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RETIREMENT SCHEMES : INTER. AND INTRA. GENERATIONAL TRANSFERS

Jijiie Anca-Stefania

UNIL | Université de Lausanne

FACULTÉ DES HAUTES ÉTUDES COMMERCIALES

DÉPARTEMENT DE SCIENCES ACTUARIELLES

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THÈSE DE DOCTORAT

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pour l'obtention du grade de
Docteure ès Sciences Actuarielles

par

Anca-Stefania JIJIIE

Directrice de thèse
Prof. Séverine Arnold

Jury

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HEC Lausanne
Le Décanat
Bâtiment Internef
CH-1015 Lausanne

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Jean-Philippe Bonardi
Doyen

HEC Lausanne
Le Décanat
Tél. +41 21 692 33 40 | Fax +41 21 692 33 05
www.hec.unil.ch | hecdoyen@unil.ch

Members of the Thesis Committee

Prof. Séverine Arnold: Thesis supervisor, Full Professor at the Department of Actuarial Science, Faculty of Business and Economics (HEC Lausanne), University of Lausanne, Switzerland.

Prof. François Dufresne: Internal Expert, Full Professor at the Department of Actuarial Science, Faculty of Business and Economics (HEC Lausanne), University of Lausanne, Switzerland.

Prof. Pierre Devolder: External Expert, Full Professor at the Institute of Statistics, Biostatistics and Actuarial Sciences, Université Catholique de Louvain, Belgium.

Prof. Carmen Boado Penas: External Expert, Senior Lecturer at the Institute for Financial and Actuarial Mathematics, University of Liverpool, United Kingdom

Prof. Felicitas Morhart: Jury President, Full Professor at the Department of Marketing, Faculty of Business and Economics (HEC Lausanne), University of Lausanne, Switzerland.

University of Lausanne
Faculty of Business and Economics

PhD in Actuarial Science

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All revisions that I or committee members
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have been addressed to my entire satisfaction.

Signature: Arnold Date: 18.09.2020

Prof. Séverine ARNOLD
Thesis supervisor

University of Lausanne
Faculty of Business and Economics

PhD in Actuarial Science

I hereby certify that I have examined the doctoral thesis of

Anca-Stefania JIJIIE

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Signature: F. Dufresne ________________________ Date: 17.09.2020 ________________

Prof. François DUFRESNE
Internal member of the doctoral committee

University of Lausanne
Faculty of Business and Economics

PhD in Actuarial Science

I hereby certify that I have examined the doctoral thesis of

Anca-Stefania JIJIIE

and have found it to meet the requirements for a doctoral thesis.
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made during the doctoral colloquium
have been addressed to my entire satisfaction.

Mc Carmen Boad

Signature: Date: ___17-09-20_______________

Dr. Carmen BOADO PENAS
External member of the doctoral committee

University of Lausanne
Faculty of Business and Economics

PhD in Actuarial Science

I hereby certify that I have examined the doctoral thesis of

Anca-Stefania JIJIIE

and have found it to meet the requirements for a doctoral thesis.
All revisions that I or committee members
made during the doctoral colloquium
have been addressed to my entire satisfaction

Signature: Date: September 16, 2020

Prof. Pierre DEVOLDER
External member of the doctoral committee

Declaration of Authorship

I, Anca Jijiie, declare that this thesis titled, Retirement schemes: Inter- and Intra-generational transfers and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
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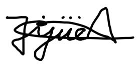
Signed: 

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Summary

This thesis focuses on the inter-generational and intra-generational transfers taking place within the public retirement systems, namely within first and second pension pillars. The current demographic and economic situation, along with the reforms put in place for many of the schemes around the world, have introduced unintended transfers, both inter- and intra-generational, which could hinder the reach of the objectives of the public pension systems. In this respect, through the unintended intra-generational transfers that take place from the poor to the rich, and not vice-versa, the solidarity is eroded. A similar effect can be observed when unintended inter-generational transfers happen, especially between the active and retired members. This thesis aims at studying these types of transfers. Hence, firstly, we analyse the inter-generational transfers that take place within the Swiss second pension pillar. For this, we build a framework that can be used by the Swiss pension funds to identify and assess the extend of their unwanted transfers between the active and retired members. Our numerical example shows that the transfers can favour both groups considered and are certainly not unilateral. With respect to the intra-generational transfers, we look at either Defined Benefit (DB) or Notional Defined Contribution (NDC) schemes from the perspective of first pension pillars, addressing those transfers that arise due to the differences in mortality between socio-economic classes. Thus we start by proposing adjusting the system parameters, such as the accrual rate or the notional rate of return, at a given retirement age according to the socio-economic class. The adjustment of the parameters would allow for more fairness within the system, lowering these types of unintended transfers and being a first step towards the reach of adequacy of the pension benefits. An alternate solution to this issue is defining the retirement age for each socio-economic class. Calculating the optimal retirement age for each class by implementing an utilitarian framework is shown to not be suitable from the point of view of policy makers, as the results are highly dependant on the choice of parameters. Consequently we propose a complementary method that aims at defining the retirement age by considering the actuarially fair framework.

Cette thèse se concentre sur les transferts inter- et intra-générationnels ayant lieu dans les systèmes de retraite publics, notamment dans les premier et deuxième piliers. La situation démographique et économique actuelle, de même que les réformes mises en place dans des nombreux plans de retraite à travers le monde, ont introduit des transferts non voulus inter-générationnels, mais aussi intra-générationnels, qui peuvent empêcher les systèmes de retraite publics d'atteindre leurs objectifs. Par conséquent, par le biais des transferts intra-générationnels non voulus des pauvres vers les riches, et non vice versa, la solidarité s'érode. Un effet similaire peut être observé quand des transferts inter-générationnels non voulus ont lieu, surtout entre les actifs et les retraités. Cette thèse vise à étudier ces types de transferts. Premièrement, nous analysons les transferts inter-générationnels ayant lieu dans le deuxième pilier en Suisse. A cette fin, nous construisons un modèle qui permet aux caisses des pensions suisses d'identifier et d'évaluer l'ampleur de leurs transferts non voulus entres les actifs et les retraités. Notre exemple numérique montre que les transferts peuvent favoriser les membres des deux groupes et qu'ils ne sont certainement pas unilatéraux. En ce qui concerne les transferts intragénérationnels dans le cadre du premier pilier, nous considérons tant les plans en primauté de prestations (DB), que les plans en primauté de cotisations (NDC) et adresson les transferts émanant des différences en terme de mortalité entre les catégories socio-économiques. De ce point de vue, nous commençons par proposer l'ajustement des paramètres du système, comme le taux d'accumulation de la rente ou le taux notionnel, en fonction de la catégorie socio-économique et ce, pour un âge de la retraite fixe. L'ajustement de ces paramètres permetterait au système d'être plus équitable, en baissant ce type de transterts et en faisant un premier pas vers des rentes plus adéquates. Une solution alternative à ce problème est de définir un âge de la retraite différent pour chaque catégorie socio-économique. Le calcul de l'âge de la retraite optimal pour chaque classe par le biais des fonctions d'utilité n'est pas adéquat du point de vue des décideurs politiques, car les résultats dépendent fortement du choix des paramètres. Par conséquant, nous proposons une méthode complementaire qui vise à définir l'âge de la retraite pour chaque classe en considerant un cadre actuariellement juste.

Chapter 1

Introduction

Modern retirement schemes, that extended past restricted circles of society, such as military personnel, have their origin in the 1880s, with the implementation of the "The Old Age and Disability Bill" by the German Chancellor Otto von Bismark[1]. The bill specified the provision of benefits for those who could no longer work due to either old age or disability. The system covered workers on a mandatory basis and it was financed by both employers and employees, as well as the state. Consequently Bismarkian systems generally aim at insuring the working population, with the contributions paid being closely linked to the benefits received.

The Beveridge Report, dating from 1942, which proposed an universal system in the United Kingdom represents another important step in the development of the retirement schemes[2]. The aim of the system proposed by William Henry Beveridge was to ensure a minimum subsistence level for the entire population. Hence the Beveridgean systems are universal and provide uniform benefits, independent of previous income, to the population.

In the 1990s, the World Bank, in their report "Averting the old age crisis: Policies to protect the old promote growth"[3], state that one pension pillar is no longer sufficient, given demographic and economical development, and propose a transition to a multi-pillar system. The proposed system would have two mandatory pillars and one voluntary. The

[1] See Filgueira and Manzi (2017), for instance.
[2] See Hills et al. (1994), for instance.
[3] See World Bank (1994).

first mandatory pillar would be financed through taxes on a Pay-As-You-Go basis and would guarantee a minimum income for all individuals. The second pillar would be fully funded and covering the workers, thus being designed to link contributions to benefits. Hence the two mandatory systems would respect the ideas behind the Bismarkian and Beveridgean systems and would have a strong foundation in solidarity. The voluntary third pillar would essentially be private insurance for those who could afford further savings for retirement. The multi-pillar framework has been adopted by many countries around the world, in one form or another. We focus in this thesis on the first and second pillar only.

Among the different issues pension systems are faced with nowadays, three are of note for the purpose of this thesis: 1) the ageing population, marked by increased life expectancy and decreased fertility rates, 2) lower benefits than those required to prevent poverty among the elderly and 3) the precarious financial sustainability of the schemes. These challenges have also been outlined by Filgueira and Manzi (2017) and OECD (2018). In order to address these issues, many countries around the world have implemented measures to reform their systems, which include, among others, increasing or adding flexibility to the retirement age, raising contribution rates or offering more incentives to workers to continue their active life[4]. However, the current demographic and economic situation, along with the reforms of the schemes have introduced unintended transfers, both inter- and intra-generational, which could hinder the reach of the objectives of the public pension systems, such as the redistribution of income from the more wealthy to those with lower incomes.

Solidarity is a key element of the first and second pension pillars. It can thus be defined as the total intended redistributions that take place between the existent groups within the system, with the purpose of helping each other. Solidarity can then appear on multiple levels, since, for instance, we can have solidarity between generations, between genders or between income classes. Van Vugt (2000) defines solidarity as the equilibrium between rights and obligations for an individual, as well as for the collectivity in general and recognises the

[4]See OECD (2019) or OECD (2015).

central place it has within the social security systems. Therefore, through the unintended intra-generational transfers that take place from the poor to the rich, and not vice-versa, the solidarity is eroded. A similar effect on the purpose of these pillars can be observed when unintended inter-generational transfers happen, especially between the active and retired members. This thesis aims at studying these types of unintended transfers, with the second chapter focusing on inter-generational transfers and the following two chapters addressing the transfers within generations.

In the next chapter of this thesis, we start by analysing the inter-generational transfers that take place within the Swiss second pension pillar. Though not the case for all countries around the world, the Swiss second pillar is mandatory and, so, it is part of the social security system of the countries. Thus analysing the inter-generational transfers for this pillar is pertinent to this thesis. The occupational pension system in Switzerland is mostly fully funded and run by pension funds that generally provide Defined Contribution schemes to employers. The law regulates a minimum standard, with the funds having the choice of being more generous in the design of their scheme. Hence we build a framework that can be used by the pension funds providing the benefits for this pillar to identify and assess the extend of their unwanted transfers between the active and retired members. Since each pension fund can tailor their schemes to a certain extent, we provide a numerical illustration of the framework using cumulative data regarding the Swiss pension funds available from the Swiss National Office of Statistics. Even though our results are mainly just an example of how our framework is supposed to be applied, they show that the transfers can favour both groups considered and are certainly not unilateral. Therefore, we reach our intended purpose, which is to widen the perspective on this topic in Switzerland and to broaden the discussions within the system. We must remark that in this case we cannot talk about a rich or poor category, about an advantaged or a disadvantaged group. Hence it is not the aim of the paper to conclude whether or not the disadvantaged group is gaining or losing, but to shed light on the fact that both active and retired can benefit from unwanted redistributions and that the magnitude of these transfers can differ from one pension fund to another and can even benefit more the active members.

With respect to the intra-generational transfers, our perspective is much broader. Indeed, we do not focus on a specific country, but we look at either Defined Benefit (DB) or Notional Defined Contribution (NDC) schemes from the perspective of a first pension pillar. In that sense, we address the intra-generational transfers that arise due to the heterogeneous increase in life expectancy within those schemes. More specifically, we focus on the differences in mortality rates by socio-economic class, as the individuals in higher classes live longer than those in lower ones. In Chapter 3, we show that, given our data from the French Office of Statistics and our pre-defined DB and NDC schemes, the transfers go in the reverse order than intended. In other words, by not considering the socio-economic mortality differences in the calculation of the pension benefits, the schemes disadvantage those in lower socio-economic classes, defined here in terms of the level of education. Conversely, those with a high education appear to gain with respect to the actuarially fair framework[5]. To compensate for this situation, we propose in this chapter to adjust the system parameters, such as the accrual rate or the notional rate of return, at a given retirement age. Besides providing a numerical example based on the data available to us, we also develop straightforward formulas for defining these parameters according to the socio-economic class and to the amount of data on socio-economic mortality rates available. The adjustment of the parameters would thus allow for more fairness within the system, lowering these types of unintended transfers and being a first step towards the reach of adequacy of the pension benefits[6]. In fact, our proposal includes the possibility to change the parameters in order to both increase the fairness of the system and achieve the adequacy level.

Another way of lowering the transfers taking place from the lower classes to the higher ones would be to adjust the retirement age for each class, but not the other parameters of the

[5]By definition, under an actuarially fair scheme, the present value at the moment of entry into the system of all contributions paid should equal the present value at the same moment of all future benefits received.

[6]Pension adequacy is achieved when the target minimum pension fixed within the system is reached.

system. We offer a numerical illustration in this respect in Chapter 4, firstly calculating the optimal retirement age for each class by implementing an utilitarian framework. We observe that, given different sets of risk aversion coefficients and individual time preference factors, the optimal ages are significantly different, but that lower socio-economic classes would, in certain scenarios, retire earlier. Although the utilitarian method is certainly good for understanding individual preferences, it is not appropriate for our purposes. We are interested in a method that would allow the policy makers to adjust the retirement ages for each socio-economic class. As such, the utility functions, which are focused on the individual risk aversion coefficient and time preference factor, are not easily implementable, since the system cannot be personalised to this extend. Although not the focus of our study, sustainability[7] is investigated in order to better assess the impact of such a method being implemented. Given our scenarios, we observe that the financial sustainability of the schemes is not necessarily guaranteed, when the retirement age is set by maximising the lifetime utility of the individuals. Therefore, we propose a complementary method that corresponds better to our goal and that aims at defining the retirement age by considering the actuarially fair framework. In other words, we propose a method based on two accounts. We compare the account holding the accumulated value at age ω of all benefits paid by either the DB or the NDC scheme with the account where we accumulate, at the same age, the benefits paid under the theoretically fair scheme. Hence the optimal retirement age for each class is set such that the values in the two accounts are close. As a result, individuals from lower socio-economic classes should benefit from a lower retirement age than individuals from higher classes, as the former have a lower life expectancy than the latter. Additionally, in this method the sustainability is improved, being implicit.

[7] A system is financially sustainable when funds are enough to pay the due benefits, on a long-term horizon. The following studies discuss the sustainability of the pension systems: Lindbeck (2006), Diamond (2004), Lindbeck and Persson (2003), Valdes-Prieto (2000), Holzmann et al. (2012b), Holzmann et al. (2012a), Holzmann et al. (2019).

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Chapter 2

Generational transfers within the Occupational Pension System in Switzerland

This chapter is based on the following paper: Séverine Arnold and Anca Jijiie. Generational transfers within the Occupational Pension System in Switzerland. European Actuarial Journal, 9(1): 109–138, 2019. https://doi.org/10.1007/s13385-018-0188-0

2.1 Introduction

Nowadays, many countries around the world are facing challenges regarding their social security systems, due to the increasing longevity and the current economical context. Ginneken (2003) defines social security systems as "benefits that society provides to individuals and households – through public and collective measures – to guarantee them a minimum standard of living and to protect them against low or declining living standards arising out of a number of basic risks and needs" and hence points out the important role solidarity plays, since these schemes are both public and collective. Van Praag and Konijn (1983) also remark that the difference between private insurance and a social security system lies in the presence of solidarity. Solidarity is therefore a key element of any social security system. It can thus be defined as the total intended redistributions that take place between the existent groups within the system, with the purpose of helping each other. Solidarity can then appear on multiple levels, since, for instance, we can have solidarity between generations, between genders or between income classes. For example, in some pension schemes, the contributions are defined as percentages of the income, while the retirement pensions are a flat amount. Similarly, the pensions can be calculated regardless of the marital status of the insured, even though only the married ones would be entitled to a survival pension. The impact and importance of solidarity has been the subject of many studies. Leitner and Lessenich (2003) perform a welfare state analysis with respect to a logic of reciprocity and a logic of solidarity. Stevens et al. (2002) analyse solidarity within the occupational pension systems in Europe and point out that without the presence of solidarity, these systems go from being second pillar to being third pillar in the three pillar framework proposed by the World Bank[1] and adopted by many countries around the world. One last example is the one of Van Vugt (2000), who defines solidarity as the equilibrium between rights and obligations for an individual, as well as for the collectivity in general and who recognises the central place it has within the social security systems, noting that in some countries the solidarity between generations is being eroded. Nevertheless, unintended redistributions also exist and Jean-Claude Ménard (2013) observes that "intergenerational fairness is violated when intergenerational transfers become unintended".

Intergenerational transfers, be they in a pay-as-you-go or a fully funded system, have been discussed in literature before, from different points of view. One common approach is the utilitarian one. In these kind of models, some studies are focused on the transfers taking place in the presence of shocks, demographic or financial (see, for example, Bucciol and Beetsma (2010), Beetsma and Bovenberg (2009) or Bohn (2009)). Other papers study

[1] World Bank (1994) proposes a three pillar framework for pension systems. The first pillar is mandatory and is meant to insure a minimum income level for all the retired people. The second pillar, also mandatory, is usually occupational and is meant to link contributions to benefits. Lastly, the third pillar is voluntary, as to give further flexibility for savings to individuals with higher income.

the transfers from the point of view of the asset allocation and portfolio risk management (see Cui et al. (2011), Gollier (2008) or Beetsma et al. (2012)).

Our paper is linked to the literature regarding the value transfers, where the utilitarian framework is not considered. From this perspective, Nelissen (1995) finds lifetime redistribution effects from old cohorts to younger ones within the first pension pillar in The Netherlands (which is financed on a pay-as-you-go basis), using a microsimulation model, but notes that the yearly redistributions appear to be more significant. Hoevenaars and Ponds (2008) create a stochastic generational accounting framework to measure transfers that ensue from a pension plan reform in a funded system and find that a transfer will inevitably take place with any policy change. Zuber et al. (2007) consider the ascendant and descendant transfers in France over the life cycle of generations, noting that there is a tendency to forget the ascending ones. The transfers from the younger generations towards the older ones are calculated in function of the difference between the contributions paid to the pay-as-you-go system and the benefits received. On the other hand, the descendant transfers, namely from the old to the young are taking into account the education received and compare the contributions paid to finance the development of the educational system and the expenses incurred while studying. Both Bonenkamp (2009) and Borsch-Supan and Reil-Held (2001) calculate the redistributions between generations, by defining them as the difference between the present values of benefits received and contributions paid. If Borsch-Supan and Reil-Held (2001) focus on the German first pillar (which is a pay-as-you-go system) and attempt to break down the system into a transfer component and an insurance part, concluding that there is a significant intergenerational redistribution, Bonenkamp (2009) studies them for the Dutch fully funded occupation pension system and finds that the uniform contribution rate used leads to redistributions not only between generations, but also within a specific generation, namely between socio-economic groups. Similarly, Bommier et al. (2010) also calculate the net present value of the difference between benefits and contributions for generations from 1850 to 2090 in the United States, for three pay-as-you-go systems, namely social security, Medicare and education, and find transfers between generations not only in all three individual systems, but also when the values are combined. Specifically, they note that the transfers are significant towards generations born before 1930. Ponds (2003) uses a value-based generational accounting approach to study the transfers between generations and notice that, for the funded Dutch pension funds, hidden transfers have taken place from the pensioners and future members to the active members. Lastly, Eling (2013) also acknowledges the presence of unintended redistributions between young and old within the Swiss occupational pension system, which is mostly fully funded, due to the use of a non-actuarial conversion rate. The transfers are projected for the period 2010-2060, by estimating the number of new pensioners, the average capital and the actuarial conversion rates.

In the scope of this paper, the term "transfers" refers to the unintended redistributions only. These transfers are not all easily identifiable or perceivable. These kind of redistributions can ensue from a lack of reform and flexibility of the system in the face of the sometimes unexpected evolution of certain parameters. In other cases, the measures adopted have consequences that were not foreseen or considered and thus, they leave the system vulnerable to transfers. Hence, such unintended redistributions affect the fairness of the system and can lead to animosities between groups. This is exactly the situation emerging in Switzerland.

The Swiss pension system is composed of three pillars. The first and second pillars are mandatory, representing the social security system, while the third pillar is both individually chosen and optional. The first pillar is a state-run pay-as-you go system meant to insure the subsistence level during retirement for all the residents in Switzerland. The second pillar, on the other hand, is mostly fully funded and concerns only the employees. It is meant to maintain the standard of living prior to retirement. As the second pillar is part of the social security system, together with the first pillar, solidarity is still present, despite the fact that the occupational pension system is mostly fully funded and mostly based on defined contribution plans.

A lack of flexibility and reform in the face of the changing demographic and financial situation is leaving more possibilities for unintended redistributions to take place within the Swiss second pillar, putting even more strain on the already problematic situation. As a result, discussions around this topic have recently intensified. As transfers are not all easily identifiable or perceivable, these discussion in Switzerland are mainly focusing on the idea that these redistributions are only favouring the pensioners, leading to increasing tensions between the active and retired members of the system. However, greater fairness can only be achieved by taking into account all sources of transfers, in particular when a reform of the system is considered.

In order to address this situation, this paper proposes a general and tractable methodology aimed at helping pension institutions to identify the main sources of transfer and quantify the corresponding amounts. Our framework can be used as a starting point to broaden the discussions around this topic by shedding light on different sources of transfers, including those favouring the active members, otherwise missing from the picture presented to the public.

The remainder of this paper is structured as follows: in Section 2.2 we give a detailed description of the Swiss occupational pension system. We then explain our method in Section 2.3, while in Section 2.4 we identify the depended and independent variables needed to compute the transfers for each of the sources identified, summarised in Figure 2.1, and provide a mathematical solution for quantifying these transfers in each case. In Section 2.5, we proceed to illustrating how our methodology can be used, by applying it to the Swiss case, basing our results on the cumulative data from pensions funds provided by the Swiss National Office of Statistics. Lastly, we summarise our results and conclusions in Section 2.6.

2.2 The Swiss Occupational Pension System

As mentioned in our introduction, the occupational pension system is meant to maintain the standard of living prior to retirement for employees. The second pillar is mandatory and hence is part of the social security system in Switzerland. The system is mostly fully funded and run by pension funds, to which employers are affiliated. Consequently, it is not the choice of the employees to which pension fund their are affiliated. The pension funds cover the benefits for retirement, as well as the risks of death and invalidity and have the liberty to define the pension plans offered, as long as they meet the minimum requirements set by law, also referred to as the minimum LPP ("Loi fédérale sur la Prévoyance Professionnelle vieillesse, survivants et invalidité") standards[2]. The salary entry level is set to $18 \cdot M$, while the maximum salary insured under the framework defined by law is $72 \cdot M$. The amount M corresponds to the minimum monthly pension from the first pillar and is equal to 1'175 CHF for the period 2015 to 2018. The retirement age is 64 for women and 65 for men. Though both defined benefit (DB) and defined contribution (DC) pension plans still exist within the Swiss system, more than 90% of pension institutions nowadays offer only DC plans (see Office Fédéral de la Statistique (OFS) (2005-2015)) and the number of pension funds offering DB schemes is slowly declining. In 2015, out of the 1782 pension funds existent, only 77 still proposed DB plans to their members, while in 2005, 2415 pension funds were offering DC plans and only 355 had DB schemes in place. For that reason, we assume here that all pension funds offer a DC scheme. Under this hypothesis, the retirement credits, defined as percentages of the insured member's salary, are accumulated in an individual savings account until retirement. The contributions perceived under the system are paid by both active members and employers, usually in equal proportions. Since the law sets the minimum framework, the retirement capital of the active members at any given age, accumulated in the saving account, is composed by a mandatory part and an extra-mandatory (or supplementary) part. The minimum interest rate that should be offered on the mandatory part of the capital is set by the Federal Counsel each year and the pension fund is allowed to be more generous and to choose the interest rate offered on the supplementary capital

[2]The rules governing the system are found in three main laws: the LPP ("Loi fédérale sur la Prévoyance Professionnelle vieillesse, survivants et invalidité"), the LFLP ("Loi fédérale sur le libre passage") and the OPP2 ("Ordonance sur la Prévoyance Professionnelle 2").

according to the performance of their investments and their financial situation.

Upon entering the retirement phase, the amount accumulated into the individual accounts is transformed into the retirement pension, $P_{th,t}$, where the subscript th indicates the pension amount is theoretical. This is achieved by applying the conversion rate $CR_{th,t}$ on the accumulated capital $CA_{x_r,t}$, as given by Equation 2.2, where x_r represents the retirement age and t the year when age x_r is reached. Hence, a higher rate will lead to a higher pension. Once awarded, the pension cannot be lowered except for rare and extreme cases and moreover, there is no obligation to index pensions under the Swiss law. The actuarial conversion rate, also called here theoretical, is equivalent to the inverse of the present value of all future benefits due to an insured person, as per Equation 2.1, in order for the capital to be sufficient to finance the necessary mathematical reserve of the individual's departure into retirement (see Equation 2.3). We define the theoretical conversion rate as per Equation 2.1, by setting the retirement pension to 1 CHF. Since the pension funds offer benefits for widow/er and for orphans in case of death after retirement, we also take into account the surviving spouse pension SPP and the orphan pension OP, which are usually defined as percentages of the retirement pension. For this study, we set the widow/er pension to 60% of the retirement pension (so $SPP = 0.6$), while the orphan pension is 20% of the same amount (so $OP = 0.2$), therefore corresponding to the minimum benefits set by the law. The accumulated capital is taken as given. Moreover, Equation 2.3 defines the theoretical mathematical reserve at time t, namely $MR_{th,t}$, as the present value of all future benefits due to the retired person.

$$CR_{th,t} = 1/\left(\ddot{a}^{(m)}_{x_r,t} + SPP \cdot \ddot{a}^{rw(m)}_{x_r,t} + OP \cdot k_{x_r,t} \cdot \ddot{a}^{(m)}_{\overline{z^*-z_{x_r,t}|}}\right) \tag{2.1}$$

$$P_{th,t} = CA_{x_r,t} \cdot CR_{th,t} \tag{2.2}$$

$$MR_{th,t} = P_{th,t} \cdot \left(\ddot{a}^{(m)}_{x_r,t} + SPP \cdot \ddot{a}^{rw(m)}_{x_r,t} + OP \cdot k_{x_r,t} \cdot \ddot{a}^{(m)}_{\overline{z^*-z_{x_r,t}|}}\right) \tag{2.3}$$

In Equation 2.1, $\ddot{a}^{(m)}_{x_r,t}$ is the present value of a monthly ($m = 12$) whole life annuity-due awarded at retirement age x_r, reached at time t, defined in Equation 2.4, with i the interest rate considered, while $\ddot{a}^{rw(m)}_{x_r,t}$ represents the present value of a reversionary monthly pension for a widow/er of a retired person, as given by Equation 2.5. Lastly, since the death probabilities for younger ages are rather small and thus taking them into account would not impact our results, we considered a certain child annuity-due, $\ddot{a}^{(m)}_{\overline{z^*-z_{x_r,t}}|}$, as expressed in Equation 2.7, which is payable until the child reaches the maximum age z^*, usually set to 25. We rely on the average age of the child for a person of age x_r at time t, given by $z_{x_r,t}$, while $k_{x_r,t}$ is the average number of children a person of age x_r has, again at time t. Moreover, in order to calculate the value of the monthly annuities, we follow Bowers et al. (1997) and apply a correction factor of $\frac{m-1}{2m}$.

$$\ddot{a}^{(m)}_{x_r,t} = \sum_{k=0}^{\infty} (1+i)^{-k} \cdot {}_kp_{x_r,t} - \frac{m-1}{2m} \tag{2.4}$$

$$\ddot{a}^{rw(m)}_{x_r,t} = \sum_{k=0}^{\infty} (1+i)^{-(k+1/2)} \cdot {}_kp_{x_r,t} \cdot q_{x_r+k,t} \cdot w_{x_r+k+1/2,t} \cdot \ddot{a}^{w(m)}_{y_{x_r+k+1/2},t} \tag{2.5}$$

$$\ddot{a}^{w(m)}_{x,t} = \sum_{k=0}^{\infty} (1+i)^{-k} \cdot {}_kp^w_{x,t} - \frac{m-1}{2m} \tag{2.6}$$

$$\ddot{a}^{(m)}_{\overline{z^*-z_{x,t}}|} = \frac{1-(1+i)^{z_{x,t}-z^*}}{1-(1+i)^{-1}} - \frac{m-1}{2 \cdot m} \cdot (1-(1+i)^{z_{x,t}-z^*}) \tag{2.7}$$

In Equation 2.5 and Equation 2.6, $\ddot{a}^{w(m)}_{x,t}$ is the monthly annuity-due, paid as long as a widow/er is alive. We assume $\ddot{a}^{w(m)}_{x+s,t}$ is linear in s, where $0 < s < 1$. Hence $\ddot{a}^{w(m)}_{x+s,t} = (1-s) \cdot \ddot{a}^{w(m)}_{x,t} + s \cdot \ddot{a}^{w(m)}_{x+1,t}$. Moreover, in Equation 2.4 and Equation 2.5, ${}_kp_{x,t}$ is the probability of surviving between the ages of x and $x+k$, calculated at time t, with ${}_1p_{x,t} = p_{x,t}$. Hence $q_{x,t} = 1 - p_{x,t}$ is the probability of dying between x and $x+1$, calculated again at time t. Similarly, ${}_kp^w_{x,t}$ is the probability for a widow/er to survive between the ages of x and $x+k$, determined at time t. We assume death occurs in the middle of the year. Furthermore, $w_{x,t}$ is the probability of being married at age x and time t, with $w_{x+s,t} = (1-s) \cdot w_{x,t} + s \cdot w_{x+1,t}$, provided that $0 < s < 1$ and $y_{x,t}$ is the average age of the partner of someone aged x at time

t, also assumed linear in s. These values are found in the actuarial life tables used by the pension funds. These life tables reflect the evolution of life expectancies and can be either periodic or generational. We consider here only periodic tables, which encompass the values for different ages at one specific point in time[3]. Therefore, the choice of tables influences both the actuarial conversion rate (and the theoretical pension amount implicitly) and the mathematical reserves to be constituted by the pension fund.

Official life tables are updated in Switzerland usually once every five years and the pension funds are at liberty to choose which ones to use and also when to adopt a new set. Nowadays, two sets of tables are used and updated: the VZ tables ("Versicherungskasse der Stadt Zürich"), which are based on the mortality experience of public pension funds, and the LPP tables ("Loi sur la Prévoyance Professionnelle"), reflecting the mortality experience of private pension funds. Since the LPP tables are more frequently used by pension funds in Switzerland, we rely upon the LPP life tables from 2010 (calibrated on data from 2005 to 2009) and 2015 (calibrated on data from 2010 to 2014). However, even if the first LPP tables date back to the years 2000, they were not available to us. To solve this issue, for the years of our study prior to 2010, we use the EVK 2000 life tables, which are based on the data from the Pension Fund of the Confederation for the period 1993 to 1998. This set of tables has been discontinued from the year 2000 forward.

The interest rate i awarded to the pensioners is called the technical interest rate. This rate is meant to estimate the future investment performance that the pension fund believes can achieve or, in other words, the rate that the fund could afford to offer as remuneration of the capital constituted for financing the future benefits. The technical interest rate is not defined by law, it is the job of the actuary to recommend the value that a pension fund can use, taking into account its structure and characteristics. However, the Swiss Chamber of Pension Actuaries defines a reference rate, which reflects the market returns over the last 20 years and the ones on the Swiss bonds over the last 10 years. Once again, the choice

[3]As opposed to the periodic table, the generational life tables show the mortality experience for one specific cohort throughout their lifetime.

of the technical interest rate influences the value of the actuarial conversion rate and of the mathematical reserves of a pension fund.

One other aspect that has to be mentioned here concerns the conversion rate. In fact, the minimum conversion rate to be applied on mandatory part of the capital is set by law in Switzerland. For example, this minimum value has been set to 6.8% for both men and women since 2014 and thus it is not adjusted to reflect the improvements in life expectancy and of the technical interest rates. However pension funds can choose to offer a better rate (or in other words a higher rate) and have no restrictions on the rate applied on the supplementary capital.

In addition, invalidity and survivors pensions are paid by the pension funds in case an active member becomes disabled or dies. In general, the invalidity pension is equal to the projected retirement pension (the pension the insured person would have received, should they have reached retirement age), while the pensions for the surviving spouse and children are defined as percentages of this invalidity pension. These pensions are usually funded under a terminal funding system. In other words, the due amounts are capitalised at the time of death or invalidity and are covered by the part of the contributions paid by the active members that is meant for risks of death and invalidity in the year when the pensions are awarded. Therefore contributions for the risks of death and invalidity of all active members collected during one year have to pay for the present value of the future benefits due to the insured members that have been awarded the pensions that same year.

As a result of all the above mentioned details, the insured capital of active members is equivalent to their accumulated retirement capital (so not the projected capital until retirement, but the actual amounts in the accounts of the active members), while the insured capital of the pensioners is composed by the mathematical reserves corresponding to each individual's benefits.

Furthermore, we mention here that pension funds in Switzerland have to constitute technical provisions if there exists the possibility that their liabilities will increase. Two important provisions are the ones for longevity and the one for the change of the technical interest rate. Since every pension fund can choose their technical interest rate, there is no common practice related to the creation of the provision for this case. However, because life tables are updated on a regular basis, the vast majority of pension funds will constitute a provision for longevity. In general, pension funds allocate 0.5% of the insured capital of the pensioners each year, since the life tables used have been published and until the tables are changed[4]. For example, let us consider a pension fund that adopts the LPP 2010 tables at the end of the year 2012. Then, under the hypothesis that they have been published at the beginning of 2010, three years have passed since the tables' publication. Therefore the pension fund should initially allocate to the provision 1.5% (0.5% for each year that has passed since the publication) of the insured capital of the pensioners. Subsequently, the pension fund will increase the provision yearly by allocating 0.5% of the corresponding yearly insured capital. Hence the provision will grow every year, until a new change will take place. Once a new table is adopted, the amount of the provision will be used to cover the costs related to the change and a new provision will start to be constituted.

Lastly, we can generally calculate the pension fund's total fortune as per Equation 2.8, where the short-term debt, the liability adjustment account, the employer contribution reserve without waiver and the non technical provisions together represent the callable liabilities. The Employer's contribution reserve without waiver is a particularity of the Swiss pension system. In fact, in Switzerland, the employers can pay contributions in advance, which are accumulated in the Employer's contribution reserves, with the amount usually limited to five times the annual contributions due. These reserves can be with waiver, meaning that they can be used by the pension fund in case of underfunding, or without waiver, when the pension fund does not have the authority to use the amount to improve its financial situation. The total fortune of the pension fund includes only the reserves with waiver.

[4]The Swiss Chamber of Experts in Pension Funds recommends in their directive (see Swiss Chamber of Experts in Pension Funds (2014)) to allocate at least 0.3% annually.

$$\begin{aligned}
&\text{Total assets from the balance sheet}\\
-&\text{Short-term debt}\\
-&\text{Liability adjustment account}\\
-&\text{Employer's contribution reserve without waiver}\\
-&\text{Non technical provisions}\\
\hline
=&\textbf{Total Fortune of the pension fund(FP)}
\end{aligned} \tag{2.8}$$

2.3 Theoretical Background: Transfers and the common pot

To calculate the transfers, or in other words the unintended redistributions, we consider a common pot belonging to both the retired and active members. Consequently, when the actives or the retired participate at the financing of the pension fund through an excess payment, this amount goes into the common pot. If, however, these groups receive an excess benefit, it will be paid from the common pot. In order to determine these excess payments or the excess benefits, we start by defining the objectives with respect to what each group has to pay or receive. These objectives ensue from the rules of the occupational pension system, from the law and from the rules of procedure of the pension funds. Thereafter, if differences appear between these objectives and what each group has actually received or paid each year, they will be classified as excess amounts and will be, depending on the case, withdrawn from or deposited into the common pot. In order to better illustrate our method, let us consider the transfers related to the interest rate awarded to the active members, which constitutes one of our sources and is presented once again in Section 2.4.1. We therefore start by noting that one of the objectives of the Swiss occupational pension system is to guarantee that the cumulated retirement pension between the first and second pillar reaches 60% of the last salary[5]. To reach this goal, the nominal interest rate credited to the actives' capital should be equal to the growth rate of salaries. In Switzerland this is referred to as the golden rule. This is referred to as the golden rule. Hence, by using this rule and the common pot method, if the credited interest rate is superior to the growth rate of salaries, then the actives receive more than intended and the funds to finance this surplus are withdrawn from the common pot. This withdrawal, benefiting the actives, is, in this case, a transfer from the retired to the active insured people. If, however, this credited interest does not reach the values of the growth rate of salaries, the actives receive less than due, and the difference that results is deposited in the common pot by the actives. Therefore, this time, we will have a transfer towards the retired.

The common pot belongs to both the actives and the retired, proportional to their respective insured capital, which is defined in Section 2.2. In other words, assuming that the actives' capital within one pension fund accounts for 80% of the total insured capital, while the remaining 20% belongs to the retirees, then if we withdraw 10 CHF from the common pot in favour of the retired, 2 belong already to this group. Hence, only 8 are transferred from the actives to the retired. On the other hand, if the actives deposit 10 CHF into the common pot, 8 already belong to them and 2 are transferred to the retired.

Finally, it is important to remark here that we define these transfers from the point of view of the active insured population. Therefore, if the total cost has a negative sign, they are benefiting the retired, while a positive result favours the actives. We can now define mathematically the transfers through the following equations:

[5]Art. 113, al. 2a of the Federal Constitution (2018) specifies that the first and second pillar together should allow the insured person to maintain the standard of living they had prior to retirement. In addition to this, the Federal Counsel in Feuille Fédérale, FF (1976) clarifies that this objective can be guaranteed when the two pensions amount to 60% of the last insured salary.

1. Funds are withdrawn from the common pot:

$$\begin{cases} T^{P}_{i,t} & = \left| \min \left\{ 0\,;\, TotC_{i,t} \cdot \dfrac{ICA_t}{ICA_t + ICP_t} \right\} \right| \\ \\ T^{A}_{i,t} & = \max \left\{ 0\,;\, TotC_{i,t} \cdot \dfrac{ICP_t}{ICA_t + ICP_t} \right\} \end{cases} \tag{2.9}$$

2. Funds are paid into the common pot:

$$\begin{cases} T^{P}_{i,t} & = \left| \min \left\{ 0\,;\, TotC_{i,t} \cdot \dfrac{ICP_t}{ICA_t + ICP_t} \right\} \right| \\ \\ T^{A}_{i,t} & = \max \left\{ 0\,;\, TotC_{i,t} \cdot \dfrac{ICA_t}{ICA_t + ICP_t} \right\} \end{cases} \tag{2.10}$$

$T^{P}_{i,t}$ and $T^{A}_{i,t}$ denote the transfers at time t towards pensioners and actives respectively, for each source of transfers i. $TotC_{i,t}$ is the total cost at time t, for the source of transfers i, where we number each source in the following sections. Lastly, ICA_t stands for the insured capital of the actives at time t and ICP_t for the insured capital of the pensioners at time t. We refer to the ratio of each of the groups' insured capital over the total sum as the actives' or retired' pro rata.

2.4 Sources of generational transfers

In this section, we identify the main sources of transfers for the Swiss system and provide for each one a mathematical method of calculating them. Since our methodology is meant to be applied by pension funds on a case-by-case basis, we first identify all the variables needed for our calculations, for each source of transfer, and we categorise them in independent variables, namely those that are not chosen by the pension funds, such as the growth rate of salaries, and dependent variables, hence the ones which every pension fund chooses and has to adapt when implementing our methodology. Figure 2.1 resumes all the information needed to compute the transfers for each identified source at a given time t, including the notations used for the variables. For example, in order to calculate the transfers related to the remuneration of the retirement capital, a pension fund will use the growth rate of salaries at the chosen time gr_t as an independent variable, since it does not depend on the financial decisions of the institution. On the other hand, the insured capital of active members at time t, ICA_t, and the interest rate credited at time t, ic_t, will be variables specific to each fund. Additionally, for each of the six sources, the final values of the transfers should be expressed as percentages of the total fortune of each pension fund (FP_t) and depend on the structure of their insured population, referred in this paper as the pro rata (as per Equation 2.9 and Equation 2.10).

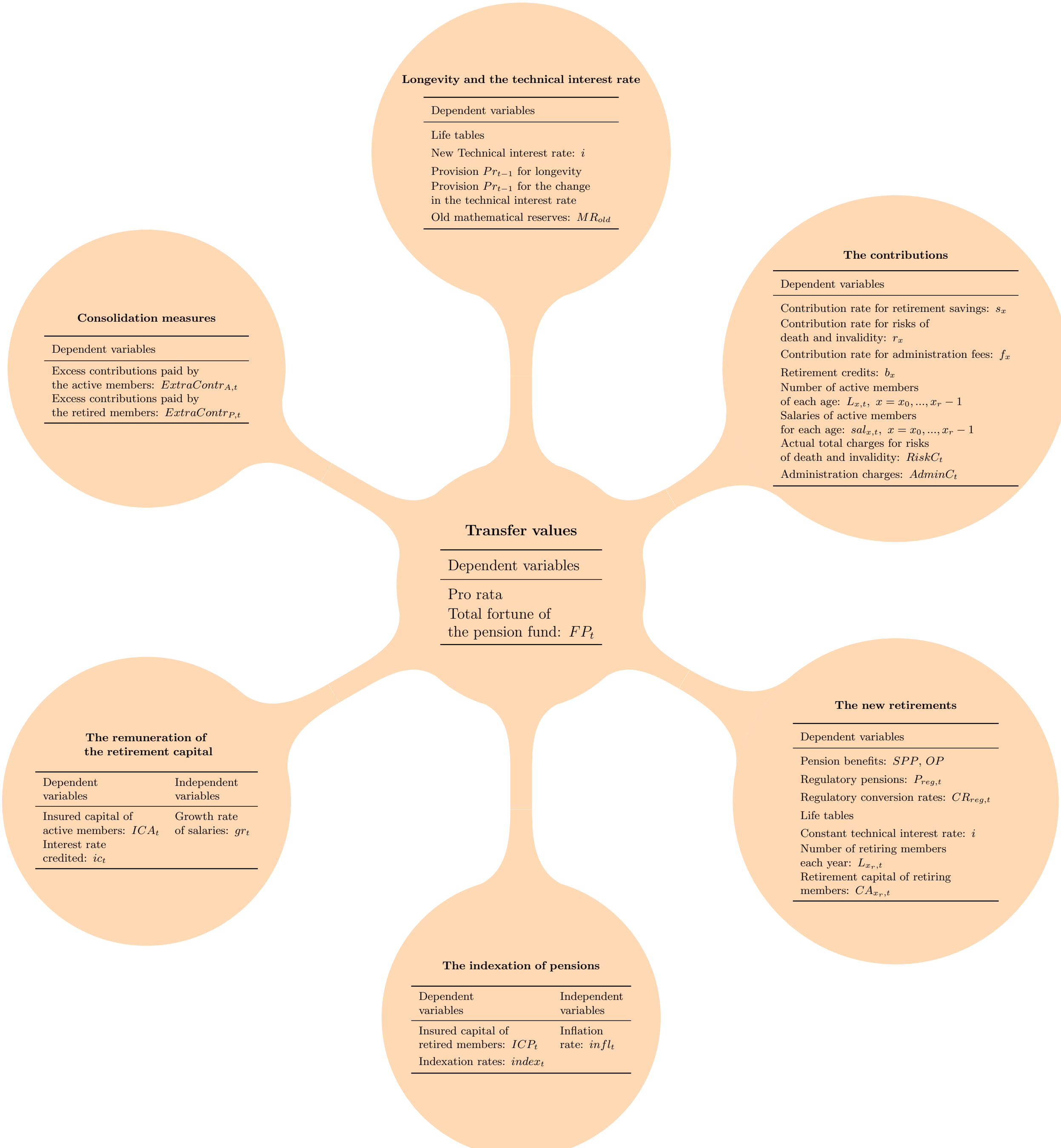


Figure 2.1: Dependent and independent variables to be considered for calculating the total transfers at time t for a given pension fund

2.4.1 The remuneration of the retirement capital of the actives

As explained in the example given in Section 2.3, for the cumulated pension to reach it's constitutional target of 60% of the last salary, we should apply the golden rule related to the interest used to accumulate the retirement capital of the active members. This rule indicates that the interest rate given to the actives should be equal to the growth rate of salaries. Therefore, if the credited interest is superior to the growth rate of salaries, then the actives have received more than due and the resulting difference is considered a transfer towards the actives, since they have withdrawn funds from the common pot. If, however, the inverse situation presents itself, then the actives receive less than what they should. They thus pay the amount into the common pot, transferring a part to the pensioners. The total cost (or the difference) in this case can be calculated as showed in Equation 2.11, with ic_t the interest rate credited by the pension fund at time t and gr_t the growth rate of salaries at time t:

$$TotC_{1,t} = ICA_t \cdot (ic_t - gr_t) \tag{2.11}$$

We must remark here that the golden rule such as defined above is specific to the Swiss second pillar. It is meant to create a simple link between the accumulated capital (and thus ultimately the pension) and the contributions paid. It should however not be confused with the rule defined in Lindbeck and Persson (2003), which requires the equality between the return and the growth rate of the aggregate wage sum in a pay-as-you-go system to avoid that gifts to one cohort are paid by subsequent cohorts.

