change ∇L_t in longevity.
Notes: Hicksian WTP (solid line) and WTA (dots) at age $t \in \{25, 45, 65\}$. Calculated from (23) and Tables 1 and 3. Upper panels (a), (b): Willingness for survival change Δ_t. Lower panels (c), (d): Willingness for longevity change ∇L_t. Left-hand panels (a), (c): One-shot: $\Delta_t = \mathbb{1}_t \Delta$. Right-hand side panels (b), (d): Permanent: $\Delta_t = \Delta, \forall t$.

First in panel a, consistent with detrimental effects of mortality (Proposition 1, claim 1.a), the WTP (resp. WTA) to attain (resp. forego) beneficial changes $\exp(-\Delta) - 1 > 0$ and to prevent (resp. accept) detrimental changes $\exp(-\Delta) - 1 < 0$ are all positive, and increasing in the magnitude of the change in survival. Second, the willingness measures are realistic compared to other estimates.[38] Third, the measures are symmetric along both the beneficial vs detrimental, and the WTP vs WTA dimensions. Beneficial and detrimental one-shot changes of equal size are thus valued equally and the buying prices (WTP) are roughly equal to the selling prices (WTA) for these changes. Fourth, consistent with Proposition 1, claim 1.c, ageing is clearly associated with lower willingness measures under the combined influences of (i) dampening effects on marginal value $\psi_t(\lambda^m)$, (ii) lower resources N_t in Fig. 5.a, and (iii) lower longevity returns to changes Δ in Fig. 2.a.

Several differences are revealed when changes in death intensity become permanent in panel b, instead of one-shot. First, as expected, the willingness measures are larger, consistent with the much more potent effects of permanent changes on k-periods ahead survival $\nabla P_{t,t+k}$ and longevity ∇L_t (Table 1) and on welfare (Proposition 1, claim 1.b). Second, they are asymmetric with both the WTP and WTA measures being larger in the gains than in the loss domain. Third, the findings confirm standard variational results linked to endowment effects, with selling prices (WTA) being larger than buying prices (WTP). Fourth, ageing once again attenuates both the levels of and the differences between WTP's and WTA's, and between detrimental and beneficial changes.

5.5. WTP/WTA for changes in longevity ∇L_t

The analysis is next re-focused in terms of changes in longevity ∇L_t instead of survival Δ_t. The two lower panels of Fig. 6 report the Hicksian willingness measures for longevity changes ∇L_t (measured in months) occurring at ages $t = 25$ (in blue), $t = 45$ (in black), and $t = 65$ (in red), again distinguishing between one-shot (panel c) and permanent (panel d) changes. For both cases, the results confirm the asymmetry displayed earlier with beneficial changes valued more than detrimental ones, and larger WTA relative to WTP. The WTP/WTA for ∇L_t are the product of the two conflicting effects of (i) age-increasing required levels of Δ (Fig. 2.a and b), and (ii) age-declining value of Δ (Fig. 6.a, and b). The results unambiguously show the dominance of the former, with age-increasing willingness over changes in longevity. Moreover, the non-indifference between one-shot (panel c) and permanent changes (panel d) is related to both discounting and preferences towards risk and towards time allocation arguments. Indeed, the higher WTP/WTA for one-shot reflects preference for immediate and certain relative to future and uncertain changes in longevity, as well as preference for early resolution of uncertainty in Non-Expected Utility settings whenever risk aversion $\gamma = 3.5382$ is larger than the inverse EIS $\varepsilon = 0.5009$.[39]

5.6. Related life valuations

Fig. 7 reports the life cycles of the related life valuations introduced in Section 4.2: VOLY (panel a), VSL (panel b), GPV (panel c) and DLV (panel d).

5.6.1. Value of life year (VOLY)

Panel a plots the life cycles of both the WTP's (in blue) and WTA's (in red) for one-shot changes in expected longevity ∇L_t corresponding to ± 12 months. First, the results compare advantageously with evidence on the willingness to pay for longevity.[40] Second, the VOLY (i) are relatively age-independent,[41] (ii) display higher WTA (dots) than WTP (lines), and (iii) are larger for beneficial gains (blue), than for detrimental losses (red) in longevity. Equivalently, attaining one year of additional longevity is valued more than losing one year throughout the life cycle, and is consistent with higher selling (WTA) than buying (WTP) prices.

5.6.2. Value of statistical life (VSL)

Panel b plots the life cycle of the two VSL measures from (24) calculated using one-shot detrimental and beneficial $\Delta = \pm 1.0e{-}03$. First, consistent with Fig. 6 on symmetric WTP and WTA for one-shot changes, both VSL^p, VSL^a are indistinguishable from one another,[42] as are the VSL measures computed for beneficial and for detrimental changes. Second, all VSL's are falling in age[43] and realistic compared to other estimates; the survival-weighted average VSL's reported in column 1 of Table 6 is close to 6.0M$, well in line with the estimates found in the reduced-form literature.[44] Third, the four VSL measures constitute a valid infra-marginal approximation to the true MRS in the case of small, one-shot changes in death risk exposure.[45]

[38] For instance, Hall et al. (2020) compute the percentage of annual consumption that agents would be willing to pay to avoid the average increase in mortality caused by COVID-19. For death risk increases corresponding to $\Delta \in \{0.0044, 0.0081\}$ across all age groups, they estimate that the WTP would respectively correspond to 28% and 41% of annual consumption. We can calculate the WTP to prevent these values for Δ and divide by the theoretical measure of consumption $c_t = \tilde{c}_t/\mu(w_t)$ using (8), (10a) and (17) to recover WTP to consumption shares of 25.3% and 45.8% respectively.

[39] See Jones-Lee et al. (2015) and Hammitt and Tunçel (2015) for evidence and discussion of non-indifference with respect to how given changes in longevity are attained (e.g. immediate versus delayed, transient versus permanent).

[40] A meta-analysis by Ryen and Svensson (2015, tab. V, p. 1295) finds mean (median) SP-based QALY estimates in 2010 Euros of 119.8 K€ = 146.9 K$ (24.2 K€= 29.7 K$) and RP-based estimates of 242.3 K€ = 297.21 K$ (109.9 K€= 134.19 K$), using the June 2010 exchange rate of 1.2266. Kniesner and Viscusi (2019, p. 14) report VSLY estimates used by the U.S. Department of Health and Human Services FDA that increased from 116 K$ to 369 K$ between 1998 and 2016.

[41] Whereas age-independence is found for one-shot changes in longevity, permanent changes result in age-increasing VOLY.

[42] See also Kniesner et al. (2014) for additional evidence on the equivalence between WTP- and WTA-based VSL estimates.