2.4.2 The indexation of pensions

One of the objectives of the pension system is to maintain the purchase power of pensioners. For this, the pensions should be indexed to inflation. However, the law does not stipulate an obligation to do so, leaving the indexation rate at the discretion of the pension fund. Moreover, diminishing the pensions is not allowed, except in rare cases and most pension funds do not index retirement pensions altogether. Therefore, if the inflation rises with respect to the indexation rate, then the retired lose a part of their purchasing power. In this case, the pensioners deposit funds into the common pot and we find a transfer towards the actives. If, on the contrary, the inflation drops under the indexation rate, the retired gain purchasing power, which translates into a withdrawal from the common pot of the difference between the ICP_t (Insured Capital of Pensioners at time t) indexed at inflation and that indexed at the rate chosen by the pension fund. Therefore, the actives transfer funds towards the retired. The total cost due to indexation is given by the Equation 2.12, where $infl_t$ is the inflation rate and $index_t$ is the indexation rate chosen by the pension fund at time t.

$$TotC_{2,t} = ICP_t \cdot (infl_t - index_t) \tag{2.12}$$

2.4.3 The new retirements

As detailed in Section 2.2, at the moment of retirement, the capital accumulated by the insured is transformed into a monthly pension by applying the conversion rate. The conversion rate should be equivalent to the inverse of the present value of all future benefits due to an insured person, in order for the capital to be sufficient to finance the necessary mathematical reserve of the individual's departure into retirement. However, the minimum conversion rate established by law to be applied to the mandatory part of the accumulated capital is 6.8% and the pension funds can use a more favourable regulatory rate. For the supplementary part, the pension funds are at liberty to choose a conversion rate, even a lower one. The slow process of reforming the system and the lack of flexibility have rendered the regulatory (and legal) rate different to the actuarial fair rate, since it cannot be adjusted to follow the evolution of the life expectancy or of the interest rates. The difference that results from applying the actuarially fair conversion rate versus the one used by the pension fund is considered, in this paper, as a transfer. If the conversion rate used by the pension fund is superior to the actuarially fair rate, then the pensions will be too high and the capital accumulated during the active life will not be sufficient to guarantee the payment of the promised benefits. In this case, the retirees withdraw from the common pot an amount equal to the present value of the difference between the two pensions (or in other terms, the difference between the two mathematical reserves). We find, in this case, a transfer from the actives towards the retired. If, on the contrary, the actuarially fair rate is the superior one, then the pensions are smaller than they should be. The retired will thus deposit the difference into the common pot and will trigger a transfer towards the actives.

Equation 2.13 shows our reasoning regarding this source of transfer. The theoretical pensions are calculated by applying the actuarially fair conversion rate, while the regulatory ones are determined by using the statutory rates.

$$TotC_{3,t} = \big(MR_{th,t} - MR_{reg,t}\big) \cdot L_{x_r,t} \tag{2.13}$$

We denote the theoretical mathematical reserves and regulatory mathematical reserves at time t with $MR_{th,t}$ and $MR_{reg,t}$ respectively, while $L_{x_r,t}$ is the number of insured people at retirement age x_r and time t.

We must specify here that the present values of the two types of pension are calculated using the same life tables and the technical interest rate should remain constant, in order to asses only the impact of the conversion rate applied[6]. We define in Equation 2.14 and Equation 2.15 the mathematical reserve at retirement age $MR_{reg,t}$, as well as the regulatory pension $P_{reg,t}$, while the theoretical conversion rate and pensions are given by Equation 2.1 and Equation 2.2.

$$P_{reg,t} = CA_{x_r,t} \cdot CR_{reg,t} \tag{2.14}$$

$$MR_{reg,t} = P_{reg,t} \cdot \left(\ddot{a}^{(m)}_{x_r,t} + SPP \cdot \ddot{a}^{rw(m)}_{x_r,t} + OP \cdot k_{x_r,t} \cdot \ddot{a}^{(m)}_{\overline{z^* - z_{x_r,t}}|} \right) \tag{2.15}$$

The theoretical and regulatory pensions at time t, given by $P_{th,t}$ and $P_{reg,t}$ respectively, are

[6]The transfers related to a change in technical interest rates are calculated in Section 2.4.5.

therefore calculated as the capital accumulated up to retirement age x_r reached at time t, namely $CA_{x_r,t}$, multiplied by the corresponding conversion rate. The regulatory conversion rate $CR_{reg,t}$ is considered as constant, set through the minimum LPP framework. As previously mentioned, in the theoretical case, the mathematical reserve will be equal to the accumulated capital, since at retirement this capital should be enough to cover all future benefits (as shown by plugging Equation 2.1 and Equation 2.2 in Equation 2.3).

2.4.4 The contributions

During their active lives, the insured pay contributions defined as percentage of their salaries. Given that this study assumes the system is based solely on defined contribution pension plans, this percentages determine, together with the performance of the pension fund's investments, the benefits, according to the rules of procedure of the pension fund. The total contribution rate for a person of age x, denoted c_x, can generally be split into three categories: a contribution for retirement savings s_x, a contribution for risks of death and invalidity r_x and a contribution for administration fees f_x.

The contribution for retirement savings does not, however, necessarily correspond to the retirement credits. In the same way, the contributions for administration charges and risks can be different than the actual costs. If any differences appear, then they are considered as transfers. For example, if the retirement credits exceed the contributions for savings, then the actives receive more than due, hence withdrawing the extra funds from the common pot. This means a transfer from the pensioners towards the actives. If the contributions for administration charges or risks are not enough to cover the incurred costs, then the resulting difference is taken from the common pot and the transfer goes also towards the actives.

The Equation 2.16 illustrates our calculations for the difference on savings, where b_x is the percentage of retirement credits defined in function of $sal_{x,t}$, the salary of a person of age x at time t. We consider here the active population of the pension fund, with active members entering at age x_0, retiring at age x_r and having $L_{x,t}$ number of actives of age x at time t.

$$TotC_{4,t} = \sum_{x=x_0}^{x_r-1} b_x \cdot sal_{x,t} \cdot L_{x,t} - \sum_{x=x_0}^{x_r-1} s_x \cdot sal_{x,t} \cdot L_{x,t} \tag{2.16}$$

The difference related to the risks of death and invalidity is given by Equation 2.17, where $RiskC_t$ represents the actual total charges for risks of death and invalidity incurred by the pension fund at time t. As per the definition of a terminal funding system, given in Section 2.2, the value $RiskC_t$ is equal to the present value of future benefits for death and invalidity that are awarded at time t.

$$TotC_{5,t} = RiskC_t - \sum_{x=x_0}^{x_r-1} r_x \cdot sal_{x,t} \cdot L_{x,t} \tag{2.17}$$

Lastly, since the administration costs are mainly driven by the entries and exists of active members, the tracking of their salary evolution and the management of their accounts, we assume that the charges for administration belong entirely to the active members. Consequently, we can calculate the difference on administration fees as per Equation 2.18, where $AdminC_t$ are the administration charges to be paid by the pension fund at time t.

$$TotC_{6,t} = AdminC_t - \sum_{x=x_0}^{x_r-1} f_x \cdot sal_{x,t} \cdot L_{x,t} \tag{2.18}$$

2.4.5 Longevity and the technical interest rate

Two of the major concerns of any pension fund are the increase in life expectancy, reflected in the choice of life tables, and the technical interest rates used. Any change related to the tables or to the technical interest rate translates into an adjustment of the mathematical reserves of the retired members. Since pensions cannot be adapted after being awarded, the increase in longevity and the decrease in the technical interest rate will increase the mathematical reserves, benefiting in this way the pensioners. The increase in the reserves due to the change of mortality tables or technical interest rates should be covered by the provisions created by the pension fund for this purpose specifically. Should the provision surpass the costs, the remaining amount would be reported to the next year and augmented by the amount to be credited during that next year. However, the pension fund might underestimate these differences, thus having insufficient provisions. In this case, the necessary amount to cover the remaining costs after the dissolution of the provisions would be withdrawn from the common pot and given to the retired. Therefore the total cost driving the transfers in this case exists only when the provisions are insufficient, which is taken into account by using the indicator function $I_{\{Pr_{t-1}<MR_{new}-MR_{old}\}}$ in Equation 2.19 below, with Pr_{t-1} the provision at time $t-1$. MR_{old} defines the amount of the mathematical reserves under the old choice of life tables or technical interest rate, while MR_{new} is calculated after the change.

$$TotC_{7,t} = \left[Pr_{t-1} - (MR_{new} - MR_{old})\right] \cdot I_{\{Pr_{t-1}<MR_{new}-MR_{old}\}} \tag{2.19}$$

2.4.6 Consolidation measures

One of the indicators of the financial situation of a pension fund is the funding ratio, defined as the ratio between assets and liabilities. Once the funding ratio falls below the minimum level, the pension funds are required to put in place consolidation measures, in order to ensure their solvency. These measures are therefore temporary and can be at the charge of the employer, the active members or the pensioners. Though the pension funds can adopt a number of different consolidation measures, the only ones that can be considered as transfers in the scope of this study are the extra contributions paid by the actives or the retired. Excess contributions can be perceived from the retired members, but only on the unplanned increases of the supplementary part of the pension (such as the indexation) and only when the regulations of the pension fund specifically allows for this option in case of underfunding. Since this measure is only temporary, the initial pension amount, calculated at the moment of retirement, is guaranteed once the funding ratio is re-established. Hence these contributions are not considered as reduction of pension and thus have no impact on the mathematical reserves constituted for the pensioners.

When either of these groups pays an excess amount, the sums are deposited into the common pot and a transfer takes place towards the other group. Therefore the total cost in this case can be defined as in the Equation 2.20, with $ExtraContr$ the excess amount paid and with the acknowledgement that it is possible that both groups pay the extra contributions during the same year, meaning the transfers could go both ways for the same period.

$$TotC_{8,t} = \begin{cases} ExtraContr_{P,t} & \text{if the pensioners pay} \\ -ExtraContr_{A,t} & \text{if the actives pay} \end{cases} \tag{2.20}$$

2.5 Numerical illustration

The aim of this section is to provide a numerical example of how our methodology works. For that, we treat the pension funds in Switzerland collectively, so in other words we consider the system as one pension fund and we refer to it from here on as the pension fund. We base our study on data from the Swiss Office of Statistics (OFS), published under the survey "Statistiques des caisses de pensions" (see Office Fédéral de la Statistique (OFS) (2005-2015)). We thus study the transfers defined in the previous section on a yearly basis for the period 2005 to 2015, in order to avoid bias related to the passage of insured members from the active state to the retired life. Hence, each year, the groups are well defined and retirement is taken only at the end of each period.

The survey mentioned previously provides us the data regarding the insured capital for both active members and retirees, according to which the actives hold on average 55% of the total insured capital, while the remaining 45% belong to the pensioners. The proportions, displayed in Figure 2.2, show a consistency throughout the eleven years studied. We also set here the hypothesis that the pro rata illustrated in Figure 2.2 corresponds to the situation when all the population resident in Switzerland is insured under the second pillar.

This hypothesis is necessary since we do not have any information on how many insured people exists in the system, for each age, or how many new retirements take place each year. Moving forward, we present the ensuing transfers for each source defined in Section 2.4, expressed as percentages of the total fortune of the system, which is calculated as explained in Equation 2.8. Once more, we remark that the following transfers are meant solely as an illustration and thus are not used to draw conclusions on the entire Swiss second pillar.

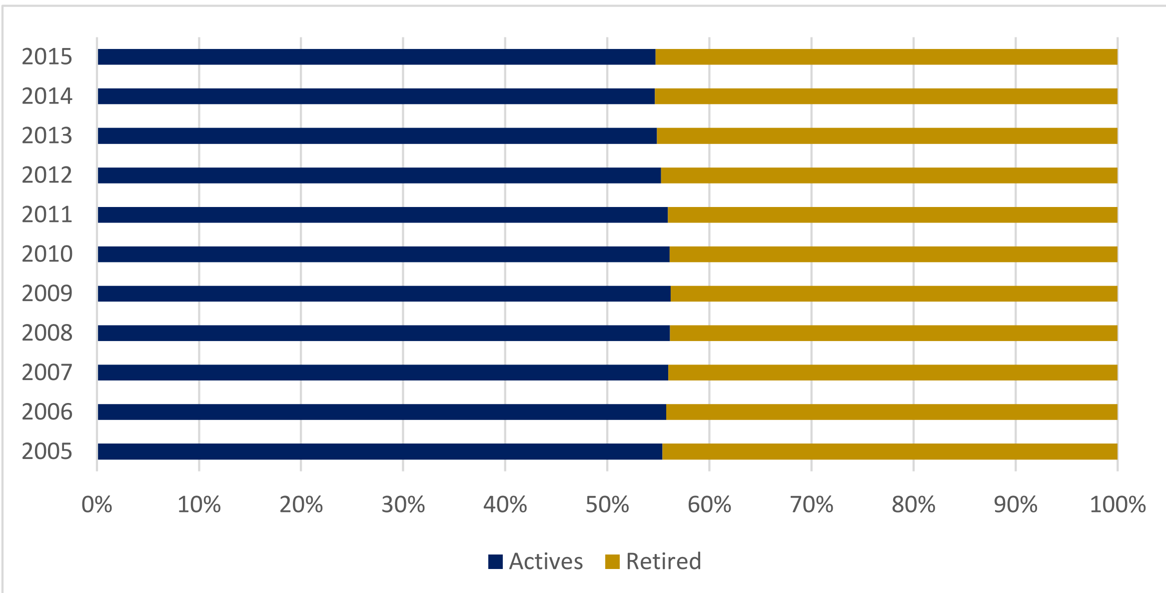


Figure 2.2: Pro Rata in the Base Scenario

2.5.1 The remuneration of the retirement capital of the actives

As described in Section 2.4.1, in order for the sum of the first and second pillar pensions to achieve the target value of 60% of the last salary, the capital accumulated for the occupational pension should be remunerated at the growth rate of salaries. However, this is not compulsory under Swiss law. Because the accumulated capital is composed by the compulsory part and the supplementary one, the pension fund must offer at least the minimum LPP rate on the mandatory part of the capital and can choose the interest rate to credit on the supplementary capital. The minimum rate is recalibrated each year in order to take into account the evolution of the financial market. Therefore, as defined in Equation 2.11, the transfers due to the remuneration of the retirement capital are driven by the difference between the credited interest rate and the growth rate of salaries.

The table below sums up the annual average growth rate of nominal salaries, as reported by the OFS and the credited interest rates, which correspond to the minimum LPP rates set by the Federal Counsel.

	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Growth rate of salaries	1.00%	1.20%	1.60%	2.00%	2.10%	0.80%	1.00%	0.80%	0.70%	0.80%	0.40%
Interest rate minimum LPP	2.50%	2.50%	2.50%	2.75%	2.00%	2.00%	2.00%	1.50%	1.50%	1.75%	1.75%

Table 2.1: The growth rate of salaries and the interest rate minimum LPP

For this source of transfer, we consider two cases. In the first one, we start by determining the entire insured capital belonging to the active members at the end of the year by applying the minimum LPP rate of interest displayed in Table 2.1 to the respective capital at the beginning of the year. We then perform a second step by using the growth rate of salaries instead of the minimum LPP rates. The differences that appear between the amounts calculated at each of the two steps are then considered transfers. In the second case, we differentiate between the mandatory part of the accumulated capital and the supplementary amount. On one hand, we remunerate the total insured capital of the actives at the growth rate of salary. On the other hand, we offer the minimum LPP rates on the mandatory part, while the supplementary capital gets a 0% interest rate. The differences that results are therefore transfers, as per Equation 2.11. This second scenario represents a limit case. Since the pension fund can choose the interest rate applied on the supplementary capital, this rate can be zero if the fund is having issues guaranteeing their benefits or reaching their target performance level. Hence we consider this limit case as a real possibility and an interesting situation to discuss here.

In the first case, during the years 2005 to 2008 and 2010 to 2015, the minimum LPP rate surpasses the growth rate of salaries, so the actives receive more than stipulated by the golden rule, thus withdrawing the difference from the common pot and triggering a transfer from the pensioners. In 2009 however, the growth rate of salaries is superior to the credited interest rate, therefore the retired are the beneficiaries of the transfers since the actives have received less than due and have thus deposited the corresponding amount into the common pot. Moreover, starting from 2010, we notice a decrease in both the growth rate of salaries and the minimum interest rate, when the minimum interest rates is influenced by the low rates on Swiss bonds as a result of the 2008 economic crisis. Figure 2.3 illustrates in blue the values of the transfers towards actives and in yellow the ones towards retired, expressed again as percentages of the fortune of the pension fund for the year in question. As expected, most of the transfers go towards the actives, as the minimum interest rate is, in general, higher than the growth rate of salaries.

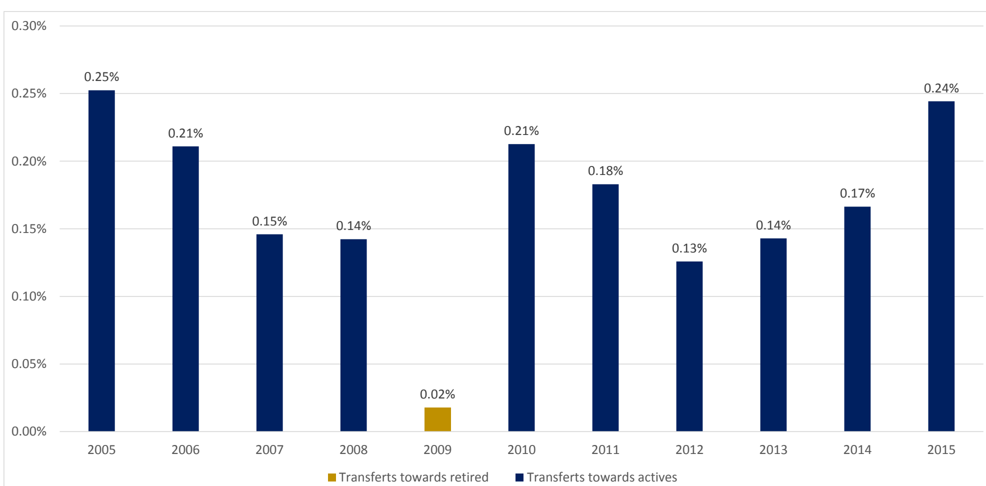


Figure 2.3: The transfers due to the remuneration of the retirement capital of active members when minimum LPP rates or the growth rate of salaries are applied on the entire capital (expressed as percentages of the fortune of the pension fund)

In the second case, the values of the transfers towards the actives decrease across the entire period. Moreover, the transfer towards the retired increases in 2009, as expected, since a part of the capital receives a 0% interest rate. One interesting year is 2008. If in the first case, we found a 0.14% transfer in favour of the actives, by differentiating between the two parts of the accumulated capital, the transfer is now directed towards the retired and is equal to 0.014% of the fortune of the pension fund. During this year, even though the minimum LPP rate surpasses the growth rate of salaries, the difference between the two rates is not enough to outweigh the impact of the 0% interest rate credited to the supplementary capital. In total, the actives receive, in this case, less than defined by the golden rule. The corresponding values are depicted in Figure 2.4.

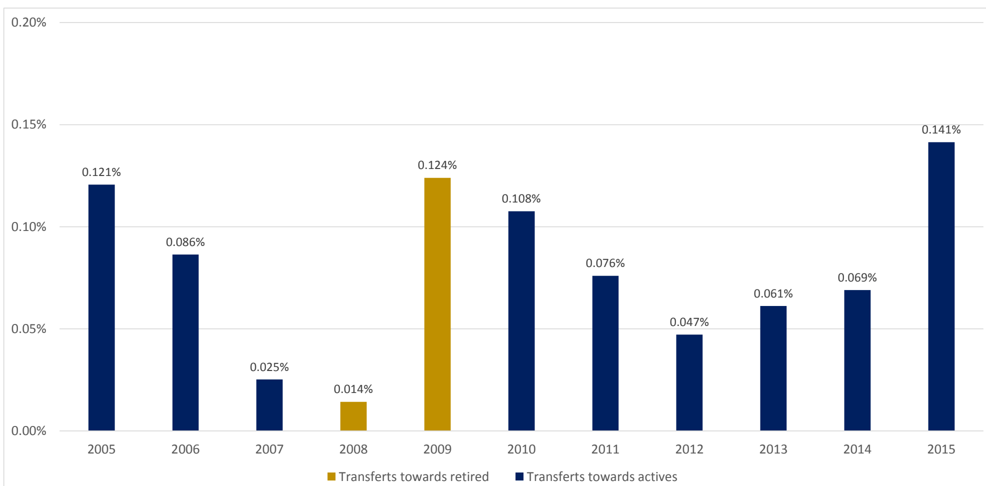


Figure 2.4: The transfers due to the remuneration of the retirement capital of active members when we differentiate between the mandatory part of the accumulated capital and the supplementary amount (expressed as percentages of the fortune of the pension fund)

2.5.2 The indexation of pensions

In Section 2.4.2, we identify the indexation of pensions as a source of transfer. In order to maintain the purchase power of pensioners, their pensions should be indexed to inflation. However, there is no obligation to do so and diminishing pensions is not allowed under the Swiss law, except in rare and extreme cases (consolidation measures). Pension funds should choose their rate according to their financial situation and possibilities. Given that the average performance of the pension funds over the period considered is of only 4% (see the statistics reported in Swisscanto Prévoyance (2016)) and that guaranteeing technical interest rates and conversion rates is putting strain on pension funds, we consider here that no indexation of pensions is offered. Table 2.2 holds the values of the inflation and the indexation for the studied period.

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Inflation	1.20%	1.10%	0.70%	2.40%	-0.50%	0.70%	0.20%	-0.70%	-0.20%	0.00%	-1.10%
Indexation	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 2.2: Values of inflation and indexation

As given by Equation 2.12, the transfers linked to the indexation of pensions ensue from the difference between the capital of the pensioners indexed at inflation and the one indexed at the rate defined by the pension fund. Since the chosen indexation rate is 0%, we will find a transfer from the actives to the retired only when the inflation is negative, situation that presents itself in 2009, 2012, 2013 and 2015. During these years, the pensioners gain purchase power and therefore withdraw the amounts from the common pot. For all the other years, except 2014, the inflation surpasses the chosen indexation rate. Therefore the retired lose purchase power and deposit the corresponding differences into the common pot, thus transferring funds to the actives. As both the inflation rate and the indexation rate are 0% in 2014, no transfer takes place during this year. Figure 2.5 shows the values of the transfers linked to this source, sustaining our reasoning above.

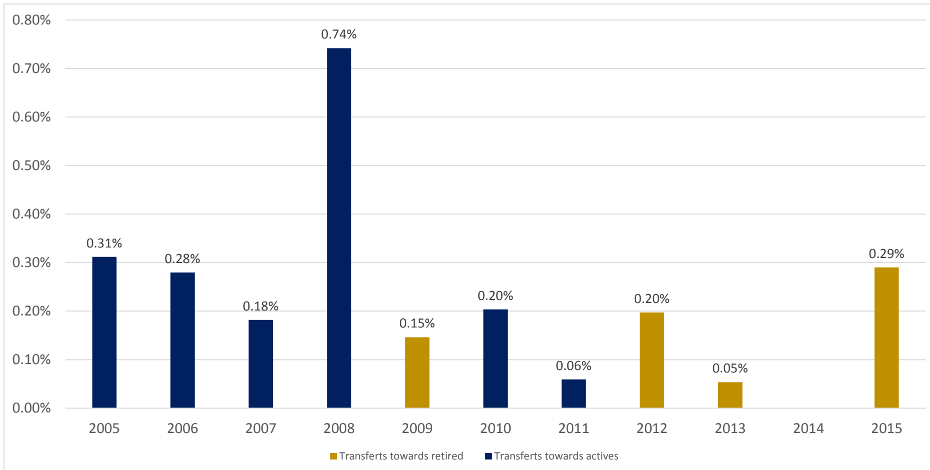


Figure 2.5: The transfers due to the indexation of pensions (expressed as percentages of the fortune of the pension fund)

The transfers towards actives, plotted in blue, reach the maximum value in 2008, when the inflation is the highest. This high value of inflation is attributed to the increase of the prices of the heating oil, as well as of the oil products in general. The effect of the economic crisis starts to be visible in 2009, when the inflation becomes negative due to the appreciation of the Swiss currency, giving way to a transfer towards the retired. The strong deflation in 2015, translated into a peak of the transfers towards retired is also due, in part, to the appreciation of the Swiss franc, caused by the decision of the Swiss National Bank to remove the currency cap on euros, and to the decrease in the petrol prices. Of course, the results presented here are highly dependent on the indexation chosen. Should the indexation rate be different, so would the value of the transfers. For instance, an indexation rate of 1% would decrease the transfers for the years for which the inflation is positive, but it would have an opposite effect on the years when the inflation is negative.

2.5.3 The new retirements

According to Section 2.4.3, the conversion rates used by the pension funds to determine the pensions of newly retired people does not necessarily correspond to the actuarially fair conversion rates. Hence we differentiate here between the regulatory pensions and conversion rates, detailed in Table 2.3 and the actuarial corresponding values given in Table 2.4. The regulatory pensions correspond to the average pensions for men and women for the studied period[7], while the regulatory conversion rates are equal to the minimum conversion rates set under the Swiss law for the compulsory part of the retirement capital. We assume here that the minimum conversion rates set by law are applicable to the supplementary capital, as well as to the mandatory part. We observe that the average pensions are much lower for women than for men. The reasons behind the big pension gender gap lie, as listed in Fluder et al. (2016), with the differences in salaries and professional career path between men and women. Indeed, for instance, the Swiss Office of Statistics[8] notes that the average gender salary gap in 2006 to 2012 is of above 15%. Fluder et al. (2016) also remark that more men than women actually receive a pension from the second pillar and that at the end of the working life, more women than men stop contributing to the pillar, thus impacting their pensions in a negative way (according to the study only 64% of women still work during the last 20 years of their active life, against 85% for men, earning on average only 35% of the average salary of men).

[7]The values are taken from the Swiss Office of Statistics.

[8]https://www.bfs.admin.ch/bfs/en/home/statistics/work-income/wages-income-employment-labour-costs/wage-levels-switzerland/wage-gap.html

	Regulatory Pension Men	Regulatory Pension Women	Regulatory Conversion rate Men	Regulatory Conversion rate Women
2005	35’706.00	18’810.00	7.15%	7.20%
2006	36’172.00	18’929.00	7.10%	7.20%
2007	36’519.00	19’080.00	7.10%	7.15%
2008	36’505.00	19’175.00	7.05%	7.10%
2009	36’509.00	18’989.00	7.05%	7.00%
2010	36’756.00	18’220.00	7.00%	6.95%
2011	36’532.00	18’332.00	6.95%	6.90%
2012	36’605.00	18’151.00	6.90%	6.85%
2013	36’437.00	18’312.00	6.85%	6.80%
2014	36’217.00	18’578.00	6.80%	6.80%
2015	35’981.00	18’313.00	6.80%	6.80%

Table 2.3: Regulatory pension amounts (data from OFS) and regulatory (legal) conversion rates

Since the regulatory pension can be calculated as in Equation 2.14, we can determine the accumulated capital by dividing the regulatory pension by the corresponding regulatory conversion rates, which values are given in Table 2.3. Having now the accumulated capital, we can determine the theoretical pensions, as given by Equation 2.2. The theoretical conversion rates (or in other words, the actuarially fair rates) necessary are displayed in Table 2.4, along with the resulting theoretical pensions.

These two types of pensions are then used to calculate the difference between the mathematical reserves for both men and women as given by Equation 2.13. The differences are multiplied by the number of people alive at retirement age (65 for men and 64 for women), leading to the transfers in Figure 2.6. Because we assume everyone alive at the retirement age will receive a pension from the second pillar, we acknowledge that we are, in fact, overestimating these transfers.

	Life Tables	Technical Interest Rate	Theoretical Conversion Rate Men	Theoretical Conversion Rate Women	Theoretical Pension Men	Theoretical Pension Women
2005	EVK 2000	2.5%	5.968%	6.242%	29'805.37	16'308.01
2006	EVK 2000	2.5%	5.968%	6.242%	30'407.00	16'411.18
2007	EVK 2000	2.5%	5.968%	6.242%	30'698.70	16'657.77
2008	EVK 2000	2.5%	5.968%	6.242%	30'904.56	16'858.60
2009	EVK 2000	2.5%	5.968%	6.242%	30'907.95	16'933.57
2010	LPP 2010	2.5%	5.734%	5.874%	30'110.93	15'400.48
2011	LPP 2010	2.5%	5.734%	5.874%	30'142.73	15'607.43
2012	LPP 2010	2.5%	5.734%	5.874%	30'421.83	15'566.13
2013	LPP 2010	2.5%	5.734%	5.874%	30'503.24	15'819.67
2014	LPP 2010	2.5%	5.734%	5.874%	30'542.00	16'049.47
2015	LPP 2015	2.5%	5.596%	5.753%	29'612.05	15'492.50

Table 2.4: Theoretical pension and actuarial conversion rates

We note here that the technical interest rate used to calculate the present value of all future benefits and subsequently the mathematical reserves, as given by Equation 2.15 and Equation 2.3, is kept constant at the level of 2.5% throughout the period, allowing us to isolate the impact of the evolution of the life expectancy on the transfers.

We observe that, for all eleven years, the theoretical conversion rate is lower than the regulatory rate. Hence the regulatory pensions are higher than they should be, meaning that the retired receive more than due and that they thus withdraw funds from the common pot. This is displayed in the Figure 2.6, where all the transfers, in yellow, favour the retired. The change in mortality tables in 2010, which reflects the increase in life expectancy, produces a spike in the transfers, since the adjustment of the regulatory conversion rate does not match the decrease in the theoretical conversion rate. In fact, the regulatory rate is lower only by 0.05%, while the theoretical conversion rate is 0.235% lower than the one of the previous year for men and 0.368% for women. The gradual decrease of the regulatory conversion rates that continues after 2010 produces a decrease of the transfers, given that the theoretical conversion rates remain constant during the period 2010-2014. Considering now the first years of our study, namely 2005-2009, the situation appears more stable, even though regulatory conversion rates decrease gradually, while actuarial rates stay constant. This might be explained by the variation of the regulatory pensions or by the decrease of the regulatory conversion rates not being sufficient to lower the transfers significantly. One last year to analyse is 2015, when the transfers are higher than the ones of 2014. This is the result of a new change of life tables, which implies an even lower theoretical conversion rate as a response to the increasing life expectancy. Since the regulatory conversion rates remain constant, the differences between pensions and thus between the reserves become larger, leading to a higher value of the transfers towards retired. Lastly, we remark that should a pension fund apply a lower conversion rate for the supplementary part of the retirement capital than the one assumed above, the value of the transfers will naturally decrease.

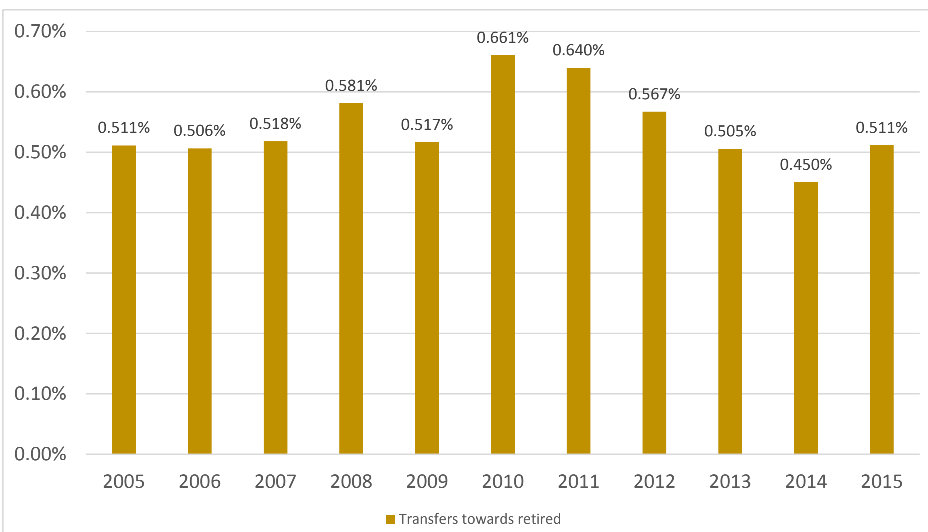


Figure 2.6: The transfers due to the conversion rate (expressed as percentages of the fortune of the pension fund)

2.5.4 The contributions

In Section 2.4.4, we define the total costs related to the retirement savings, those related to the risks of invalidity and death and those related to the administration fees. Because the charges for risks and those for administration are usually very specific to each individual pension fund, we focus here only on the total cost linked to the retirement savings, defined in Equation 2.16. The transfers in this case are driven by the difference between the retirement credits b_x and the contributions for savings s_x, both defined as percentages of salaries. We assume the entire population between the ages of 17 and 65/64 qualifies to be insured within the occupational pension system and we use historical data to extrapolate an annual average growth rate of salaries and determine the wages for the studied period.

We assume homogeneity across active members of the same age and hence, the wages for a person of age x at time $t \in [2005, 2015]$ are given by the Equation 2.21, where $t_0 = 2005$:

$$sal_{x,t} = sal_{x,t_0} \cdot (1 + g_x)^{t-t_0} \tag{2.21}$$

As the historical data is composed by the values of monthly salaries, gathered every two years starting from 1998 and until 2010, the annual growth rate of wages for age x, g_x, is calculated as below:

$$g_x = \left(\prod_{j=0}^{5} (1 + g_{x,j}) \right)^{1/12} - 1 \tag{2.22}$$

$$g_{x,j} = \frac{S_{x,1998+2(j+1)} - S_{x,1998+2j}}{S_{x,1998+2j}} \tag{2.23}$$

In Equation 2.23, $g_{x,j}$ is the growth rate of salaries over two consecutive years and $S_{x,1998+2j}$ is the monthly salary for a person of age x at time $1998 + 2j$, $j = 0, ..., 5$. This calculation is necessary since we differentiate between ages and gender and hence we cannot use the growth rate of salaries specified in Section 2.5.1. In other words, the values of gr_t are time, but not age or gender dependent. On the other hand, the values of g_x fluctuate with age, but they stay constant for that specific age throughout time, hence the accumulation factor $(1+g_x)^{t-t_0}$ used in Equation 2.21. We note here that the historical data is also provided by age groups, which leads to the growth rate of salaries being constant within the age group. The values of this variable for both men and women are displayed in Table 2.5.

	17-19	20-29	30-39	40-49	50-65
g_x for men	1.61%	1.13%	1.31%	1.31%	1.13%
g_x for women	1.97%	1.34%	1.74%	1.65%	1.41%

Table 2.5: Values of the growth rate of salaries for the different ages

The retirement credits in our study are equal to the minimum percentages set by law with respect to the mandatory part of the accumulated capital, as given in Table 2.6. As every pension fund can decide on their contributions for retirement savings, we consider two cases, also detailed in Table 2.6.

Age	Contributions for retirement savings Case a)	Contributions for retirement savings Case b)	Retirement credits
18-24	0.00%	0.00%	0.00%
25-34	7.00%	10.00%	7.00%
35-44	10.00%	10.00%	10.00%
45-54	15.00%	10.00%	15.00%
55-64/65	18.00%	10.00%	18.00%

Table 2.6: Contribution rates and retirement credits

In Case a) the contributions are equal to the retirement credits, hence no transfers take place. On the other hand, Case b) represents the limit case when contributions are constant for all ages. We remark here that even though the percentage of 10% is used by some pension funds, practice differs a lot. For that reason, the results presented hereafter are meant to be solely an illustration and will differ if other percentages are considered. We apply the specified percentages on the salaries of all active members, determined as per Equation 2.21 and then proceed to calculate the differences between contributions paid and retirement credits received. The resulting transfers are displayed in Figure 2.7. We observe that, in Case b), the actives receive, on the whole, more than they pay in terms of contributions for retirement savings. They thus withdraw the corresponding funds from the common pot and receive transfers from the retired. We also note that the transfers here are rather stable throughout the time. Moreover, should the contributions increase (e.g. if the contributions would be equal to 11% instead of 10%), the resulting transfers would decrease. In other words, if the 11% contribution would be applied, the actives would still receive more than they paid. However since they would pay 1% more compared to Case b), the gap between the amounts paid and received will decrease. It is important to note here that our results are highly dependent on the age structure of the pension fund and on the salaries. Since, in our scenario, most of the active insured members are aged 45 and above and larger contributions with respect to the retirement credits are only paid by the actives aged 25 to 34, it is not surprising that the actives are the ones receiving the transfers. Yet, if we would have a much younger pension fund, with a majority of active members belonging to the age groups below 45, the situation might be inverted, meaning that the actives might pay more contributions than retirement credits receives. Should that be the case, the actives would deposit the amounts into the common fund and we would find transfers towards the retired.

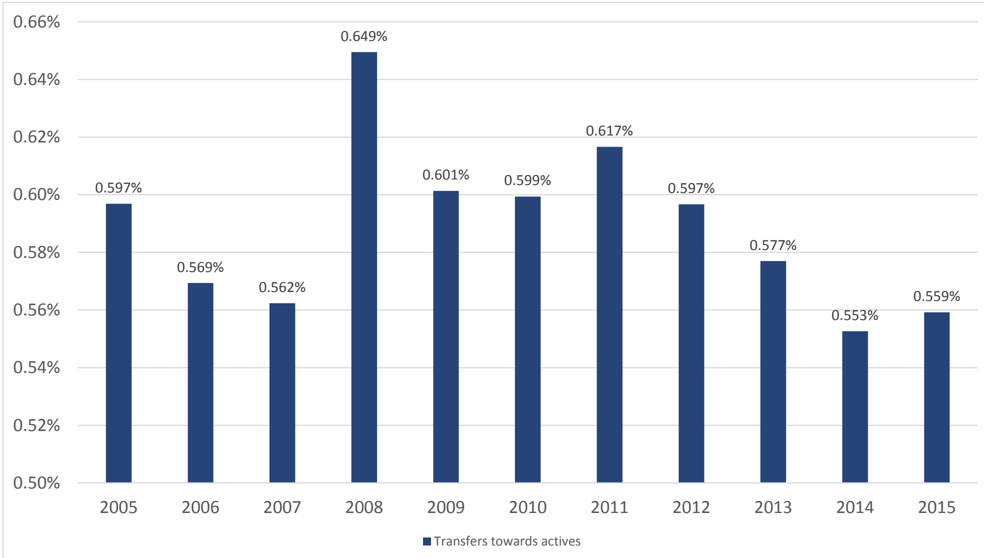


Figure 2.7: The transfers due to the contributions for retirement savings in case b) (expressed as percentages of the fortune of the pension fund)

2.5.5 Longevity and technical interest rate

The changes of life tables or technical interest rate reflect the most in the change of the mathematical reserves of the retired. In Equation 2.19, we define the total cost in this case with respect to the provisions created for this purpose. The cost is only calculated if the provisions are not sufficient to cover the increase in mathematical reserves and is equal to the resulting difference between these provisions and the amount that surpasses the old mathematical reserves. Hence the retired withdraw these differences, if existent, from the common pot and trigger a transfer in their favour. Since we consider here two separate provisions, one for longevity and one for the change of the technical interest rate, the transfers take place when either of them are insufficient. Table 2.7 sums up the life tables we use for each year, representing the most recent available sets as explained in Section 2.2, as well as the technical interest rates. These rates are defined each year by the Chamber of Experts in Pension Funds (see Swiss Chamber of Experts in Pension Funds (2015) for further details) and take into account the evolution of the financial market and the interest offered on

Swiss bonds. Therefore our table shows both when the life tables are updated and when the technical interest rate is changed. For example, in 2009, only the technical interest is lowered to 3.75%, while in 2010, both the life tables and the interest rate change. The aforementioned rate passes to 4.25%, while the new life tables are the LPP 2010.We mention here that these technical interest rates serve, in practice, as a reference for the expert actuaries and are therefore not mandatory. Since each pension fund has the liberty to decide what rate is best suited, we use the reference rates to perform our numerical illustration.

	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
Technical Interest rate	4.50%	4.50%	4.50%	4.00%	3.75%	4.25%	3.50%	3.50%	3.00%	3.00%	2.75%
Life Tables	EVK 2000	EVK 2000	EVK 2000	EVK 2000	EVK 2000	LPP 2010	LPP 2010	LPP 2010	LPP 2010	LPP 2010	LPP 2015

Table 2.7: Life Tables and the Technical Interest rate

As explained in Section 2.2, there is no common practice for creating the provision for the change of the technical interest rates, therefore we assume that the provision is zero throughout the period. The resulting transfers are showed in Figure 2.8. As expected, there are no transfers during the first three years, since the technical interest rate does not change. The same situation presents itself in 2012 and 2014, as the rates are equal to the ones used the previous year. Two interesting years are 2010, when the transfers are zero even if the technical interest rate changes, and 2011, when the transfer towards the retired is the highest. In general, mathematical provisions will increase when the technical interest rate decreases. However, in 2010 the rate increases from 3.75% to 4.25%. So the reserves suffer a decrease, leading to a zero cost related to the mathematical reserves and therefore no transfer takes place. However, the technical interest rate drops significantly in 2011, from 4.25% to 3.50%, triggering a high transfer towards the retired, as the reserves increase the most.

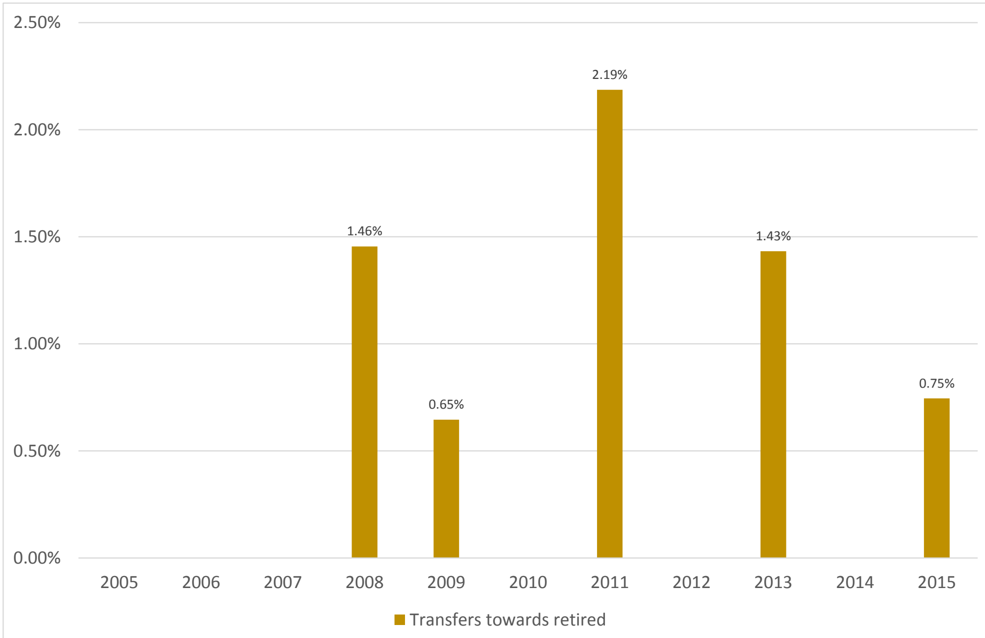


Figure 2.8: The transfers related to change in technical interest rate (expressed as percentages of the fortune of the pension fund)

The provision for longevity is constituted according to our explanation given in Section 2.2. We can formally define the initial provision for longevity $Pr^L_{t_{init}}$ as in Equation 2.24, where $\Delta_{t_{init}} = t_{init} - t_{ref}$. We define t_{ref} as the year when the life table was published and t_{init} as the year when the provision is first created.

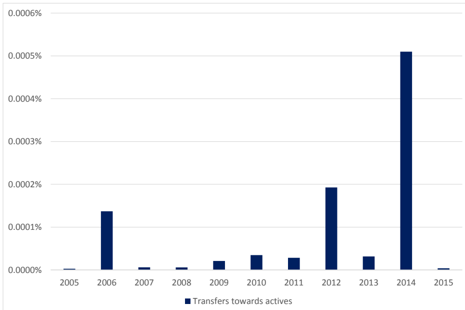
$$Pr^L_{t_{init}} = 0.5\% \cdot \Delta_{t_{init}} \cdot ICP_{t_{init}} \tag{2.24}$$

We then allocate 0.5% of the insured capital of pensioners ICP for every subsequent year when no change in the life tables used is registered. When the provision is impacted, namely in 2010 and 2015, no additional funds are attributed to the provision. In the year following a change, we restart constituting the provisions, as per Equation 2.24. After recalculating the mathematical reserves and charging the resulting difference to the provisions, we find that the respective provisions are insufficient and thus we arrive at a transfer of 0.29% in 2010, when we pass from the tables EVK 2000 to the LPP 2010, and of 0.27% in 2015, when the tables are changed to the LPP 2015.