[43] The age patterns for VSL are consistent with Murphy and Topel (2006, Fig. 3) who document falling VSL after age 30. The other empirical evidence on ageing effects on the VSL finds moderate increases followed by continuous decreases starting before mid-life. Murphy and Topel (2006) find a VSL of 2.0 M$ at age 70 while (Ketcham et al., 2021) find similar VSL of 1.0 M$ for seniors aged 67–87. See also Hammitt (2020), O'Brien (2018) and Aldy and Smyth (2014) for evidence and discussion.

[44] Guidance VSL from the US Department of Transportation were 9.6 M$ for 2016 (US Department of Transportation, 2016). Bellavance et al. (2009, Tab. 6, p. 452) report meta-analysis mean VSL values of 6.2 M$ (year 2000), whereas Doucouliagos et al. (2014) report VSL 6 M$ and 10 M$ and Robinson and

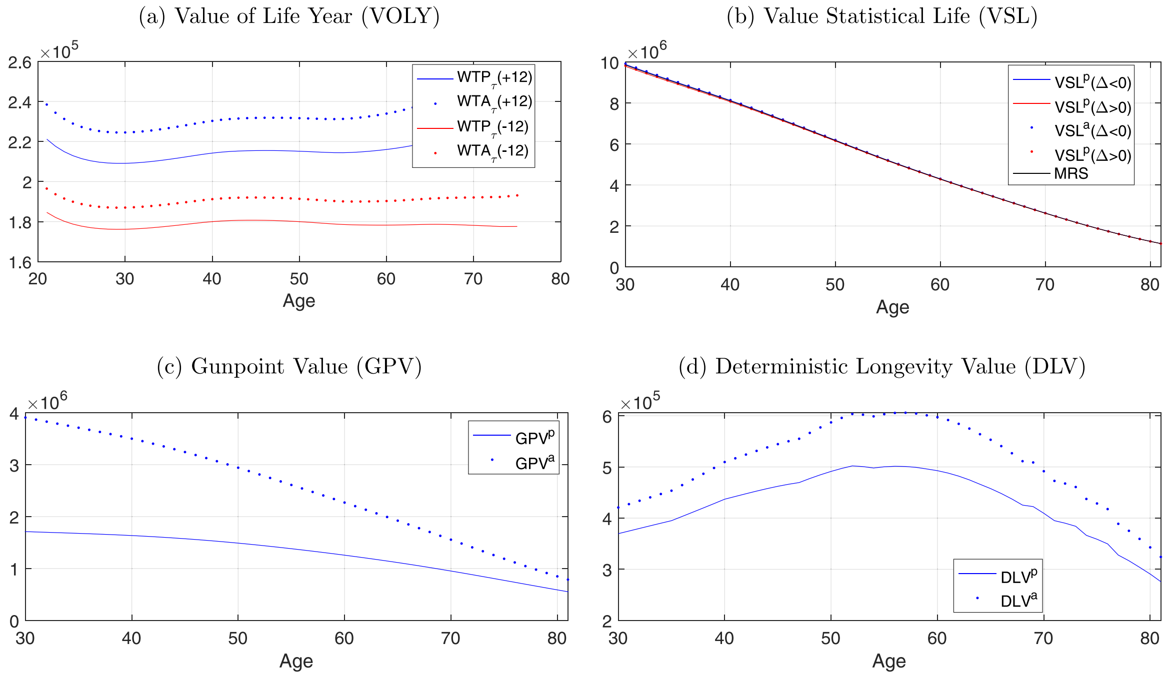

Fig. 7. Related life valuations.
Notes: (a) Value of Life Year is WTP (solid line) and WTA (dots) for one-shot change in longevity $\nabla L_t = 12$ months, by age. Calculated from Eqs. (23a) and (23b) Tables 1 and 3. (b) VSL calculated from Eq. (24) for one-shot change $\Delta_t = \mathbb{1}_t \Delta$, for beneficial and detrimental changes $\Delta = 1.0e-03$, via WTP and WTA measures. MRS approximated from VSL for one-shot change $\Delta = 1.0e-09$. (c) GPV calculated from Eq. (25) for WTP $GPV_t^p(\lambda^m, N)$, and for WTA compensation $GPV_t^a(\lambda^m, N)$ for infinite permanent detrimental change ($\Delta_t = \Delta = \infty, \forall t$) in base exposure λ^m at age t. (d) DLV calculated from Definition 1. All values at calibrated parameters along optimal path for N_t.

5.6.3. Gunpoint value of life (GPV)

Panel c plots the life cycle of the two Gunpoint values (25) for changes $\Delta = +\infty$ occurring at age t. First, both the willingness to receive compensation to accept (dots) and to pay (line) to avoid certain and impending death are falling in age. As the welfare from remaining alive V_t depicted in Fig. 5.d falls through the combined influences of falling marginal utility of wealth ψ_t (panel c) and falling total wealth N_t (panel a) so does the WTP to prevent and WTA to accept imminent death. Second, consistent the endowment effects highlighted earlier, the willingness to accept compensation is finite and larger than the willingness to pay, although the difference is attenuated by ageing.

Finally, the levels are in the same range as net total wealth with survival-weighted averages GPV^a of 2.69 M$ and GPV^p of 1.32 M$, compared to N of 2.74 M$ (see column 1 of Table 6). The WTP-based Gunpoint life values found in Hugonnier et al. (2022) are equal to their estimates for net total wealth (251 K$) and are lower than our findings. Their modeling approach is however different as it abstracts from ageing, bequests ($\tilde{b} = 0$), and leisure ($\theta = 1$). As seen below the omission of leisure lowers the utilitarian benefits from living, resulting in lower life valuations. Moreover, Hugonnier et al. (2013, 2022) allow for subsistence consumption which cannot be pledged in life valuations and must be deducted from N_t, therefore lowering the GPV.

5.6.4. Deterministic longevity value (DLV)

Finally, panel d plots the life cycle of the Deterministic Longevity Value calculated from Definition 1. First, the DLV's point towards non-negligible utilitarian costs of uncertain life horizon with mean value of 403 K$ (WTP) and 475 K$ (WTA). These costs are consistent with aversion to mean-preserving spreads whereby risk-averse ($\gamma = 3.5382$) agents would much prefer to be assured now that they will live L_t with certainty, than face an uncertain life horizon whose expected remaining duration is also equal to L_t. Second, contrary to the previous life values, the effects of ageing are non-monotone, with increasing DLV's when young, peaking values between age 50 and 60 and decreasing values afterwards. Heuristically, the cost of lifetime uncertainty is lowest for both young (high expected longevity) and for elders (high realized longevity), and highest at mid-life. In the absence of other estimates in the literature, it is difficult to assess the realism of the DLV life cycle. Nevertheless, the mid-life peak in accumulated financial wealth reserves (Fig. 5.b), as well as the high mid-life utilitarian costs of longevity uncertainty (Fig. 7.d) are both consistent with

Hammitt (2016) document VSL between 4.2 and 13.7 M$. Additional "meta-meta" analysis by Banzhaf (2021) finds a mean VSL of 7.0 M$, with 90% bounds of 2.4–11.2 M$. See also Viscusi and Masterman (2017a) and Robinson et al. (2019) for recent evidence in similar ranges.

[45] The reported one-shot MRS from (C.5) is approximated for one-shot infra-marginal $\Delta = 1.0e-09$.

Table 6
Survival-weighted population averages and comparative statics (in K$).

Δ longev. (months)	(1) Base		(2) High bequest		(3) No ℓ		(4) No H		(5) No ℓ, H		(6) VNM	
	WTP	WTA	WTP	WTA	WTP	WTA	WTP	WTA	WTP	WTA	WTP	WTA
36	666	920	399	469	276	308	462	638	191	213	246	307
30	526	670	324	369	226	247	365	465	157	171	210	253
24	401	480	253	280	178	190	278	333	123	132	172	200
18	289	327	186	199	131	138	200	227	91	96	133	149
12	185	200	121	127	86	89	128	139	60	62	91	98
6	90	93	59	61	43	43	62	64	30	30	47	49
−6	81	84	56	57	41	41	56	58	28	29	47	49
−12	154	164	108	112	80	82	107	114	55	57	94	102
−18	218	239	156	166	116	121	151	166	81	84	139	158
−24	277	311	202	218	151	160	192	216	105	111	184	218
−30	330	380	244	268	184	198	229	264	128	137	227	282
−36	378	447	284	317	216	235	262	310	150	163	269	351
VSL	5,999	6,015	3,833	3,839	2,646	2,648	4,160	4,171	1,835	1,836	2,975	2,981
MRS	6,031		3,845		2,652		4,182		1,839		2,978	
GPV	1,321	2,686	1,245	2,151	1,120	1,720	916	1,863	776	1,192	1,487	25,327
DLV	403	475	295	326	231	248	280	330	160	172	256	319
N_t	2,736		3,138		3,374		1,898		2,340		1,589	

Notes: Survival-weighted average of age-dependent life valuations. VSL calculated for $\Delta = 1.0e{-}03$. Panel (1) Baseline calibration; (2) High Bequest: High intended bequest $b^{1/(1-\gamma)}$ ↑; (3) No leisure: $\theta = \sigma = 1$; (4) No human capital: $A = B = 0$; (5) No leisure/human capital: $\theta = \sigma = 1$ and $A = B = 0$; (6) VNM: $\gamma \equiv \varepsilon$ + no bequests $b = 0$ + no leisure/human capital: $\theta = \sigma = 1$ and $A = B = 0$.

the age profile of typical annuities purchases whose primary function is to hedge lifetime uncertainty by annuitizing accumulated wealth.[46]

6. Discussion and concluding remarks

6.1. Comparative statics and robustness

The theoretical model in Section 3 contributes to standard life cycle literature in several dimensions. In particular, the continuation utility (and therefore life valuations) captures utility from bequests motives, as well as optimal decisions in market (consumption), non-market (leisure), and investment in own health. Moreover, I resort to non-expected EZW utility, instead of the more traditional VNM preferences.

To assess the role of these modeling choices, Table 6 presents survival-weighted population averages[47] for our baseline model (column 1), and for alternative parametric choices (columns 2–5) and for standard VNM preferences (column 6). The comparative statics are calculated by re-computing the optimal life cycles of (N_t, ψ_t) at the alternative parametric set, and then recalculating the implied life valuations (WTP_t, WTA_t) for beneficial increases, and detrimental decreases in expected longevity of 6 to 36 months, as well as for the Gunpoint, MRS, Statistical and Deterministic Longevity life values. Consistent with comparative statics principles, a single subset of parameters is modified at a time with others remaining at base values reported in Table 5.

Bequest motive. In column (2), I analyze the effects of increasing the intended bequest $\tilde{b} = b^{1/(1-\gamma)}$ via a 50% decrease in b. This results in two opposing forces with respect to life valuations. On the one hand, agents wish to increase bequeathed resources, resulting in an increase in net total wealth N_t from 2.74 M$ to 3.14 M$. On the other hand, the utilitarian cost of dying is attenuated by leaving bequests, thereby reducing the willingness to pay or to accept compensation for changes in longevity. The results confirm that the latter is the dominating force, leading to an overall decline in WTP's, WTA's, as well as VSL, MRS, GPV, and DLV.

Leisure motive. In column (3), I analyze the effects of the utility for leisure by removing its benefit in setting $\theta = \sigma = 1$ in (7b), resulting in $v_t = \mu_t = 1, \forall t$ in (10). Consequently, the agent inelastically supplies her full time endowment for work ($n_t = 1$) and the absence of spending on leisure ($w_t \ell_t = 0$) implies a decrease in total expenditures and an increase in total wealth from 2.74 M$ to 3.37 M$. However, this increase is more than offset by the fall in felicity from leisure activities implying a lower continuation utility and a reduction in the WTP/WTA, as well as other life values.

[46] Market evidence suggests that the best age bracket for annuity purchases is between 45 and 70, with typical buyers are older than 50 and nearing retirement (e.g. RetireGuide, 2023; Annuity.org, 2023).
[47] For any variable X_t, the survival-weighted averages are $\sum_{t=0}^{T} f_t X_t$, where the density $f_t = P_{0,t}/L_0$ uses time-0 to age-t survival rates $P_{0,t}$ and longevity L_0 in Table 1.

Human capital. Column (4) gauges the importance of human capital by removing it altogether from the optimization program, and therefore from the continuation utility and associated life valuations. This is achieved by setting both productivity A and benefit B equal to zero, thereby eliminating optimal investment and shadow value $I_t = \eta_t = V_t^H = 0, \forall t$. Unsurprisingly, net total wealth falls sharply from 2.74M$ to 1.89M$ due to the omission of the shadow value of health. Moreover, since the health-related quality of life is not longer integrated in the continuation utility, all life values fall accordingly.

Leisure and human capital. In column (5) I remove both leisure and health capital to account for quality-of-life considerations in life values. For reasons discussed above, no leisure increases net total wealth, whereas no health capital reduces it. The latter effect is dominant with a reduction in net total wealth from 2.74M$ to 2.34M$. However, the omission of both leisure and health concur to reduce the quality of living inducing a sharp fall in all the life values.