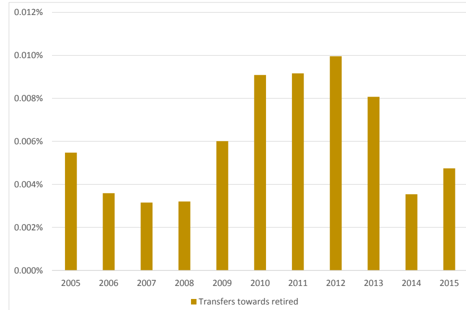
One final remark to be made here is that, in our framework, the transfers due to the change of the technical interest rate are much more frequent and have a bigger impact than the changes of life tables, leading to much higher transfers. However, a zero provision for the change of the technical interest rates represents a limit case and thus leads to higher transfers than if a provision does exist. Consequently, the situation will greatly vary depending on each pension fund's structure, on their decisions related to the rates and tables used and to their provisions. For example, a pension fund can decide to keep their technical interest rate constant for as long as their financial situation would permit it and can delay the adoption of new life tables until the provisions are enough to cover the resulting cost.

2.5.6 Consolidation measures

The last source of transfers identified regards the consolidation measures. As stated in Section 2.4.6, we only concentrate here on the extra contributions paid by the actives or the retired. The cost related to this source is defined in Equation 2.20, hence when one group pays an excess amount, the funds are deposited into the common pot and one part is transferred to the other group. The excess amounts, provided by the OFS, are translated into the transfers plotted in Figure 2.9. We note here that the two graphs in the previously mentioned picture have different scales, since the amounts of the contributions made by the two groups differ significantly and keeping a common scale would have rendered the transfers towards the actives unnoticeable. Indeed, the active members pay more than the retired, therefore the transfers towards the pensioners are higher than the ones towards actives. The transfers towards the actives are mostly insignificant. The most important values are registered in 2006 (0.0001370%), 2012 (0.0001927%) and 2014 (0.00051%). On the other hand, the transfers towards retired vary from 0.0031573%, registered in 2007, to 0.0099598%, in 2012.



(a) The transfers due to extra contributions paid by the retired



(b) The transfers due to extra contributions paid by the actives

Figure 2.9: Transfers related to excess contributions paid in the form of consolidation measures (expressed as percentages of the fortune of the pension fund)

2.5.7 Total Transfers

In light of all the results presented before, we look at the cumulated values for each year, in order to better understand which group receives the most transfers. Since we consider multiple cases for certain sources of transfer, it is natural that the total results might differ according to which situation is taken into account. We refer to these results as the base scenario and we take into account the following:

- transfers linked to the remuneration of the retirement capital when it is entirely accumulated either at the growth rate of salaries or at the minimum LPP interest rate;
- transfers linked to the indexation of pensions;
- transfers linked to new retirements;
- transfers linked to the contributions perceived as per case b) in Section 2.5.4 (contribution rates are constant for all ages, while the retirement credits increase with age)
- transfers linked to the longevity provision and the provision for change in the technical interest rate, when the latter is zero for the studied period;
- transfers linked to the consolidation measures.

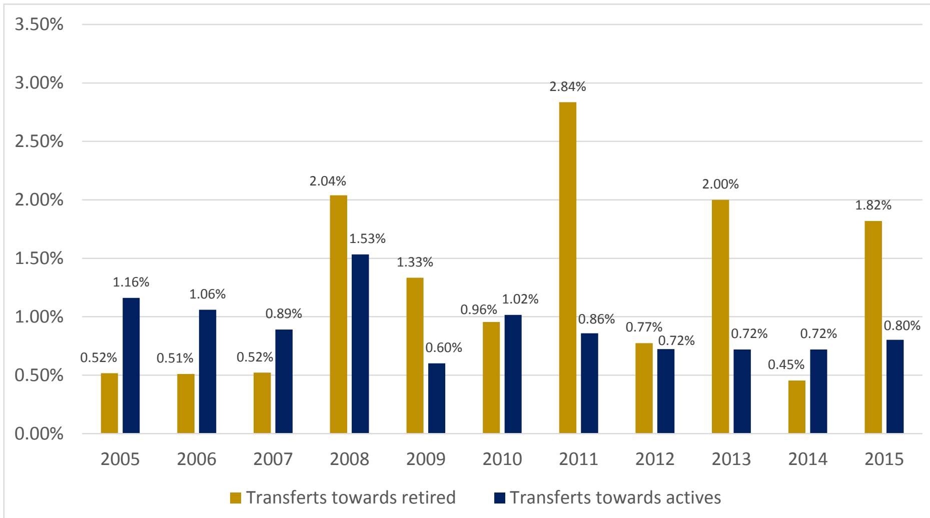


Figure 2.10: Total Transfers in the base scenario (expressed as percentages of the fortune of the pension fund)

A different split for the results is given in Appendix A. The results displayed in Figure 2.10 show that the transfers go both ways, as both actives and retired benefit from them. Moreover, the transfers towards the actives even surpass the ones for retired in 2005-2007, 2010 and 2014. The high transfer in favour of the pensioners in 2011 is mainly due the strong decrease of the technical interest rate from 4.25% to 3.5%, which causes a high increase in mathematical reserves that is not covered by the provision. The same increase in mathematical reserves explains the higher transfers towards the retired in 2008, 2009 and 2013, when the technical interest rates are lowered, as well as in 2015, when both the interest rates and life tables are changed. The transfers towards the actives are much more stable throughout the period and are mainly driven by the transfers related to the contributions. The higher value in 2008 is mainly a result of the cumulation of the transfers related to the contributions, as mentioned before, and of those linked to the indexation of pensions, since the inflation rate reaches the highest value for the studied period.

2.6 Conclusions

In our paper we study the generational transfers, so in other words the unintended redistributions, taking place within the occupational pension system in Switzerland. We first identify the main sources of transfers between active and retired members pertinent to the Defined Contribution schemes within the second pillar and develop a tractable framework that will allow each pension fund to calculate these transfers, as to shed a light on the complete picture related to this topic. We also provide a numerical illustration, by considering the entire system as one pension fund, which allows us to show that, contrary to the current opinions circulated on the topic, the retired are not necessarily the only ones to receive such transfers. Both groups might benefit from the unintended redistributions and, in fact, in our example, the active members even receive more transfers during certain years.

However, it is important to keep in mind that our results are namely just an illustration. For example, if the contribution rate is decreased by 1% with respect to case a) in Table 2.6, the transfers towards the actives increase and the total results are significantly impacted, as for certain years the transfers favouring the actives will exceed the ones towards the retirees, opposing the situation discussed in Section 2.5.7. Moreover, the amount provisioned for the change in technical interest rate or a different pro rata scenario can also change the total transfers and therefore, these total results will vary from one pension fund to another.

Each pension fund should then adapt the variables used to their specific regulations and situation, as to have a more realistic outcome. Figure 2.1 summarises the data that each pension fund should use, in order to adapt our framework and calculate the transfers ensuing from their specific case. The dependent variables refer to values that are endogenous to each pension fund, while the independent variables are exogenous to the fund. Since the data will change from one pension fund to another, the conclusions are likely to vary accordingly.

In conclusion, our study provides the tools to evaluate these transfers from a wider perspective and can be used to enlarge the discussion on the topic and to make room for better communication.

Appendices

A Splitting total transfers

In the methodology presented in this paper, the changes related to the mathematical and technical reserves are considered as yearly costs and therefore they yield transfers that can be added to the annual amounts. However, one might argue that the increases in the reserves have an impact on a long-term and thus should be considered separately. Given our numerical example, we split the transfers such that those related to the reserves (so those coming from the conversion rates, the technical interest rate and the longevity) are given in Figure A.2, while the remaining amounts (transfers linked to the remuneration of the retirement capital, to the indexation of pensions, to the contributions and to the consolidation measures) are displayed in Figure A.1. As expected, this results in a decrease of total transfers towards the retired, when comparing the results given in Figure 2.10 and those displayed in Figure A.1. When the values related to all sources are no longer summed up, the actives receive more transfers than the retired for the entire duration. Hence the impact of the reserves on the transfers towards the retired presented in our numerical example is significant.

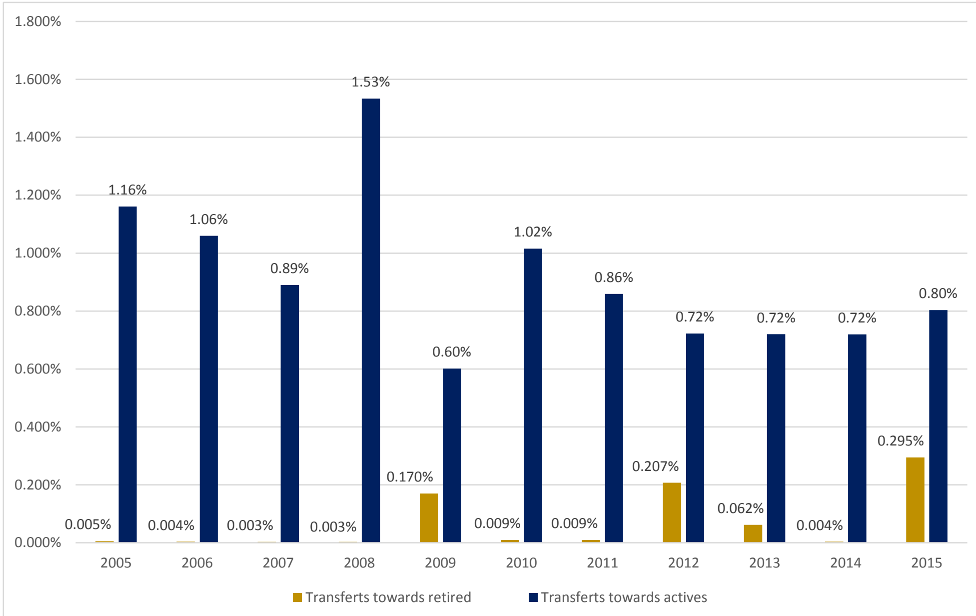


Figure A.1: Total Transfers without those pertaining to reserves (expressed as percentages of the fortune of the pension fund)

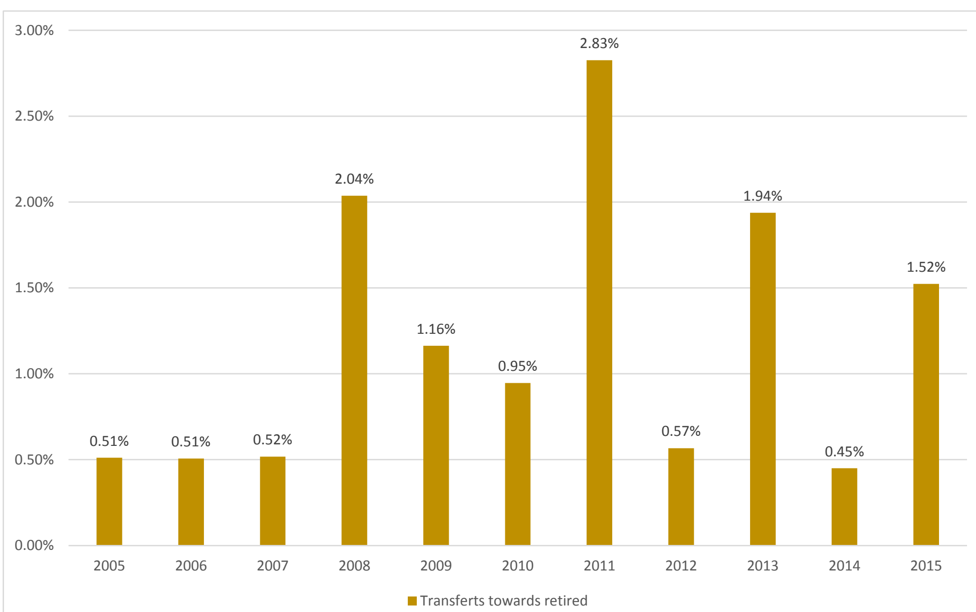


Figure A.2: Total Transfers pertaining to the reserves (expressed as percentages of the fortune of the pension fund)

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Chapter 3

Mortality by socio-economic class and its impact on the retirement schemes: How to render the systems fairer?

This chapter is based on the following working paper and is submitted for publication: Anca Jijiie, Jennifer Alonso-García and Séverine Arnold. Mortality by socio-economic class and its impact on the retirement schemes: How to render the systems fairer?. 2019. Working Paper.

3.1 Introduction

In this paper, we address the issue of actuarial fairness of pension schemes, given that socio-economic differences in mortality do exist and their impact is non-negligible. Besides discussing this matter through an example, we aim at providing an easy-to-implement solution, allowing policy-makers to not only improve the actuarial fairness of their pension schemes, but also to assess the extent to which pensions should differ depending on the socio-economic class.

Increased longevity has been a well known and well documented phenomenon in recent years, with significant impact on pension schemes around the world. For example, Oeppen and Vaupel (2002) note that the world life expectancy has roughly doubled in the course of 20 years, which has impacted the social needs of societies, among which pensions are included. OECD (2014) estimates that on average, taking into account future mortality improvements leads to higher life expectancies for both men and women than when period life tables are used (two years more for men and 2.5 years more for women at age 65, based on 2010 data). Hence the choice of mortality table becomes fundamental for pension funds and life insurance companies, with potential estimated shortfalls in reserves due to the use of period tables instead of generational ones going up to 20%. Bisetti and Favero (2014) project mortality for Italy and find that the longevity risk for the Italian pension system over the years could rise from 0.06% of the GDP in 2012 to 4.35% in 2050. Määttänen et al. (2014) also discussed the impact of increased longevity on five European countries and conclude that the cost, which is estimated as positive, would have to be paid either by the currently retired or the future generation. Moreover, they remark that the Finnish earnings-related pension system is not yet completely capable to sustain the ageing population. Lastly, Kisser et al. (2012) estimate, based on US panel data, that each additional year of life expectancy would increase the liabilities of US public and private pension funds by 3%.

With many pension schemes forming the first pension pillar still financed on a pay-as-you-go basis (the contributions perceived during one period are used to pay benefits during that same period), the burden of the increased life expectancy is far from getting any lighter (as also pointed out by Stevens (2017)). Indeed, in order to address this issue, many countries have proceeded to reforming their first pillars. OECD (2015) notes that almost all OECD countries have taken steps towards changing their systems, the most common reform measure being the increase in the minimum or legal retirement age. However, we must point out that such a measure does not account for the heterogeneity in mortality induced by socio-economic class. The relationship between socio-economic class and mortality has already been documented in the literature. For instance, Villegas and Haberman (2014) find significant differences in mortality between the most deprived and the least deprived individuals in England. Similarly, Nelissen (1999) finds 4.5 years of difference in life expectancy between individuals in the lowest social class and those in the higher social class, remarking that this impacts not only earnings, but also pension contributions and benefits. Shkolnikov et al. (2007) look into socio-economic mortality for German men, based on survey data and find that those belonging to a higher class, defined through occupation, can live more than two years longer than men in the lower class. Chetty et al. (2016) find that, in the United States, life expectancy rises with the income. Olshansky et al. (2012) also remark there is a difference in longevity in the US in function of the level of education, as well as race. On a similar note, Meara et al. (2008) observe that the gains in life expectancy have not occurred evenly for all socio-economic groups, defined in the paper by level of education, with highly educated individuals having more important improvements in life expectancy. Consequently, as lower socio-economic classes have a lower life expectancy than the higher classes, with inequalities still expected to rise (as also remarked by Ayuso et al. (2017)), increasing the retirement age would lead to individuals of lower classes spending even less time in retirement, as also pointed out by Sanzenbacher et al. (2015). Hence transfers are taking place towards those with a higher than average life expectancy, pointing out towards an unfair system, as also stated by Nelissen (1999), Barnay (2007) or Mazzaferro et al. (2012). A similar conclusion was also reached by Brown (2003), who found that when annuities are the same for all individuals, redistributions appear from the less wealthy to those that are in a better financial state. Redistributions in the pension systems, be they between the rich and the poor or between genders, have been a topic of other studies, though heterogeneity in mortality is not always considered or discussed. We do not enter into the details of such papers here, as we are focused on the transfers due to the differences in mortality by socio-economic class, but we refer the interested reader to, for example, Gustman and Steinmeier (2001), Le Garrec (2012), Coronado et al. (2002), Forteza and Ourens (2010) or Borsch-Supan and Reil-Held (2001).

Moreover, we must note that, besides not considering the socio-economic differences in mortality when increasing the retirement age, pension schemes do not take into account such differences when calculating the benefits. In particular, our paper deals with two pay-as-you-go systems: a Defined Benefit (DB) and a Notional Defined Contribution (NDC) scheme. If in a DB scheme, the benefits are fixed based on the average salary and the contribution period of an individual, with the contribution rates deriving from the benefits[1], in NDC schemes, each person has a notional account in which contributions are accumulated at a notional interest rate. At the retirement age, based on the mortality assumptions, the value accumulated into the accounts is transformed into a pension amount paid annually. Hence the benefits depend on the notional rate awarded, as well as on the mortality assumptions[2]. However, in practice, contribution rates are equal for all individuals in both DB and NDC systems, therefore not considering the socio-economic differences. Moreover, mortality by socio-economic class is not considered in determining the benefits under the NDC systems, which generally make use of unisex mortality tables. To illustrate this point, we use projected salaries and mortality by level of education[3] to calculate and compare the DB or NDC pensions with the actuarially fair pensions (in other words, what each individual should receive given their contributions and their class-specific mortality). Our numerical example shows that, under the parametrisation considered, neither one of the two schemes is fair. In fact, higher socio-economic classes seem to gain with respect to the actuarially fair pension, while lower classes would receive less than what is actuarially fair. A similar conclusion was reached by Caselli et al. (2003) and Mazzaferro et al. (2012). In other words, at a given retirement age, there exists a gap between what the individual should receive in order to maintain actuarial fairness and what is actually received. This gap should and can be filled by adjusting the parameters of the pension schemes, as it will be shown in this paper.

[1]For a more detailed description of DB schemes, see Bodie et al. (1988) or Wilcox (2006).

[2]For a more detailed description of the NDC system, see, for example, Palmer (2006), Börsch-Supan (2006), Vidal-Meliá et al. (2015) or Arnold et al. (2016).

[3]A list on the existing literature linking mortality to the level of education can be found in Ayuso et al. (2017).

Even if many studies have focused on the link between the retirement age and the socio-economic class, defined among others in function of the level of education, (see, for example, Sanzenbacher et al. (2015), Munnell et al. (2016), Rutledge et al. (2018), Venti and Wise (2015) or Stenberg and Westerlund (2013)), not enough has been said on what could be done to improve the fairness of the systems when the retirement age is fixed. In particular, the following studies are closer linked to this idea and therefore to our paper. Belloni and Maccheroni (2013) perform an analysis of the actuarial fairness of the Italian system, considering white- and blue-collar occupational differences and find that white-collar employees have a higher present value ratio[4]. Moreover, they remark that the Italian system is still unfair, even after the transition from the DB to the NDC scheme. However, the only suggested measure for improving the situation is the use of projected mortality, instead of the static mortality used by the Italian system, in the calculations of the NDC pension benefits. Bravo et al. (2017) also note the importance of considering heterogeneity in mortality, based on socio-economic factors, in the calculations of pension benefits, listing different possible interventions to mitigate its effect, including offering different accrual and accumulation rates to each socio-economic group, without going into more technical details on these two possibilities. Holzmann et al. (2019) define actuarial fairness in terms of a tax/subsidy rate for the NDC system and suggest different ways to introduce contribution rates dependent on the life expectancy of each socio-economic group. Lastly, though not specifically aiming at improving the fairness of the pension systems, Kuivalainen et al. (2018) note that socio-economic differences surpass gender differences, in terms of pension income in Finland, while Kudrna et al. (2018) propose introducing a means-tested pension in order to tip the scale towards those belonging to lower socio-economic classes.

Our paper contributes to the existing literature by offering, for the first time to the best of our knowledge, a tractable method that allows the systems to achieve greater actuarial fairness at a given retirement age, when the socio-economic differences in mortality are

[4]The present value ratio is defined as the ratio of the present value of benefits and the present value of contributions. It is a concept previously used in the literature (for instance, see Belloni and Maccheroni (2013)).

not considered by the pension schemes. This is done by adjusting the system parameters, namely the interest rate, accrual rate and notional rate, by socio-economic class, in order to compensate for the use of general mortality in the benefit calculations. The previously mentioned gap between the fair pension and the actual benefit is thus filled. Additionally to illustrating how such a process would occur based on our data by level of education, we aim at providing straightforward formulas for defining these parameters according to the socio-economic class and to the amount of data on socio-economic mortality rates available. Approximations to this formulas are also provided, in order to offer policy-makers an intuitive framework serving a double purpose. Determining such class-specific parameters will firstly allow those making decisions with respect to the pension systems to understand the importance of socio-economic mortality, by easily quantifying the extent to which pensions should differ across socio-economic class in a fairer system. Furthermore, our framework can be implemented in practice, allowing for fairer pensions even when class-specific mortality rates are not considered by the pension schemes.

The remainder of this paper is structured as follows: we define the DB and NDC pensions in Section 3.2. In Section 3.3, we assess the actuarial fairness of the DB and NDC schemes, based on our data by level of education and a defined set of parameters. We consequently illustrate, in the same section, the steps to take in order to adjust the parameters by class, along with the resulting values based on our data. We generalise our framework by providing mathematical expressions for the class-specific rates, dependant on the detail of the available data in Section 3.4. Lastly, we provide further avenues for our analysis in Section 3.5 and summarise our conclusions in Section 3.6.

3.2 The pension schemes

In this paper, we consider two pension schemes commonly used in practice, namely a Defined Benefit (DB) and a Notional Defined Contribution (NDC) scheme. Since we are interested in the social security systems (in other words, the first pillar in the three pillar pension system proposed by the World Bank (1994)), these pension schemes have a pay-as-you-go (PAYG) financing[5]. The pensions, defined hereafter, though considering the salaries per socio-economic class, do not take into account the mortality by social class. Hence, the relationship between contribution paid and benefits received might not correspond to the definition of an actuarially fair scheme. Indeed, in order to be actuarially fair, a pension scheme has to ensure, by definition, a present value ratio of one[6], so in other words that the present value at the moment of entry into the system of all contributions paid equals the present value at the same moment in time of all future benefits received, given salaries and mortality levels by socio-economic class, to account for heterogeneity. We refer to the pension satisfying this requirement as the theoretical pension. Although PAYG systems do not strive for actuarial fairness, a sustainable PAYG scheme will also be fair if the actuarial fairness is evaluated using an interest rate that is equal to the growth rate of the wage bill. Hence, the theoretical pension defined as specified above corresponds to the amount that ensures both the sustainability of the scheme and its fairness, but only if the interest rate is given by the growth rate of the wage bill. Consequently, utilising a different interest rate does not fulfil the above mentioned requirements for the PAYG systems and would thus not aid in establishing the fairness of the schemes as defined in this paper, since the sustainability of the systems cannot be guaranteed. Therefore, in order to asses the fairness of the DB and NDC schemes, we will need to compare the pensions given by each type of scheme to the theoretical pension.

[5]Though some countries have pre-funded first pillars, we focus here only on PAYG systems.

[6]The present value ratio is the ratio between the present value of benefits and the present value of contributions. It thus shows how much is returned to the worker for each monetary unit paid as contribution. A present value ratio of one indicates actuarial fairness.

As previously stated, the remainder of this section is dedicated to defining the DB and NDC pensions. For this, we allow Z socio-economic classes to coexist in the system. Individuals belong to the same class from the age of entry into the system, namely x_0^i, where i designates the class, until death. Retirement is taken at age x_r^i and the maximum lifespan is ω. Moreover, there is no unemployment or disability[7]. During their working years, individuals pay contributions as a percentage π of their salaries. To ease notation, the gender is not indicated in the given formulas through an index. However, the formulas are valid for both men and women and the subsequent analysis is split by both gender and socio-economic class.

3.2.1 The Defined Benefit (DB) Scheme

For the Defined Benefit (DB) scheme, we define the retirement benefit for an individual of socio-economic class i, retiring at age x_r^i at time t as $P_{x_r^i,t}^{i,DB}$ in Equation 3.1. Commonly, public DB pension schemes take into account the average wage over the last n working years, which we denote as $\overline{W}_t^i$. The pension is also a function of the accrual rate per year of affiliation, AR^i. In order to keep our formulas as general as possible, we consider that the accrual rate differs across the socio-economic classes. Moreover, if the individuals retire early, so before the legal retirement age x_{legal} (hence $x_r^i < x_{legal}$), a penalty of $b_{x_r^i}\%$ is applied. Similarly, if retirement is postponed ($x_r^i > x_{legal}$), a bonus of $b_{x_r^i}\%$ is awarded. We also note that the coefficients of penalty and bonus are dependent on the age at which the retirement is taken, but not on the class. In other words, postponing retirement for one year implies a different bonus percentage than postponing it for two years. These factors should be calculated actuarially, such that the equivalence between the present value of contributions and that of benefits is ensured. Finally, we note that for the purpose of this paper, the DB pension is described as

[7]Including unemployment and disability in our model would not change the conclusions of this paper. Considering these two aspects will reduce the career length and thus reduce pensions, but it will not affect the gap between the fair pension and the actual benefit that has to be filled in order to achieve greater actuarial fairness.

a function of the accrual rate AR^i.

$$P^{i,DB}_{x^i_r,t}(AR^i) = \begin{cases} \overline{W}^i_t \cdot AR^i \cdot (x^i_r - x^i_0)(1 - b_{x^i_r}\%), & \text{if } x^i_r < x_{legal} \\ \overline{W}^i_t \cdot AR^i \cdot (x^i_r - x^i_0), & \text{if } x^i_r = x_{legal} \\ \overline{W}^i_t \cdot AR^i \cdot (x^i_r - x^i_0)(1 + b_{x^i_r}\%), & \text{if } x^i_r > x_{legal} \end{cases} \tag{3.1}$$

Moreover, $\overline{W}^i_t$ is given by Equation 3.2 below, where $W^i_{x,t+x-x^i_r}$ is the salary of a person of age x at time $t + x - x^i_r$, belonging to class i, given that the retirement age x^i_r is reached at time t.

$$\overline{W}^i_t = \frac{1}{n} \sum_{x=x^i_r-n}^{x^i_r-1} W^i_{x,t+x-x^i_r} \tag{3.2}$$

Though the contribution rate in the DB scheme should ensue from the level of the pension and the mortality assumptions, that is not the case in typical social security systems. In practice, a constant contribution across time and social classes is used. This is why we adopt the same condition for the contribution rate. Hence π is a fixed percentage for all classes and genders, as well as across time and age. Nevertheless, Section 3.5.3 discusses the case when the contribution rate is different for each socio-economic class.

3.2.2 The Notional Defined Contribution (NDC) scheme

As pointed out by the World Bank (2005), the Notional Defined Contribution scheme mimics the mechanisms of a classical (funded) Defined Contribution scheme. If in a Defined Contribution scheme, each person has an individual account in which contributions are accumulated at a given interest rate, the process is similar in the NDC scheme. A notional account is created for every member in which contributions are accumulated at a notional rate nr^i (once again, for generalisation purposes, we allow the notional rate to depend on the class). However, these accounts, as well as the accumulation, are only virtual, since we are still in a PAYG setting. Moreover, the notional interest rate is based on a macroeconomic index that will ensure the sustainability of the system, such as the growth rate of GDP. It is not, therefore, an actual return on the financial market. In a NDC scheme, at the time of retirement, the present value of future pensions of a specific cohort should be, by definition, equal to the accumulated value of that cohort's savings account[8]. The pension amount is thus given by Equation 3.3 below, where $L^{unisex}_{x,t}$ is the number of people of age x alive at time t (given unisex mortality rates). In this case, the pension is calculated using unisex mortality, thus there is no difference made between classes or genders. Lastly, similarly to the DB pension, to ease the comprehension of the remainder of this paper, the NDC pension is defined as a function of the notional rate.

$$P^{i,NDC}_{x^i_r,t}(nr^i) = \frac{\pi \cdot \sum\limits_{x=x^i_0}^{x^i_r-1} L^{unisex}_{x,t-x^i_r+x} \cdot W^i_{x,t-x^i_r+x} \cdot (1+nr^i)^{x^i_r-x}}{\ddot{a}^{unisex,\beta}_{x^i_r,t}(nr^i) \cdot L^{unisex}_{x^i_r,t}} \tag{3.3}$$

Equation 3.4 provides the general definition for an annuity factor $\ddot{a}^{i,\beta}_{x_r,t}(r)$ as a function of a given interest rate r, thus following Bowers et al. (1997). Once again, i designates the class, while β is the indexation rate. Furthermore, $p^i_{x,t}$ is the class-specific survival rate, while ${}_kp^i_{x,t}$ is the probability that a person of age x at time t survives another k years. Hence, $\ddot{a}^{unisex,\beta}_{x^i_r,t}(nr^i)$, used in Equation 3.3, follows the same definition, but uses the unisex mortality and the notional rate nr^i instead.

$$\ddot{a}^{i,\beta}_{x_r,t}(r) = \sum_{k=0}^{\omega-x_r} \left(\frac{1+\beta}{1+r}\right)^k \cdot {}_kp^i_{x_r,t} \tag{3.4}$$

[8]We define here the NDC scheme such that the survival dividends (also referred to as inheritance gains) are distributed to the living individuals in the cohort at the time of retirement. For a detailed analysis of NDC schemes, please see Vidal-Meliá et al. (2015) and Arnold et al. (2016).

3.3 Assessing and improving the fairness of the pension schemes: a numerical example

In this section, we asses the fairness of both a DB and a NDC scheme for a given set of parameters. Subsequently, we optimise the parameters of each scheme in order to improve their fairness. As stated, this is a numerical example meant to illustrate how the actuarial fairness of a pension scheme can be improved in order to account for the mortality differences between socio-economic classes, given a set of original parameters, such as the contribution rates and the legal retirement age, among others.

3.3.1 The French Data

A first natural step in our example is, of course, assessing the fairness of a DB and a NDC pension scheme, when socio-economic differences are considered. For this, we use the data in function of the degree of education[9] provided by the French Office of Statistics. Table 3.1 summarises the categories for this classification, to which we attribute a category label. Hence D1 refers to the class with the highest level of education, namely people having an university degree, while D5 represents the class with no formal education. The French Office of Statistics[10] offers historical data on both salaries and mortality for these classes, which we use to project values for these two variables for the period 2016-2116[11]. The details regarding the data and the projections for salaries can be found in Appendix A, while Appendix B contains the details regarding the mortality data and projections. This classification suits our purpose, given that both salaries and mortality are provided for the same classes. In

[9]Defining socio-economic class in terms of the level of education limits the potential transitions between classes, as well as the incentives to switch class closer to the retirement age, since a higher level of education requires additional years of study. Therefore this classification allows us to make the assumption, for simplification purposes, that individuals remain in the same class throughout the years.

[10]https://www.insee.fr/en/accueil

[11]Although forecasting values on such a long period is not desirable, as it raises questions on the reliability of the values, it was in this case a necessary exercise. Because we require the salaries over the entire career of the individuals, together with their mortality for the entire lifespan, the long forecasting period was unavoidable.

addition, the historical data regarding the class-specific mortality allows us to project the mortality rates per class used for the remainder of this example.

Moreover, we define in the same table, the entry ages x_0^i for each class. Individuals with a higher level of education will enter the labour market later than the ones with lower degrees. Hence, people in category D5 enter as early as 15, while those with an university diploma will enter much later, at age 21. The entry age for the lowest class corresponds to the first age for which data is available, given the assumption that people with no formal education will start working at the earliest time possible. For the rest of the classes, we generally follow the description of the French educational system provided by Hörner et al. (2007). They note that the certificates for professional competence and studies are awarded at age 17, while those doing the Baccalaureate exam finish at 18. Once the school studies are completed, a Bachelor diploma requires another three years of studies, hence the entry age of 21 for the class D1. The only deviation from this description that we allow here is related to those having a National Diploma. Though Hörner et al. (2007) place the age of obtaining this diploma at 15, we decided to put it to 16 to allow a difference between the class D4 and D5.

Category	Descriptive	x_0^i
D1	Superior to Baccalaureate	21
D2	Baccalaureate	18
D3	CPC (Certificate of professional competence), CPS (Certificate of professional studies)	17
D4	National Diploma, CPrS (Certificate of primary studies)	16
D5	No diploma	15

Table 3.1: Socio-economic categories by level of education (France) and their entry ages into the system, adapted from Hörner et al. (2007)

3.3.2 Assessing the fairness of the pension schemes

As already indicated in the first paragraph of Section 3.2, the two pension schemes described in Section 3.2.1 and Section 3.2.2 do not necessarily ensure actuarial fairness. In fact, the differences in life expectancy across socio-economic groups affect the actuarial fairness of the system, that is, the relationship between the contributions paid and the retirement benefits received. By definition, under an actuarially fair scheme, the present value at the moment of entry into the system of all contributions paid should equal the present value at the same moment of all future benefits received. Hence, we denote by $P^{i,th}_{x^i_r,t}$, defined in Equation 3.5, the theoretical pension, that is the amount implied by an actuarially fair system for an individual from socio-economic class i and retiring at age x^i_r at time t. As before, $W^i_{x,t}$ is the salary of a person of age x at time t, belonging to class i, while r is the interest rate. Furthermore, ${}_{x-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0}$ is the probability of an individual from class i, aged x^i_0 at time $t-x^i_r+x^i_0$ (the time of entry in the system, where t corresponds to the time when retirement occurs) to survive to age x. As stated before, the interest rate r used here, should correspond to the growth rate of the wage bill (so $1+r=(1+g)(1+d)$, with g the growth rate of salaries and d the growth rate of the population).

$$P^{i,th}_{x^i_r,t}(r) = \frac{\pi \cdot \sum_{x=x^i_0}^{x^i_r-1} W^i_{x,t-x^i_r+x} \cdot (1+r)^{-(x-x^i_0)} \, {}_{x-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0}}{\ddot{a}^{i,\beta}_{x^i_r,t}(r) \cdot {}_{x^i_r-x^i_0}p^i_{x^i_0,t-x^i_r+x^i_0} \cdot (1+r)^{-(x^i_r-x^i_0)}} \tag{3.5}$$

Please note that the pension $P^{i,th}_{x^i_r,t}$ does not depend on the pension scheme studied, but that it solely depends on the life expectancy of the individual, their wages and the assumptions with regards to the interest rate r and contribution rate π. In practice, the pension actually paid will depend on the design of the public pension scheme. We can therefore compare the theoretical pension $P^{i,th}_{x^i_r,t}(r)$ to the one paid under the two different pension schemes considered here, namely the DB and the NDC scheme.

Consequently, in order to determine the fairness of each of the scheme, for each class, we use Equation 3.6 below, in which the difference in pension capitals at time t is denoted by $PV^{i,u}_{x_{ref},t}(x^i_r)$. In the previously mentioned equation, we compare the pension capital associated with a fair pension ($P^{i,th}_{x^i_r,t+x^i_r-x_{ref}}$) and the pension capital based on the (actual) amount received ($P^{i,u}_{x^i_r,t+x^i_r-x_{ref}}$, with u={DB, NDC} and i the class). The pension capital is calculated as the present value, at the fixed age x_{ref}, of future pension payments, given the retirement age $x^i_r \geq x_{ref}$ reached at time $t+x^i_r-x_{ref}$. Furthermore, as before, r is the interest rate and ${}_{x^i_r-x_{ref}}p^i_{x_{ref},t}$ the class-specific probability that a person of age x_{ref} at time t survives until age x^i_r. The annuity factor $\ddot{a}^{i,\beta}_{x^i_r,t}(r)$ is given by Equation 3.4. Consequently, a value of $PV^{i,u}_{x_{ref},t}(x^i_r)$ equal to zero means the pension received is actuarially fair, while a positive value indicates that the pension is more than fair and thus, the individuals are gaining. Conversely, a negative value means the pension is less than fair and the individuals incur losses.

$$PV^{i,u}_{x_{ref},t}(x^i_r) = (P^{i,u}_{x^i_r,t+x^i_r-x_{ref}} - P^{i,th}_{x^i_r,t+x^i_r-x_{ref}}) \cdot (1+r)^{-(x^i_r-x_{ref})} \cdot {}_{x^i_r-x_{ref}}p^i_{x_{ref},t} \cdot \ddot{a}^{i,\beta}_{x^i_r,t+x^i_r-x_{ref}}(r) \tag{3.6}$$

In order to proceed with our numerical illustration, we start by fixing the contribution rate $\pi = 14.3\%$. This rate ensures the equality between the present value of the contributions and the present value of DB benefits (as defined in Section 3.2.1) for an average individual that enters the market at age $x_0 = 17$, retires at the legal retirement age, faces unisex mortality and has average earnings (hence no class distinction is made), given an accrual rate AR of 1%. An interest rate $r = 1.8\%$[12] is used to determine the theoretical pensions for each class, as well as the value of the annuity in Equation 3.6.

[12] The variable r is the interest rate used to assess the actuarial fairness of the system. It does not correspond to an interest rate traded on the financial market, but to the growth rate of the wage bill. For our illustration, the interest rate has a value of 1.8%, which satisfies the relationship $1+r=(1+g)(1+d)$, with $g=1.4\%$ the growth rate of salaries calculated from our data and $d=0.4\%$ the growth rate of the population for the year 2016 in France.

For simplification purposes, the indexation rate β is set to zero[13]. Hereafter, we assess, in turns, the actuarial fairness of a DB and a NDC scheme, for retirement ages going from 50 to 75. Therefore, the reference age x_{ref} is the minimum retirement age considered here, namely 50.

3.3.2.1 The Defined Benefit scheme

Assessing the fairness of the DB scheme described in Section 3.2.1, process done according to Equation 3.6, starts by setting the legal retirement age x_{legal} to 65 for both men and women, for all classes, value that aligns with the policy of many OECD countries (see OECD (2017)). The accrual rate chosen is $AR = 1\%$[14], which is applied to the average salary $\overline{W}$ calculated over the entire career for all the socio-economic classes. The bonus and penalty values b_{x_r} are, just as for the contribution rate, calculated based on the average individual in the system, entering at age $x_0 = 17$, given an interest rate $r = 1.8\%$. They are therefore actuarially fair for the average individual, but not for each socio-economic class. The determined values ensure the equivalence between the present value of contributions and that of benefits and are given in Table 3.2. Hence, for example, if an individual retires at age 50, a penalty of 35.6002% is applied, while postponing the retirement to age 75 implies a bonus of 53.8640%. Since the legal retirement age is set to 65, there is no coefficient applied to this age. Given that these values are calculated based on an average person's experience, they are applied without distinction to all classes considered here and to both genders, as it is also done in practice. We calculate the value of PV, as given by Equation 3.6, for retirement ages x_r^i between 50 and 75. The results are displayed in Figure 3.1, which shows the difference in pension capitals, discounted to age $x_{ref} = 50$, given a retirement age between 50 and 75.

[13]The indexation rate does not impact the results pertinent to the NDC scheme, since it impacts both types of pensions (NDC and theoretical) in the same way. However, a positive indexation rate would shift the values related to the DB schemes, meaning that the PV values calculated according to Equation 3.6 increase with the indexation rate for all classes in the DB scheme.

[14]This rate implies that the individuals receive, depending on their class, between 44% and 50% of the average salaries over their entire careers. According to OECD (2017), among the countries offering this accrual rate (1%) we can find Hungary and Korea.

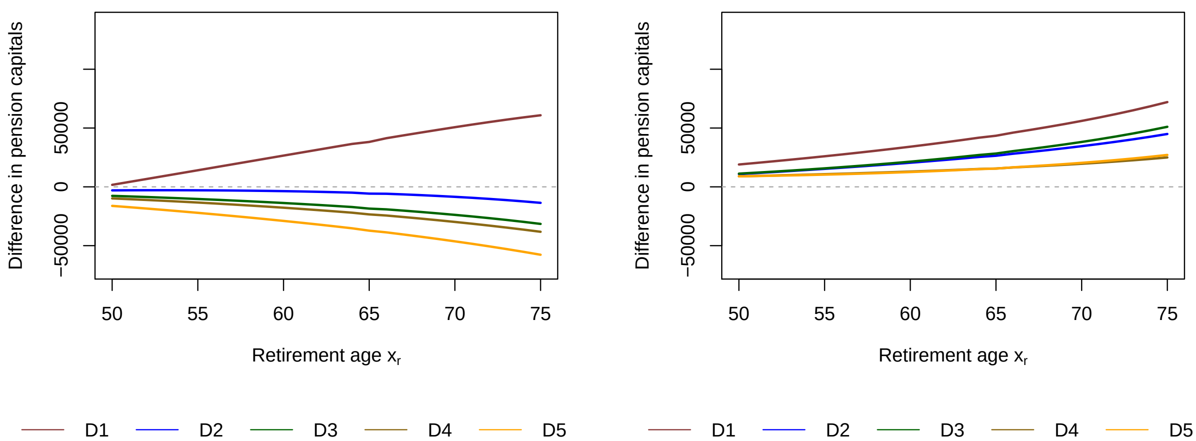
x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$
50	35.6002	57	22.4463	64	3.3061	71	27.0346
51	33.9804	58	20.1572	65	-	72	32.8732
52	32.2841	59	17.7405	66	3.8679	73	39.2397
53	30.5059	60	15.1850	67	7.8401	74	46.2077
54	28.6398	61	12.4785	68	12.1065	75	53.8640
55	26.6794	62	9.6072	69	16.7008		
56	24.6174	63	6.5556	70	21.6617		

Table 3.2: Penalty and Bonus values for the DB scheme for $x_{legal} = 65$

The results displayed in Figure 3.1 allow us to observe that such a DB scheme as the one set up here favours greatly individuals with a higher education, while the lower classes either suffer losses or do not gain as much. Though the advantage is more striking for highly educated men than for women of the same class, namely D1, the observation holds for both genders[15]. For men in class D1, postponing the retirement time translates into a higher gain with respect to the theoretical pension. In other words, the DB pension increases quicker than the theoretical pension, making it more attractive to retire late. Moreover, we notice that the penalty coefficients are insufficient for this class, since even when retirement is taken at age 50, thus 15 years before the legal retirement age, the difference in pension capitals is still positive. However, for men of lower education the situation is almost inverted. If class D2 is close to a zero difference for the interval proposed here, we note that for the remaining classes the DB pension is always smaller than the actuarially fair pension. The losses increase the more retirement is postponed, noting that these categories are at a disadvantage. The bonus of retiring later than the legal age is not enough to catch up with the increase in the theoretical pension due to the accumulation of the contributions paid and the fewer

[15]In some countries where the first pillar is DB, a cap is used with respect to the salaries insured under the system. However, because we make use of average salaries for this numerical example, we did not consider that a limit to the insured salaries was necessary. Nevertheless, in general, capping the salaries, and thus the DB pensions, would reduce the differences between high and low wage earners.

years spent in retirement. Hence those living longer are favoured by the lack of mortality consideration. The same can be said in the case of women, since we observe right from the start that all classes gain with respect to the theoretical pension. For them, the DB pension is much more generous than the actuarially fair (or theoretical) framework, even at the minimum retirement age considered. What is more, postponing the retirement age increases the gain, meaning that the increase in the DB pension surpasses the increase in the theoretical pension. Lastly, we remark that the reason for which women always gain with respect to the theoretical pension lies also in the calculation of the contribution rate, which uses unisex mortality. Due to the fact that female mortality is lower than the unisex one, the contribution rate is lower than it should be for women, leading to lower theoretical pensions. Hence the difference in pension capitals remains positive, with the DB pension being more generous than the theoretical framework.



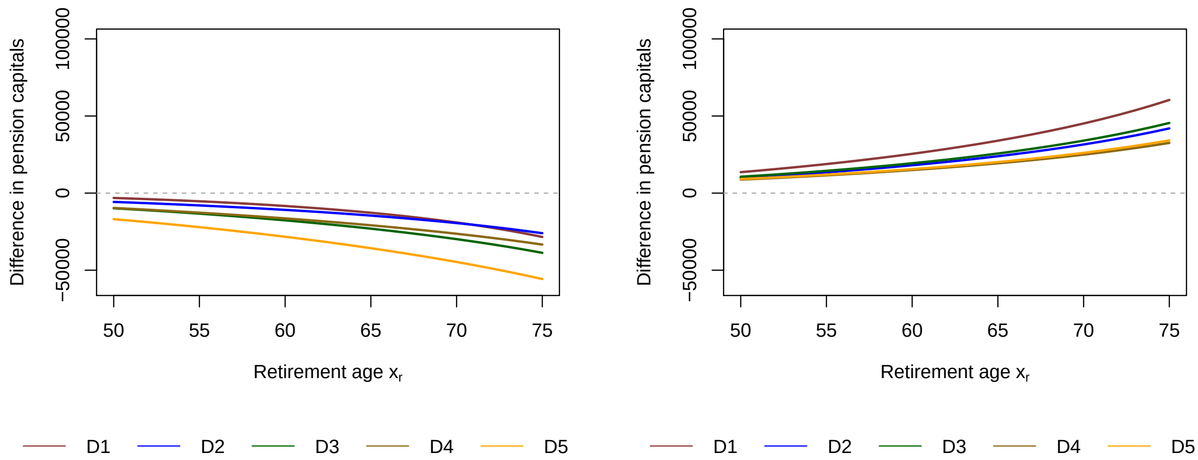
(a) $PV^{i,DB}_{x_{ref},t}(x_r)$ for men

(b) $PV^{i,DB}_{x_{ref},t}(x_r)$ for women

Figure 3.1: Difference between the DB pension capital and the theoretical pension capital, for individuals entering the system in 2016

3.3.2.2 The Notional Defined Contribution scheme

To study the fairness of the NDC scheme, described in Section 3.2.2, we keep the above mentioned contribution rate $\pi = 14.3\%$, the interest rate $r = 1.8\%$ and the indexation rate $\beta = 0\%$. We also set the notional rate of return, which we keep constant throughout time and across classes, to 1.8%. This notional rate satisfies the same relationship as the interest rate r, namely $1 + nr = (1 + g)(1 + d)$, with $g = 1.4\%$ the growth rate of salaries calculated from our data and $d = 0.4\%$ the growth rate of the population for the year 2016 in France. In Figure 3.2 we see, as for the DB scheme, that women gain with respect to the theoretical pension and this gain increases the more the retirement is postponed. This is, in fact, not surprising, since the NDC pension is calculated based on the unisex mortality, while the theoretical pension uses the corresponding class-specific female mortality, which is lower than the unisex one. However, for men the situation is reversed, with all the categories losing with respect to the actuarially fair framework. Thus, for men, the notional interest rate used to accumulate the retirement capital is not enough to compensate for the increased longevity inferred by the use of the unisex mortality, compared to the class-specific one. Even more, the difference in pension capitals decreases the more the retirement is postponed, suggesting that the cost of one year of life less spent in retirement is higher in the NDC scheme than in the actuarially fair framework. This is of more consequence to men in class D5, for whom postponing retirement to higher ages implies a higher loss than for the other classes. Lastly, it is important to note that those with the highest level of education are better off than those in the other classes. Indeed, men in class D1 lose the least (though the difference with respect to class D2 is minimal), while women in the same class gain the most, when compared to the theoretical framework.



(a) $PV^{i,NDC}_{x_{ref},t}(x_r)$ for men (b) $PV^{i,NDC}_{x_{ref},t}(x_r)$ for women

Figure 3.2: Difference between the NDC pension capital and the theoretical pension capital, for individuals entering the system in 2016

3.3.3 Improving the fairness of the pension schemes

As discussed above, for a given set of parameters, we find that neither the DB, nor the NDC scheme is fair, benefiting more the upper socio-economic classes and disadvantaging the lower classes. This is not the purpose of a social security system, which is meant to help those who really need it, namely the lower socio-economic classes. Hence, the mortality by socio-economic class should be considered in the design of the different schemes, as well as in the calculation of the actuarially fair pensions. However, in practice, the mortality rates by social class are not often used or even known. In order to improve the fairness of the system and thus compensate for the lack of use of the class-specific mortality rates, we suggest adapting the parameters that drive the pensions, namely the interest rate for the theoretical pensions, the accrual rate for the DB pensions and the notional rate for the NDC pension. Therefore, our process is done in steps, starting with the theoretical pension and so, with the interest rate, followed by the accrual rate and the notional rate for the DB and NDC schemes respectively. When socio-economic mortality differences are not considered in the calculations of actuarially fair (or theoretical) pensions, the interest rate awarded to each class should be adapted to ensure that the fair pension remains at the same level, regardless of the use of class-specific survival rates. Formally, we search for the r^i, so the interest rate for each class i that solves Equation 3.7.

$$P^{th}_{x^i_r,t}(r^i) - P^{i,th}_{x^i_r,t}(r^{fixed}) = 0 \tag{3.7}$$

In other words, we fix the interest rate r^{fixed} and calculate the theoretical pensions when the class-specific mortality rates are used. Hence $P^{i,th}_{x^i_r,t}(r^{fixed})$ is known for each class and gender. Consequently we look for the interest rate for each class that solves our equation, given that $P^{th}_{x^i_r,t}(r^i)$ utilises general mortality (so no class difference) rates[16]. Taking into account our numerical illustration provided until now, we set r^{fixed} to 1.8%, while the contribution rate remains $\pi = 14.3\%$. The entry ages into the system are those given in Table 3.1, while the retirement age is fixed at 65 for all classes and both genders, so $x^i_r = 65, \forall i$. The resulting interest rates for individuals retiring in 2066 are displayed in Table 3.3. Consequently, those belonging to class D1 reaching the age of 65 in 2066 have entered the system in 2022, while those from classes D2 to D5 have entered in 2019, 2018, 2017 and 2016 respectively.

We note that the class-specific interest rates given in Table 3.3 are unique solutions to Equation 3.7. Hence, the level of the interest rate for each class is not influenced by the type of system adopted, but is dependent on the value of r^{fixed}. Given our projections for the salaries and mortality for each class, we find that, in general the interest rates offered to lower social classes should be higher than those awarded to those with a higher education. This holds for both men and women, though the differences are slightly larger for men. Therefore, for individuals with a higher education and thus with higher survival probabilities, the use of the general mortality instead of the class specific one implies lower interest rates. If men belonging to class D1 only need an interest rate of 1.547%, we would have to offer a rate of 1.9487% to those of class D5. Similarly, women of class D1 require an interest rate of

[16]Though many alternatives exists for finding the root of our equation, we make use of the *uniroot* function in R, which is based on the bisection procedure (see https://stat.ethz.ch/R-manual/R-patched/library/stats/html/uniroot.html).

1.7585%, while for class D5 a value of 1.8214% is found. This is normal, since for lower classes, the general mortality is lower than the class-specific one, inferring lower pensions if the 1.8% interest rate would have been used. Hence, to ensure the equality a higher interest rate should be awarded. The inverse holds for higher classes. Finally, it is important to note that, in general, the gap between the rates given to the classes is smaller for women due their closer mortality and salary profiles, observation that is also visible in Figure 3.1b and Figure 3.2b.

	Male			Female		
Class	r^i	AR^i	nr^i	r^i	AR^i	nr^i
D1	1.5470	0.9070	1.9033	1.7585	0.8443	1.4198
D2	1.6620	1.0206	1.9724	1.7671	0.8649	1.4405
D3	1.7592	1.0756	2.0586	1.7803	0.8703	1.4554
D4	1.8111	1.1366	2.0958	1.7872	0.9028	1.4759
D5	1.9487	1.1872	2.2319	1.8214	0.9171	1.5127

Table 3.3: Class-specific parameters for individuals retiring at age 65 in 2066, in percentages, as obtained from Equation 3.7, Equation 3.8 and Equation 3.9

After determining the class-specific interest rates, we search for the accrual rate and the notional rate that would render the DB and the NDC pension, respectively, actuarially fair. In other words, we look for the rates that ensure the equality between the two types of pensions and the theoretical pension, respectively. Formally, this is given in Equation 3.8 and Equation 3.9 below.

$$P^{i,DB}_{x^i_r,t}(AR^i) = P^{th}_{x^i_r,t}(r^i) \tag{3.8}$$

$$P^{i,NDC}_{x^i_r,t}(nr^i) = P^{th}_{x^i_r,t}(r^i) \tag{3.9}$$

The solutions to these two equations are given in Table 3.3, alongside the values for the class-specific interest rates. We observe a similar situation for the accrual and notional rate for each class as for the interest rates. Both rates are higher for individuals with a lower education and hence lower salaries. We find that individuals with an university degree require an accrual rate of 0.907% in the case of men, while for women this value is 0.8443%. On the other hand, for those with no formal education, the accrual rate is 1.1872% for men and 0.9028% for women. We notice then that the spread between the lowest and highest class is more important for men than for women. Hence, as before, the differences are more visible for men than for women. The situation is not much different when we look at the notional rate. The highest socio-economic class should receive a notional rate of 1.9033%, in the case of men and 1.4198% in the case of women, while the lowest class is awarded a rate of 2.2319% for men and 1.5127% for women. One other remark to be made here is that the notional rate awarded to men is generally higher than the interest rate for the same gender, while for women the situation is reversed. This is due to the use of the unisex mortality rates for determining the NDC pensions. The unisex mortality is higher than the female mortality and lower than the male one. Thus, in order to preserve the equality between the actuarially fair pension and the NDC pension for the two genders, men should receive a higher notional rate to compensate for the inferred longer lifespan, while women can be awarded a lower interest rate, since the unisex mortality rates are favourable for them. Similarly, the accrual rates for women are lower than for men, since unisex mortality is utilised to determine the contribution rate used to compute the theoretical pensions, this being coupled, of course, with the higher salaries earned by men. Moreover, we must note here that in the case of the DB scheme, the effects of salaries and entry ages in the system are mixed with the effects of the socio-economic mortality rates, since the contribution rates are based on the average individual and they affect only the theoretical pension. A different contribution rate for each class, based on their corresponding salaries and entry ages would allow us to separate the two effects. Such a solution, though not necessarily possible in practice, is discussed in

Section 3.5.3.

Lastly, we can compare the obtained rates and the consequent pensions with the initial parameters and the pensions the individuals would have received (so in the case when $r = 1.8\%$, $AR = 1\%$ and $nr = 1.8\%$). We see that the accrual and notional rates for women in Table 3.3 are lower than the initial parameters. This is due to the fact that the systems were more generous for women (see Figure 3.1b and Figure 3.2b). Given that the DB and NDC pensions are increasing in the accrual and notional rates respectively, the lower rates for women mean that their pensions will decrease in order to meet the fair pensions. However, because the obtained rates are higher for lower classes, the pensions are not impacted to the same extend. For instance, decreasing the accrual rate from 1% to 0.8443% for class D1 induces a decrease in the DB pensions of 15.57%, while for the class D5 passing to a rate of 0.9171% implies a difference of only 8.29%. For the NDC pensions, the new notional rate for the women in class D1 results in a decrease of 12.5%, while for the class D5 the corresponding percentage is 10.5%. The situation is slightly different for men. Since men with higher education were advantaged by the DB scheme, while those in lower classes were loosing with respect to the fair pensions (see Figure 3.1a), the accrual rates for the upper classes decrease with respect to the initial parameters, while for lower classes they increase. Hence men in class D1 receive an accrual rate of 0.907% instead of 1%. At the other end, those in class D5 should get an accrual rate of 1.1872% instead of the initial 1%. Thus the DB pension of those in class D1 will decrease by 9.3%, while that of the individuals belonging to class D5 will increase by 18.7%. In the NDC scheme, men of all classes are at a disadvantage when compared to the theoretical framework (see Figure 3.2a). Hence men in class D1 receive a notional rate of 1.9033%, while those in class D5 are awarded a notional rate of 2.2319%, instead of the original 1.8%. Hence the increase of the pension for men in class D1 is of only 3.17%, while the increase for class D5 is of 17.8%. As stated before, the rates given in Table 3.3, through their impact on the pensions, will close the gap between the fair pension and that actually received, in order to compensate for not using socio-economic mortality rates in the pension calculations, thus reducing the transfers from the lower classes to the higher ones.

3.3.4 Extending the framework to include pension adequacy

We consider pension adequacy in terms of a minimum pension P_{min}, which is defined as a percentage RR_{target} of the mean salary in the system at time t, as given by Equation 3.10 below.

$$P_{min,t} = RR_{target} \cdot \overline{W}_t \tag{3.10}$$

As one of the goals of the social security system is to ensure a subsistence level for all individuals, it is only natural that such a target minimum pension is fixed within the system, at the legal retirement age. Depending on the chosen percentage RR_{target}, and thus on the level of the minimum pension, the interest rates, accrual rates and notional rates of those classes not reaching the intended target should be further adapted in order to allow these individuals to achieve the minimum required. To accomplish this, we look for the interest rates, accrual rates and notional rates that satisfy the equalities in Equation 3.11. The adapted rates will thus depend on the chosen target level $P_{min,t}$ and implicitly on RR_{target}.

$$P^{th}_{x^i_r,t}(r^i) = P^{i,DB}_{x^i_r,t}(AR^i) = P^{i,NDC}_{x^i_r,t}(nr^i) = P_{min,t} \tag{3.11}$$

In Switzerland, the subsistence level is defined as 40% of the mean salary in the system. Since the first pillar in France proposes a minimum pension of 37.5% of the average salary of the individual's career[17], we decided, for illustration purposes, to keep the minimum standard to 40% of the average salary in the system[18]. We start by calculating the minimum pension

[17] OECD (2015) notes that the maximum accrual rate for the state pension of 50%. The accrual rate is reduced by 1.25% for each missing quarter up to a maximum of 20 quarters. This translates into a minimum accrual rate of 37.5% ($50\% - 1.25\% \cdot 20 \cdot 50\%$).

[18] This rate has also been indicated as the minimum subsistence level in Holzmann and Hinz (2005), while Humblet and Silva (2002) remark that a replacement rate of 45% is needed to ensure the minimum living standard.

at the legal retirement age $x_{legal} = 65$, at time $t = 2066$ and we display in Table 3.4 the pensions calculated using the parameters from Table 3.3, expressed in percentage of the target minimum pension, of course at age 65.

	D1	D2	D3	D4	D5
Men	169	94	101	76	96
Women	77	53	60	45	53

Table 3.4: Pensions per class determined using the rates in Table 3.3, in percentages of the minimum pension

We see that the pensions for women are lower than those of men, because of their lower income and higher longevity. Indeed, we see that at the legal retirement age of 65, the pension for men with the highest level of education is more than 150% of the minimum, while women in the same class receive only 77% of the minimum pension. However, as expected, individuals with higher education benefit from higher pensions, and this regardless of the gender. If men in class D1 receive 169% of the minimum pension, those in class D4 only get 76% of the target pension. Similarly, women with an university degree reach 77% of the minimum pension, while the corresponding percentage for those in class D4 is 45%.

Given the percentages displayed in Table 3.4, we will need to adjust the awarded rates for women belonging to all classes, as well as for men belonging to class D2, D4 and D5. The new rates yielded by Equation 3.11 in this case are given in Table 3.5. We see, when comparing to the results in Table 3.3, that the rates to be awarded to these groups have to be increased in order to allow them to reach the intended level of 40% for the average salary in the system. Thus, for example, women in class D1 should receive an interest rate of 2.4807% instead of 1.4585% and so the accrual rate would pass from 0.8443% to 1.0972%, while the notional rate becomes 2.1646%, instead of 1.4198%. Similarly, the interest, accrual and notional rate for the class D5 are now 3.3872%, 1.7189% and 3.1233%, instead of 1.8214%, 0.9171% and 1.5127% respectively. The slightly lower rates awarded in this case to class

D3, compared to the other classes, can be anticipated from the percentage of the minimum pension that they receive, since this class is the closest to the minimum level among the four groups given here, given the data on salaries used for the projections. Men in class D2 will receive an interest, accrual and notional rate of 1.8537%, 1.0898% and 2.1585% respectively after the adjustment, as opposed to the respective original 1.662%, 1.0206% and 1.9724%. For the men in class D3 the rates pass from 1.9487%, 1.1872% and 2.2319% to 2.0547%, 1.2348% and 2.3351% for the interest rate, accrual rate and notional rate respectively. Of course, we remark once again that these results are meant to be just an illustration and thus, will depend on the minimum pension chosen and the data regarding the mortality and salaries for each class.

	Male			Female		
Class	r^i	AR^i	nr^i	r^i	AR^i	nr^i
D1	-	-	-	2.4807	1.0972	2.1646
D2	1.8537	1.0898	2.1585	3.4242	1.6218	3.1473
D3	-	-	-	3.1021	1.4394	2.8182
D4	2.5511	1.5006	2.8169	3.7824	2.0028	3.5262
D5	2.0547	1.2348	2.3351	3.3872	1.7189	3.1233

Table 3.5: Class-specific parameters for individual retiring at age 65 in 2066, adjusted given $RR_{target} = 40\%$, in percentages

3.4 Determining formally the class-specific rates

In this section, we provide easy-to-implement formulas for adjusting the parameters of the pension schemes (as illustrated by the previous section), in order to compensate for the absence of mortality by socio-economic class in the benefit calculations. Our framework allows policy-makers to render the pension system fairer, in a simple way, and to fully quantify the importance of considering socio-economic heterogeneity in mortality through the observed differences in pensions that will arise after the parameters are adjusted.

3.4.1 The general framework

As mentioned in Section 3.3, class-specific mortality might not be used in the determination of the pensions. In fact, mortality rates by socio-economic class might not be available or complete enough to yield reliable projections. This should however not impede the process of adapting the parameters of the pensions schemes as described in Section 3.3 in order to improve the fairness of the system. In this sense, it is possible to express the class-specific rates mathematically, if the relationship between the mortality of the general population and the one of the class is known and this for each gender. Hence, let us assume that the following relationship is known:

$$p^i_{x,t} = p_{x,t} \cdot M^i_{x,t} \tag{3.12}$$

In Equation 3.12 $p^i_{x,t}$ is the probability of a person of age x at time t, belonging to class i to survive to age $x+1$, $p_{x,t}$ is the general survival probability of a person also aged x at time t (hence no class distinction considered) and $M^i_{x,t}$ is an age-specific, time-specific and class-specific factor defining the relationship between the class and the general population. We note here that the gender is not specified, to ease notation, as the mathematical expressions will be identical for both genders. Given Equation 3.12, we can also express ${}_kp^i_{x,t}$, the class-specific probability for a person aged x at time t to survive to age $x+k$, in function of ${}_kp_{x,t}$ (the same probability, but without the class distinction) as below:

$${}_kp^i_{x,t} = {}_kp_{x,t} \cdot \prod_{u=0}^{k-1} M^i_{x+u,t+u} \tag{3.13}$$

In order to simplify the formulas, we drop the index i from the entry and retirement age. Hence from here onwards we refer to the entry age as x_0 and to the retirement age as x_r. However, this does not change the generalisation aspect of this section. The formulas work the same, even if these ages would be class-specific.

As in Section 3.3.3, we would like to ensure the fairness of the system by allowing a different interest rate per class r^i that would satisfy Equation 3.7, in order to compensate for the use of the general mortality, instead of a class specific one. Consequently, we follow the process described in Section 3.3 by fixing the interest rate r^{fixed} that should be used to calculate the class-specific theoretical pension $P^{i,th}_{x^i_r,t}$ and solving Equation 3.7 for the interest rate by class r^i. We can then show (the proof can be found in Appendix C) that Equation 3.7 holds for:

$$\begin{aligned} \frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{M^i_{x,t-x_r+x}}{1+r^{fixed}} \\ r^i_{x,t-x_r+x} &= \frac{1+r^{fixed}}{M^i_{x,t-x_r+x}} - 1 \end{aligned} \tag{3.14}$$

From Equation 3.14 above, we can deduce that, should the factor $M^i_{x,t-x_r+x}$ be larger than one, so in other words, should the survival probability of the class i be larger than the general gender specific survival rate, then the interest rate to be awarded, $r^i_{x,t-x_r+x}$, will be smaller than r^{fixed}. Hence those with higher than average survival rates will receive lower interest rates. Conversely, should $M^i_{x,t-x_r+x}$ be smaller than one, the interest rate awarded will be larger than r^{fixed}. Ergo, those with lower survival probabilities will receive higher rates.

Using Equation 3.8, we can easily express now the accrual rate for each class as a function of the theoretical pension as defined in Equation C.2.2, given the vector of interest rates $r^i_{vec} = \{r^i_{x0,t-x_r+x_0}, r^i_{x0+1,t-x_r+x0+1} ..., r^i_{\omega,t-x_r+\omega}\}$ found through Equation 3.14:

$$AR^i = \frac{P^{th}_{x_r,t}(r^i_{vec})}{\overline{W}^i_t \cdot (x_r - x_0)} \tag{3.15}$$

Lastly, we want to determine a formula for the notional rate of return for each class. For this, we first assume a similar relationship between the gender specific survival rate $p_{x,t}$

and the unisex rate $p^{unisex}_{x,t}$ as in Equation 3.12, therefore we have:

$$p_{x,t} = M_{x,t} \cdot p^{unisex}_{x,t} \tag{3.16}$$

Hence we find the following relationship between the interest rates and the notional rates (see Appendix D for details):

$$\begin{aligned} \frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{1}{M_{x,t-x_r+x}(1+nr^i_{x,t-x_r+x})} \\ \implies nr^i_{x,t-x_r+x} &= \frac{1+r^i_{x,t-x_r+x}}{M_{x,t-x_r+x}} - 1 = \frac{1+r^{fixed}}{M_{x,t-x_r+x} \cdot M^i_{x,t-x_r+x}} - 1 \end{aligned} \tag{3.17}$$

Similarly to the case described in Equation 3.14, should the factor $M_{x,t-x_r+x}$ be larger than one, so should the general gender-specific survival probability be larger than the corresponding unisex rate, then the notional rate awarded to class i, $nr^i_{x,t-x_r+x}$, will be smaller than the interest rate given to the same class $r^i_{x,t-x_r+x}$. Hence those that are favoured by the use of the unisex survival probabilities should receive lower notional rates. On the opposite side, should $M_{x,t-x_r+x}$ be lower than one, the notional rates will be larger than the respective interest rates.

3.4.2 A simplification

In many situations, the relationship between the survival rates by age and time, governed by $M^i_{x,t-x_r+x}$ and $M_{x,t-x_r+x}$, might not be known in such details, so by age and time. However, it might be possible to estimate average factors that would be kept constant through time and across ages or make an assumption as simple as Equation 3.18 and Equation 3.19, allowing pensions to still be adapted to increase fairness to all socio-economic classes.

$$M^i_{x,t} = y^i\% \tag{3.18}$$

$$M_{x,t} = z\% \tag{3.19}$$

With these two factors constant, the interest rates will no longer be time and age dependent, but will remain class specific. We can thus simplify the above expressions for the class-specific rates, obtaining:

$$r^i = \frac{1 + r^{fixed}}{y^i\%} - 1 \tag{3.20}$$

Consequently we obtain:

$$AR^i = \frac{P^{th}_{x_r,t}(r^i)}{\overline{W}^i_t \cdot (x_r - x_0)} \tag{3.21}$$

$$nr^i = \frac{(1 + r^i)}{z\%} - 1 = \frac{(1 + r^{fixed})}{z\% \cdot y^i\%} - 1 \tag{3.22}$$

To illustrate this, we estimate the two constant factors for the French data used in Section 3.3 by averaging across ages and across time. The values obtained are given in Table 3.6. As expected, the values for $z\%$ are the same for every class, since this factor defines the ratio between the gender specific survival rate, when no class distinction is made, and the unisex survival rates. Moreover, this rate is higher for women, due to the fact that unisex mortality is higher than the female mortality. With regards to $y^i\%$, we note that the rate decreases with the class, with the higher classes having a survival rate superior than the general one. The differences appear smaller for women than for men, congruent with our observations from Section 3.3.

	Male		Female	
Class	y^i	z	y^i	z
D1	100.54	99.82	100.29	100.91
D2	100.36	99.82	100.24	100.91
D3	100.21	99.82	100.15	100.91
D4	100.06	99.82	100.12	100.91
D5	99.82	99.82	99.90	100.91

Table 3.6: The factors governing the relationship between survival rates as given in Equation 3.18 and Equation 3.19

We then calculate the interest rates, accrual rates and notional rates according to Equation 3.20, Equation 3.21 and Equation 3.22 respectively. The results in this case are displayed in Table 3.7. We see that though the rates are different than the ones in Table 3.3, the values are in general not far from the initial ones. For instance, the difference between the interest rates r^i given in Table 3.3 and in Table 3.7 is of only 0.0384% for men in class D5, while the respective differences for the accrual and notional rate are, in this same case, 0.017% and 0.7353%. Moreover, they allow us to draw the same conclusions as in Section 3.3.3. For women in the same class, the differences between the interest rates, accrual rates and notional rates from the two tables are 0.0778%, 0.0288% and 0.5369% respectively. The lower classes require higher rates, with the spread between the newly obtained parameters being larger for men than for women. In conclusion, though not perfect, the approximation would allow providing fairer pensions, in function of the socio-economic class.

	Male			Female		
Class	r^i	AR^i	nr^i	r^i	AR^i	nr^i
D1	1.2502	0.8317	2.2232	1.5024	0.7696	0.5826
D2	1.4310	0.9432	2.4057	1.5582	0.7997	0.6379
D3	1.5876	1.0125	2.5638	1.6456	0.8272	0.7245
D4	1.7363	1.1053	2.7140	1.6789	0.8650	0.7574
D5	1.9871	1.2042	2.9672	1.8991	0.9459	0.9757

Table 3.7: Class-specific parameters for individuals retiring at age 65 in 2066, according to Equation 3.20, Equation 3.21 and Equation 3.22, in percentages

3.5 Further analysis

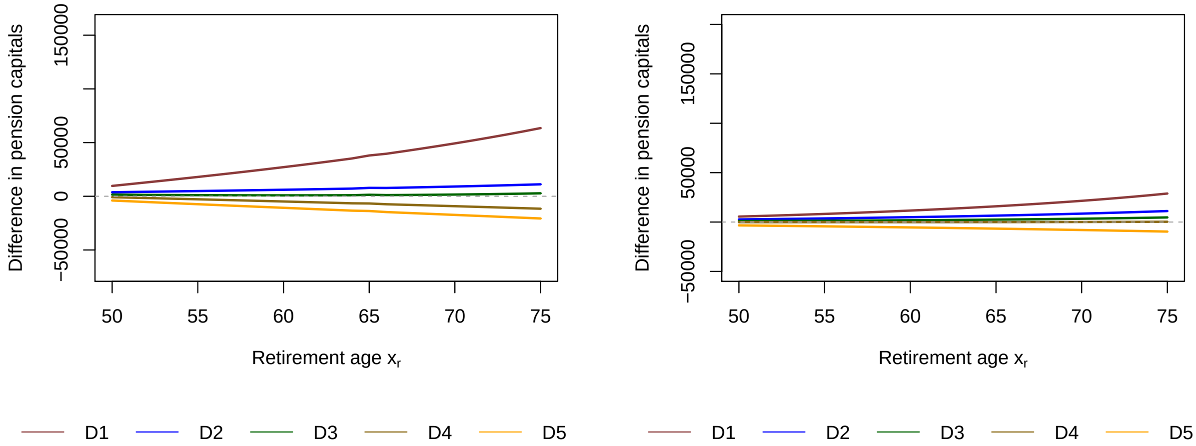
3.5.1 No gender distinction

In the European Union, differentiating between men and women is not legally allowed[19]. Hence the solution proposed in Section 3.3.3, where the interest rates, accrual rates and notional rates were adjusted for each class and gender in order to improve the actuarial fairness of the schemes, might not be possible to apply. Hence, we perform a similar analysis as done above, when no gender differences are considered. Consequently, salaries and mortality are distinct for each socio-economic class, but do not take gender into account. In setting up the salaries for each class for our numerical illustration, while disregarding gender, we assume a ratio of 51% to 49% between women and men (in other words, the salaries for each class are composed of 51% of the salaries for women and 49% of those for men of the same class).

[19]Council Directive 79/7/EEC (see Directive (1979)) specifically prohibits any discrimination between men and women in matters of social security. A famous court case in this respect is the one of Douglas Harvey Barber v Guardian Royal Exchange Assurance Group (see Shrubsall (1990)), in which the European Court of Justice ruled that occupational pensions are subject to the principles of gender equality established in the Treaty of Rome. Moreover, in 2011 the European Court of Justice ruled that discrimination by gender is prohibited in the insurance sector. Hence pricing insurance contracts must be done without considering gender.

Hence, the average salaries in the system are calculated, followed by the contribution rate and the bonus and penalty coefficients applied in the DB scheme. Consequently, the new contribution rate is 14.6%. The accrual rate is set at 1%, while the interest and notional rates are both 1.8%, as done in Section 3.3.2.

We then proceed to calculating the differences in pension capitals for the DB and the NDC scheme, as given in Equation 3.6. As in Section 3.3.2, the retirement ages considered are those from 50 to 75, with the legal retirement age being 65 and the reference age x_{ref} being set to 50. The results are displayed in Figure 3.3. Firstly, we observe, as expected, that individuals with the highest level of education are the most advantaged by either one of the schemes, with their gain increasing with age. In the case of the DB scheme, we observe that the lower classes experience a loss with respect to the actuarially fair pension, though not significant. Moreover, higher retirement ages imply an even bigger loss. The situation is slightly reversed for those in class D2 and D3, who experience a gain that increases with age. Lastly, we note that the differences between classes D2 to D5 are not big, while those with the highest level of education receive a distinct advantage from the two schemes. The situation is similar for the NDC scheme, although the gain of class D1 is less significant. Moreover, those in class D2 and D3 see their gain increase when retirement is postponed, while those in class D4 will go from suffering a loss to experiencing a small gain the more retirement is pushed. However, those without any diploma will lose with respect to the theoretical framework and their disadvantage increases with the retirement age.



(a) $PV^{i,DB}_{x_{ref},t}(x_r)$ (b) $PV^{i,NDC}_{x_{ref},t}(x_r)$

Figure 3.3: Difference between the DB or NDC pension capital and the theoretical pension capital, for individuals entering the system in 2016, when no gender differences are considered

Subsequently, we applied the methodology described in Section 3.3.3 to adjust the parameters in order to allow the system to achieve actuarial fairness. As previously stated, we consider $r^{fixed} = 1.8\%$ and we set the retirement age to 65. The results stemming from Equation 3.7, Equation 3.8 and Equation 3.9 in this case are given in Table 3.8. Of note here is that $P^{i,th}_{x_r,t}(r^{fixed})$ is calculated based on class-specific survival rate and class-specific salaries, without a gender distinction. It then follows that for $P^{th}_{x_r,t}(r^i)$ we will use class-specific salaries, but unisex mortality rates. As expected, the observations to be made here are similar to those from Section 3.3.3. The awarded rates are lower for those in higher classes and higher for those in lower classes, to compensate for the not considering class-specific mortality rates in the calculation of the pensions. Hence those in class D1 will receive an interest rate of 1.6560%, an accrual rate of 0.8889% and a notional rate of 1.6560%, while for those in class D5 the interest, accrual and notional rates are 1.8848%, 1.0704% and 1.8848% respectively. Moreover, we note that the interest rates r^i and the notional rates nr^i have the same values, since, when no gender distinction is made, the two pensions are equivalent. Lastly, we can compare the newly obtained rates with our initial parameters ($r = 1.8\%$, $AR = 1\%$ and

$nr = 1.8\%$). As expected, we see that only those in class D5 will receive an interest rate and a notional rate above the initial value of 1.8%, but that the adjusted rates are close to the initial parameters, indicating that only this class was facing a loss with respect to the theoretical pension, in 2066. The accrual rates for classes D1 to D3 are slightly lower than the initial 1%, since the individuals belonging to these classes were favoured by the DB scheme, while those in the remaining classes are awarded rates above 1% to compensate for their losses with respect to the actuarially fair pension.

Class	r^i	AR^i	nr^i
D1	1.6560	0.8898	1.6560
D2	1.7146	0.9620	1.7146
D3	1.7692	0.9922	1.7692
D4	1.7996	1.0395	1.7996
D5	1.8848	1.0704	1.8848

Table 3.8: Class-specific parameters for individuals retiring at age 65 in 2066, when no gender distinction is considered, in percentages

As in Section 3.3.4, we are also interested in making sure that the pensions reach adequacy. In other words, individuals should receive at least the minimum pension, defined in our numerical example as 40% of the average salary in the system. Given the rates presented in Table 3.8, we start by computing the corresponding pensions as percentages of the minimum pension and we display the results in Table 3.9. In this case, when no gender distinction is applied, only those in class D1 reach the intended pension target. The remaining classes are below the threshold for pension adequacy, with class D2 reaching only 75% of the minimum pension and class D4 obtaining 62% of the same reference amount. Lastly, we note that the lower pensions for classes D2 to D5 are also due to the fact that the salaries in this case are based on a slightly higher proportion of female wages than male.

	D1	D2	D3	D4	D5
Unisex	121	75	82	62	76

Table 3.9: Pensions per class determined using the rates in Table 3.8, in percentages of the minimum pension

Given the results displayed in Table 3.8, the interest, accrual and notional rates for classes D2 to D5 should be adjusted to ensure that the pensions are adequate. In other words, we repeat the process explained in Section 3.3.4 and given by Equation 3.11. The corresponding rates can be found in Table 3.10. The individuals belonging to class D2 should thus receive an interest, accrual and notional rate of 2.5255%, 1.2875% and 2.5260% respectively. For class D4 the corresponding values are 3.0567%, 1.6900% and 3.0572%. As explained, the new rates will allow the pensions for these classes to reach the minimum target pension. These results are in line with our previous observations, given in Section 3.3.

Class	r^i	AR^i	nr^i
D2	2.5255	1.2875	2.5260
D3	2.3076	1.2083	2.3081
D4	3.0567	1.6900	3.0572
D5	2.6127	1.4166	2.6132

Table 3.10: Class-specific parameters for individuals retiring at age 65 in 2066, adjusted given $RR_{target} = 40\%$, when no gender distinction is made, in percentages

3.5.2 Adjusting the parameters at retirement

In the analysis provided until now, we have considered socio-economic differences from the beginning of the working career. However, this might be difficult to implement in practice. Thus, a possible alternative that would be easier to put in practice for the policy makers consists of considering the differences in mortality based on socio-economic class only for the retirement phase. This section analyses this option in more depth. In other words, for the following analysis, all individuals are considered equal with respect to their mortality during the active life and the class-specific rates should only be applied from the retirement age onward.

Though the definition of the DB and NDC pensions does not change, our methodology for determining the class-specific rates needs to be reviewed. Moreover, in this section we add to the analysis the case of no gender distinction, as to keep in mind the limitations described in Section 3.5.1 above.

Let us begin with how the notional rate in a NDC scheme can be adjusted at the retirement age. As previously explained, in such a scheme the individuals accumulate a notional capital, which is then transformed into an annuity using unisex mortality rates. Since we are interested in considering socio-economic differences in mortality starting from the retirement time, we must differentiate between the active phase and the retirement phase for each socio-economic class. During the active phase, all classes will receive the same notional rate, while the notional rate awarded at retirement will be distinct for each class and will allow us to compensate for the use of unisex mortality rates instead of class-specific mortality rates. Hence the NDC pension at the legal retirement age x_{legal} can be rewritten as in Equation 3.23, with nr^{act} the notional rate awarded during the active life to all individuals, regardless of class. For simplification purposes, the formula assumes the notional rate during the active phase is constant for the duration. However, the methodology presented here can take into account a notional rate changing yearly. Our numerical illustration will account for both possibilities.

$$P^{i,NDC}_{x_{legal},t}(nr^i) = \frac{\pi \cdot \sum_{x=x_0^i}^{x_{legal}-1} L^{unisex}_{x,t-x_{legal}+x} \cdot W^i_{x,t-x_{legal}+x} \cdot (1+nr^{act})^{x_{legal}-x}}{\ddot{a}^{unisex,\beta}_{x_{legal},t}(nr^i) \cdot L^{unisex}_{x_{legal},t}} \tag{3.23}$$

$$= \frac{AccCap^i}{\ddot{a}^{unisex,\beta}_{x_{legal},t}(nr^i)} \tag{3.24}$$

We also redefine the theoretical pension at the legal retirement age x_{legal} as given in Equation 3.25, where $AccCap^i$ is the capital accumulated under the NDC scheme by an individual belonging to class i and r_{ret} is the interest rate at the moment of retirement (the rate for which the equation $(1+r) = (1+d) \cdot (1+g)$ holds, at the time of the retirement). Since the mortality differences are considered only at retirement, it is only logical that the accumulated capital should be the same for the NDC and the theoretical pension.

$$P^{i,th}_{x_{legal},t}(r_{ret}) = \frac{AccCap^i}{\ddot{a}^{i,\beta}_{x_{legal},t}(r_{ret})} \tag{3.25}$$

Subsequently, we look for the notional rate nr^i that solves Equation 3.26. As already explained, our goal is to determine the class-specific notional rates nr^i that will allow us to compensate for not using the mortality rates by socio-economic class in the calculation of the NDC pension.

$$P^{i,th}_{x_{legal},t}(r_{ret}) - P^{i,NDC}_{x_{legal},t}(nr^i) = 0 \tag{3.26}$$

If we develop Equation 3.26 as done in Section 3.4, using Equation 3.12 and Equation 3.16, we obtain a the same formula as given in Equation 3.17, with $x \geq x_{legal}$ and r_{ret} taking the place of r^{fixed}.

To illustrate this part of our paper, we start by assuming a constant notional rate of 1.8% for the active life, as well as an interest rate at retirement of 1.8%. This corresponds to the parameters used in Section 3.3. Hence the contribution rate stays at the level of 14.3%. The legal retirement age is considered 65, while the indexation stays zero. The solutions of

Equation 3.26 are given in Table 3.11. Because the accumulated capital is the same for the NDC and the theoretical pension, the contribution rate impacts both pensions in the same way and thus has no effect on the results stemming from Equation 3.26. We observe that the notional rates for men of all classes are higher than 1.8%, while the rates awarded to women are lower than the interest rate at retirement. This is in line with our previous analysis and the values displayed in Figure 3.2 and is due to the use of unisex mortality rates for the NDC pension. Moreover, we see once more that the rates for lower classes are higher than those for higher classes, allowing us to compensate for the use of unisex mortality rates in the calculation of the NDC pension. Hence men in class D1 are awarded a rate of 2.0415%, while those in class D5 receive 2.7528% starting at the retirement age. For women in class D1 the notional rate in this case is 0.96%, while for those with no education the corresponding value is 1.1059%. When no gender distinction is made, the notional rates are lower than the interest rate at retirement for all classes, except for class D5. Those with the highest level of education receive a notional rate of 1.5214%, while the individuals with no education are awarded a rate of 1.9550%. As for men and women, the rates for the higher classes are below those for the lower classes. The results are once again in line with the values displayed in Figure 3.3b. To further our analysis, we also take into account that at retirement, the interest rate might suffer a shock, even if the notional rate during the active period is 1.8%. If the interest rate at retirement is 1.5% and not 1.8%, the values for the class-specific notional rates are recalculated and are given in Table 3.11, alongside the results discussed above. The conclusions in this case are the same as those for the previous case, when the interest rate at retirement is 1.8%. We note that men receive higher rates than women, with the value of the interest rate at retirement of 1.5% being lower than the notional rates for men and higher than the notional rates for women. This is once more due to the use of unisex mortality rates in the calculation of the NDC pension. Moreover, individuals in higher class receive lower notional rates than those in lower classes. Hence the notional rates for class D1 are 1.7459% for men and 0.6502% for women. For those in class D5, the notional rates are 2.4598% and

0.7982% for men and women respectively. A similar conclusion can be drawn when no gender distinction is made, with the notional rate for class D1 being 1.2191% and the corresponding value for class D5 reaching a level of 1.6551%.

	$r_{ret} = 1.8$			$r_{ret} = 1.5$		
Class	nr^i men	nr^i women	nr^i unisex	nr^i men	nr^i women	nr^i unisex
D1	2.0415	0.9600	1.5214	1.7459	0.6502	1.2191
D2	2.2042	0.9766	1.6139	1.9096	0.6671	1.3123
D3	2.3835	1.0096	1.7206	2.0894	0.7006	1.4195
D4	2.4912	1.0232	1.7838	2.1978	0.7143	1.4831
D5	2.7528	1.1059	1.9550	2.4598	0.7982	1.6551

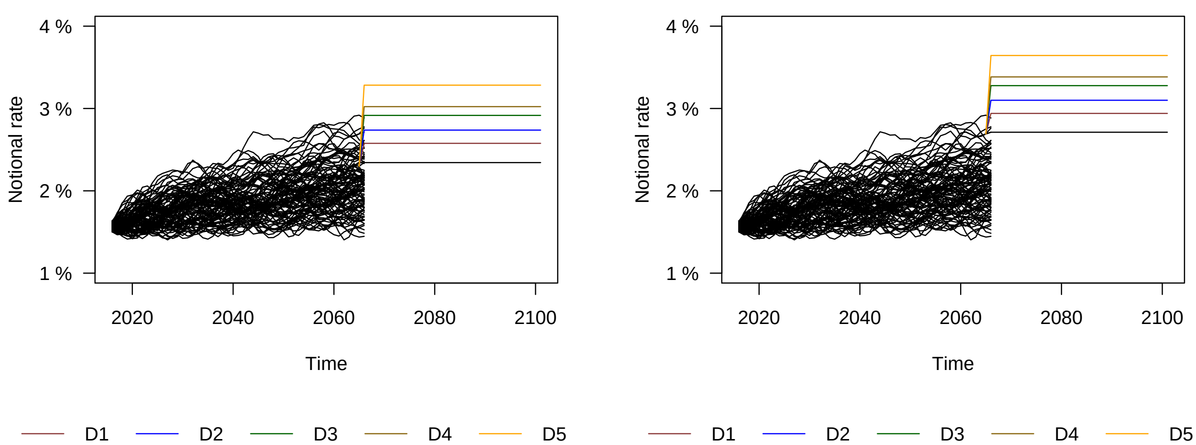
Table 3.11: Class-specific notional rates for individuals retiring at age 65 in 2066, when class distinctions in mortality are considered from retirement, in percentages

As an additional step, we allow the notional rate during the active life to differ each year. In order to accomplish this, we fix the growth rate of salaries to 1.4% and we fit an ARIMA(1,1,2) model[20] to the historical growth rate of the French population. Subsequently, we project the values of d_t by simulating 100 paths for the growth rate of population[21]. We then determine the notional rate for each year using the relationship $1+nr_t^{act} = (1+g)\cdot(1+d_t)$. At the retirement age, the interest rate r_{ret} corresponds to the notional rate of that specific year. Given the different simulations, the notional rates for each class and gender, as well as those with no gender distinction are determined according to Equation 3.26. Figure 3.4, Figure 3.5 and Figure 3.6 show the simulations for the notional rates during the active phase in black, as well as the results for the class-specific rates for men, women and when no gender distinction is made respectively, for two chosen simulations. The interest rate at retirement is 2.34% in the first depicted simulation and 2.71% in the second. Our previous observations hold in this case as well. We observe that men will receive a notional rate higher than the

[20] The model chosen presents the lowest AIC value.

[21] Only paths with positive values are considered as simulations, since it is the most reasonable assumption.

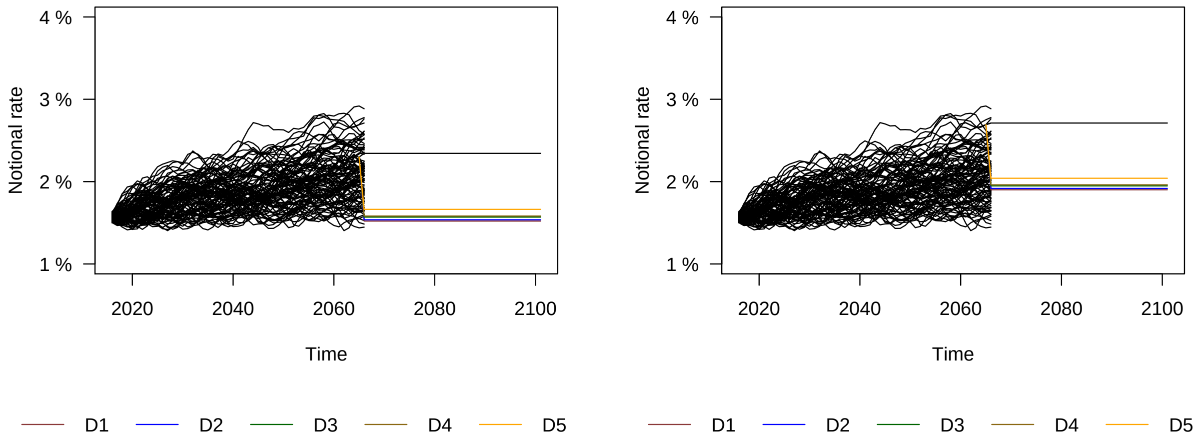
interest rate (in black in the plots), with men with the lowest level of education having a higher notional rate than the rest. For women, the notional rates are lower than the interest rate at retirement, but once again women with the lowest education receive the highest notional rates. Moreover, as before, we observe that the differences are more significant for men than for women. When no distinction for gender is made, only those in class D5 will receive a rate higher than the interest rate at retirement, with the remaining classes situated below the level. This is expected, since the values for salaries and class-specific mortality in this case are based on a gender distribution of 51% for women and 49% for men.