EZW vs VNM preferences. To assess the role of Non-Expected Utility in our results, column (6) imposes the restrictions in more traditional life cycle models introduced in Section 2, with (i) VNM preferences that force the risk aversion to be the inverse of the EIS ($\gamma \equiv \epsilon$),[48] (ii) abstract from bequests motives ($b = 0$) and (iii) omit both leisure and human capital choices ($\theta = \sigma = 1$ and $A = B = 0$). Under these restrictions, the agent's problem simplifies to:

$$\begin{aligned} V_t &= \max_{c_t} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta p_t V_{t+1}, \quad \text{s.t.} \\ W_{t+1} &= [W_t + (y - i) - c_t]R. \end{aligned}$$

As was the case in column (4), the omission of human capital results in a fall in net total wealth from 2.74M$ to 1.59M$. Moreover, the VNM case leads to lower values of life for all measures except the Gunpoint which is marginally higher in the WTP, and much larger in the WTA variant. In the absence of bequests considerations, the latter captures the trade-off between additional immediate consumption at t and lost life afterwards. Observe further that VNM/CRRA preferences imply indifference to the timing of the resolution of uncertainty; knowing the exact timing of death yields indirect utilitarian benefits related only to the elimination of longevity uncertainty for risk-averse agents. This consideration partially explains why the Deterministic Longevity Value in column (6) is lower than for the benchmark EZW preferences scenario in column (1) which explicitly assumes direct utilitarian benefits at the calibrated parameters.

6.2. Concluding remarks

This paper has proposed a rigorous methodology to adjust life valuations to both personal characteristics (e.g. preferences, age, health, morbidity, wealth, permanent income) and the distributional features of the changes in death risk (e.g. beneficial vs detrimental, immediate vs permanent, small vs large, mean vs variance). The analytical solution to a flexible life cycle problem with endogenous consumption, leisure and health decisions, and age-dependent wages, morbidity and mortality processes allowed for a precise mapping of these effects on both the Hicksian WTP/WTA measures and the associated valuations (VOLY, VSL, GPV, DLV).

These characterizations could hopefully be relied upon for policy purposes, such as for public health decisions. For example the pandemic trade-offs from reallocating resources from the general, comparatively healthy population (small, long-run detrimental effects on mortality) towards current vulnerable groups (large, immediate beneficial effects for elders and with chronic conditions) could benefit from a more precise adjustment of VOLY, VSL and GPV to both individual and risk characteristics. It could also be employed in prioritizing certain individuals for medical intervention as an optimal-health VOLY benchmark alternative or complement to the perfect health benchmark QALY-based measures. Second, public policy choices such as road safety could better adjust VSL to specific road users' characteristics (e.g. working-age drivers) against those of the general population of tax payers. Third, economic forensics could benefit from an alternative to traditional human capital methods in adjusting the life values of deceased persons to individual characteristics for litigation or tolls of death (e.g. from wars or natural catastrophes) calculation purposes. Finally, the trade-offs between innovations that alter the optimal mix between quantity and quality of life could be complemented with the Deterministic Longevity Value to assess the benefits from reducing longevity uncertainty.

Despite its advantages, several restrictive assumptions have been imposed that could be fruitfully relaxed. In particular, perfect financial markets are required in order to attain closed-form solutions. This implies that agents have access to full insurance against health shocks and can sell/buy any claim to income or expenditure streams at actuarially-fair prices. Real-life market imperfections such as borrowing constraints, limited insurability, and distortionary taxes have been voluntarily abstracted from. Moreover, I have assumed exogenous exposures to death and sickness risks. More general models allowing for health-dependent (and therefore endogenous) mortality and morbidity along the lines of Hugonnier et al. (2021) could be considered. Accounting for these caveats would likely be at the expense of tractability and involve numerical, rather than analytical solutions; these considerations are therefore left on the research agenda.

CRediT authorship contribution statement

Pascal St-Amour: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing.

[48] More precisely, I set $\gamma = \epsilon = 0.5009$. As is well known, restricting $\gamma \in (0,1)$ is required to ensure positive continuation utility and therefore guarantee that life $V_t > 0$ is preferable to death $V_t = 0$ in VNM settings (e.g. Shepard and Zeckhauser, 1984; Hugonnier et al., 2013).

Appendix A. Solution method for generalized model

The solution method relies on the following building blocks:

Leisure and consumption choices I first exploit static optimization between labor and consumption choices ($MRS_t = w_t$) to recast the agent's problem in terms of total expenses $\tilde{c}_t = c_t + w_t \ell_t$ (i.e. consumption and the opportunity cost of leisure).

Separation between human capital and financial choices Second, under the perfect markets assumption, I invoke separation properties (e.g. Bodie et al., 1992; Hugonnier et al., 2013; Palacios, 2015; Acemoglu and Autor, 2018) to solve the optimal human capital dynamics independently from financial decisions. Under perfect markets assumption, two claims can be sold on financial markets:

1. a claim to health benefits net of investment, independent of mortality risk, and over finite horizon $t = 0, \dots, T$, and whose value $V_t^H = V_t^H(H_t)$ satisfies:

$$V_t^H = \max_{I_t} \left(BH_t - I_t\right) + \exp(-r)\mathrm{E}_t V_{t+1}^H,$$

subject to health dynamics (6), which is solved by backward induction, independently from the other allocation for (c_t, ℓ_t). The shadow value of the human capital V_t^H thus corresponds to the expected net present value of its dividend stream $(BH_t - I_t)$.

2. a claim to the expected net present value of exogenous lifetime wages and fixed income, net of fixed medical expenses, independent of mortality risk, and over horizon $t = 0, \dots, T$:

$$V_0^w = \mathrm{E}_0 \sum_{t=0}^{T} \exp(-rt)(w_t + y - i).$$

Observe that V_0^w is a permanent income measure that encompasses the expected net present value of the unit of time endowment (a share ℓ_t of which is spent on leisure and is accounted for in total expenses $\tilde{c}_t$).

Equivalent total expenses and total wealth problem At the initial period $t = 0$, the agent cashes-in those two claims that are added to financial wealth W_0 to obtain (non-stochastic) net total wealth $N_0 = W_0 + V_0^w + V_0^H$, and the corresponding dynamics for the latter are adjusted accordingly. Regrouping our static optimization and separation results imply that the original problem can be equivalently recast with $V_t = V_t(\lambda_t^m, N_t)$ as follows:

$$V_t = \max_{\tilde{c}_t} \left\{ (1-\beta)(v_t \tilde{c}_t)^{1-\epsilon} + \beta \left[p_t V_{t+1}^{1-\gamma} + \left(1 - p_t\right) b\, N_{t+1}^{1-\gamma} \right]^{\frac{1-\epsilon}{1-\gamma}} \right\}^{\frac{1}{1-\epsilon}} \tag{A.1a}$$

where p_t is the one-period ahead survival probability in Table 1, and subject to

$$\begin{aligned} N_0 &\equiv W_0 + V_0^w + V_0^H(H_0), \\ N_{t+1} &= \left[N_t - \tilde{c}_t\right] R, \end{aligned} \tag{A.1b}$$

Backward iteration The dynamic optimization for both human capital investment I_t and total expenses $\tilde{c}_t$ calculates the optimal policies by backward iteration starting at maximal longevity T.

Equivalence Appendix B.3 verifies and confirms that the separable health investment and total expenses solution coincides with the direct solution method where the optimal rules for $I_t, \tilde{c}_t$ are solved simultaneously under the assumption of actuarially-fair insurance against health shock ϵ_{t+1}^h.

Appendix B. Proofs

B.1. Simplified model solution

Proof. Standard optimization shows that the Euler equation for simplified problem (1) is:

$$1 = \beta(\lambda^m) R \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$$

where $R \equiv \exp(r), \beta(\lambda^m) \equiv \exp(-\rho - \lambda^m)$. Use guess candidate $c_t = \omega(\lambda^m) N_t$ in the budget constraint $c_{t+1} = [N_t - c_t]R$, and substitute in the Euler to solve:

$$\omega(\lambda^m) = [1 + \kappa(\lambda^m)]^{-1}$$
$$\kappa(\lambda^m) \equiv \beta(\lambda^m)^{\frac{1}{\gamma}} R^{\frac{1-\gamma}{\gamma}} = \exp\left[\frac{-\rho - \lambda^m + (1-\gamma)r}{\gamma}\right]$$

substitute this solution in the objective function to obtain:

$$\begin{aligned} V_t &= \frac{\left[\omega(\lambda^m) N_t\right]^{1-\gamma}}{1-\gamma} \left\{1 + \beta(\lambda^m)[\kappa(\lambda^m) R]^{1-\gamma}\right\} \\ &= \frac{N_t^{1-\gamma}}{1-\gamma} \left\{1 + \beta(\lambda^m)^{\frac{1}{\gamma}} R^{\frac{1-\gamma}{\gamma}}\right\}^{\gamma} = \frac{N_t^{1-\gamma}}{1-\gamma} \omega(\lambda^m)^{-\gamma} \end{aligned}$$

which solves for $\psi(\lambda^m)$ in:

$$V_t = \frac{\left[\psi(\lambda^m) N_t\right]^{1-\gamma}}{1-\gamma}, \quad \text{as} \quad \psi(\lambda^m) = \omega(\lambda^m)^{\frac{\gamma}{\gamma-1}}. \quad \blacksquare$$

B.2. Generalized model: Main results

Timing convention. The optimal consumption-leisure decisions are obtained by static optimization, whereas the dynamic solutions for $I_t, \tilde{c}_t$ are obtained by backward iteration. I thus recode timing t in terms of maximal remaining survival time $s \equiv T - t = 1, 2, \ldots$ before maximal longevity T is reached. To alleviate notation I omit time subscripts for contemporary variables, use prime (′) for next-period variables, and rely on s subscripts to emphasize feedback rules calculated s-periods from T whenever appropriate.

Lemma 1. From a-temporal optimization condition (applicable in our setting), equalize wages to the marginal rate of substitution between leisure and consumption, using felicity function (7b) to obtain that the optimal leisure-consumption mix is characterized by:

$$\ell = \left[\left(\frac{1-\theta}{\theta}\right)\frac{1}{w}\right]^{\frac{1}{\sigma}} c.$$

Substitute back into the budget constraint (5a) to obtain total expenses $\tilde{c}$ in (8), and into felicity (7b) to obtain $u(c, \ell)$ in (9). $\blacksquare$

Lemma 2. The optimal investment problem (11) can be rewritten as:

$$V_s^H(H) = \max_I BH - I + \exp(-r)\left\{\exp(-\lambda^h) V_{s-1}^H(H'_+) + \left[1 - \exp(-\lambda^h)\right] V_{s-1}^H(H'_-)\right\}$$

subject to

$$H'_+ = A I^\alpha H^{1-\alpha} + (1-\delta) H$$
$$H'_- = A I^\alpha H^{1-\alpha} + (1-\delta-\phi) H$$

for $s = T - t$ periods away from maximal longevity. The candidate solutions are:

$$I_s(H) = \kappa_s H$$
$$V_s^H(H) = \eta_s H$$

$s = 1:$ Longevity being bounded leads trivially to $I_s = 0$ and $V_s^H = BH$; the initial loadings are $(\kappa_1, \eta_1) = (0, B)$.

$s = 2:$ Substituting the previous solution for $s = 1$ reveals that

$$V_s^H = \max_I BH - I + \exp(-r)\eta_{s-1}\left\{A I^\alpha H^{1-\alpha} + (1-\delta)H - [1 - \exp(-\lambda^h)]\phi H\right\}$$

with the solution to the FOC:

$$I_s = \underbrace{\left[\eta_{s-1}\exp(-r)\alpha A\right]^{\frac{1}{1-\alpha}}}_{\kappa_s} H$$

substituting back in the objective function simplifies to:

$$V_s^H = \underbrace{\left[B - \kappa_s + \eta_{s-1}\exp(-r)\left\{A\kappa_s^\alpha + (1-\delta) - [1-\exp(-\lambda^h)]\phi\right\}\right]}_{\eta_s} H$$

$s \geq 3$: It is readily verifiable that the solutions converge to the same form for the other periods.

Regrouping terms shows that the solution to (11) are:

$$I_s = \kappa_s(\lambda^h)H,$$
$$V_s^H = \eta_s(\lambda^h)H,$$

where the loadings $\{\kappa_s, \eta_s\}_{s=1}^T$ satisfy the following recursion:

$$\kappa_s(\lambda^h) = \left[\eta_{s-1}(\lambda^h)R^{-1}\alpha A\right]^{\frac{1}{1-\alpha}}$$
$$\eta_s(\lambda^h) = B - \kappa_s(\lambda^h) + \eta_{s-1}(\lambda^h)R^{-1}\left\{A\kappa_s(\lambda^h)^\alpha + (1-\delta) - [1-\exp(-\lambda^h)]\phi\right\}$$

with initial values $(\kappa_1, \eta_1) = (0, B)$. ■

Theorem 1. The candidate solutions to the optimal expenses problem (19) are:

$$\tilde{c}_s(N) = \omega_s N$$
$$V_s(N) = \psi_s N$$

$s = 1$: Since longevity is bounded, $p = \exp(-\lambda^m) = 0$, leading to the following problem:

$$V(N) = \max_{\tilde{c}} \left\{(1-\beta)(v\tilde{c}^{1-\varepsilon}) + \beta[b(N')^{1-\gamma}]^{\frac{1-\varepsilon}{1-\gamma}}\right\}^{\frac{1}{1-\varepsilon}}$$
$$= \max_{\tilde{c}} \left\{(1-\beta)(v\tilde{c}^{1-\varepsilon}) + (N')^{1-\varepsilon}\underbrace{\beta b^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_1}\right\}^{\frac{1}{1-\varepsilon}}$$

subject to $N' = (N - \tilde{C})R$. The solution to the FOC is:

$$\tilde{c}_1(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} v^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} v^{\frac{1-\varepsilon}{\varepsilon}} + \beta_1^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}}\right]}_{\omega_1} N$$