(a) Simulation 1

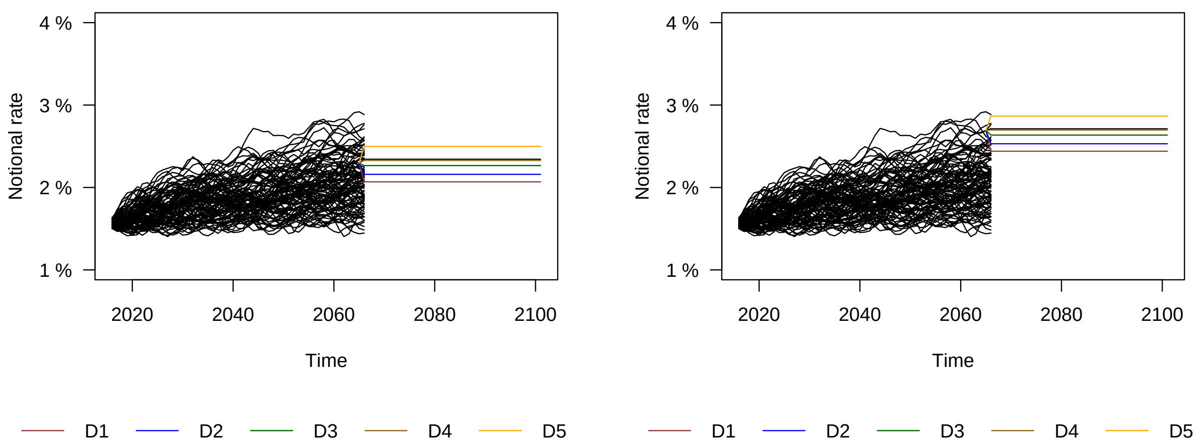
(b) Simulation 2

Figure 3.4: Class-specific notional rates for men retiring in 2066, based on simulations of the rates during the active phase



(a) Simulation 1 (b) Simulation 2

Figure 3.5: Class-specific notional rates for women retiring in 2066, based on simulations of the rates during the active phase



(a) Simulation 1 (b) Simulation 2

Figure 3.6: Class-specific notional rates for individuals retiring in 2066, when no gender distinction is made, based on simulations of the rates during the active phase

We are now interested in adjusting the accrual rate at the retirement age in a DB scheme. As expected, since there is no capital accumulation in this type of scheme, a similar reasoning as that described for the NDC scheme and given in Equation 3.25 and Equation 3.26 does not apply. We describe the process of adjusting the accrual rate at retirement in the remainder of this section. As previously explained, for an accrual rate fixed by the system, to which we will refer here as AR_{sys}, the contribution rate is calculated, based on the salary and mortality of the average individual. Hence the system assumes that the present value of benefits to be paid depends on the unisex mortality rates. However, in reality, the class-specific mortality rates will drive the present value at retirement of the benefits. Hence we are looking for the accrual rate for each class that will allow the two present values specified above to be equal. This is given in Equation 3.27, with $P^{i,DB}_{x_{legal},t}(AR_{sys}) \cdot \ddot{a}^{unisex,\beta}_{x_{legal},t}$ the amount that the system assumes the individual will receive and AR^{i}_{ret} the accrual rate that will allow the present value of what is actually paid to the individual to equate the value presumed by the system.

$$P^{i,DB}_{x_{legal},t}(AR_{sys}) \cdot \ddot{a}^{unisex,\beta}_{x_{legal},t} - P^{i,DB}_{x_{legal},t}(AR^{i}_{ret}) \cdot \ddot{a}^{i,\beta}_{x_{legal},t} = 0 \tag{3.27}$$

Developing Equation 3.27, we obtain the accrual rate for each class as given in Equation 3.28.

$$AR^{i}_{ret} = AR_{sys} \cdot \frac{\ddot{a}^{unisex,\beta}_{x_{legal},t}}{\ddot{a}^{i,\beta}_{x_{legal},t}} \tag{3.28}$$

In our numerical illustration, we keep the accrual rate set by the system to 1% (hence $AR_{sys} = 1\%$) and the indexation rate to 0. The interest rate used to calculate the values of the annuities is kept at 1.8%. The resulting accrual rates for each class are given in Table 3.12, for men, women and when no gender distinction is made. The observations to be made here are in line with our previous analysis on the notional rates. When the mortality differentials are only accounted for starting at retirement, men will receive higher rates than the fixed 1%, while the accrual rate for women is lower. For the case when no gender distinction is made, only those with no education receive a rate higher than 1%. However, the main conclusion still stands. Individuals in lower classes are awarded higher accrual rates than those in higher classes. Therefore, men in class D1 receive an accrual rate of 1.0314%, while those in class D5 are awarded a rate of 1.1263%. The corresponding rates for women are 0.8942% for class D1 and 0.9122% for class D5. When no gender distinction is made, the accrual rate for those with the higher education is 0.9643%, while for those with no education the rate awarded is 1.0201%. Lastly, we note here that a change in the interest rate used for calculating the annuities will not lead to significant changes in the results, since the rate impacts both annuities.

Class	AR^i_{ret} men	AR^i_{ret} women	AR^i_{ret} unisex
D1	1.0314	0.8942	0.9643
D2	1.0528	0.8962	0.9761
D3	1.0766	0.9003	0.9898
D4	1.0910	0.9019	0.9979
D5	1.1263	0.9122	1.0201

Table 3.12: Class-specific accrual rates for individuals retiring at age 65 in 2066, when class distinctions in mortality are considered from retirement, in percentages

Furthermore, we must remark here on the principal disadvantage of such an approach, namely considering the socio-economic differences only once the individuals reach retirement. Although this option might be easier to implement from the point of view of the policy makers, it will still slightly disadvantage those in lower socio-economic classes, as it does not account for their higher mortality during the active life. Hence the notional rates and accrual rate will not completely compensate for the lower life expectancy of those belonging to lower socio-economic classes. For instance, in the NDC scheme, since the accumulation phase does not consider the higher mortality for those groups, the survival dividends taken into account are lower than they should be. Moreover, for the DB scheme the accrual rate for those of lower classes are lower than the ones provided in Table 3.3, suggesting an insufficient compensation for the socio-economic differences. Still comparing between the two tables, we observe that the differences between the accrual rates given to each class are smaller when the socio-economic distinctions are only considered at retirement. This holds for both genders. Similarly, for the NDC scheme, as the active phase is identical between all the individuals, the differences between the classes are smaller in this case than when socio-economic differences are taken into account for the entire duration of the working life.

3.5.3 Different contribution rates

As previously explained, the choice of the contribution rates do not impact the values of the notional rates to be awarded, since both the NDC and the theoretical pension are affected by the contribution rates in the same way. However, the DB pension does not depend on the contribution rate, while the theoretical pension is impacted by it, hence a different contribution rate will lead to different accrual rates than those displayed in Table 3.3 and Table 3.10. Since the contribution rate of 14.3% is fixed based on unisex mortality, average salaries and an entry age in the system of 17, the effects of the class-specific mortality on the accrual rates of Table 3.3 and Table 3.10 are mixed with the effects of the class-specific salaries and entry ages. To remedy this situation and isolate the effect of the mortality by socio-economic class, the evolution of salaries for each class and gender, as well as the corresponding entry ages should be considered in the calculation of the contribution rates. The resulting values, using class-specific salaries and entry ages, but unisex mortality rates, are given in Table 3.13. As expected, the contribution rates for higher classes are higher, indifferent of gender, since they enter the system later and since their earning are higher. Moreover, women should pay a lower contribution rate than men to reflect their lower salaries.

Class	π^i men	π^i women	π^i unisex
D1	16.07%	14.78%	15.53%
D2	14.84%	14.48%	14.68%
D3	14.58%	14.47%	14.53%
D4	14.09%	13.97%	14.04%
D5	14.19%	13.95%	14.09%

Table 3.13: Class-specific contribution rates

Given the contribution rates given in Table 3.13, we recalculate the accrual rates to be awarded to each class. The corresponding values are given in Table 3.14. We observe that the accrual rates for those in lower classes are higher than for those with the highest level of education. Hence men in class D5 should receive an accrual rate of 1.1778%, while women in the same class should be awarded a rate of 0.8952% and when no gender distinction is considered the accrual rate is 1.0329% for the same class. For class D1 the accrual rates are 1.0191%, 0.8734% and 0.9472% for men, women and when no gender distinction is made, respectively. However, in this case the differences between classes are reduced in the case of men and the values overall are approaching those in Table 3.12. Moreover, we see that the rates for men are higher than the fixed accrual rate of 1%, while those for women are lower than the same fixed rate. When no gender distinction is made, only those with no education receive an accrual rate higher than 1%. As expected the differences between the classes are smaller in this case, when compared to the results in Table 3.3, since the distinction in contribution rates accounts for a part of the socio-economic gap. However, we note that setting the contribution rates differently for each socio-economic class to reflect the corresponding differences in salaries and entry ages does not completely explain and eliminate the discrepancies between the classes. This points towards the significant impact of the socio-economic mortality rates. Hence compensating for the lack of use of class-specific mortality rates remains an important task, in order to improve the fairness of the system and lower the disadvantage for the lower socio-economic classes.

Class	AR^i men	AR^i women	AR^i unisex
D1	1.0191	0.8734	0.9472
D2	1.0593	0.8761	0.9682
D3	1.0966	0.8809	0.9879
D4	1.1196	0.8826	0.9999
D5	1.1778	0.8952	1.0329

Table 3.14: Accrual rates for individuals retiring at age 65 in 2066, when contribution rates are different for each class, in percentages

We summarise the different scenarios considered in this paper in Table 3.15, along with the key findings for each case.

Scenario	Table	Key findings
Parameters adjusted at x_0 for each class and gender	Table 3.3	• Lower socio-economic classes should receive higher interest, accrual and notional rates. • The gap between classes is smaller for women. • Women receive lower accrual and notional rates.
Parameters adjusted at x_0 for each class with no gender distinction considered (unisex)	Table 3.8	• Awarded rates are lower for those in higher classes and higher for those in lower classes. • Interest and notional rates have the same values. • Only those in class D5 will receive an interest rate and a notional rate above the initial value of 1.8%. Accrual rates above 1% are awarded to those in class D4 and D5.
Parameters adjusted at retirement for each class and gender (or unisex)	Table 3.11 Table 3.12	• The awarded rates are higher for those with a lower level of education. • The awarded rates for men are higher than the values at retirement (notional rate of 1.8% and accrual rate of 1%). The reverse is true for women. • For the unisex case, only those with no education receive higher rates.
Accrual rates adjusted at x_0 for each class and gender (or unisex), when different contribution rates are paid by each class	Table 3.14	• The rates for those in lower classes are higher than for those with the highest level of education. • The differences between classes are reduced in the case of men. • The rates for men are higher than the fixed accrual rate of 1%. Those for women are lower than the fixed rate. For the unisex case, only those in class D5 receive an accrual rate higher than 1%.

Table 3.15: Summary of scenarios and key findings

3.6 Conclusions

In this paper, we focus on the actuarial fairness of the Defined Benefit and the Notional Defined Contribution pension scheme, when mortality rates differ by socio-economic class. We show, through a numerical example based on data by level of education from the French Office of Statistics, that these schemes can indeed be unfair. This is due to the fact that neither the DB, nor the NDC scheme incorporates mortality rates by socio-economic class. We find that not only do the DB and NDC pensions differ from the actuarially fair pension, but they also tend to advantage those with higher education. In reverse, individuals belonging to lower classes lose with respect to the actuarially fair pensions. We can thus conclude that socio-economic differences in mortality have a significant impact on the fairness of the retirement systems, be they the DB or NDC type. Therefore, mortality by socio-economic class should be included in the pension calculations. However, this is rarely done in practice. One reason for this could be the scarcity of data. Another possible explanation lies in the additional complexity introduced when considering the socio-economic mortality rates, since a new variable is added to the systems. An alternative is therefore required in order to help improve the fairness of the systems. Hence, we propose a simple methodology that allows each system to adapt its parameters, namely the interest rates, the accrual rates and the notional rates of return, for each socio-economic class. Our numerical example allows us to see that the rates should be higher for lower socio-economic groups, while individuals with higher education would receive lower rates. Subsequently, we looked beyond the fairness of each system and included pension adequacy in our framework. Hence, in order to allow all individuals to attain a given minimum pension level, the parameters for each system would need to be adapted again, for those not reaching the target value. In our example, we fix the minimum desired pension to 40% of the average salary in the system at the moment of retirement. Therefore, the class-specific rates need to be increased only for those not reaching the intended level.

We also provide simple mathematical formulas that allow us to determine the rates for each class, both when data on socio-economic level is enough to determine the relationship between class-specific survival rates and general survival probabilities, and when no data is available, but a simple hypothesis about the ratio between these two types of survival rates can be made. Our framework thus serves a double purpose. Firstly, it provides an easily-implementable tool to policy-makers that would help improve the actuarial fairness of the pension systems. Furthermore, our simplified version allows us to reduce the additional complexity listed above, since, in this version of our model, the mortality rates for each class are linked to the general gender-specific rates through a single parameter, constant throughout age and time. Although using the simplification means that the actuarial fairness is not perfectly met, it is still a step in the right direction. Secondly, our framework can be used simply to fully understand and quantify the impact of mortality by socio-economic class, since the pensions would be different by socio-economic class. Our numerical illustration already suggests that the above-mentioned impact is non-negligible and so this could be the case for all the countries around the world.

Lastly, we provide further analysis regarding the parameters to be adapted for reaching higher actuarial fairness. We offer a numerical illustration for the case when no gender distinction is made, as well as when contribution rates can be different for each class for the DB system. Our conclusions are in line with the previous analysis. Individuals in higher classes should be awarded lower rates, while those in lower classes should receive higher parameters, to compensate for not considering class-specific mortality rates in the calculations of the pensions. All the above mentioned situations are viewed from the moment of entry in the system. In other words, the parameters are adjusted for the duration of the career. However, this might be difficult to promote and implement. Hence a more accessible solution for policy makers would be to allow the parameters to be adapted at retirement, so the mortality differentials with respect to the socio-economic class are only considered starting from the retirement age. This point of view is also illustrated in our paper. The conclusions remain the same. However, it must be said that this method still disadvantages those in lower socio-economic classes, since it does not account for their higher mortality rates even before retirement. One alternative for fixing this issues would be to adjust the parameters each year, thus implementing a dynamic framework. This represents an avenue of further research stemming from the present study.

Another avenue to be explored that is closely linked to our methodology here would be how could the retirement age be adapted for each class, instead of the parameters considered here, to account for socio-economic mortality differences. Intuitively, individuals of lower socio-economic classes would retire earlier than those belonging to higher classes, since their life expectancy is lower.

The point of solidarity in a social security system is to redistribute wealth from the richer individuals to those in poorer conditions. However, as our example clearly illustrates, by not taking into account socio-economic differences in mortality the opposite might happen. Hence transfers from those in lower socio-economic classes to those in higher classes might take place, thus contradicting the aim of a social security system. In conclusion, our methodology comes as a solution to this situation, allowing fairer pensions and hence reducing the transfers from the poor to the rich. Therefore, our framework can and should be used to close the gap between the fair pensions and those actually awarded by the pension systems, and this for each socio-economic class, in order to compensate for the fact that the pension systems do not account for differences in mortality by socio-economic class.

Appendices

A The salaries

In order to project the salaries for each class, we assume homogeneity across active members of the same age. Thus, the wages for a person of age x at time $t \geq 2012$ are given by the Equation A.1, where $t_0 = 2012$:

$$W^i_{x,t} = W^i_{x,t_0} \cdot (1 + g^i_x)^{t-t_0} \tag{A.1}$$

We use the historical data for the period from 2006 to 2012[22] to calculate the annual growth rate of wages for age x and class i, g^i_x, as per Equation A.2 below:

$$g^i_x = \left(\prod_{j=0}^{5} (1 + g^i_{x,j}) \right)^{1/6} - 1 \tag{A.2}$$

$$g^i_{x,j} = \frac{S^i_{x,2006+j+1} - S^i_{x,2006+j}}{S^i_{x,2006+j}} \qquad 0 \leq j \leq 5 \tag{A.3}$$

In Equation A.3, $g^i_{x,j}$ is the growth rate of salaries from one period to the next one for each class i and $S^i_{x,2006+j}$ is the annual salary for a person of age x and class i at time $2006 + j$. The obtained values for the growth rate of wages g^i_x for men and women are presented in Table A.1.

[22] The historical data used is available with the authors upon demand.

Gender	Growth rate	D1	D2	D3	D4	D5
Men	g^i_{15-29}	2.26	1.29	1.91	1.76	2.25
	g^i_{30-49}	0.62	1.24	1.83	0.77	1.98
	g^i_{50+}	2.47	1.15	1.54	0.32	1.26
Women	g^i_{15-29}	1.95	1.47	2.19	0.59	2.47
	g^i_{30-49}	1.17	0.90	1.67	1.11	2.11
	g^i_{50+}	0.86	0.74	1.50	0.27	1.25

Table A.1: Growth rate of wages for men and women, in percentages

B Mortality

The historical mortality rates per level of education go from ages 30 to 100 for the years 1991-2013, grouped per periods. Hence we have three sets of mortality rates, namely for the periods 1991-1999, 2000-2008 and 2009-2013. Given the historical data for the period 2009-2013, we find that life expectancy at age 65 for men belonging to class D1 is 20.01 years, while for those in class D5 the value is 16.65 years. At the same age, women with the highest education (D1) are expected to live another 23.01 years, while those with no diploma have a life expectancy of only 20.6 years. Hence we see not only a significant difference between genders, with women living longer than men, but also between classes. It thus becomes important to include class differences in mortality in the calculations of pensions, alongside those of gender.

Though many models exist for projecting mortality in general, our data limitations restrict us from using classical models such as a APC model for each socio-economic class. Since we do not have the raw mortality rates or the disaggregated data per year for the number of deaths and the exposure to risk, a time trend cannot be extrapolated (as required by most models) and so a model cannot be fitted directly on the existing data. We thus proceed as suggested by Hunt and Blake (2017). We must fit a model to a larger population that represents a good reference and to which sufficient data is available. Consequently, we can estimate the difference in the level of mortality observed for each class with respect to the reference population. However, the model proposed by Hunt and Blake (2017) is much too complex for our socio-economic data. We hence use the extension of the Lee-Carter model proposed by Li and Lee (2005), also referred to as the common factor model. The chosen model allows us to fit the popular Lee-Carter model to the larger population (in this case the French population, separated by gender) and estimate in a simple manner the remaining parameter, that provides the mortality differential for each socio-economic group and gender. The common factor model is given by Equation B.1 below, in which we approximatw the force of mortality $\mu^i_{x,t}$ by the central death rate $m^i_{x,t}$, where α^i_x represents the class-specific and age-specific average mortality behaviour.

$$\log m^i_{x,t} = \alpha^i_x + \beta^p_x \kappa^p_t \tag{B.1}$$

In the bilinear term $\beta^p_x \kappa^p_t$, β^p_x corresponds to the age specific difference in mortality with respect to the average mortality for the entire population (hence the index p), while κ^p_t represents the evolution of the entire population's mortality across time. Hence the product is the same for all groups and derived by applying the modified Lee-Carted model proposed by Brouhns et al. (2002) to the French population directly. In the model described by Brouhns et al. (2002), the death count for each age and time is Poisson distributed and the mortality would be derived from Equation B.2.

$$\log m^p_{x,t} = \alpha^p_x + \beta^p_x \kappa^p_t \tag{B.2}$$

We also impose the two usual constraints:

$$\sum_x \beta_x^p = 1 \tag{B.3}$$

$$\sum_t \kappa_t^p = 0 \tag{B.4}$$

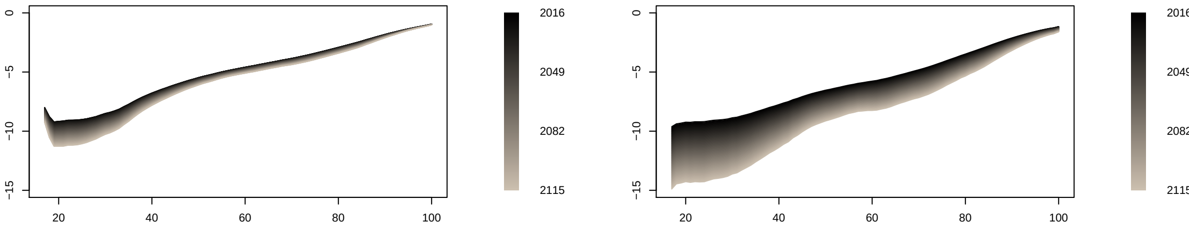
Going back to Equation B.1, we follow the framework of Li and Lee (2005) and estimate the term α_x^i by applying an OLS regression, which leads to the expression given in Equation B.5, with $T+1$ the number of periods available:

$$\alpha_x^i = \frac{\sum_{t=0}^{T} \log \hat{m}_{x,t}^i}{T+1} \tag{B.5}$$

Since we only have the values of $q_{x,t}^i$ (the mortality rate for a person of age x at time t and of class i), we determine $\hat{m}_{x,t}^i$ by following Pitacco et al. (2009) as given in Equation B.6.

$$\hat{m}_{x,t}^i = \frac{\hat{q}_{x,t}^i}{1 - 0.5 \cdot \hat{q}_{x,t}^i} \tag{B.6}$$

Therefore, we start by estimating the Lee Carter parameters for the female and male French population, using log likelihoods, fitted to the data from the Human Mortality Database for the period 1816-2015. We then use an ARIMA model to project κ_t^p for each gender (we use an ARIMA(1,1,1) for men and an ARIMA(2,2,3) for women, which correspond to minimum values of AIC) for a horizon of 100 years, in order to further determine the mortality rates for the ages 15 to 100. The projected mortality rates for men and women in this case are given in Figure B.1.



(a) Mortality projection for men (b) Mortality projection for women

Figure B.1: Mortality projections for France

By using Equation B.1, we then project mortality rates for each group from D1 to D5. For ages below 30, since we do not have class-specific mortality data, we assume that $\alpha_x^i = \alpha_x^p \cdot \frac{\alpha_{30}^i}{\alpha_{30}^p}$, for $x < 30$. These mortality rates are used in the calculations provided in this paper. Taking into account mortality improvements over time, as done here, allows us to be closer to a realistic situation regarding the evolution of mortality. However, the use of generational life tables as opposed to periodical does not impact the conclusions of our study.

C Interest rates by socio-economic class

As explained in Section 3.4.1, we want to determine the interest rates per socio-economic class that would compensate for not using the class-specific mortality rates in the pension benefit calculations, thus allowing us to achieve greater actuarial fairness. In order to simplify the formulas, we drop the index i from the entry and retirement age. Hence from here onwards we refer to the entry age as x_0 and to the retirement age as x_r.

We start by rewriting Equation 3.5 using Equation 3.13 and Equation 3.4, as well as the fixed interest rate r^{fixed}:

$$
\begin{aligned}
P^{i,th}_{x_r,t} &= \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot (1+r^{fixed})^{-(x-x_0)} \cdot {}_{x-x_0}p^i_{x_0,t-x_r+x_0}}{\ddot{a}^{i,\beta}_{x_r,t}(r^{fixed}) \cdot {}_{x_r-x_0}p^i_{x_0,t-x_r+x_0} \cdot (1+r^{fixed})^{-(x_r-x_0)}} \\
&= \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot (1+r^{fixed})^{-(x-x_0)} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} M^i_{x_0+u,t-x_r+x_0+u}}{\left(\sum_{k=0}^{\omega-x_r} \left(\frac{1+\beta}{1+r^{fixed}}\right)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} M^i_{x_r+u,t+u}\right)} \\
&\cdot \frac{1}{{}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} M^i_{x_0+u,t-x_r+x_0+u} \cdot (1+r^{fixed})^{-(x_r-x_0)}} \\
&= \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}
\end{aligned}
\tag{C.1}
$$

On the other hand, the theoretical pension when no class difference is considered for mortality rates, namely $P^{th}_{x_r,t}$, is given by Equation C.2.1.

$$
P^{th}_{x_r,t}(r^i) = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x-x_0)}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t}(1+r^i)^{-k}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x_r-x_0)}} \tag{C.2.1}
$$

We can now rewrite Equation 3.7 as follows:

$$\frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x-x_0)}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t}(1+r^i)^{-k}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot (1+r^i)^{-(x_r-x_0)}}$$
$$- \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \cdot \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}} = 0 \tag{C.3.1}$$

In order for Equation C.3.1 to hold, we require that the interest rate used to calculate Equation C.2.1 varies across age and time, in addition to the already considered socio-economic class. Hence, Equation C.2.1 becomes Equation C.2.2 below, with $r^i_{x,t}$ the interest rate dependent on the age x, time t and class i and $r^i_{vec} = \{r^i_{x_0,t-x_r+x_0}, r^i_{x_0+1,t-x_r+x_0+1} ..., r^i_{\omega,t-x_r+\omega}\}$.

$$P^{th}_{x_r,t}(r^i_{vec}) = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{1}{1+r^i_{x_r+u,t+u}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}} \tag{C.2.2}$$

Once again, we can plug Equation C.2.2 in Equation 3.7, resulting in:

$$\frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{1}{1+r^i_{x_r+u,t+u}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{1}{1+r^i_{x_0+u,t-x_r+x_0+u}}}$$

$$- \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{M^i_{x_r+u,t+u}}{1+r^{fixed}}\right) \cdot {}_{x_r-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x_r-x_0-1} \frac{M^i_{x_0+u,t-x_r+x_0+u}}{1+r^{fixed}}} = 0 \tag{C.3.2}$$

The below relationship, corresponding to Equation 3.14, is needed between the interest rates for each $x \in [x_0, \omega]$ and given that the age x_r is reached at time t, to ensure that Equation C.3.2 holds:

$$\frac{1}{1+r^i_{x,t-x_r+x}} = \frac{M^i_{x,t-x_r+x}}{1+r^{fixed}}$$
$$r^i_{x,t-x_r+x} = \frac{1+r^{fixed}}{M^i_{x,t-x_r+x}} - 1$$

D Notional rates by socio-economic class

After determining the interest rates for each socio-economic class (see Appendix C), we would like to determine the class-specific notional rates that would ensure the equality in Equation 3.9, with the purpose, as before, of reaching greater actuarial fairness.

We can firstly rewrite Equation 3.3 as follows:

$$P^{i,NDC}_{x_r,t} = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i)}}{\left(\sum_{k=0}^{\omega-x_r} (1+\beta)^k \cdot {}_kp_{x_r,t} \prod_{u=0}^{k-1} \frac{1}{M_{x_r+u,t+u}\cdot(1+nr^i)}\right) \cdot \frac{{}_{x_r-x_0}p_{x_0,t-x_r+x_0}}{\prod_{u=0}^{x_r-x_0-1} M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i)}} \tag{D.1.1}$$

According to Equation 3.9, we should determine the notional rate that ensures the equality between Equation D.1.1 and Equation C.2.2. Thus the notional rate has to evolve across age and time, as well as class, similarly to the interest rate. We rewrite Equation D.1.1 as follows, with $nr^i_{x,t}$ the notional rate for class i, at age x reached at time t:

$$P^{i,NDC}_{x_r,t} = \frac{\pi \cdot \sum_{x=x_0}^{x_r-1} W^i_{x,t-x_r+x} \cdot {}_{x-x_0}p_{x_0,t-x_r+x_0} \cdot \prod_{u=0}^{x-x_0-1} \frac{1}{M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i_{x_0+u,t-x_r+x_0+u})}}{\left(\sum_{k=0}^{\omega-x_r} \frac{(1+\beta)^k \cdot {}_kp_{x_r,t}}{\prod_{u=0}^{k-1} M_{x_r+u,t+u}\cdot(1+nr^i_{x_r+u,t+u})}\right) \cdot \frac{{}_{x_r-x_0}p_{x_0,t-x_r+x_0}}{\prod_{u=0}^{x_r-x_0-1} M_{x_0+u,t-x_r+x_0+u}\cdot(1+nr^i_{x_0+u,t-x_r+x_0+u})}} \tag{D.1.2}$$

By inserting Equation C.2.2 and Equation D.1.2 into Equation 3.9, we find the following relationship between the interest rates and the notional rates, corresponding to Equation 3.17:

$$\begin{aligned} \frac{1}{1+r^i_{x,t-x_r+x}} &= \frac{1}{M_{x,t-x_r+x}(1+nr^i_{x,t-x_r+x})} \\ \Longrightarrow \; nr^i_{x,t-x_r+x} &= \frac{1+r^i_{x,t-x_r+x}}{M_{x,t-x_r+x}} - 1 = \frac{1+r^{fixed}}{M_{x,t-x_r+x}\cdot M^i_{x,t-x_r+x}} - 1 \end{aligned}$$

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Chapter 4

Retirement ages by socio-economic class

This chapter is based on the following working paper: Séverine Arnold and Anca Jijiie. Retirement ages by socio-economic class. 2019. Working Paper.

4.1 Introduction

Increased longevity, a phenomenon well documented in the literature, is one of the major issues with which Pay-As-You-Go (PAYG) pension pillars around the world are faced (see, for instance, OECD (2014), Oeppen and Vaupel (2002), Määttänen et al. (2014) or Bisetti and Favero (2014)). Many countries have taken steps to address this problem[1], starting with increasing the retirement age for all individuals. However, the socio-economic differences in mortality are not accounted for, although they have been documented in the pertinent literature as well (see, for example, Shkolnikov et al. (2007), Villegas and Haberman (2014), Nelissen (1999), Chetty et al. (2016), Olshansky et al. (2012) or Meara et al. (2008)). Thus the universal increase in retirement ages disadvantages those in lower socio-economic classes,

[1] OECD (2015) notes that the majority of the OECD countries have initiated reforms of their first pension pillars.

as they will spend even less time in retirement. Moreover, as discussed in Jijiie et al. (2019), they are also facing losses when compared to the actuarially fair framework. Hence transfers take place from the lower classes to the higher ones. One way of compensating for this kind of situation is to allow for the retirement ages to depend on the socio-economic class of the individual.

The question that arises henceforth is how should these retirement ages by socio-economic class be defined by the system, by the policy makers. The utilitarian framework is frequently used in the literature to determine the optimal retirement age, generally in an economical context. This method takes in the point of view of the individual, as it accounts for, among other different parameters, the risk aversion of each person. Among the many models making use of the utility functions, we must specify two that are employed more often: the life-cycle model and the option value model. The option value model was introduced by Stock and Wise (1990) and it focuses on the value of postponing the retirement age. The utility value of retiring immediately is weighted against the utility derived from postponing retirement and thus earning a salary for a longer period of time. Hence the individuals retire when their retirement gain is maximum. Of note is that, in this model, the utility functions considered for the working period and for the retirement period encompass random effects that could capture the individual preference of leisure over work or the health status of the person, as well as a coefficient reflecting how much more one monetary unit with leisure is valued compared to one unit while working. The model proposed by Stock and Wise (1990) has been used in other studies. For instance, MacDonald and Cairns (2011) implement this model, but the utility levels are determined based on the ratio between consumption and current income. The model is then used to simulate retirement decisions dependent on portfolio choices, among others, for individuals with a defined contribution pension plan. Palme and Svensson (2004) also use the option value theory, but do not include the individual random effects. Other studies using this model are Lumsdaine et al. (1992), Samwick (1998), Hakola (1999), Panis et al. (2002) or Piekkola and Deschryvere (2004)[2]. One interesting variation of the option value model is the one-year model, also employed by MacDonald and Cairns (2011). In this particular variation, we would consider the gain in utility brought by delaying the retirement by one year only. Consequently, individuals can re-evaluate their decision to retire every year. The utility functions used for the active and retired period are defined in the same manner as in the classical option value model, but a constant utility value for leisure is considered in addition to the utility from the consumption during the working life and during retirement.

Utilising the life-cycle model with respect to the retirement age implies that the retirement age is chosen such that the lifetime utility of the individual is maximised. Hence we account for the utility derived from earning a salary and from receiving a retirement benefit for the entire lifespan, with the yearly utility values being discounted through an individual time preference factor. The survival probabilities from the initial age to all the subsequent ages counted in the model should also be considered when the discount factor is applied. Moreover, in order to apply this model to retirement decisions, we must account for either the disutility of work or the utility of leisure, both discounted correspondingly. The utility of consumption most commonly used is the Constant Relative Risk Aversion (CRRA) function, which remains also our choice for the remainder of this study[3]. Bloom et al. (2014) use the life-cycle model with a disutility function which is proportional to the mortality rate and conclude that the increase in life expectancy should lead to an increase in the retirement age. However this increase should be weighed against the increase in salary, as to account for the fact that a higher income can lead to a preference to retire earlier. In Knell and Nationalbank (2016) the disutility function is linear in age and the optimal retirement age is derived formally for the Notional Defined Contribution system considered and is thus dependent on the life expectancy. Rogerson and Wallenius (2009) use the utility

[2]This list is, of course, non-exhaustive.

[3]Besides the advantage of its simple form, this function fulfils the requirements imposed by the intertemporal separability (or time additivity) condition of the lifetime utility. See, for instance, Bagliano et al. (2004).

of leisure instead of the disutility of work, which depends on the number of working hours and the inter-temporal elasticity of substitution for leisure. A similar model is implemented in Ostaszewski et al. (2011), who determine that the optimal retirement age depends on the initial level of consumption. An increase in the initial consumption leads to an increase of the retirement age as well. Lacomba and Lagos (2006) also make use of the utility of leisure while retired and remark that if the contribution rates are modified by a change in the dependency ratio, a later retirement is preferred. Other studies in which the utility of leisure is employed include Samwick (1998), Sheshinski (1977), Hansen and Lonstrup (2009) and Jang et al. (2013).

Nonetheless, we are more specifically interested in retirement ages set by socio-economic class. Several studies have addressed this subject. For instance, Munnell et al. (2016) set target retirement ages for different socio-economic groups based on target replacement rates and then compare them to planned retirement ages according to their survey data. They find that there is a larger gap between the target and planned retirement ages for the lower socio-economic groups. Rutledge et al. (2018) provide a number of reasons for which less educated individuals do not choose to retire later. Among them, they note the more precarious health status, the labour market conditions and the lower gain from social security benefits due to a lower life expectancy. Similarly, Venti and Wise (2015) find that the proportion of highly educated individuals claiming early social security benefits was lower than the percentage corresponding to those with a lower education level. Hardy (1984) also find, in their study, that higher education levels are related to delayed retirement. Stenberg and Westerlund (2013) look into the impact of furthering education in adult life and conclude, based on their data, that reaching a higher education level at mid-age can indeed lead individuals to postpone the retirement phase. However, none of the papers listed above take into account the position of the government or the policy makers, nor do they consider a more actuarial approach. In particular, we note that the governing institutions have a dual objective: to provide income in retirement, while protecting those at a disadvantage, and to ensure the financial viability of the system. The primary purpose of our paper is proposing a viable method that can be used by policy makers to determine the retirement ages dependant on the socio-economic class, which would decrease the transfers from those in the lower socio-economic classes towards those in higher classes. To take into account the second purpose of policy makers listed above, we also check if the implementation of the proposed method would allow the schemes to be financially sustainable.

The contribution of our study to the existing literature is hence as follows. Firstly, we investigate the viability of using the utilitarian model for fixing the retirement ages for each class, when the point of view of the policy makers is considered. To accomplish this, by using data on mortality and salaries by level of education[4] from the French Office of Statistics, we implement the life-cycle model[5] under different scenarios regarding risk aversion and individual time preference, given a Defined Benefit (DB) and a Notional Defined Contribution (NDC) scheme, in order to find the optimal retirement ages for the classes considered. We thus observe that different combinations of parameters (risk aversion coefficient and individual time preference factor) lead to significantly different results. Although certainly an interesting and important methodology, as it offers insight into individual preferences, the utilitarian method is not practical from the point of view of the system. Since no consensus exists in the literature related to the values of these coefficients, as also pointed out by Azar (2010) and as it takes into account the individual preferences with respect to time and risk, this method appears to be volatile and thus not suitable for implementation for the institutions governing the pension schemes. Moreover, we note that, within our scenarios, most of them do not result in the financial stability of the pension schemes, adding to the

[4]Defining the socio-economic class with respect to the level of education allows us to limit the potential transitions between classes, as well as the incentives to switch classes close to retirement.

[5]Although the option value model is of particular interest for retirement decision problems, as it can account for more individual preferences, we focus in this study on the life-cycle model instead. Two reasons stand behind this choice. Firstly, the option-value model includes random factors in the construction of the utility functions. Because we are interested in social security systems, we feel that a model including random processes would veer too far from methods that could be implemented in practice. Moreover, in order to implement this type of model, individual data is required for the calibration of its many coefficients, including those pertinent to the random variables. At this moment, we do not have at our disposal the necessary data to perform a reliable study for this kind of model.

non-viability of such a methodology from the point of view of the policy makers.

Consequently, we propose an alternative method, based on the actuarially fair pension, for determining the retirement age for each class. This actuarial approach has not been yet, to the best of our knowledge, considered in the literature. In the proposed method, we utilise two accounts. In other words, we look for the retirement age that will allow the accumulated value, at age ω (the last age with survivors), of the pensions received under each system, held in one account, to be close in value to the accumulated amount should the actuarially fair pension be paid, representing the second account. The results are more stable and the financial viability of the systems is ensured, given our data, pointing to such a method being suitable for determining the retirement ages by the governing parties, as both of the systems' objectives are met. Though actuarial fairness and financial sustainability are not equivalent concepts (in other words, we can have one without the other), our alternative method builds implicit financial sustainability since the pensions are all calculated at the legal retirement age, but awarded at different times, while contributions are still paid up to the legal retirement age.

We also investigate what would be the amelioration or deterioration of mortality rates necessary for postponing or advancing retirement by one year, once again based on our data. Lastly, we provide a real case study for Switzerland, by implementing the specific scheme of the country and applying the actuarial framework proposed here to determine the optimal retirement ages. Therefore, even though our methodology is initially illustrated given the French data, in a theoretical setting, it can be adapted to other systems or countries.

The remainder of this paper is structured as follows: in Section 4.2 we define the pension schemes and the utilitarian framework and give the results ensuing from the scenarios considered and the data at our disposal. We consequently present the alternative model, referred to as the actuarial framework in Section 4.3, together with the corresponding results regarding the optimal retirement ages and with the analysis into the mortality differences driving a change in the retirement ages by one year. The case study on the Swiss pension scheme is also included in this section. Lastly, we present our conclusions in Section 4.4.

4.2 Our utilitarian framework

4.2.1 The pension schemes

As in Jijiie et al. (2019), we consider two schemes: the Defined Benefit (DB) scheme and the Notional Defined Contribution (NDC) scheme, since we are focused on the first pension pillar, usually financed on a PAYG basis. Individuals of class i enter the system at age x_0^i, which is class dependant, retire at age x_r^i, which will be optimally determined, and can live up to the maximum age ω. Though our goal is to determine optimal retirement ages for each socio-economic class, the system specifies a legal age for retiring, defined as x_{legal}. The contribution rate is equal across all socio-economic classes and is given by π. The DB benefit is defined in Equation 4.1[6]. The accrual rate AR is, in this case, equal for all classes, while $\overline{W}_t^i$ represents the average salary over the entire career of the individual of class i at time t. We also apply a bonus/penalty coefficient. Particularly, if a person postpones the retirement with respect to the legal age, a bonus is applied. Conversely, a penalty is deducted when the retirement is taken earlier. These coefficients should be determined such that the present value of future benefits equals the present value of future contributions. Moreover, the coefficients depend on the retirement age chosen, so, for example, postponing the retirement by one year implies a different bonus than waiting two years after the legal retirement age.

$$P_{x_r^i,t}^{i,DB} = \begin{cases} \overline{W}_t^i \cdot AR \cdot (x_r^i - x_0^i)(1 - b_{x_r^i}\%), & \text{if } x_r^i < x_{legal} \\ \overline{W}_t^i \cdot AR \cdot (x_r^i - x_0^i), & \text{if } x_r^i = x_{legal} \\ \overline{W}_t^i \cdot AR \cdot (x_r^i - x_0^i)(1 + b_{x_r^i}\%), & \text{if } x_r^i > x_{legal}\,. \end{cases} \tag{4.1}$$

[6]Please see Bodie et al. (1988) or Wilcox (2006) for more details on DB schemes.

The NDC pension benefit is defined in Equation 4.2. The contributions are accumulated in the notional account at the notional rate nr, with the survivor dividends being included[7]. Hence $L^{unisex}_{x,t}$ represents the number of people alive at age x and time t, given unisex mortality rates[8]. At the retirement age, the value of the account is transformed into an annuity, given unisex mortality.

$$P^{i,NDC}_{x^i_r,t} = \frac{\pi \cdot \sum_{x=x^i_0}^{x^i_r-1} L^{unisex}_{x,t-x^i_r+x} \cdot W^i_{x,t-x^i_r+x} \cdot (1+nr)^{x^i_r-x}}{\ddot{a}^{unisex,ind}_{x^i_r,t}(nr) \cdot L^{unisex}_{x^i_r,t}} \,. \tag{4.2}$$

The annuity factor $\ddot{a}^{i,ind}_{x_r,t}(r)$ is defined as a function of a given interest rate r as in Bowers et al. (1997), with ind the indexation rate. The survival probability for a person of age x at time t belonging to class i is denoted by $p^i_{x,t}$, while ${}_kp^i_{x,t}$ is the probability that a person of age x at time t survives another k years. Hence, $\ddot{a}^{unisex,ind}_{x^i_r,t}(nr)$ is calculated as per Equation 4.3, with unisex mortality and an interest rate given by nr.

$$\ddot{a}^{i,ind}_{x_r,t}(r) = \sum_{k=0}^{\omega-x_r} \left(\frac{1+ind}{1+r}\right)^k \cdot {}_kp^i_{x_r,t} \,. \tag{4.3}$$

4.2.2 The model specifications

Since it is not our purpose to study the impact of the evolution of mortality by socio-economic class over time, we consider, in this paper, that mortality stays constant across time. Mortality rates are thus differentiated only by socio-economic group. Moreover, we consider the same initial population distribution for each class (in other words, initially, the number of people of each age is the same for all classes), then derive subsequent population sizes for each group, given a population growth rate d. The salaries depend also on the class,

[7] For a more detailed description of NDC schemes, please see Palmer (2006), Börsch-Supan (2006), Vidal-Meliá et al. (2015) and Arnold et al. (2016).

[8] There are multiple ways of defining the NDC pension. Thus, we could consider only the people alive at age x and time t belonging to each class ($L^i_{x,t}$). However, since we are using unisex mortality tables for calculating the value of the annuity, we take into account all the people alive based on the unisex mortality rates, which allows us both to be closer to the practice and to ease the comprehension of our model.

but the growth rate of salaries g is the same for all classes and ages. Hence, the salaries for each class i, at age x and time t are given by Equation 4.4, with t_0 the time for which the initial level of salaries is known.

$$W^i_{x,t} = W^i_{x,t_0} \cdot (1+g)^{t-t_0}\,. \tag{4.4}$$

As mentioned in the introduction, we focus here on the life-cycle model. Since we do not look into the labour force and the labour supply, we decided to consider both the disutility of work linear in age and the utility of leisure as a constant, encompassed in two different models. Individuals consume all that they earn. This means that, during their retirement phase, consumption is equal to the pension. During their working years, individuals dispose of their salaries, after the contribution rate for the pension system is deducted. In this case, we assume that the rate π is equally divided between employer and employee. Hence, during the active life, consumption is equal to $\left(1-\frac{\pi}{2}\right)W^i_x$.

We can thus define the two models as follows:

1. **Disutility of work**

   In this case, the lifetime utility for class i, U^i_v, given the retirement age x^i_r reached at time t, is defined as:

$$\begin{aligned} U^i_v(x^i_r) = &\sum_{x=x^i_0}^{x^i_r-1} \beta^{x-x^i_0} \cdot {}_{x-x^i_0}p^i_{x^i_0} \cdot u\left(\left(1-\frac{\pi}{2}\right)W^i_{x,t-x^i_r+x}\right) + \sum_{x=x^i_r}^{x_\omega} \beta^{x-x^i_0} \cdot {}_{x-x^i_0}p^i_{x^i_0} \cdot u(P^{i,s}_{x^i_r,t}) \\ &- \sum_{x=x_0}^{x^i_r-1} \beta^{x-x^i_0} \cdot {}_{x-x^i_0}p^i_{x^i_0} \cdot v(x) \end{aligned} \tag{4.5}$$

   with s designating the system, either DB or NDC, $\beta \leq 1$ the individual time preference, the CRRA utility function $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, for the risk aversion coefficient $\gamma \neq 1$ (if $\gamma = 1$, then $u(c) = \ln(c)$) and $v(x) = v \cdot x$, where v is a constant (as done by Knell and Nationalbank (2016)). The function $u(c)$ gives the utility of consumption, while $v(x)$ represents the disutility of work.

2. **Utility of leisure**

   In the case of the utility of leisure, the lifetime utility for class i, at the retirement age x_r^i is determined as:

$$U_l^i(x_r^i) = \sum_{x=x_0^i}^{x_r^i-1} \beta^{x-x_0^i} \cdot {}_{x-x_0^i}p_{x_0^i}^i \cdot u\left(\left(1-\frac{\pi}{2}\right)W_{x,t-x_r^i+x}^i\right) + \sum_{x=x_r^i}^{x_\omega} \beta^{x-x_0^i} \cdot {}_{x-x_0^i}p_{x_0^i}^i \cdot u(P_{x_r^i,t}^{i,s}) + \sum_{x=x_r^i}^{x_\omega} \beta^{x-x_0^i} \cdot {}_{x-x_0^i}p_{x_0^i}^i \cdot v(l) \tag{4.6}$$

   with the utility of leisure $v(l) = l$ (following Lacomba and Lagos (2006)). The utility function $u(c)$ is defined as mentioned above.

Therefore, in both cases, we look for:

$$\max_{x_r^i \in \{x_{min}, x_{min}+1, \dots, x_{max}\}} U^i(x_r^i) \tag{4.7}$$

where x_{min} and x_{max} are the minimum and maximum retirement ages considered.

4.2.3 Data description and assumptions

As in Jijiie et al. (2019), we use the data on mortality and salaries from the French Office of Statistics for the five classes defined in Table 4.1. Considering socio-economic class dependant on the level of education helps us minimize as much as possible the issue of transitions between classes, since acquiring a new diploma is less frequent than, for example, changing professions.