Substituting back into the objective function implies that:

$$V_1(N) = \underbrace{\left\{(1-\beta)(v\omega_1)^{1-\varepsilon} + \beta_1\left[(1-\omega_1)R\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}}_{\psi_1} N$$

$s = 2$: Noting that $p = \exp(-\lambda^m) \neq 0$ and using our previous solution simplifies the agent's problem to:

$$V = \max_{\tilde{c}} \left\{(1-\beta)(v\tilde{c})^{1-\varepsilon} + \beta\left[p(\psi_1 N')^{1-\gamma} + (1-p)\,b\,(N')^{1-\gamma}\right]^{\frac{1-\varepsilon}{1-\gamma}}\right\}^{\frac{1}{1-\varepsilon}},$$
$$= \max_{\tilde{c}} \left\{(1-\beta)(v\tilde{c})^{1-\varepsilon} + (N')^{1-\varepsilon}\underbrace{\beta\left[p\psi_1^{1-\gamma} + (1-p)\,b\right]^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_2}\right\}^{\frac{1}{1-\varepsilon}},$$

subject to $N' = (N - \tilde{C})R$. The solution to the FOC is:

$$\tilde{c}_2(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} v^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} v^{\frac{1-\varepsilon}{\varepsilon}} + \beta_2^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}}\right]}_{\omega_2} N$$

Substituting back into the objective function implies that:

$$V_2(N) = \underbrace{\left\{(1-\beta)(\nu\omega_2)^{1-\epsilon} + \beta_2\left[(1-\omega_2)R\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}}_{\psi_2} N$$

$s \geq 3$: It is readily verifiable that the solutions converge to the same form for the other periods.

Regrouping terms shows that the solution to (19) are:

$$\tilde{c}_s = \omega_s(\lambda^m, w)N$$
$$V_s = \psi_s(\lambda^m, w)N$$

where the loadings $\{\omega_s, \psi_s\}_{s=1}^T$ satisfy the following recursion:

$$\begin{aligned}
\beta_s(\lambda^m, w) &= \beta\left\{p(\lambda^m)\psi_{s-1}(\lambda^m, w)^{1-\gamma} + \left[1-p(\lambda^m)\right] b\right\}^{\frac{1-\epsilon}{1-\gamma}}, \\
&= \beta\left\{p(\lambda^m)\psi_{s-1}(\lambda^m, w)^{1-\gamma} + \left[1-p(\lambda^m)\right] \tilde{b}^{1-\gamma}\right\}^{\frac{1-\epsilon}{1-\gamma}}, \\
\omega_s(\lambda^m, w) &= \frac{(1-\beta)^{\frac{1}{\epsilon}}\nu(w)^{\frac{1-\epsilon}{\epsilon}}}{(1-\beta)^{\frac{1}{\epsilon}}\nu(w)^{\frac{1-\epsilon}{\epsilon}} + \beta_s(\lambda^m, w)^{\frac{1}{\epsilon}} R^{\frac{1-\epsilon}{\epsilon}}} \\
\psi_s(\lambda^m, w) &= \left\{(1-\beta)\left[\nu(w)\omega_s(\lambda^m, w)\right]^{1-\epsilon} + \beta_s(\lambda^m, w)\left[\left(1-\omega_s(\lambda^m, w)\right)R\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}
\end{aligned}$$

with initial value $\beta_1(\lambda^m) = \beta b^{\frac{1-\epsilon}{1-\gamma}}$ and where (μ, ν) are given in (10a), and where net total wealth is:

$$N = N_s(W, V_s^w(w), H) = W + V_s^w(w) + \eta_s(\lambda^h)H$$

and $\tilde{b} \equiv b^{1/(1-\gamma)}$ is the share of bequeathed net total wealth. ■

Proposition 1. The two claims are proven under the maintained assumption that the inverse EIaS and EIS $\sigma, \epsilon \in (0, 1)$.

1. Effects of death intensity λ^m. Consider first a one-shot increase in mortality λ^m for which future marginal welfare $\psi_{t+1}(\lambda^m, w)$ is unaffected in the expression for β_t in (19a). A higher death risk lowers the survival probability $p(\lambda^m)$ and shifts weight away from the future marginal utility of living ψ_{t+1} towards the future marginal utility of dying $\tilde{b}$ in (19a). Under condition (21a), the former is larger than the latter and the discount factor β_t falls, thereby increasing the MPC ω_t in (19b).
   - (a) Condition (21b) implies the marginal utility of consumption $\nu(w)$ in (10b) is lower than the gross interest rate $R = \exp(r)$. The mortality-induced decrease in β_t and increase in ω_t imply a higher weight $\chi_\nu(\lambda^m, w)$ on the former and lower weight $\chi_\nu(\lambda^m, w)$ on the latter in (19d); both therefore have detrimental effects on the marginal welfare $\psi_t(\lambda^m, w)$ in (19c). Since any given net total wealth N_t was shown to be independent of λ^m, this lowers welfare $V_t = \psi_t N_t$ in (18).
   - (b) Permanent changes in λ^m re-instate the effects of lower marginal value $\psi_{t+1}(\lambda^m, w)$ on β_t in (19a) and therefore amplify the ensuing decline in discount factor, the increase in MPC and the detrimental effects on welfare.
   - (c) The post-retirement fall in wages induces an increase in the marginal felicity in (10b) whenever $\sigma \in (0, 1)$, thereby reducing the gap between $\nu(w)$ and R in condition (21b); the marginal (and total) welfare cost of higher death risk exposure is thus reduced for elders in (19c).
2. Net total wealth decumulation.
   - (a) Substituting consumption (17) and the MPC (19b) in the law of motion for net total wealth (16b) reveals that the (gross) rate of change is:

$$\frac{N'}{N} = (1-\omega_t)R = \frac{R}{\left(\frac{1-\beta}{\beta_t}\right)^{\frac{1}{\epsilon}}\left(\frac{\nu(w)}{R}\right)^{\frac{1-\epsilon}{\epsilon}} + 1} \tag{B.3}$$

     which is lower than 1 (i.e. decumulating net total wealth) if the inequality in condition (21c) holds.
   - (b) From the previous result elders face higher mortality risk λ^m which lowers β_t, and lower wages which increase marginal utility of consumption $\nu(w)$ when $\sigma \in (0, 1)$; both accelerate the decumulation rate in (B.3). ■

B.3. Separability of financial and health decisions

I now formally show that health-related and financial decisions are separable, i.e. that a joint optimization problem yields the same solutions as the ones obtained under separability.