Category	Descriptive	x_0^i
D1	Superior to Baccalaureate	21
D2	Baccalaureate	18
D3	CPC (Certificate of professional competence), CPS (Certificate of professional studies)	17
D4	National Diploma, CPrS (Certificate of primary studies)	16
D5	No diploma	15

Table 4.1: Socio-economic categories by level of education (France) and their entry ages into the system, adapted from Hörner et al. (2007)

We assume the system is put in place at time zero, which corresponds to the year 2016. Using the historical data on salaries starting from 2006 up until the year $t_0 = 2012$, we calculate an average growth rate of salaries of $g = 1.4\%$. This rate is subsequently used to determine the salaries for each class according to Equation 4.4, from 2016 forward. The indexation is defined as equal to the growth rate of salaries to ensure that pensions grow at the same rate as the wages[9]. The growth rate of the population corresponds to the official rate for the year 2016 of 0.4%. The mortality rates for the given class, as well as the unisex ones are kept constant throughout time, as already mentioned. They correspond to the projected rates for the year 2016 determined by Jijiie et al. (2019).

We set the accrual rate for the DB system to 1%. We then determine the contribution rate as the ratio between the total sum of pensions to be paid and the total sum of salaries, when the system reaches maturity, with the DB pensions and the wages calculated given an average salary and the legal retirement age of 65. The classes enter at the different ages given in Table 4.1. The contribution rate is thus $\pi = 16.3\%$. This rate will be shared

[9]Pensions indexation provides protection against price inflation, in which case the indexation rates follow the inflation rates, or against wage inflation, with the indexation following the growth rate of salaries. Both mechanisms are known and used in practice. For instance Germany, Slovenia and The Netherlands use wage inflation, while Poland, Italy or France index pensions using price inflation. Countries such as The Czech Republic, Estonia or Finland use a mix of both mechanism. In our case, the pension indexation provides protection against wage inflation and ensures that the standard of living for pensioners is maintained on par with the one of the active population.

in equal parts between employee and employer. We then calculate the bonus and penalty coefficients, such that the present value of benefits equals the present value of contributions for the average individual (earning the average salary, entering the system at age 17 and facing unisex mortality). The interest rate used to determine the present values equals the growth rate of the wage bill and satisfies the relationship $1+r=(1+g)(1+d)$. This equality is required in order for the systems to be able to reach financial sustainability[10]. Hence the interest rate is 1.8%. The bonus and penalty coefficients are given in Table 4.2, where the minimum retirement age is 50 and the maximum is 75.

x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$	x_r	$b_{x_r}(\%)$
50	41.1304%	57	25.4998%	64	1.8789%	71	37.3178%
51	39.2355%	58	22.7263%	65	-	72	45.0770%
52	37.2417%	59	19.7822%	66	7.2079%	73	53.6110%
53	35.1415%	60	16.6518%	67	12.2896%	74	63.0361%
54	32.9268%	61	13.3174%	68	17.7862%	75	73.4911%
55	30.5883%	62	9.7592%	69	23.7487%		
56	28.1163%	63	5.9550%	70	30.2360%		

Table 4.2: Penalty and Bonus values for the DB scheme for $x_{legal}=65$

In the case of the NDC system, we fix the notional rate such that it also satisfies the equation $1+nr=(1+g)(1+d)$. Hence the notional rate is also 1.8%. This rate ensures the sustainability of the system. We can now calculate the NDC and DB pensions and determine the optimal retirement age for each socio-economic group. The pensions amounts for the five classes at the legal retirement age, for both types of systems are given in Appendix C.

[10] This is in line with the findings of Samuelson (1958), but also of Magnani (2018).

4.2.4 Results

4.2.4.1 Disutility of work

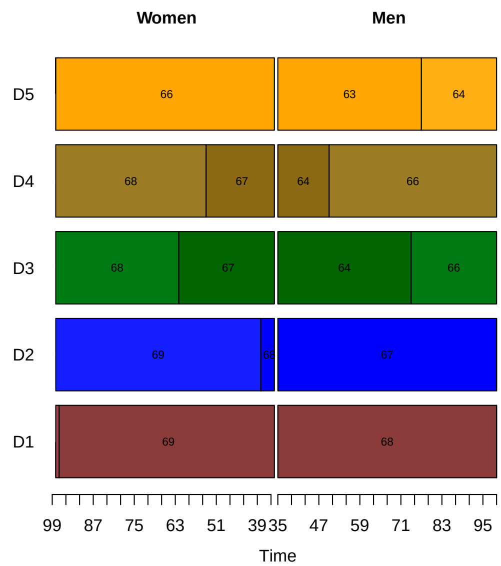
In order to solve Equation 4.7 given Equation 4.5, we need to determine the possible values of the parameter v. For this purpose, we must first define the average individual. The average person enters the system at age 17, earns the average salary in the system (no gender differences) and faces unisex mortality rates. We thus calibrate the model for each combination of γ and β and each system such that the optimal retirement age is 65, at a fixed point in time. Therefore the value of v stays constant in time, but can be different between the DB and the NDC system for the same values of the risk aversion coefficient γ and the individual time preference factor β. Table 4.3 below sums up the values used for the risk aversion coefficient and the individual time preference, as well as those for the constant v. These coefficients remain the same for each class. As pointed out before, there is no consensus regarding the values of the risk aversion coefficient or the individual time preference factor in the existing literature. Azar (2010) summarises values used in the literature for the risk aversion coefficient, which gives us insight on the range of the values used. We decide to focus on three different values for this coefficient. We take $\gamma = 0.97$ as done by Chetty (2006), $\gamma = 0.75$ as in Samwick (1998) and $\gamma = 1.25$, chosen with the purpose of testing the effect of a risk aversion coefficient larger than one. For each value of γ, we consider two individual time preference factors: $\beta = 0.97$ as done in Palme and Svensson (2004) and $\beta = 0.7$, chosen by us to study the impact of a factor that is not close to one. Hence, for instance, for a risk aversion coefficient of 0.97 and an individual time preference factor of 0.97, the corresponding value for the constant v is 0.072 in the DB scheme and 0.071 in the NDC scheme. As explained before, these are the required values for v that lead to an optimal retirement age of 65 in the DB and NDC systems respectively. The systems are put in place at time zero. We determine the optimal retirement age starting with the generations that reach the age of 50 at time 35. Because individuals enter the systems at different ages according to their classes, fixing the time at which they reach age 50 instead of the time at which they enter the system is essential for comparison purposes. We discuss here the results for the DB scheme, for the first six scenarios. These correspond to the scenarios for which the model is calibrated (hence for which the values of v are determined) given the DB scheme. The remaining results for this scheme, as well as the results for the NDC scheme can be found in Appendix A.

	γ	β	v	Scheme
Scenario 1	**0.97**	**0.97**	**0.072**	DB
Scenario 2	**0.97**	**0.7**	**260**	DB
Scenario 3	**0.75**	**0.97**	**0.61**	DB
Scenario 4	**0.75**	**0.7**	**2296.12**	DB
Scenario 5	**1.25**	**0.97**	**0.0046**	DB
Scenario 6	**1.25**	**0.7**	**16.25**	DB
Scenario 7	0.97	0.97	0.071	NDC
Scenario 8	0.97	0.7	242.89	NDC
Scenario 9	0.75	0.97	0.62	NDC
Scenario 10	0.75	0.7	2205.82	NDC
Scenario 11	1.25	0.97	0.0047	NDC
Scenario 12	1.25	0.7	16.21	NDC

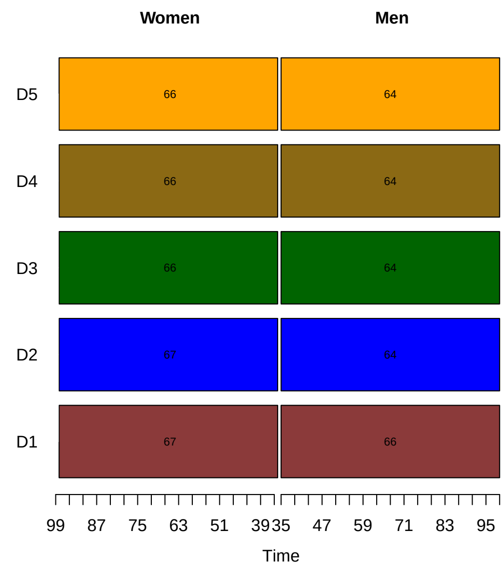
Table 4.3: Scenarios for the coefficients used in the utilitarian framework, when the disutility of work is considered

We observe in Figure 4.1 that, depending on the combination of parameters, the results differ considerably. The most interesting comparison to be made is that between Scenario 2 and Scenario 3. In Scenario 2, the retirement age stays constant throughout time for both men and women. In fact, men belonging to class D1 would choose to retire at 66, while those in classes D2 to D5 would retire at 64. Women of class D1 and D2 would retire at 67, with the remaining classes retiring at 66. Contrarily, in Scenario 3 the optimal retirement ages change more throughout time. The change over time is due to the calibration of the model being done only once, at a specific point in time. Given the growth of salaries and the indexation of pensions, in certain scenarios the disutility of work becomes too small, leading to an increase in the optimal retirement age. Moreover, we note that for class D1, the optimal retirement age rages from 72 to 75 in Scenario 3, instead of 66 for men and 67 for women from Scenario 2. The optimal ages for the remaining classes are also higher in this case. In fact, we note that a lower risk aversion coefficient ($\gamma = 0.75$), combined with a rather high individual time preference factor ($\beta = 0.97$) implies higher optimal retirement ages. This observation is natural, since the lower risk aversion coefficient implies the individuals are less risk averse, while the higher time preference model implies a view less focused on the present. In other words, the future weighs more in this case then in the case when β is 0.7. Hence postponing the retirement age becomes optimal, the individuals being willing to accept a shorter time in retirement. However, a lower time preference ($\beta = 0.7$) lowers the retirement age once more. Once again, this is to be expected since the lower time preference factor implies a view more focused on the present. Thus it is optimal to retire early. However, we must point out that Scenarios 5 and 6, for which the risk aversion coefficient is 1.25 (so when the individuals are extremely risk averse), do not yield reasonable results. Indeed, the optimal retirement ages are decreasing with time, which indicates that the disutility of work becomes too important in this case, causing individuals to retire earlier. Nevertheless, this would not be suitable scenarios for the systems, which generally aim to incentivise individuals to retire later.

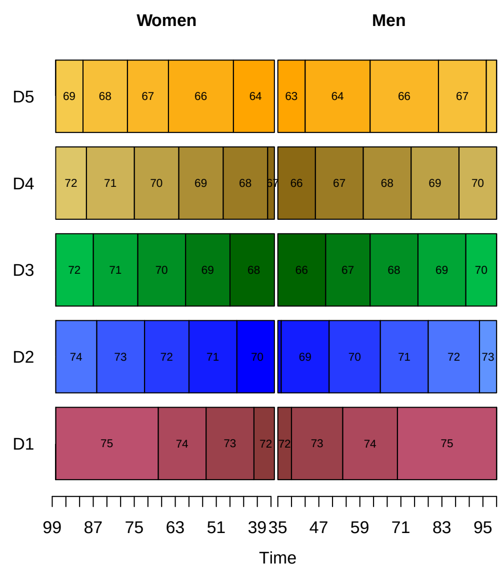
Lastly, we note that in the cases where the retirement age is not equal across all classes and time, individuals of lower classes, be they men or women, would decide to retire earlier. For example, in Scenario 3, men in class D5 would retire between the ages of 63 or 68, depending on the time, while those in class D1 would take retirement between 72 and 75. For women, the retirement age for class D5 in this case ranges from 64 to 69, while the optimal age for those with the highest level of education remains between 72 and 75. Similar observations can be drawn for the NDC scheme and the rest of our scenarios, displayed in Appendix A. This is reasonable, since the mortality rates for those of lower classes are higher. Hence, given their decreased life expectancy, it is expected that their optimal retirement age is lower.



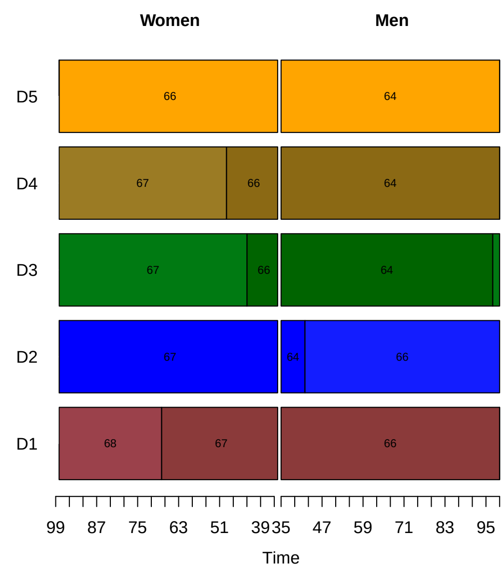
(a) Scenario 1



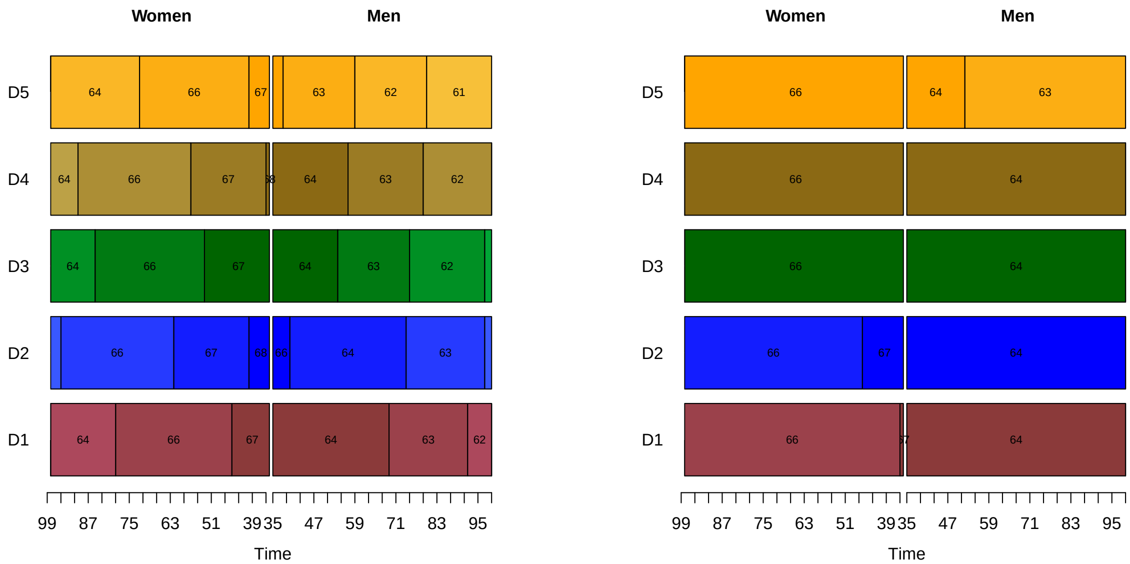
(b) Scenario 2



(c) Scenario 3



(d) Scenario 4



(e) Scenario 5 (f) Scenario 6

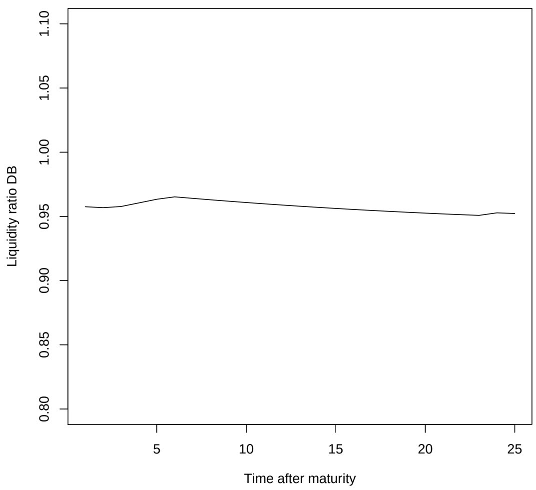
Figure 4.1: Optimal retirement ages for the DB system, when disutility of work is considered, for Scenarios 1 to 6 (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career, except for Scenarios 5 and 11, for which the opposite applies.)

We are also interested in the performance of the systems, should these retirement ages be implemented. For this we calculate the liquidity ratio as the ratio between all the contributions received and the pensions paid, once the systems are mature[11]. Since we assume constant mortality, our calculation have a fixed horizon that we consider cannot be expanded without loosing the reliability of the results. Hence we only focus on the liquidity ratio, which indicates the health of the system on an annual basis only, instead of calculating the solvency ratio. The solvency ratio would consider the sustainability of the system on a long

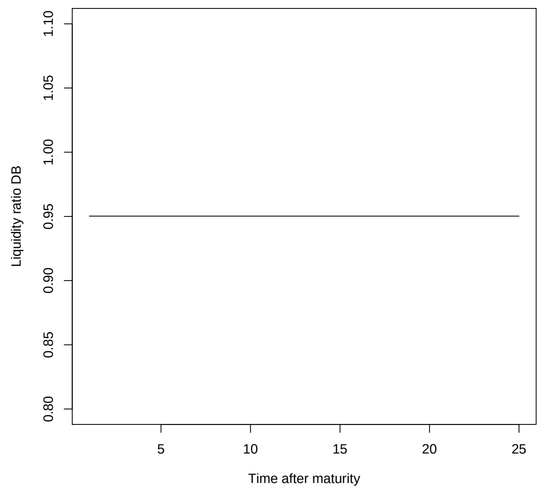
[11]The schemes are put in place at time zero, at which only those of entry age are admitted into the system and covered by it. In other words, individuals of all other ages are assumed to stay in the system they previously belonged to and are not covered within the new system. Hence, the first pensions will be paid once this first generation, also referred to as the entry generation reaches retirement. Since the liquidity ratio takes into account the contributions received and the pensions paid, the value of this indicator is only informative once we have individuals of all ages covered by the system, either paying contributions or receiving a pension. Therefore we must wait until the entry generation exits the system and the system thus becomes mature (insuring all active individuals from time zero, thus providing partial pensions for those reaching retirement age that have contributed to the system at least one year would lead to the same results since the calculations are done once the system is mature). Since in our case those belonging to class D5 will enter at age 15 and can live up to 100, the system becomes mature (or stable) after 85 years, at which point the liquidity ratio will allow us to get a picture of the financial sustainability of the systems.

term basis (typically over 50 years), but our calculations for the liquidity ratio are only done for 25 years. This time horizon is thus not appropriate for the solvency ratio. The results for the liquidity ratio for the DB system, for Scenarios 1 to 6, are presented in Figure 4.2. The remaining results can be found in Appendix A.

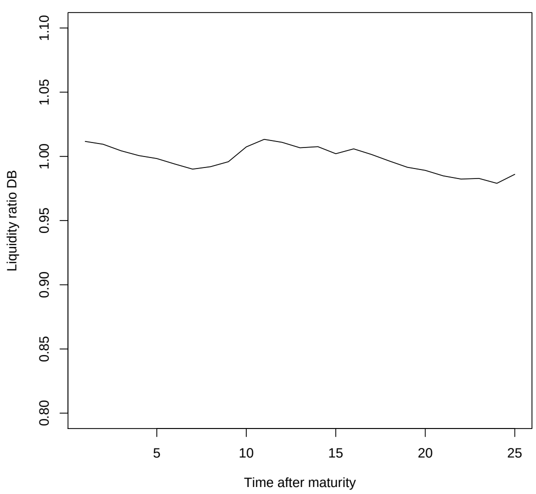
We observe that the liquidity ratios are lower than one, for all scenarios with the exception of Scenario 3. This means that implementing the retirement ages displayed above presents issues from two major points of view. On one hand, the retirement ages are determined from an individual point of view. Therefore they are rather dependent on the risk aversion coefficient and time preference factor. With no consensus on which value are appropriate, which is a natural consequence of the individualistic nature of the utilitarian method, this framework would be difficult to argue for and implement, from the systems' standpoint. Moreover, different combinations of parameters give way to more or less volatility across time, another reason for which this method is not suitable when the policy makers are concerned. On the other hand, the liquidity ratios are not equal to one. The system is either suffering a loss, not being able to pay the pensions with the collected contributions (for those scenarios, hence for the combination of parameters, for which the liquidity ratio is lower than one) or accumulate some reserves (as in Scenario 3 or Scenario 9, presented in Appendix A). In order for the systems to be viable, the liquidity ratio should be larger or equal to one. However, given the PAYG financing, reserves are not supposed to be accumulated, hence the liquidity ratio should be equal to one. Neither of the two situations present in our scenarios is in line with this condition. Of course, our results are dependant on the choice of parameters and the structure of the population considered. However, we note that none of the twelve scenarios here reaches the ideal result, pointing towards the two major reasons against this kind of approach: the retirement ages are highly dependant on the individual, so on the choice of parameters, thus not being feasible for implementation when we consider the systems (and not the people's preferences) and, additionally, they do not guarantee financial equilibrium. As such, this method would be difficult to apply in practice. The same conclusions can be drawn for the NDC scheme.



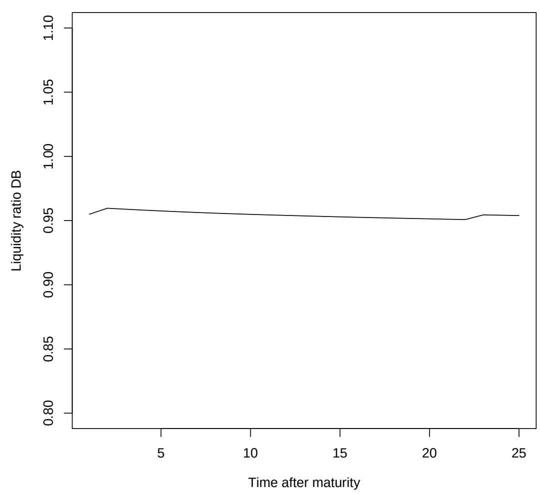
(a) Scenario 1



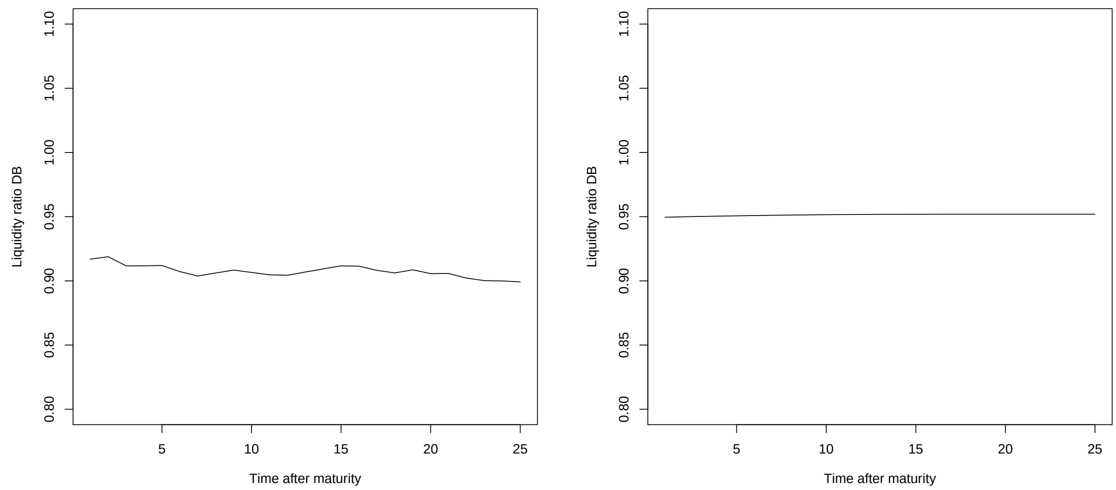
(b) Scenario 2



(c) Scenario 3



(d) Scenario 4



(e) Scenario 5 (f) Scenario 6

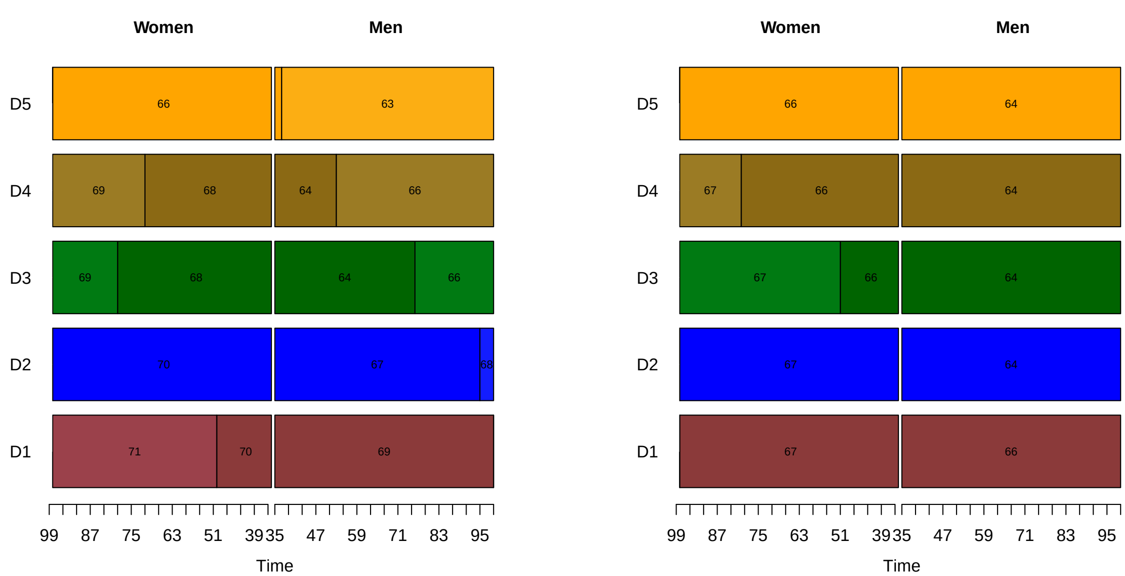
Figure 4.2: Liquidity ratios for the DB system, when disutility of work is considered, for Scenarios 1 to 6

4.2.4.2 Utility of leisure

We are now interested in solving Equation 4.7 ensuing the model presented in Equation 4.6. For this, we require the values for the constant leisure l. We perform a similar calibration for the DB and NDC systems as done in the Section 4.2.4.1. Hence the value of l is set such that the optimal retirement age for the average individual is 65, at a specific moment in time. We keep the same scenarios as in Table 4.3 with respect to the risk aversion coefficient γ and the individual time preference factor β. Because the calibration rendered identical results for both DB and NDC, for $\gamma = 1.25$, $\beta = 0.97$, we have only eleven scenarios in this section. As in the Section 4.2.4.1, we present the retirement ages for the DB system, for the first six scenarios presented in Table 4.4. The remaining results for this type of scheme, as well as those for the NDC scheme can be found in Appendix B.

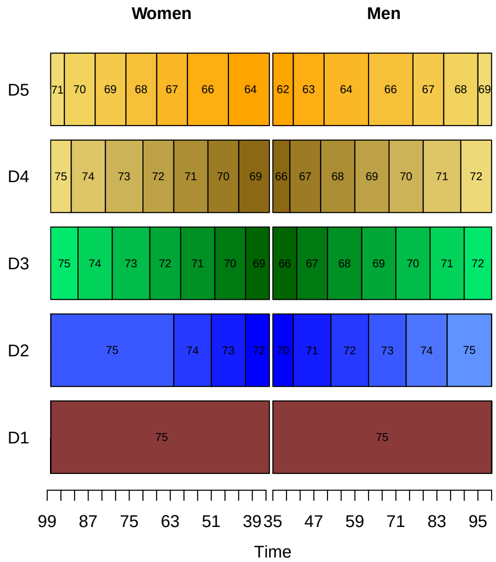
	γ	β	l	Scheme
Scenario 1	**0.97**	**0.97**	**4.65**	DB
Scenario 2	**0.97**	**0.7**	**16792.9**	DB
Scenario 3	**0.75**	**0.97**	**39.98**	DB
Scenario 4	**0.75**	**0.7**	**148300.4**	DB
Scenario 5	**1.25**	**0.97**	**0.30**	DB
Scenario 6	**1.25**	**0.7**	**1050.09**	DB
Scenario 7	0.97	0.97	4.71	NDC
Scenario 8	0.97	0.7	18750	NDC
Scenario 9	0.75	0.97	39.55	NDC
Scenario 10	0.75	0.7	166666.7	NDC
Scenario 11	1.25	0.7	937.8	NDC

Table 4.4: Scenarios for the coefficients used in the utilitarian framework, when the utility of leisure is considered

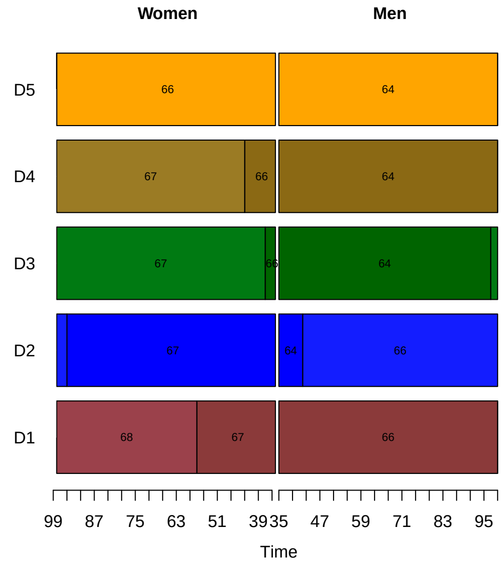


(a) Scenario 1

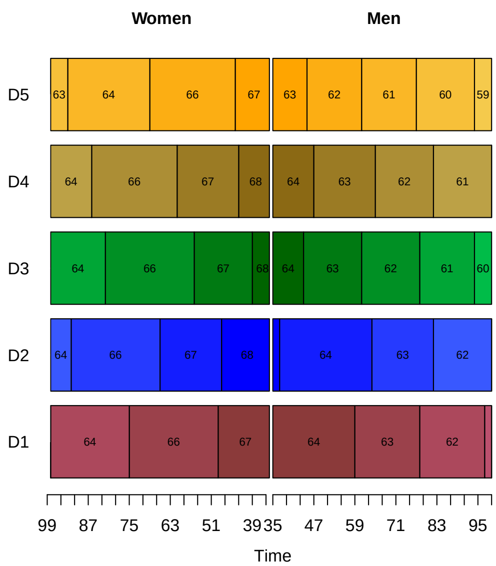
(b) Scenario 2



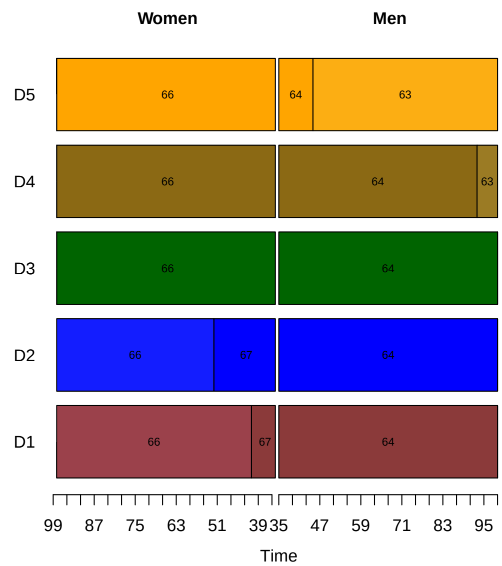
(c) Scenario 3



(d) Scenario 4



(e) Scenario 5

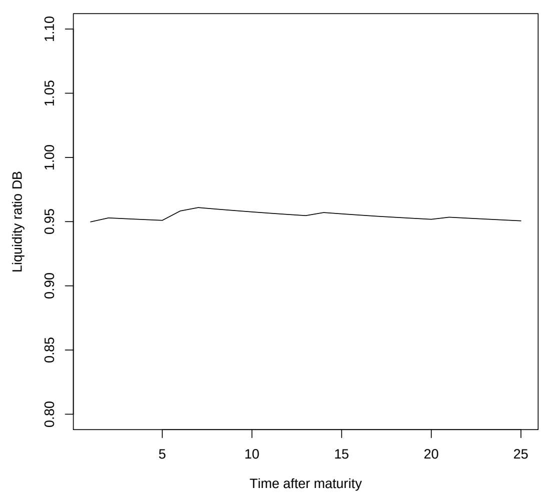


(f) Scenario 6

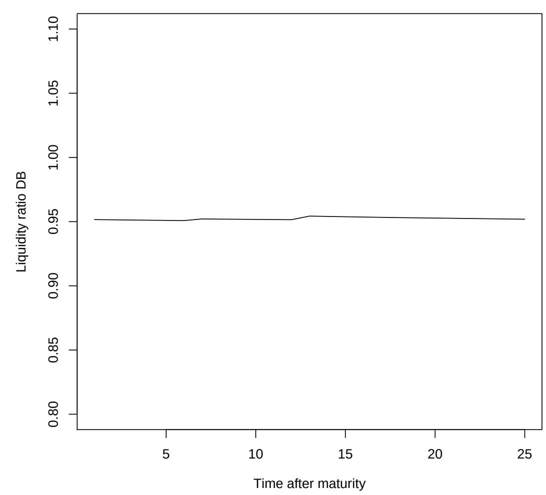
Figure 4.3: Optimal retirement ages for the DB system, when utility of leisure is considered, for Scenarios 1 to 6 (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career, except for Scenarios 5, for which the opposite applies.)

As in Section 4.2.4.1, the results are highly dependent on the choice of parameters, hence on the individual's preferences for risk and time. In other words, from the point of view of the system, the utilitarian framework is rather volatile, which implies difficulties with respect to a practical implementation. When considering the utility of leisure, the most interesting comparison to be made is, again, between Scenario 2 and Scenario 3. We observe that in Scenario 2 men of class D1 retire at 66, while those of classes D2 to D5 retire at 64. Women of classes D1 and D2 would retire at 67 in the same scenario, with those belonging to classes D3 and D4 would choose to retire either at age 66 or 67, depending on the time. Women with no diploma would retire at 66. However, in Scenario 3, individuals belonging to higher socio-economic classes choose to retire later. For instance, women of class D1 would retire at 75, while those in class D5 would retire between 64 and 71. For men the situation is rather similar. As explained in Section 4.2.4.1, the lower risk aversion coefficient, together with the higher individual time preference factor corresponding to Scenario 3 imply a vision less focused on the present, hence postponing retirement becomes more appealing. Moreover, we note that in Scenario 3 the optimal retirement age changes considerably throughout time, especially for classes D2 to D5. Once again, this is due to the method used to calibrate the model and determine the value of the constant l measuring the utility of leisure. In other words, the utility of leisure loses power over time for these classes, since the value of the constant l is only determined once, for one specific moment in time and it is not updated over the period considered. Thus the individuals postpone their retirement times, since the utility brought by the salaries is higher than that of the pension received, combined with the utility of leisure. As in the case of the disutility of work, when the risk aversion coefficient is above one, the results are not reasonable, since both men and women retire earlier as time passes. As previously mentioned, this is contrary to the desire of the systems to provide incentives for later retirement.

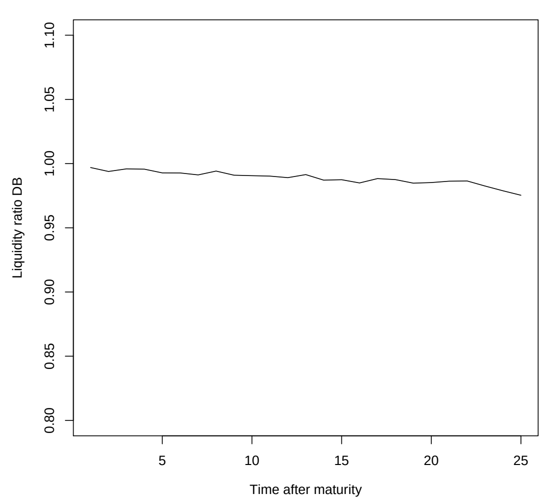
In order to complete our analysis here, we calculate the liquidity ratio, as explained in the Section 4.2.4.1. Once again, we display in Figure 4.4 only the results for the DB scheme, for Scenarios 1 to 6, while the remaining values obtained can be found in Appendix B. As in the case of the disutility of work, we note that to the main difficulty regarding the individual nature of the results, which imply a volatility depending on the parameters chosen for the system, we must add the lack of guarantee with respect to the financial equilibrium of the scheme. In fact, none of the six scenarios displayed below reaches a liquidity ratio above one. In the case of the NDC system, the situation is similar, with only Scenario 3 displaying values above one. For the remaining scenarios (Scenario 7 to 11), only Scenario 9 displays values above one for both the DB and NDC systems (see Appendix B for the corresponding graphs). Therefore, once again we can conclude that this kind of model, although interesting for understanding individual preferences, would not be suitable for setting the retirement ages by the systems and thus it would be difficult to implement in practice.



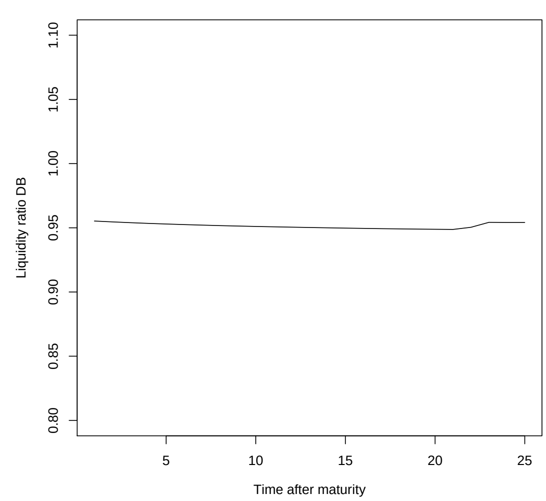
(a) Scenario 1



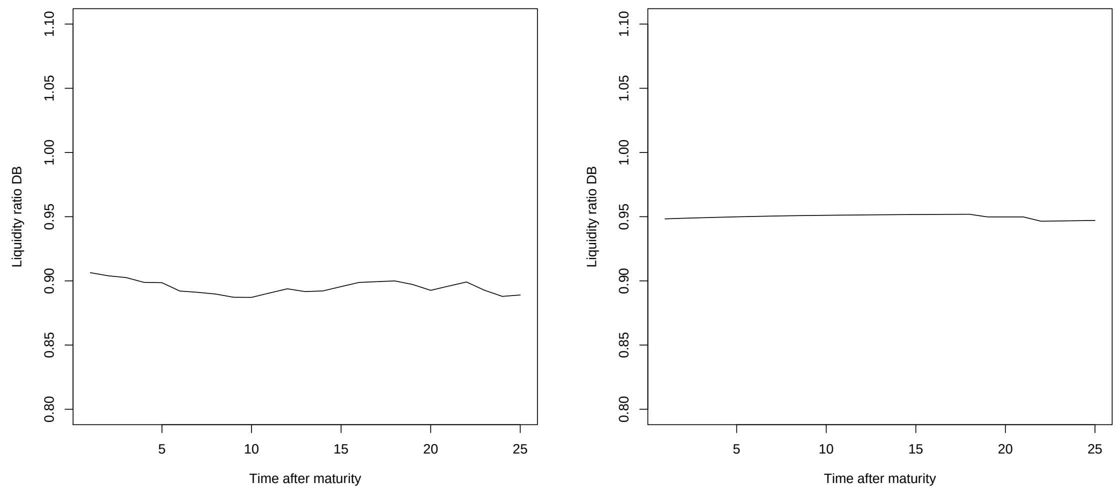
(b) Scenario 2



(c) Scenario 3



(d) Scenario 4



(e) Scenario 5 (f) Scenario 6

Figure 4.4: Liquidity ratios for the DB system, when utility of leisure is considered, for Scenarios 1 to 6

4.3 Actuarial framework

4.3.1 Optimisation problem

As pointed out in Section 4.2.4, the utilitarian framework is interesting and important for discovering individual behaviour, especially with respect to the retirement age. However, the system, represented by institutions regulated usually by state laws cannot afford such high degrees of individualism when setting retirement ages. Moreover, the use of utility functions does not guarantee the financial sustainability of the systems, thus representing another issue for policy makers. Nevertheless, since the difference in mortality by socio-economic class is non-negligible, giving way, at the status quo, to transfers from the lower classes to the higher ones, adjusting the retirement age for each class is one measure that can help decrease these transfers. Considering the point of view of the system, we propose the actuarial method described below for defining the retirement ages for the socio-economic classes. For this, we must firstly define the actuarially fair pension (also referred to as the theoretical pension) as the amount calculated such that the present value of contributions equals the present value of benefits. This is given in Equation 4.8, for class i, at the legal retirement age x_{legal} at time t. The present values are calculated for an interest rate r and using survival rates that depend on the socio-economic class.

$$P^{i,th}_{x_{legal},t} = \frac{\pi \cdot \sum_{x=x_0^i}^{x_{legal}-1} W^i_{x,t-x_{legal}+x} \cdot (1+r)^{-(x-x_0^i)} {}_{x-x_0^i}p^i_{x_0^i,t-x_{legal}+x_0^i}}{\ddot{a}^{i,ind}_{x_{legal},t}(r) \cdot {}_{x_{legal}-x_0^i}p^i_{x_0^i,t-x_{legal}+x_0^i} \cdot (1+r)^{-(x_{legal}-x_0^i)}}. \tag{4.8}$$

Although PAYG systems do not strive to be fair by definition, using the theoretical pension given in Equation 4.8 to assess fairness is pertinent when the interest rate r is equal to the growth rate of the wage bill. Indeed, a sustainable PAYG system will also be fair if the interest rate used to asses the actuarial fairness is the growth rate of the wage bill. Hence, the theoretical pension defined as specified above corresponds to the amount that ensures both the sustainability of the scheme and its fairness, but only if the interest rate is given by the growth rate of the wage bill. However, considering a different interest rate r in Equation 4.8 would not be appropriate, since the sustainability of the systems cannot be guaranteed.

After defining the theoretical pension, we consider that each individual has two savings accounts. In one account the theoretical pension is accumulated, from the time of the legal retirement until the end of the lifespan. In the other account we accumulate either the DB or the NDC pension, from the moment of retirement until the end of the lifespan (the maximum age is defined by ω). Hence the retirement age is set such that the values of the two accounts are closest to each other. Formally this is given by Equation 4.9, where $D^{i,s}_{x_r,t-x_{legal}+x_r}$, with s indicating the scheme (either DB or NDC), represents the difference between the two accounts, for an individual of class i, given a legal retirement age x_{legal} reached at time t.

$$D^{i,s}_{x_r,t-x_{legal}+x_r} = \frac{P^{i,s}_{x_{legal},t} \cdot \ddot{a}^{i,ind}_{x_r,t-x_{legal}+x_r} \cdot (1+r)^{\omega-x_r}}{{}_{\omega-x_r}p^i_{x_r} \cdot (1+ind)^{x_{legal}-x_r}} - \frac{P^{i,th}_{x_{legal},t} \cdot \ddot{a}^{i,ind}_{x_{legal},t} \cdot (1+r)^{\omega-x_{legal}}}{{}_{\omega-x_{legal}}p^i_{x_{legal}}} \tag{4.9}$$

Please note that we make the hypothesis that contributions are paid until x_{legal}, even if the retirement age is actually $x_r \neq x_{legal}$. In other words, we calculate both the NDC or DB pensions and the theoretical pension at the legal retirement age. However the accumulation can start at $x_r < x_{legal}$, if $D^{i,s}_{x_{legal},t} < 0$ or at $x_r > x_{legal}$, if $D^{i,s}_{x_{legal},t} > 0$, with the purpose of minimizing the difference defined in Equation 4.9, as stated in Equation 4.10.

$$\min_{x_r \in [x_{min}, x_{max}]} (D^{i,s}_{x_r, t - x_{legal} + x_r})^2 \tag{4.10}$$

This approach allows setting a lower retirement age for the lower socio-economic classes without reducing their pensions, as the benefits are always calculated for the legal age. Conversely, those in higher classes would have a higher retirement age without an increase in the pension benefits. Although slightly unconventional, our method is in line with the rules applied for the Swiss first pillar. Moreover, dividing by $(1 + ind)^{x_{legal} - x_r}$ allows us to avoid accounting for the indexation twice, when retirement is taken earlier than x_{legal} and to allow for growth due to indexation when x_r is larger than x_{legal}. Hence when the retirement is taken earlier than x_{legal}, the pension is diminished by the appropriate indexation factor $(1 + ind)^{x_{legal} - x_r}$, while when the retirement is taken later, the pension is augmented by the corresponding factor, to allow for a correct purchase power for pensioners. Lastly, we point out that the salary increases are considered up to the legal retirement age, even when retirement is taken before x_{legal}. This implies a necessary hypothesis on the growth rate of salaries and, in the case of the NDC system, on the notional rate, when retirement is taken before the legal age and hence no salaries are actually gained. The growth rate of salaries is assumed, in our framework, as equal to the one applied until x_r. Naturally, when retirement is taken before the legal age, a growth rate for the salaries such as described above implies higher benefits than if no increase is considered. However, since retirement is taken early by those individuals belonging to lower classes, this situation would benefit them, all the while allowing our framework to remain simple to implement.

4.3.2 Results

4.3.3 Class-specific retirement ages

In order to be consistent with our previous analysis, we keep the same parameters set for determining the optimal retirement ages in the utilitarian framework. Hence, as described in Section 4.2.3 the legal retirement age x_{legal} is set at 65, the contribution rate π is 16.3%, the indexation rate is equal to the growth rate of salaries, being 1.4%. The accrual rate AR is 1%, while the notional rate nr and the interest rate r are both equal to 1.8%. Given the optimisation problem defined in Equation 4.10, we obtain the results shown in Table 4.5. The corresponding life expectancy (LE) at the retirement ages obtained is also given in Table 4.5.

			D1	D2	D3	D4	D5
DB	Men	Age	66	64	63	62	60
		LE	24.02	25.06	25.10	25.49	25.96
	Women	Age	69	69	68	68	66
		LE	24.62	24.46	25.08	24.94	26.00
NDC	Men	Age	65	64	63	62	61
		LE	24.92	25.06	25.10	25.49	25.14
	Women	Age	69	69	68	68	67
		LE	24.62	24.46	25.08	24.94	25.05

Table 4.5: Optimal retirement ages for the DB and NDC scheme using the actuarial framework, together with the corresponding life expectancies (LE)

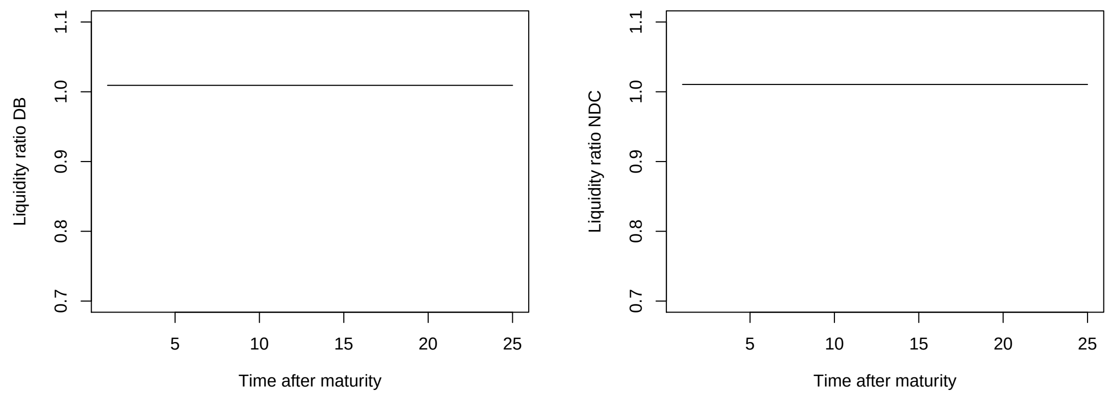
With the absence of the risk aversion coefficient and the time preference factor from this model, we eliminate, as expected, the more individualist aspects related to the utilitarian framework and thus, also the volatility throughout time of the results. Moreover, since no scenarios are required, only one set of results exists, making such a model more easily implementable for the policy makers.

We now want to analyse the results obtained. As expected, we see that women should retire later than men, which is due to the difference in mortality rates. As women live longer, the fact that their specific mortality is not considered in the calculation of the DB or NDC pensions, leads to a positive difference between these amounts and the theoretical pension at the legal retirement age. Hence, the retirement age can be postponed after the age of 65[12]. The same occurs for men in the higher classes. However, men from classes D2 to D5 retire before 65. In other words, they should be compensated for the fact that the DB and NDC pensions do not account for their lower socio-economic survival probabilities. In general, we can remark that the retirement age for lower classes is lower than that for higher classes. Lastly, we must note that the life expectancies at the optimal retirement ages are close to each other among the classes. For instance, men in class D1 retiring at age 65 in the NDC scheme have a life expectancy at that age of 24.92 years, while for those in class D5, retiring at 61 the life expectancy is 25.14. For women, those in class D1 retire at 69, when the life expectancy is 24.62, while those in class D5 retire at 67, with the corresponding life expectancy being 25.05. A similar observation holds for the DB scheme, as shown in Table 4.5.

Additionally, we plot the liquidity ratios given the retirement ages displayed in the above table. In this case, we see that the liquidity ratios are slightly larger than one (1.009 for the DB system and 1.01 for the NDC scheme). This means that both systems are viable on an yearly basis[13]. Thus, by utilising the method described here, the systems can meet their dual goal. They can define the retirement ages for each class, thus protecting those in lower classes, and maintain their financial viability.

[12]The higher retirement age for women might be considered politically incorrect. One could argue that women should retire earlier than 65 to compensate for the lower salaries earned. However, this is not in line with the actuarial fair framework, which considers the higher life expectancy of women and therefore indicates higher retirement ages. A different study is needed to provide a suitable compensation method for the wage inequality between men and women.

[13]The ratios are not exactly equal to one due to the calculation of the contribution rate by using average salaries and not class-specific wages.



(a) The DB system (b) The NDC system

Figure 4.5: Liquidity ratios for the actuarial framework

4.3.4 Further discussion

4.3.4.1 Mortality evolving throughout time

The results displayed in Table 4.5 are obtained given constant mortality rates throughout time. The use of constant mortality allows us to isolate the effect on the retirement ages of the class-specific mortality rates, as previously explained. Moreover, our data is insufficient to obtain reliable projected mortality rates (in other words projecting mortality improvements) for such a long time horizon as the one considered until now. Nevertheless, evolving mortality is an important factor in the retirement systems. This is why we also desire to compute the optimal retirement ages, considering generational mortality rates[14], as per Equation 4.9 and Equation 4.10. However, due to the above-mentioned data issues, we limit our calculation to only one generation reaching the legal retirement age of 65 at time 50. The resulting retirement ages are given in Table 4.6, together with the corresponding life expectancies at the moment of retirement. For the DB system, the retirement ages are slightly higher than the ones given in Table 4.5. However, the previous conclusions still hold. Women retire later than men, due to their higher life expectancy. Moreover, individuals from higher classes retire later than those belonging to the lower classes. Hence men in class D1 retire at 67,

[14]The generational mortality rates have been obtained as described in Section 3.3.3.

while those in class D5 would retire at 62. For women in class D1 the retirement age in this case is 72, while for those in class D5 it is 70. The corresponding life expectancies are similar among all classes and with respect to those given in Table 4.5. In the NDC scheme, we see that the retirement age for women is 69 for all classes, which differs from those given in Table 4.5, where only women in class D1 and D2 would retire at 69, while those in class D5 would retire earlier, at 67. Moreover, we note that the corresponding life expectancies are higher in this case, ranging from 27.47 years for class D1 to 26.68 years for class D5. For men, the retirement ages for classes D4 and D5 remain the same as in Table 4.5. However, those in the remaining classes retire a year earlier in this case. Thus those in class D1 retire at 64, while those in the remaining two classes retire at 63 and 62. Once more, the life expectancy in this case is higher and it is similar to the one of women. Lastly, we must note that the differences between classes in terms of life expectancies at the moment of retirement are rather stable and similar to those in Table 4.5, but we observe a more steep improvement in mortality rates for women than for men. For instance, if the life expectancy of a woman in class D1 at age 69 is 24.62 when mortality is constant throughout time, it rises to 27.47 when mortality evolves with time. However, for men the values at the same age are 21.34 and 22.66 respectively. This is due to our mortality projections (and thus implicitly on the historical data used to calibrate the mortality model) and can be also seen in Figure B.1 of Chapter 3.

			D1	D2	D3	D4	D5
DB	Men	Age	67	66	64	64	62
		LE	24.44	24.74	25.89	25.48	26.24
	Women	Age	72	72	71	71	70
		LE	24.42	24.32	25.15	25.08	25.66
NDC	Men	Age	64	63	62	62	61
		LE	27.26	27.52	27.71	27.29	27.15
	Women	Age	69	69	69	69	69
		LE	27.47	27.37	27.19	27.12	26.68

Table 4.6: Optimal retirement ages for the DB and NDC scheme using the actuarial framework, together with the corresponding life expectancies (LE), for one generation when mortality rates evolve throughout time

4.3.4.2 No gender distinction

As in the previous chapter, we must note that in some countries gender distinctions might not be allowed. Hence having different retirement age for men and women might not be possible. Nevertheless, our framework can be applied even when no gender distinction is made. We do this given constant mortality rates that differ across socio-economic classes, but not according to gender. The salaries are also only class dependent. The remaining parameters are the same as before. By applying Equation 4.9 and Equation 4.10 we obtain the retirement ages given in Table 4.7. We observe that the retirement ages for the higher classes are higher than the ones for the lower classes, compensating for the differences in mortality rates, in line with our previous analysis. If individuals with the highest level of education retire at 68 in the DB scheme and at 67 in the NDC scheme, those with no formal education retire at 63 and 64 respectively. Moreover, the life expectancies at the moment of retirement are similar among classes and also to those given in Table 4.5, except for individuals in class D1 in the DB scheme, for whom the value is under 24 years, since the retirement age for this class is the highest (thus corresponding to a lower life expectancy). Lastly, we note that the liquidity ratio for the DB scheme is in this case 1.022, while for the NDC scheme it is 1.016. Hence, as before, both systems are sustainable on an yearly basis.

			D1	D2	D3	D4	D5
DB	Unisex	Age	68	66	65	65	63
		LE	23.80	25.18	25.51	25.18	25.96
NDC	Unisex	Age	67	66	65	65	64
		LE	24.73	25.18	25.51	25.18	25.08

Table 4.7: Optimal retirement ages for the DB and NDC scheme using the actuarial framework, together with the corresponding life expectancies (LE), when no gender distinction is made

When compared to the retirement ages given in Table 4.5, we see that the differences between the classes are smaller. This is due to the mortality projections used to compute these results. Indeed, the mortality evolutions for the classes are similar, since the time trend coincides to the one of the general population. Furthermore, the differences in terms of retirement ages between the classes diminish over time, as the differences in terms of mortality also decrease.

4.3.5 Mortality differences impacting the retirement age by one year

Additionally to the class-specific retirement ages ensuing from the actuarial framework proposed here, we are interested in the impact of a mortality change on the results obtained by applying the actuarial framework. We can say that, for a given entry age, if the mortality rates of the class differ by y from the average mortality rates used by the system, with y positive, then the retirement age of that specific class should be lowered by one year. Conversely, when the difference is measured by a negative y, retirement age should be increased by one year. Hence the question that arises is what would be the corresponding value of y for which retirement should be set one year later or one year earlier? To answer this question, we focus here on the average individual (earning the average salary and facing initial average mortality), entering the system at the five different entry ages described in Table 4.1. We start by computing the optimal retirement age for these individuals, according to Equation 4.10[15]. We then consider that the mortality rates could actually be different than the average. Hence we can define $q_x^* = q_x(1+y)$, with q_x the average mortality rate for age x and y the factor driving the mortality change. Given these new mortality probabilities, we can recalculate the theoretical pension and determine new retirement ages based on Equation 4.10 once again. Thus we are interested for which values of y, so for which mortality difference, would the retirement be set one year later or one year earlier. Using the same parameters as for determining the optimal retirement ages in Table 4.5, we obtain the values of y displayed in Table 4.8.

We thus note that for individuals entering at age 21, retiring one year earlier than in the base case implies an increase in the mortality rates between 10.2% and 28%, for the DB scheme and between 8.2% and 25.7% for the NDC scheme. For those entering the systems at age 15 the corresponding intervals for the coefficient y that would induce the anticipation of retirement by one year are [8.4%, 26%] for the DB scheme and [8.1%, 25.6%] for the NDC scheme. In fact, we observe that the increase in mortality required for anticipating the retirement age by one year is less dependant on the entry age, when the NDC pension is considered. However, the results differ depending on the entry age x_0 for the DB scheme. This is due largely to the fact that the NDC scheme and the theoretical pension will react similarly to the change in the entry age, shifting the accumulation phase for both schemes in the same way. Hence the entry age will not have a big effect on the difference in mortality

[15]The contribution rate is calculated as an average, which takes into account the five different entry ages displayed in Table 4.1. Therefore it is not tailored to one specific entry age. Since in this section we define five distinct cases, considered individually, with the average individual entering at different ages, while using the contribution rates calculated as explained above, the optimal retirement age is not necessarily 65. Hence we must start by determining the initial optimality.

rates required for postponing or advancing the retirement age by one year. Conversely, the DB pension definition is not close to the theoretical pension calculation, but is dependant on the average salary over the entire working life, which in turn is highly driven by the entry age in the system. When we consider postponing the retirement age by one year, an amelioration between 5.9% and 20.1% of the mortality rates is needed, for those entering at age 21 and receiving a DB pension. For those starting at 15, the interval goes from 7.5% to 21.5%. Once again, the NDC scheme is less dependant on the entry age. The amelioration needed ranges from 7.7% to 21.7% for the entry ages of 21, 18 and 17. For the remaining entry ages the interval is [7.7%, 21.6%]. One last remark needs to be made on this part. The fact that the coefficient y is determined as a certain interval for each entry age is not surprising. Indeed, our method is based on finding the minimum with respect to the difference between two accounts in which the pensions are accumulated. Hence, for different values of y, the minimum can be found at the same age. This, of course, does not imply that the minimums themselves are equal, but that the minimal value is reached at the same point in the interval $[x_{min}, x_{max}]$.

		$x_0 = 21$	$x_0 = 18$	$x_0 = 17$	$x_0 = 16$	$x_0 = 15$
Anticipate by one year	DB	[10.2, 28.0]	[1.2, 17.9]	[14.8, 33.1]	[11.5, 29.5]	[8.4, 26.0]
	NDC	[8.2, 25.7]	[8.1, 25.7]	[8.1, 25.7]	[8.1, 25.7]	[8.1, 25.6]
Postpone by one year	DB	[-20.1, -5.9]	[-27.3, -13.9]	[-16.4, -1.8]	[-19.0, -4.7]	[-21.5, -7.5]
	NDC	[-21.7, -7.7]	[-27.7, -7.7]	[-21.7, -7.7]	[-21.6, -7.7]	[-21.6, -7.7]

Table 4.8: Values of y (in percentages) for which retirement is postponed or anticipated by one year, for the average individual

4.3.6 A real case study: the Swiss system

The goal of the Swiss first pension pillar, implemented in 1948, is to ensure the minimum standard of living to the entire retired population. Financed on a PAYG basis, it provides old-age and survivor benefits to all the residents of the country. We will focus here on the old-age pension, in line with the scope of this paper. The pension is calculated following a particular variation of the DB scheme. It is thus defined as follows:

$$R_{AVS} = c_r \cdot (k_1 \cdot M + k_2 \cdot RAMD) \tag{4.11}$$

In Equation 4.11 , c_r designates the ratio between the number of years the individual contributed and the maximum number of contributory years. The first legal contribution is due on the 1st of January of the year that follows the twentieth anniversary of the individual. Since the legal retirement age in Switzerland is 65 for men and 64 for women, the maximum number of contributory years is 44 for men and 43 for women. As specified before, should an individual choose to retire earlier, the contributions are still paid up until the legal retirement age. Moreover, postponing retirement implies paying further contributions only if the level of earnings surpasses a given threshold. However, the contributions paid in this case would not be considered in the calculation of the pension amount, but are part of the solidarity aspect of the pension system. The value of M, signifying the minimum monthly pension awarded by the system, is reviewed every two years. For 2019, the law stipulates an M equal to 1185 CHF per month. The coefficient referred to as $RAMD$ is nothing other than the average salary over the contributory period of the individual, multiplied by a revaluation factor that accounts for inflation, also already set by the system and dependant on the year of the first contribution paid and the year in which the pension is awarded. The $RAMD$ is rounded up to the highest multiple of $1.2M$ and is capped, since the maximum monthly pension paid by the system is equal to $2M$. If $RAMD > 36 \cdot M$, then $k_1 = \frac{104}{100}$ and $k_2 = \frac{8}{600}$. In the opposite case, $k_1 = \frac{74}{100}$, while $k_2 = \frac{13}{600}$. These values are also set by law. The residents pay a contribution rate of 8.4% of their salaries, with the state participating as well in the financing of this scheme. Hence this rate is lower than it should be.

We are interested in performing the same analysis as before, given this version of a DB scheme. In other words, we want firstly to compare the pension received under this first pillar scheme with the corresponding theoretical pension, at the legal retirement age. If the difference, as defined in Equation 4.9 is positive or negative, then we search for the optimal retirement age as given by Equation 4.10. In order to achieve our goal, we use the mortality rates for men and women, as well as the unisex rates from the Human Mortality Database for the year 2014.[16]. We also consider a growth rate of salaries of 1.13%, calculated from historical data from 2008 to 2014. Given a growth rate of population of 0.7% (as reported by the Swiss Office of Statistics), we use an interest rate r of 1.8%, respecting the equality defined in Section 4.2.3 (hence $1+r = (1+d)\cdot(1+g)$ for Switzerland). No indexation rate is applied, while the legal retirement age is kept at 65 for men and 64 for women. Due to data availability, our exercise is limited to the period 2009 to 2019. Hence, we are interested in those men reaching the legal retirement age in 2014 and those women reaching their respective legal age in 2013. The interval $[x_{min}, x_{max}]$ is thus $[60, 70]$. Because the legal contribution rate is too low, we calculate the rate that ensures the equality between the present value of benefits and present value of contribution for the average individual (average salary and unisex mortality rates), obtaining a value of 10.54%[17]. This rate is then used for determining the theoretical pensions for both men and women. We find that at the legal retirement age, the system is more generous towards women, compared to the theoretical framework. Hence women would have to retire at the age of 68 instead of 64. Conversely, for men, the system is less generous than the theoretical pension. Hence the retirement phase for men would begin at age 63 and not at 65. Lastly, we perform a similar analysis as in Section 4.3.5. Consequently, for the average individual (earning the average salary in Switzerland and facing unisex mortality rates), postponing the retirement by one year implies an amelioration of the mortality rates between 5.7% and 19.8%. In other words, if the mortality rates of a certain socio-economic class would be lower than the unisex mortality rates by a coefficient between 5.7% and 19.8%, their retirement age should be increased by one year. Furthermore, advancing the retirement

[16]https://www.mortality.org/

[17]Since we impose the equality $1+r = (1+g)(1+d)$, thus assuming a stable economy and demography, the contribution rate calculated here is the same as the one calculated as the ratio between benefits and salaries, as indicated by the PAYG financing method.

age by one year requires an increase in mortality rates between 11.1% and 30.4%. Hence, once again, if the mortality rates of a specific class are higher than the unisex rates by a coefficient included in the above mentioned interval, the retirement age can be lowered by one year for those individuals. Once information regarding the mortality rates by socio-economic class will be officially available in Switzerland, it will be possible to apply the methodology here to determine the retirement age for each class. In this way, the lower socio-economic classes will be better protected and the transfers taking place will be compensated for.

4.4 Conclusions

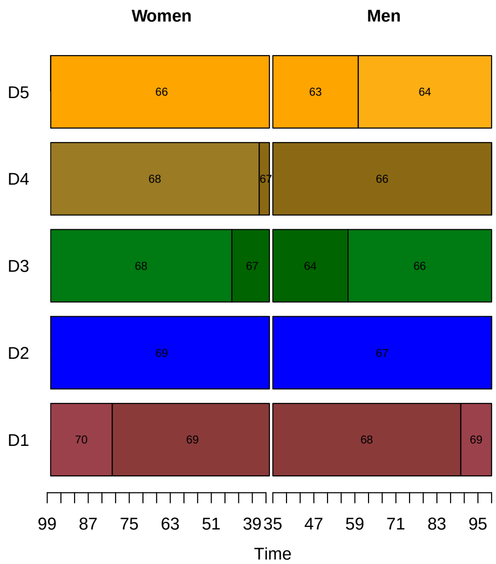
In this paper, we focus on setting the optimal retirement age for each socio-economic group. Given the mortality differences between classes that are not considered in the calculations of the pension amounts, we consider the adjustment of the retirement age according to each socio-economic class a compensation mechanism used to balance the advantages and disadvantages gained by higher and lower classes respectively (as outlined in Jijiie et al. (2019)). To achieve our goal, we first consider an utilitarian framework, with different scenarios for the risk aversion coefficient γ and the individual time preference factor β. For each scenario, we determine the optimal retirement age for men and women divided in five classes, defined by level of education, using data on mortality and salaries from the French Office of Statistics. We thus find that the retirement age of those individuals belonging to lower classes is, in general, lower than that of those with a high education. Moreover, we observe a variability in our results dependant on the scenario considered. For instance, a lower risk aversion coefficient, coupled with a rather high individual time preference factor implies higher optimal retirement ages, as individuals are less risk averse and have a broader consideration for the future. The high dependence of the optimal age to the choice of parameters, as well as the changes throughout time displayed in certain cases, lead us to conclude that such a framework is too dependant on individual preferences and thus is not suitable for setting the retirement age from the point of view of the system, which cannot be individualised. Moreover, the liquidity ratio for the scenarios considered is in general lower than one, indicating a lack of financial sustainability for the systems defined here.

Therefore, an alternative is proposed, which involves the use of the actuarially fair pension and allows the system to make decisions without such a high degree of individualism as encompassed in the utilitarian framework, all while showing better financial sustainability for the systems. In other words, we propose a method based on two accounts. We compare the account holding the accumulated value at age ω of all benefits paid by either the DB or the NDC scheme with the account where we accumulate, at the same age, the benefits paid under the theoretically fair scheme. Hence the optimal retirement age for each class is set such that the values in the two accounts are close. The results in this case are stable, since the individual traits (the risk aversion and time preference) used for the utility functions are not present. Furthermore, the financial sustainability of the systems is implicit in our framework, since the interest rate used to define the fair pension is equal to the growth rate of the wage bill and the contributions are paid until the legal retirement age, with the pensions being computed at the same age, regardless of the actual age at which retirement is taken. Thus our framework helps policy makers in reaching the two major objectives of the retirement systems by providing better protection for those in lower socio-economic classes through the decrease of transfers and by ensuring the financial sustainability of the schemes. Moreover, although the values of the retirement ages are pertinent solely to the data use, our model can be tailored to different schemes, as shown by the case study into the Swiss first pension pillar. Lastly, we also investigate the mortality differential required for advancing or postponing the retirement age by only one year, for the average individual. In other words, we display, given our data, what should be the difference between the class-specific mortality rates and the unisex mortality rates that should correspond to raising or lowering the retirement age of the specific class by one year. Once again, this analysis can be easily implemented for other schemes.

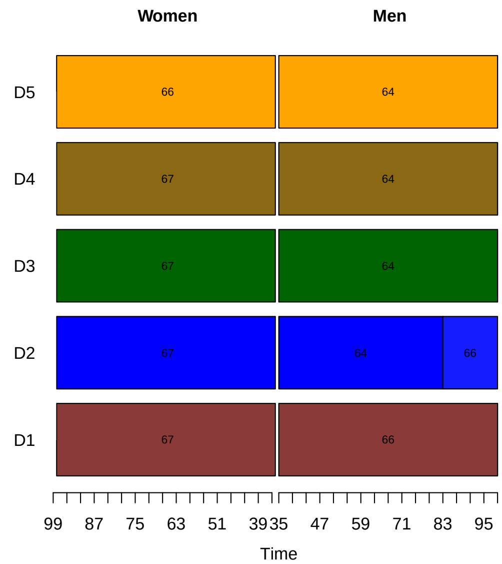
The retirement ages displayed here remain the results of a numerical exercise. However, our methodology is easily implemented in practice and should be considered by the policy makers, in order to ease the disadvantage brought to those of lower classes by an universal raise of the retirement ages.

Appendices

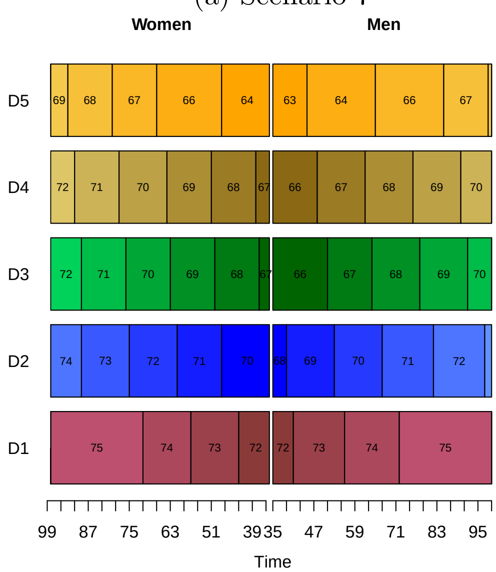
A Results for the model using the disutility of work



(a) Scenario 7



(b) Scenario 8



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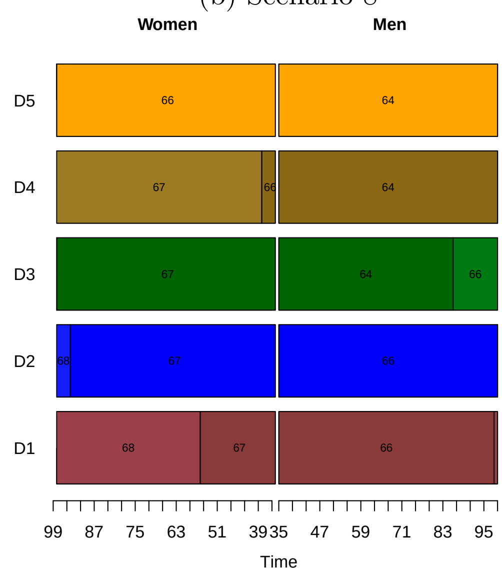

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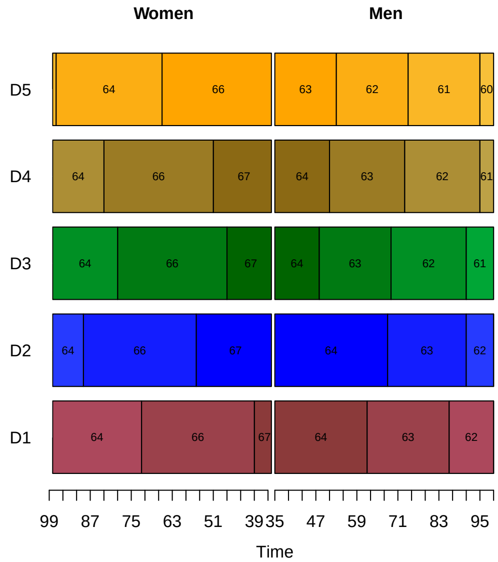
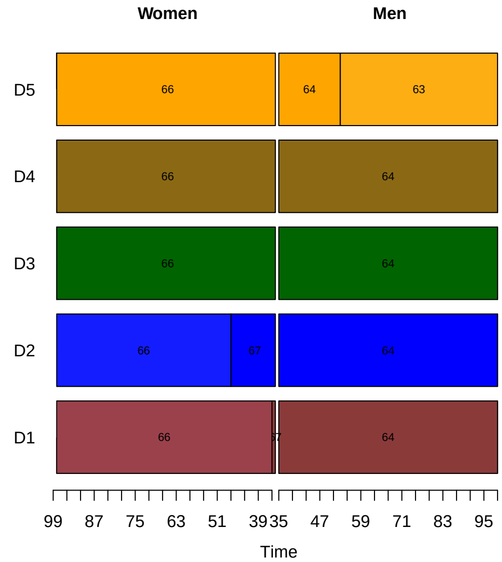
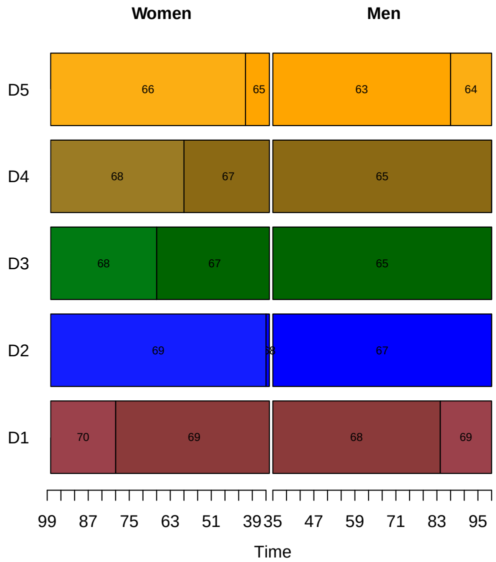


(e) Scenario 11



(f) Scenario 12

Figure A.1: Optimal retirement ages for the DB system, when disutility of work is considered, for Scenarios 7 to 12 (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career, except for Scenarios 11, for which the opposite applies.)



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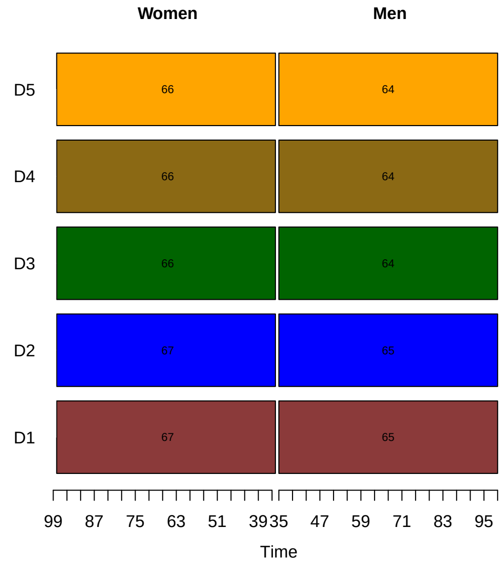

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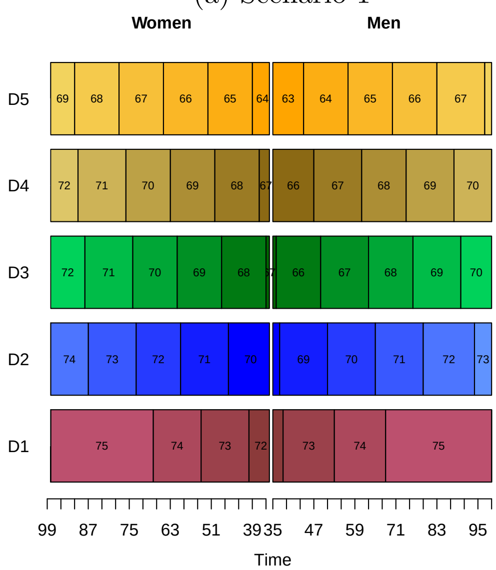
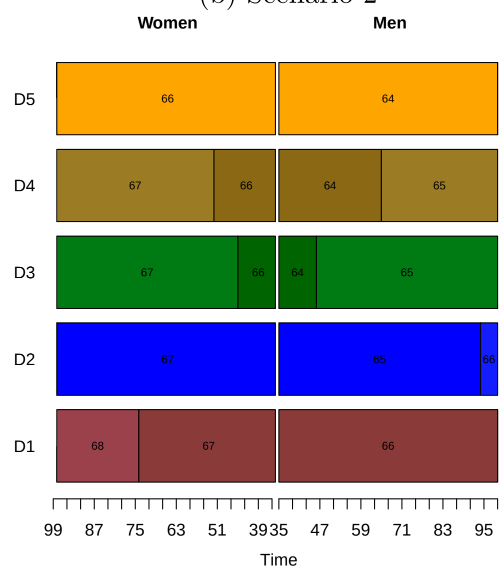
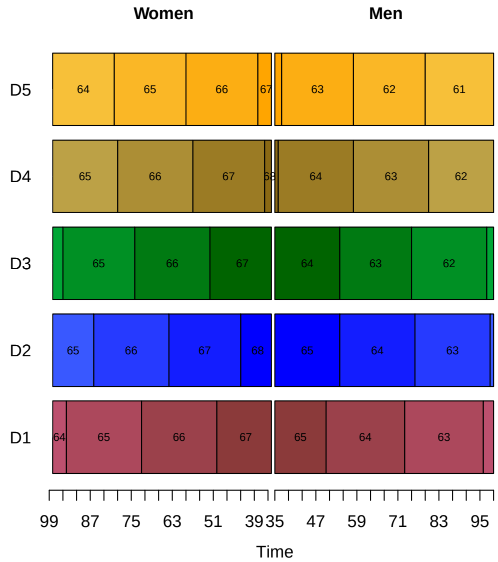


(c) Scenario 3



(d) Scenario 4



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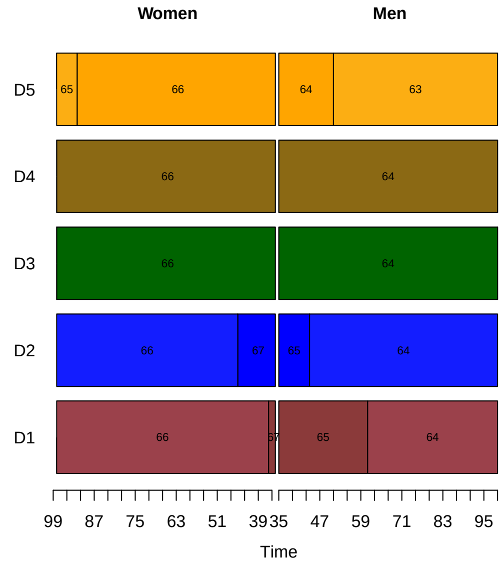
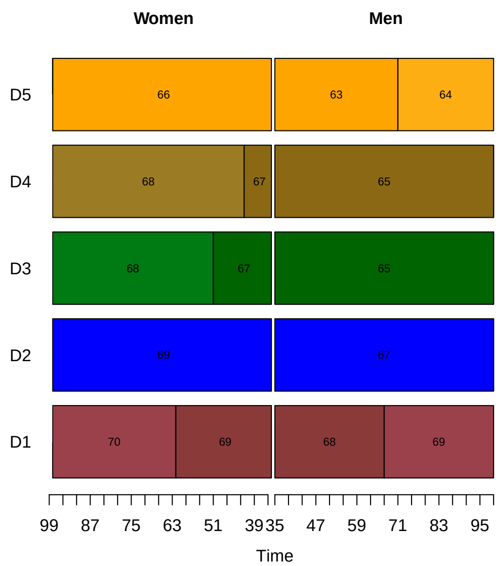
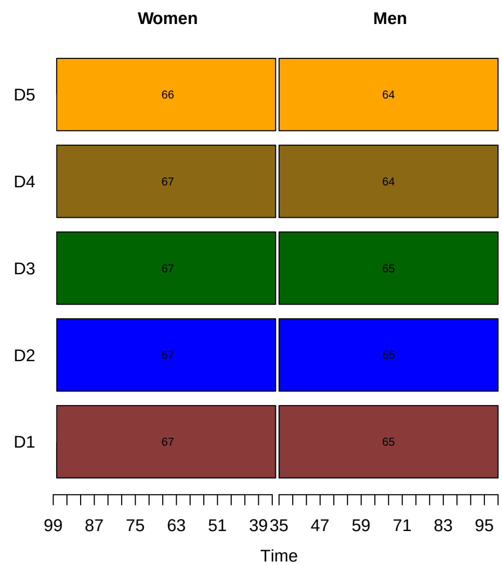


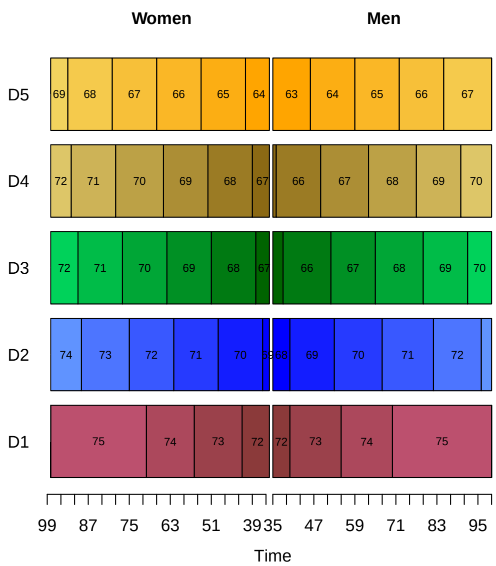
(f) Scenario 6



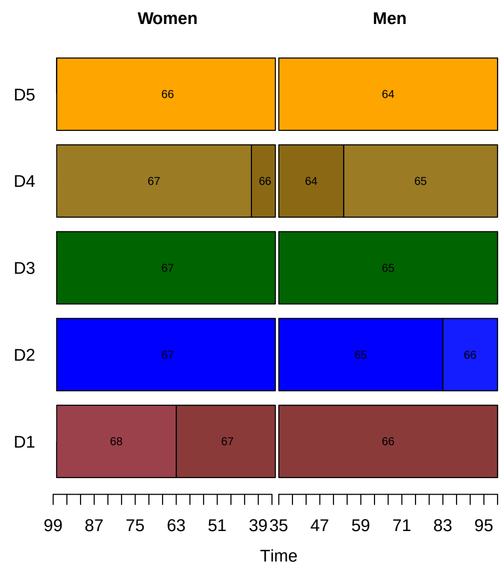
(g) Scenario 7



(h) Scenario 8



(i) Scenario 9



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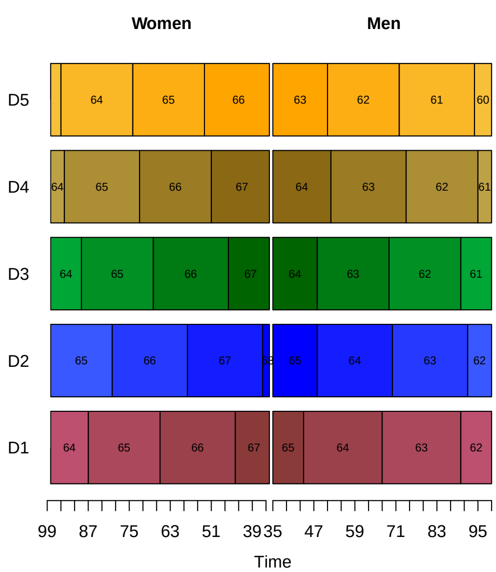

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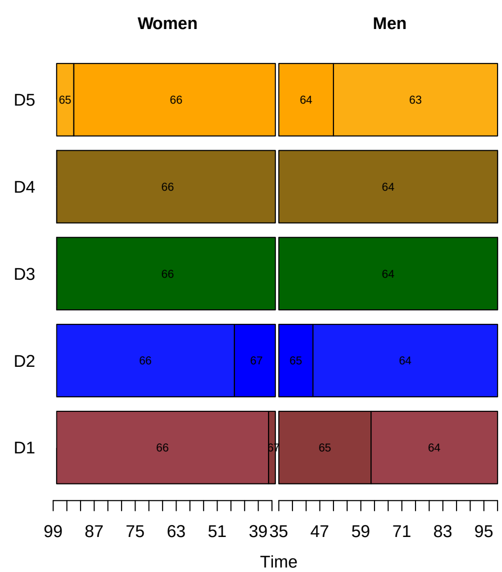
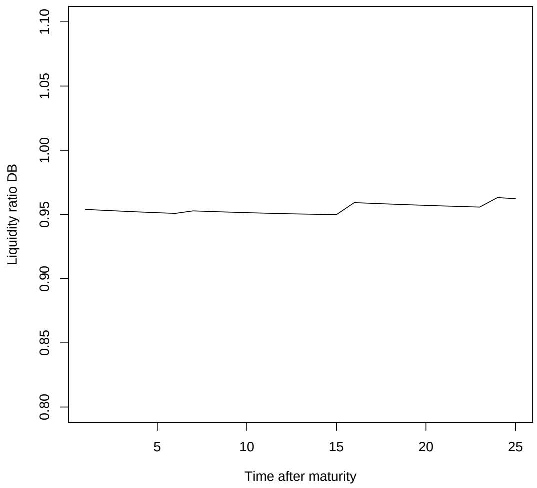
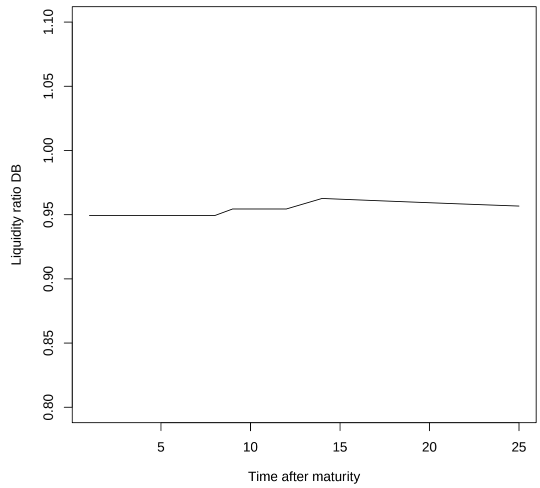


(l) Scenario 12

Figure A.2: Optimal retirement ages for the NDC system, when disutility of work is considered (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career, except for Scenarios 5 and 11, for which the opposite applies.)