Proof. First, the risk-averse agent will fully insure against health shocks ϵ^h_{t+1} at actuarially-fair prices. Consequently, the problem can be recast as a deterministic one with respect to morbidity, i.e. by setting $\epsilon^h_{t+1} = 0, \forall t$, with insurance premium calculated endogenously and deducted below from health capital value. Second, recast financial wealth as $W = W + V^w$ to include the value of the time endowment. The agent's problem can then be written as:

$$V(W,H) = \max_{\tilde{c},I} \left\{ (1-\beta)\left(v\tilde{c}_s\right)^{1-\epsilon} + \beta\left[pV(W',H')^{1-\gamma} + (1-p)b(N')^{1-\gamma}\right]^{\frac{1-\epsilon}{1-\gamma}} \right\}^{\frac{1}{1-\epsilon}}$$

subject to:

$$\begin{aligned} W' &= [W + BH - I - \tilde{c}]\,R \\ H' &= AI^{\alpha}H^{1-\alpha} + (1-\delta)H \\ N' &= W' + \eta' H'. \end{aligned}$$

The candidate solutions are the following:

$$\begin{aligned} V_s(W,H) &= V_s(N) \\ &= \psi_s N = \psi_s(W + \eta_s H) \\ I_s &= \kappa_s H \\ \tilde{c}_s &= \omega_s N \end{aligned}$$

where the age-dependent loadings $\{\psi_s, \eta_s, \kappa_s, \omega_s\}$ are determined recursively.

$s = 1$: Observing that $p = 0$ and by transversality $\eta' = \eta_0 = 0$ directly implies zero investment, i.e. $\kappa_1 = I_1 = 0$. The agent's problem simplifies to:

$$V(W,H) = \max_{\tilde{c}} \left\{ (1-\beta)\left(v\tilde{c}_s\right)^{1-\epsilon} + \underbrace{\beta b^{\frac{1-\epsilon}{1-\gamma}}}_{\beta_1} \left([\underbrace{W + BH}_{N_1} - \tilde{c}]R \right)^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}}$$

The optimal consumption and continuation utility are characterized by:

$$\tilde{c}_1(N_1) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\epsilon}} v^{\frac{1-\epsilon}{\epsilon}}}{(1-\beta)^{\frac{1}{\epsilon}} v^{\frac{1-\epsilon}{\epsilon}} + \beta_1^{\frac{1}{\epsilon}} R^{\frac{1-\epsilon}{\epsilon}}} \right]}_{\omega_1} N_1$$

and

$$V_1(W,H) = V_1(N_1) = \underbrace{\left\{ (1-\beta)(v\omega_1)^{1-\epsilon} + \beta_1\left[(1-\omega_1)R\right]^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}}}_{\psi_1} N_1$$

which is the same as under the separable problem, and establishes that the Tobin's-q in $V^H = \eta_1 H = BH$.

$s = 2$: The problem is:

$$\begin{aligned} V &= \max_{\tilde{c},I} \left\{ (1-\beta)(v\tilde{c})^{1-\epsilon} + \beta\left[p(\psi_1 N')^{1-\gamma} + (1-p)\,b\,(N')^{1-\gamma}\right]^{\frac{1-\epsilon}{1-\gamma}} \right\}^{\frac{1}{1-\epsilon}}, \\ &= \max_{\tilde{c},I} \left\{ (1-\beta)(v\tilde{c})^{1-\epsilon} + (N')^{1-\epsilon} \underbrace{\beta\left[p\psi_1^{1-\gamma} + (1-p)\,b\,\right]^{\frac{1-\epsilon}{1-\gamma}}}_{\beta_2} \right\}^{\frac{1}{1-\epsilon}}, \end{aligned}$$

subject to:

$$N' = \left\{ [W + BH - I - \tilde{c}] + R^{-1}\eta_1\left[AI^{\alpha}H^{1-\alpha} + (1-\delta)H\right] \right\} R$$

Solving for optimal investment reveals that

$$\begin{aligned} I_2 &= \underbrace{\left(R^{-1}\eta_1 \alpha A\right)^{\frac{1}{1-\alpha}}}_{\kappa_2} H \\ V_2^H &= \underbrace{\left[B - \kappa_2 + R^{-1}\eta_1\left(A\kappa_2^{\alpha} + (1-\delta)\right)\right]}_{\eta_2} H \end{aligned}$$

$$N' = \left[\underbrace{W + V_2^H}_{N_2} - \tilde{c} \right] R$$

i.e. the optimal investment is independent of mortality risk. The optimal expenditures choices solves:

$$V = \max_{\tilde{c}} \left\{ (1-\beta)(v\tilde{c})^{1-\epsilon} + \beta_2 (N')^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}},$$

The solution to the FOC is:

$$\tilde{c}_2(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\epsilon}} v^{\frac{1-\epsilon}{\epsilon}}}{(1-\beta)^{\frac{1}{\epsilon}} v^{\frac{1-\epsilon}{\epsilon}} + \beta_2^{\frac{1}{\epsilon}} R^{\frac{1-\epsilon}{\epsilon}}} \right]}_{\omega_2} N$$

Substituting back into the objective function implies that:

$$V_2(N) = \underbrace{\left\{ (1-\beta)(v\omega_2)^{1-\epsilon} + \beta_2 \left[(1-\omega_2) R \right]^{1-\epsilon} \right\}^{\frac{1}{1-\epsilon}}}_{\psi_2} N$$

$s \geq 3$: It is readily verifiable that the solutions converge to the same form for the other periods.

Health insurance The risk-averse agent purchases full insurance against health shocks ϵ_{s-1}^h when sold at actuarially-fair prices. The insurance premium is the expected loss in human capital value induced by morbidity:

$$\begin{aligned} \pi_s &= \left[1 - \exp(-\lambda^h) \right] \nabla V_s^H, \quad \text{where} \\ \nabla V_s^H &= V_s^H(\epsilon_s^h = 1) - V_s^H(\epsilon_s^h = 0) \\ &= R^{-1} \eta_{s-1} \phi H \end{aligned}$$

subtracting the insurance premium π_s from the shadow value V_s^H and regrouping terms establishes that the Tobin's-q is:

$$\eta_s = B - \kappa_s + \eta_{s-1} \left\{ R^{-1} \left[A\kappa_s^\alpha + (1-\delta) \right] - R^{-1} [1 - \exp(-\lambda^h)] \phi \right\}$$

which completes the proof that the separable and joint allocations coincide. ■

Appendix C. Additional theoretical results

C.1. Variational and MRS measures

We can substitute $v_s^e(\Delta, \lambda^m, N)$ in (22a) and $v_s^c(\Delta, \lambda^m, N)$ in (22b), take derivatives with respect to change Δ and re-arrange to obtain the marginal rate of substitution between longevity and net total wealth as:

$$MRS(\lambda^m, N) \equiv \frac{\frac{-\partial V_s(\lambda^m, N)}{\partial \lambda^m}}{\frac{\partial V_s(\lambda^m, N)}{\partial N}} = \frac{\partial v_s^e(\Delta, \lambda^m, N)}{\partial \Delta}\Big|_{\Delta=0} = \frac{-\partial v_s^c(\Delta, \lambda^m, N)}{\partial \Delta}\Big|_{\Delta=0}. \tag{C.4}$$

Hence, the shadow relative price of longevity, i.e. the required change in net total wealth to leave an agent indifferent to a marginal change in longevity is the slope of the tangent of the EV (and negative of tangent slope for the CV) evaluated at base risk $\Delta = 0$. The MRS can be calculated in closed-form as follows:

Corollary 3 (*MRS*). *The marginal rate of substitution between longevity and net total wealth solving (C.4) is given by:*

$$MRS_s(\lambda^m, N) = \frac{-\psi_s'(\lambda^m)}{\psi_s(\lambda^m)} N \tag{C.5}$$

where marginal utility is given in (19c), and where its derivative solves the following recursion:

$$\begin{aligned} \beta_s'(\lambda^m) =& \beta \left(\frac{1-\epsilon}{1-\gamma} \right) \left\{ p(\lambda^m) \psi_{s-1}(\lambda^m)^{1-\gamma} + \left[1 - p(\lambda^m) \right] b \right\}^{\frac{1-\epsilon}{1-\gamma} - 1} \\ &\times \left\{ p_s'(\lambda^m) \left[\psi_{s-1}(\lambda^m) - b \right] + \psi_{s-1}'(\lambda^m) \left[(1-\gamma) \psi_{s-1}^{-\gamma}(\lambda^m) p(\lambda_m) \right] \right\} \end{aligned} \tag{C.6a}$$

with $\psi_{s-1}'(\lambda^m) = 0$ in the case of one-shot changes,

$$\omega_s'(\lambda^m) = -\frac{(1-\beta)^{\frac{1}{\epsilon}} R^{\frac{1}{\epsilon}+1} v_s^{\frac{1}{\epsilon}+1} \beta_s'(\lambda^m) \beta_s(\lambda^m)^{\frac{1}{\epsilon}-1}}{\epsilon \left(R^{\frac{1}{\epsilon}} v_s \beta_s(\lambda^m)^{\frac{1}{\epsilon}} + R(1-\beta)^{\frac{1}{\epsilon}} v_s^{\frac{1}{\epsilon}} \right)^2} \tag{C.6b}$$

Table D.1
Identification moments.

Parameters	Identified from
	(a) Net income
	Net $\text{Income}_t^d = \beta_0 + \beta_1 H_t^d + \epsilon_t$
$m = i - y$	$-\beta_0$
and B	β_1
	(b) Health process
α, δ, ϕ (free)	$\min_{\{\alpha,\delta,\phi\}} M_h' M_h$, where
and A (fixed)	$M_{ht} = (H_t^d - H_t)/H_t^d$
	(c) Financial and labor variables
$\gamma, b, \sigma, \epsilon$ (free)	$\min_{\{\gamma,b,\sigma,\epsilon\}} M_f(:)'[\Omega^{-1} \otimes \mathcal{I}]M_f(:)$, where
and θ, ρ (fixed)	$M_{ft} = [(W_t^d - W_t)/W_t^d, (Y_t^d - Y_t)/Y_t^d, (n_t^d - n_t)/n_t^d]$

Notes: (a) Net income is labor income minus health expenditures, from PSID. (c) Feasible GLS procedure uses $\Omega = \text{Cov}(M_f)$ from first stage for weighting matrix. Time period $t \in [30, 85]$.

$$\begin{aligned}\psi_s'(\lambda^m) = &\left(\frac{1}{1-\epsilon}\right)\left\{(1-\beta)\left[v(w)\omega_s(\lambda^m)\right]^{1-\epsilon} + \beta_s(\lambda^m)\left[\left(1-\omega_s(\lambda^m)\right)R\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}-1} \\ &\times \left\{(1-\beta)v(w)^{1-\epsilon}(1-\epsilon)\omega_s^{-\epsilon}(\lambda^m)\omega_s'(\lambda^m)\right. \\ &\left.+R^{1-\epsilon}\left[\beta_s'(\lambda^m)\left(1-\omega_s(\lambda^m)\right)^{1-\epsilon} - \beta_s(\lambda^m)(1-\epsilon)\left(1-\omega_s(\lambda^m)\right)^{-\epsilon}\omega_s'(\lambda^m)\right]\right\}\end{aligned} \tag{C.6c}$$

with initial values $(\beta_1', \omega_1', \psi_1') = (0, 0, 0)$.

The proof follows directly by using the value function (18) and taking derivatives of (19) to obtain (C.6) and is therefore omitted.

Appendix D. Calibration strategy

The strategy to calculate the optimal dynamics involves five steps.

1. Given a set of parameters, the optimal recursions are solved for $\{\kappa_t, \eta_t\}_{t=0}^T$ (Lemma 2) as well as for $\{\beta_t, \omega_t, \psi_t\}_{t=0}^T$ (Theorem 1).
2. Relying on separability between financial- and health-related decisions, as well as complete markets, the value of net wages $\{V_t^w\}_{t=0}^T$ is exogenous and calculated from (15), while the human wealth $\{V_t^H\}_{t=0}^T$ is endogenous and calculated from Lemma 2.
3. For given initial financial wealth and health (W_0, H_0) set equal to their empirical counterparts, we solve initial total wealth:

   $$N_0 = W_0 + V_0^w + V_0^H(H_0).$$

   The predicted optimal paths for health and total wealth $\{H_t, N_t\}_{t=0}^T$ are solved forward for each t as:

   $$\text{E}[H'(H)] = H\left\{A\kappa_t^\alpha + (1-\delta) - [1-\exp(-\lambda^h)]\phi\right\}, \tag{D.1}$$
   $$N' = N(1-\omega_t)R. \tag{D.2}$$

4. The associated optimal paths for total expenses, and continuation utility $\{\tilde{c}_t, V_t\}$ are derived from Theorem 1. The corresponding expressions for the leisure share and salaried income can be calculated from the definitions of total expenses $\tilde{c}_t$ in (8) and $\mu(w_t)$ in (10a) as:

   $$\ell_t = \frac{\tilde{c}_t}{w_t}\left(\frac{\mu(w_t)-1}{\mu(w_t)}\right),$$
   $$Y_t = w_t(1-\ell_t).$$

5. Given net total N_t, and the human capital components V_t^w, V_t^H, the financial wealth W_t along the optimal path can be recovered as:

   $$W_t = N_t - V_t^w - V_t^H.$$

The resulting life cycle sequences for $\{H_t, W_t, Y_t, n_t\}_{t=0}^T$ are contrasted with corresponding observed counterparts $\{H_t^d, W_t^d, Y_t^d, n_t^d\}_{t=0}^T$. The weighted sum of square distances is minimized over the free parameter subset using the following identification grid:

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