(a) Scenario 7



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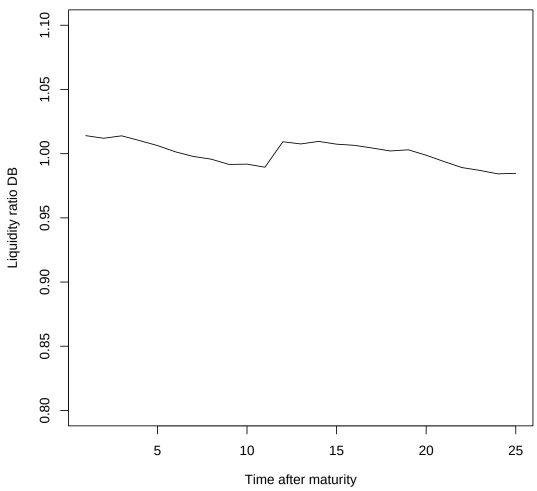

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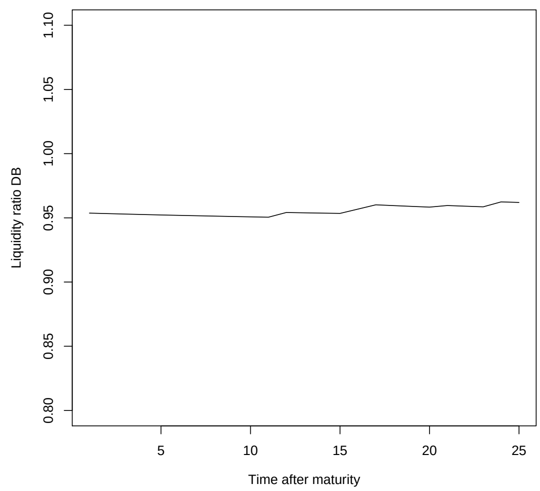

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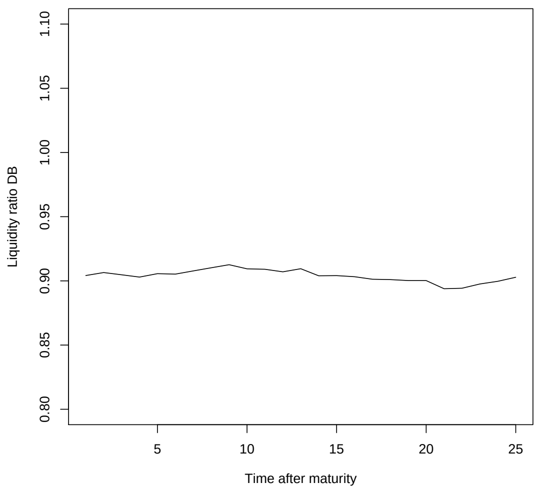

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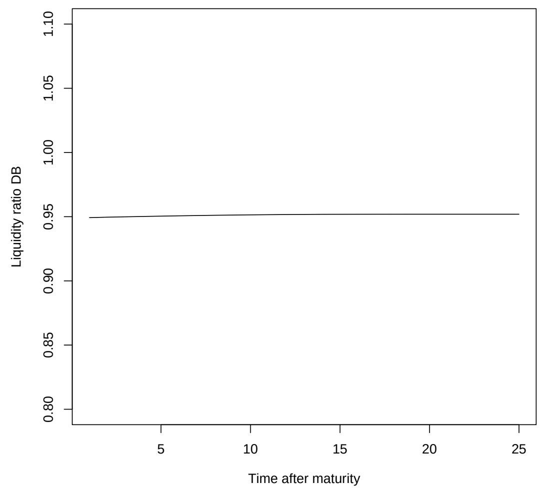
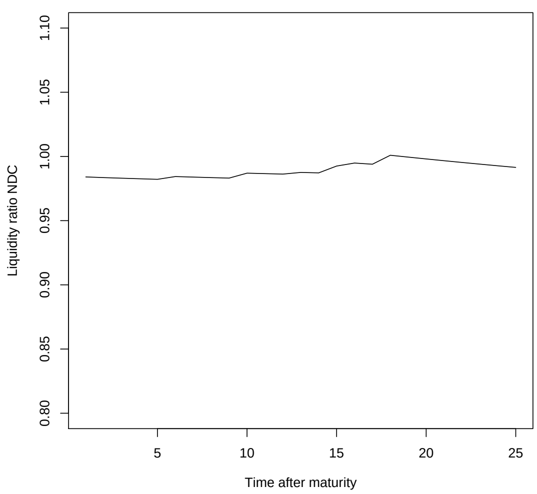
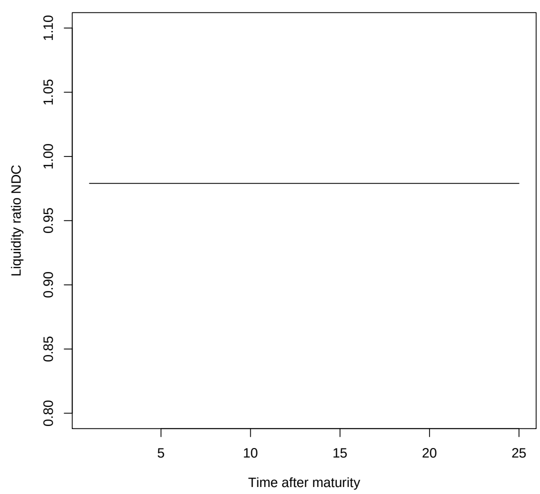


(f) Scenario 12

Figure A.3: Liquidity ratios for the DB system, when disutility of work is considered, for Scenarios 7 to 12



(a) Scenario 1



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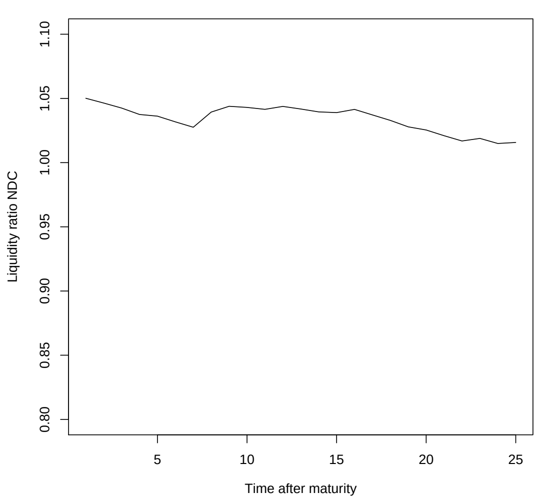
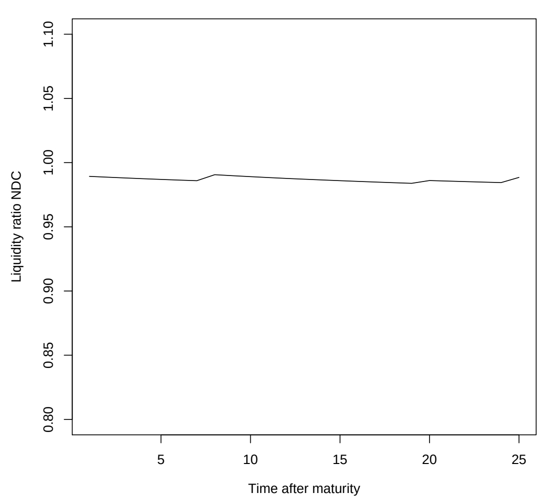
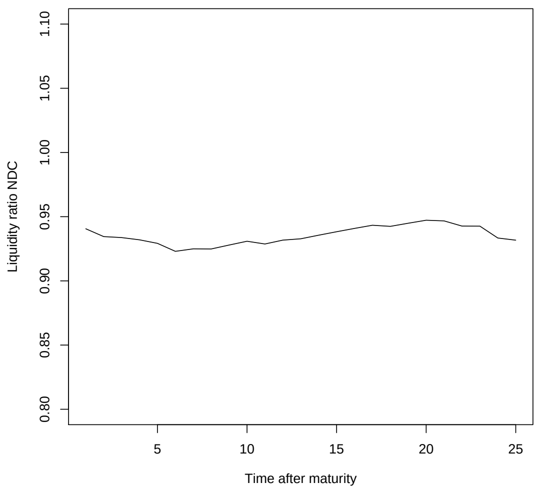


(c) Scenario 3



(d) Scenario 4



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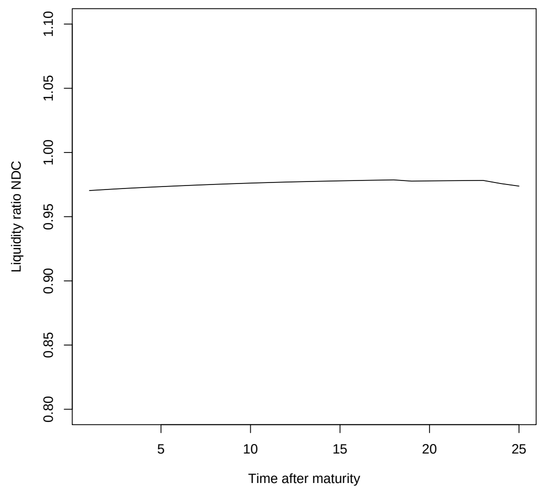

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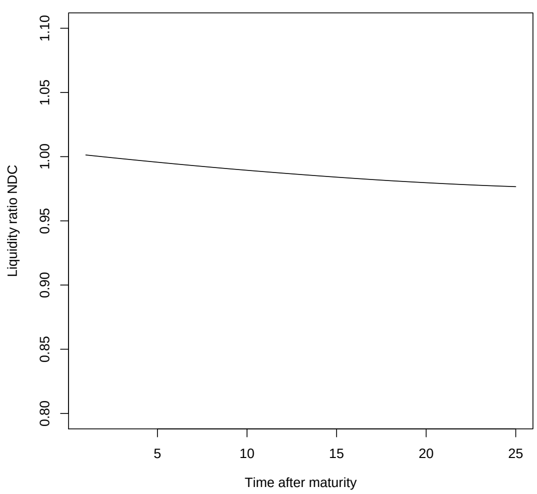

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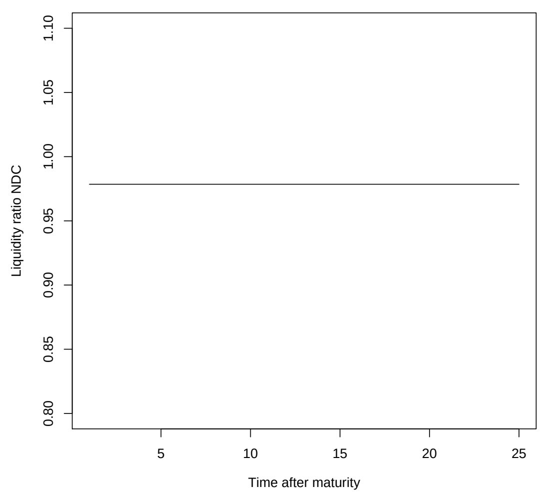

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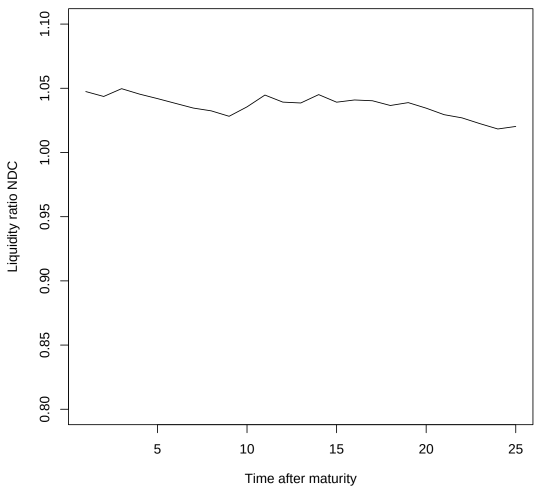

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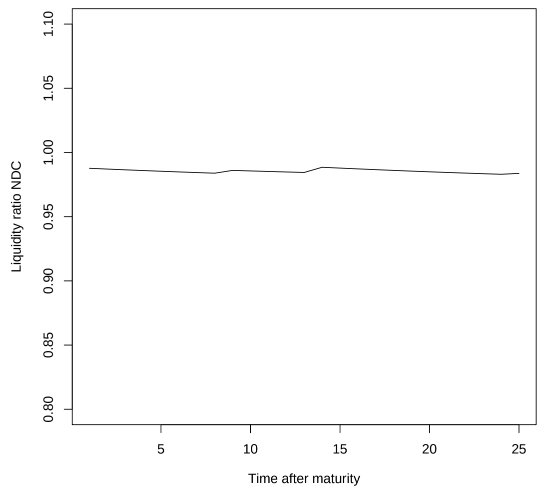

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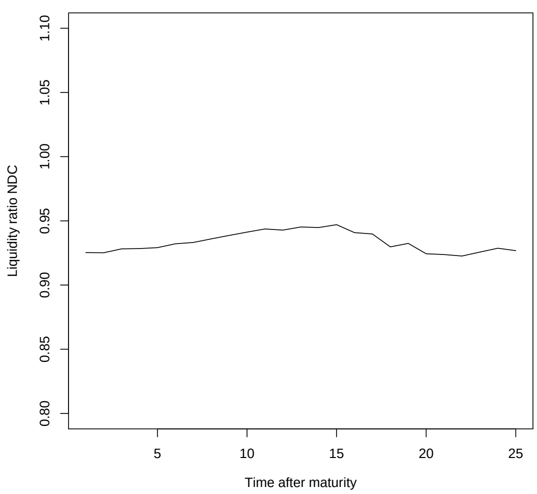

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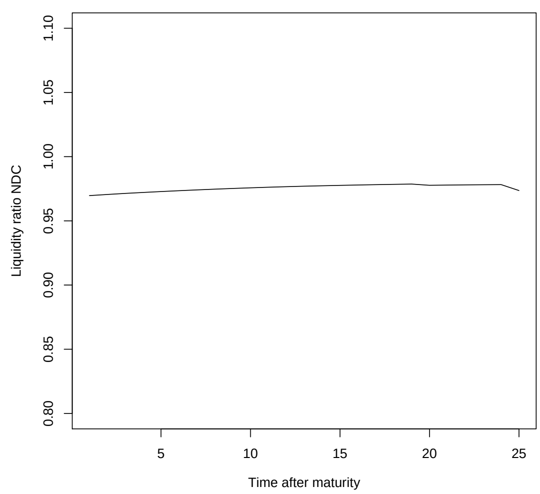
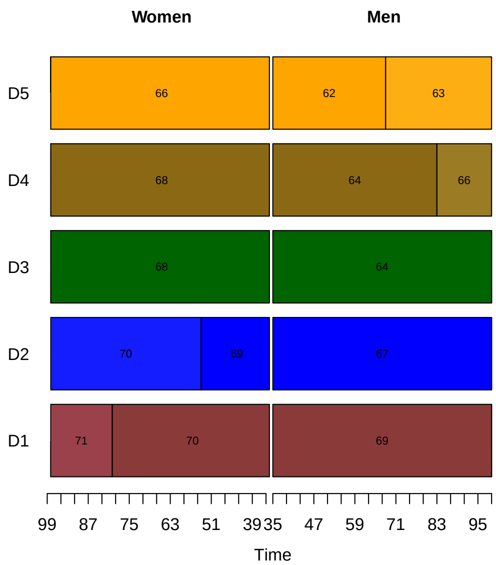


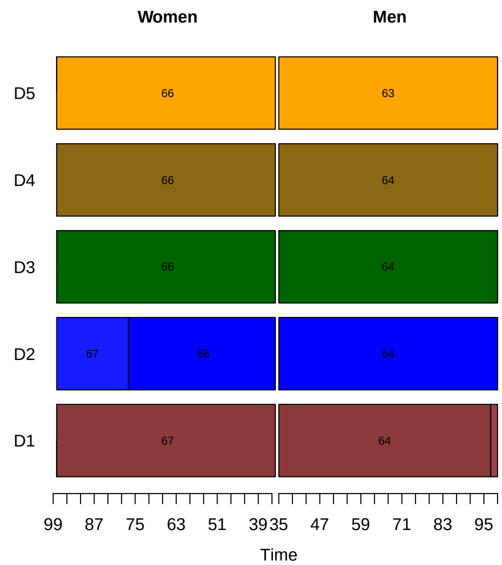
(l) Scenario 12

Figure A.4: Liquidity ratios for the NDC system, when disutility of work is considered

B Results for the model using the utility of leisure



(a) Scenario 7



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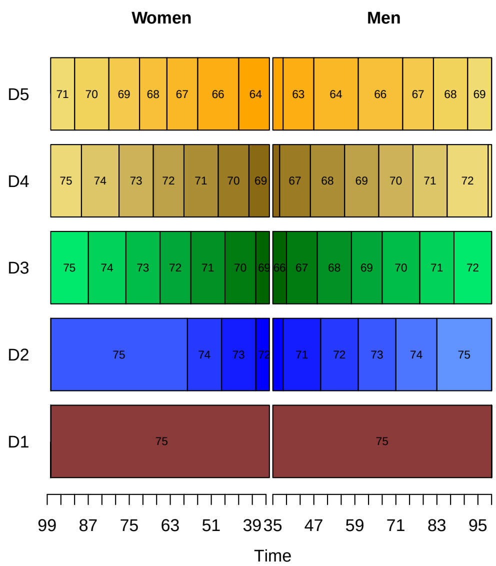
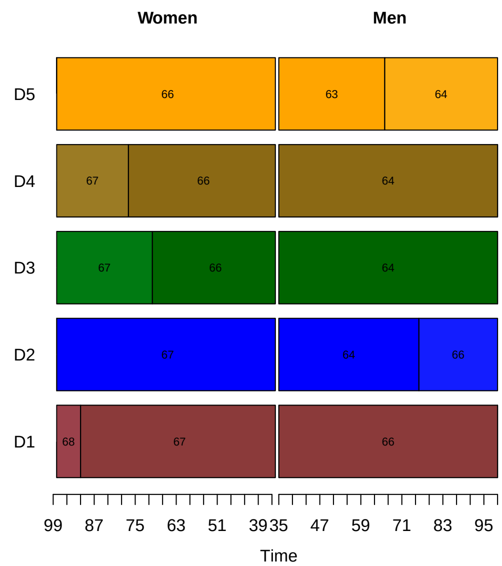
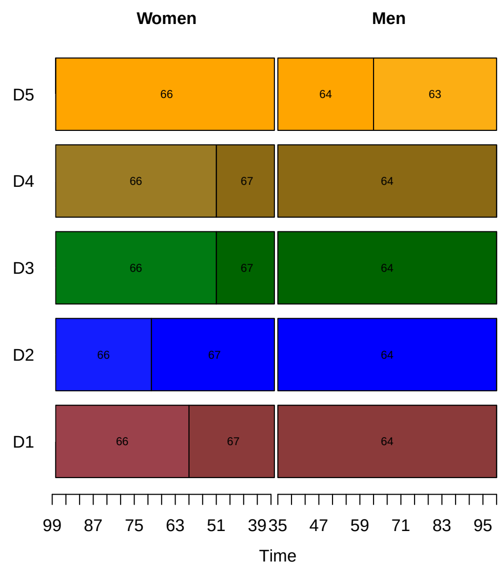


(c) Scenario 9

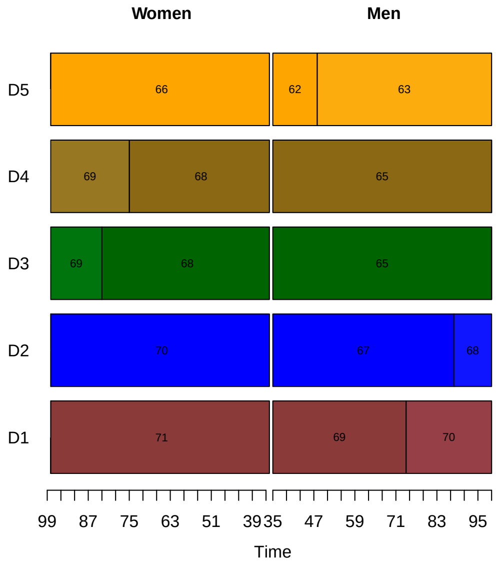


(d) Scenario 10

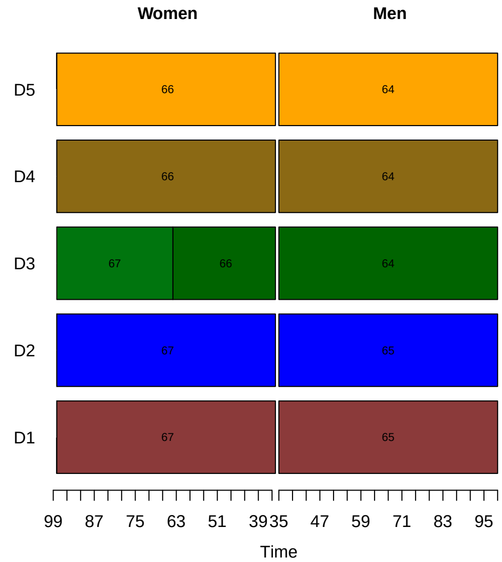


(e) Scenario 11

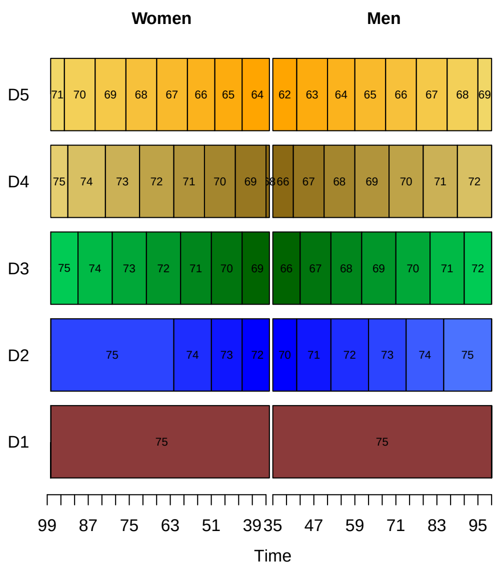
Figure B.1: Optimal retirement ages for the DB system, when utility of leisure is considered, for Scenarios 7 to 11 (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career.)



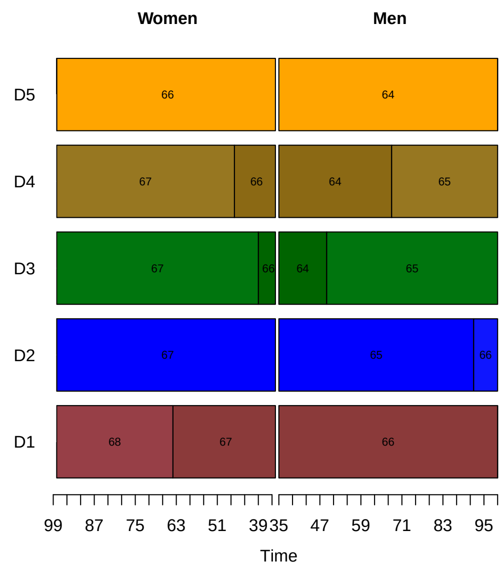
(a) Scenario 1



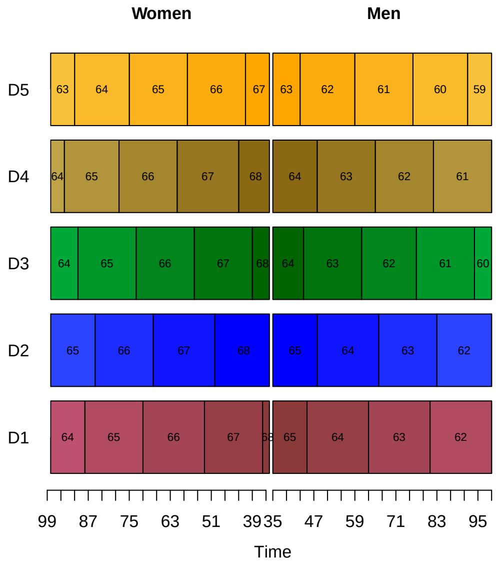
(b) Scenario 2



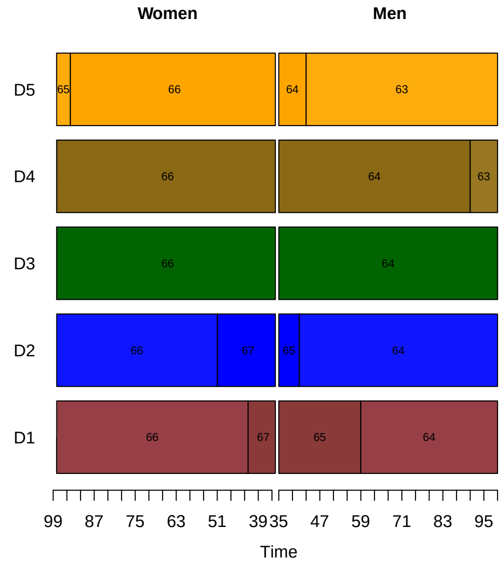
(c) Scenario 3



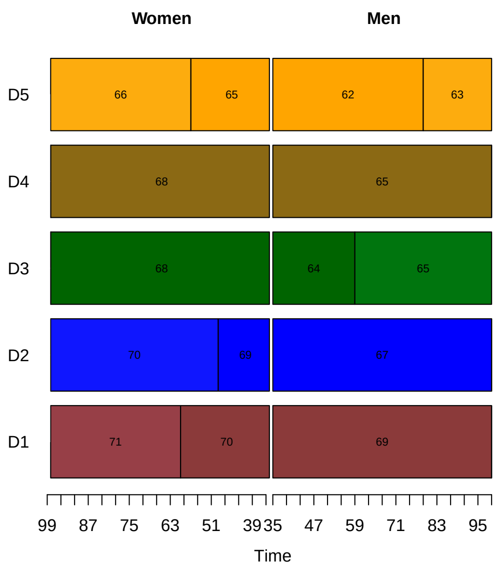
(d) Scenario 4



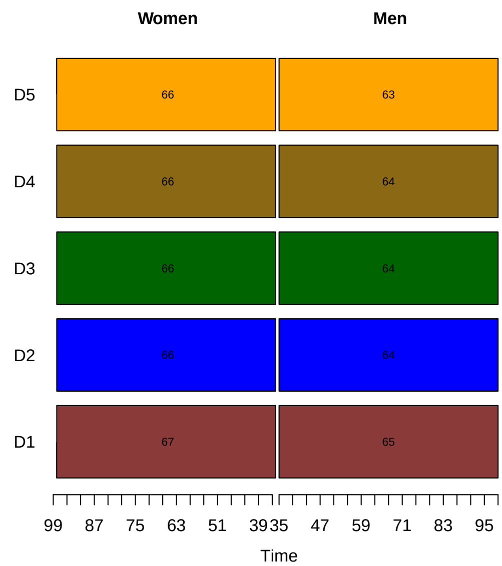
(e) Scenario 5



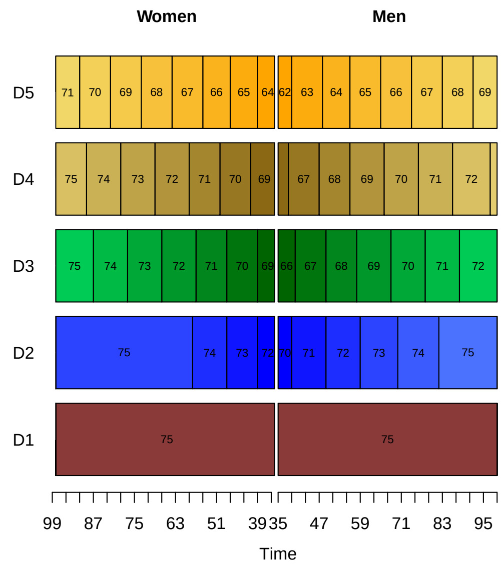
(f) Scenario 6



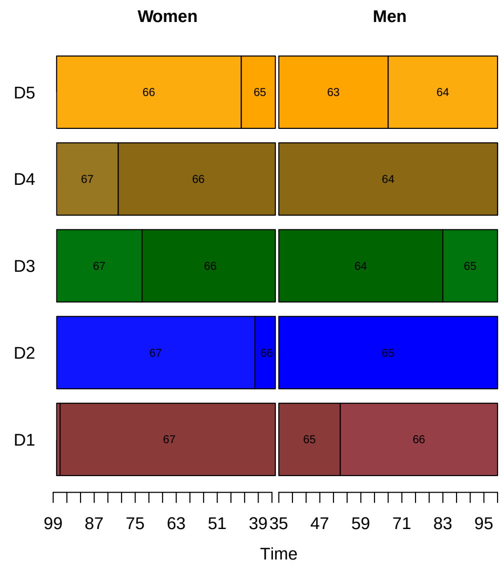
(g) Scenario 7



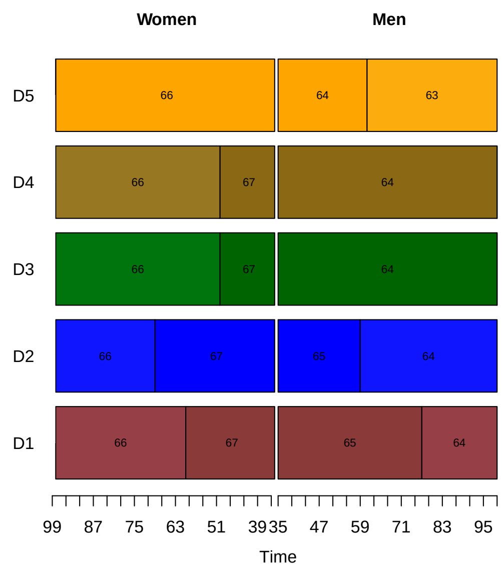
(h) Scenario 8



(i) Scenario 9 (the first retirement age for men in class D4 is 66, while the last one is 73)

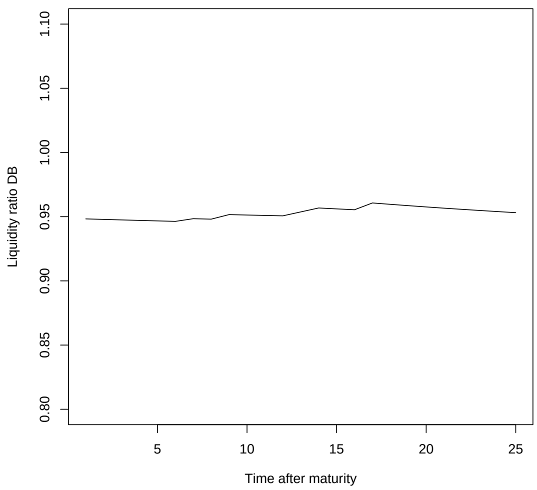


(j) Scenario 10

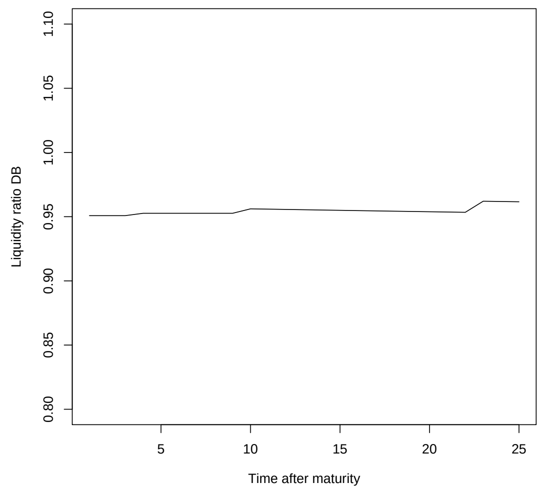


(k) Scenario 11

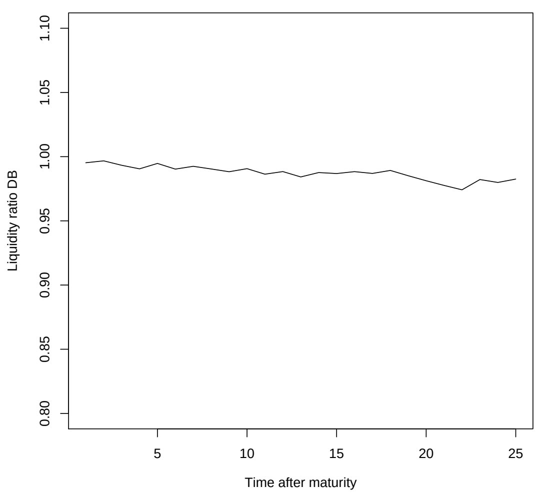
Figure B.2: Optimal retirement ages for the NDC system, when utility of leisure is considered (For the empty block, the retirement age is equal to the retirement age in the following block minus one year, for the beginning of the career and it is equal to the retirement age in the previous block plus one year, at the end of the career.)



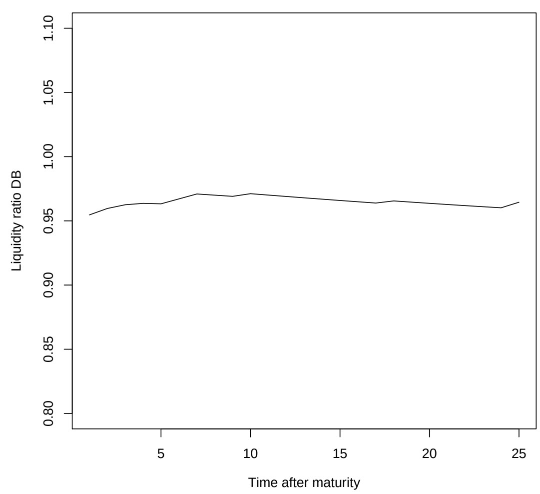
(a) Scenario 7



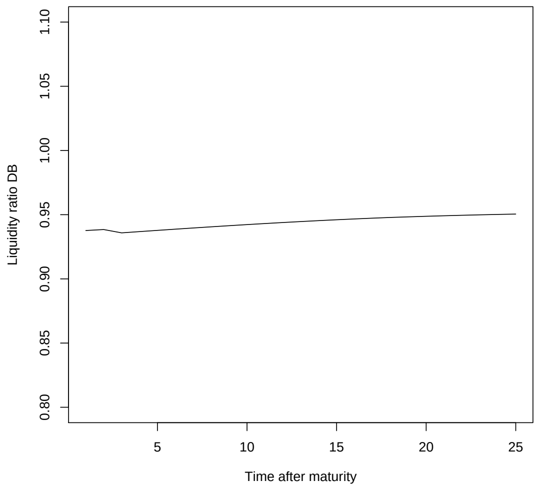
(b) Scenario 8



(c) Scenario 9

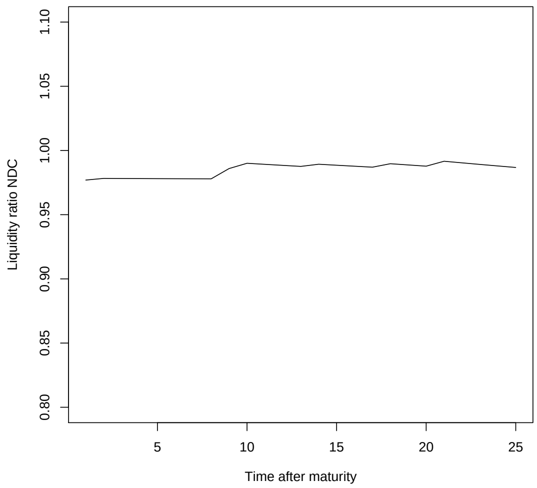


(d) Scenario 10

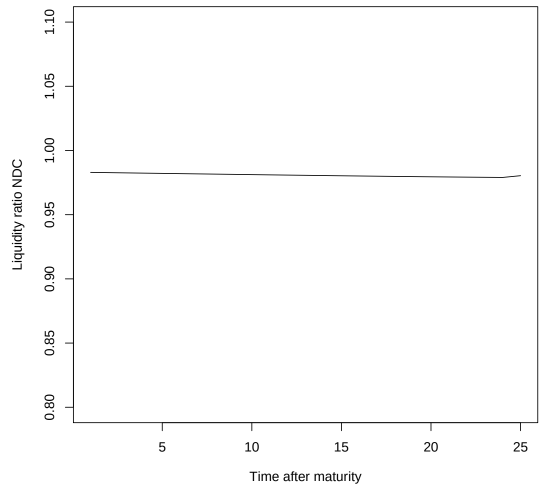


(e) Scenario 11

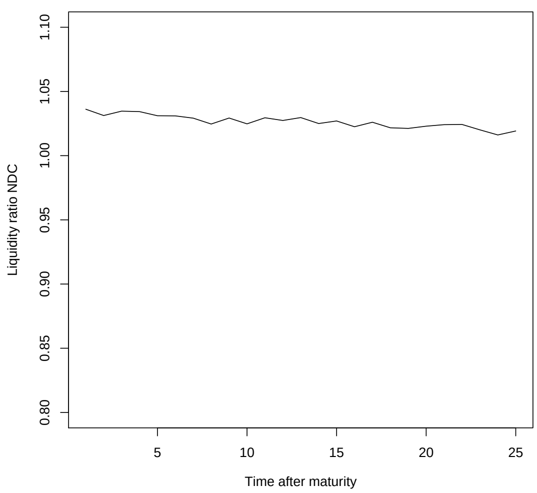
Figure B.3: Liquidity ratios for the DB system, when utility of leisure is considered, for Scenarios 7 to 11



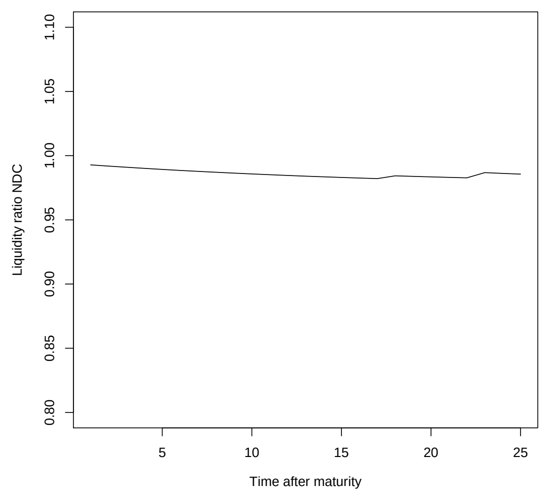
(a) Scenario 1



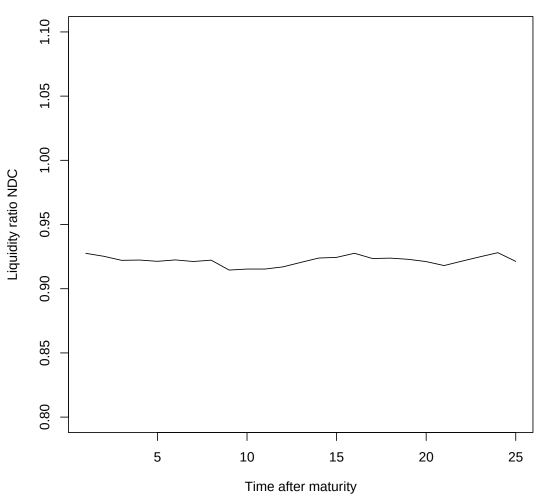
(b) Scenario 2



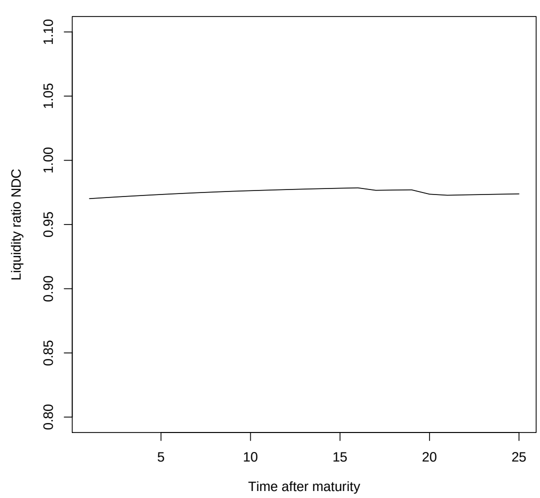
(c) Scenario 3



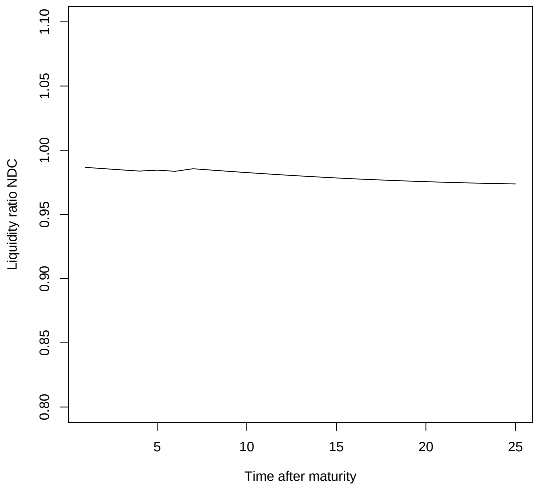
(d) Scenario 4



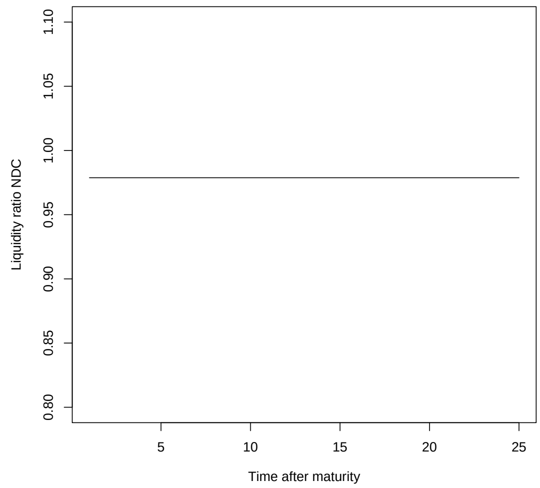
(e) Scenario 5



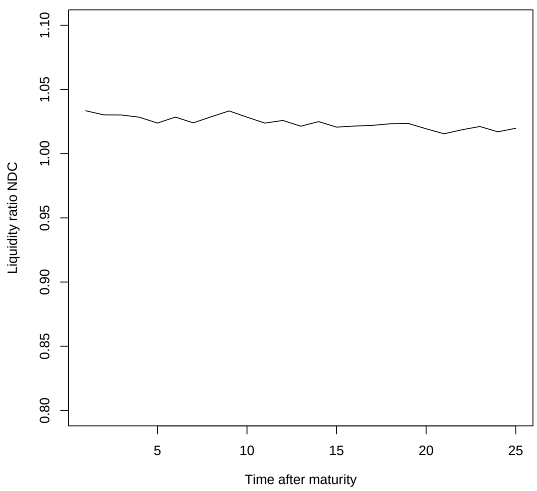
(f) Scenario 6



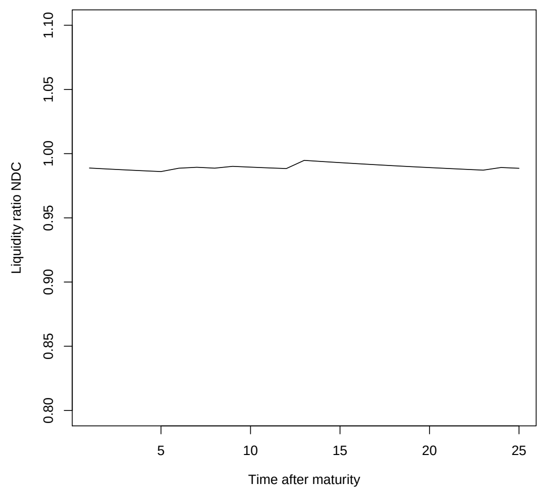
(g) Scenario 7



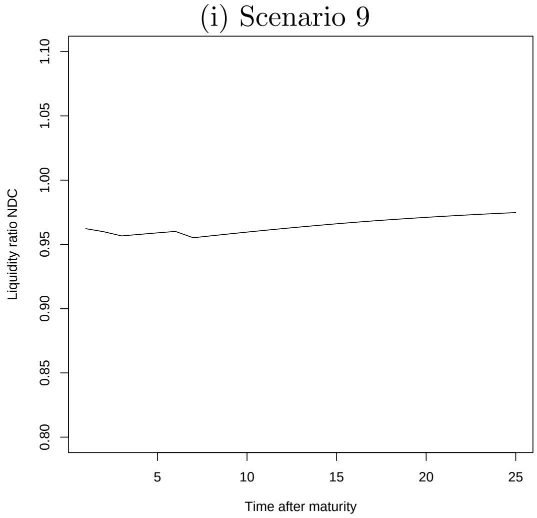
(h) Scenario 8



(i) Scenario 9



(j) Scenario 10



(k) Scenario 11

Figure B.4: Liquidity ratios for the NDC system, when utility of leisure is considered

C Pension amounts

Class		Gender	$t = 50$	$t = 60$	$t = 70$	$t = 80$	$t = 90$	$t = 100$	$t = 110$
D1	DB	Men	27669	31784.7	36512.6	41943.9	48183	55350.2	63583.4
		Women	19177.5	22030.1	25307.1	29071.5	33395.8	38363.4	44070
	NDC	Men	27434.7	31515.59	36203.51	41588.76	47775.05	54881.55	63045.14
		Women	19462.12	22357.09	25682.69	29502.97	33891.52	38932.85	44724.08
D2	DB	Men	17624	20245.5	23257	26716.5	30690.5	35255.7	40500
		Women	13212.1	15177.4	17435	20028.4	23007.6	26430	30361.4
	NDC	Men	17674.5	20303.5	23323.7	26793	30778.5	35356.7	40616
		Women	13387.7	15379.2	17666.8	20294.7	23313.5	26781.4	30765.1
D3	DB	Men	15413.7	17706.4	20340.3	23365.9	26841.5	30834.2	35420.7
		Women	11696.2	13436	15434.6	17730.5	20367.9	23397.6	26878
	NDC	Men	15848.4	18205.9	20914	24024.9	27598.6	31703.9	36419.8
		Women	12100.4	13900.3	15968	18343.2	21071.8	24206.2	27806.8
D4	DB	Men	15044	17281.8	19852.5	22805.5	26197.8	30094.7	34571.3
		Women	11029.7	12670.4	14555.1	16720.2	19207.3	22064.3	25346.4
	NDC	Men	15183.9	17442.5	20037	23017.5	26441.3	30374.5	34892.6
		Women	11272.9	12949.7	14875.9	17088.7	19630.6	22550.7	25905.1
D5	DB	Men	13370	15358.7	17643.3	20267.8	23282.6	26745.8	30724.3
		Women	9409.48	10809.1	12417	14264	16385.8	18823.1	21623
	NDC	Men	13770.1	15818.4	18171.4	20874.4	23979.4	27546.4	31643.9
		Women	9848.79	11313.8	12996.7	14930	17150.8	19701.9	22632.6

Table C.1: Pension amounts at the legal retirement age, throughout time

Table C.1 displays the yearly pension amounts for the five classes, for the DB and NDC schemes, given the legal retirement age of 65 and the parameters used in this paper ($AR = 1\%$, $nr = 1.8\%$, $\pi = 16.3\%$). The salaries are class-specific and gender specific, with the entry ages listed in Table 4.1. The pension amounts are calculated for individuals reaching the legal retirement age every ten years, namely at time 50, 60, 70, 80, 90, 100 and 110. We note that the pensions grow throughout time, which is due to the evolution of salaries. Moreover, pensions for women are lower than those for men, once again due to the salaries which are lower for this gender. As expected, pensions for those with higher education are higher, regardless of the system or gender. Lastly, we remark that although the DB and NDC systems are not equivalent, the difference between the two types of pensions is not, in our case, too substantial.

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Chapter 5

General Conclusions

This thesis is focused on the inter- and intra-generational transfers taking place within the public pension schemes, both in the first and the second pillars. Although solidarity is a key element of social security schemes, unintentional transfers such as those identified and described in this thesis erode the foundation of these pillars. Discussions and measures are thus necessary to reduce these transfers and their impact on the retirement schemes.

With this purpose in mind, we firstly identified, in Chapter 2, the sources of inter-generational transfers taking place within the occupational pension schemes in Switzerland. We identified six main sources of transfers between the active and retired members and we developed a tractable framework that will allow each pension fund to quantify the transfers pertinent to each source. Our numerical illustration shows not only how our framework works but also that the retired are not necessarily the only ones to receive such transfers, but that both groups might benefit. However, our framework should be adapted to the pension plans of each fund in order to provide a useful result that can shed a light on the measures that could be implemented to improve the situation for all parties involved.

Thereafter we considered the intra-generational transfers taking place due to the mortality differences between socio-economic classes. Specifically, in Chapter 3, we showed through a numerical example that, for a Defined Benefit and a Notional Defined Contribution scheme, the transfers can go from the lower socio-economic classes (defined in function of the education level) to the higher classes. This is due to the lack of consideration of socio-economic mortality rates in the calculation of the pension benefits and is contrary to the purpose of solidarity within the social security schemes. Consequently, we proposed a tractable methodology that allows each system to adapt its parameters, namely the interest rates, the accrual rates and the notional rates of return, for each socio-economic class, in order to improve the fairness of the systems. Our numerical example allows us to see that the rates should be higher for lower socio-economic groups, while individuals with higher education would receive lower rates. Simple mathematical formulas are then provided, allowing us to determine the rates for each class, both when data on socio-economic level is enough to determine the relationship between class-specific survival rates and general survival probabilities, and when no data is available, but a simple hypothesis about the ratio between these two types of survival rates can be made. Therefore our framework can and should be used by the policy makers to close the gap between the fair pensions and those actually awarded by the pension systems, and this for each socio-economic class, in order to compensate for the fact that the pension systems do not account for differences in mortality by socio-economic class.

Lastly, in Chapter 4, we look at the adjustment of the retirement ages for each socio-economic class as an alternative compensation mechanism with respect to the above-defined intra-generational transfers. We began by considering an utilitarian framework, with different scenarios for the risk aversion coefficient γ and the individual time preference factor β. We find, based on our data, that the retirement age of those individuals belonging to lower classes is, in general, lower than that of those with a high education. However, the utilitarian method, though important to explain individual characteristics and preferences, is not a viable tool for policy makers. Indeed, as proven by the variability of our results, the systems cannot sustain, both in terms of complexity and financial reliability, such degree of individualisation. Therefore, an alternative method is proposed with basis in the actuarially fair pension. Our proposed framework allows policy makers to set the optimal retirement age for each class such that the accumulated value at age ω of all benefits paid by either the DB or the NDC scheme is close to the value obtained should the actuarially fair pension be paid. Lastly, we investigated, given the data available, the mortality differential required for advancing or postponing the retirement age by only one year, for the average individual. Though the numerical results in this chapter are solely an illustration, our methodology is easily implemented in practice and should be considered by the policy makers, in order to ease the disadvantage brought to those of lower classes by an universal raise of the retirement ages.

This thesis gives way to several potential avenues that would extend the work presented here. Firstly, we note that both Chapter 3 and Chapter 4 consider that individuals spend their entire lifetime in the same socio-economic class. Thus the next natural and interesting step would be the inclusion of transitions between classes. Such an approach would enhance our frameworks and allow a closer link to the real situation on the job market. However, one important issue with respect to this extension lies in the complexity added to the models and the difficulty to procure appropriate data for the model implementation. Another avenue for further research lies in performing a similar analysis as presented in Chapter 2 regarding the inter-generational transfers for other countries and systems. Lastly, we would also be interested in developing a method that would allow us to quantify both inter- and intra-generational transfers within the same system and to compare said transfers in order to asses which of the two types impacts the system more.